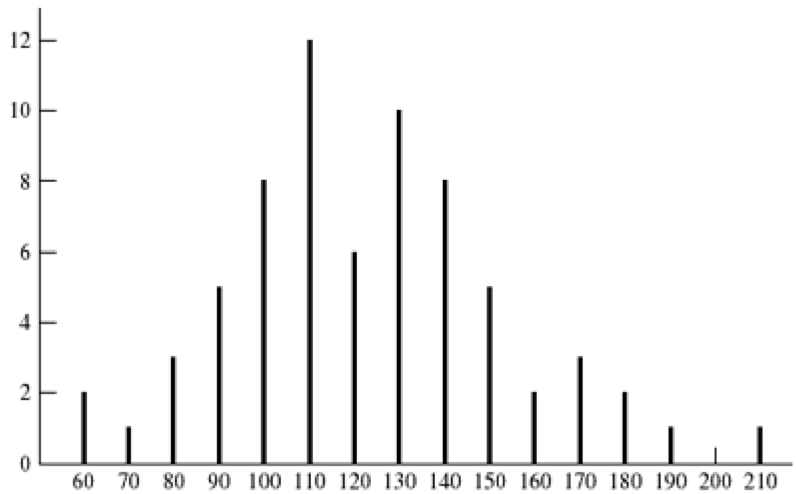
Chapter 20

20-1 (**a**)



(**b**) $f / (N\Delta x) = f / [69(10)] = f / 690$

x	f	fx	fx^2	$f / (N\Delta x)$
60	2	120	7200	0.0029
70	1	70	4900	0.0015
80	3	240	19200	0.0043
90	5	450	40500	0.0072
100	8	800	80000	0.0116
110	12	1320	145200	0.0174
120	6	720	86400	0.0087
130	10	1300	169000	0.0145
140	8	1120	156800	0.0116
150	5	750	112500	0.0174
160	2	320	51200	0.0029
170	3	510	86700	0.0043
180	2	360	64800	0.0029
190	1	130	36100	0.0015
200	0	0	0	0
210	1	210	44100	0.0015
Σ	69	8480	1 104 600	

Eq. (20-9): $\bar{x} = \dfrac{8480}{69} = 122.9$ kcycles

Eq. (20-10): $s_x = \left[\dfrac{1\,104\,600 - 8480^2/69}{69-1}\right]^{1/2} = 30.3$ kcycles *Ans.*

20-2 Data represents a 7-class histogram with $N = 197$.

x	f	fx	fx^2
174	6	1044	181 656
182	9	1638	298 116
190	44	8360	1 588 400
198	67	13 266	2 626 688
206	53	10 918	2 249 108
214	12	2568	549 552
220	6	1320	290 400
Σ	197	39 114	7 789 900

$$\bar{x} = \frac{39\,114}{197} = 198.55 \text{ kpsi} \quad Ans.$$

$$s = \left[\frac{7\,783\,900 - 39\,114^2/197}{197-1}\right]^{1/2} = 9.55 \text{ kpsi} \quad Ans.$$

20-3 Form a Table:

x	f	fx	fx^2
64	2	128	8192
68	6	408	27 744
72	6	432	31 104
76	9	684	51 984
80	19	1520	121 600
84	10	840	70 560
88	4	352	30 976
92	2	184	16 928
Σ	58	4548	359 088

$$\bar{x} = \frac{4548}{58} = 78.4 \text{ kpsi} \qquad \textit{Ans.}$$

$$s_x = \left[\frac{359\ 088 - 4548^2/58}{58-1}\right]^{1/2} = 6.57 \text{ kpsi} \qquad \textit{Ans.}$$

From Eq. 20-14

$$f(x) = \frac{1}{6.57\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-78.4}{6.57}\right)^2\right] \qquad \textit{Ans.}$$

20-4 (a)

x	f	fy	fy^2	y	$f/(Nw)$	$f(y)$	$g(y)$
5.625	1	5.625	31.64063	5.625	0.072 727	0.001 262	0.000 295
5.875	0	0	0	5.875	0	0.008 586	0.004 088
6.125	0	0	0	6.125	0	0.042 038	0.031 194
6.375	3	19.125	121.9219	6.375	0.218 182	0.148 106	0.140 262
6.625	3	19.875	131.6719	6.625	0.218 182	0.375 493	0.393 667
6.875	6	41.25	283.5938	6.875	0.436 364	0.685 057	0.725 002
7.125	14	99.75	710.7188	7.125	1.018 182	0.899 389	0.915 128
7.375	15	110.625	815.8594	7.375	1.090 909	0.849 697	0.822 462
7.625	10	76.25	581.4063	7.625	0.727 273	0.577 665	0.544 251
7.875	2	15.75	124.0313	7.875	0.145 455	0.282 608	0.273 138
8.125	1	8.125	66.015 63	8.125	0.072 727	0.099 492	0.106 720
Σ	55	396.375	2866.859				

For a normal distribution,

$$\bar{y} = 396.375/55 = 7.207, \qquad s_y = \left(\frac{2866.859 - \left(396.375^2/55\right)}{55-1}\right)^{1/2} = 0.4358$$

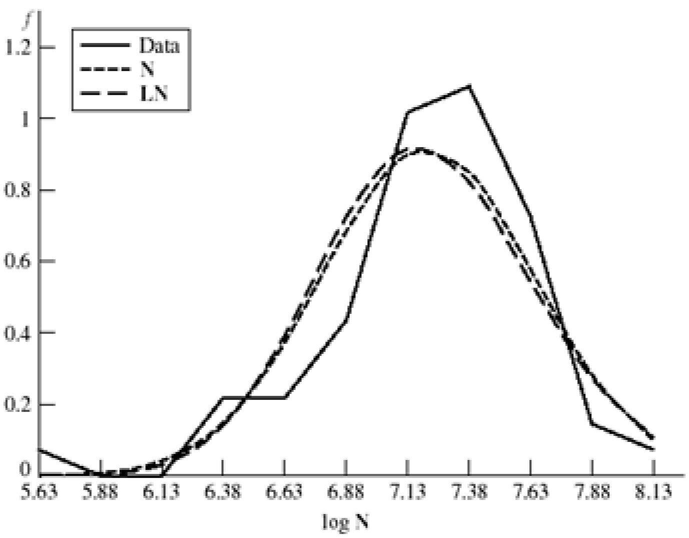
$$f(y) = \frac{1}{0.4358\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-7.207}{0.4358}\right)^2\right]$$

For a lognormal distribution,

$$\bar{x} = \ln 7.206\ 818 - \ln\sqrt{1+0.060\ 474^2} = 1.9732, \qquad s_x = \ln\sqrt{1+0.060\ 474^2} = 0.0604$$

$$g(y) = \frac{1}{x(0.0604)\left(\sqrt{2\pi}\right)} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 1.9732}{0.0604}\right)^2\right]$$

(b) Histogram



20-5 Distribution is uniform in interval 0.5000 to 0.5008 in, range numbers are $a = 0.5000$ in, $b = 0.5008$ in.

(a) Eq. (20-22)

$$\mu_x = \frac{a+b}{2} = \frac{0.5000 + 0.5008}{2} = 0.5004$$

Eq. (20-23)

$$\sigma_x = \frac{b-a}{2} = \frac{0.5008 - 0.5000}{2\sqrt{3}} = 0.000\ 231$$

(b) PDF, Eq. (20-20)

$$f(x) = \begin{cases} 1250 & 0.5000 \le \mathrm{x} \le 0.5008 \text{ in} \\ 0 & \text{otherwise} \end{cases}$$

(c) CDF, Eq. (20-21)

$$F(x) = \begin{cases} 0 & x < 0.5000 \text{ in} \\ (x - 0.5)/0.0008 & 0.5000 \le x \le 0.5008 \text{ in} \\ 1 & x > 0.5008 \text{ in} \end{cases}$$

If all smaller diameters are removed by inspection, $a = 0.5002$ in, $b = 0.5008$ in,

$$\mu_x = \frac{0.5002 + 0.5008}{2} = 0.5005 \text{ in}$$

$$\hat{\sigma}_x = \frac{0.5008 - 0.5002}{2\sqrt{3}} = 0.000\ 173 \text{ in}$$

$$f(x) = \begin{cases} 1666.7 & 0.5002 \le x \le 0.5008 \text{ in} \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0.5002 \text{ in} \\ 1666.7(x - 0.5002) & 0.5002 \le x \le 0.5008 \text{ in} \\ 1 & x > 0.5008 \text{ in} \end{cases}$$

20-6 Dimensions produced are due to tool dulling and wear. When parts are mixed, the distribution is uniform. From Eqs. (20-22) and (20-23),

$$a = \mu_x - \sqrt{3}s = 0.6241 - \sqrt{3}\left(0.000\ 581\right) = 0.6231 \text{ in}$$
$$b = \mu_x + \sqrt{3}s = 0.6241 + \sqrt{3}\left(0.000\ 581\right) = 0.6251 \text{ in}$$

We suspect the dimension was $\dfrac{0.623}{0.625}$ in *Ans.*

20-7 $F(x) = 0.555x - 33$ mm.

