GATE ECE - 2005

Explanations:

1. (A) For the given equation,

auxiliary equation is

$m^2 - 5m + 6 = 0$

$(m-3)(m-2) = 0$

$m = 3, 2$

$y = e^{2x} + e^{3x}$

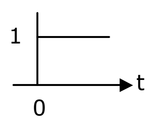
2. (C) Fourier series is defined for a periodic function.

Here $e^{-|t|}$ is not a periodic function.

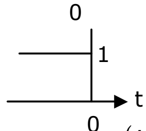
3. (D)

4. (B)

5. (A) Even part = $\frac{\mu(t) + \mu(-t)}{2}$

μ(t) = 

$= \frac{1}{2}$

μ(t) 

odd part = $\frac{\mu(t) - \mu(-t)}{2} = \frac{x(t)}{2}$

Answer is (A)

6. (C) The given sequence is a 2 sided sequence

so its ROC is a circle

answer is C;

ROC for $\left(\frac{5}{6}\right)^n \mu(n)$ is $|z| > \frac{5}{6}$

ROC for $\left(\frac{6}{5}\right)^n \mu(-n-1)$ is $|z| > \frac{6}{5}$

$\frac{5}{6} < |z| < \frac{6}{5}$

7. (C)

$u(t) = Ri + \frac{Ldi}{dt} + \frac{1}{C}\int cdt$

By differentiating, $0 = \frac{Rdi}{dt} + \frac{Ldi^2}{dt^2} + \frac{1}{C}C$

$$\frac{di^2}{dt^2}+\left(\frac{R}{L}\right)\frac{di}{dt}+\left(\frac{1}{LC}\right)i=0$$

Roots are,

$$\frac{-R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}$$

For no oscillations, $\left(\frac{R}{2L}\right)^2 \geq \frac{1}{LC}$

Answer (C). $R \geq 2\sqrt{\frac{L}{C}}$

8. (B) For an ideal transformer,

$V_1 = nV_2$

$V_2 = nV_1$

$I_1 = -\frac{1}{n}I_2$

$I_2 = -nI_1$

$A=\frac{V_1}{V_2}=n$

$B=-\frac{V_1}{I_2}=0$

$C=0$

$D=\frac{-I_1}{I_2}=-\left[\frac{-1}{n}\right]=\frac{1}{n}$

$D=\frac{1}{n}$

9. (B)
10. (C)
11. (C)
12. (B) Reverse saturation current doubles for every 10 degree rise in temperature.

I_S = 10 PA @ 20°C

I_S = 20 PA @ 30°C

I_S = 40 PA @ 40°C

13. (C)
14. (D)
15. (A)
16. ()
17. (B)
18. (B)

19. (A) No Answer ε^1 Data not sufficient

20. (B) For causal system,

h(t) = 0, for t < 0

∴ Answer is (B).

21. (D)

$$x(n) = \left(\frac{1}{2}\right)^n \mu(n); x^2(n) = \left(\frac{1}{2}\right)^{2n} \mu^2(n)$$

$$y(n) = \left(\frac{1}{4}\right)^n \mu(n); x^2(n) = \left(\frac{1}{4}\right)^n \mu(n)$$

$$Y\left(e^{j\omega}\right) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}} \Rightarrow Y\left(e^{jo}\right) = \frac{1}{1 - \frac{1}{4}}$$

$$Y\left(e^{jo}\right) = \frac{4}{3}$$

22. (D)

23. (A) S(t) = 8cos(20πt - $\frac{\pi}{2}$) + 4sin(15πt)

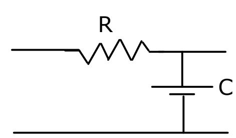
= 8sin(20πt) + 4sin(15πt)

Power in the signal = $\left(\frac{8}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 = 40$

24. (C)

25. (B)

26. (D) Lag network is an RC network



$$G(s) = \frac{1}{s + sT}$$

$$G(j\omega) = \frac{1}{1 + j\omega T} = \frac{1}{\sqrt{1 + \omega^2 T^2}} \angle - \tan^{-1} \omega T$$

$$= ML\phi$$

when ω =0, M =1 and ϕ = 0

the phasor at ω =0 has unit length ε^1 lies along the positive real axis.

As ω increases M decreases ε^1 phase angle increases negatively.

