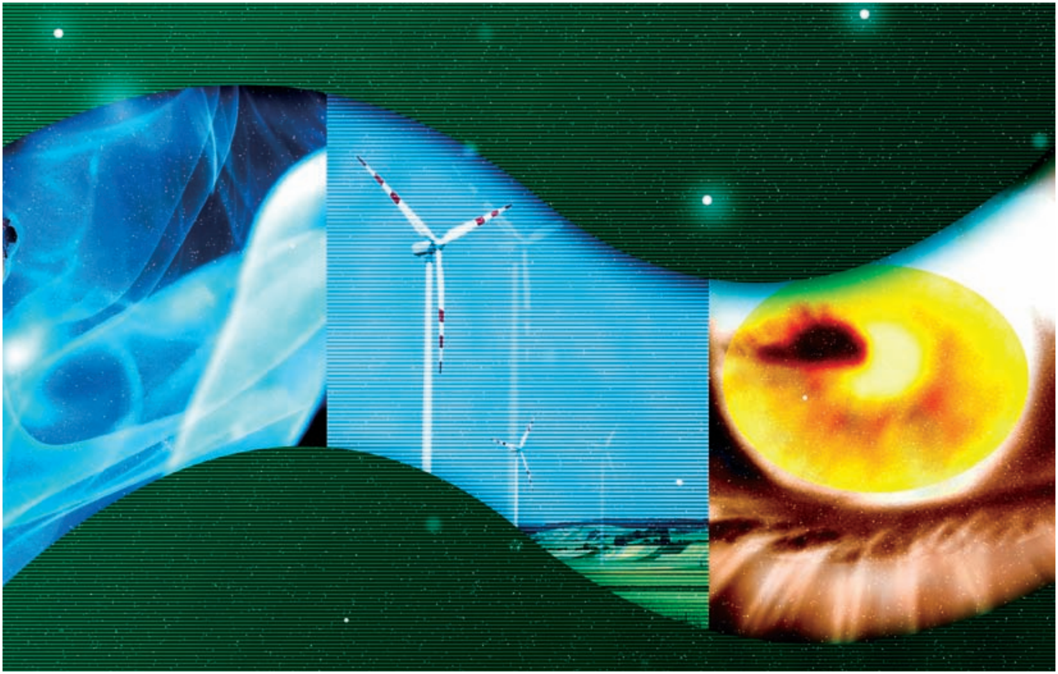
ENGINEERING THERMODYNAMICS

THIRD EDITION

SI Units Version

R. K. Rajput



ENGINEERING SERIES

ENGINEERING THERMODYNAMICS

ENGINEERING THERMODYNAMICS

[For Engineering Students of All Indian Universities and Competitive Examinations]

S.I. UNITS

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Preface to The Third Edition

I am pleased to present the **third edition** of this book. The warm reception which the previous editions and reprints of this book have enjoyed all over India and abroad has been a matter of great satisfaction to me.

The entire book has been thoroughly revised ; a large number of solved examples (questions having been selected from various universities and competitive examinations) and ample additional text have been added.

Any suggestions for the improvement of the book will be thankfully acknowledged and incorporated in the next edition.

—Author

Preface to The First Edition

Several books are available in the market on the subject of "Engineering Thermodynamics" but either they are too bulky or are miserly written and as such do not cover the syllabii of various Indian Universities effectively. Hence a book is needed which should assimilate subject matter that should primarily satisfy the requirements of the students from syllabus/examination point of view ; these requirements are completely met by this book.

The book entails the following *features* :

— The presentation of the subject matter is very systematic and language of the text is quite lucid and simple to understand.

— A number of figures have been added in each chapter to make the subject matter self speaking to a great extent.

— A large number of properly graded examples have been added in various chapters to enable the students to attempt different types of questions in the examination without any difficulty.

— Highlights, objective type questions, theoretical questions, and unsolved examples have been added at the end of each chapter to make the book a complete unit in all respects.

The author's thanks are due to his wife Ramesh Rajput for rendering all assistance during preparation and proof reading of the book. The author is thankful to Mr. R.K. Syal for drawing beautiful and well proportioned figures for the book.

The author is grateful to M/s Laxmi Publications for taking lot of pains in bringing out the book in time and pricing it moderately inspite of heavy cost of the printing.

Constructive criticism is most welcome from the readers.

—Author

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Introduction to SI Units and Conversion Factors

A. INTRODUCTION TO SI UNITS

SI, the international system of units are divided into three classes :

1. Base units
2. Derived units
3. Supplementary units.

From the scientific point of view division of SI units into these classes is to a certain extent arbitrary, because it is not essential to the physics of the subject. Nevertheless the General Conference, considering the advantages of a single, practical, world-wide system for international relations, for teaching and for scientific work, decided to base the international system on a choice of six well-defined units given in Table 1 below :

Table 1. SI Base Units

Quantity	*Name*	*Symbol*
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
luminous intensity	candela	cd
amount of substance	mole	mol

The second class of SI units contains derived units, *i.e.,* units which can be formed by combining base units according to the algebraic relations linking the corresponding quantities. Several of these algebraic expressions in terms of base units can be replaced by special names and symbols can themselves be used to form other derived units.

Derived units may, therefore, be classified under three headings. Some of them are given in Tables 2, 3 and 4.

Table 2. Examples of SI Derived Units Expressed in terms of Base Units

Quantity	*SI Units*	
	Name	*Symbol*
area	square metre	m^2
volume	cubic metre	m^3
speed, velocity	metre per second	m/s
acceleration	metre per second squared	m/s^2
wave number	1 per metre	m^{-1}
density, mass density	kilogram per cubic metre	kg/m^3
concentration (of amount of substance)	mole per cubic metre	mol/m^3
activity (radioactive)	1 per second	s^{-1}
specific volume	cubic metre per kilogram	$\text{m}^3\text{/kg}$
luminance	candela per square metre	cd/m^2

Table 3. SI Derived Units with Special Names

Quantity	*SI Units*			
	Name	*Symbol*	*Expression in terms of other units*	*Expression in terms of SI base units*
frequency	hertz	Hz	—	s^{-1}
force	newton	N	—	m.kg.s^{-2}
pressure	pascal	Pa	N/m^2	$\text{m}^{-1}\text{.kg.s}^{-2}$
energy, work, quantity of heat power	joule	J	N.m	$\text{m}^2\text{.kg.s}^{-2}$
radiant flux quantity of electricity	watt	W	J/S	$\text{m}^2\text{.kg.s}^{-3}$
electric charge	coloumb	C	A.s	s.A
electric tension, electric potential	volt	V	W/A	$\text{m}^2\text{.kg.s}^{-3}\text{.A}^{-1}$
capacitance	farad	F	C/V	$\text{m}^{-2}\text{.kg}^{-1}\text{.s}^4$
electric resistance	ohm	Ω	V/A	$\text{m}^2\text{.kg.s}^{-3}\text{.A}^{-2}$
conductance	siemens	S	A/V	$\text{m}^{-2}\text{.kg}^{-1}\text{.s}^3\text{.A}^2$
magnetic flux	weber	Wb	V.S.	$\text{m}^2\text{.kg.s}^{-2}\text{.A}^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$\text{kg.s}^{-2}\text{.A}^{-1}$
inductance	henry	H	Wb/A	$\text{m}^2\text{.kg.s}^{-2}\text{.A}^{-2}$
luminous flux	lumen	lm	—	cd.sr
illuminance	lux	lx	—	$\text{m}^{-2}\text{.cd.sr}$

Table 4. Examples of SI Derived Units Expressed by means of Special Names

Quantity	*SI Units*		
	Name	*Symbol*	*Expression in terms of SI base units*
dynamic viscosity	pascal second	Pa-s	$m^{-1}.kg.s^{-1}$
moment of force	metre newton	N.m	$m^{2}.kg.s^{-2}$
surface tension	newton per metre	N/m	$kg.s^{-2}$
heat flux density, irradiance	watt per square metre	W/m^{2}	$kg.s^{-2}$
heat capacity, entropy	joule per kelvin	J/K	$m^{2}.kg.s^{-2}.K^{-1}$
specific heat capacity, specific entropy	joule per kilogram kelvin	J/(kg.K)	$m^{2}.s^{-2}.K^{-1}$
specific energy	joule per kilogram	J/kg	$m^{2}.s^{-2}$
thermal conductivity	watt per metre kelvin	W/(m.K)	$m.kg.s^{-3}.K^{-1}$
energy density	joule per cubic metre	J/m^{3}	$m^{-1}.kg.s^{-2}$
electric field strength	volt per metre	V/m	$m.kg.s^{-3}.A^{-1}$
electric charge density	coloumb per cubic metre	C/m^{3}	$m^{-3}.s.A$
electric flux density	coloumb per square metre	C/m^{2}	$m^{-2}.s.A$
permitivity	farad per metre	F/m	$m^{-3}.kg^{-1}.s^{4}.A^{4}$
current density	ampere per square metre	A/m^{2}	—
magnetic field strength	ampere per metre	A/m	—
permeability	henry per metre	H/m	$m.kg.s^{-2}.A^{-2}$
molar energy	joule per mole	J/mol	$m^{2}.kg.s^{-2}mol^{-1}$
molar heat capacity	joule per mole kelvin	J/(mol.K)	$m^{2}.kg.s^{-2}.K^{-1}.mol^{-1}$

The SI units assigned to third class called "Supplementary units" may be regarded either as base units or as derived units. Refer Table 5 and Table 6.

Table 5. SI Supplementary Units

Quantity	*SI Units*	
	Name	*Symbol*
plane angle	radian	rad
solid angle	steradian	sr

Table 6. Examples of SI Derived Units Formed by Using Supplementary Units

Quantity	*SI Units*	
	Name	*Symbol*
angular velocity	radian per second	rad/s
angular acceleration	radian per second squared	rad/s^2
radiant intensity	watt per steradian	W/sr
radiance	watt per square metre steradian	W-m^{-2}.sr^{-1}

Table 7. SI Prefixes

Factor	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*
10^{12}	tera	T	10^{-1}	deci	d
10^{9}	giga	G	10^{-2}	centi	c
10^{6}	mega	M	10^{-3}	milli	m
10^{3}	kilo	k	10^{-6}	micro	µ
10^{2}	hecto	h	10^{-9}	nano	n
10^{1}	deca	da	10^{-12}	pico	p
			10^{-15}	fasnto	f
			10^{-18}	atto	a

B. CONVERSION FACTORS

1. Force :

$$1 \text{ newton} = \text{kg-m/sec}^2 = 0.012 \text{ kgf}$$
$$1 \text{ kgf} = 9.81 \text{ N}$$

2. Pressure :

$$1 \text{ bar} = 750.06 \text{ mm Hg} = 0.9869 \text{ atm} = 10^5 \text{ N/m}^2 = 10^3 \text{ kg/m-sec}^2$$
$$1 \text{ N/m}^2 = 1 \text{ pascal} = 10^{-5} \text{ bar} = 10^{-2} \text{ kg/m-sec}^2$$
$$1 \text{ atm} = 760 \text{ mm Hg} = 1.03 \text{ kgf/cm}^2 = 1.01325 \text{ bar}$$
$$= 1.01325 \times 10^5 \text{ N/m}^2$$

3. Work, Energy or Heat :

$$1 \text{ joule} = 1 \text{ newton metre} = 1 \text{ watt-sec}$$
$$= 2.7778 \times 10^{-7} \text{ kWh} = 0.239 \text{ cal}$$
$$= 0.239 \times 10^{-3} \text{ kcal}$$
$$1 \text{ cal} = 4.184 \text{ joule} = 1.1622 \times 10^{-6} \text{ kWh}$$
$$1 \text{ kcal} = 4.184 \times 10^3 \text{ joule} = 427 \text{ kgf-m}$$
$$= 1.1622 \times 10^{-3} \text{ kWh}$$
$$1 \text{ kWh} = 8.6042 \times 10^5 \text{ cal} = 860 \text{ kcal} = 3.6 \times 10^6 \text{ joule}$$
$$1 \text{ kgf-m} = \left(\frac{1}{427}\right) \text{ kcal} = 9.81 \text{ joules}$$

4. Power :

1 watt = 1 joule/sec = 0.860 kcal/h

1 h.p. = 75 m kgf/sec = 0.1757 kcal/sec = 735.3 watt

1 kW = 1000 watts = 860 kcal/h

5. Specific heat :

1 kcal/kg-°K = 0.4184 joules/kg-K

6. Thermal conductivity :

1 watt/m-K = 0.8598 kcal/h-m-°C

1 kcal/h-m-°C = 1.16123 watt/m-K = 1.16123 joules/s-m-K.

7. Heat transfer co-efficient :

1 watt/m^2-K = 0.86 kcal/m^2-h-°C

1 kcal/m^2-h-°C = 1.163 watt/m^2-K.

C. IMPORTANT ENGINEERING CONSTANTS AND EXPRESSIONS

Engineering constants and expressions	*M.K.S. system*	*SI Units*
1. Value of g_0	9.81 kg-m/kgf-sec^2	1 kg-m/N-sec^2
2. Universal gas constant	848 kgf-m/kg mole-°K	848 × 9.81 = 8314 J/kg-mole-°K ($\because$ 1 kgf-m = 9.81 joules)
3. Gas constant (R)	29.27 kgf-m/kg-°K for air	$\frac{8314}{29}$ = 287 joules/kg-K for air
4. Specific heats (for air)	c_v = 0.17 kcal/kg-°K c_p = 0.24 kcal/kg-°K	c_v = 0.17 × 4.184 = 0.71128 kJ/kg-K c_p = 0.24 × 4.184 = 1 kJ/kg-K
5. Flow through nozzle-Exit velocity (C_2)	$91.5\sqrt{U}$, where U is in kcal	$44.7\sqrt{U}$, where U is in kJ
6. Refrigeration 1 ton	= 50 kcal/min	= 210 kJ/min
7. **Heat transfer** The Stefan Boltzman Law is given by :	$Q = \sigma T^4$ kcal/m^2-h when $\sigma = 4.9 \times 10^{-8}$ kcal/h-m^2-°K^4	$Q = \sigma T^4$ watts/m^2-h when $\sigma = 5.67 \times 10^{-8}$ W/m^2K^4

Nomenclature

A	area
b	steady-flow availability function
C	velocity
$°C$	temperature on the celsius (or centigrade) scale
c	specific heat
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C_p	molar heat at constant pressure
C_v	molar heat at constant volume
D, d	bore ; diameter
E	emissive power ; total energy
e	base of natural logarithms
g	gravitational acceleration
H	enthalpy
h	specific enthalpy ; heat transfer co-efficient
h_f	specific enthalpy of saturated liquid (fluid)
h_{fg}	latent heat
h_g	specific enthalpy of saturated vapour ; gases
K	temperature on kelvin scale (*i.e.,* celsius absolute, compressibility)
k	thermal conductivity, blade velocity co-efficient
L	stroke
M	molecular weight
m	mass
$\dot{m}$	rate of mass flow
N	rotational speed
n	polytropic index, number of moles ; number of cylinders
P	power
p	absolute pressure
p_m	mean effective pressure
p_i	indicated mean effective pressure
p_b	brake mean effective pressure, back pressure

Q	heat, rate of heat transfer
q	rate of heat transfer per unit area
R	gas constant ; thermal resistance ; radius ; total expansion ratio in compound steam engines
R_0	universal gas constant
r	radius, expansion ratio, compression ratio
S	entropy
s	specific entropy
T	absolute temperature ; torque
t	temperature
U	internal energy ; overall heat transfer co-efficient
u	specific internal energy
V	volume
v	specific volume
W	work ; rate of work transfer ; brake load ; weight
w	specific weight ; velocity of whirl
x	dryness fraction ; length

Greek Symbols

α	absorptivity
γ	ratio of specific heats, c_p/c_v
$\in$	emissivity ; effectiveness
η	efficiency
θ	temperature difference, angle
ρ	density
σ	Stefan-Boltzmann constant
ϕ	relative humidity, angle.

1

Introduction—Outline of Some Descriptive Systems

1.1. STEAM POWER PLANT

1.1.1. Layout

Refer to Fig. 1.1. The layout of a modern steam power plant comprises of the following four circuits :

1. Coal and ash circuit.
2. Air and gas circuit.
3. Feed water and steam flow circuit.
4. Cooling water circuit.

Coal and Ash Circuit. Coal arrives at the storage yard and after necessary handling, passes on to the furnaces through the *fuel feeding device*. Ash resulting from combustion of coal collects at the back of the boiler and is removed to the ash storage yard through *ash handling equipment*.

Air and Gas Circuit. Air is taken in from atmosphere through the action of a forced or induced draught fan and passes on to the furnace through the *air preheater*, where it has been heated by the heat of flue gases which pass to the chimney *via* the preheater. The flue gases after passing around boiler tubes and superheater tubes in the furnace pass through a *dust* catching device or precipitator, then through the economiser, and finally through the air preheater before being exhausted to the atmosphere.

Feed Water and Steam Flow Circuit. In the water and steam circuit condensate leaving the condenser is first heated in a closed feed water heater through extracted steam from the lowest pressure extraction point of the turbine. It then passes through the *deaerator* and a few more water heaters before going into the boiler through *economiser*.

In the boiler drum and tubes, water circulates due to the difference between the density of water in the lower temperature and the higher temperature sections of the boiler. Wet steam from the drum is further heated up in the superheater for being supplied to the primemover. After expanding in high pressure turbine steam is taken to the reheat boiler and brought to its original dryness or superheat before being passed on to the low pressure turbine. From there it is exhausted through the condenser into the hot well. The condensate is heated in the feed heaters using the steam trapped (blow steam) from different points of turbine.

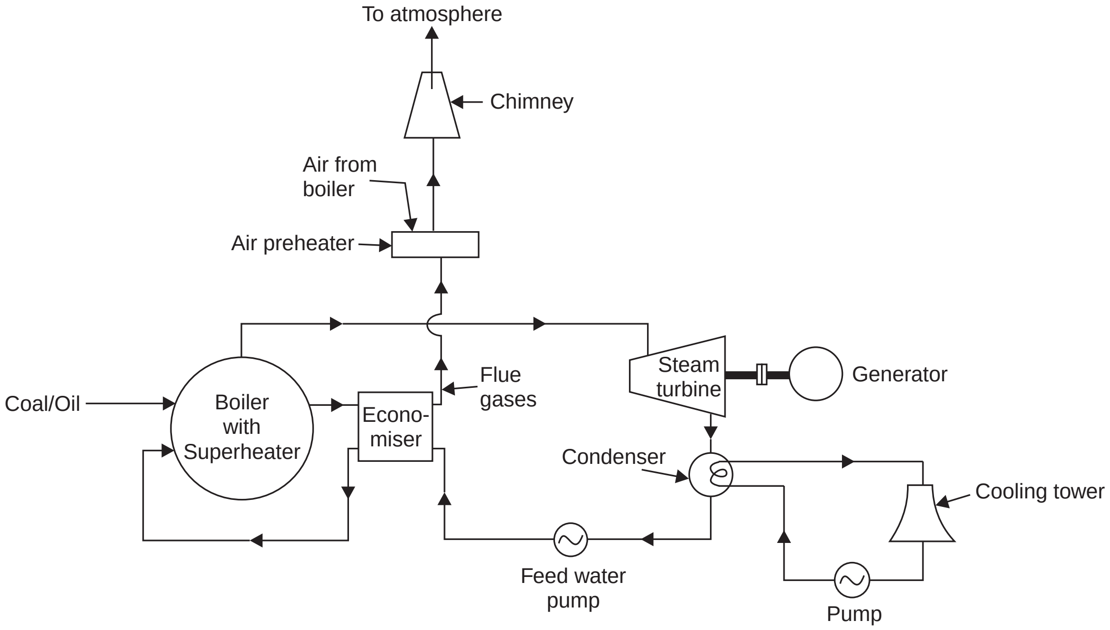


Fig. 1.1. Layout of a steam power plant.

A part of steam and water is lost while passing through different components and this is compensated by supplying additional feed water. This feed water should be purified before hand, to avoid the scaling of the tubes of the boiler.

Cooling Water Circuit. The cooling water supply to the condenser helps in maintaining a low pressure in it. The water may be taken from a natural source such as river, lake or sea or the same water may be cooled and circulated over again. In the latter case the cooling arrangement is made through spray pond or cooling tower.

1.1.2. Components of a Modern Steam Power Plant

A modern steam power plant comprises of the following *components* :

1. Boiler
   (*i*) Superheater
   (*ii*) Reheater
   (*iii*) Economiser
   (*iv*) Air-heater.
2. Steam turbine
3. Generator
4. Condenser
5. Cooling towers
6. Circulating water pump
7. Boiler feed pump
8. Wagon tippler
9. Crusher house
10. Coal mill
11. Induced draught fans
12. Ash precipitators
13. Boiler chimney
14. Forced draught fans
15. Water treatment plant
16. Control room
17. Switch yard.

Functions of some important parts of a steam power plant :

1. ***Boiler.*** Water is converted into wet steam.
2. ***Superheater.*** It converts wet steam into superheated steam.
3. ***Turbine.*** Steam at high pressure expands in the turbine and drives the generator.

4. ***Condenser.*** It condenses steam used by the steam turbine. The condensed steam (known as *condensate*) is used as a feed water.

5. ***Cooling tower.*** It cools the condenser circulating water. Condenser cooling water absorbs heat from steam. This heat is discharged to atmosphere in cooling water.

6. ***Condenser circulating water pump.*** It circulates water through the condenser and the cooling tower.

7. ***Feed water pump.*** It pumps water in the water tubes of boiler against boiler steam pressure.

8. ***Economiser.*** In economiser heat in flue gases is partially used to heat incoming feed water.

9. ***Air preheater.*** In air preheater heat in flue gases (the products of combustion) is partially used to heat incoming air.

1.2. NUCLEAR POWER PLANT

Fig. 1.2 shows schematically a *nuclear power plant.*

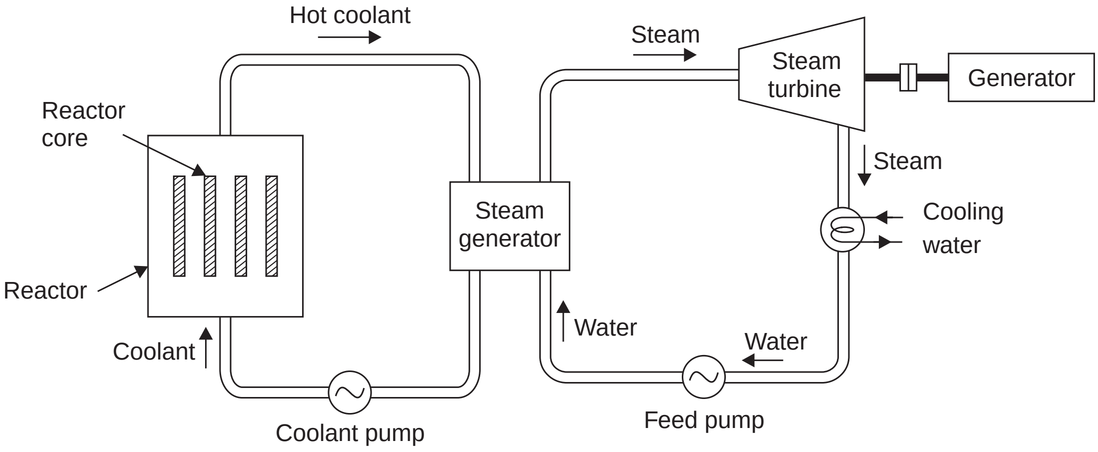


Fig. 1.2. Nuclear power plant.

The main components of a nuclear power plant are :

1. Nuclear reactor
2. Heat exchanger (steam generator)
3. Steam turbine
4. Condenser
5. Electric generator.

In a nuclear power plant the reactor performs the same function as that of the furnace of steam power plant (*i.e.*, produces heat). The heat liberated in the reactor as a result of the nuclear fission of the fuel is taken up by the coolants circulating through the reactor core. Hot coolant leaves the reactor at the top and then flows through the tubes of steam generator and passes on its heat to the feed water. The steam so produced expands in the steam turbine, producing work, and thereafter is condensed in the condenser. The steam turbine in turn runs an electric generator thereby producing electrical energy. In order to maintain the flow of coolant, condensate and feed water pumps are provided as shown in Fig. 1.2.

1.3. INTERNAL COMBUSTION ENGINES

1.3.1. Heat Engines

Any type of engine or machine which derives heat energy from the combustion of fuel or any other source and converts this energy into mechanical work is termed as a **heat engine.**

Heat engines may be *classified* into two main classes as follows :

1. External Combustion Engine.
2. Internal Combustion Engine.

1. **External Combustion Engines** (*E.C. Engines*)

In this case, combustion of fuel takes place outside the cylinder as in case of steam engines where the heat of combustion is employed to generate steam which is used to move a piston in a cylinder. Other examples of external combustion engines are *hot air engines, steam turbine* and *closed cycle gas turbine*. These engines are generally needed for driving locomotives, ships, generation of electric power etc.

2. **Internal Combustion Engines** (*I.C. Engines*)

In this case combustion of the fuel with oxygen of the air occurs within the cylinder of the engine. The internal combustion engines group includes engines employing mixtures of combustible gases and air, known as gas engines, those using lighter liquid fuel or spirit known as *petrol engines* and those using heavier liquid fuels, known as oil *compression ignition or diesel engines*.

1.3.2. Development of I.C. Engines

Many experimental engines were constructed around 1878. The first really successful engine did not appear, however until 1879, when a German engineer Dr. Otto built his famous Otto gas engine. The operating cycle of this engine was based upon principles first laid down in 1860 by a French engineer named Bea de Rochas. The majority of modern I.C. engines operate according to these principles.

The development of the well known Diesel engine began about 1883 by Rudoff Diesel. Although this differs in many important respects from the otto engine, the operating cycle of modern high speed Diesel engines is thermodynamically very similar to the Otto cycle.

1.3.3. Different parts of I.C. Engines

A cross-section of an air-cooled I.C. engines with principal parts is shown in Fig. 1.3.

A. Parts common to both petrol and diesel engines

1. Cylinder
2. Cylinder head
3. Piston
4. Piston rings
5. Gudgeon pin
6. Connecting rod
7. Crankshaft
8. Crank
9. Engine bearing
10. Crank case
11. Flywheel
12. Governor
13. Valves and valve operating mechanism.

B. Parts for petrol engines only

1. Spark plugs
2. Carburettor
3. Fuel pump.

C. Parts for Diesel engine only

1. Fuel pump.
2. Injector.

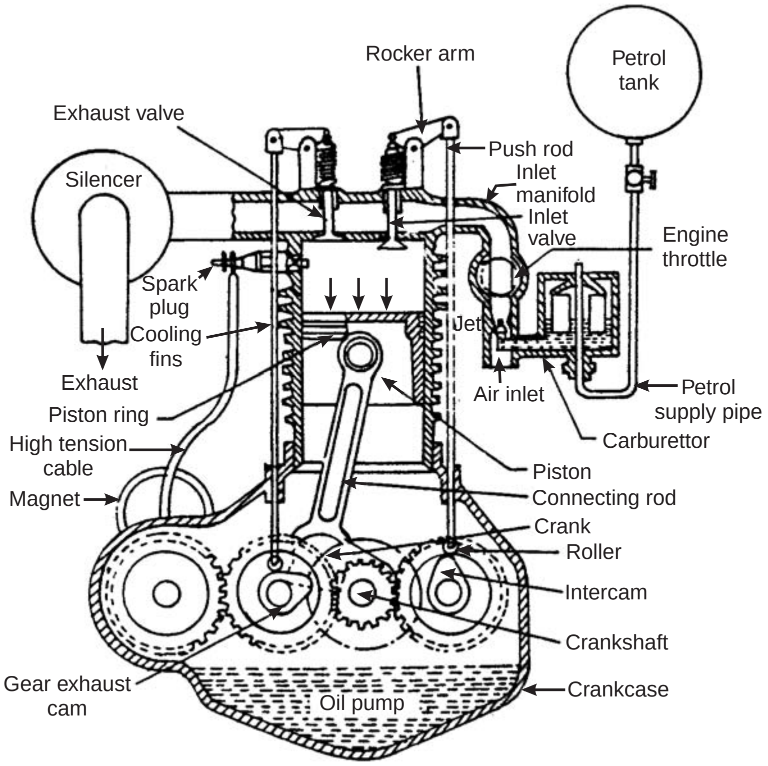


Fig. 1.3. An air-cooled four-stroke petrol engine.

1.3.4. Spark Ignition (S.I.) Engines

These engines may work on either *four stroke cycle* or *two stroke cycle*, majority of them, of course, operate on four stroke cycle.

Four stroke petrol engine :

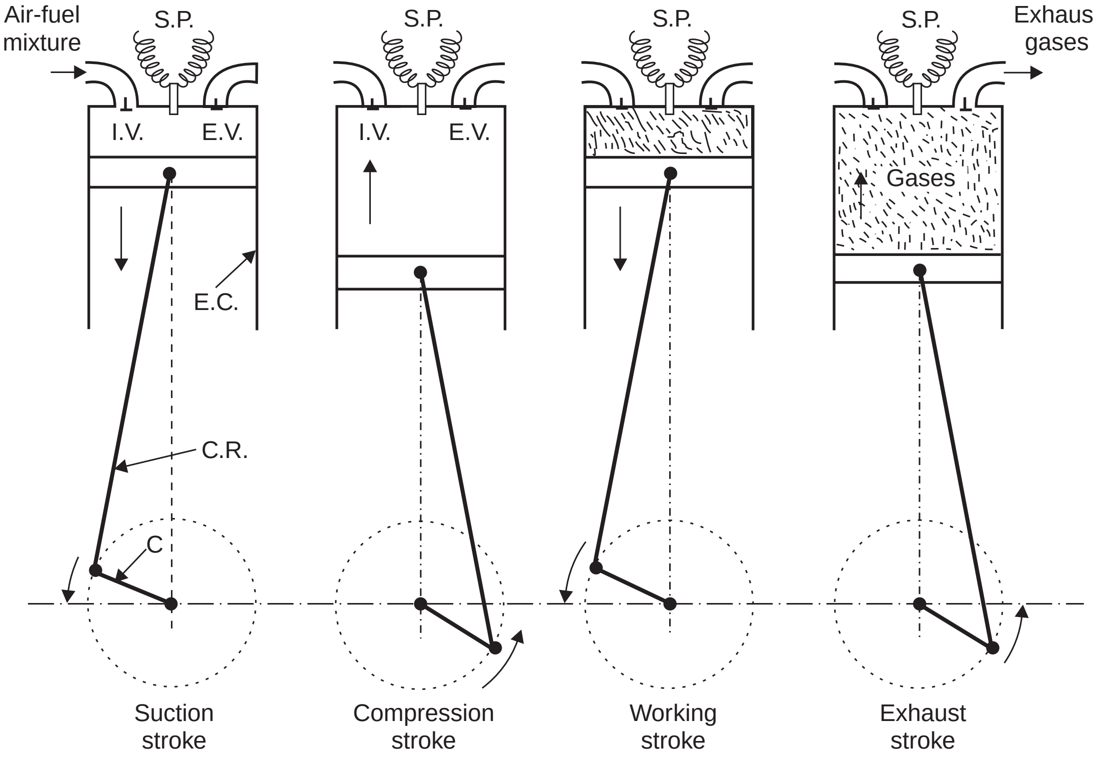
Fig. 1.4 illustrates the various strokes/series of operations which take place in a four stroke petrol (Otto cycle) engine.

Suction stroke. During *suction stroke* a mixture of air and fuel (petrol) is sucked through the inlet valve (I.V.). The exhaust valve remains closed during this operation.

Compression stroke. During *compression stroke*, both the valves remain closed, and the pressure and temperature of the mixture increase. Near the end of compression stroke, the fuel is ignited by means of an electric spark in the spark plug, causing combustion of fuel at the instant of ignition.

Working stroke. Next is the *working* (also called *power* or *expansion*) stroke. During this stroke, both the valves remain closed. Near the end of the expansion stroke, only the exhaust valve opens and the pressure in the cylinder at this stage forces most of the gases to leave the cylinder.

Exhaust stroke. Next follows the exhaust stroke, when all the remaining gases are driven away from the cylinder, while the inlet valve remains closed and the piston returns to the *top dead centre*. The cycle is then repeated.



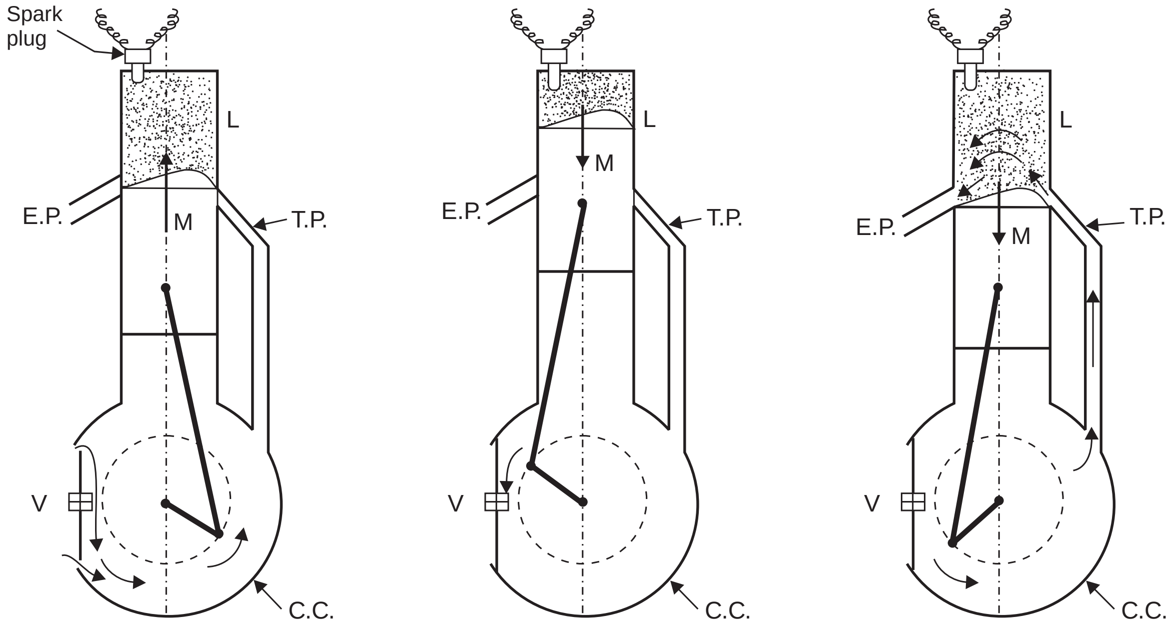
I.V = Intel valve, E.V. = Exhaust valve, E.C. = Engine cylinder,
C.R. = Connecting rod, C = Crank, S.P. = Spark plug.

Fig. 1.4. Four stroke otto cycle engine.

Two stroke petrol engine :

In 1878, Dugald-clerk, a British engineer introduced a cycle which could be completed in two strokes of piston rather than four strokes as is the case with the four stroke cycle engines. The engines using this cycle were called two stroke cycle engines. In this engine suction and exhaust strokes are eliminated. Here *instead of valves, ports are used. The exhaust gases are driven out from engine cylinder by the fresh change of fuel entering the cylinder nearly at the end of the working stroke*.

Fig. 1.5 shows a two stroke petrol engine (used in scooters, motor cycles etc.). The *cylinder L* is connected to a closed *crank chamber* C.C. During the upward stroke of the piston *M*, the gases in *L* are compressed and at the same time fresh air and fuel (petrol) mixture enters the crank chamber through the valve *V*. When the piston moves downwards, *V* closes and the mixture in the crank chamber is compressed. Refer Fig. 1.5 (*i*) the piston is moving upwards and is compressing an explosive change which has previously been supplied to *L*. Ignition takes place at the end of the stroke. The piston then travels downwards due to expansion of the gases [Fig. 1.5 (*ii*)] and near the end of this stroke the piston uncovers the *exhaust port* (E.P.) and the burnt exhaust gases escape through this port [Fig. 1.5 (*iii*)]. The *transfer port* (T.P.) then is uncovered immediately, and the compressed charge from the crank chamber flows into the cylinder and is deflected upwards by the hump provided on the head of the piston. It may be noted that the incoming air petrol mixture helps the removal of gases from the engine-cylinder ; if, in case these exhaust gases do not leave the cylinder, the fresh charge gets diluted and efficiency of the engine will decrease. The piston then again starts moving from bottom dead centre (B.D.C.) to top dead centre (T.D.C.) and the charge gets compressed when E.P. (exhaust port) and T.P. are covered by the piston ; thus the cycle is repeated.



(*i*) (*ii*) (*iii*)

L = Cylinder ; E.P. = Exhaust port ; T.P. = Transfer port ; V = Valve ; C.C. = Crank chamber

(*i*) (*ii*) (*iii*)

Fig. 1.5. Two-stroke petrol engine.

The power obtained from a two-stroke cycle engine is *theoretically twice* the power obtainable from a four-stroke cycle engine.

1.3.5. Compression Ignition (C.I.) Engines

The operation of C.I. engines (or diesel engines) is practically the same as those of S.I. engines. The cycle in both the types, consists of *suction, compression, ignition, expansion* and *exhaust*. However, the combustion process in a C.I. engine is different from that of a S.I. engine as given below :

In C.I. engine, *only air* is sucked during the stroke and the fuel is injected in the cylinder near the end of the compression stroke. Since the compression ratio is very high (between 14 : 1 to 22 : 1), the temperature of the air after compression is quite high. So when fuel is injected in the form of a spray at this stage, it ignites and burns almost as soon as it is introduced. The burnt gases are expanded and exhausted in the same way as is done in a S.I. engine.

1.4. GAS TURBINES

1.4.1. General Aspects

Probably a wind-mill was the first turbine to produce useful work, wherein there is no precompression and no combustion. The characteristic features of a gas turbine as we think of the name today include a *compression process* and an *heat addition* (or combustion) *process*. The gas turbine represents perhaps the most satisfactory way of producing very large quantities of power in a self-contained and compact unit. The gas turbine may have a future use in conjunction with the oil engine. For smaller gas turbine units, the inefficiencies in compression and expansion processes become greater and *to* improve the thermal efficiency it is necessary to use a heat exchanger. In order that a small gas turbine may compete for economy with the small oil engine or petrol engine it is necessary that a compact effective heat exchanger be used in the gas turbine cycle. The thermal efficiency of the gas turbine alone is still quite modest 20 to 30% compared with that of a modern steam turbine plant 38 to 40%. It is possible to construct combined plants whose efficiencies are of order of 45% or more. Higher efficiencies might be attained in future.

The following *are* the major fields of application of gas turbines :

1. Aviation
2. Power generation
3. Oil and gas industry
4. Marine propulsion.

The efficiency of a gas turbine is not the criteria for the choice of this plant. A gas turbine is used in aviation and marine fields because it is *self-contained, light weight, not requiring cooling water and generally fits into the overall shape of the structure.* It is selected for power generation because of its *simplicity, lack of cooling water, needs quick installation and quick starting*. It is used in oil and gas industry because of *cheaper supply of fuel and low installation cost.*

The gas turbines have the following limitations : (*i*) *They are not self-starting* ; (*ii*) *Low efficiencies at part loads* ; (*iii*) *Non-reversibility* ; (*iv*) *Higher rotor speeds ; and* (*v*) *Overall efficiency of the plant is low*.

1.4.2. Classification of Gas Turbines

The gas turbines are mainly *divided* into two groups :

1. **Constant pressure combustion gas turbine :**

(*a*) Open cycle constant pressure gas turbine

(*b*) Closed cycle constant pressure gas turbine.

2. **Constant volume combustion gas turbine.**

In almost *all the fields open cycle gas turbine plants are used. Closed cycle plants were introduced at one stage because of their ability to burn cheap fuel*. In between their progress remained slow because of availability of cheap oil and natural gas. Because of rising oil prices, now again, the attention is being paid to closed cycle plants.

1.4.3. Merits and Demerits of Gas Turbines

Merits over I.C. engines :

1. The mechanical efficiency of a gas turbine (95%) is quite high as compared with I.C. engine (85%) since the I.C. engine has a large many sliding parts.
2. A gas turbine does not require a flywheel as the torque on the shaft is continuous and uniform. Whereas a flywheel is a must in case of an I.C. engine.
3. The weight of gas turbine per H.P. developed is less than that of an I.C. engine.
4. The gas turbine can be driven at a very high speeds (40,000 r.p.m.) whereas this is not possible with I.C. engines.
5. The work developed by a gas turbine per kg of air is more as compared to an I.C. engine. This is due to the fact that gases can be expanded upto atmospheric pressure in case of a gas turbine whereas in an I.C. engine expansion upto atmospheric pressure is not possible.

6. The components of the gas turbine can be made lighter since the pressures used in it are very low, say 5 bar compared with I.C. engine, say 60 bar.
7. In the gas turbine the ignition and lubrication systems are much simpler as compared with I.C. engines.
8. Cheaper fuels such as paraffine type, residue oils or powdered coal can be used whereas special grade fuels are employed in petrol engine to check knocking or pinking.
9. The exhaust from gas turbine is less polluting comparatively since excess air is used for combustion.
10. Because of low specific weight the gas turbines are particularly suitable for use in aircrafts.

Demerits of gas turbines

1. The thermal efficiency of a simple turbine cycle is low (15 to 20%) as compared with I.C. engines (25 to 30%).
2. With wide operating speeds the fuel control is comparatively difficult.
3. Due to higher operating speeds of the turbine, it is imperative to have a speed reduction device.
4. It is difficult to start a gas turbine as compared to an I.C. engine.
5. The gas turbine blades need a special cooling system.

1.4.4. A Simple Gas Turbine Plant

A gas turbine plant may be defined as one "*in which the principal prime-mover is of the turbine type and the working medium is a permanent gas*".

Refer to Fig. 1.6. A simple gas turbine plant consists of the following :

1. *Turbine*.
2. *A compressor* mounted on the same shaft or coupled to the turbine.
3. *The combustor*.
4. *Auxiliaries* such as starting device, auxiliary lubrication pump, fuel system, oil system and the duct system etc.

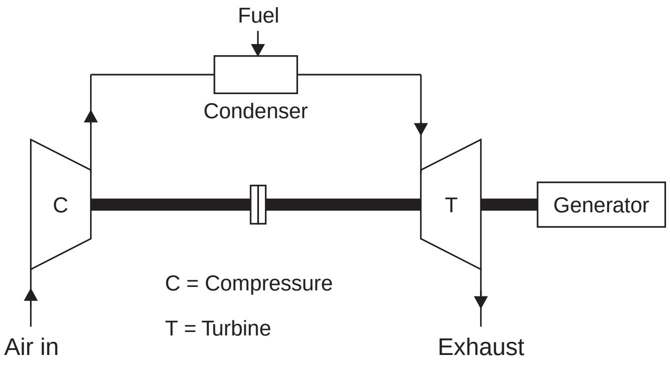


Fig. 1.6. Simple gas turbine plant.

A modified plant may have in addition to above an *intercooler, regenerator*, a *reheater* etc.

The working fluid is compressed in a compressor which is generally rotary, multistage type. Heat energy is added to the compressed fluid in the combustion chamber. This high energy fluid, at high temperature and pressure, then expands in the turbine unit thereby generating power. Part of the power generated is consumed in driving the generating compressor and accessories and the rest is utilised in electrical energy. The gas turbines work on open cycle, semiclosed cycle or closed cycle. In order to improve efficiency, compression and expansion of working fluid is carried out in multistages.

1.4.5. Energy Cycle for a Simple-Cycle Gas Turbine

Fig. 1.7 shows an energy-flow diagram for a simple-cycle gas turbine, the description of which is given below :

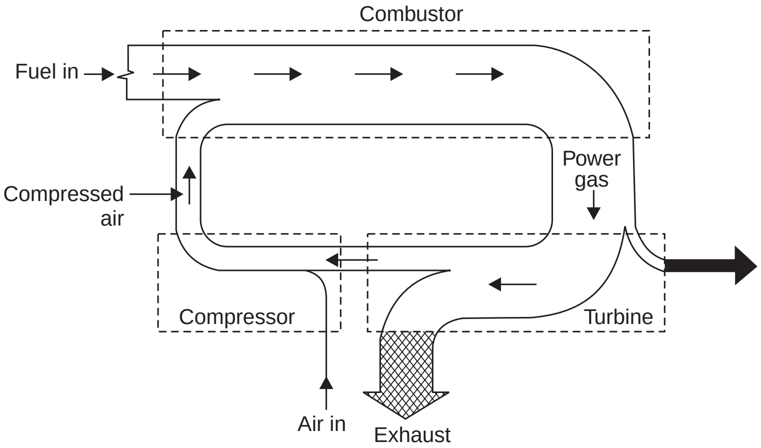


Fig. 1.7. Energy flow diagram for gas-turbine unit.

— The air brings in minute amount of energy (measured above 0°C).

— Compressor adds considerable amount of energy.

— Fuel carries major input to cycle.

— Sum of fuel and compressed-air energy leaves combustor to enter turbine.

— In turbine smallest part of entering energy goes to useful output, largest part leaves in exhaust.

Shaft energy to drive compressor is about twice as much as the useful shaft output.

Actually the shaft energy keeps circulating in the cycle as long as the turbine runs. The important comparison is *the size of the output with the fuel input*. For the simple-cycle gas turbine the output may run about 20% of the fuel input for certain pressure and temperature conditions at turbine inlet. This means 80% of the fuel energy is wasted. While the 20% thermal efficiency is not too bad, it can be improved by including *additional heat recovery apparatus*.

1.5. REFRIGERATION SYSTEMS

Refrigeration means the *cooling of or removal of heat from a system*. Refrigerators work mainly on two processes :

1. Vapour compression, and
2. Vapour absorption.

Simple Vapour Compression System :

In a simple vapour compression system the following fundamental processes are completed in one cycle :

1. Expansion 2. Vapourisation 3. Compression 4. Condensation.

The flow diagram of such a cycle is shown in Fig. 1.8.

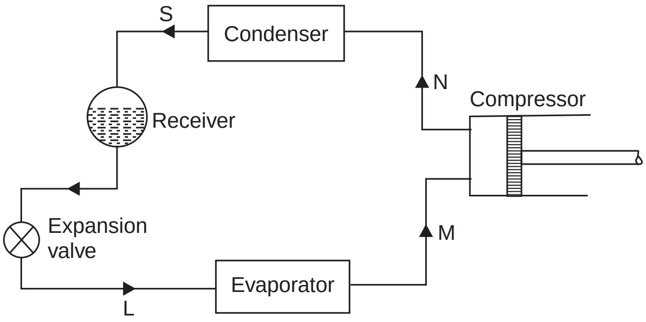


Fig. 1.8. Simple vapour compression cycle.

The *vapour at low temperature and pressure* (state '*M*') enters the *compressor* where it is compressed isoentropically and subsequently its temperature and pressure increase considerably (state '*N*'). This vapour after leaving the compressor enters the *condenser* where it is condensed into *high pressure liquid* (state '*S*') and is collected in a *receiver*. From receiver it passes through the *expansion valve*, here it is *throttled down to a lower pressure* and has *a low temperature* (state '*L*'). After finding its way through *expansion valve* it finally passes on to *evaporator* where it *extracts heat from the surroundings and vapourises to low pressure vapour* (state '*M*').

Domestic Refrigerator :

Refrigerators, these days, are becoming the common item for *house hold use, vendor's shop, hotels, motels, offices, laboratories, hospitals, chemists* and *druggists shops, studios etc*. They are manufactured in different size to meet the needs of various groups of people. They are usually rated with *internal gross volume* and the *freezer volume*. The freezer space is meant to preserve perishable products at a temperature much below 0°C such as fish, meat, chicken etc. and to produce ice and icecream as well. The refrigerators in India are available in different sizes of various makes, *i.e.*, 90, 100, 140, 160, 200, 250, 380 litres of gross volume. The freezers are usually provided at top portion of the refrigerator space occupying around one-tenth to one-third of the refrigerator volume. In some refrigerators, freezers are provided at the bottom.

A domestic refrigerator consists of the following two main parts :

1. The refrigeration system.
2. The insulated cabinet.

Fig. 1.9 shows a flow diagram of a typical refrigeration system used in a domestic refrigerator. A simple domestic refrigerator consists of a *hermetic compressor placed in the cabinet base. The condenser is installed at the back and the evaporator is placed inside the cabinet at the top*.

The working of the refrigerator is as follows :

— The low pressure and low temperature refrigerant vapour (*usually* R12) is drawn through the suction line to the compressor. The *accumulator* provided between the *suction line* and the evaporator collects liquid refrigerant coming out of the evaporator due to incomplete evaporation, if any, prevents it from entering the compressor. The *compressor* then compresses the refrigerant vapour to a high pressure and high temperature. The compressed vapour flows through the *discharge line* into condenser (vertical natural draft, wire-tube type).

— In the *condenser* the vapour refrigerant at high pressure and at high temperature is condensed to the liquid refrigerant at high pressure and low temperature.

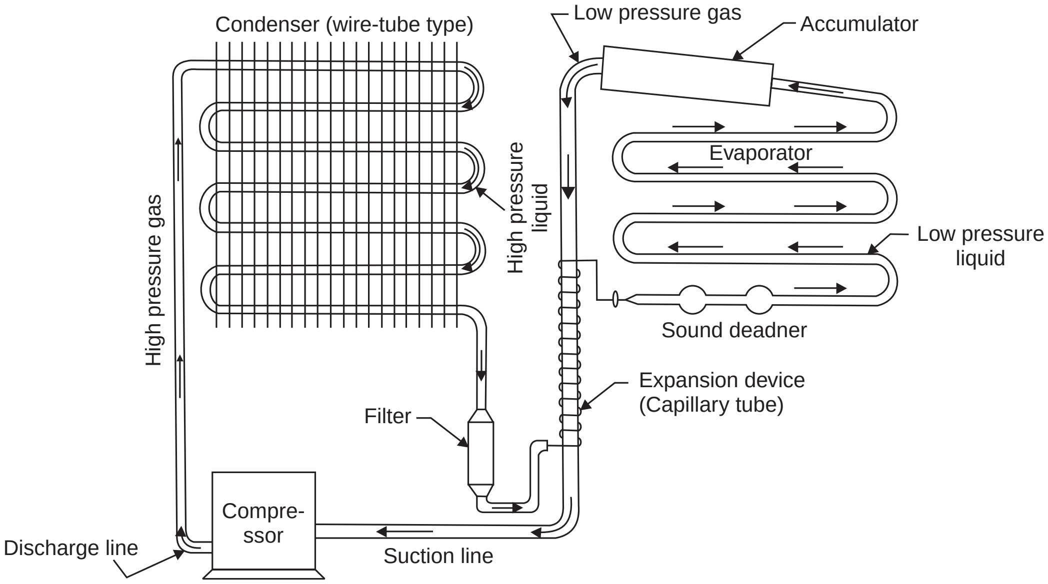


Fig. 1.9. Domestic refrigerator.

— The high pressure liquid refrigerant then flows through the *filter* and then enters the *capillary tube* (expansion device). The capillary tube is attached to the suction line as shown in Fig. 1.9. The warm refrigerant passing through the capillary tube gives some of its heat to cold suction line vapour. This increases the heat absorbing quality of the liquid refrigerant slightly and increases the superheat of vapour entering the compressor.

The capillary tube expands the liquid refrigerant at high pressure to the liquid refrigerant at low pressure so that a measured quantity of liquid refrigerant is passed into the *evaporator*.

— In the *evaporator* the liquid refrigerant gets evaporated by absorbing heat from the container/articles placed in the evaporative chamber and is sucked back into the compressor and the cycle is repeated.

HIGHLIGHTS

1. The layout of a modern steam power plant comprises of the following four circuits :
  (*i*) Coal and ash circuit
  (*ii*) Air and gas circuit
  (*iii*) Feed water and steam flow circuit
  (*iv*) Cooling water circuit.

2. Any type of engine or machine which derives heat energy from the combustion of fuel or any other source and converts this energy into mechanical work is termed as a heat engine.

3. The major fields of application of gas turbines are :
  (*i*) Aviation (*ii*) Power generation
  (*iii*) Oil and gas industry and (*iv*) Marine propulsion.

4. A simple gas turbine plant consists of the following :
  — Turbine
  — Compressor

— Combustor
— Auxiliaries such as starting device, auxiliary lubrication pump, fuel system, oil system and the duct system etc.

5. Refrigeration means the cooling or removal of heat from a system. Refrigerators work mainly on two processes
(*i*) Vapour compression and
(*ii*) Vapour absorption.

THEORETICAL QUESTIONS

1. Give the layout of a modern steam power plant and explain its various circuits.
2. List the components of a nuclear power plant.
3. Draw the cross-section of an air cooled I.C. engine and label its various parts.
4. Explain with neat sketches the working of a four stroke petrol engine.
5. How are gas turbines classified ?
6. What are the major fields of application of gas turbines ?
7. With the help of a neat diagram explain the working of a simple gas turbine plant.
8. Draw the energy cycle for a simple-cycle gas turbine.
9. Explain with a neat sketch the working of a simple vapour compression system.
10. Draw the neat diagram of a domestic refrigerator, showing its various parts. Explain its working also.

2

Basic Concepts of Thermodynamics

2.1. INTRODUCTION TO KINETIC THEORY OF GASES

The kinetic theory of gases deals with the behaviour of molecules constituting the gas. According to this theory, the molecules of all gases are in continuous motion. As a result of this they possess kinetic energy which is transferred from molecule to molecule during their collision. The energy so transferred produces a change in the velocity of individual molecules.

The complete phenomenon of molecular behaviour is quite complex. The *assumptions* are therefore made to simplify the application of theory of an ideal gas.

Assumptions :

1. The molecules of gases are assumed to be rigid, perfectly elastic solid spheres, identical in all respects such as mass, form etc.
2. The mean distance between molecules is very large compared to their own dimensions.
3. The molecules are in state of random motion moving in all directions with all possible velocities and gas is said to be in state of molecular chaos.
4. The collisions between the molecules are perfectly elastic and there are no intermolecular forces of attraction or repulsion. This means that energy of gas is all kinetic.
5. The number of molecules in a small volume is very large.
6. The time spent in collision is negligible, compared to the time during which the molecules are moving independently.
7. Between collisions, the molecules move in a straight line with uniform velocity because of frictionless motion between molecules. The distance between two collisions is called '*free path*' of the molecule, the average distance travelled by a molecule between successive collision is known as '*mean free path*'.
8. The volume of molecule is so small that it is negligible compared to total volume of the gas.

Pressure exerted by an Ideal Gas :

Let us consider a quantity of gas to be contained in a cubical vessel of side l with perfectly elastic wall and N represent the very large number of molecules in the vessel. Now let us consider a molecule which may be assumed to have a velocity C_1 in a certain direction. The velocity can be resolved into three components u_1, v_1, w_1 parallel to three co-ordinate axes X, Y and Z which are again assumed parallel to the sides of the cube as shown in Fig. 2.1.

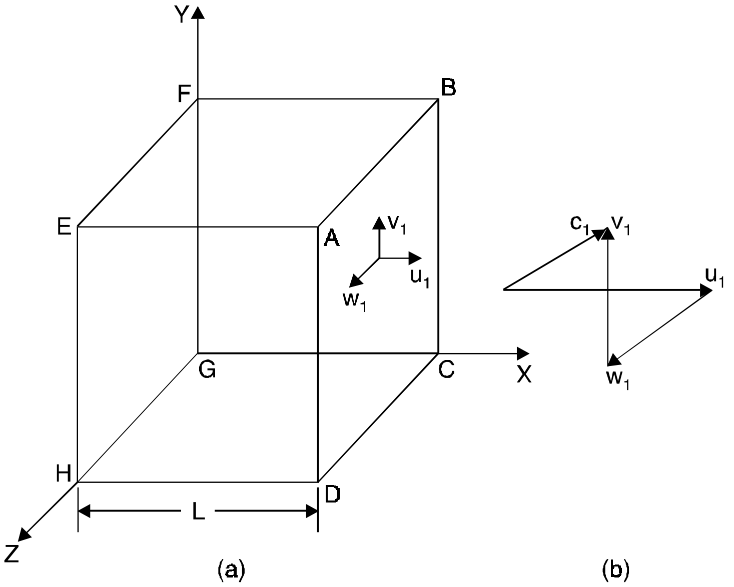


Fig. 2.1

Thus, $$C_1^2 = u_1^2 + v_1^2 + w_1^2 .$$

Let this molecule having mass m strike wall surface $ABCD$ of the cube with velocity u_1. Since the collision is perfectly elastic, the molecule will rebound from this surface with the same velocity u_1. Therefore,

The momentum of the molecule before it strikes the face $ABCD = mu_1$

The momentum of the molecule after impact $= - mu_1$.

Hence change of momentum at each impact in direction normal to the surface

$$ABCD = mu_1 - (- mu_1) = 2mu_1$$

After striking the surface $ABCD$, the molecule rebounds and travels back to the face $EFGH$, collides with it and travels back again to the face $ABCD$ covering $2l$ distance. This means molecule covers $2l$ distance to hit the same face again. Hence the time taken by the same molecule to strike the same face $ABCD$ again is $\frac{2l}{u_1}$.

Therefore, the rate of change of momentum for one molecule of the gas

$$= \frac{2mu_1}{\frac{2l}{u_1}} = \frac{mu_1^2}{l}$$

According to Newton's second law of motion the rate of change of 'momentum is the force'. If F_1 is the force due to one molecule, then

$$F_1 = \frac{mu_1^2}{l}$$

Similarly, then force F_2 due to the impact of another molecule having velocity C_2 whose components are u_2, v_2, w_2 is given by

$$F_2 = \frac{mu_2^2}{l}$$

Hence total force F_x on the face *ABCD* due to impact of N molecules is given by

$$F_x = \frac{m}{l}(u_1^2 + u_2^2 + \ldots\ldots u_N^2)$$

Since the pressure (p) is the force per unit area, hence pressure exerted on the wall *ABCD* is given by

$$p_x = \frac{F_x}{l^2} = \frac{m}{l^3}(u_1^2 + u_2^2 + \ldots\ldots u_N^2)$$

Similarly, if p_y and p_z represent the pressures on other faces which are perpendicular to the Y and Z axis respectively, we have

$$p_y = \frac{m}{l^3}(v_1^2 + v_2^2 + \ldots\ldots v_N^2)$$

and

$$p_z = \frac{m}{l^3}(w_1^2 + w_2^2 + \ldots\ldots w_N^2)$$

Since pressure exerted by the gas is the same in all directions, *i.e.,* $p_x = p_y = p_z$ the average pressure p of the gas is given by

$$p = \frac{p_x + p_y + p_z}{3}$$

or

$$p = \frac{m}{l^3}\,[(u_1^2 + v_1^2 + w_1^2) + (u_2^2 + v_2^2 + w_2^2) + \ldots\ldots(u_N^2 + v_N^2 + w_N^2)]$$

But

$$C_1^2 = (u_1^2 + v_1^2 + w_1^2)$$

$$C_2^2 = (u_2^2 + v_2^2 + w_2^2) \text{ and so on}$$

$$l^3 = V = \text{volume of gas } (m^3)$$

∴

$$p = \frac{1}{3}\,\frac{m}{v}\,(C_1^2 + C_2^2 + C_3^2 + \ldots\ldots C_N^2)$$

or

$$p = \frac{1}{3}\,\frac{m}{v}\,N\overline{C}^2 \qquad \ldots(2.1)$$

where

$$\overline{C}^2 = \left(\frac{C_1^2 + C_2^2 + C_3^2 + \ldots\ldots C_N^2}{N}\right) \text{ known as mean square velocity}$$

or

$$\overline{C} = \sqrt{\frac{C_1^2 + C_2^2 + C_3^2 + \ldots\ldots C_N^2}{N}}$$

where $\overline{C}$ is called the root *mean square velocity* of the molecules and equal to the square root of the mean of square of velocities of individual molecules which is evidently not the same as mean of velocities of different molecules

$$\left[i.e., C_{mean} = \frac{C_1 + C_2 + C_3 + \ldots\ldots C_N}{N}\right]$$

or
$$pV = \frac{1}{3}\, m\, N\overline{C}^2 \qquad ...(2.2)$$

This *equation is the fundamental equation of kinetic theory of gases* and is often referred to as **kinetic equation of gases.**

Equation (2.2) may be written as

$$pV = 2/3 \times 1/2\ m\ N\overline{C}^2$$

where $\frac{1}{2} mN\overline{C}^2$ is the *average transmission or linear kinetic energy of the system of particles.*

Equation (2.1) can be written as

$$p = 1/3\ \rho\ \overline{C}^2 \qquad ...(2.3)$$

where ρ is the density.
$$\left[\because \quad \rho = \frac{mN}{V}, i.e., \frac{\text{Total mass}}{\text{Total volume}}\right]$$

This equation expresses the pressure which any volume of gas exerts in terms of its density under the prevailing conditions and its mean square molecular speed.

From equations (2.2) and (2.3),

$$\overline{C} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3pV}{mN}}$$

Kinetic interpretation of Temperature :

If V_{mol} is the volume occupied by a gram molecule of a gas and N_0 is the number of moles in one gram molecule of gas,

$$M = \text{molecular weight} = mN_0. \qquad ...(i)$$

Since $p\ V_{mol} = R_0T$ ......Molar gas equation ...(*ii*)

From equations (2.2) and (*ii*),

$$1/3\ m\ N_0\,\overline{C} = R_0T$$

or
$$2/3 \times \tfrac{1}{2}m\ N_0\,\overline{C}^2 = R_0T$$

or
$$\tfrac{1}{2}\ m\overline{C}^2 = 3/2\ KT \qquad ...(2.4)$$

(*i.e.*, K.E. per molecule = 3/2 KT)

or
$$\overline{C} = \sqrt{\frac{3KT}{m}}$$

or
$$\overline{C} = \sqrt{\frac{3R_0T}{M}}$$

or
$$\overline{C} = \sqrt{3RT} \qquad ...(2.5)$$

R_0 = Universal gas constant

N_0 = Avogadro's number

$\frac{R_0}{N_0}$ = K (Boltzman's constant)

$$\because\ \frac{K}{m} = \frac{R_0}{N_0\, m} = \frac{R_0}{M}$$

$$\because\ R = \frac{R_0}{M}$$

where R is characteristic gas constant.

From equation (2.4) it is seen that *temperature is a measure* of the *average kinetic energy of translation possessed by molecule.* It is known as the **kinetic interpretation of temperature.** Hence, the absolute temperature of a gas is proportional to the mean translational kinetic energy of the molecules it consists. If the temperature is fixed, then the average K.E. of the molecules remains constant despite encounters.

2.2. DEFINITION OF THERMODYNAMICS

Thermodynamics may be *defined* as follows :

- *Thermodynamics is an axiomatic science which deals with the relations among heat, work and properties of system which are in equilibrium. It describes state and changes in state of physical systems.*

  Or

  Thermodynamics is *the science of the regularities governing processes of energy conversion.*

  Or

  Thermodynamics is *the science that deals with the interaction between energy and material systems.*

Thermodynamics, basically entails *four laws* or axioms known as Zeroth, First, Second and Third law of thermodynamics.

- The ***First law*** throws light on *concept of internal energy*.
- The ***Zeroth law*** deals with *thermal equilibrium* and establishes a *concept of temperature*.
- The ***Second law*** indicates the limit of *converting heat into work* and introduces the *principle of increase of entropy*.
- The ***Third law*** defines the *absolute zero of entropy*.

These laws are based on experimental observations and have no *mathematical proof*. Like all physical laws, these laws are based on *logical reasoning*.

2.3. THERMODYNAMIC SYSTEMS

2.3.1. System, Boundary and Surroundings

System. A system is *a finite quantity of matter or a prescribed region of space* (Refer Fig. 2.2)

Boundary. The *actual or hypothetical envelope enclosing the system* is the boundary of the system. The boundary may be fixed or it may move, as and when a system containing a gas is compressed or expanded. The boundary may be *real* or *imaginary.* It is not difficult to envisage a real boundary but an example of imaginary boundary would be one drawn around a system consisting of the fresh mixture about to enter the cylinder of an I.C. engine together with the remanants of the last cylinder charge after the exhaust process (Refer Fig. 2.3).

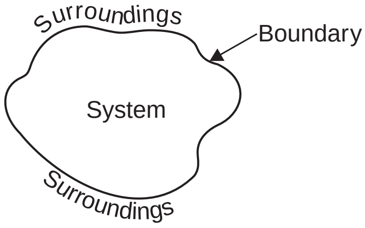


Fig. 2.2. The system.

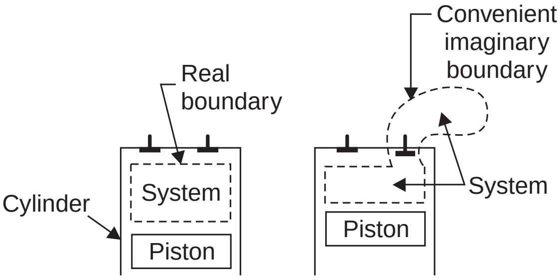


Fig. 2.3. The real and imaginary boundaries.

2.3.2. Closed System

Refer to Fig. 2.4. *If the boundary of the system is impervious to the flow of matter, it is called* a ***closed system.*** An *example* of this system is mass of gas or vapour contained in an engine cylinder, the boundary of which is drawn by the cylinder walls, the cylinder head and piston crown. Here the *boundary is continuous and no matter may enter or leave.*

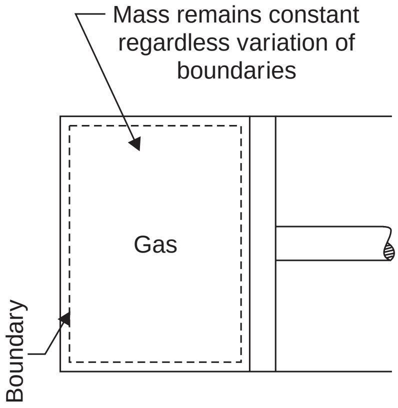


Fig. 2.4. Closed system.

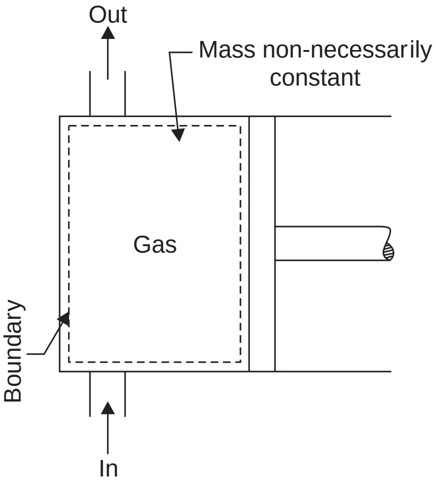


Fig. 2.5. Open system.

2.3.3. Open System

Refer to Fig. 2.5. An open system is one in which *matter flows into or out of the system*. Most of the engineering systems are open.

2.3.4. Isolated System

An isolated system is that system *which exchanges neither energy nor matter with any other system or with environment*.

2.3.5. Adiabatic System

An adiabatic system is one *which is thermally insulated from its surroundings*. It can, however, *exchange work with its surroundings*. If it does not, it becomes an isolated system.

Phase. A phase is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.

2.3.6. Homogeneous System

A system which consists of a single phase is termed as *homogeneous system.* Examples : Mixture of air and water vapour, water plus nitric acid and octane plus heptane.

2.3.7. Heterogeneous System

A system which consists of two or more phases is called a *heterogeneous system.* Examples : Water plus steam, ice plus water and water plus oil.

2.4. MACROSCOPIC AND MICROSCOPIC POINTS OF VIEW

Thermodynamic studies are undertaken by the following two different approaches.

1. Macroscopic approach—(*Macro* mean *big* or *total*)
2. Microscopic approach—(*Micro* means *small*)

These approaches are discussed (in a comparative way) below :

S. No.	*Macroscopic approach*	*Microscopic approach*
1.	In this approach a certain quantity of matter is considered *without* taking into account the events occurring at molecular level. In other words this approach to thermodynamics is concerned with *gross or overall behaviour*. This is known as *classical thermodynamics*.	The approach considers that the system is made up of a very large number of discrete particles known as *molecules*. These molecules have different velocities and energies. The values of these energies are constantly changing with time. This approach to thermodynamics which is concerned directly with the *structure of the matter* is known as *statistical thermodynamics*.
2.	The analysis of macroscopic system requires simple mathematical formulae.	The behaviour of the system is found by using statistical methods as the number of molecules is very large. So advanced statistical and mathematical methods are needed to explain the changes in the system.
3.	The values of the properties of the system are their average values. For example, consider a sample of a gas in a closed container. The *pressure* of the gas is the average value of the pressure exerted by millions of individual molecules. Similarly the *temperature* of this gas is the average value of translational kinetic energies of millions of individual molecules. These properties like *pressure* and *temperature* can be measured very easily. *The changes in properties can be felt by our senses.*	The properties like *velocity, momentum*, impulse, kinetic energy, force of impact etc. which describe the molecule *cannot be easily measured by instruments. Our senses cannot feel them.*
4.	In order to describe a system only a few properties are needed.	Large number of variables are needed to describe a system. So the approach is complicated.

Note. Although the macroscopic approach seems to be different from microscopic one, there exists a relation between them. Hence when both the methods are applied to a particular system, they give the same result.

2.5. PURE SUBSTANCE

A pure substance is one that has a homogeneous and invariable chemical composition even though there is a change of phase. In other words, it is a system which is (*a*) homogeneous in composition, (*b*) homogeneous in chemical aggregation. Examples : Liquid, water, mixture of liquid water and steam, mixture of ice and water. The mixture of liquid air and gaseous air is not a pure substance.

2.6. THERMODYNAMIC EQUILIBRIUM

A system is in *thermodynamic equilibrium* if the temperature and pressure at all points are same ; there should be no velocity gradient ; the chemical equilibrium is also necessary. Systems under temperature and pressure equilibrium but not under chemical equilibrium are sometimes said to be in metastable equilibrium conditions. *It is only under thermodynamic equilibrium conditions that the properties of a system can be fixed*.

Thus for attaining a state of *thermodynamic equilibrium* the following *three* types of equilibrium states must be achieved :

1. **Thermal equilibrium.** The temperature of the system does not change with time and has same value at all points of the system.

2. **Mechanical equilibrium.** There are no unbalanced forces within the system or between the surroundings. The pressure in the system is same at all points and does not change with respect to time.

3. **Chemical equilibrium.** No chemical reaction takes place in the system and the chemical composition which is same throughout the system does not vary with time.

2.7. PROPERTIES OF SYSTEMS

A property of a system is a characteristic of the system which depends upon its state, but not upon how the state is reached. There are two sorts of property :

1. **Intensive properties.** These properties *do not depend on the mass of the system*. *Examples* : Temperature and pressure.

2. **Extensive properties.** These properties *depend on the mass of the system*. *Example* : Volume. Extensive properties are often divided by mass associated with them to obtain the intensive properties. For example, if the volume of a system of mass m is V, then the specific volume of matter within the system is $\frac{V}{m} = v$ which is an intensive property.

2.8. STATE

State *is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of a system is called a* ***state****.*

It follows from the definition of state that each property has a single value at each state. Stated differently, all properties are *state* or *point functions.* Therefore, all properties are identical for identical states.

On the basis of the above discussion, we can determine if a given variable is *property* or not by applying the following *tests* :

— *A variable is a property, if and only if, it has a single value at each equilibrium state.*

— A variable is a property, if and only if, the change in its value between any two prescribed equilibrium states is single-valued.

Therefore, *any variable whose change is fixed by the end states is a property.*

2.9. PROCESS

A process occurs when the system undergoes a change in a state or an energy transfer at a steady state. A process may be *non-flow* in which a fixed mass within the defined boundary is undergoing a change of state. *Example* : A substance which is being heated in a closed cylinder undergoes a ***non-flow process*** (Fig. 2.4). *Closed systems undergo non-flow processes*. A process may be a ***flow process*** *in which mass is entering and leaving through the boundary of an open system*. In a steady flow process (Fig. 2.5) mass is crossing the boundary from surroundings at entry, and an equal mass is crossing the boundary at the exit so that the total mass of the system remains constant. In an open system it is necessary to take account of the work delivered from the surroundings to the system at entry to cause the mass to enter, and also of the work delivered from the system at surroundings to cause the mass to leave, as well as any heat or work crossing the boundary of the system.

Quasi-static process. Quasi means '*almost*'. A quasi-static process is also called a *reversible process. This process is a succession of equilibrium states and infinite slowness is its characteristic feature.*

2.10. CYCLE

Any process or series of processes whose end states are identical is termed a **cycle**. The processes through which the system has passed can be shown on a state diagram, but a complete section of the path requires in addition a statement of the heat and work crossing the boundary of the system. Fig. 2.6 shows such a cycle in which a system commencing at condition '1' changes in pressure and volume through a path 123 and returns to its initial condition '1'.

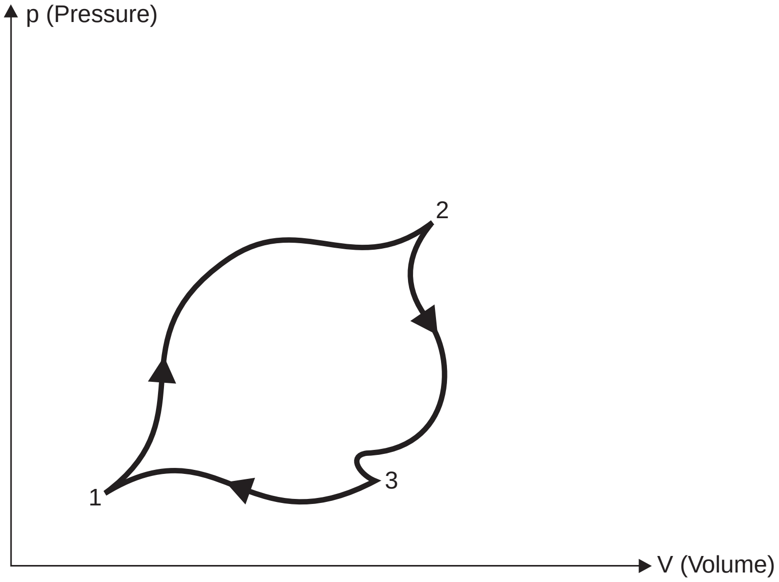


Fig. 2.6. Cycle of operations.

2.11. POINT FUNCTION

When two properties locate a point on the graph (co-ordinate axes) then those properties are called as **point function.**

Examples. Pressure, temperature, volume etc.

$$\int_1^2 dV = V_2 - V_1 \text{ (an } \textit{exact differential}\text{).}$$

2.12. PATH FUNCTION

There are certain quantities which cannot be located on a graph by a *point* but are given by the *area* or so, on that graph. In that case, the area on the graph, pertaining to the particular process, *is a function of the path of the process.* Such quantities are called **path functions.**

Examples. Heat, work etc.

Heat and work are *inexact differentials*. Their change cannot be written as difference between their end states.

Thus $\int_1^2 \delta Q \neq Q_2 - Q_1$ and is shown as ${}_1Q_2$ or Q_{1-2}

Similarly $\int_1^2 \delta W \neq W_2 - W_1$, and is shown as ${}_1W_2$ or W_{1-2}

Note. The operator δ is used to denote inexact differentials and operator d is used to denote exact differentials.

2.13. TEMPERATURE

- *The temperature is a thermal state of a body which distinguishes a hot body from a cold body*. The temperature of a body is *proportional to the stored molecular energy i.e.,* the average molecular kinetic energy of the molecules in a system. (A particular molecule does not hhave a temperature, it has energy. The gas as a system has temperature).
- Instruments for measuring *ordinary temperatures* are known as ***thermometers*** and those for measuring *high temperatures* are known as ***pyrometers.***
- It has been found that a gas will not occupy any volume at a certain temperature. This temperature is known as *absolute zero temperature*. The temperatures measured with absolute zero as basis are called *absolute temperatures*. Absolute temperature is stated in degrees centigrade. The point of absolute temperature is found to occur at 273.15°C below the freezing point of water.

Then : Absolute temperature = Thermometer reading in °C + 273.15.

Absolute temperature is degree centigrade is known as degrees kelvin, denoted by K (SI unit).

2.14. ZEROTH LAW OF THERMODYNAMICS

- **'Zeroth law of thermodynamics'** *states that if two systems are each equal in temperature to a third, they are equal in temperature to each other*.

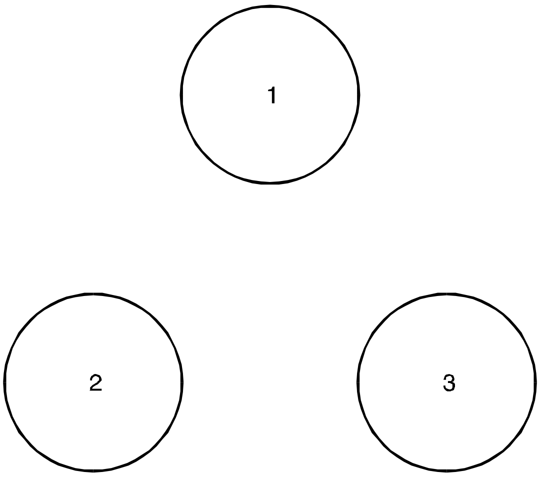


Fig. 2.7. Zeroth law of thermodynamics.

Example. Refer Fig. 2.7. System '1' may consist of a mass of gas enclosed in a rigid vessel fitted with a pressure gauge. If there is no change of pressure when this system is brought into contact with system '2' a block of iron, then the two systems are equal in temperature (assuming that the systems 1 and 2 do not react each other chemically or electrically). Experiment reveals that if system '1' is brought into contact with a third system '3' again with no change of properties then systems '2' and '3' will show no change in their properties when brought into contact provided they do not react with each other chemically or electrically. Therefore, '2' and '3' must be in equilibrium.

- This law was enunciated by R.H. Fowler in the year 1931. However, since the first and second laws already existed at that time, it was designated as ***zeroth law*** so that it *precedes* the first and second laws *to form a logical sequence*.

2.15. THE THERMOMETER AND THERMOMETRIC PROPERTY

2.15.1. Introduction

- The zeroth law of thermodynamics provides the basis for the measurement of temperature. It enables us to compare temperatures of two bodies '1' and '2' with the help of a third body '3' and say that the temperature of '1' is the same as the temperature of '2' without actually bringing '1' and '2' in thermal contact. In practice, body '3' in the zeroth law is called the **thermometer**. It is brought into thermal equilibrium with a set of standard temperature of a body '2', and is thus calibrated. Later, when any other body '1' is brought in thermal communication with the thermometer, we say that the body '1' has attained equality of temperature with the thermometer, and hence with body '2'. This way, the body '1' has the temperature of body '2' given for example by, say the height of mercury column in the thermometer '3'.
- The height of mercury column in a thermometer, therefore, becomes a **thermometric property**.

There are other methods of temperature measurement which utilize various other properties of materials, that are functions of temperature, as thermometric properties.

Six different kinds of thermometers, and the names of the corresponding thermometric properties employed are given below :

Thermometer	*Thermometric property*
1. Constant volumes gas	Pressure (p)
2. Constant pressure gas	Volume (V)
3. Alcohol or mercury-in-glass	Length (L)
4. Electric resistance	Resistance (R)
5. Thermocouple	Electromotive force (E)
6. Radiation (pyrometer)	Intensity of radiation (I or J)

2.15.2. Measurement of Temperature

Temperature can be depicted as a *thermal state which depends upon the internal or molecular energy of the body*.

2.15.2.1. Temperature Measuring Instruments

These instruments may be classified in two broad categories :

1. **Non-electrical methods :**

(*i*) By using change in volume of a liquid when its temperature is changed.

(*ii*) By using change in pressure of a gas when its temperature is changed.

(*iii*) By using changes in the vapour pressure when the temperature is changed.

2. **Electrical method :**

(*i*) By thermocouples.

(*ii*) By change in resistance of material with change in temperature.

(*iii*) By comparing the colours of filament and the object whose temperature is to be found out.

(*iv*) By ascertaining the energy received by radiation.

The **thermometers** may also be classified as follows :

1. ***Expansion thermometers***

(*i*) Liquid-in-glass thermometers (*ii*) Bimetallic thermometers.

2. ***Pressure thermometers***

(*i*) Vapour pressure thermometers (*ii*) Liquid-filled thermometers

(*iii*) Gas-filled thermometers.

3. Thermocouple thermometers
4. Resistance thermometers
5. Radiation pyrometers
6. Optical pyrometers.

1. Expansion Thermometers :

The expansion thermometers make use of the differential expansion of two different substances. Thus in liquid-in-glass thermometers, it is the difference in expansion of liquid and the containing glass. And in bimetallic thermometers, the indication is due to the difference in expansion of the two solids. These thermometers are discussed below :

(*i*) **Liquid-in-glass thermometer.** This is a very familiar type of thermometer. The mercury or other liquid fills the glass bulb and extends into the bore of the glass stem. Mercury is the most suitable liquid and is used from – 38.9°C (melting point) to about 600°C. The thermometers employed in the laboratory have the scale engraved directly on the glass stem. A usual type of mercury-in-glass thermometer is shown in Fig. 2.8. An expansion bulb is usually provided at the top of the stem to allow room for expansion of mercury, in case the thermometer is subjected to temperature above its range. The upper limit for mercury-in-glass thermometers is about 600°C. As the upper limit is far above the boiling point of mercury, some inert gas *i.e.,* nitrogen is introduced above the mercury to prevent boiling.

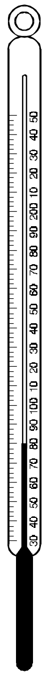


Fig. 2.8. Mercury-in-glass thermometer.

Pentane, ethyl alcohol and toluene are the other liquids which can be used for liquid-in-glass thermometers. Since these liquids are normally colourless a dye is added to facilitate reading. These liquids have a low freezing point as shown below and are suitable for low temperature thermometers.

Liquid	*Boiling point*	*Freezing point*
Pentane	36°C	– 130°C
Ethyl alcohol	78°C	– 100°C
Toluene	110°C	– 92°C

(*ii*) **Bimetallic thermometers.** In a bimetallic thermometer differential expansion of bimetallic strips is used to indicate the temperature. It has the advantage over the liquid-in-glass thermometer, that it is *less fragile* and is *easier to read*. In this type of thermometer two flat strips of different metals are placed side by side and are welded together. Many different metals can be used for this purpose. Generally one is a *low expanding metal* and the other is *high expanding metal*. The bimetal strip is coiled in the form of a spiral or helix. *Due to rise in temperature, the curvature of the strip changes. The differential expansion of a strip causes the pointer to move on the dial of the thermometer*.

2. Pressure Thermometers :

In pressure thermometers liquids, gases and vapours can all be used. The principle on which they work is quite simple. The fluid is confined in a closed system. In this case the *pressure is a function of the temperature*, so that when the fluid is heated, the pressure will rise. And the temperature can be indicated by Bourdon type pressure gauge. In general, the thermometer consists of a bulb which contains bulk of the fluid. The bulb is placed in the region whose temperature is required. A capillary tube connects the bulb to a Bourdon tube, which is graduated with a temperature scale.

Pressure thermometers are discussed below :

(*i*) **Vapour pressure thermometer.** A schematic diagram of a vapour pressure thermometer is shown in Fig. 2.9. When the bulb containing the fluid is installed in the region whose temperature is required, some of the fluid vapourizes, and increases the vapour pressure. This change of pressure is indicated on the Bourdon tube. The relation between temperature and vapour pressure of a volatile liquid is of the *exponential form*. Therefore, the scale of a vapour pressure thermometer will not be linear.

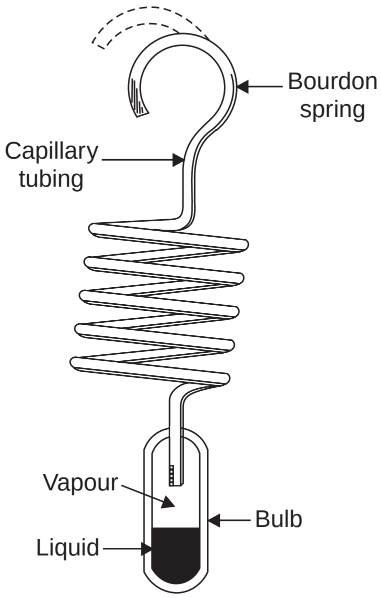


Fig. 2.9. Vapour pressure thermometer.

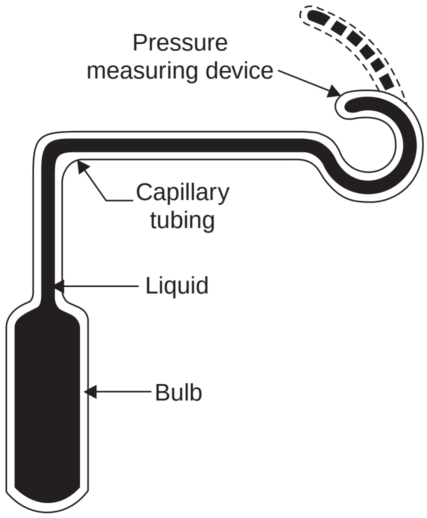


Fig. 2.10. Liquid-filled thermometer.

(*ii*) **Liquid-filled thermometer.** A liquid-filled thermometer is shown in Fig. 2.10. In this case, the *expansion of the liquid causes the pointer to move in the dial*. Therefore liquids having high co-efficient of expansion should be used. In practice many liquids *e.g.,* mercury, alcohol, toluene and glycerine have been successfully used. The operating pressure varies from about 3 to 100 bar. These type of thermometers could be used for a temperature upto 650°C in which mercury could be used as the liquid.

In actual design, the internal diameter of the capillary tube and Bourdon tube is, made much smaller than that of the bulb. This is because the capillary tube is subjected to a temperature which is quite different from that of the bulb. Therefore, to minimise the effect of variation in temperature to which the capillary tube is subjected, the *volume of the bulb is made as large as possible as compared with the volume of the capillary*. However, large volume of bulb tends to increase time lag, therefore, a *compensating device is usually built into the recording or indicating mechanism, which compensates the variations in temperature of the capillary and Bourdon tubes*.

(*iii*) **Gas-filled thermometers.** The temperature range for gas thermometer is practically the same as that of liquid filled thermometer. The gases used in the gas thermometers are *nitrogen* and *helium*. Both these gases are chemically inert, have good values for their co-efficient of expansion and have low specific heats. The construction of this type of thermometer is more or less the same as mercury-thermometer in which Bourdon spring is used. The errors are also compensated likewise. The only difference in this case is that bulb is made much larger than used in liquid-filled thermometers. For good performance the volume of the bulb should be made at least 8 times than that of the rest of the system.

These thermometers are generally used for pressures below 35 bar.

3. Thermocouple Thermometers :

For higher range of temperature *i.e.,* above 650°C, filled thermometers are unsuitable. For higher range of temperature, thermocouples and pyrometers are used.

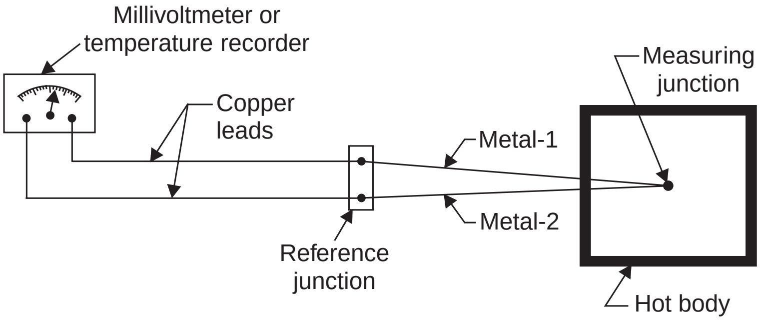


Fig. 2.11. Thermocouple.

In its simplest form a thermocouple consists of *two dissimilar metals or alloys which develop e.m.f. when the reference and measuring junctions are at different temperatures*. The reference junction or cold junction is usually maintained at some constant temperature, such as 0°C. Fig. 2.11, shows a simple circuit of a thermocouple and the temperature measuring device. In many industrial installations the instruments are equipped with automatic compensating devices for temperature changes of the reference junction, thus eliminating the necessity of maintaining this junction at constant temperature.

Table 2.1 gives the composition, useful temperatures range and temperature versus e.m.f. relationship for some commercial thermocouples.

Table 2.1. Composition, useful temperature range and e.m.f. produced for some thermocouples

S.No.	*Thermocouple*	*Composition*	*Temperature (°C)*		*Thermoelectric power*		*Remarks*
			Useful range	*Max.*	*°C*	*Millivolt*	
1.	Platinum vs Platinum-rhodium	Pure platinum vs Pt + 10 or 13% Rh	400 to 1450	1700	0 500 1000 1500	0.0 4.219 9.569 15.498	Used for high temperature measurements
2.	Chromel vs alumel	90% Ni + 10% Cr vs 95% Ni + 5% (Al + Sn) Mn	– 200 to 1200	1450	– 200 0 300 600 900 1200	– 5.75 0.0 12.21 24.90 37.36 48.85	High resistance to oxidation

3.	Iron vs constantan	Pure iron vs 45-60% Cu + 55-40% Ni	– 200 to 750	1000	– 200 0 300 600 900	– 8.27 0.0 16.59 33.27 52.29	—
4.	Copper vs constantan	Pure copper vs Cu-Ni constantan	– 200 to 350	600	– 200 0 200 400	– 5.539 0.0 9.285 20.865	Not suitable in air due to excessive oxidation

4. Resistance thermometers :

The fact that the electrical resistance of the metals increases with temperature is made use of in resistance thermometers which are purely electrical in nature. *A* ***resistance thermometer*** *is used for precision measurements below 150°C.*

A simple resistance thermometer consists of a *resistance element* or *bulb, electrical loads* and *a resistance measuring or recording instrument*. The resistance element (temperature sensitive element) is usually supplied by the manufacturers with its protecting tube and is ready for electrical connections. The resistance of the metal used as resistance element should be reproducible at any given temperature. *The resistance is reproducible if the composition or physical properties of the metal do not change with temperature. For this purpose platinum is preferred*. A *platinum resistance thermometer* can measure temperatures to within ± 0.01°C. However, because of high cost of platinum, nickel and copper are used as resistance elements for industrial purposes for low temperatures. The fine resistance wire is wound in a spiral form on a mica frame. The delicate coil is then enclosed in a porcelain or quartz tube. The change of resistance of this unit can be measured by instruments such as Wheatstone bridge, potentiometer or galvanometer.

Advantages :

The resistance thermometers possess the following *advantages* over other devices :

1. A resistance thermometer is very accurate for low ranges below 150°C.
2. It requires no reference junction like thermocouples and as such is more effective at room temperature.
3. The distance between the resistance element and the recording element can be made much larger than is possible with pressure thermometers.
4. It resists corrosion and is physically stable.

Disadvantages :

1. The resistance thermometers cost more.
2. They suffer from time lag.

5. Radiation pyrometers :

A *device which measures the total intensity of radiation emitted from a body is called* ***radiation pyrometer.***

The elements of a total radiation pyrometer are illustrated in Fig. 2.12. It collects the radiation from an object (hot body) whose temperature is required. A mirror is used to focus this radiation on a thermocouple. This energy which is concentrated on the thermocouple raises its temperature, and in turn generates an e.m.f. This e.m.f. is then measured either by the galvanometer or potentiometer method. Thus *rise of temperature is a function of the amount of radiation emitted from the object.*

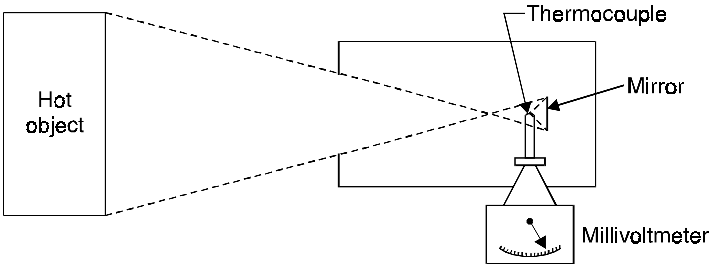


Fig. 2.12. A schematic diagram of radiation pyrometer.

Advantages of the pyrometers

1. The temperatures of moving objects can be measured.
2. A higher temperature measurement is possible than that possible by thermocouples etc.
3. The average temperatures of the extended surface can be measured.
4. The temperature of the objects which are not easily accessible can be measured.

6. Optical pyrometer :

An optical pyrometer works on the principle *that matters glow above 480°C and the colour of visible radiation is proportional to the temperature of the glowing matter*. The amount of light radiated from the glowing matter (solid or liquid) is measured and employed to determine the temperature.

Fig. 2.13 shows a *disappearing filament pyrometer*.

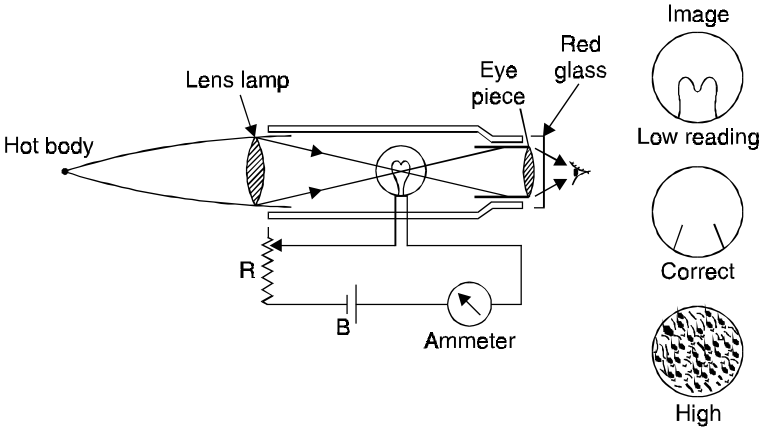


Fig. 2.13. Optical pyrometer.

Operation :

- The optical pyrometer is sighted at the hot body and focused.
- In the beginning filament will appear dark as compared to the background which is bright (being hot).
- By varying the resistance (R) in the filament circuit more and more current is fed into it, till filament becomes equally bright as the background and hence disappears.
- The current flowing in the filament at this stage is measured with the help of an ammeter which is calibrated directly in terms of temperature.
- If the filament current is further increased, the filament appears brighter as compared to the background which then looks dark.
- An optical pyrometer can measure temperatures ranging from 700 to 4000°C.

Table 2.2 gives the summary of temperature range of different instruments on next page.

Table 2.2. Summary of temperature range of different instruments

Types of instruments	*Liquid in glass thermometer*	*Bimetallic thermometer*	*Pressure thermometers*			*Thermocouples*				*Resistance thermometers*			*Radiation pyrometers*	*Optical pyrometers*
	Mercury		Vapour pressure	Liquid filled	Gas filled	Iron-Constantan	Copper-Constantan	Chromel-Alumel	Platinum Platinum-Rhodium	Copper	Nickel	Platinum	–	–
Low temperature limit	– 39°C	– 75°C	– 250°C	– 87°C	– 269°C	– 196°C	– 250°C	–184°C	0°C	– 140°C	– 184°C	– 184°C	About room temp.	760°C
High temperature limit	600°C	540°C	316°C	650°C	540°C	760°C	350°C	1260°C	1540°C	120°C	316°C	760°C	As high as desired	As high as desired
Remark	Low price ; Fairly accurate ; Easily broken	Easier to read than Liquid-in-glass thermometers	Normally scale non-linear	To be compensated for variation in ambient temperatures	Requires large bulb ; Linear scale	Widely used in Industry.	Constantan is 60% Cu 40% Ni	Chromel is 20% ; Cr 80% ; Ni-Alumel is 2% Al, 98% Ni,	Expensive	Very sensitive. Suitable for narrow range instruments			—	—

2.15.3. The International Practical Temperature Scale

For the calibration of thermometric instruments the Seventh General Conference on Weight and Measures held in 1927 formulated a convenient scale known as the *International Practical Temperature Scale*. It was revised at Thirteenth General Conference in 1968. It consists of *reproducible reference temperatures or primary fixed points* defined by a number of pure substances with assigned values of temperatures determined with precision on ideal or perfect gas temperature scale as given in Table 2.3.

Table 2.3. Fixed Points of the International Practical Temperature Scale of 1968

	Equilibrium state	*Assigned value of temperature*	
		T, K	*t°C*
1.	Triple point of hydrogen	13.81	– 259.34
2.	Boiling point of hydrogen at 33.306 kPa	17.042	– 266.108
3.	Normal boiling point of hydrogen	20.28	– 252.87
4.	Normal boiling point of neon	27.102	– 246.048
5.	Triple point of oxygen	54.361	– 218.789
6.	Normal boiling point of oxygen	90.188	– 182.962
7.	Triple point of water	273.16	0.01
8.	Normal boiling point of water	373.15	100.00
9.	Normal freezing point of antimony (antimony point)	630.74	357.59
10.	Normal freezing point of zinc (zinc point)	692.73	419.58
11.	Normal freezing point of silver (silver point)	1235.08	961.93
12.	Normal freezing point of gold (gold point)	1337.58	1064.43

It is stated here that :

— The *triple point* represents an equilibrium state between solid, liquid and vapour phases of a substance.

— *Normal boiling point* is the temperature at which the substance boils at standard atmospheric pressure of 760 mm Hg.

— *Normal freezing point* is the solidification or the melting point temperature of the substance at standard atmospheric pressure.

Based on the available method of measurement, the whole temperature scale may be divided into four ranges. The equations for interpolation for each range are as follows :

1. **From – 259.34°C (triple point of hydrogen) to 0°C :**

A *platinum resistance thermometer of a standard design* is used and a polynomial of the following form is fitted between the resistance of the wire R_t and temperature t

$$R_t = R_0 (1 + At + Bt^2 + Ct^3) \qquad ...(2.6)$$

where R_0 = resistance at the ice point.

2. **From 0°C to 630.74°C (Antimony point) :**

- It is also based on *platinum resistance thermometer*.
- The diameter of the platinum wire must lie between 0.05 and 0.2 mm.

3. **From 630.74°C to 1064.43°C (Gold point) :**

- It is based on *standard platinum versus platinum-rhodium thermocouple*.
- Following equation between e.m.f. E and temperature t is employed :

$$E = a + bt + ct^2 \qquad ...(2.7)$$

4. **Above 1064.43°C :**

- It is *based on the intensity of radiation* J_T *at temperature* T *emitted by a black body at a wavelength* λ *in the visible spectrum and by comparing this to the intensity of radiation* J_{Au} *at the same wavelength emitted by a black body at the gold point.*
- The temperature is calculated from Planck's equation for black body radiation

$$\frac{J_T}{J_{Au}} = \frac{\exp\left(\dfrac{C_2}{\lambda\, T_{Au}}\right) - 1}{\exp\left(\dfrac{C_2}{\lambda T}\right) - 1} \qquad \text{...(2.8)}$$

where $C_2 = 0.01438$ in °C, and λ = wavelength in metres.

Following points are worth noting for gas thermometers :

- *The gas thermometers are never used for the measurement of temperatures.* However, *they are ideal when used for calibration for establishing the ideal gas temperature scale, and for establishing a standard because of precision, reproducible results, and their reading being independent of the thermometric substance used.*
- *The gas thermometers can be used only for temperatures upto which gases do not liquify.*

Method in use before 1954 :

- Celsius and Fahrenheit scales are the two commonly used scales for the measurement of temperature. Symbols C and F are respectively used to denote the readings on these two scales. Until 1954 the temperature scales were based on two fixed points : (*i*) the *steam point* (boiling point of water at standard atmospheric pressure), and (*ii*) the *ice point* (freezing point of water).

The fixed points for these temperature scales are :

Temperature	**Celsius scale**	**Fahrenheit scale**
Steam point	100	212
Ice point	0	32
Interval	100	180

- The relation between a particular value C on celsius scale and F on Fahrenheit scale is found to be as mentioned below :

$$\frac{C}{100} = \frac{F-32}{180} \text{ or } \frac{C}{5} = \frac{F-32}{9} \qquad \text{...(2.9)}$$

- Further the relation between a temperature difference Δt_F on Fahrenheit scale and Δt_C on celsius scale is

$$\Delta t_F = \frac{180}{100}\Delta t_C = \frac{9}{5}\Delta t_C = 1.8\ \Delta t_C \qquad \text{...(2.10)}$$

- The use of two fixed points was found unsatisfactory and later *abandoned* because of the following *reasons* :

(*i*) It is difficult to achieve equilibrium between pure ice and air-saturated water (since when ice melts, it surrounds itself only with pure water and prevents intimate contact with air-saturated water).

(*ii*) There is extreme sensitiveness of the steam point to the change in pressure.

Method in use after 1954 :

It was suggested by Kelvin that a single fixed point only was necessary to establish a temperature. He pointed out that *triple point of water* (the state at which ice, liquid water and water vapour coexist in equilibrium) could be used as the single point. The tenth CGPM, in 1954, adopted this fixed point, and value was set at 0.01°C or 273.16 K in the Kelvin scale thus established.

Correspondingly, the ice point of 0°C on the Celsius scale becomes equal to 273.15 K on the Kelvin scale. Celsius and Kelvin scales are distinguished by using distinct symbols t and T, the relation between these two is then given by :

$$\mathrm{T(K)} = t(°\mathrm{C}) + 273.15 \quad \text{...(2.11)}$$

2.15.4. Ideal Gas

From experimental observations it has been established that an ideal gas (to a good approximation) behaves according to the simple equation

$$pV = mRT \quad \text{...(2.12)}$$

where p, V and T are the pressure, volume and temperature of gas having mass m and R is a constant for the gas known as its ***gas constant.***

Eqn. (2.10) can be written as

$$pv = RT \quad \text{...(2.13)}$$

(where v = V/m)

In reality there is no gas which can be qualified as an ideal or perfect gas. However *all gases tend to ideal or perfect gas behaviour at all temperatures as their pressure approaches* ***zero pressure.***

For two states of the gas, eqn. (2.10) can be written as,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

or

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{V_2}{V_1} \quad \text{...(2.14)}$$

With the help of this eqn. (2.12), the temperatures can be measured or compared.

2.16. PRESSURE

2.16.1. Definition of Pressure

Pressure is defined as a *force per unit area*. Pressures are exerted by gases, vapours and liquids. The instruments that we generally use, however, record pressure as the difference between two pressures. Thus, it is the *difference between the pressure exerted by a fluid of interest and the ambient atmospheric pressure.* Such devices indicate the pressure either above or below that of the atmosphere. When it is *above the atmospheric pressure*, it is termed *gauge pressure* and is *positive*. When it is *below* atmospheric, it is *negative* and is known as *vacuum*. Vacuum readings are given in millimetres of mercury or millimetres of water below the atmosphere.

It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum. *Any pressure measured above the absolute zero of pressure* is termed an '*absolute pressure*'.

A schematic diagram showing the *gauge pressure, vacuum pressure* and the *absolute pressure* is given in Fig. 2.14.

Mathematically :

(*i*) Absolute pressure = Atmospheric pressure + Gauge pressure

$$p_{\mathrm{abs.}} = p_{\mathrm{atm.}} + p_{\mathrm{gauge}}.$$

(*ii*) Vacuum pressure = Atmospheric pressure – Absolute pressure.

Vacuum is defined as the *absence of pressure*. A *perfect vacuum* is obtained when *absolute pressure is zero*, at this instant *molecular momentum is zero*.

Atmospheric pressure is measured with the help of barometer.

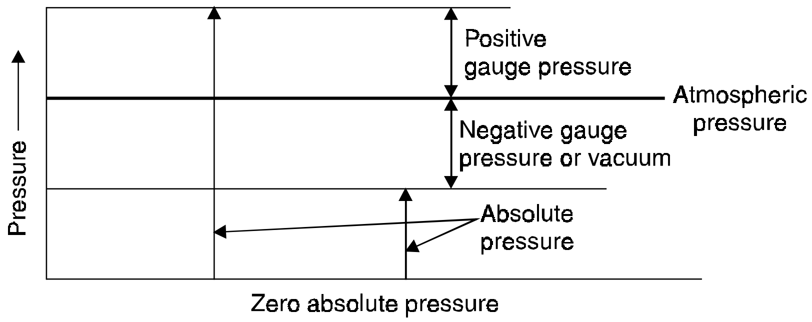


Fig. 2.14. Schematic diagram showing gauge, vacuum and absolute pressures.

2.16.2. Unit for Pressure

The fundamental SI unit of pressure is N/m^2 (sometimes called *pascal*, Pa) or bar. 1 bar = 10^5 N/m^2 = 10^5 Pa.

Standard atmospheric pressure = 1.01325 bar = 0.76 m (or 760 mm) Hg.

Low pressures are often expressed in terms of mm of water or mm of mercury. This is an abbreviated way of saying that the pressure is such that which will support a liquid column of stated height.

2.16.3. Types of Pressure Measurement Devices

The pressure may be measured by means of indicating gauges or recorders. These instruments may be mechanical, electro-mechanical, electrical or electronic in operation.

1. **Mechanical instruments.** These instruments may be classified into following two groups :

— The *first group* includes those instruments in which the *pressure* measurement is made by *balancing an unknown force with a known force*.

— The *second group* includes those employing *quantitative deformation of an elastic member for pressure measurement*.

2. **Electro-mechanical instruments.** These instruments usually *employ a mechanical means for detecting the pressure and electrical means for indicating or recording the detected pressure*.

3. **Electronic instruments.** Electronic pressure measuring instruments normally depend on some physical change that can be detected and indicated or recorded electronically.

2.16.4. Mechanical-type Instruments

The mechanical-type instruments are classified as follows :

1. **Manometer gauges**

(*i*) U-tube manometer (*ii*) Cistern manometer
(*iii*) Micro-manometer etc.

2. **Pressure gauges**

(*i*) Bourdon tube pressure gauge (*ii*) Diaphragm gauge
(*iii*) Vacuum gauge.

2.16.4.1. Liquid manometers

Low pressures are generally determined by *manometers* which employ liquid columns. It is difficult and costly to construct manometers to measure high pressures, as otherwise the liquid column will become unwieldy and temperature corrections will also be difficult. Their use is, therefore, *restricted to low pressures only*, and for such purposes they are *quite accurate*.

The liquids commonly employed for manometers are mercury and water. *Mercury is used for high and water for low pressures*. For this purpose a liquid is suitable if it has a low viscosity, so that it can adjust itself quickly, and also a low co-efficient of thermal expansion, so that density changes with temperature are minimum.

1. *U-tube manometer :*

A U-tube manometer is in the form of U-tube and is made of glass. When no pressure is applied, the height of the liquid in the two legs is the same. The pressure is then applied to one leg, whilst the other is open to the atmosphere. Under this pressure the liquid will *sink* in this leg and will *rise* in the other. As the other leg is open to the air, therefore, the pressure on this side is known, and is barometric. Now the pressure applied to the first leg can be calculated. This is explained with reference to Fig. 2.15. This consists of a water manometer.

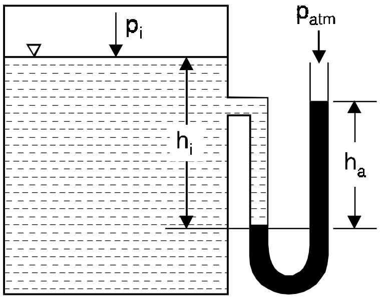


Fig. 2.15. Principle of U-tube manometer.

Considering equilibrium condition, we have

$$p_{\text{atm}} + w_a h_a = p_i + w_i h_i$$

$$\therefore \quad p_i = p_{\text{atm}} + w_a h_a - w_i h_i$$

where $p_{\text{atm.}}$ = Atmospheric pressure,

p_i = Pressure over water surface in the container,

h_a = Height of liquid in U-tube manometer,

h_i = Difference between water surface and lower surface of the liquid in manometer,

w_a = Specific weight of liquid,

w_i = Specific weight of water.

The U-tube manometer shown in Fig. 2.16 is of the simplest form. However, readings have to be taken *at two different places*. Moreover, the *deflection of the two columns may not be the same*. To avoid this difficulty cistern or well type manometer is used.

2. *Cistern manometer :*

Fig. 2.17 shows a cistern manometer. The mercury reservoir *A* is made large enough so that change of level in the reservoir is negligible. This form of manometer is generally used for measuring pressures above atmospheric. In this case, *only one reading of the level in the column is required*. However, a zero setting is necessary.

3. *Micro-manometer :*

The U-tube manometer (discussed above) is not very suitable for measuring very low pressures. Therefore, for such purposes, precision types are required. They are called *multiplying* or *micro-manometers*, because they multiply the movement of the level of the liquid. By far the most widely used type of multiplying manometer is the ***inclined manometer.*** If the tube is inclined as shown in Fig. 2.18 the sensitiveness of the U-tube manometer is increased. The inclined tube *causes a larger displacement of the liquid along the tube for a given pressure difference*.

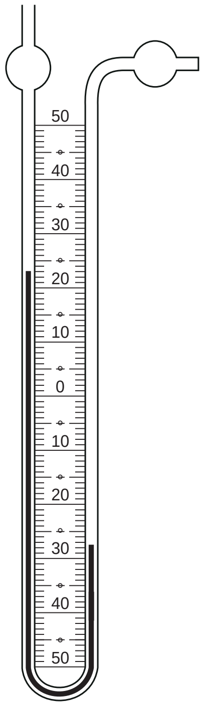


Fig. 2.16. U-tube manometer.

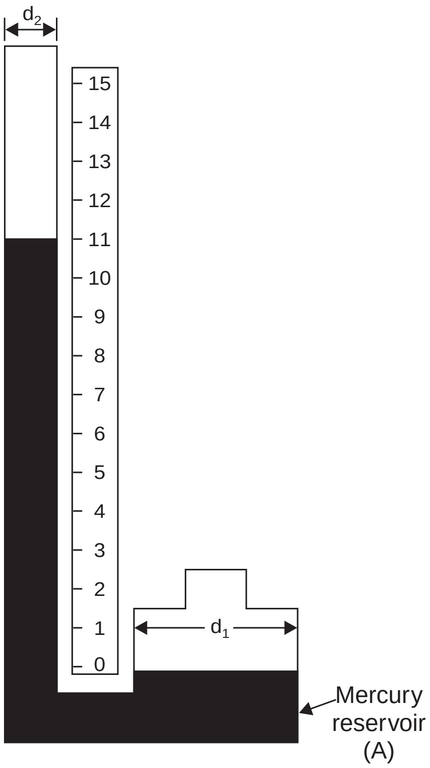


Fig. 2.17. Cistern manometer.

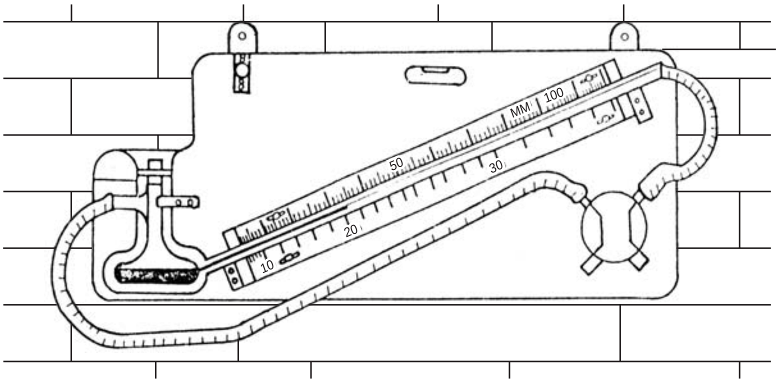


Fig. 2.18. Inclined manometer.

The principle of the inclined manometer is explained in Fig. 2.19. If pressure $p_1 = p_2$, then the level of liquid is shown by *LM*. However, when p_1 is slightly greater than p_2, the level is the reservoir sinks by h_2, whilst level in the tube rises by a greater distance h_1 as shown in the diagram. If h is the vertical distance between the two surfaces due to difference of pressure, then

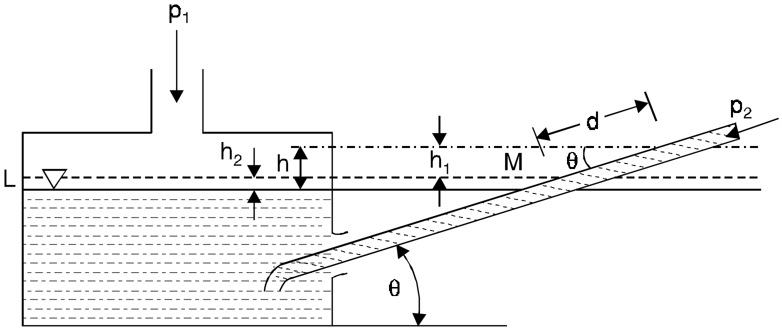


Fig. 2.19. Principle of inclined manometer.

$$h = h_1 + h_2$$

also $$h_1 = d \sin \theta$$

and $$h_2 \times A = d \times a$$

or $$h_2 = d \times \frac{a}{A}$$

where A = Area of cross-section of the reservoir, and

a = Area of cross-section of the inclined tube.

Also the pressure difference Δp *i.e.*, $(p_1 - p_2)$

$$= hw = \left(d \times \sin \theta + d \times \frac{a}{A}\right) w$$

$$= dw \left(\sin \theta + \frac{a}{A}\right)$$

where w is the specific weight of the liquid.

The sensitiveness of the instrument can be varied by *changing the slope of the inclined tube*. The position of the inclined tube is so arranged that $\left(\sin \theta + \frac{a}{A}\right)$ is round figure.

The multiplication factor of the gauge is $\frac{d}{h} = \frac{1}{\sin \theta + \frac{a}{A}}$

Thus the multiplication factor depends on θ and $\frac{a}{A}$. The *smaller the values of* θ *and* $\frac{a}{A}$, *the greater the multiplication factor*.

Advantages and disadvantages of manometers :

Advantages. The manometer claims the following advantages :

1. Relatively inexpensive.
2. Easy to fabricate.
3. Requires very little maintenance.
4. Good accuracy and sensitivity.

5. Their sensitivity can be changed by changing manometric fluids.
6. Particularly suitable to low pressures and low differential pressures.

Disadvantages. The *disadvantages* of manometers are as follows :

1. Unsuitable for recording.
2. Generally large, bulky and fragile.
3. Their calibration is affected by changes in gravitational force and density of fluids and their calibration changes with altitude and temperature.
4. Surface tension of manometric fluid creates a capillary effect and possible hysteresis.
5. A particular manometer can be used only for measurement of a particular fluid/fluids.
6. Meniscus height has to be determined by accurate means to ensure improved accuracy.

Elastic pressure elements

Elastic pressure elements or mechanical type of transducers are used for measurement of very high pressures upto about 700 MN/m^2. There are *three* main types of pressure elements.

(*a*) Bourdon tube
(*b*) Diaphragms
(*c*) Bellows.

The action of these mechanical transducers depends upon the *displacement caused by the pressure*. The displacement produced may actuate a pointer whose deflection may be direct measure of the pressure applied or the displacement is measured with the help of a secondary transducer which is electrical in nature. The output of the secondary transducer which is electrical in format is a measure of the pressure applied.

(*a*) Bourdon tubes/elements :

Fig. 2.20 shows Bourdon tube configurations.

Advantages. The Bourdon tube element has the following *advantages* :

1. Simple in construction and cheap.
2. Available in several different ranges.
3. Capability to measure gauge, absolute and differential pressures.
4. The sensitivities of Bourdon tube may be changed by changes in their dimensions.
5. Excellent sensitivity.
6. Simple and straight forward calibration with dead weight tester.
7. Easily adapted to strain, capacitance, magnetic and other electrical transducers.

Disadvantages :

1. Susceptibility to shock and vibration.
2. Inherent hysteresis and slow response to pressure changes.
3. Unsuitable for low pressure applications.

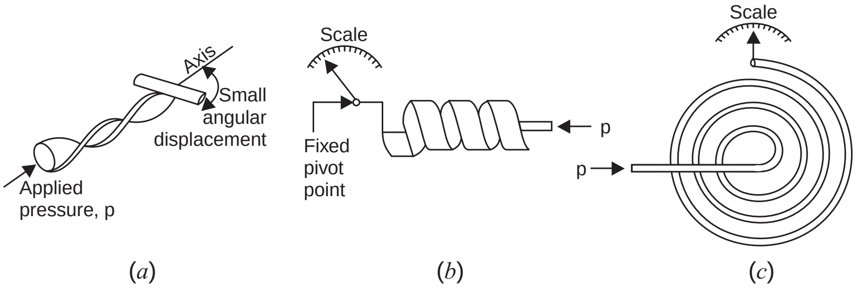


Fig. 2.20. (*a*) Twisted tube, (*b*) Helical, (*c*) Spiral.

(*b*) **Diaphragm elements :**

Fig. 2.21 shows basic diaphragm types.

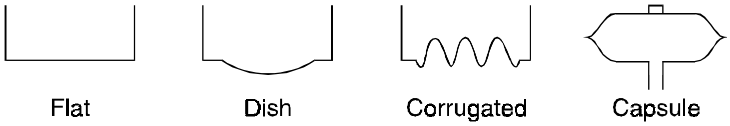


Fig. 2.21. Basic diaphragm types.

Advantages :

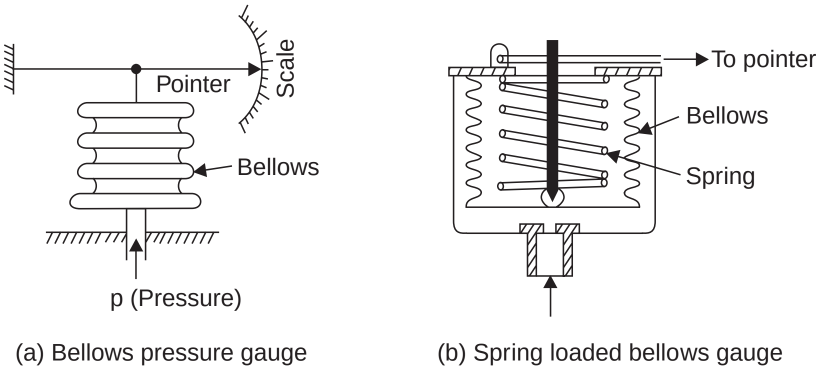
1. Small in size and moderately priced.
2. Wide linear range.
3. Can withstand high over pressures and under pressures.
4. Small hysteresis.
5. Can be used for measurement of absolute and differential pressures as also vacuum.

Disadvantages :

1. Need protection against shocks and vibrations.
2. Cannot be used to measure high pressures.
3. Difficult to repair.

(*c*) **Bellow gauges/elements :**

Fig. 2.22 shows some bellow gauges.



(a) Bellows pressure gauge (b) Spring loaded bellows gauge

Fig. 2.22. Bellow gauges.

Advantages :

1. Simple and rugged construction.
2. Useful for measurement of low and medium pressures.
3. Moderate cost.
4. Can be used for measurement of absolute, gauge and differential pressures.

Disadvantages :

1. Need spring for accurate characterisation.
2. Greater hysteresis and zero drift problems.
3. Unsuitable for transient measurements due to longer relative motion and mass.
4. Requires compensation for ambient temperature changes.

2.16.4.2. Important types of pressure gauges

The manometers and U-tubes (discussed earlier) are suitable for comparatively *low pressures.* For high pressures they become unnecessarily larger even when they are filled with heavy liquids. Therefore for measuring medium and high pressures, we make use of elastic pressure gauges. They employ different forms of elastic systems such as tubes, diaphragms or bellows etc. to measure the pressure. The elastic deformation of these elements is used to show the effect of pressure. Since these elements are deformed within the elastic limit only, therefore these gauges are sometimes called *elastic gauges.* Sometimes they are also called *secondary instruments,* which implies that they must be calibrated by comparison with primary instruments such as manometer etc.

Some of the important types of these gauges are enumerated and discussed below :

1. Bourdon tube pressure gauge
2. Diahragm gauge
3. Vacuum gauge.

1. *Bourdon tube type pressure gauge :*

A Bourdon type tube pressure gauge is used for measuring *high as well as low pressures.* A simple form of this gauge is shown in Fig. 2.23. In this case the pressure element consists of a metal tube of approximately *elliptical cross-section.* This tube is bent in the form of a segment of a circle and responds to pressure changes. When one end of the tube which is attached to the gauge case, is connected to the source of pressure, the internal pressure causes the tube to expand, whereby circumferential stress *i.e.,* hoop tension is set up. The free end of the tube moves and is in turn connected by suitable levers to a rack, which engages with a small pinion mounted on the same spindle as the pointer. Thus the pressure applied to the tube causes the rack and pinion to move. The pressure is indicated by the pointer over a dial which can be graduated in a suitable scale.

The Bourdon tubes are generally made of *bronze* or *nickel steel*. The former is generally used for low pressures and the latter for high pressures.

Depending upon the purpose for which they are required Bourdon tube gauges are made in different forms, some of them are :

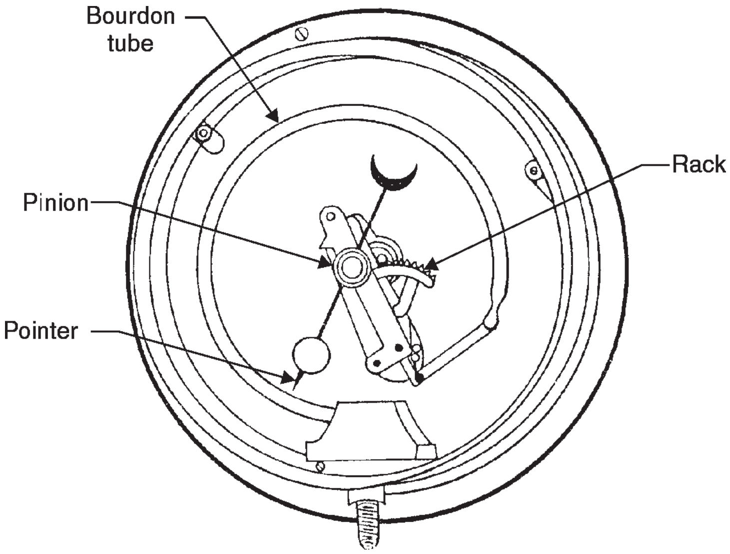


Fig. 2.23. Burdon tube pressure gauge.

(*i*) ***Compound Bourdon tube*** ...... used for measuring pressures both above and below atmospheric.

(*ii*) ***Double Bourdon tube*** ...... used where vibrations are encountered.

2. *Diaphragm gauge :*

This type of gauge employs a metallic disc or diaphragm instead of a bent tube. This disc or diaphragm is used for *actuating* the indicating device.

Refer Fig. 2.24. When pressure is applied on the lower side of the diaphragm, it is deflected upward. This movement of the diaphragm is transmitted to a rack and pinion. The latter is attached to the spindle of needle moving on a graduated dial. The dial can again be graduated in a suitable scale.

3. *Vacuum gauge :*

Bourdon gauges discussed earlier can be used to measure vacuum instead of pressure. Slight changes in the design are required for this purpose. Thus, in this case, the tube be *bent inward instead of outward* as in pressure gauge. *Vacuum gauges* are graduated in millimetres of mercury below atmospheric pressure. In such cases, therefore, absolute pressure in millimetres of mercury is the difference between barometer reading and vacuum gauge reading.

Vacuum gauges are used to measure the vacuum in the *condensers* etc. If there is leakage, the vacuum will drop.

The pressure gauge installation require the following *considerations* :

(*i*) Flexible copper tubing and compression fittings are recommended for most installations.

(*ii*) The installation of a gauge cock and tee in the line close to the gauge is recomm- ended because it permits the gauge to be removed for testing or replacement without having to shut down the system.

(*iii*) Pulsating pressures in the gauge line are not required.

(*iv*) The gauge and its connecting line is filled with an inert liquid and as such liquid seals are provided. Trapped air at any point of gauge lines may cause serious errors in pressure reading.

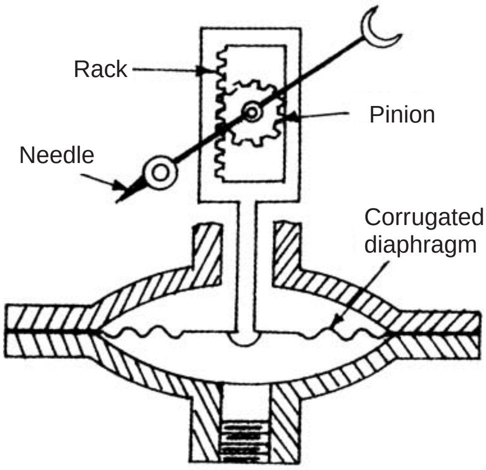


Fig. 2.24. Principle of diaphragm gauge.

2.17. SPECIFIC VOLUME

The *specific volume* of a system is the volume occupied by the unit mass of the system. The symbol used is v and units are ; for example, m^3/kg. The symbol V will be used for volume. (Note that specific volume is *reciprocal of density*).

Example 2.1. *Convert the following readings of pressure to kPa assuming that barometer reads 760 mm of Hg.*

(*i*) *80 cm of Hg* (*ii*) *30 cm Hg vacuum*

(*iii*) *1.35 m* H_2O *gauge* (*iv*) *4.2 bar.*

Solution. Assuming density of Hg, $\rho_{Hg} = 13.596 \times 1000$ kg/m^3

Pressure of 760 mm of Hg will be

$$= \rho \times g \times h = 13.596 \times 1000 \times 9.806 \times \frac{760}{1000}$$

$$= 101325 \text{ Pa} = 101.325 \text{ kPa.}$$

(*i*) **Pressure of 80 cm of Hg**

$$= \frac{800}{760} \times 101.325 = \mathbf{106.65 \text{ kPa.}} \quad \textbf{(Ans.)}$$

(*ii*) **30 cm Hg vacuum**

$$= 76 - 30 = 46 \text{ cm of Hg absolute.}$$

Pressure due to 46 cm of Hg

$$= \frac{460}{760} \times 101.325 = \mathbf{61.328 \text{ kPa.}} \quad \textbf{(Ans.)}$$

(*iii*) **Pressure due to 1.35 m H_2O gauge**

$$= 1000 \times 9.806 \times 1.35 = 13238 \text{ Pa} = \mathbf{13.238 \text{ kPa.}} \quad \textbf{(Ans.)}$$

(*iv*) **4.2 bar**

$$= 4.2 \times 10^2 \text{ kPa} = \mathbf{420 \text{ kPa.}} \quad \textbf{(Ans.)}$$

Note. Pressure of 1 atmosphere

$= 760$ mm of Hg

or $= 101325$ N/m^2.

The above values are standard. To get this value we have to use $\rho_{Hg} = 13596$ kg/m^3 and $g = 9.806$ m/s^2. When we use $\rho_{Hg} = 13600$ kg/m^3 and $g = 9.81$ m/s^2, we get $p_{atm.} = 101396$ N/m^2 which is slightly different from 101325 N/m^2. It is recommended that for pressure of 1 atm. the value 101325 N/m^2 should be used.

Example 2.2. *On a piston of 10 cm diameter a force of 1000 N is uniformly applied. Find the pressure on the piston.*

Solution. Diameter of the piston $d = 10$ cm (= 0.1 m)

Force applied on the piston, $F = 1000$ N

$\therefore$ Pressure on the piston, $$p = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{1000}{\pi / 4 \times (0.1)^2}$$

$$= 127307 \text{ N/m}^2 = \mathbf{127.307 \text{ kN/m}^2.} \quad \textbf{(Ans.)}$$

Example 2.3. *A tube contains an oil of specific gravity 0.9 to a depth of 120 cm. Find the gauge pressure at this depth (in kN/m^2).*

Solution. Specific gravity of oil = 0.9

Depth of oil in the tube, $h = 120$ cm = (1.2 m)

We know that $p = wh$

$= \rho.g.h$, ρ being the mass density

$= (0.9\ \rho_w) \times g \times h$, ρ_w being mass density of water

$$\left[\text{Specific gravity} = \frac{\rho}{\rho_w}\right]$$

$$= 0.9 \times 1000 \times 9.81 \times 1.2 \text{ N/m}^2$$

$$= 10594.8 \text{ N/m}^2 = \mathbf{10.595 \text{ kN/m}^2.} \quad \textbf{(Ans.)}$$

Example 2.4. *A vacuum recorded in the condenser of a steam power plant is 740 mm of Hg. Find the absolute pressure in the condenser in Pa. The barometric reading is 760 mm of Hg.*

Solution. Vacuum recorded in the condenser = 740 mm of Hg

Barometric reading = 760 mm of Hg

We know that,

Absolute pressure in the condenser

= Barometric reading – vacuum in the condenser

= 760 – 740 = 20 mm of Hg

$= 20 \times 133.4 \text{ N/m}^2 \qquad (\because \ 1 \text{ mm of Hg} = 133.4 \text{ N/m}^2)$

= 2668 N/m² = **2668 Pa.(Ans.)**

Example 2.5. *A vessel of cylindrical shape is 50 cm in diameter and 75 cm high. It contains 4 kg of a gas. The pressure measured with manometer indicates 620 mm of Hg above atmosphere when barometer reads 760 mm of Hg. Determine :*

(*i*) *The absolute pressure of the gas in the vessel in bar.*

(*ii*) *Specific volume and density of the gas.*

Solution. Diameter of the vessel, d = 50 cm (= 0.5 m)

Height of the vessel, h = 75 cm (= 0.75 m)

Mass of gas in the vessel, m = 4 kg

Manometer reading = 620 mm of Hg above atmosphere

Barometer reading = 760 mm of Hg

Now, volume of the vessel $= \frac{\pi}{4} d^2 \times h = \frac{\pi}{4} \times (0.5)^2 \times (0.75) = 0.147 \text{ m}^3.$

(*i*) **Total pressure in the vessel**

= 760 + 620 = 1380 mm of Hg

$= 1380 \times 133.4 \text{ N/m}^2 \qquad [\because 1 \text{ mm of Hg} = 133.4 \text{ N/m}^2]$

$= 1.841 \times 10^5 \text{ N/m}^2 =$ **1.841 bar.(Ans.)** $\qquad [\because \ 1 \text{ bar} = 10^5 \text{ N/m}^2]$

(*ii*) **Specific volume** $= \frac{0.147}{4} =$ **0.03675 m³/kg.(Ans.)**

Density $= \frac{4}{0.147} =$ **27.21 kg/m³.(Ans.)**

Example 2.6. *In a pipe line the pressure of gas is measured with a mercury manometer having one limb open to the atmosphere (Fig. 2.25). If the difference in the height of mercury in the two limbs is 550 mm, calculate the gas pressure.*

Given : Barometric reading = 761 mm of Hg

Acceleration due to gravity = 9.79 m/s²

Density of mercury = 13640 kg/m³.

Solution. At the plane LM, we have

$$p = p_0 + \rho g h$$

Now $\qquad p_0 = \rho g h_0$

where h_0 = barometric height ; ρ = density of mercury ; p_0 = atmospheric pressure

Therefore, $\qquad p = \rho g h_0 + \rho g h = \rho g (h_0 + h)$

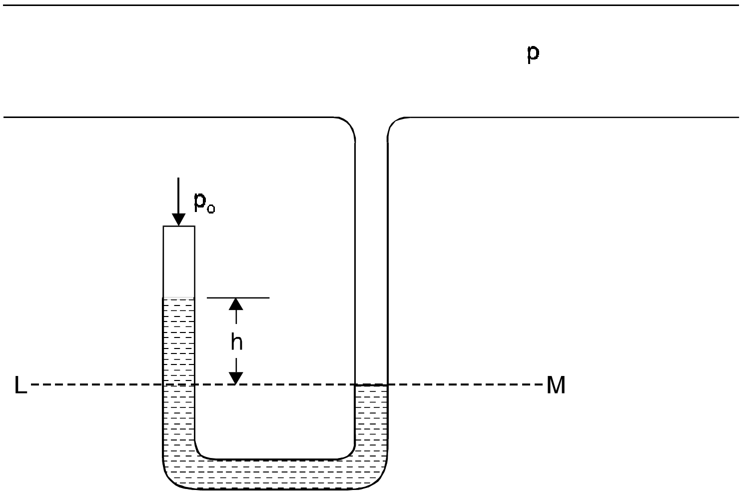


Fig. 2.25

$$= 13640 \times 9.79 \left(\frac{761}{1000} + \frac{550}{1000}\right) = 13640 \times 9.79\ (0.761 + 0.55)$$

$$= 175.065 \times 10^3 \text{ N/m}^2 = 175.065 \text{ kPa} = \mathbf{1.75\ bar.}\ \ \textbf{(Ans.)}$$

Example 2.7. *A U-tube mercury manometer with one arm open to atmosphere is used to measure pressure in a steam pipe. The level of mercury in open arm is 97.5 mm greater than that in the arm connected to the pipe. Some of steam in the pipe condenses in the manometer arm connected to the pipe. The height of this column is 34 mm. The atmospheric pressure is 760 mm of Hg. Find the absolute pressure of steam.* **(Poona University, Nov. 1997)**

Solution. Equating the pressure in mm of Hg on both arms above the line *XX* (Fig. 2.26), we get

$$p_{\text{abs.}} + p_{\text{water}} = p_{\text{Hg}} + p_{\text{atm.}}$$

Now, $p_{\text{water}} = \frac{34}{13.6} = 2.5$ mm of Hg.

$\therefore\quad p_{\text{abs}} + 2.5 = 97.5 + 760$

or $\quad p_{\text{abs}} = 97.5 + 760 - 2.5$

$= 855$ mm of Hg.