(a) Since $F(x)$ is linear, distribution is uniform at $x = a$

$$F(a) = 0 = 0.555(a) - 33$$

$\therefore$ $a = 59.46$ mm. Therefore at $x = b$

$$F(b) = 1 = 0.555b - 33$$

$\therefore$ $b = 61.26$ mm. Therefore,

$$F(x) = \begin{cases} 0 & x < 59.46 \text{ mm} \\ 0.555x - 33 & 59.46 \le x \le 61.26 \text{ mm} \\ 1 & x > 61.26 \text{ mm} \end{cases}$$

The PDF is dF/dx, thus the range numbers are:

$$f(x) = \begin{cases} 0.555 & 59.46 \le x \le 61.26 \text{ mm} \\ 0 & \text{otherwise} \end{cases} \qquad \textit{Ans.}$$

From the range numbers,

$$\mu_x = \frac{59.46 + 61.26}{2} = 60.36 \text{ mm} \qquad \textit{Ans.}$$
$$\hat{\sigma}_x = \frac{61.26 - 59.46}{2\sqrt{3}} = 0.520 \text{ mm} \qquad \textit{Ans.}$$

(***b***) σ is an uncorrelated quotient $\bar{F} = 3600$ lbf, $\bar{A} = 0.112$ in^2

$$C_F = 300/3600 = 0.083\ 33,\ C_A = 0.001/0.112 = 0.008\ 929$$

From Table 20-6, For σ

$$\bar{\sigma} = \frac{\mu_F}{\mu_A} = \frac{3600}{0.112} = 32\ 143 \text{ psi} \quad \textit{Ans.}$$

$$\hat{\sigma}_\sigma = 32\ 143\left[\frac{\left(0.08333^2 + 0.008929^2\right)}{\left(1 + 0.008929^2\right)}\right]^{1/2} = 2694 \text{ psi} \quad \textit{Ans.}$$

$$C_\sigma = 2694 / 32\ 143 = 0.0838 \quad \textit{Ans.}$$

Since **F** and **A** are lognormal, division is closed and $\boldsymbol{\sigma}$ is lognormal too.

$$\boldsymbol{\sigma} = \mathbf{LN}(32\ 143,\ 2694) \text{ psi} \quad \textit{Ans.}$$

20-8 Cramer's rule

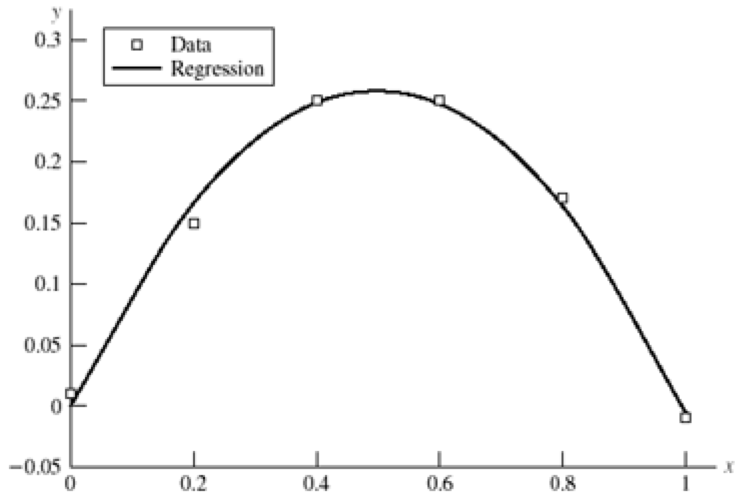
$$a_1 = \frac{\begin{vmatrix} \sum y & \sum x^2 \\ \sum xy & \sum x^3 \end{vmatrix}}{\begin{vmatrix} \sum x & \sum x^2 \\ \sum x^2 & \sum x^3 \end{vmatrix}} = \frac{\sum y \sum x^3 - \sum xy \sum x^2}{\sum x \sum x^3 - \left(\sum x^2\right)^2} \quad \textit{Ans.}$$

$$a_2 = \frac{\begin{vmatrix} \sum x & \sum y \\ \sum x^2 & \sum xy \end{vmatrix}}{\begin{vmatrix} \sum x & \sum x^2 \\ \sum x^2 & \sum x^3 \end{vmatrix}} = \frac{\sum y \sum xy - \sum y \sum x^2}{\sum x \sum x^3 - \left(\sum x^2\right)^2} \quad \textit{Ans.}$$

	x	y	x^2	x^3	xy
	0	0.01	0	0	0
	0	0.15	0.04	0.008	0.030
	0	0.25	0.16	0.064	0.100
	1	0.25	0.36	0.216	0.150
	1	0.17	0.64	0.512	0.136
	1	–0.01	1.00	1.000	–0.010
Σ	3	0.82	2.20	1.800	0.406

$a_1 = 1.040\ 714 \qquad a_2 = -1.046\ 43 \quad$ *Ans.*

	Data	Regression
x	y	y
0	0.01	0
0.2	0.15	0.166 286
0.4	0.25	0.248 857
0.6	0.25	0.247 714
0.8	0.17	0.162 857
1.0	-0.01	-0.005 710

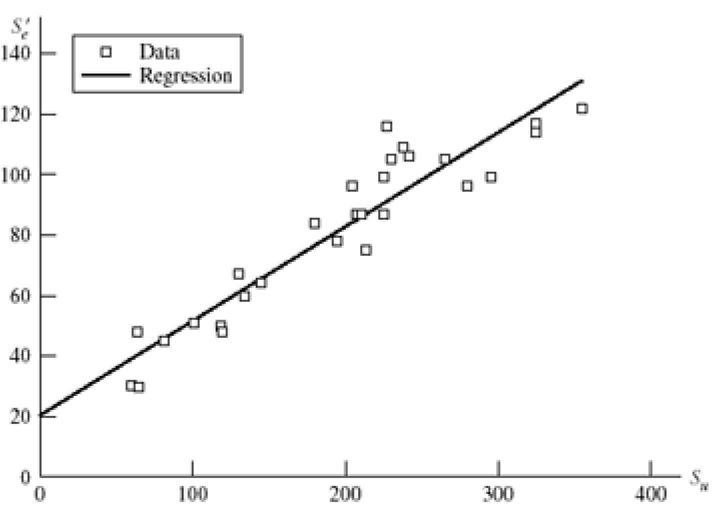


20-9

	Data	Regression		
S_u	S_e'	S_e'	S_u^2	$S_u S_e'$
0		20.356 75		
60	30	39.080 78	3 600	1 800
64	48	40.329 05	4 096	3 072
65	29.5	40.641 12	4 225	1 917.5
82	45	45.946 26	6 724	3 690
101	51	51.875 54	10 201	5 151
119	50	57.492 75	14 161	5 950
120	48	57.804 81	14 400	5 760
130	67	60.925 48	16 900	8 710
134	60	62.173 75	17 956	8 040
145	64	65.606 49	21 025	9 280
180	84	76.528 84	32 400	15 120

	195	78	81.209 85	38 025	15 210
	205	96	84.330 52	42 025	19 680
	207	87	84.954 66	42 849	18 009
	210	87	85.890 86	44 100	18 270
	213	75	86.827 06	45 369	15 975
	225	99	90.571 87	50 625	22 275
	225	87	90.571 87	50 625	19 575
	227	116	91.196 00	51 529	26 332
	230	105	92.132 20	52 900	24 150
	238	109	94.628 74	56 644	25 942
	242	106	95.877 01	58 564	25 652
	265	105	103.054 60	70 225	27 825
	280	96	107.735 60	78 400	26 880
	295	99	112.416 60	87 025	29 205
	325	114	121.778 60	105 625	37 050
	325	117	121.778 60	105 625	38 025
	355	122	131.140 60	126 025	43 310
Σ	5462	2274.5		1 251 868	501 855.5

$m = 0.312\ 067, \quad b = 20.356\ 75 \quad$ *Ans.*



20-10

$$\varepsilon = \sum \left(y - a_0 - a_2 x^2 \right)^2$$

$$\frac{\partial \varepsilon}{\partial a_o} = -2 \sum \left(y - a_0 - a_2 x^2 \right) = 0$$

$$\sum y - na_0 - a_2 \sum x^2 = 0 \quad \Rightarrow \quad \sum y = na_0 + a_2 \sum x^2$$

$$\frac{\partial \varepsilon}{\partial a_2} = 2\sum\left(y - a_0 - a_2 x^2\right)\left(2x\right) = 0 \quad \Rightarrow \quad \sum xy = a_0 \sum x + a_2 \sum x^3 \qquad \textit{Ans.}$$

