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CHAPTER

16

Columns and Struts

16.1 Introduction

A machine part subjected to an axial compressive force is called a ***strut.*** A strut may be horizontal, inclined or even vertical. But a vertical strut is known as a ***column, pillar*** or ***stanchion.*** The machine members that must be investigated for column action are piston rods, valve push rods, connecting rods, screw jack, side links of toggle jack etc. In this chapter, we shall discuss the design of piston rods, valve push rods and connecting rods.

Note: The design of screw jack and toggle jack is discussed in the next chapter on 'Power screws'.

16.2 Failure of a Column or Strut

It has been observed that when a column or a strut is subjected to a compressive load and the load is gradually increased, a stage will reach when the column will be subjected to ultimate load. Beyond this, the column will fail by crushing and the load will be known as ***crushing load.***

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It has also been experienced, that sometimes, a compression member does not fail entirely by crushing, but also by bending *i.e.* buckling. This happens in the case of long columns. It has also been observed, that all the *short columns fail due to their crushing. But, if a **long column is subjected to a compressive load, it is subjected to a compressive stress. If the load is gradually increased, the column will reach a stage, when it will start buckling. The load, at which the column tends to have lateral displacement or tends to buckle is called ***buckling load, critical load,*** or ***crippling load*** and the column is said to have developed an elastic instability. The buckling takes place about the axis having minimum radius of gyration or least moment of inertia. It may be noted that for a long column, the value of buckling load will be less than the crushing load. Moreover, the value of buckling load is low for long columns, and relatively high for short columns.

Depending on the end conditions, different columns have different crippling loads

16.3 Types of End Conditions of Columns

In actual practice, there are a number of end conditions for columns. But we shall study the Euler's column theory on the following four types of end conditions which are important from the subject point of view:

1. Both the ends hinged or pin jointed as shown in Fig. 16.1 (*a*),
2. Both the ends fixed as shown in Fig. 16.1 (*b*),
3. One end is fixed and the other hinged as shown in Fig. 16.1 (*c*), and
4. One end is fixed and the other free as shown in Fig. 16.1 (*d*).

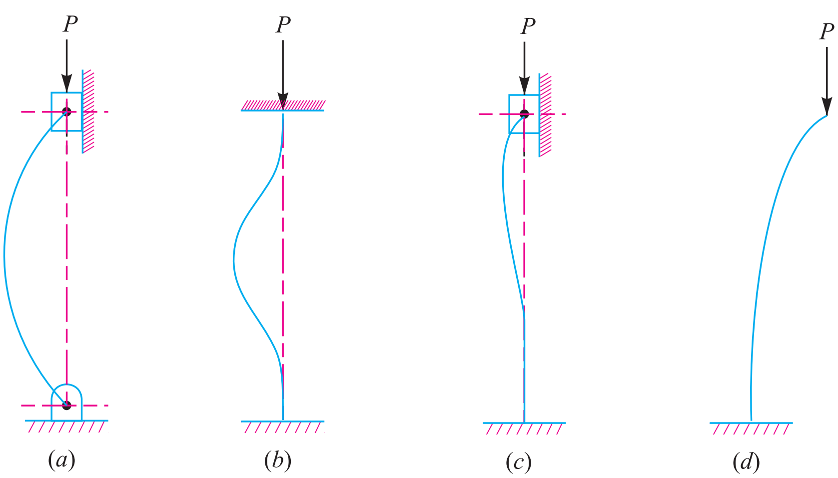

Fig. 16.1. Types of end conditions of columns.

16.4 Euler's Column Theory

The first rational attempt, to study the stability of long columns, was made by Mr. Euler. He

* The columns which have lengths less than 8 times their diameter, are called ***short columns*** (see also Art 16.8).

** The columns which have lengths more than 30 times their diameter are called ***long columns.***

derived an equation, for the buckling load of long columns based on the bending stress. While deriving this equation, the effect of direct stress is neglected. This may be justified with the statement, that the direct stress induced in a long column is negligible as compared to the bending stress. It may be noted that Euler's formula cannot be used in the case of short columns, because the direct stress is considerable, and hence cannot be neglected.

16.5 Assumptions in Euler's Column Theory

The following simplifying assumptions are made in Euler's column theory :

1. Initially the column is perfectly straight, and the load applied is truly axial.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic, and thus obeys Hooke's law.
4. The length of column is very large as compared to its cross-sectional dimensions.
5. The shortening of column, due to direct compression (being very small) is neglected.
6. The failure of column occurs due to buckling alone.
7. The weight of the column itself is neglected.

16.6 Euler's Formula

According to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is represented by a general equation,

$$W_{cr} = \frac{C\,\pi^2 E\,I}{l^2} = \frac{C\,\pi^2 E\,A\,k^2}{l^2} \qquad \ldots (\because I = A.k^2)$$

$$= \frac{C\,\pi^2 E\,A}{(l/k)^2}$$

where E = Modulus of elasticity or Young's modulus for the material of the column,

A = Area of cross-section,

k = Least radius of gyration of the cross-section,

l = Length of the column, and

C = Constant, representing the end conditions of the column or end fixity coefficient.

The following table shows the values of end fixity coefficient (C) for various end conditions.

Table 16.1. Values of end fixity coefficient (C).

S. No.	*End conditions*	*End fixity coefficient (C)*
1.	Both ends hinged	1
2.	Both ends fixed	4
3.	One end fixed and other hinged	2
4.	One end fixed and other end free	0.25

Notes : 1. The vertical column will have two moment of inertias (*viz.* I_{xx} and I_{yy}). Since the column will tend to buckle in the direction of least moment of inertia, therefore the least value of the two moment of inertias is to be used in the relation.

2. In the above formula for crippling load, we have not taken into account the direct stresses induced in the material due to the load which increases gradually from zero to the crippling value. As a matter of fact, the combined stresses (due to the direct load and slight bending), reaches its allowable value at a load lower than that required for buckling and therefore this will be the limiting value of the safe load.

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16.7 Slenderness Ratio

In Euler's formula, the ratio l / k is known as ***slenderness ratio.*** It may be defined as the ratio of the effective length of the column to the least radius of gyration of the section.

It may be noted that the formula for crippling load, in the previous article is based on the assumption that the slenderness ratio l / k is so large, that the failure of the column occurs only due to bending, the effect of direct stress (*i.e.* W / A) being negligible.

This equipment is used to determine the crippling load for axially loaded long struts.

16.8 Limitations of Euler's Formula

We have discussed in Art. 16.6 that the general equation for the crippling load is

$$W_{cr} = \frac{C\,\pi^2 E\,A}{(l/k)^2}$$

∴ Crippling stress,

$$\sigma_{cr} = \frac{W_{cr}}{A} = \frac{C\,\pi^2 E}{(l/k)^2}$$

A little consideration will show that the crippling stress will be high, when the slenderness ratio is small. We know that the crippling stress for a column cannot be more than the crushing stress of the column material. It is thus obvious that the Euler's fromula will give the value of crippling stress of the column (equal to the crushing stress of the column material) corresponding to the slenderness ratio. Now consider a mild steel column. We know that the crushing stress for mild steel is 330 N/mm^2 and Young's modulus for mild steel is 0.21×10^6 N/mm^2.

Now equating the crippling stress to the crushing stress, we have

$$\frac{C\,\pi^2 E}{(l/k)^2} = 330$$

$$\frac{1 \times 9.87 \times 0.21 \times 10^6}{(l/k)^2} = 330 \qquad \text{... (Taking } C = 1)$$

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or $(l / k)^2 = 6281$

$\therefore$ $l / k = 79.25$ say 80

Hence if the slenderness ratio is less than 80, Euler's formula for a mild steel column is not valid.

