Other reasons that have been given for the lack of general adoption for AP models include:

1. The AP modeling approach is viewed as a top-down process, whereas many organizations operate AP as a bottom-up process.
2. The assumption used in many of the models, such as linear cost structures, the aggregation of all production into a common measure, or that all workers are equal, are too simplistic or unrealistic.
3. Data requirements are too extensive or costly to obtain and maintain.
4. Decision-makers are intimidated or unwilling to deal with the complexity of the models' formulations and required analyses.

Given this, therefore, it is not surprising that few modeling approaches have been adopted in industrial settings. Although research continues on AP, there is little to indicate any significant modeling breakthrough in the near future that will dramatically change this situation.

One direction, however, is to recognize the hierarchical decision-making structure of AP and to design modeling approaches that utilize it. These systems may be different for different organizations and will be difficult to design, but currently appear to be one approach for dealing with the complexity necessary in the aggregate planning process if a modeling approach is to be followed. For a comprehensive discussion of hierarchical planning systems, see Ref. 33.

### 32.5 MATERIALS REQUIREMENTS PLANNING

Materials requirements planning (MRP) is a procedure for converting the output of the aggregate planning process, the master production schedule, into a meaningful schedule for releasing orders for component inventory items to vendors or to the production department as required to meet the delivery requirements of the master production schedule.

Materials requirements planning is used in situations where the demand for a product is irregular and highly varying as to the quantity required at a given time. In these situations, the normal inventory models for quantities manufactured or purchased do not apply. Recall that those models assume a constant demand and are inappropriate for the situation where demand is unknown and highly variable. The basic difference between the independent and dependent demand systems is the manner in which the product demand is assumed to occur. For the constant demand case, it is assumed that the daily demand is the same. For dependent demand, a forecast of required units over a planning horizon is used. Treating the dependent demand situation differently allows the business to maintain a much lower inventory level in general than would be required for the same situation under an assumed constant demand. This is so because the average inventory level will be much less in the case where MRP is applied. With MRP, the business will procure inventory to meet high demand just in advance of the requirement and at other times maintain a much lower level of average inventory.

## Definitions

Available units. Units of stock that are in inventory and are not in the category of buffer or safety stock and are not otherwise committed.
Gross requirements. The quantity of material required at a particular time that does not consider any available units.
Inventory unit. A unit of any product that is maintained in inventory.
Lead time. The time requirement for the conversion of inventory units into required subassemblies or the time required to order and receive an inventory unit.
MRP. Materials Requirements Planning: a method for converting the end item schedule for a finished product into schedules for the components that make up the final product.
MRP-II. Manufacturing Resources Planning: a procedural approach to the planning of all resource requirements for the manufacturing firm.
Net requirements. The units of a requirement that must be satisfied by either purchasing or manufacturing.
Product structure tree. A diagram representing the hierarchical structure of the product. The trunk of the tree would represent the final product as assembled from the subassemblies and inventory units that are represented by level one, which come from sub-subassemblies, and inventory units that come from the second level, and so on ad infinitum.
Scheduled receipts. Maierial that is scheduled to be delivered in a given time bucket of the planning horizon.
Time bucket. The smallest distinguishable time period of the planning horizon for which activities are coordinated.

### 32.5.1 Procedures and Required Inputs

The master production schedule is devised to meet the production requirements for a product during a given planning horizon. It is normally prepared from fixed orders in the short run and product
requirements forecasts for the time past that for which firm product orders are available. This master production schedule, together with information regarding inventory status and the product structure tree and/or the bill of materials, are used to produce a planned order schedule. An example of a master production schedule is shown in Table 32.9.

The MRP schedule is the basic document used to plan the scheduling of requirements for meeting the MPS. An example is shown in Table 32.10. Each horizontal section of this schedule is related to a single product, part, or subassembly from the product structure tree. The first section of the first form would be used for the parent product. The following sections of the form and required additional forms would be used for the children of this parent. This process is repeated until all parts and assemblies are listed.

To use the MRP schedule, it is necessary to complete a schedule first for the parent part. Upon completion of this level zero schedule, the "bottom line" becomes the input into the schedule for each child of the parent. This procedure is followed until each component, assembly, or purchased part has been scheduled for ordering or production in accordance with the time requirements and other limitations that are imposed by the problem parameters. It should be noted that if a part is used at more than one place in the assembly or manufacture of the final product, it has only one MRP schedule, which is the sum of the requirements at the various levels. The headings of the MRP schedule are as follows:

Item code. The company-assigned designation of the part or subassembly as shown on the product structure tree or the bill of materials.
Level code. The level of the product structure tree at which the item is introduced into the process. The completed product is designated level 0 , subassemblies or parts that go together to make up the completed product are level 1 , sub-subassemblies and parts that make up level 1 subassemblies are level 2, etc.
Lot size. The size of the lot that is purchased when an order is placed. This quantity may be an economic order quantity or a lot-for-lot purchase. (This later expression is used for a purchase quantity equal to the number required and no more.)
Lead time. The time required to receive an order from the time the order is placed. This order may be placed internally for manufacturing or externally for purchase.
On hand. The total of all units of stock in inventory.
Safety stock. Stock on hand that is set aside to meet emergency requirements.
Allocated (stock). Stock on hand that has been previously allocated for use, such as for repair parts for customer parts orders.

The rows related to a specific item code are designated as follows:
Gross requirements. The unit requirements for the specific item code in the specific time bucket, which are obtained from the MPS for the level 0 items. For item codes at levels other than level 0 , the gross requirements are obtained from the planned order releases for the parent item. Where an item is used at more than one level in the product, its gross requirements would be the summation of the planned order releases of the items containing the required part.
Scheduled receipts. This quantity is defined at the beginning of the planning process for products that are on order at that time. Subsequently it is not used.
Available. Those units of a given item code that are not safety stock and are not dedicated for other uses.

## Table 32.9 Example of a Master Production Schedule for a Given Product

| Part <br> Number | Quantity <br> Needed | Due <br> Date |
| :---: | :---: | ---: |
| A000 | 25 | 3 |
| A000 | 30 | 5 |
| A000 | 30 | 8 |
| A000 | 30 | 10 |
| A000 | 40 | 12 |
| A000 | 40 | 15 |

Table 32.10 Example MRP Schedule Format

| Item Code | Level Code | $\begin{aligned} & \text { Lot } \\ & \text { Size } \end{aligned}$ | Lead Time (weeks) | On Hand | Safety Stock | Allocated |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Gross requirements Scheduled receipts Available <br> Net requirements <br> Planned order receipts <br> Planned order releases |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases |  |  |  |  |  |  |  |  |  |  |  |  |



Fig. 32.6 Diagram of model car indicating all parts. (From Ref. 34.)

Net requirements. For a given item code, this is the difference between gross requirements and the quantity available.
Planned order receipts. An order quantity sufficient to meet the net requirements, determined by comparing the net requirements to the lot size (ordering quantity) for the specific item code. If the net requirements are less than the ordering quantity, an order of the size as shown as the lot size will be placed; if the lot size is LFL (lot-for-lot), a quantity equal to the net requirements will be placed.
Planned order releases. This row provides for the release of the order discussed in planned order receipts, to be released in the proper time bucket such that it will arrive appropriately to meet the need of its associated planned order receipt. Note also that this planned order release provides the input information for the requirements of those item codes that are the children of this unit in subsequent generations if such generations exist in the product structure.
Example Problem 32.6 (From Ref. 34, pp. 239-240)
If you were a Cub Scout, you may remember building and racing a little wooden race car. Such cars come 10 in a box. Each box has 10 preformed wood blocks, 40 wheels, 40 nails for axles, and a sheet of 10 vehicle number stickers. The problem is the manufacture and boxing of these race-car kits. An assembly explosion and manufacturing tree are given in Figs. 32.6 and 32.7.


Fig. 32.7 Product structure tree. (From Ref. 34.)

Studying the tree indicates four operations. The first is to cut 50 rough car bodies from a piece of lumber. The second is to plane and slot each car body. The third is to bag 40 nails and wheels. The fourth is to box materials for 10 race cars.

The information from the production structure tree for the model car, together with available information regarding lot sizes, lead time, and stock on hand, is posted to the MRP schedule format to provide information for analysis of the problem. In the problem, no safety stock was prescribed and no stock was allocated for other use.

This information allowed the input into the MRP format of all information shown below for the eight item codes of the product. The single input into the right side of the problem format is the MPS for the parent product, A000.

