CHAPTER 29
MEASUREMENTS

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29.1 STANDARDS AND ACCURACY

29.1.1 Standards

Measurement is the process by which a quantitative comparison is made between a standard and a measurand. The measurand is the particular quantity of interest—the thing that is to be quantified. The standard of comparison is of the same character as the measurand and, so far as mechanical engineering is concerned, the standards are defined by law and maintained by the National Institute of Science and Technology (NIST).* The four independent standards that have been defined are *length, time, mass* and *temperature.*[1] All other standards are derived from these four. Before 1960, the standard for length was the *international prototype meter,* kept at Sevres, France. In 1960, the meter was redefined as 1,650,763.73 wavelengths of krypton light. Then, in 1983, the 17th General Conference on Weights and Measures, adopted and immediately put into effect a new standard: "meter is the distance traveled in a vacuum by light in 1/299,792,458 seconds."[2] However, there is a copy of the international prototype meter, known as the *National Prototype Meter,* kept at the National Institute of Science and Technology. Below that level there are several bars known as National Reference Standards and below that there are the working standards. Interlaboratory standards in factories and laboratories are sent to the National Institute of Science and Technology for comparison with the working standards. These interlaboratory standards are the ones usually available to engineers.

Standards for the other three basic quantities have also been adopted by the National Institute of Science and Technology and accurate measuring devices for those quantities should be calibrated against those standards.

The *standard mass* is a cylinder of platinum–iridium, the international kilogram, also kept at Sevres, France. It is the only one of the basic standards that is still established by a prototype. In the United States, the basic unit of mass is the U.S. basic prototype kilogram No. 20. There are working copies of this standard that are used to determine the accuracy of interlaboratory standards. Force is not one of the fundamental quantities, but in the United States the standard unit of force is

*Formerly known as the "National Bureau of Standards."

Mechanical Engineers' Handbook, 2nd ed., Edited by Myer Kutz.
ISBN 0-471-13007-9

the pound, defined as the gravitational attraction for a certain platinum mass at sea level and 45° latitude.

Absolute time, or the time when some event occurred in history, is not of much interest to engineers. They are more likely to need to measure time intervals, that is, the time between two events. At one time the *second*, the basic unit for time measurements, was defined as 1/86400 of the average period of rotation of the earth on its axis, but that is not a practical standard. The period varies and the earth is slowing down. Consequently, a new standard based on the oscillations associated with a certain transition within the cesium atom has been defined and adopted. The second is now "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the fundamental state of cesium 133." [3] Thus, the cesium "clock" is the basic frequency standard, but tuning forks, crystals, electronic oscillators, and so on may be used as secondary standards. For the convenience of anyone who requires a time signal of a high order of accuracy, the National Institute of Science and Technology broadcasts continuously time signals of different frequencies from stations WWV, WWVB, and WWVL, located in Fort Collins, Colorado, and WWVH, located in Hawaii. Other nations also broadcast timing signals. For details on the time signal broadcasts, potential users should consult the National Institute of Science and Technology.[4]

Temperature is one of four fundamental quantities in the international measuring system. Temperature is fundamentally different in nature from length, time, and mass. It is an intensive quantity, whereas the others are extensive. Join two bodies that have the same temperature together and you will have a larger body at that same temperature. Join two bodies that have a certain mass and you will have one body of twice the mass of the original body. Two bodies are said to be at the same temperature if they are in thermal equilibrium. The *International Practical Temperature Scale* (IPTS-68), adopted in 1968 by the International Committee on Weights and Measurement,[5] is the one now in effect and the one with which engineers are primarily concerned. In this system, the kelvin (K) is the basic unit of temperature. It is 1/273.16 of the temperature at the triple point of water, which is the temperature at which the solid, liquid, and vapor phases of water exist in equilibrium. Degrees celsius (°C) are related to degrees kelvin by the equation

$$t = T - 273.15$$

where t = degrees celsius
T = degrees kelvin

Zero celsius is the temperature established between pure ice and air-saturated pure water at normal atmospheric pressure. The IPTS-68 established six primary fixed reference temperatures and procedures for interpolating between them. These are the temperatures and procedures used for calibrating precise temperature-measuring devices.

29.1.2 Accuracy and Precision

In measurement practice, four terms are frequently used to describe an instrument: *accuracy, precision, sensitivity,* and *linearity.* Accuracy, as applied to an instrument, is the closeness with which a reading approaches the true value. Since there is some error in every reading, the "true value" is never known. In the discussion of error analysis that follows later, methods of estimating the "closeness" with which the determination of a measured value approaches the true value will be presented. Precision is the degree to which readings agree among themselves. If the same value is measured many times and all the measurements agree very closely, the instrument is said to have a high degree of precision. It may not, however, be a very accurate instrument. Accurate calibration is necessary for accurate measurement. Measuring instruments must, for accuracy, be compared to a standard from time to time. These will usually be laboratory or company standards, which are in turn compared from time to time with a working standard at the National Institute of Science and Technology. This chain can be thought of as the pedigree of the instrument, and the calibration of the instrument is said to be traceable to NIST.

29.1.3 Sensitivity or Resolution

These two terms, as applied to a measuring instrument, refer to the smallest change in the measured quantity to which the instrument responds. Obviously, the accuracy of an instrument will depend to some extent on the sensitivity. If, for example, the sensitivity of a pressure transducer is one kilopascal, any particular reading of the transducer has a potential error of at least one kilopascal. If the readings expected are in the range of 100 kilopascals and a possible error of 1% is acceptable, then the transducer with a sensitivity of one kilopascal may be acceptable, depending upon what other sources of error may be present in the measurement. A highly sensitive instrument is difficult to use. Therefore, an instrument with a sensitivity significantly greater than that necessary to obtain the desired accuracy is no more desirable than one with insufficient sensitivity.

Many instruments in use today have digital readouts. For such instruments the concepts of sensitivity and resolution are defined somewhat differently than they are for analog-type instruments.