Sometimes, the columns whose slenderness ratio is more than 80, are known as ***long columns,*** and those whose slenderness ratio is less than 80 are known as ***short columns.*** It is thus obvious that the Euler's formula holds good only for long columns.

16.9 Equivalent Length of a Column

Sometimes, the crippling load according to Euler's formula may be written as

$$W_{cr} = \frac{\pi^2 E I}{L^2}$$

where L is the equivalent length or effective length of the column. The equivalent length of a given column with given end conditions is the length of an equivalent column of the same material and cross-section with hinged ends to that of the given column. The relation between the equivalent length and actual length for the given end conditions is shown in the following table.

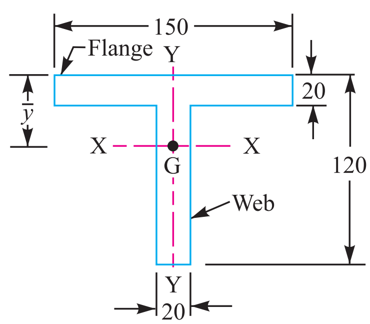
Table 16.2. Relation between equivalent length (*L*) and actual length (*l*).

S.No.	*End Conditions*	*Relation between equivalent length (L) and actual length (l)*
1.	Both ends hinged	$L = l$
2.	Both ends fixed	$L = \frac{l}{2}$
3.	One end fixed and other end hinged	$L = \frac{l}{\sqrt{2}}$
4.	One end fixed and other end free	$L = 2l$

Example 16.1. *A T-section 150 mm × 120 mm × 20 mm is used as a strut of 4 m long hinged at both ends. Calculate the crippling load, if Young's modulus for the material of the section is 200 kN/mm².*

Solution. Given : l = 4 m = 4000 mm ; E = 200 kN/mm² = 200 × 10³ N/mm²

First of all, let us find the centre of gravity (G) of the T-section as shown in Fig. 16.2.



All dimensions in mm.

Fig. 16.2

Let $\bar{y}$ be the distance between the centre of gravity (G) and top of the flange,

We know that the area of flange,

$$a_1 = 150 \times 20 = 3000 \text{ mm}^2$$

Its distance of centre of gravity from top of the flange,

$$y_1 = 20 / 2 = 10 \text{ mm}$$

Area of web, $a_2 = (120 - 20)\, 20 = 2000 \text{ mm}^2$

Its distance of centre of gravity from top of the flange,

$$y_2 = 20 + 100 / 2 = 70 \text{ mm}$$

$$\therefore \quad \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{3000 \times 10 + 2000 \times 70}{3000 + 2000} = 34 \text{ mm}$$

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We know that the moment of inertia of the section about *X-X*,

$$I_{XX} = \left[\frac{150\,(20)^3}{12} + 3000\,(34-10)^2 + \frac{20\,(100)^3}{12} + 2000\,(70-34)^2\right]$$
$$= 6.1 \times 10^6 \text{ mm}^4$$

and
$$I_{YY} = \frac{20\,(150)^3}{12} + \frac{100\,(20)^3}{12} = 5.7 \times 10^6 \text{ mm}^4$$

Since I_{YY} is less than I_{XX}, therefore the column will tend to buckle in *Y-Y* direction. Thus we shall take the value of *I* as $I_{YY} = 5.7 \times 10^6$ mm^4.

Moreover as the column is hinged at its both ends, therefore equivalent length,

$$L = l = 4000 \text{ mm}$$

We know that the crippling load,

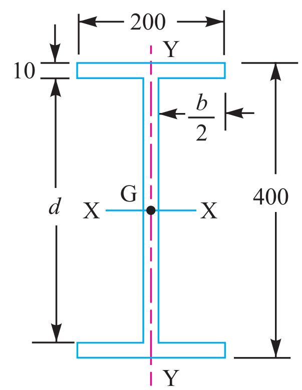
$$W_{cr} = \frac{\pi^2 E\, I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times 5.7 \times 10^6}{(4000)^2} = 703 \times 10^3 \text{ N} = 703 \text{ kN}$$ **Ans.**

Example 16.2. *An I-section 400 mm × 200 mm × 10 mm and 6 m long is used as a strut with both ends fixed. Find Euler's crippling load. Take Young's modulus for the material of the section as 200 kN/mm^2.*

Solution. Given : $D = 400$ mm ; $B = 200$ mm ; $t = 10$ mm ; $l = 6$ m $= 6000$ mm ; $E = 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2

The *I*-section is shown in Fig. 16.3.

Crippling load.



All dimensions in mm.

Fig. 16.3

We know that the moment of inertia of the *I*-section about *X-X*,

$$I_{XX} = \frac{B.D^3}{12} - \frac{b.d^3}{12}$$
$$= \frac{200\,(400)^3}{12} - \frac{(200-10)\,(400-20)^3}{12}$$
$$= 200 \times 10^6 \text{ mm}^4$$

and moment of inertia of the *I*-section about *Y-Y*,

$$I_{YY} = 2\left(\frac{t.B^3}{12}\right) + \frac{d.t^3}{12}$$

$$= 2\left[\frac{10\,(200)^3}{12}\right] + \frac{(400-20)\,10^3}{12}$$
$$= 13.36 \times 10^6 \text{ mm}^4$$

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Since I_{YY} is less than I_{XX}, therefore the section will tend to buckle about *Y-Y* axis. Thus we shall take *I* as I_{YY} = 13.36 × 10⁴ mm⁴.

Since the column is fixed at its both ends, therefore equivalent length,

$$L = l / 2 = 6000 / 2 = 3000 \text{ mm}$$

We know that the crippling load,

$$W_{cr} = \frac{\pi^2 E\ I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times 13.36 \times 10^6}{(3000)^2} = 2.93 \times 10^6 \text{ N}$$

$$= 2930 \text{ kN}$$ **Ans.**

16.10 Rankine's Formula for Columns

We have already discussed that Euler's formula gives correct results only for very long columns. Though this formula is applicable for columns, ranging from very long to short ones, yet it does not give reliable results. Prof. Rankine, after a number of experiments, gave the following empirical formula for columns.

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E} \quad ...(i)$$

where W_{cr} = Crippling load by Rankine's formula,

W_C = Ultimate crushing load for the column = $\sigma_c \times A$,

W_E = Crippling load, obtained by Euler's formula = $\dfrac{\pi^2 E\ I}{L^2}$

A little consideration will show, that the value of W_C will remain constant irrespective of the fact whether the column is a long one or short one. Moreover, in the case of short columns, the value of W_E will be very high, therefore the value of 1 / W_E will be quite negligible as compared to 1 / W_C. It is thus obvious, that the Rankine's formula will give the value of its crippling load (*i.e.* W_{cr}) approximately equal to the ultimate crushing load (*i.e.* W_C). In case of long columns, the value of W_E will be very small, therefore the value of 1 / W_E will be quite considerable as compared to 1 / W_C. It is thus obvious, that the Rankine's formula will give the value of its crippling load (*i.e.* W_{cr}) approximately equal to the crippling load by Euler's formula (*i.e.* W_E). Thus, we see that Rankine's formula gives a fairly correct result for all cases of columns, ranging from short to long columns.

From equation (*i*), we know that

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E} = \frac{W_E + W_C}{W_C \times W_E}$$

$$\therefore \quad W_{cr} = \frac{W_C \times W_E}{W_C + W_E} = \frac{W_C}{1 + \dfrac{W_C}{W_E}}$$

Now substituting the value of W_C and W_E in the above equation, we have

$$W_{cr} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c \times A \times L^2}{\pi^2 E\ I}} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c}{\pi^2 E} \times \dfrac{A.L^2}{A.k^2}} \quad ... (\because\ I = A.k^2)$$

$$= \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} = \frac{\text{Crushing load}}{1 + a\left(\dfrac{L}{k}\right)^2}$$

where σ_c = Crushing stress or yield stress in compression,

A = Cross-sectional area of the column,

a = Rankine's constant = $\dfrac{\sigma_c}{\pi^2 E}$,

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L = Equivalent length of the column, and

k = Least radius of gyration.