With this information, each of the values of the MPR schedule can be calculated. It should be noted that the output (planned order releases) of the level 0 product multiplied by the requirements per parent unit (as shown in parenthesis at the top right corner of the "child" component in the product structure tree) becomes the "gross requirements" for the (or each) "child" of the parent part.

### 32.5.2 Calculations

As previously stated, the gross requirements come either from the MPS (for the parent part) or the calculation of the planned order releases for the parent part multiplied by the per-unit requirement of the current child, per parent part. The scheduled receipts are receipts scheduled from a previous MRP plan. The available units are those on hand from a previous period plus the scheduled receipts from previous MRP. The net requirements are gross requirements less the available units. If this quantity is negative, indicating that there is more than enough, it is set to zero. If it is positive, it is necessary to include an order in a previous period of quantity equal to or greater than the lot size, sufficient to meet the current need. This is accomplished by backing up a number of periods equal to the lead time for the component and placing an order in the planned order releases now that it is equal to or greater than the lot size for the given component.

It should be noted that scheduled receipts and planned order receipts are essentially the arrival of product. The distinction between the two is that scheduled receipts are orders that were made on a previous MRP plan. The planned order receipts are those that are scheduled on the current plan.

Further, in order to keep the system operating smoothly, the MRP plan must be reworked as soon as new information becomes available regarding demand for the product for which the MPS is prepared. This essentially, provides an ability to respond and to keep materials in the "pipeline" for delivery.

Without updating, the system becomes cumbersome and unresponsive. For example, most of the component parts are exhausted at the end of the 15 -week period; hence, to respond in the 16th week would require considerable delay if the schedule were not updated.

The results of this process are shown in Tables 32.11, 32.12, and 32.13.
The planned order release schedule (Table 32.14) is the result of the MRP procedure. It is essentially the summation of the bottom lines for the individual components from the MRP schedules. It displays an overall requirement for meeting the original master production schedule.

### 32.5.3 Conclusions on MRP

It should be noted that this process is highly detailed and requires a large time commitment for even a simple product. It becomes intractable for doing by hand in realistic situations. Computerized MRP applications are available that are specifically designed for certain industries and product groups. It is suggested that should more information be required on this topic, the proper approach would be to contact software suppliers for the appropriate computer product.

### 32.5.4 Lot-Sizing Techniques

Several techniques are applicable to the determination of the lot size for the order. If there are many products and some components are used in several products, it may be that demand for that common component is relatively constant. If that is the case, EOQ models such as those used in the topic on inventory can be applied.

The POQ (periodic order quantity) is a variant of the EOQ where a nonconstant demand over a planning horizon is averaged. This average is then assumed to be the constant demand. Using this value of demand, the EOQ is calculated. The EOQ is divided into the total demand if demand is greater than EOQ. This resultant figure gives the number of inventory cycles for the planning horizon. The actual forecast is then related to the number of inventory cycles and the order sizes are determined.

## Example Problem 32.7

The requirement for a product that is purchased is given in Table 32.15. Assume that holding cost is $\$ 10$ per unit year and order cost is $\$ 25$. Calculate the POQ; no shortage is permitted.

Using the basic EOQ formula:

Table 32.11

| Item Code | Level Code | Lot Size | Lead Time (weeks) | On Hand | Safety Stock | Allocated |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A000 | 0 | 50 | 1 | 20 | 0 | 0 | Gross requirements Schedule receipts Available <br> Net requirements Planned order receipts Planned order releases | 20 | 20 <br> 50 | $\begin{array}{r} 25 \\ \hline 20 \\ 5 \\ 50 \end{array}$ | 45 | $\begin{aligned} & \frac{30}{45} \\ & \frac{1}{-} \end{aligned}$ | 15 | 15 <br> 50 | $\begin{aligned} & 30 \\ & -15 \\ & 15 \\ & 50 \end{aligned}$ | 35 | $\begin{aligned} & \frac{30}{35} \\ & - \\ & - \end{aligned}$ | $-5$ $50$ | $\begin{array}{r} 40 \\ -5 \\ 35 \\ 50 \end{array}$ | 15 | 15 <br> 50 | $\begin{aligned} & 40 \\ & 15 \\ & 25 \\ & 50 \end{aligned}$ |
| A100 | 1 | 50 | 1 | 100 | 0 | 0 | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases | $\begin{aligned} & 100 \\ & 400 \end{aligned}$ | $\begin{aligned} & 500 \\ & \\ & 100 \\ & 400 \\ & 400 \end{aligned}$ | 0 | 0 | 0 | $\begin{array}{r} 0 \\ 500 \end{array}$ | $\begin{array}{r} 500 \\ 0 \\ 500 \\ 500 \end{array}$ | 0 | 0 | $\begin{array}{r} 0 \\ 500 \end{array}$ | $\begin{array}{r} 500 \\ 0 \\ 500 \\ 500 \end{array}$ | 0 | $\begin{array}{r} 0 \\ 500 \end{array}$ | 500 0 500 500 | 0 |
| A300 | 1 | 50 | 1 | 150 | 0 | 0 | Gross requirements Scheduled receipts Available Net requirements Planned order receipts Planned order releases | 150 | $\begin{array}{r} 50 \\ 150 \\ 0 \\ - \end{array}$ | 100 | 100 | 100 | 100 | $\begin{array}{r} 50 \\ 100 \\ 0 \\ - \end{array}$ | 50 | 50 | 50 | $\begin{array}{r} 50 \\ 50 \\ 0 \end{array}$ | 0 | 0 50 | 50 0 50 50 | 0 |
| A110 | 2 | 100 | 1 | 200 | 0 | 0 | Gross requirements Scheduled receipts Available Net requirements Planned order receipts Planned order releases | $\begin{aligned} & 400 \\ & 200^{*} \\ & 400 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | 0 | 0 | 0 | $\begin{array}{r} 0 \\ 500 \end{array}$ | $\begin{array}{r} 500 \\ 0 \\ 500 \\ 500 \end{array}$ | 0 | 0 | 0 500 | $\begin{array}{r} 500 \\ 0 \\ 500 \\ 500 \end{array}$ | 0 | $\begin{array}{r} 0 \\ 500 \end{array}$ | 500 0 500 500 |  |  |

[^0]Table 32.12

| Item Code | Level Code | $\begin{array}{\|c\|} \text { Lot } \\ \text { Size } \\ \hline \end{array}$ | Lead <br> Time (weeks) | $\begin{gathered} \text { On } \\ \text { Hand } \end{gathered}$ | Safety Stock | Allocated |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A111 | 3 | 10 | 3 | 5 | 0 | 0 | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases | 5 | $5$ <br> 10 | 5 | 5 | $\begin{array}{r} 10 \\ 5 \\ 5 \\ 10 \\ \hline \end{array}$ | 5 <br> 10 | 5 | 5 | $\begin{array}{r} \hline 10 \\ 5 \\ 5 \\ 10 \\ 10 \end{array}$ | 5 | 5 | $\begin{array}{r} 10 \\ 5 \\ 5 \\ 10 \end{array}$ | 5 | 5 | 5 |
| A211 | 3 | 500 | 10 | 500 | 0 | 0 | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases | 500 | $\begin{array}{r} 50 \\ 500 \end{array}$ | 450 | 450 | 450 | 450 | $\begin{array}{r} 50 \\ 450 \end{array}$ | 400 | 400 | 400 | $\begin{array}{r} 50 \\ 400 \end{array}$ | 350 | 350 | $\begin{array}{r} 50 \\ 350 \end{array}$ | 300 |
| A311 | 3 | 500 | 2 | 300 | 0 | 0 | Gross requirements Scheduled receipts Available Net requirements Planned order receipts Planned order releases | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | 300 | $\begin{array}{r} 300 \\ 2000 \\ \hline \end{array}$ | 300 | $\begin{array}{\|r\|} \hline 2000 \\ 300 \\ 1700 \\ 2000 \end{array}$ | 300 | 300 |
| A312 | 3 | 500 | 2 | 200 | 0 | 0 | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | $\begin{gathered} 200 \\ 2000 \\ \hline \end{gathered}$ | 200 | $\left.\begin{array}{\|r\|} \hline 2000 \\ 200 \\ 1800 \\ 2000 \end{array} \right\rvert\,$ | 200 | 200 |