When ω= $\frac{1}{T}, M = \frac{1}{\sqrt{2}}, \phi = -45$

$\omega \to \infty, M \to 0, \phi = -90$

Answer is (D)

27. ()

28. (C)

29. (B) From wave equation,

$$\omega t = 50000t$$

$$\frac{2\pi}{\lambda} = 0.04$$

$$f = \frac{5\times 10^4}{2\pi}$$

$$\lambda = \frac{2\pi}{0.004}$$

$$v = f\lambda = -1.25\times 10^7 m/\sec.$$

30. (C)

31. (A)

32. (C)

33. (A)

$$A = \begin{bmatrix} 2 & \frac{-1}{10} \\ 0 & 3 \end{bmatrix}; A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & \frac{1}{10} \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0.5 & \frac{1}{60} \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$a = \frac{1}{60}, b = \frac{1}{3}$$

$$a + b = \frac{1}{60} + \frac{20}{60} = \frac{21}{60}$$

34. (A)

$$\int_0^\infty e^{\frac{-x^2}{8}} dx = \int_0^\infty e^{-bx^2 dx}$$

$$b = \frac{1}{8}, bx^2 = t, dx = \frac{dt}{2\sqrt{b}}$$

$$= \frac{1}{2\sqrt{b}}\int_0^\infty e^{-t} t^{-\frac{1}{2}} dt = \frac{T\left(\frac{1}{2}\right)}{2\sqrt{b}}$$

$T\left(\frac{1}{2}\right)$ Gamma function

$$\therefore I = \frac{1}{\sqrt{2\pi}} \times \frac{\sqrt{\pi}\sqrt{8}}{2} = \frac{\left(2\sqrt{2}\right)\sqrt{\pi}}{\left(2\sqrt{2}\sqrt{\pi}\right)} = 1$$

$$I = 1$$

35. (C) The symmetric function equation is e^{-t^2} (Simple Gaussian equation)

$$y(t) = \frac{d}{dt}\left(e^{-t^2}\right) = -2t\left[e^{-t^2}\right]$$

when t is +ve, y(t) is negative

when t is –ve, y(t) is positive.

Answer is (C).

36. (C)

37. (A) By trial ε^1 error (A) is correct

38. (A)

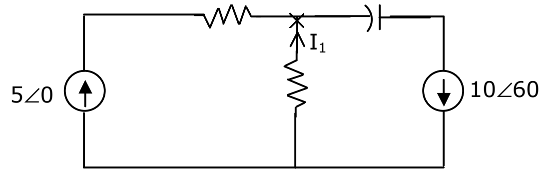
By KCL,

$5LO = -I_1 + 10\angle 60$

$I_1 = 10\angle 60 - 5$

$= 10\left(\frac{1}{2} + \frac{j\sqrt{3}}{2}\right) - 5$

$I_1 = \frac{10\sqrt{3}}{2}\angle 90$



39. (B)

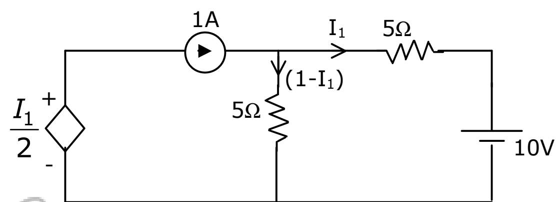
40. () By KVL,

$10 = -5I_1 + 5(1 - I_1)$

$I_1 = -\frac{1}{2}$

$V_{ab} = 5(1 - I_1)$

$V_{ab} = 7.5$



41. (C)

$I_1 = \frac{10}{2R} = \frac{5}{R}$

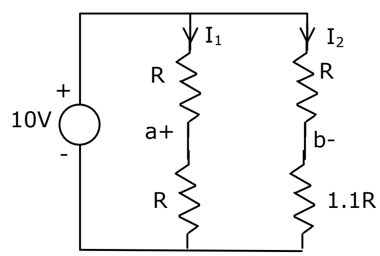
$I_2 = \frac{10}{2.1R} = 4.76/R$

$V_{ab} = -RI_1 + I_2R$

$= R\left[I_2 - I_1\right]$

$= \frac{R}{R}\left[4.76 - 5\right]$

$V_{ab} = -0.238$



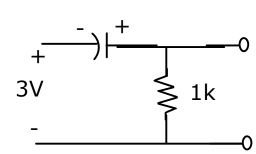
42. (D)

43. (B) Time constant = RC = 100μsec.