$= 855 \times \rho_{\text{Hg}} \times g \times 10^{-5}$ bar

$= \frac{855}{1000}\,(m) \times (13.6 \times 1000)\ (\text{kg/m}^3)$

$\times\ 9.81 \times 10^{-5}$

= **1.1407 bar. (Ans.)**

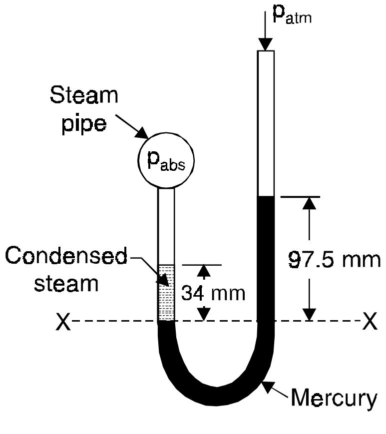


Fig. 2.26

Example 2.8. *A U-tube manometer is connected to a gas pipe. The level of the liquid in the manometer arm open to the atmosphere is 170 mm lower than the level of the liquid in the arm connected to the gas pipe. The liquid in the manometer has specific gravity of 0.8. Find the absolute pressure of the gas if the manometer reads 760 mm of Hg.*

(Poona University, Dec. 2000)

Solution. Equating pressure on both arms above the line *XX* (Fig. 2.27), we get

$$p_{\text{gas}} + p_{\text{liquid}} = p_{\text{atm.}} \qquad ...(i)$$

Now, $p_{\text{liquid}} = \rho.g.h$

$$= (0.8 \times 1000) \times 9.81 \times \frac{170}{1000}$$

$$= 1334.16 \text{ N/m}^2$$

$$= 0.0133416 \text{ bar}$$

$$p_{\text{atm.}} = 760 \text{ mm of Hg} = 1.01325 \text{ bar}$$

Substituting these value is eqn. (*i*) above, we have

$$p_{\text{gas}} + 0.0133416 = 1.01325$$

$\therefore$ $\boldsymbol{p_{\text{gas}} = 0.9999}$ **bar. (Ans.)**

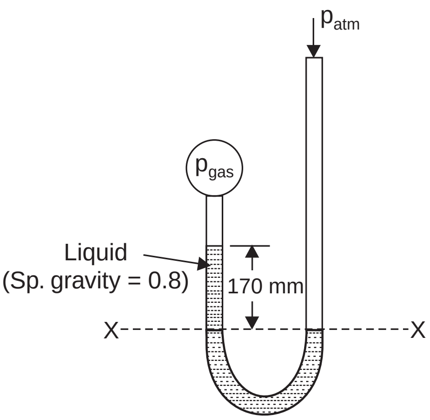


Fig. 2.27

Example 2.9. *Estimate the mass of a piston that can be supported by a gas entrapped under the piston in a 200 mm diameter vertical cylinder when a manometer indicates a difference of 117 mm of Hg column for the gas pressure.* **(Poona University, May 1996)**

Solution. Refer Fig. 2.28.

Let m = mass of the piston, kg.

p = pressure of the gas

= 117 mm of Hg column

Dia. of vertical cylinder, d = 200 mm

Now, downward force = $m.g$ ...(*i*)

and upward force = $p \times \pi/4 \ d^2$ ...(*ii*)

Equating eqns. (*i*) and (*ii*), we get

$$m.g = p \times \pi/4 \ d^2$$

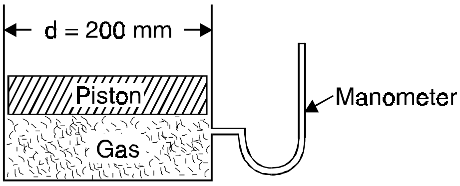


Fig. 2.28

$$m \times 9.81 = \left(13.6 \times 1000 \times 9.81 \times \frac{117}{1000}\right) \times \frac{\pi}{4} \times \left(\frac{200}{1000}\right)^2 \qquad (\because \ p = \rho g h)$$

$\therefore$ **m = 49.989 kg.(Ans.)**

2.18. REVERSIBLE AND IRREVERSIBLE PROCESSES

Reversible process. *A reversible process (also sometimes known as quasi-static process) is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states.*

This process has the following *characteristics* :

1. It must pass through the same states on the reversed path as were initially visited on the forward path.

2. This process when undone will leave no history of events in the surroundings.

3. It must pass through a continuous series of equilibrium states.

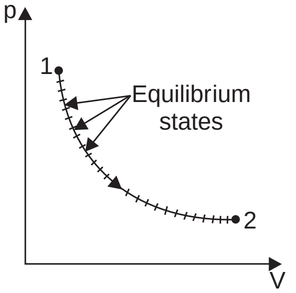


Fig. 2.29. Reversible process.

No real process is truely reversible but some processes may approach reversibility, to close approximation.

Examples. Some examples of nearly reversible processes are :

(*i*) Frictionless relative motion.

(*ii*) Expansion and compression of spring.

(*iii*) Frictionless adiabatic expansion or compression of fluid.

(*iv*) Polytropic expansion or compression of fluid.

(*v*) Isothermal expansion or compression.

(*vi*) Electrolysis.

Irreversible process. An *irreversible process* is one *in which heat is transferred through a finite temperature*.

Examples.

(*i*) Relative motion with friction (*ii*) Combustion

(*iii*) Diffusion (*iv*) Free expansion

(*v*) Throttling (*vi*) Electricity flow through a resistance

(*vii*) Heat transfer (*viii*) Plastic deformation.

An irreversible process is usually represented by a dotted (or discontinuous) line joining the end states to indicate that the intermediate states are indeterminate (Fig. 2.30).

Irreversibilities are of *two types* :

1. **External irreversibilities.** These are associated with *dissipating effects outside the working fluid*.

Example. *Mechanical friction occurring during a process due to some external source*.

2. **Internal irreversibilities.** These are associated with *dissipating effects within the working fluid*.

Example. *Unrestricted expansion of gas, viscosity and inertia of the gas*.

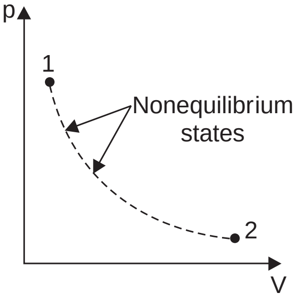


Fig. 2.30. Irreversible process.

2.19. ENERGY, WORK AND HEAT

2.19.1. Energy

Energy is a general term embracing *energy in transition* and *stored energy*. The stored energy of a substance may be in the forms of *mechanical energy* and *internal energy* (other forms of stored energy may be chemical energy and electrical energy). Part of the stored energy may take the form of either potential energy (which is the gravitational energy due to height above a chosen datum line) or kinetic energy due to velocity. The balance part of the energy is known as *internal energy*. In a ***non-flow process*** *usually there is no change of potential or kinetic energy and hence change of mechanical energy will not enter the calculations*. In a ***flow process***, however, *there may be changes in both potential and kinetic energy and these must be taken into account while considering the changes of stored energy*. ***Heat and work*** are *the forms of energy in transition*. These are the only forms in which energy can cross the boundaries of a system. *Neither heat nor work can exist as stored energy.*

2.19.2. Work and Heat

Work

Work is said to be done when a *force moves through a distance*. If a part of the boundary of a system undergoes a displacement under the action of a pressure, the work done *W* is the product of the force (pressure × area), and the distance it moves in the direction of the force. Fig. 2.31 (*a*) illustrates this with the conventional piston and cylinder arrangement, the heavy line defining the boundary of the system. Fig. 2.31 (*b*) illustrates another way in which work might be applied to a system. A force is exerted by the paddle as it changes the momentum of the fluid, and since this force moves during rotation of the paddle work is done.

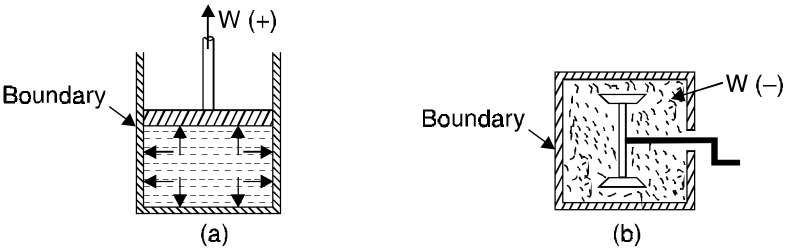


Fig. 2.31

Work is a transient quantity which only appears at the boundary while a change of state is taking place within a system. Work is 'something' which appears at the boundary when a system changes its state due to the movement of a part of the boundary under the action of a force.

Sign convention :

- If the work is done *by* the system *on* the surroundings, *e.g.,* when a fluid expands pushing a piston outwards, the work is said to be *positive*.

i.e., *Work output of the system* $= + W$

- If the work is done *on* the system *by* the surroundings, *e.g.*, when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be *negative*.

i.e., *Work input to system* $= - W$

Heat

Heat (denoted by the symbol Q), may be, defined in an analogous way to work as follows :

"*Heat is 'something' which appears at the boundary when a system changes its state due to a difference in temperature between the system and its surroundings*".

Heat, like work, is a transient quantity which only appears at the boundary while a change is taking place within the system.

It is apparent that neither δW or δQ are exact differentials and therefore any integration of the elemental quantities of work or heat which appear during a change from state 1 to state 2 must be written as

$$\int_1^2 \delta W = W_{1-2} \text{ or } {}_1W_2 \text{ (or } W\text{), and}$$

$$\int_1^2 \delta Q = Q_{1-2} \text{ or } {}_1Q_2 \text{ (or } Q\text{)}$$

Sign convention :

If the heat flows *into* a system *from* the surroundings, the quantity is said to be *positive* and, conversely, if heat flows *from* the system to the surroundings it is said to be *negative*.

In other words :

Heat received by the system $= + Q$

Heat rejected or given up by the system $= - Q$.

Comparison of Work and Heat

Similarities :

(*i*) Both are *path functions and inexact differentials*.

(*ii*) Both are boundary phenomenon *i.e.,* both are recognized at the boundaries of the system as they cross them.

(*iii*) Both are associated with a process, not a state. Unlike properties, work or heat has no meaning at a state.

(*iv*) Systems possess energy, but not work or heat.

Dissimilarities :

(*i*) In heat transfer temperature difference is required.

(*ii*) In a stable system there cannot be work transfer, however, there is no restriction for the transfer of heat.

(*iii*) The sole effect external to the system could be reduced to rise of a weight but in the case of a heat transfer other effects are also observed.

2.20. REVERSIBLE WORK

Let us consider an ideal frictionless fluid contained in a cylinder above a piston as shown in Fig. 2.32. *Assume that the pressure and temperature of the fluid are uniform and that there is no friction between the piston and the cylinder walls.*

Let A = Cross-sectional area of the piston,

p = Pressure of the fluid at any instant,

$(p - dp)\, A$ = Restraining *force exerted* by the surroundings on the piston, and

dl = The distance moved by the piston under the action of the *force exerted*.

Then work done by the fluid on the piston is given by force times the distance moved,

i.e., Work done by the fluid

$$= (pA) \times dl = pdV$$

(where dV = a small increase in volume)

Or considering unit mass

Work done = pdv (where v = specific volume)

This is only *true* when (*a*) the *process is frictionless and (b) the difference in pressure between the fluid and its surroundings during the process is infinitely small*. Hence when a reversible process takes place between state 1 and state 2, we have

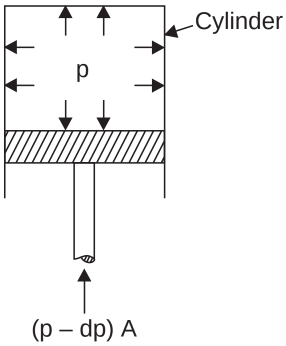


Fig. 2.32

$$\text{Work done by the unit mass of fluid} = \int_1^2 pdv \qquad ...(2.15)$$

When a fluid undergoes a reversible process a series of state points can be joined up to form a line on a diagram of properties. The work done by the fluid during any reversible process is therefore given by the area under the line of process plotted on a *p-v* diagram (Fig. 2.32).

i.e., Work done = Shaded area on Fig. 2.33

$$= \int_1^2 pdv\,.$$

When p can be expressed in terms of v then the integral, $\int_1^2 pdv$, can be evaluated.

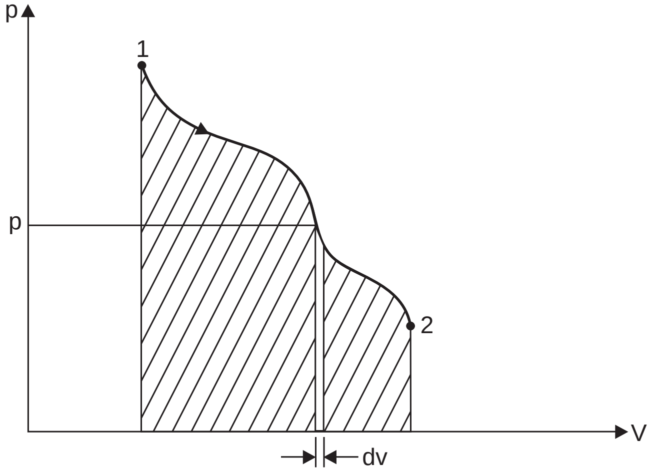


Fig. 2.33

Example 2.10. *An artificial satellite revolves round the earth with a relative velocity of 800 m/s. If acceleration due to gravity is 9 m/s² and gravitational force is 3600 N, calculate its kinetic energy.*

Solution. Relatively velocity of satellite, $v = 800$ m/s

Acceleration due to gravity, $g = 9 \text{ m/s}^2$

Gravitational force, $m.g = 3600$ N

$\therefore$ Mass, $m = \frac{3600}{g} = \frac{3600}{9} = 400$ kg.

Kinetic energy $= \frac{1}{2} mv^2 = \frac{1}{2} \times 400 \times (800)^2 \text{ J} = 128 \times 10^6$ J or **128 MJ. (Ans.)**

Example 2.11. *The specific heat capacity of the system during a certain process is given by*

$$c_n = (0.4 + 0.004\ T)\ kJ/kg°C.$$

If the mass of the gas is 6 kg and its temperature changes from 25°C to 125°C find :

(*i*) *Heat transferred* ; (*ii*) *Mean specific heat of the gas.*

Solution. Mass of the gas, $m = 6$ kg

Change in temperature of the gas = 25°C to 125°C

(*i*) **Heat transferred, Q :**

We know that heat transferred is given by,

$$Q = \int m\ c_n\ dT = 6 \int_{25}^{125} (0.4 + 0.004\ T)\ dT$$

$$= 6 \left[0.4\ T + 0.04 \left(\frac{T^2}{2} \right) \right]_{25}^{125}$$

$$= 6[0.4\ (125 - 25) + 0.002\ (125^2 - 25^2)]$$

$$= 6(40 + 30) = \textbf{420 kJ.(Ans.)}$$

(*ii*) **Mean specific heat of the gas, $\mathbf{c_n}$:**

$$Q = m.c_n.dT$$

i.e.,
$$420 = 6 \times c_n \times (125 - 25)$$

$$\therefore \quad \mathbf{c_n} = \frac{420}{6 \times 100} = \mathbf{0.7\ kJ/kg°\text{-}C.} \quad \textbf{(Ans.)}$$

Example 2.12. *A temperature scale of certain thermometer is given by the relation*

$$t = a \ln p + b$$

where a and b are constants and p is the thermometric property of the fluid in the thermometer. If at the ice point and steam point the thermometric properties are found to be 1.5 and 7.5 respectively what will be the temperature corresponding to the thermometric property of 3.5 on Celsius scale. **(Poona University, Nov. 2001)**

Solution. $t = a \ln p + b$ ...(*Given*)

On Celsius scale :

Ice point = 0°C, and

Steam point = 100°C

∴ From given conditions, we have

$$0 = a \ln 1.5 + b \quad ...(i)$$

and
$$100 = a \ln 7.5 + b \quad ...(ii)$$

i.e.,
$$0 = a \times 0.4054 + b \quad ...(iii)$$

and
$$100 = a \times 2.015 + b \quad ...(iv)$$

Subtracting (*iii*) from (*iv*), we get

$$100 = 1.61a$$

or
$$a = 62.112$$

Substituting this value in eqn. (*iii*), we get

$$b = -\ 0.4054 \times 62.112 = -\ 25.18$$

∴ When $p = 3.5$ the value of temperature is given by

$$\boldsymbol{t} = 62.112 \ln (3.5) - 25.18 = \mathbf{52.63°C.} \quad \textbf{(Ans.)}$$

Example 2.13. *A thermocouple with test junction at t°C on gas thermometer scale and reference junction at ice point gives the e.m.f. as*

$$e = 0.20\ t - 5 \times 10^{-4} t^2\ mV.$$

The millivoltmeter is calibrated at ice and steam points. What will be the reading on this thermometer where the gas thermometer reads 70°C ?

Solution. $e = 0.20\ t - 5 \times 10^{-4} t^2$ mV ...(Given)

At ice point : When t = 0°C, $e = 0$

At steam point : When t = 100°C,

$$e = 0.20 \times 100 - 5 \times 10^{-4} \times (100)^2 = 15 \text{ mV}$$

Now, when t = 70°C

$$e = 0.20 \times 70 - 5 \times 10^{-4} \times (70)^2 = 11.55 \text{ mV}$$

∴ When the gas thermometer reads 70°C the thermocouple will read

$$\boldsymbol{t} = \frac{100 \times 11.55}{15} = \mathbf{77°C.} \quad \textbf{(Ans.)}$$

☞**Example 2.14.** *Comment whether the following quantities can be called as properties or not :*

(*i*) $\int pdV$, (*ii*) $\int Vdp$,*and* (*iii*) $\int pdV + \int Vdp$.

Solution. (*i*) $\int pdV$:

p is a function of *V* and integral can only be evaluated if relation between *p* and *V* is known. It is thus an *inexact differential* and hence **not a property.**

(*ii*) $\int Vdp$:

It is **not a property** for the *same reason* as mentioned in (*i*).

(*iii*) $\int pdV + \int Vdp$:

$$\int pdV + \int Vdp = \int pdV + Vdp = \int d(pV) = pV.$$

Thus the integral can be evaluated without knowing the relation between *p* and *V*. It is an *exact differential* and hence **it is a property.**

Example 2.15. *Gas from a cylinder of compressed helium is used to inflate an inelastic flexible balloon, originally folded completely flat, to a volume 0.6 m*^{3}*. If the barometer reads 760 mm Hg, what is the amount of work done upon the atmosphere by the balloon ? Sketch the system before and after the process.*

Solution. Refer Fig. 2.34. The firm line B_1 shows the boundary of the system before the process, and dotted line B_2 shows the boundary after the process.

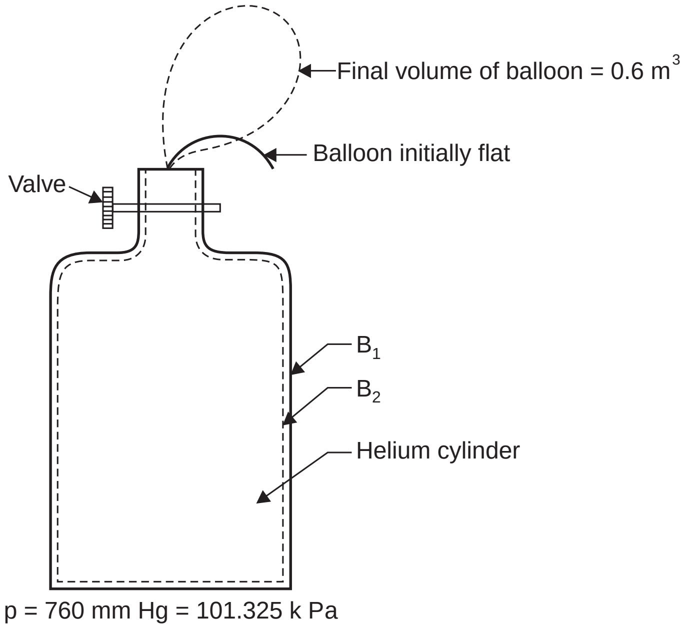


Fig. 2.34

The displacement work,

$$W_d = \int_{\text{cylinder}} pdV + \int_{\text{balloon}} pdV = 0 + \int_{\text{balloon}} pdV$$

$$= 101.325 \times 0.6 \qquad [\because \quad dV = 0.6 \text{ m}^3]$$

$$= \mathbf{60.795 \text{ kJ.}} \quad \textbf{(Ans.)}$$

This is a positive work, because the *work is done by the system*. Work done by the atmosphere is – 60.795 kJ. Since the wall of the cylinder is rigid there is no *pdV*-work involved in it.

It is assumed that the pressure in the balloon is atmospheric at all times, since the balloon fabric is light, inelastic and unstressed. If the balloon were elastic and stressed during the filling process, the work done by the gas would be *greater* than 60.795 kJ by an amount equal to the work done in stretching the balloon, although the displacement work done by atmosphere is still – 60.795 kJ. However, if the system includes both the gas and the balloon, the displacement work should be 60.795 kJ, as estimated above.

Example 2.16. *Determine the work done by the air which enters into an evacuated vessel from atmosphere when the valve is opened. The atmospheric pressure is 1.013 bar and 1.5 m³ of air at atmospheric condition enters into the vessel.*

Solution. Fig. 2.35 shows the initial and final condition of the system.

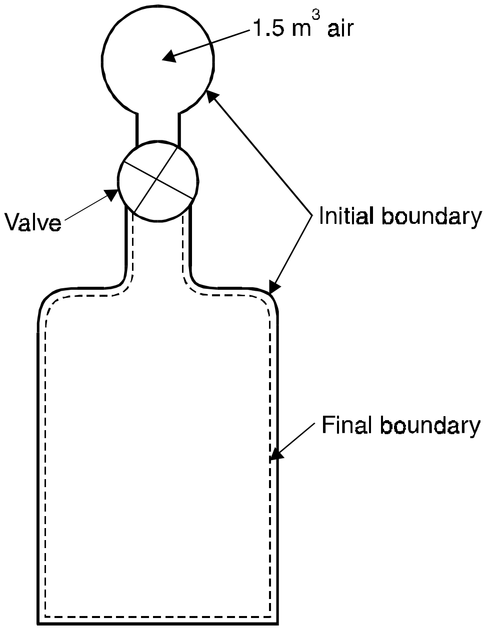


Fig. 2.35

No work is done by the boundary in contact with the vessel as the boundary does not move. Work is done by the external boundary at constant pressure.

$$\therefore \qquad W = \int_{V_1}^{V_2} p\,dV = \int_{1.5}^{0} p\,dV \qquad [\because \; V_1 = 1.5 \text{ m}^3 \text{ and } V_2 = 0]$$

$$= p(0 - 1.5) = 1.013 \times 10^5 \times (-1.5)$$

$$= -1.5195 \times 10^5 \text{ J} = -\mathbf{151.95 \text{ kJ.}} \quad \textbf{(Ans.)}$$

Since the free air boundary is contracting, the work done by the system is negative, and the surroundings do positive work upon the system.

☞ **Example 2.17.** *A piston and cylinder machine containing a fluid system has a stirring device as shown in Fig. 2.36. The piston is frictionless, and it is held down against the fluid due to atmospheric pressure of 101.3 kPa. The stirring device is turned 9500 revolutions with an average torque against the fluid of 1.25 Nm. Meanwhile the piston of 0.65 m diameter moves out 0.6 m. Find the net work transfer for the system.*

Solution. Refer Fig. 2.36.

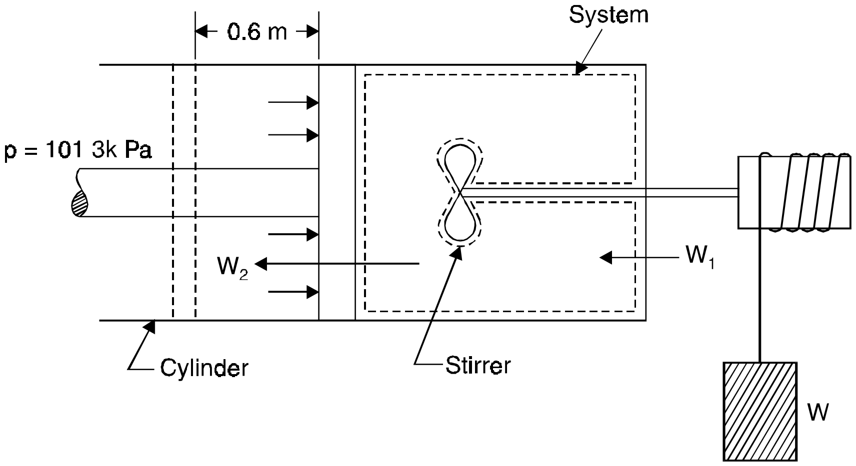


Fig. 2.36

Work done by *the stirring device* upon the system,

$$W_1 = 2\pi NT$$

where T = torque = 1.25 Nm

N = number of revolutions = 9500

$$W_1 = 2\pi \times 9500 \times 1.25 = 74622 \text{ Nm} = 74.622 \text{ kJ}$$

This is *negative work* for the system.

Work done *by the system* upon the surroundings

$$W_2 = (pA).\ L$$

where, p = Pressure = 101.3 kPa

A = Area of the piston = $\pi/4 \times (0.65)^2 = 0.3318 \text{ m}^2$, and

L = Distance moved by the piston = 0.6 m

$$W_2 = 101.3 \times 0.3318 \times 0.6 = 20.167 \text{ kJ}$$

This is a *positive work for the system*.

Hence, the net work transfer for the system

$$W_{net} = W_1 + W_2 = -\ 74.622 + 20.167 = \mathbf{-\ 54.455\ kJ.} \quad \textbf{(Ans.)}$$

Example 2.18. *A diesel engine piston which has an area of 45 cm^2 moves 5 cm during part of suction stroke. 300 cm^3 of fresh air is drawn in from the atmosphere. The pressure in the cylinder during suction stroke is 0.9 × 10^5 N/m^2 and the atmospheric pressure is 1.013 × 10^5 N/m^2. The difference between the suction and atmospheric pressure is accounted for flow resistance in the suction pipe and inlet valve. Find the net work done during the process.*

Solution. Area of diesel engine piston

$$= 45 \text{ cm}^2 = 45 \times 10^{-4} \text{ m}^2$$

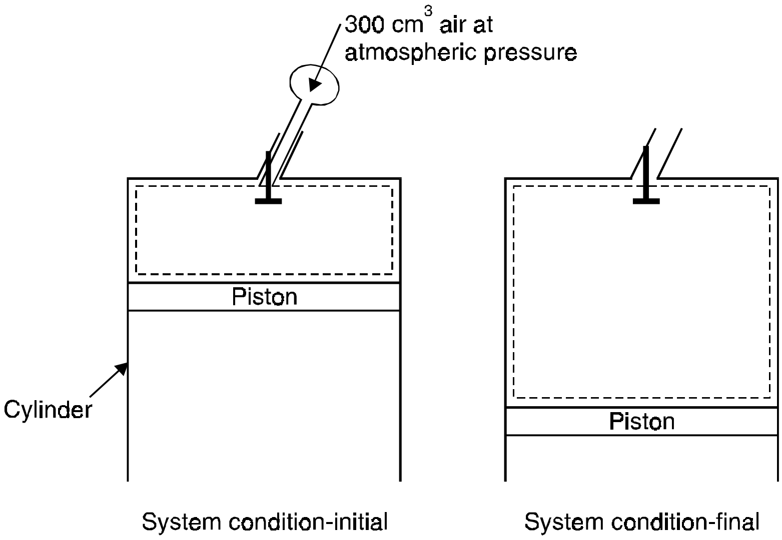


Fig. 2.37

Amount of fresh air drawn in from atmosphere

$$= 300 \text{ cm}^3 = 300 \times 10^{-6} \text{ m}^3$$

The pressure inside the cylinder during suction stroke

$$= 0.9 \times 10^5 \text{ N/m}^2$$

Atmospheric pressure $= 1.013 \times 10^5$ N/m^2

Initial and final conditions of the system are shown in Fig. 2.37.

Net work done = Work done by free air boundary + work done on the piston

The work done by the free air = – ve because *boundary contracts*

The work done by the cylinder on the piston = + ve because the *boundary expands*

∴ **Net work done** $= \int_{\text{Piston}} pdV + \int_{\substack{\text{Free air}\\ \text{boundary}}} pdV$

$$= \left[0.9 \times 10^5 \times 45 \times 10^{-4} \times \frac{5}{100} - 1.013 \times 10^5 \times 300 \times 10^{-6}\right]$$

$$= [20.25 - 30.39] = -\ \mathbf{10.14\ Nm\ or\ J.(Ans.)}$$

Example 2.19. *The properties of a closed system change following the relation between pressure and volume as pV = 3.0 where p is in bar V is in m^3. Calculate the work done when the pressure increases from 1.5 bar to 7.5 bar.*

Solution. Initial pressure, $p_1 = 1.5$ bar

Final pressure, $p_2 = 7.5$ bar

Relation between p and V, $pV = 3.0$

Work done, W :

The work done during the process is given by

$$W = \int_{V_1}^{V_2} pdV$$

$$V_1 = \frac{3.0}{p_1} = \frac{3.0}{1.5} = 2 \text{ m}^3$$

$$V_2 = \frac{3.0}{p_2} = \frac{3.0}{7.5} = 0.4 \text{ m}^3$$

$$\therefore \quad W = 10^5 \int_{2}^{0.4} \frac{3.0}{V} dV \text{ Nm} \qquad [\because \text{ 1 bar} = 10^5 \text{ N/m}^2]$$

$$= 10^5 \times 3.0 \left[\log_e V\right]_2^{0.4} = 10^5 \times 3.0(\log_e 0.4 - \log_e 2)$$

$$= -3 \times 10^5 \log_e (2/0.4) = -3 \times 10^5 \times \log_e 5 = -3 \times 10^5 \times 1.61 \text{ Nm}$$

$$= -4.83. \times 10^5 \text{ Nm} = -4.83 \times 10^5 \text{ J} = -\mathbf{483\ kJ.}\quad \textbf{(Ans.)}$$

Example 2.20. *To a closed system 150 kJ of work is supplied. If the initial volume is 0.6 m^3 and pressure of the system changes as p = 8 – 4V, where p is in bar and V is in m^3, determine the final volume and pressure of the system.*

Solution. Amount of work supplied to a closed system = 150 kJ

Initial volume = 0.6 m^3

Pressure-volume relationship, $p = 8 - 4V$

The work done during the process is given by

$$W = \int_{V_1}^{V_2} pdV$$

$$= 10^5 \int_{0.6}^{V_2} (8 - 4V)dV = 10^5 \left[8V - 4 \times \frac{V^2}{2}\right]_{0.6}^{V}$$

$$= 10^5[8(V_2 - 0.6) - 2(V_2^2 - 0.6^2)]$$

$$= 10^5[8V_2 - 4.8 - 2V_2^2 + 0.72]$$

$$= 10^5[8V_2 - 2V_2^2 - 4.08] \text{ Nm or J}$$

But this work is equal to – 150 × 10^3 J as this work is supplied to the system.

$$\therefore \quad -150 \times 10^3 = 10^5[8V_2 - 2V_2^2 - 4.08]$$

or $\quad 2V_2^2 - 8V_2 + 2.58 = 0$

$$V_2 = \frac{8 \pm \sqrt{64 - 4 \times 2 \times 2.58}}{4} = \frac{8 \pm 6.585}{4} = 0.354 \text{ m}^3$$

Positive sign is incompatible with the present problem, therefore it is not considered.

∴ **Final volume,** V_2 = **0.354 m^3. (Ans.)**

and, **final pressure,** $p_2 = 8 - 4V = 8 - 4 \times 0.354$

= 6.584 bar = **6.584 × 10^5 N/m^2 or Pa. (Ans.)**

REVERSIBLE WORK

Example 2.21. *A fluid at a pressure of 3 bar, and with specific volume of 0.18 m³/kg, contained in a cylinder behind a piston exapnds reversibly to a pressure of 0.6 bar according to a law,* $p = \frac{C}{v^2}$ *where C is a constant. Calculate the work done by the fluid on the piston.*

Solution. Refer Fig. 2.38.

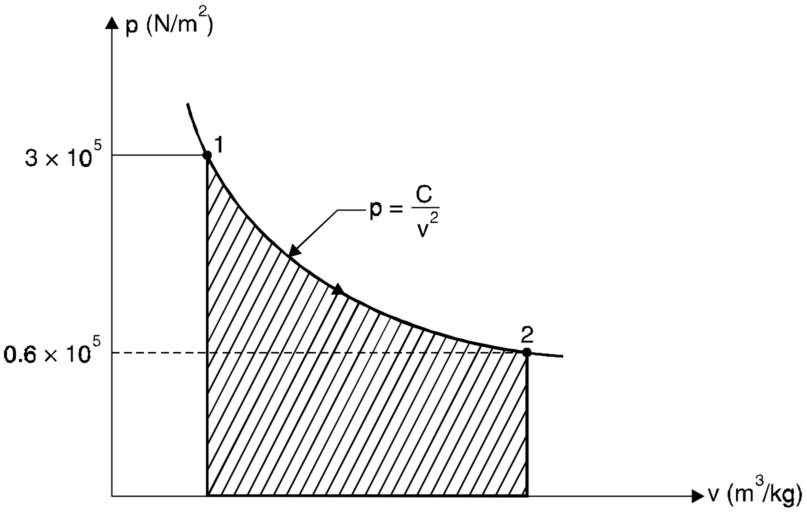


Fig. 2.38

$$p_1 = 3 \text{ bar} = 3 \times 10^5 \text{ N/m}^2$$
$$v_1 = 0.18 \text{ m}^3\text{/kg}$$

Work done = Shaded area = $\int_1^2 p\, dv$

i.e., Work done, $W = \int_1^2 \frac{C}{v^2}\,, dv = C\int_1^2 \frac{dv}{v^2} = C\left|\frac{v^{-2+1}}{-2+1}\right|_{v_1}^{v_2}$

$$= C\left[-v^{-1}\right]_{v_1}^{v_2} = C\left[-\frac{1}{v}\right]_{v_1}^{v_2} = C\left[\frac{1}{v_1} - \frac{1}{v_2}\right] \quad ...(i)$$

Also $C = pv^2 = p_1v_1^2 = 3 \times 0.18^2 = 0.0972 \text{ bar (m}^3\text{/kg)}^2$

and $v_2 = \sqrt{\frac{C}{p_2}} = \sqrt{\frac{0.0972}{0.6}} = 0.402 \text{ m}^3\text{/kg}$

Substituting the values of C, v_1 and v_2 in eqn. (*i*), we get

Work done, $W = 0.0972 \times 10^5 \left[\frac{1}{0.18} - \frac{1}{0.402}\right]$ Nm/kg

= **29840 Nm/kg. (Ans.)**

☞**Example 2.22.** *A cylinder contains 1 kg of a certain fluid at an initial pressure of 20 bar. The fluid is allowed to expand reversibly behind a piston according to a law* pV^2 *= constant until the volume is doubled. The fluid is then cooled reversibly at constant pressure until the piston regains its original position ; heat is then supplied reversibly with the piston firmly locked in position until the pressure rises to the original value of 20 bar. Calculate the net work done by the fluid, for an initial volume of 0.05* m^3.

Solution. Refer Fig. 2.39.

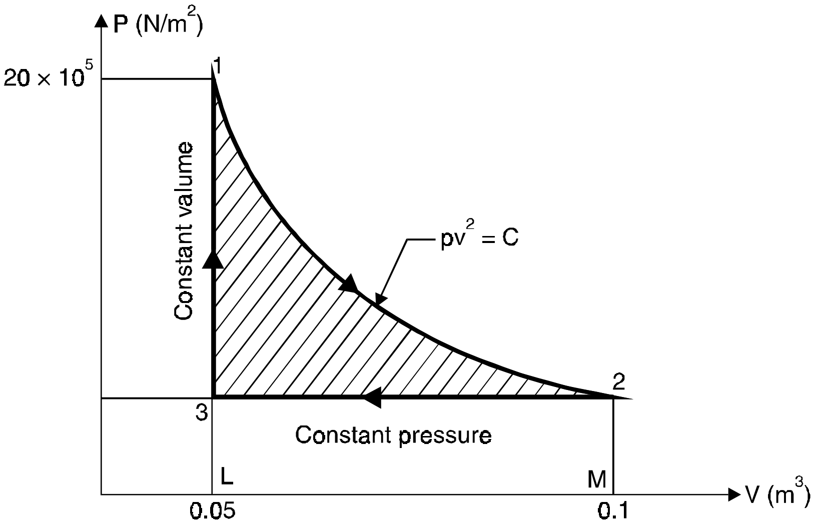


Fig. 2.39

Mass of fluid, $m = 1$ kg

$p_1 = 20 \text{ bar} = 20 \times 10^5 \text{ N/m}^2$

$V_1 = 0.05 \text{ m}^3$

Considering the process 1-2

$$p_1V_1^2 = p_2V_2^2$$

$$\therefore \quad p_2 = p_1\left(\frac{V_1}{V_2}\right)^2 = 20\left(\frac{V_1}{2V_1}\right)^2 \quad [\because \; V_2 = 2V_1 \text{ (given)}]$$

$$= \frac{20}{4} = 5 \text{ bar}$$

Work done by the fluid from 1 to 2 = Area 12 ML1 = $\int_1^2 p\,dV$

i.e., $$W_{1-2} = \int_{v_1}^{v_2} \frac{C}{V^2}\,dV \text{ , where } C = p_1V_1^2 = 20 \times 0.05^2 \text{ bar m}^6$$

$$\therefore \quad W_{1-2} = 10^5 \times 20 \times 0.0025 \left[-\frac{1}{V}\right]_{0.05}^{0.1}$$

$$= 10^5 \times 20 \times 0.0025 \left(\frac{1}{0.05} - \frac{1}{0.1}\right) = 50000 \text{ Nm}$$

Work done on fluid from 2 to 3

$= \text{Area } 32ML3 = p_2 (V_2 - V_3) = 10^5 \times 5 \times (0.1 - 0.05) = 25000 \text{ Nm}$

Work done during the process 3-1

= 0, because piston is locked in position (*i.e., Volume remains constant*)

∴ **Net work done by the fluid**

= Enclosed area 1231 = 50000 – 25000

= **25000 Nm. (Ans.)**

HIGHLIGHTS

1. *Thermodynamics* is an axiomatic science which deals with the relations among heat, work and properties of systems which are in equilibrium. It basically entails four laws or axioms known as *Zeroth, First, Second* and *Third* law of thermodynamics.
2. A *system* is a finite quantity of matter or a prescribed region of space.

   A system may be a *closed, open* or *isolated* system.
3. A *phase* is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.
4. A *homogeneous system* is one which consists of a *single phase*.
5. A *heterogeneous system* is one which consists of *two or more phases*.
6. A *pure substance* is one that has a homogeneous and invariable chemical composition even though there is a change of phase.
7. A system is in *thermodynamic equilibrium* if temperature and pressure at all points are same ; there should be no *velocity gradient*.
8. A *property of a system* is a characteristic of the system which depends upon its state, but not upon how the state is reached.

   Intensive properties do not depend on the mass of the system.

   Extensive properties depend on the mass of the system.
9. *State* is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of a system is called a state.
10. A *process* occurs when the system undergoes a change in state or an energy transfer takes place at a steady state.
11. Any process or series of processes whose end states are identical is termed a *cycle*.
12. The *pressure* of a system is the force exerted by the system on unit area of boundaries. Vacuum is defined as the absence of pressure.
13. A *reversible process* is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states.

    An *irreversible process* is one in which heat is transferred through a finite temperature.
14. Zeroth law of termodynamics states that if two systems are each equal in temperature to a third, they are equal in temperature to each other.
15. Infinite slowness is the characteristic feature of a quasi-static process. A quasi-static process is a succession of equilibrium states. It is also called a reversible process.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. A definite area or space where some thermodynamic process takes place is known as
(*a*) thermodynamic system (*b*) thermodynamic cycle
(*c*) thermodynamic process (*d*) thermodynamic law.

2. An open system is one in which
(*a*) heat and work cross the boundary of the system, but the mass of the working substance does not
(*b*) mass of working substance crosses the boundary of the system but the heat and work do not
(*c*) both the heat and work as well as mass of the working substances cross the boundary of the system
(*d*) neither the heat and work nor the mass of the working substances cross the boundary of the system.

3. An isolated system
(*a*) is a specified region where transfer of energy and/or mass take place
(*b*) is a region of constant mass and only energy is allowed to cross the boundaries
(*c*) cannot transfer either energy or mass to or from the surroundings
(*d*) is one in which mass within the system is not necessarily constant
(*e*) none of the above.

4. In an extensive property of a thermodynamic system
(*a*) extensive heat is transferred (*b*) extensive work is done
(*c*) extensive energy is utilised (*d*) all of the above
(*e*) none of the above.

5. Which of the following is an intensive property of a thermodynamic system ?
(*a*) Volume (*b*) Temperature
(*c*) Mass (*d*) Energy.

6. Which of the following is the extensive property of a thermodynamic system ?
(*a*) Pressure (*b*) Volume
(*c*) Temperature (*d*) Density.

7. When two bodies are in thermal equilibrium with a third body they are also in thermal equilibrium with each other. This statement is called
(*a*) Zeroth law of thermodyamics (*b*) First law of thermodynamics
(*c*) Second law of thermodynamics (*d*) Kelvin Planck's law.

8. The temperature at which the volume of a gas becomes zero is called
(*a*) absolute scale of temperature (*b*) absolute zero temperature
(*c*) absolute temperature (*d*) none of the above.

9. The value of one bar (in SI units) is equal to
(*a*) 100 N/m^2 (*b*) 1000 N/m^2
(*c*) 1×10^4 N/m^2 (*d*) 1×10^5 N/m^2
(*e*) 1×10^6 N/m^2.

10. The absolute zero pressure will be
(*a*) when molecular momentum of the system becomes zero
(*b*) at sea level (*c*) at the temperature of – 273 K
(*d*) under vacuum conditions (*e*) at the centre of the earth.

11. Absolute zero temperature is taken as
(*a*) – 273°C (*b*) 273°C
(*c*) 237°C (*d*) – 373°C.

12. Which of the following is *correct* ?
(*a*) Absolute pressure = gauge pressure + atmospheric pressure
(*b*) Gauge pressure = absolute pressure + atmospheric pressure
(*c*) Atmospheric pressure = absolute pressure + gauge pressure
(*d*) Absolute pressure = gauge pressure – atmospheric pressure.

13. The unit of energy in SI units is
(*a*) Joule (J)
(*b*) Joule metre (Jm)
(*c*) Watt (W)
(*d*) Joule/metre (J/m).

14. One watt is equal to
(*a*) 1 Nm/s
(*b*) 1 N/min
(*c*) 10 N/s
(*d*) 100 Nm/s
(*e*) 100 Nm/m.

15. One joule (J) is equal to
(*a*) 1 Nm
(*b*) kNm
(*d*) 10 Nm/s
(*d*) 10 kNm/s.

16. The amount of heat required to raise the temperature of 1 kg of water through 1°C is called
(*a*) specific heat at constant volume
(*b*) specific heat at constant pressure
(*c*) kilo calorie
(*d*) none of the above.

17. The heating and expanding of a gas is called
(*a*) thermodynamic system
(*b*) thermodynamic cycle
(*c*) thermodynamic process
(*d*) thermodynamic law.

18. A series of operations, which take place in a certain order and restore the initial condition is known as
(*a*) reversible cycle
(*b*) irreversible cycle
(*c*) thermodynamic cycle
(*d*) none of the above.

19. The condition for the reversibility of a cycle is
(*a*) the pressure and temperature of the working substance must not differ, appreciably, from those of the surroundings at any stage in the process
(*b*) all the processes, taking place in the cycle of operation, must be extremely slow
(*c*) the working parts of the engine must be friction free
(*d*) there should be no loss of energy during the cycle of operation
(*e*) all of the above
(*f*) none of the above.

20. In an irreversible process, there is a
(*a*) loss of heat
(*b*) no loss of heat
(*c*) gain of heat
(*d*) no gain of heat.

21. The main cause of the irreversibility is
(*a*) mechanical and fluid friction
(*b*) unrestricted expansion
(*c*) heat transfer with a finite temperature difference
(*d*) all of the above
(*e*) none of the above.

22. According to kinetic theory of heat
(*a*) temperature should rise during boiling
(*b*) temperature should fall during freezing
(*c*) at low temperature all bodies are in solid state
(*d*) at absolute zero there is absolutely no vibration of molecules
(*e*) none of the above.

23. A system comprising a single phase is called a
(*a*) closed system
(*b*) open system
(*c*) isolated system
(*d*) homogeneous system
(*e*) heterogeneous system.

Answers

1. (*a*) **2.** (*c*) **3.** (*c*) **4.** (*e*) **5.** (*b*) **6.** (*b*) **7.** (*a*)
8. (*b*) **9.** (*d*) **10.** (*a*) **11.** (*a*) **12.** (*a*) **13.** (*a*) **14.** (*a*)
15. (*a*) **16.** (*c*) **17.** (*b*) **18.** (*c*) **19.** (*e*) **20.** (*a*) **21.** (*d*)
22. (*d*) **23.** (*d*).

THEORETICAL QUESTIONS

1. Define a thermodynamic system. Differentiate between open system, closed system and an isolated system.

2. How does a homogeneous system differ from a heterogeneous system ?

3. What do you mean by a pure substance ?

4. Explain the following terms :
(*i*) State, (*ii*) Process, and (*iii*) Cycle.

5. Explain briefly zeroth law of thermodynamics.

6. What is a quasi-static process ?

7. What do you mean by 'reversible work' ?

UNSOLVED EXAMPLES

1. A cylindrical vessel of 60 cm diameter and 80 cm height contains 3.2 kg of a gas. The pressure measured with manometer indicates 60 cm of Hg above atmosphere when barometer reads 760 mm of Hg. Find : (*i*) The absolute pressure of the gas in the vessel in bar, and (*ii*) Specific volume and density of gas. [**Ans.**1.81 bar ; 0.09 m/kg ; 11.11 kg/m]

2. A force of 1600 N is applied uniformly on a piston of 8 cm diameter. Determine the pressure on the piston. [**Ans.**319 kN/m]

3. An oil of specific gravity 0.8 is contained in a tube to a depth of 80 cm. Determine the gauge pressure at this depth in kN/m. [**Ans.**6.275 kN/m]

4. A vacuum recorded in the condenser of a steam power plant is 745 mm of Hg. Find the absolute pressure in the condenser in Pa. The barometer reading is 760 mm of Hg. [**Ans.**2262 Pa]

5. The pressure of gas in a pipe line is measured with a mercury manometer having one limb open to the atmosphere. If the difference in the height of mercury in the two limbs is 562 mm, calculate the gas pressure. The barometer reads 761 mm Hg, the acceleration due to gravity is 9.79 m/s, and the density of mercury is 13640 kg/m. [**Ans.**1.77 bar]

6. A turbine is supplied with steam at a gauge pressure of 1.4 MPa. After expansion in the turbine the steam flows into a condenser which is maintained at a vacuum of 710 mm Hg. The barometric pressure is 772 mm Hg. Express the inlet and exhaust steam pressures in pascals (absolute). Take the density of mercury as 13600 kg/m. [**Ans.**1.503 MPa ; 8.27 kPa]

7. Gas from a bottle of compressed helium is used to inflate an inelastic flexible balloon, originally folded completely flat to a volume of 0.5 m. If the barometer reads 760 mm Hg, what is the amount of work done upon the atmosphere by the balloon ? [**Ans.**50.66 kJ]

8. A rubber balloon (flexible) is to be filled with hydrogen from a storage cylinder which contains hydrogen at 5 bar pressure until it has a volume of 1 m. The atmospheric pressure may be taken as 1.013 × 10 N/m. Determine the work done by the system comprising the hydrogen initially in the bottle. [**Ans.**101.3 kJ]

9. When the valve of the evacuated bottle is opened, atmospheric air rushes into it. If the atmospheric pressure is 101.325 kPa, and 1.2 m of air (measured at atmospheric conditions) enters into the bottle, calculate the work done by the air. [**Ans.**– 60.8 kJ]

10. A pump forces 1.2 m/min of water horizontally from an open well to a closed tank where the pressure is 0.9 MPa. Compute the work the pump must do upon the water in an hour just to force the water into the tank against the pressure. [**Ans.**15972 kJ]

11. 14.5 litres of gas at 172 MN/m is expanded at constant pressure until its volume becomes 130.5 litres. Determine the work done by the gas. [**Ans.**199.5 kJ]

12. 1 kg of a fluid is compressed reversibly according to a law $pv = 0.25$ where p is in bar and v is in m/kg. The final volume is 1/4 of the initial volume. Calculate the work done on the fluid and sketch the process on a p-v diagram. [**Ans.**34660 Nm]

13. A gas having a volume of 0.05 m and pressure of 6.9 bar expands reversibly in a cylinder behind a piston according to law *pv* = constant until the volume is 0.08 m. Calculate the work done by the gas. Also sketch the process on a *p-v* diagram. [**Ans.**15300 Nm]

14. A certain fluid at 10 bar is contained in a cylinder behind a piston, the initial volume being 0.05 m. Calculate the work done by the fluid when it expands reversibly, (*a*) At constant pressure to final volume of 0.2 m^3 ; (*b*) According to linear law to final volume of 0.2 m and a final pressure of 2 bar ; (*c*) According to a law *pV* = constant to a final volume of 0.1 m ; (*d*) According to law *pV* = constant to a final volume of 0.06 m. Sketch all processes on *p-V* diagram. [**Ans.**150000 ; 90000 ; 34700 ; 7640 Nm]

15. A fluid undergoes the following processes :

(*i*) Heated reversibly at a constant pressure of 1.05 bar until it has a specific volume of 0.1 m^3/kg.

(*ii*) It is then compressed reversibly according to a law *pv* = constant to a pressure of 4.2 bar.

(*iii*) It is then allowed to expand reversibly according to a law $pv^{1.3}$ = constant.

(*iv*) Finally it is heated at constant volume back to initial conditions.

The work done in the constant pressure process is 515 Nm and the mass of fluid present is 0.2 kg. Calculate the net work done on or by the fluid in the cycle and sketch the cycle on a *p-v* diagram. [**Ans.**– 422 Nm]

3

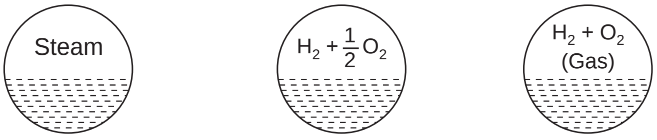
Properties of Pure Substances

3.1. DEFINITION OF THE PURE SUBSTANCE

A pure substance is a system which is (*i*) *homogeneous in composition*, (*ii*) *homogeneous in chemical aggregation*, and (*iii*) *invariable in chemical aggregation*.

— "*Homogeneous in composition*" means that the composition of each part of the system is the *same* as the composition of *every other part*. "*Composition* means the relative proportions of the chemical elements into which the sample can be analysed. It does not matter how these elements are combined.

For example in Fig. 3.1 system (*a*), comprising steam and water, is homogeneous in composition, since chemical analysis would reveal that hydrogen and oxygen atoms are presents in the ratio 2 : 1 whether the sample be taken from the steam or from the water. The same is true of system (*b*) containing uncombined hydrogen and oxygen gas in the atomic ratio 2 : 1 in the upper part, and water in the lower part. System (*c*) however, is not homogeneous in composition, for the hydrogen and oxygen are present in the ratio 1 : 1 in the upper part, but in the ratio 2 : 1 (as water) in the lower part.



(a)
Satisfies condition (i)
Satisfies condition (ii)

(b)
Satisfies condition (i)
Does not satisfy condition (ii)

(c)
Does not satisfy condition (i)

Fig. 3.1. Illustrating the definition of a pure substance.

— "*Homogeneous in chemical aggregation*" means that the chemical elements must be combined chemically in the same way in all parts of the system. Consideration of Fig. 3.1 again shows that the system (*a*) satisfies this condition also ; for steam and water consist of identical molecules. System (*b*) on the other hand is not homogeneous in chemical aggregation since in the upper part of the system the hydrogen and oxygen are not combined chemically (individual atoms of H and O are not uniquely associated), whereas in the lower part of the system the hydrogen and oxygen are combined to form water.

Note however that a uniform mixture of steam, hydrogen gas, and oxygen gas would be regarded as homogeneous in both composition and chemical aggregation whatever the relative proportions of the components.

— "*Invariable in chemical aggregation*" means that the state of chemical combination of the system does not change with *time* (condition (*ii*) referred to variation with *position*). Thus a mixture of hydrogen and oxygen, which changed into steam during the time that the system was under consideration, would not be a pure substance.

3.2. PHASE CHANGE OF A PURE SUBSTANCE

Let us consider 1 kg of liquid water at a temperature of 20°C in a cylinder fitted with a piston, which exerts on the water a constant pressure of one atmosphere (1.0132 bar) as shown in Fig. 3.2 (*i*).

— As the water is heated slowly its temperature rises until the temperature of the liquid water becomes 100°C. During the process of heating, the *volume slightly increases* as indicated by the line 1-2 on the temperature-specific volume diagram (Fig. 3.3). The piston starts moving upwards.

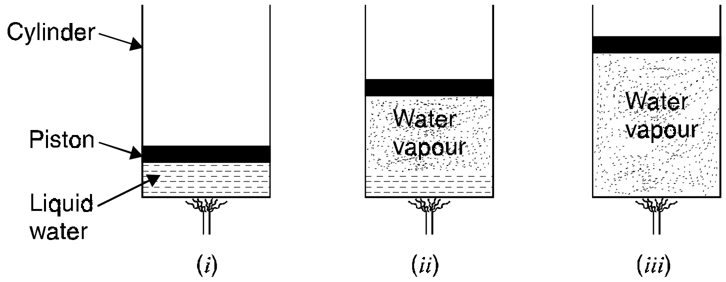


Fig. 3.2. Phase change of water at constant pressure from liquid to vapour phase.

— If the heating of the liquid, after it attains a temperature of 100°C, is continued it *undergoes a change in phase*. A portion of the liquid water changes into vapour as shown in Fig. 3.2 (*ii*). This state is described by the line 2-3 in Fig. 3.3. The amount of heat required to convert the liquid water completely into vapour under this condition is called the *heat of vapourisation*. The temperature at which vapourisation takes place at a given pressure is called the *saturation temperature* and the given pressure is called the *saturation pressure*.

During the process represented by the line 2-3 (Fig. 3.3) the volume increases rapidly and piston moves upwards Fig. 3.2 (*iii*).

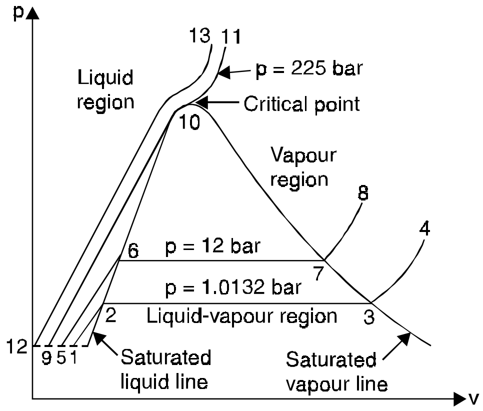


Fig. 3.3

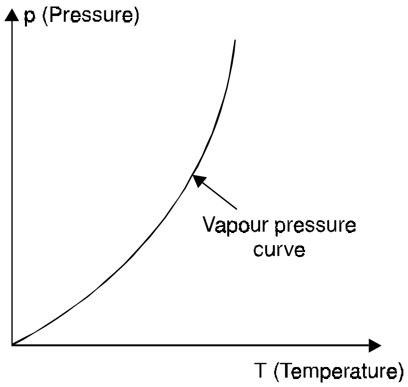


Fig. 3.4. Vapour pressure curve for water.

For a pure substance, definite relationship exists between the saturation pressure and saturation temperature as shown in Fig. 3.4, the curve so obtained is called *vapour pressure curve*.

— It may be noted that if the temperature of the liquid water on cooling becomes lower than the saturation temperature for the given pressure, the liquid water is called a *sub-cooled liquid*. The point '1' (in Fig. 3.3) illustrates this situation, when the liquid water is cooled under atmospheric pressure to a temperature of 20°C, which is below the saturation temperature (100°C).

— Further, at point '1' the temperature of liquid is 20°C and corresponding to this temperature, the saturation pressure is 0.0234 bar, which is lower than the pressure on the liquid water, which is 1 atmosphere. Thus the pressure on the liquid water is greater than the saturation pressure at a given temperature. In this condition, the liquid water is known as the *compressed liquid*.

The term *compressed liquid* or *sub-cooled liquid* is used to distinguish it from *saturated liquid. All points in the liquid region indicate the states of the compressed liquid.*

— When all the liquid has been evaporated completely and heat is further added, the *temperature of the vapour increases*. The curve 3-4 in Fig. 3.3 describes the process. When the temperature increases above the saturation temperature (in this case 100°C), the vapour is known as the *superheated vapour* and the temperature at this state is called the *superheated temperature*. There is *rapid increase in volume* and the piston moves upwards [Fig. 3.2 (*iii*)].

The difference between the superheated temperature and the saturation temperature at the given pressure is called the *degree of superheat*.

— If the above mentioned heating process is repeated at different pressures a number of curve similar to 1-2-3-4 are obtained. Thus, if the heating of the liquid water in the piston cylinder arrangement takes place under a constant pressure of 12 bar with an initial temperature of 20°C until the liquid water is converted into superheated steam, then curve 5-6-7-8 will represent the process.

— In the above heating process, it may be noted that, as the pressure increases the *length of constant temperature vapourisation gets reduced*.

From the heating process at a constant pressure of 225 bar represented by the curve 9-10-11 in Fig. 3.3, it can be seen that there is *no constant temperature vapourisation line*. The specific volume of the saturated liquid and of the saturated vapour is the same, *i.e.*, $v_f = v_g$. Such a state of the substance is called the *critical state*. The parameters like temperature, pressure, volume, etc. at such a state are called *critical parameters*.

— The curve 12-13 (Fig. 3.3) *represents a constant pressure heating process, when the pressure is greater than the critical pressure*. At this state*, the liquid water is directly converted into superheated steam*. As there is no definite point at which the liquid water changes into superheated steam, it is generally called *liquid water when the temperature is less than the critical temperature and superheated steam when the temperature is above the critical temperature*.

3.3. p-T (Pressure-Temperature) DIAGRAM FOR A PURE SUBSTANCE

If the vapour pressure of a solid is measured at various temperatures until the *triple point* is reached and then that of the liquid is measured until the critical point is reached, the result when plotted on a *p-T* diagram appears as in Fig. 3.5.

If the substance at the triple point is compressed until there is no vapour left and the pressure on the resulting mixture of liquid and solid is increased, the temperature will have to be changed for equilibrium to exist between the solid and the liquid.

Measurements of these pressures and temperatures give rise to a third curve on the *p-T* diagram, starting at the triple point and continuing indefinitely.

The points representing the coexistence of (*i*) *solid* and *vapour* lie on the '*sublimation curve*', (*ii*) *liquid* and *vapour* lie on the '*vapourisation curve*', (*iii*) *liquid* and *solid* lie on the '*fusion curve*'. In the particular case of *water*, the sublimation curve is called the *frost line*, the vapourisation curve is called the *steam line*, and the fusion curve is called the *ice line*.

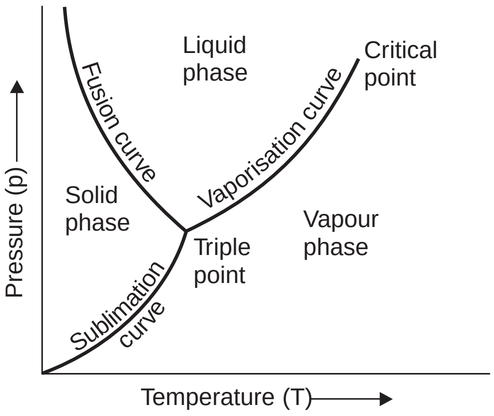


Fig. 3.5. *p*-T diagram for a substance such as water.

The slopes of sublimation and the vapourisation curves for all substances are *positive*. The slope of the fusion curve, however may be positive or negative. The fusion curve of *most substances* have a *positive slope*. Water is one of the important exceptions.

Triple point

The ***triple point*** *is merely the point of intersection of sublimation and vapourisation curves*. It must be understood that only on *p-T* diagram is the triple point represented by a *point*. On *p-V* diagram it is a *line*, and on a *U-V* diagram it is a *triangle*.

— The pressure and temperature at which all three phases of a pure substance coexist may be measured with the apparatus that is used to measure vapour pressure.

— Triple-point data for some interesting substances are given in Table 3.1.

Table 3.1. Triple-point Data

S. No.	*Substance*	*Temp., K*	*Pressure, mm Hg*
1.	Hydrogen (normal)	13.96	54.1
2.	Deuterium (normal)	18.63	128
3.	Neon	24.57	324
4.	Nitrogen	63.18	94
5.	Oxygen	54.36	1.14
6.	Ammonia	195.40	45.57
7.	Carbon dioxide	216.55	3.880
8.	Sulphur dioxide	197.68	1.256
9.	Water	273.16	4.58

3.4. p-V-T (Pressure-Volume-Temperature) SURFACE

A detailed study of the heating process reveals that the temperature of the solid rises and then during the change of phase from solid to liquid (or solid to vapour) the temperature remains constant. This phenomenon is common to all phase changes. Since the temperature is constant, pressure and temperature are not independent properties and connot be used to specify state during a change of phase.

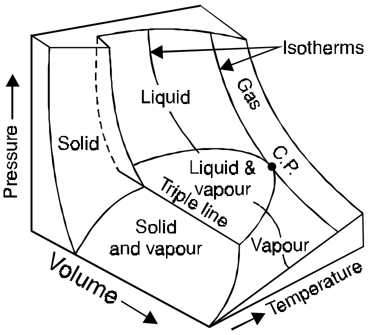


Fig. 3.6. A pressure-volume-temperature (p-V-T) surface.

The combined picture of change of pressure, specific volume and temperature may be shown on a three dimensional state model. Fig. 3.6 illustrates the equilibrium states for a pure substance which expands on fusion. Water is an example of a substance that exhibits this phenomenon.

All the equilibrium states lie on the surface of the model. States represented by the space above or below the surface are not possible. It may be seen that the *triple point appears as a line in this representation*. The point *C.P.* is called the *critical point* and no liquid phase exists at temperatures above the isotherms through this point. The term evaporation is meaningless in this situation.

At the critical point the temperature and pressure are called the critical temperature and the critical pressure respectively and when the temperature of a substance is above the critical value, it is called a gas. It is not possible to cause a phase change in a gas unless the temperature is lowered to a value less than the critical temperature. Oxygen and nitrogen are examples of gases that have critical temperatures below normal atmospheric temperature.

3.5. PHASE CHANGE TERMINOLOGY AND DEFINITIONS

Suffices :	Solid	i
	Liquid	f
	Vapour	g

Phase change	**Name**	**Process**	**Process suffix**
1. Solid-liquid	Fusion	Freezing, melting	*if*
2. Solid-vapour	Sublimation	Frosting, defrosting	*ig*
3. Liquid-vapour	Evaporation	Evaporating, Condensing	*fg*

Triple point—The only state at which the solid, liquid and vapour phases *coexist in equilibrium*.

Critical point (*C.P.*). The *limit of distinction between a liquid and vapour*.

Critical pressure. The pressure at the critical point.

Critical temperature. The temperature at the critical point.

Gas—A vapour whose *temperature is greater than the critical temperature*.

Liquid-vapour terms : Refer Fig. 3.7.

Saturation temperature. The phase change temperature corresponding to the saturation pressure. Sometimes called the *boiling temperature*.

Saturation pressure. The phase change pressure.

Compressed liquid. Liquid whose temperature is *lower than the saturation temperature*. Sometimes called a *sub-cooled liquid*.

Saturated liquid. Liquid at the saturation temperature corresponding to the saturation pressure. That is liquid about to commence evaporating, represented by the point *f* on a diagram.

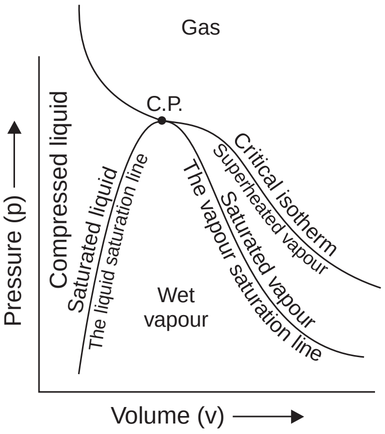


Fig. 3.7. Phase change terminology.

Saturated vapour. A term including wet and dry vapour.

Dry (saturated) vapour. Vapour which has just completed evaporation. The pressure and temperature of the vapour are the saturation values. Dry vapour is represented by a point *g* on a diagram.

Wet vapour. The mixture of saturated liquid and dry vapour during the phase change.

Superheated vapour. Vapour whose temperature is greater than the saturation temperature corresponding to the pressure of the vapour.

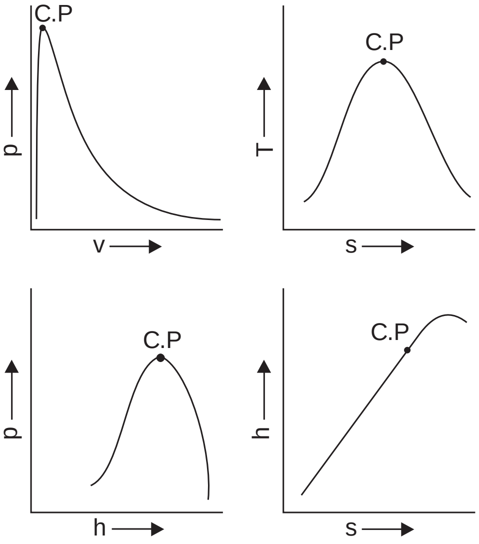
Degree of superheat. The term used for the numerical amount by which the temperature of a superheated vapour exceeds the saturation temperature.

3.6. PROPERTY DIAGRAMS IN COMMON USE

Besides *p-V* diagram which is useful because *pressure and volume are easily visualised* and the T-*s* chart which is used in *general thermodynamic work*, there are other charts which are of *practical use for particular applications*. The *specific enthalpy-specific entropy chart is used for steam plant work and the pressure-specific enthalpy chart is used in refrigeration work*. Sketches of these charts are shown in Fig. 3.8. These charts are drawn for H_2O (water and steam) and represent the correct shape of the curves for this substance.

3.7. FORMATION OF STEAM

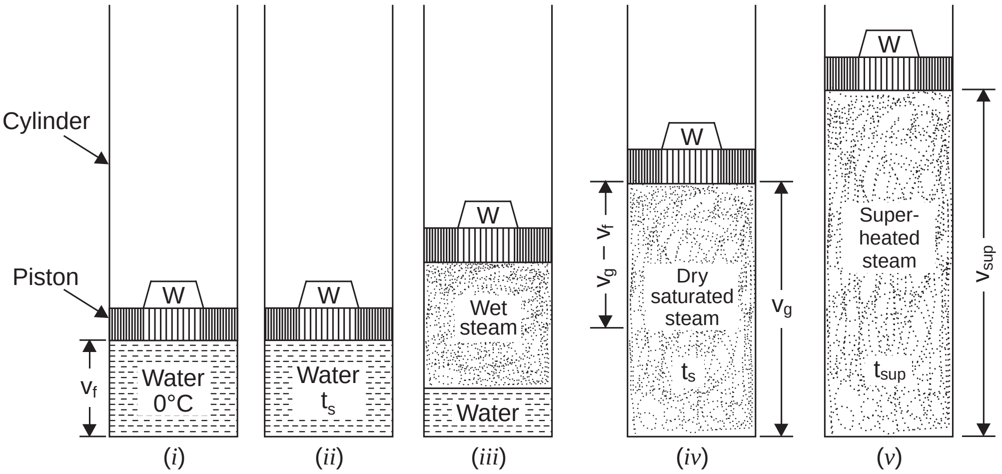
The process of formation of steam is discussed in detail in the following few pages :



C.P. = Critical point

Fig. 3.8

Consider a cylinder fitted with a piston which can move freely upwards and downwards in it. Let, for the sake of simplicity, there be 1 kg of water at 0°C with volume v_f m^3 under the piston [Fig 3.9 (*i*)]. Further let the piston is loaded with load W to ensure heating at constant pressure. Now if the heat is imparted to water, a rise in temperature will be noticed and this rise will continue till boiling point is reached. The temperature at which water starts boiling depends upon the pressure and as such for *each pressure* (under which water is heated) *there is a different boiling point*. This boiling temperature is known as the temperature of formation of steam or *saturation temperature*.



t_s = Saturation temp.
t_{sup} = Temperature of superheated steam

v_f = Volume of water
v_g = Volume of dry and saturated steam
v_{sup} = Volume of superheated steam

Fig. 3.9. Formation of steam.

It may be noted during heating up to boiling point that there will be slight increase in volume of water due to which piston moves up and hence work is obtained as shown in Fig. 3.9 (*ii*). This work, however, is so *small* that is can be *neglected*.

Now, if supply of heat to water is continued it will be noticed that rise of temperature after the boiling point is reached *nil* but piston starts moving upwards which indicates that there is increase is volume which is only possible if steam formation occurs. The heat being supplied does not show any rise of temperature but changes water into vapour state (steam) and is known as *latent heat* or *hidden heat*. So long as the steam is in contact with water, it is called *wet steam* [Fig. 3.9 (*iii*)] and if heating of steam is further progressed [as shown in Fig. 3.9 (*iv*)] such that all the water particles associated with steam are evaporated, the steam so obtained is called *dry and saturated steam*. If v_g m^3 is the volume of 1 kg of dry and saturated steam then work done on the piston will be

$$p(v_g - v_f) \qquad \text{...(3.1)}$$

where p is the constant pressure (due to weight 'W' on the piston).

Again, if supply of heat to the dry and saturated steam is continued at constant pressure there will be increase in temperature and volume of steam. The steam so obtained is called *superheated steam* and it *behaves like a perfect gas*. This phase of steam formation is illustrated in Fig. 3.9 (*v*).

Fig. 3.10 shows the graphical representation of formation of steam.

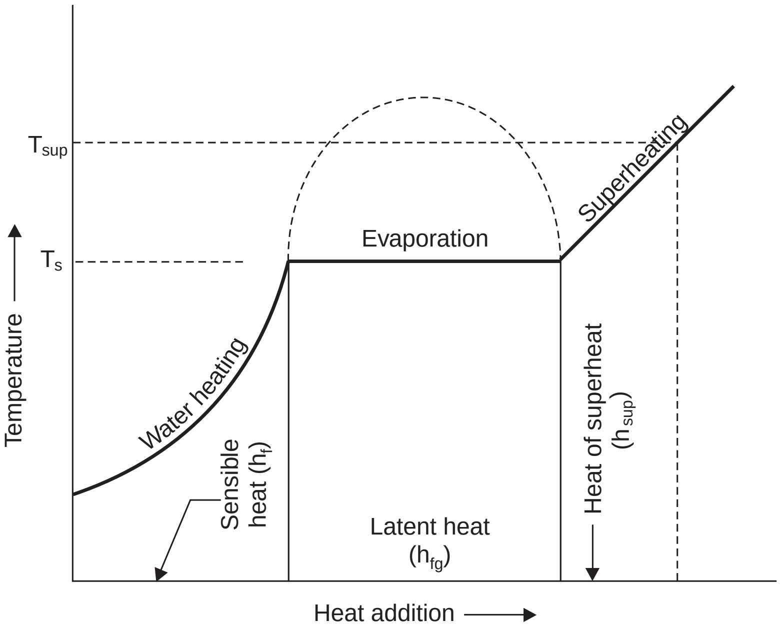


Fig. 3.10. Graphical representation of formation of steam.

3.8. IMPORTANT TERMS RELATING STEAM FORMATION

1. Sensible heat of water (h_f). *It is defined as the quantity of heat absorbed by 1 kg of water when it is heated from 0°C (freezing point) to boiling point.* It is also called total heat (or enthalpy) of water or liquid heat invariably. It is reckoned from 0°C where sensible heat is taken as zero. If 1 kg of water is heated from 0°C to 100°C the sensible heat added to it will be $4.18 \times 100 = 418$ kJ but if water is at say 20°C initially then sensible heat added will be $4.18 \times (100 - 20) = 334.4$ kJ. This type of heat is denoted by letter h_f and its value can be directly read from the steam tables.

Note. The value of specific heat of water may be taken as 4.18 kJ/kg K at low pressures but at high pressures it is different from this value.

2. Latent heat or hidden heat (h_{fg}). *It is the amount of heat required to convert water at a given temperature and pressure into steam at the same temperature and pressure.* It is expressed by the symbol h_{fg} and its value is available from steam tables. *The value of latent heat is not constant and varies according to pressure variation.*

3. Dryness fraction (x). The term dryness fraction is related with wet steam. *It is defined as the ratio of the mass of actual dry steam to the mass of steam containing it.* It is usually expressed by the symbol '*x*' or '*q*'.

If m_s = Mass of dry steam contained in steam considered, and

m_w = Weight of water particles in suspension in the steam considered,

Then,
$$x = \frac{m_s}{m_s + m_w} \qquad ...(3.2)$$

Thus if in 1 kg of wet steam 0.9 kg is the dry steam and 0.1 kg water particles then $x = 0.9$.

Note. No steam can be completely dry and saturated, so long as it is in contact with the water from which it is being formed.

4. Total heat or enthalpy of wet steam (h). *It is defined as the quantity of heat required to convert 1 kg of water at 0°C into wet steam at constant pressure*. It is the sum of total heat of water and the latent heat and this sum is also called **enthalpy.**

In other words, $$h = h_f + xh_{fg} \qquad \text{...(3.3)}$$

If steam is dry and saturated, then $x = 1$ and $h_g = h_f + h_{fg}$.

5. Superheated steam. When steam is heated after it has become dry and saturated, it is called superheated steam and the process of heating is called *superheating*. *Superheating is always carried out at constant pressure*. The additional amount of heat supplied to the steam during superheating is called as '*Heat of superheat*' and can be calculated by using the specific heat of superheated steam at constant pressure (c_{ps}), the value of which varies from 2.0 to 2.1 kJ/kg K depending upon pressure and temperature.

If T_{sup}., T_s are the temperatures of superheated steam in K and wet or dry steam, then $(T_{sup} - T_s)$ is called '*degree of superheat*'.

The total heat of superheated steam is given by

$$h_{sup} = h_f + h_{fg} + c_{ps}\,(T_{sup} - T_s) \qquad \text{...(3.4)}$$

Superheated steam behaves like a gas and therefore it follows the gas laws. The value of n for this type of steam is 1.3 and the law for the adiabatic expansion is $pv^{1.3}$ = constant.

The ***advantages*** obtained by using '*superheated*' steam are as follows :

(*i*) By superheating steam, its heat content and hence its capacity to do work is increased without having to increase its pressure.

(*ii*) Superheating is done in a superheater which obtains its heat from waste furnace gases which would have otherwise passed uselessly up the chimney.

(*iii*) High temperature of superheated steam results in an increase in thermal efficiency.

(*iv*) Since the superheated steam is at a temperature above that corresponding to its pressure, it can be considerably cooled during expansion in an engine before its temperature falls below that at which it will condense and thereby become wet. Hence, heat losses due to condensation of steam on cylinder walls etc. are avoided to a great extent.

6. Volume of wet and dry steam. If the steam has dryness fraction of x, then 1 kg of this steam will contain x kg of dry steam and $(1 - x)$ kg of water. If v_f is the volume of 1 kg of water and v_g is the volume of 1 kg of perfect dry steam (also known as specific volume), then volume of 1 kg of wet steam = volume of dry steam + volume of water.

$$= xv_g + (1 - x)v_f \qquad \text{...(3.5)}$$

Note. The volume of v_f at low pressures is very small and is generally neglected. Thus is general, the volume of 1 kg of wet steam is given by, xv_g and density $\frac{1}{xv_g}$ kg/m³.

$$\begin{aligned}
&= xv_g + v_f - xv_f \\
&= v_f + x(v_g - v_f) \\
&= v_f + xv_{fg} && \text{...[3.5 (}a\text{)]} \\
&= v_f + xv_{fg} + v_{fg} - v_{fg} \\
&= (v_f + v_{fg}) - (1 - x)\,v_{fg} \\
&= v_g - (1 - x)v_{fg} && \text{...[3.5 (}b\text{)]}
\end{aligned}$$

7. Volume of superheated steam. As superheated steam behaves like a perfect gas its volume can be found out in the same way as the gases.

If, v_g = Specific volume of dry steam at pressure p,

T_s = Saturation temperature in K,
T_{sup} = Temperature of superheated steam in K, and
v_{sup} = Volume of 1 kg of superheated steam at pressure p,

Then,
$$\frac{p\,.\,v_g}{T_s} = \frac{p\,.\,v_{\text{sup}}}{T_{\text{sup}}}$$

or,
$$v_{sup} = \frac{v_g\,T_{sup}}{T_s} \quad ...(3.6)$$

3.9. THERMODYNAMIC PROPERTIES OF STEAM AND STEAM TABLES

In engineering problem, for any fluid which is used as working fluid, the six basic thermodynamic properties required are : p (pressure), T (temperature), v (volume), u (internal energy), h (enthalpy) and s (entropy). These properties must be known at different pressure for analysing the thermodynamic cycles used for work producing devices. The values of these properties are determined theoretically or experimentally and are tabulated in the form of tables which are known as '*Steam Tables*'. The properties of wet steam are then computed from such tabulated data. Tabulated values are also available for superheated steam. It may be noted that *steam has only one saturation temperature at each pressure*.

Following are the thermodynamic properties of steam which are tabulated in the form of table :

p = Absolute pressure (bar or kPa) ;
t_s = Saturation temperature (°C) ;
h_f = Enthalpy of saturated liquid (kJ/kg) ;
h_{fg} = Enthalpy or latent heat of vapourisation (kJ/kg) ;
h_g = Enthalpy of saturated vapour (steam) (kJ/kg) ;
s_f = Entropy of saturated liquid (kJ/kg K) ;
s_{fg} = Entropy of vapourisation (kJ/kg K) ;
s_g = Entropy of saturated vapour (steam) (kJ/kg K) ;
v_f = Specific volume of saturated liquid (m^3/kg) ;
v_g = Specific volume of saturated vapour (steam) (m^3/kg).

Also,
$h_{fg} = h_g - h_f$ ...... Change of enthalpy during evaporation
$s_{fg} = s_g - s_f$ ...... Change of entropy during evaporation
$v_{fg} = v_g - v_f$ ...... Change of volume during evaporation.

The above mentioned properties at different pressures are tabulated in the form of tables as under :

The internal energy of steam ($u = h - pv$) is also tabulated in some steam tables.

STEAM TABLES

Absolute pressure	*Temperature °C*	*Specific enthalpy kJ/kg*			*Specific entropy kJ/kg K*			*Specific volume m³/kg*	
bar, p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
1.0	99.6	417.5	2257.9	2675.4	1.3027	6.0571	7.3598	0.001043	1.6934
50.0	263.9	1154.9	1639.7	2794.2	2.9206	3.0529	5.9735	0.001286	0.00394
100.0	311.1	1408.0	1319.7	2727.7	3.3605	2.2593	5.6198	0.001452	0.01811

3.10. EXTERNAL WORK DONE DURING EVAPORATION

When water is evaporated to form *saturated* steam, its volume increases from v_f to v_g at a constant pressure, and thus external work is done by steam due to increase in volume. The energy for doing the work is obtained during the absorption of latent heat. This work is called **external work of evaporation** and is given by $p(v_g - v_f)$.

i.e., External work of evaporation $= p(v_g - v_f)$ ...(3.7)

As at low pressure v_f is very small and hence neglected, work of evaporation is

$$p \,.\, v_g \quad \text{...(3.8)}$$

In case of *wet* steam with dryness fraction x, work of evaporation will be

$$pxv_g \quad \text{...(3.9)}$$

3.11. INTERNAL LATENT HEAT

The latent heat consists of true latent heat and the work of evaporation. This true latent heat is called the *internal latent heat* and may also be found as follows :

$$\text{Internal latent heat} = h_{fg} - \frac{pv_g}{J} \quad \text{...(3.10)}$$

J = 1 in SI units.

3.12. INTERNAL ENERGY OF STEAM

It is defined as the actual energy stored in the steam. As per previous articles, the total heat of steam is sum of sensible heat, internal latent heat and the external work of evaporation. Work of evaporation is not stored in the steam as it is utilised in doing external work. Hence the internal energy of steam could be found by subtracting work of evaporation from the total heat.

In other words,

$$h = \frac{pv_g}{J} + u, \text{ where } u \text{ is internal energy of 1 kg of steam at pressure } p$$

or
$$u = h - \frac{pv_g}{J}$$

In case of wet steam with dryness fraction 'x'

$$u = h - \frac{pxv_g}{J} \quad \text{...(3.11)}$$

and if steam is superheated to a volume of v_{sup} per kg.

$$h_{sup} = h_f + h_{fg} + c_{ps}\,(T_{sup} - T_s)$$

and
$$u = h_{sup} - \frac{p \,.\, v_{sup}}{J} \quad \text{...(3.12)}$$

3.13. ENTROPY OF WATER

(**Note.** For definition of entropy please refer to chapter 5.)

Consider 1 kg of water being heated from temperature T_1 to T_2 at constant pressure. The change in entropy will be given by,

$$ds = \frac{dQ}{T} = c_{pw} \,.\, \frac{dT}{T}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} c_{pw} \frac{dT}{T}$$

$$s_2 - s_1 = c_{pw} \log_e \frac{T_2}{T_1} \quad \text{...(3.13)}$$

If 0°C is taken as datum, then entropy of water per kg at any temperature T above this datum will be

$$s_f = c_{pw} \log_e \frac{T}{273} \quad \text{...(3.14)}$$

3.14. ENTROPY OF EVAPORATION

The change of entropy (ds) is given by,

$$ds = \frac{dQ}{T}$$

or $s_2 - s_1 = \dfrac{Q}{T}$, where Q is the heat absorbed.

When water is evaporated to steam completely the heat absorbed is the latent heat and this heat goes into water without showing any rise of temperature.

Then $Q = h_{fg}$

and $$s_{evap.} = \frac{h_{fg}}{T_s} \quad \text{...(3.15)}$$

However, in case of wet steam with dryness fraction x the evaporation will be partial and heat absorbed will be xh_{fg} per kg of steam. The change of entropy will be $\dfrac{xh_{fg}}{T_s}$.

3.15. ENTROPY OF WET STEAM

The total entropy of wet steam is the sum of entropy of water (s_f) and entropy of evaporation (s_{fg}).

In other words, $$s_{wet} = s_f + \frac{xh_{fg}}{T_s} \quad \text{...(3.16)}$$

where s_{wet} = Total entropy of wet steam,
s_f = Entropy of water, and
$\dfrac{xh_{fg}}{T_s}$ = Entropy of evaporation.

If steam is dry and saturated, *i.e.*, $x = 1$, then

$$s_g = s_f + \frac{h_{fg}}{T_s} \quad \text{...(3.17)}$$

3.16. ENTROPY OF SUPERHEATED STEAM

Let 1 kg of dry saturated steam at T_s (saturation temperature of steam) be heated to T_{sup}. If specific heat at constant pressure is c_{ps}, then change of entropy during superheating at constant pressure p

$$= c_{ps} \log_e \left(\frac{T_{sup}}{T_s}\right).$$

Total entropy of superheated steam above the freezing point of water.

s_{sup} = Entropy of dry saturated steam + change of entropy during superheating

$$= s_f + \frac{h_{fg}}{T_s} + c_{ps} \log_e \left(\frac{T_{sup}}{T_s}\right) = s_g + c_{ps} \log_e \left(\frac{T_{sup}}{T_s}\right) \qquad \text{...(3.18)}$$

3.17. ENTHALPY-ENTROPY (h-s) CHART OR MOLLIER DIAGRAM

Dr. Mollier, in 1904, conceived the idea of plotting total heat against entropy, and his diagram is more widely used than any other entropy diagram, since the work done on vapour cycles can be scaled from this diagram directly *as a length ; whereas on T-s diagram it is represented by an area*.

A sketch of the *h-s* chart is shown in Fig. 3.11.

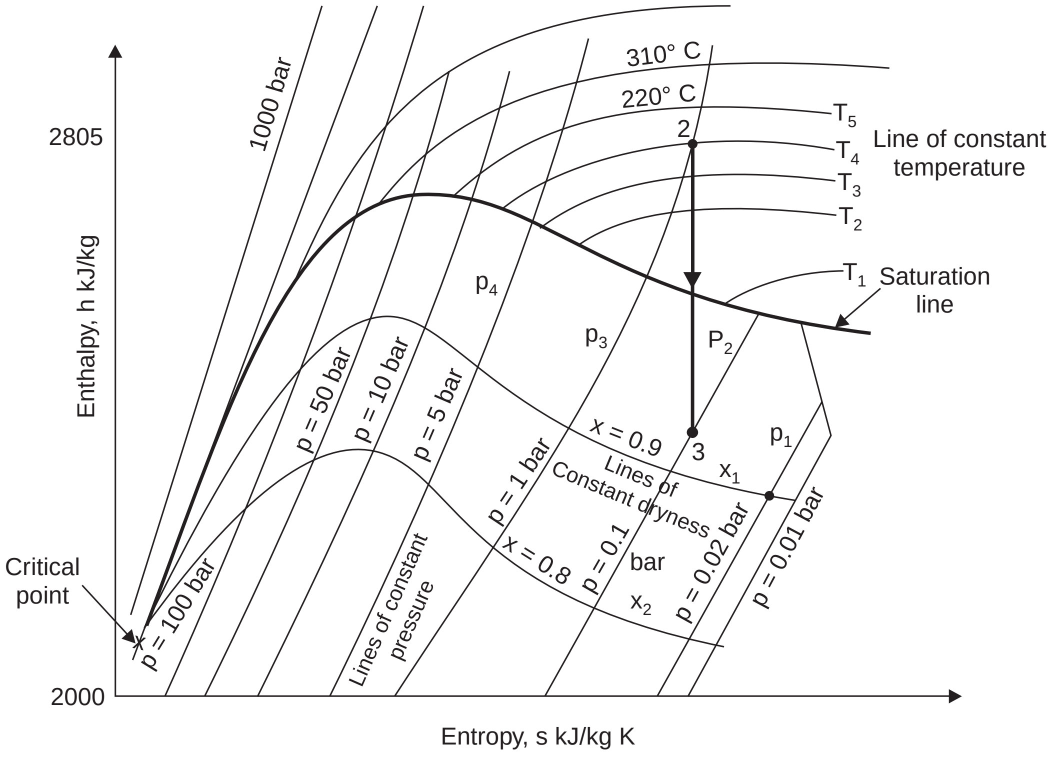


Fig. 3.11. Enthalpy-entropy (*h-s*) chart.

— Lines of constant pressure are indicated by p_1, p_2 etc., lines of constant temperature by T_1, T_2, etc.

— Any two independent properties which appear on the chart are sufficient to define the state (*e.g.*, p_1 and x_1 define state 1 and h can be read off the vertical axis).

— In the superheat region, pressure and temperature can define the state (*e.g.*, p_3 and T_4 define the state 2, and h_2 can be read off).

— A line of constant entropy between two state points 2 and 3 defines the properties at all points during an *isentropic process* between the two states.

Example 3.1. *Calculate the dryness fraction (quality) of steam which has 1.5 kg of water in suspension with 50 kg of steam.*

Solution. Mass of dry steam, m_s = 50 kg

Mass of water in suspension, m_w = 1.5 kg

$$\therefore \textbf{ Dryness fraction, } x = \frac{\text{Mass of dry steam}}{\text{Mass of dry steam + mass of water in suspension}}$$

$$= \frac{m_s}{m_s + m_w} = \frac{50}{50 + 1.5} = \mathbf{0.971.} \quad \textbf{(Ans.)}$$

☞ **Example 3.2.** *A vessel having a volume of 0.6 m^3 contains 3.0 kg of liquid water and water vapour mixture in equilibrium at a pressure of 0.5 MPa. Calculate :*

(*i*) *Mass and volume of liquid* ;

(*ii*) *Mass and volume of vapour.*

Solution. Volume of the vessel, V = 0.6 m^3

Mass of liquid water and water vapour, m = 3.0 kg

Pressure, p = 0.5 MPa = 5 bar

Thus, specific volume, $v = \frac{V}{m} = \frac{0.6}{3.0} = 0.2 \text{ m}^3/\text{kg}$

At 5 bar : From steam tables,

$$v_{fg} = v_g - v_f = 0.375 - 0.00109 = 0.3739 \text{ m}^3/\text{kg}$$

We know that, $v = v_g - (1 - x)\, v_{fg}$, where x = quality of the vapour.

$$0.2 = 0.375 - (1 - x) \times 0.3739$$

$$\therefore \quad (1 - x) = \frac{(0.375 - 0.2)}{0.3739} = 0.468$$

or $x = 0.532$

(*i*) **Mass and volume of liquid, $\mathbf{m_{liq.}}$ = ? $\mathbf{V_{liq.}}$ = ?**

$$\mathbf{m_{liq.}} = m(1 - x) = 3.0 \times 0.468 = \mathbf{1.404 \text{ kg.}} \quad \textbf{(Ans.)}$$

$$\mathbf{V_{liq.}} = m_{liq.}\, v_f = 1.404 \times 0.00109 = \mathbf{0.0015 \text{ m}^3.} \quad \textbf{(Ans.)}$$

(*ii*) **Mass and volume of vapour, $\mathbf{m_{vap.}}$ = ? $\mathbf{V_{vap.}}$ = ?**

$$\mathbf{m_{vap.}} = m.x = 3.0 \times 0.532 = \mathbf{1.596 \text{ kg.}} \quad \textbf{(Ans.)}$$

$$\mathbf{V_{vap.}} = m_{vap.}\, v_g = 1.596 \times 0.375 = \mathbf{0.5985 \text{ m}^3.} \quad \textbf{(Ans.)}$$

☞ **Example 3.3.** *A vessel having a capacity of 0.05 m^3 contains a mixture of saturated water and saturated steam at a temperature of 245°C. The mass of the liquid present is 10 kg. Find the following :*

(*i*) *The pressure,* (*ii*) *The mass,*

(*iii*) *The specific volume,* (*iv*) *The specific enthalpy,*

(*v*) *The specific entropy, and* (*vi*) *The specific internal energy.*

Solution. From steam tables, corresponding to 245°C :

$$p_{sat} = 36.5 \text{ bar}, \; v_f = 0.001239 \text{ m}^3/\text{kg}, \; v_g = 0.0546 \text{ m}^3/\text{kg}$$

$$h_f = 1061.4 \text{ kJ/kg}, \; h_{fg} = 1740.2 \text{ kJ/kg}, \; s_f = 2.7474 \text{ kJ/kg K}$$

$$s_{fg} = 3.3585 \text{ kJ/kg K.}$$

(*i*) **The pressure = 36.5 bar** (or 3.65 MPa). **(Ans.)**

(*ii*) **The mass, m :**

Volume of liquid, $V_f = m_f v_f$

$= 10 \times 0.001239 = 0.01239 \text{ m}^3$

Volume of vapour, $V_g = 0.05 - 0.01239 = 0.03761 \text{ m}^3$

∴ Mass of vapour, $m_g = \dfrac{V_g}{v_g} = \dfrac{0.03761}{0.0546} = 0.688$ kg

∴ The total mass of mixture,

$$\mathbf{m} = m_f + m_g = 10 + 0.688 = \mathbf{10.688\ kg.}\ \textbf{(Ans.)}$$

(*iii*) **The specific volume, v :**

Quality of the mixture,

$$x = \frac{m_g}{m_g + m_f} = \frac{0.688}{0.688 + 10} = 0.064$$

∴ $\mathbf{v} = v_f + x v_{fg}$

$= 0.001239 + 0.064 \times (0.0546 - 0.001239)$ $(\because\ v_{fg} = v_g - v_f)$

$= \mathbf{0.004654\ m^3/kg.}$ **(Ans.)**

(*iv*) **The specific enthalpy, h :**

$\mathbf{h} = h_f + x h_{fg}$

$= 1061.4 + 0.064 \times 1740.2 = \mathbf{1172.77\ kJ/kg.}$ **(Ans.)**

(*v*) **The specific entropy, s :**

$\mathbf{s} = s_f + x s_{fg}$

$= 2.7474 + 0.064 \times 3.3585 = \mathbf{2.9623\ kJ/kg\ K.}$ **(Ans.)**

(*vi*) **The specific internal energy, u :**

$\mathbf{u} = h - pv$

$$= 1172.77 - \frac{36.5 \times 10^5 \times 0.004654}{1000} = \mathbf{1155.78\ kJ/kg.}\ \textbf{(Ans.)}$$

Example 3.4. *Determine the amount of heat, which should be supplied to 2 kg of water at 25°C to convert it into steam at 5 bar and 0.9 dry.*

Solution. Mass of water to be converted to steam, $m_w = 2$ kg

Temperature of water, $t_w = 25°\text{C}$

Pressure and dryness fraction of steam = 5 bar, 0.9 dry

At 5 bar : From steam tables,

$h_f = 640.1$ kJ/kg ; $h_{fg} = 2107.4$ kJ/kg

Enthalpy of 1 kg of steam (above 0°C)

$h = h_f + x h_{fg}$

$= 640.1 + 0.9 \times 2107.4 = 2536.76$ kJ/kg

Sensible heat associated with 1 kg of water

$= m_w \times c_{pw} \times (t_w - 0)$

$= 1 \times 4.18 \times (25 - 0) = 104.5$ kJ

Net quantity of heat to be supplied per kg of water

$= 2536.76 - 104.5 = 2432.26$ kJ

Total amount of heat to be supplied

$$= 2 \times 2432.26 = \mathbf{4864.52\ kJ.}\quad \textbf{(Ans.)}$$

Example 3.5. *What amount of heat would be required to produce 4.4 kg of steam at a pressure of 6 bar and temperature of 250°C from water at 30°C ? Take specific heat for superheated steam as 2.2 kJ/kg K.*

Solution. Mass of steam to be produced, $m = 4.4$ kg

Pressure of steam, $p = 6$ bar

Temperature of steam, $t_{sup} = 250°C$

Temperature of water $= 30°C$

Specific heat of steam, $c_{ps} = 2.2$ kJ/kg

At 6 bar, 250°C : From steam tables,

$$t_s = 158.8°C,\ h_f = 670.4 \text{ kJ/kg},\ h_{fg} = 2085 \text{ kJ/kg}$$

Enthalpy of 1 kg superheated steam reckoned from 0°C,

$$\begin{aligned} h_{sup} &= h_f + h_{fg} + c_{ps}\,(T_{sup} - T_s) \\ &= 670.4 + 2085 + 2.2(250 - 158.8) \\ &= 2956 \text{ kJ} \end{aligned}$$

Amount of heat already with 1 kg of water

$$= 1 \times 4.18 \times (30 - 0) = 125.4 \text{ kJ}$$

Net amount of heat required to be supplied per kg

$$= 2956 - 125.4 = 2830.6 \text{ kJ}$$

Total amount of heat required

$$= 4.4 \times 2830.6 = \mathbf{12454.6\ kJ.}\quad \textbf{(Ans.)}$$

☞ **Example 3.6.** *Determine the mass of 0.15 m^3 of wet steam at a pressure of 4 bar and dryness fraction 0.8. Also calculate the heat of 1 m^3 of steam.*

Solution. Volume of wet steam, $v = 0.15$ m^3

Pressure of wet steam, $p = 4$ bar

Dryness fraction, $x = 0.8$

At 4 bar. From steam tables,

$v_g = 0.462$ m^3/kg, $h_f = 604.7$ kJ/kg, $h_{fg} = 2133$ kJ/kg

$$\therefore \qquad \text{Density} = \frac{1}{xv_g} = \frac{1}{0.8 \times 0.462} = 2.7056 \text{ kg/m}^3$$

Mass of 0.15 m^3 of steam

$$= 0.15 \times 2.7056 = \mathbf{0.4058\ kg.}\quad \textbf{(Ans.)}$$

Total heat of 1 m^3 of steam which has a mass of 2.7056 kg

$$\begin{aligned} &= 2.7056\, h \text{ (where } h \text{ is the total heat of 1 kg of steam)} \\ &= 2.7056\,(h_f + xh_{fg}) \\ &= 2.7056(604.7 + 0.8 \times 2133) \\ &= \mathbf{6252.9\ kJ.}\quad \textbf{(Ans.)} \end{aligned}$$

Example 3.7. *1000 kg of steam at a pressure of 16 bar and 0.9 dry is generated by a boiler per hour. The steam passes through a superheater via boiler stop valve where its temperature is raised to 380°C. If the temperature of feed water is 30°C, determine :*

(*i*) *The total heat supplied to feed water per hour to produce wet steam.*

(*ii*) *The total heat absorbed per hour in the superheater.*

Take specific heat for superheated steam as 2.2 kJ/kg K.

Solution. Mass of steam generated, $m = 1000$ kg/h

Pressure of steam, $p = 16$ bar

Dryness fraction, $x = 0.9$

Temperature of superheated steam,

$$T_{sup} = 380 + 273 = 653 \text{ K}$$

Temperature of feed water = 30°C

Specific heat of superheated steam, $c_{ps} = 2.2$ kJ/kg K.

At 16 bar. From steam tables,

$$t_s = 201.4°\text{C } (T_s = 201.4 + 273 = 474.4 \text{ K}) \text{ ;}$$

$$h_f = 858.6 \text{ kJ/kg ; } h_{fg} = 1933.2 \text{ kJ/kg}$$

(*i*) *Heat supplied to feed water per hour* to produce wet steam is given by :

$$\begin{aligned} H &= m\ [(h_f + xh_{fg}) - 1 \times 4.18 \times (30 - 0)] \\ &= 1000\ [(858.6 + 0.9 \times 1933.2) - 4.18 \times 30] \\ &= 1000(858.6 + 1739.88 - 125.4) \\ &= \mathbf{2473.08 \times 10^3\ kJ.} \quad \textbf{(Ans.)} \end{aligned}$$

(*ii*) *Heat absorbed by superheater per hour*

$$\begin{aligned} &= m[(1 - x)\ h_{fg} + c_{ps}\ (T_{sup} - T_s)] \\ &= 1000[(1 - 0.9) \times 1933.2 + 2.2\ (653 - 474.4)] \\ &= 1000(193.32 + 392.92) \\ &= \mathbf{586.24 \times 10^3\ kJ.} \quad \textbf{(Ans.)} \end{aligned}$$

Example 3.8. *Using steam tables, determine the mean specific heat for superheated steam :*

(*i*) *at 0.75 bar, between 100°C and 150°C* ;

(*ii*) *at 0.5 bar, between 300°C and 400°C.*

Solution. (*i*) **At 0.75 bar.** From steam tables ;

At 100°C, $h_{sup} = 2679.4$ kJ/kg

At 150°C, $h_{sup} = 2778.2$ kJ/kg

∴ $2778.2 = 2679.4 + c_{ps}\ (150 - 100)$

i.e., $$c_{ps} = \frac{2778.2 - 2679.4}{50} = \mathbf{1.976.} \quad \textbf{(Ans.)}$$

(*ii*) **At 0.5 bar.** From steam tables ;

At 300°C, $h_{sup} = 3075.5$ kJ/kg

At 400°C, $h_{sup} = 3278.9$ kJ/kg

∴ $3278.9 = 3075.5 + c_{ps}\ (400 - 300)$

i.e., $$c_{ps} = \frac{3278.9 - 3075.5}{100} = \mathbf{2.034.} \quad \textbf{(Ans.)}$$

☞ **Example 3.9.** *A pressure cooker contains 1.5 kg of saturated steam at 5 bar. Find the quantity of heat which must be rejected so as to reduce the quality to 60% dry. Determine the pressure and temperature of the steam at the new state.*

Solution. Mass of steam in the cooker = 1.5 kg

Pressure of steam, $p = 5$ bar

Initial dryness fraction of steam, $x_1 = 1$

Final dryness fraction of steam, $x_2 = 0.6$

Heat to be rejected :

Pressure and temperature of the steam at the new state :

At 5 bar. From steam tables,

$$t_s = 151.8°\text{C} \;; \qquad h_f = 640.1 \text{ kJ/kg} \;;$$
$$h_{fg} = 2107.4 \text{ kJ/kg} \;; \qquad v_g = 0.375 \text{ m}^3\text{/kg}$$

Thus, the volume of pressure cooker

$$= 1.5 \times 0.375 = 0.5625 \text{ m}^3$$

Internal energy of steam per kg at initial point 1,

$$u_1 = h_1 - p_1 v_1$$
$$= (h_f + h_{fg}) - p_1 v_{g_1} \qquad (\because \; v_1 = v_{g1})$$
$$= (640.1 + 2107.4) - 5 \times 10^5 \times 0.375 \times 10^{-3}$$
$$= 2747.5 - 187.5 = 2560 \text{ kJ/kg}$$

Also, $V_1 = V_2$ (V_2 = volume at final condition)

i.e., $$0.5625 = 1.5[(1 - x_2)\, v_{f2} + x_2 v_{g2}]$$
$$= 1.5\, x_2 v_{g2} \qquad (\because \; v_{f2} \text{ is negligible})$$
$$= 1.5 \times 0.6 \times v_{g2}$$

$$\therefore \qquad v_{g2} = \frac{0.5625}{1.5 \times 0.6} = 0.625 \text{ m}^3\text{/kg}.$$

From steam tables corresponding to 0.625 m³/kg,

$p_2 \simeq$ **2.9 bar**, t_s = **132.4°C**, $h_f = 556.5$ kJ/kg, $h_{fg} = 2166.6$ kJ/kg

Internal energy of steam per kg, at final point 2,

$$u_2 = h_2 - p_2 v_2$$
$$= (h_{f_2} + x_2 h_{fg_2}) - p_2 x v_{g2} \qquad (\because \; v_2 = x v_{g2})$$
$$= (556.5 + 0.6 \times 2166.6) - 2.9 \times 10^5 \times 0.6 \times 0.625 \times 10^{-3}$$
$$= 1856.46 - 108.75 = 1747.71 \text{ kJ/kg.}$$

Heat transferred at constant volume per kg

$$= u_2 - u_1 = 1747.71 - 2560 = -\,812.29 \text{ kJ/kg}$$

Thus, **total heat transferred**

$$= -\,812.29 \times 1.5 = -\,\mathbf{1218.43 \text{ kJ.}} \quad \textbf{(Ans.)}$$

Negative sign indicates that heat has been **rejected.**

☞ **Example 3.10.** *A spherical vessel of 0.9 m³ capacity contains steam at 8 bar and 0.9 dryness fraction. Steam is blown off until the pressure drops to 4 bar. The valve is then closed and the steam is allowed to cool until the pressure falls to 3 bar. Assuming that the enthalpy of steam in the vessel remains constant during blowing off periods, determine :*

(*i*) *The mass of steam blown off* ;

(*ii*) *The dryness fraction of steam in the vessel after cooling* ;

(*iii*) *The heat lost by steam per kg during cooling.*

Solution. Capacity of the spherical vessel, $V = 0.9\ \text{m}^3$

Pressure of the steam, $p_1 = 8$ bar

Dryness fraction of steam, $x_1 = 0.9$

Pressure of steam after blow off, $p_2 = 4$ bar

Final pressure of steam, $p_3 = 3$ bar.

(*i*) **The mass of steam blown off :**

The mass of steam in the vessel

$$m_1 = \frac{V}{x_1 v_{g_1}} = \frac{0.9}{0.9 \times 0.24} = 4.167 \text{ kg} \qquad (\because \text{ At 8 bar} : v_g = 0.24 \text{ m}^3/\text{kg})$$

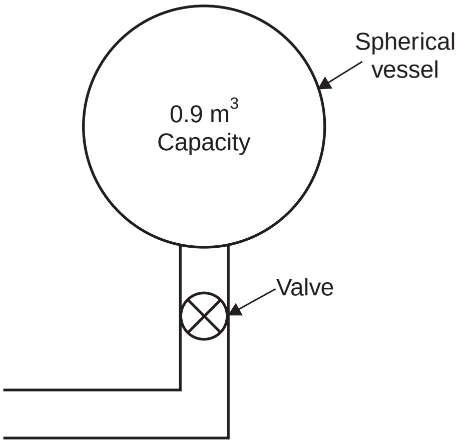


Fig. 3.12

The enthalpy of steam before blowing off (per kg)

$$= h_{f_1} + x_1 h_{fg_1} = 720.9 + 0.9 \times 2046.5 \text{ ...... at pressure } 8 \text{ bar}$$

$$= 2562.75 \text{ kJ/kg}$$

Enthalpy before blowing off = Enthalpy after blowing off

$$\therefore \quad 2562.75 = (h_{f_2} + x_2 h_{fg_2}) \text{ at pressure 4 bar}$$

$$= 604.7 + x_2 \times 2133 \text{ ...... at pressure } 4 \text{ bar}$$

$$\therefore \quad x_2 = \frac{2562.75 - 604.7}{2133} = 0.918$$

Now the mass of steam in the vessel after blowing off,

$$m_2 = \frac{0.9}{0.918 \times 0.462} = 2.122 \text{ kg} \qquad [v_{g_2} = 0.462 \text{ m}^3/\text{kg.......at 4 bar}]$$

Mass of steam blown off, $m = m_1 - m_2 = 4.167 - 2.122$

= **2.045 kg. (Ans.)**

(*ii*) **Dryness fraction of steam in the vessel after cooling, x_3 :**

As it is constant volume cooling

$$\therefore \quad x_2 v_{g_2} \text{ (at 4 bar)} = x_3 v_{g_3} \text{ (at 3 bar)}$$

$$0.918 \times 0.462 = x_3 \times 0.606$$

$$\therefore \qquad x_3 = \frac{0.918 \times 0.462}{0.606} = \mathbf{0.699.} \quad \textbf{(Ans.)}$$

(*iii*) **Heat lost during cooling :**

Heat lost during cooling = $m(u_3 - u_2)$, where u_2 and u_3 are the internal energies of steam before starting cooling or after blowing and at the end of the cooling.

$$\therefore \qquad u_2 = h_2 - p_2x_2v_{g_2} = (h_{f_2} + x_2h_{fg2}) - p_2x_2v_{g2}$$

$$= (604.7 + 0.918 \times 2133) - 4 \times 10^5 \times 0.918 \times 0.462 \times 10^{-3}$$

$$= 2562.79 - 169.65 = 2393.14 \text{ kJ/kg}$$

$$u_3 = h_3 - p_3x_3v_{g_3} = (h_{f_3} + x_3h_{fg3}) - p_3x_3v_{g3}$$

$$= (561.4 + 0.669 \times 2163.2) - 3 \times 10^5 \times 0.699 \times 0.606 \times 10^{-3}$$

$$= 2073.47 - 127.07 = 1946.4 \text{ kJ/kg}$$

∴ Heat transferred during cooling

$$= 2.045\ (1946.4 - 2393.14) = -\ 913.6 \text{ kJ.}$$

i.e., **Heat lost during cooling = 913.6 kJ. (Ans.)**

Example 3.11. *If a certain amount of steam is produced at a pressure of 8 bar and dryness fraction 0.8. Calculate :*

(*i*) *External work done during evaporation.*

(*ii*) *Internal latent heat of steam.*

Solution. Pressure of steam, $p = 8$ bar

Dryness fraction, $x = 0.8$

At 8 bar. From steam tables,

$$v_g = 0.240 \text{ m}^3\text{/kg}, \; h_{fg} = 2046.5 \text{ kJ/kg}$$

(*i*) **External work done during evaporation**

$$= pxv_g = 8 \times 10^5 \times 0.8 \times 0.24 \text{ N-m}$$

$$= \frac{8 \times 10^5 \times 0.8 \times 0.24}{10^3} = \mathbf{153.6\ kJ.} \quad \textbf{(Ans.)}$$

(*ii*) **Internal latent heat** $= xh_{fg}$ – external work done

$$= 0.8 \times 2046.5 - 153.6$$

$$= \mathbf{1483.6\ kJ.} \quad \textbf{(Ans.)}$$

Example 3.12. *A quantity of steam at 10 bar and 0.85 dryness occupies 0.15 m*^{3}*. Determine the heat supplied to raise the temperature of the steam to 300°C at constant pressure and percentage of this heat which appears as external work.*

Take specific heat of superheated steam as 2.2 kJ/kg K.

Solution. Pressure of steam, $p_1 = p_2 = 10$ bar

Dryness fraction, $x_1 = 0.85$

Volume of steam, $V_1 = 0.15 \text{ m}^3$

Final temperature of steam, $t_{sup_2} = 300°\text{C}$

Specific heat of superheated steam, $c_{ps} = 2.2$ kJ/kg K

$$\text{Mass of steam} = \frac{V_1}{x_1v_{g_1}} = \frac{0.15}{0.85 \times 0.194} = 0.909 \text{ kg} \qquad (\because \text{ At 10 bar} : v_g = 0.194 \text{ m}^3/\text{kg})$$

Heat supplied per kg of steam

$$= (1 - x_1)\, h_{fg_1} + c_{ps}\,(300 - 179.9)$$

$$= (1 - 0.85)2013.6 + 2.2 \times 120.1 \qquad \left[\because \begin{array}{l} t_s = 179.9°\text{C} \ldots \text{at 10 bar,} \\ \text{and } h_{fg} = 2013.6 \text{ kJ/kg} \end{array}\right]$$

$$= 566.26 \text{ kJ/kg}$$

Total heat supplied

$$= 0.909 \times 566.26 = \mathbf{514.7\ kJ.} \quad \textbf{(Ans.)}$$

External work done during this process

$$= p\,(v_{sup_2} - x_1 v_{g_1}) \qquad [\because p_1 = p_2 = p]$$

$$= 10 \times 10^5 \left[\left(v_{g_1} \times \frac{T_{sup_2}}{T_{s_1}}\right) - x_1 v_{g_1}\right] \times 10^{-3} \qquad \left[\because \frac{v_{g_1}}{T_{s_1}} = \frac{v_{sup_2}}{T_{sup_2}}\ i.e.,\ v_{sup_2} = \frac{v_{g_1} \times T_{sup_2}}{T_{s_1}}\right]$$

$$= 10 \times 10^5 \left[0.194 \times \frac{(300 + 273)}{(179.9 + 273)} - 0.85 \times 0.194\right] \times 10^{-3}$$

$$= \frac{10 \times 10^5}{10^3}\ (0.245 - 0.165) = 80 \text{ kJ/kg}$$

∴ **Percentage of total heat supplied (per kg) which appears as external work**

$$= \frac{80}{566.26} = 0.141 = \mathbf{14.1\%.} \quad \textbf{(Ans.)}$$

Example 3.13. *Find the specific volume, enthalpy and internal energy of wet steam at 18 bar, dryness fraction 0.85.*

Solution. Pressure of steam, $p = 18$ bar

Dryness fraction, $x = 0.85$

From steam tables corresponding to 18 bar pressure :

$h_f = 884.6$ kJ/kg, $h_{fg} = 1910.3$ kJ/kg, $v_g = 0.110$ m^3/kg, $u_f = 883$ kJ/kg, $u_g = 2598$ kJ/kg.

(*i*) **Specific volume of wet steam,**

$$v = xv_g = 0.85 \times 0.110 = \mathbf{0.0935\ m^3/kg.} \quad \textbf{(Ans.)}$$

(*ii*) **Specific enthalpy of wet steam,**

$$h = h_f + xh_{fg} = 884.6 + 0.85 \times 1910.3$$

$$= \mathbf{2508.35\ kJ/kg.} \quad \textbf{(Ans.)}$$

(*iii*) **Specific internal energy of wet steam,**

$$u = (1 - x)u_f + xu_g$$

$$= (1 - 0.85) \times 883 + 0.85 \times 2598$$

$$= \mathbf{2340.75\ kJ/kg.} \quad \textbf{(Ans.)}$$

Example 3.14. *Find the dryness fraction, specific volume and internal energy of steam at 7 bar and enthalpy 2550 kJ/kg.*

Solution. Pressure of steam, $p = 7$ bar

Enthalpy of steam, $h = 2550$ kJ

From steam tables corresponding to 7 bar pressure :

$h_f = 697.1$ kJ/kg, $h_{fg} = 2064.9$ kJ/kg, $v_g = 0.273$ m^3/kg,
$u_f = 696$ kJ/kg, $u_g = 2573$ kJ/kg.

(*i*) **Dryness fraction, x :**

At 7 bar, $h_g = 2762$ kJ/kg, hence since the actual enthalpy is given as 2550 kJ/kg, the steam must be in the wet vapour state.

Now, using the equation,

$$h = h_f + xh_{fg}$$

$$\therefore \quad 2550 = 697.1 + x \times 2064.9$$

i.e., $$x = \frac{2550 - 697.1}{2064.9} = 0.897$$

Hence, **dryness fraction = 0.897. (Ans.)**

(*ii*) **Specific volume of wet steam,**

$$v = xv_g = 0.897 \times 0.273 = \mathbf{0.2449\ m^3/kg.} \quad \textbf{(Ans.)}$$

(*iii*) **Specific internal energy of wet steam,**

$$\begin{aligned} u &= (1 - x)u_f + xu_g \\ &= (1 - 0.897) \times 696 + 0.897 \times 2573 \\ &= \mathbf{2379.67\ kJ/kg.} \quad \textbf{(Ans.)} \end{aligned}$$

Example 3.15. *Steam at 120 bar has a specific volume of 0.01721 m³/kg, find the temperature, enthalpy and the internal energy.*

Solution. Pressure of steam, $p = 120$ bar

Specific volume, $v = 0.01721$ m³/kg

(*i*) **Temperature :**

First it is necessary to decide whether the steam is wet, dry saturated or superheated.

At 120 bar, $v_g = 0.0143$ m³/kg, which is less than the actual specific volume of 0.01721 m³/kg, and hence the steam is **superheated.**

From the superheat tables at 120 bar, the specific volume is 0.01721 m³/kg at a temperature of **350°C. (Ans.)**

(*ii*) **Enthalpy :**

From the steam tables the specific enthalpy at 120 bar, 350°C,

$$h = \mathbf{2847.7\ kJ/kg.} \quad \textbf{(Ans.)}$$

(*iii*) **Internal energy :**

To find internal energy, using the equation,

$$\begin{aligned} u &= h - pv \\ &= 2847.7 - \frac{120 \times 10^5 \times 0.01721}{10^3} \\ &= \mathbf{2641.18\ kJ/kg.} \quad \textbf{(Ans.)} \end{aligned}$$

Example 3.16. *Steam at 140 bar has an enthalpy of 3001.9 kJ/kg, find the temperature, the specific volume and the internal energy.*

Solution. Pressure of steam, $p = 140$ bar

Enthalpy of steam, $h = 3001.9$ kJ/kg

(*i*) **Temperature :**

At 140 bar, $h_g = 2642.4$ kJ, which is less than the actual enthalpy of 3001.9 kJ/kg, and hence the steam is **superheated.**

From superheat tables at 140 bar, $h = 3001.9$ kJ/kg at a temperature of **400°C. (Ans.)**

(*ii*) The **specific volume,** v = **0.01722 m³/kg. (Ans.)**

$\therefore$ The **internal energy** (specific),

$$u = h - pv = 3001.9 - \frac{140 \times 10^5 \times 0.01722}{10^3}$$

$$= \mathbf{2760.82\ kJ/kg.}\quad \textbf{(Ans.)}$$

☞ **Example 3.17.** *Calculate the internal energy per kg of superheated steam at a pressure of 10 bar and a temperature of 300°C. Also find the change of internal energy if this steam is expanded to 1.4 bar and dryness fraction 0.8.*

Solution. At 10 bar, 300°C. From steam tables for superheated steam.

$$h_{sup} = 3051.2 \text{ kJ/kg } (T_{sup} = 300 + 273 = 573 \text{ K})$$

and corresponding to 10 bar (from tables of dry saturated steam)

$$T_s = 179.9 + 273 = 452.9 \text{ K} \; ; \; v_g = 0.194 \text{ m}^3\text{/kg}$$

To find $v_{sup.}$, using the relation,

$$\frac{v_g}{T_s} = \frac{v_{sup}}{T_{sup}}$$

$$\therefore \qquad v_{\text{sup}} = \frac{v_g \times T_{\text{sup}}}{T_s} = \frac{0.194 \times 573}{452.9} = 0.245 \text{ m}^3\text{/kg.}$$

Internal energy of superheated steam at 10 bar,

$$u_1 = h_{sup} - pv_{sup}$$

$$= 3051.2 - 10 \times 10^5 \times 0.245 \times 10^{-3}$$

$$= \mathbf{2806.2\ kJ/kg.}\quad \textbf{(Ans.)}$$

At 1.4 bar. From steam tables ;

$$h_f = 458.4 \text{ kJ/kg, } h_{fg} = 2231.9 \text{ kJ/kg} \; ; \; v_g = 1.236 \text{ m}^3\text{/kg}$$

Enthalpy of wet steam (after expansion)

$$h = h_f + xh_{fg}$$

$$= 458.4 + 0.8 \times 2231.9 = 2243.92 \text{ kJ.}$$

Internal energy of this steam,

$$u_2 = h - pxv_g$$

$$= 2243.92 - 1.4 \times 10^5 \times 0.8 \times 1.236 \times 10^{-3}$$

$$= 2105.49 \text{ kJ}$$

Hence change of internal energy per kg

$$u_2 - u_1 = 2105.49 - 2806.2$$

$$= -\ \mathbf{700.7\ kJ.}\quad \textbf{(Ans.)}$$

Negative sign indicates **decrease** in internal energy.

Example 3.18. *Find the internal energy of 1 kg of steam at 20 bar when*

(*i*) *it is superheated, its temperature being 400°C ;*

(*ii*) *it is wet, its dryness being 0.9.*

Assume superheated steam to behave as a perfect gas from the commencement of superheating and thus obeys Charle's law. Specific heat for steam = 2.3 kJ/kg K.

Solution. Mass of steam = 1 kg

Pressure of steam, p = 20 bar

Temperature of superheated steam = 400°C ($T_{sup} = 400 + 273 = 673$ K)

Dryness fraction, $x = 0.9$

Specific heat of superheated steam, $c_{ps} = 2.3$ kJ/kg K

(*i*) **Internal energy of 1 kg of superheated steam :**

At 20 bar. From steam tables,

$$t_s = 212.4°\text{C} \; ; \; h_f = 908.6 \text{ kJ/kg} \; ; \; h_{fg} = 1888.6 \text{ kJ/kg}, \; v_g = 0.0995 \text{ m}^3\text{/kg}$$

Now, $h_{sup} = h_f + h_{fg} + c_{ps}\,(T_{sup} - T_s)$

$= 908.6 + 1888.6 + 2.3(400 - 212.4)$

$= 3228.68$ kJ/kg

Also, $h_{sup} = u + p\,.\,v_{sup}$

or $u = h_{sup} - p\,.\,v_{sup}$

The value of v_{sup} can be found out by Charle's law

$$\frac{v_g}{T_g} = \frac{v_{sup}}{T_{sup}}$$

$$\therefore \quad v_{sup} = \frac{v_g \times T_{sup}}{T_s} = \frac{0.0995 \times 673}{(212.4 + 273)} = 0.1379 \text{ m}^3\text{/kg}$$

Hence internal energy, $u = 3228.68 - 20 \times 10^5 \times 0.1379 \times 10^{-3}$

$=$ **2952.88 kJ/kg. (Ans.)**

(*ii*) **Internal energy of 1 kg of wet steam :**

$$h = h_f + xh_{fg} = 908.6 + 0.9 \times 1888.6 = 2608.34 \text{ kJ/kg}$$

Again $h = u + p\,.\,x\,.\,v_g$

$\therefore \quad u = h - p\,.\,x\,.\,v_g = 2608.34 - 20 \times 10^5 \times 0.9 \times 0.0995 \times 10^{-3}$

$= 2429.24$ kJ/kg

Hence internal energy = 2429.24 kJ/kg. (Ans.)

Example 3.19. *Two boilers one with superheater and other without superheater are delivering equal quantities of steam into a common main. The pressure in the boilers and main is 20 bar. The temperature of steam from a boiler with a superheater is 350°C and temperature of the steam in the main is 250°C.*

Determine the quality of steam supplied by the other boiler. Take $c_{ps} = 2.25$ *kJ/kg.*

Solution. Boiler B$_1$. 20 bar, 350°C :

Enthalpy, $h_1 = h_{g_1} + c_{ps}\,(T_{sup} - T_s)$

$= 2797.2 + 2.25(350 - 212.4)$

$= 3106.8$ kJ/kg ...(*i*)

Boiler B$_2$. 20 bar (temperature not known) :

$h_2 = h_{f_2} + x_2 h_{fg_2}$

$= (908.6 + x_2 \times 1888.6)$ kJ/kg ...(*ii*)

Main. 20 bar, 250°C.

Total heat of 2 kg of steam in the steam main

$= 2[h_g + c_{ps}\,(T_{sup} - T_s)]$

$= 2[2797.2 + 2.25\,(250 - 212.4)] = 5763.6$ kJ ...(*iii*)

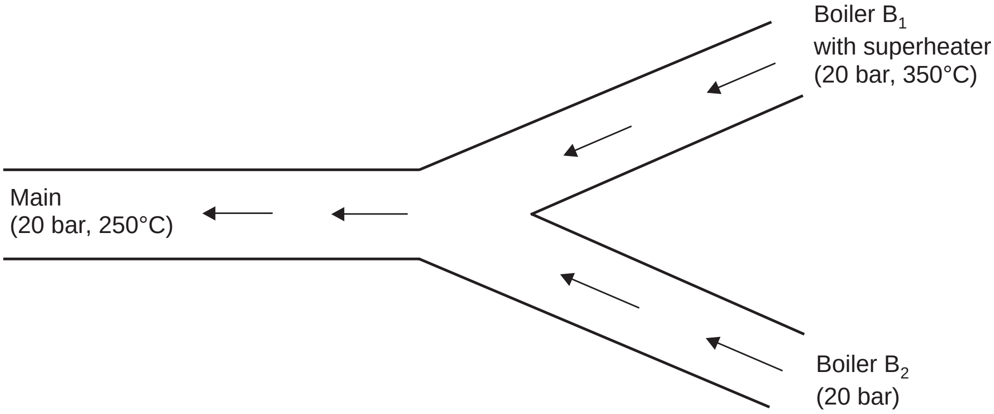


Fig. 3.13

Equating (*i*) and (*ii*) with (*iii*), we get

$$3106.8 + 908.6 + x_2 \times 1888.6 = 5763.6$$

$$4015.4 + 1888.6x_2 = 5763.6$$

$$\therefore \quad x_2 = \frac{5763.6 - 4015.4}{1888.6} = 0.925$$

Hence, **quality of steam supplied by the other boiler = 0.925. (Ans.)**

Example 3.20. *Determine the entropy of 1 kg of wet steam at a pressure of 6 bar and 0.8 dry, reckoned from freezing point (0°C).*

Solution. Mass of wet steam, $m = 1$ kg

Pressure of steam, $p = 6$ bar

Dryness fraction, $x = 0.8$

At 6 bar. From steam tables,

$$t_s = 158.8°\text{C}, \; h_{fg} = 2085 \text{ kJ/kg}$$

Entropy of wet steam is given by

$$s_{wet} = c_{pw} \log_e \frac{T_s}{273} + \frac{xh_{fg}}{T_s} \qquad \text{(where } c_{pw} = \text{specific heat of water)}$$

$$= 4.18 \log_e \left(\frac{158.8 + 273}{273}\right) + \frac{0.8 \times 2085}{(158.8 + 273)}$$

$$= 1.9165 + 3.8700 = 5.7865 \text{ kJ/kg K}$$

Hence, **entropy of wet steam = 5.7865 kJ/kg K. (Ans.)**

Example 3.21. *Steam enters an engine at a pressure 10 bar absolute and 400°C. It is exhausted at 0.2 bar. The steam at exhaust is 0.9 dry. Find :*

(*i*) *Drop in enthalpy* ;

(*ii*) *Change in entropy*.

Solution. Initial pressure of steam, $p_1 = 10$ bar

Initial temperature of steam, $t_{sup} = 400°\text{C}$

Final pressure of steam, $p_2 = 0.2$ bar

Final condition of steam, $x_2 = 0.9$

At 10 bar, 400°C. From steam tables,

$$h_{sup} = 3263.9 \text{ kJ/kg} \;;\; s_{sup} = 7.465 \text{ kJ/kg K}$$

i.e., $h_1 = h_{sup} = 3263.9$ kJ/kg and $s_1 = s_{sup} = 7.465$ kJ/kg K

At 0.2 bar. From steam tables,

$$h_f = 251.5 \text{ kJ/kg} \;;\; h_{fg} = 2358.4 \text{ kJ/kg} \;;$$
$$s_f = 0.8321 \text{ kJ/kg K} \;;\; s_g = 7.9094 \text{ kJ/kg K}$$

Also, $h_2 = h_{f_2} + x_2 h_{fg_2} = 251.5 + 0.9 \times 2358.4$
$= 2374$ kJ/kg.

Also, $s_2 = s_{f_2} + x_2 s_{fg_2}$
$= s_{f_2} + x_2(s_{g_2} - s_{f_2})$
$= 0.8321 + 0.9(7.9094 - 0.8321)$
$= 7.2016$ kJ/kg K

Hence, (*i*) **Drop in enthalpy,**

$= h_1 - h_2 = 3263.9 - 2374 =$ **889.9 kJ/kg. (Ans.)**

(*ii*) **Change in entropy**

$= s_1 - s_2 = 7.465 - 7.2016$

= **0.2634 kJ/kg K (decrease). (Ans.)**

Example 3.22. *Find the entropy of 1 kg of superheated steam at a pressure of 12 bar and a temperature of 250°C. Take specific heat of superheated steam as 2.1 kJ/kg K.*

Solution. Mass of steam, $m = 1$ kg
Pressure of steam, $p = 12$ bar
Temperature of steam, $T_{sup} = 250 + 273 = 523$ K
Specific heat of superheated steam, $c_{ps} = 2.1$ kJ/kg K

At 12 bar. From steam tables,

$$T_s = 188 + 273 = 461 \text{ K}, \; h_{fg} = 1984.3 \text{ kJ/kg}$$

∴ Entropy of 1 kg of superheated steam,

$$s_{sup} = c_{pw} \log_e \frac{T_s}{273} + \frac{h_{fg}}{T_s} + c_{ps} \log_e \frac{T_{sup}}{T_s}$$
$$= 4.18 \log_e \left(\frac{461}{273}\right) + \frac{1984.3}{461} + 2.1 \times \log_e \left(\frac{523}{461}\right)$$
$$= 2.190 + 4.304 + 0.265$$

= **6.759 kJ/kg. (Ans.)**

Example 3.23. *A piston-cylinder contains 3 kg of wet steam at 1.4 bar. The initial volume is 2.25 m^3. The steam is heated until its temperature reaches 400°C . The piston is free to move up or down unless it reaches the stops at the top. When the piston is up against the stops the cylinder volume is 4.65 m^3. Determine the amount of work and heat transfer to or from steam.*

(U.P.S.C. 1994)

Solution. Initial volume per kg of steam $= \frac{2.25}{3} = 0.75$ m^3/kg
Specific volume of steam at 1.4 bar $= 1.2363$ m^3/kg
Dryness fraction of initial steam $= \frac{0.75}{1.2363} = 0.607$

At 1.4 bar, the enthalpy of 3 kg of steam

$$= 3\left[h_f + xh_{fg}\right] = 3\ [\ 458.4 + 0.607 \times 2231.9] = 5439.5 \text{ kJ}$$

At 400°C, volume of steam per kg $= \frac{4.65}{3} = 1.55 \text{ m}^3/\text{kg}$

At 400°C, when $v_{sup} = 1.55 \text{ m}^3/\text{kg}$, from steam tables,

Pressure of steam = 2.0 bar

Saturation temperature

$$= 120.2°\text{C},\ h = 3276.6 \text{ kJ/kg}$$

Degree of superheat

$$= t_{sup} - t_s = 400 - 120.2 = 279.8°\text{C}$$

Enthalpy of superheated steam at 2.0 bar,

$$400°\text{C} = 3 \times 3276.6 = 9829.8 \text{ kg}$$

Heat *added during the process*

$$= 9829.8 - 5439.5 = \mathbf{4390.3\ kJ.}\quad \textbf{(Ans.)}$$

Internal energy of 0.607 dry steam at 1.4 bar

$$= 3 \times 1708 = 5124 \text{ kJ.}$$

Internal energy of superheated steam at 2 bar, 400°C

$$= 3(h_{sup} - pv) = 3(3276.6 - 2 \times 10^2 \times 1.55) = 8899.8 \text{ kJ}$$

$$(\because\ 1 \text{ bar} = 10^2 \text{ kPa})$$

Change in internal energy = 8899.8 – 5124 = 3775.8 kJ

Hence, **work done** = 4390.3 – 3775.8 = **614.5 kJ. (Ans.**) $(\because\ W = Q - \Delta U)$

3.18. DETERMINATION OF DRYNESS FRACTION OF STEAM

The dryness fraction of steam can be measured by using the following *calorimeters* :

1. Tank or bucket calorimeter
2. Throttling calorimeter
3. Separating and throttling calorimeter.

3.18.1. Tank or Bucket Calorimeter

The dryness fraction of steam can be found with the help of tank calorimeter as follows :

A known mass of steam is passed through a known mass of water and steam is completely condensed. The heat lost by steam is equated to heat gained by the water.

Fig. 3.14 shows the arrangement of this calorimeter.

The steam is passed through the sampling tube into the bucket calorimeter containing *a known* mass of water.

The weights of calorimeter with water before mixing with steam and after mixing the steam are obtained by weighing.

The temperature of water before and after mixing the steam are measured by *mercury thermometer*.

The pressure of steam passed through the sampling tube is measured with the help of *pressure gauge*.

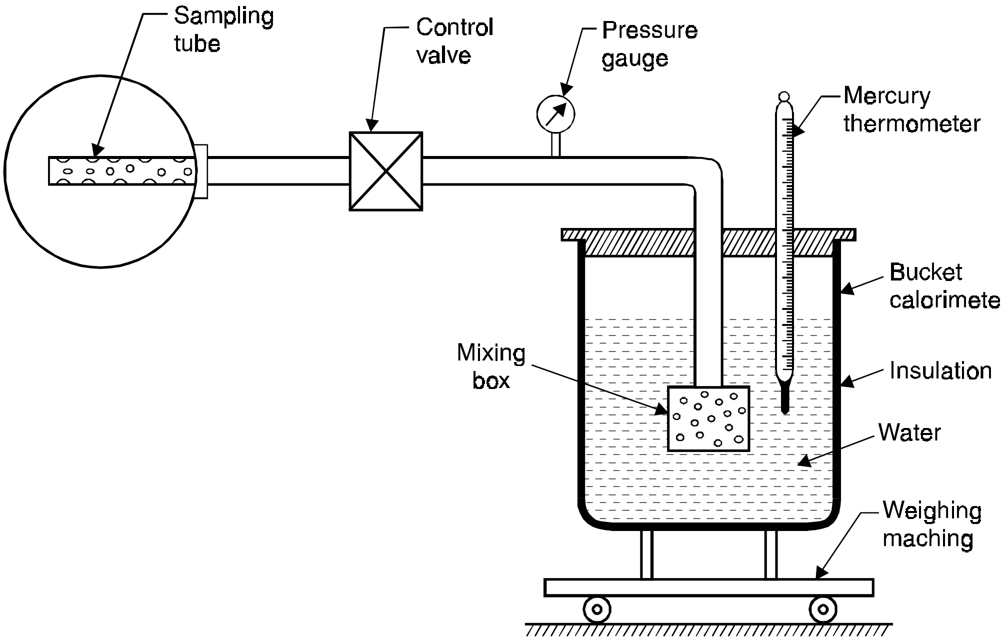


Fig. 3.14. Tank or bucket calorimeter.

Let, p_s = Gauge pressure of steam (bar),

p_a = Atmospheric pressure (bar),

t_s = Daturation temperature of steam known from steam table at pressure $(p_s + p_a)$,

h_{fg} = Latent heat of steam,

x = Dryness fraction of steam,

c_{pw} = Specific heat of water,

c_{pc} = Specific heat of calorimeter,

m_c = Mass of calorimeter, kg,

m_{cw} = Mass of calorimeter and water, kg,

$m_w = (m_{cw} - m_c)$ = Mass of water in calorimeter, kg,

m_{cws} = Mass of calorimeter, water and condensed steam, kg,

$m_s = (m_{cws} - m_{cw})$ = Mass of steam condensed in calorimeter, kg,

t_{cw} = Temperature of water and calorimeter before mixing the steam, °C, and

t_{cws} = Temperature of water and calorimeter after mixing the steam, °C.

Neglecting the losses and *assuming* that the *heat lost by steam is gained by water and calorimeter*, we have

$$(m_{cws} - m_{cw})\,[xh_{fg} + c_{pw}\,(t_s - t_{cws})] = (m_{cw} - m_c)c_{pw}\,(t_{cws} - t_{cw}) + m_c\,c_{pc}\,(t_{cws} - t_{cw})$$

$$\therefore \quad m_s[xh_{fg} + c_{pw}\,(t_s - t_{cws})] = (t_{cws} - t_{cw})\,[m_{cw} - m_c)(c_{pw} + m_c c_{pc}] \qquad ...(3.19)$$

or

$$m_s[xh_{fg} + c_{pw}\,(t_s - t_{cws})] = (t_{cws} - t_{cw})(m_w c_{pw} + m_c c_{pc})$$

The $m_c c_{pc}$ is known as *water equivalent of calorimeter*.

The value of dryness fraction 'x' can be found by solving the above equation.

The value of dryness fraction found by this method involves some *inaccuracy* since losses due to convection and radiation are *not* taken into account.

The calculated value of dryness fraction neglecting losses is *always less* than the actual value of the dryness.