Table 32.13

| $\begin{aligned} & \text { Item } \\ & \text { Code } \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline \text { Level } \\ \text { Code } \end{array}$ | $\left\lvert\, \begin{gathered} \text { Lot } \\ \text { Size } \end{gathered}\right.$ | Lead Time (weeks) | $\begin{gathered} \text { On } \\ \text { Hand } \end{gathered}$ | Safety Stock | Allocated |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A313 | 3 | 500 | 3 | 30 | 0 | 0 | Gross requirements Scheduled receipts Available <br> Net requirements <br> Planned order receipts <br> Planned order releases | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 <br> 500 | 30 | 30 | $\begin{array}{r} 50 \\ 30 \\ 20 \\ 500 \end{array}$ | 480 |
| A411 | 3 | 500 | 5 | 40 | 0 | 0 | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases | 40 | $\begin{array}{\|c\|} \hline 50 \\ 40 \\ 10 \\ 500^{*} \end{array}$ | 490 | 490 | 490 | 490 | $\begin{array}{r} 50 \\ 490 \end{array}$ | 440 | 440 | 440 | 50 440 | 390 | 390 | 50 390 | 340 |
|  |  |  |  |  |  |  | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | Gross requirements Scheduled receipts Available <br> Net requirements Planned order receipts Planned order releases |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

*Ordered on a previous schedule

Table 32.14 Planned Ordered Release Schedule

|  | Week |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| A000 |  | 50 |  |  |  |  | 50 |  |  |  | 50 |  |  | 50 |  |
| A100 | 400 |  |  |  |  | 500 |  |  |  | 500 |  |  | 500 |  |  |
| A300 |  |  |  |  |  |  |  |  |  |  |  |  |  | 50 |  |
| A110 |  |  |  |  | 500 |  |  |  | 500 |  |  |  | 500 |  |  |
| A111 |  | 10 |  |  |  | 10 |  |  | 10 |  |  |  |  |  |  |
| A211 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| A311 |  |  |  |  |  |  |  |  |  |  | 2000 |  |  |  |  |
| A312 |  |  |  |  |  |  |  |  |  |  | 2000 |  |  |  |  |
| A313 |  |  |  |  |  |  |  |  |  |  | 500 |  |  |  |  |
| A411 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Note: An advance order of 200 units of item 110 would have to have been made on a previous MRP schedule.

$$
\begin{aligned}
\hat{Q} & =\sqrt{\frac{2 C_{p} D}{C_{n}}} \\
& =\sqrt{\frac{2(\$ 25) 29(52)}{\$ 10}} \\
& =86.8 \approx 87 \\
\frac{348}{87} \frac{\text { units }}{\text { units } / \text { order }} & =4 \text { orders }
\end{aligned}
$$

Lot for Lot ( $L F L$ ) is the approach to the variable demand situation that merely requires that an order size equal to the required number of products be placed.

The first order would be $25+29+34=88$ units. The second would be $26+24+32=82$ units. The third and fourth orders would be 81 and 97 , respectively.

It is coincidental that the number of orders turned out to be an integer. Had a non-integer occurred, it could have been rounded to the nearest integer. An economic evaluation can be made, if costs are significant, of which rounding (up or down) would yield the lower cost option. Other methods exist in the area of lot-sizing.

### 32.6 JOB SEQUENCING AND SCHEDULING

Sequencing and scheduling problems are among the most common situations found in service and manufacturing facilities. Determining the order and deciding when activities or tasks should be done are part of the normal functions and responsibilities of management and increasingly of the employees themselves. These terms are often used interchangeably, but it is important to note the difference. Sequencing is determining the order of a set of activities to be performed, whereas scheduling also includes determining the specific times when each activity will be done. Thus, scheduling includes sequencing; that is, to be able to develop a schedule for a set of activities, you must also know the sequence in which those activities are to be completed.

### 32.6.1 Structure of the General Sequencing Problem

The job sequencing problem is usually stated as follows: Given $n$ jobs to be processed on $m$ machines, each job having a setup time, processing time, due date for the completion of the job, and requiring

Table 32.15

| Period <br> (week) | Demand | Price | Demand | Period | Demand |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 25 | 5 | 24 | 9 | 8 |
| 2 | 29 | 6 | 32 | 10 | 35 |
| 3 | 34 | 7 | 28 | 12 | 32 |
| 4 | 26 | 8 | 25 | 12 | 30 |

processing on one or more of the machines, determine the sequence for processing the jobs on the machines to optimize the performance criterion.

The factors, therefore, used to describe a sequencing problem are

1. The number of machines in the shop, $m$
2. The number of jobs, $n$
3. The type of shop or facility, i.e., job shop or flow shop
4. The manner in which jobs arrive at the shop, i.e., static or dynamic
5. The performance criterion used to measure the performance of the shop

Usual assumptions for the sequencing problem include the following:

1. Setup times for the jobs on each machine are independent of sequence and can be included in the processing times.
2. All jobs are available at time zero to begin processing.
3. All setup times, processing times, and due dates are known and are deterministic.
4. Once a job begins processing on a machine, it will not be preempted by another job on that machine.
5. Machines are continuously available for processing; i.e., no breakdowns occur.

Commonly used performance criteria include the following:

1. Mean flow time $(\bar{F})$ is the average time a set of jobs spends in the shop, which includes processing and waiting times.
2. Mean idle time of machines ( $\bar{I}$ ) is the average idle time for the set of machines in the shop.
3. Mean lateness of jobs $(\bar{L})$ is the difference between the actual completion time $\left(C_{j}\right)$ for a job and its due date $\left(d_{j}\right)$, i.e., $L_{j}=C_{j}-d_{j}$. A negative value means that the job is completed early. Therefore,

$$
\bar{L}=\sum_{j=1}^{n} \frac{\left(C_{j}-d_{j}\right)}{n}
$$

4. Mean tardiness of jobs $(\bar{T})$ is the maximum of 0 or its value of lateness, i.e., $T_{j}=\max \left\{O, L_{j}\right\}$. Therefore,

$$
\bar{T}=\sum_{j=1}^{n} \max \frac{\left\{0, L_{j}\right\}}{n}
$$

5. Mean number of jobs late.
6. Percentage of jobs late.
7. Mean number of jobs in the system.
8. Variance of lateness ( $s_{L}^{2}$ ), for a set of jobs and a given sequence, is the variance calculated for the corresponding $L_{j}$ 's, i.e.,

$$
\sum_{j=1}^{n} \frac{\left(L_{j}-\bar{L}\right)^{2}}{(n-1)}
$$

The following material will cover the broad range of sequencing problems, from the simple to the complex. The discussion will begin with the single-machine problem and progress through multiple machines. It will include quantitative and heuristic results for both flow shop and job shop environments.

### 32.6.2 Single-Machine Problem

In many instances, the single-machine sequencing problem is still a viable problem. For example, if one were trying to maximize production through a bottleneck operation, consideration of the bottleneck as a single machine might be a reasonable assumption. For the single-machine problem, i.e., $n$ jobs one machine, results include the following.

## Mean Flow Time

To minimize the mean flow time, jobs should be sequenced so that they are in increasing shortest processing time (SPT) order. For example, see the jobs and processing times ( $t ;$ 's) for the jobs in Table 32.16.

Table 32.16

| Job | $t_{j}$ (days) |
| :---: | :---: |
| 1 | 7 |
| 2 | 6 |
| 3 | 8 |
| 4 | 5 |

In Table 32.17, the jobs are processed in shortest processing-time order, i.e., 4, 2, 1, 3.
From Table 32.17 we conclude that $\bar{F}=60 / 4=15$ days. Any other sequence will only increase $\bar{F}$. Proof of this is available in Ref. 35.

## Mean Lateness

Note that as a result of the definition of lateness, SPT sequencing will minimize mean lateness ( $\bar{L}$ ) in the single-machine shop.

## Weighted Mean Flow Time

The above results assumed all jobs were of equal importance. What if, however, jobs should be weighted according to some measure of importance? Some jobs may be more important because of customer priority or profitability. If this importance can be measured by a weight assigned to each job, a weighted mean flow time measure, $\bar{F}_{w}$, can be defined as

$$
\bar{F}_{w}=\frac{\sum_{j=1}^{n} w_{j} F_{j}}{\sum_{j=1}^{n} w_{j}}
$$

To minimize weighted mean flow time $\left(\bar{F}_{w}\right)$, jobs should be sequenced in increasing order of weighted shortest processing time, i.e.,

$$
\frac{t_{[1]}}{w_{[1]}} \leq \frac{t_{[2]}}{w_{[2]}} \leq \ldots \leq \frac{t_{[n]}}{w_{[n]}}
$$

where the brackets indicate the first, second, etc., jobs in sequence.
As an example, consider the problem given in Table 32.18.
If jobs 2 and 6 are considered three times as important as the rest of the job, what sequence should be selected? The solution is given in Table 31.19.