Cramer's rule

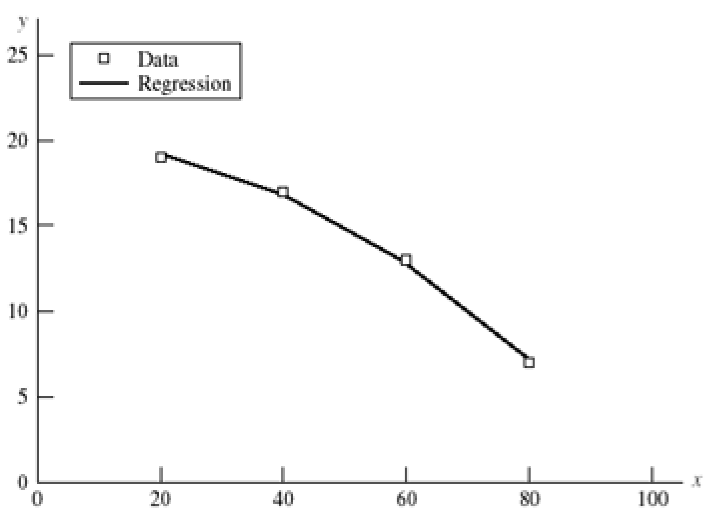
$$a_o = \frac{\begin{vmatrix} \sum y & \sum x^2 \\ \sum xy & \sum x^3 \end{vmatrix}}{\begin{vmatrix} n & \sum x^2 \\ \sum x & \sum x^3 \end{vmatrix}} = \frac{\sum x^3 \sum y - \sum x^2 \sum xy}{n\sum x^3 - \sum x \sum x^2}$$

$$a_1 = \frac{\begin{vmatrix} n & \sum y \\ \sum x & \sum xy \end{vmatrix}}{\begin{vmatrix} n & \sum x^2 \\ \sum x & \sum x^3 \end{vmatrix}} = \frac{n\sum xy - \sum x \sum y}{n\sum x^3 - \sum x \sum x^2}$$

		Data	Regression			
	x	y	y	x^2	x^3	xy
	20	19	19.2	400	8 000	380
	40	17	16.8	1600	64 000	680
	60	13	12.8	3600	216 000	780
	80	7	7.2	6400	512 000	560
Σ	200	56		12 000	800 000	2400

$$a_0 = \frac{800\ 000(56) - 12\ 000(2400)}{4(800\ 000) - 200(12\ 000)} = 20$$

$$a_1 = \frac{4(2400) - 200(56)}{4(800\ 000) - 200(12\ 000)} = -0.002$$

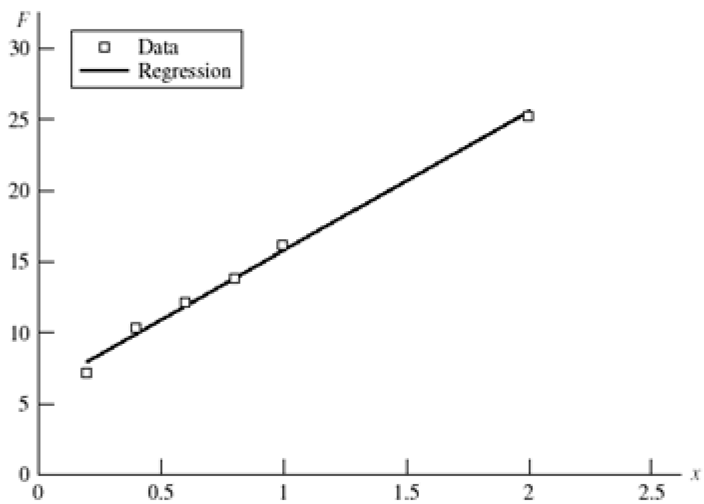


20-11

	Data	Regression					
x	y	y	x^2	y^2	xy	$x-\bar{x}$	$(x-\bar{x})^2$
0.2	7.1	7.931 803	0.04	50.41	1.42	-0.633 333	0.401 111 111
0.4	10.3	9.884 918	0.16	106.09	4.12	-0.433 333	0.187 777 778
0.6	12.1	11.838 032	0.36	146.41	7.26	-0.233 333	0.054 444 444
0.8	13.8	13.791 147	0.64	190.44	11.04	-0.033 333	0.001 111 111
1	16.2	15.744 262	1	262.44	16.2	0.166 666	0.027 777 778
2	25.2	25.509 836	4	635.04	50.4	1.166 666	1.361 111 111
Σ 5	84.7		6.2	1390.83	90.44	0	2.033 333 333

$$\hat{m} = \bar{k} = \frac{6(90.44) - 5(84.7)}{6(6.2) - (5)^2} = 9.7656$$

$$\hat{b} = \bar{F}_i = \frac{84.7 - 9.7656(5)}{6} = 5.9787$$



(a) $\bar{x} = \frac{5}{6}; \quad \bar{y} = \frac{84.7}{6} = 14.117$

Eq. (20-37):

$$s_{yx} = \sqrt{\frac{1390.83 - 5.9787(84.7) - 9.7656(90.44)}{6-2}}$$

$$= 0.556$$

Eq. (20-36):

$$s_{\hat{b}} = 0.556\sqrt{\frac{1}{6} + \frac{(5/6)^2}{2.0333}} = 0.3964 \text{ lbf}$$

$$F_i = (5.9787, 0.3964) \text{ lbf} \quad \textit{Ans.}$$

(b) Eq. (20-35):

$$s_{\hat{m}} = \frac{0.556}{\sqrt{2.0333}} = 0.3899 \text{ lbf/in}$$

$$\mathbf{k} = (9.7656, 0.3899) \text{ lbf/in} \quad \textit{Ans.}$$

20-12 The expression $\boldsymbol{\epsilon} = \boldsymbol{\delta} / \mathbf{l}$ is of the form $\mathbf{x} / \mathbf{y}$. Now $\boldsymbol{\delta}$ = (0.0015, 0.000 092) in, unspecified distribution; and $\mathbf{l}$ = (2,000, 0.008 1) in, unspecified distribution;

$$C_x = 0.000\ 092 / 0.0015 = 0.0613$$
$$C_y = 0.0081 / 2.000 = 0.004\ 05$$

Table 20-6: $\bar{\epsilon} = 0.0015 / 2.000 = 0.000\ 75$

$$\hat{\sigma}_{\epsilon} = 0.000\ 75\left[\frac{0.0613^2 + 0.004\ 05^2}{1 + 0.004\ 05^2}\right]^{1/2}$$

$$= 4.607\left(10^{-5}\right) = 0.000\ 046$$

We can predict $\bar{\epsilon}$ and $\hat{\sigma}_{\epsilon}$ but not the distribution of $\boldsymbol{\epsilon}$.

20-13 $\boldsymbol{\sigma} = \boldsymbol{\epsilon}\mathbf{E}$

$\boldsymbol{\epsilon}$ = (0.0005, 0.000 034), distribution unspecified; $\mathbf{E}$ = (29.5, 0.885) Mpsi, distribution unspecified;

$$C_x = 0.000\ 034 / 0.0005 = 0.068$$
$$C_y = 0.0885 / 29.5 = 0.03$$

$\boldsymbol{\sigma}$ is of the form $\mathbf{xy}$

Table 20-6: $\bar{\sigma} = \bar{\epsilon}\,\bar{E} = 0.0005(29.5)10^6 = 14\ 750$ psi

$$\hat{\sigma}_{\sigma} = 14\ 750\left[0.068^2 + 0.030^2 + 0.068^2\left(0.030^2\right)\right]^{1/2}$$

$$= 1096.7 \text{ psi}$$

$$C_{\sigma} = 1096.7 / 14\ 750 = 0.074\ 35$$

20-14

$$\boldsymbol{\delta} = \frac{\mathbf{Fl}}{\mathbf{AE}}$$

where $\mathbf{F}$ = (14.7, 1.3) kip, $\mathbf{A}$ = (0.226, 0.003) in^2, $\mathbf{l}$ = (1.5, 0.004) in, and $\mathbf{E}$ = (29.5, 0.885) Mpsi, distributions unspecified.

C_F = 1.3 / 14.7 = 0.0884; C_A = 0.003 / 0.226 = 0.0133; C_l = 0.004 / 1.5 = 0.00267;
C_E =0.885 / 29.5 = 0.03

$$\delta = \frac{\mathbf{Fl}}{\mathbf{AE}} = \mathbf{Fl}\left(\frac{1}{\mathbf{A}}\right)\left(\frac{1}{\mathbf{E}}\right)$$

Table 20-6:

$$\bar{\delta} = \bar{F}\,\bar{l}\,\overline{(1/A)}\;\overline{(1/E)} \doteq \bar{F}\,\bar{l}\,(1/\bar{A})(1/\bar{E})$$

$$= 14\ 700(1.5)\left(\frac{1}{0.226}\right)\left[\frac{1}{29.5(10^6)}\right] = 0.003\ 31\text{ in.} \quad \textit{Ans.}$$

For the standard deviation, using the first-order terms in Table 20-6,

$$\hat{\sigma}_\delta \doteq \frac{\bar{F}\,\bar{l}}{\bar{A}\bar{E}}\left(C_F^2 + C_l^2 + C_A^2 + C_E^2\right)^{1/2} = \bar{\delta}\left(C_F^2 + C_l^2 + C_A^2 + C_E^2\right)^{1/2}$$

$$\hat{\sigma}_\delta = 0.003\ 31\left(0.0844^2 + 0.002\ 67^2 + 0.0133^2 + 0.03^2\right)^{1/2}$$

$$= 0.000\ 313\text{ in} \quad \textit{Ans.}$$

COV: $C_\delta = \hat{\sigma}_\delta \,/\, \bar{\delta} = 0.000\ 313/0.003\ 31 = 0.0945$ *Ans.*

Force COV dominates. There is no distributional information on δ.

