Chapter 14

14-1 $$d = \frac{N}{P} = \frac{22}{6} = 3.667 \text{ in}$$

Table 14-2: $Y = 0.331$

Eq. (13-34): $$V = \frac{\pi dn}{12} = \frac{\pi(3.667)(1200)}{12} = 1152 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 1152}{1200} = 1.96$$

Eq. (13-35) : $$W^t = 33\,000\frac{H}{V} = 33\,000\frac{15}{1152} = 429.7 \text{ lbf}$$

Eq. (14-7): $$\sigma = \frac{K_v W^t P}{FY} = \frac{1.96(429.7)(6)}{2(0.331)} = 7633 \text{ psi} = 7.63 \text{ kpsi} \quad \textit{Ans.}$$

14-2 $$d = \frac{N}{P} = \frac{18}{10} = 1.8 \text{ in}$$

Table 14-2: $Y = 0.309$

Eq. (13-34): $$V = \frac{\pi dn}{12} = \frac{\pi(1.8)(600)}{12} = 282.7 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 282.7}{1200} = 1.236$$

Eq. (13-35) : $$W^t = 33\,000\frac{H}{V} = 33\,000\frac{2}{282.7} = 233.5 \text{ lbf}$$

Eq. (14-7): $$\sigma = \frac{K_v W^t P}{FY} = \frac{1.236(233.5)(10)}{1.0(0.309)} = 9340 \text{ psi} = 9.34 \text{ kpsi} \quad \textit{Ans.}$$

14-3 $$d = mN = 1.25(18) = 22.5 \text{ mm}$$

Table 14-2: $Y = 0.309$

$$V = \frac{\pi dn}{60} = \frac{\pi(22.5)(10^{-3})(1800)}{60} = 2.121 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 2.121}{6.1} = 1.348$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(0.5)}{\pi(22.5)(1800)} = 0.2358 \text{ kN} = 235.8 \text{ N}$$

Eq. (14-8): $$\sigma = \frac{K_v W^t}{FmY} = \frac{1.348(235.8)}{12(1.25)(0.309)} = 68.6 \text{ MPa} \quad \textit{Ans.}$$

14-4 $d = mN = 8(16) = 128$ mm

Table 14-2: $Y = 0.296$

$$V = \frac{\pi dn}{60} = \frac{\pi(128)(10^{-3})(150)}{60} = 1.0053 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 1.0053}{6.1} = 1.165$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(6)}{\pi(128)(150)} = 5.968 \text{ kN} = 5968 \text{ N}$$

Eq. (14-8): $$\sigma = \frac{K_v W^t}{FmY} = \frac{1.165(5968)}{90(8)(0.296)} = 32.6 \text{ MPa} \quad \textit{Ans.}$$

14-5 $d = mN = 1(16) = 16$ mm

Table 14-2: $Y = 0.296$

$$V = \frac{\pi dn}{60} = \frac{\pi(16)(10^{-3})(400)}{60} = 0.335 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 0.335}{6.1} = 1.055$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(0.15)}{\pi(16)(400)} = 0.4476 \text{ kN} = 447.6 \text{ N}$$

Eq. (14-8): $$F = \frac{K_v W^t}{\sigma mY} = \frac{1.055(447.6)}{150(1)(0.296)} = 10.6 \text{ mm}$$

From Table 13-2, use $F = 11$ mm or 12 mm, depending on availability. *Ans.*

14-6 $d = mN = 2(20) = 40$ mm

Table 14-2: $Y = 0.322$

$$V = \frac{\pi dn}{60} = \frac{\pi(40)(10^{-3})(200)}{60} = 0.419 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 0.419}{6.1} = 1.069$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(0.5)}{\pi(40)(200)} = 1.194 \text{ kN} = 1194 \text{ N}$$

Eq. (14-8): $$F = \frac{K_v W^t}{\sigma mY} = \frac{1.069(1194)}{75(2.0)(0.322)} = 26.4 \text{ mm}$$

From Table 13-2, use $F = 28$ mm. *Ans.*

14-7 $$d = \frac{N}{P} = \frac{24}{5} = 4.8 \text{ in}$$

Table 14-2: $Y = 0.337$

Eq. (13-34): $$V = \frac{\pi dn}{12} = \frac{\pi(4.8)(50)}{12} = 62.83 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 62.83}{1200} = 1.052$$

Eq. (13-35) : $$W^t = 33\,000\frac{H}{V} = 33\,000\frac{6}{62.83} = 3151 \text{ lbf}$$

Eq. (14-7): $$F = \frac{K_v W^t P}{\sigma Y} = \frac{1.052(3151)(5)}{20(10^3)(0.337)} = 2.46 \text{ in}$$

Use $F = 2.5$ in *Ans.*

14-8 $$d = \frac{N}{P} = \frac{16}{4} = 4.0 \text{ in}$$

Table 14-2: $Y = 0.296$

Eq. (13-34): $$V = \frac{\pi dn}{12} = \frac{\pi(4.0)(400)}{12} = 418.9 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 418.9}{1200} = 1.349$$

Eq. (13-35) : $$W^t = 33\,000\frac{H}{V} = 33\,000\frac{20}{418.9} = 1575.6 \text{ lbf}$$

Eq. (14-7): $$F = \frac{K_v W^t P}{\sigma Y} = \frac{1.349(1575.6)(4)}{12(10^3)(0.296)} = 2.39 \text{ in}$$

Use $F = 2.5$ in *Ans.*

14-9 Try $P = 8$ which gives $d = 18/8 = 2.25$ in and $Y = 0.309$.

Eq. (13-34): $$V = \frac{\pi dn}{12} = \frac{\pi(2.25)(600)}{12} = 353.4 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 353.4}{1200} = 1.295$$

Eq. (13-35): $$W^t = 33\,000\frac{H}{V} = 33\,000\frac{2.5}{353.4} = 233.4 \text{ lbf}$$

Eq. (14-7): $$F = \frac{K_v W^t P}{\sigma Y} = \frac{1.295(233.4)(8)}{10(10^3)(0.309)} = 0.783 \text{ in}$$

Using coarse integer pitches from Table 13-2, the following table is formed.

P	d	V	K_v	W^t	F
2	9.000	1413.717	2.178	58.356	0.082
3	6.000	942.478	1.785	87.535	0.152
4	4.500	706.858	1.589	116.713	0.240
6	3.000	471.239	1.393	175.069	0.473
8	2.250	353.429	1.295	233.426	0.782
10	1.800	282.743	1.236	291.782	1.167
12	1.500	235.619	1.196	350.139	1.627
16	1.125	176.715	1.147	466.852	2.773

Other considerations may dictate the selection. Good candidates are $P = 8$ ($F = 7/8$ in) and $P = 10$ ($F = 1.25$ in). *Ans.*

14-10 Try $m = 2$ mm which gives $d = 2(18) = 36$ mm and $Y = 0.309$.

$$V = \frac{\pi dn}{60} = \frac{\pi(36)(10^{-3})(900)}{60} = 1.696 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 1.696}{6.1} = 1.278$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(1.5)}{\pi(36)(900)} = 0.884 \text{ kN} = 884 \text{ N}$$

Eq. (14-8): $$F = \frac{1.278(884)}{75(2)(0.309)} = 24.4 \text{ mm}$$

Using the preferred module sizes from Table 13-2:

m	d	V	K_v	W^t	F
1.00	18.0	0.848	1.139	1768.388	86.917
1.25	22.5	1.060	1.174	1414.711	57.324
1.50	27.0	1.272	1.209	1178.926	40.987
2.00	36.0	1.696	1.278	884.194	24.382
3.00	54.0	2.545	1.417	589.463	12.015
4.00	72.0	3.393	1.556	442.097	7.422
5.00	90.0	4.241	1.695	353.678	5.174
6.00	108.0	5.089	1.834	294.731	3.888
8.00	144.0	6.786	2.112	221.049	2.519
10.00	180.0	8.482	2.391	176.839	1.824
12.00	216.0	10.179	2.669	147.366	1.414
16.00	288.0	13.572	3.225	110.524	0.961
20.00	360.0	16.965	3.781	88.419	0.721
25.00	450.0	21.206	4.476	70.736	0.547
32.00	576.0	27.143	5.450	55.262	0.406
40.00	720.0	33.929	6.562	44.210	0.313
50.00	900.0	42.412	7.953	35.368	0.243

$$\sigma_C = -2100\left[\frac{1.204(202.6)}{F\cos 20°}\left(\frac{1}{0.228}+\frac{1}{0.684}\right)\right]^{1/2} = -100(10^3)$$

$$F = \left(\frac{2100}{100(10^3)}\right)^2\left[\frac{1.204(202.6)}{\cos 20°}\right]\left(\frac{1}{0.228}+\frac{1}{0.684}\right) = 0.669 \text{ in}$$

Use $F = 0.75$ in *Ans.*

14-13 $d_p = 5(24) = 120$ mm, $d_G = 5(48) = 240$ mm

$$V = \frac{\pi(120)(10^{-3})(50)}{60} = 0.3142 \text{ m/s}$$

Eq. (14-6*a*): $K_v = \dfrac{3.05 + 0.3142}{3.05} = 1.103$

$$W^t = \frac{60\,000H}{\pi dn} = \frac{60(10^3)H}{\pi(120)(50)} = 3.183H$$

where H is in kW and W^t is in kN

Table 14-8: $C_p = 163\sqrt{\text{MPa}}$ [Note: Using Eq. (14-13) can result in wide variation in C_p due to wide variation in cast iron properties].

Eq. (14-12): $r_1 = \dfrac{120\sin 20°}{2} = 20.52$ mm, $r_2 = \dfrac{240\sin 20°}{2} = 41.04$ mm

Eq. (14-14): $-690 = -163\left[\dfrac{1.103(3.183)\left(10^3\right)H}{60\cos 20°}\left(\dfrac{1}{20.52}+\dfrac{1}{41.04}\right)\right]^{1/2}$

$H = 3.94$ kW *Ans.*

14-14

$d_P = 4(20) = 80$ mm, $d_G = 4(32) = 128$ mm

$$V = \frac{\pi(80)(10^{-3})(1000)}{60} = 4.189 \text{ m/s}$$

Eq. (14-6*a*): $K_v = \dfrac{3.05 + 4.189}{3.05} = 2.373$

$$W^t = \frac{60(10)(10^3)}{\pi(80)(1000)} = 2.387 \text{ kN} = 2387 \text{ N}$$

Table 14-8: $C_p = 163\sqrt{\text{MPa}}$ [Note: Using Eq. (14-13) can result in wide variation in C_p due to wide variation in cast iron properties.]