## Maximum Lateness/Maximum Tardiness

Other elementary results given without proof or example include the following. To minimize the maximum job lateness ( $L_{\text {max }}$ ) or the maximum job tardiness ( $T_{\text {max }}$ ) for a set of jobs, the jobs should be sequenced in order of non-decreasing due dates, i.e.,

$$
d_{[1]} \leq d_{[2]} \leq \ldots \leq d_{[n]}
$$

## Minimize the Number of Tardy Jobs

If the sequence above, known as the earliest due date sequence, results in zero or one tardy job, then it is also an optional sequence for the number of tardy jobs, $N_{T}$. In general, however, to find an optional sequence minimizing $N_{T}$, an algorithm attributed to Moore and Hodgson ${ }^{36}$ can be used. The

Table 32.17

| Job | $t_{j}$ (days) | $C_{j}$ |
| :---: | :---: | ---: |
| 4 | 5 | 5 |
| 2 | 6 | 11 |
| 1 | 7 | 18 |
| 3 | 8 | $\frac{26}{60}$ |
|  |  | $\Sigma C_{j}=$ |

Table 32.18

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{j}$ (days) | 20 | 27 | 16 | 6 | 15 | 24 |

algorithm divides all jobs into two sets: Set E, where all the jobs are either early or on time, and Set $T$, where all the jobs are tardy. The optional sequence then consists of Set $E$ jobs followed by Set $T$ jobs. The algorithm is as follows.

Step 1. Begin by placing all jobs in Set $E$ in non-decreasing due date order, i.e., earliest due date order. Note that Step $T$ is empty.
Step 2. If no jobs in Set $E$ are tardy, stop: the sequence in Set $E$ is optional. Otherwise, identify the first tardy job in Set E, labeling this job $k$.

Therefore, the job processing sequence should be $4,6,2,5,3$ and 1 .
Step 3. Find the job with the longest processing time among the first $k$ jobs in sequence in Set E. Remove this job from Set E and place it in Set T. Revise the job completion times of the jobs remaining in Set $E$ and go back to step 2 above. As an example in the use of this algorithm, consider the information given in Table 32.20.

The solution to the problem in Table 32.20 is:
Step 1. $E=\{3,1,4,2\} ; T=\{\phi\}$
Step 2. Job 4 is first late job
Step 3. Job 1 is removed from $E$ $\mathrm{E}=\{3,4,2\} ; \mathrm{T}=\{1\}$
Step 2. Job 2 is first late job
Step 3. Job 2 is removed from $E$ $E=\{3,4\} ; T=\{1,2\}$
Step 2. No jobs in $E$ are now late
Therefore, optional sequences are either $(3,4,1,2)$ or $(3,4,2,1)$

### 32.6.3 Flow Shops

General flow shops can be depicted as in Fig. 32.8. All products being produced through these systems flow in the same direction without backtracking. For example, in a four-machine general flow shop, product 1 may require processing on machines $1,2,3$, and 4 ; product 2 requires machines 1,3 , and 4 ; product 3 requires machines 1 and 2 only. Thus, a flow shop processes jobs much as a production line does, but, because it often processes jobs in batches, may look more like a job shop.

## Two Machines/n Jobs

The most famous result in sequencing literature is concerned with two-machine flow shops and is known as Johnson's Sequencing Algorithm. ${ }^{37}$ This algorithm will develop an optional sequence using makespan as the performance criterion. Makespan is defined as the time required to complete the set of jobs through all machines.

Steps for the algorithm are as follows:

1. List all processing times for the job set for machines 1 and 2 .
2. Find the minimum processing time for all jobs.
3. If the minimum processing time is on machine 1 , place the job first or as early as possible in the sequence. If it is on machine 2, place that job last or as late as possible in the sequence. Remove that job for further consideration.

Table 32.19

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $w_{j}$ | 1 | 3 | 1 | 1 | 1 | 3 |
| $t_{j} / w_{j}$ | 20 | 9 | 16 | 6 | 15 | 8 |

Table 32.20

| Job | $t_{j}$ (days) | $d_{j}$ (days) |
| :---: | :---: | :---: |
| 1 | 10 | 14 |
| 2 | 18 | 27 |
| 3 | 2 | 4 |
| 4 | 6 | 16 |

4. Continue, by going back to step 2, until all jobs have been sequenced.

As an example, consider the five-job problem shown in Table 32.21. Applying the algorithm will give an optional sequence of $2,4,5,3,1$ through the two machines, with a makespan of 26 time units.

## Three Machines/n Jobs

Johnson's Sequencing Algorithm can be extended to a three-machine flow shop and may generate an optional solution with makespan as the criterion.

The extension consists of creating a two-machine flow shop from the three machines by summing the processing times for all jobs for the first two machines for artificial machine 1 and, likewise, summing the processing times for all jobs for the last two machines for artificial machine 2. Johnson's Sequencing Algorithm is then used on the two-artificial-machine flow shop problem.

For example, consider the following three-machine flow shop problem given in Table 32.22. The results of forming the five-job, two-artificial-machine problem are shown in Table 32.23. Therefore, the sequence using Johnson's Sequencing Algorithm is 3, 1, 4, 5, 2. It has been shown that the sequence obtained using this extension is optimal with respect to makespan if one of the following conditions holds:

1. $\min t_{1 j} \geq \max t_{2 j}$, or
2. $\min t_{3 j} \geq \max t_{2 j}$, or
3. If the sequence using $\left\{t_{1 j}, t_{2 j}\right\}$, i.e., only the first two machines, is the same sequence as using only $\left\{t_{2 j}, t_{3 j}\right\}$, i.e., only the last two machines, as two, two-machine flow shops using Johnson's Sequencing Algorithm.

The reader should check to see that the sequence obtained above is optimal using these conditions.

## More Than Three Machines

Once the number of machines exceeds three, there are few ways to find optimal sequences in a flow shop environment. Enumeration procedures, such as branch and bound, are generally the only practical approach that has been successfully used, and then only in problems with five or fewer machines. The more usual approach is to develop heuristic procedures or using assignment rules such as priority dispatching rules. See Section 32.6 .5 for more details.

### 32.6.4 Job Shops

General job shops can be represented as in Fig. 32.9. Products being produced in these systems may begin with any machine or process, followed by a succession of processing operations on any other sequence of machines. It is the most flexible form of production, but experience has shown that it is also the most difficult to control and to operate efficiently.

## Two Machines/n Jobs

Johnson's Sequencing Algorithm can also be extended to a two-machine job shop to generate optimal schedules when makespan is the criterion. The steps to do this are as follows:

Step 1. Divide the job set into four sets, i.e.,
Set $\{\mathrm{A}\}$-jobs that require only one processing operation and that on machine 1 .
Set $\{\mathrm{B}\}$-jobs that require only 1 processing operation and that on machine 2.
Set $\{\mathrm{AB}\}$-jobs that require two processing operations, the first on machine 1 , the second on machine 2 .
Set $\{B A\}$-jobs that require two processing operations, the first on machine 2 , the second on machine 1.
Step 2. Sequence jobs in Set $\{\mathrm{AB}\}$ and Set $\{\mathrm{BA}\}$ using Johnson's Sequencing Algorithm (note that in Set \{BA\}, machine 2 is the first machine in the process).


Fig. 32.8 Product flow in a general flow shop.

Table 32.21

|  | Job | 1 | 2 | 3 | 4 | 5 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Processing | Machine 1 $\left(t_{1 j}\right)$ | 5 | 1 | 8 | 2 | 7 |
| Times | Machine 2 $\left(t_{2 j}\right)$ | 3 | 4 | 5 | 6 | 6 |

Step 3. The optional sequence with respect to makespan is
On machine $1 \Rightarrow$ Process that $\{A B\}$ jobs first, then $\{A\}$ jobs, then $\{B A\}$ jobs (note that $\{A\}$ jobs can be sequenced in any order within the set).
On machine $2 \Rightarrow$ Process the $\{B A\}$ jobs first, then $\{B\}$ jobs, then $\{A B\}$ jobs (note that $\{B\}$ jobs can be sequenced in any order within the set).