20-15 **M** = (15 000, 1350) lbf · in, distribution unspecified; **d** = (2.00, 0.005) in, distribution unspecified.

$$\boldsymbol{\sigma} = \frac{32\mathbf{M}}{\pi\mathbf{d}^3}$$

$$C_M = 1350 / 15\ 000 = 0.09, \quad C_d = 0.005 / 2.00 = 0.0025$$

σ is of the form **x/y**^{3}, Table 20-6.

Mean: $\bar{M} = 15\ 000$ lbf · in

$$\overline{\left(\frac{1}{d^3}\right)} = \frac{1}{\bar{d}^3}\left(1 + 6C_x^2\right) = \frac{1}{2^3}\left[1 + 6\left(0.0025^2\right)\right] = 0.125\text{ in}^3\ *$$

$$*\text{ Note: } \overline{\left(\frac{1}{d^3}\right)} \doteq \frac{1}{\bar{d}^3}$$

$$\bar{\sigma} = \frac{32\bar{M}}{\pi \bar{d}^3} = \frac{32(15\ 000)}{\pi}(0.125)$$

$$= 19\ 099 \text{ psi} \quad \textit{Ans.}$$

Standard Deviation:

$$\hat{\sigma}_\sigma = \bar{\sigma}\left[\left(C_M^2 + C_{d^3}^2\right)/\left(1 + C_{d^3}^2\right)\right]^{1/2}$$

Table 20-6:

$$C_{d^3} \doteq 3C_d = 3(0.0025) = 0.0075$$

$$\hat{\sigma}_\sigma = \bar{\sigma}\left[\left(C_M^2 + (3C_d)^2\right)\Big/\left(1 + (3C_d)^2\right)\right]^{1/2}$$

$$= 19\ 099\left[\left(0.09^2 + 0.0075^2\right)\Big/\left(1 + 0.0075^2\right)\right]^{1/2}$$

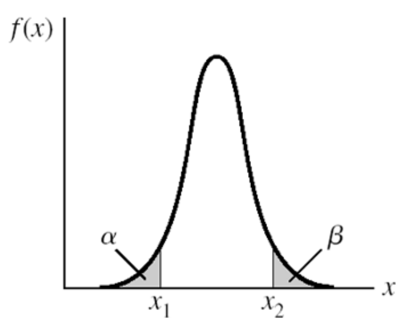
$$= 1725 \text{ psi} \quad \textit{Ans.}$$

COV:

$$C_\sigma = \frac{1725}{19\ 099} = 0.0903 \quad \textit{Ans.}$$

Stress COV dominates. No information of distribution of σ.

20-16



Fraction discarded is $\alpha + \beta$. The area under the PDF was unity. Having discarded $\alpha + \beta$ fraction, the ordinates to the truncated PDF are multiplied by *a*.

$$a = \frac{1}{1 - (\alpha + \beta)}$$

New PDF, $g(x)$, is given by

$$g(x) = \begin{cases} f(x)\Big/\left[1 - (\alpha + \beta)\right] & x_1 \le x \le x_2 \\ 0 & \text{otherwise} \end{cases}$$

A more formal proof: $g(x)$ has the property

$$1 = \int_{x_1}^{x_2} g(x)\,dx = a\int_{x_1}^{x_2} f(x)\,dx$$

$$1 = a\left[\int_{-\infty}^{\infty} f(x)\,dx - \int_{0}^{x_1} f(x)\,dx - \int_{x_2}^{\infty} f(x)\,dx\right]$$

$$1 = a\left\{1 - F(x_1) - \left[1 - F(x_2)\right]\right\}$$

$$a = \frac{1}{F(x_2) - F(x_1)} = \frac{1}{(1-\beta) - \alpha} = \frac{1}{1 - (\alpha + \beta)}$$

20-17 (**a**) **d** = **U**(0.748, 0.751)

$$\mu_d = \frac{0.751 + 0.748}{2} = 0.7495 \text{ in}$$

$$\hat{\sigma}_d = \frac{0.751 - 0.748}{2\sqrt{3}} = 0.000\ 866 \text{ in}$$

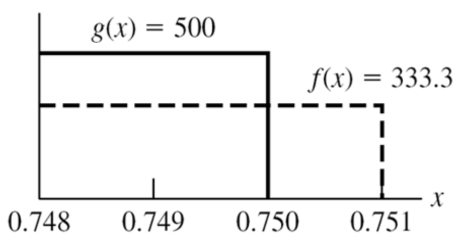
$$f(x) = \frac{1}{b-a} = \frac{1}{0.751 - 0.748} = 333.3 \text{ in}^{-1}$$

$$F(x) = \frac{x - 0.748}{0.751 - 0.748} = 333.3(x - 0.748)$$

(**b**) $F(x_1) = F(0.748) = 0$

$F(x_2) = (0.750 - 0.748)\ 333.3 = 0.6667$

If $g(x)$ is truncated, PDF becomes



$$g(x) = \frac{f(x)}{F(x_2) - F(x_1)} = \frac{333.3}{0.6667 - 0} = 500 \text{ in}^{-1}$$

$$\mu_x = \frac{a' + b'}{2} = \frac{0.748 + 0.750}{2} = 0.749 \text{ in}$$

$$\hat{\sigma}_x = \frac{b' - a'}{2\sqrt{3}} = \frac{0.750 - 0.748}{2\sqrt{3}} = 0.000\ 577 \text{ in}$$

20-18 From Table A-10, 8.1% corresponds to $z_1 = -1.4$ and 5.5% corresponds to $z_2 = +1.6$.

$$k_1 = \mu + z_1\hat{\sigma}$$

$$k_2 = \mu + z_2\hat{\sigma}$$

From which

$$\mu = \frac{z_2 k_1 - z_1 k_2}{z_2 - z_1} = \frac{1.6(9) - (-1.4)11}{1.6 - (-1.4)}$$

$$= 9.933$$

$$\hat{\sigma} = \frac{k_2 - k_1}{z_2 - z_1} = \frac{11 - 9}{1.6 - (-1.4)} = 0.6667$$

The original density function is

$$f(k) = \frac{1}{0.6667\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{k - 9.933}{0.6667} \right)^2 \right] \quad \textit{Ans.}$$

20-19 From Prob. 20-1, μ = 122.9 kcycles and $\hat{\sigma}$ = 30.3 kcycles.

$$z_{10} = \frac{x_{10} - \mu}{\hat{\sigma}} = \frac{x_{10} - 122.9}{30.3}$$

$$x_{10} = 122.9 + 30.3 z_{10}$$

From Table A-10, for 10 percent failure, $z_{10} = -1.282$

$$x_{10} = 122.9 + 30.3(-1.282)$$
$$= 84.1 \text{ kcycles} \quad \textit{Ans.}$$

20-20

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
60	2	120	7200	0.002899	0.000399
70	1	70	4900	0.001449	0.001206
80	3	240	19200	0.004348	0.003009
90	5	450	40500	0.007246	0.006204
100	8	800	80000	0.011594	0.010567
110	12	1320	145200	0.017391	0.014871
120	6	720	86400	0.008696	0.017292
130	10	1300	169000	0.014493	0.016612
140	8	1120	156800	0.011594	0.013185
150	5	750	112500	0.007246	0.008647
160	2	320	51200	0.002899	0.004685
170	3	510	86700	0.004348	0.002097
180	2	360	64800	0.002899	0.000776
190	1	190	36100	0.001449	0.000237
200	0	0	0	0	5.98E-05
210	1	210	44100	0.001449	1.25E-05
Σ	69	8480			

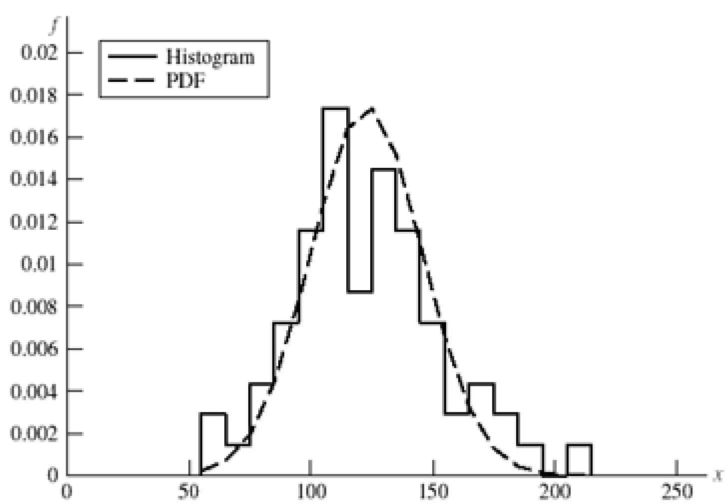
$\bar{x}$ = 122.8986 $\quad$ s_x = 22.887 19