Eq. (14-12): $r_1 = \dfrac{80\sin 20°}{2} = 13.68$ mm, $r_2 = \dfrac{128\sin 20°}{2} = 21.89$ mm

Eq. (14-14): $\sigma_C = -163\left[\dfrac{2.373(2387)}{50\cos 20°}\left(\dfrac{1}{13.68}+\dfrac{1}{21.89}\right)\right]^{1/2} = -617$ MPa *Ans.*

14-15 The pinion controls the design.

Bending $Y_P = 0.303$, $Y_G = 0.359$

$$d_P = \frac{17}{12} = 1.417 \text{ in}, \quad d_G = \frac{30}{12} = 2.500 \text{ in}$$

$$V = \frac{\pi d_P n}{12} = \frac{\pi(1.417)(525)}{12} = 194.8 \text{ ft/min}$$

Eq. (14-4*b*): $K_v = \dfrac{1200+194.8}{1200} = 1.162$

Eq. (6-8), p. 282: $S'_e = 0.5(76) = 38.0$ kpsi

Eq. (6-19), p. 287: $k_a = 2.70(76)^{-0.265} = 0.857$

$$l = \frac{2.25}{P_d} = \frac{2.25}{12} = 0.1875 \text{ in}$$

Eq. (14-3): $x = \dfrac{3Y_P}{2P} = \dfrac{3(0.303)}{2(12)} = 0.0379$ in

Eq. (b), p. 737: $t = \sqrt{4lx} = \sqrt{4(0.1875)(0.0379)} = 0.1686$ in

Eq. (6-25), p. 289: $d_e = 0.808\sqrt{hb} = 0.808\sqrt{0.875(0.1686)} = 0.310$ in

Eq. (6-20), p. 288: $k_b = \left(\dfrac{0.310}{0.3}\right)^{-0.107} = 0.996$

$$k_c = k_d = k_e = 1$$

Account for one-way bending with $k_f = 1.66$. (See Ex. 14-2.)

Eq. (6-18), p. 287: $S_e = 0.857(0.996)(1)(1)(1)(1.66)(38.0) = 53.84$ kpsi

For stress concentration, find the radius of the root fillet (See Ex. 14-2).

$$r_f = \frac{0.300}{P} = \frac{0.300}{12} = 0.025 \text{ in}$$

From Fig. A-15-6,

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.025}{0.1686} = 0.148$$

Approximate $D/d = \infty$ with $D/d = 3$; from Fig. A-15-6, $K_t = 1.68$.
From Fig. 6-20, with $S_{ut} = 76$ kpsi and $r = 0.025$ in, $q = 0.62$.

Eq. (6-32): $K_f = 1 + 0.62\,(1.68 - 1) = 1.42$

$$\sigma_{all} = \frac{S_e}{K_f n_d} = \frac{53.84}{1.42(2.25)} = 16.85 \text{ psi}$$

$$W^t = \frac{F Y_P \sigma_{all}}{K_v P_d} = \frac{0.875(0.303)(16\,850)}{1.162(12)} = 320.4 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{320.4(194.8)}{33\,000} = 1.89 \text{ hp} \quad \textit{Ans.}$$

Wear

$\nu_1 = \nu_2 = 0.292$, $E_1 = E_2 = 30(10^6)$ psi

Eq. (14-13): $$C_p = \left[\frac{1}{2\pi\left(\dfrac{1-0.292^2}{30(10^6)}\right)}\right]^{1/2} = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $$r_1 = \frac{d_P}{2}\sin\phi = \frac{1.417}{2}\sin 20° = 0.242 \text{ in}$$

$$r_2 = \frac{d_G}{2}\sin\phi = \frac{2.500}{2}\sin 20° = 0.428 \text{ in}$$

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{0.242} + \frac{1}{0.428} = 6.469 \text{ in}^{-1}$$

Eq. (6-68), p. 329: $(S_C)_{10^8} = 0.4H_B - 10 \text{ kpsi} = [0.4(149) - 10](10^3) = 49\,600 \text{ psi}$

From the discussion and equation developed on the bottom of p. 329,

$$\sigma_{C,\text{all}} = -\frac{(S_C)_{10^8}}{\sqrt{n}} = \frac{-49\,600}{\sqrt{2.25}} = -33\,067 \text{ psi}$$

Eq. (14-14): $$W^t = \left(\frac{-33\,067}{2285}\right)^2 \left[\frac{0.875\cos 20°}{1.162(6.469)}\right] = 22.6 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{22.6(194.8)}{33\,000} = 0.133 \text{ hp} \quad \textit{Ans.}$$

Rating power (pinion controls):

$H_1 = 1.89$ hp
$H_2 = 0.133$ hp

$H_{\text{all}} = (\min 1.89, 0.133) = 0.133$ hp *Ans.*

14-16 See Prob. 14-15 solution for equation numbers.

Pinion controls: $Y_P = 0.322, \quad Y_G = 0.447$

Bending $d_P = 20/3 = 6.667$ in, $d_G = 100/3 = 33.333$ in

$$V = \pi d_P n / 12 = \pi(6.667)(870) / 12 = 1519 \text{ ft/min}$$
$$K_v = (1200 + 1519) / 1200 = 2.266$$
$$S_e' = 0.5(113) = 56.5 \text{ kpsi}$$
$$k_a = 2.70(113)^{-0.265} = 0.771$$
$$l = 2.25 / P_d = 2.25 / 3 = 0.75 \text{ in}$$
$$x = 3(0.322) / [2(3)] = 0.161 \text{ in}$$
$$t = \sqrt{4(0.75)(0.161)} = 0.695 \text{ in}$$
$$d_e = 0.808\sqrt{2.5(0.695)} = 1.065 \text{ in}$$
$$k_b = (1.065 / 0.30)^{-0.107} = 0.873$$
$$k_c = k_d = k_e = 1$$
$$k_f = 1.66 \text{ (See Ex. 14-2.)}$$
$$S_e = 0.771(0.873)(1)(1)(1)(1.66)(56.5) = 63.1 \text{ kpsi}$$
$$r_f = 0.300 / 3 = 0.100 \text{ in}$$
$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.100}{0.695} = 0.144$$
$$K_t = 1.75, q = 0.85, \ K_f = 1.64$$
$$\sigma_{all} = \frac{S_e}{K_f n_d} = \frac{63.1}{1.64(1.5)} = 25.7 \text{ kpsi}$$
$$W^t = \frac{F Y_P \sigma_{\text{all}}}{K_v P_d} = \frac{2.5(0.322)(25\,700)}{2.266(3)} = 3043 \text{ lbf}$$
$$H = W^t V / 33\,000 = 3043(1519) / 33\,000 = 140 \text{ hp} \qquad \textit{Ans.}$$

Wear

Eq. (14-13):
$$C_p = \left[\frac{1}{2\pi\left(\dfrac{1-0.292^2}{30(10^6)}\right)}\right]^{1/2} = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $r_1 = (6.667/2) \sin 20° = 1.140$ in
$r_2 = (33.333/2) \sin 20° = 5.700$ in

Eq. (6-68), p. 329: $S_C = [0.4(262) - 10](10^3) = 94\,800$ psi

$$\sigma_{C,\text{all}} = -S_C / \sqrt{n_d} = -94\,800 / \sqrt{1.5} = -77\,400 \text{ psi}$$

$$W^t = \left(\frac{\sigma_{C,\text{all}}}{C_p}\right)^2 \frac{F\cos\phi}{K_v}\left(\frac{1}{1/r_1 + 1/r_2}\right)$$

$$= \left(\frac{-77\ 400}{2285}\right)^2 \left(\frac{2.5\cos 20°}{2.266}\right)\left(\frac{1}{1/1.140 + 1/5.700}\right)$$

$$= 1130 \text{ lbf}$$

$$H = \frac{W^t V}{33\ 000} = \frac{1130(1519)}{33\ 000} = 52.0 \text{ hp} \quad \textit{Ans.}$$

For 10^8 cycles (revolutions of the pinion), the power based on wear is 52.0 hp.
Rating power (pinion controls):

$$H_1 = 140 \text{ hp}$$
$$H_2 = 52.0 \text{ hp}$$
$$H_{\text{rated}} = \min(140,\ 52.0) = 52.0 \text{ hp} \quad \textit{Ans.}$$

14-17 See Prob. 14-15 solution for equation numbers.
Given: $\phi = 20°$, $n = 1145$ rev/min, $m = 6$ mm, $F = 75$ mm, $N_P = 16$ milled teeth, $N_G = 30T$, $S_{ut} = 900$ MPa, $H_B = 260$, $n_d = 3$, $Y_P = 0.296$, and $Y_G = 0.359$.

Pinion bending

$$d_P = mN_P = 6(16) = 96 \text{ mm}$$
$$d_G = 6(30) = 180 \text{ mm}$$
$$V = \frac{\pi d_P n}{60} = \frac{\pi(96)(10^{-3})(1145)}{(60)} = 5.76 \text{ m/s}$$
$$K_v = \frac{6.1 + 5.76}{6.1} = 1.944$$
$$S_e' = 0.5(900) = 450 \text{ MPa}$$
$$k_a = 4.51(900)^{-0.265} = 0.744$$
$$l = 2.25m = 2.25(6) = 13.5 \text{ mm}$$
$$x = 3Ym/2 = 3(0.296)6/2 = 2.664 \text{ mm}$$
$$t = \sqrt{4lx} = \sqrt{4(13.5)(2.664)} = 12.0 \text{ mm}$$
$$d_e = 0.808\sqrt{75(12.0)} = 24.23 \text{ mm}$$
$$k_b = \left(\frac{24.23}{7.62}\right)^{-0.107} = 0.884$$
$$k_c = k_d = k_e = 1$$
$$k_f = 1.66 \text{ (See Ex. 14-2)}$$
$$S_e = 0.744(0.884)(1)(1)(1)(1.66)(450) = 491.3 \text{ MPa}$$
$$r_f = 0.300m = 0.300(6) = 1.8 \text{ mm}$$
$$r/d = r_f/t = 1.8/12 = 0.15,\ K_t = 1.68,\ q = 0.86,\ K_f = 1.58$$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{491.3}{1.58(1.3)} = 239.2 \text{ MPa}$$

Eq. (14-8): $$W^t = \frac{FYm\sigma_{\text{all}}}{K_v} = \frac{75(0.296)(6)(239.2)}{1.944} = 16\,390 \text{ N}$$

Eq. (13-36): $$H = \frac{W^t \pi dn}{60\,000} = \frac{16.39\pi(96)(1145)}{60\,000} = 94.3 \text{ kW} \quad \textit{Ans.}$$