The following table gives the values of crushing stress and Rankine's constant for various materials.

Table 16.3. Values of crushing stress (σ_c) and Rankine's constant (a) for various materials.

S.No.	*Material*	σ_c *in MPa*	$a = \dfrac{\sigma_c}{\pi^2 E}$
1.	Wrought iron	250	$\dfrac{1}{9000}$
2.	Cast iron	550	$\dfrac{1}{1600}$
3.	Mild steel	320	$\dfrac{1}{7500}$
4.	Timber	50	$\dfrac{1}{750}$

16.11 Johnson's Formulae for Columns

Prof. J.B. Johnson proposed the following two formula for short columns.

1. ***Straight line formula.*** According to straight line formula proposed by Johnson, the critical or crippling load is

$$W_{cr} = A\left[\sigma_y - \frac{2\,\sigma_y}{3\pi}\left(\frac{L}{k}\right)\sqrt{\frac{\sigma_y}{3C \times E}}\right] = A\left[\sigma_y - C_1\left(\frac{L}{k}\right)\right]$$

where A = Cross-sectional area of column,

σ_y = Yield point stress,

$$C_1 = \frac{2\,\sigma_y}{3\pi}\sqrt{\frac{\sigma_y}{3C.E}}$$

= A constant, whose value depends upon the type of material as well as the type of ends, and

$\dfrac{L}{k}$ = Slenderness ratio.

If the safe stress (W_{cr} / A) is plotted against slenderness ratio (L / k), it works out to be a straight line, so it is known as straight line formula.

2. ***Parabolic formula.*** Prof. Johnson after proposing the straight line formula found that the results obtained by this formula are very approximate. He then proposed another formula, according to which the critical or crippling load,

$$W_{cr} = A \times \sigma_y\left[1 - \frac{\sigma_y}{4\,C\,\pi^2\,E}\left(\frac{L}{k}\right)^2\right] \text{ with usual notations.}$$

If a curve of safe stress (W_{cr} / A) is plotted against (L / k), it works out to be a parabolic, so it is known as parabolic formula.

Fig. 16.4 shows the relationship of safe stress (W_{cr} / A) and the slenderness ratio (L / k) as given by Johnson's formula and Euler's formula for a column made of mild steel with both ends hinged (*i.e.* C = 1), having a yield strength, σ_y = 210 MPa. We see from the figure that point A (the point of tangency between the Johnson's straight line formula and Euler's formula) describes the use of two formulae. In other words, Johnson's straight line formula may be used when $L / k < 180$ and the Euler's formula is used when $L / k > 180$.

Similarly, the point *B* (the point of tangency between the Johnson's parabolic formula and Euler's formula) describes the use of two formulae. In other words, Johnson's parabolic formula is used when $L / k < 140$ and the Euler's formula is used when $L / k > 140$.

Note : For short columns made of ductile materials, the Johnson's parabolic formula is used.

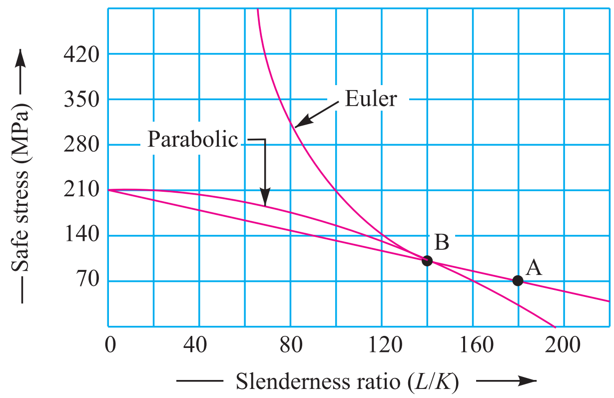

Fig. 16.4. Relation between slendeness ratio and safe stress.

16.12 Long Columns Subjected to Eccentric Loading

In the previous articles, we have discussed the effect of loading on long columns. We have always referred the cases when the load acts axially on the column (*i.e.* the line of action of the load coincides with the axis of the column). But in actual practice it is not always possible to have an axial load on the column, and eccentric loading takes place. Here we shall discuss the effect of eccentric loading on the Rankine's and Euler's formula for long columns.

Consider a long column hinged at both ends and subjected to an eccentric load as shown in Fig. 16.5.

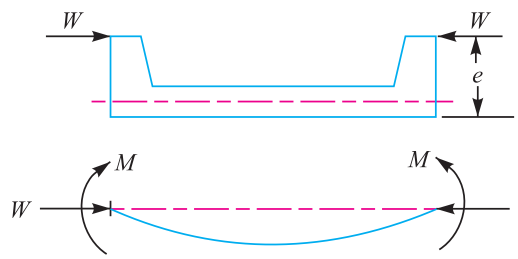

Fig. 16.5. Long column subjected to eccentric loading.

Let W = Load on the column,

A = Area of cross-section,

e = Eccentricity of the load,

Z = Section modulus,

y_c = Distance of the extreme fibre (on compression side) from the axis of the column,

k = Least radius of gyration,

I = Moment of inertia = $A.k^2$,

E = Young's modulus, and

l = Length of the column.

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We have already discussed that when a column is subjected to an eccentric load, the maximum intensity of compressive stress is given by the relation

$$\sigma_{max} = \frac{W}{A} + \frac{M}{Z}$$

The maximum bending moment for a column hinged at both ends and with eccentric loading is given by

$$M = W.e.\sec\frac{l}{2}\sqrt{\frac{W}{E.I}} = W.e.\sec\frac{l}{2\,k}\sqrt{\frac{W}{E.A}} \qquad \ldots (\because\ I = A.k^2)$$

$$\therefore \qquad \sigma_{max} = \frac{W}{A} + \frac{W.e.\sec\dfrac{l}{2\,k}\sqrt{\dfrac{W}{E.A}}}{Z}$$

$$= \frac{W}{A} + \frac{W.e.y_c.\sec\dfrac{l}{2\,k}\sqrt{\dfrac{W}{E.A}}}{A.k^2} \qquad \ldots (\because\ Z = I/y_c = A.k^2/y_c)$$

$$= \frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec\frac{l}{2\,k}\sqrt{\frac{W}{E.A}}\right]$$

$$= {}^{*}\frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec\frac{L}{2\,k}\sqrt{\frac{W}{E.A}}\right]$$

... (Substituting $l = L$, equivalent length for both ends hinged).

16.13 Design of Piston Rod

Since a piston rod moves forward and backward in the engine cylinder, therefore it is subjected to alternate tensile and compressive forces. It is usually made of mild steel. One end of the piston rod is secured to the piston by means of tapered rod provided with nut. The other end of the piston rod is joined to crosshead by means of a cotter.

Piston rod is made of mild steel.

* The expression $\sigma_{max} = \frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec\frac{L}{2k}\sqrt{\frac{W}{E.A}}\right]$ may also be written as follows:

$$\sigma_{max} = \frac{W}{A} + \frac{W}{A} \times \frac{e.y_c}{\dfrac{I}{A}}\sec\frac{L}{2k}\sqrt{\frac{W}{E \times \dfrac{I}{k^2}}} \qquad \ldots\left(\text{Substituting } k^2 = \frac{I}{A} \text{ and } A = \frac{I}{k^2}\right)$$

$$= \frac{W}{A} + \frac{W.e}{Z}\sec\frac{L}{2}\sqrt{\frac{W}{E.I}}$$

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Let p = Pressure acting on the piston,
D = Diameter of the piston,
d = Diameter of the piston rod,
W = Load acting on the piston rod,
W_{cr} = Buckling or crippling load = W × Factor of safety,
σ_t = Allowable tensile stress for the material of rod,
σ_c = Compressive yield stress,
A = Cross-sectional area of the rod,
l = Length of the rod, and
k = Least radius of gyration of the rod section.