For example, see Table 32.24. The solution is:
Optional Sequence:
Machine $1 \Rightarrow 6,5,4,1,8,9,7,10$
Machine $2 \Rightarrow 8,9,7,10,2,3,6,5,4$
giving a makespan of 42 time units.

## m Machines/n Jobs

Once the problem size exceeds two machines in a job shop, optimal sequences are difficult to develop even with makespan as the criterion. If optimal sequences are desired, the only options are usually enumeration techniques, such as branch and bound, that attempt to take account of the special structure that may exist in the particular problem. However, because of the complexity involved in these larger problems, sequencing attention generally turns away from seeking the development of optimal schedules to the development of feasible schedules through the use of heuristic decision rules called priority dispatching rules.

### 32.6.5 Heuristics/Priority Dispatching Rules

A large number of these rules have evolved, each with their proponents, given certain shop conditions and desired performance criteria. Some of the more commonly found ones are

FCFS-select the job on a first-come, first-served basis
SPT-select the job with the shortest processing time
$E D D$-select the job with the earliest due date
$S T O P$-select the job with the smallest ratio of remaining slack time to the number of remaining operations
LWKR-select the job with the least amount of work remaining to be done
Rules such as these are often referred to as either local or global rules. A local rule is applied from each machine's or processing operation's perspective, whereas a global view is applied from the overall shop's perspective. For example, SPT is a local rule, since deciding which of the available jobs to next process will be determined by each machine or process operator. On the other hand, LWKR is a global rule, since it considers all remaining processing that must be done on the job. Therefore, LWKR can be considered the global equivalent of SPT. Some rules, such as FCFS, can be used in either local or global applications. The choice of local or global use is often a matter of whether shop scheduling is done in a centralized or decentralized manner and whether the information

Table 32.22

|  | Processing Times |  |  |
| :---: | :---: | :---: | :---: |
| Job | $t_{1 j}$ | $t_{2 j}$ | $t_{3 j}$ |
| 1 | 1 | 3 | 8 |
| 2 | 4 | 1 | 3 |
| 3 | 1 | 2 | 3 |
| 4 | 7 | 2 | 7 |
| 5 | 6 | 1 | 5 |

Table 32.23

|  | Processing <br> Times |  |
| :---: | :---: | ---: |
| Job | $\boldsymbol{t}^{1}{ }_{A j}$ | $\boldsymbol{t}_{B j}{ }_{B j}$ |
| $\mathbf{1}$ | 4 | 11 |
| 2 | 5 | 4 |
| 3 | 3 | 5 |
| 4 | 9 | 9 |
| 5 | 7 | 6 |

system will support centralized scheduling. Implicit within these concepts is the fact that centralized scheduling requires more information to be distributed to individual workstations and is inherently a more complex scheduling environment requiring more supervisory oversight. Global scheduling intuitively should produce better system's schedules, but empirical evidence seems to indicate that local rules are generally more effective.

Whichever rule may be selected, the use of priority assignments is to resolve conflicts. As an example, consider the three-machine, four-job sequencing problem given in Table 32.25. Assuming all jobs are available at time zero, the initial job loading is shown in Fig. 32.10. As shown, there is no conflict on machines 1 and 2, so the first operation for jobs 1 and 2 would be assigned to these machines. However, there is a conflict on machine 3. If SPT were being used, the first operation for job 3 would be assigned on machine 3 , and the earliest that the first operation of job 4 could be assigned to machine 3 is at time equal to 2 days.

Continuing this example following these assignments, the situation shown in Fig. 32.11 would then exist. If we continue to use SPT, we would assign the second operation of job 2 to machine 1 to resolve the conflict. (Note: if FCFS were being used, the second operation of job 3 would have been assigned.) Even though there is no conflict at this stage on machine 2, no job would normally


Fig. 32.9 Product flow in a general-job shop.

Table 32.24

|  |  | Processing <br> Times |  |
| :---: | :---: | :---: | ---: |
|  | Job Set | Job | $t_{1 j}$ |
| $\{\mathrm{t}\}$ | 1 | 3 |  |
| $\{\mathrm{~B}\}$ | 2 |  | 2 |
|  | 3 |  | 4 |
| $\{\mathrm{AB}\}$ | 4 | 4 | 2 |
|  | 5 | 6 | 5 |
|  | 6 | 3 | 7 |
| $\{\mathrm{BA}\}$ | 7 | 3 | 8 |
|  | 8 | 4 | 1 |
|  | 9 | 7 | 9 |
|  | 10 | 2 | 4 |

be assigned at this time because it would introduce idle time unnecessarily within the schedule. The first operation for job 4, however, would be assigned to machine 3.

With these assignments, the schedule now appears in Fig. 32.12. The assignments made at this stage would include

1. Second operation, job 4 to machine 2
2. Third operation, job 2 to machine 3 , since no other job could be processed on machine 3 during the idle time from 3-6 days

Note that the second operation for job 3 may or may not be scheduled at this time because the third operation for job 4 would also be available to begin processing on machine 1 at the 6 th day. Because of this, there would be a conflict at the beginning of the sixth day, and if SPT is being used, the third operation for job 4 would be selected over the second operation for job 3.

Several observations for this partial example can be made:

1. Depending upon the order in which conflicts are resolved, two people using the same priority dispatching rule may develop different schedules.
2. Developing detailed schedules is a complex process. Almost all large-scale scheduling environments would benefit from the use of computer aids.
3. Determining the effectiveness of a dispatching rule is difficult in the schedule-generating process, because of the precedence relationships which must be maintained in processing.

Testing the effectiveness of dispatching rules is most often done by means of simulation studies. Such studies are used to establish the conditions found in the shop of interest for testing various sequencing strategies that management may believe to be worth investigating. For a good historical discussion of these as well as attempted general conclusions, see Ref. 38.

Two broad classifications of priority dispatching rules seem to have emerged:

1. Those trying to reduce the flowtime in which a job spends in the system, i.e., increasing the speed going through the shop or reducing the waiting time
2. Those that are due date-based rules, which may also be manifested in trying to reduce the variation associated with the selected performance measure

Table 32.25

| Job | Processing Times (days) |  |  | Job | Operation Sequence |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $t_{1}$ | $t_{2 j}$ | $t_{3 j}$ |  | M/C1 | M/C2 | M/C3 |
| 1 | 4 | 6 | 8 | 1 | 1 | 2 | 3 |
| 2 | 2 | 3 | 4 | 2 | 2 | 1 | 3 |
| 3 | 4 | 2 | 1 | 3 | 2 | 3 | 1 |
| 4 | 3 | 3 | 2 | 4 | 3 | 2 | 1 |



Fig. 32.10 Example Problem: Stage 1.

Although simulation has proven to be effective in evaluating the effectiveness of dispatching rules in a particular environment, few general conclusions have been drawn. When maximum throughput or speed is the primary criterion, SPT is often a good rule to use, even in situations when the quality of information is poor concerning due dates and processing times. When due date rules are of interest, selection is much more difficult. Results have been developed showing that when shop loads are heavy, SPT still may do well; when shop loads are moderate, STOP was preferable. Other research has shown that the manner in which due dates are set, as well the tightness of the due dates, can greatly affect the performance of the rule. Overall, the conclusions that can be drawn are:

1. It is generally more difficult to select an effective due date-based rule than a flowtime-based rule.
2. If time and resources are available, the best course of action is to develop a valid model of the particular shop of interest, and experiment with the various candidate rules to determine which are most effective, given that situation.

### 32.6.6 Assembly Line Balancing

Assembly lines are viewed as one of the purest forms of production lines. A usual form is visualized as shown in Fig. 32.14, where work moves continuously by means of a powered conveyor through a series of workstations where the assigned work is performed.

## Definitions

Cycle time ( $\boldsymbol{C}$ ). The time available for a workstation to perform its assigned work, assumed to be the same for each workstation. The cycle time must be greater than or equal to the longest


Fig. 32.11 Example Problem: Stage 2.


Fig. 32.12 Example Problem: Stage 3.
work element for the product. Note that it is also the time between successive completions of units of product on the line.
balance delay of a workstation. The difference between the cycle time $(C)$ and the station time $\left(S_{j}\right)$ for a workstation, i.e., the idle time for the station $=\left(C-S_{j}\right)$.
Station time ( $S_{j}$ ). The total amount of work assigned to station $j$, which consists of one or more of the work elements necessary for completion of the product. Note that each $S_{j}$ must be less than or equal to $C$.
Work element (i). An amount of work necessary in the completion of a unit of product. It is usually considered indivisible. $I$ is the total number of work elements necessary to complete one unit of product.
Work element time $\left(t_{i}\right)$. The amount of time required to complete work element $i$. Therefore, the sum of all of the work elements, i.e., the total work content,

$$
T=\sum_{i=1}^{I} t_{i}
$$

is the time necessary to complete one unit of product.
Workstation ( $j$ ). A location on the line where assigned work elements on the product are performed ( $1 \leq j \leq J$ ).