Wear: Pinion and gear

Eq. (14-12): $r_1 = (96/2) \sin 20° = 16.42$ mm
$r_2 = (180/2) \sin 20° = 30.78$ mm

Eq. (14-13): $$C_p = \left[\frac{1}{2\pi\left(\dfrac{1-0.292^2}{207\left(10^3\right)}\right)}\right]^{1/2} = 190\sqrt{\text{MPa}}$$

Eq. (6-68), p. 329: $S_C = 6.89[0.4(260) - 10] = 647.7$ MPa

$$\sigma_{C,\text{all}} = -S_C / \sqrt{n_d} = \frac{-647.7}{\sqrt{1.3}} = -568 \text{ MPa}$$

Eq. (14-14): $$W^t = \left(\frac{\sigma_{C,\text{all}}}{C_p}\right)^2 \frac{F\cos\phi}{K_v}\left(\frac{1}{1/r_1 + 1/r_2}\right)$$

$$= \left(\frac{-568}{190}\right)^2 \left(\frac{75\cos 20^\circ}{1.944}\right)\left(\frac{1}{1/16.42 + 1/30.78}\right) = 3469 \text{ N}$$

Eq. (13-36): $$H = \frac{W^t \pi dn}{60\,000} = \frac{3.469\pi(96)(1145)}{60\,000} = 20.0 \text{ kW}$$

Thus, wear controls the gearset power rating; $H = 20.0$ kW. *Ans.*

14-18 $N_P = 17$ teeth, $N_G = 51$ teeth

$$d_P = \frac{N}{P} = \frac{17}{6} = 2.833 \text{ in}$$

$$d_G = \frac{51}{6} = 8.500 \text{ in}$$

$$V = \pi d_P n / 12 = \pi(2.833)(1120) / 12 = 830.7 \text{ ft/min}$$

Eq. (14-4*b*): $K_v = (1200 + 830.7)/1200 = 1.692$

$$\sigma_{\text{all}} = \frac{S_y}{n_d} = \frac{90\ 000}{2} = 45\ 000 \text{ psi}$$

Table 14-2: $Y_P = 0.303$, $Y_G = 0.410$

Eq. (14-7): $$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P} = \frac{2(0.303)(45\ 000)}{1.692(6)} = 2686 \text{ lbf}$$

Eq. (13-35): $$H = \frac{W^t V}{33\ 000} = \frac{2686(830.7)}{33\ 000} = 67.6 \text{ hp}$$

Based on yielding in bending, the power is 67.6 hp.

(a) Pinion fatigue

Bending

Eq. (2-121), p. 41: $S_{ut} = 0.5\ H_B = 0.5(232) = 116$ kpsi

Eq. (6-8), p. 282: $S'_e = 0.5 S_{ut} = 0.5(116) = 58$ kpsi

Eq. (6-19), p. 287: $k_a = 2.70(116)^{-0.265} = 0.766$

Table 13-1, p. 696: $$l = \frac{1}{P_d} + \frac{1.25}{P_d} = \frac{2.25}{P_d} = \frac{2.25}{6} = 0.375 \text{ in}$$

Eq. (14-3): $$x = \frac{3Y_P}{2P} = \frac{3(0.303)}{2(6)} = 0.0758 \text{ in}$$

Eq. (*b*), p. 737: $t = \sqrt{4lx} = \sqrt{4(0.375)(0.0758)} = 0.337$ in

Eq. (6-25), p. 289: $d_e = 0.808\sqrt{F\,t} = 0.808\sqrt{2(0.337)} = 0.663$ in

Eq. (6-20), p. 288: $$k_b = \left(\frac{0.663}{0.30}\right)^{-0.107} = 0.919$$

$$k_c = k_d = k_e = 1$$

Account for one-way bending with $k_f = 1.66$. (See Ex. 14-2.)

Eq. (6-18): $S_e = 0.766(0.919)(1)(1)(1)(1.66)(58) = 67.8$ kpsi

For stress concentration, find the radius of the root fillet (See Ex. 14-2).

$$r_f = \frac{0.300}{P} = \frac{0.300}{6} = 0.050 \text{ in}$$

Fig. A-15-6: $$\frac{r}{d} = \frac{r_f}{t} = \frac{0.05}{0.338} = 0.148$$

Estimate $D/d = \infty$ by setting $D/d = 3$, $K_t = 1.68$.

Fig. 6-20, p. 295: $q = 0.86$
Eq. (6-32), p. 295: $K_f = 1 + (0.86)(1.68 - 1) = 1.58$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{67.8}{1.58(2)} = 21.5 \text{ kpsi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.303)(21\,500)}{1.692(6)} = 1283 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{1283(830.7)}{33\,000} = 32.3 \text{ hp} \quad \textit{Ans.}$$

(b) Pinion fatigue

Wear

Eq. (14-13): $$C_p = \left\{\frac{1}{2\pi[(1 - 0.292^2)/30(10^6)]}\right\}^{1/2} = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $$r_1 = \frac{d_P}{2}\sin\phi = \frac{2.833}{2}\sin 20^\circ = 0.485 \text{ in}$$

$$r_2 = \frac{d_G}{2}\sin\phi = \frac{8.500}{2}\sin 20^\circ = 1.454 \text{ in}$$

$$\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{0.485} + \frac{1}{1.454} = 2.750 \text{ in}$$

Eq. (6-68): $$(S_C)_{10^8} = 0.4H_B - 10 \text{ kpsi}$$

In terms of gear notation

$$\sigma_C = [0.4(232) - 10]10^3 = 82\,800 \text{ psi}$$

We will introduce the design factor of $n_d = 2$ and because it is a contact stress apply it to the load W^t by dividing by $\sqrt{2}$. (See p. 329.)

$$\sigma_{C,\text{all}} = -\frac{\sigma_c}{\sqrt{2}} = -\frac{82\,800}{\sqrt{2}} = -58\,548 \text{ psi}$$

Solve Eq. (14-14) for W^t:

$$W^t = \left(\frac{-58\,548}{2285}\right)^2\left[\frac{2\cos 20^\circ}{1.692(2.750)}\right] = 265 \text{ lbf}$$

$$H_{\text{all}} = \frac{W^t V}{33\,000} = \frac{265(830.7)}{33\,000} = 6.67 \text{ hp} \quad \textit{Ans.}$$

For 10^8 cycles (turns of pinion), the allowable power is 6.67 hp.

(c) Gear fatigue due to bending and wear

Bending

Eq. (14-3): $x = \dfrac{3Y_G}{2P} = \dfrac{3(0.4103)}{2(6)} = 0.1026$ in

Eq. (*b*), p. 737: $t = \sqrt{4lx} = \sqrt{4(0.375)(0.1026)} = 0.392$ in ±

Eq. (6-25): $d_e = 0.808\sqrt{F\,t} = 0.808\sqrt{2(0.392)} = 0.715$ in

Eq. (6-20): $k_b = \left(\dfrac{0.715}{0.30}\right)^{-0.107} = 0.911$

$k_c = k_d = k_e = 1$

$k_f = 1.66$. (See Ex. 14-2.)

Eq. (6-18): $S_e = 0.766(0.911)(1)(1)(1)(1.66)(58) = 67.2$ kpsi

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.050}{0.392} = 0.128$$

Approximate $D/d = \infty$ by setting $D/d = 3$ for Fig. A-15-6; $K_t = 1.80$.
Fig. 6-20: $q = 0.82$
Eq. (6-32): $K_f = 1 + (0.82)(1.80 - 1) = 1.66$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{67.2}{1.66(2)} = 20.2 \text{ kpsi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.4103)(20\,200)}{1.692(6)} = 1633 \text{ lbf}$$

$$H_{\text{all}} = \frac{W^t V}{33\,000} = \frac{1633(830.7)}{33\,000} = 41.1 \text{ hp} \quad \textit{Ans.}$$

The gear is thus stronger than the pinion in bending.

Wear

Since the material of the pinion and the gear are the same, and the contact stresses are the same, the allowable power transmission of both is the same. Thus, $H_{\text{all}} = 6.67$ hp for 10^8 revolutions of each. As yet, we have no way to establish S_C for $10^8/3$ revolutions.

(d)

Pinion bending: $H_1 = 32.3$ hp
Pinion wear: $H_2 = 6.67$ hp
Gear bending: $H_3 = 41.1$ hp
Gear wear: $H_4 = 6.67$ hp

Power rating of the gear set is thus

$H_{\text{rated}} = \min(32.3, 6.67, 41.1, 6.67) = 6.67$ hp *Ans.*

14-19 $d_P = 16/6 = 2.667$ in, $d_G = 48/6 = 8$ in

$$V = \frac{\pi(2.667)(300)}{12} = 209.4 \text{ ft/min}$$

$$W^t = \frac{33\,000(5)}{209.4} = 787.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$.

Eq. (14-28): $Q_v = 6, \quad B = 0.25(12 - 6)^{2/3} = 0.8255$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $K_v = \left(\dfrac{59.77 + \sqrt{209.4}}{59.77}\right)^{0.8255} = 1.196$

Table 14-2: $Y_P = 0.296, \quad Y_G = 0.4056$

From Eq. (a), Sec. 14-10 with $F = 2$ in

$$(K_s)_P = 1.192\left(\frac{2\sqrt{0.296}}{6}\right)^{0.0535} = 1.088$$

$$(K_s)_G = 1.192\left(\frac{2\sqrt{0.4056}}{6}\right)^{0.0535} = 1.097$$

From Eq. (14-30) with $C_{mc} = 1$

$$C_{pf} = \frac{2}{10(2.667)} - 0.0375 + 0.0125(2) = 0.0625$$

$$C_{pm} = 1, \quad C_{ma} = 0.093 \quad \text{(Fig. 14 - 11)}, \quad C_e = 1$$

$$K_m = 1 + 1[0.0625(1) + 0.093(1)] = 1.156$$

Assuming constant thickness of the gears → $K_B = 1$

$$m_G = N_G/N_P = 48/16 = 3$$

With N (pinion) = 10^8 cycles and N (gear) = $10^8/3$, Fig. 14-14 provides the relations:

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8 / 3)^{-0.0178} = 0.996$$

Fig. 14-6: $J_P = 0.27, \quad J_G \doteq 0.38$

Table 14-10: $K_R = 0.85$

$K_T = C_f = 1$

Eq. (14-23): $I = \dfrac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\dfrac{3}{3+1}\right) = 0.1205$