Example 3.24. *Steam at a pressure of 5 bar passes into a tank containing water where it gets condensed. The mass and temperature in the tank before the admission of steam are 50 kg and 20°C respectively. Calculate the dryness fraction of steam as it enters the tank if 3 kg of steam gets condensed and resulting temperature of the mixture becomes 40°C. Take water equivalent of tank as 1.5 kg.*

Solution. Pressure of steam, $p = 5$ bar

Mass of water in the tank = 50 kg

Initial temperature of water = 20°C

Amount of steam condensed, $m_s = 3$ kg

Final temperature after condensation of steam = 40°C

Water equivalent of tank = 1.5 kg

Dryness fraction of steam, x :

At 5 bar. From steam tables,

$$h_f = 640.1 \text{ kJ/kg} \; ; \; h_{fg} = 2107.4 \text{ kJ/kg}$$

Total mass of water, m_w = mass of water in the tank + water equivalent of tank

= 50 + 1.5 = 51.5 kg

Also, heat lost by steam = heat gained by water

$$m_s \, [(h_f + xh_{fg}) - 1 \times 4.18\,(40 - 0)] = m_w[1 \times 4.18\,(40 - 20)]$$

or $$3[(640.1 + x \times 2107.4) - 4.18 \times 40] = 51.5 \times 4.18 \times 20$$

or $$3(472.9 + 2107.4x) = 4305.4$$

or $$472.9 + 2107.4x = 1435.13$$

$$\therefore \quad x = \frac{1435.13 - 472.9}{2107.4} = 0.456.$$

Hence **dryness fraction of steam = 0.456. (Ans.)**

Example 3.25. *Steam at a pressure of 1.1 bar and 0.95 dry is passed into a tank containing 90 kg of water at 25°C. The mass of tank is 12.5 kg and specific heat of metal is 0.42 kJ/kg K. If the temperature of water rises to 40°C after the passage of the steam, determine the mass of steam condensed. Neglect radiation and other losses.*

Solution. Pressure of steam, $p = 1.1$ bar

Dryness fraction of steam, $x = 0.95$

Mass of water in the tank = 90 kg

Initial temperature of water in the tank = 25°C

Mass of tank = 12.5 kg

Specific heat of metal = 0.42 kJ/kg K

Final temperature of water = 40°C.

Mass of steam condensed, m_s :

Since the radiation losses are neglected,

∴ Heat lost by steam = Heat gained by water

or $$m_s[(h_f + xh_{fg}) - 1 \times 4.18\,(40 - 0)] = m[1 \times 4.18(40 - 25)]$$

But $$m = m_1 + m_2$$

where, m_1 = Mass of cold water in the vessel before steam supply, and

m_2 = Water equivalent of vessel = 0.42 × 12.5 = 5.25 kg

At 1.1 bar. From steam tables,

$$h_f = 428.8 \text{ kJ/kg} \; ; \; h_{fg} = 2250.8 \text{ kJ/kg}$$

$$\therefore \quad m_s\,[(428.8 + 0.95 \times 2250.8) - 1 \times 4.18 \times 40]$$
$$= (90 + 5.25)\,[1 \times 4.18 \times (40 - 25)]$$

$$m_s\,[2567.06 - 167.20] = 95.25 \times 62.7$$

i.e., $$2399.86 m_s = 5972.17$$

$$\therefore \quad m_s = 2.488 \text{ kg}$$

Hence, **mass of steam condensed = 2.488 kg. (Ans.)**

3.18.2. Throttling Calorimeter

The dryness fraction of wet steam can be determined by using a throttling calorimeter which is illustrated diagrammatically in Fig. 3.15.

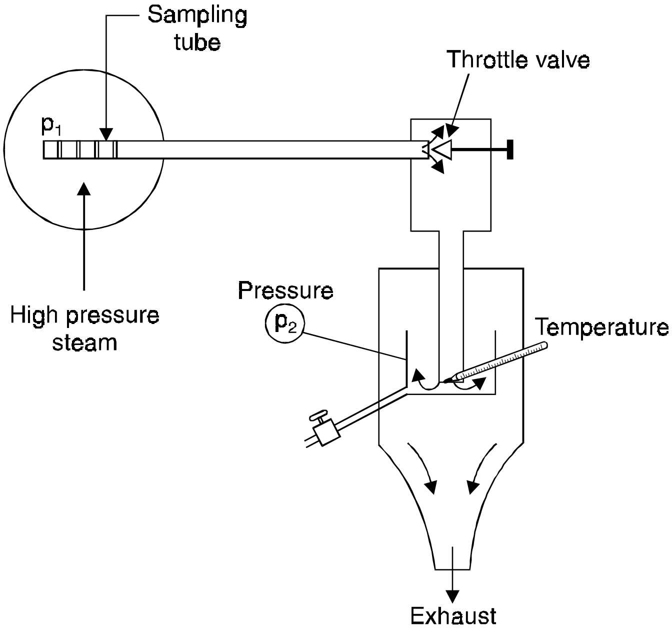


Fig. 3.15. Throttling calorimeter.

The steam to be sampled is taken from the pipe by means of suitable positioned and dimensioned sampling tube. It passes into an insulated container and is throttled through an orifice to atmospheric pressure. Here the temperature is taken and the steam ideally should have about 5.5 K of superheat.

The throttling process is shown on *h-s* diagram in Fig. 3.16 by the line 1-2. If steam initially wet is throttled through a sufficiently large pressure drop, then the steam at state 2 will become superheated. State 2 can then be defined by the *measured pressure and temperature*. The enthalpy, h_2 can then be found and hence

$$h_2 = h_1 = (h_{f_1} + x_1 h_{fg_1}) \text{ at } p_1$$

[where $h_2 = h_{f_2} + h_{fg_2} + c_{ps}\,(T_{sup_2} - T_{s_2})$]

$$\therefore \quad x_1 = \frac{h_2 - h_{f_1}}{h_{fg_1}} \qquad \text{...(3.20)}$$

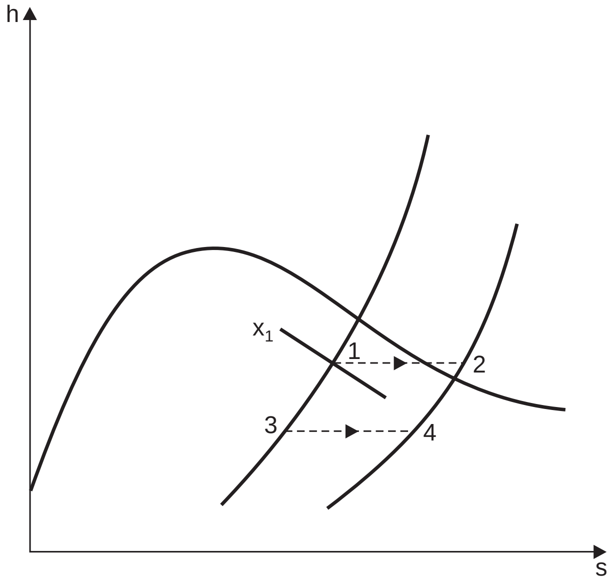


Fig. 3.16. Throttling process.

Hence the dryness fraction is determined and state 1 is defined.

Example 3.26. *A throttling calorimeter is used to measure the dryness fraction of the steam in the steam main which has steam flowing at a pressure of 8 bar. The steam after passing through the calorimeter is at 1 bar pressure and 115°C.*

Calculate the dryness fraction of the steam in the main. Take c_{ps} *= 2.1 kJ/kg K.*

Solution. Condition of steam *before throttling* :

$$p_1 = 8 \text{ bar}, x_1 = ?$$

Condition of steam *after throttling* :

$$p_2 = 1 \text{ bar}, \ t_2 = t_{sup_2} = 115°\text{C}$$

As throttling is a *constant enthalpy* process

$$\therefore \quad h_1 = h_2$$

i.e.,

$$h_{f_1} + x_1 h_{gf_1} = h_{f_2} + h_{fg_2} + c_{p_s}\,(T_{\text{sup}_2} - T_{s_2}) \quad [\because \ T_{sup_2} = 115 + 273 = 388 \text{ K}$$

$$T_{s_2} = 99.6 + 273 = 372.6 \text{ K (at 1 bar)}]$$

$$720.9 + x_1 \times 2046.5 = 417.5 + 2257.9 + 2.1(388 - 372.6)$$

$$720.9 + 2046.5\, x_1 = 2707.7$$

$$\therefore \quad x_1 = \frac{2707.7 - 720.9}{2046.5} = 0.97$$

Hence, **dryness fraction of steam in the main = 0.97. (Ans.)**

3.18.3. Separating and Throttling Calorimeter

If the steam whose dryness fraction is to be determined is *very wet* then throttling to atmospheric pressure *may not be sufficient to ensure superheated steam at exit*. In this case it is *necessary to dry the steam partially, before throttling*. This is done by *passing the steam sample from the main through a separating calorimeter* as shown in Fig. 3.17. The steam is made to change direction suddenly, and the water, being denser than the dry steam is separated out. The quantity of water which is separated out (m_w) is measured at the separator, the steam remaining, which *now has a higher dryness fraction*, is passed through the *throttling calorimeter*. With the combined separating and throttling calorimeter it is *necessary* to condense the steam after throttling and measure the amount of condensate (m_s). If a throttling calorimeter only is sufficient, there is no need to measure condensate, the pressure and temperature measurements at exit being sufficient.

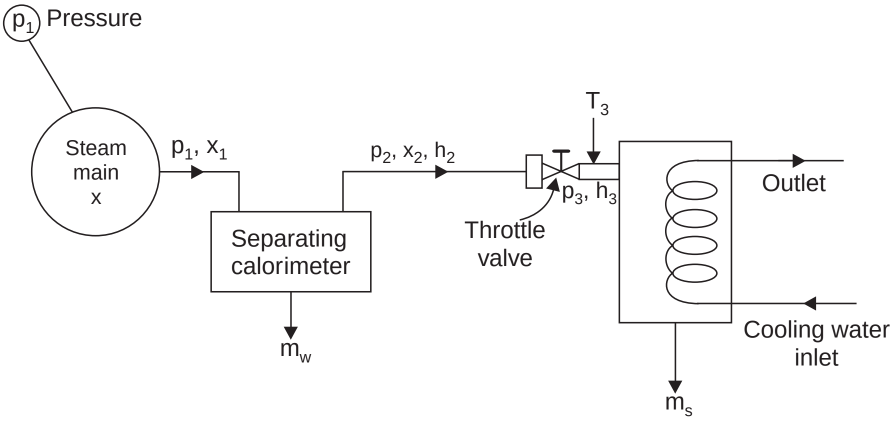


Fig. 3.17. Separating and throttling calorimeter.

Dryness fraction at 2 is x_2, therefore, the mass of dry steam leaving the separating calorimeter is equal to $x_2 m_s$ and this must be the mass of dry vapour in the sample drawn from the main at state 1.

Hence fraction in main, $x_1 = \dfrac{\text{Mass of dry vapour}}{\text{Total mass}} = \dfrac{x_2\, m_s}{m_w + m_s}$.

The dryness fraction, x_2, can be determined as follows :

$$^*h_3 = h_2 = h_{f_2} + x_2 h_{fg_2} \text{ at } p_2 \ [^*h_3 = h_{f_3} + h_{fg_3} + c_{ps}\,(T_{sup_3} - T_{s_3}) \text{ at pressure } p_3]$$

or
$$x_2 = \frac{h_3 - h_{f_2}}{h_{fg_2}}$$

The values of h_{f_2} and h_{fg_2} are read from steam tables at pressure p_2. The pressure in the separator is small so that p_1 is approximately equal to p_2.

Example 3.27. *The following observations were taken with a separating and a throttling calorimeter arranged in series :*

Water separated = 2 kg, steam discharged from the throttling calorimeter = 20.5 kg, temperature of steam after throttling = 110°C, initial pressure = 12 bar abs., barometer = 760 mm of Hg, final pressure = 5 mm of Hg.

Estimate the quality of steam supplied.

Solution. Quantity of water separated out, m_w = 2 kg

Steam (condensate) discharged from the throttling calorimeter, m_s = 20.5 kg

Temperature of steam after throttling, t_{sup} = 110°C

Initial pressure of steam, p_1 = 12 bar abs.

Final pressure of steam, $p_3 = 760 + 5 = 765$ mm

$$= \frac{765}{1000} \times 1.3366 \qquad (\because \;\; 1 \text{ m Hg} = 1.3366 \text{ bar})$$

$$\simeq 1 \text{ bar}$$

From steam tables :

At $p_1 = p_2 =$ **12 bar :** $h_f = 798.4$ kJ/kg, $h_{fg} = 1984.3$ kJ/kg

At $p_3 =$ **1 bar :** $t_s = 99.6$°C, $h_f = 417.5$ kJ/kg, $h_{fg} = 2257.9$ kJ/kg

$t_{sup} = 110$°C (given)

Also $h_3 = h_2$

$$(h_{f_3} + h_{fg_3}) + c_{ps}(T_{sup_3} - T_{s_3}) = hf_2 + x_2 h_{fg_2}$$

Taking $c_{ps} = 2$ kJ/kg K, we get

$$417.5 + 2257.9 + 2[(110 + 273) - (99.6 + 273)] = 798.4 + x_2 \times 1984.3$$

$$2696.2 = 798.4 + 1984.3\, x_2$$

$$\therefore \qquad x_2 = \frac{2696.2 - 798.4}{1984.3} = 0.956$$

Now, **quality of steam supplied,**

$$x_1 = \frac{x_2 m_s}{m_w + m_s} = \frac{0.956 \times 20.5}{2 + 20.5}$$

$$= \mathbf{0.87.} \quad \textbf{(Ans.)}$$

☞ **Example 3.28.** *The following data were obtained in a test on a combined separating and throttling calorimeter :*

Pressure of steam sample = 15 bar, pressure of steam at exit = 1 bar, temperature of steam at the exit = 150°C, discharge from separating calorimeter = 0.5 kg/min, discharge from throttling calorimeter = 10 kg/min.

Determine the dryness fraction of the sample steam.

Solution. Pressure of steam sample, $p_1 = p_2 = 15$ bar

Pressure of steam at the exit, $p_3 = 1$ bar

Temperature of steam at the exit, $t_{sup_3} = 150$°C

Discharge from separating calorimeter, $m_w = 0.5$ kg/min

Discharge from throttling calorimeter, $m_s = 10$ kg/min

From steam tables :

At $p_1 = p_2 =$ **15 bar** : $h_{f_2} = 844.7$ kJ/kg, $h_{fg_2} = 1945.2$ kJ/kg

At $p_3 =$ **1 bar and 150°C** : $h_{sup_3} = 2776.4$ kJ/kg

Also, $h_2 = h_3$

$$h_{f_2} + x_2 h_{fg_2} = h_{sup_3}$$

$$844.7 + x_2 \times 1945.2 = 2776.4$$

$$\therefore \qquad x_2 = \frac{2776.4 - 844.7}{1945.2} = 0.993$$

Now, **quality of steam supplied**,

$$x_1 = \frac{x_2 m_s}{m_s + m_w} = \frac{0.993 \times 10}{10 + 0.5} = \mathbf{0.946.} \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. A pure substance is a system which is (*i*) homogeneous in composition, (*ii*) homogeneous in chemical aggregation, (*iii*) invariable in chemical aggregation.
2. The triple point is merely the point of intersection of sublimation and vapourisation curves. It must be understood that only on *p-T* diagram is the triple point represented by a point. On *p-V* diagram it is a line, and on a *U-V* diagram it is a triangle.
3. Steam as a vapour does not obey laws of perfect gases unless and until it is highly in super dry condition.
4. *Dryness fraction* is the ratio of the mass of actual dry steam to the mass of steam containing it.

   i.e., $$\text{Dryness fraction} = \frac{m_s}{m_s + m_w}$$

   where, m_s = Mass of dry steam contained in steam considered

   m_w = Mass of water particles in suspension in the steam considered.
5. Superheated steam behaves like a gas and therefore, it follows gas laws. The law for adiabatic expansion is $pv^{1.3} = C$.
6. External work of evaporation $= p(v_g - v_f)$

   Internal latent heat $= h_{fg} - \dfrac{pv_g}{J}$

   Internal energy of steam, u :

   (*i*) For wet steam : $u = h - \dfrac{p \, . \, x \, . \, v_g}{J}$

   (*ii*) For superheated steam : $u = h_{sup} - \dfrac{p \, . \, v_{sup}}{J}$. (J = 1, in SI Units)
7. Entropy of water when heated from temperature T_1 to T_2 ;

   $$s_2 - s_1 = c_{pw} \log_e \frac{T_2}{T_1}$$

   If 0°C is taken as the datum then entropy of water at any temperature T, above this datum will be

   $$s_f = c_{pw} \log_e \frac{T}{273}$$

   Entropy of evaporation :

   $$s_{evap.} = \frac{h_{fg}}{T_s} \quad \text{...(when water is evaporated to steam completely)}$$

   $$= \frac{xh_{fg}}{T_s} \quad \text{...(when water is evaporated partially and dryness fraction of steam is } x)$$

   Entropy of steam :

   $$s_{wet} = s_f + \frac{xh_{fg}}{T_s} \quad \text{...[wet steam } (x < 1)]$$

   $$s_g = s_f + \frac{h_{fg}}{T_s} \quad \text{...[Dry and saturated steam } (x = 1)]$$

   $$s_{sup} = s_f + \frac{h_{fg}}{T_s} + c_{ps} \log_e \frac{T_{sup}}{T_s} \quad \text{...(Superheated steam)}$$

8. Mollier chart/diagram is more widely used than any other entropy diagram, since the work done on vapour cycles can be scaled from this diagram directly as a length, whereas on *T-s* diagram it is represented by an area.

9. *Different processes :*

(*i*) Constant volume heating or cooling

$$x_1 v_{g_1} = x_2 v_{g_2}$$

$$x_1 v_{g_1} = v_{sup_2} = v_{g_2} \cdot \frac{T_{sup_2}}{T_{s_2}}$$

(*ii*) Constant pressure heating or cooling

$$Q = h_2 - h_1$$

(*iii*) Isentropic expansion (non-flow process)

$$W = (u_1 - u_2) \text{ and } s_1 = s_2$$

(*iv*) Throttling

$$h_{f_1} + x_1 h_{fg_1} = h_{f_2} + x_2 h_{fg_2} \quad \text{...(For wet condition)}$$

$$= h_{f_2} + h_{fg_2} + c_{ps}\,(T_{sup} - T_{s_2}) \quad \text{...(For superheated condition)}$$

10. Dryness fraction of steam can determined by the following methods :

(*i*) Bucket calorimeter

(*ii*) Throttling calorimeter

(*iii*) Separating and throttling calorimeter.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer :

1. (*a*) Specific volume of water decreases on freezing
(*b*) Boiling point of water decreases with increasing pressure
(*c*) Specific volume of CO_2 increases on freezing
(*d*) Freezing temperature of water decreases with increasing pressure.

2. (*a*) The slope of vapourisation curve is always negative
(*b*) The slope of vapourisation curve is always positive
(*c*) The slope of sublimation curve in negative for all pure substances
(*d*) The slope of fusion curve is positive for all pure substances.

3. (*a*) The process of passing from liquid to vapour is condensation
(*b*) An isothermal line is also a constant pressure line during wet region
(*c*) Pressure and temperature are independent during phase change
(*d*) The term dryness fraction is used to describe the fraction by mass of liquid in the mixture of liquid water and water vapour.

4. The latent heat of vapourisation at critical point is
(*a*) less than zero (*b*) greater than zero
(*c*) equal to zero (*d*) none of the above.

5. (*a*) Critical point involves equilibrium of solid and vapour phases
(*b*) Critical point involves equilibrium of solid and liquid phases
(*c*) Critical point involves equilibrium of solid, liquid and vapour phases
(*d*) Triple point involves equilibrium of solid, liquid and vapour phases.

6. With the increase in pressure
(*a*) boiling point of water increases and enthalpy of evaporation increases
(*b*) boiling point of water increases and enthalpy of evaporation decreases
(*c*) boiling point of water decreases and enthalpy of evaporation increases.

7. With increase in pressure
(*a*) enthalpy of dry saturated steam increases
(*b*) enthalpy of dry saturated steam decreases
(*c*) enthalpy of dry saturated steam remains same
(*d*) enthalpy of dry saturated steam first increases and then decreases.

8. Dryness fraction of steam is defined as
(*a*) mass of water vapour in suspension/(mass of water vapour in suspension + mass of dry steam)
(*b*) mass of dry steam/mass of water vapour in suspension
(*c*) mass of dry steam/(mass of dry steam + mass of water vapour in suspension)
(*d*) mass of water vapour in suspension/mass of dry steam.

9. The specific volume of water when heated at 0°C
(*a*) first increases and then decreases (*b*) first decreases and then increases
(*c*) increases steadily (*d*) decreases steadily.

10. Only throttling calorimeter is used for measuring
(*a*) very low dryness fraction upto 0.7 (*b*) very high dryness fraction upto 0.98
(*c*) dryness fraction of only low pressure steam (*d*) dryness fraction of only high pressure steam.

11. Heat of superheated steam is given by
(*a*) $h_{sup} = h_f + h_{fg} + c_{ps} \log_e \frac{T_{sup}}{T_s}$ (*b*) $h_{sup} = h_f + xh_{fg}$
(*c*) $h_{sup} = h_f + h_{fg}$ (*d*) $h_{sup} = h_f + xh_{fg} + c_{ps} \log_e \frac{T_s}{273}$.

12. Volume of wet steam (per kg) with dryness fraction x is given by
(*a*) $x^3 v_g$ (*b*) xv_f
(*c*) $x^2(v_g - v_f)$ (*d*) $x^2 v_g$
(*e*) none of the above.

13. Internal latent heat is given by
(*a*) $h_{fg} - \frac{pv_g}{J}$ (*b*) $h_g - \frac{pv_g}{J}$
(*c*) $h_{sup} - \frac{pv_f}{J}$ (*d*) $h_{fg} + \frac{pv_g}{J}$
(*e*) none of the above.

14. Entropy of 1 kg of water at T K is given by
(*a*) $c_{pw} \log_e \frac{T}{273}$ (*b*) $c_{pw} \log_e \frac{T_2}{T_1}$
(*c*) $c_{pw} \log_{10} \frac{T}{273}$ (*d*) $c_{pw} \log_e \frac{T_2}{273}$
(*e*) none of the above.

15. Entropy of wet steam (1 kg) is given by
(*a*) $s_f + \frac{xh_{fg}}{T_s}$ (*b*) $s_g + \frac{xh_{fg}}{T_s}$
(*c*) $s_f + \frac{h_{fg}}{T_s}$ (*d*) $s_f + c_{ps} \log_e \frac{T_{sup}}{T_s}$
(*e*) none of the above.

16. In throttling process

(*a*) $h_1^2 = h_2$ (*b*) $h_1 = h_2$

(*c*) $h_1 = h_2 + \dfrac{h_{fg}}{T_s}$ (*d*) $h_2 = h_1 + \dfrac{h_{fg}}{T_s}$

(*e*) none of the above.

17. In isentropic process

(*a*) $W = 2(u_2 - u_1)$ (*b*) $W = (u_2 - u_1)^2$

(*c*) $W = u_2 - u_1$ (*d*) $W = (u_2 - u_1)^{1/2}$

(*e*) none of the above.

Answers

1. (*d*) **2.** (*a*) **3.** (*b*) **4.** (*c*) **5.** (*d*) **6.** (*b*) **7.** (*b*)
8. (*c*) **9.** (*b*) **10.** (*b*) **11.** (*a*) **12.** (*e*) **13.** (*a*) **14.** (*a*)
15. (*a*) **16.** (*b*) **17.** (*c*).

THEORETICAL QUESTIONS

1. What is a pure substance ?

2. Draw and explain a *p*-*T* (pressure-temperature) diagram for a pure substance.

3. What is a triple point ?

4. Explain with a neat diagram *p*-V-*T* surface.

5. Does wet steam obey laws of perfect gases ?

6. Describe the process of formation of steam and give its graphical representation also.

7. Explain the following terms relating to steam formation :

(*i*) Sensible heat of water, (*ii*) Latent heat of steam,
(*iii*) Dryness fraction of steam, (*iv*) Enthalpy of wet steam, and
(*v*) Superheated steam.

8. What advantages are obtained if superheated steam is used in steam prime movers ?

9. What do you mean by the following :

(*i*) Internal latent heat (*ii*) Internal energy of steam
(*iii*) External work of evaporation (*iv*) Entropy of evaporation
(*v*) Entropy of wet steam (*vi*) Entropy of superheated steam.

10. Write a short note on Mollier chart.

11. Draw a neat sketch of throttling calorimeter and explain how dryness fraction of steam is determined ; clearly explain its limitations.

12. Describe with a neat sketch a separating-throttling calorimeter for measuring the dryness fraction of steam.

UNSOLVED EXAMPLES

1. Find the specific volume, enthalpy and internal energy of wet steam at 18 bar, dryness fraction 0.9.
[**Ans.** 0.0994 m^3/kg ; 2605.8 kJ/kg ; 2426.5 kJ/kg]

2. Find the dryness fraction, specific volume and internal energy of steam at 7 bar and enthalpy 2600 kJ/kg.
[**Ans.** 0.921 ; 0.2515 m^3/kg, 2420 kJ/kg]

3. Steam at 110 bar has a specific volume of 0.0196 m^3/kg, find the temperature, the enthalpy and the internal energy. [**Ans.** 350°C ; 2889 kJ/kg ; 2673.4 kJ/kg]

4. Steam at 150 bar has an enthalpy of 3309 kJ/kg, find the temperature, the specific volume and the internal energy. [**Ans.** 500°C ; 0.02078 m^3/kg ; 2997.3 kJ/kg]

5. Steam at 19 bar is throttled to 1 bar and the temperature after throttling is found to be 150°C. Calculate the initial dryness fraction of the steam. [**Ans.** 0.989]

6. Find the internal energy of one kg of steam at 14 bar under the following conditions :

(*i*) When the steam is 0.85 dry ;

(*ii*) When steam is dry and saturated ; and

(*iii*) When the temperature of steam is 300°C. Take c_{ps} = 2.25 kJ/kg K.

[**Ans.** (*i*) 2327.5 kJ/kg ; (*ii*) 2592.5 kJ/kg ; (*iii*) 2784 kJ/kg]

7. Calculate the internal energy of 0.3 m^3 of steam at 4 bar and 0.95 dryness. If this steam is superheated at constant pressure through 30°C, determine the heat added and change in internal energy.

[**Ans.** 2451 kJ/kg ; 119 kJ ; 107.5 kJ/kg]

8. Water is supplied to the boiler at 15 bar and 80°C and steam is generated at the same pressure at 0.9 dryness. Determine the heat supplied to the steam in passing through the boiler and change in entropy.

[**Ans.** 2260.5 kJ/kg ; 4.92 kJ/kg K]

9. A cylindrical vessel of 5 m^3 capacity contains wet steam at 1 bar. The volume of vapour and liquid in the vessel are 4.95 m^3 and 0.05 m^3 respectively. Heat is transferred to the vessel until the vessel is filled with saturated vapour. Determine the heat transfer during the process. [**Ans.** 104.93 MJ]

10. A pressure cooker contains 1.5 kg of steam at 5 bar and 0.9 dryness when the gas was switched off. Determine the quantity of heat rejected by the pressure cooker when the pressure in the cooker falls to 1 bar. [**Ans.** – 2355 kJ]

11. A vessel of spherical shape having a capacity of 0.8 m^3 contains steam at 10 bar and 0.95 dryness. Steam is blown off until the pressure drops to 5 bar. The valve is then closed and the steam is allowed to cool until the pressure falls to 4 bar. Assuming that the enthalpy of steam in the vessel remains constant during blowing off periods, determine :

(*i*) The mass of steam blown-off,

(*ii*) The dryness fraction of steam in the vessel after cooling, and

(*iii*) The heat lost by steam per kg during cooling. [**Ans.** (*i*) 2.12 kg ; (*ii*) 0.78 ; (*iii*) – 820 kJ]

12. Two boilers one with superheater and other without superheater are delivering equal quantities of steam into a common main. The pressure in the boilers and the main is 15 bar. The temperature of the steam from a boiler with a superheater is 300°C and temperature of the steam in the main is 200°C. Determine the quality of steam supplied by the other boiler. [**Ans.** 0.89]

13. A tank of capacity 0.5 m^3 is connected to a steam pipe through a valve which carries steam at 14 bar and 300°C. The tank initially contains steam at 3.5 bar and saturated condition. The valve in the line connecting the tank is opened and the steam is allowed to pass into the tank until the pressure in the tank becomes 14 bar.

Determine the mass of steam that entered into the tank. [**Ans.** 1.565 kg]

4

First Law of Thermodynamics

4.1. INTERNAL ENERGY

It is the heat energy stored in a gas. If a certain amount of heat is supplied to a gas the result is that temperature of gas may increase or volume of gas may increase thereby doing some external work or both temperature and volume may increase ; but it will be decided by the conditions under which the gas is supplied heat. *If during heating of the gas the temperature increases its internal energy will also increase.*

Joule's law of internal energy states that internal energy of a perfect gas is a *function of temperature only*. In other words, internal energy of a gas is dependent on the temperature change only and is not affected by the change in pressure and volume.

We do not know how to find the absolute quantity of internal energy in any substance ; however, what is needed in engineering is the change of internal energy (ΔU).

4.2. LAW OF CONSERVATION OF ENERGY

In the early part of nineteenth century the scientists developed the concept of energy and hypothesis that it can be neither created nor destroyed ; this came to be known as the *law of the conservation of energy*. The first law of thermodynamics is merely one statement of this general law/principle with particular reference to heat energy and mechanical energy *i.e.,* work.

4.3. FIRST LAW OF THERMODYNAMICS

It is observed that when a system is made to undergo a complete cycle then net work is done *on* or *by* the system. Consider a cycle in which net work is done by the system. Since energy cannot be created, this mechanical energy must have been supplied from some source of energy. Now the system has been returned to its initial state : Therefore, its *intrinsic* energy is unchanged, and hence the mechanical energy has not been provided by the system itself. The only other energy involved in the cycle is the heat which was supplied and rejected in various processes. Hence, by the law of conservation of energy, the net work done by the system is equal to the net heat supplied to the system. The First Law of Thermodynamics can, therefore, be stated as follows :

"When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to net work done by the system on its surroundings.

or
$$\oint dQ = \oint dW$$

where $\oint$ represents the sum for a complete cycle.

The first law of Thermodynamics *cannot be proved analytically, but experimental evidence has repeatedly confirmed its validity*, and since no phenomenon has been shown to contradict it, the first law is accepted as a *law of nature.* It may be remarked that no restriction was imposed which limited the application of first law to reversible energy transformation. Hence the first law applies to reversible as well as irreversible transformations : For non-cyclic process, a more general formulation of first law of thermodynamics is required. A new concept which involves a term called *internal energy* fulfills this need.

— The First Law of Thermodynamics may also be stated as follows :

"Heat and work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant".

Or

— **"No machine can produce energy without corresponding expenditure of energy, *i.e.*, it is impossible to construct a perpetual motion machine of first kind".**

Fig. 4.1 shows the experiment for checking first law of thermodynamics.

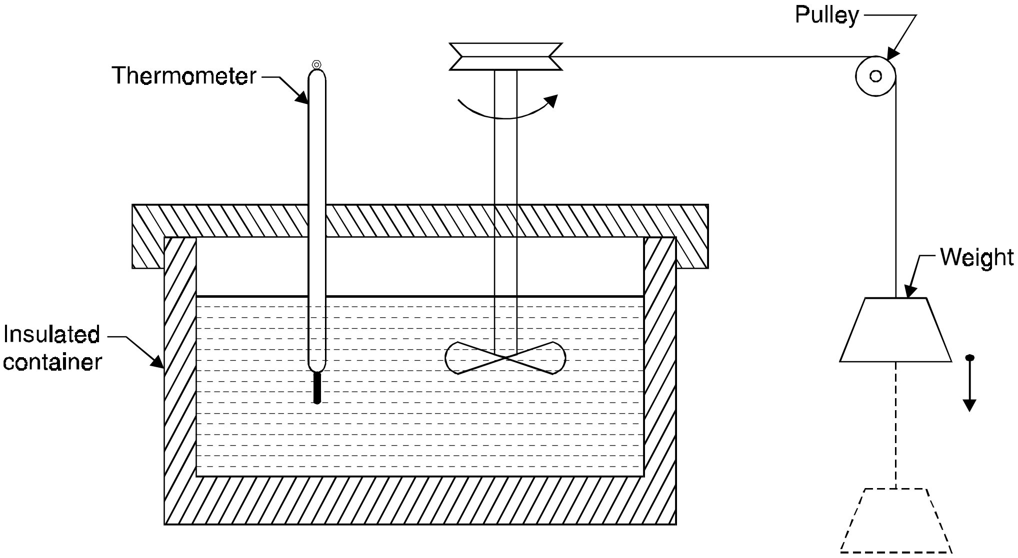


Fig. 4.1. Heat and work.

The work input to the paddle wheel is measured by the fall of weight, while the corresponding temperature rise of liquid in the insulated container is measured by the thermometer. It is already known to us from experiments on heat transfer that temperature rise can also be produced by heat transfer. The experiments show : (*i*) A definite quantity of work is always required to accomplish the same temperature rise obtained with a unit amount of heat. (*ii*) Regardless of whether the temperature of liquid is raised by work transfer or heat transfer, the liquid can be returned by heat transfer in opposite direction to the identical state from which it started. The above results lead to the inference that *work and heat* are different forms of something more general, which is called *energy*.

— It can be stated as an invariable experience that whenever a physical system passes through a complete cycle the algebraic sum of the work transfers during the cycle $\oint dW$ bears a definite ratio to the algebraic sum of the heat transfers during the cycle, $\oint dQ$. This may be expressed by the equation,

$$\oint dW = J \oint dQ \qquad \text{...(4.1)}$$

where J is the proportionality constant and is known as *Mechanical Equivalent of heat*. In S.I. units its value is unity, *i.e.*, 1 Nm/J.

4.4. APPLICATION OF FIRST LAW TO A PROCESS

When a process is executed by a system, the *change in stored energy of the system is numerically equal to the net heat interactions minus the net work interaction during the process.*

$$\therefore \qquad E_2 - E_1 = Q - W$$

$$\therefore \qquad \Delta E = Q - W \qquad [\text{or } Q = \Delta E + W]$$

or

$$\int_1^2 d(Q - W) = \Delta E = E_2 - E_1 \qquad \text{...(4.2)}$$

where E represents the *total internal energy*.

If the electric, magnetic and chemical energies are *absent* and changes in potential and kinetic energy for a *closed system* are neglected, the above equation can be written as

$$\int_1^2 d(Q - W) = \Delta U = U_2 - U_1 \qquad \text{...(4.3)}$$

$$\therefore \qquad Q - W = \Delta U = U_2 - U_1 \qquad \text{...(4.4)}$$

Generally, when heat is added to a system its temperature rises and external work is performed due to increase in volume of the system. *The rise in temperature is an indication of increase of internal energy*.

Heat *added* to the system will be considered as *positive* and the heat *removed* or *rejected*, from the system, as *negative*.

4.5. ENERGY—A PROPERTY OF SYSTEM

Consider a system which changes its state from state 1 to state 2 by following the path L, and returns from state 2 to state 1 by following the path M (Fig. 4.2). So the system undergoes a cycle. Writing the first law for path L

$$Q_L = \Delta E_L + W_L \qquad \text{...(4.5)}$$

and for path M

$$Q_M = \Delta E_M + W_M \qquad \text{...(4.6)}$$

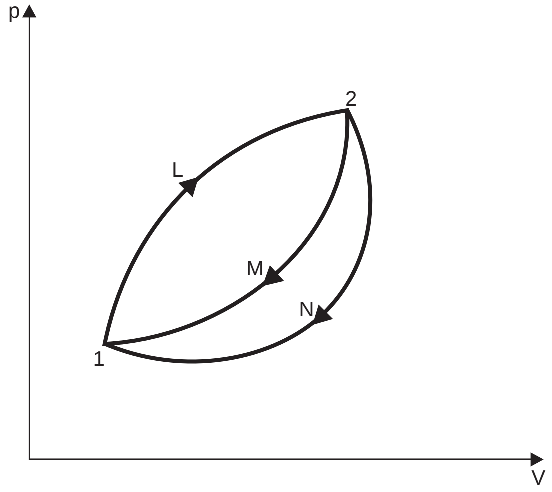


Fig. 4.2. Energy—a property of system.

The processes L and M together constitute a cycle, for which

$$\oint dW = \oint dQ$$

$$W_L + W_M = Q_L + Q_M$$

or $$Q_L - W_L = W_M - Q_M \qquad \text{...(4.7)}$$

From equations (4.5), (4.6) and (4.7), it yields

$$\Delta E_L = - \Delta E_M \qquad \text{...(4.8)}$$

Similarly, had the system returned from state 2 to state 1 by following the path N instead of path M

$$\Delta E_L = - \Delta E_N \qquad \text{...(4.9)}$$

From equations (4.8) and (4.9),

$$\Delta E_M = \Delta E_N \qquad \text{...(4.10)}$$

Thus, it is seen that the *change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state.* If some arbitrary value of energy is assigned to state 2, the value of energy at state 1 is fixed independent of the path the system follows. Therefore, *energy has a definite value for every state of the system.* Hence, it is *a point function and a property of the system.*

4.6. PERPETUAL MOTION MACHINE OF THE FIRST KIND—PMM 1

The first law of thermodynamics states the general principle of the conservation of energy. Energy is neither created nor destroyed, but only gets transformed from one form to another. *There can be no machine which would continuously supply mechanical work without some form of energy disappearing simultaneously* (Fig. 4.3). Such a *fictitious machine* is called a ***perpetual motion machine of the first kind,*** or in brief, PMM 1. A PMM 1 is thus ***impossible.***

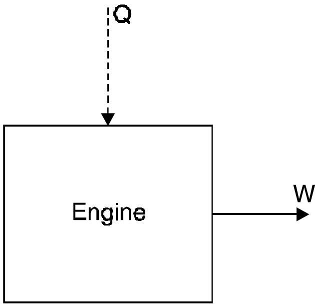


Fig. 4.3. A PPM 1.

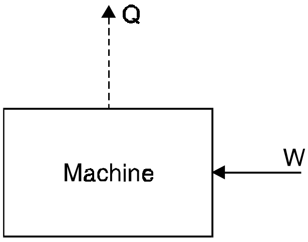


Fig. 4.4. The converse of PMM 1.

— The converse of the above statement is also true, *i.e.*, there can be no machine which would continuously consume work without some other form of energy appearing simultaneously (Fig. 4.4).

4.7. ENERGY OF AN ISOLATED SYSTEM

An isolated system is one in which there is no interaction of the system with the surroundings.

For an isolated system,

$$dQ = 0,\ dW = 0$$

The first law of thermodynamics gives

$$dE = 0$$

or

$$E = \text{constant}$$

The energy of an isolated system is always constant.

4.8. THE PERFECT GAS

4.8.1. The Characteristic Equation of State

— At temperatures that are considerably in excess of critical temperature of a fluid, and also at very low pressure, the vapour of fluid tends to obey the equation

$$\frac{pv}{T} = \text{constant} = R$$

In practice, no gas obeys this law rigidly, but many gases tend towards it.

An imaginary ideal gas which obeys this law is called a *perfect gas*, and the equation $\frac{pv}{T} = R$, is called the *characteristic equation of a state of a perfect gas*. The constant R is called the *gas constant*. Each perfect gas has a different gas constant.

Units of R are Nm/kg K or kJ/kg K.

Usually, the characteristic equation is written as

$$pv = RT \qquad ...(4.11)$$

or for m kg, occupying V m^3

$$pV = mRT \qquad ...(4.12)$$

— The characteristic equation in *another form*, can be derived by using kilogram-mole as a unit.

The *kilogram-mole* is defined as a quantity of a gas equivalent to M kg of the gas, where M is the molecular weight of the gas (*e.g.,* since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen).

As per definition of the kilogram-mole, for m kg of a gas, we have

$$m = nM \qquad ...(4.13)$$

where n = number of moles.

Note. Since the standard of mass is the kg, kilogram-mole will be written simply as mole.

Substituting for m from eqn. (4.13) in eqn. (4.12) gives

$$pV = nMRT$$

or

$$MR = \frac{pV}{nT}$$

According to *Avogadro's hypothesis* the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure. Therefore, $\frac{V}{n}$ is the same for all gases at the same value of p and T. That is the quantity $\frac{pV}{nT}$ is a *constant* for all gases. This constant is called *universal gas constant*, and is given the symbol, R_0.

i.e.,

$$MR = R_0 = \frac{pV}{nT}$$

or

$$pV = nR_0T \qquad ...(4.14)$$

Since $MR = R_0$, then

$$R = \frac{R_0}{M} \qquad ...(4.15)$$

It has been found experimentally that the volume of 1 mole of any perfect gas at 1 bar and 0°C is approximately 22.71 m^3.

Therefore from eqn. (4.14),

$$R_0 = \frac{pV}{nT} = \frac{1 \times 10^5 \times 22.71}{1 \times 273.15}$$

$$= 8314.3 \text{ Nm/mole K}$$

Using eqn. (4.15), the gas constant for any gas can be found when the molecular weight is known.

Example. For oxygen which has a molecular weight of 32, the gas constant

$$R = \frac{R_0}{M} = \frac{8314}{32} = 259.8 \text{ Nm/kg K.}$$

4.8.2. Specific Heats

The **specific heat** of a solid or liquid is usually defined as the *heat required to raise unit mass through one degree temperature rise.*

For small quantities, we have

$$dQ = mcdT$$

where m = mass,
c = specific heat, and
dT = temperature rise.

For a gas there are an infinite number of ways in which heat may be added between any two temperatures, and hence a *gas could have an infinite number of specific heats.* However, only two specific heats for gases are defined.

Specific heat at constant volume, c_v

and, *Specific heat at constant pressure,* c_p.

We have

$dQ = m\ c_p\ dT$ For a reversible non-flow process at *constant pressure* ...(4.16)

and, $dQ = m\ c_v\ dT$ For a reversible non-flow process at *constant volume* ...(4.17)

The values of c_p and c_v, for a perfect gas, are constant for any one gas at all pressures and temperatures. Hence, integrating eqns. (4.16) and (4.17), we have

Flow of heat in a reversible constant pressure process

$$= mc_p\ (T_2 - T_1) \quad ...(4.18)$$

Flow of heat in a reversible constant volume process

$$= mc_v\ (T_2 - T_1) \quad ...(4.19)$$

In case of *real gases,* c_p and c_v *vary with temperature,* but a suitable *average value* may be used for most practical purposes.

4.8.3. Joule's Law

Joule's law states as follows :

"The internal energy of a perfect gas is a function of the absolute temperature only."

i.e., $u = f(T)$

To evaluate this function let 1 kg of a perfect gas be heated at constant volume.

According to non-flow energy equation,

$$dQ = du + dW$$

$dW = 0$, since volume remains constant

$$\therefore \quad dQ = du$$

At constant volume for a perfect gas, from eqn. (4.17), for 1 kg

$$dQ = c_v dT$$

$$\therefore \quad dQ = du = c_v dT$$

and integrating $u = c_v\ T + K$, K being constant.

According to Joule's law $u = f(T)$, which means that internal energy varies linearly with absolute temperature. Internal energy can be made zero at any arbitrary reference temperature. For a perfect gas it can be assumed that $u = 0$ when $T = 0$, hence constant K is zero.

i.e., Internal energy, $u = c_v\ T$ for a perfect gas ...(4.20)

or For mass m, of a perfect gas

Internal energy, $U = mc_v\ T$ ...(4.21)

For a perfect gas, in any process between states 1 and 2, we have from Eqn. (4.21)

Gain in internal energy,

$$U_2 - U_1 = mc_v\ (T_2 - T_1) \quad ...(4.22)$$

Eqn. (4.22) gives the gains of internal energy for a perfect gas between two states *for any process, reversible* or *irreversible.*

4.8.4. Relationship Between Two Specific Heats

Consider a perfect gas being heated at constant pressure from T_1 to T_2.

According to non-flow equation,

$$Q = (U_2 - U_1) + W$$

Also for a perfect gas,

$$U_2 - U_1 = mc_v\ (T_2 - T_1)$$

$$Q = mc_v\ (T_2 - T_1) + W$$

In a constant pressure process, the work done by the fluid,

$$W = p(V_2 - V_1)$$

$$= mR(T_2 - T_1) \qquad \left[\begin{aligned} \because\ p_1V_1 &= mRT_1 \\ p_2V_2 &= mRT_2 \\ p_1 = p_2 &= p \text{ in this case} \end{aligned}\right]$$

On substituting

$$Q = mc_v\,(T_2 - T_1) + mR\,(T_2 - T_1) = m(c_v + R)\,(T_2 - T_1)$$

But for a constant pressure process,

$$Q = mc_p\,(T_2 - T_1)$$

By equating the two expressions, we have

$$m(c_v + R)(T_2 - T_1) = mc_p(T_2 - T_1)$$

$$\therefore \qquad c_v + R = c_p$$

or

$$c_p - c_v = R \qquad \text{...(4.23)}$$

Dividing both sides by c_v, we get

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}$$

$$\therefore \qquad c_v = \frac{R}{\gamma - 1} \qquad \text{...[4.23 (a)]}$$

(where $\gamma = c_p/c_v$)

Similarly, dividing both sides by c_p, we get

$$c_p = \frac{\gamma R}{\gamma - 1} \qquad \text{...[4.23 (b)]}$$

$$\left[\begin{aligned} &\text{In M.K.S. units} : c_p - c_v = \frac{R}{J};\ c_v = \frac{R}{J(\gamma - 1)},\ c_p = \frac{\gamma R}{(\gamma - 1)J} \\ &\text{In SI units the value of } J \text{ is unity.} \end{aligned}\right]$$

4.8.5. Enthalpy

— One of the fundamental quantities which occur invariably in thermodynamics is the sum of internal energy (u) and pressure volume product (pv). This sum is called **Enthalpy** (h).

i.e.,

$$h = u + pv \qquad \text{...(4.24)}$$

— The enthalpy of a fluid is the property of the fluid, since it consists of the sum of a property and the product of the two properties. Since enthalpy is a property like internal energy, pressure, specific volume and temperature, it can be introduced into any problem whether the process is a flow or a non-flow process.

The total enthalpy of mass, m, of a fluid can be

$$H = U + pV, \text{ where } H = mh.$$

For a **perfect gas,**

Referring equation (4.24),

$$h = u + pv$$

$$= c_v T + RT \qquad [\because \quad pv = RT]$$

$$= (c_v + R)T$$

$$= c_p T \qquad [\because \quad c_p = c_v + R]$$

i.e., $h = c_p T$

and $H = mc_p T.$

(Note that, since it has been assumed that $u = 0$ at $T = 0$, then $h = 0$ at $T = 0$).

4.8.6. Ratio of Specific Heats

The ratio of specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma).

i.e., $$\gamma = \frac{c_p}{c_v} \qquad ...(4.25)$$

Since $c_p = c_v + R$, it is clear that c_p must be *greater* than c_v for any perfect gas. It follows, therefore, that the ratio, $\frac{c_p}{c_v} = \gamma$ is *always greater than unity.*

In general, the approximate values of γ are as follows :

For *monoatomic* gases such as *argon, helium* = 1.6.

For *diatomic* gases such as *carbon monoxide, hydrogen, nitrogen* and *oxygen* = 1.4.

For triatomic gases such as *carbon dioxide* and *sulphur dioxide* = 1.3.

For some hydro-carbons the value of γ is quite low.

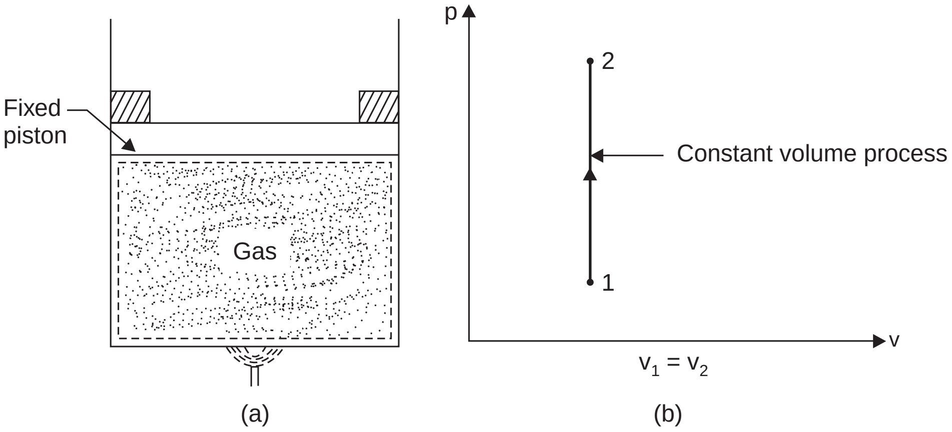
[*e.g.*, for ethane $\gamma = 1.22$, and for isobutane $\gamma = 1.11$]

4.9. APPLICATION OF FIRST LAW OF THERMODYNAMICS TO NON-FLOW OR CLOSED SYSTEM

1. Reversible Constant Volume (or Isochoric) Process (v = constant) :

In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle-wheel work input. It will be assumed that '*constant volume*' implies zero work unless stated otherwise.

Fig. 4.5 shows the system and states before and after the heat addition at constant volume.



(a) (b)

Fig. 4.5. Reversible constant volume process.

Considering mass of the working substance *unity* and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W \quad \text{...(4.26)}$$

The work done $W = \int_1^2 pdv = 0$ as $dv = 0$.

$$\therefore \quad Q = (u_2 - u_1) = c_v(T_2 - T_1) \quad \text{...[4.27 (a)]}$$

where c_v = Specific heat at constant volume.

For mass, m, of working substance

$$Q = U_2 - U_1 = mc_v(T_2 - T_1) \quad \text{...[4.27 (b)]}$$

$[\because \quad mu = U]$

2. Reversible Constant Pressure (or Isobaric) Process (p = constant).

It can be seen from Fig. 4.5 (*b*) that when the boundary of the system is *inflexible* as in a constant volume process, then the pressure rises when heat is supplied. Hence for a constant pressure process, the boundary must move against an external resistance as heat is supplied ; for instance a gas [Fig. 4.6 (*a*)] in a cylinder behind a piston can be made to undergo a constant pressure process. Since the *piston is pushed through a certain distance* by the force exerted by the gas, then the work is done by the gas on its surroundings.

Fig. 4.6 shows the system and states before and after the heat addition at constant pressure.

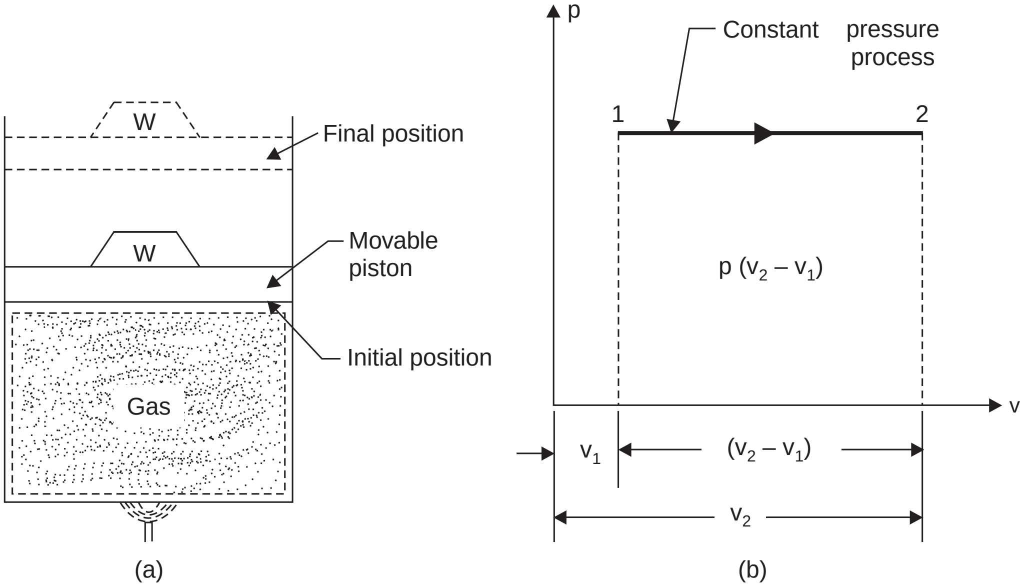


Fig. 4.6. Reversible constant pressure process.

Considering unit mass of working substance and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W$$

The work done, $W = \int_1^2 pdv = p(v_2 - v_1)$

$$\therefore \quad Q = (u_2 - u_1) + p(v_2 - v_1) = u_2 - u_1 + pv_2 - pv_1$$

$$= (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1 \qquad [\because \ h = u + pv]$$

or
$$Q = h_2 - h_1 = c_p\,(T_2 - T_1) \qquad ...(4.28)$$

where h = Enthalpy (specific), and

c_p = Specific heat at constant pressure.

For mass, m, of working substance

$$Q = H_2 - H_1 = mc_p\,(T_2 - T_1) \qquad ...[4.28\,(a)]$$

$$[\because \ mh = H]$$

3. Reversible Temperature (or Isothermal) Process (pv = constant, T = constant) :

A *process at a constant temperature is called an isothermal process.* When a working substance in a cylinder behind a piston expands from a high pressure to a low pressure there is a tendency for the temperature to fall. In an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from the working substance continuously during the process.

Fig. 4.7 shows the system and states before and after the heat addition at constant temperature.

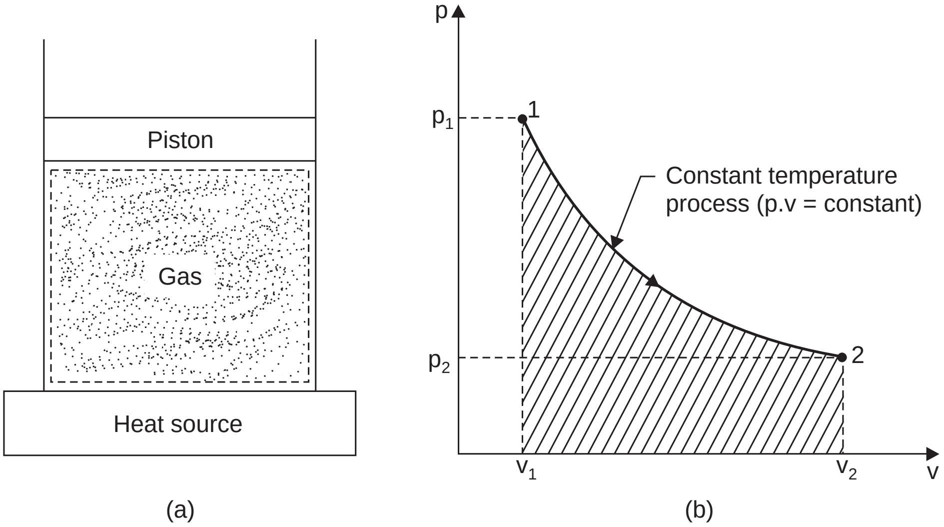


Fig. 4.7. Reversible isothermal process.

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W$$
$$= c_v\,(T_2 - T_1) + W$$
$$= 0 + W \qquad [\because \ T_2 = T_1]$$

The work done, $W = \int_1^2 p\,dv$

In this case pv = constant or $p = \dfrac{C}{v}$ (where C = constant)

$$\therefore \qquad W = \int_{v_1}^{v_2} C\frac{dv}{v} = C\big[\log_e v\big]_{v_1}^{v_2} = C\log_e\frac{v_2}{v_1}$$

The constant C can either be written as p_1v_1 or as p_2v_2, since

$$p_1v_1 = p_2v_2 = \text{constant, } C$$

i.e., $W = p_1v_1 \log_e \dfrac{v_2}{v_1}$ per unit mass of working substance

or $W = p_2v_2 \log_e \dfrac{v_2}{v_1}$ per unit mass of working substance

$$\therefore \quad Q = W = p_1v_1 \log_e \frac{v_2}{v_1} \quad \text{...(4.29)}$$

For mass, m, of the working substance

$$Q = p_1V_1 \log_e \frac{V_2}{V_1} \quad \text{...[4.29 (a)]}$$

or

$$Q = p_1V_1 \log_e \frac{p_1}{p_2} \quad \left[\because \frac{V_2}{V_1} = \frac{p_1}{p_2}\right] \quad \text{...[4.29 (b)]}$$

4. Reversible Adiabatic Process (pv^{γ} = constant) :

An **adiabatic process** *is one in which no heat is transferred to or from the fluid during the process.* Such a process can be reversible or irreversible. The reversible adiabatic non-flow process will be considered in this section.

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W$$

$$O = (u_2 - u_1) + W$$

or $W = (u_1 - u_2)$ for any adiabatic process ...(4.30)

Eqn. (4.30) is true for an adiabatic process whether the process is reversible or not. In an adiabatic expansion, the work done W by the fluid is at the expense of a reduction in the internal energy of the fluid. Similarly in an adiabatic compression process all the work done on the fluid goes to increase the internal energy of the fluid.

For an adiabatic process to take place, perfect thermal insulation for the system must be available.

To derive the law pv$^{\gamma}$ = constant :

To obtain a law relating p and v for a reversible adiabatic process let us consider the non-flow energy equation in differential form,

$$dQ = du + dW$$

For a *reversible process*

$$dW = pdv$$

$$\therefore \quad dQ = du + pdv = 0$$

(Since for an adiabatic process $Q = 0$)

Also for a perfect gas

$$pv = RT \text{ or } p = \frac{RT}{v}$$

Hence substituting,

$$du + \frac{RTdv}{v} = 0$$

Also $u = c_vT$ or $du = c_vdT$

$$\therefore \quad c_vdT + \frac{RTdv}{v} = 0$$

Dividing both sides by T, we get

$$c_v \frac{dT}{T} + \frac{Rdv}{v} = 0$$

Integrating

$$c_v \log_e T + R \log_e v = \text{constant}$$

Substituting $T = \frac{pv}{R}$

$$c_v \log_e \frac{pv}{R} + R \log_e v = \text{constant}$$

Dividing throughout both sides by c_v

$$\log_e \frac{pv}{R} + \frac{R}{c_v} . \log_e v = \text{constant}$$

Again $$c_v = \frac{R}{(\gamma - 1)} \quad \text{or} \quad \frac{R}{c_v} = \gamma - 1$$

Hence substituting

$$\log_e \frac{pv}{R} + (\gamma - 1) \log_e v = \text{constant}$$

$$\therefore \qquad \log_e \frac{pv}{R} + \log_e v^{\gamma - 1} = \text{constant}$$

$$\log_e \frac{pv \times v^{\gamma - 1}}{R} = \text{constant}$$

i.e., $$\log_e \frac{pv^{\gamma}}{R} = \text{constant}$$

i.e., $$\frac{pv^{\gamma}}{R} = e^{\text{constant}} = \text{constant}$$

or $$pv^{\gamma} = \text{constant} \qquad ...(4.31)$$

Expression for work W :

A reversible adiabatic process for a perfect gas is shown on a *p-v* diagram in Fig. 4.8 (*b*).

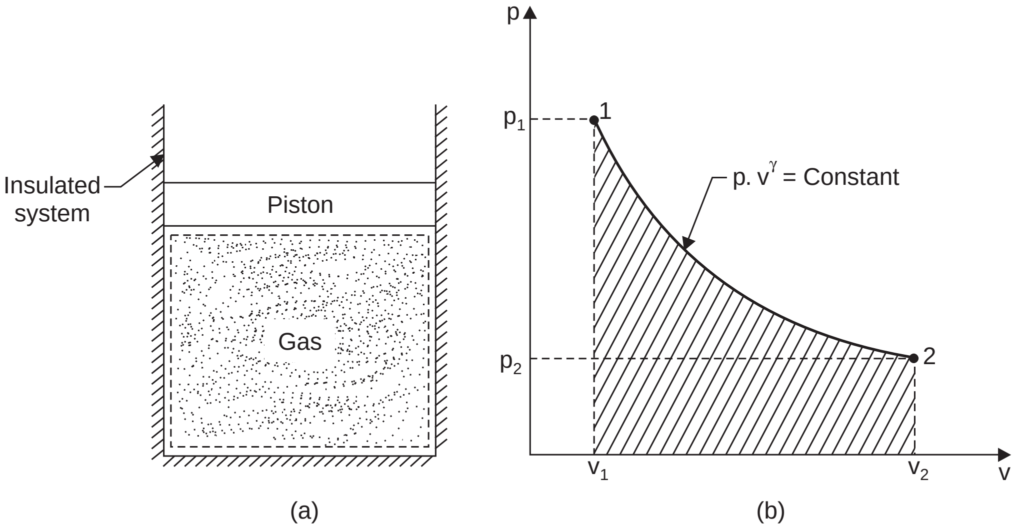


Fig. 4.8. Reversible adiabatic process.

The work done is given by the shaded area, and this area can be evaluated by integration.

i.e., $$W = \int_{v_1}^{v_2} p\,dv$$

Therefore, since pv^{γ} = constant, C, then

$$W = \int_{v_1}^{v_2} C\,\frac{dv}{v^{\gamma}} \qquad \left[\because\ p = \frac{C}{v^{\gamma}}\right]$$

i.e., $$W = C\int_{v_1}^{v_2} \frac{dv}{v^{\gamma}} = C\left|\frac{v^{-\gamma+1}}{-\gamma+1}\right|_{v_1}^{v_2}$$

$$= C\left(\frac{{v_2}^{-\gamma+1} - {v_1}^{-\gamma+1}}{1-\gamma}\right) = C\left(\frac{{v_1}^{-\gamma+1} - {v_2}^{-\gamma+1}}{\gamma-1}\right)$$

The constant in this equation can be written as $p_1{v_1}^{\gamma}$ or as $p_2{v_2}^{\gamma}$. Hence,

$$W = \frac{p_1\,{v_1}^{\gamma}{v_1}^{-\gamma+1} - p_2\,{v_2}^{\gamma}{v_2}^{-\gamma+1}}{\gamma-1} = \frac{p_1v_1 - p_2v_2}{\gamma-1}$$

i.e., $$W = \frac{p_1v_1 - p_2v_2}{\gamma-1} \qquad ...(4.32)$$

or $$W = \frac{R(T_1 - T_2)}{\gamma-1} \qquad ...(4.33)$$

Relationship between T and v, and T and p :

By using equation $pv = RT$, the relationship between T and v, and T and p, may by derived as follows :

i.e., $$pv = RT$$

$$\therefore \quad p = \frac{RT}{v}$$

Putting this value in the equation pv^{γ} = constant

$$\frac{RT}{v}\,.\,v^{\gamma} = \text{constant}$$

i.e., $$Tv^{\gamma-1} = \text{constant} \qquad ...(4.34)$$

Also $v = \dfrac{RT}{p}$; hence substituting in equation pv^{γ} = constant

$$p\left(\frac{RT}{p}\right)^{\gamma} = \text{constant}$$

$$\therefore \quad \frac{T^{\gamma}}{p^{\gamma-1}} = \text{constant}$$

or $$\frac{T}{(p)^{\frac{\gamma-1}{\gamma}}} = \text{constant} \qquad ...(4.35)$$

Therefore, for a reversible adiabatic process for a perfect gas between states 1 and 2, we can write :

From Eqn. (4.31),

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \qquad ...(4.36)$$

From Eqn. (4.34),

$$T_1 v_1^{\gamma - 1} = T_2 v_2^{\gamma - 1} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} \qquad ...(4.37)$$

From Eqn. (4.35),

$$\frac{T_1}{(p_1)^{\frac{\gamma - 1}{\gamma}}} = \frac{T_2}{(p_2)^{\frac{\gamma - 1}{\gamma}}} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} \qquad ...(4.38)$$

From eqn. (4.30), the work done in an adiabatic process per kg of gas is given by $W = (u_1 - u_2)$. The gain in internal energy of a perfect gas is given by equation :

$$u_2 - u_1 = c_v (T_2 - T_1) \qquad \text{(for 1 kg)}$$

$$\therefore \qquad W = c_v (T_1 - T_2)$$

Also, we know that

$$c_v = \frac{R}{\gamma - 1}$$

Hence substituting, we get

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

Using equation, $pv = RT$

$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

This is the same expression obtained before as eqn. (4.32).

5. Polytropic Reversible Process (pv^n = constant) :

It is found that many processes in practice approximate to a reversible law of form pv^n = constant, where n is a constant. Both vapours and perfect gases obey this type of law closely in many non-flow processes. Such processes are *internally reversible*.

We know that for any reversible process,

$$W = \int p \, dv$$

For a process in pv^n = constant, we have

$$p = \frac{C}{v^n}, \text{ where } C \text{ is a constant}$$

$$\therefore \qquad W = C \int_{v_1}^{v_2} \frac{dv}{v^n} = C \left| \frac{v^{-n+1}}{-n+1} \right| = C \left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1} \right)$$

i.e.,

$$W = C \left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1} \right) = \frac{p_1 v_1^n v_1^{-n+1} - p_2 v_2^n v_2^{-n+1}}{n-1}$$

(since the constant C, can be written as $p_1 v_1^n$ or as $p_2 v_2^n$)

i.e., Work done, $$W = \frac{p_1v_1 - p_2v_2}{n-1} \quad ...(4.39)$$

or $$W = \frac{R(T_1 - T_2)}{n-1} \quad ...(4.40)$$

Eqn. (4.39) is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process, we can write

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \quad ...(4.41)$$

The following relations can be derived (following the same procedure as was done under reversible adiabatic process)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \quad ...(4.42)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad ...(4.43)$$

Heat transfer during polytropic process (for perfect gas pv = RT) :

Using non-flow energy equation, the heat flow/transfer during the process can be found,

i.e., $$Q = (u_2 - u_1) + W$$

$$= c_v(T_2 - T_1) + \frac{R(T_1 - T_2)}{n-1}$$

i.e., $$Q = \frac{R(T_1 - T_2)}{n-1} - c_v\,(T_1 - T_2)$$

Also $$c_v = \frac{R}{(\gamma - 1)}$$

On substituting,

$$Q = \frac{R}{n-1}(T_1 - T_2) - \frac{R}{(\gamma-1)}\,(T_1 - T_2)$$

i.e., $$Q = R(T_1 - T_2)\left(\frac{1}{n-1} - \frac{1}{\gamma-1}\right)$$

$$= \frac{R\,(T_1 - T_2)(\gamma - 1 - n + 1)}{(\gamma-1)(n-1)} = \frac{R(T_1 - T_2)(\gamma - n)}{(\gamma-1)(n-1)}$$

$\therefore$ $$Q = \frac{(\gamma - n)}{(\gamma - 1)}\,\frac{R(T_1 - T_2)}{(n-1)}$$

or $$Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W \qquad \left[\because\ W = \frac{R(T_1 - T_2)}{(n-1)}\right] \quad ...(4.44)$$

In a polytropic process, the *index n depends only on the heat and work quantities* during the process. The various processes considered earlier are special cases of polytropic process for a perfect gas. For example,

When $n = 0$ pv° = constant *i.e.,* p = constant

When $n = \infty$ pv^{∞} = constant

or $p^{1/\infty}\, v$ = constant, *i.e.,* v = constant

When $n = 1$ pv = constant, *i.e.,* T = constant

[since $(pv)/T$ = constant for a perfect gas]

When $n = \gamma$ pv^{γ} = constant, *i.e.,* reversible adiabatic

This is illustrated on a *p-v* diagram in Fig. 4.9.

(*i*) State 1 to state *A* is *constant pressure cooling* ($n = 0$).

(*ii*) State 1 to state *B* is *isothermal compression* ($n = 1$).

(*iii*) State 1 to state *C* is *reversible adiabatic compression* ($n = \gamma$).

(*iv*) State 1 to state *D* is *constant volume heating* ($n = \infty$).

Similarly,

(*i*) State 1 to state *A′* is *constant pressure heating* ($n = 0$).

(*ii*) State 1 to state *B′* is *isothermal expansion* ($n = 1$).

(*iii*) State 1 to state *C′* is *reversible adiabatic expansion* ($n = \gamma$).

(*iv*) State 1 to state *D′* is *constant volume cooling* ($n = \propto$).

It may be noted that, since γ is always greater than unity, than process 1 to *C* must lie between processes 1 to *B* and 1 to *D* ; similarly, process 1 to *C′* must lie between processes 1 to *B′* and 1 to *D′*.

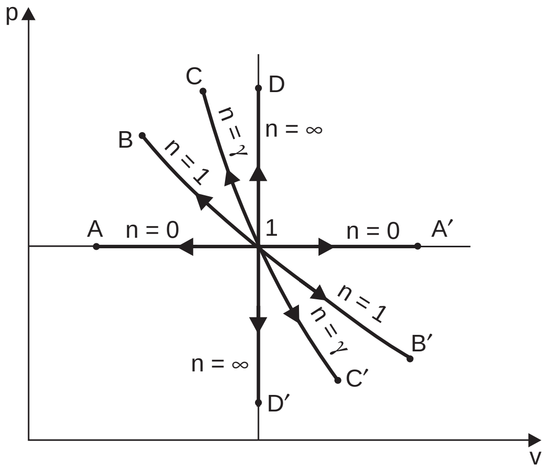


Fig. 4.9

6. Free Expansion

Consider two vessels 1 and 2 interconnected by a short pipe with a valve *A*, and perfectly thermally insulated [Fig. 4.10]. Initially let the vessel 1 be filled with a fluid at a certain pressure, and let 2 be completely evacuated. When the valve *A* is opened the fluid in 1 will expand rapidly to fill both vessels 1 and 2. The pressure finally will be lower than the initial pressure in vessel 1. This is known as *free or unresisted expansion*. The process is *highly irreversible* ; since the fluid is eddying continuously during the process. Now applying first law of thermodynamics (or non-flow energy equation) between the initial and final states,

$$Q = (u_2 - u_1) + W$$

In this process, no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well lagged. The process is therefore, *adiabatic but irreversible*.

i.e., $$u_2 - u_1 = 0 \quad \text{or} \quad u_2 = u_1$$

In a free expansion, therefore, the internal energy initially equals the initial energy finally.

For a perfect gas,

$$u = c_v T$$

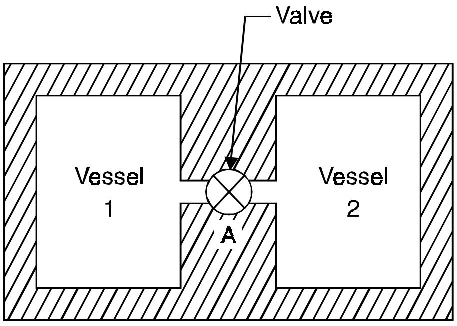


Fig. 4.10. Free expansion.

∴ For a free expansion of a perfect gas,

$$c_vT_1 = c_vT_2 \quad i.e., \quad T_1 = T_2$$

That is, for a perfect gas undergoing a free expansion, the initial temperature is equal to the final temperature.

Table 4.1
Summary of Processes for Perfect Gas (Unit mass)

Process	*Index* n	*Heat added*	$\int_1^2 pdv$	*p, v, T relations*	*Specific heat, c*
Constant pressure	$n = 0$	$c_p(T_2 - T_1)$	$p(v_2 - v_1)$	$\frac{T_2}{T_1} = \frac{v_2}{v_1}$	c_p
Constant volume	$n = \infty$	$c_v(T_2 - T_1)$	0	$\frac{T_1}{T_2} = \frac{p_1}{p_2}$	c_v
Constant temperature	$n = 1$	$p_1v_1 \log_e \frac{v_2}{v_1}$	$p_1v_1 \log_e \frac{v_2}{v_1}$	$p_1v_1 = p_2v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\frac{p_1v_1 - p_2v_2}{\gamma - 1}$	$p_1v_1^{\gamma} = p_2v_2^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$ $= \left(\frac{p_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$	0
Polytropic	$n = n$	$c_n(T_2 - T_1)$ $= c_v\left(\frac{\gamma - n}{1 - n}\right) \times (T_2 - T_1)$ $= \frac{\gamma - n}{\gamma - 1} \times$ work done (non–flow)	$\frac{p_1v_1 - p_2v_2}{n - 1}$	$p_1v_1^{n} = p_2v_2^{n}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n - 1}$ $= \left(\frac{p_2}{p_1}\right)^{\frac{n - 1}{n}}$	$c_n = c_v\left(\frac{\gamma - n}{1 - n}\right)$

Note. Equations must be used keeping dimensional consistence.

Example 4.1. *In an internal combustion engine, during the compression stroke the heat rejected to the cooling water is 50 kJ/kg and the work input is 100 kJ/kg.*

Calculate the change in internal energy of the working fluid stating whether it is a gain or loss.

Solution. Heat rejected to the cooling water, $Q = -\,50$ kJ/kg

(–ve sign since heat is rejected)

Work input, $W = -\,100$ kJ/kg

(–ve sign since work is supplied to the system)

Using the relation, $Q = (u_2 - u_1) + W$

$$-\,50 = (u_2 - u_1) - 100$$

or $$u_2 - u_1 = -\,50 + 100 = 50 \text{ kJ/kg}$$

Hence, **gain in internal energy = 50 kJ/kg. (Ans.)**

Example 4.2. *In an air motor cylinder the compressed air has an internal energy of 450 kJ/kg at the beginning of the expansion and an internal energy of 220 kJ/kg after expansion. If the work done by the air during the expansion is 120 kJ/kg, calculate the heat flow to and from the cylinder.*

Solution. Internal energy at beginning of the expansion,

$$u_1 = 450 \text{ kJ/kg}$$

Internal energy after expansion,

$$u_2 = 220 \text{ kJ/kg}$$

Work done by the air during expansion,

$$W = 120 \text{ kJ/kg}$$

Heat flow, Q :

Using the relation, $Q = (u_2 - u_1) + W$

$\therefore$ $$Q = (220 - 450) + 120$$

$$= -\,230 + 120 = -\,110 \text{ kJ/kg}$$

Hence, **heat rejected by air = 110 kJ/kg. (Ans.)**

Example 4.3. *0.3 kg of nitrogen gas at 100 kPa and 40°C is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 1 MPa and temperature becomes 160°C. The work done during the process is 30 kJ.*

Calculate the heat transferred from the nitrogen to the surroundings.

c_v *for nitrogen = 0.75 kJ/kg K.*

Solution. Mass of nitrogen, $m = 0.3$ kg

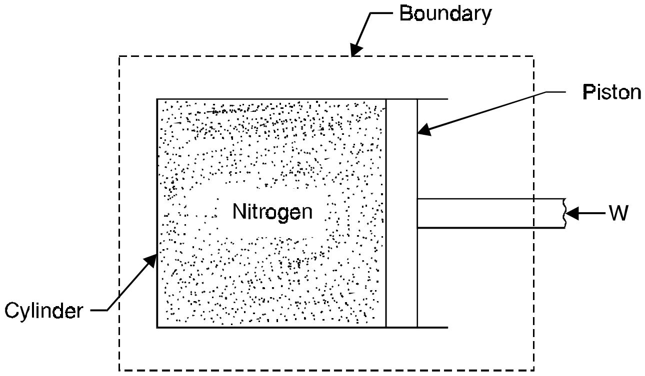


Fig. 4.11

Temperature before compression = 40°C or 313 K

Temperature after compression = 160°C or 433 K

The work done during the compression process, $W = -\ 30$ kJ

According to first law of thermodynamics,

$$Q = \Delta U + W = (U_2 - U_1) + W$$
$$= mc_v\ (T_2 - T_1) + W$$
$$= 0.3 \times 0.75(433 - 313) - 30 = -\ 3 \text{ kJ}$$

Hence, **heat 'rejected' during the process = 3 kJ. (Ans.)**

Note. Work, W has been taken –ve because it has been supplied from outside.

Example 4.4. *When a stationary mass of gas was compressed without friction at constant pressure its initial state of 0.4 m³ and 0.105 MPa was found to change to final state of 0.20 m³ and 0.105 MPa. There was a transfer of 42.5 kJ of heat from the gas during the process.*

How much did the internal energy of the gas change ?

Solution.

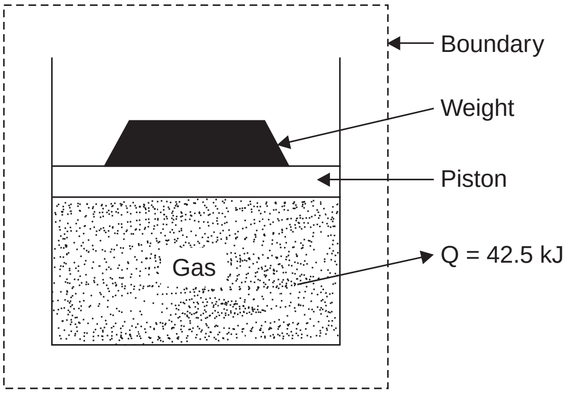


Fig. 4.12

Initial state

Pressure of gas, $p_1 = 0.105$ MPa

Volume of gas, $V_1 = 0.4$ m³

Final state

Pressure of gas, $p_2 = 0.105$ MPa

Volume of gas, $V_2 = 0.20$ m³

Process used : *Constant pressure*

Heat transferred, $Q = -\ 42.5$ kJ

(–ve sign indicates that heat is rejected)

Change in internal energy, $\Delta U = U_2 - U_1$:

First law for a stationary system in a process gives

$$Q = \Delta U + W$$

or
$$Q_{1-2} = (U_2 - U_1) + W_{1-2} \qquad ...(i)$$

Here
$$W_{1-2} = \int_{V_1}^{V_2} pdV = p(V_2 - V_1)$$

$= 0.105(0.20 - 0.40)$ MJ $= -21$ kJ [$\because$ 1 MJ = 10^3 kJ]

Substituting this value of W_{1-2} in equation (*i*), we get

$$-42.5 = (U_2 - U_1) - 21$$

$$\therefore \quad U_2 - U_1 = -42.5 + 21 = -21.5 \text{ kJ}$$

Hence **'decrease' in internal energy = 21.5 kJ. (Ans.)**

Example 4.5. *A container is divided into compartments by a partition. The container is completely insulated so that there is no heat transfer. One portion contains gas at temperature* T_1 *and pressure* p_1 *while the other portion also has the same gas but at temperature* T_2 *and pressure* p_2.

How will the First Law of Thermodynamics conclude the result if partition is removed ?

Solution. Refer Fig. 4.13.

According to First Law of Thermodynamics,

$$\delta Q = \delta U + \delta W$$

When partition removed, $\delta Q = 0$

$$\delta W = 0$$

$$\therefore \quad \delta U = 0.$$

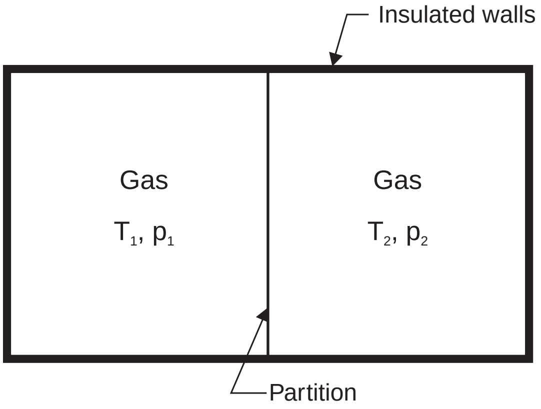


Fig. 4.13

Conclusion. *There is conservation of internal energy.*

Example 4.6. *Air enters a compressor at* 10^5 *Pa and 25°C having volume of 1.8* m^3/kg *and is compressed to* 5×10^5 *Pa isothermally.*

Determine : (*i*) *Work done ;*

(*ii*) *Change in internal energy ; and*

(*iii*) *Heat transferred.*

Solution. Initial pressure of air, $p_1 = 10^5$ Pa

Initial temperature of air, $T_1 = 25 + 273 = 298$ K

Final pressure of air, $p_2 = 5 \times 10^5$ Pa

Final temperature of air, $T_2 = T_1 = 298$ K (isothermal process)

Since, it is a closed steady state process, we can write down the first law of thermodynamics as,

$$Q = (u_2 - u_1) + W \text{ ......per kg}$$

(*i*) *For isothermal process :*

$$W_{1-2} = \int_1^2 p \, . dv = p_1 v_1 \log_e \left(\frac{p_1}{p_2}\right)$$

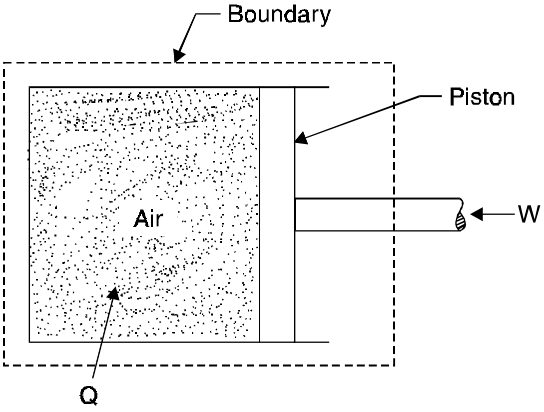


Fig. 4.14

as $p_1 v_1 = p_2 v_2$ for isothermal process

$$\therefore \quad W_{1-2} = -10^5 \times 1.8 \log_e \left(\frac{1 \times 10^5}{5 \times 10^5}\right)$$

$$= -2.897 \times 10^5 = -289.7 \text{ kJ/kg.}$$

(– ve sign indicates that the work is supplied to the air)

∴ **Work done on the air = 289.7 kJ/kg. (Ans.)**

(*ii*) Since temperature is constant,

$$\therefore \quad u_2 - u_1 = 0$$

∴ **Change in internal energy = zero. (Ans.)**

(*iii*) Again, $Q_{1-2} = (u_2 - u_1) + W$

$$= 0 + (-289.7) = -289.7 \text{ kJ}$$

(– ve sign indicates that heat is lost from the system to the surroundings)

∴ **Heat rejected = 289.7 kJ/kg. (Ans.)**

Example 4.7. *A cylinder containing the air comprises the system. Cycle is completed as follows :*

(*i*) *82000 N-m of work is done by the piston on the air during compression stroke and 45 kJ of heat are rejected to the surroundings.*

(*ii*) *During expansion stroke 100000 N-m of work is done by the air on the piston.*

Calculate the quantity of heat added to the system.

Solution. Refer Fig. 4.15.

Compression stroke. Process 1-2 :

Work done by the piston *on the air,* $W_{1-2} = -82000$ N-m (= – 82 kJ)

Heat rejected to the system, $Q_{1-2} = -45$ kJ

Now, $Q_{1-2} = (U_2 - U_1) + W$

$$-45 = (U_2 - U_1) + (-82)$$

$$\therefore \quad (U_2 - U_1) = 37 \text{ kJ} \qquad ...(i)$$

Expansion stroke. Process 2-1 :

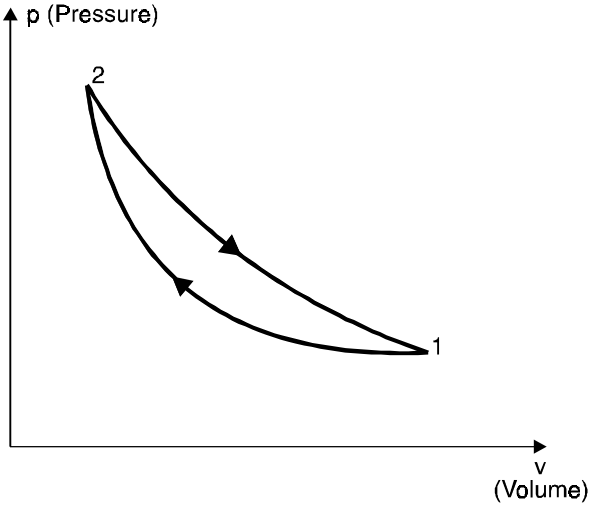


Fig. 4.15

Work done by air on the piston, W_{2-1} = 100000 N-m (= 100 kJ)

Now, $Q_{2-1} = (U_1 - U_2) + W$

$= -37 + 100 \text{ kJ} = 63 \text{ kJ}$

Hence, **quantity of heat added to the system = 63 kJ. (Ans.)**

☞**Example 4.8.** *A tank containing air is stirred by a paddle wheel. The work input to the paddle wheel is 9000 kJ and the heat transferred to the surroundings from the tank is 3000 kJ.*

Determine : (*i*) *Work done ;*

(*ii*) *Change in internal energy of the system.*

Solution. Refer Fig. 4.16.

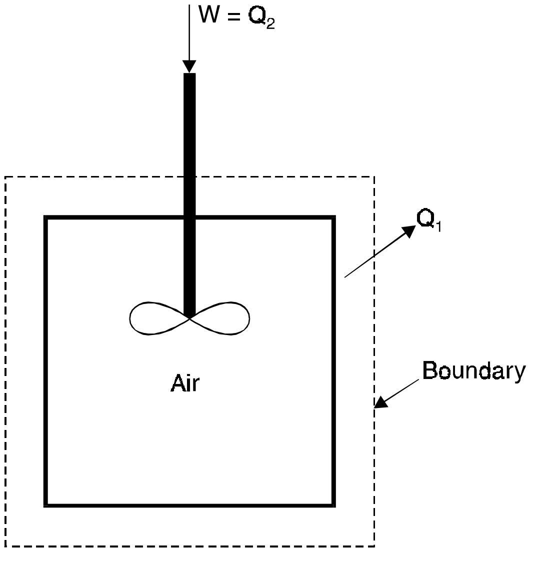


Fig. 4.16

Work input to the paddle wheel = 9000 kJ

Heat transferred to the surroundings from the tank = 3000 kJ

As it is a closed system, the first law of thermodynamics can be written as

$$U_1 - Q + W = U_2 \qquad \text{...}(i)$$

The *work* enters into the tank in the form of energy only so this should be considered as *heat input*.

$$\therefore \qquad Q = Q_1 - Q_2$$
$$= 3000 - 9000 = -6000 \text{ kJ.}$$

(*i*) Since volume does not change (being constant volume process)

$\therefore$ Work done, $W = 0$

Putting the value of $W = 0$ in equation (*i*), we get

(*ii*) $U_1 - (-6000) + 0 = U_2$

$\therefore \qquad U_2 - U_1 = 6000$ kJ

Hence, **change in internal energy (increase) = 6000 kJ. (Ans.)**

Example 4.9. *A stone of 20 kg mass and a tank containing 200 kg water comprise a system. The stone is 15 m above the water level initially. The stone and water are at the same temperature initially. If the stone falls into water, then determine* ΔU, ΔPE, ΔKE, *Q and W, when*

(*i*) *the stone is about to enter the water,*

(*ii*) *the stone has come to rest in the tank, and*

(*iii*) *the heat is transferred to the surroundings in such an amount that the stone and water come to their initial temperature.*

Solution. Refer Fig. 4.17.

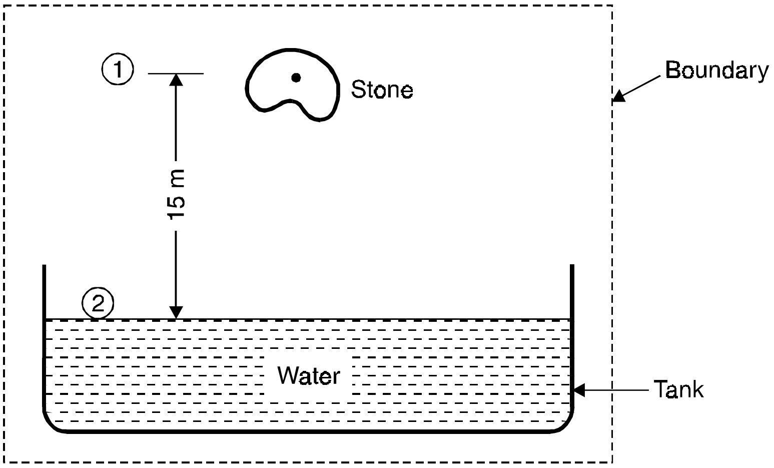


Fig. 4.17

Mass of stone = 20 kg

Mass of water in the tank = 200 kg

Height of stone above water level = 15 m

Applying the first law of thermodynamics,

$$Q = (U_2 - U_1) + m\left[\frac{C_2^2 - C_1^2}{2}\right] + mg\,(Z_2 - Z_1) + W$$
$$= \Delta U + \Delta KE + \Delta PE + W \qquad \text{...(1)}$$

Here Q = Heat leaving the boundary.

(*i*) When the stone is about to enter the water,

$$Q = 0,\ W = 0,\ \Delta U = 0$$

$$\because \quad -\Delta KE = \Delta PE = mg\,(Z_2 - Z_1)$$

$$= 20 \times 9.81\,(0 - 15) = -2943 \text{ J}$$

$$\therefore \quad \Delta KE = 2943 \text{ J}$$

and $\Delta PE = \mathbf{-2943\ J.}$ **(Ans.)**

(*ii*) When the stone dips into the tank and comes to rest

$$Q = 0,\ W = 0,\ \Delta KE = 0$$

Substituting these values in eqn. (1), we get

$$0 = \Delta U + 0 + \Delta PE + 0$$

$\therefore \quad \Delta U = -\Delta PE = -(-2943) =$ **2943 J. (Ans.)**

This shows that the internal energy (temperature) of the system increases.

(*iii*) When the water and stone come to their initial temperature,

$$W = 0,\ \Delta KE = 0$$

Substituting these values in eqn. (1), we get

$\therefore \quad Q = -\Delta U = -$ **2943 J. (Ans.)**

The *negative* sign shows that the *heat is lost from the system to the surroundings.*

☞**Example 4.10.** *When a system is taken from state l to state m, in Fig. 4.18, along path lqm, 168 kJ of heat flows into the system, and the system does 64 kJ of work :*

(*i*) *How much will be the heat that flows into the system along path lnm if the work done is 21 kJ ?*

(*ii*) *When the system is returned from m to l along the curved path, the work done on the system is 42 kJ. Does the system absorb or liberate heat, and how much of the heat is absorbed or liberated ?*

(*iii*) *If $U_l = 0$ and $U_n = 84$ kJ, find the heat absorbed in the processes ln and nm.*

Solution. Refer Fig. 4.18.

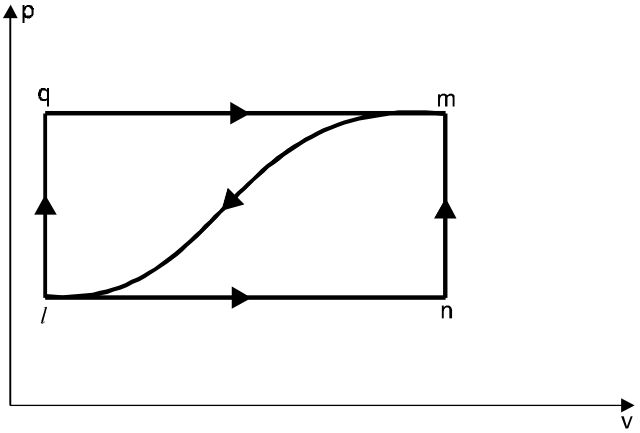


Fig. 4.18

$$Q_{l-q-m} = 168 \text{ kJ}$$

$$W_{l-q-m} = 64 \text{ kJ}$$

We have, $Q_{l-q-m} = (U_m - U_l) + W_{l-q-m}$

$168 = (U_m - U_l) + 64$

$\therefore$ $U_m - U_l =$ **104 kJ. (Ans.)**

(*i*) $Q_{l-n-m} = (U_m - U_l) + W_{l-n-m}$

$= 104 + 21 =$ **125 kJ. (Ans.)**

(*ii*) $Q_{m-l} = (U_l - U_m) + W_{m-l}$

$= -104 + (-42) =$ **– 146 kJ. (Ans.)**

The system liberates 146 kJ.

(*iii*) $W_{l-n-m} = W_{l-n} + W_{n-m} = W_{l-m} = 21$ kJ

[$\because$ $W_{n-m} = 0$, since volume does not change.]

$\therefore$ $Q_{l-n} = (U_n - U_l) + W_{l-n}$

$= (84 - 0) + 21 =$ **105 kJ. (Ans.)**

Now $Q_{l-m-n} = 125 \text{ kJ} = Q_{l-n} + Q_{n-m}$

$\therefore$ $Q_{n-m} = 125 - Q_{l-n}$

$= 125 - 105 =$ **20 kJ. (Ans.)**

Example 4.11. *In a system, executing a non-flow process, the work and heat per degree change of temperature are given by*

$$\frac{dW}{dT} = 200 \text{ W-s/°C and } \frac{dQ}{dT} = 160 \text{ J/°C}$$

What will be the change of internal energy of the system when its temperature changes from

$$T_1 = 55°C \text{ to } T_2 = 95°C\ ?$$

Solution. Initial temperature, $T_1 = 55$°C ; Final temperature, $T_2 = 95$°C

$$\frac{dW}{dT} = 200 \text{ W-s/°C} ; \frac{dQ}{dT} = 160 \text{ J/°C}.$$

Change of internal energy :

Now, $\dfrac{dW}{dT} = 200$ W-s/°C

$$\therefore \quad W = \int_{T_1}^{T_2} 200\, dT = \Big| 200\, T \Big|_{T_1}^{T_2} = \Big| 200\, T \Big|_{55}^{95}$$

$= 200\ (95 - 55) = 8000 \text{ W-s} = 8000$ J [$\because$ 1 W-s = 1 J]

Also, $\dfrac{dQ}{dT} = 160$ J/°C

$$\therefore \quad Q = \int_{T_1}^{T_2} 160\, dT = \Big| 160\, T \Big|_{T_1}^{T_2}$$

$$= \Big| 160\, T \Big|_{55}^{95} = 160\ (95 - 55) = 6400 \text{ J}$$

Applying the first law of thermodynamics to the given non-flow system,

$$Q = \Delta U + W$$

$\therefore$ $6400 = \Delta U + 8000$

or $\Delta U = -1600\ \text{J} = \mathbf{-1.6\ kJ.}$ **(Ans.)**

The –ve sign indicates that there is *decrease* in internal energy.

Example 4.12. *A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is – 340 kJ. The system completes 200 cycles per min.*

Complete the following table showing the method for each item, and compute the net rate of work output in kW.

Process	*Q (kJ/min)*	*W (kJ/min)*	*ΔE (kJ/min)*
1—2	*0*	*4340*	—
2—3	*42000*	*0*	—
3—4	*– 4200*	—	*– 73200*
4—1	—	—	—

Solution. Sum of all heat transferred during the cycle = – 340 kJ.

Number of cycles completed by the system = 200 cycles/min.

Process 1—2 :

$$Q = \Delta E + W$$

$$0 = \Delta E + 4340$$

$\therefore$ $\Delta E = -4340$ kJ/min.

Process 2—3 :

$$Q = \Delta E + W$$

$$42000 = \Delta E + 0$$

$$\Delta E = 42000 \text{ kJ/min.}$$

Process 3—4 :

$$Q = \Delta E + W$$

$$-4200 = -73200 + W$$

$\therefore$ $W = 69000$ kJ/min.

Process 4—1 :

$$\sum_{\text{cycle}} Q = -340 \text{ kJ}$$

The system completes 200 cycles/min

$\because$ $Q_{1-2} = Q_{2-3} + Q_{3-4} + Q_{4-1} = -340 \times 200 = -68000$ kJ/min

or $0 + 42000 + (-4200) + Q_{4-1} = -68000$

$$Q_{4-1} = -105800 \text{ kJ/min.}$$

Now, $\int dE = 0$, since cyclic integral of any property is zero.

$$\Delta E_{1-2} + \Delta E_{2-3} + \Delta E_{3-4} + \Delta E_{4-1} = 0$$

$$-4340 + 42000 + (-73200) + \Delta E_{4-1} = 0$$

$\therefore$ $\Delta E_{4-1} = 35540$ kJ/min.

$\therefore$ $W_{4-1} = Q_{4-1} - \Delta E_{4-1}$

$$= -105800 - 35540 = -141340 \text{ kJ/min}$$

The completed table is given below :

Process	*Q(kJ/min)*	*W(kJ/min)*	ΔE*(kJ/min)*
1—2	0	4340	– 4340
2—3	42000	0	42000
3—4	– 4200	69000	– 73200
4—1	– 105800	– 141340	35540

Since
$$\sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$$

Rate of work output = – 68000 kJ/min $= -\dfrac{68000}{60}$ kJ/s or kW

= **1133.33 kW. (Ans.)**

Example 4.13. *The power developed by a turbine in a certain steam plant is 1200 kW. The heat supplied to the steam in the boiler is 3360 kJ/kg, the heat rejected by the system to cooling water in the condenser is 2520 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 6 kW.*

Calculate the steam flow round the cycle in kg/s.

Solution. The power developed by the turbine = 1200 kW

The heat supplied to the steam in the boiler = 3360 kJ/kg

The heat rejected by the system to cooling water = 2520 kJ/kg

Feed pump work = 6 kW

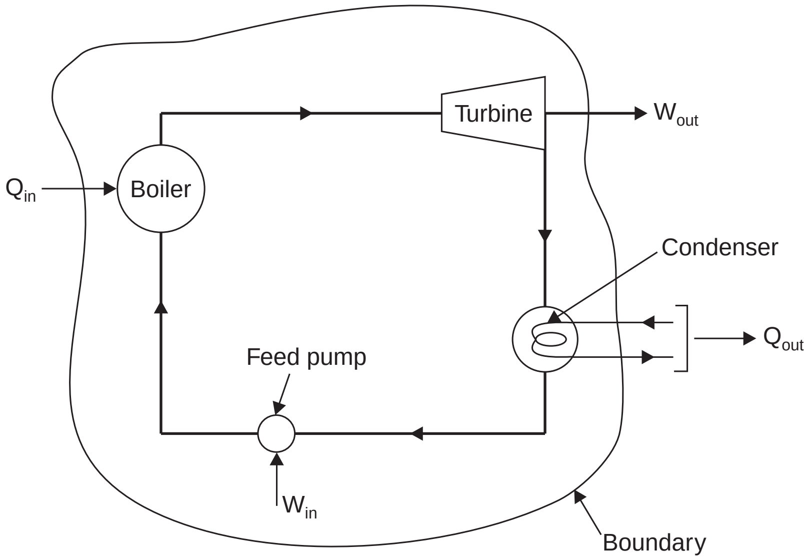


Fig. 4.19

Fig. 4.19 shows the cycle. A boundary is shown which encompasses the entire plant. Strictly, this boundary should be thought of as encompassing the working fluid only.

$$\oint dQ = 3360 - 2520 = 840 \text{ kJ/kg}$$

Let the system flow be in kg/s.

$$\therefore \quad \oint dQ = 840\ \dot{m}\ \text{kJ/s}$$

$$\oint dW = 1200 - 6 = 1194\ \text{kJ/s}$$

But $$\oint dQ = \oint dW$$

i.e., $$840\ \dot{m} = 1194$$

$$\therefore \quad \dot{m} = \frac{1194}{840} = 1.421\ \text{kg/s}$$

$\because$ **Steam flow round the cycle = 1.421 kg/s. (Ans.)**

Example 4.14. *A closed system of constant volume experiences a temperature rise of 25°C when a certain process occurs. The heat transferred in the process is 30 kJ. The specific heat at constant volume for the pure substance comprising the system is 1.2 kJ/kg°C, and the system contains 2.5 kg of this substance. Determine :*

(*i*) *The change in internal energy ;*

(*ii*) *The work done.*

Solution. Temperature rise, $(T_2 - T_1) = 25°\text{C}$

The heat transferred in the process, $Q = 30$ kJ

Specific heat at constant volume, $c_v = 1.2$ kJ/kg°C

Mass of the substance, $m = 2.5$ kg

Now, $$\Delta U = m \int_{T_1}^{T_2} c_v\, dT$$

$$= 2.5 \int_{T_1}^{T_2} 1.2\, dT = 3.0 \times (T_2 - T_1)$$

$$= 3.0 \times 25 = \mathbf{75\ kJ}$$

Hence, **the change in internal energy is 75 kJ. (Ans.)**

According to the first law of thermodynamics,

$$Q = \Delta U + W$$

$$\therefore \quad 30 = 75 + W$$

$$\therefore \quad W = 30 - 75 = -45\ \text{kJ}$$

Hence, **the work done = – 45 kJ. (Ans.)**

It may be observed that *even though the volume is constant the work is not zero.* Clearly, the process is **irreversible.**

Example 4.15. *A system receives 50 kJ of heat while expanding with volume change of 0.14 m^3 against an atmosphere of 1.2 × 10^5 N/m^2. A mass of 90 kg in the surroundings is also lifted through a distance of 5.5 metres.*

(*i*) *Find the change in energy of the system.*

(*ii*) *The system is returned to its initial volume by an adiabatic process which requires 110 kJ of work. Find the change in energy of the system.*

(iii) For the combined processes of (i) and (ii) determine the change in energy of the system.

Solution. Heat received by the system,

$$Q = 50 \text{ kJ}$$

Change in volume $\Delta V = 0.14 \text{ m}^3$

Pressure $= 1.2 \times 10^5 \text{ N/m}^2$

Mass lifted in the surroundings $= 90$ kg

Distance through which lifted $= 5.5$ m

Work done during adiabatic process = – 110 kJ.

(*i*) $$Q = \Delta E + W \quad ...(i)$$

Now, $$W = p.\Delta V + W_{net}$$

$$= \left(\frac{1.2 \times 10^5 \times 0.14}{1000}\right) + \left(\frac{90 \times 5.5 \times 9.8}{1000}\right) \text{kJ}$$

$$= 16.8 + 4.85 = 21.65 \text{ kJ}$$

But [from (*i*)], $$\Delta E = Q - W$$

$$= 50 - 21.65 = \mathbf{28.35\ kJ.} \quad \textbf{(Ans.)}$$

(*ii*) Since the process is adiabatic,

$$Q = 0$$

and $$\Delta E = -W$$

$$= -(-110) = \mathbf{110\ kJ.} \quad \textbf{(Ans.)}$$

(*iii*) Change in internal energy,

$$\Delta E = Q - W$$

$$= 50 - [(-110) + 21.65] = \mathbf{138.35\ kJ.} \quad \textbf{(Ans.)}$$

☞**Example 4.16.** *A fluid system undergoes a non-flow frictionless process following the pressure-volume relation as* $p = \frac{5}{V} + 1.5$ *where p is in bar and V is in* m^3*. During the process the volume changes from 0.15* m^3 *to 0.05* m^3 *and the system rejects 45 kJ of heat. Determine :*

(*i*) *Change in internal energy ;*

(*ii*) *Change in enthalpy.*

Solution. Pressure-volume relation : $p = \frac{5}{V} + 1.5$

Initial volume, $V_1 = 0.15 \text{ m}^3$

Final volume, $V_2 = 0.05 \text{ m}^3$

Heat *rejected* by the system, $Q = -45$ kJ

Work done is given by,

$$W = \int_1^2 p \,.\, dV = \int_{V_1}^{V_2} \left(\frac{5}{V} + 1.5\right) dV$$

$$= \int_{0.15}^{0.05} \left(\frac{5}{V} + 1.5\right) dV = \left[5 \log_e \frac{V_2}{V_1} + 1.5\,(V_2 - V_1)\right]_{0.15}^{0.05} \times 10^5 \text{ N-m}$$

$$= 10^5 \left[5 \log_e \frac{0.05}{0.15} + 1.5\,(0.05 - 0.15)\right] = 10^5(-5.49 - 0.15) \text{ N-m}$$

$$= -5.64 \times 10^5 \text{ N-m} = -5.64 \times 10^5 \text{ J} \qquad [\because \ 1 \text{ Nm} = 1 \text{ J}]$$

$$= -564 \text{ kJ.}$$

(*i*) Applying the first law energy equation,

$$Q = \Delta U + W$$

$$-45 = \Delta U + (-564)$$

$$\therefore \quad \mathbf{\Delta U = 519\ kJ.} \quad \textbf{(Ans.)}$$

This shows that the internal energy is **increased.**

(*ii*) Change in enthalpy,

$$\Delta H = \Delta U + \Delta (pV)$$

$$= 519 \times 10^3 + (p_2V_2 - p_1V_1)$$

$$p_1 = \frac{5}{V_1} + 1.5 = \frac{5}{0.15} + 1.5 = 34.83 \text{ bar}$$

$$= 34.83 \times 10^5 \text{ N/m}^2$$

$$p_2 = \frac{5}{V_2} + 1.5 = \frac{5}{0.05} + 1.5$$

$$= 101.5 \text{ bar} = 101.5 \times 10^5 \text{ N/m}^2$$

$$\therefore \quad \Delta H = 519 \times 10^3 + (101.5 \times 10^5 \times 0.05 - 34.83 \times 10^5 \times 0.15)$$

$$= 519 \times 10^3 + 10^3(507.5 - 522.45)$$

$$= 10^3(519 + 507.5 - 522.45) = 504 \text{ kJ}$$

∴ **Change in enthalpy = 504 kJ. (Ans.)**

☞**Example 4.17.** *The following equation gives the internal energy of a certain substance*

$$u = 3.64\, pv + 90$$

where u is kJ/kg, p is in kPa and v is in m^3/kg.

A system composed of 3.5 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.25 m^3 *to a final pressure 100 kPa in a process in which pressure and volume are related by* $pv^{1.25}$ *= constant.*

(*i*) *If the expansion is quasi-static, find Q,* ΔU *and W for the process.*

(*ii*) *In another process, the same system expands according to the same pressure-volume relationship as in part (i), and from the same initial state to the same final state as in part (i), but the heat transfer in this case is 32 kJ. Find the work transfer for this process.*

(*iii*) *Explain the difference in work transfer in parts (i) and (ii).*

Solution. Internal energy equation : $u = 3.64\, pv + 90$

Initial volume, $V_1 = 0.25 \text{ m}^3$

Initial pressure, $p_1 = 500$ kPa

Final pressure, $p_2 = 100$ kPa

Process : $pv^{1.25}$ = constant.

(*i*) Now, $u = 3.64\, pv + 90$

$$\Delta u = u_2 - u_1$$

$$= 3.64\,(p_2v_2 - p_1v_1) \quad \text{...per kg}$$

$$\therefore \quad \Delta U = 3.64\,(p_2V_2 - p_1V_1) \quad \text{...for 3.5 kg}$$

Now, $p_1V_1^{1.25} = p_2V_2^{1.25}$

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{1/1.25} = 0.25 \left(\frac{500}{100}\right)^{1/1.25}$$

$$= 0.906 \text{ m}^3$$

$\therefore$ $\Delta U = 3.64\ (100 \times 10^3 \times 0.906 - 500 \times 10^3 \times 0.25)$ J [$\because$ 1 Pa = 1 N/m^2]

$= 3.64 \times 10^5\ (0.906 - 5 \times 0.25)$ J

$= -\ 3.64 \times 10^5 \times 0.344$ J $= -\ 125.2$ kJ

i.e., **ΔU = – 125.2 kJ. (Ans.)**

For a **quasi-static process**

$$W = \int p dV = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{(500 \times 10^3 \times 0.25 - 100 \times 10^3 \times 0.906)}{(1.25 - 1)} = \frac{125 - 90.6}{0.25} \text{ kJ} = 137.6 \text{ kJ}$$

$\therefore$ $Q = \Delta U + W$

$= -\ 125.2 + 137.6 = 12.4$ kJ

i.e., **Q = 12.4 kJ. (Ans.)**

(*ii*) Here $Q = 32$ kJ

Since the end states are the same, ΔU would remain the same as in (*i*)

$\therefore$ $W = Q - \Delta U = 32 - (-\ 125.2)$

= **157.2 kJ. (Ans.)**

(*iii*) The work in (*ii*) is **not equal** to $\int p\ dV$ since the process is **not quasi-static.**

☞**Example 4.18.** *The properties of a system, during a reversible constant pressure non-flow process at p = 1.6 bar, changed from v_1 = 0.3 m^3/kg, T_1 = 20°C to v_2 = 0.55 m^3/kg, T_2 = 260°C. The specific heat of the fluid is given by*

$$c_p = \left(1.5 + \frac{75}{T + 45}\right) \text{ kJ/kg°C, where } T \text{ is in °C.}$$

Determine : (*i*) *Heat added/kg ;* (*ii*) *Work done/kg ;*
(*iii*) *Change in internal energy/kg ;* (*iv*) *Change in enthalpy/kg.*

Solution. Initial volume, $v_1 = 0.3$ m^3/kg

Initial temperature, $T_1 = 20$°C

Final volume, $v_2 = 0.55$ m^3/kg

Final temperature, $T_2 = 260$°C

Constant pressure, $p = 1.6$ bar

Specific heat at constant pressure, $c_p = \left(1.5 + \frac{75}{T + 45}\right)$ kJ/kg°C

(*i*) The **heat added** per kg of fluid is given by

$$Q = \int_{T_1}^{T_2} c_p\ dT = \int_{20}^{260} \left(1.5 + \frac{75}{T + 45}\right) dT$$

$$= \left| 1.5\,T + 75 \log_e (T + 45) \right|_{20}^{260}$$

$$= 1.5\ (260 - 20) + 75 \times \log_e \left(\frac{260 + 45}{20 + 45}\right) = 475.94 \text{ kJ}$$

$\therefore$ **Heat added** **= 475.94 kJ/kg. (Ans.)**

(*ii*) The *work done* per kg of fluid is given by

$$W = \int_{v_1}^{v_2} p\,dv = p(v_2 - v_1) = 1.6 \times 10^5(0.55 - 0.3) \text{ N-m}$$
$$= 40 \times 10^3 \text{ J} = 40 \text{ kJ}$$

∴ **Work done = 40 kJ/kg. (Ans.)**

(*iii*) **Change in internal energy,**

$$\Delta u = Q - W = 475.94 - 40 = \mathbf{435.94\ kJ/kg.}\quad \textbf{(Ans.)}$$

(*iv*) **Change in enthalpy,** (for **non-flow process**)

$$\Delta h = Q = \mathbf{475.94\ kJ/kg.}\quad \textbf{(Ans.)}$$

Example 4.19. *1 kg of gaseous* CO_2 *contained in a closed system undergoes a reversible process at constant pressure. During this process 42 kJ of internal energy is decreased. Determine the work done during the process.*

Take c_p *= 840 J/kg°C and* c_v *= 600 J/kg°C.*

Solution. Mass CO_2, m = 1 kg

Decrease in internal energy, $\Delta u = -42 \text{ kJ} = -42 \times 10^3$ J

Specific heat at constant pressure, c_p = 840 J/kg°C

Specific heat at constant volume, c_v = 600 J/kg°C

Let, initial temperature of CO_2 = T_1

Final temperature of CO_2 = T_2

Now change in internal energy,

$$\Delta U = m \times c_v(T_2 - T_1)$$
$$-42 \times 10^3 = 1 \times 600(T_2 - T_1)$$
$$\therefore \quad T_2 - T_1 = -\frac{42 \times 10^3}{600} = -70°\text{C}$$

The heat supplied or rejected,

$$Q = mc_p(T_2 - T_1)$$
$$= 1 \times 840 \times (-70) = -58800 \text{ J or } -58.8 \text{ kJ}$$

Applying first law to the process,

$$Q = \Delta U + W$$
$$-58.8 = -42 + W \text{ or } W = -16.8 \text{ kJ}$$

∴ **Work done during the process = – 16.8 kJ. (Ans.)**

☞**Example 4.20.** *A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume (p = a + bV). The internal energy of the fluid is given by the following equation*

$$U = 42 + 3.6\,pV$$

where U is in kJ, p in kPa, and V in cubic metre. If the fluid changes from an initial state of 190 kPa, 0.035 m^3 *to a final state of 420 kPa, 0.07* m^3*, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.*

Solution. Relation between pressure and volume, $p = a + bV$.

Equation of internal energy : $U = 42 + 3.6pV$

Initial pressure, p_1 = 190 kPa

Initial volume, V_1 = 0.035 m^3

Final pressure, p_2 = 420 kPa

Final volume, V_2 = 0.07 m^3

The change in internal energy of the fluid during the process

$$\begin{aligned} U_2 - U_1 &= (42 + 3.6p_2V_2) - (42 + 3.6p_1V_1) \\ &= 3.6(p_2V_2 - p_1V_1) \\ &= 3.6(4.2 \times 10^5 \times 0.07 - 1.9 \times 10^5 \times 0.035) \text{ J} \\ &= 360(4.2 \times 0.07 - 1.9 \times 0.035) \text{ kJ} \\ &= 81.9 \text{ kJ} \end{aligned}$$

Now, $p = a + bV$

$$190 = a + b \times 0.035 \quad \text{...}(i)$$

$$420 = a + b \times 0.07 \quad \text{...}(ii)$$

Subtracting (*i*) from (*ii*), we get

$$230 = 0.035\ b \text{ or } b = \frac{230}{0.035} = 6571 \text{ kN/m}^5$$

and $a = -\,40 \text{ kN/m}^2$

Work transfer involved during the process

$$W_{1-2} = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} (a + bV)\,dV = a(V_2 - V_1) + b\left(\frac{{V_2}^2 - {V_1}^2}{2}\right)$$

$$= (V_2 - V_1)\left[a + \frac{b}{2}(V_1 + V_2)\right]$$

$$= (0.07 - 0.035)\left[-\,40 \text{ kN/m}^2 + \frac{6571}{2} \text{ kN/m}^5 \,(0.035 + 0.07)\right] = 10.67 \text{ kJ}$$

∴ **Work done by the system = 10.67 kJ. (Ans.)**

Heat transfer involved,

$$Q_{1-2} = (U_2 - U_1) + W_{1-2} = 81.9 + 10.67 = 92.57 \text{ kJ.}$$

92.57 kJ of heat flow into the system during the process. (Ans.)

Example 4.21. *90 kJ of heat are supplied to a system at a constant volume. The system rejects 95 kJ of heat at constant pressure and 18 kJ of work is done on it. The system is brought to original state by adiabatic process. Determine :*

(*i*) *The adiabatic work ;*

(*ii*) *The values of internal energy at all end states if initial value is 105 kJ.*

Solution. Refer Fig. 4.20.

Heat supplied at constant volume = 90 kJ

Heat rejected at constant pressure = – 95 kJ

Work done *on* the system = – 18 kJ

Initial value of internal energy, $U_l = 105$ kJ

Process *l–m* (*constant volume*) :

$$W_{l-m} = 0$$

$$Q_{l-m} = 90 = U_m - U_l$$

$$\therefore \quad U_m = U_l + 90 = 105 + 90 = 195 \text{ kJ}$$

Process *m–n* (*constant pressure*) :

$$Q_{m-n} = (U_n - U_m) + W_{m-n}$$

$$-\,95 = (U_n - U_m) - 18$$

$$\therefore \quad U_n - U_m = -\,77 \text{ kJ}$$

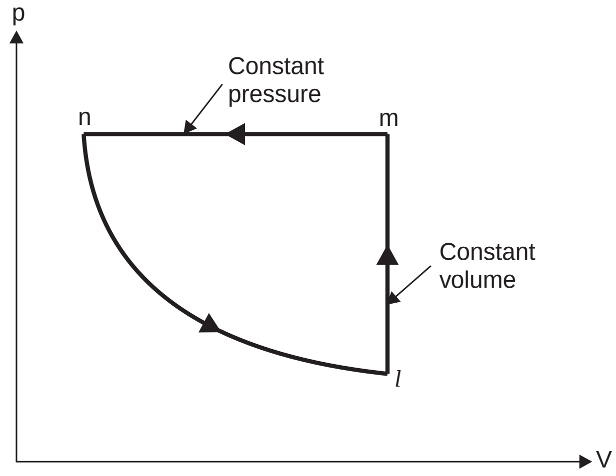


Fig. 4.20

$$\therefore \qquad U_n = 195 - 77 = 118 \text{ kJ}$$

$Q_{n-l} = 0$ being adiabatic process

$$\therefore \qquad \oint \delta Q = 90 - 95 = -5 \text{ kJ}$$

and
$$\oint \delta W = -18 + W_{n-l} = -5$$

$$\therefore \qquad W_{n-l} = -5 + 18 = 13 \text{ kJ}$$

Hence, $\mathbf{W_{n-l} = 13}$ **kJ ;** $\mathbf{U_l = 105}$ **kJ ;** $\mathbf{U_m = 195}$ **kJ ;** $\mathbf{U_n = 118}$ **kJ. (Ans.)**

Example 4.22. *A movable frictionless piston closes a fully insulated cylinder on one side and offers a constant resistance during its motion. A paddle work is drawn into the cylinder and does work on the system.*

Prove that the paddle work is equal to change in enthalpy.

Solution. Refer Fig. 4.21.

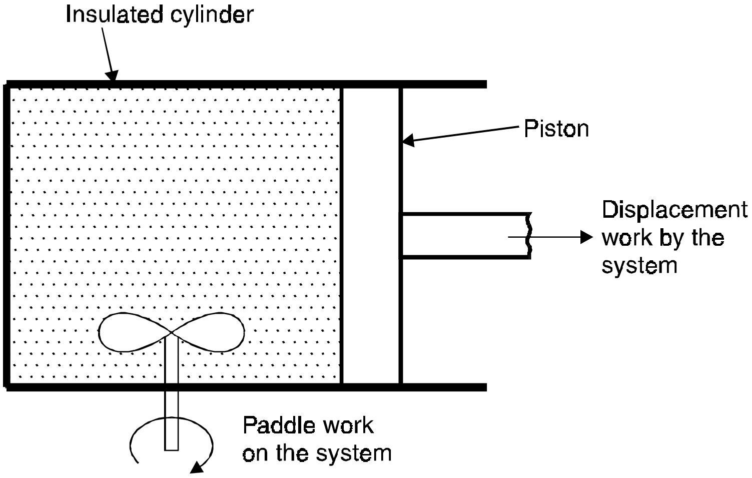


Fig. 4.21

$$Q = W_{paddle} = \Delta U + p\Delta V = \Delta U + \Delta(pV) = \Delta(U + pV) = \Delta H$$

Hence **paddle work is equal to change in enthalpy. (Ans.)**

☞**Example 4.23.** *0.2 m^3 of air at 4 bar and 130°C is contained in a system. A reversible adiabatic expansion takes place till the pressure falls to 1.02 bar. The gas is then heated at constant pressure till enthalpy increases by 72.5 kJ. Calculate :*

(*i*) *The work done ;*

(*ii*) *The index of expansion, if the above processes are replaced by a single reversible polytropic process giving the same work between the same initial and final states.*

Take c_p = 1 kJ/kg K, c_v = 0.714 kJ/kg K.

Solution. Refer Fig. 4.22.

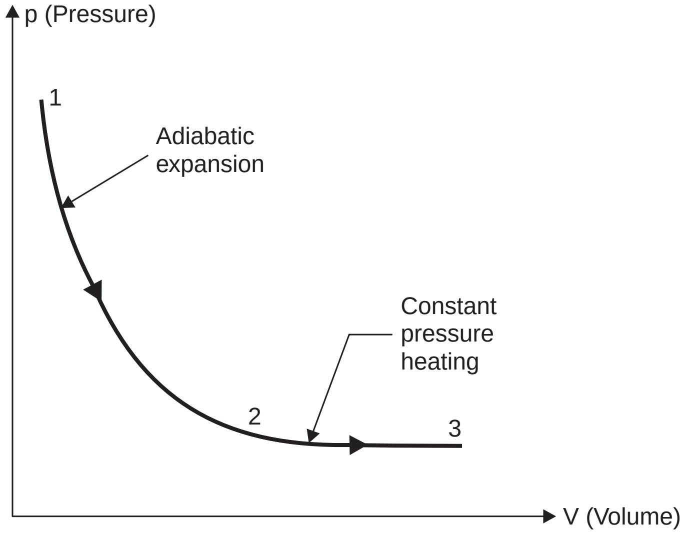


Fig. 4.22

Initial volume, $V_1 = 0.2 \text{ m}^3$

Initial pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 130 + 273 = 403 \text{ K}$

Final pressure after adiabatic expansion,

$$p_2 = 1.02 \text{ bar} = 1.02 \times 10^5 \text{ N/m}^2$$

Increase in enthalpy during constant pressure process

= 72.5 kJ.

(*i*) **Work done :**

Process 1-2 : *Reversibe adiabatic process* :

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

Also $$\gamma = \frac{c_p}{c_v} = \frac{1}{0.714} = 1.4$$

$\therefore$ $$V_2 = 0.2 \times \left(\frac{4 \times 10^5}{1.02 \times 10^5}\right)^{\frac{1}{1.4}} = 0.53 \text{ m}^3$$

Also, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$\therefore$ $$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$= 403 \left(\frac{1.02 \times 10^5}{4 \times 10^5}\right)^{\frac{1.4 - 1}{1.4}} = 272.7 \text{ K}$$

Mass of the gas,

$$m = \frac{p_1 V_1}{R T_1} \qquad [\because \; pV = mRT]$$

where, $R = (c_p - c_v) = (1 - 0.714)$ kJ/kg K

= 0.286 kJ/kg K = 286 J/kg K or 286 Nm/kg K

$\therefore$ $$m = \frac{4 \times 10^5 \times 0.2}{286 \times 403} = 0.694 \text{ kg.}$$

Process 2-3. *Constant pressure* :

$$Q_{2-3} = mc_p (T_3 - T_2)$$

$$72.5 = 0.694 \times 1 \times (T_3 - 272.7)$$

$\therefore$ $$T_3 = \frac{72.5}{0.694} + 272.7 = 377 \text{ K}$$

Also, $$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

or $$\frac{0.53}{272.7} = \frac{V_3}{377}$$

$\therefore$ $$V_3 = \frac{0.53 \times 377}{272.7} = 0.732 \text{ m}^3$$

Work done by the path 1-2-3 is given by

$$W_{1-2-3} = W_{1-2} + W_{2-3}$$

$$= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} + p_2 (V_3 - V_2)$$

$$= \frac{4 \times 10^5 \times 0.2 - 1.02 \times 10^5 \times 0.53}{1.4 - 1} + 1.02 \times 10^5 (0.732 - 0.53)$$

$$= \frac{10^5 (4 \times 0.2 - 1.02 \times 0.53)}{0.4} + 1.02 \times 10^5 (0.732 - 0.53)$$

$$= 64850 + 20604 = 85454 \text{ Nm or J}$$

Hence, **total work done = 85454 Nm or J. (Ans.)**

(*ii*) **Index of expansion, n :**

If the work done by the polytropic process is the same,

$$W_{1-2-3} = W_{1-3} = \frac{p_1V_1 - p_3V_3}{n-1}$$

$$85454 = \frac{4 \times 10^5 \times 0.2 - 1.02 \times 10^5 \times 0.732}{(n-1)} = \frac{5336}{n-1}$$

$$\therefore \qquad n = \frac{5336}{85454} + 1$$

i.e., $n = 1.062$

Hence, **value of index = 1.062. (Ans.)**

Example 4.24. *The following is the equation which connects u, p and v for several gases*

$$u = a + bpv$$

where a and b are constants. Prove that for a reversible adiabatic process,

$$pv^{\gamma} = constant, \text{ where } \gamma = \frac{b+1}{b}.$$

Solution. Consider a unit mass.

For a reversible adiabatic process, first law gives

$$0 = du + pdv$$

$$\therefore \qquad \frac{du}{dv} = -p \qquad ...(i)$$

Also, $u = a + bpv$

$$\therefore \qquad \frac{du}{dv} = \frac{d\,(a + bpv)}{dv} = bv\,\frac{dp}{dv} + bp$$

$$= b\left(p + v\,.\,\frac{dp}{dv}\right) \qquad ...(ii)$$

Equating (*i*) and (*ii*), we get

$$b\left(p + v\,.\,\frac{dp}{dv}\right) = -p$$

$$bp + b\,.\,v\,.\,\frac{dp}{dv} = -p$$

$$bp + p + bv.\,\frac{dp}{dv} = 0$$

$$p(b+1) + bv.\,\frac{dp}{dv} = 0$$

Multiplying both sides by $\frac{dv}{bpv}$, we get

$$\left(\frac{b+1}{b}\right)\frac{dv}{v} + \frac{dp}{p} = 0$$

or

$$\frac{dp}{p} + \left(\frac{b+1}{b}\right)\frac{dv}{v} = 0$$

$$d(\log_e p) + \left(\frac{b+1}{b}\right) d(\log_e v) = 0$$

Also, $\frac{b+1}{b} = \gamma$ ...(Given)

$\therefore$ $d(\log_e p) + \gamma d\,(\log_e v) = 0$

Integrating, we get $pv^\gamma = \text{constant}.$

Example 4.25. *A 15 cm diameter vertical cylinder, closed by a piston contains a combustible mixture at a temperature of 30°C. The piston is free to move and its weight is such that the mixture pressure is 3 bar. Upper surface of the piston is exposed to the atmosphere. The mixture is ignited. As the reaction proceeds, the piston moves slowly upwards and heat transfer to the surroundings takes place. When the reaction is complete and the contents have been reduced to the initial temperature of 30°C, it is found that the piston has moved upwards a distance of 8.5 cm and the magnitude of heat transfer is 4 kJ. Evaluate :*

(*i*) *The work ;*

(*ii*) *Decrease in internal energy of the system.*

Solution. Diameter of vertical cylinder, d = 15 cm (or 0.15 m)

Temperature of combustible mixture = 30°C (or 303 K)

Pressure of the mixture = 3 bar = 3×10^5 N/m^2

Upward displacement of the system = 8.5 cm (or 0.085 m)

Magnitude of heat transfer, $Q = -4$ kJ ...(*i*)

(*i*) Work done by the system,

$$W = \int p\,dv$$

$$= 3 \times 10^5 \int dv \qquad [\because \; p = \text{constant} = 3 \times 10^5 \text{ N/m}^2]$$

$$= 3 \times 10^5 \left[\frac{\pi}{4} \times (0.15)^2 \times 0.085\right] \text{ N-m}$$

$$= 450.62 \text{ N-m or J} = 0.4506 \text{ kJ}$$

$$\therefore \quad W = 0.4506 \text{ kJ.}$$

(*ii*) By first law of thermodynamics,

$$Q = \Delta U + W$$

$$-4 = \Delta U + 0.4506$$

$$\therefore \quad \Delta U = -4.4506 \text{ kJ}$$

$\therefore$ **Decrease in internal energy = 4.4506 kJ. (Ans.)**

Example 4.26. *A house wife, on a warm summer day, decides to beat the heat by closing the windows and doors in the kitchen and opening the refrigerator door. At first she feels cool and refreshed, but after a while the effect begins to wear off.*

Evaluate the situation as it relates to First Law of Thermodynamics, considering the room including the refrigerator as the system.

Solution. Initially, the temperature of air in the room falls when it communicates with the cool refrigerator with its door open. This makes the house wife feel cool.

Considering the room and its contents as the system, and assuming the walls, windows and doors non-conducting, we find, $Q = 0$.

To operate the refrigerator, *electricity* is supplied from outside and hence *external work W is done on the system*.

Applying the first law to the system,

$$Q = \Delta U + W$$

$$0 = \Delta U + (-W)$$

$$\therefore \quad \Delta U = W$$

The right hand side is a positive figure indicating the increase in energy of the system with time. As the energy is increasing the temperature of air increases and hence the effect of coolness gradually begins to wear off.

It may be pointed out here that in this case the energy rise manifests itself in a rise in temperature.

☞**Example 4.27.** *A cylinder contains 0.45 m³ of a gas at 1 × 10⁵ N/m² and 80°C. The gas is compressed to a volume of 0.13 m³, the final pressure being 5 × 10⁵ N/m². Determine :*

(*i*) *The mass of gas ;*

(*ii*) *The value of index 'n' for compression ;*

(*iii*) *The increase in internal energy of the gas ;*

(*iv*) *The heat received or rejected by the gas during compression.*

Take $\gamma = 1.4$, $R = 294.2$ *J/kg°C.*

Solution. Initial volume of gas, $V_1 = 0.45 \text{ m}^3$

Initial pressure of gas, $p_1 = 1 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 80 + 273 = 353 \text{ K}$

Final volume after compression, $V_2 = 0.13 \text{ m}^3$

The final pressure, $p_2 = 5 \times 10^5 \text{ N/m}^2$.

(*i*) To find mass '*m*' using the relation

$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.45}{294.2 \times 353} = \mathbf{0.433\ kg.}\quad \textbf{(Ans.)}$$

(*ii*) To find index '*n*' using the relation

$$p_1V_1^{\,n} = p_2V_2^{\,n}$$

or
$$\left(\frac{V_1}{V_2}\right)^n = \frac{p_2}{p_1}$$

$$\left(\frac{0.45}{0.13}\right)^n = \left(\frac{5 \times 10^5}{1 \times 10^5}\right) = 5$$

or
$$(3.46)^n = 5$$

Taking log on both sides, we get

$$n \log_e 3.46 = \log_e 5$$

$$n = \log_e 5/\log_e 3.46 = \mathbf{1.296.}\quad \textbf{(Ans.)}$$

(*iii*) In a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{0.45}{0.13}\right)^{1.296-1} = 1.444$$

$$\therefore \quad T_2 = 353 \times 1.444 = 509.7 \text{ K}$$

Now, increase in internal energy,

$$\Delta U = mc_v(T_2 - T_1)$$

$$= 0.433 \times \frac{R}{(\gamma - 1)}(T_2 - T_1) \qquad \left[\because c_v = \frac{R}{(\gamma - 1)}\right]$$

$$= 0.433 \times \frac{294.2}{(1.4 - 1)\,1000}(509.7 - 353)$$

$$= \mathbf{49.9\ kJ.}\quad \textbf{(Ans.)}$$

(*iv*) $Q = \Delta U + W$

Now, $W = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$

$= \frac{0.433 \times 294.2(353 - 509.7)}{1.296 - 1}$

$= -67438$ N-m or -67438 J $= -67.44$ kJ

∴ $Q = 49.9 + (-67.44) = -17.54$ kJ

∴ **Heat rejected = 17.54 kJ. (Ans.)**

Example 4.28. *Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m*^{3}*, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate :*

(*i*) *The final temperature ;*

(*ii*) *The final volume ;*

(*iii*) *The work done.*

Solution. Initial pressure, $p_1 = 1.02$ bar

Initial temperature, $T_1 = 22 + 273 = 295$ K

Initial volume, $V_1 = 0.015$ m^3

Final pressure, $p_2 = 6.8$ bar

Law of compression : $pv^\gamma = C$

(*i*) **Final temperature :**

Using the relation,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{295} = \left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}} \qquad [\because \gamma \text{ for air} = 1.4]$$

∴ $$T_2 = 295\left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}} = 507.24 \text{ K}$$

i.e., **Final temperature** = 507.24 – 273 = **234.24°C. (Ans.)**

(*ii*) **Final volume :**

Using the relation,

$$p_1V_1^\gamma = p_2V_2^\gamma$$

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^\gamma \quad \text{or} \quad \frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

∴ $$V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.015 \times \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}} = 0.00387 \text{ m}^3$$

i.e., **Final volume = 0.00387 m^3. (Ans.)**

Now, work done *on the air*,

$$W = \frac{mR(T_1 - T_2)}{(\gamma - 1)} \qquad ...(i)$$

where m is the mass of air and is found by the following relation,

$$pV = mRT$$

$$\therefore \quad m = \frac{p_1V_1}{RT_1} = \frac{1.02 \times 10^5 \times 0.015}{0.287 \times 10^3 \times 295} \qquad [\because \ R \text{ for air} = 0.287 \times 10^3]$$

$$= 0.01807 \text{ kg}$$

$$\therefore \quad W = \frac{0.01807 \times 0.287 \times 10^3(295 - 507.24)}{(1.4 - 1)} = -2751 \text{ J or } -2.751 \text{ kJ}$$

i.e., **Work done = 2.751 kJ. (Ans.)**

(– ve sign indicates that work is done on the air).

Example 4.29. *0.44 kg of air at 180°C expands adiabatically to three times its original volume and during the process, there is a fall in temperature to 15°C. The work done during the process is 52.5 kJ. Calculate* c_p *and* c_v.

Solution. Refer Fig. 4.23.

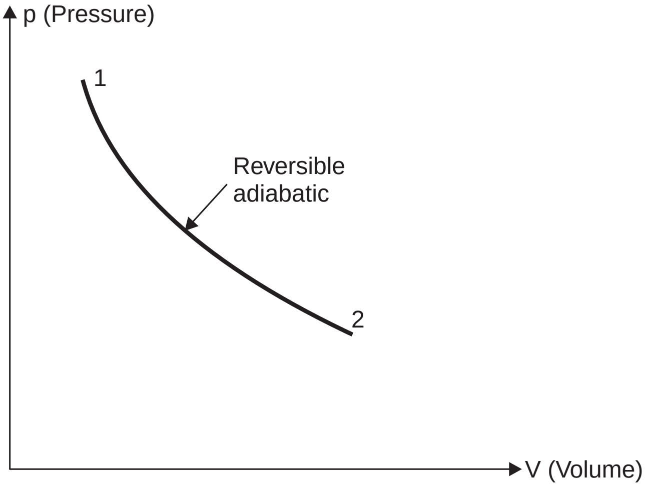


Fig. 4.23

Mass of air, $m = 0.44$ kg

Initial temperature, $T_1 = 180 + 273 = 453$ K

$$\text{Ratio} = \frac{V_2}{V_1} = 3$$

Final temperature, $T_2 = 15 + 273 = 288$ K

Work done during the process, $W_{1-2} = 52.5$ kJ

$$\boldsymbol{c_p} = ?, \ \boldsymbol{c_v} = ?$$

For adiabatic process, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$\frac{288}{453} = \left(\frac{1}{3}\right)^{\gamma - 1} \qquad \text{or} \qquad 0.6357 = (0.333)^{\gamma-1}$$

or Taking log on both sides, we get

$$\log_e (0.6357) = (\gamma - 1) \log_e (0.333)$$

$$- 0.453 = (\gamma - 1) \times (- 1.0996)$$

$$\therefore \quad \gamma = \frac{0.453}{1.0996} + 1 = 1.41$$

Also, $\dfrac{c_p}{c_v} = \gamma = 1.41$

Work done during adiabatic process,

$$W_{1-2} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$\therefore \quad 52.5 = \frac{0.44\, R(453 - 288)}{(1.41 - 1)}$$

$$\therefore \quad R = \frac{52.5\,(1.41 - 1)}{0.44\,(453 - 288)} = 0.296$$

$$\therefore \quad c_p - c_v = 0.296 \qquad [\because \; R = c_p - c_v]$$

Also $\dfrac{c_p}{c_v} = 1.41$ or $c_p = 1.41\, c_v$

$$\therefore \quad 1.41\, c_v - c_p = 0.296$$

or $c_v =$ **0.722 kJ/kg K. (Ans.)**

and $c_p =$ **1.018 kJ/kg K. (Ans.)**

☞ **Example 4.30.** *1 kg of ethane (perfect) gas is compressed from 1.1 bar, 27°C according to a law $pV^{1.3}$ = constant, until the pressure is 6.6 bar. Calculate the heat flow to or from the cylinder walls.*

Given : Molecular weight of ethane = 30, c_p = 1.75 kJ/kg K.