Eq. (20-14):

$$f(x) = \frac{1}{\hat{\sigma}_x\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\hat{\sigma}_x}\right)^2\right]$$

$$= \frac{1}{22.88719\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-122.8986}{22.88719}\right)^2\right]$$

x	$f\ /\ (Nw)$	$f(x)$	x	$f\ /\ (Nw)$	$f(x)$
55	0	0.000 214	145	0.011 594	0.010 935
55	0.002 899	0.000 214	145	0.007 246	0.010 935
65	0.002 899	0.000 711	155	0.007 246	0.006 518
65	0.001 449	0.000 711	155	0.002 899	0.006 518
75	0.001 449	0.001 951	165	0.002 899	0.002 21
75	0.004 348	0.001 951	165	0.004 348	0.003 21
85	0.004 348	0.004 425	175	0.004 348	0.001 306
85	0.007 246	0.004 425	175	0.002 899	0.001 306
95	0.007 246	0.008 292	185	0.002 899	0.000 439
95	0.011 594	0.008 292	185	0.001 449	0.000 439
105	0.011 594	0.012 839	195	0.001 449	0.000 122
105	0.017 391	0.012 839	195	0	0.000 122
115	0.017 391	0.016 423	205	0	2.8E-05
115	0.008 696	0.016 423	205	0.001 499	2.8E-05
125	0.008 696	0.017 357	215	0.001 499	5.31E-06
125	0.014 493	0.017 357	215	0	5.31E-06
135	0.014 493	0.015 157			
135	0.011 594	0.015 157			

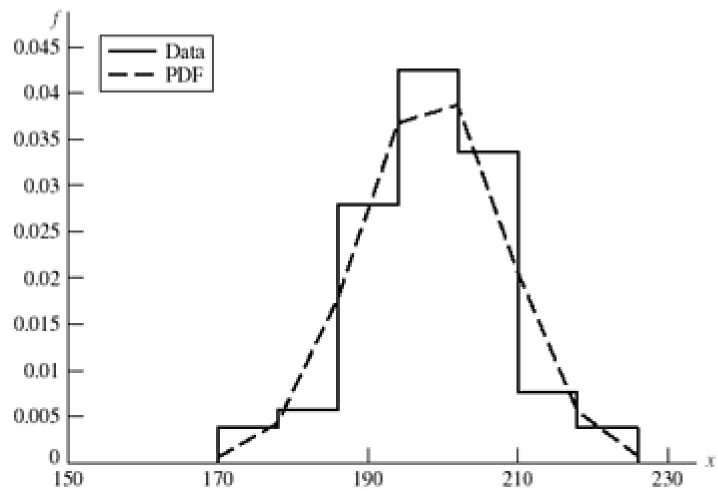


20-21

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
174	6	1044	181656	0.003807	0.001642
182	9	1638	298116	0.005711	0.009485
190	44	8360	1588400	0.027919	0.027742
198	67	13266	2626668	0.042513	0.041068
206	53	10918	2249108	0.033629	0.030773
214	12	2568	549552	0.007614	0.011671
222	6	1332	295704	0.003807	0.002241
1386	197	39126	7789204		

$\bar{x} = 198.6091$ $s_x = 9.695\ 071$

x	$f/(Nw)$	$f(x)$
170	0	0.000529
170	0.003807	0.000529
178	0.003807	0.004297
178	0.005711	0.004297
186	0.005711	0.017663
186	0.027919	0.017663
194	0.027919	0.036752
194	0.042513	0.036752
202	0.042513	0.038708
202	0.033629	0.038708
210	0.033629	0.020635
210	0.007614	0.020635
218	0.007614	0.005568
218	0.003807	0.005568
226	0.003807	0.00076
226	0	0.00076

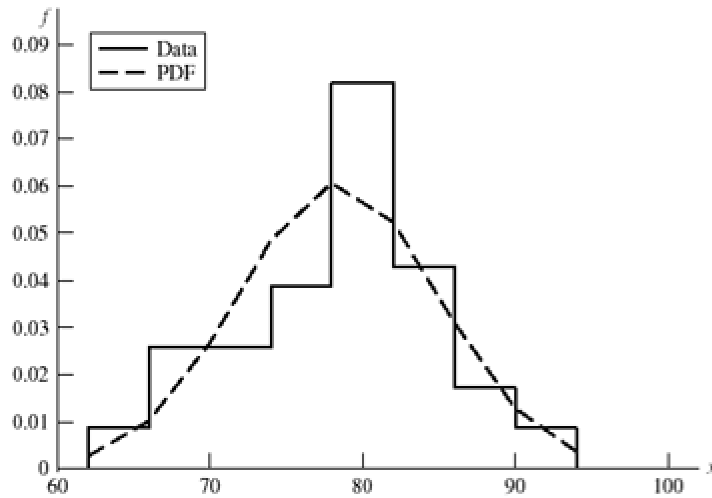


20-22

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
64	2	128	8192	0.008621	0.00548
68	6	408	27744	0.025862	0.017299
72	6	432	31104	0.025862	0.037705
76	9	684	51984	0.038793	0.056742
80	19	1520	121600	0.081897	0.058959
84	10	840	70560	0.043103	0.042298
88	4	352	30976	0.017241	0.020952
92	2	184	16928	0.008621	0.007165
624	58	4548	359088		

$\bar{x} = 78.041379 \qquad s_x = 6.572\,229$

x	$f/(Nw)$	$f(x)$
62	0	0.002684
62	0.008621	0.002684
66	0.008621	0.010197
66	0.025862	0.010197
70	0.025862	0.026749
70	0.025862	0.026749
74	0.025862	0.048446
74	0.038793	0.048446
78	0.038793	0.060581
78	0.0381897	0.060581
82	0.081897	0.052305
82	0.043103	0.052305
86	0.043103	0.03118
86	0.017241	0.03118
90	0.017241	0.012833
90	0.008621	0.012833
94	0.008621	0.003647
94	0	0.003647



20-23

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(40)}{\pi\left(1^2\right)} = 50.93 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{4\hat{\sigma}_P}{\pi d^2} = \frac{4(8.5)}{\pi\left(1^2\right)} = 10.82 \text{ kpsi}$$

$$\hat{\sigma}_{S_y} = 5.9 \text{ kpsi}$$

For no yield, $m = S_y - \sigma \geq 0$

$$z = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - \mu_m}{\hat{\sigma}_m} = -\frac{\mu_m}{\hat{\sigma}_m}$$

$$\mu_m = \bar{S}_y - \bar{\sigma} = 78.4 - 50.93 = 27.47 \text{ kpsi}$$

$$\hat{\sigma}_m = \left(\hat{\sigma}_\sigma^2 + \sigma_{S_y}^2\right)^{1/2} = \left(10.82^2 + 5.9^2\right)^2 = 12.32 \text{ kpsi}$$

$$z = -\frac{\mu_m}{\hat{\sigma}_m} = -\frac{27.47}{12.32} = -2.230$$

Table A-10, $p_f = 0.0129$

$$R = 1 - p_f = 1 - 0.0129 = 0.987 \quad \textit{Ans.}$$

20-24 For a lognormal distribution,

Eq. (20-18) $\mu_y = \ln \mu_x - \ln \sqrt{1 + C_x^2}$

Eq. (20-19) $\hat{\sigma}_y = \sqrt{\ln\left(1 + C_x^2\right)}$

From Prob. (20-23)

$$\mu_m = \bar{S}_y - \bar{\sigma} = \mu_x$$

$$\mu_y = \left(\ln \bar{S}_y - \ln \sqrt{1 + C_{S_y}^2}\right) - \left(\ln \bar{\sigma} - \ln \sqrt{1 + C_\sigma^2}\right)$$

$$= \ln\left[\frac{\bar{S}_y}{\bar{\sigma}}\sqrt{\frac{1 + C_\sigma^2}{1 + C_{S_y}^2}}\right]$$

$$\hat{\sigma}_y = \left[\ln\left(1 + C_{S_y}^2\right) + \ln\left(1 + C_\sigma^2\right)\right]^{1/2}$$

$$= \sqrt{\ln\left[\left(1 + C_{S_y}^2\right)\left(1 + C_\sigma^2\right)\right]}$$

$$z = -\frac{\mu}{\hat{\sigma}} = -\frac{\ln\left(\frac{\bar{S}_y}{\bar{\sigma}}\sqrt{\frac{1 + C_\sigma^2}{1 + C_{S_y}^2}}\right)}{\sqrt{\ln\left[\left(1 + C_{S_y}^2\right)\left(1 + C_\sigma^2\right)\right]}}$$