Table 14-8: $C_p = 2300\sqrt{\text{psi}}$

Strength: Grade 1 steel with $H_{BP} = H_{BG} = 200$

Fig. 14-2: $(S_t)_P = (S_t)_G = 77.3(200) + 12\,800 = 28\,260$ psi

Fig. 14-5: $(S_c)_P = (S_c)_G = 322(200) + 29\,100 = 93\,500$ psi

Fig. 14-15: $(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$
$(Z_N)_G = 1.4488(10^8/3)^{-0.023} = 0.973$

Sec. 14-12: $H_{BP}/H_{BG} = 1 \quad \therefore\ C_H = 1$

Pinion tooth bending

Eq. (14-15):
$$(\sigma)_P = W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}$$
$$= 787.8(1)(1.196)(1.088)\left(\frac{6}{2}\right)\left[\frac{(1.156)(1)}{0.27}\right]$$
$$= 13\,170 \text{ psi} \quad \textit{Ans.}$$

Eq. (14-41):
$$(S_F)_P = \left[\frac{S_t Y_N\ /\ (K_T K_R)}{\sigma}\right]$$
$$= \frac{28\,260(0.977)\ /\ [(1)(0.85)]}{13\,170} = 2.47 \quad \textit{Ans.}$$

Gear tooth bending

Eq. (14-15):
$$(\sigma)_G = 787.8(1)(1.196)(1.097)\left(\frac{6}{2}\right)\left[\frac{(1.156)(1)}{0.38}\right] = 9433 \text{ psi} \quad \textit{Ans.}$$

Eq. (14-41):
$$(S_F)_G = \frac{28\,260(0.996)\ /\ [(1)(0.85)]}{9433} = 3.51 \quad \textit{Ans.}$$

Pinion tooth wear

Eq. (14-16):
$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$
$$= 2300\left[787.8(1)(1.196)(1.088)\left(\frac{1.156}{2.667(2)}\right)\left(\frac{1}{0.1205}\right)\right]^{1/2}$$
$$= 98\,760 \text{ psi} \quad \textit{Ans.}$$

Eq. (14-42):
$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \left\{\frac{93\,500(0.948)/[(1)(0.85)]}{98\,760}\right\} = 1.06 \quad \textit{Ans.}$$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.097}{1.088}\right)^{1/2}(98\ 760) = 99\ 170 \text{ psi} \quad \textit{Ans.}$$

$$(S_H)_G = \frac{93\ 500(0.973)(1)/[(1)(0.85)]}{99\ 170} = 1.08 \quad \textit{Ans.}$$

The hardness of the pinion and the gear should be increased.

14-20 $d_P = 2.5(20) = 50$ mm, $d_G = 2.5(36) = 90$ mm

$$V = \frac{\pi d_P n_P}{60} = \frac{\pi(50)(10^{-3})(100)}{60} = 0.2618 \text{ m/s}$$

$$W^t = \frac{60(120)}{\pi(50)(10^{-3})(100)} = 458.4 \text{ N}$$

With no specific information given to indicate otherwise, assume

$$K_B = K_o = Y_\theta = Z_R = 1$$

Eq. (14-28): $Q_v = 6$, $B = 0.25(12 - 6)^{2/3} = 0.8255$
$A = 50 + 56(1 - 0.8255) = 59.77$

Eq. (14-27): $$K_v = \left[\frac{59.77 + \sqrt{200(0.2618)}}{59.77}\right]^{0.8255} = 1.099$$

Table 14-2: $Y_P = 0.322$, $Y_G = 0.3775$

Similar to Eq. (*a*) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_b} = 0.8433\left(mF\sqrt{Y}\right)^{0.0535}$$

$$(K_s)_P = 0.8433\left[2.5(18)\sqrt{0.322}\right]^{0.0535} = 1.003 \quad \text{use } 1$$

$$(K_s)_G = 0.8433\left[2.5(18)\sqrt{0.3775}\right]^{0.0535} = 1.007 \quad \text{use } 1$$

$$C_{mc} = C_e = C_{pm} = 1$$

$$F = 18/25.4 = 0.709 \text{ in}, \ C_{pf} = \frac{18}{10(50)} - 0.025 = 0.011$$

$$C_{ma} = 0.247 + 0.0167(0.709) - 0.765(10^{-4})(0.709^2) = 0.259$$

$$K_H = 1 + 1[0.011(1) + 0.259(1)] = 1.27$$

Fig. 14-14: $(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$
$(Y_N)_G = 1.3558(10^8/1.8)^{-0.0178} = 0.987$

Fig. 14-6: $(Y_J)_P = 0.33$, $(Y_J)_G = 0.38$
Eq. (14-38): $Y_Z = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$

Eq. (14-23): $Z_I = \dfrac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\dfrac{1.8}{1.8+1}\right) = 0.103$

Table 14-8: $Z_E = 191\sqrt{\text{MPa}}$

Strength Grade 1 steel, $H_{BP} = H_{BG} = 200$

Fig. 14-2: $(S_t)_P = (S_t)_G = 0.533(200) + 88.3 = 194.9$ MPa

Fig. 14-5: $(S_c)_P = (S_c)_G = 2.22(200) + 200 = 644$ MPa

Fig. 14-15: $(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$

$(Z_N)_G = 1.4488(10^8 / 1.8)^{-0.023} = 0.961$

Fig. 14-12: $H_{BP} / H_{BG} = 1 \quad \therefore \; Z_W = C_H = 1$

Pinion tooth bending

Eq. (14-15): $(\sigma)_P = \left(W^t K_o K_v K_s \dfrac{1}{bm_t}\dfrac{K_H K_B}{Y_J}\right)_P$

$$= 458.4(1)(1.099)(1)\left[\frac{1}{18(2.5)}\right]\left[\frac{1.27(1)}{0.33}\right] = 43.08 \text{ MPa} \quad \textit{Ans.}$$

Eq. (14-41) for SI: $(S_F)_P = \left(\dfrac{S_t}{\sigma}\dfrac{Y_N}{Y_\theta Y_Z}\right)_P = \dfrac{194.9}{43.08}\left[\dfrac{0.977}{1(0.885)}\right] = 4.99 \quad \textit{Ans.}$

Gear tooth bending

$$(\sigma)_G = 458.4(1)(1.099)(1)\left[\frac{1}{18(2.5)}\right]\left[\frac{1.27(1)}{0.38}\right] = 37.42 \text{ MPa} \quad \textit{Ans.}$$

$$(S_F)_G = \frac{194.9}{37.42}\left[\frac{0.987}{1(0.885)}\right] = 5.81 \quad \textit{Ans.}$$

Pinion tooth wear

Eq. (14-16): $(\sigma_c)_P = \left(Z_E\sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b}\dfrac{Z_R}{Z_I}}\right)_P$

$$= 191\sqrt{458.4(1)(1.099)(1)\left[\frac{1.27}{50(18)}\right]\left[\frac{1}{0.103}\right]} = 501.8 \text{ MPa} \quad \textit{Ans.}$$

Eq. (14-42) for SI: $(S_H)_P = \left(\dfrac{S_c}{\sigma_c}\dfrac{Z_N Z_W}{Y_\theta Y_Z}\right)_P = \dfrac{644}{501.8}\left[\dfrac{0.948(1)}{1(0.885)}\right] = 1.37 \quad \textit{Ans.}$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1}{1}\right)^{1/2}(501.8) = 501.8 \text{ MPa} \quad \textit{Ans.}$$

$$(S_H)_G = \frac{644}{501.8}\frac{0.961(1)}{1(0.885)} = 1.39 \quad \textit{Ans.}$$

14-21

$$P_t = P_n \cos\psi = 6\cos 30° = 5.196 \text{ teeth/in}$$
$$d_P = \frac{16}{5.196} = 3.079 \text{ in}, \quad d_G = \frac{48}{16}(3.079) = 9.238 \text{ in}$$
$$V = \frac{\pi(3.079)(300)}{12} = 241.8 \text{ ft/min}$$
$$W^t = \frac{33\,000(5)}{241.8} = 682.3 \text{ lbf}, \quad K_v = \left(\frac{59.77 + \sqrt{241.8}}{59.77}\right)^{0.8255} = 1.210$$

From Prob. 14-19:

$$Y_P = 0.296, \quad Y_G = 0.4056$$
$$(K_s)_P = 1.088, \quad (K_s)_G = 1.097, \quad K_B = 1$$
$$m_G = 3, \quad (Y_N)_P = 0.977, \quad (Y_N)_G = 0.996, \quad K_R = 0.85$$
$$(S_t)_P = (S_t)_G = 28\,260 \text{ psi}, \quad C_H = 1, \quad (S_c)_P = (S_c)_G = 93\,500 \text{ psi}$$
$$(Z_N)_P = 0.948, \quad (Z_N)_G = 0.973, \quad C_p = 2300\sqrt{\text{psi}}$$

The pressure angle is:

$$\phi_t = \tan^{-1}\left(\frac{\tan 20°}{\cos 30°}\right) = 22.80°$$
$$(r_b)_P = \frac{3.079}{2}\cos 22.8° = 1.419 \text{ in}, \quad (r_b)_G = 3(r_b)_P = 4.258 \text{ in}$$
$$a = 1/P_n = 1/6 = 0.167 \text{ in}$$

Eq. (14-25):

$$Z = \left[\left(\frac{3.079}{2} + 0.167\right)^2 - 1.419^2\right]^{1/2} + \left[\left(\frac{9.238}{2} + 0.167\right)^2 - 4.258^2\right]^{1/2}$$
$$-\left(\frac{3.079}{2} + \frac{9.238}{2}\right)\sin 22.8°$$
$$= 0.9479 + 2.1852 - 2.3865 = 0.7466 \quad \text{Conditions } \textit{O.K.} \text{ for use}$$

$$p_N = p_n \cos\phi_n = \frac{\pi}{6}\cos 20° = 0.4920 \text{ in}$$

Eq. (14-21): $$m_N = \frac{p_N}{0.95Z} = \frac{0.492}{0.95(0.7466)} = 0.6937$$

Eq. (14-23): $I = \left[\dfrac{\sin 22.8^\circ \cos 22.8^\circ}{2(0.6937)}\right]\left(\dfrac{3}{3+1}\right) = 0.193$

Fig. 14-7: $J'_P \doteq 0.45, \quad J'_G \doteq 0.54$

Fig. 14-8: Corrections are 0.94 and 0.98.

$$J_P = 0.45(0.94) = 0.423, \quad J_G = 0.54(0.98) = 0.529$$
$$C_{mc} = 1, \quad C_{pf} = \frac{2}{10(3.079)} - 0.0375 + 0.0125(2) = 0.0525$$
$$C_{pm} = 1, \quad C_{ma} = 0.093, \quad C_e = 1$$
$$K_m = 1 + (1)[0.0525(1) + 0.093(1)] = 1.146$$