Solution. Mass of ethane gas, $m = 1$ kg

Initial pressure, $p_1 = 1.1$ bar

Initial temperature, $T_1 = 27 + 273 = 300$ K

Final pressure, $p_2 = 6.6$ bar

Law of compression, $pV^{1.3} = C$

Quantity of heat transferred, Q :

Now, characteristic gas constant,

$$R = \frac{\text{Universal gas constart } (R_0)}{\text{Molecular weight } (M)}$$

$$= \frac{8314}{30} = 277.13 \text{ N-m/kg K} = 277.31 \text{ J/kg K}$$

$$= 0.277 \text{ kJ/kg K}$$

Also $c_p - c_v = R$

$$\therefore \quad c_v = c_p - R = 1.75 - 0.277 = 1.473 \text{ kJ/kg K}$$

$$\gamma = \frac{c_p}{c_v} = \frac{1.75}{1.473} = 1.188$$

In case of a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{6.6}{1.1}\right)^{\frac{1.3-1}{1.3}} = 1.5119$$

$\therefore$ $T_2 = 300 \times 1.5119 = 453.6$ K

Now, work done, $W = \dfrac{R\,(T_1 - T_2)}{n-1} = \dfrac{0.277(300 - 453.6)}{1.3 - 1} = -\ 141.8$ kJ/kg

To find heat flow, using the relation,

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W = \left(\frac{1.188 - 1.3}{1.188 - 1}\right) \times -\ 141.8 = +\ 84.5 \text{ kJ/kg}$$

i.e., **Heat supplied = 84.5 kJ/kg. (Ans.)**

Example 4.31. *0.1 m^3 of an ideal gas at 300 K and 1 bar is compressed adiabatically to 8 bar. It is then cooled at constant volume and further expanded isothermally so as to reach the condition from where it started. Calculate :*

(*i*) *Pressure at the end of constant volume cooling.*

(*ii*) *Change in internal energy during constant volume process.*

(*iii*) *Net work done and heat transferred during the cycle. Assume* $c_p = 14.3$ *kJ/kg K and* $c_v = 10.2$ *kJ/kg K.*

Solution. *Given :* $V_1 = 0.1$ m^3 ; $T_1 = 300$ K ; $p_1 = 1$ bar ; $c_p = 14.3$ kJ/kg K ; $c_v = 10.2$ kJ/kg K.

Refer to Fig. 4.24.

(*i*) **Pressure at the end of constant volume cooling, p_3 :**

$$\gamma = \frac{c_p}{c_v} = \frac{14.3}{10.2} = 1.402$$

Characteristic gas constant,

$$R = c_p - c_v = 14.3 - 10.2 = 4.1 \text{ kJ/kg K}$$

Considering *process 1-2*, we have :

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.1 \times \left(\frac{1}{8}\right)^{\frac{1}{1.402}} = 0.0227 \text{ m}^3$$

Also, $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{8}{1}\right)^{\frac{1.402-1}{1.402}} = 1.815$

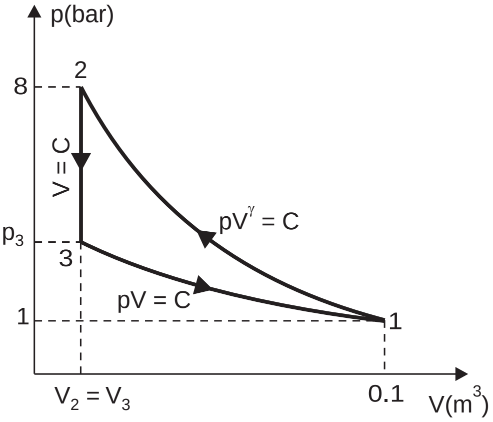


Fig. 4.24

or $T_2 = T_1 \times 1.815 = 300 \times 1.815 = 544.5$ K

Considering *process 3–1*, we have

$$p_3 V_3 = p_1 V_1$$

$\therefore$ $p_3 = \dfrac{p_1 V_1}{V_3} = \dfrac{1 \times 0.1}{0.0227} =$ **4.4 bar. (Ans.)** $(\because\ V_3 = V_2)$

(*ii*) **Change in internal energy during constant volume process, $(U_3 - U_2)$:**

Mass of gas, $m = \dfrac{p_1 V_1}{R T_1} = \dfrac{(1 \times 10^5) \times 0.1}{(4.1 \times 1000) \times 300} = 0.00813$ kg

∴ Change in internal energy during *constant volume process 2–3,*

$$U_3 - U_2 = mc_v(T_3 - T_2)$$
$$= 0.00813 \times 10.2\,(300 - 544.5) \qquad (\because \quad T_3 = T_1)$$
$$= \mathbf{-\,20.27\ kJ.} \quad \textbf{(Ans.)}$$

(– ve sign means *decrease* in internal energy)

- During constant volume cooling process, temperature and hence internal energy is *reduced*. This *decrease in internal energy equals to heat flow to surroundings since work done is zero.*

(*iii*) **Net work done and heat transferred during the cycle :**

$$W_{1-2} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$
$$= \frac{0.00813 \times 4.1(300 - 544.5)}{1.402 - 1} = -\,20.27 \text{ kJ}$$

$$W_{2-3} = 0 \qquad \text{... since volume remains constant}$$

$$W_{3-1} = p_3V_3 \log_e\left(\frac{V_1}{V_3}\right) = p_1V_1 \log_e\left(\frac{p_3}{p_1}\right) \qquad (\because \quad p_3V_3 = p_1V_1)$$
$$= (1 \times 10^5) \times 0.1 \times \log_e\left(\frac{4.4}{1}\right)$$
$$= 14816 \text{ Nm (or J) or } 14.82 \text{ kJ}$$

∴ **Net work done** $= W_{1-2} + W_{2-3} + W_{3-1}$
$= (-\,20.27) + 0 + 14.82 = \mathbf{-\,5.45\ kJ}$

–ve sign indicates that work has been done *on the system.* **(Ans.)**

For a cyclic process : $\oint \delta Q = \oint \delta W$

∴ Heat transferred during the complete cycle = – **5.45 kJ**

–ve sign means heat has been *rejected i.e., lost from the system.* **(Ans.)**

Example 4.32. *0.15 m^3 of an ideal gas at a pressure of 15 bar and 550 K is expanded isothermally to 4 times the initial volume. It is then cooled to 290 K at constant volume and then compressed back polytropically to its initial state.*

Calculate the net work done and heat transferred during the cycle.

Solution. *Given :* $V_1 = 0.15 \text{ m}^3$; $p_1 = 15$ bar ; $T_1 = T_2 = 550$ K ; $\frac{V_2}{V_1} = 4$; $T_3 = 290$ K

Refer to Fig. 4.25.

Considering the *isothermal process* 1–2, we have

$$p_1V_1 = p_2V_2 \quad \text{or} \quad p_2 = \frac{p_1V_1}{V_2}$$

or,
$$p_2 = \frac{15 \times 0.15}{(4 \times 0.15)} = 3.75 \text{ bar}$$

Work done, $W_{1-2} = p_1V_1 \log_e\left(\frac{V_2}{V_1}\right)$

$$= (15 \times 10^5) \times 0.15 \times \log_e (4)$$

$$= 311916 \text{ J} = 311.9 \text{ kJ}$$

Considering *constant volume process 2–3*, we get

$$V_2 = V_3 = 4 \times 0.15 = 0.6 \text{ m}^3$$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or, $$p_3 = p_2 \times \frac{T_3}{T_2} = 3.75 \times \frac{290}{550} = 1.98 \text{ bar}$$

$$W_{2-3} = 0$$

... since volume remains constant

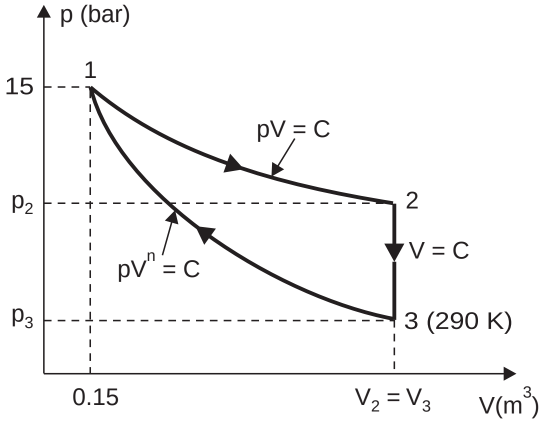


Fig. 4.25

Consider *polytropic process 3–1 :*

$$p_3V_3^n = p_1V_1^n \quad \text{or} \quad \frac{p_1}{p_3} = \left(\frac{V_3}{V_1}\right)^n$$

Taking log on both sides, we get

$$\log_e (p_1/p_3) = n \log_e (V_3/V_1)$$

or, $$n = \frac{\log_e(p_1/p_3)}{\log_e(V_3/V_1)} = \frac{\log_e (15/1.98)}{\log_e (4)} = 1.46$$

$$W_{3-1} = \frac{p_3V_3 - p_1V_1}{n - 1} = \frac{1.98 \times 10^5 \times 0.6 - 15 \times 10^5 \times 0.15}{(1.46 - 1)}$$

$$= -\,230869 \text{ J} \quad \text{or} \quad -\,230.87 \text{ kJ}$$

$\therefore$ *Net work done* $= W_{1-2} + W_{2-3} + W_{3-1}$

$= 311.9 + 0 + (-\,230.87) =$ **81.03 kJ. (Ans.)**

For a cyclic process, $\oint \delta Q = \oint \delta W$

$\therefore$ Heat *transferred during the cycle* = **81.03 kJ. (Ans.)**

Example 4.33. *A system consisting of 1 kg of an ideal gas at 5 bar pressure and 0.02 m*^{3} *volume executes a cyclic process comprising the following three distinct operations :* (*i*) *Reversible expansion to 0.08 m*^{3} *volume, 1.5 bar pressure, presuming pressure to be a linear function of volume* ($p = a + bV$), (*ii*) *Reversible cooling at constant pressure and* (*iii*) *Reversible hyperbolic compression according to law pV = constant. This brings the gas back to initial conditions.*

(*i*) *Sketch the cycle on p-V diagram.*

(*ii*) *Calculate the work done in each process starting whether it is done on or by the system and evaluate the net cyclic work and heat transfer.*

Solution. *Given :* $m = 1$ kg ; $p_1 = 5$ bar ; $V_1 = 0.02$ m^3 ; $V_2 = 0.08$ m^3 ; $p_2 = 1.5$ bar.

(*i*) **p-V diagram :** *p-V* diagram of the cycle is shown in Fig. 4.26.

(*ii*) **Work done and heat transfer :**

- *Process 1-2* (*Linear law*) :

$$p = a + bV \qquad \text{...(Given)}$$

The values of constants *a* and *b* can be determined from the values of pressure and volume at the state points 1 and 2.

$$5 = a + 0.02b \qquad ...(i)$$

$$1.5 = a + 0.08b \qquad ...(ii)$$

From (*i*) and (*ii*) we get, $b = -58.33$ and $a = 6.167$

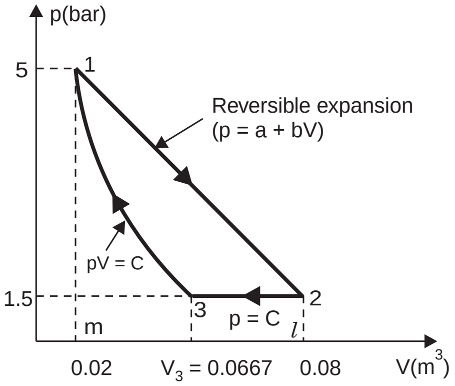


Fig. 4.26. *p-V* diagram.

$$W_{1-2} = \int_1^2 p dV = \int_1^2 (a + bV)\, dV$$

$$= \int_1^2 (6.167 - 58.33V)\, dV$$

$$= 10^5 \left| 6.167\, V - 58.33 \times \frac{V^2}{2} \right|_{0.02}^{0.08}$$

$$= 10^5 \left| 6.167\,(0.08 - 0.02) - 58.33 \times \frac{(0.08^2 - 0.02^2)}{2} \right| \times 10^{-3} \text{ kJ} = \mathbf{19.5\ kJ}$$

This is work done **by the system. (Ans.)**

Alternatively : W_{1-2} = Area under the process line 1–2
= Area of trapezium 1–2–*l*–*m*

$$= \left[\frac{5 + 1.5}{2} \times 10^5 \right] \times (0.08 - 0.02) = 19.5 \text{ kJ}$$

- *Process 2 – 3* (*constant pressure*) :

$$p_3 = p_2 = 1.5 \text{ bar}$$

The volume V_3 can be worked out from the hyperbolic compression 3–1, as follows :

$$p_1V_1 = p_3V_3 \quad \text{or} \quad V_3 = \frac{p_1V_1}{p_3} = \frac{5 \times 0.02}{1.5} = 0.0667 \text{ m}^3$$

$$\therefore \qquad W_{2-3} = p_2(V_3 - V_2) = 1.5 \times 10^5\,(0.0667 - 0.08) \times 10^{-3} \text{ kJ} = -1.995 \text{ kJ}$$

- *Process 3 – 1* (*hyperbolic process*) :

$$W_{3-1} = p_3V_3 \log_e \left(\frac{V_1}{V_3} \right)$$

$$= (10^5 \times 1.5) \times 0.0667 \log_e \left(\frac{0.02}{0.0667} \right) \times 10^{-3} \text{ kJ} = \mathbf{-12.05\ kJ.}$$

This is the work **done on the system. (Ans.)**

Net work done, $W_{net} = W_{1-2} + W_{2-3} + W_{3-1}$
$= 19.5 + (-1.995) + (-12.05) =$ **5.445 kJ. (Ans.)**

Heat transferred during the complete cycle, $\oint \delta Q = \oint \delta W =$ **5.455 kJ. (Ans.)**

Example 4.34. *Fig. 4.27 shows a cylinder of 8 cm inside diameter having a piston loaded with a spring (stiffness = 150 N/cm of compression). The initial pressure, volume and temperature of air in the cylinder are 3×10^5 N/m^2, 0.000045 m^3 and 20°C respectively. Determine the amount of heat added to the system so that piston moves by 3.5 cm.*

Assume $c_v = 0.71$ kJ/kg K and $R = 0.287$ kJ/kg K.

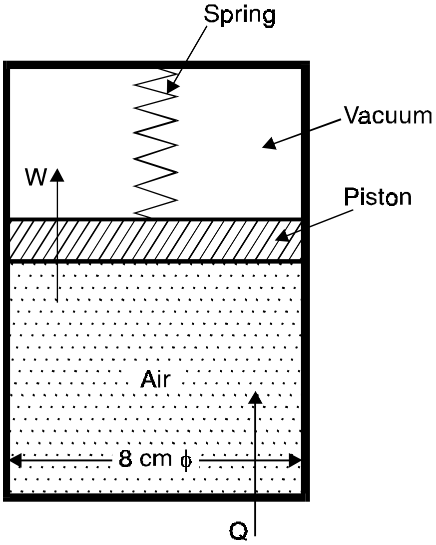


Fig. 4.27

Solution. Insider diameter of the cylinder
$= 8$ cm

Stiffness of the spring, $S = 150$ N/cm

Initial pressure of air,
$p_1 = 3 \times 10^5$ N/m^2 or 30 N/cm^2

Initial volume of air,
$V_1 = 0.000045$ m^3 = 45 cm^3

Initial temperature of air,
$T_1 = 20 + 273 = 293$ K

Specific heat at constant volume,
$c_v = 0.71$ kJ/kg K

Characteristic constant for air,
$R = 0.287$ kJ/kg K

Refer Fig. 4.28.

Let, oo = An arbitrary datum from which the position of the lower face of the piston is to be measured,

y = Distance of the lower face of the piston,

$y = y_0$, when spring length is its free length, and

p = Pressure of air within the cylinder when $y = y_0$.

Now, force balance for the piston is given by

$$Ap = S\,(y - y_0) \qquad ...(i)$$

where, A = The area of the piston, and

S = Stiffness of the spring.

With heat transfer to the air, let the pressure inside the cylinder increase by dp forcing the piston to move upward by distance dy. Now the force balance for the piston is

$$A(p + dp) = S(y + dy - y_0) \qquad ...(ii)$$

From eqns. (*i*) and (*ii*), we have

$$Adp = Sdy \qquad ...(iii)$$

The increase in volume dV of the gas for the piston displacement is given by

$$dV = Ady \qquad ...(iv)$$

$$\therefore \quad dp = \frac{S}{A^2}\,dy \qquad ...(v)$$

$$\therefore \quad p = \frac{S}{A^2}\,V + C \qquad ...(vi)$$

The p-V relationship for the process is a straight line (Fig. 4.29) having a slope of $\frac{S}{A^2}$ and pressure axis intercept of C. The value of C can be found out from the knowledge of pressure and volume at any state point.

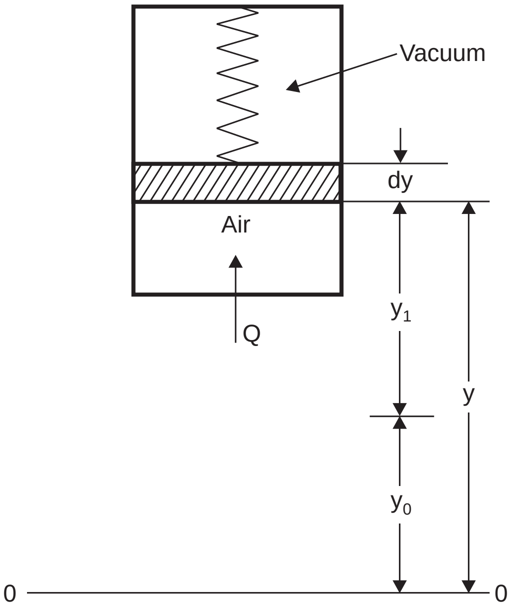


Fig. 4.28

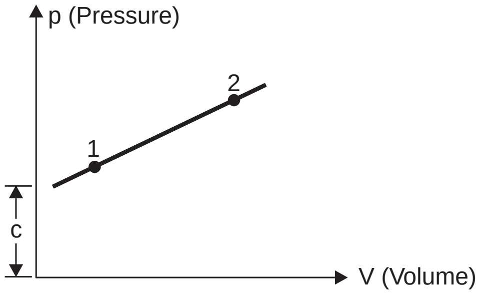


Fig. 4.29

Now, substituting the values of p_1, V_1, A in eqn. (*vi*), we get

$$p = \frac{150}{\left(\frac{\pi}{4} \times 8^2\right)^2} V + C$$

or $$p = 0.0594\ V + C \qquad \text{...(vii)}$$

where p is in N/cm^2 and V is in cm^3.

$$\therefore \qquad p_1 = 0.0594\ V_1 + C$$

$$30 = 0.0594 \times 45 + C$$

$$\therefore \qquad C = 27.33$$

Hence, p-V relationship for the process is,

$$p = 0.0594\ V + 27.33 \qquad \text{...(viii)}$$

During the process the piston is moved by a distance of 3.5 cm.

This increases the volume of gas by

$$3.5 \times A^2 = 3.5 \times \left(\frac{\pi}{4} \times 8^2\right) = 175.9 \text{ cm}^3$$

Hence, the final volume of air,

$$V_2 = 45 + 175.9 = 220.9 \text{ cm}^3$$

Substituting this value in equation (*viii*), we get

$$p(= p_2) = 0.0594 \times 220.9 + 27.33 = 40.45 \text{ N/cm}^2$$

The work done W during the process is given by

$$W = \int pdV = \int_{p_1}^{p_2} \frac{A^2}{S}\, pdp$$

$$= \frac{A^2}{S}\left(\frac{{p_2}^2 - {p_1}^2}{2}\right) = \frac{A^2}{S}\left(\frac{p_2 + p_1}{2}\right)\left(\frac{p_2 - p_1}{2}\right)$$

$$= \frac{A^2}{S}\left(\frac{p_2 + p_1}{2}\right)\frac{S}{A^2}(V_2 - V_1)$$

or $$W = \left(\frac{p_2 + p_1}{2}\right)(V_2 - V_1) \quad \text{...}(ix)$$

= (Mean pressure) × (Change in volume)

$$W = \left(\frac{40.45 + 30}{2}\right) \times (220.9 - 45)$$

= 6196 N-cm or 61.96 N-m

It may be noted that work done does not cross the system boundary when spring and cylinder are considered system.

Now, to find T_2, using the relation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\therefore \quad T_2 = \frac{p_2V_2T_1}{p_1V_1} = \frac{40.45 \times 220.9 \times 293}{30 \times 45} = 1939.3 \text{ K}$$

Also, $$m = \frac{p_1v_1}{R_1T_1} = \frac{30 \times 45}{(0.287 \times 10^3) \times 293} = 0.0001605 \text{ kg}$$

Now, change in internal energy,

$$\Delta U = m \times c_v \times (T_2 - T_1)$$

$$= 0.0001605 \times 0.71 \times (1939.3 - 293) = 0.1876 \text{ kJ}$$

According to first law,

$$Q_{1-2} = \Delta U + W$$

$$= 0.1876 + 61.96 \times 10^{-3} = 0.2495 \text{ kJ}$$

∴ **Amount of heat added to the system = 0.2495 kJ. (Ans.)**

4.10. APPLICATION OF FIRST LAW TO STEADY FLOW PROCESS

Steady Flow Energy Equation (S.F.E.E.)

In many practical problems, the rate at which the fluid flows through a machine or piece of apparatus is constant. This type of flow is called *steady flow.*

Assumptions :

The following *assumptions* are made in the system analysis :

(*i*) The mass flow through the system remains constant.

(*ii*) Fluid is uniform in composition.

(*iii*) The only interaction between the system and surroundings are work and heat.

(*iv*) The state of fluid at any point remains constant with time.

(*v*) In the analysis only potential, kinetic and flow energies are considered.

Fig. 4.30 shows a schematic flow process for an open system. An open system is one in which both mass and energy may cross the boundaries. A wide interchange of energy may take place within an open system. Let the system be an automatic engine with the inlet manifold at the first state point and exhaust pipe as the second point. There would be an interchange of chemical energy in the fuel, kinetic energy of moving particles, internal energy of gas and heat transferred and shaft work within the system. From Fig. 4.30 it is obvious that if there is no variation of flow of mass or energy with time across the boundaries of the system the steady flow will prevail. The conditions may pass through the cyclic or non-cyclic changes within the system. As a result the mass entering the system equals the mass leaving, also energy entering the system equals energy leaving.

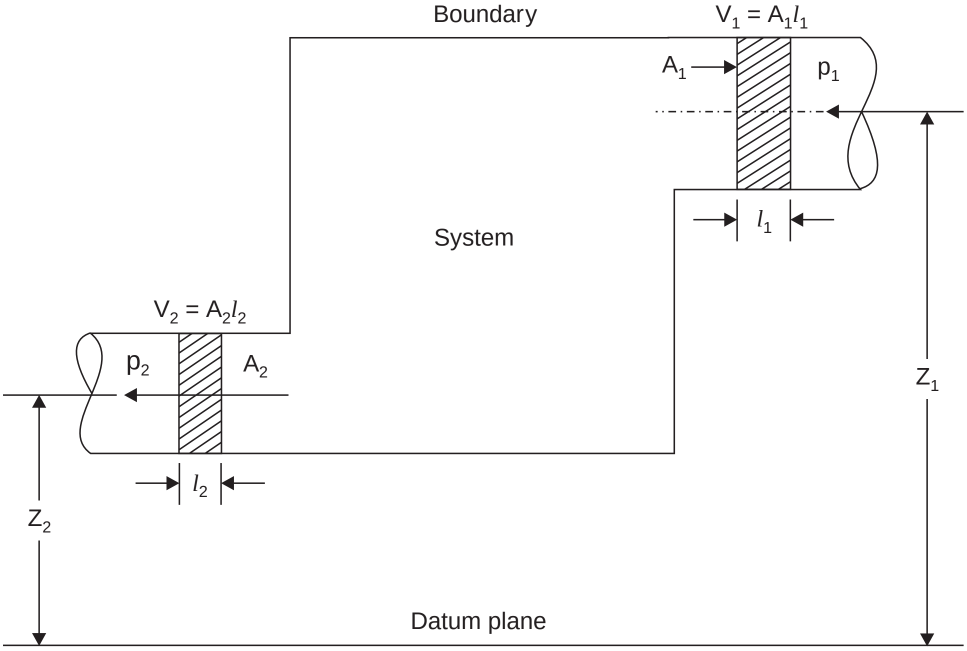


Fig. 4.30

The steady flow equation can be expressed as follows :

$$u_1 + \frac{C_1^2}{2} + Z_1 g + p_1 v_1 + Q = u_2 + \frac{C_2^2}{2} + Z_2 g + p_2 v_2 + W \qquad \text{...(4.45)}$$

$$(u_1 + p_1 v_1) + \frac{C_1^2}{2} + Z_1 g + Q = (u_2 + p_2 v_2) + \frac{C_2^2}{2} + Z_2 g + W$$

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \qquad [\because \quad h = u + pv]$$

If Z_1 and Z_2 are neglected, we get

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \qquad \text{...[4.45 (a)]}$$

where, Q = Heat supplied (or entering the boundary) per kg of fluid,

W = Work done by (or work coming out of the boundary) 1 kg of fluid,

C = Velocity of fluid ,
Z = Height above datum,
p = Pressure of the fluid,
u = Internal energy per kg of fluid, and
pv = Energy required for 1 kg of fluid.

This equation is applicable to any medium in any steady flow. It is applicable not only to rotary machines such as centrifugal fans, pumps and compressors but also to reciprocating machines such as steam engines.

In a steady flow the rate of mass flow of fluid at any section is the same as at any other section. Consider any section of cross-sectional area A, where the fluid velocity is C, the rate of volume flow past the section is CA. Also, since mass flow is volume flow divided by specific volume,

$$\therefore \quad \text{Mass flow rate, } \dot{m} = \frac{CA}{v} \qquad ...(4.46)$$

(where v = *Specific volume* at the section)

This equation is known as the **continuity of mass equation**.

With reference to Fig. 4.30.

$$\therefore \quad \dot{m} = \frac{C_1 A_1}{v_1} = \frac{C_2 A_2}{v_2} \qquad ...[4.46\ (a)]$$

4.11. ENERGY RELATIONS FOR FLOW PROCESS

The energy equation (m kg of fluid) for a steady flow system is given as follows :

$$m\left(u_1 + \frac{C_1^2}{2} + Z_1 g + p_1 v_1\right) + Q = m\left(u_2 + \frac{C_2^2}{2} + Z_2 g + p_2 v_2\right) + W$$

i.e.,
$$Q = m\left[(u_2 - u_1) + (Z_2 g - Z_1 g) + \left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right) + (p_2 v_2 - p_1 v_1)\right] + W$$

i.e.,
$$Q = m\left[(u_2 - u_1) + g(Z_2 - Z_1) + \left(\frac{C_2^2 - C_1^2}{2}\right) + (p_2 v_2 - p_1 v_1)\right] + W$$

$$= \Delta U + \Delta PE + \Delta KE + \Delta\,(pv) + W$$

where
$\Delta U = m\,(u_2 - u_1)$
$\Delta PE = mg\,(Z_2 - Z_1)$
$\Delta KE = m\left(\frac{C_2^2 - C_1^2}{2}\right)$
$\Delta pv = m(p_2 v_2 - p_1 v_1)$

$$\therefore \quad Q - \Delta U = [\Delta PE + \Delta KE + \Delta(pV) + W] \qquad ...(4.47)$$

For non-flow process,

$$Q = \Delta U + W = \Delta U + \int_1^2 p dV$$

i.e.,
$$Q - \Delta U = \int_1^2 p.dV \qquad ...(4.48)$$

The internal energy is a function of temperature only and it is a point function. Therefore, for the same two temperatures, change in internal energy is the same whatever may be the process, non-flow, or steady flow, reversible or irreversible.

For the same value of Q transferred to non-flow and steady flow process and for the same temperature range, we can equate the values of eqns. (4.47) and (4.48) for $(Q - \Delta U)$.

$$\therefore \qquad \int_1^2 p.dV = \Delta PE + \Delta KE + \Delta (pV) + W \qquad ...(4.49)$$

where, W = Work transfer in flow process

and $\int_1^2 p.dV$ = Total change in mechanical energy of reversible steady flow process.

Property Relations for Energy Equations

We know that

$$h = u + pv$$

Differentiating above equation

$$dh = du + pdv + vdp$$

But $dQ = du + p.dv$ (as per first law applied to closed system)

or $du = dQ - p.dv$

Substituting this value of du in the above equation, we get

$$dh = dQ - p.dv + pdv + vdp$$

$$= dQ + vdp$$

$$\therefore \qquad vdp = dh - dQ$$

$$\therefore \qquad -\int_1^2 vdp = Q - \Delta h \qquad ...(4.50)$$

where $-\int_1^2 vdp$ represents on a p-v diagram the area behind 1-2 as shown in Fig. 4.31 (b).

The eqn. (4.47) for a unit mass flow can be written as

$$dQ = d(PE) + d(KE) + du + d(pv) + dW$$

Substituting the value of $dQ = du + p.dv$ in the above equation, we get

$$du + pdv = d(PE) + d(KE) + du + pdv + vdp + dW$$

$$\therefore \qquad -vdp = d(PE) + d(KE) + dW$$

$$\therefore \qquad -\int_1^2 vdp = \Delta PE + \Delta KE + W \qquad ...[4.50\ (a)]$$

If $\Delta PE = 0$ (as in most of thermodynamic systems)

$$-\int_1^2 vdp = \Delta KE + W \qquad ...[4.50\ (b)]$$

If $W = 0$, the area behind the curve represents ΔKE and if $\Delta KE = 0$, area behind the curve represents W which is shaft work.

$-\int_1^2 vdp$ is a *positive quantity and represents work done by the system.*

If $\Delta PE = 0$ and $W = 0$, then

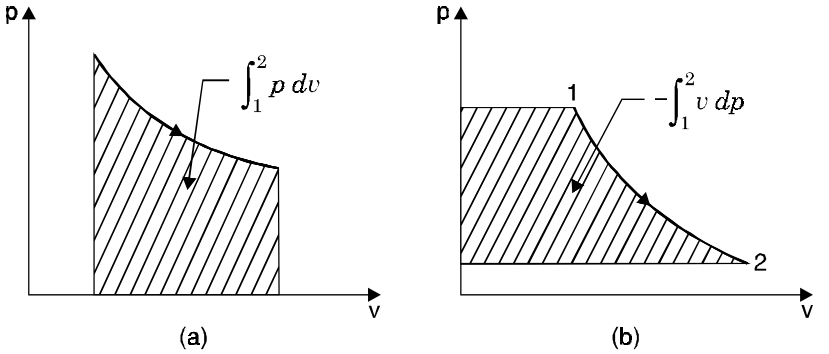
$-\int_1^2 vdp = \Delta KE$, this is applicable in case of a *nozzle.*

i.e., $$\int_1^2 vdp = \frac{C^2}{2} \text{ in the case of a nozzle.}$$

If $\Delta PE = 0$ and $\Delta KE = 0$, as in case of a *compressor*, $-\int_1^2 vdp = W$

or $$W = \int_1^2 vdp \text{ in the case of a } compressor.$$

The integral $\int_1^2 pdv$ and $\int_1^2 vdp$ are shown in Fig. 4.31 (*a*) and (*b*).



(*a*) Work done in non-flow process. (*b*) Work done in flow process.

Fig. 4.31. Representation of work on *p-v* diagram.

The work done during *non-flow process* is given by

$$\int_1^2 pdv = Q - \Delta u \qquad \text{...[4.50 (c)]}$$

For isothermal process, we have

$$\Delta u = 0 \text{ and } \Delta h = 0.$$

Substituting these values in (equations) 4.50 and [4.50 (*c*)]

$$-\int_1^2 vdp = Q \text{ and } \int_1^2 pdv = Q$$

$$\therefore \qquad \int_1^2 pdv = -\int_1^2 vdp$$

The above equation indicates that the *area under both curves is same for an isothermal process.*

Note. In all the above equations '*v*' represents volume per unit mass as mass flow is considered unity.

Now let us find out expressions for work done for different flow processes as follows :

(*i*) **Steady flow constant pressure process :**

$$W = -\int_1^2 v\,.\,dp = 0 \qquad [\because \quad dp = 0] \qquad \text{...(4.51)}$$

(*ii*) **Steady flow constant volume process :**

$$W = -\int_1^2 Vdp = -V(p_2 - p_1) = V(p_1 - p_2)$$

i.e.,

$$W = V(p_1 - p_2) \quad ...(4.52)$$

(*iii*) **Steady flow constant temperature process :**

The constant temperature process is represented by

$$pV = p_1V_1 = p_2V_2 = C \text{ (constant)}$$

$$\therefore \quad W = -\int_1^2 Vdp$$

$$= -\int_1^2 \frac{C}{p}\,dp \qquad \left[\because V = \frac{C}{p}\right]$$

$$= -C\int_1^2 \frac{dp}{p} = -C\,\left|\log_e p\right|_1^2$$

$$= -C\log_e \frac{p_2}{p_1} = C\log_e \frac{p_1}{p_2}$$

i.e.,

$$W = p_1V_1 \log_e\left(\frac{p_1}{p_2}\right) \quad ...(4.53)$$

Now substituting the values of W in the equation (4.49), considering unit mass flow :

(*a*) The energy equation for *constant pressure flow process*

$$dQ = \Delta\, PE + \Delta\, KE + \Delta\, h$$

$$= \Delta\, h \text{ (if } \Delta PE = 0 \text{ and } \Delta\, KE = 0).$$

(*b*) The energy equation for constant volume flow process

$$dQ = -\int_1^2 vdp + \Delta\, PE + \Delta\, KE + \Delta\, u + pdv + vdp$$

$$= \Delta\, PE + \Delta\, KE + \Delta\, u \qquad \left[\because pdv = 0 \text{ and } v\,.\,dp = \int_1^2 vdp\right]$$

$$\therefore \quad dQ = \Delta\, u \text{ (if } \Delta\, PE = 0 \text{ and } \Delta\, KE = 0)$$

4.12. ENGINEERING APPLICATIONS OF STEADY FLOW ENERGY EQUATION (S.F.E.E.)

4.12.1. Water Turbine

Refer to Fig. 4.32. In a water turbine, water is supplied from a height. The potential energy of water is converted into kinetic energy when it enters into the turbine and part of it is converted into useful work which is used to generate electricity.

Considering centre of turbine shaft as *datum*, the energy equation can be written as follows :

$$\left(u_1 + p_1v_1 + Z_1g + \frac{{C_1}^2}{2}\right) + Q = \left(u_2 + p_2v_2 + Z_2g + \frac{{C_2}^2}{2}\right) + W$$

In this case,

$$Q = 0$$

$$\Delta\, u = u_2 - u_1 = 0$$

$$\therefore \quad v_1 = v_2 = v$$

$$Z_2 = 0$$

$$\therefore \qquad \left(p_1 v + Z_1 g + \frac{C_1^2}{2}\right) = \left(p_2 v + Z_2 g + \frac{C_2^2}{2}\right) + W \qquad \text{...(4.54)}$$

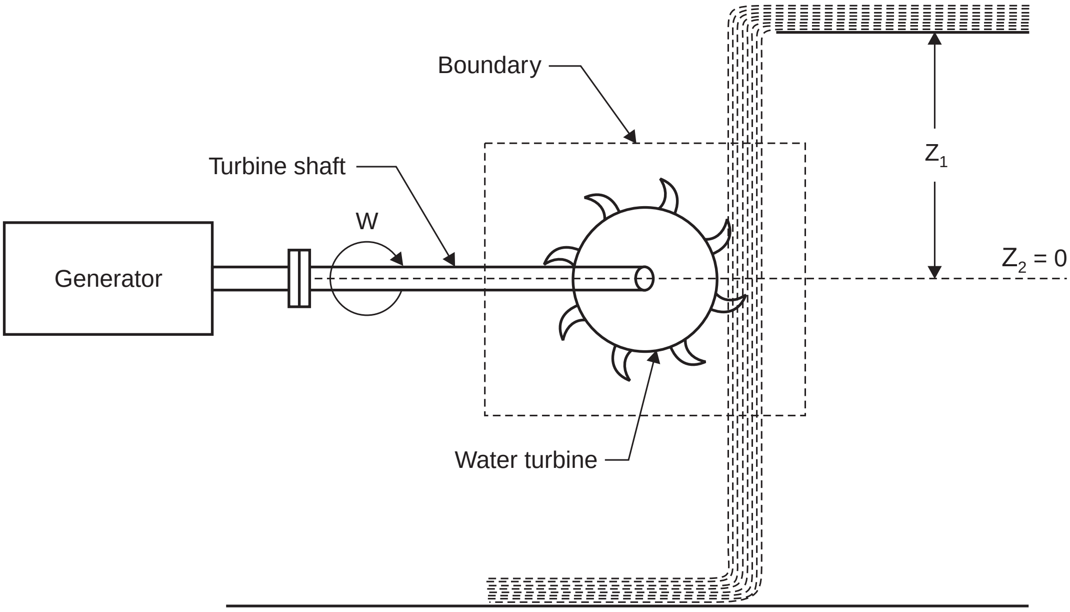


Fig. 4.32. Water turbine.

W is *positive* because work is done by the system (or work comes out of the boundary).

4.12.2. Steam or Gas Turbine

In a steam or gas turbine steam or gas is passed through the turbine and part of its energy is converted into work in the turbine. This output of the turbine runs a generator to produce electricity as shown in Fig. 4.33. The steam or gas leaves the turbine at lower pressure or temperature.

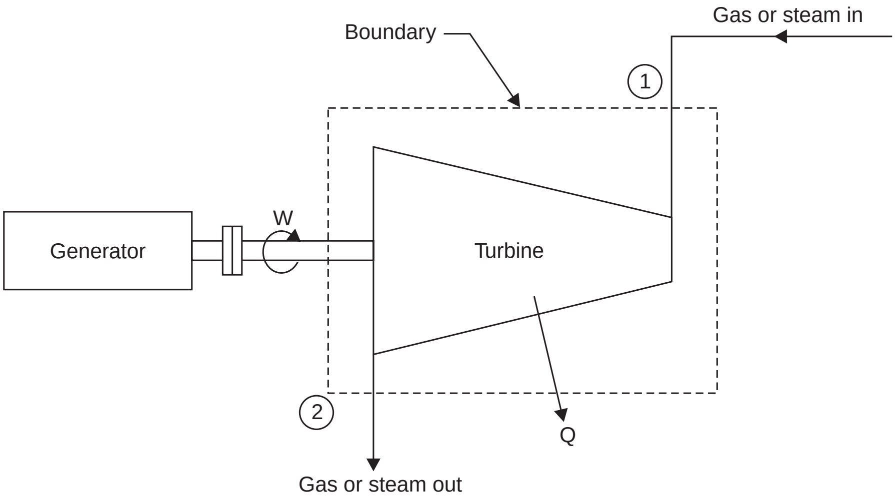


Fig. 4.33. Steam or gas turbine.

Applying energy equation to the system.

Here, $Z_1 = Z_2$ (*i.e.,* $\Delta Z = 0$)

$$h_1 + \frac{C_1^2}{2} - Q = h_2 + \frac{C_2^2}{2} + W \qquad \text{...(4.55)}$$

The sign of Q is *negative* because heat is *rejected* (or comes out of the boundary).

The sign of W is *positive* because work is done by the system (or work comes out of the boundary).

4.12.3. Centrifugal Water Pump

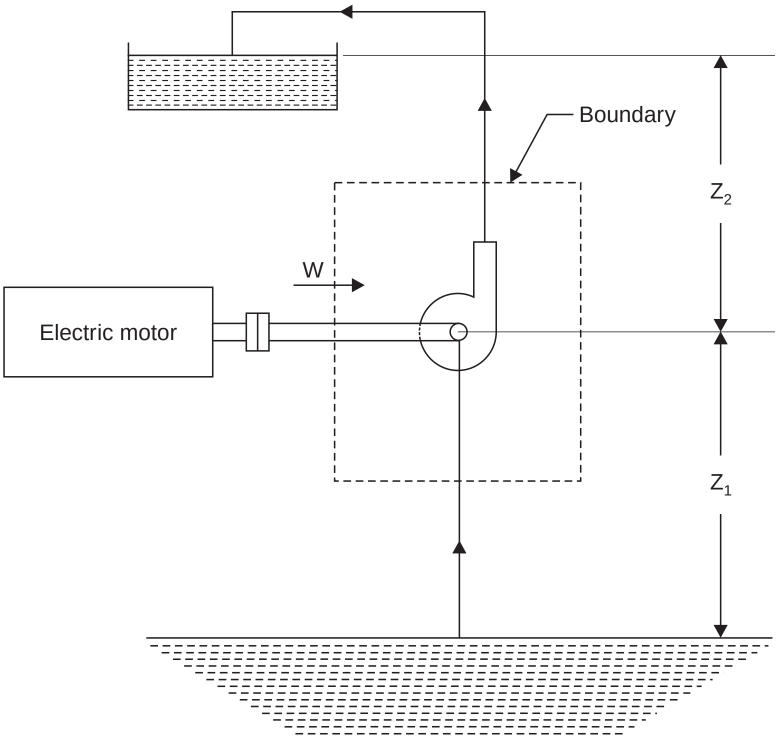


Fig. 4.34. Centrifugal water pump.

A centrifugal water pump draws water from a lower level and pumps to higher level as shown in Fig. 4.34. Work is required to run the pump and this may be supplied from an external source such as an electric motor or a diesel engine.

Here $Q = 0$ and $\Delta u = 0$ as there is no change in temperature of water ; $v_1 = v_2 = v$.

Applying the energy equation to the system

or
$$p_1v_1 + Z_1g + \frac{C_1^2}{2} = p_2v_2 + Z_2g + \frac{C_2^2}{2} - W \qquad \text{...(4.56)}$$

The sign of W is *negative* because *work is done on the system* (or *work enters the boundary*).

4.12.4. Centrifugal Compressor

Refer to Fig. 4.35. A centrifugal compressor compresses air and supplies the same at moderate pressure and in large quantity.

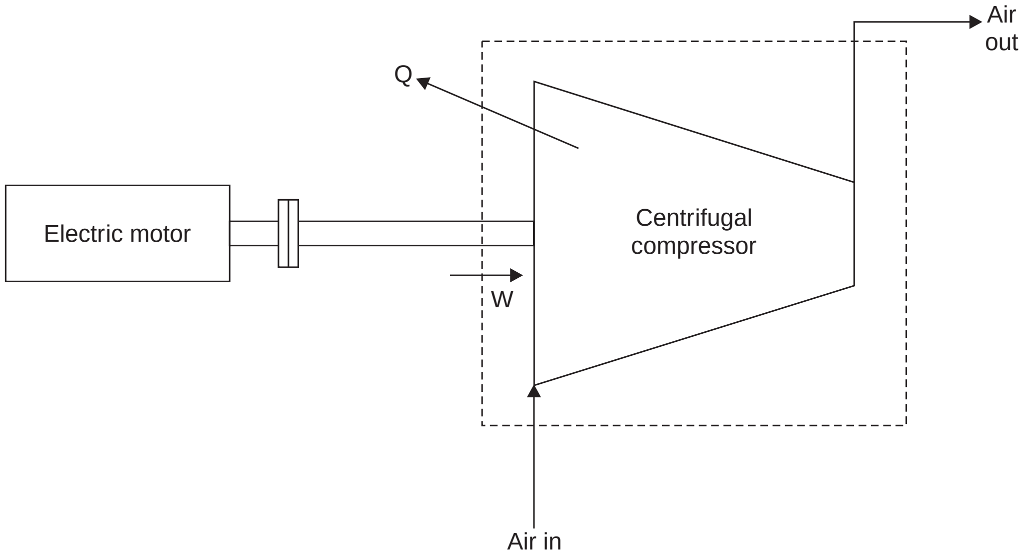


Fig. 4.35. Centrifugal compressor.

Applying energy equation to the system (Fig. 4.35)

$$\Delta Z = 0 \text{ (generally taken)}$$

$$\left(h_1 + \frac{C_1^2}{2}\right) - Q = \left(h_2 + \frac{C_1^2}{2}\right) - W$$

The Q is taken as *negative* as heat is *lost* from the system and W is taken as *negative* as work is *supplied* to the system.

or
$$\left(h_1 + \frac{C_1^2}{2}\right) - Q = \left(h_2 + \frac{C_2^2}{2}\right) - W \qquad \text{...(4.57)}$$

4.12.5. Reciprocating Compressor

Refer Fig. 4.36. The reciprocating compressor draws in air from atmosphere and supplies at a considerable higher pressure in small quantities (compared with centrifugal compressor). The reciprocating compressor can be considered as steady flow system *provided the control volume includes the receiver which reduces the fluctuations of flow considerably.*

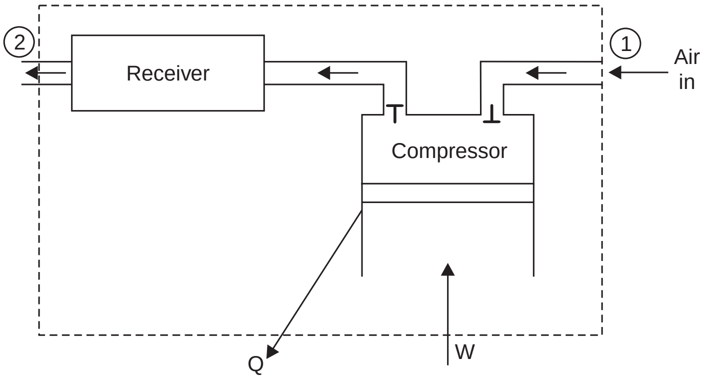


Fig. 4.36. Reciprocating compressor.

Applying energy equation to the system, we have :

$\Delta PE = 0$ and $\Delta KE = 0$ since these changes are negligible compared with other energies.

$$\therefore \qquad h_1 - Q = h_2 - W \qquad ...(4.58)$$

4.12.6. Boiler

A boiler transfers heat to the incoming water and generates the steam. The system is shown in Fig. 4.37.

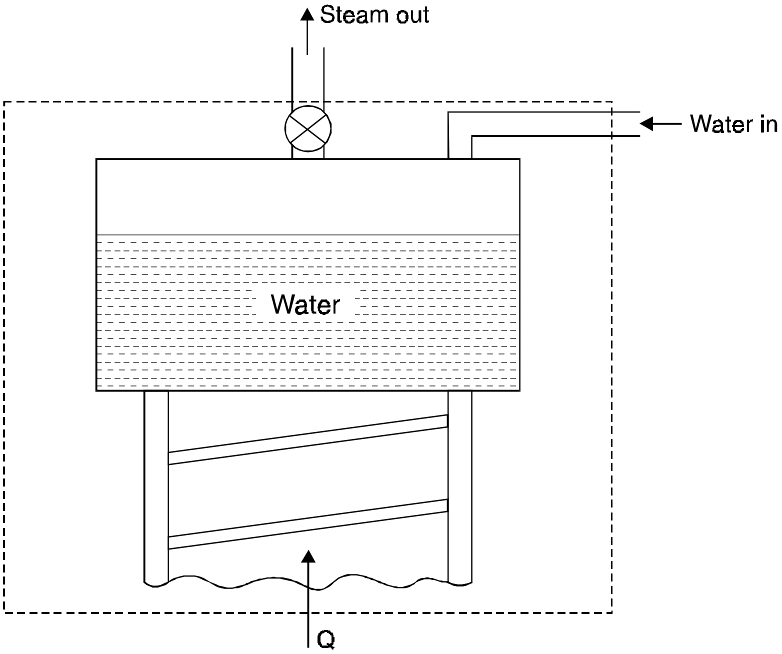


Fig. 4.37. Boiler.

For this system, $\Delta Z = 0$ and $\Delta \left(\frac{C_2^2}{2}\right) = 0$

$W = 0$ since neither any work is developed nor absorbed.

Applying energy equation to the system

$$h_1 + Q = h_2 \qquad ...(4.59)$$

4.12.7. Condenser

The condenser is used to condense the steam in case of steam power plant and condense the refrigerant vapour in the refrigeration system using water or air as cooling medium. Fig. 4.38 shows the system.

For this system :

$\Delta PE = 0$, $\Delta KE = 0$ (as their values are very small compared with enthalpies)

$W = 0$ (since neither any work is developed nor absorbed)

Using energy equation to steam flow

$$h_1 - Q = h_2 \qquad ...[4.60\ (a)]$$

where Q = Heat lost by 1 kg of steam passing through the condenser.

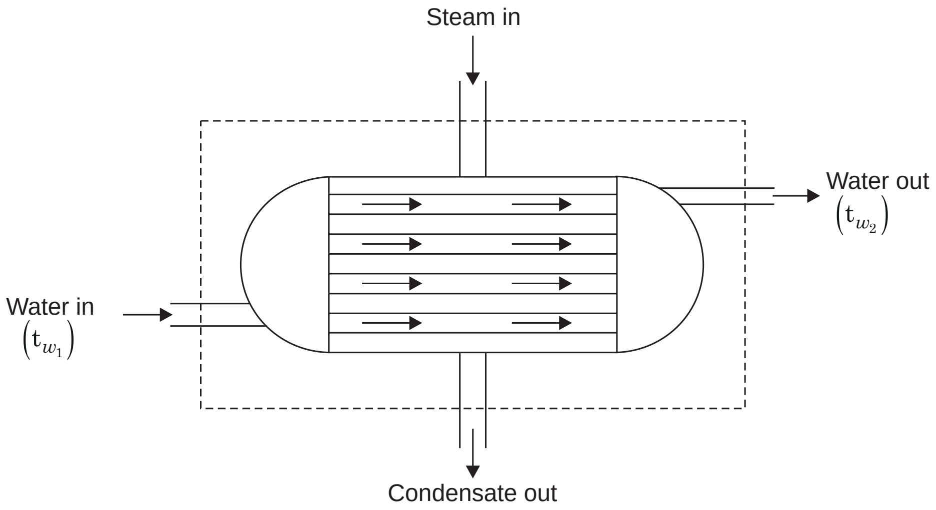


Fig. 4.38. Condenser.

Assuming there are no other heat interactions except the heat transfer between steam and water, then

$$Q = \text{Heat gained by water passing through the condenser}$$

$$= m_w (h_{w2} - h_{w1}) = m_w c_w (t_{w2} - t_{w1})$$

Substituting this value of Q in eqn. [4.60 (*a*)], we get

$$h_1 - h_2 = m_w (h_{w2} - h_{w1}) = m_w c_w (t_{w2} - t_{w1}) \qquad \text{...[4.60 (b)]}$$

where, m_w = Mass of cooling water passing through the condenser, and

c_w = Specific heat of water.

4.12.8. Evaporator

An evaporator is an equipment used in refrigeration plant to carry heat from the refrigerator to maintain the low temperature. Here the refrigerant liquid is passed through the evaporator and it comes out as vapour absorbing its latent heat from the surroundings of the evaporator. Fig. 4.39 shows the system. For this system

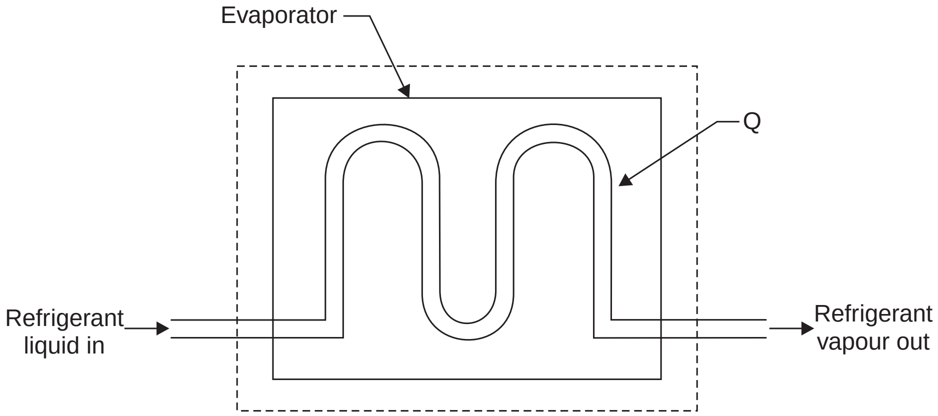


Fig. 4.39. Evaporator.

$$\Delta PE = 0,\ \Delta KE = 0$$

$$W = 0 \qquad [\because \text{ No work is absorbed or supplied}]$$

Applying the energy equation to the system

$$h_1 + Q = h_2 \qquad ...(4.61)$$

Q is taken as + ve because heat *flows from the surroundings to the system* as the temperature in the system is lower than the surroundings.

4.12.9. Steam Nozzle

In case of a nozzle as the enthalpy of the fluid decreases and pressure drops simultaneously the flow of fluid is *accelerated*. This is generally used to convert the part of the energy of steam into *kinetic energy of steam* supplied to the turbine.

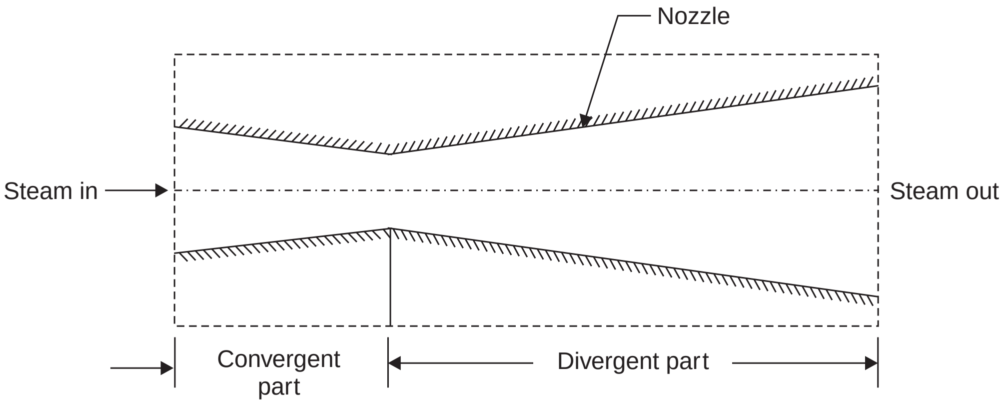


Fig. 4.40. Steam nozzle.

Fig. 4.40 shows a commonly used convergent-divergent nozzle.

For this system,

$$\Delta PE = 0$$

$$W = 0$$

$$Q = 0$$

Applying the energy equation to the system,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

or

$$\frac{C_2^2}{2} - \frac{C_1^2}{2} = h_1 - h_2 \quad \text{or} \quad C_2^2 - C_1^2 = 2(h_1 - h_2)$$

or

$$C_2^2 = C_1^2 + 2(h_1 - h_2)$$

$$\therefore \quad C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)} \qquad ...(4.62)$$

where velocity C is in m/s and enthalpy h in joules.

If $C_1 << C_2$, then

$$C_2 = \sqrt{2(h_1 - h_2)} \qquad ...[4.63\ (a)]$$

$$\therefore \quad C_2 = \sqrt{2\Delta h}\ . \qquad ...[4.63\ (b)]$$

4.13. THROTTLING PROCESS AND JOULE-THOMPSON POROUS PLUG EXPERIMENT

Throttling process involves the *passage of a higher pressure fluid through a narrow constriction. The effect is the reduction in pressure and increase in volume.* This process is *adiabatic* as no heat flows from and to the system, but it is *not reversible.* It is *not an isentropic process.* The *entropy* of the fluid actually *increases.*

Such a process occurs in a flow through a *porous plug,* a *partially closed valve* and a *very narrow orifice.* The porous plug is shown in Fig. 4.41.

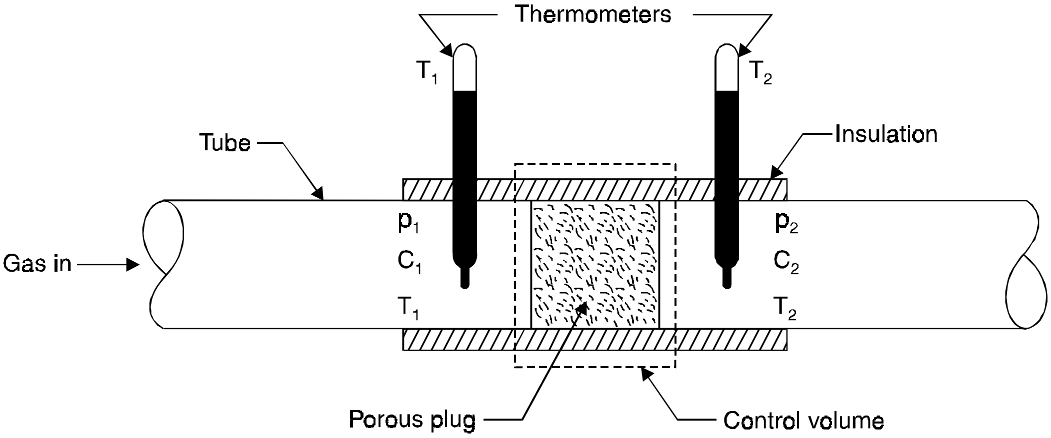


Fig. 4.41. The Joule-Thomson porous plug experiment.

In this system,

$$Q = 0 \qquad (\because \text{ System is isolated})$$

$$W = 0 \qquad (\because \text{ There is no work interaction})$$

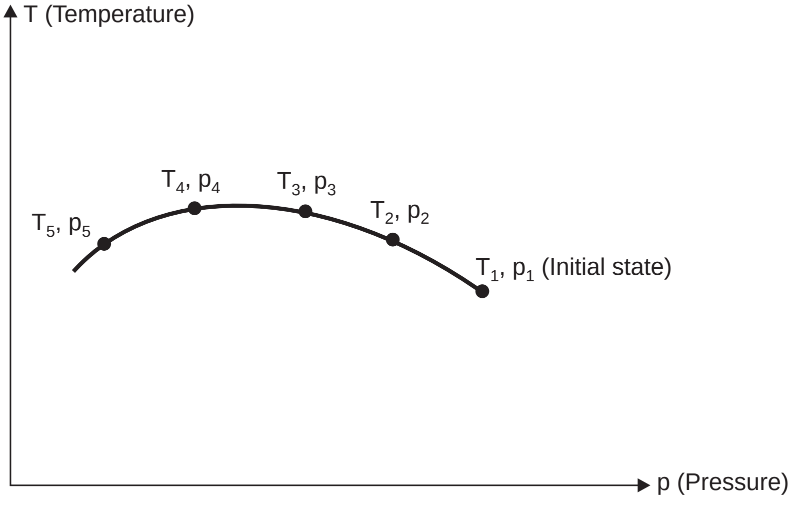


Fig. 4.42. Constant enthalpy curve.

$$\Delta PE = 0 \qquad (\because \text{ Inlet and outlet are at the same level})$$

$$\Delta KE = 0 \qquad (\because \text{ Kinetic energy does not change significantly})$$

Applying the energy equation to the system

$$h_1 = h_2$$

This shows that *enthalpy remains constant during adiabatic throttling process.*

The throttling process is commonly used for the following purposes :

(*i*) For determining the condition of steam (dryness fraction).

(*ii*) For controlling the speed of the turbine.

(*iii*) Used in refrigeration plant for reducing the pressure of the refrigerant before entry into the evaporator.

Throttling process frequently encountered in practice was investigated by Joule and Thompson (Lord Kelvin) in their famous *porous plug experiment* (Fig. 4.41). A stream of gas at pressure p_1 and temperature T_1 is forced continuously through a *porous plug* in a tube from which it emerges at a lower pressure p_2 and temperature T_2. The whole apparatus is *thermally insulated.*

In this process (as earlier stated)

$$h_1 = h_2$$

Whether the temperature and internal energy change in a throttling process depends on whether the fluid behaves as an ideal gas or not. Since the enthalpy of an ideal gas is a function of temperature alone, it follows that

$$T_1 = T_2 \text{ for (throttling process)}_{ideal\ gas} \qquad ...(4.64)$$

and, therefore, $u_1 = u_2$

For an *ideal gas,* therefore, the *throttling* process takes place at

(*i*) constant enthalpy,

(*ii*) constant temperature, and

(*iii*) constant internal energy.

The *enthalpy of a real gas is not a function of temperature alone.* In this case

$$T_1 \neq T_2 \qquad ...(4.65)$$

Also since the *pv* product may be different before and after throttling, the change in internal energy is not zero, as it is in *free expansion*, but is given by

$$u_2 - u_1 = p_1v_1 - p_2v_2 \qquad ...(4.66)$$

Joule-Thompson and Joule Co-efficients

When a real gas undergoes a throttling process a change in temperature takes place. Let us perform a series of the experiments on the same gas, keeping p_1 and T_1 constant, by varying the pressure downstream of the plug to various values p_2, p_3, p_4 etc. After throttling let T_1, T_2, T_3, T_4 etc. be the corresponding temperatures. Now if a graph is plotted between p and T (Fig. 4.42), a smooth curve drawn through these points will be a curve of *constant enthalpy* because $h_1 = h_2 = h_3 = h_4$ etc.

It may be noted that this curve does *not* represent the process executed by the gas in passing through the plug, since the process is *irreversible* and the gas does not pass through a sequence of equilibrium states.

The *slope* of a constant enthalpy line or a *p-T* diagram at a particular state may be *positive, zero* or *negative* value. The slope is called ***Joule-Thompson co-efficient,*** μ and is given by

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h \qquad ...(4.67)$$

$$= 0 \text{ for ideal gas.}$$

If we carry out other series of experiments similar to described above starting from different initial states, we can obtain a family of constant enthalpy curves as shown in Fig. 4.43. The states where $\mu = 0$ are called '***inversion states***' and locus of these states is called the ***inversion curve***.

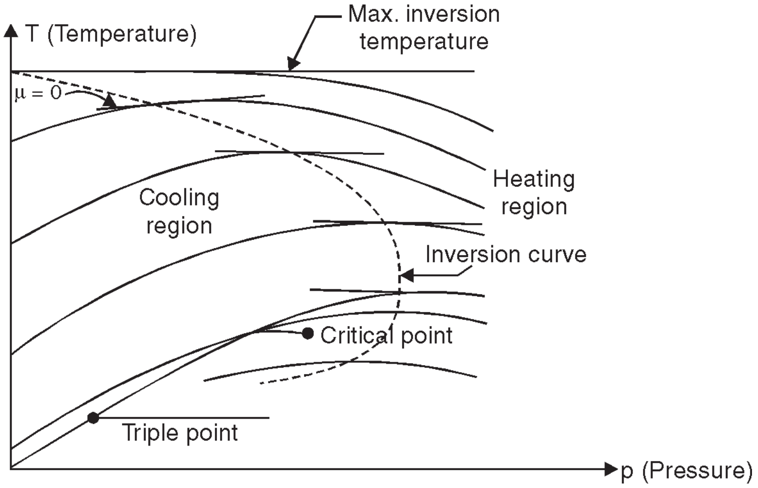


Fig. 4.43. Inversion curve.

The region inside the inversion curve is the cooling region since μ is *positive*, and temperature falls with fall in pressure.

The *region outside the inversion curve is the heating region* since μ is *negative* and temperature rises with fall in pressure.

Cooling can take place only if the initial temperature before throttling is below the *maximum inversion temperature*. This temperature is about $5T_c$.

The *maximum inversion temperatures* of some gases are given below :

(*i*) He = 24 K
(*ii*) H_2 = 195 K
(*iii*) Air = 603 K
(*iv*) N_2 = 261 K
(*v*) A = 732 K
(*vi*) CO_2 = 1500 K.

The free expansion process is also a Joule (not Joule-Thompson). The Joule co-efficient is defined by

$$\text{Joule co-efficient} = \eta = -\left(\frac{\partial T}{\partial v}\right)_u \qquad ...(4.68)$$

For free expansion of gases the experimental data obtained is limited. From the data available it appears that η is positive (*i.e.,* cooling accompanies a fall in pressure or increase in specific volume).

Note. The throttling process is used in the liquification of gases where μ is positive.

STEADY FLOW SYSTEMS

Example 4.35. *10 kg of fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are : $p_1 = 1.5$ bar, $\rho_1 = 26$ kg/m^3, $C_1 = 110$ m/s and $u_1 = 910$ kJ/kg and at the exit are $p_2 = 5.5$ bar, $\rho_2 = 5.5$ kg/m^3, $C_2 = 190$ m/s and $u_2 = 710$ kJ/kg. During the passage, the fluid rejects 55 kJ/s and rises through 55 metres. Determine :*

(*i*) *The change in enthalpy* (Δh) ;

(*ii*) *Work done during the process* (*W*).

Solution. Flow of fluid = 10 kg/min

Properties of fluid at the **inlet** :

Pressure, $p_1 = 1.5 \text{ bar} = 1.5 \times 10^5 \text{ N/m}^2$

Density, $\rho_1 = 26 \text{ kg/m}^3$

Velocity, $C_1 = 110 \text{ m/s}$

Internal energy, $u_1 = 910 \text{ kJ/kg}$

Properties of the fluid at the **exit** :

Pressure, $p_2 = 5.5 \text{ bar} = 5.5 \times 10^5 \text{ N/m}^2$

Density, $\rho_2 = 5.5 \text{ kg/m}^3$

Velocity, $C_2 = 190 \text{ m/s}$

Internal energy, $u_2 = 710 \text{ kJ/kg}$

Heat *rejected* by the fluid,

$$Q = 55 \text{ kJ/s}$$

Rise is elevation of fluid = 55 m.

(*i*) The change in enthalpy,

$$\Delta h = \Delta u + \Delta(pv) \quad \text{...}(i)$$

$$\Delta(pv) = \frac{p_2v_2 - p_1v_1}{1}$$

$$= \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} = \frac{5.5 \times 10^5}{5.5} - \frac{1.5 \times 10^5}{26}$$

$$= 1 \times 10^5 - 0.0577 \times 10^5$$

$$= 10^5 \times 0.9423 \text{ Nm or J} = 94.23 \text{ kJ}$$

$$\Delta u = u_2 - u_1 = (710 - 910) = -200 \text{ kJ/kg}$$

Substituting the value in eqn. (*i*), we get

$$\Delta h = -200 + 94.23 = -\mathbf{105.77 \ kJ/kg.} \quad \textbf{(Ans.)}$$

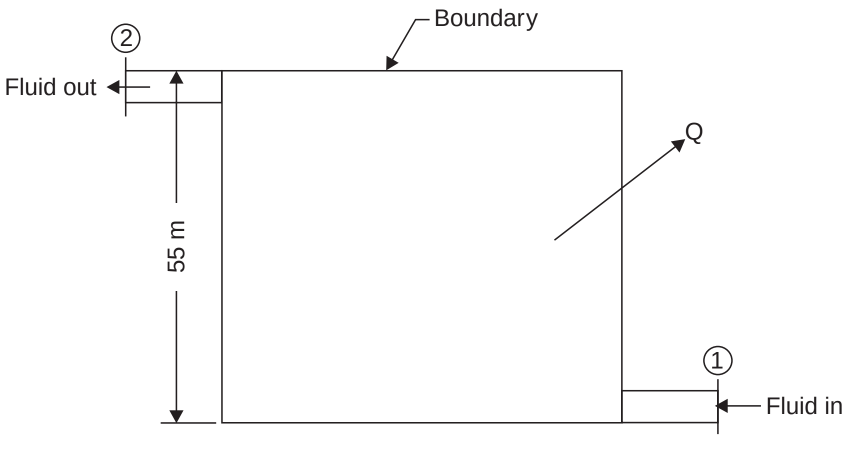


Fig. 4.44

(*ii*) The steady flow equation for unit mass flow can be written as

$$Q = \Delta \, KE + \Delta \, PE + \Delta \, h + W$$

where Q is the heat transfer per kg of fluid

$$Q = 55 \text{ kJ/s} = \frac{55 \text{ kJ/s}}{\frac{10}{60} \text{ kg/s}} = 55 \times 6 = 330 \text{ kJ/kg}$$

Now, $$\Delta KE = \frac{C_2^{\,2} - C_1^{\,2}}{2} = \frac{(190)^2 - (110)^2}{2} \text{ Nm or J} = 12000 \text{ J or } 12 \text{ kJ/kg}$$

$$\Delta PE = (Z_2 - Z_1)\, g = (55 - 0) \times 9.81 \text{ Nm or J} = 539.5 \text{ J or} \approx 0.54 \text{ kJ/kg}$$

Substituting the value in steady flow equation,

$$-330 = 12 + 0.54 - 105.77 + W \quad \text{or} \quad W = -236.77 \text{ kJ/kg.}$$

Work done per second $= -236.77 \times \frac{10}{60} = -39.46$ kJ/s $= -$ **39.46 kW. (Ans.)**

Example 4.36. *In a gas turbine unit, the gases flow through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and 400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are 50 m/s and 110 m/s respectively. Calculate :*

(*i*) *The rate at which heat is rejected to the turbine, and*

(*ii*) *The area of the inlet pipe given that the specific volume of the gases at the inlet is 0.45 m*^{3}*/kg.*

Solution. Rate of flow of gases, $\dot{m} = 15$ kg/s

Volume of gases at the inlet, $v = 0.45$ m^3/kg

Power developed by the turbine, $P = 12000$ kW

∴ Work done, $W = \frac{12000}{15} = 800$ kJ/kg

Enthalpy of gases at the inlet, $h_1 = 1260$ kJ/kg

Enthalpy of gases at the oulet, $h_2 = 400$ kJ/kg

Velocity of gases at the inlet, $C_1 = 50$ m/s

Velocity of gases at the outlet, $C_2 = 110$ m/s.

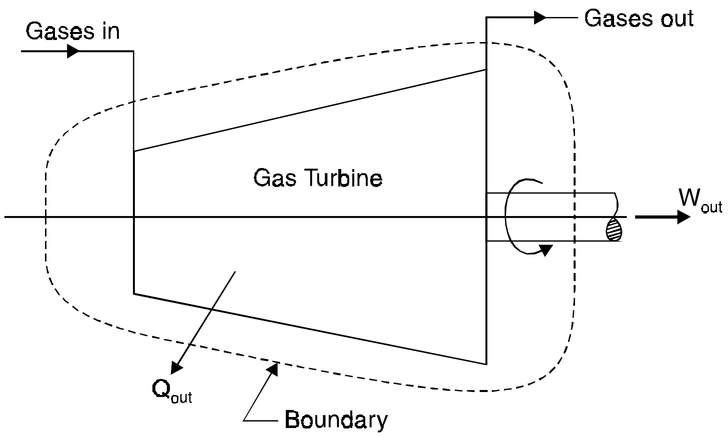


Fig. 4.45

(*i*) **Heat rejected, Q :**

Using the flow equation,

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \qquad ...(i) \quad [\because \; Z_1 = Z_2]$$

$$\text{Kinetic energy at inlet} = \frac{C_1^2}{2} = \frac{50^2}{2} \text{ m}^2/\text{s}^2 = \frac{50^2 \text{ kg m}^3}{2 \text{ s}^2\text{kg}} = 1250 \text{ Nm/kg} = 1.25 \text{ kJ/kg}$$

$$\text{Kinetic energy at outlet} = \frac{C_1^2}{2} = \frac{110^2}{2 \times 1000} = 6.05 \text{ kJ/kg}$$

Substituting these values in eqn. (*i*), we get

$$1260 + 1.25 + Q = 400 + 6.05 + 800$$

$$\therefore \qquad Q = -\,55.2 \text{ kJ/kg}$$

i.e., **Heat rejected** $= +\,55.2$ kJ/kg $= 55.2 \times 15$ kJ/s $=$ **828 kW. (Ans.)**

(*ii*) **Inlet area, A :**

Using the relation,

$$\dot{m} = \frac{CA}{v}$$

$$\therefore \qquad A = \frac{v\dot{m}}{C} = \frac{0.45 \times 15}{50} = \mathbf{0.135 \ m^2}. \quad \textbf{(Ans.)}$$

☞**Example 4.37.** *In an air compressor air flows steadily at the rate of 0.5 kg/s through an air compressor. It enters the compressor at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m*^{3}*/kg and leaves at 5 m/s with a pressure of 7 bar and a specific volume of 0.16 m*^{3}*/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 60 kJ/s. Calculate :*

(*i*) *The power required to drive the compressor ;*

(*ii*) *The inlet and output pipe cross-sectional areas.*

Solution. Air flow rate through the compressor, $\dot{m} = 0.5$ kg/s

Velocity of air at the inlet to compressor, $C_1 = 6$ m/s

Velocity of air at the outlet of compressor, $C_2 = 5$ m/s

Pressure of air at the inlet to the compressor, $p_1 = 1$ bar

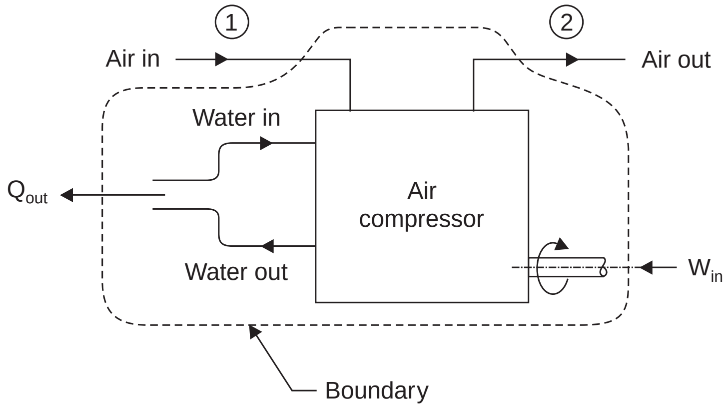


Fig. 4.46

Pressure of air at outlet to the compressor, $p_2 = 7$ bar

Specific volume of air at inlet to the compressor, $v_1 = 0.85$ m^3/kg

Specific volume of air at outlet to the compressor, $v_2 = 0.16$ m^3/kg

Difference of internal energy at the outlet and inlet of the compressor,

$$(u_2 - u_1) = 90 \text{ kJ/kg}$$

Heat rejected by air (to cooling water),

$$Q = -\frac{60}{0.5} = -120 \text{ kJ/kg.}$$

(*i*) **Power required to drive the compressor :**

Using the steady flow energy equation,

$$u_1 + \frac{C_1^2}{2} + p_1v_1 + Q = u_2 + \frac{C_2^2}{2} + p_2v_2 + W$$

$$\therefore \qquad W = (u_1 - u_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right) + (p_1v_1 - p_2v_2) + Q$$

$$= -90 + \frac{1}{1000}\left(\frac{6^2}{2} - \frac{5^2}{2}\right) + \frac{10^5}{1000}(1 \times 0.85 - 7 \times 0.16) + (-120)$$

$$= -90 + 0.0055 - 27 - 120 = -237 \text{ kJ/kg (app.).}$$

(Note that the change in kinetic energy is negligibly small in comparison with the other terms).

i.e., Work input required = 237 kJ/kg = 237 × 0.5 kJ/s = 118.5 kW

Hence, **power required to drive the compressor = 118.5 kW. (Ans.)**

(*ii*) **Inlet and outlet pipe cross-sectional areas, A_1 and A_2 :**

Using the relation,

$$\dot{m} = \frac{CA}{v}$$

$$\therefore \qquad A_1 = \frac{\dot{m}v_1}{C_1} = \frac{0.5 \times 0.85}{6} \text{m}^2 = 0.0708 \text{ m}^2$$

i.e., Inlet pipe cross-sectional area, A_1 = **0.0708 m^2. (Ans.)**

Again, $$A_2 = \frac{\dot{m}v_2}{C_2} = \frac{0.5 \times 0.16}{5} \text{m}^2 = 0.016 \text{ m}^2.$$

i.e., Outlet pipe cross-sectional area, A_2 = **0.016 m^2. (Ans.)**

Note. In this example, the steady flow energy equation has been used, despite the fact the compression consists of : suction of air ; compression in a closed cylinder ; and discharge of air. The steady flow equation can be used because the cycle of processes takes place many times in a minute, and therefore, average effect is steady flow of air through the machine.

Example 4.38. *In a steam plant, 1 kg of water per second is supplied to the boiler. The enthalpy and velocity of water entering the boiler are 800 kJ/kg and 5 m/s. The water receives 2200 kJ/kg of heat in the boiler at constant pressure. The steam after passing through the turbine comes out with a velocity of 50 m/s, and its enthalpy is 2520 kJ/kg. The inlet is 4 m above the turbine exit. Assuming the heat losses from the boiler and the turbine to the surroundings are*

20 kJ/s, calculate the power developed by the turbine. Consider the boiler and turbine as single system.

Solution. Enthalpy of water entering the boiler, $h_1 = 800$ kJ/kg

Velocity of water entering the boiler, $C_1 = 5$ m/s

Enthalpy of steam at the outlet of the turbine, $h_2 = 2520$ kJ/kg

Velocity of steam at the outlet of the turbine, $C_2 = 50$ m/s

Elevation difference, $(Z_1 - Z_2) = 4$ m

Net heat added to the water in the boiler, $Q = 2200 - 20 = 2180$ kJ/kg

Power developed by the turbine :

Using the flow equation,

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

$$\therefore \quad W = (h_1 - h_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right) + (Z_1 - Z_2)\, g + Q$$

$$= (800 - 2520) + \frac{1}{1000}\left[\frac{5^2}{2} - \frac{50^2}{2}\right] + \frac{4 \times 9.81}{1000} + 2180$$

$$= -\,1720 + \frac{1}{1000}(12.5 - 1250) + \frac{39.24}{1000} + 2180$$

$$= -\,1720 - 1.2375 + 0.03924 + 2180$$

$$= 458.8 \text{ kJ/kg} = 458.8 \text{ kJ/s} = 458.8 \text{ kW}$$

Hence, **power developed by the turbine = 458.8 kW. (Ans.)**

Example 4.39. *A turbine, operating under steady-flow conditions, receives 4500 kg of steam per hour. The steam enters the turbine at a velocity of 2800 m/min, an elevation of 5.5 m and a specific enthalpy of 2800 kJ/kg. It leaves the turbine at a velocity of 5600 m/min, an elevation of 1.5 m and a specific enthalpy of 2300 kJ / kg. Heat losses from the turbine to the surroundings amout to 16000 kJ/h.*

Determine the power output of the turbine.

Solution. Quantity of steam supplied to the turbine, $m = 4500$ kg/h

Steam velocity at the entrance to the turbine, $C_1 = 2800$ m/min

Elevation at the entrance, $Z_1 = 5.5$ m

Specific enthalpy at the entrance, $h_1 = 2800$ kJ/g

Steam velocity at the exit, $C_2 = 5600$ m/min

Elevation at the exit, $Z_2 = 1.5$ m

Specific enthalpy at the exit, $h_2 = 2300$ kJ/kg

Heat losses from the turbine to the surroundings, $Q = -\,16000$ kJ/h

Applying the steady flow energy equation at entry (1) and exit (2)

$$m\left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) + Q = m\left(h_2 + \frac{C_2^2}{2} + Z_2 g\right) + W$$

$$\therefore \quad Q - W = m\left[(h_2 - h_1) + \left(\frac{C_2^2 - C_2^2}{2}\right) + (Z_2 - Z_1)g\right]$$

$$-\frac{16000}{3600} - W = \frac{4500}{3600}\left[(2300 - 2800) + \left\{\frac{\left(\frac{5600}{60}\right)^2 - \left(\frac{2800}{60}\right)^2}{2 \times 1000}\right\} + \frac{(1.5 - 5.5) \times 9.81}{1000}\right]$$

$-4.44 - W = 1.25\ (500 + 3.26 - 0.039)$ or $W = 633.44$ kJ/s

∴ **Power output of the turbine = 633.44 kW. (Ans.)**

Example 4.40. *Steam at a 6.87 bar, 205°C, enters in an insulated nozzle with a velocity of 50 m/s. It leaves at a pressure of 1.37 bar and a velocity of 500 m/s.*

Determine the final enthalpy of steam.

Solution. Pressure of steam at the entrance, $p_1 = 6.87$ bar

The velocity with which steam enters the nozzle, $C_1 = 50$ m/s

Pressure of steam at the exit, $p_2 = 1.37$ bar

Velocity of steam at the exit, $C_2 = 500$ m/s.

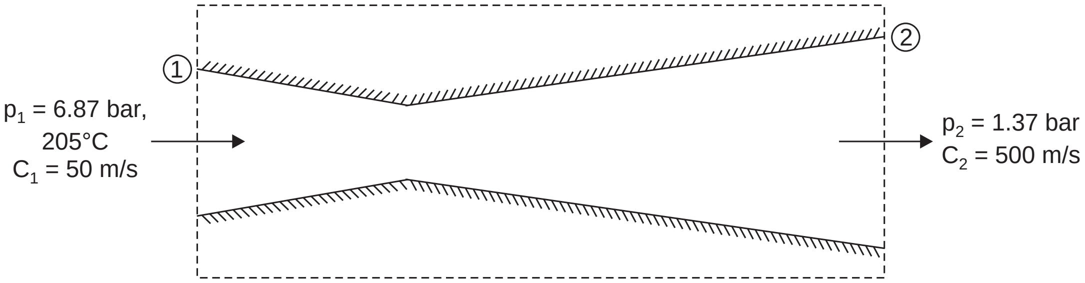


Fig. 4.47

The steady flow energy equation is given by

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \qquad \text{...}(i)$$

Considering the nozzle as an open system, it is evident that :

— there is *no work transfer* across the boundary of the system (*i.e.,* $W = 0$)

— there is *no heat transfer* because the nozzle is insulated (*i.e.,* $Q = 0$).

— the change in potential energy is negligible since there is no significant difference in elevation between the entrance and exit of the nozzle [*i.e.* $(Z_2 - Z_1)\, g = 0$].

Thus eqn. (*i*) reduces to

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$\therefore \qquad (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} = 0$$

From steam table corresponding to 6.87 bar, $h_1 = 2850$ kJ/kg

$$\therefore \qquad (h_2 - 2850) + \frac{(500)^2 - (50)^2}{2 \times 1000} = 0$$

or $h_2 - 2850 + 123.75 = 0$ or $h_2 = 2726.25$ kJ

Hence final enthalpy of steam = 2726.25 kJ. (Ans.)

Example 4.41. *The working fluid, in a steady flow process flows at a rate of 220 kg/min. The fluid rejects 100 kJ/s passing through the system. The conditions of the fluid at inlet and outlet are given as :* $C_1 = 320$ *m/s,* $p_1 = 6.0$ *bar,* $u_1 = 2000$ *kJ/kg,* $v_1 = 0.36$ *m*^{3}*/kg and* $C_2 = 140$ *m/s,* $p_2 = 1.2$ *bar,* $u_2 = 1400$ *kJ/kg,* $v_2 = 1.3$ *m*^{3}*/kg. The suffix 1 indicates the condition at inlet and 2 indicates at outlet of the system.*

Determine the power capacity of the system in MW.

The change in potential energy may be neglected.

Solution. Refer Fig. 4.48.

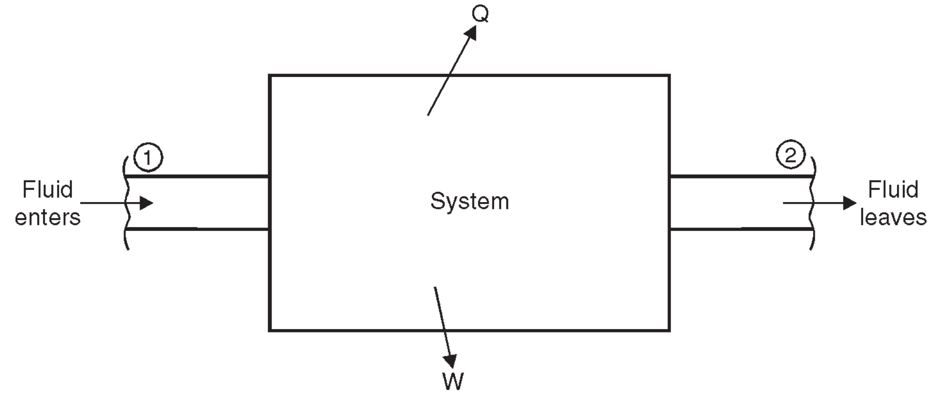


Fig. 4.48

Conditions of the fluid at point 1 :

Velocity, $C_1 = 320$ m/s
Pressure, $p_1 = 6.0$ bar $= 6 \times 10^5$ N/m^2
Internal energy, $u_1 = 2000$ kJ/kg
Specific volume, $v_1 = 0.36$ m^3/kg.

Conditions of the fluid at point 2 :

Velocity, $C_2 = 140$ m/s
Pressure, $p_2 = 1.2$ bar $= 1.2 \times 10^5$ N/m^2
Internal energy, $u_2 = 1400$ kJ/kg
Specific volume, $v_2 = 1.3$ m^3/kg
Heat rejected by the fluid, $Q = 100$ kJ/s (–).

Power capacity of the system :

Applying the energy equation at '1' and '2', we get

$$m\left[u_1 + p_1v_1 + \frac{C_1^2}{2} + Z_1g\right] \pm Q = m\left[u_1 + p_2v_2 + \frac{C_2^2}{2} + Z_2g\right] \pm W$$

Taking –ve sign for Q as the system *rejects* heat and +ve sign for W as the system *develops work*.

$$m\left[u_1 + p_1v_1 + \frac{C_1^2}{2} + Z_1g\right] - Q = m\left[u_2 + p_2v_2 + \frac{C_2^2}{2} + Z_2g\right] + W$$

$$\therefore \quad W = m\left[(u_1 - u_2) + (p_1v_1 - p_2v_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)\right] - Q\,. \qquad [\because \ (Z_1 - Z_2)\,g = 0]$$

In the above equation :

— the mass flow is in kg/s

— velocity in m/s

— internal energy in J/kg

— pressure in N/m^2

— specific volume m^3/kg

— the value of Q is in J/s

Then the unit of W will be J/s.

$$\therefore \qquad W = \frac{220}{60}\left[(2000 - 1400) \times 10^3 + 10^5\,(6 \times 0.36 - 1.2 \times 1.3) + \left(\frac{320^2 - 140^2}{2}\right)\right] - 100 \times 10^3$$

$$= \frac{220}{60}\,[600 \times 10^3 + 10^5 \times 0.6 + 41.4 \times 10^3] - 100 \times 10^3$$

$$= \frac{220}{60}\,[600 \times 10^3 + 60 \times 10^3 + 41.4 \times 10^3] - 100 \times 10^3$$

$$= 10^3 \times 2471.8 \text{ J/s} \qquad [\because \; 1 \text{ kJ} = 10^3 \text{ J}]$$

$$= 2471.8 \text{ kJ/s or kW} = 2.4718 \text{ MW}$$

Hence **power capacity of the system = 2.4718 MW. (Ans.)**

☞**Example 4.42.** *A stream of gases at 7.5 bar, 750°C and 140 m/s is passed through a turbine of a jet engine. The stream comes out of the turbine at 2.0 bar, 550°C and 280 m/s. The process may be assumed adiabatic. The enthalpies of gas at the entry and exit of the turbine are 950 kJ/kg and 650 kJ/kg of gas respectively.*

Determine the capacity of the turbine if the gas flow is 5 kg/s.

Solution. Refer Fig. 4.49.

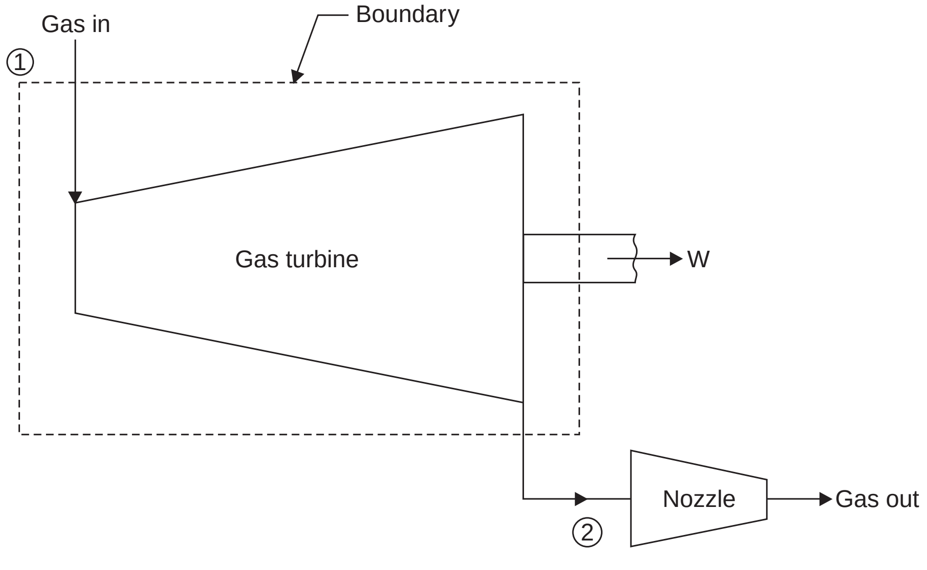


Fig. 4.49

Conditions at '1' :

Pressure, p_1 = 7.5 bar = 7.5 × 10^5 N/m^2, 750°C

Velocity, C_1 = 140 m/s

Enthalpy, h_1 = 950 kJ/kg

Conditions at '2' :

Pressure, p_2 = 2.0 bar = 2 × 10^5 N/m^2, 550°C

Velocity, C_2 = 280 m/s

Enthalpy, h_2 = 650 kJ/kg

Gas flow, m = 5 kg/s.

Capacity of the turbine :

Considering the flow of gas as 1 kg and neglecting the change in potential energy, we can write the steady flow energy equation for the turbine as

$$h_1 + \frac{C_1^2}{2} \pm Q = h_2 + \frac{C_2^2}{2} \pm W$$

Q = 0 as the system is adiabatic and W should be taken as +ve since it develops work.

$$\therefore \qquad h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} + W$$

$$\therefore \qquad W = (h_1 - h_2) + \frac{C_1^2 - C_2^2}{2} = 10^3\,(950 - 650) + \frac{140^2 - 280^2}{2}$$

$$= 10^3 \times 300 - 29.4 \times 10^3$$

$$= 270.6 \times 10^3 \text{ J/kg} = 270.6 \text{ kJ/kg.}$$

Power capacity of the turbine

$= \dot{m}W$ = 5 × 270.6 = 1353 kJ/s

= **1353 kW. (Ans.)**

☞**Example 4.43.** *12 kg of air per minute is delivered by a centrifugal air compressor. The inlet and outlet conditions of air are C_1 = 12 m/s, p_1 = 1 bar, v_1 = 0.5 m^3/kg and C_2 = 90 m/s, p_2 = 8 bar, v_2 = 0.14 m^3/kg. The increase in enthalpy of air passing through the compressor is 150 kJ/kg and heat loss to the surroundings is 700 kJ/min.*

Find : (i) Motor power required to drive the compressor ;

(ii) Ratio of inlet to outlet pipe diameter.

Assume that inlet and discharge lines are at the same level.

Solution. Quantity of air delivered by the compressor, $m = \frac{12}{60}$ = 0.2 kg/s.

Conditions of air at the inlet 1 :

Velocity, C_1 = 12 m/s

Pressure, p_1 = 1 bar = 1 × 10^5 N/m^2

Specific volume, v_1 = 0.5 m^3/kg

Conditions of air at the outlet 2 :

Velocity, C_2 = 90 m/s

Pressure, p_2 = 8 bar = 8 × 10^5 N/m^2

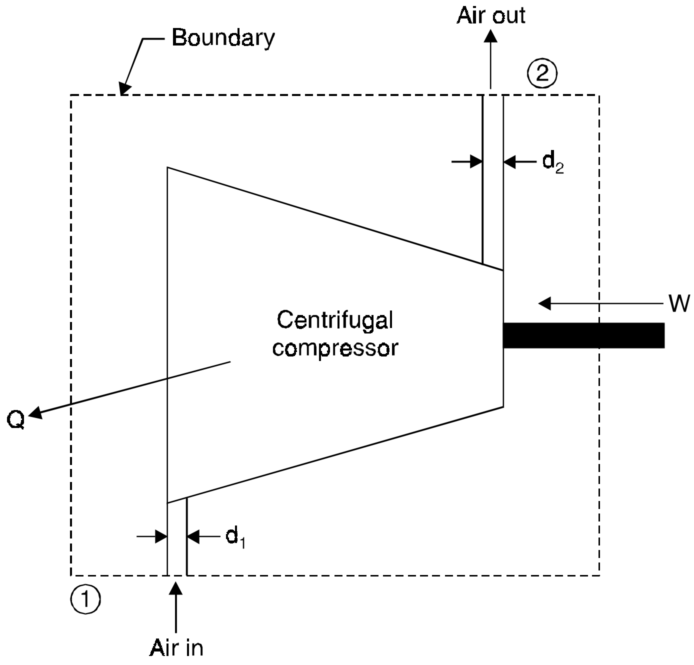


Fig. 4.50

Specific volume, $v_2 = 0.14\ \text{m}^3/\text{kg}$

Increase in enthalpy of air passing through the compressor,

$$(h_2 - h_1) = 150\ \text{kJ/kg}$$

Heat lost to the surroundings,

$$Q = -\ 700\ \text{kJ/min} = -\ 11.67\ \text{kJ/s}.$$

(*i*) **Motor power required to drive the compressor :**

Applying energy equation to the system,

$$m\left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) + Q = m\left(h_2 + \frac{C_2^2}{2} + Z_2 g\right) + W$$

Now $Z_1 = Z_2$ (given)

$$\therefore \quad m\left(h_1 + \frac{C_1^2}{2}\right) + Q = m\left(h_2 + \frac{C_2^2}{2}\right) + W$$

$$W = m\left[(h_1 - h_2) + \frac{C_1^2 - C_2^2}{2}\right] + Q$$

$$= 0.2\left[-\ 150 + \frac{12^2 - 90^2}{2 \times 1000}\right] + (-\ 11.67)$$

$$= -\ 42.46\ \text{kJ/s} = -\ 42.46\ \text{kW}$$

∴ **Motor power required (or work done on the air) = 42.46 kW. (Ans.)**

(*ii*) **Ratio of inlet to outlet pipe diameter, $\mathbf{\frac{d_1}{d_2}}$:**

The mass of air passing through the compressor is given by

$$m = \frac{A_1C_1}{v_1} = \frac{A_2C_2}{v_2}$$

$$\therefore \qquad \frac{A_1}{A_2} = \frac{C_2}{C_1} \times \frac{v_1}{v_2} = \frac{90}{12} \times \frac{0.5}{0.14} = 26.78$$

$$\therefore \qquad \left(\frac{d_1}{d_2}\right)^2 = 26.78 \quad \text{or} \quad \frac{d_1}{d_2} = 5.175$$

Hence ratio of inlet to outlet pipe diameter = 5.175. (Ans.)

Example 4.44. *In a test of water cooled air compressor, it is found that the shaft work required to drive the compressor is 175 kJ/kg of air delivered and the enthalpy of air leaving is 70 kJ/kg greater than that entering and that the increase in enthalpy of circulating water is 92 kJ/kg.*

Compute the amount of heat transfer to the atmosphere from the compressor per kg of air.

Solution. Refer Fig. 4.51.

Shaft work required to drive the compressor, $W = -\ 175$ kJ/kg

Increase in enthalpy of air passing through the compressor, $(h_2 - h_1) = 70$ kJ/kg

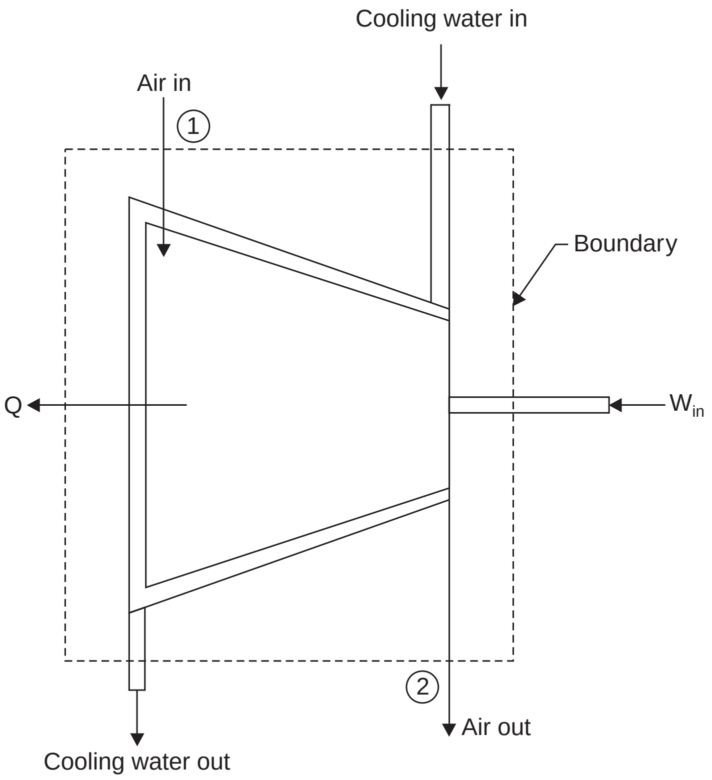


Fig. 4.51

Increase in enthalpy of circulating water, $Q_{\text{water}} = -92$ kJ/kg

Amount of heat transferred to atmosphere, $Q_{\text{atm.}} = ?$

Applying steady-flow energy equation at '1' and '2', we get

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

or
$$Q = (h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2}\right) + (Z_2 - Z_1)g + W$$

Assuming change in P.E. and K.E. to be negligible.

$\therefore$ $Q = (h_2 - h_1) + W = 70 + (-175) = -105$ kJ

But $Q = Q_{\text{atm}} + Q_{\text{water}}$ or $-105 = Q_{\text{atm}} + (-92)$

$\therefore$ $Q_{\text{atm}} = -13$ kJ/kg.

Thus heat transferred to atmosphere = 13 kJ/kg. (Ans.)

☞**Example 4.45.** *At the inlet to a certain nozzle the enthalpy of fluid passing is 2800 kJ/kg, and the velocity is 50 m/s. At the discharge end the enthalpy is 2600 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.*

(*i*) *Find the velocity at exit of the nozzle.*

(*ii*) *If the inlet area is 900 cm^2 and the specific volume at inlet is 0.187 m^3/kg, find the mass flow rate.*

(*iii*) *If the specific volume at the nozzle exit is 0.498 m^3/kg, find the exit area of nozzle.*

Solution. Refer Fig. 4.52.

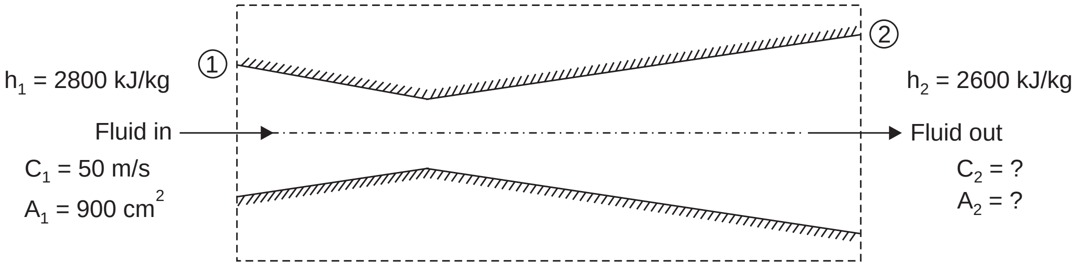


Fig. 4.52

Conditions of fluid at inlet (1) :

Enthalpy, $h_1 = 2800$ kJ/kg

Velocity, $C_1 = 50$ m/s

Area, $A_1 = 900 \text{ cm}^2 = 900 \times 10^{-4} \text{ m}^2$

Specific volume, $v_1 = 0.187$ m^3/kg

Conditions of fluid at exit (2) :

Enthalpy, $h_2 = 2600$ kJ/kg

Specific volume, $v_2 = 0.498$ m^3/kJ

Area, $A_2 = ?$

Mass flow rate, $\dot{m} = ?$

(*i*) **Velocity at exit of the nozzle, $\mathbf{C_2}$:**

Applying energy equation at '1' and '2', we get

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

Here $Q = 0,\ W = 0,\ Z_1 = Z_2$

$$\therefore \quad h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$\frac{C_2^2}{2} = (h_1 - h_2) + \frac{C_1^2}{2}$$

$$= (2800 - 2600) \times 1000 + \frac{50^2}{2} = 201250 \text{ N-m}$$

$$\therefore \quad C_2^2 = 402500$$

$$\therefore \quad \mathbf{C_2 = 634.4\ m/s.} \quad \textbf{(Ans.)}$$

(*ii*) **Mass flow rate $\dot{\mathbf{m}}$:**

By continuity equation,

$$\dot{m} = \frac{AC}{v} = \frac{A_1 C_1}{v_1} = \frac{900 \times 10^{-4} \times 50}{0.187} \text{ kg/s} = 24.06 \text{ kg/s}$$

∴ **Mass flow rate = 24.06 kg/s. (Ans.)**

(*iii*) **Area at the exit, $\mathbf{A_2}$:**

Now, $$\dot{m} = \frac{A_2\, C_2}{v_2}$$

$$24.06 = \frac{A_2 \times 634.4}{0.498}$$

$$\therefore \quad A_2 = \frac{24.06 \times 0.498}{634.4} = 0.018887 \text{ m}^2 = 188.87 \text{ cm}^2$$

Hence, **area at the exit = 188.87 cm^2. (Ans.)**

Example 4.46. *In one of the sections of the heating plant in which there are no pumps enters a steady flow of water at a temperature of 50°C and a pressure of 3 bar (h = 240 kJ/kg). The water leaves the section at a temperature of 35°C and at a pressure of 2.5 bar (h = 192 kJ/kg). The exit pipe is 20 m above the entry pipe.*

Assuming change in kinetic energy to be negligible, evaluate the heat transfer from the water per kg of water flowing.

Solution. Refer Fig. 4.53.

Enthalpy at '1', $h_1 = 240$ kJ/kg

Enthalpy at '2', $h_2 = 192$ kJ/kg

$Z_2 - Z_1 = 20$ m

Applying steady flow energy equation,

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

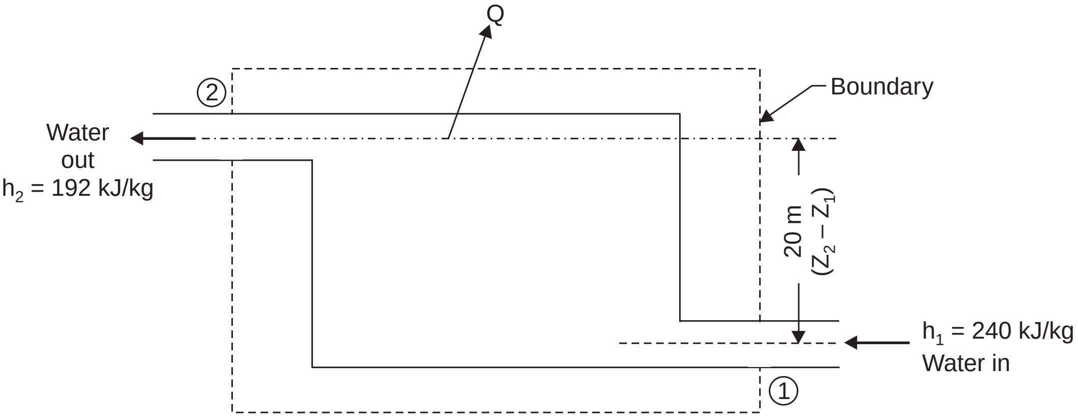


Fig. 4.53

$$Q = (h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2}\right) + (Z_2 - Z_1)\,g + W$$

Here $W = 0$ (no pumps)

$$\frac{C_2^2 - C_1^2}{2} = 0 \qquad \text{(given)}$$

$$\therefore \qquad Q = (192 - 240) + \frac{20 \times 9.81}{1000} = -\,47.8 \text{ kJ/kg}$$

∴ **Heat transfer from water/kg = 47.8 kJ/kg. (Ans.)**

Example 4.47. *The gas leaving the turbine of a gas turbine jet engine flows steadily into the engine jet pipe at a temperature of 900°C, a pressure of 2 bar and a velocity of 300 m/s relative to the pipe. Gas leaves the pipe at a temperature of 820°C and a pressure of 1.1 bar. Heat transfer from the gas is negligible. Using the following data evaluate the relative velocity of gas leaving the jet pipe. For the gas at t = 820°C, h = 800 kJ/kg and at 910°C, 915 kJ/kg.*

Solution. Pressure at entry to the engine jet pipe, $p_1 = 2$ bar

Velocity relative to the pipe, $C_1 = 300$ m/s

Heat transfer from gas, $Q = 0$

At temperature, $t_1 = 910°\text{C}$, $h_1 = 915$ kJ/kg

At temperature, $t_2 = 820°\text{C}$, $h_2 = 800$ kJ/kg

Relative velocity of gas leaving the jet pipe, C_2 :

Steady flow energy equation is given by :

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

$$Q = 0$$

$$W = 0$$

$$Z_1 = Z_2 \text{ (assumed)}$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$\frac{C_2^{\,2}}{2} = (h_1 - h_2) + \frac{C_1^{\,2}}{2} = (915 - 800) \times 1000 + \frac{300^2}{2}$$

$\therefore$ $C_2^{\,2} = 320000$ or $C_2 = 565.7$ m/s.

Hence relative velocity of gas leaving the jet pipe = 565.7 m/s. (Ans.)

Example 4.48. *A centrifugal pump delivers 50 kg of water per second. The inlet and outlet pressures are 1 bar and 4.2 bar respectively. The suction is 2.2 m below the centre of the pump and delivery is 8.5 m above the centre of the pump. The suction and delivery pipe diameters are 20 cm and 10 cm respectively.*

Determine the capacity of the electric motor to run the pump.

Solution. Refer Fig. 4.54.

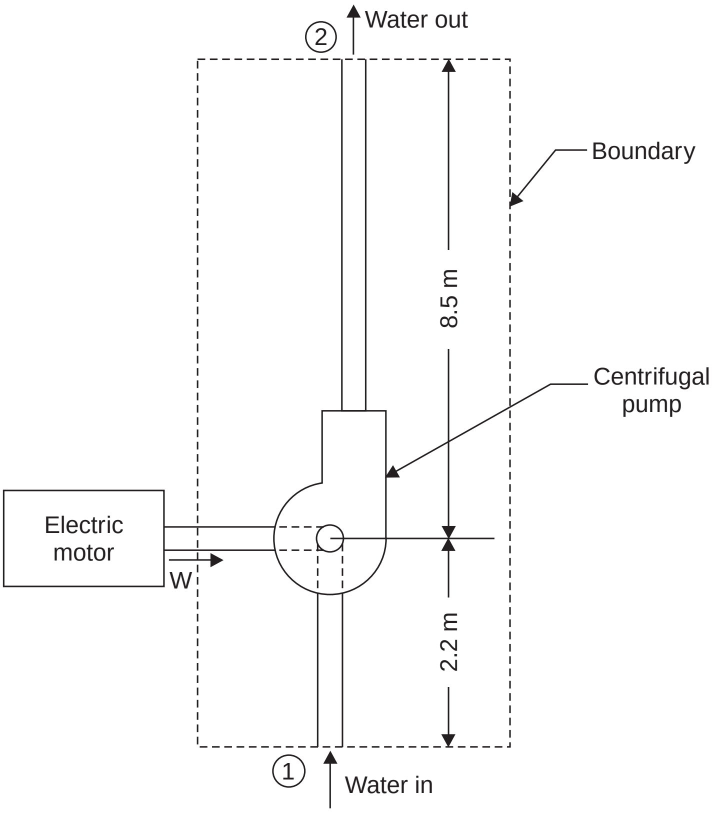


Fig. 4.54

Quantity of water delivered by the pump, $m_w = 50$ kg/s

Inlet pressure, $p_1 = 1$ bar $= 1 \times 10^5$ N/m^2

Outlet pressure, $p_2 = 4.2$ bar $= 4.2 \times 10^5$ N/m^2

Suction-below the centre of the pump = 2.2 m

Delivery-above the centre of the pump = 8.5 m

Diameter of suction pipe, $d_1 = 20$ cm $= 0.2$ m

Diameter of delivery pipe, $d_2 = 10$ cm $= 0.1$ m

Capacity of electric motor :

Steady flow energy equation is given by

$$m_w\left(u_1 + p_1v_1 + \frac{C_1^2}{2} + Z_1g\right) + Q = m_w\left(u_2 + p_2v_2 + \frac{C_2^2}{2} + Z_2g\right) + W \quad ...(i)$$

Considering the datum from suction 1, as shown

$Z_1 = 0,\ Z_2 = 8.5 + 2.2 = 10.7$ m

$u_2 - u_1 = 0\ ;\ Q = 0$

Thus eqn. (*i*) reduces to

$$W = m_w\left[(p_1v_1 - p_2v_2) + (Z_1 - Z_2)g + \left(\frac{C_1^2 - C_2^2}{2}\right)\right] \quad ...(ii)$$

As water is incompressible fluid.

$$\therefore \quad v_2 = v_1 = v = \frac{1}{\rho} = \frac{1}{1000}$$

The mass flow through inlet and exit pipe is given by

$$m_w = \frac{\pi}{4} \times d_1^2 \times C_1 \times \rho = \frac{\pi}{4} \times d_2^2 \times C_2 \times \rho \text{ as } \rho_1 = \rho_2 = \rho \text{ (for water)}$$

$$\therefore \quad 50 = \frac{\pi}{4} \times (0.2)^2 \times C_1 \times 1000$$

$$\therefore \quad C_1 = \frac{50 \times 4}{\pi \times (0.2)^2 \times 1000} = 1.59 \text{ m/s}$$

and

$$C_2 = \frac{50 \times 4}{\pi \times (0.1)^2 \times 1000} = 6.37 \text{ m/s}$$

Substituting the values in eqn. (*ii*), we get

$$W = 50\left[\left(1 \times 10^5 \times \frac{1}{1000} - 4.2 \times 10^5 \times \frac{1}{1000}\right) + (0 - 10.7) \times 9.81 + \left(\frac{1.59^2 - 6.37^2}{2}\right)\right]$$

$$= 50[-320 - 104.96 - 19.02]$$

$$= 22199 \text{ J/s or } 22.199 \text{ kJ/s} \simeq 22.2 \text{ kW.}$$

Hence capacity of electric motor **= 22.2 kW. (Ans.)**

Example 4.49. *During flight, the air speed of a turbojet engine is 250 m/s. Ambient air temperature is – 14°C. Gas temperature at outlet of nozzle is 610°C. Corresponding enthalpy values for air and gas are respectively 250 and 900 kJ/kg. Fuel air ratio is 0.0180. Chemical energy of fuel is 45 MJ/kg. Owing to incomplete combustion 6% of chemical energy is not released in the reaction. Heat loss from the engine is 21 kJ/kg of air.*

Calculate the velocity of the exhaust jet.

Solution. Refer Fig. 4.55.

Air speed of turbojet engine, $C_a = 250$ m/s

Ambient air temperature $= -14$°C

Gas temperature at outlet of nozzle $= 610$°C

Enthalpy of air, $h_a = 250$ kJ/kg

Enthalpy of gas, $h_g = 900$ kJ/kg

Fuel air ratio = 0.0180

$$\begin{bmatrix}\text{If, mass of air, } m_a = 1\,\text{kg, then mass of fuel, } m_f = 0.018\,\text{kg}\\ \text{and mass of gas} = 1 + .018 = 1.018\,\text{kg}\end{bmatrix}$$

Chemical energy of the fuel = 45 MJ/kg.

Heat loss from the engine, Q = 21 kJ/kg of air

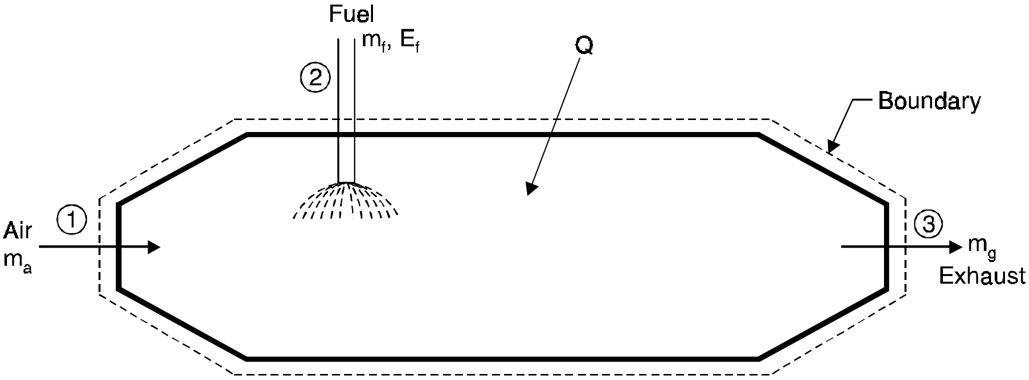


Fig. 4.55

Velocity of the exhaust gas jet, C_g :

Energy equation for turbojet engine is given by,

$$m_a\left(h_a + \frac{{C_a}^2}{2}\right) + m_f E_f + Q = m_g\left(h_g + \frac{{C_g}^2}{2} + E_g\right)$$

$$1\left(250 + \frac{250^2}{2\times 1000}\right) + 0.018 \times 45 \times 10^3 + (-21)$$

$$= 1.018\left[900 + \frac{{C_g}^2}{2\times 1000} + 0.06 \times \frac{0.018}{1.018} \times 45 \times 10^3\right]$$

$$281.25 + 810 - 21 = 1.018\left(900 + \frac{{C_g}^2}{2000} + 47.74\right)$$

$$1070.25 = 1.018\left(947.74 + \frac{{C_g}^2}{2000}\right)$$

$\therefore$ C_g = 455.16 m/s

Hence, **velocity of exhaust gas jet = 455.16 m/s. (Ans.)**

☞**Example 4.50.** *Air at a temperature of 20°C passes through a heat exchanger at a velocity of 40 m/s where its temperature is raised to 820°C. It then enters a turbine with same velocity of 40 m/s and expands till the temperature falls to 620°C. On leaving the turbine, the air is taken at a velocity of 55 m/s to a nozzle where it expands until the temperature has fallen to 510°C. If the air flow rate is 2.5 kg/s, calculate :*

(*i*) *Rate of heat transfer to the air in the heat exchanger ;*

(*ii*) *The power output from the turbine assuming no heat loss ;*

(*iii*) *The velocity at exit from the nozzle, assuming no heat loss.*

Take the enthalpy of air as $h = c_p t$, *where* c_p *is the specific heat equal to 1.005 kJ/kg°C and t the temperature.*

Solution. Refer Fig. 4.56.

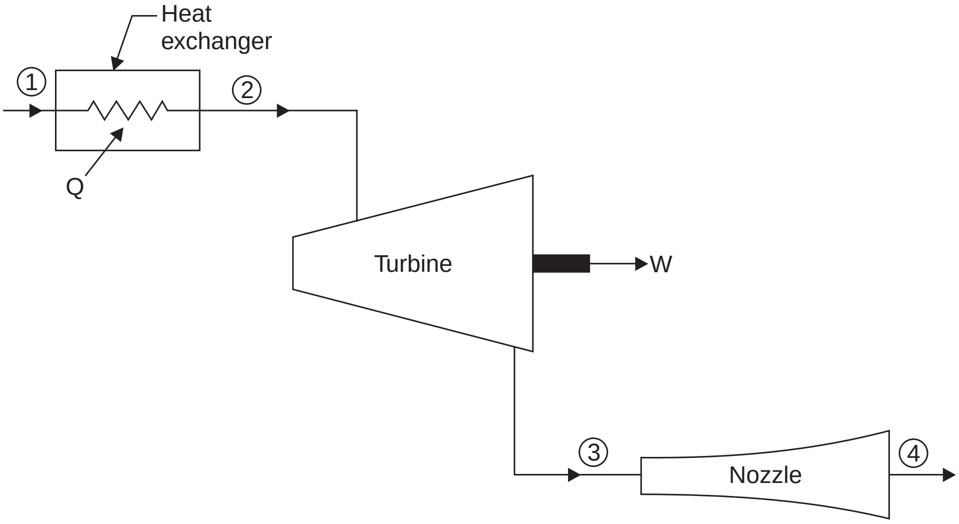


Fig. 4.56

Temperature of air, $t_1 = 20°\text{C}$

Velocity of air, $C_1 = 40$ m/s.

Temperature of air after passing the heat exchanger, $t_2 = 820°\text{C}$

Velocity of air at entry to the turbine, $C_2 = 40$ m/s

Temperature of air after leaving the turbine, $t_3 = 620°\text{C}$

Velocity of air at entry to nozzle, $C_3 = 55$ m/s

Temperature of air after expansion through the nozzle, $t_4 = 510°\text{C}$

Air flow rate, $\dot{m} = 2.5$ kg/s.

(*i*) **Heat exchanger :**

Rate of heat transfer :

Energy equation is given as,

$$m\left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) + Q_{1-2} = m\left(h_2 + \frac{C_2^2}{2} + Z_2 g\right) + W_{1-2}$$

Here, $Z_1 = Z_2,\ C_1,\ C_2 = 0,\ W_{1-2} = 0$

$\therefore \quad mh_1 + Q_{1-2} = mh_2$

or $\quad Q_{1-2} = m(h_2 - h_1)$

$\quad = mc_p\,(t_2 - t_1) = 2.5 \times 1.005\,(820 - 20) = 2010$ kJ/s.

Hence, **rate of heat transfer = 2010 kJ/s. (Ans.)**

(*ii*) **Turbine :**

Power output of turbine :

Energy equation for turbine gives

$$m\left(h_2 + \frac{C_2^2}{2}\right) = m\left(h_3 + \frac{C_3^2}{2}\right) + W_{2-3} \qquad [\because \; Q_{2-3} = 0,\ Z_1 = Z_2]$$

$$\therefore \qquad W_{2-3} = m\left(h_2 + \frac{C_2^2}{2}\right) - m\left(h_3 + \frac{C_3^2}{2}\right)$$

$$= m\left[(h_2 - h_3) + \left(\frac{C_2^2 - C_3^2}{2}\right)\right]$$

$$= m\left[c_p\,(t_2 - t_3) + \frac{C_2^2 - C_3^2}{2}\right]$$

$$= 2.5\left[1.005\,(820 - 620) + \frac{(40)^2 - (55)^2}{2 \times 1000}\right]$$

$$= 2.5\ [201 + 0.7125] = 504.3 \text{ kJ/s or } 504.3 \text{ kW}$$

Hence, **power output of turbine = 504.3 kW. (Ans.)**

(*iii*) **Nozzle :**

Velocity at exit from the nozzle :

Energy equation for nozzle gives,

$$h_3 + \frac{C_3^2}{2} = h_4 + \frac{C_4^2}{2} \qquad [\because \; W_{3-4} = 0,\ Q_{3-4} = 0,\ Z_1 = Z_2]$$

$$\frac{C_4^2}{2} = (h_3 - h_4) + \frac{C_3^2}{2} = c_p(t_3 - t_4) + \frac{C_3^2}{2}$$

$$= 1.005(620 - 510) + \frac{55^2}{2 \times 1000} = 112.062 \times 10^3 \text{ J}$$

$$\therefore \qquad C_4 = 473.4 \text{ m/s.}$$

Hence, **velocity at exit from the nozzle = 473.4 m/s. (Ans.)**

4.14. HEATING-COOLING AND EXPANSION OF VAPOURS

The basic energy equations for non-flow and flow processes are *also valid for vapours.*

∴ When $\Delta KE = 0$ and $\Delta PE = 0$

$dQ = du + p.dv$ ......for *non-flow process.*

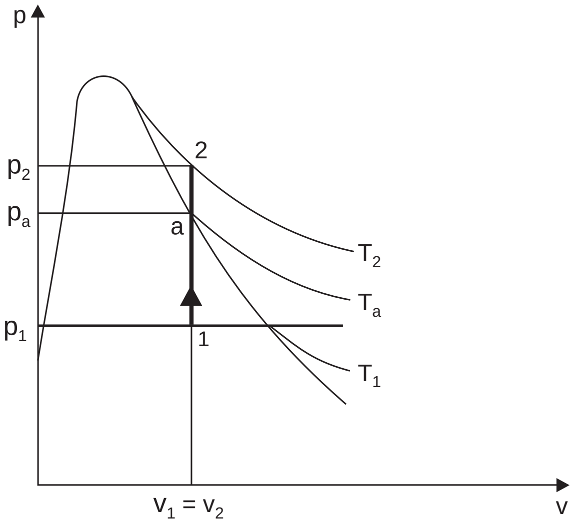
$dQ = dh - v.dp$ ......for *flow process.*

The various processes using vapour are discussed below :

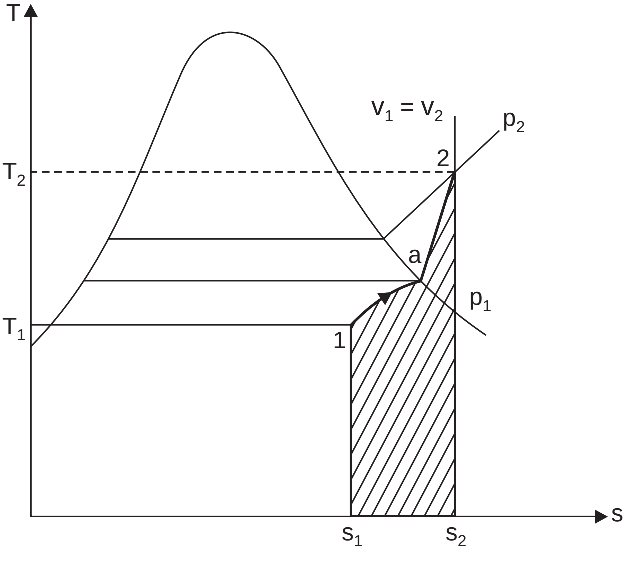
1. **Constant Volume Heating or Cooling.** The constant volume heating process is represented on *p-v*, *T-s* and *h-s* diagram as shown in Fig. 4.57 (*a*), (*b*), (*c*) respectively. It is assumed that the steam is in wet condition before heating at pressure p_1, becomes superheated after heating and pressure increases from p_1 to p_2.

Since the mass of steam, *m*, remains constant during the heating process,

$$\therefore \qquad m = \frac{V}{x_1 v_{g_1}} = \frac{V}{v_{sup_2}}$$, where *V* is the total constant volume of steam



(*a*)



(*b*)

Also $$\frac{v_{sup_2}}{v_{g_2}} = \frac{T_{sup_2}}{T_{s_2}}$$

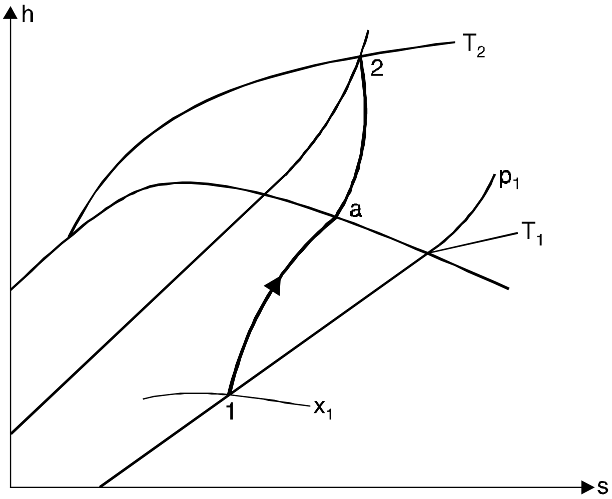
$$\therefore \quad \frac{T_{sup_2}}{T_{s_2}} = \frac{x_1 v_{g_1}}{v_{g_2}} \qquad \text{...(4.69)}$$

v_{g_2} and T_{s_2} can be found from the steam tables corresponding to pressure p_2 and then T_{sup_2} can be calculated by using the above equation. When the final condition is known, the change in all other properties can be found easily.

If after cooling the condition of steam remains wet, then the mass fraction be obtained as follows :

$$\frac{V}{x_1 v_{g_1}} = \frac{V}{x_2 v_{g_2}}$$

$$x_2 = \frac{x_1 v_{g_1}}{v_{g_2}} \qquad \text{...(4.70)}$$



(*c*)

Fig. 4.57. Constant volume process.

where v_{g_2} can be found from the steam tables corresponding to pressure p_2.

Applying the first law of thermodynamics, we have

$$Q = \Delta u + \int_1^2 p \, . \, dv = \Delta u \text{ as } \int_1^2 pdv = 0$$

$$= u_2 - u_1$$

i.e.,

$$Q_1 = [h_2 - p_2 v_{sup_2}] - [h_1 - p_1(x_1 v_{g_1})] \qquad \text{...(4.71)}$$

In case the condition of steam remains wet after heating, then

$$Q = (u_2 - u_1) = [h_2 - p_2(x_2 v_{g_2})] - [h_1 - p_1(x_1 v_{g_1})] \qquad \text{...(4.72)}$$

In the *cooling process,* the same equations are used except that the suffixes 1, 2 are interchanged.

☞**Example 4.51.** *A rigid cylinder of volume 0.028 m³ contains steam at 80 bar and 350°C. The cylinder is cooled until the pressure is 50 bar. Calculate :*

(*i*) *The state of steam after cooling ;*

(*ii*) *The amount of heat rejected by the steam.*

Solution. Volume of rigid cylinder = 0.028 m³

Pressure of steam before cooling, p_1 = 80 bar

Temperature of steam before cooling = 350°C

Pressure of steam after cooling, p_2 = 50 bar

Steam at 80 bar and 350°C is in a superheated state, and the specific volume from tables is 0.02995 m^3/kg. Hence the mass of steam in the cylinder is given by

$$m = \frac{0.028}{0.02995} = 0.935 \text{ kg}$$

Internal energy at state 1, (*80 bar, 350°C*),

$$u_1 = h_1 - p_1 v_1$$

$$= 2987.3 - \frac{80 \times 10^5 \times 0.02995}{10^3} \quad \text{or} \quad u_1 = 2747.7 \text{ kJ/kg.}$$

(*i*) **State of steam after cooling :**

At state 2, p_2 = 50 bar and v_2 = 0.02995 m^3/kg, therefore, steam is wet, and dryness fraction is given by,

$$x_2 = \frac{v_2}{v_{g_2}} = \frac{0.02995}{0.0394} = 0.76.$$

(*ii*) **Heat rejected by the steam :**

Internal energy at state 2 (50 bar),

$$u_2 = (1 - x_2)\ u_{f_2} + x_2 u_{g_2}$$

$$= (1 - 0.76) \times 1149 + 0.76 \times 2597 = 2249.48 \text{ kJ/kg}$$

At constant volume, $Q = U_2 - U_1 = m(u_2 - u_1)$

$$= 0.935(2249.48 - 2747.7) = -\ 465.5 \text{ kJ}$$

i.e., **Heat rejected** = **465.5 kJ. (Ans.)**

Fig. 4.58 shows the process drawn on *T-s* diagram, the shaded area representing the heat rejected by the system.

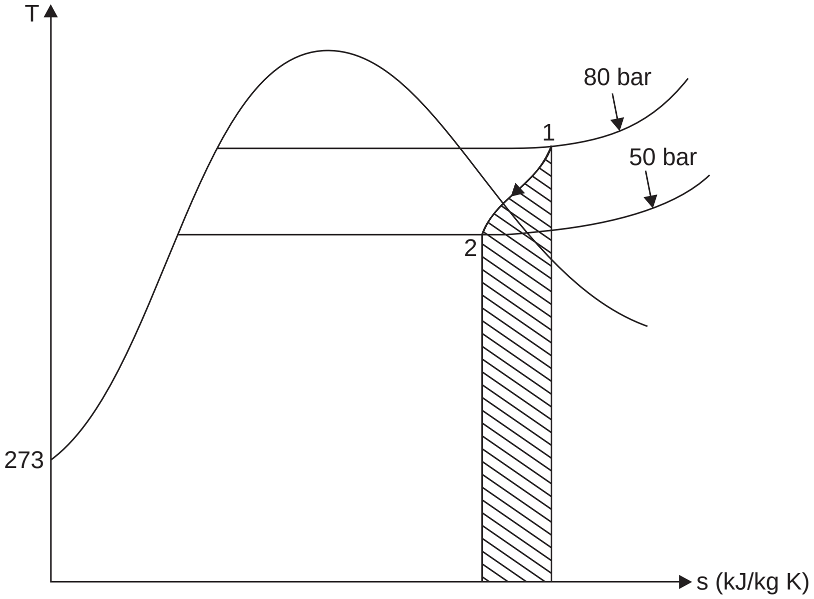


Fig. 4.58

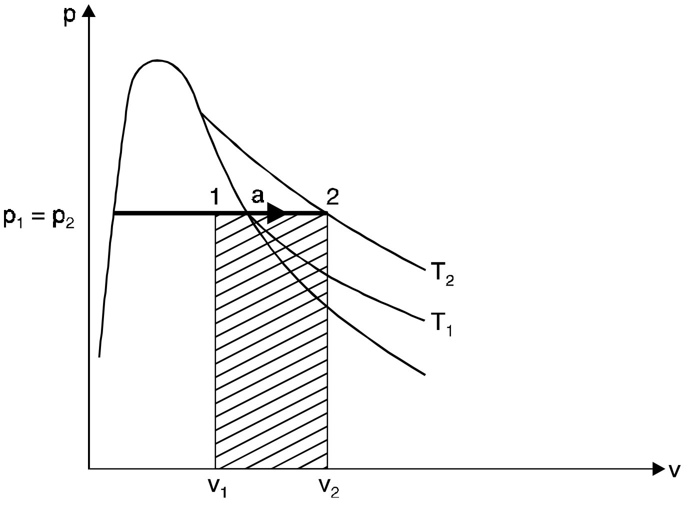
2. **Constant pressure Heating or Cooling.** Fig. 4.59 (*a*), (*b*) and (*c*) shows the constant pressure heating process on *p-v*, *T-s* and *h-s* diagrams respectively.

Generation of steam in the boilers is an example of constant pressure heating.

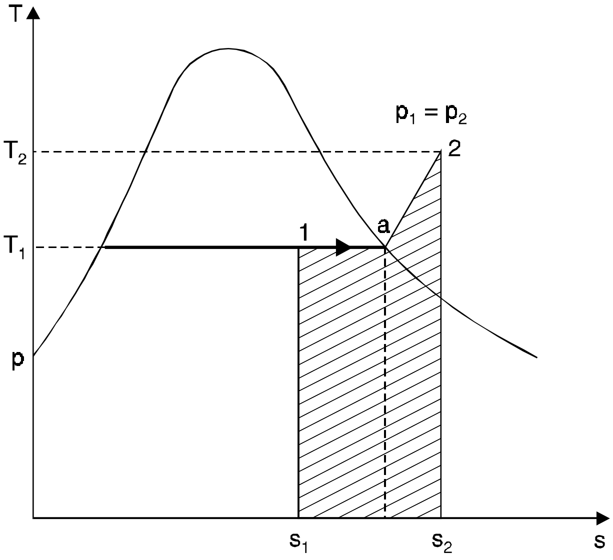
Applying first law of thermodynamics, we have

$$Q = \Delta u + \int_1^2 p\,.\,dv$$

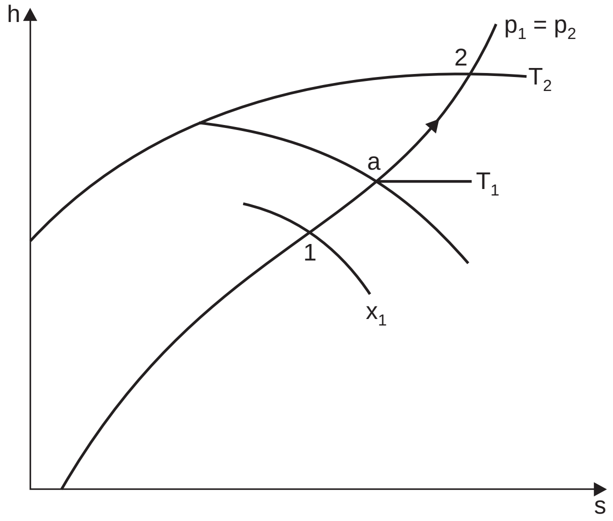
$$= (u_2 - u_1) + p(v_2 - v_1) = (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1$$



(*a*)



(*b*)



(c)

Fig. 4.59. Constant pressure process.

If the initial condition of steam is wet and final condition is superheated, then

$$Q = (u_2 + pv_{sup_2}) - (u_1 + p \, . \, x_1 \, v_{g_1})$$

$$= (h_2 - h_1) \quad ...(4.73)$$

here h_1 and h_2 are the actual enthalpies of steam per kg before and after heating.

The heat added during the constant pressure process is equal to the change in enthalpy of steam during the process. When the steam is wet before heating and becomes superheated after heating the work done,

$$W = p\,(v_{sup_2} - x_1 v_{g_1}) \quad ...(4.74)$$

Example 4.52. *0.08 kg of dry steam is heated at a constant pressure of 2 bar until the volume occupied is 0.10528 m³. Calculate :*

(*i*) *Heat supplied ;*

(*ii*) *Work done.*

Solution. Mass of steam, m = 0.08 kg

Pressure of steam, p = 2 bar

Volume occupied after heating = 0.10528 m³

Initially the steam is dry saturated at 2 bar, hence

$$h_1 = h_g \text{ (at 2 bar)} = 2706.3 \text{ kJ/kg}$$

Finally the steam is at 2 bar and the specific volume is given by

$$v_2 = \frac{0.10528}{0.08} = 1.316 \text{ m}^3\text{/kg}$$

Hence the steam is *superheated finally* (since the value of v_g at 2 bar = 0.885 m³/kg). From superheat tables at 2 bar and 1.316 m³/kg the temperature of steam is 300°C, and the enthalpy, h_2 = 3071.8 kJ/kg.