For example, the resolution of a digital voltmeter depends on the "bit" specification and the voltage range. The relationship between the two is expressed by the equation[6]

$$\epsilon = V/2^n$$

where V = voltage range
n = number of bits

Thus, an 8-bit instrument on a one-volt scale would have a resolution of 1/256, or 0.004 volts. On a ten-volt scale that would increase to 0.04 volts. As in analog instruments, the higher the resolution, the more difficult it is to use the instrument, so if the choice is available, one should take the instrument which just gives the desired resolution and no more.

29.1.4 Linearity

The calibration curve for an instrument does not have to be a straight line. However, conversion from a scale reading to the corresponding measured value is most convenient if it can be done by multiplying by a constant rather than by referring to a nonlinear calibration curve, or by computing from an equation. Consequently, instrument manufacturers generally try to produce instruments with a linear readout, and the degree to which an instrument approaches this ideal is indicated by its "linearity." Several definitions of "linearity" are used in instrument-specification practice.[7] So-called "independent linearity" is probably the most commonly used in specifications. For this definition, the data for the instrument readout versus the input are plotted and then a "best straight line" fit is made using the method of least squares. Linearity is then a measure of the maximum deviation of any of the calibration points from this straight line. This deviation can be expressed as a percentage of the actual reading or a percentage of the full scale reading. The latter is probably the most commonly used, but it may make an instrument appear to be much more linear than it actually is. A better specification is a combination of the two. Thus, linearity = $\pm A\%$ of reading or $\pm B\%$ of full scale, whichever is greater.

Sometimes the term *independent linearity* is used to describe linearity limits based on actual readings. Since both are given in terms of a fixed percentage, an instrument with $A\%$ proportional linearity is much more accurate at low reading values than an instrument with $A\%$ independent linearity.

It should be noted that although specifications may refer to an instrument as having $A\%$ linearity, what is really meant is $A\%$ nonlinearity. If the linearity is specified as independent linearity, the user of the instrument should try to minimize the error in readings by selecting a scale, if that option is available, such that the actual reading is close to full scale. Never take a reading near the low end of a scale if it can possibly be avoided.

For instruments that use digital processing, linearity is still an issue since the analog to digital converter used can be nonlinear. Thus linearity specifications are still essential.

29.2 IMPEDANCE CONCEPTS[7]

A basic question that must be considered when any measurement is made is how the measured quantity has been affected by the instrument used to measure it. Is the quantity the same as it would have been had the instrument not been there? If the answer to the question is no, the effect of the instrument is called "loading." To characterize the loading, the concepts of "stiffness" and "input impedance" are used. At the input of each component in a measuring system there exists a variable q_{i1}, which is the one we are primarily concerned with in the transmission of information. At the same point, however, there is associated with q_{i1} another variable q_{i2} such that the product $q_{i1}q_{i2}$ has the dimensions of power and represents the rate at which energy is being withdrawn from the system. When these two quantities are identified, the generalized input impedance Z_{gi} can be defined by

$$Z_{gi} = q_{i1}/q_{i2} \tag{29.1}$$

if q_{i1} is an "effort variable." The effort variable is also sometimes called the "across variable." The quantity q_{i2} is called the "flow variable" or "through variable."

The application of these concepts is illustrated by the example in Fig. 29.1. The output of the linear network in blackbox (*a*) is the open circuit voltage E_0 until the load Z_L is attached across the terminals *A–B*. If Thevenin's theorem is applied after the load Z_L is attached, the system in Fig. 29.1*b* is obtained. For that system the current is given by

$$i_m = E_0/[Z_{AB} + Z_L] \tag{29.2}$$

and the voltage E_L across Z_L is

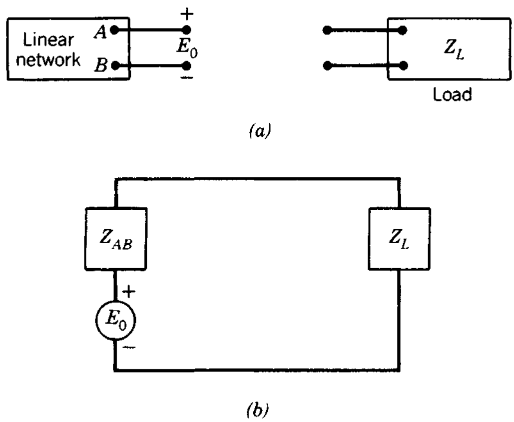

Fig. 29.1 Application of Thevenin's theorem.

$$E_L = i_m Z_L = E_0 Z_L / [Z_{AB} + Z_L]$$

or

$$E_L = E_0/[1 + Z_{AB}/Z_L] \tag{29.3}$$

In a measurement situation, E_L would be voltage indicated by the voltmeter, Z_L would be the input impedance of the voltmeter, and Z_{AB} would be the output impedance of the linear network. The true output voltage, E_0, has been reduced by the voltmeter, but it can be computed from the voltmeter reading if Z_{AB} and Z_L are known. From Eq. (29.3) it is seen that the effect of the voltmeter on the reading is minimized by making Z_L as large as possible.

If the generalized input and output impedances Z_{gi} and Z_{go} are defined for nonelectrical systems as well as electrical systems, Eq. (29.3) can be generalized to

$$q_{im} = q_{iu}/[1 + Z_{go}/Z_{gi}] \tag{29.4}$$

where q_{im} is the measured value of the effort variable and q_{iu} is the undisturbed value of the effort variable. The output impedance Z_{go} is not always defined or easy to determine; consequently, Z_{gi} should be large. If it is large enough, knowing Z_{go} is unimportant. However, Z_{go} and Z_{gi} can be measured[8] and Eq. 29.4 can be applied.