Pinion tooth bending

$$(\sigma)_P = 682.3(1)(1.21)(1.088)\left(\frac{5.196}{2}\right)\frac{1.146(1)}{0.423} = 6323 \text{ psi} \quad \textit{Ans.}$$
$$(S_F)_P = \frac{28\ 260(0.977)\ /\ [1(0.85)]}{6323} = 5.14 \quad \textit{Ans.}$$

Gear tooth bending

$$(\sigma)_G = 682.3(1)(1.21)(1.097)\left(\frac{5.196}{2}\right)\frac{1.146(1)}{0.529} = 5097 \text{ psi} \quad \textit{Ans.}$$
$$(S_F)_G = \frac{28\ 260(0.996)\ /\ [1(0.85)]}{5097} = 6.50 \quad \textit{Ans.}$$

Pinion tooth wear

$$(\sigma_c)_P = 2300\left\{682.3(1)(1.21)(1.088)\left[\frac{1.146}{3.078(2)}\right]\left(\frac{1}{0.193}\right)\right\}^{1/2} = 67\ 700 \text{ psi} \quad \textit{Ans.}$$
$$(S_H)_P = \frac{93\ 500(0.948)\ /\ [(1)(0.85)]}{67\ 700} = 1.54 \quad \textit{Ans.}$$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{1.097}{1.088}\right]^{1/2}(67\ 700) = 67\ 980 \text{ psi} \quad \textit{Ans.}$$
$$(S_H)_G = \frac{93\ 500(0.973)/[(1)(0.85)]}{67\ 980} = 1.57 \quad \textit{Ans.}$$

14-22 Given: $R = 0.99$ at 10^8 cycles, $H_B = 232$ through-hardening Grade 1, core and case, both gears. $N_P = 17T$, $N_G = 51T$,
Table 14-2: $Y_P = 0.303$, $Y_G = 0.4103$

Fig. 14-6: $J_P = 0.292$, $J_G = 0.396$

$d_P = N_P / P = 17 / 6 = 2.833$ in, $d_G = 51 / 6 = 8.500$ in.

Pinion bending

From Fig. 14-2:

$$_{0.99}(S_t)_{10^7} = 77.3H_B + 12\ 800$$
$$= 77.3(232) + 12\ 800 = 30\ 734 \text{ psi}$$

Fig. 14-14: $Y_N = 1.6831(10^8)^{-0.0323} = 0.928$

$$V = \pi d_P n / 12 = \pi(2.833)(1120 / 12) = 830.7 \text{ ft/min}$$
$$K_T = K_R = 1, \quad S_F = 2, \quad S_H = \sqrt{2}$$
$$\sigma_{\text{all}} = \frac{30\ 734(0.928)}{2(1)(1)} = 14\ 261 \text{ psi}$$
$$Q_v = 5, \quad B = 0.25(12 - 5)^{2/3} = 0.9148$$
$$A = 50 + 56(1 - 0.9148) = 54.77$$
$$K_v = \left(\frac{54.77 + \sqrt{830.7}}{54.77}\right)^{0.9148} = 1.472$$
$$K_s = 1.192\left(\frac{2\sqrt{0.303}}{6}\right)^{0.0535} = 1.089 \Rightarrow \text{use } 1$$
$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$
$$C_{mc} = 1$$
$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F$$
$$= \frac{2}{10(2.833)} - 0.0375 + 0.0125(2) = 0.0581$$
$$C_{pm} = 1$$
$$C_{ma} = 0.127 + 0.0158(2) - 0.093(10^{-4})(2^2) = 0.1586$$
$$C_e = 1$$
$$K_m = 1 + 1[0.0581(1) + 0.1586(1)] = 1.217$$
$$K_B = 1$$

Eq. (14-15):
$$W^t = \frac{FJ_P\sigma_{\text{all}}}{K_oK_vK_sP_dK_mK_B}$$
$$= \frac{2(0.292)(14\ 261)}{1(1.472)(1)(6)(1.217)(1)} = 775 \text{ lbf}$$
$$H = \frac{W^tV}{33\ 000} = \frac{775(830.7)}{33\ 000} = 19.5 \text{ hp}$$

Pinion wear

Fig. 14-15: $Z_N = 2.466N^{-0.056} = 2.466(10^8)^{-0.056} = 0.879$

$m_G = 51\,/\,17 = 3$

Eq. (14-23): $I = \dfrac{\cos 20^\circ \sin 20^\circ}{2}\left(\dfrac{3}{3+1}\right) = 1.205, \quad C_H = 1$

Fig. 14-5: $_{0.99}(S_c)_{10^7} = 322H_B + 29\ 100$

$$= 322(232) + 29\ 100 = 103\ 804 \text{ psi}$$

$$\sigma_{c,\text{all}} = \frac{103\ 804(0.879)}{\sqrt{2}(1)(1)} = 64\ 519 \text{ psi}$$

Eq. (14-16): $W^t = \left(\dfrac{\sigma_{c,\text{all}}}{C_p}\right)^2 \dfrac{Fd_P I}{K_o K_v K_s K_m C_f}$

$$= \left(\frac{64\ 519}{2300}\right)^2 \left[\frac{2(2.833)(0.1205)}{1(1.472)(1)(1.2167)(1)}\right]$$

$$= 300 \text{ lbf}$$

$$H = \frac{W^t V}{33\ 000} = \frac{300(830.7)}{33\ 000} = 7.55 \text{ hp}$$

The pinion controls, therefore $H_{\text{rated}} = 7.55$ hp *Ans.*

14-23 $l = 2.25/\,P_d, \quad x = 3Y\,/\,2P_d$

$$t = \sqrt{4lx} = \sqrt{4\left(\frac{2.25}{P_d}\right)\left(\frac{3Y}{2P_d}\right)} = \frac{3.674}{P_d}\sqrt{Y}$$

$$d_e = 0.808\sqrt{F\,t} = 0.808\sqrt{F\left(\frac{3.674}{P_d}\right)\sqrt{Y}} = 1.5487\sqrt{\frac{F\sqrt{Y}}{P_d}}$$

$$k_b = \left(\frac{1.5487\sqrt{F\sqrt{Y}\ /\ P_d}}{0.30}\right)^{-0.107} = 0.8389\left(\frac{F\sqrt{Y}}{P_d}\right)^{-0.0535}$$

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P_d}\right)^{0.0535} \qquad \textit{Ans.}$$

14-24 $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.345$, $J_G = 0.410$, $K_o = 1.25$. The service conditions are adequately described by K_o. Set $S_F = S_H = 1$.

$d_P = 22\,/\,4 = 5.500$ in

$d_G = 60\,/\,4 = 15.000$ in

$$V = \frac{\pi(5.5)(1145)}{12} = 1649 \text{ ft/min}$$

Pinion bending

$$_{0.99}(S_t)_{10^7} = 77.3H_B + 12\,800 = 77.3(250) + 12\,800 = 32\,125 \text{ psi}$$
$$Y_N = 1.6831[3(10^9)]^{-0.0323} = 0.832$$

Eq. (14-17): $$(\sigma_{\text{all}})_P = \frac{32\,125(0.832)}{1(1)(1)} = 26\,728 \text{ psi}$$
$$B = 0.25(12 - 6)^{2/3} = 0.8255$$
$$A = 50 + 56(1 - 0.8255) = 59.77$$
$$K_v = \left(\frac{59.77 + \sqrt{1649}}{59.77}\right)^{0.8255} = 1.534$$
$$K_s = 1, \quad C_m = 1$$
$$C_{mc} = \frac{F}{10d} - 0.0375 + 0.0125F$$
$$= \frac{3.25}{10(5.5)} - 0.0375 + 0.0125(3.25) = 0.0622$$
$$C_{ma} = 0.127 + 0.0158(3.25) - 0.093(10^{-4})(3.25^2) = 0.178$$
$$C_e = 1$$
$$K_m = C_{mf} = 1 + (1)[0.0622(1) + 0.178(1)] = 1.240$$
$$K_B = 1, \quad K_T = 1$$

Eq. (14-15): $$W_1^t = \frac{26\,728(3.25)(0.345)}{1.25(1.534)(1)(4)(1.240)} = 3151 \text{ lbf}$$
$$H_1 = \frac{3151(1649)}{33\,000} = 157.5 \text{ hp}$$

Gear bending By similar reasoning, $W_2^t = 3861$ lbf and $H_2 = 192.9$ hp

Pinion wear

$$m_G = 60 / 22 = 2.727$$
$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\left(\frac{2.727}{1 + 2.727}\right) = 0.1176$$
$$_{0.99}(S_c)_{10^7} = 322(250) + 29\,100 = 109\,600 \text{ psi}$$
$$(Z_N)_P = 2.466[3(10^9)]^{-0.056} = 0.727$$
$$(Z_N)_G = 2.466[3(10^9) / 2.727]^{-0.056} = 0.769$$
$$(\sigma_{c,\text{all}})_P = \frac{109\,600(0.727)}{1(1)(1)} = 79\,679 \text{ psi}$$

$$W_3^t = \left(\frac{\sigma_{c,\text{all}}}{C_p}\right)^2 \frac{F d_P I}{K_o K_v K_s K_m C_f}$$

$$= \left(\frac{79\ 679}{2300}\right)^2 \left[\frac{3.25(5.5)(0.1176)}{1.25(1.534)(1)(1.24)(1)}\right] = 1061 \text{ lbf}$$

$$H_3 = \frac{1061(1649)}{33\ 000} = 53.0 \text{ hp}$$

Gear wear

Similarly, $W_4^t = 1182 \text{ lbf}, \quad H_4 = 59.0 \text{ hp}$

Rating

$$H_{\text{rated}} = \min(H_1, H_2, H_3, H_4)$$
$$= \min(157.5, 192.9, 53, 59) = 53 \text{ hp} \quad \textit{Ans.}$$

Note differing capacities. Can these be equalized?