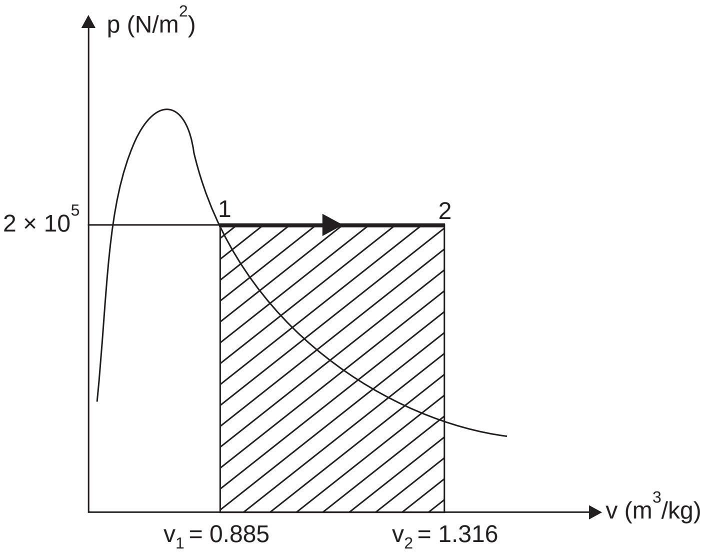


Fig. 4.60

(*i*) **Heat supplied :**

Heat supplied, $Q = H_2 - H_1 = m(h_2 - h_1)$

$= 0.08(3071.8 - 2706.3)$

$=$ **29.24 kJ. (Ans.)**

(*ii*) **Work done :**

The process is shown on a *p-v* diagram in Fig. 4.60. The work done is given by the shaded area

i.e., $W = p(v_2 - v_1)$ Nm/kg

Here $v_1 = v_g$ at 2 bar = 0.885 m^3/kg

and v_2 = 1.316 m^3/kg

∴ $W = 2 \times 10^5 (1.316 - 0.885) = 2 \times 10^5 \times 0.431$ Nm/kg

Now work done by the total mass of steam (0.08 kg) present

$= 0.08 \times 2 \times 10^5 \times 0.431 \times 10^{-3}$ kJ

$=$ **6.896 kJ. (Ans.)**

Example 4.53. *1 kg of steam at 8 bar, entropy 6.55 kJ/kg K, is heated reversibly at constant pressure until the temperature is 200°C. Calculate the heat supplied, and show on a T-s diagram the area which represents the heat flow.*

Solution. Mass of steam, m = 1 kg

Pressure of steam, p = 8 bar

Entropy of steam (at 8 bar), s = 6.55 kJ/kg K

Temperature after heating = 200°C

At 8 bar, s_g = 6.66 kJ/kg K, hence steam is *wet*, since the actual entropy, s, is less than s_g. To find the dryness fraction x_1, using the relation,

$$s_1 = s_{f_1} + x_1 s_{fg_1}$$

$$6.55 = 2.0457 + x_1 \times 4.6139$$

$$\therefore \qquad x_1 = \frac{6.55 - 2.0457}{4.6139} = 0.976$$

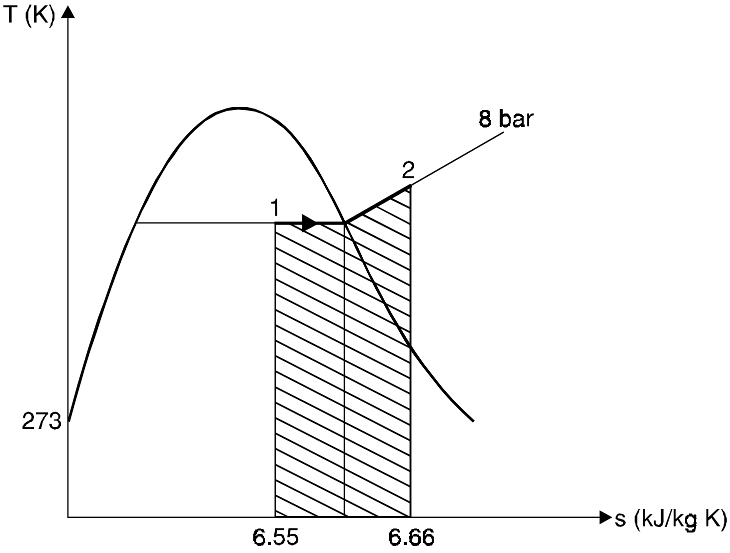


Fig. 4.61

Now, *initial enthalpy* (at 8 bar),

$$h_1 = h_{f_1} + x_1 h_{fg_1}$$

$$= 720.9 + 0.976 \times 2046.5 = 2718.28 \text{ kJ/kg}$$

Final enthalpy, h_2 : At state 2 the steam is at 200°C at 8 bar and is therefore, superheated.

From superheated tables, $h_2 = 2839.3$ kJ/kg

Now, $Q = h_2 - h_1 = 2839.3 - 2718.28 = 121.02$ kJ/kg

i.e., **Heat supplied = 121.02 kJ/kg. (Ans.)**

The *T-s* diagram showing the process is given in Fig. 4.61, the shaded area representing the heat flow.

3. Constant Temperature or Isothermal Expansion. Fig. 4.62 (*a*), (*b*) and (*c*) shows the constant temperature or isothermal expansion on *p-v, T-s* and *h-s* diagrams respectively.

In the wet region, the constant temperature process is also a constant pressure process during evaporation and as well as condensation. When the steam becomes saturated it behaves like a gas and constant temperature process in superheated region becomes hyperbolic (pv = constant).

When the wet steam is heated at constant temperature till it becomes *dry and saturated,* then the heat transfer (Q) is given by :

$$Q = h_2 - h_1$$

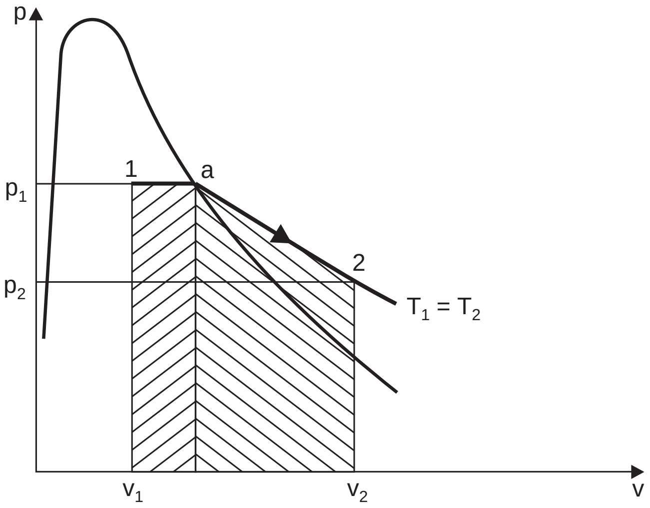
and work done, $$W = p_1 (v_{g_2} - x_1 v_{g_1})$$

$$= p v_{g_1} (1 - x_1)$$

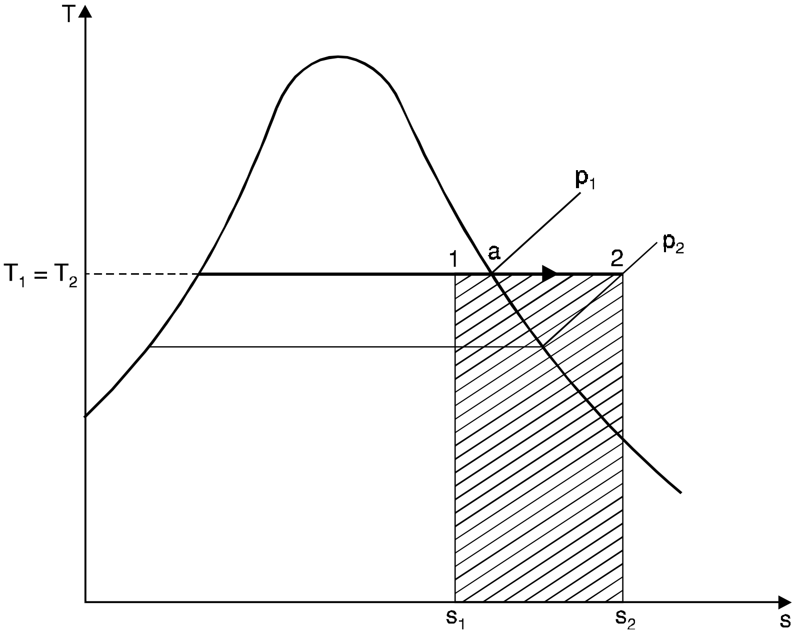
$$[\because \quad v_{g_2} = v_{g_1} \text{ as pressure remains constant during this process}]$$

This process is limited to *wet region only*.

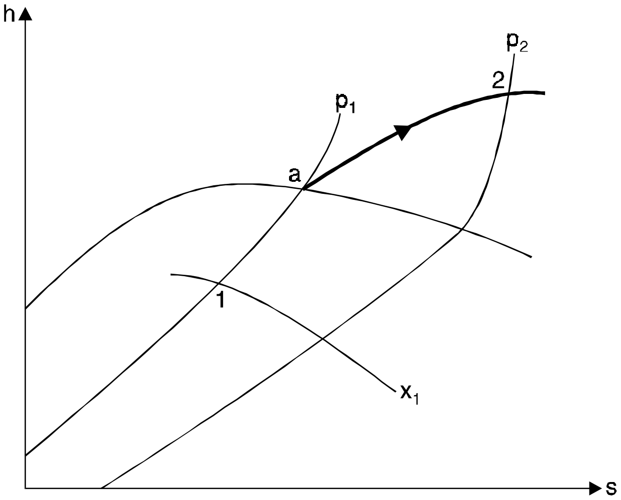
Hyperbolic process (pv = constant) is also an isothermal process in the superheat region as the steam behaves like a gas in this region. The work done during the hyperbolic expansion in a *non-flow system* is given by



(*a*)



(*b*)



(c)

Fig. 4.62. Constant temperature or isothermal expansion.

$$W = \int_1^2 p dv = \int_1^2 \frac{C}{v} dv = C \log_e \left(\frac{v_2}{v_1}\right)$$

$$= p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right) \qquad \text{...(4.75)}$$

where v_1 and v_2 are the specific volumes of steam before and after expansion.

Applying first law of energy equation,

$$Q = \Delta u + \int_1^2 p \cdot dv$$

$$= (u_2 - u_1) + p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$$

$$= (h_2 - p_2 v_2) - (h_1 - p_1 v_1) + p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$$

Since $p_1 v_1 = p_2 v_2$

$$\therefore \quad Q = (h_2 - h_1) + p_1 v_1 \log_e \frac{v_2}{v_1} \qquad \text{...(4.76)}$$

☞**Example 4.54.** *Steam at 7 bar and dryness fraction 0.95 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. The heat supplied during the process is found to be 420 kJ/kg. Calculate per kg :*

(*i*) *The change of internal energy ;* (*ii*) *The change of enthalpy ;*

(*iii*) *The work done.*

Solution. Initial pressure of steam, p_1 = 7 bar = 7 × 10^5 N/m^2

Final pressure of steam, p_2 = 1.5 bar = 1.5 ×10^5 N/m^2

Heat supplied during the process, Q = 420 kJ/kg.

The process is shown in Fig. 4.63. The saturation temperature corresponding to 7 bar is 165°C. Therefore, the steam is *superheated at the state 2.*

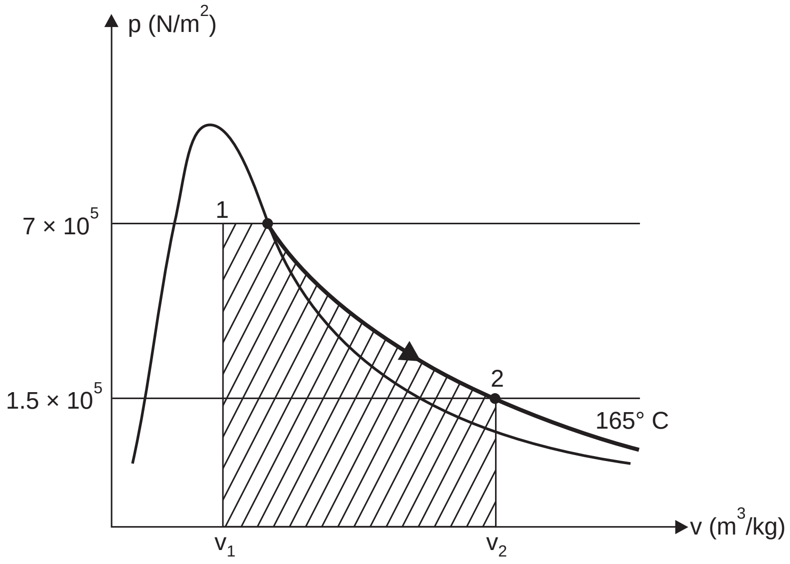


Fig. 4.63

(*i*) **Change of internal energy :**

The internal energy at state 1 is found by using the relation :

$$u_1 = (1 - x)\, u_f + xu_g$$
$$= (1 - 0.95)\, 696 + (0.95 \times 2573)$$
$$\therefore \quad u_1 = 2479.15 \text{ kJ/kg}$$

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$u_2 = 2580 + \frac{15}{50}\,(2856 - 2580)$$
$$= 2602.8 \text{ kJ/kg}$$

∴ **Gain in internal energy,**

$$u_2 - u_1 = 2602.8 - 2479.15 = \mathbf{123.65\ kJ/kg.} \quad \textbf{(Ans.)}$$

(*ii*) **Change of enthalpy :**

Enthalpy at state 1 (7 bar),

$$h_1 = h_{f_1} + x_1 h_{fg_1}$$

At 7 bar. $h_f = 697.1$ kJ/kg and $h_{fg} = 2064.9$ kJ/kg

$$\therefore \quad h_1 = 697.1 + 0.95 \times 2064.9 = 2658.75 \text{ kJ/kg}$$

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$h_2 = 2772.6 + \frac{15}{50}\,(2872.9 - 2772.6) = 2802.69 \text{ kJ/kg}$$

∴ Change of enthalpy

$$= h_2 - h_1 = 2802.69 - 2658.75 = \mathbf{143.94\ kJ/kg.} \quad \textbf{(Ans.)}$$

(*iii*) **Work done :**

From non-flow energy equation,

$$Q = (u_2 - u_1) + W$$

$$\therefore \quad W = Q - (u_2 - u_1) = 420 - 123.65 = 296.35 \text{ kJ/kg}$$

i.e., **Work done by the steam = 296.35 kJ/kg. (Ans.)**

Note. The work done is also given by the area on the Fig. 4.60 $\left(\int_{v_1}^{v_2} pdv\right)$, this can only be evaluated graphically.

Example 4.55. *In a steam engine cylinder the steam expands from 5.5 bar to 0.75 bar according to a hyperbolic law, pv = constant. If the steam is initially dry and saturated, calculate per kg of steam :*

(*i*) *Work done ;*

(*ii*) *Heat flow to or from the cylinder walls.*

Solution. Initial pressure of steam, p_1 = 5.5 bar = 5.5 × 10^5 N/m^2

Initial condition of steam, $x_1 = 1$

Final pressure of steam, p_2 = 0.75 bar = 0.75 bar × 10^5 N/m^2

At 5.5 bar, $v_1 = v_g = 0.3427$ m^3/kg

Also $p_1v_1 = p_2v_2$

$$\therefore \quad v_2 = \frac{p_1v_1}{p_2} = \frac{5.5 \times 0.3427}{0.75} = 2.513 \text{ m}^3/\text{kg}$$

At 0.75 bar, $v_g = 2.217$ m^3/kg.

Since $v_2 > v_g$ (at 0.75 bar), therefore, the steam is *superheated* at state 2.

Interpolating from superheat tables at 0.75 bar, we have

$$u_2 = 2510 + \left(\frac{2.513 - 2.271}{2.588 - 2.271}\right)(2585 - 2510)$$

$$= 2510 + \frac{0.242}{0.317} \times 75 = 2567.25 \text{ kJ/kg.}$$

For dry saturated steam at 5.5 bar

$$u_1 = u_g = 2565 \text{ kJ/kg}$$

Hence, gain in internal energy

$$= u_2 - u_1 = 2567.25 - 2565 = 2.25 \text{ kJ/kg}$$

The process is shown on a *p-v* diagram in Fig. 4.64, the shaded area representing the work done.

Now,
$$W = \int_{v_1}^{v_2} p\,dv$$

$$= \int_{v_1}^{v_2} \left(\frac{\text{constant}}{v}\right) dv \qquad \left[\because \ pv = \text{constant, and } p = \frac{\text{constant}}{v}\right]$$

$$= \text{constant}\left[\log_e v\right]_{v_1}^{v_2}$$

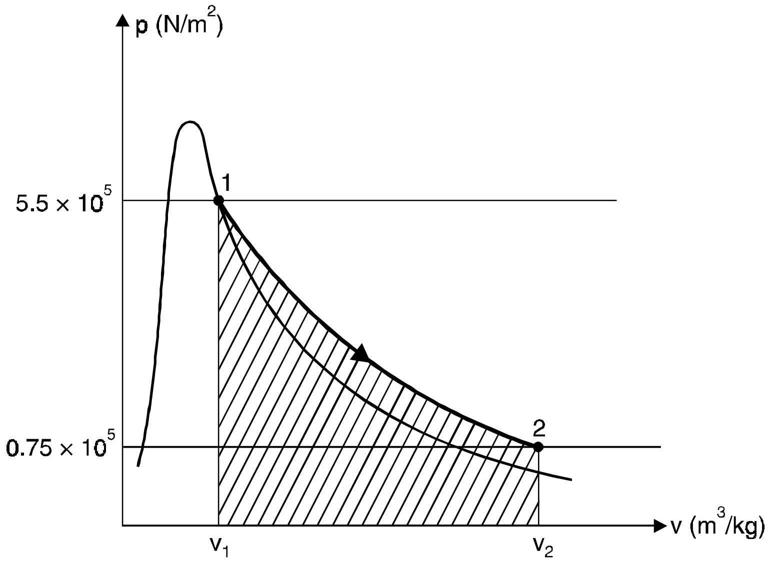


Fig. 4.64

The constant is either p_1v_1 or p_2v_2

i.e.,
$$W = 5.5 \times 10^5 \times 0.3427 \times \log_e \frac{p_1}{p_2} \qquad \left[\because \; p_1v_1 = p_2v_2 \;\; \text{or} \;\; \frac{v_2}{v_1} = \frac{p_1}{p_2}\right]$$

$$= 5.5 \times 10^5 \times 0.3427 \times \log_e \left(\frac{5.5}{0.75}\right) = 375543 \text{ N-m/kg.}$$

Using non-flow energy equation, we get

$$Q = (u_2 - u_1) + W$$

$$= 2.25 + \frac{375543}{10^3} = 378 \text{ kJ/kg}$$

i.e., **Heat supplied** **= 378 kJ/kg. (Ans.)**

Example 4.56. *Dry saturated steam at 100 bar expands isothermally and reversibly to a pressure of 10 bar. Calculate per kg of steam :*

(*i*) *The heat supplied ;*

(*ii*) *The work done.*

Solution. Initial pressure of steam, p_1 = 100 bar

Final pressure of steam, p_2 = 10 bar

The process is shown in Fig. 4.65, the shaded area representing the heat supplied.

At 100 bar, dry saturated : From steam tables,

$$s_1 = s_g = 5.619 \text{ kJ/kg K and } t_{s_1} = 311°\text{C}$$

At 10 bar and 311°C the steam is superheated, hence interpolating

$$s_2 = 7.124 + \left(\frac{311-300}{350-300}\right)(7.301 - 7.124) \qquad \text{or} \qquad s_2 = 7.163 \text{ kJ/kg K.}$$

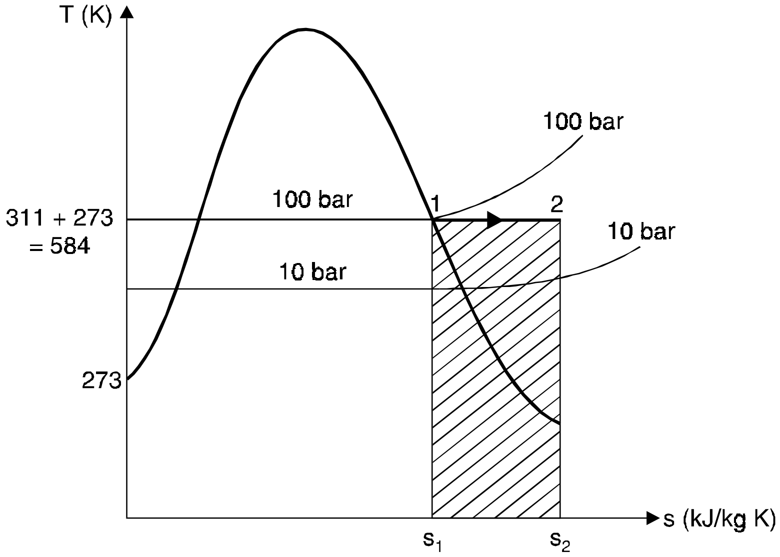


Fig. 4.65

(*i*) **Heat supplied :**

Now, heat supplied, Q = shaded area = $T(s_2 - s_1)$

$= 584(7.163 - 5.619) =$ **901.7 kJ/kg. (Ans.)**

(*ii*) **Work done :**

To find work done, applying non-flow energy equation,

$$Q = (u_2 - u_1) + W$$

or $$W = Q - (u_2 - u_1)$$

From steam tables at 100 bar, dry saturated,

$$u_1 = u_g = 2545 \text{ kJ/kg}$$

At 10 bar 311°C, interpolating,

$$u_2 = 2794 + \left(\frac{311 - 300}{350 - 300}\right)(2875 - 2794)$$

i.e., $$u_2 = 2811.8 \text{ kJ/kg}$$

Then, $$W = Q - (u_2 - u_1)$$

$$= 901.7 - (2811.8 - 2545) = 634.9 \text{ kJ/kg}$$

Hence, **work done by the steam = 634.9 kJ/kg. (Ans.)**

4. Reversible Adiabatic or Isentropic Process. Fig. 4.66 (*a*), (*b*) and (*c*) shows the isentropic process on *p-v*, *T-s* and *h-s* diagrams respectively.

Let us consider that the process is non-flow reversible adiabatic. Now applying first law energy equation, we have

$$Q = \Delta u + \int_1^2 p\,.dv = (u_2 - u_1) + W$$

As for adiabatic process, $Q = 0$

$$\therefore \quad W = (u_1 - u_2) \qquad ...(4.77)$$

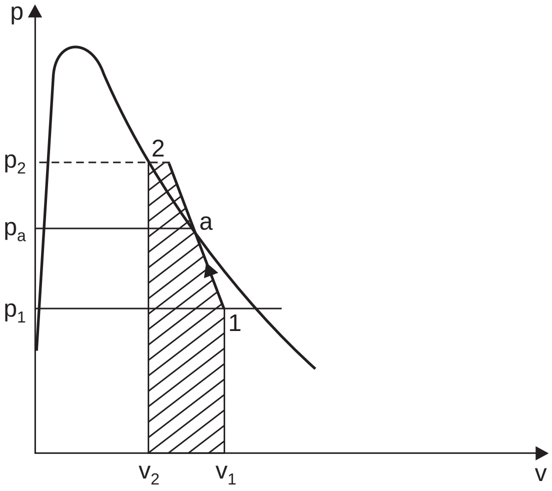
In case the process is steady flow reversible adiabatic, then first law energy equation can be written as

$$u_1 + p_1v_1 + Q = u_2 + p_2v_2 + W$$

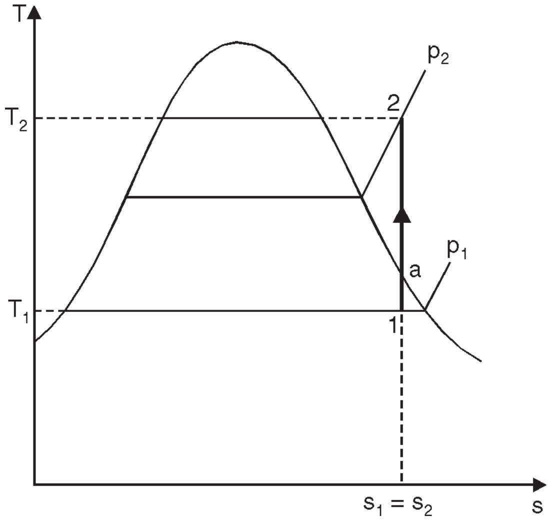
where v_1 and v_2 are the specific volumes of steam before and after executing the process.

$$\therefore \qquad h_1 + 0 = h_2 + W \qquad (\because \quad Q = 0)$$

$$\therefore \qquad W = (h_1 - h_2) \qquad ...(4.78)$$



(*a*)



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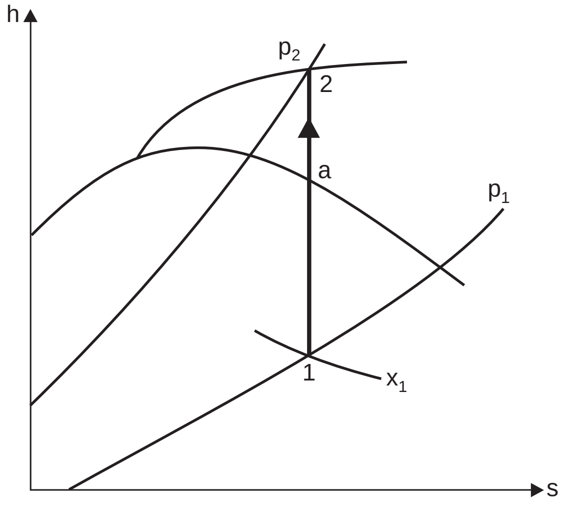


(*c*)

Fig. 4.66. Reversible adiabatic or isentropic process.

☞**Example 4.57.** *1 kg of steam at 120 bar and 400°C expands reversibly in a perfectly thermally insulated cylinder behind a piston until the pressure is 38 bar and the steam is then dry saturated. Calculate the work done by the steam.*

Solution. Mass of steam, $m = 1$ kg

Initial pressure of steam, $p_1 = 120 \text{ bar} = 120 \times 10^5 \text{ N/m}^2$

Initial temperature of steam, $t_1 = 400°\text{C}$.

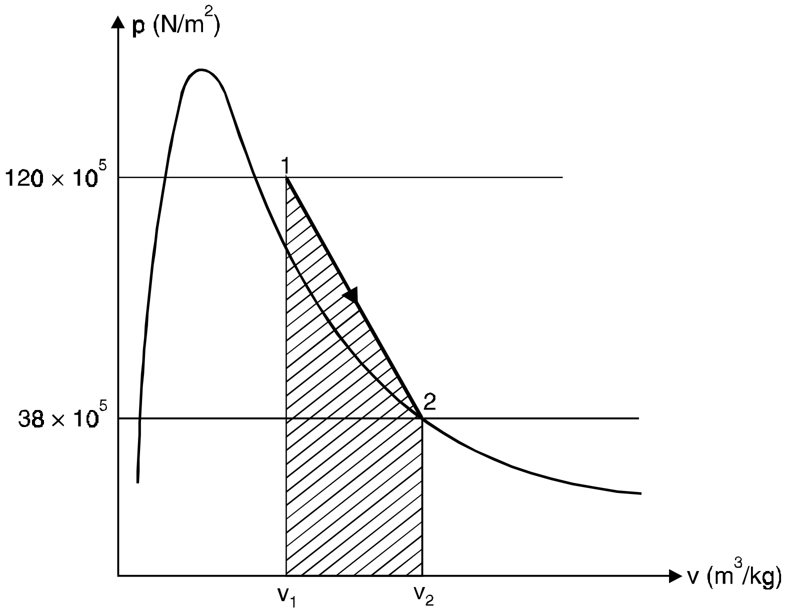


Fig. 4.67

Final pressure of steam, $p_2 = 38$ bar

From superheat tables, at *120 bar and 400°C*

$h_1 = 3051.3$ kJ/kg and $v_1 = 0.02108$ m^3/kg

Now, using the equation :

$$u = h - pv$$

$$\therefore \quad u_1 = 3051.3 - \frac{120 \times 10^5 \times 0.02108}{10^3} = 2798.34 \text{ kJ/kg}$$

Also, $u_1 = u_g$ at 38 bar = 2602 kJ/kg.

Since the cylinder is perfectly thermally insulated then no heat flows to or from the steam during the expansion, the process therefore is adiabatic.

$\therefore$ **Work done by the steam,** $W = u_1 - u_2$

$= 2798.34 - 2602 =$ **196.34 kJ/kg. (Ans.)**

The process is shown on *p-v* diagram in Fig. 4.67, the shaded area representing the work done.

5. Polytropic process. In this process, the steam follows the law pv^n = constant. This process on *p-v, T-s* and *h-s* diagrams is shown in Fig. 4.68 (*a*), (*b*) and (*c*).

The work done during this process is given by

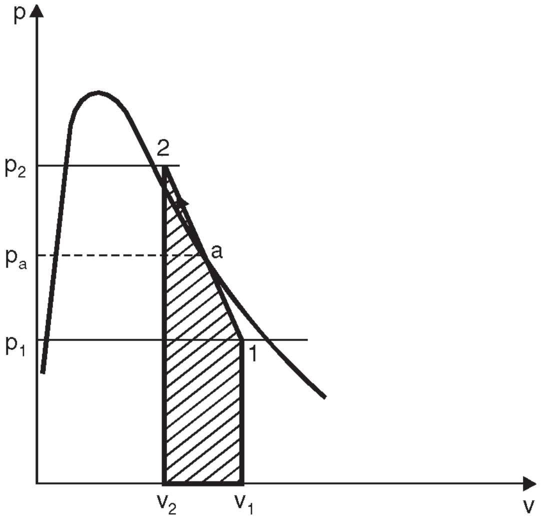
$$W = \left(\frac{p_1v_1 - p_2v_2}{n-1}\right) \text{ N-m/kg}$$

Applying the first law energy equation to non-flow process, we have

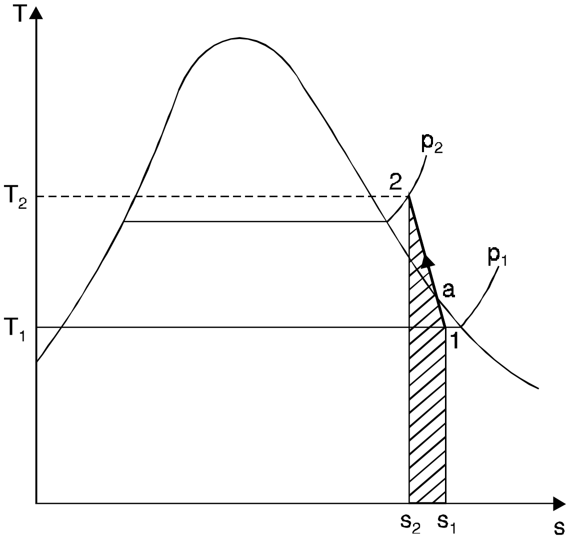
$$Q = \Delta u + W$$

$$= (u_2 - u_1) + \left(\frac{p_1v_1 - p_2v_2}{n-1}\right)$$

$$= (h_2 - p_2v_2) - (h_1 - p_1v_1) + \left(\frac{p_1v_1 - p_2v_2}{n-1}\right)$$



(*a*)



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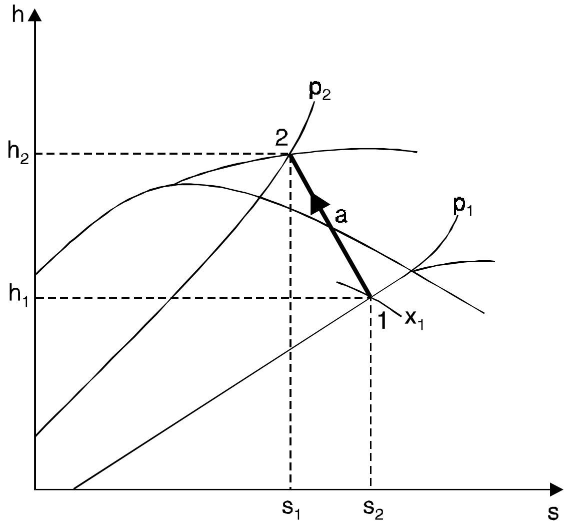


(*c*)

Fig. 4.68. Polytropic process.

$$= (h_2 - h_1) + (p_1v_1 - p_2v_2) \left(1 + \frac{1}{n-1}\right)$$

$$= (h_2 - h_1) + \frac{n}{n-1}\ (p_1v_1 - p_2v_2) \qquad \text{...(4.79)}$$

In adiabatic process $Q = 0$ and if $\Delta s \neq 0$ then the process behaves like *adiabatic process* and *not isentropic*. Such a process with steam will be a particular case of the law pv^n = constant. The index n in this case will be that particular index which will satisfy the condition :

$$Q = 0$$

$$\therefore \quad 0 = \Delta u + W$$

$$\therefore \quad W = -\Delta u = -(u_2 - u_1) = (u_1 - u_2)$$

i.e., $$W = (u_1 - u_2) \quad \text{...(4.80)}$$

Adiabatic process (not reversible) is also a polytropic process with an index n. The appropriate value of n for adiabatic compression of steam are

n = 1.13 for wet steam

n = 1.3 for superheated steam

When the initial condition and end condition are *both in wet region* then $p_1v_1^n = p_2\ v_2^n$ reduces to :

$$p_1(x_1v_{g_1})^n = p_2(x_2v_{g_2})^n.$$

As p_1, x_1, n and p_2 are specified the value of x_2 can be calculated.

When the end condition is *superheated*, then

$$p_1(x_1v_{g_1})^n = p_2(v_{sup_2})^n.$$

Solving for v_2, then using

$$\frac{v_2}{v_{g_2}} = \frac{T_{sup_2}}{T_{s_2}}$$

T_{sup_2} can be calculated. Knowing T_{s_2} and T_{sup} all properties at the end condition can be calculated.

☞**Example 4.58.** *In a steam engine the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.98 and expansion follows the law* $pv^{1.1}$ *= constant, down to a pressure of 0.34 bar. Calculate per kg of steam :*

(*i*) *The work done during expansion ;*

(*ii*) *The heat flow to or from the cylinder walls during the expansion.*

Solution. Initial pressure of steam, $p_1 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$

Dryness fraction, $x_1 = 0.98$

Law of expansion, $pv^{1.1}$ = constant

Final pressure of steam, $p_2 = 0.34 \text{ bar} = 0.34 \times 10^5 \text{ N/m}^2$.

At 7 bar : $v_g = 0.273 \text{ m}^3\text{/kg}$

$\therefore \quad v_1 = x_1v_g = 0.98 \times 0.273 = 0.267 \text{ m}^3\text{/kg}$

Also, $p_1v_1^n = p_2v_2^n$

i.e., $$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/n}$$

$$\therefore \quad \frac{v_2}{0.267} = \left(\frac{7}{0.34}\right)^{\frac{1}{1.1}} \quad \text{or} \quad v_2 = 0.267\left(\frac{7}{0.34}\right)^{\frac{1}{1.1}} = 4.174 \text{ m}^3\text{/kg}.$$

(*i*) **Work done by the steam during the process :**

$$W = \frac{p_1v_1 - p_2v_2}{n-1} = \frac{7 \times 10^5 \times 0.267 - 0.34 \times 10^5 \times 4.174}{(1.1 - 1)}$$

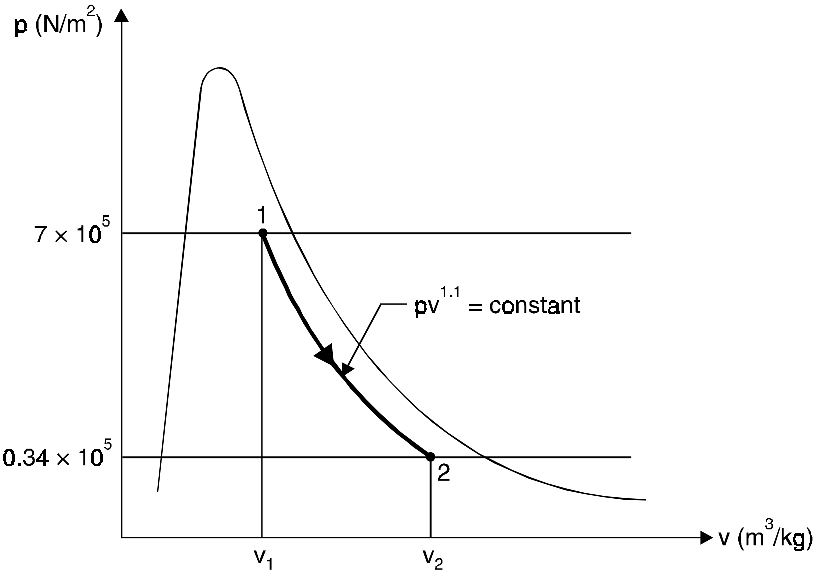


Fig. 4.69

$$= \frac{10^5}{0.1} (1.869 - 1.419) = 10^5 \times 4.5 \text{ N-m/kg}$$

i.e., **Work done** $= \frac{10^5 \times 4.5}{10^3}$ = **450 kJ/kg. (Ans.)**

At 0.34 bar : v_{g2} = 4.65 m^3/kg, therefore, steam is *wet* at state 2 (since $v_2 < v_{g2}$).

Now, $v_2 = x_2 v_{g2}$, where x_2 = dryness fraction at pressure p_2 (0.34 bar)

$$4.174 = x_2 \times 4.65 \quad \text{or} \quad x_2 = \frac{4.174}{4.65} = 0.897$$

The expansion is shown on a *p-v* diagram in Fig. 4.69, the area under 1-2 represents the work done per kg of steam.

(*ii*) **Heat transferred :**

Internal energy of steam at initial state 1 per kg,

$$u_1 = (1 - x_1)u_{f1} + x_1 u_{g_1} = (1 - 0.98)\ 696 + 0.98 \times 2573 = 2535.46 \text{ kJ/kg}$$

Internal energy of steam at final state 2 per kg,

$$u_2 = (1 - x_2)\ u_{f2} + x_2 u_{g2}$$

$$= (1 - 0.897)\ 302 + 0.897 \times 2472 = 2248.49 \text{ kJ/kg}$$

Using the non-flow energy equation,

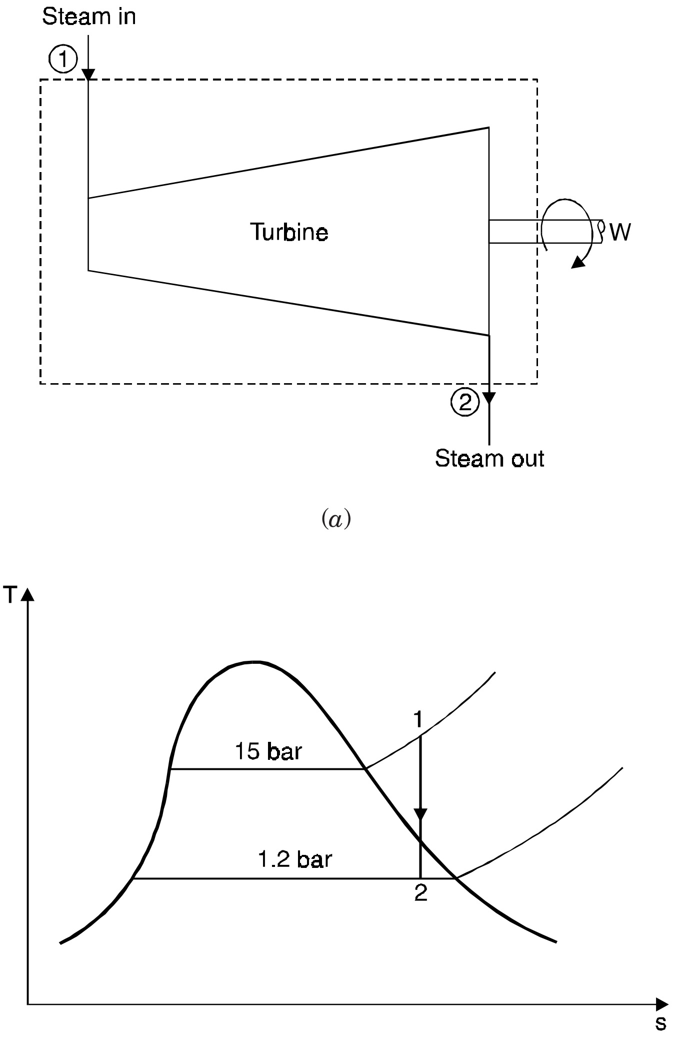
$$Q = (u_2 - u_1) + W$$

$$= (2248.49 - 2535.46) + 450 = 163.03 \text{ kJ/kg}$$

i.e., **Heat supplied** **= 163.03 kJ/kg. (Ans.)**

Example 4.59. *Steam enters a steam turbine at a pressure of 15 bar and 350°C with a velocity of 60 m/s. The steam leaves the turbine at 1.2 bar and with a velocity of 180 m/s. Assuming the process to be reversible adiabatic, determine the work done per kg of steam flow through the turbine.*

Neglect the change in potential energy.



(*a*)

(*b*)

Fig. 4.70

Solution. Initial pressure of steam, p_1 = 15 bar

Initial temperature of steam, $t_1 = t_{sup}$ = 350°C

Initial velocity of steam, C_1 = 60 m/s

Final pressure, p_2 = 1.2 bar

Final velocity, C_2 = 180 m/s

Process of expansion : *Reversible adiabatic*

As the process is reversible adiabatic, it will be represented by a vertical line on *T-s* diagram by 1-2 as it is also constant entropy process.

The condition at point '2' can be calculated by equating the entropy at point '1' and point '2',

i.e., $s_1 = s_2$ .....per kg of steam

$$7.102 = s_{f_2} + x_2(s_{g_2} - s_{f_2})$$

$$= 1.3609 + x_2(7.2984 - 1.3609)$$

$$\therefore \quad x_2 = \frac{7.102 - 1.3609}{7.2984 - 1.3609} = 0.967$$

$$h_2 = h_{f_2} + x_2 h_{fg_2} = 439.4 + 0.967 \times 2244.1 = 2609.44 \text{ kJ/kg}$$

h_1 (at 15 bar and 350°C) = 3147.5 kJ/kg

Applying the first law energy equation for steady flow process,

$$h_1 + \frac{C_1^{\,2}}{2} = h_2 + \frac{C_2^{\,2}}{2} + W$$

i.e.,
$$W = (h_1 - h_2) + \left(\frac{C_1^{\,2} - C_2^{\,2}}{2}\right)$$

$$= 3147.5 - 2609.44 + \left(\frac{60^2 - 180^2}{2 \times 10^3}\right)$$

$$= 3147.5 - 2609.44 - 14.4 = 523.66 \text{ kJ/kg.}$$

Hence **work done per kg of steam = 523.66 kJ/kg. (Ans.)**

Example 4.60. *Steam at 10 bar and 200°C enters a convergent divergent nozzle with a velocity of 60 m/s and leaves at 1.5 bar and with a velocity of 650 m/s. Assuming that there is no heat loss, determine the quality of the steam leaving the nozzle.*

Solution. Initial pressure of steam, $p_1 = 10$ bar

Initial temperature of steam, $t_1 = t_{sup} = 200°\text{C}$

Initial velocity, $C_1 = 60$ m/s

Final velocity, $C_2 = 650$ m/s

Final pressure, $p_2 = 1.5$ bar

Heat loss = nil

Quality of steam at the outlet :

It is a steady-state non-work developing system. Applying the steady flow energy equation to the process, we get

$$h_1 + \frac{C_1^{\,2}}{2} = h_2 + \frac{C_2^{\,2}}{2} \qquad (\because \quad Q = 0,\ W = 0)$$

$$\therefore \quad h_2 = h_1 + \left(\frac{C_1^{\,2} - C_2^{\,2}}{2}\right)$$

At 10 bar, 250ºC : h_1 = 2827.9 kJ/kg (from steam tables)

$$\therefore \quad h_2 = 2827.9 + \left[\frac{60^2 - 650^2}{2 \times 10^3}\right] = 2618.45 \text{ kJ/kg}$$

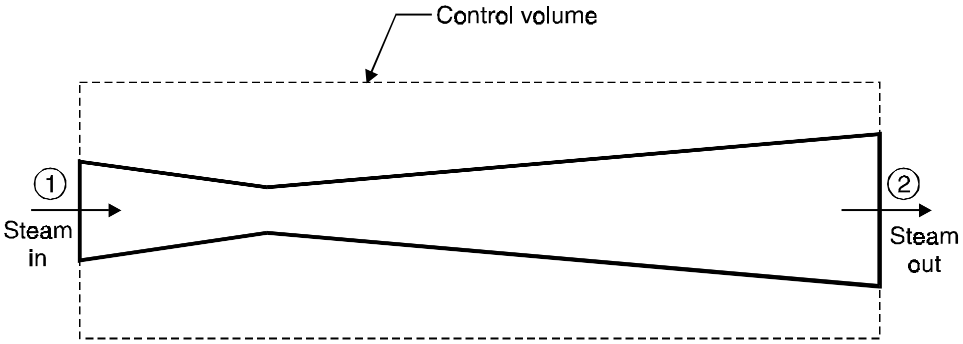


Fig. 4.71

As the enthalpy and pressure of steam at the exit of the nozzle are known, we can find out quality of steam,

$$h_{g_2} \text{ (at 1.5 bar)} = 2693.4 \text{ kJ/kg}$$

As $h_2 < h_{g_2}$, the steam is *wet*.

The enthalpy of wet steam is given by

$$h_2 = h_{f_2} + x_2 h_{fg_2}$$

$$2618.45 = 467.1 + x_2 \times 2226.2$$

$$\therefore \quad x_2 = \frac{2618.45 - 467.1}{2226.2} = 0.966.$$

Hence **the condition of steam leaving the nozzle is 96.6% dry. (Ans.)**

6. Throttling. A flow of fluid is said to be throttled when there is some *restriction to the flow*, when the velocities before and after the restriction are either equal or negligibly small, and when there is a *negligible heat loss to the surroundings.*

The restriction to the flow can be :

(*i*) partly open valve

(*ii*) an orifice or

(*iii*) any other sudden reduction in the cross-section of the flow.

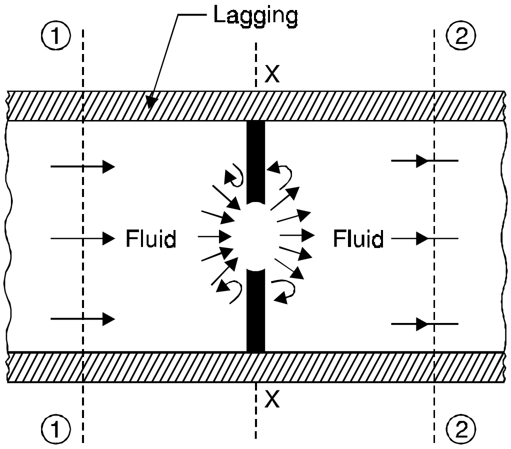


Fig. 4.72. Throttling.

An example of throttling is shown in Fig. 4.72. It is represented on *T-s* and *h-s* diagrams as shown in Figs. 4.73 and 4.74 respectively. The fluid (say steam) flowing steadily along a well-lagged pipe, passes through an orifice at section *X*. Since the pipe is well-lagged it can be assumed that no heat flows to or from the fluid.

Applying flow equation between any two sections of the flow, we have

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W$$

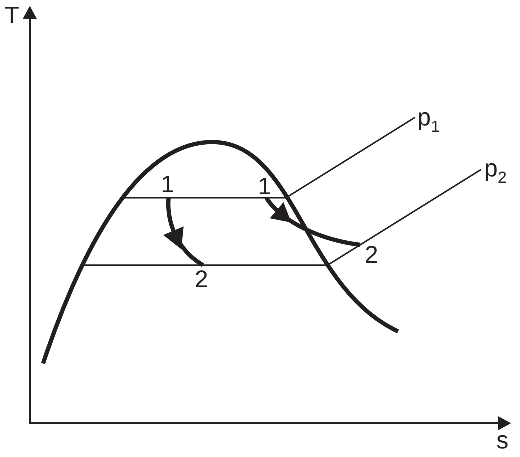


Fig. 4.73. *T-s* diagram.

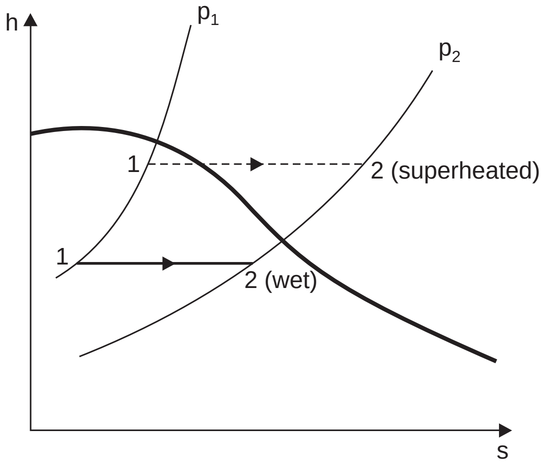


Fig. 4.74. *h-s* diagram.

Now since $Q = 0$, and $W = 0$, then

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

When the velocities C_1 and C_2 are small, or when C_1 is very nearly equal to C_2, then the K.E. terms may be neglected.

Then $$h_1 = h_2 \qquad ...(4.81)$$

i.e., For a throttling process :

Initial enthalpy = Final enthalpy.

The process is adiabatic but highly irreversible because of the eddying of the fluid around the orifice at X. Between sections 1 and *X* the enthalpy drops and K.E. increases as the fluid accelerates through the orifice. Between sections *X* and 2 the *enthalpy increases as K.E. is destroyed by fluid eddies*.

During throttling *pressure always falls*.

The throttling process is *used* for the following *purposes* :

1. To *determine the dryness fraction of steam.*
2. To *control the speed of the engine and turbine.*
3. To *reduce* the pressure and temperature of the liquid refrigerant from the condenser condition to evaporator condition in a *refrigeration system*.

☞**Example 4.61.** *Steam at 18 bar is throttled to 1 bar and the temperature after throttling is found to be 150°C. Calculate the initial dryness fraction of the steam.*

Solution. Pressure of steam before throttling, p_1 = 18 bar

Pressure of steam after throttling = 1 bar

Temperature after throttling = 150°C

Initial dryness fraction, x_1 :

From superheat tables at **1 bar and 150°C,** we have

$$h_2 = 2776.4 \text{ kJ/kg}$$

Then for throttling, $h_1 = h_2 = 2776.4$

But $$h_1 = h_{f_1} + x_1 h_{fg_1}$$

At 18 bar : $h_f = 884.6$ kJ/kg, $h_{fg} = 1910.3$ kJ/kg

$$\therefore \qquad 2776.4 = 884.6 + x_1 \times 1910.3$$

or
$$x_1 = \frac{2776.4 - 884.6}{1910.3} = 0.99$$

i.e., **Initial, dryness fraction = 0.99. (Ans.)**

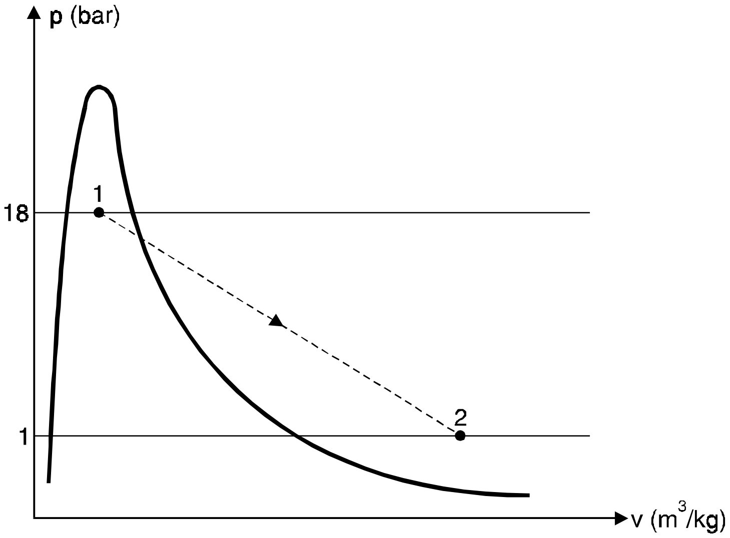


Fig. 4.75

The process is shown on a *p-v* diagram in Fig. 4.75. States 1 and 2 are fixed, but the intermediate states are indeterminate ; the process must be drawn dotted, as shown. No work is done during the process, and the area under the line 1-2 is **not equal to work done.**

Example 4.62. *Steam at 10 bar and 0.9 dryness fraction is throttled to a pressure of 2 bar. Determine the exit condition of steam using* **Mollier chart.**

Solution. Refer to Fig. 4.76.

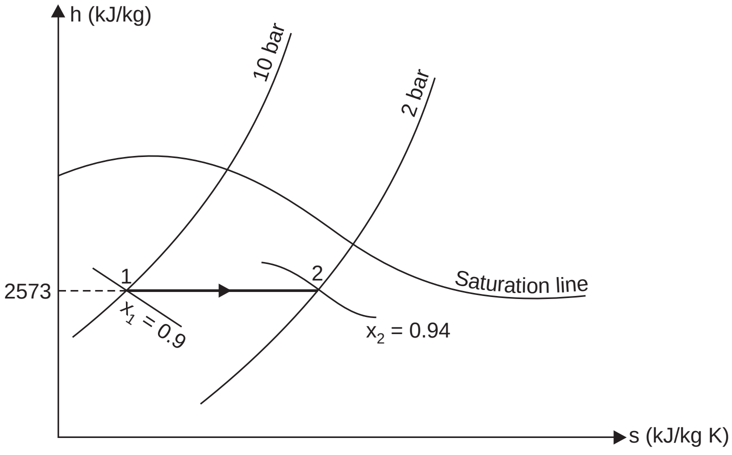


Fig. 4.76

Locate the point '1' at an intersection of the 10 bar pressure line and 0.9 dryness fraction line.

Throttling is a constant enthalpy line so draw a line parallel to *X*-axis till it cuts the 2 bar line and locate the point 2. The dryness fraction of steam at point 2 is 0.94.

(The total enthalpy before and after throttling = 2573 kJ/kg)

Hence **exit condition of steam = 0.94. (Ans.)**

Note. This process occurs during the control of flow of steam supplied to a turbine to take care of the varying load.

☞**Example 4.63.** *Steam initially at a pressure of 15 bar and 0.95 dryness expands isentropically to 7.5 bar and is then throttled until it is just dry. Determine per kg of steam :*

(*i*) *Change in entropy ;*

(*ii*) *Change in enthalpy ;*

(*iii*) *Change in internal energy.*

Using : (*a*) *Steam tables*

(*b*) *Mollier chart.*

Is the entire process reversible ? Justify your statement.

Solution. (*a*) **Using steam tables**

Condition 1 : 15 bar, 0.95 dryness

$$h_{f_1} = 844.7 \text{ kJ/kg} ;\ t_{s_1} = 198.3°\text{C},\ s_{f_1} = 2.3145 \text{ kJ/kg K},$$

$$s_{g_1} = 6.4406 \text{ kJ/kg K},\ v_{g_1} = 0.132 \text{ m}^3\text{/kg}$$

$$h_1 = h_{f_1} + x_1 h_{fg_1} = 844.7 + 0.95 \times 1945.2 = 2692.64 \text{ kJ/kg}$$

$$s_1 = s_{f_1} + x_1 (s_{g_1} - s_{f_1}) = 2.3145 + 0.95(6.4406 - 2.3145) = 6.2343 \text{ kJ/kg K.}$$

Condition 2 : 7.5 bar

$$h_{f_2} = 709.3 \text{ kJ/kg},\ t_{s_2} = 167.7°\text{C},\ h_{fg_2} = 2055.55 \text{ kJ/kg},\ s_{f_2} = 2.0195 \text{ kJ/kg K}$$

$$s_{g_2} = 6.6816 \text{ kJ/kg K},\ v_{g_2} = 0.255 \text{ m}^3\text{/kg.}$$

Consider **isentropic expansion 1-2 :**

(*i*) **Change in entropy** = 0

i.e., Entropy at 1 = entropy at 2

$$\therefore \quad s_1 = s_2$$

$$6.2343 = s_{f_2} + x_2 (s_{g_2} - s_{f_2})$$

$$= 2.0195 + x_2(6.6816 - 2.0195)$$

$$\therefore \quad x_2 = \frac{6.2343 - 2.0195}{6.6816 - 2.0195} = 0.9$$

Now, enthalpy at point 2,

$$h_2 = h_{f_2} + x_2 h_{fg_2} = 709.3 + 0.9 \times 2055.55 = 2559.29 \text{ kJ/kg.}$$

(*ii*) **Change in enthalpy** $= h_2 - h_1$

$= 2559.29 - 2692.64 = -$ **133.35 kJ/kg. (Ans.)**

(–ve sign indicates *decrease*).

(*iii*) **Change in internal energy :**

Internal energy at point 1,

$$u_1 = h_1 - p_1 x_1 v_{g_1}$$

$$= 2692.64 - 15 \times 10^5 \times 0.95 \times 0.132 \times 10^{-3} = 2504.54 \text{ kJ/kg}$$

Internal energy at point 2,

$$u_2 = h_2 - p_2 x_2 v_{g_2}$$

$$= 2559.29 - 7.5 \times 10^5 \times 0.9 \times 0.255 \times 10^{-3} = 2387.16 \text{ kJ/kg}$$

∴ Change in internal energy

$$= u_2 - u_1 = 2387.16 - 2504.54 = -117.38 \text{ kJ/kg}$$

(–ve sign indicates *decrease*)

Consider the **throttling expansion 2-3 :**

Entropy at point 2,

$$s_2 = (s_1) = 6.2343 \text{ kJ/kg K}$$

Entropy at point 3,

$$s_3 = s_{f_3} + x_3 (s_{g_3} - s_{f_3})$$

The pressure at point 3 can be read from *h-s* chart ($p_3 = 0.06$ bar) and the corresponding values of sf_3 and h_{fg_3} from steam tables.

Condition 3. At 0.06 bar, $x_3 = 1$. From steam tables,

$$s_{f_3} = 0.521 \text{ kJ/kg K}, \quad s_{g_3} = 8.330 \text{ kJ/kg K}$$

∴ $s_3 = 0.521 + 1 \times (8.330 - 0.521) = 8.330$ kJ/kg K

Change in entropy $= s_3 - s_2$

$= 8.330 - 6.2343 = 2.0957$ kJ/kg K

Change in enthalpy $= 0$

i.e., $h_2 = h_3$

Change in internal energy = 0

i.e., $u_3 = u_2$

Combining the results obtained from isentropic and throttling expansion, we get during the entire process :

(*i*) **Change in entropy = 2.0957 kJ/kg K (increase). (Ans.)**

(*ii*) **Change in enthalpy = 133.35 kJ/kg K (decrease). (Ans.)**

(*iii*) **Change in internal energy = 117.38 kJ/kg (decrease). (Ans.)**

Only the expansion of steam from point 1 to 2 (*i.e., isentropic expansion*) is *reversible* because of unresisted flow whereas the expansion from point 2 to point 3 (*i.e., throttling expansion*) is *irreversible* because of frictional resistance to flow. *Increase of entropy also shows that expansion from point 2 to point 3 is irreversible.*

(*b*) **Using Mollier chart.**

Refer to Fig. 4.77.

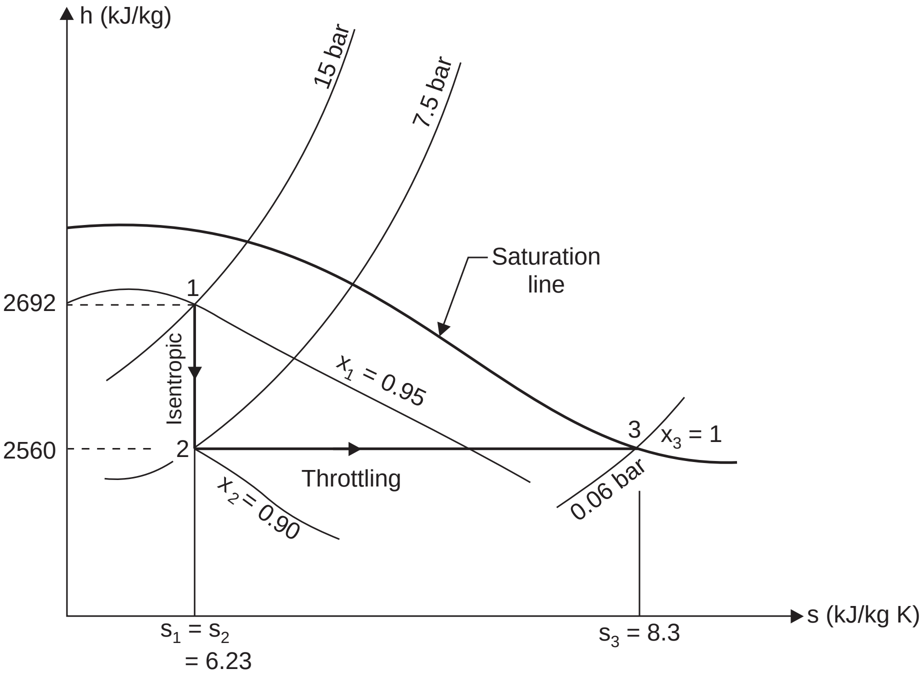


Fig. 4.77

- Locate point 1 at an intersection of 15 bar pressure line and 0.95 dryness fraction line.
- Draw vertical line from point 1 intersecting 7.5 bar pressure line at point 2. Line 1-2 represents *isentropic expansion.*
- From point 2 draw a horizontal line intersecting at the saturation line at point 3. Line 2-3 then represents *throttling expansion.*

From *Mollier chart* :

$$h_1 = 2692 \text{ kJ/kg}, \qquad h_2 = 2560 \text{ kJ/kg}$$

$$s_1 = s_2 = 6.23 \text{ kJ/kg}, \qquad s_3 = 8.3 \text{ kJ/kg K}$$

∴ (*i*) **Change in entropy** $= s_3 - (s_1 \text{ or } s_2)$

$= 8.3 - 6.23 =$ **2.07 kJ/kg K (increase). (Ans.)**

(*ii*) **Change in enthalpy** $= h_2 \text{ (or } h_3) - h_1$

$= 2560 - 2692 = -132$ kJ/kg

= **132 kJ/kg (decrease). (Ans.)**

4.15. UNSTEADY FLOW PROCESSES

In engineering practice, the variable flow process applications are as common as the steady flow process. The *rate of energy and mass transfer into and out of the control volume are not same in the case of unstable* (or variable or transient) *flow process.*

Following two cases only will be discussed :

1. Filling a tank.
2. Emptying a tank or tank discharge.

1. Filling a tank :

Let m_1 = Initial mass of fluid,

p_1 = Initial pressure,

v_1 = Initial specific volume,

T_1 = Initial temperature,

u_1 = Initial specific internal energy,

and m_2 = Final mass of fluid,

p_2 = Final pressure,

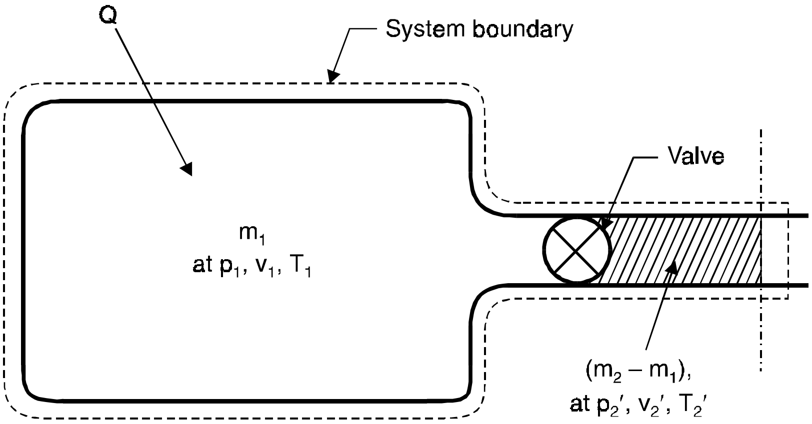


Fig. 4.78

v_2 = Final specific volume,

T_2 = Final temperature,

u_2 = Final specific internal energy,

Also, let p' = Entering fluid pressure,

v' = Entering fluid specific volume,

T' = Entering fluid temperature,

C' = Entering fluid velocity,

u' = Entering specific internal energy of fluid, and

h' = Entering specific enthalpy of fluid.

The quantity of fluid entering

$$= m_2 - m_1$$

Energy of entering fluid

$$= (m_2 - m_1)\left(u' + p'v' + \frac{C'^2}{2}\right) \quad ...(4.82)$$

$$= (m_2 - m_1)\left(h' + \frac{C'^2}{2}\right) \quad ...(4.83)$$

If Q = Heat transferred into the control volume, we have

$$(m_2 - m_1)\left(h' + \frac{C'^2}{2}\right) + Q = m_2u_2 - m_1u_1 \quad ...(4.84)$$

When the tank is fully insulated and thus no heat transfer takes place,

$$Q = 0$$

and

$$(m_2 - m_1)\left(h' + \frac{C'^2}{2}\right) = m_2u_2 - m_1u_1 \quad ...(4.85)$$

Also, if the tank is empty initially and fully insulated for heat transfer,

$$m_1 = 0$$

Thus

$$h' + \frac{C'^2}{2} = u_2 \quad ..(4.86)$$

Also, if kinetic energy in the pipe line is neglected

$$h' = u_2 \quad ...(4.87)$$

2. Emptying a tank :

Analogous to the filling of the tank, the equation can be written as

$$(m_1 - m_2)\left(h' + \frac{C'^2}{2}\right) - Q = m_1u_1 - m_2u_2 \quad ...(4.88)$$

where h' = Specific enthalpy of leaving fluid, and

C' = Velocity of leaving fluid.

For fully emptying the tank and no heat transfer and negligible exit velocity,

$$h' = u_1 \quad ...(4.89)$$

Example 4.64. *An air receiver of volume 5.5 m³ contains air at 16 bar and 42°C. A valve is opened and some air is allowed to blow out to atmosphere. The pressure of the air in the receiver drops rapidly to 12 bar when the valve is then closed.*

Calculate the mass of air which has left the receiver.

Solution. Initial volume of air, $V_1 = 5.5 \text{ m}^3$

Initial pressure of air, $p_1 = 16$ bar

Initial temperature of air, $T_1 = 42 + 273 = 315$ K

Final volume of air, $V_2 = V_1 = 5.5 \text{ m}^3$

Final pressure of air, $p_2 = 12$ bar

Mass of air which left the receiver :

Mass of air in the *initial* condition,

$$m_1 = \frac{p_1V_1}{RT_1} = \frac{16 \times 10^5 \times 5.5}{(0.287 \times 10^3) \times 315} = 97.34 \text{ kg.}$$

Assuming that the mass in the receiver undergoes a reversible adiabatic process, then

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{315} = \left(\frac{12}{16}\right)^{\frac{1.4 - 1}{1.4}} = \left(\frac{12}{16}\right)^{0.286} \quad \text{or} \quad T_2 = 315 \times \left(\frac{12}{16}\right)^{0.286} = 290 \text{ K}$$

Now mass of air in the receiver in *final* condition,

$$m_2 = \frac{p_2V_2}{RT_2} = \frac{12 \times 10^5 \times 5.5}{(0.287 \times 10^3) \times 290} = 79.3 \text{ kg.}$$

∴ **Mass of air which left the receiver,**

$$m = m_1 - m_2 = 97.34 - 79.3 = \mathbf{18.04\ kg.}\quad \textbf{(Ans.)}$$

Example 4.65. *A 1.6 m^3 tank is filled with air at a pressure of 5 bar and a temperature of 100°C. The air is then let off to the atmosphere through a valve. Assuming no heat transfer, determine the work obtainable by utilising the kinetic energy of the discharge air to run a frictionless turbine.*

Take : Atmospheric pressure = 1 bar ;

c_p *for air = 1 kJ/kg K ;*

c_v *for air = 0.711 kJ/kg K.*

Solution. Initial volume of air, $V_1 = 1.6\ \text{m}^3$

Initial pressure of air, $p_1 = 5\ \text{bar} = 5 \times 10^5\ \text{N/m}^2$

Initial temperature of air, $T_1 = 100 + 273 = 373\ \text{K}$

Final pressure of air, $p_2 = 1\ \text{bar} = 1 \times 10^5\ \text{N/m}^2$

Now, initial quantity of air in the tank before discharge,

$$m_1 = \frac{p_1V_1}{RT_1} = \frac{5 \times 10^5 \times 1.6}{(0.287 \times 10^3) \times 373} = 7.47\ \text{kg.}$$

Assuming that system undergoes a reversible adiabatic expansion

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

where T_2 is the final temperature of air in the tank.

$$\therefore \qquad \frac{T_2}{373} = \left(\frac{1}{5}\right)^{\frac{1.4 - 1}{1.4}} = 0.631$$

$T_2 = 373 \times 0.631 = 235.4$ K (*i.e.*, finally in the line)

The final quantity of air remaining in the tank is

$$m_2 = \frac{p_2V_2}{RT_2} = \frac{1 \times 10^5 \times 1.6}{(0.287 \times 10^3) \times 235.4} = 2.368\ \text{kg.}$$

With $Q = 0$, kinetic energy is found from,

$$(m_1 - m_2)\left(h' + \frac{C'^2}{2}\right) = m_1u_1 - m_2u_2$$

or
$$(m_1 - m_2)\,h' + (m_1 - m_2)\,\frac{C'^2}{2} = m_1u_1 - m_2u_2$$

∴ Kinetic energy,

$$(m_1 - m_2)\,\frac{C'^2}{2} = (m_1u_1 - m_2u_2) - (m_1 - m_2)\,h'$$
$$= m_1c_vT_1 - m_2c_vT_2 - (m_1 - m_2)\,c_pT_2$$
$$= 7.47 \times 0.771 \times 373 - 2.368 \times 0.711 \times 235.4 - (7.47 - 2.368) \times 1 \times 235.4$$
$$= 2148.24 - 396.33 - 1201 = \mathbf{550.9\ kJ.}\quad \textbf{(Ans.)}$$

☞**Example 4.66.** *A frictionless piston is free to move in a closed cylinder. Initially there is 0.035 m^3 of oxygen at 4.5 bar, 60°C on one side of the piston and 0.07 m^3 of methane at 4.5 bar* *and – 12°C on the other side. The cylinder walls and piston may be regarded as perfect thermal insulators but the oxygen may be heated electrically. Heating takes place so that the volume of oxygen doubles. Find :*

(*i*) *Final state condition ;* (*ii*) *Work done by the piston ;*

(*iii*) *Heat transferred to oxygen.*

Treat both gases as perfect and take :

For oxygen $c_p = 0.88$ *kJ/kg K,* $R = 0.24$ *kJ/kg K*

For methane $c_p = 1.92$ *kJ/kg K,* $R = 0.496$ *kJ/kg K.*

Solution. For oxygen :

Initial volume, $V_1 = 0.035\text{ m}^3$

Initial pressure, $p_1 = 4.5\text{ bar}$

Initial temperature, $T_1 = 60 + 273 = 333\text{ K}$

For methane :

Initial volume, $V_1 = 0.07\text{ m}^3$

Final volume, $V_2 = 0.035\text{ m}^3$

Initial pressure, $p_1 = 4.5\text{ bar}$

Initial temperature of methane,

$$T_1 = -12 + 273 = 261\text{ K.}$$

For Methane :

$$c_p = R \times \frac{\gamma}{\gamma - 1} \quad \text{or} \quad 1.92 = 0.496\left(\frac{\gamma}{\gamma - 1}\right)$$

or
$$\frac{1.92}{0.496} = \frac{\gamma}{\gamma - 1} \quad \text{or} \quad 1.92\,(\gamma - 1) = 0.496\,\gamma$$

$\therefore$
$$\gamma = \frac{1.92}{(1.92 - 0.496)} = 1.348 \text{ say } 1.35$$

For Oxygen :

$$c_v = c_p - R = 0.88 - 0.24 = 0.64\text{ kJ/kg K.}$$

(*i*) According to problem ; for *methane*

$$pV^{\gamma} = \text{constant holds good}$$

$\therefore$
$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma} = 4.5\,(2)^{1.35} = \mathbf{11.47\ bar.}\quad \textbf{(Ans.)}$$

Also,
$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

or
$$T_2 = \frac{p_2 V_2 T_1}{p_1 V_1} = \frac{11.47 \times 0.035 \times 261}{4.5 \times 0.07} = \mathbf{332.6\ K.}\quad \textbf{(Ans.)}$$

$\therefore$
$$\text{Work done} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{4.5 \times 10^5 \times 0.07 - 11.47 \times 10^5 \times 0.035}{(1.35 - 1)}\text{ J}$$

$$= \frac{10^5\,(4.5 \times 0.07 - 11.47 \times 0.035)}{0.35 \times 1000}\text{ kJ}$$

$$= -24.7\text{ kJ } (\textit{done on the methane})$$

(*ii*) **The piston will be in virtual equilibrium and hence zero work is effected by the piston. (Ans.)**

(*iii*) Work done by oxygen = work done on methane and expansion of oxygen is effected in the system

$$\therefore \quad W_{oxygen} = +\ 24.7 \text{ kJ}$$

and
$$Q = (U_2 - U_1) + W$$

Amount of oxygen present
$$= \frac{p_1V_1}{RT_1} = \frac{4.5 \times 10^5 \times 0.035}{0.24 \times 1000 \times 333} = 0.197 \text{ kg}$$

and
$$T_2 = \frac{p_2V_2}{p_1V_1} \times T_1 = \frac{11.47 \times 0.07 \times 333}{4.5 \times 0.035} = \mathbf{1697.5\ K.} \quad \textbf{(Ans.)}$$

(As the piston is free, the final pressure of oxygen and methane will be **same**).

$$\therefore \quad Q = (U_2 - U_1) + W$$
$$= mc_v\ (T_2 - T_1) + W$$
$$= 0.197 \times 0.64\ (1697.5 - 333) + 24.7 = \mathbf{196.7\ kJ.} \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. *Internal energy* is the heat energy stored in a gas. The internal energy of a perfect gas is a function of *temperature* only.

2. First law of thermodynamics states :

— Heat and work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant.

Or

— No machine can produce energy without corresponding expenditure of energy, *i.e.,* it is impossible to construct a perpetual motion machine of first kind.

First law can be expressed as follows :

$$Q = \Delta E + W$$

$$Q = \Delta U + W$$ ... if electric, magnetic, chemical energies are absent and changes in potential and kinetic energies are neglected.

3. There can be no machine which would continuously supply mechanical work without some form of energy disappearing simultaneously. Such a fictitious machine is called a perpetual motion machine of the first kind, or in brief, PMM1. A PMM1 is thus impossible.

4. The energy of an isolated system is always constant.

5. In case of

(*i*) **Reversible constant volume process** (v = constant)

$$\Delta u = c_v(T_2 - T_1)\ ;\ W = 0\ ;\ Q = c_v\ (T_2 - T_1)$$

(*ii*) **Reversible constant pressure process** (p = constant)

$$\Delta u = c_v(T_2 - T_1)\ ;\ W = p(v_2 - v_1)\ ;\ Q = c_p\ (T_2 - T_1)$$

(*iii*) **Reversible temperature or isothermal process** (pv = constant)

$$\Delta u = 0,\ W = p_1V_1 \log_e r,\ Q = W$$

where r = expansion or compression ratio.

(*iv*) **Reversible adiabatic process** (pv^γ = constant)

$$\pm\Delta u = \mp\ W = \frac{R(T_1 - T_2)}{\gamma - 1}\ ;\ Q = 0\ ;\ \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

(*v*) **Polytropic reversible process** (pv^n = constant)

$$\Delta u = c_v (T_2 - T_1) \; ; \; W = \frac{R(T_1 - T_2)}{n - 1} \; ; \; Q = \Delta u + W \; ;$$

and $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{and} \quad Q = \left(\frac{\gamma - n}{n - 1}\right) \times W.$$

6. Steady flow equation can be expressed as follows :

$$u_1 + \frac{C_1^2}{2} + Z_1 g + p_1 v_1 + Q = u_2 + \frac{C_2^2}{2} + Z_2 g + p_2 v_2 + W \quad ...(i)$$

or $$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W, \text{ neglecting } Z_1 \text{ and } Z_2 \quad ...(ii)$$

where, Q = Heat supplied per kg of fluid ; W = Work done by 1 kg of fluid ;
C = Velocity of fluid ; Z = Height above datum ;
p = Pressure of the fluid ; u = Internal energy per kg of fluid ;
pv = Energy required per kg of fluid.

This equation is applicable to any medium in any steady flow.

7. During adiabatic *throttling process* enthalpy remains constant. The slope of a constant enthalpy line on a *p-T* diagram is called Joule-Thompson co-efficient, μ.

8. In unsteady-flow processes, the rates at which mass and energy enter the control volume may not be the same as the rate of flow of mass and energy moving out of the control volume. The filling of a tank is an example of unsteady flow process.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. If all the variables of a stream are independent of time it is said to be in
(*a*) steady flow (*b*) unsteady flow
(*c*) uniform flow (*d*) closed flow
(*e*) constant flow.

2. A control volume refers to
(*a*) a fixed region in space (*b*) a specified mass
(*c*) an isolated system (*d*) a reversible process only
(*e*) a closed system.

3. Internal energy of a perfect gas depends on
(*a*) temperature, specific heats and pressure (*b*) temperature, specific heats and enthalpy
(*c*) temperature, specific heats and entropy (*d*) temperature only.

4. In reversible polytropic process
(*a*) true heat transfer occurs (*b*) the entropy remains constant
(*c*) the enthalpy remains constant (*d*) the internal energy remains constant
(*e*) the temperature remains constant.

5. An isentropic process is always
(*a*) irreversible and adiabatic (*b*) reversible and isothermal
(*c*) frictionless and irreversible (*d*) reversible and adiabatic
(*e*) none of the above.

6. The net work done per kg of gas in a polytropic process is equal to

(*a*) $p_1v_1 \log_e \frac{v_2}{v_1}$
(*b*) $p_1 (v_1 - v_2)$
(*c*) $p_2 \left(v_2 - \frac{v_1}{v_2} \right)$
(*d*) $\frac{p_1v_1 - p_2v_2}{n - 1}$
(*e*) $\frac{p_2v_1 - p_2v_2}{n - 1}$.

7. Steady flow occurs when
(*a*) conditions do not change with time at any point
(*b*) conditions are the same at adjacent points at any instant
(*c*) conditions change steadily with the time
(*d*) $\left(\frac{\partial v}{\partial t} \right)$ is constant.

8. A reversible process requires that
(*a*) there be no heat transfer
(*b*) newton's law of viscosity be satisfied
(*c*) temperature of system and surroundings be equal
(*d*) there be no viscous or coloumb friction in the system
(*e*) heat transfer occurs from surroundings to system only.

9. The first law of thermodynamics for steady flow
(*a*) accounts for all energy entering and leaving a control volume
(*b*) is an energy balance for a specified mass of fluid
(*c*) is an expression of the conservation of linear momentum
(*d*) is primarily concerned with heat transfer
(*e*) is restricted in its application to perfect gases.

10. The characteristic equation of gases $pV = mRT$ holds good for
(*a*) monoatomic gases
(*b*) diatomic gas
(*c*) real gases
(*d*) ideal gases
(*e*) mixture of gases.

11. A gas which obeys kinetic theory perfectly is known as
(*a*) monoatomic gas
(*b*) diatomic gas
(*c*) real gas
(*d*) pure gas
(*e*) perfect gas.

12. Work done in a free expansion process is
(*a*) zero
(*b*) minimum
(*c*) maximum
(*d*) positive
(*e*) negative.

13. Which of the following is not a property of the system ?
(*a*) Temperature
(*b*) Pressure
(*c*) Specific volume
(*d*) Heat
(*e*) None of the above.

14. In the polytropic process equation pv^n = constant, if $n = 0$, the process is termed as
(*a*) constant volume
(*b*) constant pressure
(*c*) constant temperature
(*d*) adiabatic
(*e*) isothermal.

15. In the polytropic process equation pv^n = constant, if n is infinitely large, the process is termed as
(*a*) constant volume
(*b*) constant pressure
(*c*) constant temperature
(*d*) adiabatic
(*e*) isothermal.

16. The processes or systems that do not involve heat are called

(*a*) isothermal processes (*b*) equilibrium processes
(*c*) thermal processes (*d*) steady processes
(*e*) adiabatic processes.

17. In a reversible adiabatic process the ratio (T_1/T_2) is equal to

(*a*) $\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$ (*b*) $\left(\frac{v_1}{v_2}\right)^{\frac{\gamma-1}{\gamma}}$

(*c*) $(v_1 v_2)^{\frac{\gamma-1}{2\gamma}}$. (*d*) $\left(\frac{v_2}{v_1}\right)^{\gamma}$.

18. In isothermal process

(*a*) temperature increases gradually (*b*) volume remains constant
(*c*) pressure remains constant (*d*) enthalpy change is maximum
(*e*) change in internal energy is zero.

19. During throttling process

(*a*) internal energy does not change (*b*) pressure does not change
(*c*) entropy does not change (*d*) enthalpy does not change
(*e*) volume change is negligible.

20. When a gas is to be stored, the type of compression that would be ideal is

(*a*) isothermal (*b*) adiabatic
(*c*) polytropic (*d*) constant volume
(*e*) none of the above.

21. If a process can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states, it is known as

(*a*) adiabatic process (*b*) isothermal process
(*c*) ideal process (*d*) frictionless process
(*e*) energyless process.

22. The state of a substance whose evaporation from its liquid state is complete, is known as

(*a*) vapour (*b*) perfect gas
(*c*) air (*d*) steam.

23. In SI units, the value of the universal gas constant is

(*a*) 0.8314 J/mole/K (*b*) 8.314 J/mole/K
(*c*) 83.14 J/mole/K (*d*) 831.4 J/mole/K
(*e*) 8314 J/mole/K.

24. When the gas is heated at constant pressure, the heat supplied

(*a*) increases the internal energy of the gas (*b*) increases the temperature of the gas
(*c*) does some external work during expansion (*d*) both (*b*) and (*c*)
(*e*) none of the above.

25. The gas constant (R) is equal to the

(*a*) sum of two specific heats (*b*) difference of two specific heats
(*c*) product of two specific heats (*d*) ratio of two specific heats.

26. The heat absorbed or rejected during a polytropic process is

(*a*) $\left(\frac{\gamma-n}{\gamma-1}\right)$ × work done (*b*) $\left(\frac{\gamma-n}{\gamma-1}\right)^2$ × work done

(*c*) $\left(\frac{\gamma-n}{\gamma-1}\right)^{1/2}$ × work done (*d*) $\left(\frac{\gamma-n}{\gamma-1}\right)^3$ × work done.

Answers

1.	(*a*)	**2.**	(*a*)	**3.**	(*d*)	**4.**	(*a*)	**5.**	(*d*)	**6.**	(*d*)	**7.**	(*a*)
8.	(*d*)	**9.**	(*a*)	**10.**	(*c*)	**11.**	(*e*)	**12.**	(*a*)	**13.**	(*d*)	**14.**	(*b*)
15.	(*a*)	**16.**	(*e*)	**17.**	(*a*)	**18.**	(*e*)	**19.**	(*d*)	**20.**	(*a*)	**21.**	(*c*)
22.	(*b*)	**23.**	(*e*)	**24.**	(*d*)	**25.**	(*b*)	**26.**	(*a*).				

THEORETICAL QUESTIONS

1. Define 'internal energy' and prove that it is a property of a system.

2. Explain the First Law of Thermodynamics as referred to closed systems undergoing a cyclic change.

3. State the First Law of Thermodynamics and prove that for a non-flow process, it leads to the energy equation $Q = \Delta U + W$.

4. What is the mechanical equivalent of heat ? Write down its value when heat is expressed in kJ and work is expressed in N-m.

5. What do you mean by "Perpetual motion machine of first kind-PMM 1" ?

6. Why only in constant pressure non-flow process, the enthalpy change is equal to heat transfer ?

7. Prove that the rate of change of heat interchange per unit change of volume when gas is compressed or expanded is given by $\frac{\gamma - n}{\gamma - 1} \times \frac{pdv}{J}$.

8. Write down the general energy equation for steady flow system and simplify when applied for the following systems :

(*i*) Centrifugal water pump (*ii*) Reciprocating air compressor

(*iii*) Steam nozzle (*iv*) Steam turbine

(*v*) Gas turbine.

9. Explain clearly the difference between a non-flow and a steady flow process.

10. For isothermal flow and non-flow steady processes, prove that

$$\int_1^2 pdv = -\int_1^2 v \, . \, dp$$

Also state the assumptions made.

UNSOLVED EXAMPLES

Closed Systems

1. In a cyclic process, heat transfers are + 14.7 kJ, – 25.2 kJ, – 3.56 kJ and + 31.5 kJ. What is the net work for this cyclic process ? [**Ans.** 17.34 kJ]

2. A domestic refrigerator is loaded with food and the door closed. During a certain period the machine consumes 1 kWh of energy and the internal energy of the system drops by 5000 kJ. Find the net heat transferred in the system. [**Ans.** – 8.6 MJ]

3. 1.5 kg of liquid having a constant specific heat of 2.5 kJ/kg°C is stirred in a *well-insulated* chamber causing the temperature to rise by 15°C. Find :

(*i*) Change in internal energy, and

(*ii*) Work done for the process. [**Ans.** (*i*) 56.25 kJ, $W = -\,56.25$ kJ]

4. A system is composed of a stone having a mass of 10 kg and a bucket containing 100 kg of water. Initially the stone and water are at the same temperature, the stone then falls into the water. Determine ΔU, ΔKE, ΔPE, ΔQ and ΔW for the following cases :

(*i*) At the instant the stone is about to enter the water.

(*ii*) Just after the stone comes to rest in the bucket.

[**Ans.** (*i*) $\Delta Q = \Delta W = \Delta E = 0$, $\Delta KE = 4.184$ kJ, $\Delta PE = -4.184$ kJ ;
(*ii*) $\Delta Q = 0$, $\Delta W = 0$, $\Delta KE = 0$, $\Delta U = +4.184$ kJ, $\Delta PE = -4.184$ kJ]

5. A closed system of constant volume experiences a temperature rise of 20°C when a certain process occurs. The heat transferred in the process is 18 kJ. The specific heat at constant volume for the pure substance comprising the system is 1.2 kJ/kg°C, and the system contains 2 kg of this substance. Determine the change in internal energy and the work done. [**Ans.** $\Delta U = 48$ kJ ; $W = -30$ kJ]

6. A stationary mass of gas is compressed without friction from an initial state of 2 m^3 and 2×10^5 N/m^2 to a final state of 1 m^3 and 2×10^5 N/m^2, the pressure remaining the same. There is a transfer of 360 kJ of heat from the gas during the process. How much does the internal energy of the gas change ? [**Ans.** $\Delta U = -160$ kJ]

7. The internal energy of a certain substance is given by the following equation :

$$u = pv + 84$$

where u is given in kJ/kg, p is in kPa and v is in m^3/kg.

A system composed of 3 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.22 m^3 to a final pressure 100 kPa in a process in which pressure and volume are related by $pv^{1.2}$ = constant.

(*i*) If the expansion is quasi-static, find Q, ΔU and W for the process.

(*ii*) In another process the same system expands according to the same pressure-volume relationship as in part (*i*) and from the same initial state to the same final state as in part (*i*) but the heat transfer in this case is 30 kJ. Find the work transfer for this process.

(*iii*) Explain the difference in work transfer in parts (*i*) and (*ii*).

[**Ans.** (*i*) $\Delta U = -91$ kJ, $W = 127.5$ kJ, $Q = 36.5$ kJ
(*ii*) $W = 121$ kJ, (*iii*) The work in (*ii*) in not equal
to $\int pdV$ since the process is not quasi-static.]

8. A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is linear function of the volume ($p = a + bV$). The internal energy of the fluid is given by the following equation

$$U = 34 + 3.15\, pV$$

where U is in kJ, p in kPa and V in cubic metre. If the fluid changes from an initial state of 170 kPa, 0.03 m^3 to a final state of 400 kPa, 0.06 m^3, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

[**Ans.** $W_{1-2} = 10.35$ kJ ;
$Q_{1-2} = 69.85$ kJ (heat flows into the system during the process)]

9. A piston cylinder arrangement has a gas in the cylinder space. During a constant pressure expansion to a larger volume the work effect for the gas are 1.6 kJ, the heat added to the gas and cylinder arrangement is 3.2 kJ and the friction between the piston and cylinder wall amounts to 0.24 kJ. Determine the change in internal energy of the entire apparatus (gas, cylinder, piston). [**Ans.** 1.84 kJ]

10. A system receives 42 kJ of heat while expanding with volume change of 0.123 m^3 against an atmosphere of 12 N/cm^2. A mass of 80 kg in the surroundings is also lifted through a distance of 6 metres.

(*i*) Find the change in energy of the system.

(*ii*) The system is returned to its initial volume by an adiabatic process which requires 100 kJ of work. Find the change in energy of system.

(*iii*) Determine the total change in energy of the system. [**Ans.** (*i*) 22.54 kJ, (*ii*) 100 kJ, (*iii*) 122.54 kJ]

11. A thermally insulated battery is being discharged at atmospheric pressure and constant volume. During a 1 hour test it is found that a current of 50 A and 2 V flows while the temperature increases from 20°C to 32.5°C. Find the change in internal energy of the cell during the period of operation. [**Ans.** -36×10^4 J]

12. In a certain steam plant the turbine develops 1000 kW. The heat supplied to the steam in the boiler is 2800 kJ/kg, the heat received by the system from cooling water in the condenser is 2100 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 5 kW. Calculate the steam flow round the cycle in kg/s. [**Ans.** 1.421 kg/s]

13. In the compression stroke of an internal-combustion engine the heat rejected to the cooling water is 45 kJ/kg and the work input is 90 kJ/kg. Calculate the change in internal energy of the working fluid stating whether it is a gain or a loss. [**Ans.** 45 kJ/kg (gain)]

14. 85 kJ of heat are supplied to a system at constant volume. The system rejects 90 kJ of heat at constant pressure and 20 kJ of work is done on it. The system is brought to its original state by adiabatic process. Determine the adiabatic work. Determine also the value of internal energy at all end states if initial value is 100 kJ. [**Ans.** W = 15 kJ ; U_1 = 100 kJ, U_2 = 185 kJ ; U_3 = 115 kJ]

15. A closed system undergoes a reversible process at a constant pressure process of 3.5 bar and its volume changes from 0.15 m^3 to 0.06 m^3. 25 kJ of heat is rejected by the system during the process. Determine the change in internal energy of the system. [**Ans.** 6.5 kJ (increase)]

16. An air compressor takes in air at 10^5 Pa and 27°C having volume of 1.5 m^3/kg and compresses it to 4.5×10^5 Pa. Find the work done, heat transfer and change in internal energy if the compression is isothermal.

[**Ans.** – 225 kJ ; – 225 kJ ; $\Delta U = 0$]

17. A cylinder fitted with piston contains 0.2 kg of N_2 at 100 kPa and 30°C. The piston is moved compressing N_2 until the pressure becomes 1 MPa and temperature becomes 150°C. The work done during the process is 20 kJ. Determine the heat transferred from N_2 to the surroundings. Take c_v = 0.75 kJ/kg K for N_2.

[**Ans.** – 2 kJ]

18. A closed system consisting of 1 kg of gaseous CO_2 undergoes a reversible process at constant pressure causing a decrease of 30 kJ in internal energy. Determine the work done during the process. Take c_p = 840 J/kg°C and c_v = 600 J/kg°C. [**Ans.** – 12 kJ]

19. The specific heat at constant pressure of one kg fluid undergoing a non-flow constant pressure process is given by

$$c_p = \left[2.5 + \frac{40}{T + 20}\right] \text{ kg/kg°C}$$

where T is in °C.

The pressure during the process is maintained at 2 bar and volume changes from 1 m^3 to 1.8 m^3 and temperature changes from 50°C to 450°C. Determine :

(*i*) Heat added (*ii*) Work done

(*iii*) Change in internal energy (*iv*) Change in enthalpy.

[**Ans.** (*i*) 1076 kJ ; (*ii*) 160 kJ ; (*iii*) 916 kJ ; (*iv*) 1076 kJ]

20. 1 kg of nitrogen (molecular weight 28) is compressed reversibly and isothermally from 1.01 bar, 20°C to 4.2 bar. Calculate the work done and the heat flow during the process. Assume nitrogen to be a perfect gas.

[**Ans.** W = 124 kJ/kg ; Q = – 124 kJ/kg]

21. Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m^3, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate :

(*i*) The final temperature (*ii*) The final volume

(*iii*) The work done on the mass of air in the cylinder. [**Ans.** (*i*) 234.5°C, (*ii*) 0.00388 m^3 ; (*iii*) 2.76 kJ]

22. 1 kg of a perfect gas is compressed from 1.1 bar, 27°C according to a law $pv^{1.3}$ = constant, until the pressure is 6.6. bar. Calculate the heat flow to or from the cylinder walls,

(*i*) When the gas is ethane (molecular weight 30), which has

c_p = 1.75 kJ/kg K.

(*ii*) When the gas is argon (molecular weight 40), which has

c_p = 0.515 kJ/kg K. [**Ans.** (*i*) 84.5 kJ/kg, (*ii*) – 59.4 kJ/kg]

23. 1 kg of air at 1 bar, 15°C is compressed reversibly and adiabatically to a pressure of 4 bar. Calculate the final temperature and the work done on the air. [**Ans.** 155°C ; 100.5 kJ/kg]

24. A certain perfect gas is compressed reversibly from 1 bar, 17°C to a pressure of 5 bar in a perfectly thermally insulated cylinder, the final temperature being 77°C. The work done on the gas during the compression is 45 kJ/kg. Calculate γ, c_v, R and the molecular weight of the gas.

[**Ans.** 1.132 ; 0.75 kJ/kg K ; 0.099 kJ/kg K ; 84]

25. 1 kg of air at 1.02 bar, 20°C is compressed reversibly according to a law $pv^{1.3}$ = constant, to a pressure of 5.5 bar. Calculate the work done on the air and heat flow to or from the cylinder walls during the compression. [**Ans.** 133.5 kJ/kg ; – 33.38 kJ/kg]

26. 0.05 kg of carbon dioxide (molecular weight 44), occupying a volume of 0.03 m^3 at 1.025 bar, is compressed reversibly until the pressure is 6.15 bar. Calculate final temperature, the work done on the CO_2, the heat flow to or from the cylinder walls,

(*i*) When the process is according to law $pv^{1.4}$ = constant,

(*ii*) When the process is isothermal,

(*iii*) When the process takes place in a perfectly thermally insulated cylinder.

Assume CO_2 to be a perfect gas, and take $\gamma = 1.3$. [**Ans.** 270°C ; 5.138 kJ ; 1.713 kJ ; 52.6°C ; 5.51 kJ ; – 5.51 kJ ; 219°C ; 5.25 kJ ; 0 kJ]

27. Oxygen (molecular weight 32) is compressed reversibly and polytropically in a cylinder from 1.05 bar, 15°C to 4.2 bar in such a way that one-third of the work input is rejected as heat to the cylinder walls. Calculate the final temperature of the oxygen.

Assume oxygen to be a perfect gas and take c_v = 0.649 kJ/kg K. [**Ans.** 113°C]

28. A cylinder contains 0.5 m^3 of a gas at 1×10^5 N/m^2 and 90°C. The gas is compressed to a volume of 0.125 m^3, the final pressure being 6×10^5 N/m^2. Determine :

(*i*) The mass of gas.

(*ii*) The value of index '*n*' for compression.

(*iii*) The increase in internal energy of gas.

(*iv*) The heat received or rejected by the gas during compression.

($\gamma = 1.4$, R = 294.2 Nm/kg°C). [**Ans.** 0.468 kg ; 1.292 ; 62.7 kJ ; – 22.67 kJ]

Steady Flow Systems

29. 12 kg of a fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are p_1 = 1.4 bar, ρ_1 = 25 kg/m^3, C_1 = 120 m/s and u_1 = 920 kJ/kg and at the exit are p_2 = 5.6 bar, ρ_2 = 5 kg/m^3, C_2 = 180 m/s and u_2 = 720 kJ/kg. During the passage, the fluid rejects 60 kJ/s and rises through 60 metres. Determine : (*i*) the change in enthalpy (Δh) and (*ii*) work done during the process (W).

[**Ans.** Δh = – 93.6 kJ/kg ; W = – 44.2 kW]

30. In the turbine of a gas turbine unit the gases flow through the turbine is 17 kg/s and the power developed by the turbine is 14000 kW. The enthalpies of the gases at inlet and outlet are 1200 kJ/kg and 360 kJ/kg respectively, and the velocities of the gases at inlet and outlet are 60 m/s and 150 m/s respectively. Calculate the rate at which the heat is rejected from the turbine. Find also the area of the inlet pipe given that the specific volume of the gases at inlet is 0.5 m^3/kg. [**Ans.** 119.3 kW (heat rejected) ; 0.142 m^3]

31. Air flows steadily at the rate of 0.4 kg/s through an air compressor, entering at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m^3/kg, and leaving at 4.5 m/s with a pressure of 6.9 bar and a specific volume of 0.16 m^3/kg. The internal energy of air leaving is 88 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 59 kJ/s. Calculate the power required to drive the compressor and the inlet and outlet pipe cross-sectional areas.

[**Ans.** 104.4 kW ; 0.057 m^2 ; 0.014 m^2]

32. A turbine operating under steady flow conditions receives steam at the following state : pressure 13.8 bar ; specific volume 0.143 m^3/kg ; internal energy 2590 kJ/kg ; velocity 30 m/s. The state of the steam leaving the turbine is : pressure 0.35 bar ; specific volume 4.37 m^3/kg ; internal energy 2360 kJ/kg ; velocity 90 m/s. Heat is lost to the surroundings at the rate of 0.25 kJ/s. If the rate of steam flow is 0.38 kg/s, what is the power developed by the turbine ? [**Ans.** 102.8 kW]

33. A nozzle is a device for increasing the velocity of a steadily flowing stream of fluid. At the inlet to a certain nozzle the enthalpy of the fluid is 3025 kJ/kg and the velocity is 60 m/s. At the exit from the nozzle the enthalpy is 2790 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.

(*i*) Find the velocity at the nozzle exit.

(*ii*) If the inlet area is 0.1 m^2 and specific volume at inlet is 0.19 m^3/kg, find the rate of flow of fluid.

(*iii*) If the specific volume at the nozzle exit is 0.5 m^3/kg, find the exit area of the nozzle.

[**Ans.** 688 m/s ; 31.6 kg/s ; 0.0229 m^2]

34. A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa and an enthalpy of 534.5 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor.

(*i*) Evaluate the external work done per unit mass of gas assuming the gas velocities at entry and exit to be negligible.

(*ii*) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 80 m/s and that at exit is 160 m/s. [**Ans.** 143.3 kJ/kg, 152.9 kJ/kg]

35. A turbine, operating under steady-flow conditions, receives 5000 kg of steam per hour. The steam enters the turbine at a velocity of 3000 m/min, an elevation of 5 m and a specific enthalpy of 2787 kJ/kg. It leaves the turbine at a velocity of 6000 m/min, an elevation of 1 m and a specific enthalpy of 2259 kJ/kg. Heat losses from the turbine to the surroundings amount to 16736 kJ/h.

Determine the power output of the turbine. [**Ans.** 723 kW]

36. In a steady flow process, the working fluid flows at a rate of 240 kg/min. The fluid rejects 120 kJ/s passing through the system. The conditions of fluid at inlet and outlet are given as : C_1 = 300 m/s, p_1 = 6.2 bar, u_1 = 2100 kJ/kg, v_1 = 0.37 m^3/kg and C_2 = 150 m/s, p_2 = 1.3 bar, u_2 = 1500 kJ/kg, v_2 = 1.2 m^3/kg. The suffix 1 indicates the conditions at inlet and 2 indicates at outlet of the system. Neglecting the change in potential energy, determine the power capacity of the system in MW. [**Ans.** 2.7086 MW]

37. Steam enters a turbine at 20 m/s and specific enthalpy of 3000 kJ/kg and leaves the turbine at 40 m/s and specific enthalpy of 2500 kJ/kg. Heat lost to the surroundings is 25 kJ/kg of steam as the steam passes through the turbine. If the steam flow rate is 360000 kg/h, determine the output from the turbine in MW.

[**Ans.** 47.44 MW]

38. A stream of gases at 7.5 bar, 800°C and 150 m/s is passed through a turbine of a jet engine. The stream comes out of the turbine at 2.0 bar, 600°C and 300 m/s. The process may be assumed adiabatic. The enthalpies of gas at the entry and exit of the turbine are 960 kJ/kg and 700 kJ/kg gas respectively.

Determine the capacity of the turbine if the gas flow is 4 kg/s. [**Ans.** 905 kW]

39. In a steam power plant 1.5 kg of water is supplied per second to the boiler. The enthalpy and velocity of water entering into the boiler are 800 kJ/kg and 10 m/s. Heat at the rate of 2200 kJ per kg of water is supplied to the water. The steam after passing through the turbine comes out with a velocity of 50 m/s and enthalpy of 2520 kJ/kg. The boiler inlet is 5 m above the turbine exit. The heat loss from the boiler is 1800 kJ/min and from the turbine 600 kJ/min.

Determine the power capacity of the turbine, considering boiler and turbine as single unit.

[**Ans.** 678 kW]

40. 15 kg of air per minute is delivered by a centrifugal compressor. The inlet and outlet conditions of air are : C_1 = 10 m/s, p_1 = 1 bar, v_1 = 0.5 m^3/kg and C_2 = 80 m/s, p_2 = 7 bar, v_2 = 0.15 m^3/kg. The increase in enthalpy of air passing through the compressor is 160 kJ/kg, and heat loss to the surroundings is 720 kJ/min. Assuming that inlet and discharge lines are at the same level, find :

(*i*) Motor power required to drive the compressor.

(*ii*) Ratio of inlet to outlet pipe diameter.

$$\left[\textbf{Ans. } (i)\ 52.78 \text{ kW } (ii)\ \frac{d_1}{d_2} = 5.16\right]$$

41. A centrifugal air compressor used in gas turbine receives air at 100 kPa and 300 K and it discharges air at 400 kPa and 500 K. The velocity of air leaving the compressor is 100 m/s. Neglecting the velocity at the entry of the compressor, determine the power required to drive the compressor if the mass flow rate is 15 kg/s. Take c_p (air) = 1 kJ/kg K and assume that there is no heat transfer from the compressor to the surroundings.

[**Ans.** 3075 kW]

42. In a water cooled compressor 0.5 kg of air is compressed per second. A shaft input of 60 kW is required to run the compressor. Heat lost to the cooling water is 30 per cent of input and 10 per cent of the input is lost in bearings and other frictional effects. Air enters the compressor at 1 bar and 20°C. Neglecting the changes in kinetic energy and potential energy, determine the exit air temperature. Take c_p = 1 kJ/kg°C air.

Consider steady flow process. [**Ans.** 92°C]

43. Steam at 7 bar and 200°C enters an insulated convergent divergent nozzle with a velocity of 60 m/s. It leaves the nozzle at a pressure of 1.4 bar and enthalpy of 2600 kJ/kg.

Determine the velocity of the steam at exit. [**Ans.** 701 m/s]

44. A petrol engine develops 50 kW brake power. The fuel and air-flow rates are 10 kg and 107 kg/h. The temperature of fuel-air mixture entering the engine is 20°C and temperature of gases leaving the engine is 500°C. The heat transfer rate from the engine to the jacket cooling water is 50 kJ/s and that to the surroundings is 10 kJ/s.

Evaluate the increase in the specific enthalpy of the mixture as it flows through the engine.

[**Ans.** – 110 kJ/s]

45. A compressor takes air at 100 kN/m^2 and delivers the same at 550 kN/m^2. The compressor discharges 16 m^3 of free air per minute. The densities of air at inlet and exit are 1.25 kg/m^3 and 5 kg/m^3. The power of the motor driving the compressor is 40 kW. The heat lost to the cooling water circulated around the compressor is 30 kJ/kg of air passing through the compressor.

Neglecting changes in P.E. and K.E. determine the change in specific internal energy. [**Ans.** 60 kJ/kg]

46. A centrifugal pump operating under steady flow conditions delivers 3000 kg of water per minute at 20°C. The suction pressure is 0.8 bar and delivery pressure is 3 bar. The suction pipe diameter is 15 cm and discharge pipe diameter is 10 cm. Find the capacity of the drive motor.

Neglect the change in internal energy and assume that the suction and discharge are at same level.

[**Ans.** 11.8 kW]

47. 60 kg of water is delivered by a centrifugal pump per second. The inlet and outlet pressures are 1 bar and 4 bar respectively. The suction is 2 m below the centre of the pump and delivery is 8 m above the centre of the pump. Determine the capacity of the electric motor to run the pump. The suction and delivery pipe diameters are 20 cm and 10 cm and respectively. [**Ans.** 27.15 kW]

48. The air speed of a turbojet engine in flight is 270 m/s. Ambient air temperature is – 15°C. Gas temperature outlet of the nozzle is 600°C. Corresponding enthalpy values for air and gas are respectively 260 and 912 kJ/kg. Fuel air ratio is 0.0190. Chemical energy of the fuel is 44.5 MJ/kg. Owing to incomplete combustion 5% of the chemical energy is not released in the reaction. Heat loss from the engine is 21 kJ/kg of air.

Calculate the velocity of exhaust jet. [**Ans.** 560 m/s]

49. Air at a temperature of 15°C passes through a heat exchanger at a velocity of 30 m/s, where its temperature is raised to 800°C. It then enters a turbine with the same velocity of 30 m/s and expands until the temperature falls to 650°C. On leaving the turbine, the air is taken at a velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C. If the air flow rate is 2 kg/s, calculate (*i*) the rate of heat transfer to the air, (*ii*) the power output from the turbine assuming no heat loss, and (*iii*) the velocity at exit from nozzle, assuming no heat loss.

Take the enthalpy of air as $h = c_p t$, where c_p is the specific heat equal to 1.005 kJ/kg°C and t the temperature.

[**Ans.** 1580 kJ/s ; 298.8 kW ; 554 m/s]

Vapour (Steam)

50. 0.05 kg of steam is heated at a constant pressure of 2 bar until the volume occupied is 0.0658 m^3. Calculate the heat supplied and work done. [**Ans.** 18.25 kJ ; 4.304 kJ]

51. Steam at 7 bar and dryness fraction 0.9 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. Calculate the change of internal energy and the change of enthalpy per kg of steam. The heat supplied during the process is found to be 400 kJ/kg. Calculate the work done per kg of steam.

[**Ans.** 217.5 kJ/kg (gain) ; 245.7 kJ/kg ; 182.5 kJ/kg]

52. 1 kg of steam at 100 bar and 375°C expands reversibly in a perfectly thermally insulated cylinder behind a piston until pressure is 38 bar and the steam is then saturated.

Calculate the work done by the steam. [**Ans.** 169.7 kJ/kg]

53. In a steam engine the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.95, and the expansion follows the law $pv^{1.1}$ = constant, down to a pressure of 0.34 bar. Calculate the work done per kg of steam during the expansion, and the heat flow per kg of steam to or from the cylinder walls during the expansion. [**Ans.** 436 kJ/kg ; 155.6 kJ/kg (heat supplied)]

54. Steam at 19 bar is throttled to 1 bar and the temperature after throttling is found to be 150°C. Calculate the initial dryness fraction of the steam. [**Ans.** 0.989]

55. 1 kg of steam at 7 bar, entropy 6.5 kJ/kg K, is heated reversibly at constant pressure until the temperature is 250°C. Calculate the heat supplied, and show on a *T-s* diagram the area which represents the heat flow.
[**Ans.** 283 kJ/kg]

56. 1 kg of steam at 20 bar, dryness fraction 0.9, is heated reversibly at constant pressure to a temperature of 300°C.

Calculate the heat supplied and change of entropy and show the process on a *T-s* diagram, indicating the area which represents the heat flow. [**Ans.** 415 kJ/kg ; 0.8173 kJ/kg K]

57. Steam at 0.05 bar, 100°C is to be condensed completely by a reversible constant pressure process.

Calculate the heat to be removed per kg of steam and the change of entropy. Sketch the process on a *T-s* diagram and shade in the area which represents the heat flow. [**Ans.** 2550 kJ/kg ; 8.292 kJ/kg K]

58. 0.05 kg of steam at 10 bar, dryness fraction 0.84, is heated reversibly in a rigid vessel until the pressure is 20 bar.

Calculate the change of entropy and the heat supplied. Show the area which represents the heat supplied on a *T-s* diagram. [**Ans.** 0.0704 kJ/kg K ; 36.85 kJ]

59. 1 kg of steam undergoes a reversible isothermal process from 20 bar and 250°C to a pressure of 30 bar. Calculate the heat flow, stating whether it is supplied or rejected and sketch the process on a *T-s* diagram.
[**Ans.** – 135 kJ/kg]

60. Steam at 5 bar, 250°C, expands isentropically to a pressure of 0.7 bar. Calculate the final condition of steam.
[**Ans.** 0.967]

61. Steam expands reversibly in a cylinder behind a piston from 6 bar dry saturated, to a pressure of 0.65 bar. Assuming that the cylinder is perfectly thermally insulated, calculate the work done during the expansion per kg of steam. Sketch the process on a *T-s* diagram. [**Ans.** 323.8 kJ/kg]

62. A steam engine receives steam at 4 bar, dryness fraction 0.8, and expands it according to a law $pv^{1.05}$ = constant to a condenser pressure of 1 bar. Calculate the change of entropy per kg of steam during the expansion, and sketch the process on a *T-s* diagram. [**Ans.** 0.381 kJ/kg K]

63. Steam at 15 bar is throttled to 1 bar and a temperature of 150°C. Calculate the initial dryness fraction and the change of entropy. Sketch the process on a *T-s* diagram and state the assumptions made in the throttling process. [**Ans.** 0.992, 1.202 kJ/kg K]

64. Steam enters a turbine at 70 bar, 500°C and leaves at 2 bar in a dry saturated state. Calculate the isentropic efficiency and effectiveness of the process. Neglect changes of kinetic and potential energy and assume that the process is adiabatic.

The atmospheric temperature is 17°C. [**Ans.** 84.4% ; 88%]

65. Steam at 10 bar and 250°C expands until the pressure becomes 2.75 bar. The dryness fraction of the steam at the end of expansion is 0.95. Determine the change in internal energy. [**Ans.** – 273 kJ/kg]

66. Calculate the quantity of heat required to form 2.5 kg of dry steam at 11 bar from water at 20°C. Also determine the amount of heat removed at constant pressure to cause the steam to become 0.95 dry. Calculate the specific volume at the respective conditions
[**Ans.** 6740 kJ ; 250 kJ ; 0.1775 m^3/kg ; 0.167 m^3/kg]

67. Steam at 10 bar and 0.95 dryness is available. Determine the final condition of steam in each of the following operations :

(*i*) 160 kJ of heat is removed at constant pressure ;

(*ii*) It is cooled at constant volume till the temperature inside falls to 140°C.

(*iii*) Steam expands isentropically in a steam turbine developing 300 kJ of work per kg of steam when the exit pressure of steam is 0.5 bar. [**Ans.** (*i*) 0.874 ; (*ii*) 0.367 ; (*iii*) 0.882]

68. Calculate the internal energy of 0.3 m^3 of steam at 4 bar and 0.95 dryness. If this steam is superheated at constant pressure through 30°C, determine the heat added and change in internal energy.
[**Ans.** 2451 kJ/kg ; 119 kJ ; 107.5 kJ/kg]

69. 1 kg of water at 30°C and 1 bar is heated at constant pressure until it becomes saturated vapour. Determine the change in volume, and internal energy during the process. [**Ans.** 1.694 m^3/kg (app.) ; 2380.6 kJ/kg]

70. Water is supplied to the boiler at 15 bar and 80°C and steam is generated at the same pressure at 0.9 dryness. Determine the heat supplied to the steam in passing through the boiler and change in entropy.
[**Ans.** 2260.5 kJ/kg ; 4.92 kJ/kg K]

71. A cylindrical vessel of 5 m^3 capacity contains wet steam at 1 bar. The volume of vapour and liquid in the vessel are 4.95 m^3 and 0.05 m^3 respectively. Heat is transferred to the vessel until the vessel is filled with saturated vapour. Determine the heat transfer during the process. [**Ans.** 104.93 MJ]

72. A closed vessel of 0.5 m^3 capacity contains dry saturated steam at 3.5 bar. The vessel is cooled until the pressure is reduced to 2 bar. Calculate :

(*i*) The mass of steam in the vessel.

(*ii*) Final dryness fraction of the steam, and

(*iii*) The amount of heat transferred during the process. [**Ans.** (*i*) 0.955 kg ; (*ii*) 0.582 ; (*iii*) – 828 kJ]

73. A closed vessel of 0.3 m^3 capacity contains steam at 8 bar and 200°C ;

(*i*) Determine the mass of the steam in the vessel.

(*ii*) The vessel is cooled till the steam becomes just dry and saturated. What will be the pressure of the steam in the vessel at this stage ?

(*iii*) The vessel is further cooled till the temperature drops to 158.85°C. Determine the pressure and condition of the steam. [**Ans.** (*i*) 1.2 kg ; (*ii*) 7.362 bar ; (*iii*) 6 bar, 0.826]

74. 0.5 kg of steam at 4 bar is contained in a cylinder fitted with a piston. The initial volume of steam is 0.1 m^3. Heat is transferred to the steam at constant pressure until the temperature becomes 300°C. Determine the heat transfer and work done during the process. [**Ans.** 771 kJ ; 91 kJ]

75. A quantity of steam at 13 bar and 0.8 dryness occupies 0.1 m^3. Determine the heat supplied to raise the temperature of the steam to 250°C at constant pressure and percentage of this heat which appears as external work. Take specific heat for superheated steam as 2.2 kJ/kg K. [**Ans.** 423 kJ/kg ; 15.3%]

76. A certain quantity of dry and saturated steam at 1.5 bar occupies initially a volume of 2.32 m^3. It is compressed until the volume is halved :

(*i*) Isothermally,

(*ii*) As per the law pv = constant, determine the final condition of steam in each case.

Also determine the heat rejected during the isothermal compression process.

[**Ans.** (*i*) 0.5, 2226.5 kJ ; (*ii*) 0.956]

77. Steam enters a turbine at a pressure of 10 bar and 300°C with a velocity of 50 m/s. The steam leaves the turbine at 1.5 bar and with a velocity of 200 m/s. Assuming the process to be reversible adiabatic and neglecting the change in potential energy, determine the work done per kg of steam flow through the turbine. [**Ans.** 375.55 kJ/kg]

78. Steam at 10 bar and 300°C passing through a convergent divergent nozzle expands reversibly and adiabatically till the pressure falls to 2 bar. If the velocity of steam entering into the nozzle is 50 m/s, determine the exit velocity of the steam. [**Ans.** 832 m/s]

Unsteady Flow Processes

79. An air receiver of volume 6 m^3 contains air at 15 bar and 40.5°C. A valve is opened and some air is allowed to blow out to atmosphere. The pressure of the air in the receiver drops rapidly to 12 bar when the valve is then closed. Calculate the mass of air which has left the receiver. [**Ans.** 14.7 kg]

80. The internal energy of air is given, at ordinary temperatures, by $u = u_0 + 0.718t$

where u is in kJ/kg, u_0 is any arbitrary value of u at 0°C, kJ/kg and t is temperature in °C.

Also for air, $pv = 0.287\ (t + 273)$, where p is in kPa and v is in m^3/kg.

(*i*) An evacuated bottle is fitted with a valve through which air from the atmosphere, at 760 mm Hg and 25°C, is allowed to flow slowly to fill the bottle. If no heat is transferred to or from the air in the bottle, what will its temperature be when the pressure in the bottle reaches 760 mm Hg ?

(*ii*) If the bottle initially contains 0.03 m^3 of air at 400 mm Hg and 25°C, what will the temperature be when the pressure in the bottle reaches 760 mm of Hg ? [**Ans.** (*i*) 144.2°C ; (*ii*) 71.6°C]

5

Second Law of Thermodynamics and Entropy

5.1. LIMITATIONS OF FIRST LAW OF THERMODYNAMICS AND INTRODUCTION TO SECOND LAW

It has been observed that *energy can flow* from a system in the form of *heat* or *work*. The first law of thermodynamics sets no limit to the amount of the total energy of a system which can be caused to flow out as work. A limit is imposed, however, as a result of the principle enunciated in the second law of thermodynamics which states that heat will flow naturally from one energy reservoir to another at a lower temperature, but not in opposite direction without assistance. This is very important because a heat engine operates between two energy reservoirs at different temperatures.

Further the first law of thermodynamics *establishes equivalence between the quantity of heat used and the mechanical work but does not specify the conditions under which conversion of heat into work is possible, neither the direction in which heat transfer can take place*. This gap has been *bridged* by the second law of thermodynamics.

5.2. PERFORMANCE OF HEAT ENGINES AND REVERSED HEAT ENGINES

Refer Fig. 5.1 (*a*). A *heat engine* is used to produce the maximum work transfer from a given positive heat transfer. The measure of success is called the *thermal efficiency* of the engine and is defined by the ratio :

$$\text{Thermal efficiency, } \eta_{th} = \frac{W}{Q_1} \qquad ...(5.1)$$

where, W = Net work transfer from the engine, and

Q_1 = Heat transfer to engine.

For a *reversed heat engine* [Fig. 5.1 (*b*)] acting as a *refrigerator* when the purpose is to achieve the maximum heat transfer from the cold reservoir, the measure of success is called the *co-efficient of performance* (C.O.P.). It is defined by the ratio :

Co-efficient of performance, $(\text{C.O.P.})_{ref.} = \dfrac{Q_2}{W}$ ...(5.2)

where, Q_2 = Heat transfer *from cold reservoir*, and

W = The net work transfer to the refrigerator.

For a **reversed heat engine** [Fig. 5.1 (*b*)] acting as a *heat pump*, the measure of success is again called the *co-efficient of performance*. It is defined by the ratio :

Co-efficient of performance, $(\text{C.O.P.})_{heat\ pump} = \dfrac{Q_1}{W}$ ...(5.3)

where, Q_1 = Heat transfer *to hot reservoir*, and

W = Net work transfer to the heat pump.

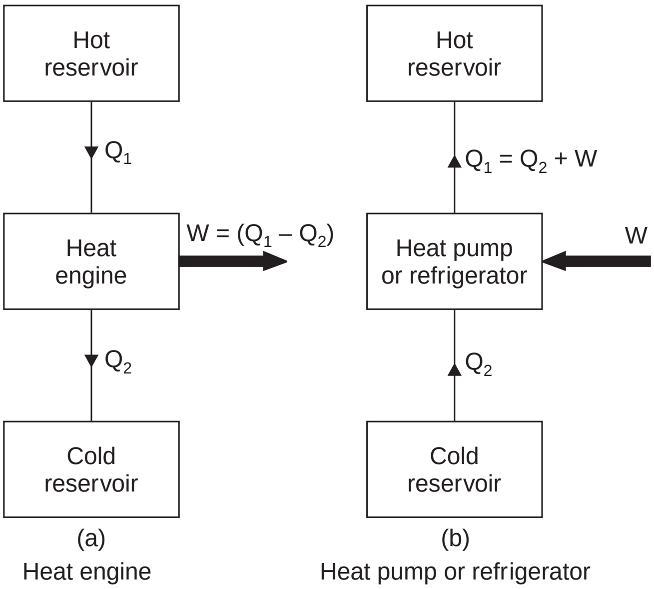


Fig. 5.1

In all the above three cases application of the first law gives the relation $Q_1 - Q_2 = W$, and this can be used to rewrite the expressions for thermal efficiency and co-efficient of performance solely in terms of the heat transfers.

$$\eta_{th} = \frac{Q_1 - Q_2}{Q_1} \qquad ..(5.4)$$

$$(C.O.P.)_{ref} = \frac{Q_2}{Q_1 - Q_2} \qquad ...(5.5)$$

$$(C.O.P.)_{heat\ pump} = \frac{Q_1}{Q_1 - Q_2} \qquad ..(5.6)$$

It may be seen that η_{th} is *always less than unity* and $(C.O.P.)_{heat\ pump}$ is *always greater than unity*.

5.3. REVERSIBLE PROCESSES

A reversible process should fulfill the following *conditions* :

1. The process should not involve friction of any kind.
2. Heat transfer should not take place with finite temperature difference.

3. The energy transfer as heat and work during the forward process should be identically equal to energy transfer as heat and work during the reversal of the process.
4. There should be no free or unrestricted expansion.
5. There should be no mixing of the fluids.
6. The process must proceed in a series of equilibrium states.

Some examples of *ideal reversible processes* are :

(*i*) Frictionless adiabatic expansion or compression ;
(*ii*) Frictionless isothermal expansion or compression ;
(*iii*) Condensation and boiling of liquids.

Some examples of *irreversible processes are* :

(*i*) Combustion process ;
(*ii*) Mixing of two fluids ;
(*iii*) All processes involving friction ;
(*iv*) Flow of electric current through a resistance ;
(*v*) Heat flow from a higher temperature to lower temperature.

Reversible processes are preferred because the devices which produce work such as engines and turbines, reversible process of the working fluid delivers *more work* than the corresponding irreversible processes. Also in case of fans, compressors, refrigerators and pumps *less power input* is required when *reversible processes* are used in place of corresponding irreversible ones.

In thermodynamic analysis concept of reversibility, though hypothetical, is very important because a reversible process is the *most efficient process*. Only reversible processes can be truely represented on property diagrams. Thermodynamic reversibility can only be approached but can *never* be achieved. Thus the main task of the engineer is to design the system which will evolve approximate reversible processes.

5.4. STATEMENTS OF SECOND LAW OF THERMODYNAMICS

The second law of thermodynamics has been enunciated meticulously by Clausius, Kelvin and Planck in slightly different words although both statements are basically identical. Each statement is based on an *irreversible process*. The *first considers transformation of heat between two thermal reservoirs* while the *second considers the transformation of heat into work*.

5.4.1. Clausius Statement

"*It is impossible for a self acting machine working in a cyclic process unaided by any external agency, to convey heat from a body at a lower temperature to a body at a higher temperature*".

In other words, heat of, itself, cannot flow from a colder to a hotter body.

5.4.2. Kelvin-Planck Statement

"*It is impossible to construct an engine, which while operating in a cycle produces no other effect except to extract heat from a single reservoir and do equivalent amount of work*".

Although the Clausius and Kelvin-Planck statements appear to be different, they are really equivalent in the sense that a *violation of either statement implies violation of other*.

5.4.3. Equivalence of Clausius Statement to the Kelvin-Planck Statement

Refer Fig. 5.2. Consider a higher temperature reservoir T_1 and low temperature reservoir T_2. Fig. 5.2 shows a heat pump which requires no work and transfers an amount of Q_2 from a low temperature to a higher temperature reservoir (in violation of the Clausius statement). Let an amount of heat Q_1 (greater than Q_2) be transferred from high temperature reservoir to heat engine which devolops a net work, $W = Q_1 - Q_2$ and rejects Q_2 to the low temperature reservoir. Since there is no heat interaction with the low temperature, it can be eliminated. The combined system of the heat engine and heat pump acts then like a heat engine exchanging heat with a single reservoir, which is the violation of the Kelvin-Planck statement.

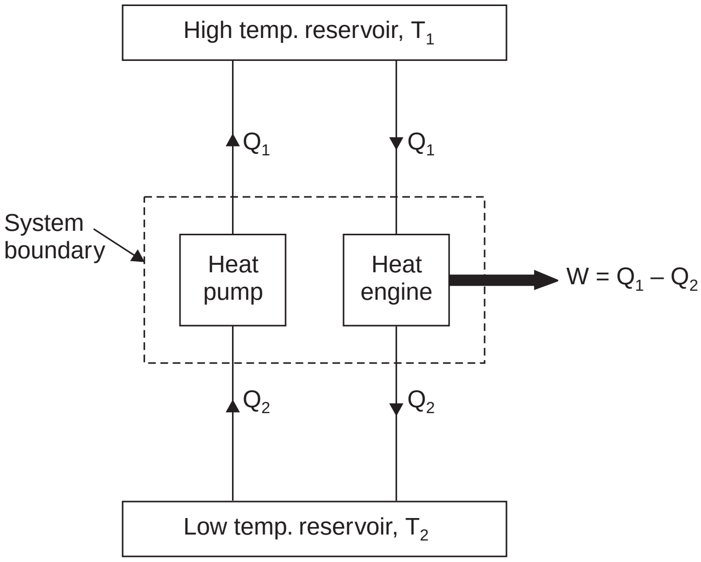


Fig. 5.2. Equivalence of Clausius statement to Kelvin-Planck statement.

5.5. PERPETUAL MOTION MACHINE OF THE SECOND KIND

— A machine which voilates the first law of thermodynamics is called the perpetual motion machine of the first kind (PMM1). Such a machine creates its own energy from nothing and *does not exist*.

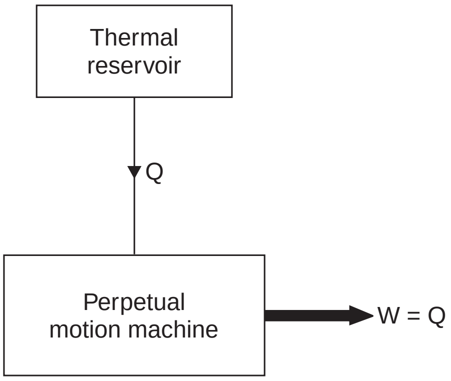


Fig. 5.3. Perpetual motion machine of second kind (PMM2).

— Without violating the first law, a machine can be imagined which would continuously absorb heat from a single thermal *reservoir and would convert this heat completely into work. The efficiency* of such a machine would be *100 per cent*. This machine is called the *perpetual motion machine of the second kind (PMM2)*.

Fig. 5.3 shows the perpetual motion machine of the second kind. *A machine of this kind will evidently violates the second law of thermodynamics*.

5.6. THERMODYNAMIC TEMPERATURE

Take the case of reversible heat engine operating between two reservoirs. Its thermal efficiency is given by the eqn. (5.4),

$$\eta_{th} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

The temperature of a reservoir remains uniform and fixed irrespective of heat transfer. This means that reservoir has only one property defining its state and the heat transfer from a reservoir is some function of that property, *temperature*. Thus $Q = \phi\ (K)$, where K is the temperature of reservoir. The choice of the function is universally accepted to be such that the relation,

$$\frac{Q_1}{Q_2} = \frac{\phi(K_1)}{\phi(K_2)} \text{ becomes } \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad ...(5.7)$$

where T_1 and T_2 are the thermodynamic temperatures of the reservoirs. Zero thermodynamic temperature (that temperature to which T_2 tends, as the heat transfer Q_2 tends to zero) has never been attained and *one form of third law of thermodynamics is the statement* :

"The temperature of a system cannot be reduced to zero in a finite number of processes."

After establishing the concept of a zero thermodynamic temperature, a reference reservoir is chosen and assigned a numerical value of temperature. Any other thermodynamic temperature may now be defined in terms of reference value and the heat transfers that would occur with reversible engine,

$$T = T_{ref.} \frac{Q}{Q_{ref.}} \qquad ...(5.8)$$

The determination of thermodynamic temperature cannot be made in this way as it is not possible to build a reversible engine. Temperatures are determined by the application of thermodynamic relations to other measurements.

The SI unit of thermodynamic temperature is the kelvin (*K*). The relation between thermodynamic temperature and celsius scale, which is in common use is :

Thermodynamic temperature = Celsius temperature + 273.15°.

The kelvin unit of thermodynamic temperature is the fraction $\frac{1}{273.15}$ of thermodynamic temperature of 'Triple point' of water.

5.7. CLAUSIUS INEQUALITY

When a reversible engine uses more than two reservoirs the third or higher numbered reservoirs will not be equal in temperature to the original two. Consideration of expression for efficiency of the engine indicates that for maximum efficiency, all the heat transfer should take place at maximum or minimum reservoir temperatures. Any intermediate reservoir used will, therefore, lower the efficiency of the heat engine. Practical engine cycles often involve continuous changes of temperature during heat transfer. A relationship among processes in which these sort of changes occur is necessary. The ideal approach to a cycle in which temperature continually changes is to consider the system to be in communication with a large number of reservoirs in procession. Each reservoir is considered to have a temperature differing by a small amount from the previous one. In such a model it is possible to imagine that each reservoir is replaced by a reversible heat engine in communication with standard reservoirs at same temperature T_0. Fig. 5.4 shows one example to this substitution.

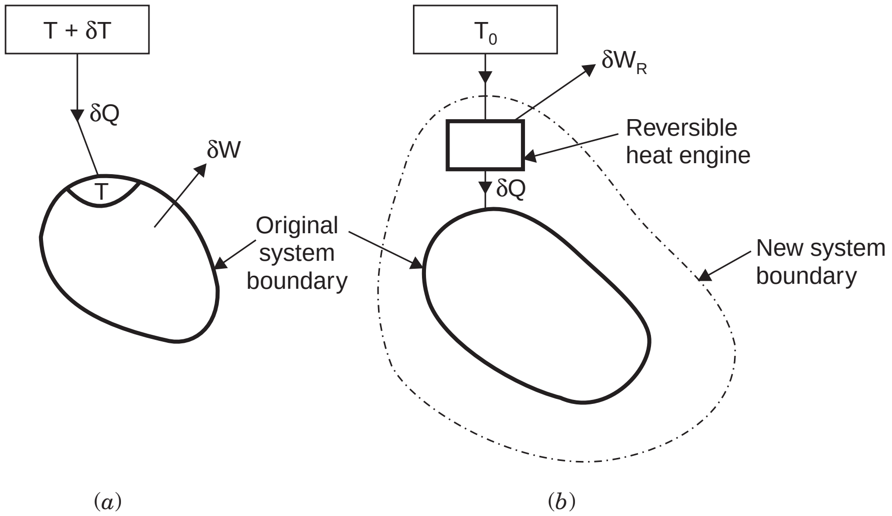


Fig. 5.4. The clausius inequality.

The system to which the heat transfer is effected is neither concerned with the source of energy it receives nor with the method of transfer, save that it must be reversible. Associated with the small heat transfer dQ to the original system is a small work transfer dW and for this system the first law gives

$$\sum_{\text{cycle}} (\delta Q - \delta W) = 0 \qquad ...(5.9)$$

Now consider the engine replacing the reservoirs and apply the second law to the new system in Fig. 5.4 (*b*). If the new system is not a perpetual motion machine of second kind, no positive work transfer is possible with a single reservoir.

Therefore,
$$\sum_{\text{cycle}} (\delta W - \delta W_R) \le 0 \qquad ...(5.10)$$

But by the definition of thermodynamic temperature in equation (5.8)

$$\frac{\delta W_R}{\delta Q} = \frac{\delta Q_0 - \delta Q}{\delta Q} = \frac{T_0 - T}{T} \qquad ...(5.11)$$

and by combination of eqns. (5.9), (5.10) and (5.11)

$$T_0 \sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \text{ but } T_0 \ne 0 \text{ and therefore ;}$$

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \qquad ..(5.12)$$

This is known as **Clausius inequality**.

Let us now consider the case of a reversible engine for which

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \text{ ,}$$

reverse the engine and for the reversible heat pump obtained it is possible to develop the expression,

$$-\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0$$

The *negative sign indicates that the heat transfers have all reversed in direction when the engine was reversed*. This means that for the same machine we have two relations which are only satisfied if in the reversible case,

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \qquad \text{...(5.13)}$$

For a reversible case, as the number of reservoirs used tends to infinity, the limiting value of the summation will be

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) = 0$$

In words, the Clausius inequality may be expressed as follows :

"*When a system performs a reversible cycle, then*

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) = 0,$$

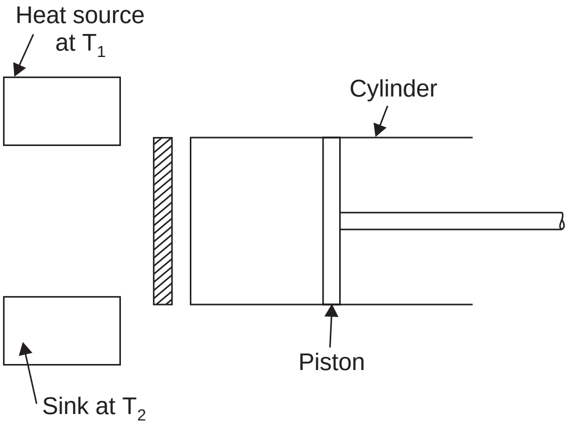
but when the cycle is not reversible

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) < 0 \text{''}.$$

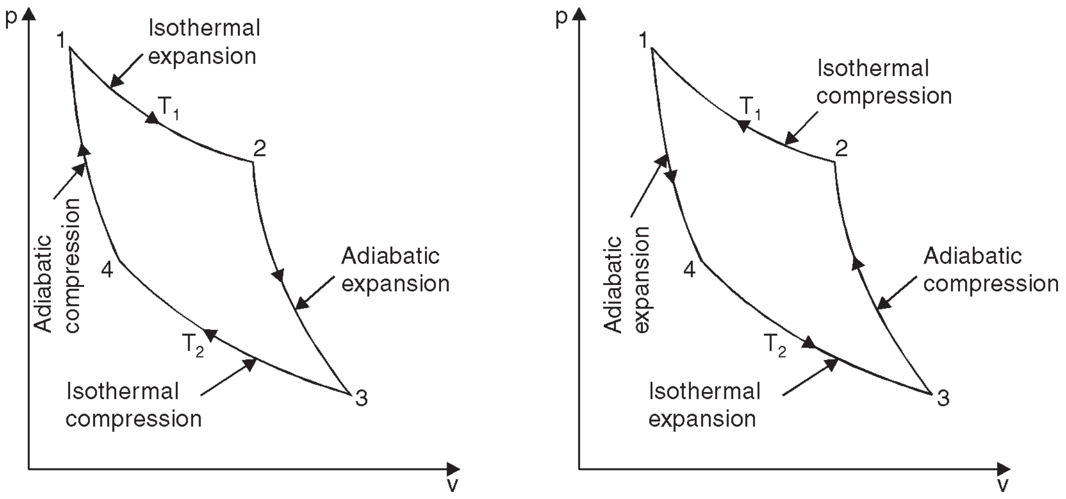
5.8. CARNOT CYCLE

The cycle was first suggested by a French engineer Sadi Carnot in 1824 which works on reversible cycle and is known as *Carnot cycle*.