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(30)}{\pi\left(1^2\right)} = 38.197 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{4\hat{\sigma}_P}{\pi d^2} = \frac{4(5.1)}{\pi\left(1^2\right)} = 6.494 \text{ kpsi}$$

$$C_\sigma = \frac{6.494}{38.197} = 0.1700$$

$$C_{S_y} = \frac{3.81}{49.6} = 0.076\ 81$$

$$z = -\frac{\ln\left[\frac{49.6}{38.197}\sqrt{\frac{1 + 0.170^2}{1 + 0.07681^2}}\right]}{\sqrt{\ln\left[\left(1 + 0.076\ 81^2\right)\left(1 + 0.170^2\right)\right]}} = -1.470$$

Table A-10

$$p_f = 0.0708$$

$$R = 1 - p_f = 0.929 \quad \textit{Ans.}$$

20-25

x	n	nx	nx^2
93	19	1767	164 311
95	25	2375	225 625
97	38	3686	357 542
99	17	1683	166 617
101	12	1212	122 412
103	10	1030	106 090
105	5	525	55 125
107	4	428	45 796
109	4	436	47 524
111	2	222	24 642
	136	13 364	1 315 704

$\bar{x}$ = 13 364/136 = 98.26 kpsi

$$s_x = \left(\frac{1\ 315\ 704 - 13\ 364^2/136}{136-1}\right)^{1/2} = 4.30 \text{ kpsi}$$

Under normal hypothesis,

$$\begin{aligned} z_{0.01} &= (x_{0.01} - 98.26)/4.30 \\ x_{0.01} &= 98.26 + 4.30 z_{0.01} \\ &= 98.26 + 4.30(-2.3267) \\ &= 88.26 \doteq 88.3 \text{ kpsi} \quad \textit{Ans.} \end{aligned}$$

20-26 From Prob. 20.25, μ_x = 98.26 kpsi, and $\hat{\sigma}_x = 4.30$ kpsi.

$$C_x = \hat{\sigma}_x / \mu_x = 4.30/98.26 = 0.043\ 76$$

From Eqs. (20-18) and (20-19),

$$\mu_y = \ln(98.26) - 0.043\ 76^2/2 = 4.587$$

$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.043\ 76^2)} = 0.043\ 74$$

For a yield strength exceeded by 99% of the population,

$$z_{0.01} = (\ln x_{0.01} - \mu_y)/\hat{\sigma}_y \quad \Rightarrow \quad \ln x_{0.01} = \mu_y + \hat{\sigma}_y z_{0.01}$$

From Table A-10, for 1% failure, $z_{0.01} = -2.326$. Thus,

$$\ln x_{0.01} = 4.587 + 0.043\ 74(-2.326) = 4.485$$
$$x_{0.01} = 88.7\text{ kpsi} \quad \textit{Ans.}$$

The normal PDF is given by Eq. (20-14) as

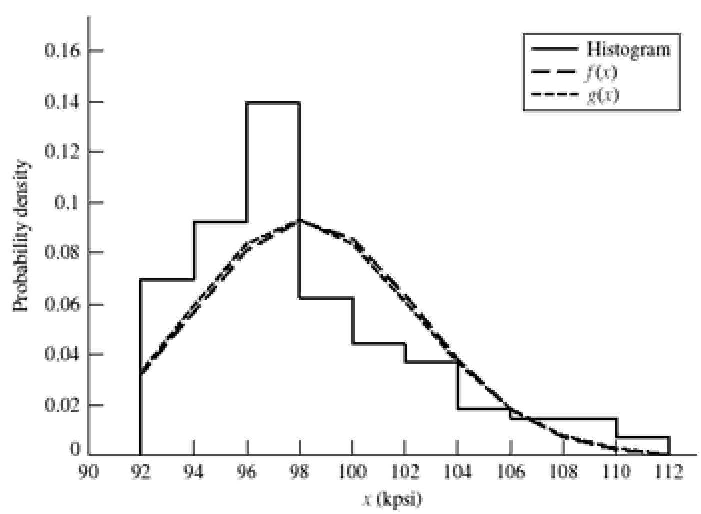
$$f(x) = \frac{1}{4.30\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-98.26}{4.30}\right)^2\right]$$

For the lognormal distribution, from Eq. (20-17), defining $g(x)$,

$$g(x) = \frac{1}{x(0.043\ 74)\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln x - 4.587}{0.043\ 74}\right)^2\right]$$

x(kpsi)	$f/(Nw)$	$f(x)$	$g(x)$	x(kpsi)	$f/(Nw)$	$f(x)$	$g(x)$
92	0.000 00	0.032 15	0.032 63	102	0.036 76	0.063 56	0.061 34
92	0.069 85	0.032 15	0.032 63	104	0.036 76	0.038 06	0.037 08
94	0.069 85	0.056 80	0.058 90	104	0.018 38	0.038 06	0.037 08
94	0.091 91	0.056 80	0.058 90	106	0.018 38	0.018 36	0.018 69
96	0.091 91	0.080 81	0.083 08	106	0.014 71	0.018 36	0.018 69
96	0.139 71	0.080 81	0.083 08	108	0.014 71	0.007 13	0.007 93
98	0.139 71	0.092 61	0.092 97	108	0.014 71	0.007 13	0.007 93
98	0.062 50	0.092 61	0.092 97	110	0.014 71	0.002 23	0.002 86
100	0.062 50	0.085 48	0.083 67	110	0.007 35	0.002 23	0.002 86
100	0.044 12	0.085 48	0.083 67	112	0.007 35	0.000 56	0.000 89
102	0.044 12	0.063 56	0.061 34	112	0.000 00	0.000 56	0.000 89

Note: rows are repeated to draw histogram



The normal and lognormal are almost the same. However, the data is quite skewed and perhaps a Weibull distribution should be explored. For a method of establishing the

Weibull parameters see Shigley, J. E., and C. R. Mishke, *Mechanical Engineering Design*, McGraw-Hill, 5th ed., 1989, Sec. 4-12.

20-27 $\mathbf{x} = \left(\mathbf{S}'_{fe}\right)_{10^4}$ $x_0 = 79$ kpsi, $\theta = 86.2$ kpsi, $b = 2.6$

Eq. (20-28):

$$\begin{aligned}\bar{x} &= x_0 + \left(\theta - x_0\right)\Gamma(1+1/b)\\ &= 79 + (86.2-79)\Gamma(1+1/2.6)\\ &= 79 + 7.2\Gamma(1.38)\end{aligned}$$

From Table A-34, $\Gamma(1.38) = 0.888\ 54$

$$\bar{x} = 79 + 7.2(0.888\ 54) = 85.4 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (20-29)

$$\begin{aligned}\hat{\sigma}_x &= \left(\theta - x_0\right)\left[\Gamma\left(1+2/b\right) - \Gamma^2\left(1+1/b\right)\right]^{1/2}\\ &= \left(86.2-79\right)\left[\Gamma\left(1+2/2.6\right) - \Gamma^2\left(1+1/2.6\right)\right]^{1/2}\\ &= 7.2\left[0.923\ 76 - 0.888\ 54^2\right]^{1/2}\\ &= 2.64 \text{ kpsi} \quad \textit{Ans.}\end{aligned}$$

$$C_x = \frac{\hat{\sigma}_x}{\bar{x}} = \frac{2.64}{85.4} = 0.031 \quad \textit{Ans.}$$

20-28 $\mathbf{x} = \mathbf{S}_{ut}$ $x_0 = 27.7$ kpsi, $\theta = 46.2$ kpsi, $b = 4.38$

$$\begin{aligned}\mu_x &= 27.7 + \left(46.2-27.7\right)\Gamma\left(1+1/4.38\right)\\ &= 27.7 + 18.5\Gamma(1.23)\\ &= 27.7 + 18.5(0.910\ 75)\\ &= 44.55 \text{ kpsi} \quad \textit{Ans.}\end{aligned}$$

$$\begin{aligned}\hat{\sigma}_x &= \left(46.2-27.7\right)\left[\Gamma\left(1+2/4.38\right) - \Gamma^2\left(1+1/4.38\right)\right]^{1/2}\\ &= 18.5\left[\Gamma(1.46) - \Gamma^2(1.23)\right]^{1/2}\\ &= 18.5\left[0.8856 - 0.920\ 75^2\right]^{1/2}\\ &= 4.38 \text{ kpsi} \quad \textit{Ans.}\end{aligned}$$

$$C_x = \frac{4.38}{44.55} = 0.098 \quad \textit{Ans.}$$

From the Weibull survival equation

$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right] = 1 - p$$

$$R_{40} = \exp\left[-\left(\frac{x_{40} - x_0}{\theta - x_0}\right)^b\right] = 1 - p_{40}$$

$$= \exp\left[-\left(\frac{40 - 27.7}{46.2 - 27.7}\right)^{4.38}\right] = 0.846$$

$$p_{40} = 1 - R_{40} = 1 - 0.846 = 0.154 = 15.4\% \quad \textit{Ans.}$$

20-29 $\mathbf{x} = \mathbf{S}_{ut}$, $x_0 = 151.9$ kpsi, $\theta = 193.6$ kpsi, $b = 8$