14-25 From Prob. 14-24:

$$W_1^t = 3151 \text{ lbf}, \quad W_2^t = 3861 \text{ lbf},$$
$$W_3^t = 1061 \text{ lbf}, \quad W_4^t = 1182 \text{ lbf}$$
$$W^t = \frac{33\ 000 K_o H}{V} = \frac{33\ 000(1.25)(40)}{1649} = 1000 \text{ lbf}$$

Pinion bending: The factor of safety, based on load and stress, is

$$(S_F)_P = \frac{W_1^t}{1000} = \frac{3151}{1000} = 3.15$$

Gear bending based on load and stress

$$(S_F)_G = \frac{W_2^t}{1000} = \frac{3861}{1000} = 3.86$$

Pinion wear

based on load: $n_3 = \dfrac{W_3^t}{1000} = \dfrac{1061}{1000} = 1.06$

based on stress: $(S_H)_P = \sqrt{1.06} = 1.03$

Gear wear

based on load: $n_4 = \dfrac{W_4^t}{1000} = \dfrac{1182}{1000} = 1.18$

based on stress: $(S_H)_G = \sqrt{1.18} = 1.09$

Factors of safety are used to assess the relative threat of loss of function 3.15, 3.86, 1.06, 1.18 where the threat is from pinion wear. By comparison, the AGMA safety factors

$$(S_F)_P, (S_F)_G, (S_H)_P, (S_H)_G$$

are

$$3.15, 3.86, 1.03, 1.09 \quad \text{or} \quad 3.15, 3.86, 1.06^{1/2}, 1.18^{1/2}$$

and the threat is again from pinion wear. Depending on the magnitude of the numbers, using S_F and S_H as defined by AGMA, does not *necessarily* lead to the same conclusion concerning threat. Therefore be cautious.

14-26 Solution summary from Prob. 14-24: $n = 1145$ rev/min, $K_o = 1.25$, Grade 1 materials, $N_P = 22T$, $N_G = 60T$, $m_G = 2.727$, $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.345$, $J_G = 0.410$, $P_d = 4T$ /in, $F = 3.25$ in, $Q_v = 6$, $(N_c)_P = 3(10^9)$, $R = 0.99$, $K_m = 1.240$, $K_T = 1$, $K_B = 1$, $d_P = 5.500$ in, $d_G = 15.000$ in, $V = 1649$ ft/min, $K_v = 1.534$, $(K_s)_P = (K_s)_G = 1$, $(Y_N)_P = 0.832$, $(Y_N)_G = 0.859$, $K_R = 1$

Pinion H_B: 250 core, 390 case
Gear H_B: 250 core, 390 case

Bending

$$(\sigma_{\text{all}})_P = 26\ 728 \text{ psi} \qquad (S_t)_P = 32\ 125 \text{ psi}$$
$$(\sigma_{\text{all}})_G = 27\ 546 \text{ psi} \qquad (S_t)_G = 32\ 125 \text{ psi}$$
$$W_1^t = 3151 \text{ lbf}, \qquad H_1 = 157.5 \text{ hp}$$
$$W_2^t = 3861 \text{ lbf}, \qquad H_2 = 192.9 \text{ hp}$$

Wear

$$\phi = 20^\circ, \quad I = 0.1176, \quad (Z_N)_P = 0.727$$
$$(Z_N)_G = 0.769, \quad C_P = 2300\sqrt{\text{psi}}$$
$$(S_c)_P = S_c = 322(390) + 29\ 100 = 154\ 680 \text{ psi}$$
$$(\sigma_{c,\text{all}})_P = \frac{154\ 680(0.727)}{1(1)(1)} = 112\ 450 \text{ psi}$$
$$(\sigma_{c,\text{all}})_G = \frac{154\ 680(0.769)}{1(1)(1)} = 118\ 950 \text{ psi}$$
$$W_3^t = \left(\frac{112\ 450}{79\ 679}\right)^2 (1061) = 2113 \text{ lbf}, \qquad H_3 = \frac{2113(1649)}{33\ 000} = 105.6 \text{ hp}$$
$$W_4^t = \left(\frac{118\ 950}{109\ 600(0.769)}\right)^2 (1182) = 2354 \text{ lbf}, \quad H_4 = \frac{2354(1649)}{33\ 000} = 117.6 \text{ hp}$$

Rated power

$$H_{\text{rated}} = \min(157.5, 192.9, 105.6, 117.6) = 105.6 \text{ hp} \quad \textit{Ans.}$$

Prob. 14-24:

$$H_{\text{rated}} = \min(157.5, 192.9, 53.0, 59.0) = 53 \text{ hp}$$

The rated power approximately doubled.

14-27 The gear and the pinion are 9310 grade 1, carburized and case-hardened to obtain Brinell 285 core and Brinell 580–600 case.

Table 14-3: $\quad {}_{0.99}(S_t)_{10^7} = 55\ 000 \text{ psi}$

Modification of S_t by $(Y_N)_P = 0.832$ produces

$$(\sigma_{\text{all}})_P = 45\ 657 \text{ psi},$$

Similarly for $(Y_N)_G = 0.859$

$$(\sigma_{\text{all}})_G = 47\ 161 \text{ psi}, \quad \text{and}$$
$$W_1^t = 4569 \text{ lbf}, \quad H_1 = 228 \text{ hp}$$
$$W_2^t = 5668 \text{ lbf}, \quad H_2 = 283 \text{ hp}$$

From Table 14-8, $C_p = 2300\sqrt{\text{psi}}$. Also, from Table 14-6:

$${}_{0.99}(S_c)_{10^7} = 180\ 000 \text{ psi}$$

Modification of S_c by Y_N produces

$$(\sigma_{c,\text{all}})_P = 130\ 525 \text{ psi}$$
$$(\sigma_{c,\text{all}})_G = 138\ 069 \text{ psi}$$

and

$$W_3^t = 2489 \text{ lbf}, \quad H_3 = 124.3 \text{ hp}$$
$$W_4^t = 2767 \text{ lbf}, \quad H_4 = 138.2 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(228, 283, 124, 138) = 124 \text{ hp} \quad \textit{Ans.}$$

14-28 Grade 2, 9310 carburized and case-hardened to 285 core and 580 case in Prob. 14-27.

Summary:

Table 14-3: $\quad {}_{0.99}(S_t)_{10^7} = 65\ 000$ psi

$$(\sigma_{\text{all}})_P = 53\ 959 \text{ psi}$$
$$(\sigma_{\text{all}})_G = 55\ 736 \text{ psi}$$

and it follows that

$$W_1^t = 5400 \text{ lbf}, \qquad H_1 = 270 \text{ hp}$$
$$W_2^t = 6699 \text{ lbf}, \qquad H_2 = 335 \text{ hp}$$

From Table 14-8, $C_p = 2300\sqrt{\text{psi}}$. Also, from Table 14-6:

$$S_c = 225\ 000 \text{ psi}$$
$$(\sigma_{c,\text{all}})_P = 181\ 285 \text{ psi}$$
$$(\sigma_{c,\text{all}})_G = 191\ 762 \text{ psi}$$

Consequently,

$$W_3^t = 4801 \text{ lbf}, \quad H_3 = 240 \text{ hp}$$
$$W_4^t = 5337 \text{ lbf}, \quad H_4 = 267 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(270, 335, 240, 267) = 240 \text{ hp.} \quad \textit{Ans.}$$

14-29 Given: $n = 1145$ rev/min, $K_o = 1.25$, $N_P = 22T$, $N_G = 60T$, $m_G = 2.727$, $d_P = 2.75$ in, $d_G = 7.5$ in, $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.335$, $J_G = 0.405$, $P = 8T$ /in, $F = 1.625$ in, $H_B = 250$, case and core, both gears. $C_m = 1$, $F/d_P = 0.0591$, $C_f = 0.0419$, $C_{pm} = 1$, $C_{ma} = 0.152$, $C_e = 1$, $K_m = 1.1942$, $K_T = 1$, $K_B = 1$, $K_s = 1$, $V = 824$ ft/min, $(Y_N)_P = 0.8318$, $(Y_N)_G = 0.859$, $K_R = 1$, $I = 0.117\ 58$

$$ {}_{0.99}(S_t)_{10^7} = 32\ 125 \text{ psi}$$
$$(\sigma_{\text{all}})_P = 26\ 668 \text{ psi}$$
$$(\sigma_{\text{all}})_G = 27\ 546 \text{ psi}$$

and it follows that

$$W_1^t = 879.3 \text{ lbf}, \quad H_1 = 21.97 \text{ hp}$$
$$W_2^t = 1098 \text{ lbf}, \quad H_2 = 27.4 \text{ hp}$$

For wear

$$W_3^t = 304 \text{ lbf}, \quad H_3 = 7.59 \text{ hp}$$
$$W_4^t = 340 \text{ lbf}, \quad H_4 = 8.50 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(21.97, 27.4, 7.59, 8.50) = 7.59 \text{ hp}$$

In Prob. 14-24, $H_{\text{rated}} = 53$ hp. Thus,

$$\frac{7.59}{53.0} = 0.1432 = \frac{1}{6.98}, \quad \text{not } \frac{1}{8} \quad \textit{Ans.}$$

The transmitted load rating is

$$W_{\text{rated}}^t = \min(879.3,\ 1098,\ 304,\ 340) = 304 \text{ lbf}$$

In Prob. 14-24

$$W_{\text{rated}}^t = 1061 \text{ lbf}$$

Thus

$$\frac{304}{1061} = 0.2865 = \frac{1}{3.49}, \quad \text{not } \frac{1}{4} \quad \textit{Ans.}$$

14-30 $S_P = S_H = 1, \quad P_d = 4, \quad J_P = 0.345, \quad J_G = 0.410, \quad K_o = 1.25$

Bending

Table 14-4: $\ {}_{0.99}(S_t)_{10^7} = 13\,000$ psi

$$(\sigma_{\text{all}})_P = (\sigma_{\text{all}})_G = \frac{13\,000(1)}{1(1)(1)} = 13\,000 \text{ psi}$$
$$W_1^t = \frac{\sigma_{\text{all}} F J_P}{K_o K_v K_s P_d K_m K_B} = \frac{13\,000(3.25)(0.345)}{1.25(1.534)(1)(4)(1.24)(1)} = 1533 \text{ lbf}$$
$$H_1 = \frac{1533(1649)}{33\,000} = 76.6 \text{ hp}$$
$$W_2^t = W_1^t J_G / J_P = 1533(0.410) / 0.345 = 1822 \text{ lbf}$$
$$H_2 = H_1 J_G / J_P = 76.6(0.410) / 0.345 = 91.0 \text{ hp}$$

Wear

Table 14-8: $\ C_p = 1960\sqrt{\text{psi}}$

Table 14-7: $\ {}_{0.99}(S_c)_{10^7} = 75\,000 \text{ psi} = (\sigma_{c,\text{all}})_P = (\sigma_{c,\text{all}})_G$

$$W_3^t = \left[\frac{(\sigma_{c,\text{all}})_P}{C_p}\right]^2 \frac{Fd_pI}{K_oK_vK_sK_mC_f}$$

$$W_3^t = \left(\frac{75\ 000}{1960}\right)^2 \frac{3.25(5.5)(0.1176)}{1.25(1.534)(1)(1.24)(1)} = 1295 \text{ lbf}$$

$$W_4^t = W_3^t = 1295 \text{ lbf}$$

$$H_4 = H_3 = \frac{1295(1649)}{33\ 000} = 64.7 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(76.7, 94.4, 64.7, 64.7) = 64.7 \text{ hp} \quad \textit{Ans.}$$

Notice that the balance between bending and wear power is improved due to CI's more favorable S_c/S_t ratio. Also note that the life is 10^7 pinion revolutions which is (1/300) of $3(10^9)$. Longer life goals require power de-rating.