If q_{i1} is a flow variable rather than an effort variable (current is a flow variable, voltage an effort variable), it is better to define an input admittance

$$Y_{gi} = q_{i1}/q_{i2} \tag{29.5}$$

rather than the generalized input impedance

$$Z_{gi} = \text{effort variable/flow variable}$$

The power drain of the instrument is

$$P = q_{i1}q_{i2} = q_{i2}^2/Y_{gi} \tag{29.6}$$

Hence, to minimize power drain, Y_{gi} must be large. For an electrical circuit

$$I_m = I_u/[1 + Y_o/Y_i] \tag{29.7}$$

where I_m = measured current
I_u = actual current
Y_o = output admittance of the circuit
Y_i = input admittance of the meter

When the power drain is zero, as in structures in equilibrium—as, for example, when deflection is to be measured—the concepts of impedance and admittance are replaced with the concepts of "static stiffness" and "static compliance." Consider the idealized structure in Fig. 29.2.

To measure the force in member K_2, an elastic link with a spring constant K_m is inserted in series with K_2. This link would undergo a deformation proportional to the force in K_2. If the link is very soft in comparison with K_1, no force can be transmitted to K_2. On the other hand, if the link is very stiff, it does not affect the force in K_2 but it will not provide a very good measure of the force. The measured variable is an effort variable and in general when it is measured it is altered somewhat. To apply the impedance concept a flow variable whose product with the effort variable gives power is selected. Thus,

$$\text{flow variable} = \text{power/effort variable}$$

Mechanical impedance is then defined as force divided by velocity, or

$$Z = \text{force/velocity}$$

This is the equivalent of electrical impedance. However, if the static mechanical impedance is calculated for the application of a constant force, the impossible result

$$Z = \text{force}/0 = \infty$$

is obtained.

This difficulty is overcome if energy rather than power is used in defining the variable associated with the measured variable. In that case, the static mechanical impedance becomes the "stiffness" and

$$\text{stiffness} = S_g = \text{effort}/\int \text{flow } dt$$

In structures,

$$S_g = \text{effort variable/displacement}$$

When these changes are made the same formulas used for calculating the error caused by the loading of an instrument in terms of impedances can be used for structures by inserting S for Z. Thus

$$q_{im} = q_{iu}/(1 + S_{go}/S_{gi}) \tag{29.8}$$

where q_{im} = measured value of the effort variable
q_{iu} = undisturbed value of the effort variable
S_{go} = static output stiffness of the measured system
S_{gi} = static stiffness of the measuring system

For an elastic-force-measuring device such as a load cell, S_{gi} is the spring constant K_m. As an example, consider the problem of measuring the reactive force at the end of a propped cantilever beam, as in Fig. 29.3.

According to Eq. 29.8, the force indicated by the load cell will be

$$F_m = F_u/(1 + S_{go}/S_{gi})$$
$$S_{gi} = K_m \quad \text{and} \quad S_{go} = 3EI/L^3$$

The latter is obtained by noting that the deflection at the tip of a tip-loaded cantilever is given by

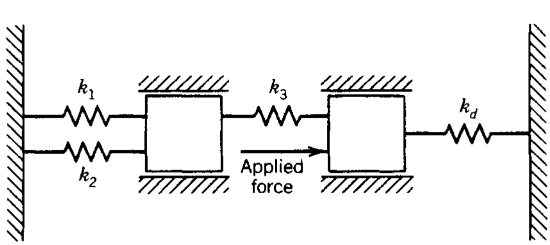

Fig. 29.2 Idealized elastic structure.

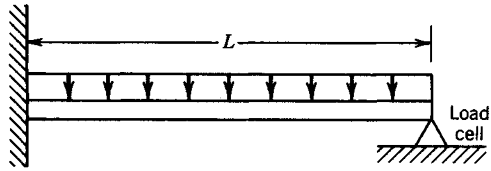

Fig. 29.3 Measuring the reactive force at the tip.

$$\delta = PL^3/3EI$$

where P = tip load
E = modulus of elasticity of the beam material
I = moment of inertia of the beam cross section

The stiffness is the quantity by which the deflection must be multiplied to obtain the force producing the deflection.

For the cantilever beam, then,

$$F_m = F_u/(1 + 3EI/K_mL^3) \tag{29.9}$$

or

$$F_u = F_m\,(1 + 3EI/K_mL^3) \tag{29.10}$$

Clearly, if $K_m >> 3EI/L^3$, the effect of the load cell on the measurement will be negligible.

To measure displacement rather than force, introduce the concept of compliance and define it as

$$C_g = \text{flow variable}/\textstyle\int \text{effort variable } dt$$

then

$$q_m = q_u/(1 + C_{go}/C_{gi}) \tag{29.11}$$

If displacements in an elastic structure are considered, the compliance becomes the reciprocal of stiffness, or the quantity by which the force must be multiplied to obtain the displacement caused by the force. The cantilever beam in Fig. 29.4 again provides a simple illustrative example.

If the deflection at the tip of this cantilever is to be measured using a dial gage with a spring constant K_m,

$$C_{gi} = 1/K_m \qquad \text{and} \qquad C_{go} = L^3/3EI$$

Thus,

$$\delta_m = \delta_u\,(1 + K_mL^3/3EI) \tag{29.12}$$

Not all interactions between a system and a measuring device lend themselves to this type of analysis. A pitot tube, for example, inserted into a flow field distorts the flow field but does not

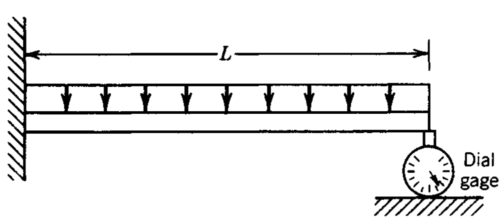

Fig. 29.4 Measuring the tip deflection.

extract energy from the field. Impedance concepts cannot be used to determine how the flow field will be affected.

There are also applications in which it is not desirable for a force-measuring system to have the highest possible stiffness. A subsoil pressure gage, for example, if it is much stiffer than the surrounding soil, will take a disproportionate share of the total load and will consequently indicate a higher pressure than would have existed in the soil if the gage had not been there.