$$\mu_x = 151.9 + (193.6 - 151.9)\Gamma(1 + 1/8)$$

$$= 151.9 + 41.7\,\Gamma(1.125)$$

$$= 151.9 + 41.7(0.941\ 76)$$

$$= 191.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\hat{\sigma}_x = (193.6 - 151.9)\left[\Gamma(1 + 2/8) - \Gamma^2(1 + 1/8)\right]^{1/2}$$

$$= 41.7\left[\Gamma(1.25) - \Gamma^2(1.125)\right]^{1/2}$$

$$= 41.7\left[0.906\ 40 - 0.941\ 76^2\right]^{1/2}$$

$$= 5.82 \text{ kpsi} \quad \textit{Ans.}$$

$$C_x = \frac{5.82}{191.2} = 0.030$$

20-30 $\mathbf{x} = \mathbf{S}_{ut}$, $x_0 = 47.6$ kpsi, $\theta = 125.6$ kpsi, $b = 11.4$

$$\bar{x} = 47.6 + (125.6 - 47.6)\Gamma(1 + 1/11.84)$$

$$= 47.6 + 78\,\Gamma(1.08)$$

$$= 47.6 + 78(0.959\ 73) = 122.5 \text{ kpsi}$$

$$\hat{\sigma}_x = (125.6 - 47.6)\left[\Gamma(1 + 2/11.84) - \Gamma^2(1 + 1/11.84)\right]^{1/2}$$

$$= 78\left[\Gamma(1.08) - \Gamma^2(1.17)\right]^{1/2}$$

$$= 78\left[0.959\ 73 - 0.936\ 70^2\right]^{1/2} = 22.4 \text{ kpsi}$$

From Prob. 20-28,

$$p = 1 - \exp\left[-\left(\frac{x - x_0}{\theta - \theta_0}\right)^b\right] = 1 - \exp\left[-\left(\frac{100 - 47.6}{125.6 - 47.6}\right)^{11.84}\right] = 0.0090 \quad \textit{Ans.}$$

$\mathbf{y} = \mathbf{S}_y$, $y_0 = 64.1$ kpsi, $\theta = 81.0$ kpsi, $b = 3.77$

$$\bar{y} = 64.1 + (81.0 - 64.1)\Gamma\left(1 + 1/3.77\right)$$
$$= 64.1 + 16.9\,\Gamma(1.27)$$
$$= 64.1 + 16.9(0.902\ 50) = 79.35 \text{ kpsi}$$
$$\sigma_y = (81 - 64.1)\left[\Gamma(1 + 2/3.77) - \Gamma\left(1 + 1/3.77\right)\right]^{1/2}$$
$$= 16.9\left[\left(0.887\ 57\right) - 0.902\ 50^2\right]^{1/2} = 4.57 \text{ kpsi}$$
$$p = 1 - \exp\left[-\left(\frac{y - y_0}{\theta - y_0}\right)^{3.77}\right]$$
$$= 1 - \exp\left[-\left(\frac{70 - 64.1}{81 - 64.1}\right)^{3.77}\right] = 0.019 \quad \textit{Ans.}$$

20-31 $\mathbf{x} = \mathbf{S}_{ut} = \mathbf{W}[122.3, 134.6, 3.64]$ kpsi, $p(x > 120) = 1 = 100\%$ since $x_0 > 120$ kpsi

$$p(x > 133) = \exp\left[-\left(\frac{133 - 122.3}{134.6 - 122.3}\right)^{3.64}\right]$$
$$= 0.548 = 54.8\% \quad \textit{Ans.}$$

20-32 Using Eqs. (20-28) and (20-29) and Table A-34,

$$\mu_n = n_0 + \left(\theta - n_0\right)\Gamma\left(1 + 1/b\right) = 36.9 + \left(133.6 - 36.9\right)\Gamma\left(1 + 1/2.66\right)$$
$$= 122.85 \text{ kcycles}$$
$$\hat{\sigma}_n = \left(\theta - n_0\right)\left[\Gamma\left(1 + 2/b\right) - \Gamma^2\left(1 + 1/b\right)\right] = 34.79 \text{ kcycles}$$

For the Weibull density function, Eq. (20-27),

$$f_W(n) = \frac{2.66}{133.6 - 36.9}\left(\frac{n - 36.9}{133.6 - 36.9}\right)^{2.66-1} \exp\left[-\left(\frac{n - 36.9}{133.6 - 36.9}\right)^{2.66}\right]$$

For the lognormal distribution, Eqs. (20-18) and (20-19) give,

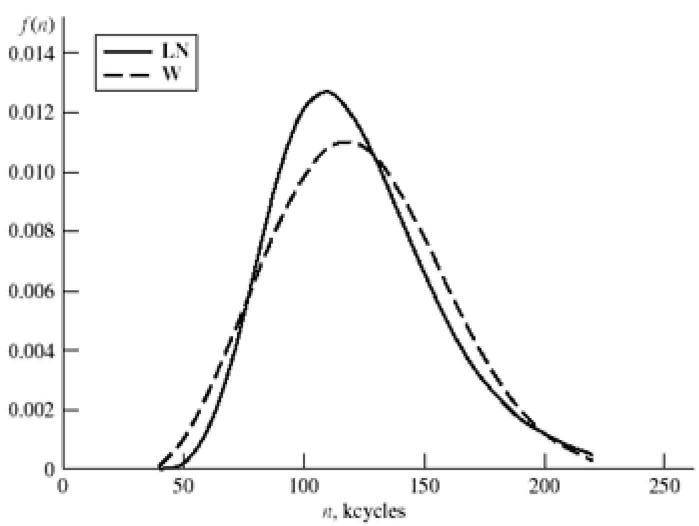
$$\mu_y = \ln\left(122.85\right) - \left(34.79/122.85\right)^2/2 = 4.771$$
$$\hat{\sigma}_y = \sqrt{\left[1 + \left(34.79/122.85\right)^2\right]} = 0.2778$$

From Eq. (20-17), the lognormal PDF is

$$f_{\mathrm{LN}}(n) = \frac{1}{0.2778n\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln n - 4.771}{0.2778} \right)^2 \right]$$

We form a table of densities $f_W(n)$ and $f_{LN}(n)$ and plot.

n(kcycles)	$f_W(n)$	$f_{LN}(n)$
40	9.1E-05	1.82E-05
50	0.000 991	0.000 241
60	0.002 498	0.001 233
70	0.004 380	0.003 501
80	0.006 401	0.006 739
90	0.008 301	0.009 913
100	0.009 822	0.012 022
110	0.010 750	0.012 644
120	0.010 965	0.011 947
130	0.010 459	0.010 399
140	0.009 346	0.008 492
150	0.007 827	0.006 597
160	0.006 139	0.004 926
170	0.004 507	0.003 564
180	0.003 092	0.002 515
190	0.001 979	0.001 739
200	0.001 180	0.001 184
210	0.000 654	0.000 795
220	0.000 336	0.000 529



The Weibull L10 life comes from Eq. (20-26) with reliability of $R = 0.90$. Thus,

$$n_{0.10} = 36.9 + (133.6 - 36.9)\left[\ln(1/0.90)\right]^{1/2.66} = 78.4 \text{ kcycles} \quad \textit{Ans.}$$

The lognormal L10 life comes from the definition of the z variable. That is,

$$\ln n_0 = \mu_y + \hat{\sigma}_y z \quad \text{or} \quad n_0 = \exp\left(\mu_y + \hat{\sigma}_y z\right)$$

From Table A-10, for $R = 0.90$, $z = -1.282$. Thus,

$$n_0 = \exp\left[4.771 + 0.2778\left(-1.282\right)\right] = 82.7 \text{ kcycles} \qquad \textit{Ans.}$$

20-33 Form a table

i	x $(10^{-5})L$	f_i	$f_i x \cdot (10^{-5})$	$f_i x^2 \cdot (10^{-10})$	$g(x) \cdot (10^5)$
1	3.05	3	9.15	27.9075	0.0557
2	3.55	7	24.85	88.2175	0.1474
3	4.05	11	44.55	180.4275	0.2514
4	4.55	16	72.80	331.24	0.3168
5	5.05	21	106.05	535.5525	0.3216
6	5.55	13	72.15	400.4325	0.2789
7	6.05	13	78.65	475.8325	0.2151
8	6.55	6	39.30	257.415	0.1517
9	7.05	2	14.10	99.405	0.1000
10	7.55	0	0	0	0.0625
11	8.05	4	32.20	259.21	0.0375
12	8.55	3	25.65	219.3075	0.0218
13	9.05	0	0	0	0.0124
14	9.55	0	0	0	0.0069
15	10.05	1	10.05	101.0025	0.0038
		100	529.50	2975.95	

$$\bar{x} = 529.5(10^5)/100 = 5.295(10^5) \text{ cycles} \quad \textit{Ans.}$$

$$s_x = \left[\frac{2975.95(10^{10}) - \left[529.5(10^5)\right]^2/100}{100-1}\right]^{1/2}$$