14-31 From Table A-24*a*, $E_{av} = 11.8(10^6)$ Mpsi
For $\phi = 14.5°$ and $H_B = 156$

$$S_C = \sqrt{\frac{1.4(81)}{2\sin 14.5° \,/\, [11.8(10^6)]}} = 51\ 693 \text{ psi}$$

For $\phi = 20°$

$$S_C = \sqrt{\frac{1.4(112)}{2\sin 20° \,/\, [11.8(10^6)]}} = 52\ 008 \text{ psi}$$

$$S_C = 0.32(156) = 49.9 \text{ kpsi}$$

The first two calculations were approximately 4 percent higher.

14-32 Programs will vary.

14-33

$$(Y_N)_P = 0.977, \quad (Y_N)_G = 0.996$$

$$(S_t)_P = (S_t)_G = 82.3(250) + 12\ 150 = 32\ 725 \text{ psi}$$

$$(\sigma_{\text{all}})_P = \frac{32\ 725(0.977)}{1(0.85)} = 37\ 615 \text{ psi}$$

$$W_1^t = \frac{37\ 615(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 1558 \text{ lbf}$$

$$H_1 = \frac{1558(925)}{33\ 000} = 43.7 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{32\ 725(0.996)}{1(0.85)} = 38\ 346 \text{ psi}$$

$$W_2^t = \frac{38\ 346(1.5)(0.5346)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 2007 \text{ lbf}$$

$$H_2 = \frac{2007(925)}{33\ 000} = 56.3 \text{ hp}$$

$$(Z_N)_P = 0.948, \quad (Z_N)_G = 0.973$$

Table 14-6: $_{0.99}(S_c)_{10^7} = 150\ 000 \text{ psi}$

$$(\sigma_{c,\text{allow}})_P = 150\ 000\left[\frac{0.948(1)}{1(0.85)}\right] = 167\ 294 \text{ psi}$$

$$W_3^t = \left(\frac{167\ 294}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 2074 \text{ lbf}$$

$$H_3 = \frac{2074(925)}{33\ 000} = 58.1 \text{ hp}$$

$$(\sigma_{c,\text{allow}})_G = \frac{0.973}{0.948}(167\ 294) = 171\ 706 \text{ psi}$$

$$W_4^t = \left(\frac{171\ 706}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.052)}\right] = 2167 \text{ lbf}$$

$$H_4 = \frac{2167(925)}{33\ 000} = 60.7 \text{ hp}$$

$$H_{\text{rated}} = \min(43.7,\ 56.3,\ 58.1,\ 60.7) = 43.7 \text{ hp} \quad \textit{Ans.}$$

Pinion bending is controlling.

14-34

$$(Y_N)_P = 1.6831(10^8)^{-0.0323} = 0.928$$

$$(Y_N)_G = 1.6831(10^8\ /\ 3.059)^{-0.0323} = 0.962$$

Table 14-3: $S_t = 55\ 000$ psi

$$(\sigma_{\text{all}})_P = \frac{55\ 000(0.928)}{1(0.85)} = 60\ 047 \text{ psi}$$

$$W_1^t = \frac{60\ 047(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 2487 \text{ lbf}$$

$$H_1 = \frac{2487(925)}{33\ 000} = 69.7 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{0.962}{0.928}(60\ 047) = 62\ 247 \text{ psi}$$

$$W_2^t = \frac{62\ 247}{60\ 047}\left(\frac{0.5346}{0.423}\right)(2487) = 3258 \text{ lbf}$$

$$H_2 = \frac{3258}{2487}(69.7) = 91.3 \text{ hp}$$

Table 14-6: $S_c = 180\ 000$ psi

$$(Z_N)_P = 2.466(10^8)^{-0.056} = 0.8790$$

$$(Z_N)_G = 2.466(10^8 / 3.059)^{-0.056} = 0.9358$$

$$(\sigma_{c,\text{all}})_P = \frac{180\ 000(0.8790)}{1(0.85)} = 186\ 141 \text{ psi}$$

$$W_3^t = \left(\frac{186\ 141}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 2568 \text{ lbf}$$

$$H_3 = \frac{2568(925)}{33\ 000} = 72.0 \text{ hp}$$

$$(\sigma_{c,\text{all}})_G = \frac{0.9358}{0.8790}(186\ 141) = 198\ 169 \text{ psi}$$

$$W_4^t = \left(\frac{198\ 169}{186\ 141}\right)^2\left(\frac{1.043}{1.052}\right)(2568) = 2886 \text{ lbf}$$

$$H_4 = \frac{2886(925)}{33\ 000} = 80.9 \text{ hp}$$

$$H_{\text{rated}} = \min(69.7,\ 91.3,\ 72,\ 80.9) = 69.7 \text{ hp} \quad \textit{Ans.}$$

Pinion bending controlling

14-35 $(Y_N)_P = 0.928$, $(Y_N)_G = 0.962$ (See Prob. 14-34)

Table 14-3: $S_t = 65\ 000$ psi

$$(\sigma_{\text{all}})_P = \frac{65\ 000(0.928)}{1(0.85)} = 70\ 965 \text{ psi}$$

$$W_1^t = \frac{70\ 965(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)} = 2939 \text{ lbf}$$

$$H_1 = \frac{2939(925)}{33\ 000} = 82.4 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{65\ 000(0.962)}{1(0.85)} = 73\ 565 \text{ psi}$$

$$W_2^t = \frac{73\ 565}{70\ 965}\left(\frac{0.5346}{0.423}\right)(2939) = 3850 \text{ lbf}$$

$$H_2 = \frac{3850}{2939}(82.4) = 108 \text{ hp}$$

Table 14-6: $S_c = 225\ 000$ psi

$$(Z_N)_P = 0.8790, \quad (Z_N)_G = 0.9358$$

$$(\sigma_{c,\text{all}})_P = \frac{225\ 000(0.879)}{1(0.85)} = 232\ 676 \text{ psi}$$

$$W_3^t = \left(\frac{232\ 676}{2300}\right)^2 \left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 4013 \text{ lbf}$$

$$H_3 = \frac{4013(925)}{33\ 000} = 112.5 \text{ hp}$$

$$(\sigma_{c,\text{all}})_G = \frac{0.9358}{0.8790}(232\ 676) = 247\ 711 \text{ psi}$$

$$W_4^t = \left(\frac{247\ 711}{232\ 676}\right)^2 \left(\frac{1.043}{1.052}\right)(4013) = 4509 \text{ lbf}$$

$$H_4 = \frac{4509(925)}{33\ 000} = 126 \text{ hp}$$

$$H_{\text{rated}} = \min(82.4,\ 108,\ 112.5,\ 126) = 82.4 \text{ hp} \quad \textit{Ans.}$$

The bending of the pinion is the controlling factor.

14-36 $P = 2$ teeth/in, $d = 8$ in, $N = dP = 8\,(2) = 16$ teeth

$$F = 4p = 4\left(\frac{\pi}{P}\right) = 4\left(\frac{\pi}{2}\right) = 2\pi$$

$$\sum M_x = 0 = 10(300)\cos 20^\circ - 4F_B \cos 20^\circ$$

$$F_B = 750 \text{ lbf}$$

$$W^t = F_B \cos 20^\circ = 750 \cos 20^\circ = 705 \text{ lbf}$$

$$n = 2400 / 2 = 1200 \text{ rev/min}$$

$$V = \frac{\pi dn}{12} = \frac{\pi(8)(1200)}{12} = 2513 \text{ ft/min}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Fig. 14-2: $S_t = 102(300) + 16\ 400 = 47\ 000$ psi

Fig. 14-5: $S_c = 349(300) + 34\ 300 = 139\ 000$ psi

Fig. 14-6: $J = 0.27$

Eq. (14-23): $I = \dfrac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\dfrac{2}{2+1}\right) = 0.107$

Table 14-8: $C_p = 2300\sqrt{\text{psi}}$

Assume a typical quality number of 6.

Eq. (14-28): $B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{2513}}{59.77}\right)^{0.8255} = 1.65$$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, $Y = 0.296$. From Eq. (a), Sec. 14-10,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{2\pi\sqrt{0.296}}{2}\right)^{0.0535} = 1.23$$

The load distribution factor is applicable for straddle-mounted gears, which is not the case here since the gear is mounted outboard of the bearings. Lacking anything better, we will use the load distribution factor as a rough estimate.

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $$C_{pf} = \frac{2\pi}{10(8)} - 0.0375 + 0.0125(2\pi) = 0.1196$$

Eq. (14-33): $C_{pm} = 1.1$

Fig. 14-11: $C_{ma} = 0.23$ (commercial enclosed gear unit)

Eq. (14-35): $C_e = 1$

Eq. (14-30): $K_m = 1 + 1[0.1196(1.1) + 0.23(1)] = 1.36$

For the stress-cycle factors, we need the desired number of load cycles.

$N = 15\,000$ h (1200 rev/min)(60 min/h) = 1.1 (10^9) rev

Fig. 14-14: $Y_N = 0.9$

Fig. 14-15: $Z_N = 0.8$

Eq. 14-38: $K_R = 0.658 - 0.0759 \ln(1 - R) = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$

With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = C_f = 1$

Tooth bending

Eq. (14-15): $$\sigma = W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}$$

$$= 705(1)(1.65)(1.23)\left(\frac{2}{2\pi}\right)\left[\frac{(1.36)(1)}{0.27}\right] = 2294 \text{ psi}$$

Eq. (14-41): $$S_F = \left[\frac{S_t Y_N / (K_T K_R)}{\sigma}\right]$$

$$= \frac{47\,000(0.9) / [(1)(0.885)]}{2294} = 20.8 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $\sigma_c = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)^{1/2}$

$$= 2300\left[705(1)(1.65)(1.23)\left(\frac{1.36}{8(2\pi)}\right)\left(\frac{1}{0.107}\right)\right]^{1/2}$$

$$= 43\,750 \text{ psi}$$

Since gear B is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_H = \frac{S_c Z_N\ /\ (K_T K_R)}{\sigma_c}$$

$$= \left\{\frac{139\,000(0.8)\ /\ [(1)(0.885)]}{43\,750}\right\} = 2.9 \quad \textit{Ans}$$

14-37 m = 18.75 mm/tooth, d = 300 mm

$N = d/m = 300 / 18.75 = 16$ teeth

$F = b = 4p = 4(\pi m) = 4\pi(18.75) = 236$ mm

$\sum M_x = 0 = 300(11)\cos 20^\circ - 150 F_B \cos 25^\circ$

F_B = 22.81 kN

$W^t = F_B \cos 25^\circ = 22.81 \cos 25^\circ = 20.67$ kN

n = 1800 / 2 = 900 rev/min

$$V = \frac{\pi dn}{60} = \frac{\pi(0.300)(900)}{60} = 14.14 \text{ m/s}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Fig. 14-2: $S_t = 0.703(300) + 113 = 324$ MPa

Fig. 14-5: $S_c = 2.41(300) + 237 = 960$ MPa

Fig. 14-6: $J = Y_J = 0.27$

Eq. (14-23): $I = Z_I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\frac{5}{5+1}\right) = 0.134$

Table 14-8: $Z_E = 191\sqrt{\text{MPa}}$

Assume a typical quality number of 6.