29.3 ERROR ANALYSIS

29.3.1 Introduction

It may be accepted as axiomatic that there will always be errors in measured values. Thus, if a quantity X is measured, the correct value q and X will differ by some amount e. Hence,

$$\pm(q - X) = e$$

or

$$q = X \pm e \tag{29.13}$$

It is essential, therefore, in all measurement work that a realistic estimate of e be made. Without such an estimate, the measurement of X is of no value. There are two ways of estimating the error in a measurement. The first is the external estimate or ϵ_E, where $\epsilon = e/q$. This estimate is based on knowledge of the experiment and measuring equipment, and to some extent on the internal estimate ϵ_I.

The internal estimate is based on an analysis of the data using statistical concepts.

29.3.2 Internal Estimates

If a measurement is repeated many times, the repeat values will not, in general, be the same. Engineers, it may be noted, do not usually have the luxury of repeating measurements many times. Nevertheless, the standardized means for treating results of repeated measurements are useful, even in the error analysis for a single measurement.[9]

If some quantity is measured many times and it is assumed that the errors occur in a completely random manner, that small errors are more likely to occur than large errors, and that errors are just as likely to be positive as negative, the distribution of errors can be represented by the well-known bell-shaped error curve. The equation of the curve is

$$F(X) = Y_0 e^{-(X-\bar{X})/2\sigma^2} \tag{29.14}$$

where $F(X)$ = number of measurements for a given value of $(X - \bar{X})$
Y_0 = maximum height of the curve or the number of measurements for which $X = \bar{X}$
$\bar{X}$ = value of X at the point where maximum height of the curve occurs
σ determines the lateral spread of the curve

This curve is the normal, or Gaussian, frequency distribution. The area under the curve between X and δX represents the number of data points which fall between these limits and the total area under the curve denotes the total number of measurements made. If the normal distribution is defined so that the area between X and $X + \delta X$ is the probability that a data point will fall between those limits, the total area under the curve will be unity and

$$F(X) = \frac{\exp - (X - \bar{X})^2/2\sigma^2}{\sigma\sqrt{2\pi}} \tag{29.15}$$

and

$$P_x = \int_{-x}^{x} \frac{\exp - (X - \bar{X})^2/2\sigma^2}{\sigma\sqrt{2\pi}}\, dx \tag{29.16}$$

Now, if $\bar{X}$ is defined as the average of all the measurements and σ as the standard deviation, then

$$\sigma = [\Sigma\, (X - \bar{X})^2/N]^{1/2} \tag{29.17}$$

where N is the total number of measurements. Actually, this definition is used as the best estimate for a universe standard deviation, that is, for a very large number of measurements. For smaller subsets of measurements, the best estimate of σ is given by

$$\sigma = [\Sigma\,(X - \overline{X})^2/(n - 1)]^{1/2} \tag{29.18}$$

where n is the number of measurements in the subset. Obviously the difference between the two values of σ becomes negligible as n becomes very large (or as $n \rightarrow N$).

The probability curve based on these definitions is shown in Fig. 29.5.

The area under this curve between $-\sigma$ and $+\sigma$ is 0.68. Hence, 68% of the measurements can be expected to have errors that fall in the range of $\pm\sigma$. Thus, the chances are 68:32, or better than 2:1, that the error in a measurement will fall in this range. For the range $\pm 2\sigma$ the area is 0.95. Hence, 95% of all the measurement errors will fall in this range and the odds are about 20:1 that a reading will be within this range. The odds are about 384:1 that any given error will be in the range of $\pm 3\sigma$.

Some other definitions related to the normal distribution curve are:

1. *Probable error.* The error likely to be exceeded in half of all the measurements and not reached in the other half of the measurements. This error in Fig. 29.5 is about 0.67σ.
2. *Mean error.* The arithmetic mean of all the errors regardless of sign. This is about 0.8σ.
3. *Limit of error.* The error that is so large it is most unlikely ever to occur. It is usually taken as 4σ.

29.3.3 Use of Normal Distribution to Calculate the Probable Error in *X*

The foregoing statements apply strictly only if the number of measurements is very large. Suppose that n measurements have been made. That is, a sample of n data points out of an infinite number. From that sample, $\overline{X}$ and σ are calculated as above. How good are these numbers? To determine that, proceed as follows:

Let

$$\overline{X} = F\,(X_1, X_2, X_3, \ldots, X_n) = (\Sigma X_i)/n \tag{29.19}$$

$$e_{\overline{X}} = \sum_{i=1}^{n} \frac{\partial F}{\partial X_i}\, e_{x_i} \tag{29.20}$$

where $e_{\bar{x}}$ = the error in $\overline{X}$
e_{xi} = the error in X_i

$$(e_{\bar{x}})^2 = \sum_{i=1}^{n} (\partial F/\partial X_i e_{xi})^2 + \sum_{i=1,j=1}^{n} (\partial F/\partial X_i e_{xi})\,(\partial F/\partial X_j e_{xj}) \tag{29.21}$$

where $i \neq j$

If the errors e_i to e_n are independent and symmetrical, the cross-product terms will tend to disappear and

$$(e_{\bar{x}})^2 = \sum_{i=1}^{n} (\partial F/\partial X_i e_{xi})^2 \tag{29.22}$$

Since $\qquad \partial F/\partial X_i = 1/n$

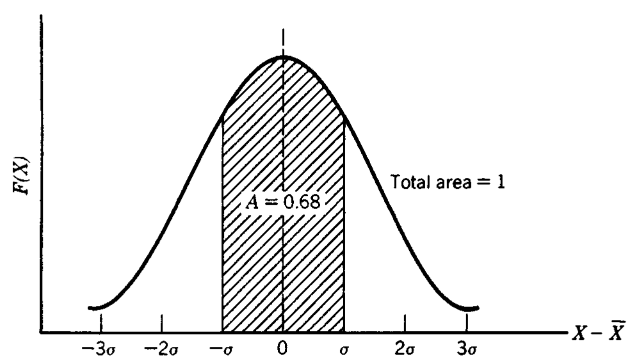

Fig. 29.5 Probability curve.