The diameter of the piston rod is obtained as discussed below:

1. When the length of the piston rod is small *i.e.* when slenderness ratio (l / k) is less than 40, then the diameter of piston rod may be obtained by equating the load acting on the piston rod to its tensile strength, *i.e.*

$$W = \frac{\pi}{4} \times d^2 \times \sigma_t$$

or
$$\frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times d^2 \times \sigma_t$$

∴
$$d = D\sqrt{\frac{p}{\sigma_t}}$$

2. When the length of the piston rod is large, then the diameter of the piston rod is obtained by using Euler's formula or Rankine's formula. Since the piston rod is securely fastened to the piston and cross head, therefore it may be considered as fixed ends. The Euler's formula is

$$W_{cr} = \frac{\pi^2 E\, I}{L^2}$$

and Rankine's formula is,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2}$$

Example 16.3. *Calculate the diameter of a piston rod for a cylinder of 1.5 m diameter in which the greatest difference of steam pressure on the two sides of the piston may be assumed to be 0.2 N/mm². The rod is made of mild steel and is secured to the piston by a tapered rod and nut and to the crosshead by a cotter. Assume modulus of elasticity as 200 kN/mm² and factor of safety as 8. The length of rod may be assumed as 3 metres.*

Solution. Given : D = 1.5 m = 1500 mm ; p = 0.2 N/mm² ; E = 200 kN/mm² = 200 × 10³ N/mm² ; l = 3 m = 3000 mm

We know that the load acting on the piston,

$$W = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}\,(1500)^2 \times 0.2 = 353\,475 \text{ N}$$

∴ Buckling load on the piston rod,

$$W_{cr} = W \times \text{Factor of safety} = 353\,475 \times 8 = 2.83 \times 10^6 \text{ N}$$

Since the piston rod is considered to have both ends fixed, therefore from Table 16.2, the equivalent length of the piston rod,

$$L = \frac{l}{2} = \frac{3000}{2} = 1500 \text{ mm}$$

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Let d = Diameter of piston rod in mm, and

I = Moment of inertia of the cross-section of the rod $= \dfrac{\pi}{64} \times d^4$

According to Euler's formula, buckling load (W_{cr}),

$$2.83 \times 10^6 = \frac{\pi^2 E\, I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times \pi\, d^4}{(1500)^2 \times 64} = 0.043\, d^4$$

$\therefore$ $d^4 = 2.83 \times 10^6 / 0.043 = 65.8 \times 10^6$ or $d = 90$ mm

According to Rankine's formula, buckling load,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} \qquad \text{...}(i)$$

We know that for mild steel, the crushing stress,

$$\sigma_c = 320 \text{ MPa} = 320 \text{ N/mm}^2, \text{ and } a = \frac{1}{7500}$$

and least radius of gyration for the piston rod section,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi\, d^4}{64} \times \frac{4}{\pi\, d^2}} = \frac{d}{4}$$

Substituting these values in the above equation ***(i)***, we have

$$2.83 \times 10^6 = \frac{320 \times \dfrac{\pi\, d^2}{4}}{1 + \dfrac{1}{7500}\left(\dfrac{1500 \times 4}{d}\right)^2} = \frac{251.4\, d^2}{1 + \dfrac{4800}{d^2}} = \frac{251.4\, d^4}{d^2 + 4800}$$

$$251.4\, d^4 - 2.83 \times 10^6\, d^2 - 2.83 \times 10^6 \times 4800 = 0$$

or $$d^4 - 11\,257\, d^2 - 54 \times 10^6 = 0$$

$\therefore$ $$d^2 = \frac{11\,250 \pm \sqrt{(11\,257)^2 + 4 \times 1 \times 54 \times 10^6}}{2} = \frac{11\,257 \pm 18\,512}{2}$$

$= 14\,885$ ... (Taking +ve sign)

or $d = 122$ mm

Taking larger of the two values, we have

$d = 122$ mm **Ans.**

16.14 Design of Push Rods

The push rods are used in overhead valve and side valve engines. Since these are designed as long columns, therefore Euler's formula should be used. The push rods may be treated as pin end columns because they use spherical seated bearings.

Let W = Load acting on the push rod,

D = Diameter of the push rod,

d = Diameter of the hole through the push rod,

I = Moment of inertia of the push rod,

$= \dfrac{\pi}{64} \times D^4$, for solid rod

These rods are used in overhead valve and side valve engines.

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$$= \frac{\pi}{64} (D^4 - d^4), \text{ for tubular section}$$

l = Length of the push rod, and

E = Young's modulus for the material of push rod.

If m is the factor of safety for the long columns, then the critical or crippling load on the rod is given by

$$W_{cr} = m \times W$$

Now using Euler's formula, $W_{cr} = \frac{\pi^2 EI}{L^2}$, the diameter of the push rod (D) can be obtained.

Notes: 1. Generally the diameter of the hole through the push rod is 0.8 times the diameter of push rod, *i.e.*

$$d = 0.8\, D$$

2. Since the push rods are treated as pin end columns, therefore the equivalent length of the rod (L) is equal to the actual length of the rod (l).

Example 16.4. *The maximum load on a petrol engine push rod 300 mm long is 1400 N. It is hollow having the outer diameter 1.25 times the inner diameter. Spherical seated bearings are used for the push rod. The modulus of elasticity for the material of the push rod is 210 kN/mm². Find a suitable size for the push rod, taking a factor of safety of 2.5.*

Solution. Given : l = 300 mm ; W = 1400 N ; D = 1.25 d ; E = 210 kN/mm² = 210 × 10³ N/mm² ; m = 2.5

Let d = Inner diameter of push rod in mm, and

D = Outer diameter of the push rod in mm = 1.25 d ...(Given)

∴ Moment of inertia of the push rod section,

$$I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} [(1.25\, d)^4 - d^4] = 0.07\, d^4 \text{ mm}^4$$

We know that the crippling load on the push rod,

$$W_{cr} = m \times W = 2.5 \times 1400 = 3500 \text{ N}$$

Now according to Euler's formula, crippling load (W_{cr}),

$$3500 = \frac{\pi^2 E\, I}{L^2} = \frac{9.87 \times 210 \times 10^3 \times 0.07\, d^4}{(300)^2} = 1.6\, d^4 \quad \text{...}(\because L = l)$$

∴ $d^4 = 3500 / 1.6 = 2188$ or d = 6.84 mm **Ans.**

and $D = 1.25\, d = 1.25 \times 6.84 = 8.55$ mm **Ans.**

16.15 Design of Connecting Rod

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore the cross-section of the connecting rod is designed as a strut and the Rankine's formula is used.

A connecting rod subjected to an axial load W may buckle with X-axis as neutral axis (*i.e.* in the plane of motion of the connecting rod) or Y-axis as neutral axis (*i.e.* in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis. A connecting rod should be equally strong in buckling about either axes.

Let A = Cross-sectional area of the connecting rod,

l = Length of the connecting rod,

σ_c = Compressive yield stress,

W_{cr} = Crippling or buckling load,

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I_{xx} and I_{yy} = Moment of inertia of the section about X-axis and Y-axis respectively, and

k_{xx} and k_{yy} = Radius of gyration of the section about X-axis and Y-axis respectively.

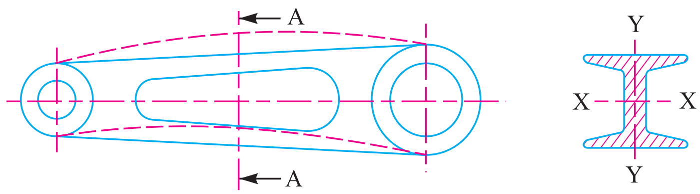

Fig. 16.6. Buckling of connecting rod.