T = 2 sec >> 100μsec.

So, C is fully charged (opposite polarity)

$V_{out} = -3$



44. (B) Conductivity is proportional to poping concentration

NA = 10^{18}, NB = 10^{18}, $\frac{\mu A}{\mu B} = \frac{1}{3}$

σ = n q μ (n concentration)

$$\frac{\sigma A}{\sigma B} = \left(\frac{\mu A}{\mu B}\right) = \frac{1}{(3)}$$

$$\frac{\sigma A}{\sigma B} = \frac{1}{3}$$

45. (B)

$$C = \frac{\varepsilon_o \varepsilon_r}{t} = \frac{11.7 \times 8.85 \times 10^{-12}}{10 \times 10^{-6}}$$

$$C = 10.35 \mu F$$

46. (C) It is high pass,

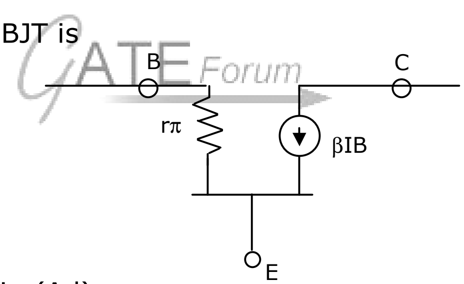
$$f = \frac{1}{2\pi RC}; R = 1k, C = 1\mu F$$

$$\omega = \frac{1}{1K \times 1\mu F} = 1000 rad / \sec$$

47. ()

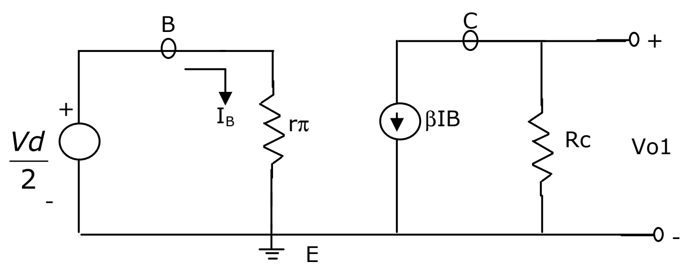
48. (A)

49. (D) Equivalent circuit of BJT is



Differential mode gain (Ad):

Small signal model for difference mode



$$Ad = \frac{V_{01}}{V_d} = \frac{-\beta Rc}{2r\pi} = -\frac{gmRc}{2}$$

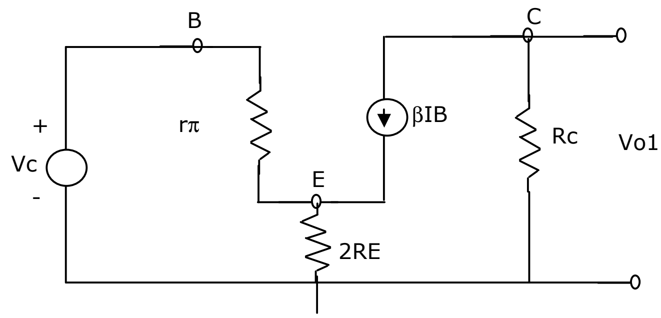
$$Ad = \left(\frac{-gmRc}{2}\right)$$

Common mode gain (Ac):

AC =

$$\frac{V_{01}}{Vc} = \frac{-\beta Rc}{2(HB)RE + r\pi}$$

$$Ac = \frac{-Rc}{2RE}$$



for $\beta >> 1$ $r\pi << 2R_E$

decrease Ac with $R_E \uparrow$

$$CMRR = \frac{Ad}{Ac}$$

$$CMRR = gmRE$$

50. (C) VDs = 5 – 1 = 4V

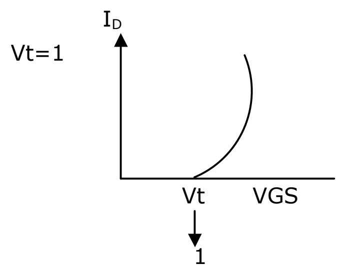
VGs = 3 – 1 = 2

VGs – Vt = 2 – 1 = 1

VDs >, VGs – if

4 >, 1

so device in saturation.