$$e_{\bar{x}} = \left[\sum_{i=1}^{n} (1/n)^2 e_{x_i}^2 \right]^{1/2} \tag{29.23}$$

or

$$e_{\bar{x}} = \left[(1/n)^2 \sum_{i=1}^{n} (e_{x_i})^2 \right]^{1/2} \tag{29.24}$$

from the definition of σ

$$\sum (e_{x_i})^2 = n\sigma^2 \tag{29.25}$$

and

$$e_{\bar{x}} = \sigma/\sqrt{n}$$

This equation must be corrected because the real errors in X are not known. If the number n were to approach infinity, the equation would be correct. Since n is a finite number, the corrected equation is written as

$$e_{\bar{x}} = \sigma/(n - 1)^{1/2} \tag{29.26}$$

and

$$q = \bar{X} \pm \sigma/(n - 1)^{1/2} \tag{29.27}$$

This says that if one reading is likely to differ from the true value by an amount σ, then the average of 10 readings will be in error by only $\sigma/3$ and the average of 100 readings will be in error by $\sigma/10$. To reduce the error by a factor of 2, the number of readings must be increased by a factor of 4.

29.3.4 External Estimates

In almost all experiments, several steps are involved in making a measurement. It may be assumed that in each measurement there will be some error, and if the measuring devices are adequately calibrated, errors are as likely to be positive as negative. The worst condition insofar as accuracy of the experiment is concerned would be for all errors to have the same sign. In that case, assuming the errors are all much less than one, the resultant error will be the sum of the individual errors, that is,

$$\epsilon_E = \epsilon_1 + \epsilon_2 + \epsilon_3 + \cdots \tag{29.28}$$

It would be very unusual for all errors to have the same sign. Likewise, it would be very unusual for the errors to be distributed in such a way that

$$\epsilon_E = 0$$

Following is a general method for treating problems that involve a combination of errors to determine what error is to be expected as a result of the combination:

Suppose that

$$V = F\,(a, b, c, d, e, \ldots, x, y, z) \tag{29.29}$$

where $a, b, c, \ldots, x, y, z$ represent quantities that must be individually measured to determine V.

$$\delta V = \sum_{n=a}^{z} (\partial F/\partial n)\delta n$$

$$\epsilon_E = \sum_{n=a}^{z} (\partial F/\partial n)e_n \tag{29.30}$$

The sum of the squares of the error contributions is given by

$$e_E^2 = \left(\sum_{n=a}^{z} (\partial F/\partial n)e_n \right)^2 \tag{29.31}$$

Now, as in the discussion of internal errors, assume that errors e_n are independent and symmetrical. This justifies taking the sum of the cross products as zero.

$$\sum_{n=a,m=a}^{z} (\partial F/\partial n)\,(\partial F/\partial m)\; e_n e_m = 0 \qquad (29.32)$$

$$n \neq m$$

Hence,

$$(e_E)^2 = \sum_{n=a}^{z} (\partial F/\partial N)^2 e_n^2$$

or

$$e_E = \left[\sum_{n=a}^{z} (\partial F/\partial n)^2 e_n^2\right]^{1/2} \qquad (29.33)$$

This is the "most probable value" of e_E. It is much less than the worst case

$$\epsilon_e = [|\epsilon_a| + |\epsilon_b| + |\epsilon_c| + \ldots + |\epsilon_g|] \qquad (29.34)$$

As an application, the determination of g, the local acceleration of gravity, by use of a simple pendulum will be considered

$$g = 4\pi^2 L/T^2 \qquad (29.35)$$

where L = the length of the pendulum
T = the period of the pendulum

If an experiment is performed to determine g, the length L and the period T would be measured. To determine how the accuracy of g will be influenced by errors in measuring L and T, write

$$\partial g/\partial L = 4\pi^2/T^2 \qquad \text{and} \qquad \partial g/\partial T = -8\pi^2 L/T^3 \qquad (29.36)$$

The error in g is the variation in g, written as follows:

$$\delta g = (\partial g/\partial L)\,\Delta L + (\partial g/\partial T)\,\Delta T \qquad (29.37)$$

or

$$\delta g = (4\pi^2/T^2)\,\Delta L - (8\pi^2 L/T^3)\,\Delta T \qquad (29.38)$$

It is always better to write the errors in terms of percentages. Consequently, Eq. (29.38) is rewritten

$$\delta g = (4\pi^2 L/T^2)\,\Delta L/L - 2(4\pi^2 L/T^2)\,\Delta T/T \qquad (29.39)$$

or

$$\delta g/g = \Delta L/L - 2\Delta T/T \qquad (29.40)$$

then

$$e_g = [e_L^2 + (2e_T)^2]^{1/2} \qquad (29.41)$$

where e_g is the "most probable error" in the measured value of g. That is to say,

$$g = 4\pi^2 L/T^2 \pm e_g \qquad (29.42)$$

where L and T are the measured values. Note that even though a positive error in T causes a negative error in the calculated value of g, the contribution of the error in T to the "most probable error" is taken as positive. Note also that an error in T contributes four times as much to the "most probable error" as an error in L contributes. It is fundamental in measurements of this type that those quantities which appear in the functional relationship raised to some power greater than unity contribute more heavily to the "most probable error" than other quantities and must, therefore, be measured with greater care.