According to Rankine's formula,

$$W_{cr} \text{ about } X\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} \quad \text{... } (\because \text{ For both ends hinged, } L = l)$$

and
$$W_{cr} \text{ about } Y\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{yy}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{2k_{yy}}\right)^2} \quad \text{...}\left(\because \text{ For both ends fixed, } L = \frac{l}{2}\right)$$

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, *i.e.*

$$\frac{\sigma_c \times A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{2\,k_{yy}}\right)^2} \quad \text{or} \quad \left(\frac{l}{k_{xx}}\right)^2 = \left(\frac{l}{2\,k_{yy}}\right)^2$$

$$\therefore \quad k^2_{xx} = 4\,k^2_{yy} \quad \text{or} \quad I_{xx} = 4\,I_{yy} \quad \text{... } (\because I = A \times k^2)$$

This shows that the connecting rod is four times strong in buckling about Y-axis than about X-axis. If $I_{xx} > 4\,I_{yy}$, then buckling will occur about Y-axis and if $I_{xx} < 4\,I_{yy}$, buckling will occur about X-axis. In actual practice, I_{xx} is kept slightly less than $4\,I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis. The design will alwyas be satisfactory for buckling about Y-axis.

The most suitable section for the connecting rod is I-section with the proportions as shown in Fig. 16.7 (*a*).

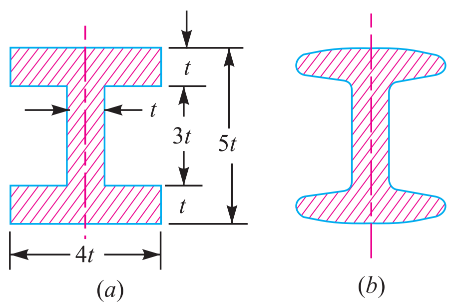

Fig. 16.7. I-section of connecting rod.

Area of the section

$$= 2\,(4\,t \times t) + 3\,t \times t = 11\,t^2$$

$\therefore$ Moment of inertia about X-axis,

$$I_{xx} = \frac{1}{12}\left[4\,t\,(5\,t)^3 - 3\,t\,(3\,t)^3\right] = \frac{419}{12}\,t^4$$

and moment of inertia about Y-axis,

$$I_{yy} = \left[2 \times \frac{1}{12}\,t \times (4\,t)^3 + \frac{1}{12}\,(3\,t)\,t^3\right] = \frac{131}{12}\,t^4$$

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$$\therefore \qquad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore *I*-section chosen is quite satisfactory.

Notes : 1. The *I*-section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible. It can also withstand high gas pressure.

2. Sometimes a connecting rod may have rectangular section. For slow speed engines, circular sections may be used.

3. Since connecting rod is manufactured by forging, therefore the sharp corners of *I*-section are rounded off as shown in Fig. 16.7 (*b*) for easy removal of the section from the dies.

Example 16.5. *A connecting rod of length l may be considered as a strut with the ends free to turn on the crank pin and the gudgeon pin. In the directions of the axes of these pins, however, it may be considered as having fixed ends. Assuming that Euler's formula is applicable, determine the ratio of the sides of the rectangular cross-section so that the connecting rod is equally strong in both planes of buckling.*

Solution. The rectangular cross-section of the connecting rod is shown in Fig. 16.8.

Let b = Width of rectangular cross-section, and

h = Depth of rectangular cross-section.

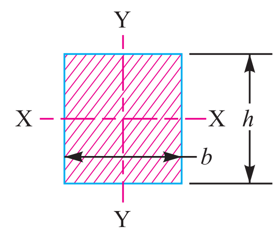

Fig. 16.8

∴ Moment of inertia about *X-X*,

$$I_{xx} = \frac{b.h^3}{12}$$

and moment of inertia about *Y-Y*,

$$I_{yy} = \frac{h.b^3}{12}$$

According to Euler's formula, buckling load,

$$W_{cr} = \frac{\pi^2 E I}{L^2}$$

∴ Buckling load about *X-X*,

$$W_{cr}\,(X\text{-axis}) = \frac{\pi^2 E I_{xx}}{l^2} \qquad \ldots (\because L = l, \text{ for both ends free to turn})$$

and buckling load about *Y-Y*,

$$W_{cr}\,(Y\text{-axis}) = \frac{\pi^2 E I_{yy}}{(l/2)^2} = \frac{4\pi^2 E I_{yy}}{l^2} \qquad \ldots (\because L = l/2, \text{ for both ends fixed})$$

In order to have the connecting rod equally strong in both the planes of buckling,

$$W_{cr}\,(X\text{-axis}) = W_{cr}\,(Y\text{-axis})$$

$$\frac{\pi^2 E I_{xx}}{l^2} = \frac{4\pi^2 E I_{yy}}{l^2} \qquad \text{or} \qquad I_{xx} = 4 I_{yy}$$

$$\therefore \qquad \frac{b\,h^3}{12} = \frac{4\,h\,b^3}{12} \qquad \text{or} \qquad h^2 = 4 b^2$$

and $h^2 / b^2 = 4$ or $h / b = 2$ **Ans.**

16.16 Forces Acting on a Connecting Rod

A connecting rod is subjected to the following forces :

1. Force due to gas or steam pressure and inertia of reciprocating parts, and

2. Inertia bending forces.

We shall now derive the expressions for the forces acting on a horizontal engine, as discussed below:

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1. ***Force due to gas or steam pressure and inertia of reciprocating parts***

Consider a connecting rod PC as shown in Fig. 16.9.

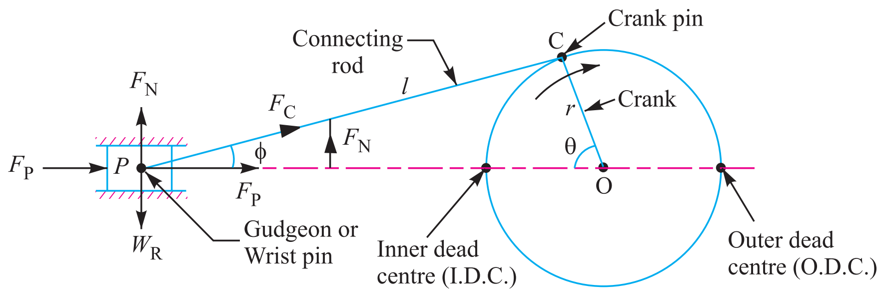

Fig. 16.9. Forces on a connecting rod.

Let
p = Pressure of gas or steam,
A = Area of piston,
m_R = Mass of reciprocating parts,
= Mass of piston, gudgeon pin etc. + $\frac{1}{3}$ rd mass of connecting rod,
ω = Angular speed of crank,
ϕ = Angle of inclination of the connecting rod with the line of stroke,
θ = Angle of inclination of the crank from inner dead centre,
r = Radius of crank,
l = Length of connecting rod, and
n = Ratio of length of connecting rod to radius of crank = l / r.

We know that the force on the piston due to pressure of gas or steam,

$$F_L = \text{Pressure} \times \text{Area} = p \times A$$

and inertia force of reciprocating parts,

$$F_I = \text{Mass} \times {}^*\text{Acceleration} = m_R \times \omega^2 \times r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

It may be noted that in a horizontal engine, reciprocating parts are accelerated from rest during the first half of the stroke (*i.e.* when the piston moves from inner dead centre to outer dead centre). It is then retarted during the latter half of the stroke (*i.e.* when the piston moves from outer dead centre to inner dead centre). The inertia force due to the acceleration of reciprocating parts, opposes the force on the piston. On the other hand, the inertia force due to retardation of the reciprocating parts, helps the force on the piston.