51. (B)

$\beta = 50$, $\alpha = \frac{50}{51} = 0.98039$

$V_{BE} = 0.7V$

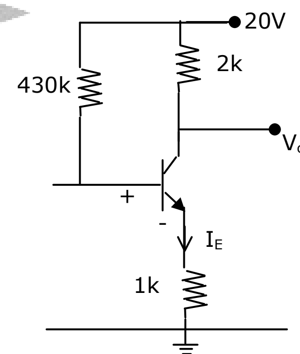
20 V (430 K)I_B + 0.7 + I_E(1k)

20 = (430 K)I_B + 0.7 + (51I_B)1k

$I_B = 40\mu A$

$I_C = \beta I_B = 2000\ \mu A$

$V_o = 20 - 2k \times 2000\mu A = 16V$



52. (A) For the zener diode to be operated in the breakdown region, the maximum load current is

$$i_{L(\max)} = \frac{V_s(\min) - V_{20}}{R_s} - I_2 k$$

$$= \frac{20 - 5.8}{1k} - \left(\frac{1}{2}\right) mA$$

$$i_{L(\max)} = 13.7mA$$

53. (C) V_o is at logic o. So transistor is in saturation.

V_{BE}= 0.75 V

$$\therefore I_R = \frac{0.75}{1k} = 0.75mA$$

I_R = 0.75mA

54. (A)

$$f = A'BC + ABC'$$
$$= B\left[A'C + AC'\right] = B\left[\left(A + C\right)\left(A' + C'\right)\right]$$
$$f = B\left[A + C\right]\left[A' + C'\right]$$

55. (C)

$$I_{DS}\left(T_1\right) = I_{DS}\left(T_2\right)$$
$$\frac{1}{2}K_1\left(VGs_1 - V_t\right)^2 = \frac{1}{2}K_2\left(VGs_2 - V_t\right)^2$$
$$36\left(5 - V_o - 1\right)^2 = 9\left(V_o - 1\right)^2$$
$$6\left(4 - V_o\right) = 3\left(V_o - 1\right) \Rightarrow 2\left(4 - V_o\right) = V_o - 1$$
$$8 + 1 = 3V_o = 9$$
$$V_o = 3$$

56. (C)

JK	Q_{n+1}
00	Q(n)
01	0
10	1
11	$[Q(n)]^1$

If J = 1,

$Q(n+1) = 1$ (or) $[Q(n)]^1$

$= 1$ (or) $(0)^1$

Q(n+1) = 1 [given Q(n) = 0]

57. (A) T Flip-flop will toggle if T = 1

Q_0 1 → 0

Q_1 1 → 0

Q_2 0 → 1

Answer is 100

58. (D) Address range of chip 1 ε^1 chip 2 →

A0 – A7	A9A8A7 – A0	
∞ - FF	00FF – 00	
	01FF – 00	$\bar{A}9$ A8
	10FF – 00	A9 $\bar{A}8$
	11FF – 00	

A8 $\bar{A}9$ → selecting chip #1 [Address range is 100 – 1FF]

$\bar{A}8$A9 → selecting chip #2 [200 – 2FF]

Address range is 100 – 2FF

Since A10 – A15 are not used, the address range can be 100 – 2FF (or) 500 – 6FF (or) 900 – AFF, ….

Answer is (D).

For the address space F800 – F9FF chip #1 ε^{1} chip #2 are not selected.

59. (A)

$$x(t - to) \leftrightarrow e^{-j\omega to} \times (\omega)$$

$$Y(\omega) = 0.5e^{-j\omega(td-T)} \times (\omega) + e^{-j\omega td} \times (\omega) + 0.5e^{-j\omega c(td+T)}$$

$$= e^{-j\omega td} \times (\omega)\left[1 + 0.5\left(2\cos(\omega T)\right)\right]$$

$$\frac{Y(\omega)}{X(\omega)} = e^{-j\omega td}\left[1 + \cos\omega T\right] = H(\omega)$$

60. (C)