Any fluid may be used to operate the Carnot cycle (Fig. 5.5) which is performed in an engine cylinder the head of which is supposed alternatively to be perfect conductor or a perfect insulator of a heat. Heat is caused to flow into the cylinder by the application of high temperature energy source to the cylinder head during expansion, and to flow from the cylinder by the application of a lower temperature energy source to the head during compression.



(*a*)



(*b*) Carnot engine cycle

(*c*) Carnot heat pump cycle

Fig. 5.5

The ***assumptions*** made for describing the working of the Carnot engine are as follows :

(*i*) The piston moving in a cylinder does not develop any friction during motion.

(*ii*) The walls of piston and cylinder are considered as perfect insulators of heat.

(*iii*) The cylinder head is so arranged that it can be a perfect heat conductor or perfect heat insulator.

(*iv*) The transfer of heat does not affect the temperature of source or sink.

(*v*) Working medium is a perfect gas and has constant specific heat.

(*vi*) Compression and expansion are reversible.

Following are the *four stages* of Carnot cycle :

Stage 1. (Process 1-2). Hot energy source is applied. Heat Q_1 is taken in whilst the fluid expands isothermally and reversibly at constant high temperature T_1.

Stage 2. (Process 2-3). The cylinder becomes a perfect insulator so that no heat flow takes place. The fluid expands adiabatically and reversibly whilst temperature falls from T_1 to T_2.

Stage 3. (Process 3-4). Cold energy source is applied. Heat Q_2 flows from the fluid whilst it is compressed isothermally and reversibly at constant lower temperature T_2.

Stage 4. (Process 4-1). Cylinder head becomes a perfect insulator so that no heat flow occurs. The compression is continued adiabatically and reversibly during which temperature is raised from T_2 to T_1.

The work delivered from the system during the cycle is represented by the enclosed area of the cycle. Again for a closed cycle, according to first law of the thermodynamics the work obtained is equal to the difference between the heat supplied by the source (Q_1) and the heat rejected to the sink (Q_2).

$$\therefore \qquad W = Q_1 - Q_2$$

Also, thermal efficiency, $\eta_{th} = \dfrac{\text{Work done}}{\text{Heat supplied by the source}} = \dfrac{Q_1 - Q_2}{Q_1}$

$$= 1 - \frac{Q_2}{Q_1} \left(= 1 - \frac{T_2}{T_1} \right) \qquad \left[\begin{array}{c} \because \quad Q_1 = m\, c_p\, T_1 \\ Q_2 = m\, c_p\, T_2 \\ \text{where, } m = \text{mass of fluid.} \end{array} \right]$$

Such an engine since it consists entirely of reversible processes, can operate in the reverse direction so that it follows the cycle shown in Fig. 5.5 (*b*) and operates as a **heat pump.** Q_2 is being taken in at the lower temperature T_2 during the isothermal expansion (process 4-3) and heat Q_1 is being rejected at the upper temperature T_1 (process 2-1). Work W will be needed to drive the pump. Again, the enclosed area represents this work which is exactly equal to that flowing from it when used as engine.

The Carnot cycle *cannot be performed in practice* because of the following **reasons** :

1. It is imposible to perform a frictionless process.
2. It is impossible to transfer the heat without temperature potential.
3. Isothermal process can be achieved only if the piston moves very slowly to allow heat transfer so that the temperature remains contant. Adiabatic process can be achieved only if the piston moves as fast as possible so that the heat transfer is negligible due to very short time available. The isothermal and adiabatic processes take place during the same stroke therefore the piston has to move very slowly for part of the stroke and it has to move very fast during remaining stroke. This variation of motion of the piston during the same stroke is not possible.

5.9. CARNOT'S THEOREM

"It states that of all engines operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine".

Refer Fig. 5.6.

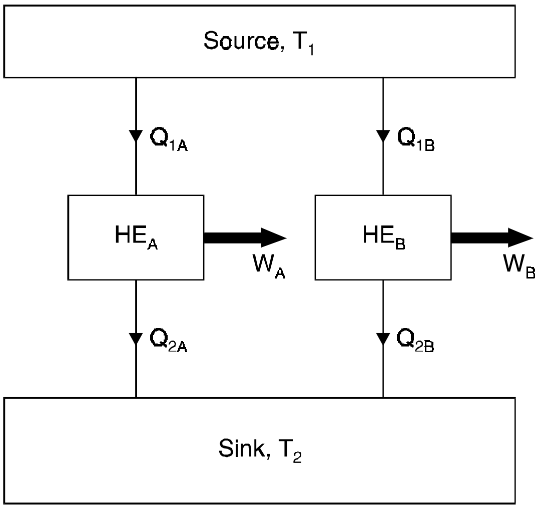


Fig. 5.6. Two cyclic heat engines HE_A and HE_B operating between the same source and sink, of which HE_B is reversible.

HE_A and HE_B are the two engines operating between the given source at temperature T_1 and the given sink at temperature T_2.

Let HE_A be *any* heat engine and HE_B be *any reversible* heat engine. We have to prove that efficiency of HE_B is more than that of HE_A. Let us assume that $\eta_A > \eta_B$. Let the rates of working of the engines be such that

$$Q_{1A} = Q_{1B} = Q_1$$

Since $$\eta_A > \eta_B$$

$$\frac{W_A}{Q_{1A}} > \frac{W_B}{Q_{1B}}$$

$\therefore$ $$W_A > W_B$$

Now, let HE_B be reversed. Since HE_B is a reversible heat engine, the magnitudes of heat and work transfer quantities will remain the same, but their directions will be reversed, as shown in Fig. 5.7. Since $W_A > W_B$, some part of W_A (equal to W_B) may be fed to drive the reversed heat engine $\exists H_B$. Since $Q_{1A} = Q_{1B} = Q_1$, the heat discharged by $\exists H_B$ may be supplied to HE_A. The source may, therefore, be eliminated (Fig. 5.8). The net result is that HE_A and $\exists H_B$ together constitute a heat engine which, operating in a cycle produces net work $W_A - W_B$ while exchanging heat with a single reservoir at T_2. This violates the Kelvin-Planck statement of the second law. Hence the assumption that $\eta_A > \eta_B$ is wrong.

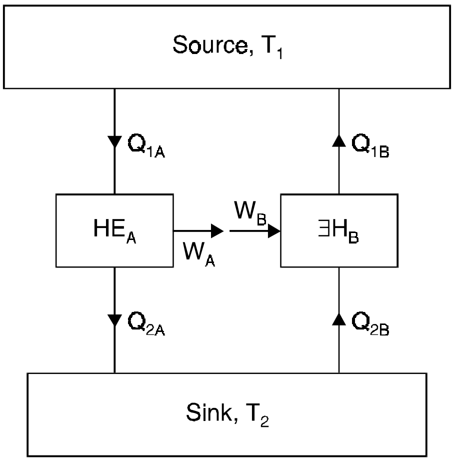


Fig. 5.7. HE_B is reversed.

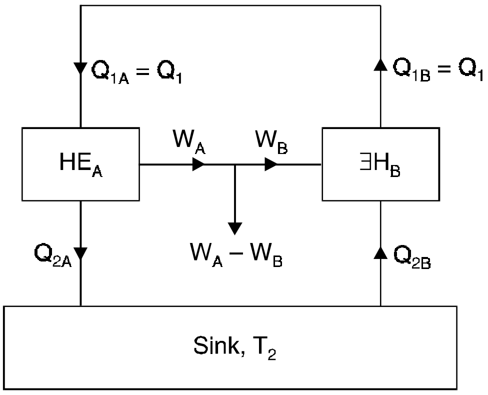


Fig. 5.8. HE_A and $\exists H_B$ together violate the Kelvin-Planck statement.

$\therefore$ $$\eta_B \geq \eta_A.$$

5.10. COROLLARY OF CARNOT'S THEOREM

"The efficiency of all reversible heat engines operating between the same temperature levels is the same".

Refer Fig. 5.6. Let both the heat engines HE_A and HE_B be *reversible*. Let us assume $\eta_A > \eta_B$. Similar to the procedure outlined in the Article 5.9, if HE_B is reversed to run say, as a heat pump using some part of the work output (W_A) of engine HE_A, we see that the combined system of heat pump HE_B and engine HE_A, becomes a *PMM*2. So η_A cannot be greater than η_B. Similary, if we assume $\eta_B > \eta_A$ and reverse the engine HE_A, we observe that η_B cannot be greater than η_A

$$\therefore \qquad \eta_A = \eta_B.$$

Since the efficiencies of all reversible engines operating between the same heat reservoirs are the same, *the efficiency of a reversible engine is independent of the nature or amount of the working substance undergoing the cycle.*

5.11. EFFICIENCY OF THE REVERSIBLE HEAT ENGINE

The efficiency of a reversible heat engine in which heat is received solely at T_1 is found to be

$$\eta_{rev.} = \eta_{max} = 1 - \left(\frac{Q_2}{Q_1}\right)_{rev.} = 1 - \frac{T_2}{T_1}$$

or

$$\eta_{rev.} = \frac{T_1 - T_2}{T_1}$$

From the above expression, it may be noted that as T_2 decreases and T_1 increases, the efficiency of the reversible cycle increases.

Since η is always less than unity, T_2 is always greater than zero and + ve.

The C.O.P. of a refrigerator is given by

$$(\text{C.O.P.})_{ref.} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

For a *reversible refrigerator*, using

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$(\text{C.O.P.})_{rev.} = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$\therefore \qquad [(\text{C.O.P.})_{ref.}]_{rev.} = \frac{T_2}{T_1 - T_2} \qquad ...(5.14)$$

Similarly, for a *reversible heat pump*

$$[(\text{C.O.P.})_{heat\ pump}]_{rev.} = \frac{T_1}{T_1 - T_2} \qquad ...(5.15)$$

Example 5.1. *A heat engine receives heat at the rate of 1500 kJ/min and gives an output of 8.2 kW. Determine :*

(*i*) *The thermal efficiency ;* (*ii*) *The rate of heat rejection.*

Solution. Heat received by the heat engine,

$$Q_1 = 1500 \text{ kJ/min}$$

$$= \frac{1500}{60} = 25 \text{ kJ/s}$$

Work output, $W = 8.2 \text{ kW} = 8.2 \text{ kJ/s}$.

(*i*) Thermal efficiency, $\eta_{th} = \frac{W}{Q_1}$

$$= \frac{8.2}{25} = 0.328 = 32.8\%$$

Hence, **thermal efficiency = 32.8%. (Ans.)**

(*ii*) Rate of heat rejection,

$$Q_2 = Q_1 - W = 25 - 8.2$$

$$= 16.8 \text{ kJ/s}$$

Hence, **the rate of heat rejection = 16.8 kJ/s. (Ans.)**

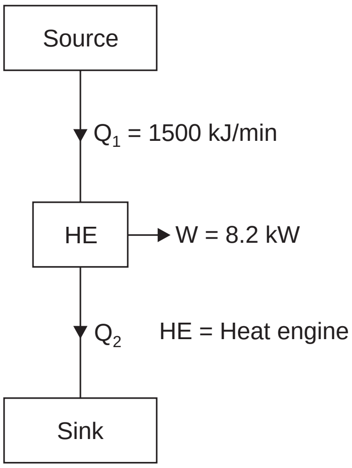


Fig. 5.9

☞**Example 5.2.** *During a process a system receives 30 kJ of heat from a reservoir and does 60 kJ of work. Is it possible to reach initial state by an adiabatic process ?*

Solution. Heat received by the system = 30 kJ

Work done = 60 kJ

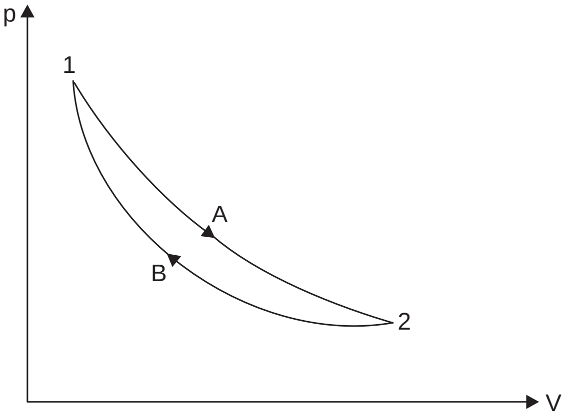


Fig. 5.10

Process 1-2 : By first law of thermodynamics,

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

$$30 = (U_2 - U_1) + 60 \qquad \therefore \qquad (U_2 - U_1) = -30 \text{ kJ}.$$

Process 2-1 : By first law of thermodynamics,

$$Q_{2-1} = (U_1 - U_2) + W_{2-1}$$

$$\therefore \qquad 0 = 30 + W_{2-1} \qquad \therefore \qquad W_{2-1} = -30 \text{ kJ}.$$

Thus 30 kJ work has to be done *on the system* to restore it to original state, by adiabatic process.

Example 5.3. *Find the co-efficient of performance and heat transfer rate in the condenser of a refrigerator in kJ/h which has a refrigeration capacity of 12000 kJ/h when power input is 0.75 kW.*

Solution. Refer Fig. 5.11.

Refrigeration capacity, Q_2 = 12000 kJ/h

Power input, W = 0.75 kW (= 0.75 × 60 × 60 kJ/h)

Co-efficient of performance, C.O.P. :

Heat transfer rate :

$$(\text{C.O.P.})_{refrigerator} = \frac{\text{Heat absorbed at lower temperature}}{\text{Work input}}$$

$$\therefore \qquad \text{C.O.P.} = \frac{Q_2}{W} = \frac{12000}{0.75 \times 60 \times 60} = 4.44$$

Hence **C.O.P. = 4.44. (Ans.)**

Hence transfer rate in condenser = Q_1

According to the first law

$Q_1 = Q_2 + W$ = 12000 + 0.75 × 60 × 60 = 14700 kJ/h

Hence, **heat transfer rate = 14700 kJ/h. (Ans.)**

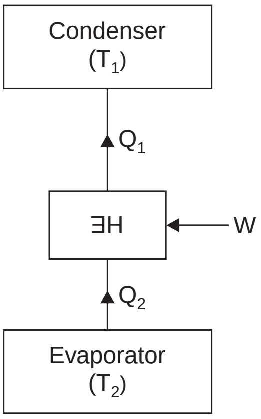


Fig. 5.11

Example 5.4. *A domestic food refrigerator maintains a temperature of – 12°C. The ambient air temperature is 35°C. If heat leaks into the freezer at the continuous rate of 2 kJ/s determine the least power necessary to pump this heat out continuously.*

Solution. Freezer temperature,

$$T_2 = -12 + 273 = 261 \text{ K}$$

Ambient air temperature,

$$T_1 = 35 + 273 = 308 \text{ K}$$

Rate of heat leakage into the freezer = 2 kJ/s

Least power required to pump the heat :

The refrigerator cycle removes heat from the freezer at the same rate at which heat leaks into it (Fig. 5.12).

For minimum power requirement

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}$$

$$\therefore \qquad Q_1 = \frac{Q_2}{T_2} \times T_1 = \frac{2}{261} \times 308 = 2.36 \text{ kJ/s}$$

$$\therefore \qquad W = Q_1 - Q_2$$

$$= 2.36 - 2 = 0.36 \text{ kJ/s} = 0.36 \text{ kW}$$

Hence, *least power required to pump the heat continuously*

= **0.36 kW. (Ans.)**

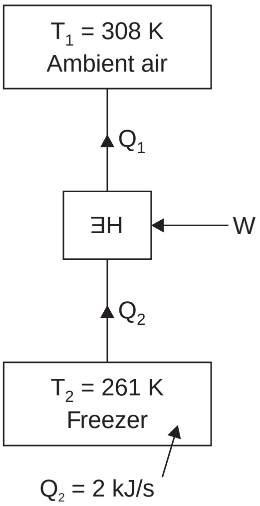


Fig. 5.12

Example 5.5. *A house requires 2 × 10^5 kJ/h for heating in winter. Heat pump is used to absorb heat from cold air outside in winter and send heat to the house. Work required to operate the heat pump is 3 × 10^4 kJ/h. Determine :*

(*i*) *Heat abstracted from outside ;*

(*ii*) *Co-efficient of performance.*

Solution. (*i*) Heat requirement of the house, Q_1 (or heat rejected)

$$= 2 \times 10^5 \text{ kJ/h}$$

Work required to operate the heat pump,

$$W = 3 \times 10^4 \text{ kJ/h}$$

Now, $$Q_1 = W + Q_2$$

where Q_2 is the heat abstracted from outside.

$$\therefore \quad 2 \times 10^5 = 3 \times 10^4 + Q_2$$

Thus $$Q_2 = 2 \times 10^5 - 3 \times 10^4$$
$$= 200000 - 30000 = 170000 \text{ kJ/h}$$

Hence, **heat abstracted from outside = 170000 kJ/h. (Ans.)**

(*ii*) $$(\text{C.O.P.})_{heat\ pump} = \frac{Q_1}{Q_1 - Q_2}$$
$$= \frac{2 \times 10^5}{2 \times 10^5 - 170000} = 6.66$$

Hence, **co-efficient of performance = 6.66. (Ans.)**

Note. If the heat requirements of the house were the same but this amount of heat had to be abstracted from the house and rejected out, *i.e., cooling of the house in summer*, we have

$$(\text{C.O.P.})_{refrigerator} = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W}$$
$$= \frac{170000}{3 \times 10^4} = \mathbf{5.66}$$

Thus the same device has two values of C.O.P. depending upon the objective.

Example 5.6. *What is the highest possible theoretical efficiency of a heat engine operating with a hot reservoir of furnace gases at 2100°C when the cooling water available is at 15°C ?*

Solution. Temperature of furnace gases, $T_1 = 2100 + 273 = 2373$ K

Temperature of cooling water, $T_2 = 15 + 273 = 288$ K

Now, $$\eta_{max} \ (= \eta_{carnot}) = 1 - \frac{T_2}{T_1} = 1 - \frac{288}{2373} = \mathbf{0.878 \text{ or } 87.8\%}. \quad \textbf{(Ans.)}$$

Note. It should be noted that a system in practice operating between similar temperatures (*e.g.,* a steam generating plant) would have a thermal efficiency of about 30%. The discrepency is *due to irreversibility in the actual plant*, and *also because of deviations from the ideal Carnot cycle made for various practical reasons*.

Example 5.7. *A Carnot cycle operates between source and sink temperatures of 250°C and – 15°C. If the system receives 90 kJ from the source, find :*

(*i*) *Efficiency of the system ;* (*ii*) *The net work transfer ;*

(*iii*) *Heat rejected to sink.*

Solution. Temperature of source, $T_1 = 250 + 273 = 523$ K

Temperature of sink, $T_2 = -15 + 273 = 258$ K

Heat received by the system, $Q_1 = 90$ kJ

(*i*) $$\eta_{\mathbf{carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{258}{523} = 0.506 = \mathbf{50.6\%}. \quad \textbf{(Ans.)}$$

(*ii*) **The net work transfer,** $W = \eta_{carnot} \times Q_1$ $\left[\because \ \eta_{carnot} = \dfrac{W}{Q_1}\right]$

$= 0.506 \times 90 =$ **45.54 kJ. (Ans.)**

(*iii*) **Heat rejected to the sink,** $Q_2 = Q_1 - W$ $[\because \ W = Q_1 - Q_2]$

$= 90 - 45.54 =$ **44.46 kJ. (Ans.)**

Example 5.8. *An inventor claims that his engine has the following specifications :*

Temperature limits	...... *750°C and 25°C*
Power developed	...... *75 kW*
Fuel burned per hour	...... *3.9 kg*
Heating value of the fuel	...... *74500 kJ/kg*

State whether his claim is valid or not.

Solution. Temperature of source, $T_1 = 750 + 273 = 1023$ K

Temperature of sink, $T_2 = 25 + 273 = 298$ K

We know that the thermal efficiency of Carnot cycle is the maximum between the specified temperature limits.

Now, $$\eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{298}{1023} = 0.7086 \quad \text{or} \quad 70.86\%$$

The actual thermal efficiency claimed,

$$\eta_{thermal} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{75 \times 1000 \times 60 \times 60}{3.9 \times 74500 \times 1000} = 0.9292 \quad \text{or} \quad 92.92\%.$$

Since $\eta_{thermal} > \eta_{carnot}$, therefore claim of the inventor is **not valid** (or possible). **(Ans.)**

Example 5.9. *A cyclic heat engine operates between a source temperature of 1000°C and a sink temperature of 40°C. Find the least rate of heat rejection per kW net output of the engine ?*

Solution. Temperature of source,

$$T_1 = 1000 + 273 = 1273 \text{ K}$$

Temperature of sink,

$$T_2 = 40 + 273 = 313 \text{ K}$$

Least rate of heat rejection per kW net output :

For a reversible heat engine, the rate of heat rejection will be minimum (Fig. 5.13)

$$\eta_{max} = \eta_{rev.} = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{313}{1273} = 0.754$$

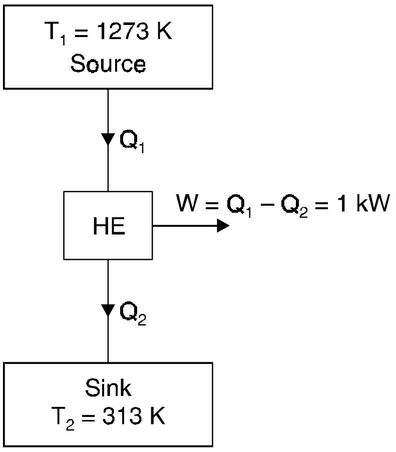


Fig. 5.13

Now $$\frac{W_{net}}{Q_1} = \eta_{max} = 0.754$$

$$\therefore \quad Q_1 = \frac{W_{net}}{0.754} = \frac{1}{0.754} = 1.326 \text{ kW}$$

Now $$Q_2 = Q_1 - W_{net} = 1.326 - 1 = 0.326 \text{ kW}$$

Hence, **the least rate of heat rejection = 0.326 kW. (Ans.)**

Example 5.10. *A fish freezing plant requires 40 tons of refrigeration. The freezing temperature is – 35°C while the ambient temperature is 30°C. If the performance of the plant is 20% of the theoretical reversed Carnot cycle working within the same temperature limits, calculate the power required.*

Given : 1 ton of refrigeration = 210 kJ/min.

Solution. Cooling required = 40 tons = 40 × 210

= 8400 kJ/min

Ambient temperature, $T_1 = 30 + 273 = 303$ K

Freezing temperature, $T_2 = -35 + 273 = 238$ K

Performance of plant = 20% of the theoretical reversed Carnot cycle

$$(\text{C.O.P.})_{refrigerator} = \frac{T_2}{T_1 - T_2} = \frac{238}{303 - 238} = 3.66$$

∴ Actual C.O.P = 0.20 × 3.66 = 0.732

Now work needed to produce cooling of 40 tons is calculated as follows :

$$(\text{C.O.P.})_{actual} = \frac{\text{Cooling reqd.}}{\text{Work needed}}$$

$$0.732 = \frac{8400}{W} \quad \text{or} \quad W = \frac{8400}{0.732} \text{ kJ/min} = 191.25 \text{ kJ/s} = 191.25 \text{ kW}$$

Hence, **power required = 191.25 kW. (Ans.)**

Example 5.11. *Source 1 can supply energy at the rate of 12000 kJ/min at 320°C. A second source 2 can supply energy at the rate of 120000 kJ/min at 70°C. Which source (1 or 2) would you choose to supply energy to an ideal reversible heat engine that is to produce large amount of power if the temperature of the surroundings is 35°C ?*

Solution. Source 1 :

Rate of supply of energy = 12000 kJ/min

Temperature, $T_1 = 320 + 273 = 593$ K.

Source 2 :

Rate of supply of energy = 120000 kJ/min

Temperature, $T_1 = 70 + 273 = 343$ K

Temperature of the surroundings, $T_2 = 35°\text{C} + 273 = 308$ K

Let the Carnot engine be working in the two cases with the two source temperatures and the single sink temperature. The efficiency of the cycle will be given by :

$$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{308}{593} = 0.4806 \quad \text{or} \quad 48.06\%$$

$$\eta_2 = 1 - \frac{T_2}{T_1} = 1 - \frac{308}{343} = 0.102 \quad \text{or} \quad 10.2\%$$

∴ The work delivered in the two cases is given by

$$W_1 = 12000 \times 0.4806 = 5767.2 \text{ kJ/min}$$

and $$W_2 = 120000 \times 0.102 = 12240 \text{ kJ/min.}$$

Thus, choose **source 2**. **(Ans.)**

Note. The source 2 is selected even though *efficiency* in this case is *lower*, because the *criterion for selection is the larger output.*

☞**Example 5.12.** *A reversible heat engine operates between two reservoirs at temperatures 700°C and 50°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 50°C and – 25°C. The heat transfer to the engine is 2500 kJ and the net work output of the combined engine refrigerator plant is 400 kJ.*

(*i*) *Determine the heat transfer to the refrigerant and the net heat transfer to the reservoir at 50°C ;*

(*ii*) *Reconsider (i) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 45 per cent of their maximum possible values.*

Solution. Refer Fig. 5.14.

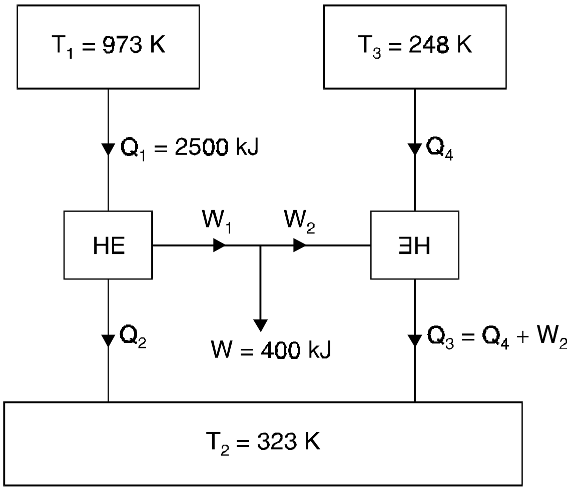


Fig. 5.14

Temperature, $T_1 = 700 + 273 = 973$ K

Temperature, $T_2 = 50 + 273 = 323$ K

Temperature, $T_3 = -25 + 273 = 248$ K

The heat transfer to the heat engine, $Q_1 = 2500$ kJ

The network output of the combined engine refrigerator plant,

$$W = W_1 - W_2 = 400 \text{ kJ.}$$

(*i*) Maximum efficiency of the heat engine cycle is given by

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{323}{973} = 0.668$$

Again, $$\frac{W_1}{Q_1} = 0.668$$

∴ $$W_1 = 0.668 \times 2500 = 1670 \text{ kJ}$$

$$(\text{C.O.P.})_{max} = \frac{T_3}{T_2 - T_3} = \frac{248}{323 - 248} = 3.306$$

Also, $$\text{C.O.P.} = \frac{Q_4}{W_2} = 3.306$$

Since, $W_1 - W_2 = W = 400$ kJ

$W_2 = W_1 - W = 1670 - 400 = 1270$ kJ

$\therefore$ $Q_4 = 3.306 \times 1270 = 4198.6$ kJ

$Q_3 = Q_4 + W_2 = 4198.6 + 1270 = 5468.6$ kJ

$Q_2 = Q_1 - W_1 = 2500 - 1670 = 830$ kJ.

Heat rejection to the 50°C reservoir

$= Q_2 + Q_3 = 830 + 5468.6 =$ **6298.6 kJ. (Ans.)**

(*ii*) Efficiency of actual heat engine cycle,

$\eta = 0.45\ \eta_{max} = 0.45 \times 0.668 = 0.3$

$\therefore$ $W_1 = \eta \times Q_1 = 0.3 \times 2500 = 750$ kJ

$\therefore$ $W_2 = 750 - 400 = 350$ kJ

C.O.P. of the actual refrigerator cycle,

$$\text{C.O.P.} = \frac{Q_4}{W_2} = 0.45 \times 3.306 = 1.48$$

$\therefore$ $Q_4 = 350 \times 1.48 =$ **518 kJ. (Ans.)**

$Q_3 = 518 + 350 = 868$ kJ

$Q_2 = 2500 - 750 = 1750$ kJ

Heat rejected to 50°C reservoir

$= Q_2 + Q_3 = 1750 + 868 =$ **2618 kJ. (Ans.)**

☞**Example 5.13.** (*i*) *A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 25°C. If the heat removal rate from the refrigerator is 1440 kJ/min, determine the C.O.P. of the machine and work input required.*

(*ii*) *If the required input to run the pump is developed by a reversible engine which receives heat at 380°C and rejects heat to atmosphere, then determine the overall C.O.P. of the system.*

Solution. Refer Fig. 5.15 (*a*).

(*i*) Temperature, $T_1 = 25 + 273 = 298$ K

Temperature, $T_2 = 0 + 273 = 273$ K

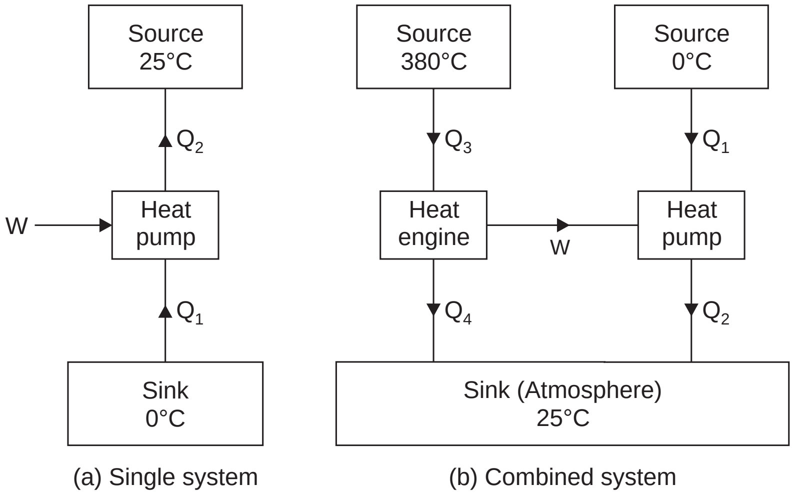


Fig. 5.15

Heat removal rate from the refrigerator,

$$Q_1 = 1440 \text{ kJ/min} = 24 \text{ kJ/s}$$

Now, co-efficient of performance, for reversible heat pump,

$$\textbf{C.O.P.} = \frac{T_1}{T_1 - T_2} = \frac{298}{(298 - 273)} = \textbf{11.92.} \quad \textbf{(Ans.)}$$

$$\therefore \qquad (\text{C.O.P.})_{ref.} = \frac{T_2}{T_1 - T_2} = \frac{273}{298 - 273} = 10.92$$

Now, $$10.92 = \frac{Q_1}{W} = \frac{24}{W}$$

$$\therefore \qquad W = 2.2 \text{ kW}$$

i.e., **Work input required = 2.2 kW. (Ans.)**

$$Q_2 = Q_1 + W = 24 + 2.2 = 26.2 \text{ kJ/s}$$

(*ii*) Refer Fig. 5.15 (*b*).

The overall C.O.P. is given by,

$$\text{C.O.P.} = \frac{\text{Heat removed from the refrigerator}}{\text{Heat supplied from the source}}$$

$$= \frac{Q_1}{Q_3} \qquad \ldots(i)$$

For the reversible engine, we can write

$$\frac{Q_3}{T_3} = \frac{Q_4}{T_4}$$

or $$\frac{Q_4 + W}{T_3} = \frac{Q_4}{T_4}$$

or $$\frac{Q_4 + 2.2}{(380 + 273)} = \frac{Q_4}{(25 + 273)}$$

or $$\frac{Q_4 + 2.2}{653} = \frac{Q_4}{298}$$

or $$298(Q_4 + 2.2) = 653\ Q_4$$

or $$Q_4(653 - 298) = 298 \times 2.2$$

or $$Q_4 = \frac{298 \times 2.2}{(653 - 298)} = 1.847 \text{ kJ/s}$$

$$\therefore \qquad Q_3 = Q_4 + W = 1.847 + 2.2 = 4.047 \text{ kJ/s}$$

Substituting this value in eqn. (*i*), we get

$$\textbf{C.O.P.} = \frac{24}{4.047} = \textbf{5.93.} \quad \textbf{(Ans.)}$$

If the purpose of the system is to supply the heat to the sink at 25°C, then

$$\textbf{Overall C.O.P.} = \frac{Q_2 + Q_4}{Q_3} = \frac{26.2 + 1.847}{4.047} = \textbf{6.93.} \quad \textbf{(Ans.)}$$

Example 5.14. *An ice plant working on a reversed Carnot cycle heat pump produces 15 tonnes of ice per day. The ice is formed from water at 0°C and the formed ice is maintained at 0°C. The heat is rejected to the atmosphere at 25°C. The heat pump used to run the ice plant is* *coupled to a Carnot engine which absorbs heat from a source which is maintained at 220°C by burning liquid fuel of 44500 kJ/kg calorific value and rejects the heat to the atmosphere. Determine :*

(*i*) *Power developed by the engine ;*

(*ii*) *Fuel consumed per hour.*

Take enthalpy of fusion of ice = 334.5 kJ/kg.

Solution. (*i*) Fig. 5.16 shows the arrangement of the system.

Amount of ice produced per day = 15 tonnes.

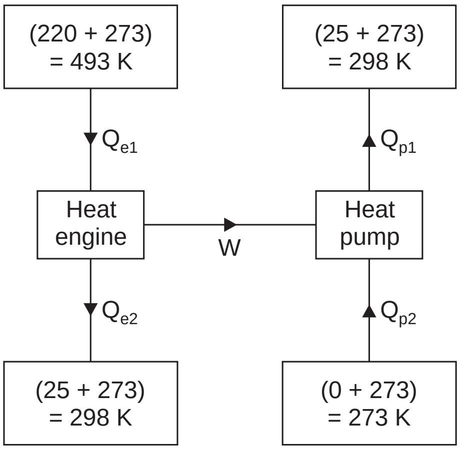


Fig. 5.16

∴ The amount of heat removed by the heat pump,

$$Q_{p_2} = \frac{15 \times 1000 \times 334.5}{24 \times 60} = 3484.4 \text{ kJ/min}$$

$$\text{C.O.P. of the heat pump } = \frac{Q_{p_2}}{W} = \frac{273}{298 - 273}$$

$$\therefore \qquad W = Q_{p_2} \times \frac{298 - 273}{273} = 3484.4 \times \frac{25}{273} = 319.08 \text{ kJ/min}$$

This work must be developed by the Carnot engine,

$$W = \frac{319.08}{60} = 5.3 \text{ kJ/s} = 5.3 \text{ kW}$$

Thus **power developed by the engine = 5.3 kW. (Ans.)**

(*ii*) The efficiency of Carnot engine is given by

$$\eta_{carnot} = \frac{W}{Q_{e_1}} = 1 - \frac{298}{493} = 0.396$$

$$\therefore \qquad Q_{e_1} = \frac{W}{0.396} = \frac{5.3}{0.396} = 13.38 \text{ kJ/s}$$

$$\therefore \qquad Q_{e_1(per\ hour)} = 13.38 \times 60 \times 60 = 48168 \text{ kJ}$$

∴ **Quantity of fuel consumed/hour**

$$= \frac{48168}{44500} = \mathbf{1.082 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

Example 5.15. *Two Carnot engines work in series between the source and sink temperatures of 550 K and 350 K. If both engines develop equal power determine the intermediate temperature.*

Solution. Fig. 5.17 shows the arrangement of the system.

Temperature of the source, $T_1 = 550$ K

Temperature of the sink, $T_3 = 350$ K

Intermediate temperature, T_2 :

The efficiencies of the engines HE_1 and HE_2 are given by

$$\eta_1 = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{W}{Q_2 + W} \quad ...(i)$$

$$\eta_2 = \frac{W}{Q_2} = \frac{T_2 - T_3}{T_2} = \frac{W}{Q_3 + W} \quad ...(ii)$$

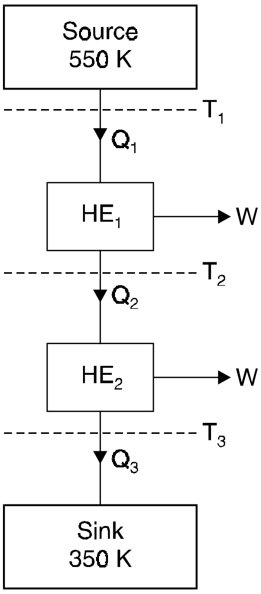


Fig. 5.17

From eqn. (*i*), we get

$$W = (Q_2 + W)\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \quad W\left[1 - \left(\frac{T_1 - T_2}{T_1}\right)\right] = Q_2\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \quad W\left(\frac{T_2}{T_1}\right) = Q_2\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \quad W = Q_2\left(\frac{T_1 - T_2}{T_2}\right) \quad ...(iii)$$

From eqn. (*ii*), we get

$$W = Q_2\left(\frac{T_2 - T_3}{T_2}\right) \quad ...(iv)$$

Now from eqns. (*iii*) and (*iv*), we get

$$T_1 - T_2 = T_2 - T_3$$

$$2T_2 = T_1 + T_3 = 550 + 350$$

$$\therefore \quad T_2 = 450 \text{ K}$$

Hence **intermediate temperature = 450 K. (Ans.)**

Example 5.16. *A Carnot heat engine draws heat from a reservoir at temperature T_1 and rejects heat to another reservoir at temperature T_3. The Carnot forward cycle engine drives a Carnot reversed cycle engine or Carnot refrigerator which absorbs heat from reservoir at temperature T_2 and rejects heat to a reservoir at temperature T_3. If the high temperature T_1 = 600 K and low temperature T_2 = 300 K, determine :*

(*i*) *The temperature T_3 such that heat supplied to engine Q_1 is equal to the heat absorbed by refrigerator Q_2.*

(*ii*) *The efficiency of Carnot engine and C.O.P. of Carnot refrigerator.*

Solution. Refer Fig. 5.18.

Temperature, $T_1 = 600$ K

Temperature, $T_2 = 300$ K

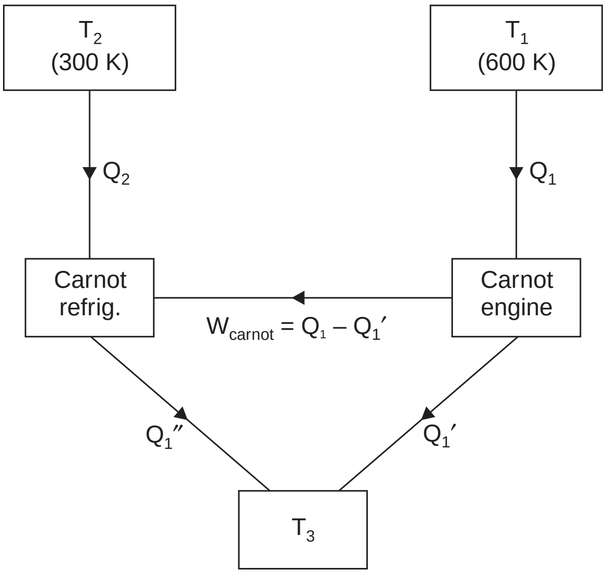


Fig. 5.18

Efficiency of Carnot engine,

$$\eta_{carnot\ engine} = \frac{Q_1 - Q_1'}{Q_1} = \frac{T_1 - T_3}{T_1}$$

$$= \frac{\text{Work of Carnot engine}}{\text{Heat supplied to the Carnot engine}} = \frac{W_{carnot}}{Q_1}$$

or $$W_{carnot} = Q_1\left(\frac{T_1 - T_3}{T_1}\right) \quad ...(i)$$

Also $$\text{C.O.P.}_{(carnot\ refrigerator)} = \frac{Q_2}{Q_1'' - Q_2} = \frac{T_2}{T_3 - T_2}$$

$$= \frac{\text{Heat absorbed}}{W_{carnot}} = \frac{Q_2}{W_{carnot}}$$

or $$W_{carnot} = Q_2\left(\frac{T_3 - T_2}{T_2}\right) \quad ...(ii)$$

(*i*) **Temperature, T_3 :**

From eqns. (*i*) and (*ii*), we get

$$Q_1\left(\frac{T_1 - T_3}{T_1}\right) = Q_2\left(\frac{T_3 - T_2}{T_2}\right)$$

∴ $$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}\left(\frac{T_1 - T_3}{T_3 - T_2}\right)$$

or $$\frac{Q_2}{Q_1} = 1 = \frac{300}{600}\left(\frac{600 - T_3}{T_3 - 300}\right)$$

or $$600 - T_3 = 2(T_3 - 300)$$

$$600 - T_3 = 2T_3 - 600 \quad \text{or} \quad T_3 = 400 \text{ K}$$

Hence, **temperature, T_3 = 400 K. (Ans.)**

(*ii*) Efficiency of Carnot engine,

$$\eta_{carnot\ engine} = \frac{T_1 - T_3}{T_1} = \frac{600 - 400}{600} = 0.3333 = \mathbf{33.33\%.} \quad \textbf{(Ans.)}$$

$$\text{C.O.P.}_{refrigerator} = \frac{T_2}{T_3 - T_2} = \frac{300}{400 - 300} = \mathbf{3.} \quad \textbf{(Ans.)}$$

Example 5.17. *A heat pump working on a reversed carnot cycle takes in energy from a reservoir maintained at 5°C and delivers it to another reservoir where temperature is 77°C. The heat pump derives power for its operation from a reversible engine operating within the higher and lower temperatures of 1077°C and 77°C. For 100 kJ/kg of energy supplied to reservoir at 77°C, estimate the energy taken from the reservoir at 1077°C.* **(U.P.S.C., 1994)**

Solution. *Given :* $T_3 = 5 + 273 = 278$ K ; $T_2 = T_4 = 77 + 273 = 350$ K ;
$T_1 = 273 + 1077 = 1350$ K ;

Energy taken from the revervoir at 1077°C, Q_1 :

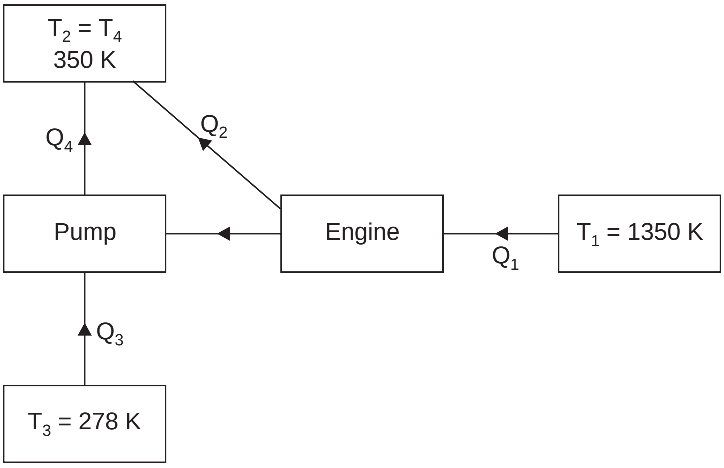


Fig. 5.19

For reversible engine, $\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$ ...(*i*)

or $$1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$\therefore$ $$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

For reversible heat pump, C.O.P. $= \frac{Q_4}{Q_4 - Q_3} = \frac{T_4}{T_4 - T_3}$ ...(*ii*)

Since work for running the pump is being supplied by the engine

$\therefore$ $$Q_1 - Q_2 = Q_4 - Q_3$$

or $$\frac{Q_1}{T_1}(T_1 - T_2) = \frac{Q_4}{T_4}(T_4 - T_3)$$ [From (*i*) and (*ii*)]

∴ $$\frac{Q_1}{Q_4} = \frac{T_1}{T_4}\left(\frac{T_4 - T_3}{T_1 - T_2}\right)$$

$$= \frac{1350}{350}\left(\frac{350 - 278}{1350 - 350}\right) = 0.278$$

or $$Q_4 = \frac{Q_1}{0.278} = 3.6\ Q_1$$

and $$Q_2 = \frac{T_2}{T_1} \times Q_1 = \frac{350}{1350} Q_1 = 0.259\ Q_1$$

∴ $$Q_4 + Q_2 = (3.6 + 0.259)\ Q_1 = 100$$

∴ $$\mathbf{Q_1 = \frac{100}{3.6 + 0.259} = 25.9\ kJ.}\quad \textbf{(Ans.)}$$

CLAUSIUS INEQUALITY

☞**Example 5.18.** *300 kJ/s of heat is supplied at a constant fixed temperature of 290°C to a heat engine. The heat rejection takes place at 8.5°C. The following results were obtained :*

(*i*) *215 kJ/s are rejected.*

(*ii*) *150 kJ/s are rejected.*

(*iii*) *75 kJ/s are rejected.*

Classify which of the result report a reversible cycle or irreversible cycle or impossible results.

Solution. Heat supplied at 290°C = 300 kJ/s

Heat rejected at 8.5°C : (*i*) 215 kJ/s, (*ii*) 150 kJ/s, (*iii*) 75 kJ/s.

Applying *Clausius inequality* to the cycle or process, we have :

(*i*) $$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{300}{290 + 273} - \frac{215}{8.5 + 273}$$
$$= 0.5328 - 0.7637 = -0.2309 < 0.$$

∴ **Cycle is irreversible. (Ans.)**

(*ii*) $$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{300}{290 + 273} - \frac{150}{8.5 + 273}$$
$$= 0.5328 - 0.5328 = 0$$

∴ **Cycle is reversible. (Ans.)**

(*iii*) $$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{300}{290 + 273} - \frac{75}{8.5 + 273}$$
$$= 0.5328 - 0.2664 = 0.2664 > 0.$$

This **cycle is impossible** by second law of thermodynamics, *i.e., Clausius inequality*. **(Ans.)**

Example 5.19. *A steam power plant operates between boiler temperature of 160°C and condenser temperature of 50°C. Water enters the boiler as saturated liquid and steam leaves the boiler as saturated vapour. Verify the Clausius inequality for the cycle.*

Given : Enthalpy of water entering boiler = 687 kJ/kg.

Enthalpy of steam leaving boiler = 2760 kJ/kg

Condenser pressure = $0.124 \times 10^5\ N/m^2$.

Solution. Boiler temperature, $T_1 = 160 + 273 = 433$ K

Condenser temperature, $T_2 = 50 + 273 = 323$ K

From steam tables :

Enthalpy of water entering boiler, $h_{f_1} = 687$ kJ/kg

Enthalpy of steam leaving boiler, $h_2 = 2760$ kJ/kg

Condenser pressure = 0.124×10^5 N/m^2

Boiler pressure = 6.18×10^5 N/m^2 ......(corresponding to 160°C)

Enthalpy of vapour leaving the turbine, $h_3 = 2160$ kJ/kg (assuming isentropic expansion)

Enthalpy of water leaving the condenser, $h_{f4} = 209$ kJ/kg

Now $$Q_{boiler},\ Q_1 = h_2 - h_{f1} = 2760 - 687 = 2073 \text{ kJ/kg}$$

and $$Q_{condenser},\ Q_2 = h_{f4} - h_3 = 209 - 2160 = -1951 \text{ kJ/kg}$$

$\therefore$ $$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{2073}{433} + \left(\frac{-1951}{323}\right)$$

$$= -1.25 \text{ kJ/kg K}$$

$$< \mathbf{0}. \quad \text{......} \quad \textbf{Proved.}$$

☞**Example 5.20.** *In a power plant cycle, the temperature range is 164°C to 51°C, the upper temperature being maintained in the boiler where heat is received and the lower temperature being maintained in the condenser where heat is rejected. All other processes in the steady flow cycle are adiabatic. The specific enthalpies at various points are given in Fig. 5.20.*

Verify the Clausius Inequality.

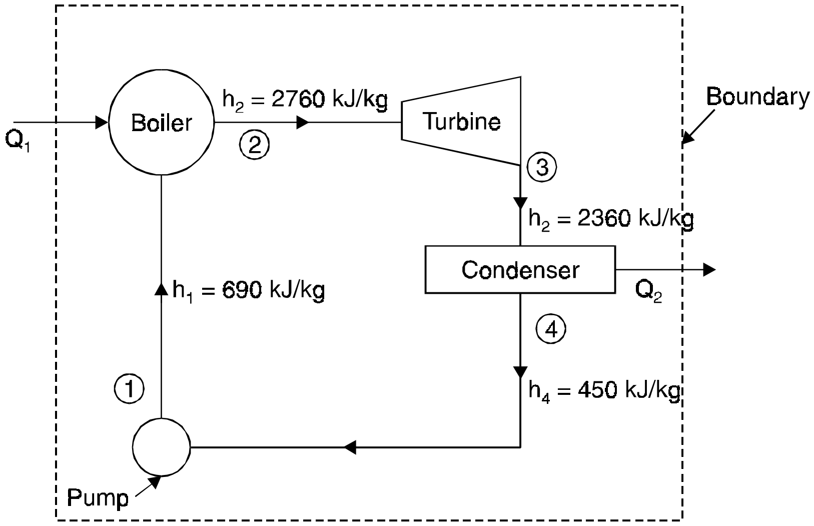


Fig. 5.20

Solution. Temperature maintained in boiler, $T_1 = 164 + 273 = 437$ K

Temperature maintained in condenser, $T_2 = 51 + 273 = 324$ K

Heat transferred in the boiler per kg of fluid,

$$Q_1 = h_2 - h_1 = 2760 - 690 = 2070 \text{ kJ/kg}$$

Heat transferred out at the condenser per kg of fluid,

$$Q_2 = h_4 - h_3 = 450 - 2360 = -1910 \text{ kJ/kg}$$

Since there is no transfer of heat at any other point, we have per kg

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{2070}{437} + \left(\frac{-1910}{324}\right)$$

$$= 4.737 - 5.895$$

$$= -1.158 \text{ kJ/kg K} < 0.$$

The Clausius Inequality is proved. The steady flow cycle is obviously irreversible.

If the cycle is reversible between the same temperature limits and the heat supplied at higher temperature is same, the heat rejected can be calculated as follows :

$$\eta_{reversible} = 1 - \frac{T_2}{T_1} = 1 - \frac{324}{437} = 0.2586 \text{ or } 25.86\%$$

∴ Heat rejected per kg is given by

$$Q_2 = (1 - 0.2586) \times Q_1 = (1 - 0.2586) \times 2070 = 1534.7 \text{ kJ/kg}$$

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{2070}{437} - \frac{1534.7}{324} = 4.73 - 4.73 = 0$$

i.e.,

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_{added}}{T_{source}} = \frac{Q_{rejected}}{T_{sink}} = 0$$

Thus **Clausius Equality sign for a reversible engine is verified.**

5.12. ENTROPY

5.12.1. Introduction

In heat engine theory, the term entropy plays a vital role and leads to important results which by other methods can be obtained much more laboriously.

It may be noted that all heat is not equally valuable for converting into work. Heat that is supplied to a substance at high temperature has a greater possibility of conversion into work than heat supplied to a substance at a lower temperature.

"***Entropy** is a function of a quantity of heat which shows the possibility of conversion of that heat into work. The increase in entropy is small when heat is added at a high temperature and is greater when heat addition is made at a lower temperature. Thus for maximum entropy, there is minimum availability for conversion into work and for minimum entropy there is maximum availability for conversion into work.*"

5.12.2. Entropy—a Property of a System

Refer Fig. 5.21. Let us consider a system undergoing a reversible process from state 1 to state 2 along path *L* and then from state 2 to the original state 1 along path *M*. Applying the Clausius theorem to this reversible cyclic process, we have

$$\oint_R \frac{\delta Q}{T} = 0$$

(where the subscript designates a reversible cycle)

Hence when the system passes through the cycle 1-*L*-2-*M*-1, we have

$$\int_{1(L)}^{2} \frac{\delta Q}{T} + \int_{2(M)}^{1} \frac{\delta Q}{T} = 0 \qquad ...(5.16)$$

Now consider another reversible cycle in which the system changes from state 1 to state 2 along path *L*, but returns from state 2 to the original state 1 along a different path *N*. For this reversible cyclic process, we have

$$\int_{1(L)}^{2} \frac{\delta Q}{T} + \int_{2(N)}^{1} \frac{\delta Q}{T} = 0 \qquad ...(5.17)$$

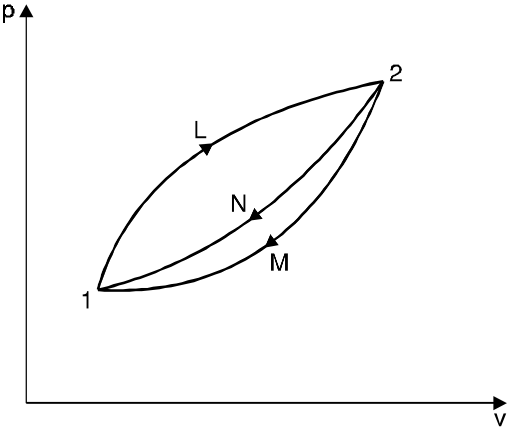


Fig. 5.21. Reversible cyclic process between two fixed end states.

Subtracting equation (5.17) from equation (5.16), we have

$$\int_{2(M)}^{1} \frac{\delta Q}{T} - \int_{2(N)}^{1} \frac{\delta Q}{T} = 0$$

or

$$\int_{1}^{2(M)} \frac{\delta Q}{T} = \int_{1}^{2(N)} \frac{\delta Q}{T}$$

As no restriction is imposed on paths *L* and *M*, except that they must be reversible, the *quantity* $\frac{\delta Q}{T}$ *is a function of the initial and final states of the system and is independent of the path of the process. Hence it represents a property of the system.* This property is known as the **"entropy".**

5.12.3. Change of Entropy in a Reversible Process

Refer Fig. 5.21.

Let S_1 = Entropy at the initial state 1, and

S_2 = Entropy at the final state 2.

Then, the change in entropy of a system, as it undergoes a change from state 1 to 2, becomes

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_R \qquad ...(5.18)$$

Lastly, if the two equilibrium states 1 and 2 are infinitesimal near to each other, the integral sign may be omitted and $S_2 - S_1$ becomes equal to dS.

Hence equation (5.18) may be written as

$$dS = \left(\frac{\delta Q}{T}\right)_R \qquad \text{...(5.19)}$$

where dS is an exact differential.

Thus, from equation (5.19), we find that the change of entropy in a reversible process is equal to $\frac{\delta Q}{T}$. *This is the mathematical formulation of the second law of thermodynamics.*

Equation (5.19) indicates that when an *inexact* differential δQ is divided by an integrating factor T during a reversible process, it becomes an *exact differential*.

The ***third law of thermodynamics*** states "*When a system is at zero absolute temperature, the entropy of system is zero*".

It is clear from the above law that the absolute value of entropy corresponding to a given state of the system could be determined by integrating $\left(\frac{\delta Q}{T}\right)_R$ between the state at absolute zero and the given state. *Zero entropy, however, means the absence of all molecular, atomic, electronic and nuclear disorders.*

As it is not practicable to get data at zero absolute temperature, the change in entropy is calculated either between two known states or by selecting some convenient point at which the entropy is given an arbitrary value of zero. For steam, the reference point at which the entropy is given an arbitrary value of zero is 0°C and for refrigerants like ammonia, Freon-12, carbon dioxide etc. the reference point is – 40°C, at which the entropy it taken as zero.

Thus, in practice we can *determine the change in entropy and not the absolute value of entropy*.

5.13. ENTROPY AND IRREVERSIBILITY

We know that change in entropy in a *reversible process* is equal to $\left(\frac{\delta Q}{T}\right)_R$ (eqn. 5.19). Let us now find the change in entropy in an *irreversible process.*

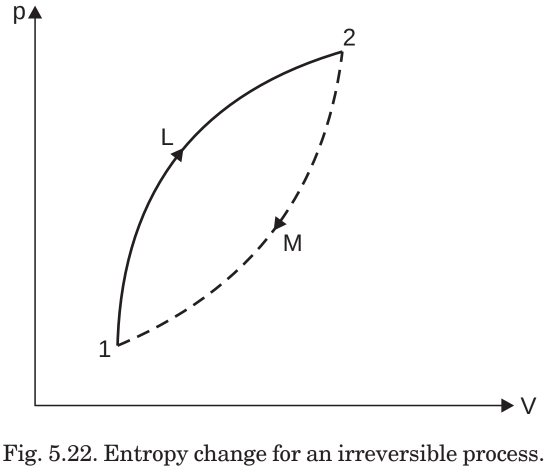


Fig. 5.22. Entropy change for an irreversible process.

Consider a closed system undergoing a change from state 1 to state 2 by a reversible process 1-*L*-2 and returns from state 2 to the initial state 1 by an irreversible process 2-*M*-1 as shown in Fig. 5.22 on the thermodynamic coordinates, pressure and volume.

Since entropy is a thermodynamic property, we can write

$$\oint dS = \int_{1(L)}^{2} (dS)_R + \int_{2(M)}^{1} (dS)_I = 0 \qquad ...(5.20)$$

(Subscript I represents the irreversible process).

Now for a reversible process, from eqn. (5.19), we have

$$\int_{1(L)}^{2} (dS)_R = \int_{1(L)}^{2} \left(\frac{\delta Q}{T}\right)_R \qquad ...(5.21)$$

Substituting the value of $\int_{1(L)}^{2} (dS)_R$ in eqn. (5.20), we get

$$\int_{1(L)}^{2} \left(\frac{\delta Q}{T}\right)_R + \int_{2(M)}^{1} (dS)_I = 0 \qquad ...(5.22)$$

Again, since in eqn. (5.20) the processes 1-L-2 and 2-M-1 together form an irreversible cycle, applying Clausius equality to this expression, we get

$$\oint \frac{\delta Q}{T} = \int_{1(L)}^{2} \left(\frac{\delta Q}{T}\right)_R + \int_{2(M)}^{1} \left(\frac{\delta Q}{T}\right)_I < 0 \qquad ...(5.23)$$

Now subtracting eqn. (5.23) from eqn. (5.22), we get

$$\int_{2(M)}^{1} (dS)_I > \int_{2(M)}^{1} \left(\frac{\delta Q}{T}\right)_I$$

which for infinitesimal changes in states can be written as

$$(dS)_I > \int \left(\frac{\delta Q}{T}\right)_I \qquad ...(5.24)$$

Eqn. (5.24) states that the *change in entropy in an irreversible process is greater than* $\frac{\delta Q}{T}$.

Combining eqns. (5.23) and (5.24), we can write the equation in the general form as

$$dS \geq \frac{\delta Q}{T} \qquad ...(5.25)$$

where *equality sign stands for the reversible process and inequality sign stands for the irreversible process.*

It may be noted here that the *effect of irreversibility is always to increase the entropy of the system.*

Let us now consider an *isolated system.* We know that in an isolated system, matter, work or heat cannot cross the boundary of the system. Hence according to first law of thermodynamics, the internal energy of the system will remain constant.

Since for an isolated system, $\delta Q = 0$, from eqn. (5.25), we get

$$(dS)_{isolated} \geq 0 \qquad ...(5.26)$$

Eqn. (5.26) states that the *entropy of an isolated system either increases or remains constant.* This is a corollary of the second law. It explains the *principle of increase in entropy.*

5.14. CHANGE IN ENTROPY OF THE UNIVERSE

We know that the entropy of an isolated system either increase or remains constant, *i.e.,*

$$(dS)_{isolated} \geqq 0$$

By including any system and its surrounding within a single boundary, as shown in Fig. 5.23, an isolated system can be formed. The combination of the system and the surroundings within a single boundary is sometimes called the **Universe.** Hence, applying the principle of increase in entropy, we get

$$(dS)_{universe} \geqq 0$$

where $(dS)_{universe} = (dS)_{system} + (dS)_{surroundings}$.

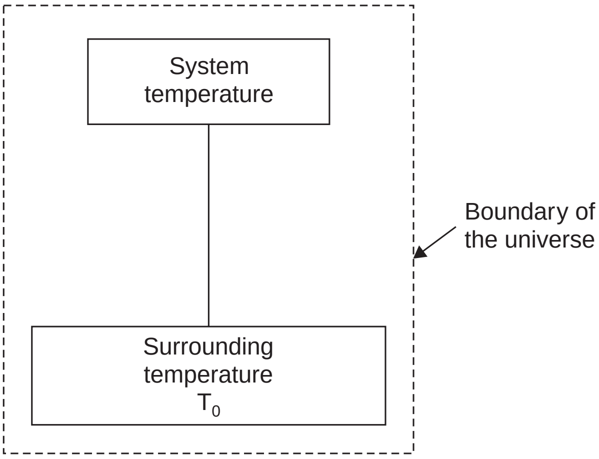


Fig. 5.23. Entropy change of universe.

In the combined closed system consider that a quantity of heat δQ is transferred from the system at temperature T to the surroundings at temperature T_0. Applying eqn. (5.24) to this process, we can write

$$(dS)_{system} > -\frac{\delta Q}{T}$$

(–ve sign indicates that heat is transferred from the system).

Similarly, since an amount of heat δQ is absorbed by the surroundings, for a reversible process, we can write

$$(dS)_{surroundings} = \frac{\delta Q}{T_0}$$

Hence, the total change in entropy for the combined system

$$(dS)_{system} + (dS)_{surroundings} \geq -\frac{\delta Q}{T} + \frac{\delta Q}{T_0}$$

or

$$(dS)_{universe} \geq dQ\left(-\frac{1}{T} + \frac{1}{T_0}\right)$$

The same result can be obtained in the case of an open system.

For both closed and open systems, we can write

$$(dS)_{universe} \geq 0 \qquad ...(5.27)$$

Eqn. (5.27) states that the process involving the interaction of a system and the surroundings takes place only if the net entropy of the combined system increases or in the limit remains constant. *Since all natural processes are irreversible, the entropy is increasing continually.*

The entropy attains its maximum value when the system reaches a stable equilibrium state from a non-equilibrium state. This is the state of maximum disorder and is one of maximum thermodynamic probability.

5.15. TEMPERATURE-ENTROPY DIAGRAM

If entropy is plotted-horizontally and absolute temperature vertically the diagram so obtained is called *temperature-entropy* (T-s) diagram. Such a diagram is shown in Fig. 5.24. If working fluid receives a small amount of heat dQ in an elementary portion ab of an operation AB when temperature is T, and if dQ is represented by the shaded area of which T is the mean ordinate, the width of the figure must be $\dfrac{dQ}{T}$. This is called '*increment of entropy*' and is denoted by dS. The total heat received by the operation will be given by the area under the curve AB and $(S_B - S_A)$ will be corresponding increase of entropy.

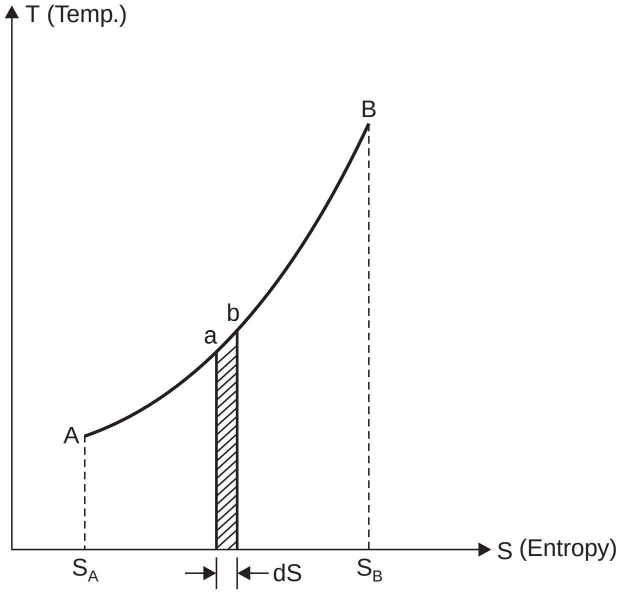


Fig. 5.24. Temperature-entropy diagram.

From above we conclude that :

$$\text{Entropy change,} \quad dS = \frac{\text{Heat change } (Q)}{\text{Absolute temperature } (T)} .$$

"***Entropy*** *may also be defined as the thermal property of a substance which remains constant when substance is expanded or compressed adiabatically in a cylinder*".

Note. 's' stands for specific entropy whereas 'S' means total entropy (*i.e.,* $S = ms$).

5.16. CHARACTERISTICS OF ENTROPY

The characteristics of entropy in a summarised form are given below :

1. It increases when heat is supplied irrespective of the fact whether temperature changes or not.

2. It decrease when heat is removed whether temperature changes or not.

3. It remains unchanged in all adiabatic frictionless processes.

4. It increases if temperature of heat is lowered without work being done as in a throttling process.

5.17. ENTROPY CHANGES FOR A CLOSED SYSTEM

5.17.1. General Case for Change of Entropy of a Gas

Let 1 kg of gas at a pressure p_1, volume v_1, absolute temperature T_1 and entropy s_1, be heated such that its final pressure, volume, absolute temperature and entropy are p_2, v_2, T_2 and s_2 respectively. Then by law of conservation of energy,

$$dQ = du + dW$$

where, dQ = Small change of heat,
du = Small internal energy, and
dW = Small change of work done (pdv).

Now $$dQ = c_v dT + pdv$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = \frac{c_v dT}{T} + \frac{pdv}{T}$$

But $$\frac{dQ}{T} = ds$$

and as $$pv = RT$$

$$\therefore \quad \frac{p}{T} = \frac{R}{v}$$

Hence $$ds = \frac{c_v dT}{T} + R\frac{dv}{v}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R\int_{v_1}^{v_2} \frac{dv}{v}$$

or $$(s_2 - s_1) = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1} \qquad \text{...(5.28)}$$

This expression can be reproduced in the following way :

According to the gas equation, we have

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}$$

or $$\frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{v_2}{v_1}$$

Substituting the value of $\frac{T_2}{T_1}$ in eqn. (5.28), we get

$$s_2 - s_1 = c_v \log_e \frac{p_2}{p_1} \times \frac{v_2}{v_1} + R \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + c_v \log_e \frac{v_2}{v_1} + R \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + (c_v + R) \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}$$

$$\therefore \quad s_2 - s_1 = c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1} \quad ...(5.29)$$

Again, from gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

Putting the value of $\frac{v_2}{v_1}$ in eqn. (5.28), we get

$$(s_2 - s_1) = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

$$= c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{p_1}{p_2} + R \log_e \frac{T_2}{T_1}$$

$$= (c_v + R) \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

$$= c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

$$\therefore \quad s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}. \quad ...(5.30)$$

5.17.2. Heating a Gas at Constant Volume

Refer Fig. 5.25. Let 1 kg of gas be heated at constant volume and let the change in entropy and absolute temperature be from s_1 to s_2 and T_1 to T_2 respectively.

Then $Q = c_v(T_2 - T_1)$

Differentiating to find small increment of heat dQ corresponding to small rise in temperature dT.

$$dQ = c_v dT$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = c_v \cdot \frac{dT}{T}$$

or $$ds = c_v \cdot \frac{dT}{T}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_v \int_{T_1}^{T_2} \frac{dT}{T}$$

or $$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} \quad ...(5.31)$$

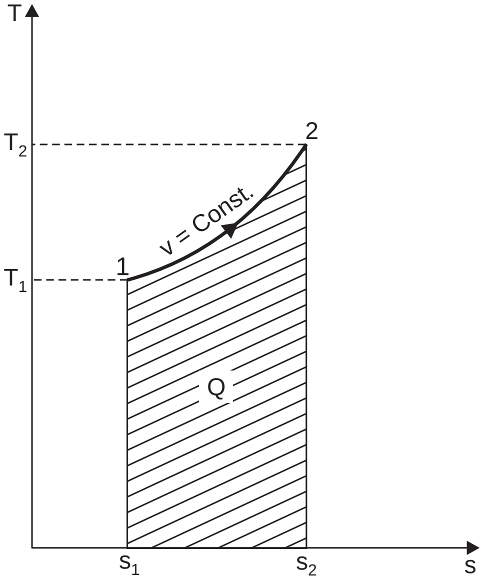


Fig. 5.25. *T-s* diagram : Constant volume process

5.17.3. Heating a Gas at Constant Pressure

Refer Fig. 5.26. Let 1 kg of gas be heated at constant pressure, so that its absolute temperature changes from T_1 to T_2 and entropy s_1 to s_2.

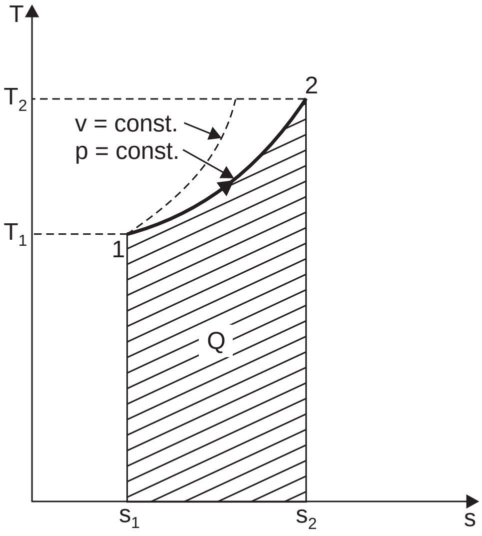


Fig. 5.26. *T-s* diagram : Constant pressure process.

Then, $Q = c_p(T_2 - T_1)$.

Differentiating to find small increase in heat, dQ of this gas when the temperature rise is dT.

$$dQ = c_p \,.\, dT$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = c_p \,.\, \frac{dT}{T}$$

or

$$ds = c_p \,.\, \frac{dT}{T}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_p \int_{T_1}^{T_2} \frac{dT}{T}$$

$$s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} \qquad ...(5.32)$$

5.17.4. Isothermal Process

An isothermal expansion 1-2 at constant temperature T is shown in Fig. 5.27.

Entropy changes from s_1 to s_2 when gas absorbs heat during expansion. The heat taken by the gas is given by the area under the line 1-2 which also represents the work done during expansion. In other words, $Q = W$.

But $$Q = \int_{s_1}^{s_2} T ds = T(s_2 - s_1)$$

and $$W = p_1 v_1 \log_e \frac{v_2}{v_1} = RT_1 \log_e \frac{v_2}{v_1} \text{ per kg of gas} \qquad [\because \; p_1 v_1 = RT_1]$$

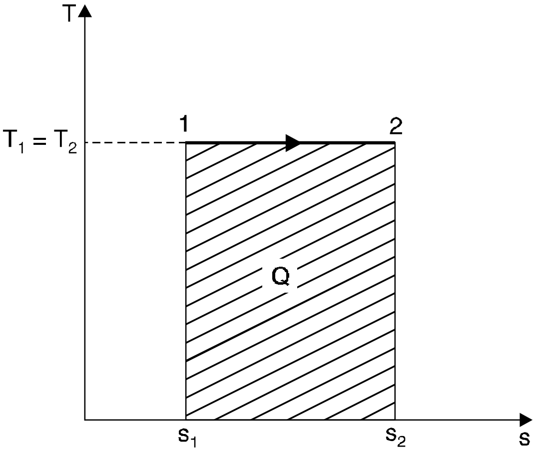


Fig. 5.27. *T-s* diagram : Isothermal process.

$$\therefore \qquad T(s_2 - s_1) = RT_1 \log_e \frac{v_2}{v_1}$$

or $$s_2 - s_1 = R \log_e \frac{v_2}{v_1}. \qquad [\because \quad T_1 = T_2 = T] \qquad ...(5.33)$$

5.17.5. Adiabatic Process (Reversible)

During an adiabatic process as heat is neither supplied nor rejected,

$$dQ = 0$$

or $$\frac{dQ}{dT} = 0$$

or $$ds = 0 \qquad ...(5.34)$$

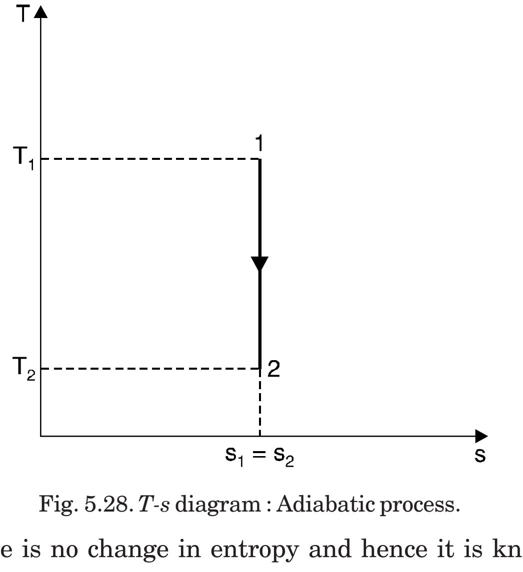


Fig. 5.28. *T-s* diagram : Adiabatic process.

This shows that there is no change in entropy and hence it is known as *isentropic process.*

Fig. 5.28 represents an adiabatic process. It is a vertical line (1-2) and therefore area under this line is nil ; hence heat supplied or rejected and entropy change is zero.

5.17.6. Polytropic Process

Refer Fig. 5.29.

The expression for 'entropy change' in polytropic process (pv^n = constant) can be obtained from eqn. (5.28)

i.e.,
$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$$

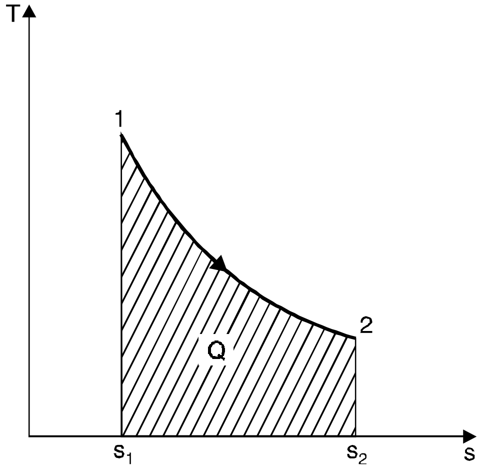


Fig. 5.29. *T-s* diagram : Polytropic process.

Also
$$p_1v_1^n = p_2v_2^n$$

or
$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n \qquad ...(i)$$

Again, as
$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}$$

or
$$\frac{p_1}{p_2} = \frac{v_2}{v_1} \times \frac{T_1}{T_2} \qquad ...(ii)$$

From (*i*) and (*ii*), we get
$$\left(\frac{v_2}{v_1}\right)^n = \frac{v_2}{v_1} \times \frac{T_1}{T_2}$$

or
$$\left(\frac{v_2}{v_1}\right)^{n-1} = \frac{T_1}{T_2}$$

or
$$\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

Substituting the value of $\frac{v_2}{v_1}$ in eqn. (5.28), we get
$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}} = c_v \log_e \frac{T_2}{T_1} + R \left(\frac{1}{n-1}\right) \log_e \frac{T_1}{T_2}$$

$$= c_v \log_e \frac{T_2}{T_1} - R\left(\frac{1}{n-1}\right) \log_e \frac{T_2}{T_1}$$

$$= c_v \log_e \frac{T_2}{T_1} - (c_p - c_v) \times \left(\frac{1}{n-1}\right) \log_e \frac{T_2}{T_1} \qquad [\because \ R = c_p - c_v]$$

$$= c_v \log_e \frac{T_2}{T_1} - (\gamma \,.\, c_v - c_v) \times \left(\frac{1}{n-1}\right) \log_e \frac{T_2}{T_1} \qquad [\because \ c_p = \gamma \,.\, c_v]$$

$$= c_v\left[1 - \left(\frac{\gamma-1}{n-1}\right)\right] \log_e \frac{T_2}{T_1} = c_v\left[\frac{(n-1)-(\gamma-1)}{(n-1)}\right] \log_e \frac{T_2}{T_1}$$

$$= c_v\left(\frac{n-1-\gamma+1)}{n-1}\right) \log_e \frac{T_2}{T_1}$$

$$= c_v \,.\, \left(\frac{n-\gamma}{n-1}\right) \log_e \frac{T_2}{T_1} \text{ per kg of gas}$$

$$\therefore \qquad s_2 - s_1 = c_v \left(\frac{n-\gamma}{n-1}\right) \log_e \frac{T_2}{T_1} \text{ per kg of gas} \qquad ...(5.35)$$

5.17.7. Approximation For Heat Absorbed

The curve *LM* shown in the Fig. 5.30 is obtained by heating 1 kg of gas from initial state *L* to final state *M*. Let temperature during heating increases from T_1 to T_2. Then heat absorbed by the gas will be given by the area (shown shaded) under curve *LM*.

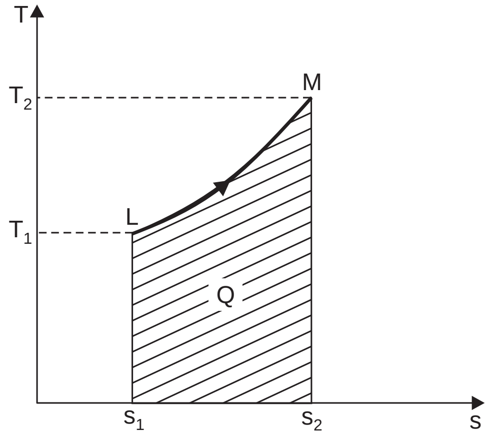


Fig. 5.30

As the curve on *T-s* diagram which represents the heating of the gas, usually has very slight curvature, it can be assumed a straight line for a small temperature range. Then,

Heat absorbed = Area under the curve *LM*

$$= (s_2 - s_1)\left(\frac{T_1 + T_2}{2}\right) \qquad ...(5.36)$$

In other words, heat absorbed *approximately* equals the product of change of entropy and mean absolute temperature.

Table 5.1. Summary of Formulae

S. No.	*Process*	*Change of entropy (per kg)*
1.	General case	(*i*) $c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$ (in terms of T and v) (*ii*) $c_v \log_e \frac{p_2}{p_1} + c_v \log_e \frac{v_2}{v_1}$ (in terms of p and v) (*iii*) $c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$ (in terms of T and p)
2.	Constant volume	$c_v \log_e \frac{T_2}{T_1}$
3.	Constant pressure	$c_p \log_e \frac{T_2}{T_1}$
4.	Isothermal	$R \log_e \frac{v_2}{v_1}$
5.	Adiabatic	Zero
6.	Polytropic	$c_v \left(\frac{n-\gamma}{n-1}\right) \log_e \frac{T_2}{T_1}$

5.18. ENTROPY CHANGES FOR AN OPEN SYSTEM

In an open system, as compared with closed system, there is additional change of entropy due to the mass crossing the boundaries of the system. *The net change of entropy of a system due to mass transport is equal to the difference between the product of the mass and its specific entropy at the inlet and at the outlet of the system.* Therefore, the total change of entropy of the system during a small interval is given by

$$dS \geq \frac{dQ}{T_0} + \Sigma s_i \,.\, dm_i - \Sigma s_0 \,.\, dm_0$$

where,
T_0 = Temperature of the surroundings,
s_i = Specific entropy at the inlet,
s_0 = Specific entropy at the outlet,
dm_i = Mass entering the system, and
dm_0 = Mass leaving the system.

(Subscripts i and 0 refer to inlet and outlet conditions)

The above equation in general form can be written as

$$dS \geq \frac{dQ}{T_0} + \sum s \,.\, dm \qquad ...(5.37)$$

In equation (5.37) entropy flow *into* the system is considered *positive* and entropy *out-flow* is considered *negative.* The *equality sign is applicable to reversible process* in which the heat interactions and mass transport to and from the system is accomplished reversibly. The *inequality sign is applicable to irreversible processes.*

If equation (5.37) is divided by dt, then it becomes a rate equation and is written as

$$\frac{dS}{dt} \geq \frac{1}{T_0} \cdot \frac{dQ}{dt} + \sum s \cdot \frac{dm}{dt} \qquad ...(5.38)$$

In a steady-state, steady flow process, the rate of change of entropy of the system $\left(\frac{dS}{dt}\right)$ becomes zero.

$$\therefore \qquad O \geq \frac{1}{T_0} \frac{dQ}{dt} + \sum s \cdot \frac{dm}{dt}$$

or

$$\frac{1}{T_0} \dot{Q} + \Sigma s \, . \, \dot{m} \leq 0 \qquad ...(5.39)$$

where $\dot{Q} = \frac{dQ}{dt}$

and $\dot{m} = \frac{dm}{dt}$.

For *adiabatic steady flow process,* $\dot{Q} = 0$

$$\sum s \, . \, \dot{m} \leq 0 \qquad ...(5.40)$$

If the process is *reversible adiabatic*, then

$$\sum s \, . \, \dot{m} = 0 \qquad ...(5.41)$$

5.19. THE THIRD LAW OF THERMODYNAMICS

- The third law of thermodynamics is stated as follow :

  "*The entropy of all perfect crystalline solids is zero at absolute zero temperature*".
- The third law of thermodynamics, often referred to as *Nernst Law*, provides the basis for the calculation of absolute entropies of substances.

According to this law, if the entropy is zero at $T = 0$, the absolute entropy $s_{ab.}$ of a substance at any temperature T and pressure p is expressed by the expression.

$$s_{ab} = \int_0^{T_s = T_{f1}} c_{ps} \frac{dT}{T} + \frac{h_{sf}}{T_s} + \int_{T_s}^{T_{f2} = T_g} c_{pf} \frac{dT}{T} + \frac{h_{fg}}{T_g} \int_{T_g}^{T} c_{pg} \frac{dT}{T} \qquad ...(5.42)$$

where $T_s = T_{f1} = T_{sf} = T_{sat}$ ...... for fusion,

$T_{f2} = T_g = T_{fg} = T_{sat}$ ......for vaporisation

c_{ps}, c_{pf}, c_{pg} = Constant pressure specific heats for solids, liquids and gas,

h_{sf}, h_{fg} = Latent heats of fusion and vaporisation.

Thus by putting $s = 0$ at $T = 0$, one may integrate zero kelvin and standard state of 278.15 K and 1 atm., and find the entropy difference.

- Further, it can be shown that the entropy of a crystalline substance at $T = 0$ is not a function of pressure, *viz.,*

$$\left(\frac{\partial s}{\partial p}\right)_{T = 0} = 0$$

However, *at temperatures above absolute zero, the entropy is a function of pressure also*. The absolute entropy of a substance at 1 atm pressure can be calculated using eqn. (5.42) ; for pressures different from 1 atm, necessary corrections have to be applied.

ENTROPY

Exmaple 5.21. *An iron cube at a temperature of 400°C is dropped into an insulated bath containing 10 kg water at 25°C. The water finally reaches a temperature of 50°C at steady state. Given that the specific heat of water is equal to 4186 J/kg K. Find the entropy changes for the iron cube and the water. Is the process reversible ? If so why ?* **(GATE, 1996)**

Solution. *Given* **:** Temperature of iron cube = 400°C = 673 K
Temperature of water = 25°C = 298 K
Mass of water = 10 kg
Temperature of water and cube after equilibrium = 50°C = 323 K
Specific heat of water, c_{pw} = 4186 J/kg K

Entropy changes for the iron cube and the water :

Is the process reversible ?

Now, Heat lost by iron cube = Heat gained by water

$$m_i\, c_{pi}\,(673 - 323) = m_w\, c_{pw}\,(323 - 298)$$
$$= 10 \times 4186\,(323 - 298)$$

$$\therefore \qquad m_i\, c_{pi} = \frac{10 \times 4186\,(323 - 298)}{(623 - 323)} = 2990$$

where, m_i = Mass of iron, kg, and
c_{pi} = Specific heat of iron, J/kg K

$$\text{Entropy of iron at } 673\text{ K} = m_i\, c_{pi} \ln\left(\frac{673}{273}\right)$$
$$= 2990 \ln\left(\frac{673}{273}\right)$$
$$= 2697.8 \text{ J/K} \qquad \text{[Taking 0°C as datum]}$$

$$\text{Entropy of water at } 298\text{ K} = m_w\, c_{pw} \ln\left(\frac{298}{273}\right)$$
$$= 10 \times 4186 \ln\left(\frac{298}{273}\right) = 3667.8 \text{ J/K}$$

$$\text{Entropy of iron at } 323\text{ K} = 2990 \times \ln\left(\frac{323}{273}\right) = 502.8 \text{ J/K}$$

$$\text{Entropy water at } 323\text{ K} = 10 \times 4186 \ln\left(\frac{323}{273}\right) = 7040.04 \text{ J/K}$$

Changes in entropy of iron = 502.8 – 2697.8 = – 2195 J/K
Change in entropy of water = 7040.04 – 3667.8 = 3372.24 J/K
Net change in entropy = 3372.24 – 2195 = 1177.24 J/K

Since $\Delta S > 0$ hence the process is **irrevesible. (Ans.)**

Example 5.22. *An ideal gas is heated from temperature T_1 to T_2 by keeping its volume constant. The gas is expanded back to its initial temperature according to the law pv^n = constant. If the entropy change in the two processes are equal, find the value of n in terms of the adiabatic index γ.* **(U.P.S.C., 1997)**

Solution. Change in entropy during constant volume process

$$= m\, c_v \ln \left(\frac{T_2}{T_1}\right) \qquad \ldots(i)$$

Change in entropy during polytropic process (pv^n = constant)

$$= m\, c_v \left(\frac{\gamma - n}{n - 1}\right) \ln \left(\frac{T_2}{T_1}\right) \qquad \ldots(ii)$$

For the same entropy, equating (*i*) and (*ii*), we have

$$\frac{\gamma - n}{n - 1} = 1, \qquad \text{or} \qquad (\gamma - n) = (n - 1) \qquad \text{or} \qquad 2n = \gamma + 1$$

$$\therefore \qquad \mathbf{n = \frac{\gamma + 1}{2}}. \quad \textbf{(Ans).}$$

Example 5.23. *Air at 20°C and 1.05 bar occupies 0.025 m³. The air is heated at constant volume until the pressure is 4.5 bar, and then cooled at constant pressure back to original temperature. Calculate :*

(*i*) *The net heat flow from the air.*

(*ii*) *The net entropy change.*

Sketch the process on T-s diagram.

Solution. The processes are shown on a *T-s* diagram in Fig. 5.31.

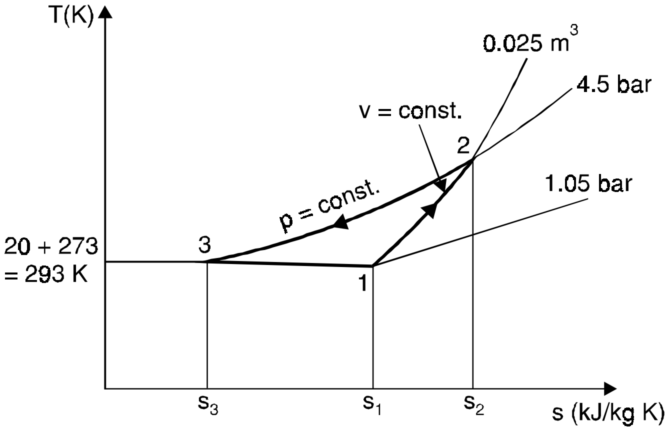


Fig. 5.31

For air :

Temperature, $T_1 = 20 + 273 = 293$ K

Volume, $V_1 = V_3 = 0.025$ m³

Pressure, $p_1 = 1.05$ bar $= 1.05 \times 10^5$ N/m²

Pressure, $p_2 = 4.5$ bar $= 4.5 \times 10^5$ N/m².

(*i*) **Net heat flow :**

For a perfect gas (corresponding to point 1 of air),

$$m = \frac{p_1 V_1}{R T_1} = \frac{1.05 \times 10^5 \times 0.025}{0.287 \times 10^3 \times 293} = 0.0312 \text{ kg}$$

For a perfect gas at *constant volume*,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$\frac{1.05}{293} = \frac{4.5}{T_2} \quad \text{or} \quad T_2 = \frac{4.5 \times 293}{1.05} = 1255.7 \text{ K.}$$

At **constant volume,**

$$Q = mc_v\,(T_2 - T_1) = 0.0312 \times 0.718\ (1255.7 - 293)$$

i.e., $Q_{1-2} = 21.56$ kJ.

Also, at **constant pressure,**

$$Q = m \times c_p \times (T_3 - T_2) = 0.0312 \times 1.005\ (293 - 1255.7)$$

i.e., $Q_{2-3} = -\,30.18$ kJ

$\therefore$ **Net heat flow** $= Q_{1-2} + Q_{2-3} = 21.56 + (-\,30.18) = -\,8.62$ kJ

i.e., **Heat rejected = 8.62 kJ. (Ans.)**

(*ii*) **Net entropy change :**

Referring to Fig. 5.31.

Net decrease in entropy,

$$S_1 - S_2 = (S_2 - S_3) - (S_2 - S_1)$$

At **constant pressure**, $dQ = mc_p\,dT$, hence

$$m(s_2 - s_3) = \int_{293}^{1255.7} \frac{mc_p dT}{T}$$

$$= 0.0312 \times 1.005 \times \log_e \frac{1255.7}{293}$$

i.e., $S_2 - S_3 = 0.0456$ kJ/K

At **constant volume**, $dQ = mc_v\,dT$, hence

$$m(s_2 - s_1) = \int_{293}^{1255.7} \frac{mc_v dT}{T}$$

$$= 0.0312 \times 0.718 \times \log_e \frac{1255.7}{293} = 0.0326 \text{ kJ/K}$$

i.e., $S_2 - S_1 = 0.0326$ kJ/K

$$\therefore \quad m(s_1 - s_3) = S_1 - S_3 = (S_2 - S_3) - (S_2 - S_1)$$

$$= 0.0456 - 0.0326 = 0.013 \text{ kJ/K}$$

Hence, **decrease in entropy = 0.013 kJ/K. (Ans.)**

Note that since *entropy is a property*, the decrease in entropy is given by $S_1 - S_3$, is *independent of the process undergone between states 1 and 3.*

Example 5.24. *0.04 m³ of nitrogen contained in a cylinder behind a piston is initially at 1.05 bar and 15°C. The gas is compressed isothermally and reversibly until the pressure is 4.8 bar. Calculate :*

(*i*) *The change of entropy*,

(*ii*) *The heat flow*, *and*

(*iii*) *The work done.*

Sketch the process on a p-v and T-s diagram.

Assume nitrogen to act as a perfect gas. Molecular weight of nitrogen = 28.

Solution. Refer Fig. 5.32.

Initial pressure, $p_1 = 1.05 \text{ bar} = 1.05 \times 10^5 \text{ N/m}^2$

Initial volume, $V_1 = 0.04 \text{ m}^3$

Temperature, $T_1 = 15 + 273 = 288 \text{ K}$

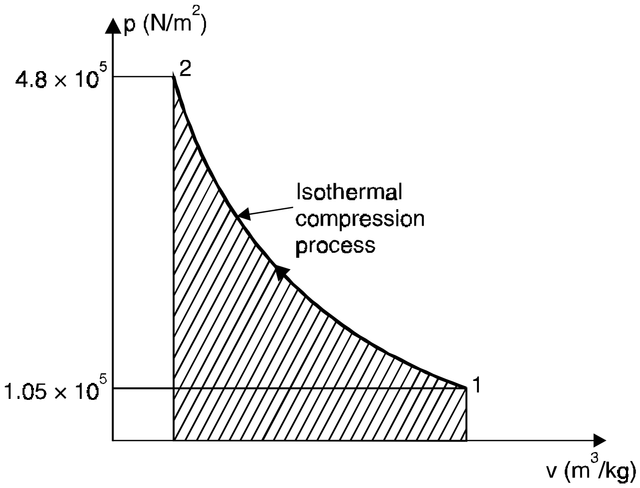
Final pressure, $p_2 = 4.8 \text{ bar} = 4.8 \times 10^5 \text{ N/m}^2$

Final temperature, $T_2 = T_1 = 288 \text{ K}$.

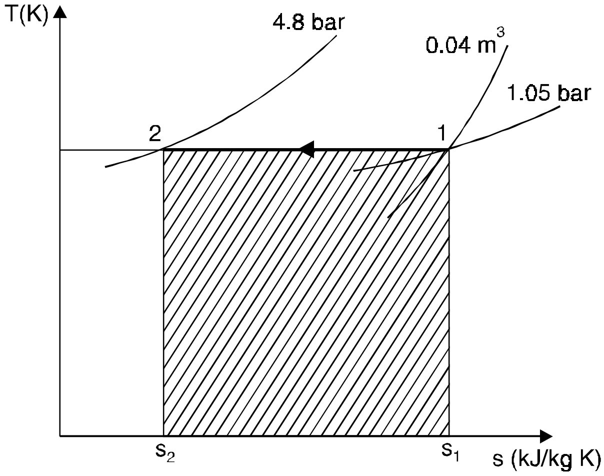
The process is shown on a *p-v* and a *T-s* diagram in Figs. 5.32 (*a*) and 5.32 (*b*) respectively. The shaded area in Fig. 5.32 (*a*) represents **work input**, and the shaded area on Fig. 5.32 (*b*) represents **heat rejected.**

Characteristic gas constant,

$$R = \frac{\text{Universal gas constant, } R_0}{\text{Molecular weight, } M} = \frac{8314}{28} = 297 \text{ Nm/kg K}$$



(*a*)



(*b*)

Fig. 5.32

Now, using characteristic gas equation (to find mass 'm' of nitrogen), we have :

$$p_1V_1 = mRT_1$$

$$m = \frac{p_1V_1}{RT_1} = \frac{1.05 \times 10^5 \times 0.04}{297 \times 288} = 0.0491 \text{ kg}$$

(*i*) **The change of entropy,**

$$S_2 - S_1 = mR \log_e \frac{p_1}{p_2}$$

$$= 0.0491 \times \frac{297}{10^3} \log_e \left(\frac{1.05}{4.8}\right)$$

i.e., $S_2 - S_1 = -0.02216$ kJ/K.

∴ **Decrease in entropy, $S_1 - S_2$ = 0.02216 kJ/K. (Ans.)**

(*ii*) **Heat rejected** = Shaded area on Fig. 5.32 (*b*)

$$= T(S_1 - S_2) = 288 \times 0.02216 = \textbf{6.382 kJ. (Ans.)}$$

(*iii*) For an isothermal process for a perfect gas,

$$W = Q = 6.382 \text{ kJ}$$

Hence, **the work done on air = 6.382 kJ. (Ans.)**

Example 5.25. *1 kg of gas enclosed in an isolated box of volume v_1, temperature T_1 and pressure p_1 is allowed to expand freely till volume increases to $v_2 = 2v_1$.*

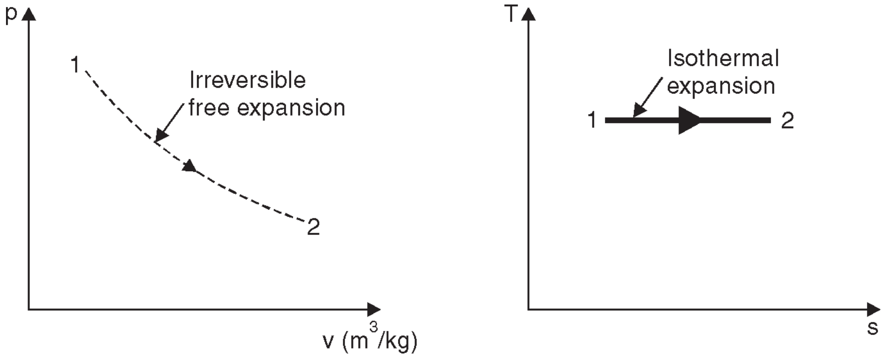
Determine the change in entropy.

Take R for gas as 287 kJ/kg K.

Solution. During the process of free expansion in an isolated box,

$$\Delta U = 0, W = 0 \text{ and } Q = \Delta U + W = 0$$

The process is represented by dotted line on *p-v* diagram as shown in Fig. 5.33 (*a*) where $v_2 = 2v_1$.



(*a*) (*b*)

Fig. 5.33

To calculate the entropy change, assume that the irreversible free expansion process is replaced by a reversible isothermal process as temperature in free expansion remains constant, in such a way that the volume increases to double of its original as shown in Fig. 5.33 (*b*). As the work is developed by the system and heat is given to the system at constant temperature, during isothermal reversible system then as per first law of thermodynamics :

$$\Delta U = 0, \ Q = W$$

i.e.,

$$Q = \int_{v_1}^{v_2} p \, . \, dv$$

$$= \int_{v_1}^{v_2} \frac{RT}{v} \, . \, dv \qquad \left[\because \ pv = RT \text{ and } p = \frac{RT}{v}\right]$$

$$= RT \log_e \frac{v_2}{v_1}$$

$$\therefore \qquad \frac{Q}{T} = R \log_e \frac{v_2}{v_1}$$

But this is the expression for change in entropy of the system. Entropy being the property of the system, its change is same whether it is reversible or irreversible process.

∴ For the given process,

$$\Delta s = R \log_e \left(\frac{v_2}{v_1}\right)$$

$$= 287 \log_e (2) \qquad [\because \ v_2 = 2v_1 \text{ (given)}]$$

$$= 198.9 \text{ kJ/kg K}$$

Hence **change in entropy = 198.9 kJ/kg K. (Ans.)**

Example 5.26. *0.04 kg of carbon dioxide (molecular weight = 44) is compressed from 1 bar, 20°C, until the pressure is 9 bar, and the volume is then 0.003 m^3. Calculate the change of entropy.*

Take c_p for carbon dioxide as 0.88 kJ/kg K, and assume carbon dioxide to be a perfect gas.

Solution. Mass of carbon dioxide, $m = 0.04$ kg

Molecular weight, $M = 44$

Initial pressure, $p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 20 + 273 = 293 \text{ K}$

Final pressure, $p_2 = 9$ bar

Final volume, $V_2 = 0.003 \text{ m}^3$

c_p for carbon dioxide $= 0.88$ kJ/kg K

Change of entropy :

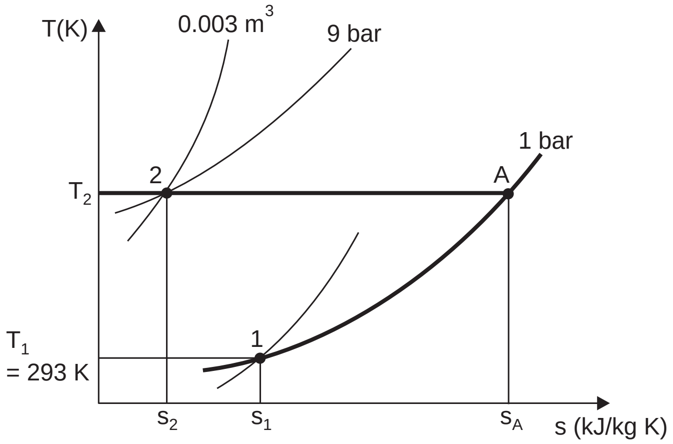


Fig. 5.34

Characteristics gas constant,

$$R = \frac{R_0}{M} = \frac{8314}{44} = 189 \text{ Nm/kg K}$$

To find T_2, using the relation,

$$p_2V_2 = mRT_2$$

$$\therefore \quad T_2 = \frac{p_2V_2}{mR} = \frac{9 \times 10^5 \times 0.003}{0.04 \times 189} = 357 \text{ K}$$

Now $$s_A - s_2 = R \log_e \frac{p_2}{p_1} = \frac{189}{10^3} \log_e \left(\frac{9}{1}\right)$$
$$= 0.4153 \text{ kJ/kg K}$$

Also at constant pressure from 1 to A

$$s_A - s_1 = c_p \log_e \frac{T_2}{T_1} = 0.88 \log_e \left(\frac{357}{293}\right)$$
$$= 0.1738 \text{ kJ/kg K}$$

Then $$(s_1 - s_2) = (s_A - s_2) - (s_A - s_1)$$
$$= 0.4153 - 0.1738 = 0.2415 \text{ kJ/kg K}$$

Hence for 0.04 kg of carbon dioxide **decrease in entropy,**

$$S_1 - S_2 = m(s_1 - s_2) = 0.04 \times 0.2415$$
$$= \mathbf{0.00966 \text{ kJ/K.}} \quad \textbf{(Ans.)}$$

Note. In short, the change of entropy can be found by using the following relation :

$$(s_2 - s_1) = c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1} = 0.88 \log_e \left(\frac{357}{293}\right) - \frac{189}{10^3} \log_e \left(\frac{9}{1}\right)$$
$$= 0.1738 - 0.4153 = -0.2415 \text{ kJ/kg K}$$

$$\therefore \quad S_2 - S_1 = m(s_2 - s_1) = 0.04 \times (-0.2415)$$
$$= -0.00966 \text{ kJ/K}$$

(– ve sign means *decrease* in entropy)

or $$S_1 - S_2 = 0.00966 \text{ kJ/K.}$$

Example 5.27. *Calculate the change of entropy of 1 kg of air expanding polytropically in a cylinder behind a piston from 7 bar and 600°C to 1.05 bar. The index of expansion is 1.25.*

Solution. The process is shown on a T-s diagram in Fig. 5.35.

Initial pressure, $p_1 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 600 + 273 = 873 \text{ K}$

Final pressure, $p_2 = 1.05 \text{ bar} = 1.05 \times 10^5 \text{ N/m}^2$

Index of expansion, $n = 1.25$

Mass of air $= 1$ kg

To find T_2, using the relation,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \quad \frac{T_2}{873} = \left(\frac{1.05}{7}\right)^{\frac{1.25-1}{1.25}}$$

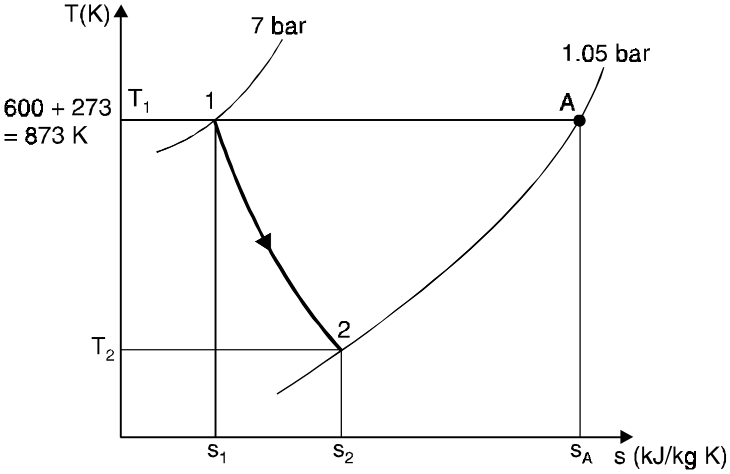


Fig. 5.35

or $$T_2 = 873 \times \left(\frac{1.05}{7}\right)^{\frac{0.25}{1.25}} = 873 \times (0.15)^{0.2} = 597.3 \text{ K}.$$

Now replace the process 1 to 2 by processes, 1 to *A* and *A* to 2.

Then at *constant temperature* from 1 to *A*,

$$s_A - s_1 = R \log_e \frac{v_2}{v_1} = R \log_e \frac{p_1}{p_2} = 0.287 \log_e \left(\frac{7}{1.05}\right) = 0.544 \text{ kJ/kg K}.$$

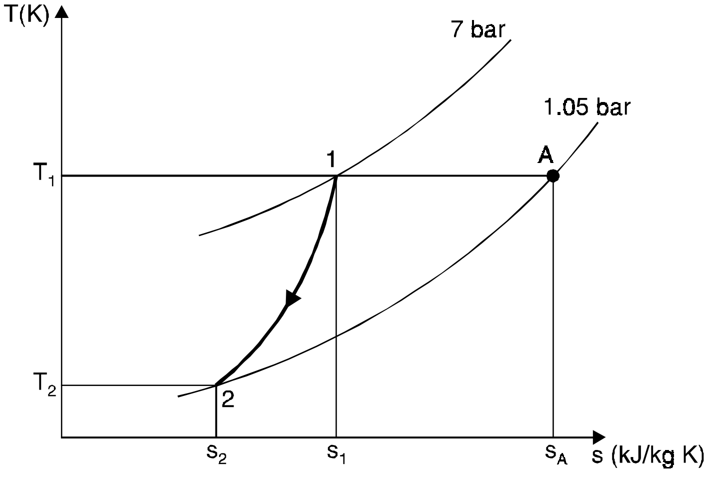


Fig. 5.36

At *constant pressure* from *A* to 2

$$s_A - s_2 = c_p \log_e \frac{T_1}{T_2} = 1.005 \log_e \frac{873}{597.3} = 0.3814 \text{ kJ/kg K}$$

Then $$s_2 - s_1 = 0.544 - 0.3814 = 0.1626 \text{ kJ/kg K}$$

i.e., **Increase in entropy = 0.1626 kJ/kg K. (Ans.)**

Note that if in this problem $s_A - s_2$ happened to be greater than $s_A - s_1$, this would mean that s_1 was greater than s_2, and the process should appear as in Fig. 5.36.

Note. The change of entropy can also be found by using the following relation :

$$s_2 - s_1 = c_v \left(\frac{n-\gamma}{n-1}\right) \log_e \frac{T_2}{T_1}$$

$$= 0.718 \left(\frac{1.25 - 1.399}{1.25 - 1}\right) \log_e \left(\frac{597.3}{873}\right) \qquad \left[\because \gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.399\right]$$

$$= 0.718 \times (-0.596) \times (-0.3795) = 0.1626 \text{ kJ/kg K (increase)}.$$

Example 5.28. *In an air turbine the air expands from 7 bar and 460°C to 1.012 bar and 160°C. The heat loss from the turbine can be assumed to be negligible.*

(*i*) *Show that the process is irreversible ;*

(*ii*) *Calculate the change of entropy per kg of air.*

Solution. Refer Fig. 5.37.

Initial pressure, $p_1 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 460 + 273 = 733 \text{ K}$

Final pressure, $p_2 = 1.012 \text{ bar} = 1.012 \times 10^5 \text{ N/m}^2$

Final temperature, $T_2 = 160 + 273 = 433 \text{ K}$

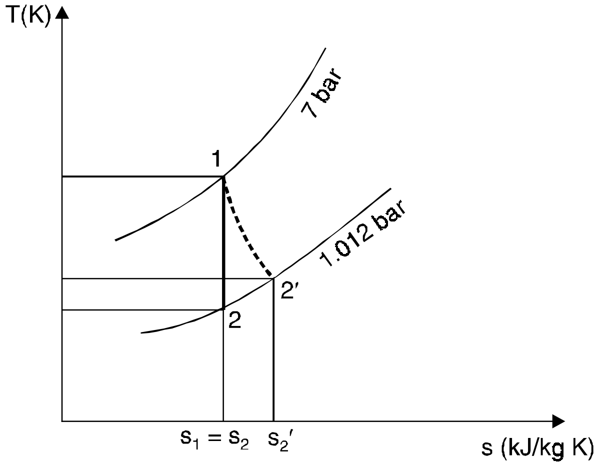


Fig. 5.37

(*i*) **To prove that the process is irreversible :**

Since the heat loss is negligible, the process is ***adiabatic.***

For a reversible adiabatic process for a perfect gas, using the following equation, we have :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{733} = \left(\frac{1.012}{7}\right)^{\left(\frac{1.4-1}{1.4}\right)}$$

$$\therefore \qquad T_2 = 733 \times \left(\frac{1.012}{7}\right)^{\frac{0.4}{1.4}} = 733 \times (0.1446)^{0.286} = 421.6 \text{ K}$$

$$= 421.6 - 273 = 148.6°\text{C}.$$

But the actual temperature is 160°C at the pressure of 1.012 bar, hence the process is irreversible. **Proved.**

(*ii*) **Change of entropy per kg of air :**

The change of entropy $s_2' - s_1$, can be found by considering a reversible constant pressure process between 2 and 2′.

$$\therefore \qquad s_2' - s_2 = c_p \log_e \frac{T_2'}{T_2} = 1.005 \log_e \frac{433}{421.6} = 0.02681 \text{ kJ/kg K}$$

i.e., **Increase of entropy, $s_2' - s_1$ = 0.02681 kJ/kg K. (Ans.)**

☞**Example 5.29.** *A fluid undergoes a reversible adiabatic compression from 4 bar, 0.3 m^3 to 0.08 m^3 according to the law, $pv^{1.25}$ = constant.*

Determine : (*i*) *Change in enthalpy ;* (*ii*) *Change in internal energy ;*
(*iii*) *Change in entropy ;* (*iv*) *Heat transfer ;*
(*v*) *Work transfer.*

Solution. Refer Fig. 5.38.

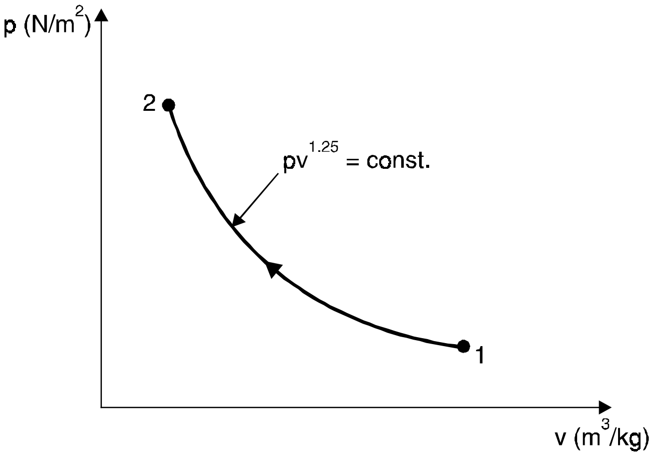


Fig. 5.38

Initial volume, $V_1 = 0.3 \text{ m}^3$

Initial pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

Final volume, $V_2 = 0.08 \text{ m}^3$

Law of compression : $pv^{1.25}$ = constant.

For reversible adiabatic process,

$$p_1 V_1^n = p_2 V_2^n$$

or

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n$$

$$\therefore \qquad p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^n = 4 \times \left(\frac{0.3}{0.08}\right)^{1.25} = 20.87 \text{ bar.}$$

(*i*) **Change in enthalpy, $H_2 - H_1$:**

We know that, $$\int_{H_1}^{H_2} dH = \int_{p_1}^{p_2} V dp \quad ...(i)$$

Also $$p_1V_1^n = pV^n$$

$$\therefore \quad V = \left(\frac{p_1V_1^n}{p}\right)^{1/n}$$

Substituting this value of V in eqn. (*i*), we get

$$\int_{H_1}^{H_2} dH = \int_{p_1}^{p_2} \left[\left(\frac{p_1V_1^n}{p}\right)\right]^{1/n} dp$$

$$= (p_1V_1^n)^{1/n} \int_{p_1}^{p_2} p^{-1/n}\, dp$$

$$= (p_1V_1^n)^{1/n} \left[\frac{p^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\right]_{p_1}^{p_2}$$

$$= [(p_1V_1^n)]^{1/n} \left[\frac{p_2^{\left(1-\frac{1}{n}\right)} - p_1^{\left(1-\frac{1}{n}\right)}}{1-\frac{1}{n}}\right]$$

$$= (p_1V_1^n)^{1/n} \times \frac{n}{(n-1)}\left[(p_2)^{\left(1-\frac{1}{n}\right)} - (p_1)^{\left(1-\frac{1}{n}\right)}\right]$$

$$= \frac{n(p_2V_2 - p_1V_1)}{(n-1)} \quad [\because \; p_1V_1^n = p_2V_2^n]$$

$$= \frac{1.25}{(1.25-1)\times 10^3}\,[20.87 \times 10^5 \times 0.08 - 4 \times 10^5 \times 0.3] \text{ kJ}$$

$$= \frac{1.25}{0.25 \times 10^3} \times 10^5\,(20.87 \times 0.08 - 4 \times 0.3) \text{ kJ} = 234.8 \text{ kJ.}$$

Hence, **change in enthalpy = 234.8 kJ. (Ans.)**

(*ii*) **Change in internal energy, $U_2 - U_1$:**

$$H_2 - H_1 = (U_2 + p_2V_2) - (U_1 + p_1V_1)$$

$$= (U_2 - U_1) + (p_2V_2 - p_1V_1)$$

$$\therefore \quad U_2 - U_1 = (H_2 - H_1) - (p_2V_2 - p_1V_1)$$

$$= 234.8 - \left(\frac{20.87 \times 10^5 \times 0.08 - 4 \times 10^5 \times 0.3}{10^3}\right) \text{kJ}$$

$$= 234.8 - 46.96 = 187.84 \text{ kJ.}$$

Hence, **change in internal energy = 187.84 kJ. (Ans.)**

(*iii*) **Change in entropy, $S_2 - S_1 = 0$. (Ans.)**

(*iv*) **Heat transfer, $Q_{1-2} = 0$. (Ans.)**

(*v*) **Work transfer, W_{1-2} :**

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

$$\therefore \quad W_{1-2} = Q_{1-2} - (U_2 - U_1)$$

$$= 0 - 187.84 = - 187.84 \text{ kJ}$$

Hence, **work done on the fluid = 187.84 kJ. (Ans.)**

☞**Example 5.30.** *An insulated cylinder of volume capacity 4 m³ contains 20 kg of nitrogen. Paddle work is done on the gas by stirring it till the pressure in the vessel gets increased from 4 bar to 8 bar. Determine :*

(*i*) *Change in internal energy,*

(*ii*) *Work done,*

(*iii*) *Heat transferred, and*

(*iv*) *Change in entropy.*

Take for nitrogen : c_p = 1.04 kJ/kg K, and c_v = 0.7432 kJ/kg K.

Solution. Pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

Pressure, $p_2 = 8 \text{ bar} = 8 \times 10^5 \text{ N/m}^2$

Volume, $V_1 = V_2 = 4 \text{ m}^3$

and it is constant for both end states.

Now, $$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

or $$\frac{T_2}{T_1} = \frac{p_2}{p_1} = \frac{8 \times 10^5}{4 \times 10^5} = 2$$

Also, $R = c_p - c_v = 1.04 - 0.7432 = 0.2968$ kJ/kg K.

The mass of the gas in the cylinder is given by

$$m = \frac{pV}{RT} \quad \text{or} \quad mT = \frac{pV}{R}$$

$$\therefore \quad mT_1 = \frac{p_1V_1}{R} = \frac{4 \times 10^5 \times 4}{0.2968 \times 1000} = 5390.8 \text{ kg K}$$

and $$mT_2 = \frac{p_2V_2}{R} = \frac{8 \times 10^5 \times 4}{0.2968 \times 1000} = 10781.6 \text{ kg K.}$$

(*i*) **Change in internal energy,**

$$\Delta U = (U_2 - U_1)$$

$$= mc_v (T_2 - T_1) = c_v (mT_2 - mT_1)$$

$$= 0.7432 (10781.6 - 5390.8) = \mathbf{4006.4 \text{ kJ.}} \quad \textbf{(Ans.)}$$

(*ii*) **Work done, W :**

Energy in the form of paddle work crosses into the system, but there is no change in system boundary or *pdv* work is absent. No heat is transferred to the system. We have

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

But $$Q_{1-2} = 0$$

$$\therefore \quad W_{1-2} = -(U_2 - U_1) = - \mathbf{4006.4 \text{ kJ or kN-m.}} \quad \textbf{(Ans.)}$$

(*iii*) **Heat transferred,** $Q_{1-2} = 0$. **(Ans.)**

(*iv*) **Change in entropy,**

$$S_2 - S_1 = mc_v \log_e \frac{T_2}{T_1}$$

For constant volume process

$$= 20 \times 0.7432 \log_e 2 = \mathbf{10.3\ kJ/K.}\quad \textbf{(Ans.)}$$

Example 5.31. *5 m^3 of air at 2 bar, 27°C is compressed up to 6 bar pressure following $pv^{1.3}$ = constant. It is subsequently expanded adiabatically to 2 bar. Considering the two processes to be reversible, determine the net work. Also plot the processes on T-S diagram.*

Solution. Refer Fig. 5.39.

Given : $V_1 = 5\ m^3$; $p_1 = 2$ bar ; $T_1 = 27 + 273 = 300$ K ; $p_2 = 6$ bar ; $p_3 = 2$ bar

Net work :

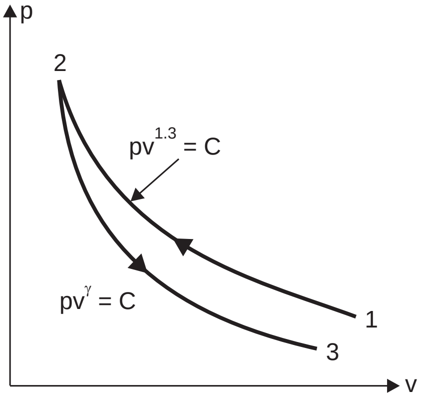


Fig. 5.39. *p-V* diagram.

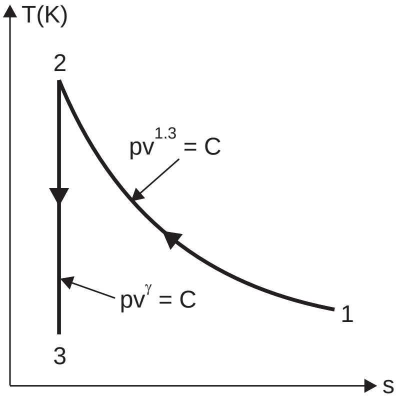


Fig. 5.40. *T-s* diagram.

Mass of air, $m = \frac{p_1V_1}{RT_1} = \frac{2 \times 10^5 \times 5}{287 \times 300} = 11.61$ kg.

Considering polytropic compression process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad \frac{T_2}{300} = \left(\frac{6}{2}\right)^{\frac{13-1}{13}} \quad \text{or} \quad T_2 = 386.5 \text{ K.}$$

Considering isentropic process 2-3, we get

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{2}\right)^{\frac{14-1}{1.4}} = 1.369 \qquad (\because \ p_3 = p_1)$$

$$\therefore \qquad T_3 = \frac{T_2}{1.369} = \frac{386.5}{1.369} = 282.3 \text{ K}$$

Now, work done during polytropic *compression* 1-2,

$$W_{1-2} = \frac{mR(T_1 - T_2)}{n - 1} = \frac{11.61 \times 0.287\,(300 - 386.5)}{13 - 1} = -\ 960.7 \text{ kJ}$$

and, work done during *adiabatic expansion* 2-3,

$$W_{2-3} = \frac{mR(T_2 - T_3)}{\gamma - 1} = \frac{11.61 \times 0.287\,(386.5 - 282.3)}{1.4 - 1} = 868 \text{ kJ}$$

$\therefore$ Net work done = $W_{1-2} + W_{2-3} = -\ 960.7 + 868 = -\ 92.7$ kJ

Hence **net work done on the air = 92.7 kJ. (Ans.)**

The process plotted on *T-s* diagram is shown in Fig. 5.40.

Example 5.32. *A rigid cylinder containing 0.004 m^3 of nitrogen at 1 bar and 300 K is heated reversibly until temperature becomes 400 K. Determine :*

(*i*) *The heat supplied.* (*ii*) *The entropy change.*

Assume nitrogen to be perfect gas (*molecular mass = 28*) *and take* $\gamma = 1.4$.

Solution. *Given :* $V_1 = 0.004$ m^3 ; $p_1 = 1$ bar ; $T_1 = 300$ K ; $T_2 = 400$ K ; M for $N_2 = 28$; $\gamma = 1.4$.

(*i*) **The heat supplied :**

$$\text{Gas constant } R = \frac{R_0 \text{ (Universal gas constant)}}{M \text{ (Molecular mass)}} = \frac{8.314}{28} = 0.297 \text{ kg/kg K}$$

$$\text{Mass, } m = \frac{p_1V_1}{RT_1} = \frac{(1 \times 10^5) \times 0.004}{(0.297 \times 1000) \times 300} = 0.00449 \text{ kg}$$

$$c_v = \frac{R}{\gamma - 1} = \frac{0.297}{1.4 - 1} = 0.742 \text{ kJ/kg K}$$

$$\therefore \quad \text{Heat supplied} = m\, c_v(T_2 - T_1)$$

$$= 0.00449 \times 0.742(400 - 300) = \mathbf{0.333\ kJ.} \quad \textbf{(Ans.)}$$

(*ii*) **The entropy change :**

$$\text{The entropy change, } S_2 - S_1 = m\, c_v \log_e \left(\frac{T_2}{T_1}\right)$$

$$= 0.00449 \times 0.742 \times \log_e \left(\frac{400}{300}\right) = \mathbf{9.584 \times 10^{-4}\ kJ/kg\ K.} \quad \textbf{(Ans.)}$$

Example 5.33. *A piston-cylinder arrangement contains 0.05 m^3 of nitrogen at 1 bar and 280 K. The piston moves inwards and the gas is compressed isothermally and reversibly until the pressure becomes 5 bar. Determine :*

(*i*) *Change in entropy.* (*ii*) *Work done.*

Assume nitrogen to be a perfect gas.

Solution. *Given :* $V_1 = 0.05$ m^3 ; $p_1 = 1$ bar ; $T_1 = 280$ K ; $p_2 = 5$ bar.

(*i*) **Change in entropy, ($S_2 - S_1$) :**

$$\text{Gas constant,} \quad R = \frac{R_0}{M} = \frac{8.314}{28} = 0.297 \text{ kJ/kg K}$$

$$\text{Mass of the gas,} \quad m = \frac{p_1V_1}{RT_1} = \frac{(1 \times 10^5) \times 0.05}{(0.297 \times 1000) \times 280} = 0.06 \text{ kg}$$

$$\therefore \quad \text{Change in entropy } S_2 - S_1 = mR \log_e \left(\frac{p_1}{p_2}\right)$$

$$= 0.06 \times 0.297 \log_e \left(\frac{1}{5}\right) = \mathbf{-\ 0.0287\ kJ/K.} \quad \textbf{(Ans.)}$$

$$\text{Heat interaction,} \quad Q = T(S_2 - S_1)$$

$$= 280 \times (-\ 0.0287) = -\ 8.036 \text{ kJ}$$

$$\therefore \quad \text{Work done,} \quad W = Q = \mathbf{-\ 8.036\ kJ.} \quad \textbf{(Ans.)} \qquad (\because \text{ In its other process, } W = Q)$$

$$\left[\begin{aligned} \textit{Alternatively}: W &= p_1V_1 \log_e \left(\frac{V_2}{V_1}\right) = p_1V_1 \log_e \left(\frac{p_1}{p_2}\right) \\ &= 1 \times 10^5 \times 0.05 \times \log_e \left(\frac{1}{5}\right) \times 10^{-3} \text{ kJ} = 8.04 \text{ kJ} \end{aligned}\right]$$

Example 5.34. *1 kg of air initially at 8 bar pressure and 380 K expands polytropically* ($pv^{1\cdot 2}$ *= constant) until the pressure is reduced to one-fifth value. Calculate :*

(*i*) *Final specific volume and temperature.*

(*ii*) *Change of internal energy, work done and heat interaction.*

(*iii*) *Change in entropy.*

Take : $R = 0.287$ *kJ/kg K and* $\gamma = 1.4$.

Solution. *Given :* $m = 1$ kg ; $p_1 = 8$ bar ; $T_1 = 380$ K ; Law of expansion : $pv^{1.2}$ = constant ;

$$p_2 = \frac{p_1}{5} = \frac{8}{5} = 1.6 \text{ bar} \; ; \; R = 0.287 \text{ kJ/kg K} \; ; \; \gamma = 0.4.$$

(*i*) **Final specific volume and temperature, v_2, T_2 :**

$$p_1 v_1 = RT_1$$

or,

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.287 \times 10^3) \times 380}{8 \times 10^5} = 0.1363 \text{ m}^3\text{/kg.}$$

Also,

$$p_1 v_1^{\,n} = p_2 v_2^{\,n} \quad \text{or} \quad v_2 = v_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

$\therefore$

$$v_2 = 0.1363 \times \left(\frac{8}{1.6}\right)^{\frac{1}{1.2}} = \mathbf{0.5211\ m^3/kg.} \quad \textbf{(Ans.)}$$

Again,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad \frac{T_2}{380} = \left(\frac{1}{5}\right)^{\frac{1.2-1}{1.2}}$$

$$\therefore \quad T_2 = 290.6 \text{ K}$$

$$\left[\textbf{Alternatively : } T_2 = \frac{p_2 v_2}{R} = \frac{1.6 \times 10^5 \times 0.5211}{287} = 290.6 \text{ K}\right]$$

(*ii*) **Change of internal energy, work done and heat interaction :**

Change of internal energy

$$\mathbf{u_2 - u_1} = c_v(\mathrm{T}_2 - T_1) = \frac{R}{\gamma - 1}(T_2 - T_1)$$

$$= \frac{0.287}{1.4 - 1}(290.6 - 380) = \mathbf{-\ 64.14\ kJ/kg} \ \textit{(Decrease)}. \quad \textbf{(Ans.)}$$

Work done,

$$\mathbf{W_{1\text{-}2}} = \frac{p_1 v_1 - p_2 v_2}{n - 1} = \frac{\mathrm{R}(T_1 - T_2)}{n - 1}$$

$$= \frac{0.287(380 - 290.6)}{1.2 - 1} = \mathbf{128.29\ kJ/kg} \ \textit{(Work done by air)}. \quad \textbf{(Ans.)}$$

Heat interaction, $Q_{1-2} = (u_2 - u_1) + W = -\ 64.14 + 128.29 =$ **64.15 kJ/kg** (*Heat received*). **(Ans.)**

$$\left[\textit{Alternatively} : Q = \frac{\gamma - n}{\gamma - 1} \times W = \frac{1.4 - 1.2}{1.4 - 1} \times 128.29 = 64.15 \text{ kJ/kg}\right]$$

(*iii*) **Change in entropy, $(s_2 - s_1)$:**

$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$$

$$= \frac{0.287}{1.4-1} \times \log_e \left(\frac{290.6}{380}\right) + 0.287 \log_e \left(\frac{0.5211}{0.1363}\right)$$

$= -\ 0.192 + 0.385 =$ **0.193 kJ/kg K** *(increase)*. **(Ans.)**

Example 5.35. (*a*) *Show that approximate change of entropy during a polytropic process equals the quantity of heat transferred divided by the mean absolute temperature.*

(*b*) *One kg of air at 290 K is compressed in a cylinder according to the polytropic law* $pv^{1.3}$ = *constant. If the compression ratio is 16, calculate the entropy change of air during the compression process stating whether it is an increase or decrease.*

What would be the percentage error if the entropy change is calculated by dividing the quantity of heat exchanged by the mean absolute temperature during the process ?

Take $\gamma = 1.4$ *and* $c_v = 0.718$ *kJ/kg K.*

Solution. (*a*) In Fig. 5.41 curve 1-2 represents the polytropic process ($pv^n = c$) from state point 1 to state point 2. The area under the process curve 1-2 on *T-S* diagram represents the heat transferred during the process. The slope of the curve 1-2 is usually *small* and can be considered to be a straight line (shotted dotted).

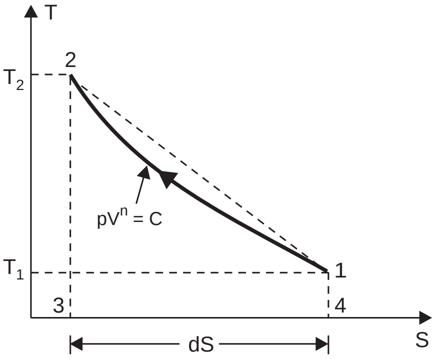


Fig. 5.41

Heat transferred = Area of trapezium 1-2-3-4

= Base × mean ordinate

$= dS \times \left(\frac{T_1 + T_2}{2}\right)$

= Entropy change × mean absolute temperature during the process

or, Entropy change $= \dfrac{\text{Heat transferred}}{\text{Mean absolute temperature}}$

(*b*) *Given :* $m = 1$ kg ; $T_1 = 290$ K ; $pv^{1.3}$ = constant ; $r = 16$; $\gamma = 1.4$; $c_v = 0.718$ kJ/kg K

For a polytropic process : $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{n-1}$

or, $T_2 = 290 \times (16)^{1.3-1} = 666.2$ K

Now, $s_2 - s_1 = c_v \left(\dfrac{n-\gamma}{n-1}\right) \log_e \left(\dfrac{T_2}{T_1}\right)$ ... per kg

$= 0.718 \left(\dfrac{1.3-1.4}{1.3-1}\right) \log_e \left(\dfrac{666.2}{290}\right) =$ **– 0.199 kJ/kg K. (Ans.)**

The –ve sign indicates *decrease* in entropy.

Heat transferred during the process is given by,

$$Q = \frac{\gamma - n}{\gamma - 1} \times W = \frac{\gamma - n}{\gamma - 1} \times \frac{R(T_1 - T_2)}{n - 1} = c_v \left(\frac{\gamma - n}{\gamma - 1}\right)(T_1 - T_2) \text{ ...per kg}$$

$$= 0.718 \left(\frac{1.4-1.3}{1.3-1}\right)(290 - 666.2) = -\ 90.04 \text{ kJ/kg} \qquad \left(\because c_v = \frac{R}{\gamma - 1}\right)$$

Mean absolute temperature, $T_{\text{mean}} = \dfrac{T_1 + T_2}{2} = \dfrac{290 + 666.2}{2} = 478.1$ K

Approximate change of entropy $= \dfrac{Q}{T_{\text{mean}}} = \dfrac{-90.04}{478.1} = -0.188$ kJ/kg K

$\therefore$ % age error $= \dfrac{0.199 - 0.188}{0.199} \times 100 =$ **5.53%. (Ans.)**

- The approximate value of entropy change is lower, because in the relation $Q = T_{av} \times dS$ actual value of heat transferred is substituted instead of approximate value (*i.e.*, Area under the straight line) which is higher.

Example 5.36. *1.2 m³ of air is heated reversibly at constant pressure from 300 K to 600 K, and is then cooled reversibly at constant volume back to initial temperature. If the initial pressure is 1 bar, calculate :*

(*i*) *The net heat flow.*

(*ii*) *The overall change in entropy.*

Represent the processes on T-S plot.

Take $c_p = 1.005$ kJ/kg K and R = 0.287 kJ/kg K

Solution. *Given :* $V_1 = 1.2$ m³ ; $p_1 = p_2$; $T_1 = 300$ K ; $T_2 = 600$ K ; $p_1 = 1$ bar ;
$c_p = 1.005$ kJ/kg K ; $R = 0.287$ kJ/kg K

Fig. 5.42 shows the *T-S* plot of the processes.

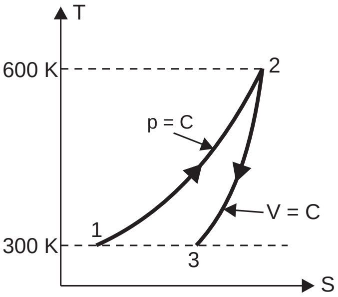


Fig. 5.42

(*i*) **The net heat flow, Q :**

Mass of air, $m = \dfrac{p_1V_1}{RT_1} = \dfrac{1 \times 10^5 \times 1.2}{(0.287 \times 1000) \times 300} = 1.394$ kg

$$Q = Q_{1\text{-}2} + Q_{2\text{-}3}$$
$$= mc_p(T_2 - T_1) + mc_v(T_3 - T_2)$$
$$= mc_p(T_2 - T_1) + mc_v(T_1 - T_2) \quad ...(\because T_1 = T_3)$$
$$= m(T_2 - T_1)(c_p - c_v) = m(T_2 - T_1) \times R$$
$$= 1.394(600 - 300) \times 0.287 = \textbf{120 kJ.} \quad \textbf{(Ans.)}$$

(*ii*) **The overall change in entropy :**

Entropy change during *constant pressure process 1-2*,

$$S_2 - S_1 = mc_p \log_e \left(\frac{T_2}{T_1}\right)$$

$$= 1.394 \times 1.005 \log_e \left(\frac{600}{300}\right) = 0.9711 \text{ kJ/K}$$

Entropy change during *constant volume process 2-3 ;*

$$S_3 - S_2 = mc_v \log_e \left(\frac{T_3}{T_2}\right) = m(c_p - R) \log_e \left(\frac{T_1}{T_2}\right)$$

$$= 1.394 \times (1.005 - 0.287) \log_e \left(\frac{300}{600}\right) = -0.6938 \text{ kJ/K}$$

∴ Overall change of entropy

$$= (S_2 - S_1) + (S_3 - S_2)$$

$$= 0.9771 + (-0.6938) = \mathbf{0.2833\ kJ/K.} \quad \textbf{(Ans.)}$$

Example 5.37. *A closed system contains air at a pressure 1 bar, temperature 300 K and volume 0.018* m^3*. This system undergoes a thermodynamic cycle consisting of the following three processes in series :* (*i*) *Constant volume heat addition till pressure becomes 5 bar,* (*ii*) *Constant pressure cooling, and* (*iii*) *Isothermal heating to initial state.*

Represent the cycle on T-S and p-V plots and evaluate the change in entropy for each process.

Take $c_p = 0.718$ *kJ/kg K and* $R = 0.287$ *kJ/kg K.*

Solution. *Given :* $p_1 = 1$ bar ; $T_1 = 300$ K ; $V_1 = 0.018$ m^3 ; $p_2 = 5$ bar ;

$c_v = 0.718$ kJ/kg K ; $R = 0.287$ kJ/kg K.