The determination of the "most probable error" is simple and straightforward. The question is, how are the errors, such as $\Delta L/L$ and $\Delta T/T$, determined. If the measurements could be repeated often enough, the statistical methods discussed in the internal error evaluation could be used to arrive at a value. Even in that case it would be necessary to choose some representative error, such as the standard deviation or the mean error. Unfortunately, as was noted previously, in engineering experiments it usually is not possible to repeat measurements enough times to make statistical treatments meaningful. Engineers engaged in making measurements will have to use what knowledge they have of the measuring instruments and the conditions under which the measurements are made to make a reasonable estimate of the accuracy of each measurement. When all of this has been done and a "most probable error" has been calculated, it should be remembered that the result is not the actual error in the quantity being determined, but is, rather, the engineer's best estimate of the magnitude of the uncertainty in the final result.[10,11]

Consider again the problem of determining g. Suppose that the length L of the pendulum has been determined by means of a meter stick with 1-mm calibration marks and the error in the calibration is considered negligible in comparison with other errors. Suppose the value of L is determined to be 91.7 cm. Since the calibration marks are 1 mm apart, it can be assumed that ΔL is no greater than 0.5 mm. Hence, maximum $\Delta L/L = 5.5 \times 10^{-4}$. Suppose T is determined with the pendulum swinging in a vacuum with an arc of $\pm 5°$ using a stopwatch that has an inherent accuracy of 1 part in 10,000. (If the arc is greater than $\pm 5°$, a nonisochronous swing error enters the picture.) This means that the error in the watch reading will be no more than 10^{-4} sec. However, errors are introduced in the period determination by human error in starting and stopping the watch as the pendulum passes a selected point in the arc. This error can be minimized by selecting the highest point in the arc because the pendulum has zero velocity at that point, and timing a large number of swings so as to spread the error out over that number of swings. Human reaction time may vary from as low as 0.2 sec to as high as 0.7 sec. A value of 0.5 sec will be assumed. Thus, the estimated maximum error in starting and stopping the watch will be 1 sec (± 0.5 sec at the start and ± 0.5 sec at the stop). A total of 100 swings will be timed. Thus, the estimated maximum error in the period will be 1/100 sec. If the period is determined to be 1.92 sec, the estimated maximum error will be $0.01/1.92 = .005$. Compared to this, the error in the period due to the inherent inaccuracy of the watch is negligible. The nominal value of g calculated from the measured values of L and T is 982.03 cm/sec^2. The "most probable error" is

$$[4(0.005)^2 + (5.5 \times 10^{-4})^2]^{1/2} = 0.01 \tag{29.43}$$

The uncertainty in the value of g is then ± 9.82 cm/sec^2, or, in other words, the value of g will be somewhere between 972.21 and 991.85 cm/sec^2.

Often it is necessary for the engineer to determine in advance how accurately the measurements must be made in order to achieve a given accuracy in the final calculated result. For example, in the pendulum problem it may be noted that the contribution of the error in T to the "most probable error" is more than 300 times the contribution of the error in the length measurement. This suggests, of course, that the uncertainty in the value of g could be greatly reduced if the error in T could be reduced. Two possibilities for doing this might be (1) to find a way to do the timing that does not involve human reaction time, or (2) if that is not possible, to increase the number of cycles timed. If the latter alternative is selected and the other factors remain the same, the error in T, timed over 200 swings, is 1/200 or 0.005 sec. As a percentage the error is $0.005/1.92 = 0.0026$. The "most probable error" in g then becomes

$$e_g = [4 \times (2.6 \times 10^{-3})^2 + (5.5 \times 10^{-4})^2]^{1/2} = 0.005 \tag{29.44}$$

This is approximately one-half of the "most probable error" in the result obtained by timing just 100 swings. With this new value of e_g, the uncertainty in the value of g becomes ± 4.91 cm/sec^2 and g then can be said to be somewhere between 977.12 and 986.94 cm/sec^2. The procedure for reducing this uncertainty still further is now self-evident.

Clearly the value of this type of error analysis depends upon the skill and objectivity of the engineer in estimating the errors in the individual measurements. Such skills are acquired only by practice and careful attention to all the details of the measurements.

29.4 APPENDIX

29.4.1 Vibration Measurement

See Section 23.3.

29.4.2 Acceleration Measurement

For acceleration measurements see Section 23.4. For the measurement of mechanical shock and vibration and acceleration see Chapter 23 on vibration and shock of this handbook.

29.4.3 Shock Measurement

See Section 23.5.

29.4.4 Sound Measurement

Introduction

Sound[12] is an oscillation about the mean of pressure or stress in a fluid or solid medium. Sound is also an auditory sensation produced by those oscillations. Acoustics[12] is the science of sound, which includes its generation, transmission, and effects. Most appliances, machines, and vehicles generate unwanted sound, called acoustic noise.[12] It is the purpose here to present methods for the accurate measurement of acoustic noise as a means for determining noise exposure of people in work spaces, as an aid in reducing noise, and for its use in diagnosing vibration problems.

Sound pressure is commonly determined by a microphone that converts pressure to an electrical signal, which is processed and presented on a visual display. Such devices are known as *sound level meters.* Three types of handheld meters, specified in an American National Standard,[13] are commercially available. The type required depends on the intended use. For example, the type 1 meter is required for precision laboratory measurements, while the type 3 is intended for sound surveys where high accuracy is unwarranted. Sound level meters are calibrated to give accurate measures of sound pressure and means are provided in some cases for a calibration check before making measurements.

Because of the extremely large range of pressures that can be sensed by the human ear, sound pressures are usually measured and expressed in logarithmic units. The *decibel notation* is used and the measured quantity is compared to a reference pressure. Logarithmic qualities are referred to as *levels. Sound pressure level* (SPL) is defined as follows and denoted by the symbol *Lp*

$$\mathrm{Lp} = 20 \log_{10} \frac{P_x}{P_{ref}} \tag{29.45}$$

where P_x is the measured pressure and P_{ref} is 20×10^{-6} Pa (N/m). The units of Lp are dB referenced to 20 μPa.

Since acoustic noise has a major impact on people, measurements are usually made in the frequency range of hearing. This is normally taken as 20 Hz to 20 kHz. The knowledge of the distribution of sound energy with frequency is important when dealing with noise impact on people and using radiated sound to identify machine vibrations. Thus, sound level meters have some means of determining the frequency distribution of the spectral components of sound pressure.