61. (B) y(n) = Ax(n – no)

$$Y\left(e^{j\omega}\right) = Ae^{-j\omega no} \times \left(e^{j\omega}\right)$$

$$H\left(e^{j\omega}\right) = \frac{Y\left(e^{j\omega}\right)}{X\left(e^{j\omega}\right)} = Ae^{-j\omega no} = \left|H\left(e^{j\omega}\right)\right| \angle H\left(e^{j\omega}\right)$$

$$\angle H\left(e^{j\omega}\right) = -\omega no$$

$$\angle H\left(e^{j\omega}\right) = -\omega_o no + 2\pi k$$

General solution; k is a arbitrary integer

62. (B) Inverse Fourier transform of

$$x(ab + b) \leftrightarrow \frac{1}{a} e^{-j2\pi\left(\frac{b}{a}\right)t} x\left(\frac{t}{a}\right)$$

a = 3, b = 2 $\leftrightarrow \frac{1}{3} x\left(\frac{t}{3}\right) e^{-j\frac{4\pi t}{3}}$

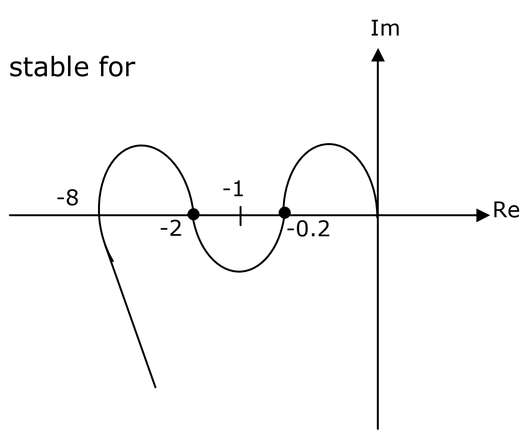
63. (B) The closed loop system is stable for

8K < 1 → K < $\frac{1}{8}$

0.2K < 1 → K < 5

2K > 1 → K > $\frac{1}{2}$

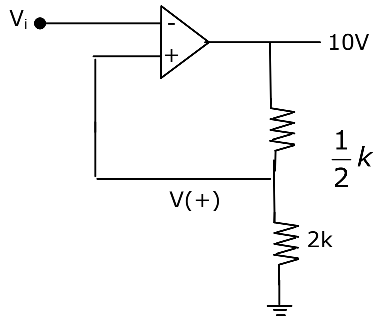
K < $\frac{1}{8}$ or $\frac{1}{2}$<K<5



64. (C)

65. () $V_{out} = \pm 10V$; as $V_{supply} = \pm 10V$

When $V_o = 10V$

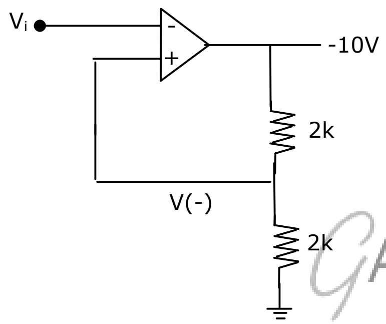


$$V+ = \frac{2k}{\left(2+\frac{1}{2}\right)k} \times 10$$

$\frac{1}{2}k$

$$V+ = 8V$$

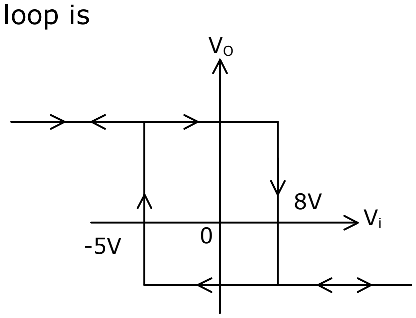
When $V_o = -10V$



$$V- = \frac{2k}{4k} \times (-10)$$

$$V- = -5V$$

Hysteresis loop is



66. (A)

67. ()

68. (C) The Rough sketch of the root locus is a circle.

The breakaway point is $\frac{d}{ds}\left[G(s)H(s)\right] = 0;$

$$\frac{-s(s+3)-(1-s)(2s+3)}{s(s+3)} = 0 \Rightarrow S = 3 \text{ or } -1$$

Answer is C

69. (C)

70. (C) FM equation is

A cos [ωct + βsinωmt]

$$\beta = \frac{\Delta f}{fm} = \frac{90}{5} = 18 = A\cos\left[\omega_c t + 18\sin\omega_m t\right]$$

$$y(t) = x^2(t) = A^2\cos^2\left[\omega_c t + 18\sin\omega_m t\right]$$

$$= A^2\left[\frac{1}{2} + \frac{1}{2}\cos\left(2\omega_c t + 36\sin\omega_m t\right)\right]$$