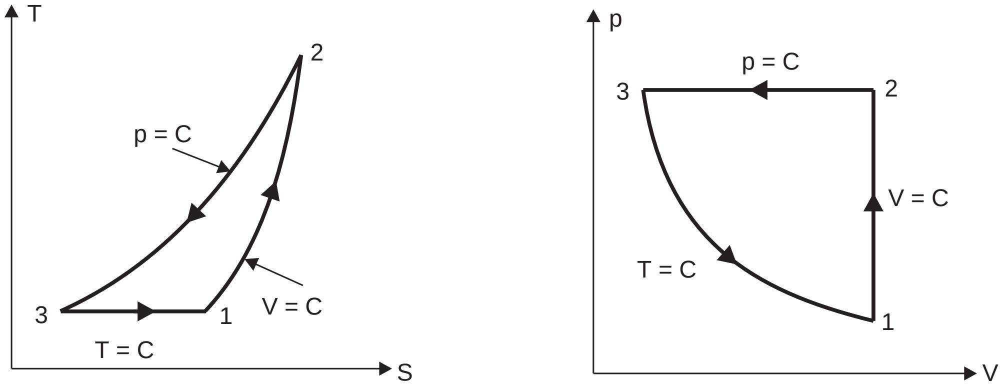


Fig. 5.43. *T-S* and *p-V* diagrams.

$$\text{Mass of air, } m = \frac{p_1V_1}{RT_1} = \frac{(1 \times 10^5) \times 0.018}{(0.287 \times 1000) \times 300} = 0.0209 \text{ kg}$$

Refer to Fig. 5.43.

- *Constant volume process 1-2 :*

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \text{or} \quad T_2 = T_1 \times \frac{p_2}{p_1} = 300 \times \frac{5}{1} = 1500 \text{ K}$$

∴ Change in entropy, $\mathbf{S_2 - S_1} = mc_v \log_e \left(\frac{T_2}{T_1}\right)$

$$= 0.0209 \times 0.718 \times \log_e \left(\frac{1500}{300}\right) = \mathbf{0.0241\ kJ/K.} \quad \textbf{(Ans.)}$$

- *Constant pressure process 2-3 :*

$$T_3 = T_1 = 300 \text{ K}$$

Now, change in entropy,

$$\mathbf{S_3 - S_2} = mc_p \log_e \left(\frac{T_3}{T_2}\right) = m(c_v + R) \log_e \left(\frac{T_3}{T_2}\right)$$

$$= 0.0209 \times (0.718 + 0.287) \times \log_e \left(\frac{300}{1500}\right)$$

$$= -\ \mathbf{0.0338\ kJ/K.}\quad \textbf{(Ans.)}$$

- *Constant temperature (isothermal) process 3-1* :

$$p_3 = p_2 = 5 \text{ bar}$$

Change in entropy,

$$\mathbf{S_1 - S_3} = mR \log_e \left(\frac{p_3}{p_1}\right)$$

$$= 0.0209 \times 0.287 \times \log_e \left(\frac{5}{1}\right) = \mathbf{0.00965\ kJ/K.}\quad \textbf{(Ans.)}$$

Example 5.38. *Derive expressions for entropy change with variable specific heat.*

Solution. Let us assume that the specific heats of a gas vary with temperature according to the linear relations :

$$c_p = a + kT, \quad \text{and} \quad c_v = b + kT$$

where, a, b and k = Constants, and T = Temperature, K.

For *unit mass* of gas, $Q = T\,ds = du + pdv$

$$= c_v dT + pdv$$

or,
$$ds = c_v \frac{dT}{T} + \frac{pdv}{T} = c_v \frac{dT}{T} + R\frac{dv}{v} \qquad (\because\ pv = RT)$$

Now,
$$R = c_p - c_v = (a + kT) - (b + kT) = a - b$$

$\therefore$
$$ds = (b + kT)\frac{dT}{T} + (a - b)\frac{dv}{v}$$

This is the differential form of entropy change.

Integrating both sides between limits 1 and 2, we get

$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) + (a - b) \log_e \left(\frac{v_2}{v_1}\right) \qquad ...(1)$$

For the entropy change the following expressions can be obtained by suitable manipulations to *eqn. (1)* :

1. *Expression for entropy change in terms of* **temperature** *only* :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$$

or,
$$\log_e \frac{T_2}{T_1} = (n-1)\log_e \left(\frac{v_1}{v_2}\right) = -(n-1)\log_e \left(\frac{v_2}{v_1}\right)$$

$\therefore$
$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) - \left(\frac{a-b}{n-1}\right)\log_e \left(\frac{T_2}{T_1}\right) \qquad ...\text{[From eqn. (1)]}$$

or,
$$\mathbf{s_2 - s_1} = \left(b - \frac{a-b}{n-1}\right)\log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) \qquad ...(i)$$

2. *Expression for entropy change in terms of* **pressure, volume and temperature.**

From eqn. (1), we have

$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) + a \log_e \left(\frac{v_2}{v_1}\right) - b \log_e \left(\frac{v_2}{v_1}\right)$$

$$= a \log_e \left(\frac{v_2}{v_1}\right) + b \log_e \left(\frac{T_2}{T_1} \times \frac{v_1}{v_2}\right) + k(T_2 - T_1)$$

or,

$$\mathbf{s_2 - s_1} = a \log_e \left(\frac{v_2}{v_1}\right) + b \log_e \left(\frac{p_2}{p_1}\right) + k(T_2 - T_1) \quad ...(ii)$$

3. *Expression for entropy change in terms of* **pressure and temperature only.**

Again, from eqn. (1), we have

$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1}\right) + k\,(T_2 - T_1) + a \log_e \left(\frac{v_2}{v_1}\right) - b \log_e \left(\frac{v_2}{v_1}\right)$$

$$= a \log_e \left(\frac{T_2}{T_1} \times \frac{p_1}{p_2}\right) + b \log_e \left(\frac{T_2}{T_1} \times \frac{v_1}{v_2}\right) + k\,(T_2 - T_1)$$

$$= a \log_e \left(\frac{T_2}{T_1}\right) - a \log_e \left(\frac{p_2}{p_1}\right) + b \log_e \left(\frac{p_2}{p_1}\right) + k(T_2 - T_1)$$

or,

$$\mathbf{s_2 - s_1} = a \log_e \left(\frac{T_2}{T_1}\right) + (b - a) \log_e \left(\frac{p_2}{p_1}\right) + k(T_2 - T_1) \quad ...(iii)$$

- Derivation of the formula $T^b v^{a-b} e^{kT}$ = constant for the *adiabatic expansion of gas* :

We know that, $ds = (a - b)\dfrac{dv}{v} + (b + kT)\dfrac{dT}{T}$

$$\left.\begin{aligned} s_2 - s_1 &= a \log_e v - b \log_e v + b \log_e T + kT \\ &= a \log_e v + b \log_e \left(\frac{T}{v}\right) + kT \\ &= a \log_e v + b \log_e \left(\frac{p}{R}\right) + kT \\ &= a \log_e v + \log_e \left(\frac{p}{a - b}\right)^b + kT \end{aligned}\right\} = 0 \text{ for adiabatic expansion}$$

This gives :

$$v^a\, p^b\, e^{kT} = \text{constant}$$
$$p v^{a-b}\, e^{kT} = \text{constant}$$
$$T^b\, v^{a-b} e^{kT} = \text{constant}$$

The above expressions can be obtained by taking kT on right-side and taking the antilog of the resulting expressions.

Example 5.39. *Determine the entropy change of 4 kg of a perfect gas whose temperature varies from 127°C to 227°C during a constant volume process. The specific heat varies linearly with absolute temperature and is represented by the relation :*

$$c_v = (0.48 + 0.0096\, T) \text{ kJ/kg K.}$$

Solution. *Given :* $m = 4$ kg ; $T_1 = 127 + 273 = 400$ K ; $T_2 = 227 + 273 = 500$ K ;

$$c_v = (0.48 + 0.0096\, T) \text{ kJ/kg K.}$$

Entropy variation for a constant volume process is given by :

$$dS = mc_v \frac{dT}{T}, \quad \text{or,} \quad dS = 4 \times (0.48 + 0.0096T) \frac{dT}{T}$$

Integrating both sides, we get,

$$S_2 - S_1 = 4 \times 0.48 \int_{T_1}^{T_2} dT + 4 \times 0.0096 \int_{T_1}^{T_2} dT$$

$$= 1.92 \log_e \left(\frac{T_2}{T_1}\right) + 0.0384\,(T_2 - T_1)$$

$$= 1.92 \log_e \left(\frac{500}{400}\right) + 0.0384(500 - 400) = 4.268 \text{ kJ/K}$$

i.e., $\mathbf{S_2 - S_1 = 4.268}$ **kJ. (Ans.)**

Example 5.40. *The specific heats of a gas vary linearly with absolute temperature according to the following relations :*

c_p *= (0.85 + 0.00025 T) kg/kg K, and*

c_v *= (0.56 + 0.00025 T) kJ/kg K*

If the entropy of the gas at 1 bar pressure and 273 K is zero, find the entropy of the gas at 25 bar and 750 K temperature.

Solution. *Given :* $c_p = (0.85 + 0.00025\ T)$ kJ/kg K ; $c_v = (0.56 + 0.00025\ T)$ kJ/kg K ;

$p_1 = 1$ bar ; $T_1 = 273$ K ; $p_2 = 25$ bar ; $T_2 = 750$ K.

We know that, $$ds = c_v \frac{dT}{T} + \frac{p}{T} dv = c_v \frac{dT}{T} + R \frac{dv}{v}$$

Integrating both sides, we get,

$$s_2 - s_1 = \int c_v \frac{dT}{T} + R \log_e \left(\frac{v_2}{v_1}\right)$$

$$= \int c_v \frac{dT}{T} + (c_p - c_v) \log_e \left(\frac{p_1}{p_2} \times \frac{T_2}{T_1}\right)$$

$$= \int \left(\frac{0.56}{T} + 0.00025\right) dT + 0.29 \log_e \left(\frac{1}{25} \times \frac{750}{273}\right)$$

$$= \left[0.56 \log_e \left(\frac{T_2}{T_1}\right) + 0.00025\,(T_2 - T_1)\right] - 0.6404$$

$$= 0.56 \log_e \left(\frac{750}{273}\right) + 0.00025(750 - 273) - 0.6404 = 0.0448 \text{ kJ/kg K}$$

i.e., $\mathbf{s_2 - s_1 = 0.0448}$ **kJ/kg K. (Ans.)**

Example 5.41. *An insulated vessel of 0.5 m*^{3} *capacity is divided by a rigid conducting diaphragm into two chambers A and B, each having a capacity of 0.25 m*^{3}*. Chamber A contains air at 1.4 bar pressure and 290 K temperature and the corresponding parameters for air in chamber B are 4.2 bar and 440 K. Calculate :*

(*i*) *Final equilibrium temperature,*

(*ii*) *Final pressure on each side of the diaphragm, and*

(*iii*) *Entropy change of system.*

For air take c_v *= 0.715 kJ/kg K and R = 0.287 kJ/kg K.*

Solution. *Given :* $V_A = 0.25\ \text{m}^3$; $p_{Ai} = 1.4$ bar ; $T_{Ai} = 290$ K ; $V_B = 0.25\ \text{m}^3$; $p_{Bi} = 4.2$ bar ; $T_{Bi} = 440$ K ; $c_v = 0.715$ kJ/kg K.

(*i*) **Final equilibrium temperature, T_f :**

Mass of air, $$m_A = \frac{p_{Ai}V_A}{RT_{Ai}} = \frac{(1.4 \times 10^5) \times 0.25}{(0.287 \times 1000) \times 290} = 0.4205 \text{ kg}$$

$$m_B = \frac{p_{Bi}V_B}{RT_{Bi}} = \frac{(4.2 \times 10^5) \times 0.25}{(0.287 \times 1000) \times 440} = 0.8315 \text{ kg}$$

Let T_f be the final equilibrium temperature (K). Since the diaphragm is conducting,

Heat gained by air in chamber A = Heat lost by air in chamber B

$$m_A\, c_v\, (T_f - 290) = m_B\, c_v (440 - T_f)$$

or, $$0.4205 \times (T_f - 290) = 0.8315(440 - T_f)$$

or, $$0.4205\ T_f - 121.94 = 365.86 - 0.8315\ T_f$$

$\therefore$ $T_f =$ **389.6 K. (Ans.)**

(*ii*) **Final pressure on each side of the diaphragm : p_{Af} ; p_{Bf}**

$$p_{Af} = \frac{1.4 \times 389.6}{290} = \mathbf{1.88\ bar.\ \ (Ans.)}$$

$$p_{Bf} = \frac{4.2 \times 389.6}{440} = \mathbf{3.72\ bar.\ \ (Ans.)}$$

(*iii*) **Entropy change of the system :**

Entropy change for chamber $A = m_A\, c_v \log_e \left(\frac{T_f}{T_{Ai}}\right)$

$$= 0.4205 \times 0.715 \times \log_e \left(\frac{389.6}{290}\right) = 0.0888 \text{ kJ/K}$$

Entropy change for chamber $B = m_B\, c_v \log_e \left(\frac{T_f}{T_{Bi}}\right)$

$$= 0.8315 \times 0.715 \times \log_e \left(\frac{389.6}{440}\right) = -\ 0.0723 \text{ kJ/K}$$

$\therefore$ Net change of entropy = 0.0888 + (– 0.0723) = **0.0165 kJ/K. (Ans.)**

Example 5.42. *A certain gas has a specific heat at constant volume of 1.25 kJ/kg K. When it is expanded reversibly and adiabatically from a specific volume of 0.0624 m³/kg and a temperature of 530 K to a specific volume of 0.186 m³/kg its temperature falls by 165 K. When it is expanded into an evacuated space from the same initial condition to the same final specific volume its temperature falls only by 25 K.*

Find the change in entropy in each of the adiabatic processes.

Solution. Refer Fig. 5.44.

Specific heat of gas at constant volume, $c_v = 1.25$ kJ/kg K

Initial specific volume, $v_1 = 0.0624\ \text{m}^3/\text{kg}$

Initial temperature, $T_1 = 530$ K

Final specific volume, $v_2 = 0.186\ \text{m}^3/\text{kg}$

Temperature fall when expanded reversibly and adiabatically = 165 K
Temperature fall when expanded into an evacuated space = 25 K

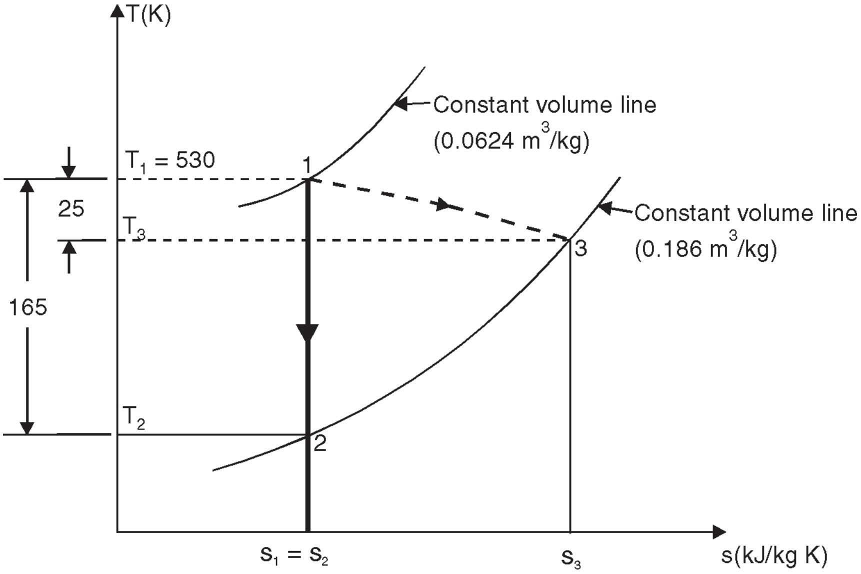


Fig. 5.44

Change in entropy :

Path 1-2 : *Reversible adiabatic process.*

Change in entropy, $(s_2 - s_1) = 0$.

Path 1-3 : *Adiabatic process such that* $v_3 = 0.186\ m^3/kg\ (= v_2)$

(States 2 and 3 lie on the same constant volume line on *T-s* diagram)

and $T_1 - T_3 = 25$ K.

Change in entropy during this adiabatic process = $s_3 – s_1$.

To calculate $(s_3 – s_1)$ a reversible path has to be selected joining 3 and 1.

This is achieved by selecting the reversible adiabatic path 1-2 and the reversible constant volume process 2-3.

$$s_3 - s_1 = (s_3 - s_2) + (s_2 - s_1)$$
$$= (s_3 - s_2) + 0 = (s_3 - s_2)$$
$$= c_v \log_e \frac{T_3}{T_2} = 1.25 \log_e \left(\frac{530 - 25}{530 - 165}\right)$$
$$= 1.25 \log_e \left(\frac{505}{365}\right) = \mathbf{0.4058\ kJ/kg\ K.} \quad \textbf{(Ans.)}$$

Example 5.43. *A heat pump operates between two identical bodies which are at temperature* T_1 *and cools one of the bodies to a temperature* T_2 $(T_2 < T_1)$*. Prove that for this operation the minimum work required by the heat pump is given by*

$$W = c_p \left(\frac{T_1^2}{T_2^2} + T_2 - 2T_1\right)$$

where c_p *is the specific heat which is same for both the bodies.*

Solution. The arrangement is shown in Fig. 5.45.

For the minimum work absorbed by the heat pump, the heat pump must be reversed Carnot cycle engine and the required condition is

$$\oint \left(\frac{dQ}{T}\right) = 0$$

For infinitely small changes, we can write

$$c_p \frac{dT_1}{T_1} + c_p \frac{dT_2}{T_2} = 0$$

If T_1' is the final temperature of the high level reservoir, then the above equation can be written as

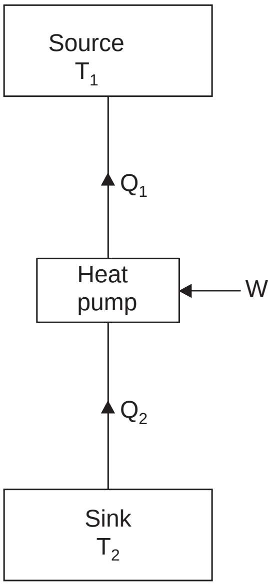


Fig. 5.45

$$c_p \int_{T_1}^{T_1'} \left(\frac{dT_1}{T_1}\right) + c_p \int_{T_1}^{T_2} \left(\frac{dT_2}{T_2}\right) = 0$$

$$\therefore \quad \log_e \left(\frac{T_1'}{T_1}\right) + \log_e \left(\frac{T_2}{T_1}\right) = 0 = \log_e (1)$$

$$\therefore \quad \log_e \left(\frac{T_1' T_2}{T_1^2}\right) = \log_e (1)$$

$$\therefore \quad T_1' = \frac{T_1^2}{T_2}$$

Now the work given to the heat pump

= Heat rejected at higher level temperature

– heat picked up at lower level temperature

$$\therefore \quad W = c_p \int_{T_1}^{T_1'} (dT) - c_p \int_{T_2}^{T_1} (dT)$$

$$= c_p \, [(T_1' - T_1) - (T_1 - T_2)] = c_p \, (T_1' + T_2 - 2T_1)$$

Now substituting the value of T_1' in the above equation in terms of T_1 and T_2

$$W = c_p \left[\frac{T_1^2}{T_2} + T_2 - 2T_1\right]. \quad \textbf{Proved.}$$

☞**Example 5.44.** *The connections of a reversible engine to three sources at 500 K, 400 K and 300 K are shown in Fig. 5.46. It draws 1500 kJ/min of energy from the source at 800 K and develops 200 kJ/min of work.*

(*i*) *Determine the heat interactions with the other two sources of heat.*

(*ii*) *Evaluate the entropy change due to each heat interaction with the engine.*

(*iii*) *Total entropy change during the cycle.*

Solution. Refer Fig. 5.46.

Temperature of source 1 = 500 K

Temperature of source 2 = 400 K

Temperature of source 3 = 300 K

Heat energy drawn from source 1, Q_1 = 1500 kJ/min

Work developed, W = 200 kJ/min.

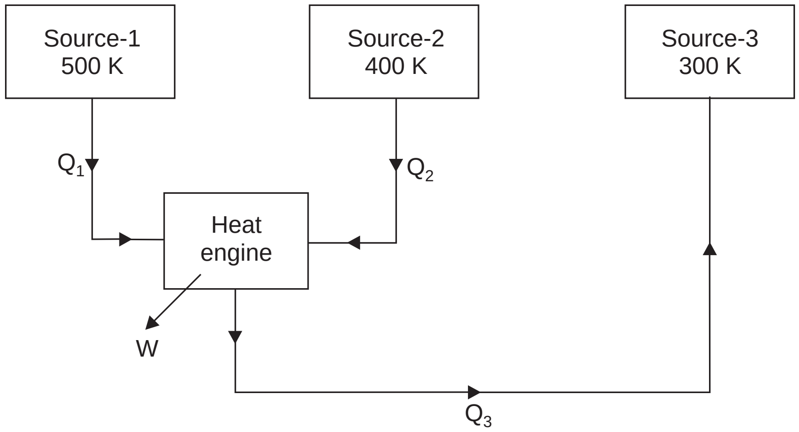


Fig. 5.46

(*i*) The direction of heat flow from source 1 is known as given in the problem. Assume that the quantities of heats Q_2 and Q_3 are taken from heat sources and their directions are arbitrarily chosen.

For the cyclic operation of the engine

$$\oint \left(\frac{dQ}{T}\right) = 0$$

$$\therefore \qquad \frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = 0$$

and

$$Q_1 + Q_2 - Q_3 = W$$

$$\therefore \qquad \frac{1500}{500} + \frac{Q_2}{400} - \frac{Q_3}{300} = 0 \qquad \text{...}(i)$$

and

$$1500 + Q_2 - Q_3 = 200 \qquad \text{...}(ii)$$

Solving eqns. (*i*) and (*ii*), we get

$$Q_2 = -\ 1600 \text{ kJ/min and } Q_3 = -\ 300 \text{ kJ/min.}$$

The above values indicate that the direction of Q_2 and Q_3 are reversed. Since Q_2 should be +v and Q_3 also must be +ve but both are –ve therefore, their assumed directions should be reversed. The arrangement is shown in Fig. 5.47.

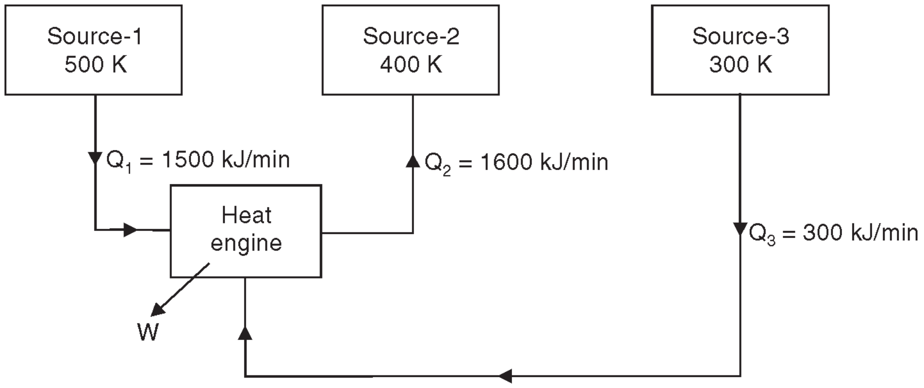


Fig. 5.47

(*ii*) *Entropy change of source 1* $= \dfrac{-Q_1}{T_1} = \dfrac{-1500}{500} = -$ **3 kJ/K. (Ans.)**

Entropy change of sink 2 $= \dfrac{Q_2}{T_2} = \dfrac{1600}{400} =$ **4 kJ/K. (Ans.)**

Entropy change of source 3 $= \dfrac{-Q_3}{T_3} = \dfrac{-300}{300} = -$ **1 kJ/K. (Ans.)**

(*iii*) Net change of the entropy $= -3 + 4 - 1 = 0$

As *the cycle is completed, the net change in entropy must be zero because entropy is a property.*

It may be observed from the new arrangement that the engine takes heat from source 1 and source 3 and rejects to source 2, only *i.e.*, the equipment does both *a heat engine and a heat pump function.*

Example 5.45. *The heat capacity of a system at constant volume is given by* $C_v = ZT^2$ *where* $Z = 0.045$ *J/K*^{3}

A system is originally at 250 K, and a thermal reservoir at 125 K is available. Determine the maximum amount of work that can be recovered as the system is cooled down to the temperature of the reservoir.

Solution. Refer Fig. 5.48.

Initial temperature of the system, $T_1 = 250$ K

Temperature of the reservoir, $T_2 = 125$ K

Heat capacity of the system at constant volume is

$$C_v = ZT^2 = 0.045\ T^2$$

$$\left[\begin{array}{l}\text{The product of mass and specific heat } (mc) \text{ is called the} \\ \textit{heat capacity} \text{ of the substance. The capital letter } C, C_p \\ \text{or } C_v \text{ is used for heat capacity}\end{array}\right]$$

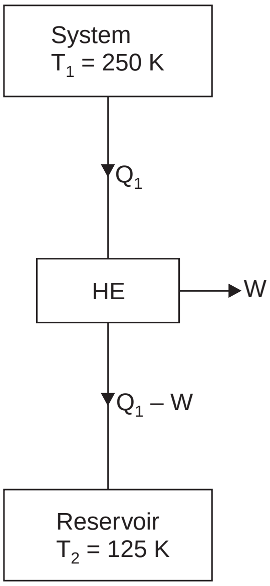


Fig. 5.48

Heat removed from the system

$$Q_1 = \int_{T_1}^{T_2} C_v\, dT = \int_{250}^{125} 0.045\ T^2\, dT$$

$$= 0.045 \left[\frac{T^3}{3}\right]_{250}^{125} = \frac{0.045}{3}\,(125^3 - 250^3)$$

$$= -205.08 \times 10^3 \text{ J}$$

$$(\Delta S)_{system} = \int_{250}^{125} C_v \frac{dT}{T} = \int_{250}^{125} 0.045\, T^2 \frac{dT}{T}$$

$$= 0.045 \int_{250}^{125} T dT = 0.045 \left[\frac{T^2}{2}\right]_{250}^{125}$$

$$= \frac{0.045}{2}\,(125^2 - 250^2) = -1054.7 \text{ J/K}$$

$$(\Delta S)_{\text{reservoir}} = \frac{Q_1 - W}{T_{\text{reservoir}}} = \frac{205.08 \times 10^3 - W}{125} \text{ J/K}$$

$$(\Delta S)_{\text{working fluid in } HE} = 0$$

$$\therefore \quad (\Delta S)_{\text{universe}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{reservoir}} = -1054.7 + \frac{205.08 \times 10^3 - W}{125}$$

Since $\quad (\Delta S)_{\text{universe}} \geq 0$

$$\therefore \quad -1054.7 + \frac{205.08 \times 10^3 - W}{125} \geq 0$$

or $\quad -1054.7 + 1640.64 - \frac{W}{125} \geq 0 \quad$ or $\quad 585.94 - \frac{W}{125} \geq 0$

or $\quad 585.94 \geq \frac{W}{125} \quad$ or $\quad \frac{W}{125} \leq 585.94$

i.e., $\quad W_{(\text{max.})} = 585.94 \times 125 =$ **73.24 kJ. (Ans.)**

Example 5.46. *In an insulated duct air is flowing steadily. The pressure and temperature measurements of the air at two stations A and B are given below :*

Station	*Pressure*	*Temperature*
A	*140 kPa*	*60°C*
B	*110 kPa*	*15°C*

Establish the direction of the flow of the air in the duct.

For air assume that :

$$c_p = 1.005 \text{ kJ/kg K}$$

$$h = c_p T \text{ and } \frac{v}{T} = \frac{0.287}{p}$$

where p, v and T are pressure (in kPa), volume (in m^3/kg) and temperature (in K) respectively.

Solution. From property relation,

$$Tds = dh - vdp$$

$$ds = \frac{dh}{T} - \frac{vdp}{T}$$

For two states at A and B the entropy changes of the system is given by

$$\int_{s_A}^{s_B} ds = \int_{T_A}^{T_B} \frac{c_p dT}{T} - \int_{p_A}^{p_B} 0.287 \frac{dp}{p}$$

$$\therefore \quad s_B - s_A = 1.005 \log_e \frac{T_B}{T_A} - 0.287 \log_e \frac{110}{140}$$

$$= 1.005 \log_e \left(\frac{15 + 273}{60 + 273}\right) - 0.287 \log_e \frac{110}{140}$$

$$= -0.1459 + 0.0692 = -0.0767 \text{ kJ/kg K}$$

$$(\Delta s)_{\text{system}} = -0.0767 \text{ kJ/kg K}$$

Since the duct is insulated, $(\Delta s)_{\text{surroundings}} = 0$

$$\therefore \quad (\Delta s)_{\text{universe}} = -0.0767 \text{ kJ/kg K.}$$

This is impossible.

So, **the flow is from B to A. (Ans.)**

Example 5.47. *3 kg of water at 80°C is mixed with 4 kg of water at 15°C in an isolated system. Calculate the change of entropy due to mixing process.*

Solution. Fig. 5.49 shows the isolated system before mixing. When barrier is removed, the water from two compartments mix each other. Let t_m is the final equilibrium temperature after mixing.

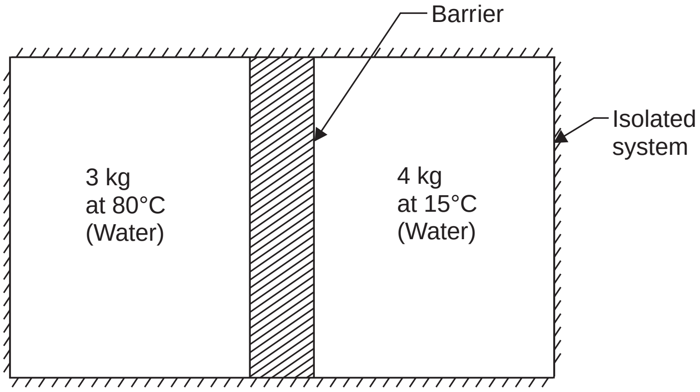


Fig. 5.49

Applying first law of thermodynamics to the isolated system :

Total energy before mixing = Total energy after mixing

$$\therefore \quad 3c_{pw}\,(80 - 0) + 4c_{pw}\,(15 - 0) = 7\,c_{pw}\,(t_m - 0)$$

[c_{pw} = Specific heat of water at constant pressure]

$$\text{or} \quad 240c_{pw} + 60c_{pw} = 7c_{pw}\,t_m$$

$$\text{or} \quad 240 + 60 = 7\,t_m$$

$$\therefore \quad t_m = \frac{300}{7} = 42.85\text{°C}$$

Initial entropy of the system,

$$= 3c_{pw}\log_e\left(\frac{80 + 273}{273}\right) + 4c_{pw}\log_e\left(\frac{15 + 273}{273}\right)$$

$$= 0.7709c_{pw} + 0.2139\,c_{pw} = 0.9848\,c_{pw}$$

Final entropy of the system

$$= (3 + 4)\,c_{pw}\log_e\left(\frac{42.85 + 273}{273}\right) = 1.0205\,c_{pw}$$

Net change in entropy,

$$\Delta S = \text{Final entropy} - \text{Initial entropy}$$

$$= 1.0205c_{pw} - 0.9848\,c_{pw} = 0.0357\,c_{pw}$$

$$= 0.0357 \times 4.187 \text{ kJ/K} \qquad [\because \; c_{pw} = 4.187 \text{ kJ/kg K}]$$

$$= 0.1495 \text{ kJ/K}$$

Hence, **net change in entropy = 0.1495 kJ/K. (Ans.)**

Example 5.48. *A mass 'm' of fluid at temperature* T_1 *is mixed with an equal mass of the same fluid at* T_2*. Prove that the resultant change of entropy of the universe is* $2\,mc\,\dfrac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$ *and also prove that it is always positive.*

Solution. Mean temperature of the mixture = $\left(\frac{T_1 + T_2}{2}\right)$.

Thus change in entropy is given by

$$\Delta S = mc\int_{T_1}^{(T_1+T_2)/2} \frac{dT}{T} - mc\int_{(T_1+T_2)/2}^{T_2} \frac{dT}{T}$$

$$= mc \log_e \left(\frac{T_1 + T_2}{2T_1}\right) - mc \log_e \left(\frac{2T_2}{T_1 + T_2}\right)$$

$$= mc \log_e \left(\frac{T_1 + T_2}{2T_1}\right) + mc \log_e \left(\frac{T_1 + T_2}{2T_2}\right) = mc \log_e \frac{(T_1 + T_2)^2}{4T_1 T_2}$$

$$= mc \log_e \left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right]^2 = 2\ mc \log_e \left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right]$$

$$= 2\ mc \log_e \left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}\right]$$

i.e., **Resultant change of entropy of universe**

$$\mathbf{= 2\ mc \log_e \left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}\right]} \quad \textbf{...Proved}$$

The arithmetic mean $\frac{T_1 + T_2}{2}$ is greater than geometric mean $\sqrt{T_1 T_2}$.

Therefore, $\log_e \frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$ is + ve.

Thus, **the entropy of the universe increases.**

☞**Example 5.49.** (*a*) *One kg of water at 0°C is brought into contact with a heat reservoir at 90°C. When the water has reached 90°C, find :*

(*i*) *Entropy change of water ;*

(*ii*) *Entropy change of the heat reservoir ;*

(*iii*) *Entropy change of the universe.*

(*b*) *If water is heated from 0°C to 90°C by first bringing it in contact with a reservoir at 40°C and then with a reservoir at 90°C, what will the entropy change of the universe be ?*

(*c*) *Explain how water might be heated from 0°C to 90°C with almost no change in the entropy of the universe.*

Solution. Mass of water, $m = 1$ kg

Temperature of water, $T_1 = 0 + 273 = 273$ K

Temperature of the heat reservoir, $T_2 = 90 + 273 = 363$ K.

(*a*) Refer Fig. 5.50. Water is being heated through a finite temperature difference. The entropy of water would increase and that of the reservoir would decrease so that net entropy change of the water (system) and the reservoir together would be positive definite. *Water is being heated, irreversibly,* and to find the entropy change of water, we have to assume a reversible path between the end states, which are at equilibrium.

(*i*) *Entropy of water,*

$$(\Delta s)_{water} = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc \log_e \frac{T_2}{T_1}$$

(c = Specific heat of water)

$$= 1 \times 4.187 \times \log_e \frac{363}{273}$$

= **1.193 kJ/kg K. (Ans.)**

(*ii*) The temperature of the reservoir remains *constant* irrespective of the amount of the heat withdrawn.

Amount of heat absorbed by the system from the reservoir,

$$Q = 1 \times 4.187 \times (363 - 273) = 376.8 \text{ kJ}$$

∴ *Entropy change of the reservoir,*

$$(\Delta s)_{reservoir} = -\frac{Q}{T} = -\frac{376.8}{363}$$

= – **1.038 kJ/K. (Ans.)**

(–ve sign indicates decrease in entropy).

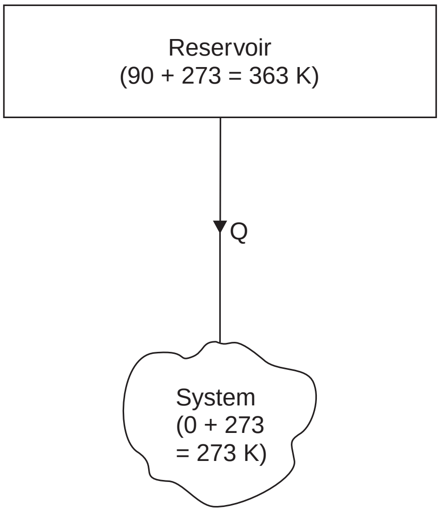


Fig. 5.50

(*iii*) *Entropy change of the universe,*

$$(\Delta s)_{universe} = (\Delta s)_{water\ (or\ system)} + (\Delta s)_{reservoir}$$

= 1.193 + (– 1.038) = **0.155 kJ/K. (Ans.)**

(*b*) The heating of water is being carried out in two stages, first from 0°C (273 K) to 40°C (*i.e.,* 313 K) by bringing in contact with a reservoir at 40°C (313 K), and then from 40°C (313 K) to 90°C (363 K) by bringing in contact with a second reservoir at 90°C (363 K).

$$(\Delta s)_{water} = \int_{273}^{313} mc \frac{dT}{T} + \int_{313}^{363} mc \frac{dT}{T} = 1 \times 4.187 \left(\log_e \frac{313}{273} + \log_e \frac{363}{313} \right)$$

$$= 4.187\ (0.1367 + 0.1482) = 1.1928 \text{ kJ/K.}$$

$$(\Delta s)_{res.\ I} = -\frac{1 \times 4.187 \times (313 - 273)}{313} = -0.535 \text{ kJ/K}$$

$$(\Delta s)_{res.\ II} = -\frac{1 \times 4.187 \times (363 - 313)}{363} = -0.576 \text{ kJ/K}$$

$$\therefore \quad (\Delta s)_{univ.} = (\Delta s)_{water} + (\Delta s)_{res.\ I} + (\Delta s)_{res.\ II}$$

$$= 1.1928 + (-0.535) + (-0.576) = 0.0818 \text{ kJ/K.}$$

i.e., **Entropy change of universe = 0.0818 kJ/K. (Ans.)**

(*c*) The entropy change of universe would be *less and less*, if the water is heated in *more and more stages*, by bringing the water in contact successively with *more and more heat reservoirs,* each succeeding reservoir being at a *higher temperature* than the preceding one. When water is heated in infinite steps, by bringing in contact with an infinite number of reservoirs in succession, so that at any instant the temperature difference between the water and the reservoir in contact is infinitesimally small, then the entropy change of the universe would be *zero* and the water would be *reversibly* heated.

Example 5.50. *1 kg of ice at – 5°C is exposed to the atmosphere which is at 25°C. The ice melts and comes into thermal equilibrium.*

(*i*) *Determine the entropy increase of the universe.*

(ii) What is the minimum amount of work necessary to convert the water back into ice at – 5°C ?

Take : c_p *of ice = 2.093 kJ/kg°C*

Latent heat of fusion of ice = 333.33 kJ/kg.

Solution. Mass of ice, m = 1 kg

Temperature of ice = – 5°C (= – 5 + 273 = 268 K)

Temperature of atmosphere = 25°C (= 25 + 273 = 298 K)

Heat absorbed by ice from the atmosphere (Fig. 5.51)

= Heat absorbed in solid phase + latent heat + heat absorbed in liquid phase

= 1 × 2.093 × [0 – (– 5)] + 1 × 333.33 + 1 × 4.187 × (25 – 0)

= 10.46 + 333.33 + 104.67 = 448.46 kJ.

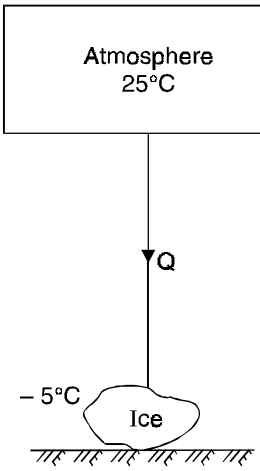


Fig. 5.51

(*i*) **Entropy increase of the universe,** $(\Delta s)_{universe}$:

Entropy change of the atmosphere,

$$(\Delta s)_{atm.} = -\frac{Q}{T} = -\frac{448.46}{298} = -1.5049 \text{ kJ/K}$$

Entropy change of system (ice) as it gets heated from – 5°C to 0°C,

$$(\Delta s_I)_{system} = \int_{268}^{273} mc_p \frac{dT}{T} = 1 \times 2.093 \log_e \frac{273}{268} = 0.0386 \text{ kJ/K}$$

Entropy change of the system as ice melts at 0°C to become water at 0°C.

$$(\Delta s_{II})_{system} = \frac{333.33}{273} = 1.2209 \text{ kJ/K}$$

Entropy change of water as it gets heated from 0°C to 25°C

$$(\Delta s_{III})_{system} = \int_{273}^{298} mc_p \frac{dT}{T} = 1 \times 4.187 \log_e \left(\frac{298}{273}\right) = 0.3668 \text{ kJ/K}$$

Total entropy change of ice as it melts into water

$$(\Delta s)_{total} = \Delta s_I + \Delta s_{II} + \Delta s_{III}$$
$$= 0.0386 + 1.2209 + 0.3668 = 1.6263 \text{ kJ/K}$$

Then temperature-entropy diagram for the system as ice at – 5°C converts to water at 25°C is shown in Fig. 5.52.

∴ **Entropy increase of the universe,**

$$(\Delta s)_{univ.} = (\Delta s)_{system} + (\Delta s)_{atm.}$$
$$= 1.6263 + (-1.5049) = \mathbf{0.1214 \text{ kJ/K.}} \quad \textbf{(Ans.)}$$

(*ii*) **Minimum amount of work necessary to convert the water back into ice at – 5°C,** $\mathbf{W_{min.}}$ **:**

To convert 1 kg of water at 25°C to ice at – 5°C, 448.46 kJ of heat have to be removed from it, and the system has to be brought from state 4 to state 1 (Fig. 5.52). A refrigerator cycle, as shown in Fig. 5.53, is assumed to accomplish this. The entropy change of the system would be the same, *i.e.*, $s_4 - s_1$, with the only difference that its sign will be negative, because heat is removed from the system (Fig. 5.52).

$$(\Delta s)_{system} = s_1 - s_4 \ (negative)$$

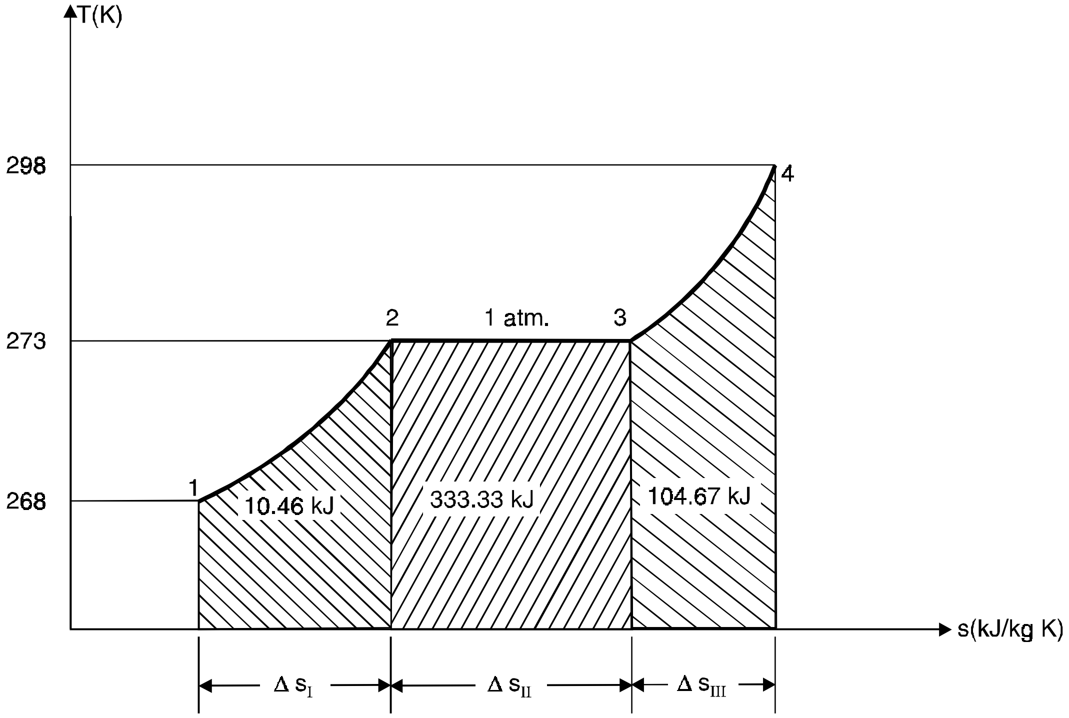


Fig. 5.52

The entropy change of the working fluid in the refrigerator would be zero, since it is operating in a cycle, *i.e.*,

$$(\Delta s)_{refrigerator} = 0.$$

The entropy change of the atmosphere (*positive*),

$$(\Delta s)_{atm.} = \frac{Q+W}{T}$$

∴ Entropy change of the universe

$$(\Delta s)_{univ.} = (\Delta s)_{system} + (\Delta s)_{refrigerator} + (\Delta s)_{atm.}$$

$$= (s_1 - s_4) + 0 + \frac{Q+W}{T} = (s_1 - s_4) + \frac{Q+W}{T}$$

By the principle of increase of entropy

$$(\Delta s)_{univ.\ or\ isolated\ system} \geq 0$$

$$\therefore \qquad \left[(s_1 - s_4) + \frac{Q+W}{T}\right] \geq 0$$

$$\therefore \qquad \frac{Q+W}{T} \geq (s_4 - s_1)$$

$$W \geq T\,(s_4 - s_1) - Q$$

$$W_{(min)} = T\,(s_4 - s_1) - Q$$

Here $Q = 448.46$ kJ

$T = 298$ K

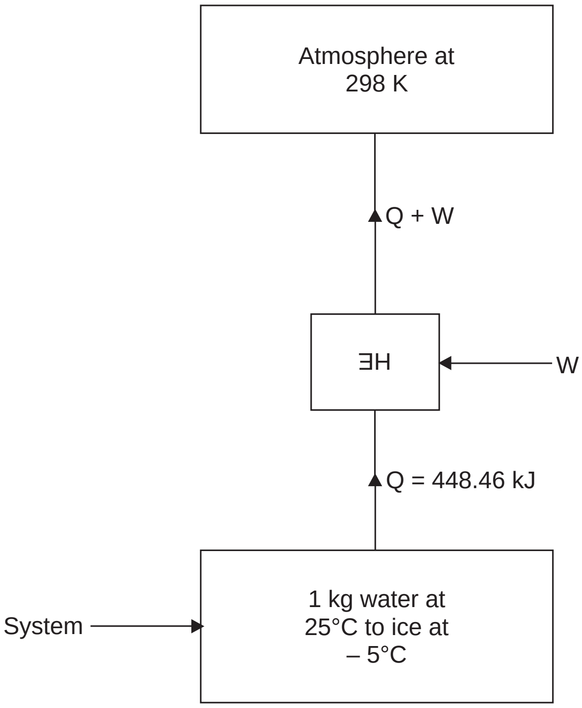


Fig. 5.53

$$s_4 - s_1 = 1.6263 \text{ kJ/K}$$

$$\therefore \quad \mathbf{W}_{(min)} = 298 \times 1.6263 - 448.46 = \mathbf{36.17\ kJ.} \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. **Clausius statement :**

   "It is impossible for a self-acting machine working in a cyclic process, unaided by any external agency, to convey heat from a body at a lower temperature to a body at a higher temperature."

   Kelvin-Planck statement :

   "It is impossible to construct an engine, which while operating in a cycle produces no other effect except to extract heat from a single reservoir and do equivalent amount of work".

   Although above statements of second law of thermodynamic appear to be different, they are really equivalent in the sense that violation of either statement implies violation of other.
2. **Perpetual motion machine of second kind** (*PMM*2) is that imaginary machine which would continuously absorb heat from a single thermal reservoir and convert this heat completely into work. The efficiency of such a machine would be 100%.
3. Clausius inequality is given by,

$$\sum_{\text{Cycle}} \left(\frac{\delta Q}{T}\right) \le 0$$

"When a system performs a reversible cycle, then

$$\sum_{\text{Cycle}} \left(\frac{\delta Q}{T}\right) = 0,$$

but when the cycle is not reversible

$$\sum_{\text{Cycle}} \left(\frac{\delta Q}{T}\right) < 0.”$$

4. **‘Entropy’** is a function of a quantity of heat which shows the possibility of conversion of that heat into work. The increase in entropy is small when heat is added at a high temperature and is greater when heat addition is made at lower temperature. Thus for maximum entropy, there is a minimum availability for conversion into work and for minimum entropy there is maximum availability for conversion into work.

5. **Entropy changes for a closed system (per kg) :**

(*i*) *General case* :

(*a*) $c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$ (in terms of T and v)

(*b*) $c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}$ (in terms of p and v)

(*c*) $c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$ (in terms of T and p)

(*ii*) *Constant volume* : $c_v \log_e \frac{T_2}{T_1}$

(*iii*) *Constant pressure* : $c_p \log_e \frac{T_2}{T_1}$

(*iv*) *Isothermal* : $R \log_e \frac{v_2}{v_1}$

(*v*) *Adiabatic* : zero

(*vi*) *Polytropic* : $c_v \left(\frac{n-\gamma}{n-1}\right) \log_e \frac{T_2}{T_1}$.

6. **Entropy change for an open system**

$$dS \geq \frac{dQ}{T_0} + \Sigma s_i \,.\, dm_i - \Sigma s_0 \,.\, dm_0$$

where, T_0 = Temperature of the surroundings.

Subscripts i and 0 refer to inlet and outlet conditions.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. Second law of thermodynamics defines
   (*a*) heat (*b*) work (*c*) enthalpy
   (*d*) entropy (*e*) internal energy.
2. For a reversible adiabatic process, the change in entropy is
   (*a*) zero (*b*) minimum (*c*) maximum
   (*d*) infinite (*e*) unity.
3. For any reversible process, the change in entropy of the system and surroundings is
   (*a*) zero (*b*) unity (*c*) negative
   (*d*) positive (*e*) infinite.
4. For any irreversible process the net entropy change is
   (*a*) zero (*b*) positive (*c*) negative
   (*d*) infinite (*e*) unity.

5. The processes of a Carnot cycle are
(*a*) two adiabatic and two constant volume
(*b*) one constant volume and one constant pressure and two isentropics
(*c*) two adiabatics and two isothermals (*d*) two constant volumes and two isothermals
(*e*) two isothermals and two isentropics.

6. Isentropic flow is
(*a*) irreversible adiabatic flow (*b*) ideal fluid flow (*c*) perfect gas flow
(*d*) frictionless reversible flow (*e*) reversible adiabatic flow.

7. In a Carnot engine, when the working substance gives heat to the sink
(*a*) the temperature of the sink increases
(*b*) the temperature of the sink remains the same
(*c*) the temperature of the source decreases
(*d*) the temperatures of both the sink and the source decrease
(*e*) changes depend on the operating conditions.

8. If the temperature of the source is increased, the efficiency of the Carnot engine
(*a*) decreases (*b*) increases
(*c*) does not change (*d*) will be equal to the efficiency of a practical engine
(*e*) depends on other factors.

9. The efficiency of an ideal Carnot engine depends on
(*a*) working substance (*b*) on the temperature of the source only
(*c*) on the temperature of the sink only
(*d*) on the temperatures of both the source and the sink
(*e*) on the construction of engine.

10. The efficiency of a Carnot engine using an ideal gas as the working substance is
(*a*) $\frac{T_1 - T_2}{T_1}$ (*b*) $\frac{T_1}{T_1 - T_2}$ (*c*) $\frac{T_1T_2}{T_1 - T_2}$
(*d*) $\frac{T_1 - T_2}{T_1T_2}$ (*e*) $\frac{T_2(T_1 - T_2)}{T_1(T_1 + T_2)}$.

11. In a reversible cycle, the entropy of the system
(*a*) increases (*b*) decreases
(*c*) does not change (*d*) first increases and then decreases
(*e*) depends on the properties of working substance.

12. A frictionless heat engine can be 100% efficient only if its exhaust temperature is
(*a*) equal to its input temperature (*b*) less than its input temperature
(*c*) 0°C (*d*) 0°K (*e*) – 100°C.

13. Kelvin-Planck's law deals with
(*a*) conservation of energy (*b*) conservation of heat (*c*) conservation of mass
(*d*) conversion of heat into work (*e*) conversion of work into heat.

14. Which of the following statements is *correct* according to Clausius statement of second law of thermodynamics ?
(*a*) It is impossible to transfer heat from a body at a lower temperature to a body at a higher temperature
(*b*) It is impossible to transfer heat from a body at a lower temperature to a body at a higher temperature, without the aid of an external source.
(*c*) It is possible to transfer heat from a body at a lower temperature to a body at a higher temperature by using refrigeration cycle
(*d*) None of the above.

15. According to Kelvin-Planck's statement of second law of thermodynamics

(*a*) It is impossible to construct an engine working on a cyclic process, whose sole purpose is to convert heat energy into work

(*b*) It is possible to construct an engine working on a cyclic process, whose sole purpose is to convert the heat energy into work

(*c*) It is impossible to construct a device which while working in a cyclic process produces no effect other than the transfer of heat from a colder body to a hotter body

(*d*) When two dissimilar metals are heated at one end and cooled at the other, the e.m.f. developed is proportional to the difference of their temperatures at the two end.

(*e*) None of the above.

16. The property of a working substance which increases or decreases as the heat is supplied or removed in a reversible manner is known as

(*a*) enthalpy (*b*) internal energy

(*c*) entropy (*d*) external energy.

17. The entropy may be expressed as a function of

(*a*) pressure and temperature (*b*) temperature and volume

(*c*) heat and work (*d*) all of the above

(*e*) none of the above.

18. The change of entropy, when heat is absorbed by the gas is

(*a*) positive (*b*) negative (*c*) positive or negative.

19. Which of the following statements is *correct* ?

(*a*) The increase in entropy is obtained from a given quantity of heat at a low temperature

(*b*) The change in entropy may be regarded as a measure of the rate of the availability of heat for transformation into work

(*c*) The entropy represents the maximum amount of work obtainable per degree drop in temperature

(*d*) All of the above.

20. The condition for the reversibility of a cycle is

(*a*) the pressure and temperature of working substance must not differ, appreciably from those of the surroundings at any stage in the process

(*b*) all the processes taking place in the cycle of operation, must be extremely slow

(*c*) the working parts of the engine must be friction free

(*d*) there should be no loss of energy during the cycle of operation

(*e*) all of the above.

21. In an irreversible process there is a

(*a*) loss of heat (*b*) no loss of work

(*c*) gain of heat (*d*) no gain of heat.

22. The main cause for the irreversibility is

(*a*) mechanical and fluid friction (*b*) unrestricted expansion

(*c*) heat transfer with a finite temperature difference

(*d*) all of the above.

23. The efficiency of the Carnot cycle may be increased by

(*a*) increasing the highest temperature (*b*) decreasing the highest temperature

(*c*) increasing the lowest temperature (*d*) decreasing the lowest temperature

(*e*) keeping the lowest temperature constant.

24. Which of the following is the *correct* statement ?
  (*a*) All the reversible engines have the same efficiency
  (*b*) All the reversible and irreversible engines have the same efficiency
  (*c*) Irreversible engines have maximum efficiency
  (*d*) All engines are designed as reversible in order to obtain maximum efficiency.

Answers

1.	(*d*)	**2.**	(*a*)	**3.**	(*a*)	**4.**	(*b*)	**5.**	(*e*)	**6.**	(*e*)	**7.**	(*b*)
8.	(*b*)	**9.**	(*d*)	**10.**	(*a*)	**11.**	(*c*)	**12.**	(*d*)	**13.**	(*d*)	**14.**	(*b*)
15.	(*e*)	**16.**	(*c*)	**17.**	(*a*)	**18.**	(*a*)	**19.**	(*d*)	**20.**	(*e*)	**21.**	(*a*)
22.	(*d*)	**23.**	(*d*)	**24.**	(*a*).								

THEORETICAL QUESTIONS

1. State the limitations of first law of thermodynamics.
2. What is the difference between a heat engine and a reversed heat engine ?
3. Enumerate the conditions which must be fulfilled by a reversible process. Give some examples of ideal reversible processes.
4. What is an irreversible process ? Give some examples of irreversible processes.
5. Give the following statements of second law of thermodynamics.
  (*i*) Clausius statement
  (*ii*) Kelvin-Planck statement.
6. Define heat engine, refrigerator and heat pump.
7. What is the perpetual motion machine of the second kind ?
8. What do you mean by 'Thermodynamic temperature' ?
9. What do you mean by 'Clausius inequality' ?
10. Describe the working of a Carnot cycle.
11. Derive an expression for the efficiency of the reversible heat engine.
12. What do you mean by the term 'Entropy' ?
13. What are the characteristics of entropy ?
14. Prove that entropy is a property of a system.
15. Derive an expression for the change in entropy of the universe.
16. What is a temperature-entropy diagram ?
17. Derive expressions for entropy changes for a closed system in the following cases :
  (*i*) General case for change of entropy of a gas
  (*ii*) Heating a gas at constant volume
  (*iii*) Heating a gas at constant pressure
  (*iv*) Polytropic process.
18. Give an expression for entropy changes for an open system.

UNSOLVED EXAMPLES

Heat Engines and Reversible Heat Engines

1. A cyclic heat engine operates between a source temperature of 800°C and a sink temperature of 30°C. What is the least rate of heat rejection per kW net output of the engine ? [**Ans.** 0.392 kW]

2. A domestic food freezer maintains a temperature of – 15°C. The ambient air temperature is 30°C. If heat leaks into the freezer at the continuous rate of 1.75 kJ/s what is the least power necessary to pump this heat out continuously. [**Ans.** 0.31 kW]

3. A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and – 20°C. The heat transfer to the heat engine is 2000 kJ and the net work output for the combined engine refrigerator is 360 kJ. (*i*) Calculate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C. (*ii*) Reconsider (*i*) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 40 per cent of their maximum possible values.

[**Ans.** Heat rejection to 40°C reservoir (*i*) 5539 kJ ; (*ii*) 1899.6 kJ]

4. A heat engine is supplied heat at the rate of 1700 kJ/min and gives an output of 9 kW. Determine the thermal efficiency and the rate of heat rejection. [**Ans.** 31.76% ; 9.333 kJ/s]

5. What is the highest possible theoretical efficiency of a heat engine operating with a hot reservoir of furnace gases at 2000°C when the cooling water available is at 10°C ? [**Ans.** 87.54%]

6. A Carnot cycle operates between source and sink temperatures of 260°C and – 17.8°C. If the system receives 100 kJ from the source, find (*i*) efficiency of the system, (*ii*) the net work transfer, (*iii*) heat rejected to the sink. [**Ans.** 52.2% ; 52.2 kJ ; 47.8 kJ]

7. Source *A* can supply energy at a rate of 11000 kJ/min at 320°C. A second source *B* can supply energy at a rate of 110000 kJ/min at 68°C. Which source *A* or *B*, would you choose to supply energy to an ideal reversible engine that is to produce large amount of power if the temperature of the surroundings is 40°C ? [**Ans.** Source *B*]

8. A fish freezing plant requires 50 tons of refrigeration. The freezing temperature is – 40°C while the ambient temperature is 35°C. If the performance of the plant is 15% of the theoretical reversed Carnot cycle working within the same temperature limits, calculate the power required. [**Ans.** 375 kW]

Take 1 ton = 210 kJ/ min.

Clausius Inequality

9. A heat engine is supplied with 278 kJ/s of heat at a constant fixed temperature of 283°C and the heat rejection takes place at 5°C. The following results were reported : (*i*) 208 kJ/s are rejected, (*ii*) 139 kJ/s are rejected, (*ii*) 70 kJ/s are rejected.

Classify which of the results report a reversible cycle or irreversible cycle or impossible results.

[**Ans.** (*i*) Irreversible (*ii*) Reversible (*iii*) Impossible]

Entropy

10. Air at 15°C and 1.05 bar occupies 0.02 m^3. The air is heated at constant volume until the pressure is 4.2 bar, and then cooled at constant pressure back to the original temperature. Calculate the net heat flow to or from the air and the net entropy change.

Sketch the process on a *T-s* diagram.

[**Ans.** Heat rejected = 6.3 kJ, decrease in entropy of air = 0.0101 kJ/K]

11. 0.03 m^3 of nitrogen contained in a cylinder behind a piston is initially at 1.05 bar and 15°C. The gas is compressed isothermally and reversibly until the pressure is 4.2 bar. Calculate the change of entropy, the heat flow, and the work done, and sketch the process on a *p-v* and *T-s* diagrams. Assume nitrogen to act as a perfect gas. Molecular weight of nitrogen = 28.

[**Ans.** 0.01516 kJ/K (decrease) ; 4.37 kJ (heat rejected) ; 4.37 kJ]

12. Calculate the change of entropy of 1 kg of air expanding polytropically in a cylinder behind a piston from 6.3 bar and 550°C to 1.05 bar. The index of expansion is 1.3. [**Ans.** 0.1 kJ/kg K (increase)]

13. 0.05 kg of carbon dioxide (molecular weight = 44) is compressed from 1 bar, 15°C, until the pressure is 8.3 bar, and the volume is then 0.004 m^3. Calculate the change of entropy. Take c_p for carbon dioxide as 0.88 kJ/kg K, and assume carbon dioxide to be a perfect gas. [**Ans.** 0.0113 kJ/K (decrease)]

14. In an air turbine the air expands from 6.8 bar and 430°C to 1.013 bar and 150°C. The heat loss from the turbine can be assumed to be negligible. Show that the process is irreversible, and calculate the change of entropy per kg of air. [**Ans.** 0.0355 kJ/kg K (increase)]

15. (*i*) One kg of water at 273 K is brought into contact with a heat reservoir at 373 K. When the water has reached 373 K, find the entropy change of the water of the heat reservoir, and of the universe.

(*ii*) If water is heated from 273 K to 373 K by first bringing it in contact with a reservoir at 323 K and then with reservoir at 373 K, what will the entropy change of the universe be ?

[**Ans.** (*i*) 0.183 kJ/K ; (*ii*) 0.098 kJ/K]

16. One kg of ice at – 5°C is exposed to the atmosphere which is at 20°C. The ice melts and comes into thermal equilibrium with the atmosphere.

(*i*) Determine the entropy increase of the universe.

(*ii*) What is the minimum amount of work necessary to convert the water back into ice at – 5°C ? c_p of ice is 2.093 kJ/kg °C and the latent heat of fusion of ice is 333.3 kJ/kg.

[**Ans.** (*i*) 0.0949 kJ/K (increase) (*ii*) 28.5 kJ]

17. A system has a heat capacity at constant volume $C_v = AT^2$, where $A = 0.042$ J/K^3. The system is originally at 200 K and a thermal reservoir at 100 K is available. What is the maximum amount of work that can be recovered as the system is cooled down to the temperature of the reservoir ? [**Ans.** 35 kJ]

18. A fluid undergoes a reversible adiabatic compression from 0.5 MPa, 0.2 m^3 to 0.05 m^3 according to the law, $pv^{1.3}$ = constant. Determine the change in enthalpy, internal energy and entropy, and the heat transfer and work transfer during the process. [**Ans.** 223.3 kJ ; 171.77 kJ, ; zero ; zero ; – 171.77 kJ]

19. A rigid cylinder containing 0.006 m^3 of nitrogen (molecular weight 28) at 1.04 bar, 15°C, is heated reversibly until the temperature is 90°C. Calculate the change of entropy and the heat supplied. Sketch the process on *T-s* diagram. Take the isentropic index, γ, for nitrogen as 1.4, and assume that nitrogen is a perfect gas.

[**Ans.** 0.00125 kJ/K ; 0.407 kJ]

20. 1 m^3 of air is heated reversibly at constant pressure from 15°C to 300°C, and is then cooled reversibly at constant volume back to the initial temperature. The initial pressure is 1.03 bar. Calculate the net heat flow and overall change of entropy, and sketch the process on a *T-s* diagram. [**Ans.** 101.5 kJ ; 0.246 kJ/K]

21. 1 kg of air is allowed to expand reversibly in a cylinder behind a piston in such a way that the temperature remains constant at 260°C while the volume is doubled. The piston is then moved in, and heat is rejected by the air reversibly at constant pressure until the volume is the same as it was initially. Calculate the net heat flow and the overall change of entropy. Sketch the processes on a *T-s* diagram.

[**Ans.** – 161.9 kJ/kg ; – 0.497 kJ/kg K]

22. 1 kg of air at 1.013 bar, 17°C, is compressed according to a law $pv^{1.3}$ = constant, until the pressure is 5 bar. Calculate the change of entropy and sketch the process on a *T-s* diagram, indicating the area, which represents the heat flow. [**Ans.** – 0.0885 kJ/kg K]

23. 0.06 m^3 of ethane (molecular weight 30), at 6.9 bar and 60°C, is allowed to expand isentropically in a cylinder behind a piston to a pressure of 1.05 bar and a temperature of 107°C. Calculate γ, R, c_p, c_v for ethane, and calculate the work done during the expansion. Assume ethane to be a perfect gas.

The same mass of ethane at 1.05 bar, 107°C, is compressed to 6.9 bar according to a law $pv^{1.4}$ = constant. Calculate the final temperature of the ethane and the heat flow to or from the cylinder walls during the compression. Calculate also the change of entropy during the compression, and sketch both processes on *p-v* and *T-s* diagrams.

[**Ans.** 1.219 ; 0.277 kJ/kg K ; 1.542 kJ/kg K ; 1.265 kJ/kg K ; 54.2 kJ ; 378°C ; 43.4 kJ ; 0.0867 kJ/K]

24. In a reversible process the rate of heat transfer to the system per unit temperature rise is given by $\frac{dQ}{dT}$ = 0.5 kJ/°C. Find the change in entropy of the system if its temperature rises from 500 K to 800 K.

[**Ans.** 0.235 kJ/kg°C]

25. In a reversible cycle 100 kJ of heat is received at 500 K ; then an adiabatic expansion occurs to 400 K, at which temperature 50 kJ of heat is received, then a further adiabatic expansion to 300 K at which temperature 100 kJ of heat is rejected :

(*i*) Find the change in entropy which occurs as the system is restored to its initial state in the remainder of the cycle.

(*ii*) If during the remainder of the cycle heat is transferred only at 400 K, how much heat is transferred and in what direction ? [**Ans.** 0.008 kJ/K ; + 3.2 kJ]

26. 1 kg of air is compressed according to the law $pv^{1.25}$ = constant from 1.03 bar and 15°C to 16.45 bar. Calculate the change in entropy. [**Ans.** 0.255 kJ/kg K]

27. A quantity of gas (mean molecular weight 36.2) is compressed according to the law pv^n = constant, the initial pressure and volume being 1.03 bar and 0.98 m^3 respectively. The temperature at the start of compression is 17°C and at the end it is 115°C. The amount of heat rejected during compression is 3.78 kJ, c_p = 0.92. Calculate :

(*i*) Value of *n*, (*ii*) Final pressure, (*iii*) Change in entropy.

[**Ans.** (*i*) 1.33 ; (*ii*) 1.107 bar ; (*iii*) 0.228 kJ/kg K]

6

Availability and Irreversibility

6.1. AVAILABLE AND UNAVAILABLE ENERGY

There are many forms in which an energy can exist. But even under ideal conditions all these forms cannot be converted completely into work. This indicates that energy has two parts :

— *Available part.*

— *Unavailable part.*

'***Available energy***' is the *maximum portion of energy which could be converted into useful work by ideal processes which reduce the system to a dead state* (a state in equilibrium with the earth and its atmosphere). Because there can be only one value for maximum work which the system alone could do while descending to its dead state, it follows immediately that *'Available energy' is a property*.

A system which has a pressure difference from that of surroundings, work can be obtained from an expansion process, and if the system has a different temperature, heat can be transferred to a cycle and work can be obtained. But when the temperature and pressure becomes equal to that of the earth, transfer of energy ceases, and although the system contains internal energy, this energy is ***unavailable.***

Summarily available energy denote, the latent capability of energy to do work, and in this sense it can be applied to energy in the system or in the surroundings.

The theoretical maximum amount of work which can be obtained from a system at any state p_1 *and* T_1 *when operating with a reservoir at the constant pressure and temperature* p_0 *and* T_0 *is called* '**availability**'.

6.2. AVAILABLE ENERGY REFERRED TO A CYCLE

The ***available energy (A.E.)*** or the available part of the energy supplied is the maximum work output obtainable from a certain heat input in a cyclic heat engine (Fig. 6.1). The minimum energy that has to be rejected to the sink by the second law is called the ***unavailable energy (U.E.),*** or the unavailable part of the energy supplied.

$$\therefore \qquad Q_1 = \text{A.E.} + \text{U.E.}$$

or

$$W_{max} = \text{A.E.} = Q_1 - \text{U.E.}$$

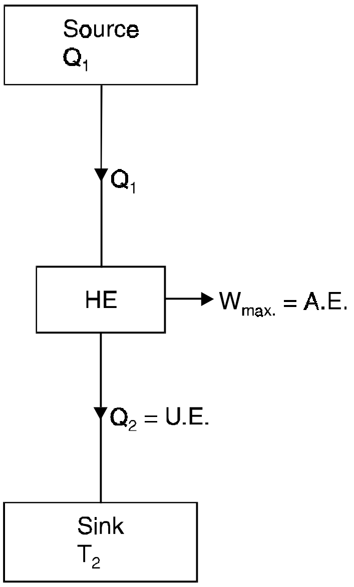


Fig. 6.1. Available and unavailable energy in a cycle.

For the given values of the source temperature T_1 and sink temperature T_2, the reversible efficiency,

$$\eta_{rev.} = 1 - \frac{T_2}{T_1}$$

For a given T_1, $\eta_{rev.}$ will increase with the decrease of T_2. *The lowest practicable temperature of heat rejection is the temperature of the surroundings,* T_0.

$$\therefore \qquad \eta_{max} = 1 - \frac{T_0}{T_1}$$

and

$$W_{max} = \left(1 - \frac{T_0}{T_1}\right) Q_1$$

Consider a finite process *l-m*, in which heat is supplied reversibly to a heat engine (Fig. 6.2). Taking an elementary cycle, if dQ_1 is the heat received by the engine reversibly at T_1,

Then

$$dW_{max} = \frac{T_1 - T_0}{T_1}\, dQ_1$$

$$= dQ_1 - \frac{T_0}{T_1} dQ_1 = \text{A.E.}$$

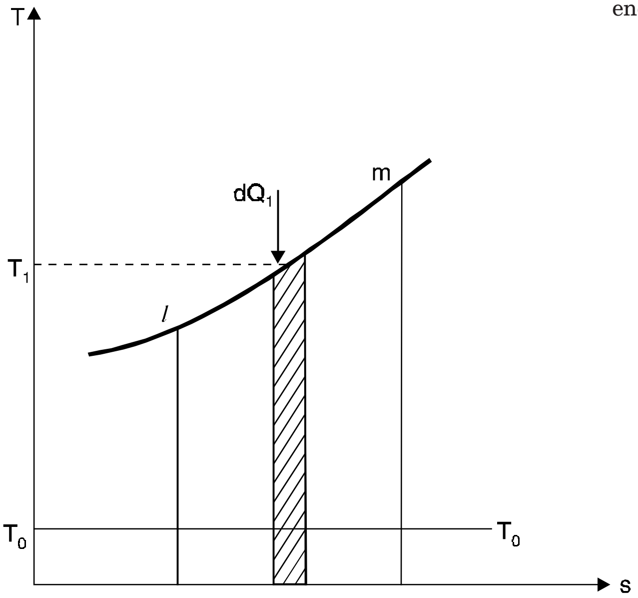


Fig. 6.2. Availability of energy.

For the heat engine receiving heat for the whole process *l-m*, and rejecting heat at T_0

$$\int_l^m dW_{max} = \int_l^m dQ_1 - \int_l^m \frac{T_0}{T_1}\, dQ_1$$

$$\therefore \qquad W_{max} = \text{A.E.} = Q_{l-m} - T_0\,(s_l - s_m) \qquad ...(6.1)$$

or unavailable energy,

$$\text{U.E.} = Q_{l-m} - W_{max}$$

or

$$\text{U.E.} = T_0\,(s_l - s_m)$$

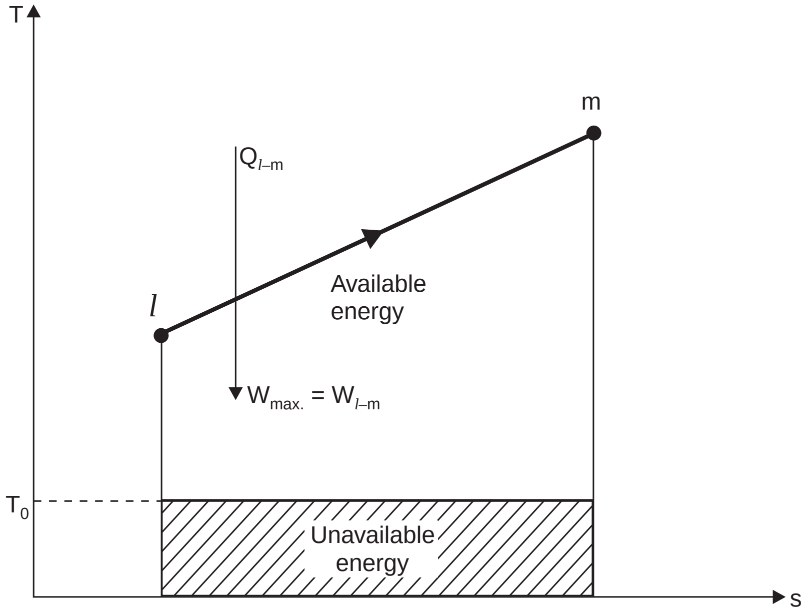


Fig. 6.3. Unavailable energy by the second law of thermodynamics.

Thus unavailable energy is the product of the lowest temperature of heat rejection, and the change of entropy of the system during the process of supplying heat (Fig. 6.3).

6.3. DECREASE IN AVAILABLE ENERGY WHEN HEAT IS TRANSFERRED THROUGH A FINITE TEMPERATURE DIFFERENCE

When transfer of heat takes place through a finite temperature difference, there is a decrease in the availability of energy so transferred. Consider a reversible heat engine operating between temperatures T_1 and T_0 (Fig. 6.4). Then

$$Q_1 = T \,.\, \Delta s \;;$$

$$Q_2 = T_0 \, \Delta s \;;$$

and $$W = \text{A.E.} = [T_1 - T_0] \, \Delta s.$$

Assume that heat Q_1 is transferred through a finite temperature difference from the reservoir or source at T_1 to the engine absorbing heat at T_1', lower than T_1 (Fig. 6.4). The availability of Q_1 as received by the engine at T_1' can be found by allowing the engine to operate reversibly in a cycle between T_1' and T_0 receiving Q_1 and rejecting Q_2'.

Now, $$Q_1 = T_1 \Delta s = T_1' \Delta s'$$

$\because$ $$T_1 > T_1'$$

$\therefore$ $$\Delta s' > \Delta s$$

$$Q_2 = T_0 \, \Delta s$$

$$Q_2' = T_0 \, \Delta s'$$

$\because$ $$\Delta s' > \Delta s$$

$\therefore$ $$Q_2' > Q_2$$

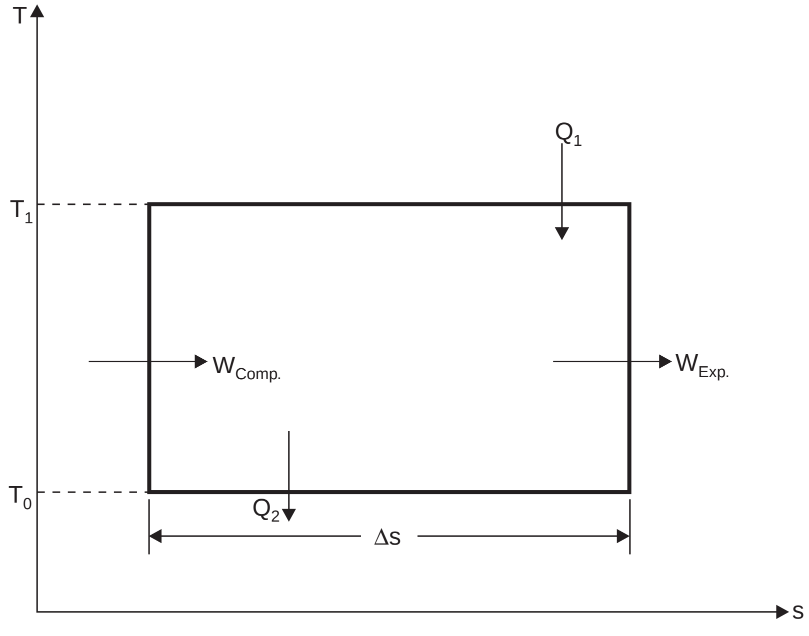


Fig. 6.4. Carnot-cycle.

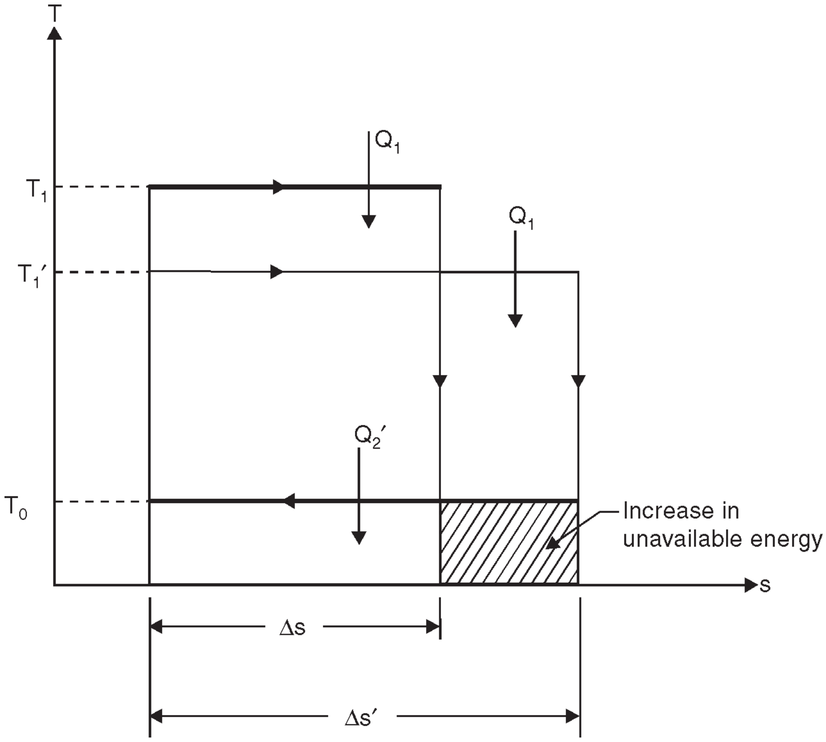


Fig. 6.5. Increase in unavailable energy due to heat transfer through a finite temperature difference.

$\therefore \qquad W' = Q_1 - Q_2' = T_1' \,\Delta s' - T_0 \,\Delta s'$

and $\qquad W = Q_1 - Q_2 = T_1 \,\Delta s - T_0 \,\Delta s$

$\therefore \qquad W' < W$, because $Q_2' > Q_2$

The *loss of available energy* due to irreversible heat transfer through finite temperature difference between the source and the working fluid during the heat addition process is given as :

$$W - W' = Q_2' - Q_2$$
$$= T_0\,(\Delta s' - \Delta s)$$

i.e., Decrease in available energy, A.E.

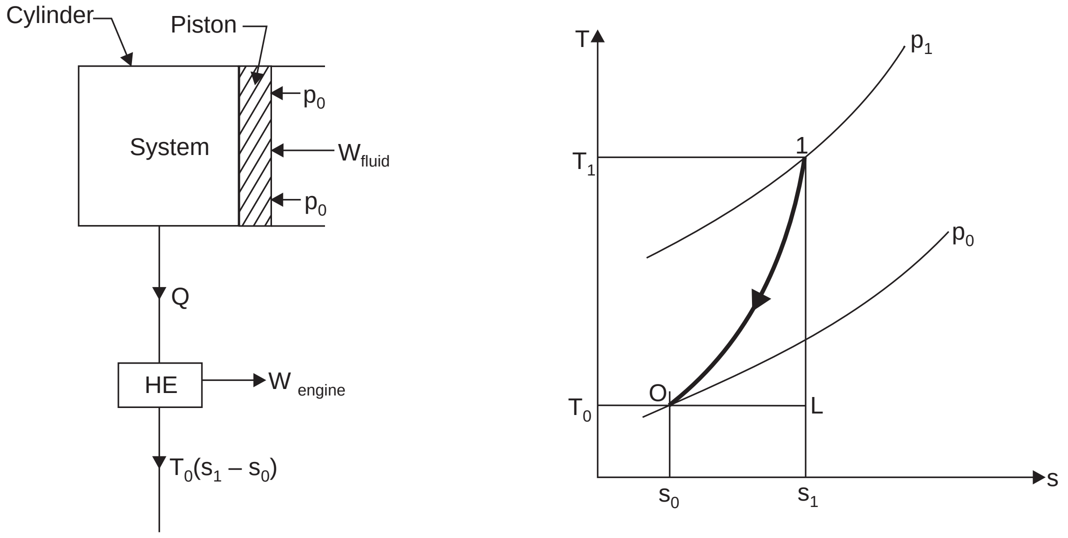
$$= T_0\,(\Delta s' - \Delta s) \qquad \text{...(6.2)}$$

Thus the decrease in A.E. is the product of the lowest feasible temperature of heat rejection and the additional entropy change in the system while receiving heat irreversibly, compared to the case of reversible heat transfer from the same source. *The greater is the temperature difference* $(T_1 - T_1')$, *the greater is the heat rejection* Q_2' *and the greater will be the unavailable part of the energy supplied* (Fig. 6.5).

Energy is said to be *degraded* each time it flows through a finite temperature difference. That is, why the second law of thermodynamics is sometimes called the *law of the degradation of energy*, and energy is said to '*run down hill*'.

6.4. AVAILABILITY IN NON-FLOW SYSTEMS

Let us consider a system consisting of a fluid in a cylinder behind a piston, the fluid expanding reversibly from initial condition of p_1 and T_1 to final atmospheric conditions of p_0 and T_0. Imagine also that the system works in conjunction with a reversible heat engine which receives heat reversibly from the fluid in the cylinder such that the working substance of the heat engine follows the cycle *O1LO* as shown in Fig. 6.6, where $s_1 = s_L$ and $T_0 = T_L$ (the only possible way in which this could occur would be if an infinite number of reversible heat engines were arranged in parallel, each operating on a Carnot cycle, each one receiving heat at a different constant temperature and each one rejecting heat at T_0). The work done by the engine is given by :



(*a*) (*b*)

Fig. 6.6

W_{engine} = Heat supplied – Heat rejected

$$= Q - T_0 (s_1 - s_0) \qquad ...(i)$$

The heat supplied to the engine is equal to the heat rejected by the fluid in the cylinder. Therefore, for the fluid in the cylinder undergoing the process 1 to 0, we have

$$- Q = (u_0 - u_1) + W_{fluid}$$

i.e.,
$$W_{fluid} = (u_1 - u_0) - Q \qquad ...(ii)$$

Adding eqns. (*i*) and (*ii*), we get

$$W_{fluid} + W_{engine} = [(u_1 - u_0) - Q] + [Q - T_0 (s_1 - s_0)]$$
$$= (u_1 - u_0) - T_0 (s_1 - s_0)$$

The work done by the fluid on the piston is less than the total work done by the fluid, since there is no work done on the atmosphere which is at constant pressure p_0

i.e., Work done on atmosphere $= p_0 (v_0 - v_1)$

Hence, *maximum work available*

$$= (u_1 - u_0) - T_0 (s_1 - s_0) - p_0 (v_0 - v_1)$$

Note. When a fluid undergoes a complete cycle then the net work done on the atmosphere is zero.

$$W_{max} = (u_1 + p_0v_1 - T_0s_1) - (u_0 + p_0v_0 - T_0s_0) \qquad ...(6.3)$$

$$\therefore \quad W_{max} = a_1 - a_0 \qquad ...[6.3\ (a)]$$

The property, $a = u + p_0v - T_0s$ (per unit mass) is called the **non-flow availability function.**

6.5. AVAILABILITY IN STEADY FLOW SYSTEMS

Consider a fluid flowing steadily with a velocity C_1 from a reservoir in which the pressure and temperature remain constant at p_1 and T_1 through an apparatus to atmospheric pressure of p_0. Let the reservoir be at a height Z_1 from the datum, which can be taken at exit from the apparatus, *i.e.,* $Z_0 = 0$. For maximum work to be obtained from the apparatus the exit velocity, C_0, must be zero. It can be shown as for article 6.4 that a reversible heat engine working between the limits would reject $T_0 (s_1 - s_0)$ units of heat, where T_0 is the atmospheric temperature. Thus, we have

$$W_{max} = \left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) - h_0 - T_0 (s_1 - s_0)$$

In several thermodynamic systems the kinetic and potential energy terms are negligible

i.e.,
$$W_{max} = (h_1 - T_0s_1) - (h_0 - T_0s_0)$$
$$= b - b_0$$

The property, $b = h - T_0s$ (per unit mass) is called the **steady-flow availability function.**

[In the equation $b = h - T_0s$; the function 'b' (like the function 'a') is a composite property of a system and its environment ; this is also known as ***Keenan function***].

Note 1. The alternative names for availability and unavailable quantity $T_0\Delta s$ are ***energy*** and a ***energy*** respectively.

2. The only difference between $a = u + p_0v - T_0s$ function and $b = (h - T_0s) = (u + pv - T_0s)$ function is in pressure only.

6.6. HELMHOLTZ AND GIBBS FUNCTIONS

The work done in a non-flow reversible system (per unit mass) is given by :

$$W = Q - (u_0 - u_1)$$
$$= T.ds - (u_0 - u_1)$$

$$= T\,(s_0 - s_1) - (u_0 - u_1)$$

i.e., $$W = (u_1 - Ts_1) - (u_0 - Ts_0) \quad \text{...(6.4)}$$

The term $(u - Ts)$ is known as *Helmholtz function*. This gives maximum possible output when the heat Q is transferred at constant temperature and is the case with a very large source.

If work against atmosphere is equal to $p_0\,(v_0 - v_1)$, then the maximum work available,

$$\begin{aligned} W_{max} &= W - \text{work against atmosphere} \\ &= W - p_0\,(v_0 - v_1) \\ &= (u_1 - Ts_1) - (u_0 - Ts_0) - p_0\,(v_0 - v_1) \\ &= (u_1 + p_0v_1 - Ts_1) - (u_0 + p_0v_0 - Ts_0) \\ &= (h_1 - Ts_1) - (h_0 - Ts_0) \end{aligned}$$

i.e., $$W_{max} = g_1 - g_0 \quad \text{...(6.5)}$$

where $g = h - T.s$ is known as ***Gibb's function* or *free energy function*.**

The maximum possible available work when system changes from 1 to 2 is given by

$$W_{max} = (g_1 - g_0) - (g_2 - g_0) = g_1 - g_2 \quad \text{...(6.6)}$$

Similarly, for *steady flow system* the maximum work available is given by

$$W_{max} = (g_1 - g_2) + (\text{KE}_1 - \text{KE}_2) + (\text{PE}_1 - \text{PE}_2) \quad \text{...(6.7)}$$

where K.E. and P.E. represent the kinetic and potential energies.

It may be noted that Gibb's function $g = (h - Ts)$ is a property of the system where availability function $a = (u + p_0v - T_0s)$ is a composite property of the system and surroundings.

Again, $$\begin{aligned} a &= u + p_0v - T_0s \\ b &= u + pv - T_0s \\ g &= u + pv - Ts \end{aligned}$$

When state 1 proceeds to *dead state* (zero state)

$$a = b = g.$$

6.7. IRREVERSIBILITY

The actual work which a system does is always *less* than the idealized reversible work, and the difference between the two is called the *irreversibility of the process*.

Thus, Irreversibility, $I = W_{max} - W$ ...(6.8)

This is also sometimes referred to as '***degradation***' or '***dissipation***'.

For a *non-flow process* between the equilibrium states, when the system exchanges heat only with environment, irreversibility (*per unit mass*),

$$\begin{aligned} i &= [(u_1 - u_2) - T_0(s_1 - s_2)] - [(u_1 - u_2) + Q] \\ &= T_0\,(s_2 - s_1) - Q \\ &= T_0\,(\Delta s)_{system} + T_0\,(\Delta s)_{surr.} \end{aligned}$$

i.e., $$i = T_0\,[(\Delta s)_{system} + (\Delta s)_{surr.}] \quad \text{...(6.9)}$$

$\therefore$ $$i \geq 0$$

Similarly, for *steady flow-process*

$$\begin{aligned} i &= W_{max} - W \text{ (per unit mass)} \\ &= \left[\left(b_1 + \frac{C_1^2}{2} + gZ_1\right) - \left(b_2 + \frac{C_2^2}{2} + gZ_2\right)\right] \\ &\quad - \left[\left(h_1 + \frac{C_1^2}{2} + gZ_1\right) - \left(h_2 + \frac{C_2^2}{2} + gZ_2\right) + Q\right] \end{aligned}$$

$$= T_0\,(s_2 - s_1) - Q$$

$$= T_0\,(\Delta s)_{system} + T_0\,(\Delta s)_{surr.}$$

i.e., $$i = T_0\,(\Delta s_{system} + \Delta s_{surr.})$$

The same expression for irreversibility applies to both flow and non-flow processes.

The quantity $T_0\,(\Delta s_{system} + \Delta s_{surr.})$ represents (per unit mass) an increase in unavailable energy (or energy).

6.8. EFFECTIVENESS

Effectiveness *is defined as the ratio of actual useful work to the maximum useful work.* The useful output of a system is given by the increase of availability of the surroundings.

Effectiveness, $$\in = \frac{\text{Increase of availability of surroundings}}{\text{Loss of availability of the system}} \qquad \text{...(6.10)}$$

For a compression or heating process the effectiveness is given by

$$\in = \frac{\text{Increase of availability of the system}}{\text{Loss of availability of the surroundings}}$$

or $$\in = \frac{W_{useful}}{W_{max\,.\,useful}} \qquad \text{...[6.10 (a)]}$$

The *effectiveness of an actual process is always less than unity*. Thus effectiveness of a process is the measure of the extent to which advantage has been taken of an opportunity to obtain useful work.

Example 6.1. *One kg of air is compressed polytropically from 1 bar pressure and temperature of 300 K to a pressure of 6.8 bar and temperature of 370 K. Determine the irreversibility if the sink temperature is 293 K. Assume R = 0.287 kJ/kg K,* $c_p = 1.004$ *kJ/kg K and* $c_v = 0.716$ *kJ/kg K.* **(U.P.S.C.)**

Solution. Irreversibility $I = W_{max} - W_{act}$

$$- W_{max} = \text{Change in internal energy} - T_0 \times \text{Change in entropy}$$

or $$- W_{max} = (u_2 - u_1) - T_0(s_2 - s_1) = W_{rev}$$

or $$- W_{max} = c_v(T_2 - T_1) - T_0[c_p \ln (T_2/T_1) - R \ln (p_2/p_1)]$$

$$= 0.716(370 - 300) - 293 \times [1.005 \ln (370/300) - 0.287 \ln (6.8/1)]$$

or $$W_{max} = -149.53 \text{ kJ/kg} = W_{rev}$$

(negative sign indicates that work is done on air)

The index of compression '*n*' is given by

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{[(n-1)/n]}$$

or $$\frac{n-1}{n} = \frac{\ln (T_2/T_1)}{\ln (p_2/p_1)} = \frac{\ln (370/300)}{\ln (6.8/1)}$$

or $$n = 1.123$$

$$W_{actual} = \frac{mR(T_1 - T_2)}{n-1} = \frac{1 \times 0.287(300 - 370)}{1.123 - 1} = -\,163.33 \text{ kJ/kg}$$

$$I = W_{rev} - W_{act} = -\,149.53 - (-\,163.33) = \mathbf{13.8\ kJ/kg.} \quad \textbf{(Ans.)}$$

Example 6.2. *A system at 500 K receives 7200 kJ/min from a source at 1000 K. The temperature of atmosphere is 300 K. Assuming that the temperatures of system and source remain constant during heat transfer find out :*

(*i*) *The entropy produced during heat transfer ;*
(*ii*) *The decrease in available energy after heat transfer.*

Solution. Refer Fig. 6.7.

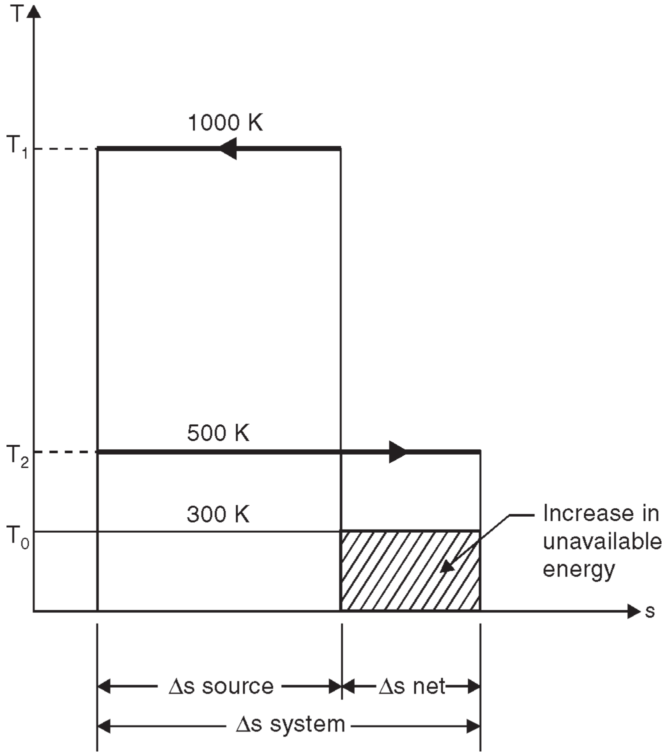


Fig. 6.7

Temperature of source, $T_1 = 1000$ K
Temperature of system, $T_2 = 500$ K
Temperature of atmosphere, $T_0 = 300$ K
Heat received by the system, $Q = 7200$ kJ/min.

(*i*) **Net change of entropy :**

Change in entropy of the *source* during heat transfer

$$= \frac{-Q}{T_1} = \frac{-7200}{1000} = -7.2 \text{ kJ/min-K}$$

Change in entropy of the *system* during heat transfer

$$= \frac{Q}{T_2} = \frac{7200}{500} = 14.4 \text{ kJ/min-K}$$

The net change of entropy, $\Delta S = -7.2 + 14.4 =$ **7.2 kJ/min-K. (Ans.)**

(*ii*) **Decrease in available energy :**

Available energy with source

$$= (1000 - 300) \times 7.2 = 5040 \text{ kJ}$$

Available energy with the system

$$= (500 - 300) \times 14.4 = 2880 \text{ kJ}$$

∴ **Decrease in available energy** = 5040 – 2880 = **2160 kJ. (Ans.)**

$$\left[\begin{array}{l}\text{Also, increase in available energy} \\ \quad = T_0(S_2 - S_1) = T_0\ \Delta S \\ \quad = 300 \times 7.2 = 2160 \text{ kJ}\end{array}\right]$$

Example 6.3. *8 kg of air at 650 K and 5.5 bar pressure is enclosed in a closed system. If the atmosphere temperature and pressure are 300 K and 1 bar respectively, determine :*

(*i*) *The availability if the system goes through the ideal work producing process.*

(*ii*) *The availability and effectiveness if the air is cooled at constant pressure to atmospheric temperature without bringing it to complete dead state. Take* c_v *= 0.718 kJ/kg K ;* c_p*= 1.005 kJ/kg K.*

Solution. Mass of air, $m = 8$ kg

Temperature, $T_1 = 650$ K

Pressure, $p_1 = 5.5$ bar

Atmospheric pressure, $p_0 = 1$ bar

Atmospheric temperature, $T_0 = 300$ K

For air : $c_v = 0.718$ kJ/kg K ; $c_p = 1.005$ kJ/kg K.

(*i*) Change in available energy (for bringing the system to dead state),

$$= m[(u_1 - u_0) - T_0\Delta s]$$

Also
$$\Delta s = c_v \log_e \left(\frac{T_1}{T_0}\right) + R \log_e \frac{v_1}{v_0}$$

Using the ideal gas equation,

$$\frac{p_1v_1}{T_1} = \frac{p_0v_0}{T_0}$$

$$\therefore \quad \frac{v_0}{v_1} = \frac{p_1}{p_0} \cdot \frac{T_0}{T_1} = \frac{5.5}{1} \times \frac{300}{650} = 2.54$$

$$\therefore \quad \Delta s = 0.718 \log_e \left(\frac{650}{300}\right) + 0.287 \log_e \left(\frac{1}{2.54}\right)$$

$$= 0.555 + (-0.267) = 0.288 \text{ kJ/kg K}$$

∴ Change in available energy

$$= m[(u_1 - u_0) - T_0\Delta s] = m[c_v(T_1 - T_0) - T_0\Delta s]$$

$$= 8[0.718(650 - 300) - 300 \times 0.288] = 1319.2 \text{ kJ}$$

Loss of availability per unit mass during the process

$$= p_0\ (v_0 - v_1) \text{ per unit mass}$$

Total loss of availability = $p_0(V_0 - V_1)$

But
$$V_1 = \frac{mRT_1}{p_1} = \frac{8 \times 287 \times 650}{5.5 \times 10^5} = 2.713 \text{ m}^3 \qquad \left[\because\ pV = mRT \text{ or } V = \frac{mRT}{p}\right]$$

and
$$V_0 = 2.54 \times 2.713 = 6.891 \text{ m}^3$$

∴ **Loss of availability** $= \dfrac{1 \times 10^5}{10^3}(6.891 - 2.713) =$ **417.8 kJ. (Ans.)**

(*ii*) Heat transferred during cooling (constant pressure) process

$$= m.\ c_p\ (T_1 - T_0)$$

$$= 8 \times 1.005\ (650 - 300) = 2814 \text{ kJ}$$

Change in entropy during cooling

$$\Delta s = mc_p \log_e \left(\frac{T_1}{T_0}\right)$$

$$= 8 \times 1.005 \times \log_e \left(\frac{650}{300}\right) = 6.216 \text{ kJ/K}$$

Unavailable energy $= T_0\ \Delta S$

$$= 300 \times 6.216 = 1864.8 \text{ kJ}$$

Available energy = 2814 – 1864.8 = **949.2 kJ. (Ans.)**

Effectiveness, $\in = \dfrac{\text{Available energy}}{\text{Change in available energy}}$

$$= \frac{949.2}{1319.2} = \mathbf{0.719.}\ \ \textbf{(Ans.)}$$

☞**Example 6.4.** *In a power station, the saturated steam is generated at 200°C by transferring the heat from hot gases in a steam boiler. Find the increase in total entropy of the combined system of gas and water and increase in unavailable energy due to irreversible heat transfer. The gases are cooled from 1000°C to 500°C and all the heat from gases goes to water. Assume water enters the boiler at saturated condition and leaves as saturated steam.*

Take : c_{pg} *(for gas) = 1.0 kJ/kg K,* h_{fg} *(latent heat of steam at 200°C) = 1940.7 kJ/kg.*

Atmospheric temperature = 20°C.

Obtain the results on the basis of 1 kg of water.

Solution. Refer Fig. 6.8.

Temperature of saturation steam = 200 + 273 = 473 K

Initial temperature of gases = 1000 + 273 = 1273 K

Final temperature of gases = 500 + 273 = 773 K

For gases : c_{pg} = 1 kJ/kg K

Latent heat of steam of 200°C saturation temperature, h_{fg} = 1940.7 kJ/kg

Atmospheric temperature = 20 + 273 = 293 K

Heat lost by gases = Heat gained by 1 kg saturated water when it is converted to steam at 200°C.

$$\therefore \quad m_g c_{pg}\ (1273 - 773) = 1940.7$$

[where m_g = mass of gases, c_{pg} = specific heat of gas at constant pressure]

i.e., $$m_g = \frac{1940.7}{1.0 \times (1273 - 773)} = 3.88 \text{ kg}$$

Change of entropy of m_g kg of gas,

$$(\Delta S)_g = m_g\ c_{pg} \log_e \left(\frac{773}{1273}\right)$$

$$= 3.88 \times 1.0 \times \log_e \left(\frac{773}{1273}\right) = -\ 1.935 \text{ kJ/K}$$

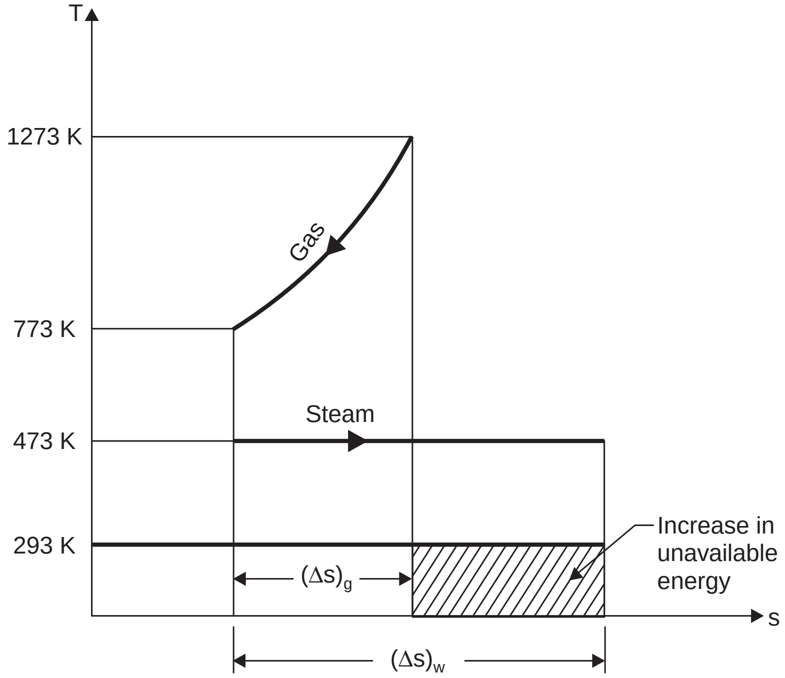


Fig. 6.8

Change of entropy of water (per kg) when it is converted into steam,

$$(\Delta s)_w = \frac{h_{fg}}{T_s} = \frac{1940.7}{(200+273)} = 4.103 \text{ kJ/kg K.}$$

Net change in entropy due to heat transfer

$$= -1.935 + 4.103 = \mathbf{2.168\ kJ/K.} \quad \textbf{(Ans.)}$$

Increase in unavailable energy due to heat transfer

$= 293 \times 2.168$, *i.e.*, cross hatched area

= 635.22 kJ per kg of steam formed. (Ans.)

Example 6.5. *3 kg of gas (c_v = 0.81 kJ/kg K) initially at 2.5 bar and 400 K receives 600 kJ of heat from an infinite source at 1200 K. If the surrounding temperature is 290 K, find the loss in available energy due to above heat transfer.*

Solution. Refer Fig. 6.9.

Mass of gas, $m_g = 3$ kg

Initial pressure of gas = 2.5 bar

Initial temperature, $T_1' = 400$ K

Quantity of heat received by gas, $Q = 600$ kJ

Specific heat of gas, $c_v = 0.81$ kJ/kg K

Surrounding temperature = 290 K

Temperature of infinite source, $T_1 = 1200$ K

Heat received by the gas is given by,

$$Q = m_g c_v (T_2' - T_1')$$

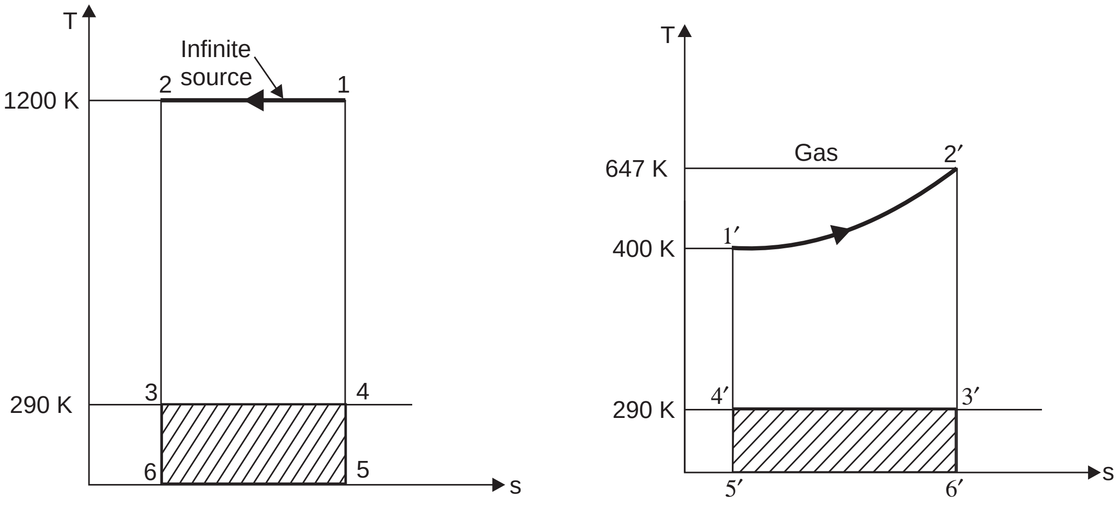


Fig. 6.9

$$600 = 3 \times 0.81\,(T_2' - 400)$$

$$\therefore \qquad T_2' = \frac{600}{3 \times 0.81} + 400 = 646.9 \text{ K say } 647 \text{ K}$$

Available energy with the source

$$= \text{area } 1\text{-}2\text{-}3\text{-}4\text{-}1$$

$$= (1200 - 290) \times \frac{600}{1200} = 455 \text{ kJ}$$

Change in entropy of the gas

$$= m_g c_v \log_e \left(\frac{T_2'}{T_1'}\right) = 3 \times 0.81 \times \log_e \left(\frac{647}{400}\right) = 1.168 \text{ kJ/K}$$

Unavailability of the gas $= \text{area } 3'\text{-}\ 4'\text{-}\ 5'\text{-}\ 6'\text{-}\ 3'$

$$= 290 \times 1.168 = 338.72 \text{ kJ}$$

Available energy with the gas

$$= 600 - 338.72 = 261.28 \text{ kJ}$$

$\therefore$ **Loss in available energy due to heat transfer**

$$= 455 - 261.28 = \mathbf{193.72 \text{ kJ.}} \quad \textbf{(Ans.)}$$

☞**Example 6.6.** *Calculate the unavailable energy in 60 kg of water at 60°C with respect to the surroundings at 6°C, the pressure of water being 1 atmosphere.*

Solution. Refer Fig. 6.10.

Mass of water, $m = 60$ kg

Temperature of water, $T_1 = 60 + 273 = 333$ K

Temperature of surroundings, $T_0 = 6 + 273 = 279$ K

Pressure of water, $p = 1$ atm.

If the water is cooled at a constant pressure of 1 atm. from 60°C to 6°C the heat given up may be used as a *source for a series of Carnot engines each using the surroundings as a sink.* It is assumed that the amount of energy received by any engine is small relative to that in the source and temperature of the source does not change while heat is being exchanged with the engine.

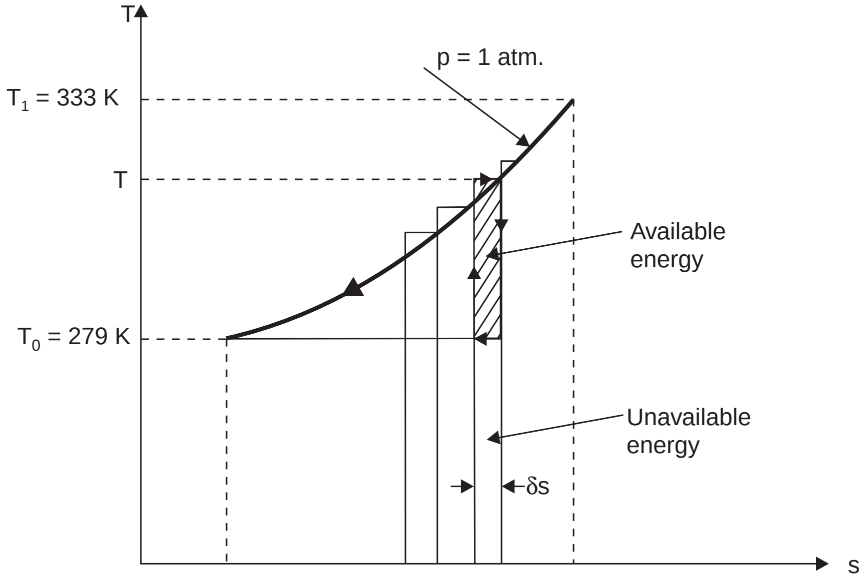


Fig. 6.10

Consider that the source has fallen to temperature T, at which level there operates a Carnot engine which takes in heat at this temperature and rejects heat at T_0 = 279 K. If δs is the entropy change of water, the work obtained is

$$\delta W = -\, m(T - T_0)\, \delta s$$

where δs is negative.

$$\therefore \qquad \delta W = -\, 60\, (T - T_0)\, \frac{c_p \delta T}{T} = -\, 60\, c_p \left(1 - \frac{T_0}{T}\right) \delta T$$

With a very great number of engines in the series, the total work (maximum) obtainable when the water is cooled from 333 K to 279 K would be

$$W_{max} = \text{Available energy}$$

$$= -\,\text{lim.} \sum_{333}^{279} 60\, c_p \left(1 - \frac{T_0}{T}\right) \delta T$$

$$= \int_{279}^{333} 60\, c_p \left(1 - \frac{T_0}{T}\right) dT$$

$$= 60\, c_p \left[(333 - 279) - 279 \log_e \left(\frac{333}{279}\right)\right]$$

$$= 60 \times 4.187\,(54 - 49.36) = 1165.7 \text{ kJ}$$

Also, $Q_1 = 60 \times 4.187 \times (333 - 279) = 13565.9$ kJ

∴ **Unavailable energy** = $Q_1 - W_{max}$

= 13565.9 – 1165.7 = **12400.2 kJ. (Ans.)**

Example 6.7. *15 kg of water is heated in an insulated tank by a churning process from 300 K to 340 K. If the surrounding temperature is 300 K, find the loss in availability for the process.*

Solution. Mass of water, m = 15 kg

Temperature, T_1 = 340 K

Surrounding temperature, $T_0 = 300$ K

Specific heat of water, $c_p = 4.187$ kJ/kg K

Loss in availability :

Work added during churning

$$= \text{Increase in enthalpy of the water}$$

$$= 15 \times 4.187 \times (340 - 300) = 2512.2 \text{ kJ}$$

Now the energy in the water = 2512.2 kJ

The availability out of this energy is given by

$$m[(u_1 - u_0) - T_0\,\Delta s]$$

where $$\Delta s = c_p \log_e \left(\frac{T_1}{T_0}\right)$$

$$\therefore \quad \Delta s = 4.187 \log_e \left(\frac{340}{300}\right) = 0.524 \text{ kJ/kg K}$$

∴ Available energy

$$= m\,[c_v\,(T_1 - T_0) - T_0\,\Delta s]$$

$$= 15\,[4.187\,(340 - 300) - 300 \times 0.524] = 158.7 \text{ kJ}$$

∴ **Loss in availability**

= 2508 – 158.7 = **2349.3 kJ. (Ans.)**

This shows that *conversion of work into heat is highly irreversible process* (since out of 2512.2 kJ of work energy supplied to increase the temperature, only 158.7 kJ will be available again for conversion into work).

Example 6.8. *5 kg of air at 550 K and 4 bar is enclosed in a closed system.*

(*i*) *Determine the availability of the system if the surrounding pressure and temperature are 1 bar and 290 K respectively.*

(*ii*) *If the air is cooled at constant pressure to the atmospheric temperature, determine the availability and effectiveness.*

Solution. Mass of air, $m = 5$ kg

Temperature, $T_1 = 550$ K

Pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

Temperature, $T_2 = T_0 = 290$ K

Pressure, $p_2 = p_0 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$.

(*i*) **Availability of the system :**

Availability of the system is

$$= m[(u_1 - u_0) - T_0(s_1 - s_0)] = m[c_v(T_1 - T_0) - T_0\,\Delta s]$$

$$\Delta s = c_p \log_e \frac{T_1}{T_0} - R \log_e \left(\frac{p_1}{p_0}\right)$$

$$= 1.005 \log_e \left(\frac{550}{290}\right) - 0.287 \log_e \left(\frac{4}{1}\right)$$

$$= 0.643 - 0.397 = 0.246 \text{ kJ/kg K}$$

∴ **Availability of the system**

$$= m\,[c_v\,(T_1 - T_0) - T_0\Delta s]$$

= 5[0.718 (550 – 290) – 290 × 0.246] = **576.7 kJ. (Ans.)**

(*ii*) Heat transferred during cooling

$$Q = m \times c_p \times (T_1 - T_0)$$
$$= 5 \times 1.005 \times (550 - 290)$$
$$= 1306.5 \text{ kJ} \quad \text{...... heat lost by the system}$$

Change of entropy during cooling

$$\Delta S = m \times c_p \times \log_e \left(\frac{T_1}{T_0}\right)$$
$$= 5 \times 1.005 \times \log_e \left(\frac{550}{290}\right) = 3.216 \text{ kJ/K}$$

Unavailable portion of this energy

$$= T_0 \,.\, (\Delta S) = 290 \times 3.216 = 932.64 \text{ kJ}$$

∴ **Available energy** $= 1306.5 - 932.64 =$ **373.86 kJ. (Ans.)**

Effectiveness, $\epsilon = \dfrac{\text{Available energy}}{\text{Availability of the system}} = \dfrac{373.86}{576.7}$

$=$ **0.648 or 64.8%. (Ans.)**

Example 6.9. *Air at the rate of 25 kg/min is compressed in a centrifugal air compressor from 1 bar to 2 bar. The temperature increases from 15°C to 100°C during compression. Determine actual and minimum power required to run the compressor. The surrounding air temperature is 15°C.*

Neglect the heat interaction between the compressor and surroundings and changes in potential and kinetic energy.

Take for air, $c_p = 1.005$ *kJ/kg K,* $R = 0.287$ *kJ/kg K.*

Solution. Rate of flow of air, $m = 25$ kg/min.

Initial pressure, $p_1 = 1$ bar

Final pressure, $p_2 = 2.0$ bar

Initial temperature, $T_1 = T_0 = 15 + 273 = 288$ K

Final temperature, $T_2 = 100 + 273 = 373$ K.

Applying energy equation to compressor,

$$W_{actual} = h_2 - h_1 \qquad [\text{as } Q = 0,\ \Delta PE = 0,\ \Delta KE = 0]$$
$$= c_p\,(T_2 - T_1) = 1.005\,(373 - 288) = 85.4 \text{ kJ/kg}$$

Total actual work done/min

$$= 25 \times 85.4 = 2135 \text{ kJ/min}$$
$$= \frac{2135}{60} = 35.58 \text{ kJ/s} = 35.58 \text{ kW}$$

The *minimum work required is given by the increase in availability of the air stream.*

$$W_{min} = b_2 - b_1 = (h_2 - h_1) - T_0\,(s_2 - s_1)$$
$$s_2 - s_1 = c_p \log_e \left(\frac{T_2}{T_1}\right) - R \log_e \left(\frac{p_2}{p_1}\right) \qquad \text{...per unit mass}$$
$$= 1.005 \log_e \left(\frac{373}{288}\right) - 0.287 \log_e \left(\frac{2.0}{1}\right)$$
$$= 0.2599 - 0.1989 = 0.061 \text{ kJ/kg K}$$

∴ $$W_{min} = (h_2 - h_1) - T_0(s_2 - s_1)$$

$$= 85.4 - 288 \times 0.061 = 67.8 \text{ kJ/kg}$$

$\therefore$ **Minimum work required**

$$\frac{25 \times 67.8}{60} = 28.25 \text{ kJ/s} = \mathbf{28.25 \text{ kW.}} \quad \textbf{(Ans.)}$$

Example 6.10. *1 kg of oxygen at 1 bar and 450 K is mixed with 1 kg of hydrogen at the same temperature and pressure by removing the diaphragm as shown in Fig. 6.11. Determine the loss in availability if the surrounding temperature is 290 K.*

Assume that the system is fully isolated.

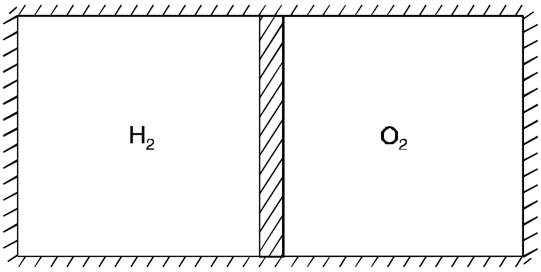


Fig. 6.11

Solution. Mass of oxygen, $m_{O_2} = 1$ kg

Mass of hydrogen, $m_{H_2} = 1$ kg

Pressure, $p = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Temperature, $T_{O_2} = T_{H_2} = 450$ K

Surrounding temperature $= 290$ K

Characteristic gas constant of O_2,

$$R_{O_2} = \frac{R_0}{M_{O_2}} = \frac{8314}{32} = 259.6 \text{ J/kg K}$$

Now to find volume of O_2 using the relation,

$$pv = mRT$$

$$v = \frac{mRT}{p} = \frac{1 \times 259.6 \times 450}{1 \times 10^5} = 1.168 \text{ m}^3$$

i.e., $v_{O_2} = 2.336 \text{ m}^3$

Characteristic gas constant of H_2,

$$R_{H_2} = \frac{R_0}{M_{H_2}} = \frac{8314}{2} = 4157 \text{ J/kg K}$$

Volume of H_2, $v_{H_2} = \frac{1 \times 4157 \times 450}{1 \times 10^5} = 18.706 \text{ m}^3$

Total volume after mixing

$$= v_{O_2} + v_{H_2} = 1.168 + 18.706$$
$$= 19.874 \text{ m}^3$$

The partial pressure of each gas changes after the mixing even though the temperature is the same due to increase in volume.

Change in entropy of *oxygen*

$$= R_{O_2} \log_e \frac{v_2}{v_1}$$

$$= 259.6 \log_e \left(\frac{19.874}{1.168}\right) = 735.7 \text{ J/K}$$

Change in entropy of hydrogen

$$= R_{H_2} \log_e \frac{v_2}{v_1} = 4157 \log_e \left(\frac{19.874}{18.706}\right)$$

$$= 251.78 \text{ J/K}$$

Net change in entropy,

$$\Delta S = 735.7 + 251.78 = 987.48 \text{ J/K}$$

Loss in availability

$$= T_0 \Delta S = 290 \times 987.48 \text{ J} = \frac{290 \times 987.48}{10^3} \text{ kJ} = 286.36 \text{ kJ}$$

i.e., **Loss in availability = 286.36 kJ. (Ans.)**

☞**Example 6.11.** *Calculate the decrease in available energy when 20 kg of water at 90°C mixes with 30 kg of water at 30°C, the pressure being taken as constant and the temperature of the surroundings being 10°C.*

Take c_p of water as 4.18 kJ/kg K.

Solution. Temperature of surrounding, $T_0 = 10 + 273 = 283$ K

Specific heat of water, $c_p = 4.18$ kJ/kg K

The available energy of a system of mass m, specific heat c_p, and at temperature T, is given by,

Available energy, $$\text{A.E.} = mc_p \int_{T_0}^{T} \left(1 - \frac{T_0}{T}\right) dT$$

Now, available energy of 20 kg of water at 90°C,

$$(\text{A.E.})_{20 \text{ kg}} = 20 \times 4.18 \int_{(10+273)}^{(90+273)} \left(1 - \frac{283}{T}\right) dT$$

$$= 83.6 \left[(363 - 283) - 283 \log_e \left(\frac{363}{283}\right)\right]$$

$$= 83.6\ (80 - 70.45) = 798.38 \text{ kJ}$$

Available energy of 30 kg of water at 30°C,

$$(\text{A.E.})_{30 \text{ kg}} = 30 \times 4.18 \int_{(10+273)}^{(30+273)} \left(1 - \frac{283}{T}\right) dT$$

$$= 30 \times 4.18 \left[(303 - 283) - 283 \log_e \left(\frac{303}{283}\right)\right]$$

$$= 125.4\ (20 - 19.32) = 85.27 \text{ kJ}$$

Total available energy,

$$(\text{A.E.})_{\text{total}} = (\text{A.E.})_{20 \text{ kg}} + (\text{A.E.})_{30 \text{ kg}}$$

$$= 798.38 + 85.27 = 883.65 \text{ kJ}$$

If t°C is the final temperature after mixing, then

$$20 \times 4.18 \times (90 - t) = 30 \times 4.18\ (t - 30)$$

or $$20(90 - t) = 30\,(t - 30)$$

$$\therefore \qquad t = \frac{20 \times 90 + 30 \times 30}{20 + 30} = 54°\text{C}$$

Total mass after mixing = 20 + 30 = 50 kg

Available energy of 50 kg of water at 54°C

$$(\text{A.E.})_{50\ \text{kg}} = 50 \times 4.18 \left[(327 - 283) - 283 \log_e \left(\frac{327}{283}\right)\right]$$

$$= 209\,(44 - 40.89) = 649.99 \text{ kJ}$$

∴ **Decrease in available energy due to mixing**

= Total energy before mixing – Total energy after mixing

= 883.65 – 649.99 = **233.66 kJ. (Ans.)**

☞ **Example 6.12.** *In an heat exchanger (parallel flow type) waters enter at 50°C and leaves at 70°C while oil (specific gravity = 0.82, specific heat = 2.6 kJ/kg K) enters at 240°C and leaves at 90°C. If the surrounding temperature is 27°C determine the loss in availability on the basis of one kg of oil per second.*

Solution. Refer Fig. 6.12.

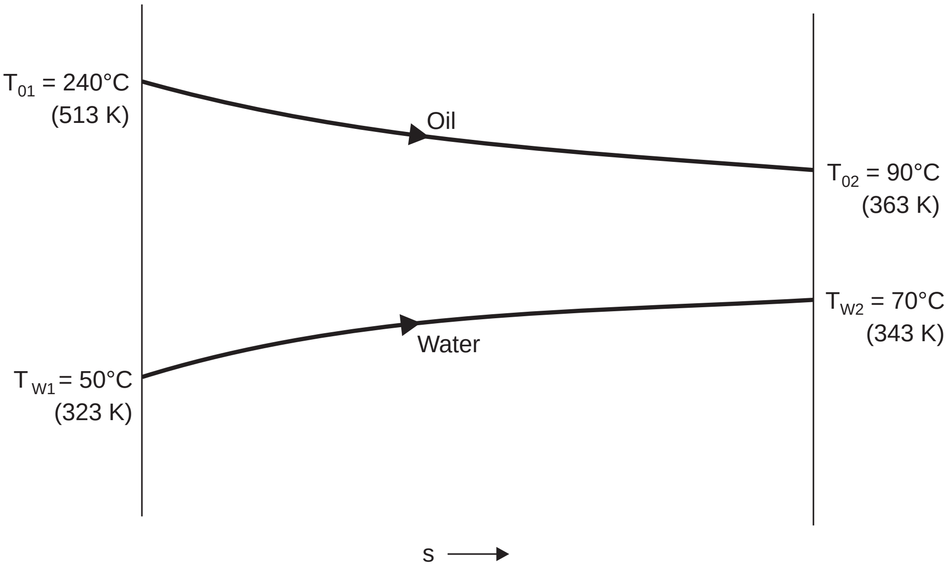


Fig. 6.12

Inlet temperature of water, $T_{w_1} = 50°\text{C} = 323 \text{ K}$

Outlet temperature of water, $T_{w_2} = 70°\text{C} = 343 \text{ K}$

Inlet temperature of oil, $T_{o_1} = 240°\text{C} = 513 \text{ K}$

Outlet temperature of oil, $T_{o_2} = 90°\text{C} = 363 \text{ K}$

Specific gravity of oil = 0.82

Specific heat of oil = 2.6 kJ/kg K

Surrounding temperature, $T_0 = 27 + 273 = 300$ K.

Loss in availability :

Consider one kg of oil.

Heat lost by oil = Heat gained by water

$$m_0 \times c_{po} \times (T_{o_1} - T_{o_2}) = m_w \times c_{pw} \times (T_{w_2} - T_{w_1})$$

where c_{po} = Specific heat of oil (2.6 kJ/kg K),

c_{pw} = Specific heat of water (4.18 kJ/kg K), and

m_o = Mass of oil (= 1 kg).

m_w = Mass of water (= ?)

$\therefore \quad 1 \times 2.6 \times (513 - 363) = m_w \times 4.18 \times (343 - 323)$

or $\quad 390 = 83.6\ m_w$ or $m_w = 4.66$ kg

Entropy change of water

$$= m_w\, c_{pw} \log_e \frac{T_{w_2}}{T_{w_1}} = 4.66 \times 4.18 \times \log_e \left(\frac{343}{323}\right) = 1.17 \text{ kJ/K}$$

Entropy change of oil

$$= m_o c_{po} \log_e \left(\frac{T_{o_2}}{T_{o_1}}\right) = 1 \times 2.6 \log_e \left(\frac{363}{513}\right) = -\,0.899 \text{ kJ/K}$$

Change in availability of water

$$= m_w[c_{pw}(T_{w_2} - T_{w_1})] - T_o\,(\Delta S)_w$$

$$= 4.66[(4.18\ (343 - 323)] - 300 \times 1.17 = 38.57 \text{ kJ}$$

+ve sign indicates an *increase in availability*

Change in availability of oil

$$= m_o[c_{po}(T_{o_2} - T_{o_1})] - T_0\,(\Delta S)_0]$$

$$= 1[2.6(363 - 513)] - 300 \times (-\,0.899) = -\,120.3 \text{ kJ/K}$$

$\therefore$ **Loss in availability**

$$= -\,120.3 + 38.57 = -\,\mathbf{81.73\ kJ.} \quad \textbf{(Ans.)}$$

(–ve sign indicates the **loss**).

Example 6.13. *1 kg of ice at 0°C is mixed with 12 kg of water at 27°C. Assuming the surrounding temperature as 15°C, calculate the net increase in entropy and unavailable energy when the system reaches common temperature :*

Given : Specific heat of water = 4.18 kJ/kg K ; specific heat of ice = 2.1 kJ/kg K and enthalpy of fusion of ice (latent heat) = 333.5 kJ/kg.

Solution. Mass of ice, m_{ice} = 1 kg

Temperature of ice, T_{ice} = 0 + 273 = 273 K

Mass of water, m_{water} = 12 kg

Temperature of water, T_{water} = 27 + 273 = 300 K

Surrounding temperature, T_0 = 15 + 273 = 288 K

Specific heat of water = 4.18 kJ/kg K

Specific heat of ice = 2.1 kJ/kg K

Latent heat of ice = 333.5 kJ/kg

Let T_c = *common temperature* when heat flows between ice and water stops.

Heat lost by water = Heat gained by ice

i.e., $$12 \times 4.18(300 - T_c) = 4.18(T_c - 273) + 333.5$$

or $$15048 - 50.16T_c = 4.18T_c - 1141.14 + 333.5$$

or $$54.34\ T_c = 15855.64$$

$$\therefore \quad T_c = 291.8 \text{ K or } 18.8°\text{C}.$$

Change of entropy of water $= 12 \times 4.18 \log_e \left(\dfrac{291.8}{300}\right) = -1.39$ kJ/K

Change of entropy of ice $= 1 \times 4.18 \log_e \left(\dfrac{291.8}{273}\right) + \dfrac{333.5}{273} = 1.499$ kJ/K

Net change of entropy, $\Delta S = -1.39 + 1.499 = 0.109$ kJ/K

Hence, **net increase in entropy = 0.109 kJ/K. (Ans.)**

Increase in unavailable energy = $T_0 \Delta S = 288 \times 0.109 =$ **31.39 kJ. (Ans.)**

☞**Example 6.14.** *A vapour, in a certain process, while condensing at 400°C, transfers heat to water at 200°C. The resulting steam is used in a power cycle which rejects heat at 30°C.*

What is the fraction of the available energy in the heat transferred from the process vapour at 400°C that is lost due to the irreversible heat transfer at 200°C ?

Solution. Refer Fig. 6.13.

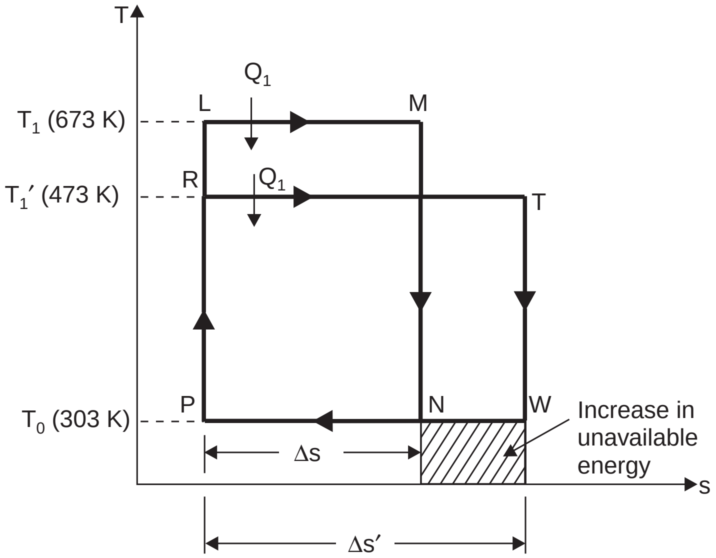


Fig. 6.13

Temperature of vapour, $T_1 = 400 + 273 = 673$ K

Temperature of water, $T_2 = 200 + 273 = 473$ K

Temperature at which heat is rejected, $T_0 = 30 + 273 = 303$ K.

LMNP (Fig. 6.13) would have been the power cycle, if there was no temperature difference between the vapour condensing and the vapour evaporating, and the area under *NP* would have been the unavailable energy. *RTWP* is the power cycle when the vapour condenses at 400°C and the water evaporates at 200°C. The unavailable energy becomes the area under *PW*. Therefore, the increase in unavailable energy due to irreversible heat transfer is represented by the area under *NW*.

Now, $$Q_1 = T_1 \Delta s = T_1' \Delta s'$$

$$\frac{\Delta s'}{\Delta s} = \frac{T_1}{T_1'}$$

$$W = \text{Work done in cycle } LMNP$$

$$= (\mathrm{T}_1 - T_0)\,\Delta s \qquad \text{...per unit mass}$$

$$W' = \text{Work done in cycle } RTWP$$

$$= (T_1 - T_0)\,\Delta s' \qquad \text{...per unit mass}$$

The fraction of energy that becomes unavailable due to irreversible heat transfer,

$$\frac{W - W'}{W} = \frac{(T_1 - T_0)\Delta s - (T_1 - T_0)\Delta s'}{(T_1 - T_0)\Delta s} = \frac{T_0\,(\Delta s' - \Delta s)}{(T_1 - T_0)\Delta s}$$

$$= \frac{T_0\left(\frac{\Delta s'}{\Delta s} - 1\right)}{(T_1 - T_0)} = \frac{T_0\left(\frac{T_1}{T_1'} - 1\right)}{(T_1 - T_0)}$$

$$= \frac{T_0\,(T_1 - T_1')}{T_1'\,(T_1 - T_0)} = \frac{303(673 - 473)}{473(673 - 303)} = 0.346$$

Hence **the fraction of energy that becomes unavailable = 0.346 or 34.6%. (Ans.)**

Example 6.15. *A liquid is heated at approximately constant pressure from 20°C to 80°C by passing it through tubes which are immersed in a furnace. The furnace temperature is constant at 1500°C. Calculate the effectiveness of the heating process when the atmospheric temperature is 15°C.*

Take specific heat of liquid as 6.3 kJ/kg K.

Solution. Initial temperature of fluid, $T_1 = 20 + 273 = 293$ K

Final temperature of fluid, $T_2 = 80 + 273 = 353$ K

Temperature of the furnace, $T_f = 1500 + 273 = 1773$ K

Atmospheric temperature, $T_0 = 15 + 273 = 288$ K

Specific heat of liquid, $c_{pl} = 6.35$ kJ/kg K

Increase of availability of the liquid

$$= b_2 - b_1 = (h_2 - h_1) - T_0(s_2 - s_1)$$

i.e., $$b_2 - b_1 = c_{pl}\,(T_2 - T_1) - T_0 \times c_{pl} \log_e \frac{T_2}{T_1}$$

$$= 6.3\,(353 - 293) - 288 \times 6.3 \times \log_e \left(\frac{353}{293}\right) = 39.98 \text{ kJ/kg}$$

Now, the heat rejected by the furnace = Heat supplied to the liquid, $(h_2 - h_1)$.

If this quantity of heat were supplied to a heat engine operating on the Carnot cycle its thermal efficiency would be,

$$\eta_{th} = \left(1 - \frac{T_0}{T_f}\right) = \left(1 - \frac{288}{1773}\right) = 0.837 \text{ (or } 83.7\%)$$

∴ Work which could be obtained from a heat engine

= Heat supplied × Thermal efficiency

i.e., Possible work of heat engine = $(h_2 - h_1) \times 0.837$

The possible work from a heat engine is a measure of the loss of availability of the furnace.

∴ Loss of availability of surroundings

$$= (h_2 - h_1) \times 0.837 = c_{pl}\,(T_2 - T_1) \times 0.837$$
$$= 6.3\ (353 - 293) \times 0.837 = 316.38 \text{ kJ/kg}$$

Then, *effectiveness of the heating process*,

$$\in = \frac{\text{Increase of availability of the liquid}}{\text{Loss of availability of surroundings}}$$
$$= \frac{39.98}{316.38} = \mathbf{0.1263} \text{ or } \mathbf{12.63\%.} \quad \textbf{(Ans.)}$$

Note. *The very low value of effectiveness reflects the irreversibility of the transfer of heat through a large temperature difference.* If the furnace temperature were much lower then process would be much more effective, although the heat transferred to the liquid would remain the same.

Example 6.16. *Air at 20°C is to be heated to 50°C by mixing it in steady flow with a quantity of air at 100°C. Assuming that the mixing process is adiabatic and neglecting changes in kinetic and potential energy, calculate :*

(*i*) *The ratio of mass flow of air initially at 100°C to that initially at 20°C.*

(*ii*) *The effectiveness of heating process, if the atmospheric temperature is 20°C.*

Solution. (*i*) Let, x = ratio of mass flows.