The basic configuration of a sound level meter is that shown in Fig. 29.6. The weighting networks are used to shape the noise spectrum so that a measure that relates to the hearing characteristic can be made. The display may be an analog meter, numeric display, or digital read out. The frequency-versus-amplitude characteristics of the weighting networks are shown in Fig. 29.7. The A weighting that produced a level referred to as L_A in dBA is shaped as approximately the inverse of the loudness contour of the ear at the threshold of hearing. The B and C weightings are matched to successively

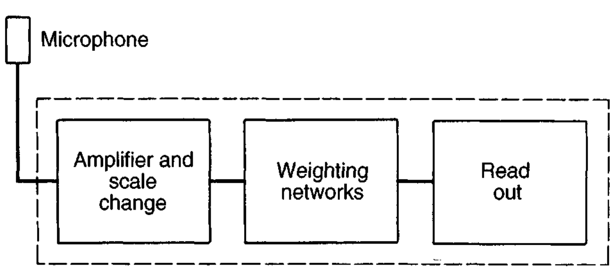

Fig. 29.6 Sound level meter.

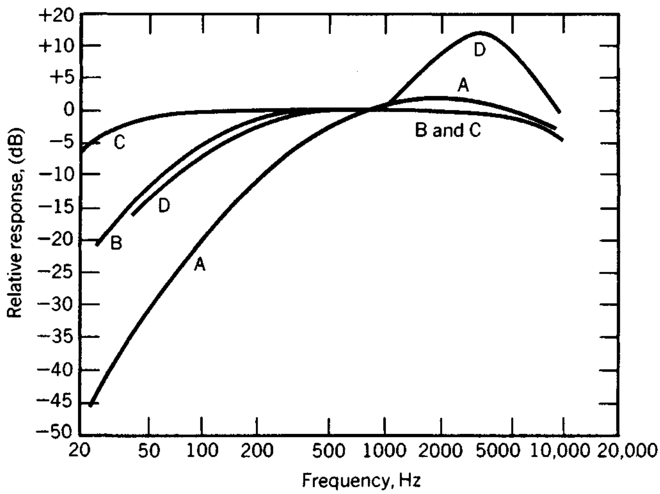

Fig. 29.7 Frequency characteristics for weighting functions of sound level meters.

higher loudness levels; however, most noise regulations require data to be presented in dBA. Some meters may have the D weighting, which is used as a measure of annoyance due to noise from aircraft flyovers. Some meters may have a "flat" weighting position, which eliminates the weighting networks. Then the frequency response is limited only by the characteristics of the microphone and amplifiers. In addition, some meters provide for an external filter or spectrum analyzer for a more complete frequency analysis.

Several other types of sound level meters are available that process the acoustic signal for special purposes. There are meters that will respond to instantaneous peak pressures and "hold" for convenient readout. Some have a response function that gives increased accuracy for impulsive repetitive noise. Others will integrate over time to give an equivalent sound level (L_{eq}) for noise with large and random variations with time. A final type worn by a worker gives a continuous readout of the accumulated dose of excessive noise.

Measurements in Open Spaces[14]

In this situation, the sound source is usually not inside a building and the acoustic waves propagate freely over appreciable distances. The measuring microphone is not in an enclosure. The noise impact may concern

1. Community noise
2. Transportation noise
3. Industrial noise

In open spaces, the microphone will sense the pressure produced by a passing acoustic wave. At high frequencies, when the microphone size is an appreciable fraction of the acoustic wavelength, waves are diffracted by the microphone and it becomes directional. For a 1-in. diameter microphone, these effects occur at frequencies about 2 kHz and higher. Under these conditions, one should use a microphone designed for *free-field* conditions[15] and correct for angle of wave arrival.

An accurate measure of the sound pressure level of an acoustic wave cannot be made when there are reflecting surfaces nearby. An operator holding a sound level meter can cause reflections. Holding the microphone at arm's length to the side of the body minimizes this problem; however, a microphone on a remote cable placed on the ground is recommended.

Large plane surfaces near the propagation path or behind the operator can cause serious interference effects. At a particular frequency, a direct and reflected path difference of one half-wavelength can cause almost complete cancellation. Thus, errors will depend on the frequency content of the sound wave.

The ground is a large reflecting surface and is usually present. However, if the microphone is placed directly on the ground, only the direct wave will be measured. A relatively open space without diffracting objects with the microphone placed a very few inches above the surface is recommended for accurate measurements.

Open-space measurements usually imply exposure to all the elements of weather. Wind speeds of a few miles per hour can cause microphone noise that will mask the acoustic signal to be measured. *Wind screens* made of porous materials greatly reduce this effect. Ordinary sound level meters should not be exposed to temperature and moisture extremes. However, microphone and measuring systems can be obtained that operate over extreme weather conditions.

The sound radiated from a small sound source, one whose physical dimensions are small compared to the acoustic wavelength, spreads or diverges in all directions. This divergence is spherical, which results in the sound pressure decaying inversely with distance. The SPL will decrease 6 dB when the measuring distance is doubled. Acoustically large sources will not radiate equally well in all directions. However, in any radial direction the SPL will still decay 6 dB with a doubling of distance.

When acoustic measurements are to be made in an out-of-doors location, a sound pressure level will be present due to local and remote sound sources. This level is referred to as the *ambient*, and it will provide a lower limit for other measurements. The energy adds in unrelated or uncorrelated sound fields. Thus, when the noise to be measured has a sound pressure equal to the ambient, the measured SPL will be 3 dB above the ambient.

When the ambient SPL and the addition of a sound source raises the SPL above the ambient, a correct source level can be obtained. Since the energy in an acoustic wave is proportional to sound pressure squared, the correct pressure squared is

$$P_s^2 = P_m^2 - P_a^2 \tag{29.46}$$

where P_s is the correct source pressure, P_m is the measured pressure, and P_a is the ambient. P can be obtained from Lp as follows:

$$P = P_{\text{ref}} 10^{(Lp/20)} \tag{29.47}$$

where P_{ref} is 20 μPa. The corrected Lp is

$$(\text{Lp})_s = 20 \log^{10} \frac{P_s}{P_{\text{ref}}} \tag{29.48}$$

When it is necessary to do a significant analysis on a sound source, a simple level correction as above is not adequate. When the frequency distribution of energy and statistical amplitude distributions are desired, the sound pressure level should be a least 20 dB above the ambient noise. Thus, a measuring distance must be chosen to ensure this signal-to-noise ratio.