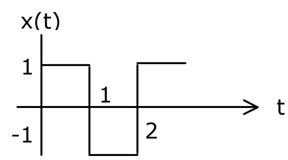
$$= \frac{A^2}{2} + \frac{A^2}{2}\cos\left(2\omega_c t + 36\sin\omega_m t\right)$$

$$\beta = 36 = \frac{\Delta f}{fm} \Rightarrow \Delta f = 36 \times 5 = 180$$

By carsons rule bandwidth = 2(Δf +fm) = 2(180+5)

Bω=370

71. (C) $x(t) = \mu(t) - 2\mu(t-1) + \mu(t-2)$ for $x(t)$



The impulse of a matched filter is delayed version of the input.

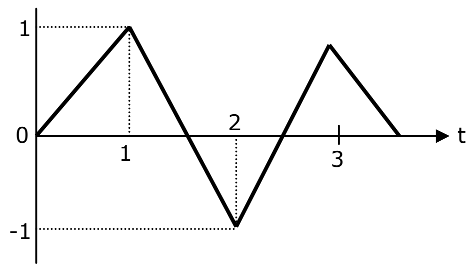
Now delay not given, so assumed to be zero.

$\therefore$y(t) = x(t)*x(t) [n(t) = x(t)] where * represents convolution.

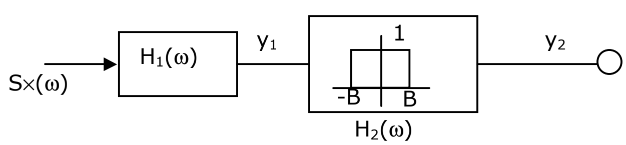
y(t) =r(t) – 4r(t-1)+6r(t-2) –4r(t-3)+r(t-4)

Here μ(t) * μ(t) = r(t) Ramp function.

y(t) →



72. (A)



PSD of input S × (ω) = No W/H2. $H_1(\omega) = 2e^{-j\omega td}$

$Sy_1(\omega)$ $\left|H_1(\omega)\right|^2 S \times (\omega) = 4No$ W/Hz

$Sy_2(\omega)$ $\left|H_2(\omega)\right|^2 Sy_1(\omega) = 4No$ W/Hz,-B $\le \omega \le$ B

Noise power = $\int_{-B}^{B} sy_2(\omega)\,dw = 2 \times 4NoB = 8NoB$

73. ()

74. (C) From plot, P(V) = $\frac{k}{4}V$,

$$\int_0^4 P(V)\,dV = 1, \int_0^4 \frac{K}{4} V dV = 1 \Rightarrow K = \frac{1}{2}$$

mean square value of V = $\int_0^4 VP(V)\,dV = \int_0^4 V.\frac{K}{4} V dV$

mean square value = 8.

75. (D)

76. (D)

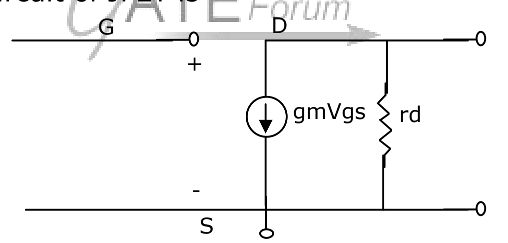
$$Z_0 = \sqrt{Z_o c^2 Sc}$$

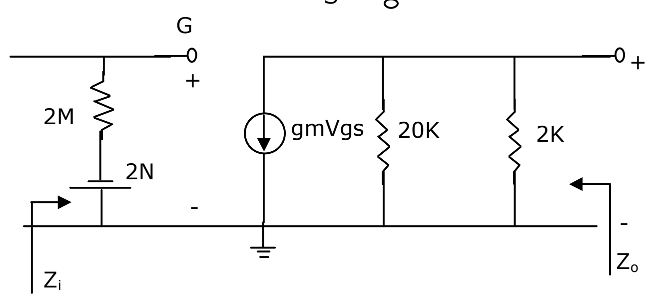
$$Z_{Sc} = \frac{Z_o^{\ 2}}{Z_{oc}} = \frac{50^2}{100 + j150} = 7.69 - j11.54$$

$$Z_{Sc} = 7.69 - j11.54$$

77. (B)