Eq. (14-28): $B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$

$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$

Eq. (14-27): $K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{200(14.14)}}{59.77}\right)^{0.8255} = 1.69$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, Y = 0.296.
Similar to Eq. (a) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_b} = 0.8433\left(mF\sqrt{Y}\right)^{0.0535}$$

$$K_s = 0.8433\left[18.75(236)\sqrt{0.296}\right]^{0.0535} = 1.28$$

Convert the diameter and facewidth to inches for use in the load-distribution factor equations. $d = 300/25.4 = 11.81$ in, $F = 236/25.4 = 9.29$ in

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $C_{pf} = \frac{9.29}{10(11.81)} - 0.0375 + 0.0125(9.29) = 0.1573$

Eq. (14-33): $C_{pm} = 1.1$

Fig. 14-11: $C_{ma} = 0.27$ (commercial enclosed gear unit)

Eq. (14-35): $C_e = 1$

Eq. (14-30): $K_m = K_H = 1 + 1[0.1573(1.1) + 0.27(1)] = 1.44$

For the stress-cycle factors, we need the desired number of load cycles.

$$N = 12\,000 \text{ h } (900 \text{ rev/min})(60 \text{ min/h}) = 6.48\,(10^8) \text{ rev}$$

Fig. 14-14: $Y_N = 0.9$

Fig. 14-15: $Z_N = 0.85$

Eq. 14-38: $K_R = 0.658 - 0.0759\ln(1-R) = 0.658 - 0.0759\ln(1-0.98) = 0.955$

With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = Z_R = 1$.

Tooth bending

Eq. (14-15): $$\sigma = W^t K_o K_v K_s \frac{1}{bm_t}\frac{K_H K_B}{Y_J}$$

$$= 20\,670(1)(1.69)(1.28)\left[\frac{1}{236(18.75)}\right]\left[\frac{(1.44)(1)}{0.27}\right] = 53.9 \text{ MPa}$$

Eq. (14-41): $$S_F = \left[\frac{S_t Y_N / (K_T K_R)}{\sigma}\right]$$

$$= \frac{324(0.9) / [(1)(0.955)]}{53.9} = 5.66 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $$\sigma_c = Z_E\left(W^t K_o K_v K_s \frac{K_H}{d_{w1} b}\frac{Z_R}{Z_I}\right)^{1/2}$$

$$= 191\left[20\,670(1)(1.69)(1.28)\left(\frac{1.44}{300(236)}\right)\left(\frac{1}{0.134}\right)\right]^{1/2}$$
$$= 498 \text{ MPa}$$

Since gear B is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_H = \frac{S_c Z_N / (K_T K_R)}{\sigma_c}$$
$$= \frac{960(0.85) / [(1)(0.955)]}{498} = 1.72 \quad \textit{Ans}$$

14-38 From the solution to Prob. 13-40, n = 191 rev/min, W^t = 1600 N, d = 125 mm, N = 15 teeth, m = 8.33 mm/tooth.

$$F = b = 4p = 4(\pi m) = 4\pi(8.33) = 105 \text{ mm}$$
$$V = \frac{\pi d n}{60} = \frac{\pi(0.125)(191)}{60} = 1.25 \text{ m/s}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Table 14-3: S_t = 65 kpsi = 448 MPa
Table 14-6: S_c = 225 kpsi = 1550 MPa
Fig. 14-6: $J = Y_J = 0.25$

Eq. (14-23): $$I = Z_I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\frac{2}{2+1}\right) = 0.107$$

Table 14-8: $Z_E = 191\sqrt{\text{MPa}}$

Assume a typical quality number of 6.

Eq. (14-28): $$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$
$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{200(1.25)}}{59.77}\right)^{0.8255} = 1.21$$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, Y = 0.290. Similar to Eq. (a) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_b} = 0.8433\left(mF\sqrt{Y}\right)^{0.0535}$$
$$K_s = 0.8433\left[8.33(105)\sqrt{0.290}\right]^{0.0535} = 1.17$$

Convert the diameter and facewidth to inches for use in the load-distribution factor equations. $d = 125/25.4 = 4.92$ in, $F = 105/25.4 = 4.13$ in

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $$C_{pf} = \frac{4.13}{10(4.92)} - 0.0375 + 0.0125(4.13) = 0.0981$$

Eq. (14-33): $C_{pm} = 1$

Fig. 14-11: $C_{ma} = 0.32$ (open gearing)

Eq. (14-35): $C_e = 1$

Eq. (14-30): $$K_m = K_H = 1 + 1[0.0981(1) + 0.32(1)] = 1.42$$

For the stress-cycle factors, we need the desired number of load cycles.

$N = 12\,000$ h (191 rev/min)(60 min/h) $= 1.4\,(10^8)$ rev

Fig. 14-14: $Y_N = 0.95$

Fig. 14-15: $Z_N = 0.88$

Eq. 14-38: $$K_R = 0.658 - 0.0759\ln(1-R) = 0.658 - 0.0759\ln(1-0.95) = 0.885$$

With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = Z_R = 1$.

Tooth bending

Eq. (14-15): $$\sigma = W^t K_o K_v K_s \frac{1}{bm_t}\frac{K_H K_B}{Y_J}$$

$$= 1600(1)(1.21)(1.17)\left[\frac{1}{105(8.33)}\right]\left[\frac{(1.42)(1)}{0.25}\right] = 14.7 \text{ MPa}$$

Since gear is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_F = \left[\frac{S_t Y_N / (K_T K_R)}{\sigma}\right]$$

$$= \frac{448(0.95) / [(1)(0.885)]}{14.7} = 32.7 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $$\sigma_c = Z_E\left(W^t K_o K_v K_s \frac{K_H}{d_{w1} b}\frac{Z_R}{Z_I}\right)^{1/2}$$

$$= 191\left[1600(1)(1.21)(1.17)\left(\frac{1.42}{125(105)}\right)\left(\frac{1}{0.107}\right)\right]^{1/2}$$

$$= 289 \text{ MPa}$$

Eq. (14-42): $$S_H = \left[\frac{S_c Z_N / (K_T K_R)}{\sigma_c}\right]$$

$$= \left\{\frac{1550(0.88) / [(1)(0.885)]}{289}\right\} = 5.33 \quad \textit{Ans}$$

14-39 From the solution to Prob. 13-41, $n = 2(70) = 140$ rev/min, $W^t = 180$ lbf, $d = 5$ in
$N = 15$ teeth, $P = 3$ teeth/in.

$$F = 4p = 4\left(\frac{\pi}{P}\right) = 4\left(\frac{\pi}{3}\right) = 4.2 \text{ in}$$

$$V = \frac{\pi dn}{12} = \frac{\pi(5)(140)}{12} = 183.3 \text{ ft/min}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Table 14-3: $S_t = 65$ kpsi
Table 14-6: $S_c = 225$ kpsi
Fig. 14-6: $J = 0.25$

Eq. (14-23): $I = \dfrac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\dfrac{2}{2+1}\right) = 0.107$

Table 14-8: $C_p = 2300\sqrt{\text{psi}}$

Assume a typical quality number of 6.

Eq. (14-28): $B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$
$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$

Eq. (14-27): $K_v = \left(\dfrac{A + \sqrt{V}}{A}\right)^B = \left(\dfrac{59.77 + \sqrt{183.3}}{59.77}\right)^{0.8255} = 1.18$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, $Y = 0.290$.
From Eq. (*a*), Sec. 14-10,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{4.2\sqrt{0.290}}{3}\right)^{0.0535} = 1.17$$

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $C_{pf} = \dfrac{4.2}{10(5)} - 0.0375 + 0.0125(4.2) = 0.099$

Eq. (14-33): $C_{pm} = 1$
Fig. 14-11: $C_{ma} = 0.32$ (Open gearing)
Eq. (14-35): $C_e = 1$
Eq. (14-30): $K_m = 1 + 1[0.099(1) + 0.32(1)] = 1.42$

For the stress-cycle factors, we need the desired number of load cycles.
$N = 14\,000$ h (140 rev/min)(60 min/h) $= 1.2\,(10^8)$ rev
Fig. 14-14: $Y_N = 0.95$
Fig. 14-15: $Z_N = 0.88$
Eq. 14-38: $K_R = 0.658 - 0.0759\ln(1-R) = 0.658 - 0.0759\ln(1-0.98) = 0.955$
With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = C_f = 1$.

Tooth bending

Eq. (14-15): $\sigma = W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J}$

$$= 180(1)(1.18)(1.17)\left(\frac{3}{4.2}\right)\left[\frac{(1.42)(1)}{0.25}\right] = 1010 \text{ psi}$$

Eq. (14-41): $S_F = \left[\dfrac{S_t Y_N \ / \ (K_T K_R)}{\sigma}\right]$

$$= \frac{65\,000(0.95) \ / \ [(1)(0.955)]}{1010} = 64.0 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $\sigma_c = C_p \left(W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I} \right)^{1/2}$

$$= 2300\left[180(1)(1.18)(1.17)\left(\frac{1.42}{5(4.2)}\right)\left(\frac{1}{0.107}\right)\right]^{1/2}$$

$$= 28\,800 \text{ psi}$$

Since gear B is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_H = \left[\frac{S_c Z_N \ / \ (K_T K_R)}{\sigma_c}\right]$$

$$= \left\{\frac{225\,000(0.88) \ / \ [(1)(0.955)]}{28\,800}\right\} = 7.28 \quad \textit{Ans}$$