∴ Net force acting on the piston pin (or gudgeon or wrist pin),

$$F_P = \text{Force due to pressure of gas or steam} \pm \text{Inertia force}$$
$$= F_L \pm F_I$$

The –ve sign is used when the piston is accelerated and +ve sign is used when the piston is retarted.

* Acceleration of reciprocating parts = $\omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$

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The force F_P gives rise to a force F_C in the connecting rod and a thrust F_N on the sides of the cylinder walls (or normal reaction on crosshead guides). From Fig. 16.9, we see that force in the connecting rod at any instant.

$$F_C = \frac{F_P}{\cos\phi} = \frac{{}^{*}\,F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

The force in the connecting rod will be maximum when the crank and the connecting rod are perpendicular to each other (*i.e.* when $\theta = 90°$). But at this position, the gas pressure would be decreased considerably. Thus, for all practical purposes, the force in the connecting rod (F_C) is taken equal to the maximum force on the piston due to pressure of gas or steam (F_L), neglecting piston inertia effects.

Connecting rod.

2. *Inertia bending forces*

Consider a connecting rod *PC* and a crank *OC* rotating with uniform angular velocity ω rad /s. In order to find the acceleration of various points on the connecting rod, draw the Klien's acceleration diagram *CQNO* as shown in Fig. 16.10 (*a*). *CO* represents the acceleration of *C* towards *O* and *NO* represents the acceleration of *P* towards *O*. The acceleration of other points such as *D*, *E*, *F* and *G* etc. on the connecting rod *PC* may be found by drawing horizontal lines from these points to intersect *CN* at *d*, *e*, *f* and *g* respectively. Now *dO*, *eO*, *fO* and *gO* represents the acceleration of *D*, *E*, *F* and *G* all towards *O*. The inertia force acting on each point will be as follows :

Inertia force at $C = m \times \omega^2 \times CO$

Inertia force at $D = m \times \omega^2 \times dO$

Inertia force at $E = m \times \omega^2 \times eO$, and so on.

The inertia forces will be opposite to the direction of acceleration or centrifugal forces. The inertia forces can be resolved into two components, one parallel to the connecting rod and the other perpendicular to the rod. The parallel (or longitudinal) components adds up algebraically to the force acting on the connecting rod (F_C) and produces thrust on the pins. The perpendicular (or transverse) components produces bending action (also called whipping action) and the stress induced in the connecting rod is called ***whipping stress.***

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* For derivation, please refer to author's popular book on **'Theory of Machines'.**

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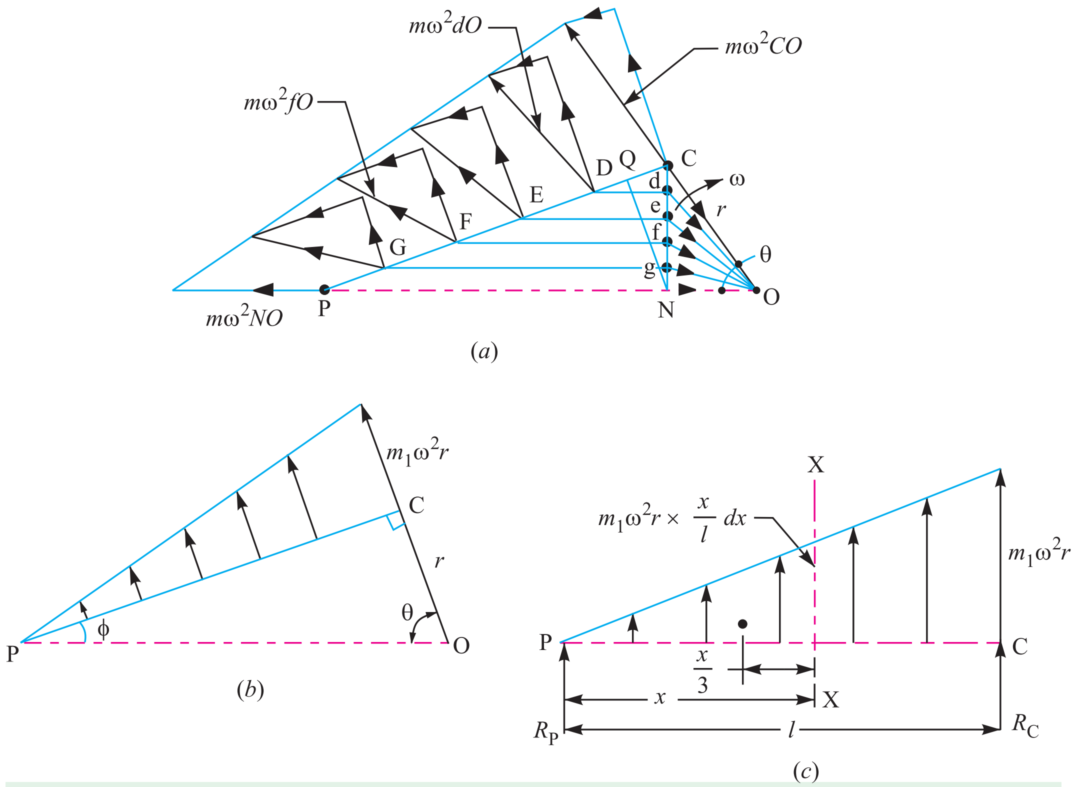

Fig. 16.10. Inertia bending forces.

A little consideration will show that the perpendicular components will be maximum, when the crank and connecting rod are at right angles to each other.

The variation of the inertia force on the connecting rod is linear and is like a simply supported beam of variable loading as shown in Fig. 16.10 (*b*) and (*c*). Assuming that the connecting rod is of uniform cross-section and has mass m_1 kg per unit length, therefore

Inertia force per unit length at the crank pin

$$= m_1 \times \omega^2 r$$

and inertia force per unit length at the gudgeon pin

$$= 0$$

Inertia forces due to small element of length dx at a distance x from the gudgeon pin P,

$$dF_I = m_1 \times \omega^2 r \times \frac{x}{l} \times dx$$

∴ Resultant inertia force,

$$F_I = \int_0^l m_1 \times \omega^2 r \times \frac{x}{l} \times dx = \frac{m_1 \times \omega^2 r}{l}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{m_1 \times l}{2} \times \omega^2 r = \frac{m}{2} \times \omega^2 r \qquad \text{... (Substituting } m_1.l = m)$$

This resultant inertia force acts at a distance of $2l / 3$ from the gudgeon pin P.

Since it has been assumed that $\frac{1}{3}$ rd mass of the connecting rod is concentrated at gudgeon pin P (*i.e.* small end of connecting rod) and $\frac{2}{3}$ rd at the crank pin (*i.e.* big end of connecting rod),

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therefore the reactions at these two ends will be in the same proportion, *i.e.*

$$R_P = \frac{1}{3} F_I, \text{ and } R_C = \frac{2}{3} F_I$$

Now the bending moment acting on the rod at section *X-X* at a distance *x* from *P*,

$$M_X = R_P \times x - {}^{*}m_1 \times \omega^2 r \times \frac{x}{l} \times \frac{1}{2} x \times \frac{x}{3}$$

$$= \frac{1}{3} F_I \times x - \frac{m_1 . l}{2} \times \omega^2 \, r \times \frac{x^3}{3\,l^2} \qquad \ldots \left(\because R_P = \frac{1}{3} F_I \right)$$

... (Multiplying and dividing the latter expression by *l*)

$$= \frac{F_I \times x}{3} - F_I \times \frac{x^3}{3l^2} = \frac{F_I}{3} \left(x - \frac{x^3}{l^2} \right) \qquad \ldots(i)$$