78. () The equivalent circuit of JFET is





Zi = 2M; Z_{out} = 20k||8k = $\left(\frac{20}{11}\right) k\Omega$

79. (A) $I_D = I_{DSS}\left(1 - \frac{VGs}{V_P}\right)^2 = 10\left(1 - \frac{2}{8}\right)^2 = 5.625$

I_D = 5.625 mA

VDs = 20 – 2k × 5.625 = 8.75 V

VDs = 8.75 V

80. (B)

$$gm = \frac{-2I_{DSS}}{V_P}\left[1 - \frac{VGs}{V_p}\right] = \frac{-2 \times 10mA}{-8}\left[1 - \frac{-2}{-8}\right]$$

$gm = 1.875$ msiemens

$$\frac{V_0}{Vgs} = V_{gain} = -\left[\frac{20}{11}\right]KAgm = -3.41$$

$AV = -3.41$

81. (A) (C)

(B) (C)

82. (A) ( ) Gain crossover frequency

gain of

$$\frac{3e^{-2s}}{s(s+2)} = \left|\frac{3e^{-2s}}{s(s+2)}\right| = 1$$

$$e^{-2s} = \cos(2\omega) - j\sin 2\omega; \left|e^{-2s}\right| = 1$$

$$\left|\frac{3}{j\omega(2+j\omega)}\right| = 1; \left|\frac{3}{-\omega^2 + j2\omega}\right| = 1$$

$\omega = 1.26$

phase crossover frequency

phase angle = $-90-\tan^{-1}\left(\frac{\omega}{2}\right) - 2\omega\left(\frac{180}{\pi}\right)$

phase = -180 ⇒ $-90 - \tan^{-1}\left(\frac{\omega}{2}\right) - 2\omega\left(\frac{180}{\pi}\right) = -180$

$\omega = 0.632$

(B) ( )

Gain margin

a = Gain = $\left|\frac{3e^{-2s}}{s(s+2)}\right|_{\omega=0.632} = 2.2631$

GM = $20\log\left(\frac{1}{a}\right) = -7.094dB$

Phase margin = 180 + [$-90 - \tan^{-1}\left(\frac{\omega}{2}\right) - 2\omega\left(\frac{180}{\pi}\right)$

$\omega = 1.26$

Phase margin = -86.59°

83. (A) (B) Area of region 1 = Area of region 2 = Area of region 3

$$2a \times \frac{1}{4} = 2 \times \frac{1}{8} + \frac{(1-a)}{4} \Rightarrow a = \frac{2}{3}$$

(B) ( ) Quantization noise power between –a to +a = $\int_{-\frac{2}{3}}^{\frac{2}{3}} x^2 p(x)\,dx$

$$= \int_{-\frac{2}{3}}^{\frac{2}{3}} \frac{x^2}{4}\,dx = \frac{x^3}{12}\int_{-\frac{2}{3}}^{\frac{2}{3}}$$

Quantization noise power = $\frac{4}{81}$

84. (A) (B)

V_{max} = 4V

V_{min} = 1V V_{max} = $|V_i| + |V_r|$ [V_i = Vincident, V_r = Vreflected]

V_{min} = $|V_i| - |V_r|$

V_i = 2.5, V_r = 1.5

$$K = \frac{V_r}{V_i} = \frac{1.5}{2.5} = 0.6$$

$$K = \frac{Z_R - 20}{Z_R + 20} = \frac{3}{5} = \frac{Z_R - 50}{Z_R + 50} \Rightarrow Z_R = 200\Omega$$

(B) (C) $K = 0.6 \rightarrow K = \frac{|V_r|}{|V_i|} = 0.6(+ve)$

85. (A) (A) y(2) = x(0) =2

y(4) = x(1) =1

Answer is (A)

(B) (C) y(2n) $\left\{\frac{1}{2}, \underset{\underset{n=0}{\uparrow}}{1}, 2, 1, \frac{1}{2}, \ldots\right\}$

Fourier transform of y(n) is $\sum_{n=-\infty}^{\infty} y(n) e^{-j\omega n}$

$$y(2n) \;\; \frac{1}{2}e^{j\omega} + 1 + 2e^{-j\omega} + e^{j2\omega} + \frac{1}{2}e^{-j3\omega}$$

$$y(2n) \;\; e^{j\omega}\left[\cos 2\omega + 2\cos\omega + 2\right]$$