Measurement in Enclosed Spaces[14]

In many noisy situations, a sound source radiates into a completely enclosed space, such as a room with closed doors and windows. Since sound waves travel at about 330 m/sec, it only takes a few milliseconds for a large number of acoustic rays to be traversing the paths between reflecting surfaces and filling the room with a *reverberant sound field*. The sound pressure level continues to increase until the absorbing surfaces in the room absorb the acoustic energy at the same rate as the source is supplying it. Thus, there will be a region near the source where the directly radiated sound will dominate over the reverberant sound, and the pressure will decay inversely with distance. At some distance, determined by the acoustic absorption in the room, the direct sound will equal the reverberant sound. In the rest of the room the reverberant sound will be relatively uniform and independent of the distance from the source. An exception to this uniformity occurs when large single-frequency components produce dominant standing waves in the room. More details on acoustic characteristics of rooms can be found in the acoustic literature.[16,17]

The sound pressure levels measured in enclosed spaces depend on many factors. The sound sources and the acoustical absorption in the room will determine the levels in the direct field close to the sources and in the reverberant field. All the sources in the room will contribute to the reverberant field. If the sound pressure level near a noisy machine is greater than the reverberant sound pressure level, the measurement is characteristic of that machine. If it is lower, no information is obtained. If other noise sources can be turned off, a single machine can be studied.

When the noise exposure of workers in a noisy environment is to be determined, it is essential to measure the sound pressure level at the work station at ear level. If the worker stays at one place, a single measurement may suffice. However, for a mobile worker, an acoustic dosimeter worn by the individual is essential.

Acoustic Intensity—Diagnosis of Vibrations

The acoustic power radiated by a noisy machine and the frequency spectrum of that power are very useful quantities. When the directive nature of radiated power is known, the source pressure level in the vicinity of such a machine placed in an open space can be determined. When such a machine is in an enclosed space, the reverberant sound field can be predicted. In this case, the total absorption in the room controls the reverberant field and the directive nature is not needed.

When the spectral content of the sound power of a noisy machine is measured, sources of vibrations may be located. Rotating unbalanced parts, oscillating structures, and repetitive impacts will cause vibration of machine surfaces that radiate sound. The period of a dominant frequency noise component can be related to the rotational, oscillating, or repetition frequency or a harmonic of that frequency. Such correlation can be used to locate problems due to excess wear in an old machine and locating the trouble areas in the design of quiet machines. For example, worn bearings and gears can be located by the characteristic noise they generate.

Vibrational diagnosis can be done with a single microphone used to measure sound pressure. The "ac" signal out of a sound level meter may be directed to a spectrum analyzer or filter set for this purpose. The determination of sound power, however, requires the quantitative measure of acoustic intensity. Intensity is the vector product of the real part of acoustic pressure times acoustic particle velocity:

$$\mathrm{I} = \mathrm{Re}\,(p\mathrm{v})\mathrm{w/m^2} \tag{29.49}$$

Intensity is thus the power per unit area flowing through a surface surrounding a noisy machine. Since intensity is a directed quantity, the component in the direction away from the machine will be radiated as acoustic power. When the machine surface is subdivided into small areas as in Fig. 29.8, the average intensity times that area plus the products for the rest of the small areas give total power radiated:

$$w = \sum_{i}^{n} I_i A_i \ \mathrm{w} \tag{29.50}$$

Small areas should be selected that have small intensity variations; then the average value can be used to calculate power flow through the area.

Since measurements can be made near to a noise source and the product of pressure and velocity tend to discriminate against uncorrelated signals, sound power determination can be made in enclosed spaces. This is particularly advantageous for large machines that cannot be placed in anechoic rooms or reverberation chambers.

The acoustic variables are measured by two microphones, as shown in Fig. 29.9. The average of the two measured pressures is the pressure between them. The acoustic partial velocity in the direction of the microphone axes is related to the pressure gradient. Then

$$v_x = \frac{1}{\rho_0 d} \int (p_1 - p_2)\, dt \ \mathrm{m/sec} \tag{29.51}$$

where ρ_0 is air density and d is the microphone spacing. Then the average intensity in the x direction becomes

$$I_x = \frac{1}{T} \int_0^T p_{\mathrm{av}}\, v_x\, dt \tag{29.52}$$

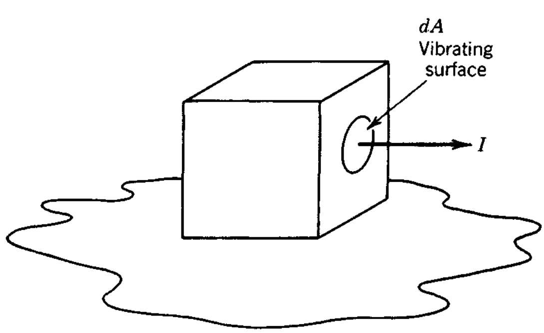

Fig. 29.8 Intensity and power measurements.

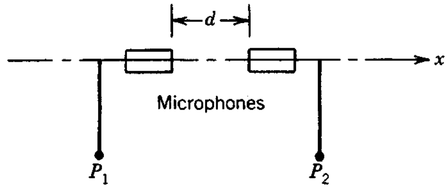

Fig. 29.9 Intensity measuring probe.

where averaging time T is long compared to the time scale of p and v. Some commercial instruments measure intensity in this way.

When a two-channel spectrum analyzer is available, intensity can be determined from the two microphone signals in another way. The frequency-domain representation of intensity can be obtained by the Fourier transform of the two pressure signals.[18,19]

$$I_x(f) = \frac{\mathrm{Im}(S_{12})}{2\rho_0 d\omega} \tag{29.53}$$

where $\mathrm{Im}(S_{12})$ is the imaginary part of the cross-spectral density $S_{12} = P_1(f)\, P_2^*(f)$ (* designates the complex conjugate). The angular frequency is ω. The frequency-domain representation is very informative, since the frequency of predominant energy components is displayed. Vibration diagnosis is then readily accomplished.

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