## Structure of the Assembly Line Balancing Problem

The objective of assembly line balancing is to assign work elements to the workstations so as to minimize the total balance delay (total idle time) on the line. The problem is normally presented by means of a listing of the work elements and a precedence diagram showing the relationships that must be maintained in the assembling of the product. See Table 32.26 and Figs. 32.15 and 32.16. In designing the assembly line, therefore, the work elements must be assigned to the workstations while adhering to these precedence relationships.


Fig. 32.13 Typical assembly line configuration.


Fig. 32.14 Automobile back-up light assembly. (From Ref. 39.)

Note that if the balance delay is summed up over the entire production line, the total balance delay (total idle time) is equal to

$$
\sum_{j=1}^{J}\left(C-S_{j}\right)
$$

So, to minimize the total balance delay is the same as

$$
\begin{aligned}
\operatorname{Min} . \sum_{j=1}^{J}\left(C-S_{j}\right) & =J C-\sum_{j=1}^{J} S_{j} \\
& =J C-\text { total work content for one unit of product } \\
& =J C-\text { a constant }
\end{aligned}
$$

Therefore, minimizing the total balance delay is equivalent to

Table 32.26

| Workstation | Work Elements <br> Assigned | Station <br> Time | Balance <br> Delay <br> Time |
| :---: | :---: | :---: | :---: |
| 1 | 10 | .104 |  |
| 2 | 20 | $\frac{.105}{.209}$ | 0.016 |
|  | 30 | .102 |  |
| 3 | 70 | $\underline{.081}$ | 0.042 |
|  | 40 | .183 | 0.00 |
|  | 50 | .053 |  |
|  | 60 | $\underline{.048}$ |  |
|  |  | .201 | 0.024 |
|  | 90 | .112 |  |
|  |  | $\underline{.097}$ | 0.209 |



Fig. 32.15 Precedence diagram for automobile backup-light assembly. (From Ref. 39.)

1. Keeping the number of workstations constant and minimizing the cycle time, or
2. Keeping the cycle time constant and minimizing the number of workstations, or
3. Jointly trying to minimize the product of cycle time and number of workstations

Which approach might be followed could depend upon the circumstances. For example, if production space were constrained, approach 1 above might be used to estimate the volume the line would be capable of producing. Approach 2 might be used if the primary concern were ensuring a certain volume of product could be produced in a certain quantity of time. Approach 3 could be used in developing alternative assignments by trading off faster line speed (shorter cycle times, more workstations and greater production) for slower line speeds (fewer workstations, longer cycle times, and less production).

## Designing the Assembly Line

Given the above structure and definitions, the following must hold.

1. $\left\{\max t_{j}\right\} \leq C \leq T$.
2. Minimum number of work stations $=[T / C]$, where the brackets indicate the value is rounded to the next largest integer.
3. $C_{\max }=\frac{\text { production time available }}{\text { production volume required }}\left(C_{\text {max }}\right.$ is the maximum value the cycle time can be, if the line is to generate the specified quantity in the specified time.)

As an example, consider the data provided in Table 32.25 and Figs. 32.14 and 32.15. Designing a line to produce 2000 units in seven-and-a-half-hour shift would give the following:

From condition 3:

$$
C_{\max }=\frac{(71 / 2 \mathrm{hr} / \mathrm{shift})(60 \mathrm{~min} / \mathrm{hr})}{2000 \mathrm{units}(\mathrm{shift})}=.225 \mathrm{~min} / \mathrm{unit}
$$

From condition 2:

$$
\text { minimum no. of workstations }=\frac{.802}{.225}=[3.56]=4
$$

Also note condition 1 is satisfied, i.e.,

$$
.112 \leq .225 \leq .802
$$

## Line Balancing Techniques

Efforts have been made to optimally model variations of these problems, but currently no procedures exist that guarantee optimal solutions to these types of problems. Practitioners, therefore, have developed a variety of heuristic procedures (for examples, see Ref. 40). A general approach in making the assignment of work elements to workstations is to select a cycle time and to start assigning work elements where precedence restrictions are satisfied to the first workstation. Combinations of work elements may be explored in order to reduce the idle time present to the lowest level possible before going to the next workstation and repeating the procedure. This process is continued until all work elements have been assigned.


Fig. 32.16 Example of a two-card Kanban system.

Applying this procedure to the data given in Table 32.25 and Fig. 32.14 would give the following solution, for a cycle time of 0.225 min . Also note that the cycle time could be reduced to 0.209 min with this assignment and would theoretically reduce the total balance delay by $.016 \mathrm{~min} \times 4$ workstations $=.064 \mathrm{~min}$, resulting in a production increase to a total of

$$
\frac{(71 / 2 \mathrm{hr} / \mathrm{shift})(60 \mathrm{~min} / \mathrm{shift})}{.209 \mathrm{~min}}=2153 \mathrm{units} / \mathrm{shift}
$$

## Mixed Model Assembly Lines

The above discussion is predicated on the premise that only one product is being manufactured on the line. Many production lines, such as those found in the automobile industry, are designed to produce a variety of products. Good examples of these are assembly lines that may produce several models of the same automobile with a wide variety of options. These are often referred to as mixed model assembly lines.

Mixed model assembly lines are significantly more complex to design than the single model line because of two problem areas:

1. The assignment of work elements to the workstations
2. The ordering or sequencing of models on the line

One usual approach taken in designing a mixed model line uses the same general objective of minimizing the total balance delay (or idle time) in the assignment of work elements to workstations. However, in the mixed model case, a production period has to be defined and the assignments are made so as to minimize the total amount of idle time for all stations and models for the production period, rather than for the cycle time, as in the single model case. To use this approach, the designer must define all of the work elements for all of the models and determine the quantity of each of the models being assembled within the specified production period. Thus, knowing the total work content and the time allowed for production, work elements are assigned to workstations usually based upon similarity of the work elements, tooling or equipment required, and time to perform the tasks. If the stations on the mixed model line are not tightly linked and small in-process inventory buffers are allowed to exist between workstations, this approach seems satisfactory. However, if the stations are tightly linked where no in-process inventory is allowed between stations, or if the line is operating as a JIT system, this approach may not produce satisfactory assignments without analysts being especially diligent in determining the sequence of models being produced.

Determining the order of models to produce on the line is generally more difficult than the assignment of work elements to workstations problem, because it has to be done in a constantly changing environment. This difficulty is further due to two interrelated subproblems:

1. Trying to fully utilize the resources of the line so that no station is idled due to a bottleneck or lack of product to work on
2. Trying to even out the flow of component parts to the line from "upstream" manufacturing or subassembly operations feeding the line so that these resources are also fully utilized or have a relatively constant amount of work

The first of these, that is, fully utilizing the resources of the line, is the easier of the two, especially if some flexibility is present in the system, such as producing in a make-to-stock environment, or allowing small-buffer, in-process inventories between workstations, or variable-time workstations on the line. Examples of how some of this flexibility can be built in can be found in Ref. 41.

Smoothing out the flow of components into the assembly line is a very difficult problem, especially if the facility is operating in a JIT environment. One of the earliest approaches that discussed how this problem was handled in the Toyota company is known as a goal chasing method. ${ }^{42}$ The procedure has since evolved into a newer version, and since it has the capability of handling multiple goals, it is called a goals-coordinating method. ${ }^{43}$ This procedure has two main components:

## 1. Appearance ratio control

2. Continuation and interval controls

Appearance ratio control is a heuristic that determines the sequence of models on the line by attempting to minimize the variances of the components used for those products, that is, minimize the actual variation in component usage around a calculated average usage. A production schedule of end products is built by starting with the first end product to be scheduled, then working toward the last end product. For each step in determining the sequence, the following is calculated for each product, with the minimum $D$ determining the next product to be produced:

$$
D_{K i}=\sqrt{\sum_{j=1}^{\beta}\left(\frac{K N_{j}}{Q}-X_{j . K-1}-b_{i j}\right)^{2}}
$$

where $D_{K i}=$ the distance to be minimized for sequence number $K$ and for end product $i$
$\beta=$ the number of different components required
$K=$ the sequence number of the current end product in the schedule
$N_{j}=$ the total number of component $j$ required for all products in the final schedule
$Q=$ the total production quantity of all end products in the final schedule
$X_{i, K}=$ the cumulative number of component $j$ actually used through assembly sequence $K$
$b_{i j}=$ the number of component $j$ required to make one unit of end product $i$
However, while this approach results in a smoothed production for the majority of the schedule, it will potentially cause very uneven use of components during the final phases of the day's schedule. To prevent this, continuation and interval controls are applied as constraints that may override the appearance ratio control and introduce other type models on the line. Continuation controls ensure that no more than a designated number of consecutive end products that use a particular component are scheduled (a maximum sequencing number condition), whereas interval controls ensure that at least a designated minimum number of certain end products are scheduled between other end products that require a particular component (a minimum sequencing condition).