For maximum bending moment, differentiate M_X with respect to *x* and equate to zero, *i.e.*

$$\frac{d_{MX}}{dx} = 0 \quad \text{or} \quad \frac{F_I}{3} \left[1 - \frac{3x^2}{l^2} \right] = 0$$

$$\therefore \quad 1 - \frac{3x^2}{l^2} = 0 \quad \text{or} \quad 3x^2 = l^2 \quad \text{or} \quad x = \frac{l}{\sqrt{3}}$$

Substituting this value of *x* in the above equation *(i)*, we have maximum bending moment,

$$M_{max} = \frac{F_I}{3} \left[\frac{l}{\sqrt{3}} - \frac{\left(\frac{l}{\sqrt{3}} \right)^3}{l^2} \right] = \frac{F_I}{3} \left[\frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}} \right]$$

$$= \frac{F_I}{3} \times \frac{2l}{3\sqrt{3}} = \frac{2\, F_I \times l}{9\sqrt{3}}$$

$$= 2 \times \frac{m}{2} \times \omega^2 \, r \times \frac{l}{9\sqrt{3}} = m \times \omega^2 \, r \times \frac{l}{9\sqrt{3}} \qquad \ldots \left(\because F_I = \frac{m}{2} \times \omega^2 r \right)$$

and the maximum bending stress, due to inertia of the connecting rod,

$$\sigma_{max} = \frac{M_{max}}{Z}$$

where Z = Section modulus.

From above we see that the maximum bending moment varies as the square of speed, therefore, the bending stress due to high speed will be dangerous. It may be noted that the maximum axial force and the maximum bending stress do not occur simultaneously. In an I.C. engine, the maximum gas load occurs close to top dead centre whereas the maximum bending stress occurs when the crank angle θ = 65° to 70° from top dead centre. The pressure of gas falls suddenly as the piston moves from dead centre. In steam engines, even though the pressure is maintained till cut off occurs, the speed is low and therefore the bending stress due to inertia is small. Thus the general practice is to design a connecting rod by assuming the force in the connecting rod (F_C) equal to the maximum force on the piston due to pressure of gas or steam (F_L), neglecting piston inertia effects and then checked for bending stress due to inertia force (*i.e.* whipping stress).

* B.M. due to variable loading from $\left(0 \text{ to } m_1 \; \omega^2 \, r \times \frac{x}{l} \right)$ is equal to the area of triangle multiplied by distance of C.G. from *X-X* $\left(i.e. \; \frac{x}{3} \right)$.

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Example 16.6. *Determine the dimensions of an I-section connecting rod for a petrol engine from the following data :*

Diameter of the piston = 110 mm
Mass of the reciprocating parts = 2 kg
Length of the connecting rod from centre to centre = 325 mm
Stroke length = 150 mm
R.P.M. = 1500 with possible overspeed of 2500
Compression ratio = 4 : 1
Maximum explosion pressure = 2.5 N/mm²


Connecting rod of a petrol engine.

Solution. Given : D = 110 mm = 0.11 m ; m_R = 2 kg ; l = 325 mm = 0.325 m ; Stroke length = 150 mm = 0.15 m ; N_{min} = 1500 r.p.m. ; N_{max} = 2500 r.p.m. ; *Compression ratio = 4 : 1 ; p = 2.5 N/mm^2

We know that the radius of crank,

$$r = \frac{\text{Stroke length}}{2} = \frac{150}{2} = 75 \text{ mm} = 0.075 \text{ m}$$

and ratio of the length of connecting rod to the radius of crank,

$$n = \frac{l}{r} = \frac{325}{75} = 4.3$$

We know that the maximum force on the piston due to pressure,

$$F_L = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (110)^2\, 2.5 = 23\,760 \text{ N}$$

and maximum angular speed,

$$\omega_{max} = \frac{2\pi \times N_{max}}{60} = \frac{2\pi \times 2500}{60} = 261.8 \text{ rad/s}$$

We know that maximum inertia force of reciprocating parts,

$$F_I = m_R\,(\omega_{max})^2\, r \left(\cos\theta + \frac{\cos 2\theta}{n}\right) \quad \text{... } (i)$$

The inertia force of reciprocating parts is maximum, when the crank is at inner dead centre, *i.e.* when θ = 0°.

$$\therefore \quad F_I = m_R\,(\omega_{max})^2 r \left(1 + \frac{1}{n}\right) \quad \text{... [From equation } (i)]$$

$$= 2(261.8)^2\, 0.075 \left(1 + \frac{1}{4.3}\right) = 12\,672 \text{ N}$$

Since the connecting rod is designed by taking the force in the connecting rod (F_C) equal to the maximum force on the piston due to gas pressure (F_L), therefore

Force in the connecting rod,

$$F_C = F_L = 23\,760 \text{ N}$$

* Superfluous data.

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Consider the *I*-section of the connecting rod with the proportions as shown in Fig. 16.11. We have discussed in Art. 16.15 that for such a section

$$\frac{I_{xx}}{I_{yy}} = 3.2$$

or $$\frac{k^2_{xx}}{k^2_{yy}} = 3.2, \text{ which is satisfactory.}$$

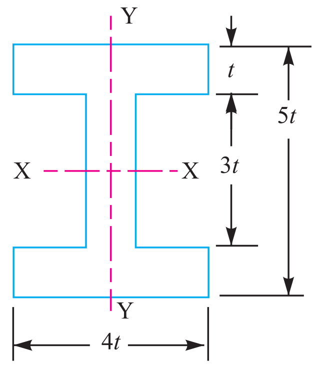

Fig. 16.11

We have also discussed that the connecting rod is designed for buckling about *X*-axis (*i.e.* in a plane of motion of the connecting rod), assuming both ends hinged. Taking a factor of safety as 6, the buckling load,

$$W_{cr} = F_C \times 6 = 23\ 760 \times 6 = 142\ 560 \text{ N}$$

and area of cross-section,

$$A = 2\,(4t \times t) + t \times 3t = 11\, t^2 \text{ mm}^2$$

Moment of inertia about *X*-axis,

$$I_{xx} = \left[\frac{4\,t\,(5\,t)^3}{12} - \frac{3\,t\,(3\,t)^3}{12}\right] = \frac{419\,t^4}{12} \text{ mm}^4$$

∴ Radius of gyration,

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419\,t^4}{12} \times \frac{1}{11\,t^2}} = 1.78\,t$$

We know that equivalent length of the rod for both ends hinged,

$$L = l = 325 \text{ mm}$$

Taking for mild steel, σ_c = 320 MPa = 320 N/mm^2 and a = 1 / 7500, we have from Rankine's formula,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

$$142\ 560 = \frac{320 \times 11\,t^2}{1 + \frac{1}{7500}\left(\frac{325}{1.78\,t}\right)^2}$$

$$40.5 = \frac{t^2}{1 + \frac{4.44}{t^2}} = \frac{t^4}{t^2 + 4.44}$$

or $$t^4 - 40.5\,t^2 - 179.8 = 0$$

∴ $$t^2 = \frac{40.5 \pm \sqrt{(40.5)^2 + 4 \times 179.8}}{2} = \frac{40.5 \pm 48.6}{2} = 44.55$$

... (Taking +ve sign)

or $$t = 6.67 \text{ say } 6.8 \text{ mm}$$

Therefore, dimensions of cross-section of the connecting rod are

Height $= 5\,t = 5 \times 6.8 = 34$ mm **Ans.**

Width $= 4\,t = 4 \times 6.8 = 27.2$ mm **Ans.**

Thickness of flange and web

$= t = 6.8$ mm $= 0.0068$ m **Ans.**

Now let us find the bending stress due to inertia force on the connecting rod.