$$= 1.319(10^5) \text{ cycles} \quad \textit{Ans.}$$

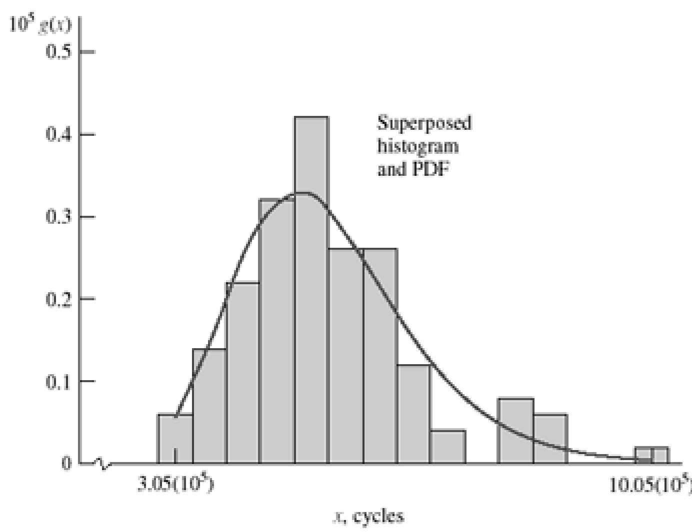
$$C_x = s/\bar{x} = 1.319/5.295 = 0.249$$

$$\mu_y = \ln 5.295(10^5) - 0.249^2/2 = 13.149$$

$$\hat{\sigma}_y = \sqrt{\ln(1+0.249^2)} = 0.245$$

$$g(x) = \frac{1}{x\hat{\sigma}_y\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_y}{\hat{\sigma}_y}\right)^2\right]$$

$$= \frac{1.628}{x}\exp\left[-\frac{1}{2}\left(\frac{\ln x - 13.149}{0.245}\right)^2\right]$$



20-34 $\mathbf{X} = \mathbf{S}_u = \mathbf{W}[70.3, 84.4, 2.01]$

$$\mu_x = 70.3 + (84.4 - 70.3)\Gamma(1 + 1/2.01)$$

Eq. (2-28): $= 70.3 + (84.4 - 70.3)\Gamma(1.498)$

$$= 82.8 \text{ kpsi} \quad \textit{Ans.}$$

$$\hat{\sigma}_x = (84.4 - 70.3)\left[\Gamma(1+2/2.01) - \Gamma^2(1+1/2.01)\right]^{1/2}$$

Eq. (2-29):

$$\hat{\sigma}_x = 14.1\left[0.997\ 91 - 0.886\ 17^2\right]^{1/2}$$

$$= 6.502 \text{ kpsi}$$

$$C_x = \frac{6.502}{82.8} = 0.079 \quad \textit{Ans.}$$

20-35 Take the Weibull equation for the standard deviation

$$\hat{\sigma}_x = \left(\theta - x_0\right)\left[\Gamma(1+2/b) - \Gamma^2(1+1/b)\right]^{1/2}$$

and the mean equation solved for $\bar{x} - x_0$

$$\bar{x} - x_0 = \left(\theta - x_0\right)\Gamma\left(1+1/b\right)$$

and divide the first by the second,

$$\frac{\hat{\sigma}_x}{\bar{x} - x_0} = \frac{\left[\Gamma\left(1+2/b\right) - \Gamma^2\left(1+1/b\right)\right]^{1/2}}{\Gamma\left(1+1/b\right)}$$

$$\frac{4.2}{49 - 33.8} = \sqrt{\frac{\Gamma\left(1+2/b\right)}{\Gamma^2\left(1+1/b\right)} - 1} = \sqrt{R} = 0.2763$$

Make a table and solve for *b* iteratively

b	1 + 2/*b*	1 + 1/*b*	$\Gamma\left(1+2/b\right)$	$\Gamma\left(1+1/b\right)$	$\sqrt{R}$
3	1.67	1.33	0.903 30	0.893 38	0.363
4	1.5	1.25	0.886 23	0.906 40	0.280
4.1	1.49	1.24	0.885 95	0.908 52	0.271

$b \doteq 4.068$ Using MathCad *Ans.*

$$\theta = x_0 + \frac{\bar{x} - x_0}{\Gamma(1+1/b)} = 33.8 + \frac{49 - 33.8}{\Gamma\left(1+1/4.068\right)}$$

$$= 49.8 \text{ kpsi} \quad \textit{Ans.}$$

20-36 **x** = $\mathbf{S}_y$ = **W**[34.7, 39, 2.93] kpsi

$$\bar{x} = 34.7 + (39 - 34.7)\Gamma(1 + 1/2.93) = 34.7 + 4.3\Gamma(1.34)$$
$$= 34.7 + 4.3(0.892\ 22) = 38.5 \text{ kpsi}$$
$$\hat{\sigma}_x = (39 - 34.7)\left[\Gamma(1 + 2/2.93) - \Gamma^2(1 + 1/2.93)\right]^{1/2}$$
$$= 4.3\left[\Gamma(1.68) - \Gamma^2(1.34)\right]^{1/2}$$
$$= 4.3\left[0.905\ 00 - 0.892\ 22^2\right]^{1/2} = 1.42 \text{ kpsi} \quad \textit{Ans.}$$
$$C_x = 1.42/38.5 = 0.037 \quad \textit{Ans.}$$

20-37

	x (Mrev)	f	fx	fx^2
	1	11	11	11
	2	22	44	88
	3	38	114	342
	4	57	228	912
	5	31	155	775
	6	19	114	684
	7	15	105	735
	8	12	96	768
	9	11	99	891
	10	9	90	900
	11	7	77	847
	12	5	60	720
Sum	78	237	1193	7673

$$\mu_x = 1193(10^6)/237 = 5.034(10^6) \text{ cycles}$$
$$\hat{\sigma}_x = \sqrt{\frac{7673(10^{12}) - \left[1193(10^6)\right]^2/237}{237 - 1}} = 2.658(10^6) \text{ cycles}$$
$$C_x = 2.658/5.034 = 0.528$$

From Eqs. (20-18) and (20-19),

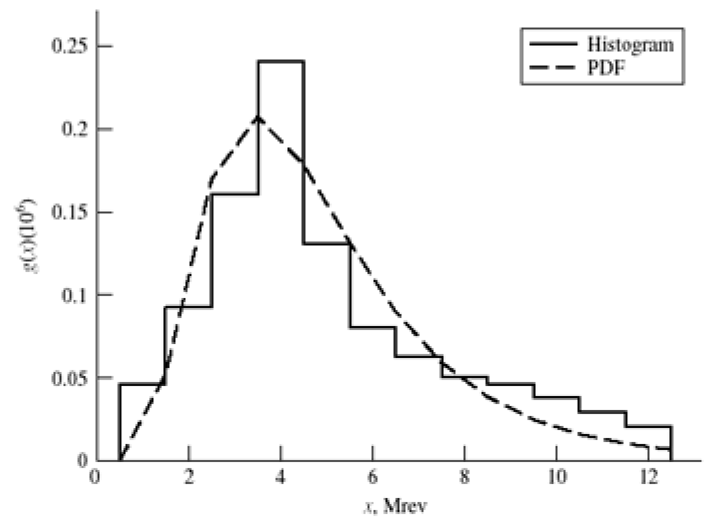
$$\mu_y = \ln\left[5.034(10^6)\right] - 0.528^2/2 = 15.292$$
$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.528^2)} = 0.496$$

From Eq. (20-17), defining $g(x)$,

$$g(x) = \frac{1}{x(0.496)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 15.292}{0.496}\right)^2\right]$$

x (Mrev)	$f/(Nw)$	$g(x)\cdot(10^6)$
0.5	0.000 00	0.000 11

0.5	0.046414	0.000 11
1.5	0.046414	0.052 03
1.5	0.092827	0.052 03
2.5	0.092827	0.169 92
2.5	0.160338	0.169 92
3.5	0.160338	0.207 54
3.5	0.240506	0.207 54
4.5	0.240506	0.178 47
4.5	0.130802	0.178 47
5.5	0.130802	0.131 58
5.5	0.080 17	0.13158
6.5	0.080 17	0.090 11
6.5	0.063 29	0.090 11
7.5	0.063 29	0.059 53
7.5	0.050 63	0.059 53
8.5	0.050 63	0.038 69
8.5	0.046 41	0.038 69
9.5	0.046 41	0.025 01
9.5	0.037 97	0.025 01
10.5	0.037 97	0.016 18
10.5	0.029 54	0.016 18
11.5	0.029 54	0.010 51
11.5	0.021 10	0.010 51
12.5	0.021 10	0.006 87
12.5	0.000 00	0.006 87



$$z = \frac{\ln x - \mu_y}{\hat{\sigma}_y} \Rightarrow \ln x = \mu_y + \hat{\sigma}_y z = 15.292 + 0.496z$$

L_{10} life, where 10% of bearings fail, from Table A-10, $z = -1.282$. Thus,

$$\ln x = 15.292 + 0.496(-1.282) = 14.66$$
$$\therefore\ x = 2.33\ (10^6)\ \text{rev} \quad \textit{Ans.}$$