The overall sequencing selection process then works as follows.
Step 1. Appearance ratio control is used to determine the first (or next) end product in sequence.
Step 2. If the selected end product also satisfies the continuation and interval controls, the end product is assigned that position in the sequence. Unless all end products have been scheduled, go to step 1. Otherwise, stop; the schedule is complete.
Step 3. If the selected end product does not satisfy both the continuation and interval controls, the appearance ratio control is applied to the remaining end products, while ignoring the component that violated the continuation and/or interval controls. Out of the end products that do not require the component in question, the end product that minimizes the amount of total deviation in the following formula would be selected as the next ( $K$ th) in sequence ( $j=$ component number).

$$
\sum_{j=1}^{n}\left|\begin{array}{ccc}
\text { total number of } \\
\text { end product of } \\
\text { the specified component } i \\
\text { total number of } \\
\text { end product }
\end{array} \quad \begin{array}{c}
\text { accumulated }
\end{array} \begin{array}{c}
\text { number of } \\
\text { component } j \text { of } \\
\text { number of component } j \\
\text { up to }(k-1) \text { th additional } \\
\text { end product }
\end{array}\right|
$$

Unless all end products have been scheduled, go to step 1. Otherwise, stop; the schedule is complete.

As the number of models and components increase, the difficulties of developing satisfactory solutions for leveling production for mixed model lines also increase. As this occurs, the response is often to shorten the scheduled time period from, say, a day to every hour, to reduce the number of alternatives being investigated. While this may seem desirable, particularly if the facility is operating in a JIT environment, there is a danger that the resulting schedules may become so inefficient that they degrade the overall performance of the line. The leveling of production on mixed model lines remains an active research topic, with much of the research focusing on developing better or more efficient heuristic scheduling procedures. ${ }^{44,45}$

### 32.7 OTHER RELATED TOPICS

Within the arena of manufacturing, a number of new approaches have revolutionized thinking toward designing and controlling manufacturing organizations. Foremost have been the Japanese, who have developed and perfected a whole new philosophy. Some of the more important concepts related to production planning and control are presented below.

### 32.7.1 Japanese Manufacturing Philosophy

The central concepts are to design manufacturing systems as simply as possible and then to design simple control procedures to control them. This does not mean that Japanese manufacturing systems are simple, but that the design is well engineered to perform the required functions and the system is neither overdesigned nor underdesigned.

Central to this philosophy is the Just in Time (JIT) concept. JIT is a group of beliefs and management practices that attempt to eliminate all forms of "waste" in a manufacturing enterprise, where
waste is defined as anything not necessary in the manufacturing organization. Waste in practice may include inventories, waiting times, equipment breakdowns, scrap, defective products, and excess equipment changeover times. The elimination of waste and the resulting simplification of the manufacturing organization are the results of implementing the following related concepts usually considered as defining or making up JIT.

1. Kanban (the word means "card") is used to control the movement and quantity of inventory through the shop, since a kanban card must be attached to each container of parts. The amount of production and in-process inventory, therefore, is controlled by the number of cards that are issued to the plant floor. An additional, major benefit of using Kanban is the very significant reduction in the information system that has to be used to control production.

Various forms of Kanban exist, but the most frequently encountered are variations of the single-card or two-card system. One example of a two-card Kanban system is presented in Fig. 32.16. This example consists of two workstations, $K$ and N. For simplicity, it is assumed that the production from Workstation K is used at Workstation N . The containers that move between these workstations have been sized to hold only a certain quantity of product. The two different types of Kanbans used are a withdrawal and a production Kanban. To control the amount of production for a given period of time, say one day, Workstation N is issued a predetermined number of withdrawal kanbans. The system operates as follows:
a. When Workstation N needs parts, the operator takes an empty container, places a withdrawal kanban on it, and takes it to the storage area.
b. The full containers in the storage area each have a production kanban on them. He removes the production kanban from a full container and places it on the empty container, and removes the withdrawal kanban and places it on the full container.
c. He then transports the full container (now with the withdrawal kanban) back to Workstation N .
d. Workstation K checks the production kanbans (on the empty containers) when checking for work to do. If a production kanban is present, this is his signal to begin production. If no production kanbans are present, Workstation K does not continue to produce parts.
For this system to work, certain rules have to be adhered to:
i. Each workstation works as long as there are parts to work on and a container in which to put them. If one or the other is missing, production stops.
ii. There must be the same number of kanban cards as there are containers.
iii. Containers are conveyed either full, with only their standard quantities, or empty.
2. Lot size reduction is used to reduce the amount of in-process inventory in concert with Kanban, by selecting the proper-sized containers to use, and increase the flexibility of the shop to change from one product to another. Overall benefits from using reduced lot sizes include shorter throughput times for product and thus smaller leadtimes required in satisfying customer orders.
3. Scheduling is used to schedule small lot production to increase the flexibility of the shop to be able to react to changes in demand and to produce the quantity of goods at just the time they are needed.
4. Setup time reduction is used to reduce the times required for machines to change from one product to another so as to allow lot size reduction and JIT scheduling. Reducing changeover times between products is critical to operating the production facility more like a flow shop and less like a job shop.
5. Total quality management and maintenance is used to reduce the disturbances to the manufacturing system by attempting to eliminate the making of defective products and breakdown of equipment. Central to the Japanese manufacturing philosophy is an obsession with maintenance and quality issues. For such a tightly controlled system to work, it is imperative that equipment function when it is supposed to and that components and products be produced that meet or exceed customers' requirements. Unexpected breakdowns or the production of bad parts is considered waste, and causes of such happenings are always high on the list for elimination in the quest for continuous improvement of the manufacturing processes.
6. Employee cross training is used to provide flexibility in the workforce to allow the organization to be able to react to changes in product demand and its resultant effect on the type and quantity of employee skills required. Multiskilled workers are necessary prerequisites in any form of JIT implementation.

### 32.7.2 Time-Based Competition

Following on the heels of JIT and the Japanese manufacturing philosophy is a business strategy called time-based competition (TBC). The successes of these earlier approaches were primarily grounded in providing the customer with better, more consistent-quality products that might also be less ex-
pensive in certain cases. Quality and cost were the major attributes of competitiveness for the organization that successfully employed these techniques. Although being competitive in quality and cost will always be important, some industries are finding that they alone are not enough to maintain an edge over their competitors, since many of their competitors also have gained these benefits by implementing JIT and related concepts. A third element is being introduced-that of time. TBC seeks a competitive advantage by the reduction of lead times associated with getting product to customers. TBC attempts to achieve reductions in the times required to design, manufacture, sell, and deliver products for its customers by analyzing and redesigning the processes that perform these functions.

TBC is seen as a natural evolution of JIT in that the implementation of JIT was most often found in production. Realizing that time spent on the shop floor represents less than one-half of the time it takes to get a product to the customer for most industries, TBC is a form of extension of JIT to the rest of the manufacturing organization, including such areas as design, sales, and distribution. Wherever in the organization lead times exist that lengthen the time it takes to get the desired product to the customer, the TBC approach seeks to reduce them.

Two forms of TBC exist: first to market for new products (FM) and first to customer for existing products (FC). Companies that seek to gain a competitive advantage through FM tend to be in dynamic industries such as automobile manufacturers and consumer products. For these industries, new innovations, developments, and improvements are important for their product's image, and are necessary to maintain and increase product sales. Companies employing FC as a competitive advantage tend to be in more stable industries, where innovations and new product developments are less frequent and dramatic. Thus, the products that competitors sell are very similar and competitive in terms of features, price, and quality. Here the emphasis is on speed-reducing the time it takes to get the product in the customer's hands from the time it was ordered is key. There is nothing, of course, preventing a company from employing both FM and FC approaches, and in the continuous improvement context, both approaches will be necessary if the full benefits of TBC are to be realized.