We know that the mass of the connecting rod per metre length,

$$m_1 = \text{Volume} \times \text{density} = \text{Area} \times \text{length} \times \text{density}$$

$$= A \times l \times \rho = 11\,t^2 \times l \times \rho \qquad ..(\because A = 11\,t^2)$$

$$= 11(0.0068)^2\ 1 \times 7800 = 3.97 \text{ kg} \qquad \text{... (Taking } \rho = 7800 \text{ kg / m}^3)$$

∴ Maximum bending moment,

$$M_{max} = m\,\omega^2\,r \times \frac{l}{9\sqrt{3}} = m_1\,\omega^2\,r \times \frac{l^2}{9\sqrt{3}} \qquad \text{... } (\because m = m_1.l)$$

$$= 3.97\,(261.8)^2\,(0.075) \times \frac{(0.325)^2}{9\sqrt{3}} = 138.3 \text{ N-m}$$

and section modulus,

$$Z_{xx} = \frac{I_{xx}}{5\,t/2} = \frac{419\,t^4}{12} \times \frac{2}{5\,t} = \frac{419}{30}\,t^3$$

$$= \frac{419}{30}\,(0.0068)^3 = 4.4 \times 10^{-6} \text{ m}^3$$

∴ Maximum bending or whipping stress due to inertia bending forces,

$$\sigma_{b(max)} = \frac{M_{max}}{Z_{xx}} = \frac{138.3}{4.4 \times 10^{-6}} = 31.4 \times 10^6 \text{ N/m}^2$$

$$= 31.4 \text{ MPa, which is safe}$$

Note : The maximum compressive stress in the connecting rod will be,

$$\sigma_{c(max)} = \text{Direct compressive stress} + \text{Maximum bending stress}$$

$$= \frac{320}{6} + 31.4 = 84.7 \text{ MPa}$$

EXERCISES

1. Compare the ratio of strength of a solid steel column to that of a hollow column of internal diameter equal to 3/4th of its external diameter. Both the columns have the same cross-sectional areas, lengths and end conditions. **[Ans. 25/7]**

2. Find the Euler's crippling load for a hollow cylindrical steel column of 38 mm external diameter and 35 mm thick. The length of the column is 2.3 m and hinged at its both ends. Take $E = 200$ GN/m^2. Also determine the crippling load by Rankine's formula, using

$$\sigma_c = 320 \text{ MPa ; and } a = \frac{1}{7500}$$

[Ans. 17.25 kN ; 17.4 kN]

3. Determine the diameter of the pistion rod of the hydraulic cylinder of 100 mm bore when the maximum hydraulic pressure in the cylinder is limited to 14 N/mm^2. The length of the piston rod is 1.2 m. The factor of safety may be taken as 5 and the end fixity coefficient as 2. **[Ans. 45 mm]**

4. Find the diameter of a piston rod for an engine of 200 mm diameter. The length of the piston rod is 0.9 m and the stroke is 0.5 m. The pressure of steam is 1 N/mm^2. Assume factor of safety as 5. **[Ans. 31 mm]**

5. Determine the diameter of the push rod made of mild steel of an I.C. engine if the maximum force exerted by the push rod is 1500 N. The length of the push rod is 0.5 m. Take the factor of safety as 2.5 and the end fixity coefficient as 2. **[Ans. 10 mm]**
6. The eccentric rod to drive the D-slide valve mechanism of a steam engine carries a maximum compressive load of 10 kN. The length of the rod is 1.5 m. Assuming the eccentric rod hinged at both the ends, find
   (*a*) diameter of the rod, and
   (*b*) dimensions of the cross-section of the rod if it is of rectangular section. The depth of the section is twice its thickness.
   Take factor of safety = 40 and E = 210 kN/mm^2. **[Ans. 60 mm ; 30 × 60 mm]**

Screwjacks

7. Determine the dimensions of an *I*-section connecting rod for an internal combustion engine having the following specifications :

Diameter of the piston	=	120 mm
Mass of reciprocating parts piston	=	350 kg/m^2 of area
Length of connecting rod	=	350 mm
Engine revolutions per minute	=	1800
Maximum explosion pressure	=	3 N/mm^2
Stroke length	=	180 mm

   The flange width and the depth of the *I*-section rod are in the ratio of 4 t : 6 t where t is the thickness of the flange and web. Assume yield stress in compression for the material as 330 MPa and a factor of safety as 6. **[Ans. t = 7.5 mm]**
8. The connecting rod of a four stroke cycle Diesel engine is of circular section and of length 550 mm. The diameter and stroke of the cylinder are 150 mm and 240 mm respectively. The maximum combustion pressure is 4.7 N/mm^2. Determine the diameter of the rod to be used, for a factor of safety of 3 with a material having a yield point of 330 MPa.
   Find also the maximum bending stress in the connecting rod due to whipping action if the engine runs at 1000 r.p.m. The specific weight of the material is 7800 kg/m^3.
   [Ans. 33.2 mm ; 48 MPa]

QUESTIONS

1. What do you understand by a column or strut ? Explain the various end conditions of a column or strut.
2. State the assumptions used in Euler's column theory.
3. Define 'slenderness ratio'. How it is used to define long and short columns ?
4. What is equivalent length of a column ? Write the relations between equivalent length and actual length of a column for various end conditions.
5. Explain Johnson's formula for columns. Describe the use of Johnson's formula and Euler's formula.
6. Write the formula for obtaining a maximum stress in a long column subjected to eccentric loading.

7. How the piston rod is designed ?
8. Explain the design procedure of valve push rods.
9. Why an *I*-Section is usually preferred to a round section in case of connecting rods?

OBJECTIVE TYPE QUESTIONS

1. A machine part is designed as a strut, when it is subjected to
   (*a*) an axial tensile force (*b*) an axial compressive force
   (*c*) a tangential force (*d*) any one of these
2. Slenderness ratio is the ratio of
   (*a*) maximum size of a column to minimum size of column
   (*b*) width of column to depth of column
   (*c*) effective length of column to least radius of gyration of the column
   (*d*) effective length of column to width of column
3. A connecting rod is designed as a
   (*a*) long column (*b*) short column
   (*c*) strut (*d*) any one of these
4. Which of the following formula is used in designing a connecting rod ?
   (*a*) Euler's formula (*b*) Rankine's formula
   (*c*) Johnson's straight line formula (*d*) Johnson's parabolic formula
5. A connecting rod subjected to an axial load may buckle with
   (*a*) *X*-axis as neutral axis (*b*) *Y*-axis as neutral axis
   (*c*) *X*-axis or *Y*-axis as neutral axis (*d*) *Z*-axis
6. In designing a connecting rod, it is considered like .... for buckling about *X*-axis.
   (*a*) both ends hinged (*b*) both ends fixed
   (*c*) one end fixed and the other end hinged (*d*) one end fixed and the other end free
7. A connecting rod should be
   (*a*) strong in buckling about *X*-axis
   (*b*) strong in buckling about *Y*-axis
   (*c*) equally strong in buckling about *X*-axis and *Y*-axis
   (*d*) any one of the above
8. The buckling will occur about *Y*-axis,if
   (*a*) $I_{xx} = I_{yy}$ (*b*) $I_{xx} = 4I_{yy}$
   (*c*) $I_{xx} > 4I_{yy}$ (*d*) $I_{xx} < 4I_{yy}$
9. The connecting rod will be equally strong in buckling about X-axis and Y-axis, if
   (*a*) $I_{xx} = I_{yy}$ (*b*) $I_{xx} = 2I_{yy}$
   (*c*) $I_{xx} = 3I_{yy}$ (*d*) $I_{xx} = 4I_{yy}$
10. The most suitable section for the connecting rod is
   (*a*) *L*-section (*b*) *T*-section
   (*c*) *I*-section (*d*) *C*-section

ANSWERS

1. (*b*)	**2.** (*c*)	**3.** (*c*)	**4.** (*b*)	**5.** (*c*)
6. (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*c*)