## REFERENCES

1. W. A. Silver and R. Peterson, Decision Systems for Inventory Management and Production Planning, Wiley, New York, 1985.
2. H. A. Taha, Operations Research, 4th ed., Macmillan, New York, 1987.
3. D. P. Gover and G. L. Thompson, Programming and Probability Models for Operations Research, Wadsworth, Belmont, CA, 1973.
4. A. M. Wagner, Principles of Management Science, Prentice-Hall, Englewood Cliffs, NJ, 1970.
5. W. J. Fabrycky, P. M. Ghare, and P. E. Torgersen, Applied Operations Research and Management Science, Prentice-Hall, Englewood Cliffs, NJ, 1984.
6. S. Nam and R. Logendran, "Aggregate Production Planning-A Survey of Models and Methodologies," European Journal of Operations Research 61, 255-272 (1992).
7. E. H. Bowman, "Production Scheduling by the Transportation Method of Linear Programming," Operations Research 4, 100-103 (1956).
8. A. Charnes, W. W. Cooper, and B. Mellon, "A Model for Optimizing Production by Reference to Cost Surrogates," Econometrics 23, 307-323 (1955).
9. M. E. Posner and W. Szwarc, "A Transportation Type Aggregate Production Model with Backordering," Management Science 29, 188-199 (1983).
10. K. Sighal and V. Adlakha, "Cost and Shortage Trade-offs in Aggregate Production Planning," Decision Science 20, 158-164 (1989).
11. C. C. Holt, F. Modigliani, and H. A. Simon, "A Linear Decision Rule for Production and Employment Scheduling," Management Science 2, 1-30 (1955).
12. C. C. Holt, F. Modigliani, and J. F. Muth, "Derivation of a Linear Decision Rule for Production and Employment," Management Science 2, 159-177 (1956).
13. C. C. Holt, F. Modigliani, J. F. Muth, and H. A. Simon. Planning Production Inventories and Work Force, Prentice-Hall, Englewood Cliffs, NJ, 1960.
14. A. S. Manne, "Programming of Economic Lot Sizes," Management Science 4, 115-135 (1958).
15. H. M. Wagner and T. M. Whitin, "Dynamic Version of the Economic Lot Size Model," Management Science 5, 89-96 (1958).
16. S. Gorenstein, "Planning Tire Production," Management Science 17, B72-B82 (1970).
17. L. S. Lasdon and R. C. Terjung, "An Efficient Algorithm for Multi-item Scheduling," Operations Research 19, 946-966 (1971).
18. S. M. Lee and L. J. Moore, "A Practical Approach to Production Scheduling," Journal of Production and Inventory Management 15, 79-92 (1974).
19. R. F. Deckro and J. E. Hebert, "Goal Programming Approaches to Solving Linear Decision Rule Based Aggregate Production Planning Models," IIE Transactions 16, 308-315 (1984).
20. D. P. Gaver, "Operating Characteristics of a Simple Production, Inventory-Control Model," Operations Research 9, 635-649 (1961).
21. W. I. Zangwill, "A Deterministic Multiproduct, Multifacility Production and Inventory Model," Operations Research 14, 486-507 (1966).
22. W. I. Zangwill, "A Deterministic Multiperiod Production Scheduling Model with Backlogging," Management Science 13, 105-119 (1966).
23. W. I. Zangwill, "Production Smoothing of Economic Lot Sizes with Non-Decreasing Requirements," Management Science 13, 191-209 (1966).
24. G. D. Eppen and F. J. Gould, "A Lagrangian Application to Production Models," Operations Research 16, 819-829 (1968).
25. D. R. Lee and D. Orr, "Further Results on Planning Horizons in the Production Smoothing Problem," Management Science 23, 490-498 (1977).
26. C. H. Jones, "Parametric Production Planning," Management Science 13, 843-866 (1967).
27. A. D. Flowers and S. E. Preston, "Work Force Scheduling with the Search Decision Rule," OMEGA 5, 473-479 (1977).
28. W. B. Lee and B. M. Khumawala, "Simulation Testing of Aggregate Production Planning Models in an Implementation Methodology," Management Science 20, 903-911 (1974).
29. J. M. Mellichamp and R. M. Love, "Production Switching Heuristics for the Aggregate Planning Problem," Management Science 24, 1242-1251 (1978).
30. H. Hwang and C. N. Cha, "An Improved Version of the Production Switching Heuristic for the Aggregate Production Planning Problems," International Journal of Production Research 33, 2567-2577 (1995).
31. E. H. Bowman, "Consistency and Optimality in Management Decision Making," Management Science 9, 310-321 (1963).
32. S. Eilon, "Five Approaches to Aggregate Production Planning," AIIE Transactions 7, 118-131 (1975).
33. A. C. Hax and D. Candea, Production and Inventory Management, Prentice-Hall, Englewood Cliffs, NJ, 1984.
34. D. D. Bedworth and J. E. Bailey, Integrated Production Control Systems, Wiley, New York, 1982.
35. K. Baker, Introduction to Sequencing and Scheduling, Wiley, New York, 1974.
36. J. M. Moore, "Sequencing in Jobs on One Machine to Minimize the Number of Tardy Jobs," Management Science 17(1), 102-109 (1968).
37. S. M. Johnson, "Optional Two-and-Three Stage Production Schedules with Setup Times Included," Naval Research Logistics Quarterly 1, (1) (1954).
38. S. S. Panwalker and W. Iskander, "A Survey of Scheduling Rules," Operations Research 25, 45-61 (1977).
39. J. Lorenz and D. Poock, "Assembly Line Balancing," in Production Handbook, 4th ed., J. White (ed.), Wiley, New York, 1987, pp. 3.176-3.189.
40. E. Elsayed and T. Boucher, Analysis and Control of Production Systems, Prentice-Hall, Englewood Cliffs, NJ, 1994.
41. N. Thomopoulos, "Mixed Model Line Balancing with Smoothed Station Assignments," Management Science 16, 593-603 (1970).
42. Y. Moden, Toyota Production System: Practical Approach to Production Management, Industrial Engineering and Management Press, Atlanta, 1983.
43. Y. Moden, Toyota Production System: An Integrated Approach to Just-In-Time, 2nd ed., Industrial Engineering and Management Press, Atlanta, 1993.
44. R. T. Sumichrast and R. S. Russell, "Evaluating Mixed-Model Assembly Line Sequencing Heuristics for Just-in-Time Production Systems," Journal of Operation Management 9, 371-386 (1990).
45. J. F. Bard, A. Shtub, and S. B. Joshi, "Sequencing Mixed-Model Assembly Lines to Level Parts Usage and Minimize Line Length," International Journal of Production Research 32, 2431-2454 (1994).

## BIBLIOGRAPHY

Adam, E. E., Jr., and R. J. Ebert, Production and Operations Management, 4th ed., Prentice-Hall, Englewood Cliffs, NJ, 1989.
Carter, P., S. Melnyk, and R. Handfield, "Identifying the Basic Process Strategies for Time Based Competition," Production and Inventory Management Journal 36(1), 1st Qtr., 65-70 (1995).
Eilon, S., "Aggregate Production Scheduling," in Handbook of Industrial Engineering, G. Salvendy (ed.), Wiley Interscience, New York, 11.3.1-11.3.23.

French, S., Sequencing and Scheduling: An Introduction to the Mathematics of the Job Shop, Halsted Press, New York, 1982.
Gaither, N., Production and Operations Management, 6th ed., Dryden Press, Fort Worth, 1992.
Hax, A. C., "Aggregate Production Planning," in Production Handbook, 4th ed., J. White (ed.), Wiley, New York, 1987, pp. 3.116-3.127.
Johnson, L. A., and D. C. Montgomery, Operations Research in Production Planning, Scheduling and Inventory Control, Wiley, New York, 1974.
Nahmias, S., Production and Operations Analysis, 2nd ed., Irwin, Hinsdale, IL, 1993.
Schonberger, R. J., Japanese Manufacturing Techniques, Free Press, New York, 1983, pp. 219-245.
Vollmann, T. E., W. L. Berry, and D. C. Whybark, Manufacturing Planning and Control Systems, 3rd ed., Irwin, Homewood, IL, 1992.


[^0]:    *Order on a previous schedule