Stream 1 = air at 20°C ($T_1 = 20 + 273 = 293$ K)
Stream 2 = air at 100°C ($T_2 = 100 + 273 = 373$ K)
Stream 3 = air at 50°C ($T_3 = 50 + 273 = 323$ K)
If, c_p = Specific heat of air constant pressure

Then $c_pT_1 + xc_pT_2 = (1 + x)c_pT_3$

or $c_p\mathrm{T}_1 + xc_pT_2 = c_pT_3 + xc_pT_3$

or $xc_p(T_2 - T_3) = c_p(T_3 - T_1)$

i.e., $xc_p(373 - 323) = c_p(323 - 293)$

∴ $x = \dfrac{30}{50} = \mathbf{0.6.}$ **(Ans.)**

(*ii*) Let the system considered be a stream of air of unit mass, heated from 20°C to 50°C.

Increase of availability of system

$$= b_3 - b_1 = (h_3 - h_1) - T_0(s_3 - s_1) = c_p(T_3 - T_1) - T_0(s_3 - s_1)$$
$$= 1.005(323 - 293) - 293(s_3 - s_1) \qquad [\because \ T_0 = 20 + 273 = 293 \text{ K}]$$

Also, $s_3 - s_1 = c_p \log_e \dfrac{T_3}{T_1} = 1.005 \log_e \dfrac{323}{293} = 0.0979$ kJ/kg K

∴ *Increase of availability of system*

$$= 1.005 \times 30 - 293 \times 0.0979 = 1.465 \text{ kJ/kg.}$$

The system, which is the air being heated, is 'surrounded' by the air stream being cooled. Therefore, the loss of availability of the surroundings is given by, $x(b_2 - b_3)$.

i.e., *Loss of availability of surroundings*

$$= x[(h_2 - h_3) - T_0(s_2 - s_3)] = 0.6[c_p(T_2 - T_3) - T_0(s_2 - s_3)]$$
$$= 0.6\left[1.005\,(373 - 323) - 293 \times 1.005 \log_e\left(\frac{373}{323}\right)\right]$$

$$= 0.6[50.25 - 42.38] = 4.722 \text{ kJ/kg}$$

$$\therefore \qquad \textit{Effectiveness} = \frac{\text{Increase of availability of system}}{\text{Loss of availability of surroundings}}$$

$$= \frac{1.465}{4.722} = \mathbf{0.31} \quad \text{or} \quad \mathbf{31\%.} \quad \textbf{(Ans.)}$$

The *low* figure for the effectiveness is an indication of the *highly irreversible nature of the mixing process.*

☞**Example 6.17.** *2.5 kg of air at 6 bar, 90°C expands adiabatically is a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 1 bar, 5°C. For this process determine :*

(*i*) *The maximum work ;*

(*ii*) *The change in availability ;*

(*iii*) *The irreversibility.*

For air take : $c_v = 0.718$ *kJ/kg K,* $R = 0.287$ *kJ/kg K.*

Solution. Mass of air, $m = 2.5$ kg

Initial pressure of air, $p_1 = 6 \text{ bar} = 6 \times 10^5 \text{ N/m}^2$

Ratio of final to initial volume, $\frac{V_2}{V_1} = 2$

Initial temperature of air, $T_1 = 90 + 273 = 363$ K

Final pressure of air, $p_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Final temperature of air, $T_2 = T_0 = 5 + 273 = 278$ K

From the property relation

$$TdS = dU + pdV$$

or

$$dS = \frac{dU}{T} + \frac{pdV}{T}$$

$$dS = \frac{mc_v dT}{T} + \frac{mRdV}{V} \quad \left[\because u = c_v\, dT \text{ and } pV = mRT \text{ or } \frac{p}{T} = \frac{mR}{V}\right]$$

∴ The entropy change of air between the initial and final states is

$$\int_1^2 dS = \int_1^2 \frac{mc_v dT}{T} + \int_1^2 \frac{mRdV}{V}$$

or

$$S_2 - S_1 = mc_v \log_e \frac{T_2}{T_1} + mR \log_e \frac{V_2}{V_1}$$

(*i*) **The maximum work, W_{max} :**

Also, $$W_{max} = (U_1 - U_2) - T_0(S_1 - S_2)$$

$$= m\left[c_v(T_1 - T_2) + T_0\left(c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}\right)\right]$$

$$= 2.5\left[0.718(363 - 278) + 278\left(0.718 \log_e\left(\frac{278}{363}\right) + 0.287 \log_e 2\right)\right]$$

$$= 2.5[61.03 + 278(-0.1915 + 0.1989)] = 157.7 \text{ kJ}$$

Hence, **maximum work = 157.7 kJ. (Ans.)**

(*ii*) **The change in availability :**

The change in availability is given by,

$$A_1 - A_2 = (U_1 - U_2) - T_0(S_1 - S_2) + p_0(V_1 - V_2)$$

$$= W_{max} + p_0\,(V_1 - V_2)$$

$$= 157.7 + p_0\,(V_1 - 2V_1) = 157.7 - P_0V_1 \qquad \left[\begin{array}{l} p_1V_1 = mRT_1 \\ \because\ V_1 = \dfrac{mRT_1}{p_1} \end{array}\right]$$

$$= 157.7 - \frac{1 \times 10^5}{10^3}\left[\frac{2.5 \times (0.287 \times 1000) \times 363}{6 \times 10^5}\right] = 114.29 \text{ kJ}$$

Hence **change in availability = 114.29 kJ. (Ans.)**

(*iii*) **The irreversibility ; I :**

The irreversibility is given by

$$I = W_{max.\ useful} - W_{actual}$$

From the first law of thermodynamics,

$$W_{actual} = Q - \Delta U = -\Delta U = U_1 - U_2 \qquad [\because\ Q = 0 \text{ ... adiabatic process}]$$

$$I = (U_1 - U_2) - T_0(S_1 - S_2) - (U_1 - U_2)$$

$$= T_0(S_2 - S_1)$$

$$= T_0(\Delta S)_{system}$$

For adiabatic process, $(\Delta S)_{surr.} = 0$

$$I = T_0\left[mc_v \log_e \frac{T_2}{T_1} + mR \log_e \frac{v_2}{v_1}\right]$$

$$= 278 \times 2.5\left(0.718 \log_e\left(\frac{278}{363}\right) + 0.287 \log_e 2\right)$$

$$= 695(-0.1915 + 0.1989) = 5.143 \text{ kJ}$$

Hence, **the irreversibility = 5.143 kJ. (Ans.)**

☞**Example 6.18.** *In a turbine the air expands from 7 bar, 600°C to 1 bar, 250°C. During expansion 9 kJ/kg of heat is lost to the surroundings which is at 1 bar, 15°C. Neglecting kinetic energy and potential energy changes, determine per kg of air :*

(*i*) *The decrease in availability ;*

(*ii*) *The maximum work ;*

(*iii*) *The irreversibility.*

For air, take : $c_p = 1.005$ *kJ/kg K,* $h = c_pT$*, where* c_p *is constant.*

Solution. Mass of air considered = 1 kg

Pressure, $p_1 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$

Temperature, $T_1 = 600 + 273 = 873 \text{ K}$

Pressure, $p_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Temperature, $T_2 = 250 + 273 = 523 \text{ K}$

Surrounding temperature, $T_0 = 15 + 273 = 288 \text{ K}$

Heat lost to the surroundings during expansion,

$$Q = 9 \text{ kJ/kg.}$$

(*i*) From the property relation,

$$TdS = dH - Vdp$$

$$dS = \frac{dH}{T} - \frac{Vdp}{T}$$

$$= \frac{m.c_p dT}{T} - \frac{mRdp}{p}$$

or $$\int_1^2 dS = \int_1^2 \frac{m.c_p dT}{T} - \int_1^2 \frac{mRdp}{p}$$

or $$S_2 - S_1 = mc_p \log_e \frac{T_2}{T_1} - mR \log_e \frac{p_2}{p_1}$$

$$\left\{ \begin{array}{l} \because\ h = c_p T,\ dh = c_p dT \\ dH = mc_p dT \text{ and } pV = mRT \\ \text{or } V = \dfrac{mRT}{p} \end{array} \right\}$$

For 1 kg of air

$$s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

Now, the change in availability is given by

$$b_1 - b_2 = (h_1 - T_0 s_1) - (h_2 - T_0 s_2)$$

$$= (h_1 - h_2) - T_0(s_1 - s_2)$$

$$= c_p\ (T_1 - T_2) - T_0 \left(R \log_e \frac{p_2}{p_1} - c_p \log_e \frac{T_2}{T_1} \right)$$

$$= 1.005(873 - 523) - 288 \left[0.287 \log_e \left(\frac{1}{7} \right) - 1.005 \log_e \left(\frac{523}{873} \right) \right]$$

$$= 351.75 - 288(-0.5585 + 0.5149) = 364.3 \text{ kJ/kg}$$

i.e., **Decrease in availability = 364.3 kJ/kg. (Ans.)**

(*ii*) **The maximum work,**

W_{max} = Change in availability = **364.3 kJ/kg. (Ans.)**

(*iii*) From steady flow energy equation,

$$Q + h_1 = W + h_2$$

$$W = (h_1 - h_2) + Q$$

$$= c_p\ (T_1 - T_2) + Q$$

$$= 1.005(873 - 523) + (-9) = 342.75 \text{ kJ/kg}$$

The irreversibility,

$$I = W_{max} - W$$

$= 364.3 - 342.75 =$ **21.55 kJ/kg. (Ans.)**

$$\left[\begin{array}{l} \text{Alternatively, } I = T_0\ (\Delta S_{system} + \Delta S_{surr.}) \\ \quad = 288 \left[1.005 + \log_e \left(\dfrac{523}{873} \right) - 0.287 \log_e \left(\dfrac{1}{7} \right) + \dfrac{9}{288} \right] \\ \quad = 288\ [-0.5149 + 0.5585 + 0.03125] \\ \quad = 21.55 \text{ kJ/kg.} \end{array} \right]$$

Example 6.19. *1 kg of air undergoes a polytropic compression from 1 bar and 290 K to 6 bar and 400 K. If the temperature and pressure of the surroundings are 290 K and 1 bar respectively, determine :*

(*i*) *The irreversibility ;*

(*ii*) *The effectiveness.*

Take for air : $c_p = 1.005$ *kJ/kg K,* $c_v = 0.718$ *kJ/kg K,* $R = 0.287$ *kJ/kg K.*

Solution. Mass of air, $m = 1$ kg

Initial temperature, $T_1 = T_0 = 290$ K

Final temperature, $T_2 = 400$ K

Initial pressure, $p_1 = p_0 = 1$ bar

Final pressure, $p_2 = 6$ bar.

(*i*) **The irreversibility, I :**

We know that reversible work,

$$W_{rev.} = \text{Change in internal energy} - T_0 \times \text{Change in entropy}$$

$$= (u_2 - u_1) - T_0\left[c_p \log_e\left(\frac{T_2}{T_1}\right) - R\log_e\left(\frac{p_2}{p_1}\right)\right]$$

$$= c_v\,(T_2 - T_1) - T_0\left[c_p \log_e\left(\frac{T_2}{T_1}\right) - R\log_e\left(\frac{p_2}{p_1}\right)\right]$$

$$= 0.718\,(400 - 290) - 290\left[1.005\log_e\left(\frac{400}{290}\right) - 0.287\log_e\left(\frac{6}{1}\right)\right]$$

$$= 78.98 - 290\,(0.3232 - 0.5142)$$

$= 134.37$ kJ/kg (–) [–ve sign indicates that work is done on the air]

Actual work is given by

$$W_{actual} = \frac{mR(T_1 - T_2)}{n-1} = \frac{R(T_1 - T_2)}{n-1} \text{ as } m = 1 \text{ kg.}$$

The index n for the compression is given by :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \quad \frac{400}{290} = \left(\frac{6}{1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad 1.379 = (6)^{\frac{n-1}{n}}$$

$$\log_e 1.379 = \left(\frac{n-1}{n}\right)\log_e 6 \quad \text{or} \quad 0.3213 = 1.7917\left(\frac{n-1}{n}\right)$$

$$\therefore \quad \frac{n-1}{n} = \frac{0.3213}{1.7917} = 0.1793 \quad \text{or} \quad n = 1.218$$

$$\therefore \quad W_{actual} = \frac{0.287\,(290 - 400)}{(1.218 - 1)} = -\,144.8 \text{ kJ}$$

(–ve sign means that the work is done on the air)

Now, **irreversibility** = $W_{rev} - W_{actual}$

$= -\,134.37 - (-\,144.8) =$ **10.43 kJ. (Ans.)**

(*ii*) **The effectiveness, $\in$:**

$$\in = \frac{W_{rev.}}{W_{actual}} = \frac{-\,134.37}{-\,144.8}$$ **= 0.928** or **92.8%. (Ans.)**

Example 6.20. *A flywheel whose moment of inertia is 0.62 kg m^2 rotates at a speed 2500 r.p.m. in a large heat insulated system, the temperature of which is 20°C.*

(*i*) *If the K.E. of the flywheel is dissipated as frictional heat at the shaft bearings which have a water equivalent of 1.9 kg, find the rise in the temperature of the bearings when the flywheel has come to rest.*

(*ii*) *Calculate the greatest possible amount of the above heat which may be returned to the flywheel as high-grade energy, showing how much of the original K.E. is now unavailable. What would be the final r.p.m. of the flywheel, if it is set in motion with this available energy ?*

Solution. Moment of inertia of the flywheel, I = 0.62 kg m^2

Initial angular velocity of the flywheel, $\omega_1 = \dfrac{2\pi N_1}{60} = \dfrac{2\pi \times 2500}{60}$ = 261.8 rad/s.

Temperature of insulated system, T_0 = 20 + 273 = 293 K

Water equivalent of shaft bearings = 1.9 kg

(*i*) Initial available energy of the flywheel,

$$(\text{K.E.})_{initial} = \frac{1}{2} I\omega_1^2$$

$$= \frac{1}{2} \times 0.62 \times (261.8)^2 = 2.12 \times 10^4 \text{ N.m} = 21.2 \text{ kJ.}$$

When this K.E. is dissipated as frictional heat, if Δt is the temperature rise of the bearings, we have

Water equivalent of bearings × rise in temperature = 21.2

i.e., $(1.9 \times 4.18)\ \Delta t = 21.2$

or $$\Delta t = \frac{21.2}{1.9 \times 4.18} = 2.67°\text{C}$$

Hence, **rise in temperature of bearings = 2.67°C. (Ans.)**

∴ **Final temperature of the bearings = 20 + 2.67 = 22.67°C.**

(*ii*) *The maximum amount of energy which may be returned to the flywheel as high-grade energy is*,

$$\text{A.E.} = 1.9 \times 4.18 \int_{293}^{295.67} \left(1 - \frac{293}{T}\right) dT$$

$$= 1.9 \times 4.18 \left[(295.67 - 293) - 293 \log_e \left(\frac{295.67}{293}\right)\right] = \mathbf{0.096\ kJ.}\ \ \textbf{(Ans.)}$$

The amount of energy rendered unavailable is

$$\text{U.E.} = (\text{A.E.})_{\text{initial}} - (\text{A.E.})_{\text{returnable as high grade energy}}$$

$$= 21.2 - 0.096 = 21.1 \text{ kJ.}$$

Since the amount of energy returnable to the flywheel is 0.096 kJ, if ω_2 is the final angular velocity, and the flywheel is set in motion with this energy, then

$$0.096 \times 10^3 = \frac{1}{2} \times 0.62 \times \omega_2^2$$

∴ $$\omega_2^2 = \frac{0.096 \times 10^3 \times 2}{0.62} = 309.67 \quad \text{or} \quad \omega_2 = 17.59 \text{ rad/s.}$$

If N_2 is the final r.p.m. of the flywheel, then

$$\omega_2 = \frac{2\pi N_2}{60} \quad \text{or} \quad N_2 = \frac{17.59 \times 60}{2\pi} = 168 \text{ r.p.m.}$$

i.e., **Final r.p.m. of the flywheel = 168 r.p.m. (Ans.)**

☞**Example 6.21.** *The air, in a steady flow, enters the system at a pressure of 8 bar and 180°C with a velocity of 80 m/s and leaves at 1.4 bar and 20°C with a velocity of 40 m/s. The temperature of the surroundings is 20°C and pressure is 1 bar. Determine :*

(*i*) *Reversible work and actual work assuming the process to be adiabatic ;*

(*ii*) *Irreversibility and effectiveness of the system on the basis of 1 kg of air flow.*

Take for air : $c_p = 1.005$ *kJ/kg K ;* $R = 0.287$ *kJ/kg K.*

Solution. Initial pressure of air, $p_1 = 8$ bar

Initial temperature of air, $T_1 = 180 + 273 = 453$ K

Final pressure of air, $p_2 = 1.4$ bar

Final temperature, $T_2 = T_0 = 20 + 273 = 293$ K

Surroundings' pressure, $p_0 = 1$ bar

Mass of air = 1 kg

Initial velocity of air, $C_1 = 80$ m/s

Final velocity of air, $C_2 = 40$ m/s.

(*i*) **Reversible work and actual work :**

Availability of air at the inlet

$$= (h_1 - h_0) - T_0 (s_1 - s_0) + \frac{C_1^2}{2}$$

$$= c_p (T_1 - T_0) - T_0 (s_1 - s_0) + \frac{C_1^2}{2}$$

$$(s_1 - s_0) = c_p \log_e \left(\frac{T_1}{T_0}\right) - R \log_e \left(\frac{p_1}{p_0}\right)$$

$$= 1.005 \log_e \left(\frac{453}{293}\right) - 0.287 \log_e \left(\frac{8}{1}\right)$$

$$= 0.437 - 0.596 = -0.159 \text{ kJ/kg K}$$

∴ Availability of air at the inlet

$$= 1.005 (453 - 293) - 293 (-0.159) + \frac{80^2}{2 \times 10^3}$$

$$= 160.8 + 46.58 + 3.2 = 210.58 \text{ kJ}$$

Availability at the exit

$$= (h_2 - h_0) - T_0 (s_2 - s_0) + \frac{C_2^2}{2}$$

$$= -T_0 (s_2 - s_0) + \frac{C_2^2}{2}$$

as $h_2 = h_0$ because $T_2 = T_0 = 293$ K

Now $s_2 - s_0 = -R \log_e \left(\frac{p_2}{p_0}\right) = -0.287 \log_e \left(\frac{1.4}{1}\right) = -0.09656$ kJ/kg K

∴ Availability at the exit

$$= -293 (-0.09656) + \frac{40^2}{2 \times 10^3} = 29.09 \text{ kJ/kg}$$

Reversible/theoretical work which must be available,

$$W_{rev} = 210.58 - 29.09 = \mathbf{181.49\ kJ.\ (Ans.)}$$

Actual work developed can be calculated by using the energy equation for adiabatic steady flow process as follows :

$$W_{actual} = (h_1 - h_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)$$

$$= c_p\,(T_1 - T_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)$$

$$= 1.005(453 - 293) + \left(\frac{80^2 - 40^2}{2 \times 10^3}\right)$$

$$= 160.8 + 2.4 = \mathbf{163.2\ kJ/kg.\quad (Ans.)}$$

(*ii*) **Irreversibility and effectiveness :**

Irreversibility, $I = W_{rev} - W_{actual}$

$$= 181.49 - 163.2 = \mathbf{18.29\ kJ/kg.\quad (Ans.)}$$

Effectiveness, $\in = \dfrac{W_{actual}}{W_{rev}} = \dfrac{163.2}{181.49}$

$$= \mathbf{0.899} \text{ or } \mathbf{89.9\%}.\quad \mathbf{(Ans.)}$$

Example 6.22. *Steam expands adiabatically in a turbine from 20 bar, 400°C to 4 bar, 250°C. Calculate :*

(*i*) *The isentropic efficiency of the process ;*

(*ii*) *The loss of availability of the system assuming an atmospheric temperature of 20°C ;*

(*iii*) *The effectiveness of the process ;*

The changes in K.E. and P.E. may be neglected.

Solution. Initial pressure of steam, $p_1 = 20$ bar

Initial temperature of steam, $t_1 = 400°C$

Final pressure of steam, $p_2 = 4$ bar

Final temperature of steam, $t_2 = 250°C$

Atmospheric temperature, $= 20°C$ (= 293 K).

Initial state 1 : 20 bar, 400°C ; From steam tables,

$$h_1 = 3247.6 \text{ kJ/kg} ;\ s_1 = 7.127 \text{ kJ/kg K}$$

Final state 2 : 4 bar 250°C ; From steam tables,

$$h_2' = 2964.2 \text{ kJ/kg},\ s_2' = 7.379 \text{ kJ/kg K}$$

The process is shown as 1 to 2′ in Fig. 6.14

$$s_1 = s_2 = 7.127 \text{ kJ/kg K}$$

Hence, interpolating,

$$h_2 = 2752.8 + \left(\frac{7.127 - 6.930}{7.171 - 6.930}\right)(2860.5 - 2752.8)$$

$$= 2752.8 + \frac{0.197}{0.241} \times 107.7 = 2840.8 \text{ kJ/kg.}$$

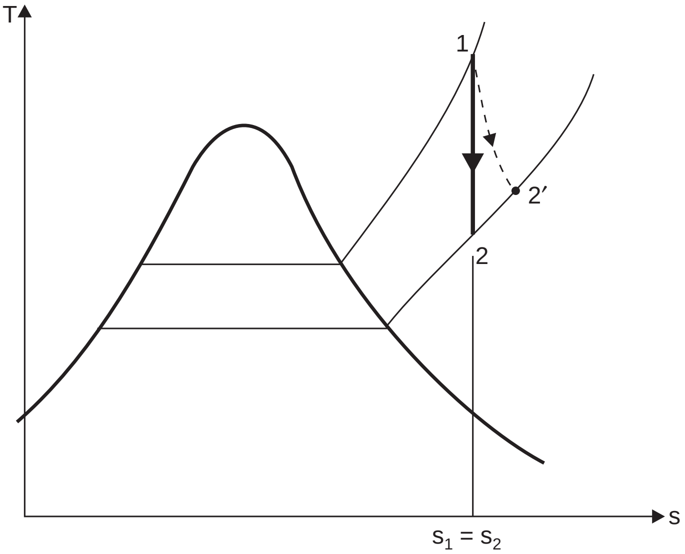


Fig. 6.14

(*i*) **Isentropic efficiency :**

$$\eta_{isen.} = \frac{\text{Actual work output}}{\text{Isentropic work}}$$

$$= \frac{h_1 - h_2'}{h_1 - h_2} = \frac{3247.6 - 2964.2}{3247.6 - 2840.8}$$

$$= \frac{283.4}{406.8} = \mathbf{0.6966} \text{ or } \mathbf{69.66\%}. \quad \textbf{(Ans.)}$$

(*ii*) **Loss of availability :**

Loss of availability $= b_1 - b_2$

$= h_1 - h_2' + T_0 (s_2' - s_1')$

$= 3247.6 - 2964.2 + 293 (7.379 - 7.127)$

$= 283.4 + 73.83 =$ **357.23 kJ/kg. (Ans.)**

(*iii*) **Effectiveness :**

Effectiveness, $$\in = \frac{W}{b_1 - b_2'} = \frac{h_1 - h_2'}{b_1 - b_2'}$$

$$= \frac{3247.6 - 2964.2}{357.9} = \mathbf{0.7918 \text{ or } 79.18\%}. \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. '*Available energy*' is the maximum portion of the energy which could be converted into useful work by ideal processes which reduce the system to a dead state.
2. The theoretical maximum amount of work which can be obtained from a system at any state p_1 and T_1 when operating with a reservoir at the constant pressure and temperature p_0 and T_0 is called '*availability*'.

3. Energy is said to be *degraded* each time it flows through a finite temperature difference. That is, why the second law of thermodynamics is sometimes called the *law of the degradation of energy*, and energy is said to '*run down hill*'.

4. In non-flow systems :

Maximum work available,

$$\begin{aligned} W_{max} &= (u_1 - u_0) - T_0(s_1 - s_0) - p_0(v_0 - v_1) \\ &= (u_1 + p_0v_1 - T_0s_1) - (u_0 + p_0v_0 - T_0s_0) \\ &= a_1 - a_0 \qquad \text{...per unit mass} \end{aligned}$$

The property $a = u + p_0v - T_0s$ is called the *non-flow availability function*.

5. In steady-flow systems :

Maximum work available,

$$\begin{aligned} W_{max} &= (h_1 - T_0s_1) - (h_0 - T_0s_0) \\ &= b - b_0 \qquad \text{...per unit mass} \end{aligned}$$

The property, $b = h - T_0s$ is called the *steady-flow availability function*.

6. It may be noted that Gibb's function $g = (h - Ts)$ is a property of the system where availability function $a = u + p_0v - T_0s$ is a composite property of the system and surroundings.

Again,
$$\begin{aligned} a &= u + p_0v - T_0s \\ b &= u + pv - T_0s \\ g &= u + pv - Ts \end{aligned}$$

When state 1 proceeds to dead state (zero state)

$$a = b = g.$$

7. The actual work which a system does is always less than the idealized reversible work, and the difference between the two is called the *irreversibility of the process*. This is also sometimes referred to as *degradation* or *dissipation*.

Effectiveness is defined as the ratio of actual useful work to the maximum useful work.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer :

1. Gibb's function is expressed as

(*a*) $(u + pv - Ts)$	(*b*) $(u + pv - Tds)$
(*c*) $(u + pdv - Tds)$	(*d*) $(u + pv - sdT)$.

2. Availability function is expressed as

(*a*) $a = (u + p_0v - T_0s)$	(*b*) $a = (u + p_0dv + T_0ds)$
(*c*) $a = (du + p_0dv - T_0ds)$	(*d*) $a = (u + p_0v + T_0s)$.

3. To increase work capacity of energy transferred by heat transfer from high temperature to low temperature

(*a*) lower temperature should be lowered keeping temperature difference same

(*b*) higher temperature should be increased keeping temperature difference same

(*c*) temperature difference should be increased (*d*) temperature difference should be decreased.

4. Helmholtz function is expressed as

(*a*) $(u - Ts)$	(*b*) $(h - Ts)$
(*c*) $(- sdT + vdp)$	(*d*) $(u + pv)$.

5. If a heat source at temperature T_1 transfers heat to a system at temperature $T_2 (T_1 > T_2)$, state which of the following statements is *not true* ?

(*a*) Δ_{system} decreases	(*b*) Δ_{source} decreases
(*c*) $(\Delta_{\text{system}} + \Delta_{\text{source}})$ decreases	(*d*) $(\Delta_{\text{system}} + \Delta_{\text{source}})$ increases.

Answers

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*a*) **5.** (*d*).

THEORETICAL QUESTIONS

1. Explain the concept of available and unavailable energy. When does the system become dead ?
2. Define the term 'availability'.
3. Is the availability function same for a non-flow and a flow process ?
4. Define availability function and find the relationship between availability function and change in availability.
5. How are the concepts of entropy and unavailable energy related to each other ?
6. Derive an expression for availability in non-flow systems.
7. Derive an expression for availability in steady flow systems.
8. Differentiate between availability function and Gibb's energy function.
9. Derive an expression for decrease in available energy when heat is transferred through a finite temperature difference.
10. Derive a general expression for irreversibility in (*i*) non-flow process, (*ii*) steady flow process.
11. What is the effectiveness of a system and how does it differ from efficiency ?

UNSOLVED EXAMPLES

1. A system receives 10000 kJ of heat at 500 K from a source at 1000 K. The temperature of the surroundings is 300 K. Assuming that the temperature of the system and source remains constant during heat transfer, find :

(*i*) The entropy production due to above mentioned heat transfer ;

(*ii*) Decrease in available energy. [**Ans.** (*i*) 10 kJ/K ; (*ii*) 3000 kJ]

2. In a power station, saturated steam is generated at 252°C by transferring heat from the hot gases generated in the combustion chamber. The gases are cooled from 1100°C to 550°C during transferring the heat for steam generation. Determine the increase in total entropy of the combined system of gas and steam and increase in unavailable energy on the basis of one kg of steam generated. Assume water enters the boiler at saturated condition and leaves as saturated steam. [**Ans.** 1.99 kJ/K ; 597 kJ/kg of steam formed]

3. Air at 15°C is to be heated to 40°C by mixing it in steady flow with a quantity of air at 90°C. Assuming that the mixing process is adiabatic and neglecting changes in kinetic and potential energy, calculate the ratio of the mass flow of air initially at 90°C to that initially at 15°C. Calculate also the effectiveness of the heating process, if the atmospheric temperature is 15°C. [**Ans.** 0.5, 0.327 or 32.7%]

4. A liquid of specific heat 6.3 kJ/kg K is heated at approximately constant pressure from 15°C to 70°C by passing it through tubes which are immersed in a furnace. The furnace temperature is constant at 1400°C. Calculate the effectiveness of the heating process when the atmospheric temperature is 10°C.

[**Ans.** 0.121 or 12.1%]

5. 500 kJ of heat from an infinite source at 1000 K is supplied to 2 kg of gas initially at 2 bar and 350 K in a closed tank. Find the loss in available energy due to above heat transfer. Take : c_v (gas) = 0.8 kJ/kg K and surrounding temperature = 300 K. [**Ans.** 157.5 kJ]

6. In an heat exchanger of parallel flow type, water enters at 60°C and leaves at 80°C while oil of specific gravity 0.8 enters at 250°C and leaves at 100°C. The specific heat of oil is 2.5 kJ/kg K and surrounding temperature is 300 K. Determine the loss in availability on the basis of one kg of oil flow per second.

[**Ans.** – 59.9 kJ]

7. 1 kg of ice at 0°C is mixed with 10 kg of water at 30°C. Determine the net increase in the entropy and unavailable energy when the system reaches common temperature. Assume that surrounding temperature is 10°C. Take, specific heat of water = 4.18 kJ/kg K ; specific heat of ice = 2.1 kJ/kg K ; latent heat of ice = 333.5 kJ/kg. [**Ans.** 0.114 kJ/K, 32.30 kJ]

8. 1000 kJ of heat leaves hot gases at 1400°C from a fire box of a boiler and goes to steam at 250°C. Atmospheric temperature is 20°C. Divide the energy into available and unavailable portions :

(*i*) As it leaves the hot gases.

(*ii*) As it enters the steam. [**Ans.** (*i*) 825 kJ, 175 kJ ; (*ii*) 440 kJ, 580 kJ]

9. In a certain process, a vapour, while condensing at 420°C, transfers heat to water evaporating at 250°C. The resulting steam is used in a power cycle which rejects heat at 35°C. What is the fraction of the available energy in the heat transferred from the process vapour at 420°C that is lost due to the irreversible heat transfer at 250°C. [**Ans.** 0.26]

10. In a steam boiler, hot gases from a fire transfer heat to water which vapourizes at constant temperature. In certain case, the gases are cooled from 1100°C to 550°C while the water evaporates at 220°C. The specific heat of gases is 1.005 kJ/kg K, and the latent heat of water at 220°C is 1858.5 kJ/kg. All the heat transferred from the gases goes to the water. How much does the total entropy of the combined system of gas and water increase as a result of irreversible heat transfer ? Obtain the result on the basis of 1 kg of water evaporated. If the temperature of the surroundings is 30°C find the increase in unavailable energy due to irreversible heat transfer. [**Ans.** 2.045 kJ/K, 620 kJ]

11. Calculate the unavailable energy in 40 kg of water at 75°C with respect to the surroundings at 5°C, the pressure being 1 atmosphere. [**Ans.** 10420 kJ]

12. Calculate the decrease in available energy when 25 kg of water at 95°C mixes with 35 kg of water at 35°C, the pressure being taken as constant and the temperature of the surroundings being 15°C.

Take c_p of water = 4.18 kJ/kg K. [**Ans.** 281.81 kJ]

13. 2 kg of air at 5 bar, 80°C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 1 bar, 5°C. For this process, determine :

(*i*) The maximum work ;

(*ii*) The change in availability ;

(*iii*) The irreversibility.

For air take : $c_v = 0.718$ kJ/kg K, $u = c_vT$, where c_v is constant and $pV = mRT$, where p is in bar, V volume in m^3, m mass in kg, R is constant equal to 0.287 kJ/kg K, and T temperature in K.

[**Ans.** (*i*) 122.72 kJ ; (*ii*) 82.2 kJ ; (*iii*) 15.2 kJ]

14. One kg of air at a pressure p, and temperature 900 K is mixed with 1 kg of air at the same pressure and 500 K.

Determine the loss in availability if the surrounding temperature is 300 K. [**Ans.** 54 kJ]

15. 10 kg of water is heated in an insulated tank by a churning process from 300 K to 350 K. Determine the loss in availability for the process if the surrounding temperature is 300 K. [**Ans.** 1968 kJ]

16. A closed system contains 10 kg of air at 600 K and 5 bar. Determine the availability of the system if the surrounding pressure and temperature are 1 bar and 300 K respectively.

If the air is cooled at constant pressure to the atmospheric temperature, determine the availability and effectiveness. [**Ans.** 1464 kJ ; 921 kJ ; 0.63]

17. In a turbine air expands from 5 bar, 520°C to 1 bar, 300°C. During expansion 10 kJ/kg of heat is lost to the surroundings which is at 0.98 bar, 20°C. Neglecting kinetic and potential energy changes, determine per kg of air :

(*i*) The decrease in availability ;

(*ii*) The maximum work ;

(*iii*) The irreversibility.

For air take : $c_p = 1.005$ kJ/kg K ; $h = c_pT$, where c_p is constant.

[**Ans.** (*i*) 260.7 kJ/kg ; (*ii*) 260.7 kJ/kg ; (*iii*) 49.6 kJ/kg]

18. A centrifugal air compressor compresses air at the rate of 20 kg/min from 1 bar to 2 bar. The temperature increases from 20°C to 120°C during the compression. Determine actual and minimum power required to run the compressor. The surrounding air temperature is 20°C.

Neglect the heat interaction between the compressor and surroundings and changes in potential and kinetic energy. [**Ans.** 24.2 kW]

19. 1 kg of air is compressed polytropically from 1 bar and 300 K to 7 bar and 380 K. Determine the irreversibility and effectiveness, assuming temperature and pressure as 300 K and 1 bar.

Take for air : c_p = 1.005 kJ/kg K, c_p = 0.718 kJ/kg K, R = 0.287 kJ/kg K. [**Ans.** 1.25 kJ ; 0.924]

20. The moment of inertia of a flywheel is 0.54 kg-m^2 and it rotates at a speed of 3000 r.p.m. in a large heat insulated system, the temperature of which is 15°C. If the kinetic energy of the flywheel is dissipated as frictional heat at the shaft bearings which have a water equivalent of 2 kg, find the rise in the temperature of the bearings when the flywheel has come to rest.

Calculate the greatest possible amount of the above heat which may be returned to the flywheel as high-grade energy, showing how much of the original kinetic energy is now unavailable. What would be the final r.p.m. of the flywheel, if it is set in motion with this available energy ?

[**Ans.** 3.19°C ; 0.1459 kJ ; 222 r.p.m.]

21. In a steady flow air enters the system at a pressure of 10 bar and 200°C with a velocity of 100 m/s and leaves at 1.5 bar and 25°C with a velocity of 50 m/s. The temperature of the surroundings is 25°C and pressure is 1 bar. Determine reversible work and actual work assuming the process to be adiabatic.

Determine also the irreversibility and effectiveness of the system on the basis of one kg of air flow.

Take for air : c_p = 1 kJ/kg K ; R = 287 J/kg K. [**Ans.** 200.65 kJ ; 178.75 kJ/kg, 21.90 kJ/kg ; 0.894]

7

Thermodynamic Relations

7.1. General aspects. 7.2. Fundamentals of partial differentiation. 7.3. Some general thermodynamic relations. 7.4. Entropy equations (*Tds* equations). 7.5. Equations for internal energy and enthalpy. 7.6. Measurable quantities : Equation of state, co-efficient of expansion and compressibility, specific heats, Joule-Thomson co-efficient 7.7. Clausius-Claperyon equation—Highlights—Objective Type Questions—Exercises.

7.1. GENERAL ASPECTS

In this chapter, some important thermodynamic relations are deduced ; principally those which are useful when tables of properties are to be compiled from limited experimental data, those which may be used when calculating the work and heat transfers associated with processes undergone by a liquid or solid. It should be noted that the relations only apply to a substance in the solid phase when the stress, *i.e.* the pressure, is uniform in all directions ; if it is not, a single value for the pressure cannot be alloted to the system as a whole.

Eight properties of a system, namely *pressure* (p), *volume* (v), *temperature* (T), *internal energy* (u), *enthalpy* (h), *entropy* (s), *Helmholtz function* (f) and *Gibbs function* (g) have been introduced in the previous chapters. h, f and g are sometimes referred to as **thermodynamic potentials.** Both f and g are useful when considering chemical reactions, and the former is of fundamental importance in statistical thermodynamics. The Gibbs function is also useful when considering processes involving a change of phase.

Of the above eight properties only the first three, *i.e.*, p, v and T are directly measurable. We shall find it convenient to introduce other combination of properties which are relatively easily measurable and which, together with measurements of p, v and T, enable the values of the remaining properties to be determined. These combinations of properties might be called '**thermodynamic gradients**' ; they are all defined as the rate of change of one property with another while a third is kept constant.

7.2. FUNDAMENTALS OF PARTIAL DIFFERENTIATION

Let three variables are represented by x, y and z. Their functional relationship may be expressed in the following forms :

$$f(x, y, z) = 0 \quad \text{...}(i)$$

$$x = x(y, z) \quad \text{...}(ii)$$

$$y = y(x, z) \quad \text{...}(iii)$$

$$z = z(x, y) \quad \text{...}(iv)$$

Let x is a function of two independent variables y and z

$$x = x(y, z) \quad \text{...(7.1)}$$

Then the differential of the dependent variable x is given by

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz \qquad ...(7.2)$$

where dx is called an **exact differential**.

If $\left(\frac{\partial x}{\partial y}\right)_z = M$ and $\left(\frac{\partial x}{\partial z}\right)_y = N$

Then $$dx = Mdy + Ndz \qquad ...(7.3)$$

Partial differentiation of M and N with respect to z and y, respectively, gives

$$\frac{\partial M}{\partial z} = \frac{\partial^2 x}{\partial y \partial z} \quad \text{and} \quad \frac{\partial N}{\partial y} = \frac{\partial^2 x}{\partial z \partial y}$$

or $$\frac{\partial M}{\partial z} = \frac{\partial N}{\partial y} \qquad ...(7.4)$$

dx is a perfect differential when eqn. (7.4) is satisfied for any function x.

Similarly if $y = y(x, z)$ and $z = z(x, y)$ ...(7.5)

then from these two relations, we have

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz \qquad ...(7.6)$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \qquad ...(7.7)$$

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x \left[\left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy\right]$$

$$= \left[\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y\right] dx + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$= \left[\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y\right] dx + dy$$

or $$\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = 0$$

or $$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -\left(\frac{\partial y}{\partial x}\right)_z$$

or $$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x = -1 \qquad ...(7.8)$$

In terms of p, v and T, the following relation holds good

$$\boxed{\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial v}{\partial T}\right)_p = -1} \qquad ...(7.9)$$

7.3. SOME GENERAL THERMODYNAMIC RELATIONS

The first law applied to a closed system undergoing a reversible process states that

$$dQ = du + pdv$$

According to second law,

$$ds = \left(\frac{dQ}{T}\right)_{\text{rev.}}$$

Combining these equations, we get

$$Tds = du + pdv$$

or
$$du = Tds - pdv \quad \text{...(7.10)}$$

The properties *h, f* and *g* may also be put in terms of T, s, p and v as follows :

$$dh = du + pdv + vdp = Tds + vdp$$

Helmholtz free energy function,

$$df = du - Tds - sdT \quad \text{...(7.11)}$$

$$= -\ pdv - sdT \quad \text{...(7.12)}$$

Gibb's free energy function,

$$dg = dh - Tds - sdT = vdp - sdT \quad \text{...(7.13)}$$

Each of these equations is a *result of the two laws of thermodynamics.*

Since *du, dh, df* and *dg* are the *exact differentials,* we can express them as

$$du = \left(\frac{\partial u}{\partial s}\right)_v ds + \left(\frac{\partial u}{\partial v}\right)_s dv,$$

$$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_s dp,$$

$$df = \left(\frac{\partial f}{\partial v}\right)_T dv + \left(\frac{\partial f}{\partial T}\right)_v dT,$$

$$dg = \left(\frac{\partial g}{\partial p}\right)_T dp + \left(\frac{\partial g}{\partial T}\right)_p dT.$$

Comparing these equations with (7.10) to (7.13) we may equate the corresponding co-efficients. For example, from the two equations for du, we have

$$\left(\frac{\partial u}{\partial s}\right)_v = T \text{ and } \left(\frac{\partial u}{\partial v}\right)_s = -\ p$$

The complete group of such relations may be summarised as follows :

$$\left(\frac{\partial u}{\partial s}\right)_v = T = \left(\frac{\partial h}{\partial s}\right)_p \quad \text{...(7.14)}$$

$$\left(\frac{\partial u}{\partial v}\right)_s = -\ p = \left(\frac{\partial f}{\partial v}\right)_T \quad \text{...(7.15)}$$

$$\left(\frac{\partial h}{\partial p}\right)_s = v = \left(\frac{\partial g}{\partial p}\right)_T \quad \text{...(7.16)}$$

$$\left(\frac{\partial f}{\partial T}\right)_v = -\ s = \left(\frac{\partial g}{\partial T}\right)_p \quad \text{...(7.17)}$$

Also,

$$\boxed{\begin{aligned}\left(\frac{\partial T}{\partial v}\right)_s &= -\left(\frac{\partial p}{\partial s}\right)_v & \quad ...(7.18)\\ \left(\frac{\partial T}{\partial p}\right)_s &= \left(\frac{\partial v}{\partial s}\right)_p & \quad ...(7.19)\\ \left(\frac{\partial p}{\partial T}\right)_v &= \left(\frac{\partial s}{\partial v}\right)_T & \quad ...(7.20)\\ \left(\frac{\partial v}{\partial T}\right)_p &= -\left(\frac{\partial s}{\partial p}\right)_T & \quad ...(7.21)\end{aligned}}$$

The equations (7.18) to (7.21) are known as **Maxwell relations.**

It must be emphasised that eqns. (7.14) to (7.21) do not refer to a process, but simply express relations between properties which must be satisfied when any system is in a state of equilibrium. Each partial differential co-efficient can itself be regarded as a property of state. The state may be defined by a point on a three dimensional surface, the surface representing all possible states of stable equilibrium.

7.4. ENTROPY EQUATIONS (Tds Equations)

Since entropy may be expressed as a function of any other two properties, *e.g.* temperature T and specific volume v,

$$s = f(T, v)$$

i.e.,

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

or

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_v dT + T\left(\frac{\partial s}{\partial v}\right)_T dv \quad ...(7.22)$$

But for a reversible constant volume change

$$dq = c_v\,(dT)_v = T(ds)_v$$

or

$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v \quad ...(7.23)$$

But,

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v \quad \text{[Maxwell's eqn. (7.20)]}$$

Hence, substituting in eqn. (7.22), we get

$$\boxed{Tds = c_v dT + T\left(\frac{\partial p}{\partial T}\right)_v dv} \quad ...(7.24)$$

This is known as the *first form of entropy equation* or the **first Tds equation.**

Similarly, writing

$$s = f(T, p)$$

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp \quad ...(7.25)$$

where $c_p = T \left(\frac{\partial s}{\partial T}\right)_p$ ...(7.26)

Also $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$ [Maxwell's eqn. (7.21)]

whence, substituting in eqn. (7.25)

$$\boxed{Tds = c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp} \quad \text{...(7.27)}$$

This is known as the *second form of entropy equation* or the **second Tds equation.**

7.5. EQUATIONS FOR INTERNAL ENERGY AND ENTHALPY

(*i*) Let $u = f(T, v)$

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv = c_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv \quad \text{...(7.28)}$$

To evaluate $\left(\frac{\partial u}{\partial v}\right)_T$ let $u = f(s, v)$

Then $du = \left(\frac{\partial u}{\partial s}\right)_v ds + \left(\frac{\partial u}{\partial v}\right)_s dv$

or $\left(\frac{\partial u}{\partial v}\right)_T = \left(\frac{\partial u}{\partial s}\right)_v \left(\frac{\partial s}{\partial v}\right)_T + \left(\frac{\partial u}{\partial v}\right)_s$

But $\left(\frac{\partial u}{\partial s}\right)_v = T,\ \left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial s}{\partial T}\right)_v, \left(\frac{\partial u}{\partial v}\right)_s = -p$

Hence $\boxed{\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p}$ ...(7.29)

This is sometimes called the **energy equation.**

From equation (7.28), we get

$$\boxed{du = c_v dT + \left\{T\left(\frac{\partial p}{\partial T}\right)_v - p\right\} dv} \quad \text{...(7.30)}$$

(*ii*) To evaluate *dh* we can follow similar steps as under

$$h = f(T, p)$$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$= c_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp \quad \text{...(7.31)}$$

To find $\left(\frac{\partial h}{\partial p}\right)_T$; let $h = f(s, p)$

Then,
$$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_s dp$$

$$\left(\frac{\partial h}{\partial p}\right)_T = \left(\frac{\partial h}{\partial s}\right)_p \left(\frac{\partial s}{\partial p}\right)_T + \left(\frac{\partial h}{\partial p}\right)_s$$

But
$$\left(\frac{\partial h}{\partial s}\right)_p = T, \left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial p}\right)_p, \left(\frac{\partial h}{\partial p}\right)_s = v$$

Hence
$$\boxed{\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p} \quad ...(7.32)$$

From eqn. (7.31), we get

$$\boxed{dh = c_p\, dT + \left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\} dp} \quad ...(7.33)$$

7.6. MEASURABLE QUANTITIES

Out of eight thermodynamic properties, as earlier stated, only p, v and T are directly measurable. Let us now examine the information that can be obtained from measurements of these primary properties, and then see what other easily measurable quantities can be introduced.

The following will be discussed :

(*i*) Equation of state
(*ii*) Co-efficient of expansion and compressibility
(*iii*) Specific heats
(*iv*) Joule-Thomson co-efficient.

7.6.1. Equation of State

Let us imagine a series of experiments in which the volume of a substance is measured over a range of temperatures while the pressure is maintained constant, this being repeated for various pressures. The results might be represented graphically by a three-dimensional surface, or by a family of constant pressure lines on a v-T diagram. It is useful if an equation can be found to express the relation between p, v and T, and this can always be done over a limited range of states. No single equation will hold for all phases of a substance, and usually more than one equation is required even in one phase if the accuracy of the equation is to match that of the experimental results. Equations relating p, v and T are called *equations of state* or *characteristic equations.* Accurate equations of state are usually complicated, a typical form being

$$pv = A + \frac{B}{v} + \frac{C}{v^2} + ...$$

where A, B, C, ... are functions of temperature which differ for different substances.

An equation of state of a particular substance is an empirical result, and *it cannot be deduced from the laws of thermodynamics.* Nevertheless the general form of the equation may be predicted from hypotheses about the microscopic structure of matter. This type of prediction has been developed to a high degree of precision for gases, and to a lesser extent for liquids and solids. The simplest postulates about the molecular structure of gases lead to the concept of the perfect gas which has the equation of state $pv = RT$. Experiments have shown that the behaviour of real gases at low pressure with high temperature agrees well with this equation.

7.6.2. Co-efficient of Expansion and Compressibility

From p-v-T measurements, we find that an equation of state is not the only useful information which can be obtained. When the experimental results are plotted as a series of constant pressure lines on a v-T diagrams, as in Fig. 7.1 (a), the slope of a constant pressure line at any given state is $\left(\frac{\partial v}{\partial T}\right)_p$. If the gradient is divided by the volume at that state, we have a value of a property of the substance called its co-efficient of cubical expansion β. That is,

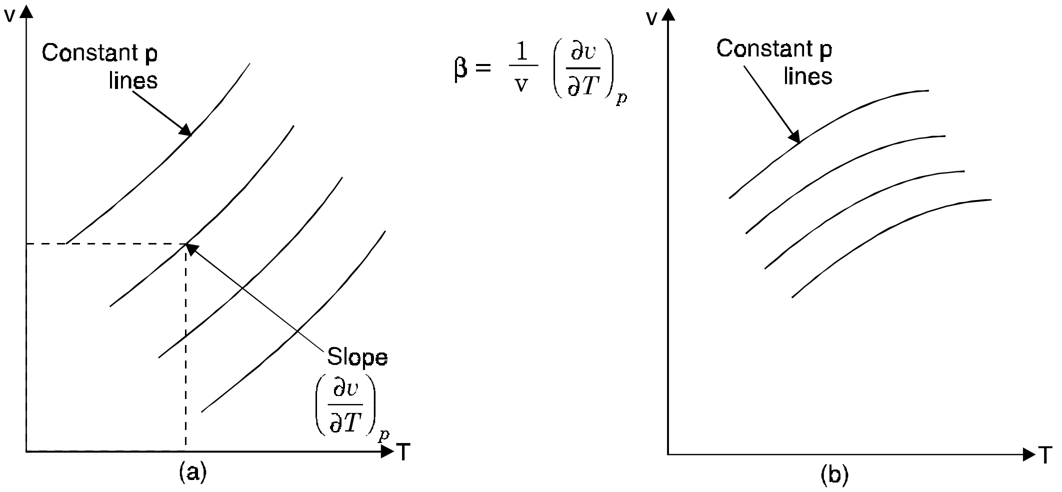


Fig. 7.1. Determination of co-efficient of expansion from p-v-T data.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \qquad ...(7.34)$$

Value of β can be tabulated for a range of pressures and temperatures, or plotted graphically as in Fig. 7.2 (b). For solids and liquids over the normal working range of pressure and temperature, the variation of β is small and can often be neglected. In tables of physical properties β is usually quoted as an average value over a small range of temperature, the pressure being atmospheric. This average co-efficient may be symbolised by β and it is defined by

$$\beta = \frac{v_2 - v_1}{v_1 (T_2 - T_1)} \qquad ...(7.35)$$

Fig. 7.2 (a) can be replotted to show the variation of volume with pressure for various constant values of temperature. In this case, the gradient of a curve at any state is $\left(\frac{\partial v}{\partial p}\right)_T$. When this gradient is divided by the volume at that state, we have a property known as the *compressibility K of the substance*. Since this gradient is always *negative, i.e., the volume of a substance always decreases* with *increase of pressure* when the *temperature is constant*, the compressibility is usually made a positive quantity by defining it as

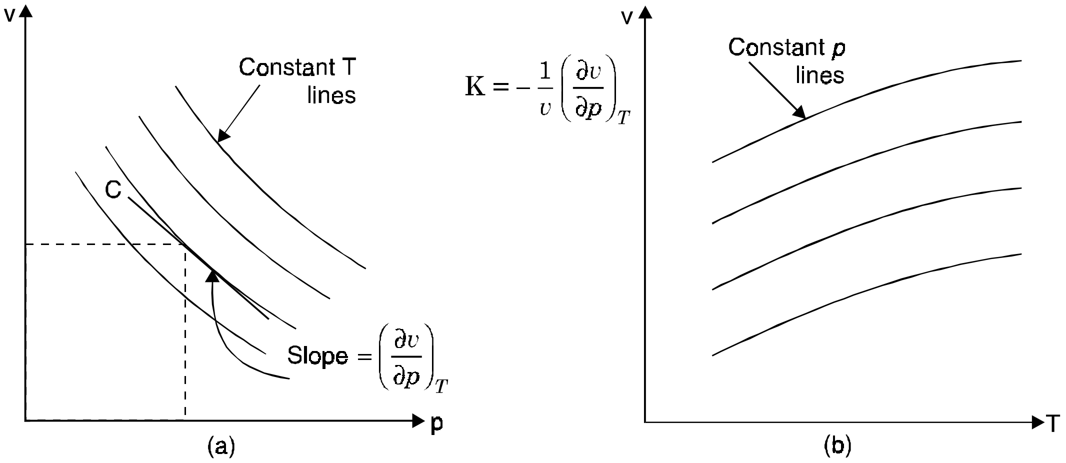


Fig. 7.2. Determination of compressibility from p-T data.

$$K = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \qquad ...(7.36)$$

K can be regarded as a constant for many purposes for solids and liquids. In tables of properties it is often quoted as an average a value over a small range of pressure at atmospheric temperature, *i.e.,*

$$\overline{K} = -\frac{v_2 - v_1}{v_1(p_2 - p_1)}$$

When β and K are known, we have

$$\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_p \left(\frac{\partial v}{\partial p}\right)_T = -1$$

Since $$\left(\frac{\partial v}{\partial T}\right)_p = \beta v \quad \text{and} \quad \left(\frac{\partial v}{\partial p}\right)_T = -Kv,$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{\beta}{K} \qquad ...(7.37)$$

When the equation of state is known, the co-efficient of cubical expansion and compressibility can be found by differentiation. For a perfect gas, for example, we have

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \quad \text{and} \quad \left(\frac{\partial v}{\partial p}\right)_T = \frac{RT}{p^2}$$

Hence $$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{pv} = \frac{1}{T},$$

and $$K = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = \frac{RT}{p^2 v} = \frac{1}{p} .$$

7.6.3. Specific Heats

Following are the three differential co-efficients which can be relatively easily determined experimentally.

Consider the first quantity $\left(\frac{\partial u}{\partial T}\right)_v$. During a process at constant volume, the first law informs us that an increase of internal energy is equal to heat supplied. If a calorimetric experiment is conducted with a known mass of substance at constant volume, the quantity of heat Q required to raise the temperature of unit mass by ΔT may be measured. We can then write : $\left(\frac{\Delta u}{\Delta T}\right)_v = \left(\frac{Q}{\Delta T}\right)_v$. The quantity obtained this way is known as the mean *specific heat at constant volume* over the temperature range ΔT. It is found to vary with the conditions of the experiment, *i.e.,* with the temperature range and the specific volume of the substance. As the temperature range is reduced the value approaches that of $\left(\frac{\partial u}{\partial T}\right)_v$, and the *true specific heat at constant volume* is defined by $c_v = \left(\frac{\partial u}{\partial T}\right)_v$. This is a property of the substance and in general its value varies with the state of the substance, *e.g.,* with temperature and pressure.

According to first law of thermodynamics the heat supplied is equal to the increase of enthalpy during a reversible constant pressure process. Therefore, a calorimetric experiment carried out with a substance at constant pressure gives us, $\left(\frac{\Delta h}{\Delta T}\right)_p = \left(\frac{Q}{\Delta T}\right)_p$ which is the mean specific heat at constant pressure. As the range of temperature is made infinitesimally small, this becomes the rate of change of enthalpy with temperature at a particular state defined by T and p, and this is true *specific heat at constant pressure* defined by $c_p = \left(\frac{\partial h}{\partial T}\right)_p$. c_p also varies with the state, *e.g.*, with pressure and temperature.

The description of experimental methods of determining c_p and c_v can be found in texts on physics. When solids and liquids are considered, it is not easy to measure c_v owing to the stresses set up when such a substance is prevented from expanding. However, a relation between c_p, c_v, β and K can be found as follows, from which c_v may be obtained if the remaining three properties have been measured.

The First Law of Thermodynamics, for a reversible process states that

$$dQ = du + p\,dv$$

Since we may write $u = \phi(T, v)$, we have

$$du = \left(\frac{du}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$\therefore \qquad dQ = \left(\frac{\partial u}{\partial T}\right)_v dT + \left\{p + \left(\frac{\partial u}{\partial v}\right)_T\right\} dv = c_v\,dT + \left\{p + \left(\frac{\partial u}{\partial v}\right)_T\right\} dv$$

This is true for any reversible process, and so, for a reversible constant pressure process,

$$dQ = c_p(dT)_p = c_v(dT)_p + \left\{p + \left(\frac{\partial u}{\partial v}\right)_T\right\} (dv)_p$$

Hence

$$c_p - c_v = \left\{p + \left(\frac{\partial u}{\partial v}\right)_T\right\} \left(\frac{\partial v}{\partial T}\right)_p$$

Also

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T = \frac{1}{T}\left\{p + \left(\frac{\partial u}{\partial v}\right)_T\right\}, \text{ and therefore}$$

$$c_p - c_v = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p$$

Now, from eqns. (7.34) and (7.37), we have

$$c_p - c_v = \frac{\beta^2 T v}{K} \qquad \text{...(7.38)}$$

Thus at any state defined by T and v, c_v can be found if c_p, β and K are known for the substance at that state. The values of T, v and K are always positive and, although β may sometimes be negative (*e.g.,* between 0° and 4°C water contracts on heating at constant pressure), β^2 is always positive. It follows that c_p *is always greater than* c_v.

The other expressions for c_p and c_v can be obtained by using the equation (7.14) as follows :

Since $$c_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial u}{\partial s}\right)_v \left(\frac{\partial s}{\partial T}\right)_v$$

We have $$c_v = T\left(\frac{\partial s}{\partial T}\right)_v \qquad \text{...(7.39)}$$

Similarly, $$c_p = \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{\partial h}{\partial s}\right)_p \left(\frac{\partial s}{\partial T}\right)_p$$

Hence, $$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \qquad \text{...(7.40)}$$

Alternative Expressions for Internal Energy and Enthalpy

(*i*) Alternative expressions for equations (7.29) and (7.32) can be obtained as follows :

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \qquad \text{...(7.29)}$$

But $$\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)\left(\frac{\partial v}{\partial p}\right)_T = -1$$

or $$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T = +\frac{\beta v}{K v} = \frac{\beta}{K}$$

Substituting in eqn. (7.29), we get

$$\boxed{\left(\frac{\partial u}{\partial v}\right)_T = T\frac{\beta}{K} - p} \qquad \text{...(7.41)}$$

Thus, $$\boxed{du = c_v dT + \left(\frac{T\beta}{K} - p\right) dv} \qquad \text{...[7.28 (a)]}$$

Similarly, $$\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{...(7.32)}$$

But by definition, $\left(\frac{\partial u}{\partial T}\right)_p = \beta v$

Hence $$\boxed{\left(\frac{\partial h}{\partial p}\right)_T = v(1 - \beta T)} \qquad \text{...(7.42)}$$

Thus $$\boxed{dh = c_p\, dT + v(1 - \beta T)\, dp} \qquad \text{...[7.31 }(a)]$$

(*ii*) Since $$u = h - pv$$

or $$\left(\frac{\partial u}{\partial p}\right)_T = \left(\frac{\partial h}{\partial p}\right)_T - p\left(\frac{\partial v}{\partial p}\right)_T - v$$

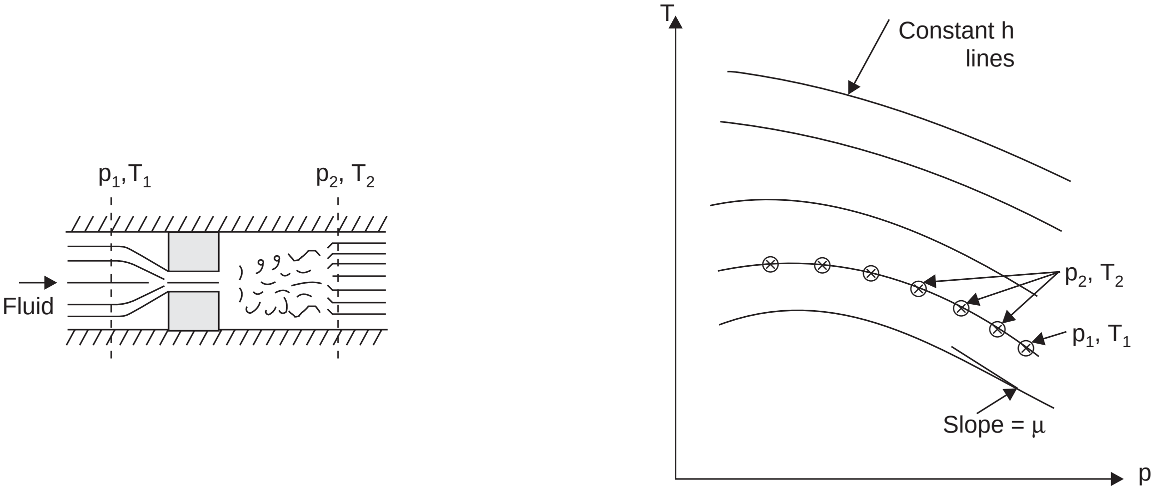
$$= v - v\beta T + pKv - v$$

Hence $$\left(\frac{\partial u}{\partial p}\right)_T = pKv - v\beta T \qquad \text{...(7.43)}$$

7.6.4. Joule-Thomson Co-efficient

Let us consider the partial differential co-efficient $\left(\frac{\partial T}{\partial p}\right)_h$. We know that if a fluid is flowing through a pipe, and the pressure is reduced by a throttling process, the enthalpies on either side of the restriction may be equal.

The throttling process is illustrated in Fig. 7.3 (*a*). The velocity increases at the restriction, with a consequent decrease of enthalpy, but this increase of kinetic energy is dissipated by friction, as the eddies die down after restriction. The steady-flow energy equation implies that the enthalpy of the fluid is restored to its initial value if the flow is adiabatic and if the velocity before restriction is equal to that downstream of it. These conditions are very nearly satisfied in the following experiment which is usually referred to as the Joule-Thomson experiment.



(*a*) (*b*)

Fig. 7.3. Determination of Joule-Thomson co-efficient.

Through a *porous plug* (inserted in a pipe) a fluid is allowed to flow steadily from a high pressure to a low pressure. The pipe is well lagged so that any heat flow to or from the fluid is negligible when steady conditions have been reached. Furthermore, the velocity of the flow is kept low, and any difference between the kinetic energy upstream and downstream of the plug is negligible. *A porous plug is used because the local increase of directional kinetic energy, caused by the restriction, is rapidly converted to random molecular energy by viscous friction in fine passages of the plug. Irregularities in the flow die out in a very short distance* downstream of the plug, and temperature and pressure measurements taken there will be values for the fluid in *a state of thermodynamic equilibrium*.

By keeping the upstream pressure and temperature constant at p_1 and T_1, the downstream pressure p_2 is reduced in steps and the corresponding temperature T_2 is measured. The fluid in the successive states defined by the values of p_2 and T_2 must always have the same value of the enthalpy, namely the value of the enthalpy corresponding to the state defined by p_1 and T_1. From these results, points representing equilibrium states of the same enthalpy can be plotted on a *T-s* diagram, and joined up to form a curve of constant enthalpy. The curve does *not* represent the throttling process itself, which is irreversible. During the actual process, the fluid undergoes first a decrease and then an increase of enthalpy, and no single value of the specific enthalpy can be ascribed to all elements of the fluid. If the experiment is repeated with different values of p_1 and T_1, a family of curves may be obtained (covering a range of values of enthalpy) as shown in Fig. 7.3 (*b*).

The slope of a curve [Fig. 7.3 (*b*)] at any point in the field is a function only of the state of the fluid, it is the Joule-Thomson co-efficient μ, defined by $\mu = \left(\frac{\partial T}{\partial p}\right)_h$. The change of temperature due to a throttling process is small and, if the fluid is a gas, it may be an increase or decrease. At any particular pressure there is a temperature, the *temperature of inversion*, above which a gas can never be cooled by a throttling process.

Both c_p and μ, as it may be seen, are defined in terms of *p*, *T* and *h*. The third partial differential co-efficient based on these three properties is given as follows :

$$\left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_h \left(\frac{\partial T}{\partial h}\right)_p = -1$$

Hence

$$\left(\frac{\partial h}{\partial p}\right)_T = -\mu c_p \qquad \text{...(7.44)}$$

μ may be expressed in terms of c_p, *p*, *v* and *T* as follows :

The property relation for *dh* is $dh = T\,ds + v\,dp$

From second *T ds* equation, we have

$$Tds = c_p\,dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$\therefore \quad dh = c_p\,dT - \left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] dp \qquad \text{...(7.45)}$$

For a constant enthalpy process $dh = 0$. Therefore,

$$0 = (c_p\,dT)_h + \left[\left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\} dp\right]_h$$

or

$$(c_p\,dT)_h = \left[\left\{T\left(\frac{\partial v}{\partial T}\right)_p - v\right\} dp\right]_h$$

$$\therefore \quad \mu = \left(\frac{\partial T}{\partial p}\right)_h = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \qquad \text{...(7.46)}$$

For an ideal gas, $pv = RT \; ; \quad v = \frac{RT}{p}$

or
$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} = \frac{v}{T}$$

∴
$$\mu = \frac{1}{c_p}\left(T \times \frac{v}{T} - v\right) = 0.$$

Therefore, if an ideal gas is throttled, there will not be any change in temperature.

Let
$$h = f(p, T)$$

Then
$$dh = \left(\frac{\partial h}{\partial p}\right)_T dp + \left(\frac{\partial h}{\partial T}\right)_p dT \quad ...(7.47)$$

But
$$\left(\frac{\partial h}{\partial T}\right)_p = c_p$$

∴
$$dh = \left(\frac{\partial h}{\partial p}\right)_T dp + c_p\, dT$$

For throttling process, $dh = 0$

∴
$$0 = \left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_h + c_p \quad ...(7.48)$$

or
$$c_p = -\frac{1}{\mu}\left(\frac{\partial h}{\partial p}\right)_T \quad ...(7.49)$$

$\left(\frac{\partial h}{\partial p}\right)_T$ is known as the *constant temperature co-efficient.*

7.7. CLAUSIUS-CLAPERYON EQUATION

Clausius-Claperyon equation is a relationship between the *saturation pressure*, *temperature*, the *enthalpy of evaporation*, and the *specific volume* of the *two phases* involved. This equation provides a basis for calculations of properties in a two-phase region. It gives the *slope* of a curve separating the two phases in the *p-T* diagram.

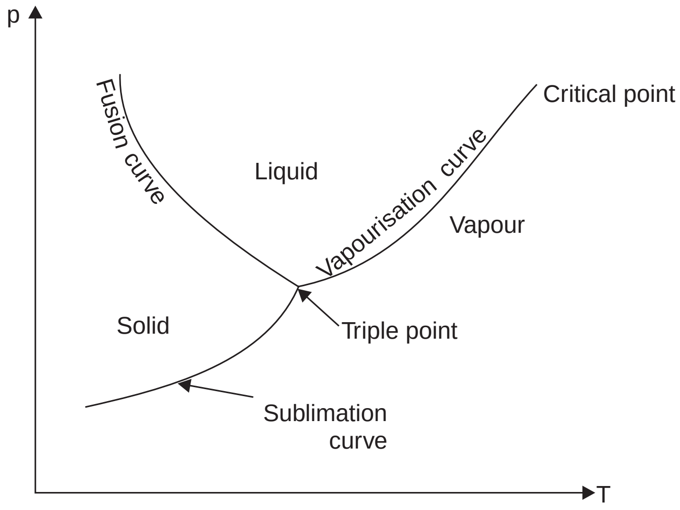


Fig. 7.4. *p-T* diagram.

The Clausius-Claperyon equation can be derived in different ways. The method given below involves the use of the Maxwell relation [eqn. (7.20)]

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

Let us consider the change of state from saturated liquid to saturated vapour of a pure substance which takes place at constant temperature. During the evaporation, the pressure and temperature are independent of volume.

$$\therefore \qquad \left(\frac{dp}{dT}\right) = \frac{s_g - s_f}{v_g - v_f}$$

where, s_g = Specific entropy of saturated vapour,

s_f = Specific entropy of saturated liquid,

v_g = Specific volume of saturated vapour, and

v_f = Specific volume of saturated liquid.

Also, $$s_g - s_f = s_{fg} = \frac{h_{fg}}{T}$$

and $$v_g - v_f = v_{fg}$$

where s_{fg} = Increase in specific entropy,

v_{fg} = Increase in specific volume, and

h_{fg} = Latent heat added during evaporation at saturation temperature T.

$$\therefore \qquad \frac{dp}{dT} = \frac{s_g - s_f}{v_g - v_f} = \frac{s_{fg}}{v_{fg}} = \frac{h_{fg}}{T\,.v_{fg}} \qquad ...(7.50)$$

This is known as **Clausius-Claperyon or Claperyon equation** for *evaporation of liquids*. The derivative $\frac{dp}{dT}$ is the slope of vapour pressure versus temperature curve. Knowing this slope and the specific volume v_g and v_f from experimental data, we can determine the enthalpy of evaporation, $(h_g - h_f)$ which is relatively difficult to measure accurately.

Eqn. (7.50) is also valid for the change from a solid to liquid, and from solid to a vapour.

At very low pressures, if we assume $v_g \simeq v_{fg}$ and the equation of the vapour is taken as $pv = RT$, then eqn. (7.50) becomes

$$\frac{dp}{dT} = \frac{h_{fg}}{Tv_g} = \frac{h_{fg}\,p}{RT^2} \qquad ...(7.51)$$

or $$h_{fg} = \frac{RT^2}{p}\frac{dp}{dT} \qquad ...(7.52)$$

Eqn. (7.52) may be used to obtain the enthalpy of vapourisation. This equation can be rearranged as follows :

$$\frac{dp}{p} = \frac{h_{fg}}{R}\cdot\frac{dT}{T^2}$$

Integrating the above equation, we get

$$\int\frac{dp}{p} = \frac{h_{fg}}{R}\int\frac{dT}{T^2}$$

$$\ln\frac{p_2}{p_1} = \frac{h_{fg}}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right] \qquad ...(7.53)$$

Knowing the vapour pressure p_1 at temperature T_1 we can find the vapour pressure p_2 corresponding to temperature T_2 from eqn. (7.53).

From eqn. (7.50), we see that the *slope of the vapour pressure curve is always +ve*, since $v_g > v_f$ and h_{fg} is always +ve. Consequently, the vapour pressure of any simple compressible substance increases with temperature.

— It can be shown that the slope of the *sublimation curve is also +ve for any pure substance*.

— However, the slope of the *melting curve could be +ve or –ve.*

— *For a substance that contracts on freezing, such as water, the slope of the melting curve will be negative.*

☞**Example 7.1.** *For a perfect gas, show that*

$$c_p - c_v = \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right]\left(\frac{\partial v}{\partial T}\right)_p = pv\beta + v\beta\left(\frac{\partial u}{\partial v}\right)_T$$

where β *is the co-efficient of cubical/volume expansion.*

Solution. The first law of thermodynamics applied to a closed system undergoing a reversible process states as follows :

$$dQ = du + pdv \qquad ...(i)$$

As per second law of thermodynamics,

$$ds = \left(\frac{dQ}{T}\right)_{rev.} \qquad ...(ii)$$

Combining these equations (*i*) and (*ii*), we have

$$Tds = du + pdv$$

Also, since $h = u + pv$

$\therefore$ $dh = du + pdv + vdp = Tds + vdp$

Thus, $Tds = du + pdv = dh - vdp$

Now, writing relation for u taking T and v as independent, we have

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$= c_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

Similarly, writing relation for h taking T and p as independent, we have

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$= c_p\, dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

In the equation for Tds, substituting the value of du and dh, we have

$$c_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv + pdv = c_p\, dT + \left(\frac{\partial h}{\partial p}\right)_T dp - vdp$$

or

$$c_v\, dT + \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right] dv = c_p\, dT - \left[v - \left(\frac{\partial h}{\partial p}\right)_T\right] dp$$

Since the above equation is true for any process, therefore, it will also be true for the case when $dp = 0$ and hence

$$(c_p - c_v)\ (dT)_p = \left[p + \left(\frac{\partial u}{\partial v} \right)_T \right] (dv)_p$$

or
$$(c_p - c_p) = \left[p + \left(\frac{\partial u}{\partial v} \right)_T \right] \left(\frac{\partial v}{\partial T} \right)_p$$

By definition,
$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p$$

∴ The above equation becomes,

$$c_p - c_v = \left[p + \left(\frac{\partial u}{\partial v} \right)_T \right] v\beta$$

or
$$= pv\beta + v\beta \left(\frac{\partial u}{\partial v} \right)_T$$ **Proved.**

☞**Example 7.2.** *Find the value of co-efficient of volume expansion* β *and isothermal compressibility K for a Van der Waals' gas obeying*

$$\left(p + \frac{a}{v^2} \right)(v - b) = RT.$$

Solution. Van der Waals equation is

$$\left(p + \frac{a}{v^2} \right)(v - b) = RT$$

Rearranging this equation, we can write

$$p = \frac{RT}{v - b} - \frac{a}{v^2}$$

Now for β we require $\left(\frac{\partial u}{\partial T} \right)_p$. This can be found by writing the cyclic relation,

$$\left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial T}{\partial p} \right)_v \left(\frac{\partial p}{\partial v} \right)_T = -1$$

Hence
$$\left(\frac{\partial v}{\partial T} \right)_p = - \frac{\left(\frac{\partial p}{\partial T} \right)_v}{\left(\frac{\partial p}{\partial v} \right)_T}$$

From the Van der Waals equation,

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{R}{v - b}$$

Also
$$\left(\frac{\partial p}{\partial v} \right)_T = - \frac{RT}{(v - b)^2} + \frac{2a}{v^3}$$

Hence
$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p = \frac{1}{v} \left[- \frac{\left(\frac{\partial p}{\partial T} \right)_v}{\left(\frac{\partial p}{\partial v} \right)_T} \right]$$

or
$$\beta = \frac{1}{v}\left[-\frac{\frac{R}{v-b}}{\frac{-RT}{(v-b)^2}+\frac{2a}{v^3}}\right] \cdot \frac{Rv^2(v-b)}{RTv^3-2a(v-b)^2} \quad . \textbf{ (Ans.)}$$

Also,
$$K = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = -\frac{1}{v}\left[\frac{1}{\frac{2a}{v^3}-\frac{RT}{(v-b)^2}}\right] = \frac{v^2(v-b)^2}{RTv^3-2a(v-b)^2} \; . \textbf{ (Ans.)}$$

Example 7.3. *Prove that the internal energy of an ideal gas is a function of temperature alone.*

Solution. The equation of state for an ideal gas is given by

$$p = \frac{RT}{v}$$

But
$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \qquad \text{[Eqn. (7.29)]}$$

$$= T\,\frac{R}{v} - p = p - p = 0.$$

Thus, if the temperature remains constant, there is no change in internal energy with volume (and therefore also with pressure). Hence internal energy (u) is a function of temperature (T) alone. **...Proved.**

Example 7.4. *Prove that specific heat at constant volume (c_v) of a Van der Waals' gas is a function of temperature alone.*

Solution. The Van der Waals equation of state is given by,

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

or
$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

or
$$\left(\frac{\partial^2 p}{\partial T^2}\right)_v = 0$$

Now
$$\left(\frac{dc_v}{dv}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Hence
$$\left(\frac{\partial c_v}{\partial v}\right)_T = 0$$

Thus c_v of a Van der Waals gas is independent of volume (and therefore of pressure also). Hence it is a function of temperature alone.

☞**Example 7.5.** *Determine the following when a gas obeys Van der Waals' equation,*

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

(*i*) *Change in internal energy ;* (*ii*) *Change in enthalpy ;*

(*iii*) *Change in entropy.*

Solution. (*i*) **Change in internal energy :**

The change in internal energy is given by

$$du = c_v dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv$$

But, $$\left(\frac{\partial p}{\partial T}\right)_v = \left[\frac{\partial}{\partial T}\left\{\frac{RT}{v-b} - \frac{a}{v^2}\right\}\right]_v = \frac{R}{v-b}$$

$\therefore$ $$\int_1^2 du = c_v \int_1^2 dT + \int_1^2 \left[T\left(\frac{R}{v-b}\right) - p\right] dv$$

$$= c_v \int_1^2 dT + \int_1^2 \left[T\left(\frac{R}{v-b}\right) - \left\{\frac{RT}{v-b} - \frac{a}{v^2}\right\}\right] dv$$

$$= c_v \int_1^2 dT + \int_1^2 \left[\frac{RT}{v-b} - \frac{RT}{v-b} + \frac{a}{v^2}\right] dv$$

$$= c_v \int_1^2 dT + \int_1^2 \frac{a}{v^2} . dv$$

$\therefore$ $$u_2 - u_1 = \mathbf{c_v(T_2 - T_1) + a \left(\frac{1}{v_1} - \frac{1}{v_2}\right)}. \quad \textbf{(Ans.)}$$

(*ii*) **Change in enthalpy :**

The change in enthalpy is given by

$$dh = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$\left(\frac{\partial h}{\partial p}\right)_T = 0 + v - T\left(\frac{\partial v}{\partial T}\right)_p \quad \text{...(1)}$$

Let us consider $p = f(v, T)$

$\therefore$ $$dp = \left(\frac{\partial p}{\partial v}\right)_T dv + \left(\frac{\partial p}{\partial T}\right)_v dT$$

$\therefore$ $$(dp)_T = \left(\frac{\partial p}{\partial v}\right)_T dv + 0 \quad \text{as } dT = 0 \quad \text{...(2)}$$

From equation (1),

$$(dh)_T = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right](dp)_T.$$

Substituting the value of $(dp)_T$ from eqn. (2), we get

$$(dh)_T = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]\left(\frac{\partial p}{\partial v}\right)_T dv$$

$$= \left[v\left(\frac{\partial p}{\partial v}\right)_T - T\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T\right] dv \quad \text{...(3)}$$

Using the cyclic relation for p, v, T which is

$$\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial p}{\partial v}\right)_T = -1$$

$\therefore$ $$\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T = -\left(\frac{\partial p}{\partial T}\right)_v$$

Substituting this value in eqn. (3), we get

$$(dh)_T = \left[v \left(\frac{\partial p}{\partial v} \right)_T + T \left(\frac{\partial p}{\partial T} \right)_v \right] dv \qquad ...(4)$$

For Van der Waals equation

$$\left(\frac{\partial p}{\partial v} \right)_T = \frac{\partial}{\partial v} \left[\left(\frac{RT}{v-b} \right) - \frac{a}{v^2} \right]_T$$

$$= - \frac{RT}{(v-b)^2} + \frac{2a}{v^3} \qquad ...(5)$$

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{\partial}{\partial T} \left[\left(\frac{RT}{v-b} - \frac{a}{v^2} \right) \right]_v = \frac{R}{v-b} \qquad ...(6)$$

Substituting the values of eqns. (5) and (6) in equation (1), we get

$$(dh)_T = \left[v \left\{ - \frac{RT}{(v-b)^2} + \frac{2a}{v^3} \right\} + T \left(\frac{R}{v-b} \right) \right] dv$$

$$\therefore \quad \int_1^2 (dh)_T = - RT \int_1^2 \frac{v}{(v-b)^2}\, dv + 2a \int_1^2 \frac{dv}{v^2} + RT \int_1^2 \frac{dv}{(v-b)}$$

$$\therefore \quad (h_2 - h_1)_T = - RT \left[\log_e \left(\frac{v_2 - b}{v_1 - b} \right) - b \left\{ \frac{1}{v_2 - b} - \frac{1}{v_1 - b} \right\} \right]$$

$$- 2a \left(\frac{1}{v_2} - \frac{1}{v_1} \right) + RT \log_e \left(\frac{v_2 - b}{v_1 - b} \right)$$

$$= \mathbf{bRT} \left[\frac{\mathbf{1}}{\mathbf{(v_2 - b)}} - \frac{\mathbf{1}}{\mathbf{(v_1 - b)}} \right] - \mathbf{2a} \left[\frac{\mathbf{1}}{\mathbf{v_2}} - \frac{\mathbf{1}}{\mathbf{v_1}} \right]. \quad \textbf{(Ans.)}$$

(*iii*) **Change in entropy :**

The change in entropy is given by

$$ds = c_p \frac{dT}{T} + \left(\frac{\partial p}{\partial T} \right)_v . \, dv$$

For Van der Waals equation,

$$\left(\frac{\partial p}{\partial T} \right)_v = \frac{R}{v-b} \qquad ...\text{as per eqn. (6)}$$

$$\therefore \quad ds = c_v \frac{dT}{T} + \frac{R}{v-b}\, dv$$

$$\therefore \quad \int_1^2 ds = c_v \int_1^2 \left[\frac{dT}{T} \right] + R \int_1^2 \frac{dv}{(v-b)}$$

$$\therefore \quad \mathbf{s_2 - s_1 = c_v \log_e \left[\frac{T_2}{T_1} \right] + R \log_e \left[\frac{v_2 - b}{v_1 - b} \right]}. \quad \textbf{(Ans.)}$$

Example 7.6. *The equation of state in the given range of pressure and temperature is given by*

$$v = \frac{RT}{p} - \frac{C}{T^3}$$

where C is constant.

Derive an expression for change of enthalpy and entropy for this substance during an isothermal process.

Solution. The general equation for finding dh is given by

$$dh = c_p\, dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$\int_1^2 dh = 0 + \int_1^2\left[\left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\}\right]_T$$

as $dT = 0$ for isothermal change.

From the given equation of state, we have

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3C}{T^4} \quad \ldots(i)$$

$$\therefore \qquad h_2 - h_1 = \int_1^2\left[\left\{\left(\frac{RT}{p} - \frac{C}{T^3}\right) - \frac{RT}{p} - \frac{3C}{T^3}\right\} dp\right]_T$$

$$= \left[\int_1^2\left(-\frac{4C}{T^3}\right) dp\right]_T = -\frac{4C}{T^3}\,[(p_2 - p_1)]_T$$

The general equation for finding ds is given by

$$ds = c_p\,\frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dp$$

$$\int_1^2 ds = \left[-\int_1^2\left(\frac{\partial v}{\partial T}\right)_p dp\right]_T$$

as $dT = 0$ for isothermal change.

Substituting the value from eqn. (i), we get

$$(s_2 - s_1) = \left[\int_1^2 -\left(\frac{R}{p} + \frac{3C}{T^4}\right) dp\right]_T$$

$$= \mathbf{-\,R\,\log_e\left(\frac{p_2}{p_1}\right) - \left(\frac{3C}{T^4}\right)(p_2 - p_1)} \quad \textbf{(Ans.)}$$

Example 7.7. *For a perfect gas obeying* $pv = RT$, *show that* c_v *and* c_p *are independent of pressure.*

Solution. Let $s = f(T, v)$

Then $\qquad ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$

Also $\qquad u = f(T, v)$

Then $\qquad du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv = c_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv$

Also, $\qquad du = Tds - pdv$

$$Tds - pdv = c_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$ds = c_v\,\frac{dT}{T} + \frac{1}{T}\left[\left(\frac{\partial u}{\partial v}\right)_T + p\right] dv$$

Equating the co-efficients of dT in the two equations of ds, we have

$$\frac{c_v}{T} = \left(\frac{\partial s}{\partial T}\right)_v$$

$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\,\frac{\partial^2 s}{\partial T \partial v}$$

From eqn. (7.20),

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

$$\frac{\partial^2 s}{\partial v \partial T} = \left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Also $$p = \frac{RT}{v} \qquad \text{...(Given)}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v}$$

$$\left(\frac{\partial^2 p}{\partial T^2}\right)_v = 0 \quad \text{or} \quad \left(\frac{\partial c_v}{\partial v}\right)_T = 0$$

This shows that c_v *is a function of T alone,* or c_v *is independent of pressure.*

Also, $$c_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = T\,\frac{\partial^2 s}{\partial T \partial p}$$

From eqn. (7.21), $$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

$$\frac{\partial^2 s}{\partial p \partial T} = -\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

Again, $$v = \frac{R}{p} \qquad \text{...(Given)}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}$$

and $$\left(\frac{\partial^2 v}{\partial T^2}\right)_p = 0\ ; \qquad \left(\frac{\partial c_p}{\partial p}\right)_T = 0$$

This shows that c_p *is a function of T alone or* c_p *is independent of pressure.*

Example 7.8. *Using the first Maxwell equation, derive the remaining three.*

Solution. The *first Maxwell relation* is as follows :

$$\boxed{\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v} \quad \text{...(}i\text{) (Eqn. 7.18)}$$

(1) Using the cyclic relation

$$\left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial s}\right)_T \cdot \left(\frac{\partial s}{\partial T}\right)_v = -1$$

$$\therefore \quad \left(\frac{\partial s}{\partial v}\right)_T = -\left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial s}{\partial T}\right)_v \quad \text{...(}ii\text{)}$$

Substituting the value from eqn. (*i*) in eqn. (*ii*), we get

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial s}\right)_v \cdot \left(\frac{\partial s}{\partial T}\right)_v \quad \text{...(}iii\text{)}$$

Using the chain rule,

$$\left(\frac{\partial p}{\partial s}\right)_v \cdot \left(\frac{\partial s}{\partial T}\right)_v \cdot \left(\frac{\partial T}{\partial p}\right)_v = 1 \quad \text{...(}iv\text{)}$$

Substituting the value of eqn. (*iv*) in eqn. (*iii*), we get

$$\boxed{\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v}$$

This is *Maxwell Third relation.*

(2) Again using the cyclic relation

$$\left(\frac{\partial s}{\partial p}\right)_v \cdot \left(\frac{\partial p}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial s}\right)_p = -1$$

$$\therefore \quad \left(\frac{\partial v}{\partial s}\right)_p = -\left(\frac{\partial p}{\partial s}\right)_v \cdot \left(\frac{\partial v}{\partial p}\right)_s \quad \text{...(}v\text{)}$$

Substituting the value from eqn. (*i*) into eqn. (*v*)

$$\left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial p}\right)_s \quad \text{...(}vi\text{)}$$

Again using the chain rule,

$$\left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial p}\right)_s \cdot \left(\frac{\partial p}{\partial T}\right)_s = 1$$

Substituting the value of (*vi*) into (*v*), we get

$$\boxed{\left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s}$$

This is *Maxwell second relation.*

(3) $$\left(\frac{\partial v}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_v \cdot \left(\frac{\partial p}{\partial v}\right)_T = -1$$

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial p}{\partial T}\right)_v \cdot \left(\frac{\partial v}{\partial p}\right)_T$$

$$= -\left(\frac{\partial p}{\partial s}\right)_v \left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial s}{\partial p}\right)_T \left(\frac{\partial v}{\partial s}\right)_T$$

Substituting the value from eqn. (*i*), we get

$$\left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial T}{\partial v}\right)_S \left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial s}{\partial p}\right)_T \left(\frac{\partial v}{\partial s}\right)_T$$

$$= \left\{\left(\frac{\partial T}{\partial v}\right)_S \cdot \left(\frac{\partial v}{\partial s}\right)_T \cdot \left(\frac{\partial s}{\partial T}\right)_v\right\} \left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial s}{\partial p}\right)_T$$

$$\therefore \qquad \boxed{\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T}$$

This is *Maxwell fourth relation.*

Example 7.9. *Derive the following relations :*

(*i*) $u = a - T\left(\frac{\partial a}{\partial T}\right)_v$ (*ii*) $h = g - T\left(\frac{\partial g}{\partial T}\right)_p$

(*iii*) $c_v = -T\left(\frac{\partial^2 a}{\partial T^2}\right)_v$ (*iv*) $c_p = -T\left(\frac{\partial^2 g}{\partial T^2}\right)_p$

where a = *Helmholtz function (per unit mass), and*

g = *Gibbs function (per unit mass).*

Solution. (*i*) Let $a = f(v, T)$

Then $da = \left(\frac{\partial a}{\partial v}\right)_T dv + \left(\frac{\partial a}{\partial T}\right)_v dT$

Also $da = -pdv - sdT$

Comparing the co-efficients of dT, we get

$$\left(\frac{\partial a}{\partial T}\right)_v = -s$$

Also $a = u - Ts$

or $u = a + Ts = a - T\left(\frac{\partial a}{\partial T}\right)_v$

Hence $u = a - T\left(\frac{\partial a}{\partial T}\right)_v$. **(Ans.)**

(*ii*) Let $g = f(p, T)$

Then $dg = \left(\frac{\partial g}{\partial p}\right)_T dp + \left(\frac{\partial g}{\partial T}\right)_p dT$

Also $dg = vdp - sdT$

Comparing the co-efficients of dT, we get

$$\left(\frac{\partial g}{\partial T}\right)_p = -s$$

Also $$h = g + Ts = g - T\left(\frac{\partial g}{\partial T}\right)_p$$

Hence $$h = g - T\left(\frac{\partial g}{\partial T}\right)_p. \quad \textbf{(Ans.)}$$

(*iii*) From eqn. (7.23), we have

$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v \qquad ...(i)$$

Also $$\left(\frac{\partial a}{\partial T}\right)_v = -s$$

or $$\left(\frac{\partial s}{\partial T}\right)_v = -\left(\frac{\partial^2 a}{\partial T^2}\right)_v \qquad ...(ii)$$

From eqns. (*i*) and (*ii*), we get

$$c_v = -T\left(\frac{\partial^2 a}{\partial T^2}\right)_v. \quad \textbf{(Ans.)}$$

(*iv*) From eqn. (7.26), we have

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \qquad ...(i)$$

Also $$\left(\frac{\partial g}{\partial T}\right)_p = -s$$

or $$\left(\frac{\partial s}{\partial T}\right)_p = -\left(\frac{\partial^2 g}{\partial T^2}\right)_p \qquad ...(ii)$$

From eqns. (*i*) and (*ii*), we get

$$c_p = -\mathrm{T}\left(\frac{\partial^2 g}{\partial T^2}\right)_p. \quad \textbf{(Ans.)}$$

Example 7.10. *Find the expression for ds in terms of dT and dp.*
Solution. Let $s = f(T, p)$

Then $$ds = \left(\frac{\partial s}{\partial T}\right)_p . dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

As per Maxwell relation (7.21)

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Substituting this in the above equation, we get

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT - \left(\frac{\partial v}{\partial T}\right)_p . dp \qquad ...(i)$$

The enthalpy is given by

$$dh = c_p dT = Tds + vdp$$

Dividing by dT at constant pressure

$$\left(\frac{\partial h}{\partial T}\right)_p = c_p = T\left(\frac{\partial s}{\partial T}\right)_p + 0 \qquad \text{(as } dp = 0 \text{ when pressure is constant)}$$

Now substituting this in eqn. (*i*), we get

$$ds = c_p \frac{dT}{T} - \left(\frac{\partial s}{\partial T}\right)_p . dp \qquad \text{...(}ii\text{)}$$

But
$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

Substituting this in eqn. (*ii*), we get

$$\boxed{ds = c_p \frac{dT}{T} - \beta v dp} \quad \textbf{(Ans.)}$$

Example 7.11. *Derive the following relations :*

(*i*) $\left(\frac{\partial T}{\partial p}\right)_s = \frac{Tv\beta}{c_p}$ (*ii*) $\left(\frac{\partial T}{\partial v}\right)_s = -\frac{T\beta}{c_v K}$.

where β = *Co-efficient of cubical expansion, and*
K = *Isothermal compressibility.*

Solution. (*i*) Using the Maxwell relation (7.19), we have

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial s}\right)_p$$

Also
$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

From eqn. (7.34),
$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{\beta v T}{c_p}$$

i.e.,
$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{Tv\beta}{c_p}. \quad \textbf{(Ans.)}$$

(*ii*) Using the Maxwell relation (7.18)

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v = -\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial s}\right)_v$$

Also
$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v \qquad \text{(Eqn. 7.23)}$$

$$K = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \qquad \text{(Eqn. 7.36)}$$

Then
$$\left(\frac{\partial T}{\partial v}\right)_s = -\frac{T}{c_v}\left(\frac{\partial p}{\partial T}\right)_v$$

Also
$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

i.e.,
$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{1}{-vK}\right)\beta v = \frac{\beta}{K}$$

$$\therefore \qquad \left(\frac{\partial T}{\partial v}\right)_s = \frac{-T\beta}{c_v K}. \quad \textbf{(Ans.)}$$

☞**Example 7.12.** *Derive the third Tds equation*

$$Tds = c_v \left(\frac{\partial T}{\partial p}\right)_v dp + c_p \left(\frac{\partial T}{\partial v}\right)_p dv$$

and also show that this may be written as :

$$Tds = \frac{c_v}{\beta} Kdp + \frac{c_p}{\beta v} dv.$$

Solution. Let $s = f(p, v)$

Then
$$ds = \left(\frac{\partial s}{\partial p}\right)_v dp + \left(\frac{\partial s}{\partial v}\right)_p dv$$

or
$$Tds = T\left(\frac{\partial s}{\partial p}\right)_v dp + T\left(\frac{\partial s}{\partial v}\right)_p dv$$

$$= T\left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial T}{\partial p}\right)_v + T\left(\frac{\partial s}{\partial T}\right)_p \left(\frac{\partial T}{\partial v}\right)_p dv$$

But
$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c_v}{T} \quad \text{and} \quad \left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T}$$

Hence
$$\mathbf{Tds} = \mathbf{c_v}\left(\frac{\partial \mathbf{T}}{\partial \mathbf{p}}\right)_{\mathbf{v}} \mathbf{dp} + \mathbf{c_p}\left(\frac{\partial \mathbf{T}}{\partial \mathbf{v}}\right)_{\mathbf{p}} \mathbf{dv} \qquad \textbf{...Proved.}$$

Also
$$\left(\frac{\partial T}{\partial p}\right)_v = \frac{-1}{\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p} = -\left(\frac{\partial T}{\partial v}\right)_p \left(\frac{\partial v}{\partial p}\right)_T = \frac{K}{\beta}$$

and
$$\left(\frac{\partial T}{\partial v}\right)_p = \frac{1}{\beta v}$$

Substituting these values in the above *Tds* equation, we get

$$\mathbf{Tds} = \frac{\mathbf{c_v K}}{\beta} \mathbf{dp} + \frac{\mathbf{c_p}}{\beta \mathbf{v}} \mathbf{dv} \qquad \textbf{...Proved.}$$

Example 7.13. *Using Maxwell relation derive the following Tds equation*

$$Tds = c_p\, dT - T\left(\frac{\partial v}{\partial T}\right)_p dp. \qquad \textbf{(U.P.S.C. 1988)}$$

Solution. $s = f(T, p)$

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp \qquad ...(i)$$

where $c_p = T\left(\frac{\partial s}{\partial T}\right)_p$

Also,
$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{......Maxwell relation}$$

Substituting these in eqn. *(i)*, we get

$$T_{ds} = c_p\, dT - T\left(\frac{\partial v}{\partial T}\right)_p dp. \quad \textbf{(Ans.)}$$

Example 7.14. *Derive the following relations*

$$\left(\frac{\partial T}{\partial v}\right)_u = \frac{T\left(\frac{\partial p}{\partial T}\right)_v - p}{c_v} .$$

Solution. $\left(\frac{\partial T}{\partial v}\right)_u$ can be expressed as follows :

$$\left(\frac{\partial T}{\partial v}\right)_u = \frac{-\left(\frac{\partial T}{\partial u}\right)_v}{\left(\frac{\partial v}{\partial u}\right)_T} = -\frac{\left(\frac{\partial u}{\partial v}\right)_T}{\left(\frac{\partial u}{\partial T}\right)_v}$$

Also $\quad Tds = du + pdv$

or $\quad du = Tds - pdv$

or $$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T - p\left(\frac{\partial u}{\partial v}\right)_T$$

or $$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T - p \qquad ...(i)$$

or $$\left(\frac{\partial u}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v \qquad ...(ii)$$

Dividing eqn. (*i*) by eqn. (*ii*), we get

$$\left(\frac{\partial T}{\partial v}\right)_u = \frac{T\left(\frac{\partial s}{\partial v}\right)_T - p}{T\left(\frac{\partial s}{\partial T}\right)_v} \qquad ...(iii)$$

Also $$c_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

and $$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v \qquad \text{... Maxwell relation}$$

Substituting these value in eqn. (*iii*), we get

$$\left(\frac{\partial T}{\partial v}\right)_u = \frac{T\left(\frac{\partial p}{\partial T}\right)_v - p}{c_v} \qquad \text{...}\textbf{Proved.}$$

☞**Example 7.15.** *Prove that for any fluid*

(*i*) $\left(\frac{\partial h}{\partial v}\right)_T = v\left(\frac{\partial p}{\partial v}\right)_T + T\left(\frac{\partial p}{\partial T}\right)_v$ (*ii*) $\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p$

Show that for a fluid obeying van der Waal's equation

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

where R, a and b are constants

$$h\ (enthalpy) = \frac{RTb}{v-b} - \frac{2a}{v} + f(T)$$

where f(T) is arbitrary.

Solution. We know that

$$ds = \frac{c_v}{T} dT + \left(\frac{\partial p}{\partial T}\right)_v dv \qquad \text{[Eqn. (7.24)]}$$

Also $$dh = Tds + vdp = T\left[\frac{c_v}{T} dT + \left(\frac{\partial p}{\partial T}\right)_v dv\right] + vdp$$

i.e., $$dh = c_v dT + T\left(\frac{\partial p}{\partial T}\right)_v + dv + vdp$$

Putting $dT = 0$, we get

$$\left(\frac{\partial \mathbf{h}}{\partial \mathbf{v}}\right)_{\mathbf{T}} = \mathbf{T}\left(\frac{\partial \mathbf{p}}{\partial \mathbf{T}}\right)_{\mathbf{v}} + \mathbf{v}\left(\frac{\partial \mathbf{p}}{\partial \mathbf{v}}\right)_{\mathbf{T}} \quad \textbf{...Proved.}$$

(*ii*) $$\left(\frac{\partial h}{\partial p}\right)_T = \left(\frac{\partial h}{\partial v}\right)_T \left(\frac{\partial v}{\partial p}\right)_T = \left[T\left(\frac{\partial p}{\partial T}\right)_v + v\left(\frac{\partial p}{\partial v}\right)_T\right]\left(\frac{\partial v}{\partial p}\right)_T$$

i.e., $$\left(\frac{\partial h}{\partial p}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial p}\right)_T + v \qquad ...(i)$$

Also $$\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

∴ Eqn. (*i*) becomes

$$\left(\frac{\partial \mathbf{h}}{\partial \mathbf{p}}\right)_{\mathbf{T}} = \mathbf{v} - \mathbf{T}\left(\frac{\partial \mathbf{v}}{\partial \mathbf{T}}\right)_{\mathbf{p}} \quad \textbf{...Proved.}$$

Now $$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

and $$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

∴ $$\left(\frac{\partial h}{\partial v}\right)_T = v\left[\frac{-RT}{(v-b)^2} + \frac{2a}{v^3}\right] + T\left(\frac{R}{v-b}\right)$$

$$= -\frac{RTv}{(v-b)^2} + \frac{2a}{v^2} + \frac{RT}{v-b} = \frac{-RTv}{(v-b)^2} + \frac{RT}{v-b} + \frac{2a}{v^2}$$

$$= \frac{-RTv + RT(v-b)}{(v-b)^2} + \frac{2a}{v^2} = \frac{-RTv + RTv - RTb}{(v-b)^2} + \frac{2a}{v^2}$$

i.e., $$\left(\frac{\partial h}{\partial v}\right)_T = \frac{-RTb}{(v-b)^2} + \frac{2a}{v^2}$$

or $$\mathbf{h} = \frac{\mathbf{RTb}}{\mathbf{v}-\mathbf{b}} - \frac{\mathbf{2a}}{\mathbf{v}} + \mathbf{f(T)} \quad \textbf{...Proved.}$$

This shows h depends on T and v.

Example 7.16. *Derive the following relations :*

(*i*) $\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p = -c_p\left(\frac{\partial T}{\partial p}\right)_h$ (*ii*) $\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p$

With the aid of eqn. (ii) show that

$$\left(\frac{\partial u}{\partial p}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_p - p\left(\frac{\partial v}{\partial p}\right)_T$$

The quantity $c_p\left(\frac{\partial T}{\partial p}\right)_h$ *is known as Joule-Thomson cooling effect. Show that this cooling effect for a gas obeying the equation of state* $(v - b) = \frac{RT}{p} - \frac{C}{T^2}$ *is equal to* $\left(\frac{3C}{T^2}\right) - b$.

Solution. We know that

$$\left(\frac{\partial h}{\partial p}\right)_T = -\mu c_p \qquad \text{...[Eqn. (7.44)]}$$

Also $$\mu = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \qquad \text{...[Eqn. (7.46)]}$$

$\therefore$ $$\left(\frac{\partial h}{\partial p}\right)_T = -\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = v - T\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{... } \textbf{Proved.}$$

Also $$\mu = \left(\frac{\partial T}{\partial p}\right)_h$$

$\therefore$ $$\left(\frac{\partial h}{\partial p}\right)_T = -c_p\left(\frac{\partial T}{\partial p}\right)_h .$$

(*ii*) Let $$u = f(T, v)$$

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$= c_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv \qquad \text{...}(i)$$

Also $$du = Tds - pdv$$

Substituting the value of Tds [from eqn. 7.24], we get

$$du = c_v\, dT + T\left(\frac{\partial p}{\partial T}\right)_v dv - pdv$$

$$= c_v\, dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv \qquad \text{...}(ii)$$

From (*i*) and (*ii*), we get

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \qquad \text{...}\textbf{Proved.}$$

Also $$\left(\frac{\partial u}{\partial p}\right)_T = \left(\frac{\partial u}{\partial v}\right)_T\left(\frac{\partial v}{\partial p}\right)_T$$

or $$\left(\frac{\partial u}{\partial p}\right)_T = \left(\frac{\partial v}{\partial p}\right)_T\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]$$

or $$\left(\frac{\partial u}{\partial p}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial v}{\partial p}\right)_T - p\left(\frac{\partial v}{\partial p}\right)_T \qquad \text{...}\textbf{Proved.}$$

We know that $$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

or $$\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Also $$\left(\frac{\partial u}{\partial p}\right)_T = -T\left(\frac{\partial v}{\partial T}\right)_p - p\left(\frac{\partial v}{\partial p}\right)_T$$ ...Already proved.

and $$\mu = \frac{1}{c_p}\left[T\left(\frac{\partial u}{\partial T}\right)_p - v\right]$$ ...[Eqn. (7.46)]

Now $$v - b = \frac{RT}{p} - \frac{C}{T^2}$$ ...[*Given*]

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{2C}{T^3}$$

Substituting this value in the expression of μ above, we get

$$\mu = \frac{1}{c_p}\left[T\left(\frac{R}{p} + \frac{2C}{T^3}\right) - v\right]$$

or $$\mu c_p = T\left(\frac{R}{p} + \frac{2C}{T^3}\right) - \frac{RT}{p} + \frac{C}{T^2} - b = \frac{3C}{T^2} - b$$

or $$c_p\left(\frac{\partial T}{\partial p}\right)_h = \frac{3C}{T^2} - b$$ ...**Proved.**

Example 7.17. *The pressure on the block of copper of 1 kg is increased from 20 bar to 800 bar in a reversible process maintaining the temperature constant at 15°C. Determine the following :*

(*i*) *Work done on the copper during the process,*
(*ii*) *Change in entropy,* (*iii*) *The heat transfer,*
(*iv*) *Change in internal energy, and* (*v*) $(c_p - c_v)$ *for this change of state.*

Given : β *(Volume expansivity* = 5×10^{-5}/K, K *(thermal compressibility)* = 8.6×10^{-12} m²/N *and v (specific volume)* = 0.114×10^{-3} m³/kg.

Solution. (*i*) **Work done on the copper, W :**

Work done during isothermal compression is given by

$$W = \int_1^2 p\,dv$$

The isothermal compressibility is given by

$$K = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

$$\therefore \quad dv = -K(v.dp)_T$$

$$\therefore \quad W = -\int_1^2 pKv.dp = -vK\int_1^2 p\,dp$$

Since *v* and *K* remain essentially constant

$$\therefore \quad W = -\frac{vK}{2}(p_2^2 - p_1^2)$$

$$= -\frac{0.114 \times 10^{-3} \times 8.6 \times 10^{-12}}{2}\left[(800 \times 10^5)^2 - (20 \times 10^5)^2\right]$$

$$= -\frac{0.114 \times 8.6 \times 10^{-15}}{2} \times 10^{10} [(800)^2 - (20)^2]$$

$$= -\frac{0.114 \times 8.6 \times 10^{-5}}{2} (640000 - 400) = -\ \mathbf{3.135\ J/kg.} \quad \textbf{(Ans.)}$$

The *negative sign indicates that the work is done on the copper block.*

(*ii*) **Change in entropy :**

The change in entropy can be found by using the following Maxwell relation :

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p = -\frac{v}{v}\left(\frac{\partial v}{\partial T}\right)_p = -v\beta \text{ as } \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \beta$$

$\therefore \qquad (ds)_T = -v\beta\ (dp)_T$

Integrating the above equation, assuming v and β remaining constant, we get

$$\mathbf{s_2 - s_1} = -v\beta\ (p_2 - p_1)_T$$

$$= -0.114 \times 10^{-3} \times 5 \times 10^{-5} [800 \times 10^5 - 20 \times 10^5]$$

$$= -0.114 \times 10^{-3} \times 5\ (800 - 20) = -\ \mathbf{0.446\ J/kg\ K.} \quad \textbf{(Ans.)}$$

(*iii*) **The heat transfer, Q :**

For a reversible isothermal process, the heat transfer is given by :

$$\mathbf{Q} = T(s_2 - s_1) = (15 + 273)(-0.4446) = -\ \mathbf{128\ J/kg.} \quad \textbf{(Ans.)}$$

(*iv*) **Change in internal energy, du :**

The change in internal energy is given by :

$$\mathbf{du} = Q - W$$

$$= -128 - (-3.135) = -\ \mathbf{124.8\ J/kg.} \quad \textbf{(Ans.)}$$

(*v*) $\mathbf{c_p - c_v}$ **:**

The difference between the specific heat is given by :

$$\mathbf{c_p - c_v} = \frac{\beta^2 T v}{K} \qquad \text{... [Eqn. (7.38)]}$$

$$= \frac{(5 \times 10^{-5})^2 \times (15 + 273) \times 0.114 \times 10^{-3}}{8.6 \times 10^{-12}} = \mathbf{9.54\ J/kg\ K.} \quad \textbf{(Ans.)}$$

Example 7.18. *Using Clausius-Claperyon's equation, estimate the enthalpy of vapourisation. The following data is given :*

At 200°C : $v_g = 0.1274\ m^3/kg\ ;\ v_f = 0.001157\ m^3/kg\ ;\ \left(\frac{dp}{dT}\right) = 32\ kPa/K.$

Solution. Using the equation

$$\left(\frac{dp}{dT}\right) = \frac{h_{fg}}{T_s(v_g - v_f)}$$

where, h_{fg} = Enthalpy of vapourisation.

Substituting the various values, we get

$$32 \times 10^3 = \frac{h_{fg}}{(200 + 273)(0.1274 - 0.001157)}$$

$\therefore \qquad \mathbf{h_{fg}} = 32 \times 10^3\ (200 + 273)(0.1274 - 0.001157)\ \text{J}$

$$= 1910.8 \times 10^3\ \text{J/kg} = \mathbf{1910.8\ kJ/kg.} \quad \textbf{(Ans.)}$$

Example 7.19. *An ice skate is able to glide over the ice because the skate blade exerts sufficient pressure on the ice that a thin layer of ice is melted. The skate blade then glides over this thin melted water layer. Determine the pressure an ice skate blade must exert to allow smooth ice skate at – 10°C.*

The following data is given for the range of temperatures and pressures involved :

$$h_{fg(ice)} = 334 \text{ kJ/kg} \; ; \; v_{liq.} = 1 \times 10 \text{ m}^3\text{/kg} \; ; \; v_{ice} = 1.01 \times 10^3 \text{ m}^3\text{/kg}.$$

Solution. Since it is a problem of phase change from solid to liquid, therefore, we can use Clausius-Claperyon equation given below :

$$\frac{dp}{dT} = \frac{h_{fg}}{v_{fg}} \cdot \frac{1}{T}$$

Multiplying both the sides by dT and integrating, we get

$$\int_{p_1}^{p_2} dp = \frac{h_{fg}}{v_{fg}} \int_{T_1}^{T_2} \frac{dT}{T}$$

or

$$(p_2 - p_1) = \frac{h_{fg}}{v_{fg}} \log_e \left(\frac{T_2}{T_1}\right) \quad ...(i)$$

But at $p_1 = 1$ atm., $t_1 = 0$°C

Thus, $p_1 = 1.013$ bar, $T_1 = 0 + 273 = 273$ K

$p_2 = ?$, $T_2 = -10 + 273 = 263$ K

Substituting these values in eqn. (*i*), we get

$$(p_2 - 1.013 \times 10^5) = \frac{334 \times 10^3}{(1 - 1.01)} \times \log_e \left(\frac{263}{273}\right)$$

$$= \frac{334 \times 10^3}{0.01} \times \log_e \left(\frac{273}{263}\right) = 12.46 \times 10^5 \text{ N/m}^2$$

or

$$p_2 = 12.46 \times 10^5 + 1.013 \times 10^5$$

$$= 13.47 \times 10^5 \text{ N/m}^2 \text{ or } \mathbf{13.47 \text{ bar.} \quad (Ans.)}$$

This pressure is considerably high. It can be achieved with ice skate blade by having only a small portion of the blade surface in contact with the ice at any given time. If *the temperature drops lower than – 10°C, say – 15°C, then it is not possible to generate sufficient pressure to melt the ice and conventional ice skating will not be possible.*

Example 7.20. *For mercury, the following relation exists between saturation pressure (bar) and saturation temperature (K) :*

$$\log_{10} p = 7.0323 - 3276.6/T - 0.652 \log_{10} T$$

Calculate the specific volume v_g of saturation mercury vapour at 0.1 bar.

Given that the latent heat of vapourisation at 0.1 bar is 294.54 kJ/kg.

Neglect the specific volume of saturated mercury liquid.

Solution. Latent heat of vapourisation, $h_{fg} = 294.54$ kJ/kg (at 0.1 bar) ...(given)

Using Clausius-Claperyon equation

$$\frac{dp}{dT} = \frac{h_{fg}}{v_{fg}T} = \frac{h_{fg}}{(v_g - v_f)T} \quad ...(i)$$

Since v_f is neglected, therefore eqn. (*i*) becomes

$$\frac{dp}{dT} = \frac{h_{fg}}{v_g T}$$

Now,

$$\log_{10} p = 7.0323 - \frac{3276.6}{T} - 0.652 \log_{10} T$$

Differentiating both sides, we get

$$\frac{1}{2.302p} \cdot \frac{dp}{dT} = \frac{3276.6}{T^2} - \frac{0.652}{2.302\,T}$$

or $$\frac{dp}{dT} = 2.302 \times 3276.6 \times \frac{p}{T^2} - 0.652\ \frac{p}{T} \qquad ...(ii)$$

From (*i*) and (*ii*), we have

or $$\frac{h_{fg}}{v_g T} = 2.302 \times 3276.6 \times \frac{p}{T^2} - 0.652\ \frac{p}{T} \qquad ...(iii)$$

We know that $$\log_{10} p = 7.0323 - \frac{3276.6}{T} - 0.652 \log_{10} T \qquad \text{... (given)}$$

At $p = 0.1$ bar,

$$\log_{10} (0.1) = 7.0332 - \frac{3276.6}{T} - 0.652 \log_{10} T$$

$$-1 = 7.0323 - \frac{3276.6}{T} - 0.652 \log_{10} T$$

or $$0.652 \log_{10} T = 8.0323 - \frac{3276.6}{T}$$

or $$\log_{10} T = 12.319 - \frac{5025.4}{T}$$

Solving by hit and trial method, we get

$$T = 523 \text{ K}$$

Substituting this value in eqn. (*iii*), we get

$$\frac{294.54 \times 10^3}{v_g \times 523} = 2.302 \times 3276.6 \times \frac{0.1 \times 10^5}{(523)^2} - 0.652 \times \frac{0.1 \times 10^5}{523}$$

$$\frac{563.17}{v_g} = 275.75 - 12.46$$

i.e., $$\mathbf{v_g = 2.139\ m^3/kg.} \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. Maxwell relations are given by

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v \ ; \left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \ ; \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T .$$

2. The specific heat relations are

$$c_p - c_v = \frac{vT\beta^2}{K} \ ; c_v = T\left(\frac{\partial s}{\partial T}\right)_v \ ; c_p = T\left(\frac{\partial s}{\partial T}\right)_p .$$

3. Joule-Thomson co-efficient is expressed as

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h .$$

4. Entropy equations (*Tds* equations) :

$$Tds = c_v dT + T \left(\frac{\partial p}{\partial T}\right)_v dv \qquad \text{...(1)}$$

$$Tds = c_p dT - T \left(\frac{\partial v}{\partial T}\right)_p dp \qquad \text{...(2)}$$

5. Equations for *internal energy* and *enthalpy* :

$$\left(\frac{\partial u}{\partial v}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_v - p \qquad \text{...(1)}$$

$$du = c_v dT + \left\{T\left(\frac{\partial p}{\partial T}\right)_v - p\right\} dv \qquad \text{...[1 (a)]}$$

$$\left(\frac{\partial h}{\partial p}\right)_T = v - T \left(\frac{\partial v}{\partial T}\right)_p \qquad \text{...(2)}$$

$$dh = c_p dT + \left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\} dp \qquad \text{...[2 (a)]}$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The specific heat at constant pressure (c_p) is given by

(*a*) $c_p = T \left(\frac{\partial s}{\partial T}\right)_p$ (*b*) $c_p = T \left(\frac{\partial T}{\partial s}\right)_p$

(*c*) $c_p = T \left(\frac{\partial v}{\partial T}\right)_p$ (*d*) $c_p = T \left(\frac{\partial v}{\partial T}\right)_p$.

2. The specific heat relation is

(*a*) $(c_p - c_v) = \frac{vT\beta^2}{K}$ (*b*) $(c_p - c_v) = \frac{vTK}{\beta^2}$

(*c*) $(c_p - c_v) = \frac{pTK}{\beta^2}$ (*d*) $(c_p - c_v) = \frac{v^2T\beta}{K}$.

3. The relation of internal energy is

(*a*) $du = \left(\frac{K}{\beta} c_v\right) dp + \left(\frac{c_p}{v\beta} - p\right) dv$ (*b*) $du = \left(\frac{K}{\beta} c_v\right) dp + \left(\frac{c_p}{v\beta} + p\right) dv$

(*c*) $du = \left(\frac{K}{\beta} c_p\right) dp + \left(\frac{c_p}{v\beta} - v\right) dv$ (*d*) $du = \left(\frac{K}{\beta} c_p\right) dp + \left(\frac{c_v}{v\beta} - p\right) dv$.

4. *Tds* equation is

(*a*) $Tds = c_v dT + \frac{T\beta}{K} dv$ (*b*) $Tds = c_p dT - \frac{T\beta}{K} dv$

(*c*) $Tds = c_v dT + \frac{TK}{\beta} dv$ (*d*) $Tds = c_v dT + \frac{T\beta}{K} dp$.

Answers

1. (*a*) **2.** (*a*) **3.** (*a*) **4.** (*a*).

EXERCISES

1. Define the co-efficient of :

(*i*) Volume expansion (*ii*) Isothermal compressibility

(*iii*) Adiabatic compressibility.

2. Derive the Maxwell relations and explain their importance in thermodynamics.

3. Show that the equation of state of a substance may be written in the form

$$\frac{dv}{v} = -\,Kdp + \beta dT.$$

4. A substance has the volume expansivity and isothermal compressibility :

$$\beta = \frac{1}{T}\ ;\ K = \frac{1}{p}$$

Find the equation of state. $\left[\textbf{Ans.}\ \ \frac{pv}{T} = \text{constant}\right]$

5. For a perfect gas, show that the difference in specific heats is

$$c_p - c_v = \frac{R}{T}\ .$$

6. For the following given differential equations,

$$du = Tds - pdv$$

and

$$dh = Tds + vdp$$

prove that for perfect gas equation,

$$\left(\frac{\partial u}{\partial p}\right)_T = 0 \quad \text{and} \quad \left(\frac{\partial h}{\partial p}\right)_T = 0.$$

7. Using the cyclic equation, prove that

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{\beta}{KT}\ .$$

8. Prove that the change in entropy is given by

$$ds = \frac{c_v}{T}\left[\frac{KT}{\beta}\,.\,dp + \frac{c_p}{\beta v}\right]dv.$$

9. Deduce the following thermodynamic relations :

(*i*) $\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p = -\,c_p\left(\frac{\partial T}{\partial p}\right)_h$ (*ii*) $\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p.$

10. Show that for a Van der Waals gas

$$c_p - c_v = \frac{R}{1 - 2a\,(v-b)^2 / RTv^3}\ .$$

11. A gas obeys $p(v - b) = RT,$ where b is positive constant. Find the expression for the Joule-Thomson co-efficient of this gas. Could this gas be cooled effectively by throttling ?

12. The pressure on the block of copper of 1 kg is increased from 10 bar to 1000 bar in a reversible process maintaining the temperature constant at 15°C. Determine :

(*i*) Work done on the copper during the process (*ii*) Change in entropy

(*iii*) The heat transfer (*iv*) Change in internal energy

(*v*) $(c_p - c_v)$ for this change of state.

The following data may be assumed :

Volume expansivity (β) = 5×10^{-5}/K

Isothermal compressibility (K) = 8.6×10^{-12} m^2/N

Specific volume (v) = 0.114×10^{-3} m^3/kg

[**Ans.** (*i*) – 4.9 J/kg ; (*ii*) – 0.57 J/kg K ; (*iii*) – 164 J/kg ; (*iv*) – 159.1 J/kg ; 9.5 J/kg K]

8

Ideal and Real Gases

8.1. INTRODUCTION

An '*ideal gas*' is defined as a *gas having no forces of intermolecular attraction*. The gases which *follow the gas laws at all ranges of pressures and temperatures* are considered as "*ideal gases*". However, '*real gases*' *follow these laws at low pressures or high temperatures or both*. This is because the forces of attraction between molecules *tend to be very small* at reduced pressures and elevated temperatures.

An *ideal gas* obeys the law $pv = RT$. The specific heat capacities *are not constant but are functions of temperature*. A *perfect gas* obeys the law $pv = RT$ and has *constant specific heat capacities*.

A perfect gas is well suited to mathematical manipulation and is therefore a most useful model to use for analysis of practical machinery which uses real gases as a working substance.

In *reality there is no ideal or perfect gas*. At a very low pressure and at a very high temperature, real gases like hydrogen, oxygen, nitrogen, helium etc. behave nearly the same way as perfect gases. These gases are called *semi-perfect* or *permanent gases*. The term semi-perfect has the implication that the behaviour of the gases are nearly the same as that of a perfect gas. The term '*permanent*' was used for these gases by earlier chemists who thought that these gases did not change their phase (*i.e.,* did not condense to a liquid state). Hence they are called permanent gases. There is no gas which does not change phase, and there is no permanent gas in the real sense. However, these gases can be changed into a liquid phase only if they are subjected to a great decrease in temperature and increase in pressure.

All gases behave in nearly in a similar way, especially at pressures considerably lower than the critical pressure, and at temperatures above the critical temperature. *The relation between the independent properties, such as pressure, specific volume and temperature for a pure substance* is known as the '*equation of state*'. For engineering calculations, the equation of state for perfect gases can be used for real gases so long as the pressures are well below their critical pressure and the temperatures are above the critical temperature.

8.2. THE EQUATION OF STATE FOR A PERFECT GAS

Boyle's law. It states that *volume of a given mass of a perfect gas varies inversely as the absolute pressure when temperature is constant*.

If p is the absolute pressure of the gas and V is the volume occupied by the gas, then

$$V \propto \frac{1}{p}$$

or $\quad pV$ = Constant, so long as the temperature is constant ...(8.1)

Fig. 8.1 shows the graphical representation of Boyle's law. The curves are rectangular hyperbolas asymptotic to the p-v axis. Each curve corresponds to a different temperature. For any two points on the curve,

$$\frac{p_1}{p_2} = \frac{V_2}{V_1} \qquad \text{...(8.2)}$$

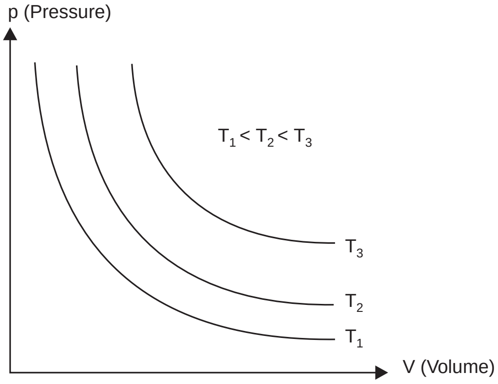


Fig. 8.1. p-V relation of a perfect gas at constant temperature.

Charle's law. It states that *if any gas is heated at constant pressure, its volume changes directly as its absolute temperature.*

In other words, $V \propto T$

or $\quad \frac{V}{T}$ = Constant, so long as pressure is constant ...(8.3)

If a gas changes its volume from V_1 to V_2 and absolute temperature from T_1 to T_2 without any change of pressure, then

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \text{...(8.4)}$$

Fig. 8.2 gives the graphical representation of Charle's law.

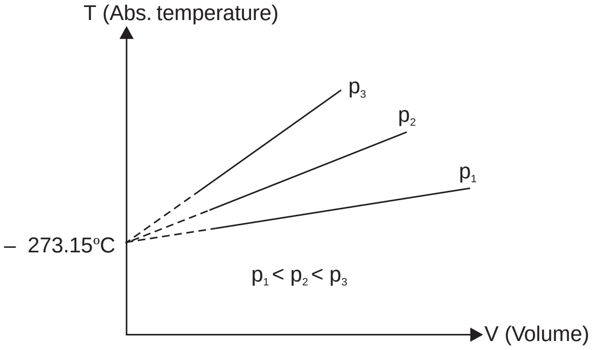


Fig. 8.2. T-v relation of a perfect gas constant pressure.

To derive the *equation of state* for a perfect gas let us consider a *unit mass* of a perfect gas to change its state in the following two successive processes (Fig. 8.3)

(*i*) Process 1-2′ at constant pressure, and

(*ii*) Process 2′-2 at constant temperature.

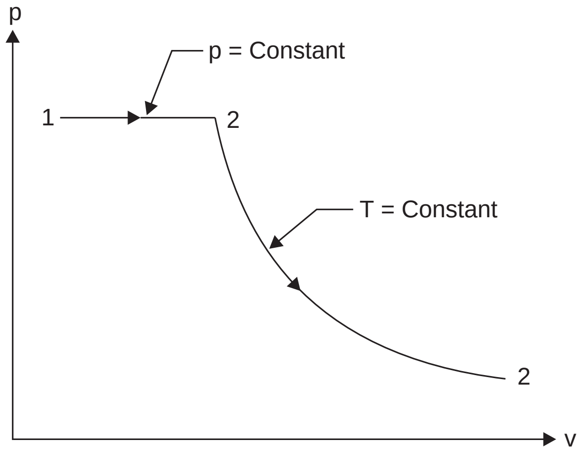


Fig. 8.3. Formulation of equation of state of a perfect gas.

For the *process 1-2′*, applying Charle's law

$$\frac{v_1}{T_1} = \frac{v_2{}'}{T_2{}'}$$

and since $T_2{}' = T_2$, we may write

$$\frac{v_1}{T_1} = \frac{v_2{}'}{T_2} \qquad ...(i)$$

For the *process 2′-2*, using Boyle's law

$$p_2{}'v_2{}' = p_2v_2$$

and since $p_2{}' = p_1$

$$p_1v_2{}' = p_2v_2$$

i.e.,

$$v_2{}' = \frac{p_2v_2}{p_1} \qquad ...(ii)$$

Substituting the value of $v_2{}'$ from eqn. (*ii*) in eqn. (*i*), we get

$$\frac{v_1}{T_1} = \frac{p_2v_2}{p_1T_2}$$

or

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}$$

i.e.,

$$\frac{pv}{T} = \text{constant} \qquad ...(8.5)$$

The magnitude of this constant depends upon the particular gas and it is denoted by R, where R is called the **specific gas constant**. Then

$$\frac{pv}{T} = R$$

The equation of the state for a perfect gas is thus given by the equation

$$pv = RT \qquad ...(8.6)$$

or for m kg, occupying V m^3,

$$pV = mRT \qquad ...(8.7)$$

If the mass is chosen to be numerically equal to the molecular weight of the gas then 1 mole of the gas has been considered, *i.e.,* 1 kg mole of oxygen is 32 kg oxygen, or 1 kg mole of hydrogen is 2 kg hydrogen.

The equation may be written as

$$pV_0 = MRT \qquad ...(8.8)$$

where V_0 = Molar volume, and

M = Molecular weight of the gas.

Avogadro discovered that V_0 *is the same for all gases at the same pressure and temperature* and therefore it may be seen that MR = a constant ; R_0 and thus

$$pV_0 = R_0T \qquad ...(8.9)$$

R_0 is called the **molar or universal gas constant** and its value is **8.3143 kJ/kg mol K.**

If there are n moles present then the ideal gas equation may be written as

$$pV = nR_0T \qquad ...(8.10)$$

where V is the volume occupied by n moles at pressure p and temperature T.

8.3. p-v-T SURFACE OF AN IDEAL GAS

The equation of state of an ideal gas is a relationship between the variables pressure (p), volume (V) and temperature (T). On plotting these variables along three mutually perpendiculars axes, we get a surface which represents the equation of state ($pv = RT$). Such a surface is called *p-v-T surface*. These surfaces represent the fundamental properties of a substance and *provide a tool to study the thermodynamic properties and processes of that substance.* Fig. 8.4 shows a portion of a *p-v-T* surface for an *ideal gas*. *Each point on this surface represents an equilibrium state and a line on the surface represents a process*. The Fig. 8.4 also shows the constant pressure, constant volume and constant temperature lines.

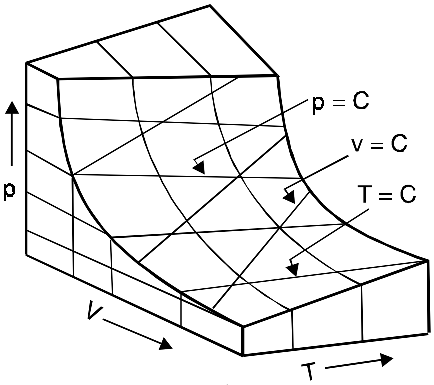


Fig. 8.4

8.4. INTERNAL ENERGY AND ENTHALPY OF A PERFECT GAS

Joule's Law. *Joule's law states that the specific internal energy of a gas depends only on the temperature of the gas and is independent of both pressure and volume.*

i.e., $u = f(T)$

Joule concluded this result from a series of experiments conducted with an apparatus similar to the one shown in Fig. 8.5.

— Two tanks connected by a valve were submerged in a bath of water.

— Initially one tank was evacuated and the other was filled with air under high pressure.

— A thermometer was placed in the water bath.

— After the tank and water had attained the same temperature, the valve between the two tanks was opened to pass air slowly from high pressure tank to the evacuated tank. Time was allowed for equilibrium to be attained.

Joule observed that there was *no change in temperature of water during or after the process*. Since there was no change in the temperature of water, he concluded that there *was no heat transfer between air and water i.e.,* $\delta Q = 0$. And since there was no work during the process, *i.e.,* $\delta W = 0$, from the first law of thermodynamics, $\delta Q = dE + \delta W$, Joule *concluded that change in internal energy of the air is zero, i.e.,* $dE = 0$.

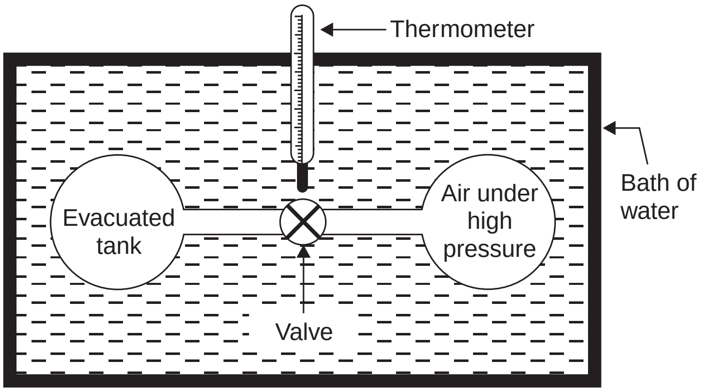


Fig. 8.5. Apparatus for demonstration of Joule's law.

Again, since both pressure and volume changed during the process, he remarked that *internal energy was a function only of temperature ; since during the process temperature did not change, the internal energy remained constant*.

Later on when experiments were conducted with more refined instruments, it was found that there was a *very small change in temperature of water,* indicating that *for real gases internal energy was not a function of temperature alone.* However, at low pressure and high temperature where real gases behave like semi-perfect gases and where the equation of state for a semi-perfect gas, $pv = RT$, is sufficiently accurate, Joule's law holds equally good in that range.

From definition of enthalpy,

$$h = u + pv$$

Also $$pv = RT$$

∴ $$h = u + RT \qquad ...(8.11)$$

Since u is a function of temperature only, h is a function of temperature,

i.e., $$h = f(T) \qquad ...(8.12)$$

8.5. SPECIFIC HEAT CAPACITIES OF AN IDEAL GAS

The specific heat capacity at constant volume of any substance is defined by

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

It may be seen that as Joule's law for an ideal gas states $u = f(T)$, then

$$c_v = \frac{du}{dT} \qquad \text{...(8.13)}$$

Since $h = u + pv$, Boyle's law, $pV = f(T)$ and Joule's law $u = f(T)$ together show, $h = f(T)$ and by similar argument to the above it may be seen that :

$$c_p = \frac{dh}{dT} \qquad \text{...(8.14)}$$

Further as $h = u + pv$, then $h = u + RT$ and by differentiation

$$\frac{dh}{dT} = \frac{du}{dT} + R$$

Substitution from eqns. (8.13) and (8.14) gives,

$$c_p = c_v + R \quad \text{i.e.,} \quad c_p - c_v = R \qquad \text{...(8.15)}$$

If expressed in terms of molar quantities then eqn. (8.15) becomes

$$C_p - C_v = R_0 \qquad \text{...(8.16)}$$

where C_p and C_v are *molar specific heat capacities.*

Equations for specific heat capacities of ideal gases

Since both u and h are functions of temperature, the equations to c_p and c_v must also be functions of temperature. They are usually expressed in a form :

$$c_p = a + KT + K_1T^2 + K_2\, T^3 \qquad \text{...(8.17)}$$

$$c_v = b + KT + K_1T^2 + K_2T^3 \qquad \text{...(8.18)}$$

where a, b, K, K_1 and K_2 are constants. Values of specific enthalpy etc. are then obtained by integration.

8.6. REAL GASES

It has been observed that when experiments are performed at relatively low pressures and temperatures most of the real gases obey Boyle's and Charle's laws quite closely. But the actual behaviour of real gases at elevated pressures and at low temperatures deviates considerably.

The ideal gas equation $pv = RT$ can be derived analytically using the kinetic theory of gases by making the following **assumptions** :

(*i*) A finite volume of gas contains large number of molecules.

(*ii*) The collision of molecules with one another and with the walls of the container are perfectly elastic.

(*iii*) The molecules are separated by large distances compared to their own dimensions.

(*iv*) The molecules do not exert forces on one another except when they collide.

As long as the above assumptions are valid the behaviour of a real gas approaches closely that of an ideal gas.

8.7. VAN DER WAALS' EQUATION

Van der Waals' equation (for a real gas) may be written as :

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \qquad \text{...[8.19 (a)]}$$

The constants a and b are specific constants and depend upon the type of the fluid considered, 'v' represents the volume per unit mass and R is the gas constant.

If the volume of one mole is considered then the above equation can be written as

$$\left(p + \frac{a}{v^{-2}}\right) \; (\bar{v} - b) \; = R_0T \qquad \text{...[8.19 } (b)]$$

The units of p, $\bar{v}$, T, R, a and b are as follows :

p (N/m^2), $\bar{v}$ (m^3/kg-mol), T (K) and R = 8314 Nm/kg mol K, a [Nm^4/(kg-mol)^2], b (m^3/kg mol).

Table 8.1. Constants of Van der Waals' Equation

S.No.	*Substance*	*a* *Nm⁴/(kg-mol)²*	*b* *m³/kg-mol*
1.	Hydrogen (H_2)	25105	0.0262
2.	Oxygen (O_2)	139250	0.0314
3.	Carbon dioxide (CO_2)	362850	0.0423
4.	Helium (He)	3417620	0.0228
5.	Air	135522	0.0362
6.	Water (H_2O) vapour	551130	0.0300
7.	Mercury (Hg) vapour	2031940	0.0657

Van der Waals equation was proposed in 1873 for the gaseous and liquid states of a fluid, and accounts qualitatively for many important properties, but quantitatively it fails in many particulars.

The characteristic equation for a perfect gas is obtained by neglecting the finite size of the molecules. If this be taken into account it is obvious that the equation must be modified, for the distance travelled by a molecule between two successive encounters will be less than if the molecules were point spheres. Let the average distance traversed by a molecule between two successive encounters be denoted by λ, the mean free path. In Fig. 8.6 suppose L and M to be the two molecules of diameter 'd' at a distance λ apart. If these molecules were to impinge along the line of centres the path moved over would be less by an amount 'd' than if the molecules were point spheres. Now all the encounters between molecules are not direct, so their mean free paths will be lessened by an amount kd, where k is a fraction. That is, the mean free path is diminished in the ratio $(\lambda - kd) : \lambda$ or $\left(1 - \frac{kd}{\lambda}\right)$: 1.

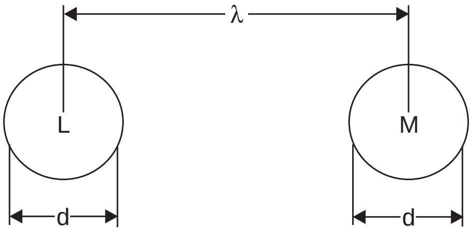


Fig. 8.6

If the mean free path is lessened in this ratio, the encounters per second will be increased in the ratio $1 : 1 - \frac{kd}{\lambda}$. But the pressure of the gas depends upon the encounters per second with the wall of the containing vessel. Hence the new pressure is given by

$$p = \frac{1}{3} \, \rho\overline{C}^2 \cdot \frac{1}{1 - \dfrac{kd}{\lambda}} \qquad \text{...(8.20)}$$

(where ρ is the density and $\overline{C}$ is the average velocity).

The mean free path is inversely proportional to the density of the gas, for if the volume were halved, *i.e.,* the density doubled, there would be twice as many molecules in the same space, and therefore any molecule would only have to travel approximately half as far before encountering another molecule. Hence writing v for $\frac{1}{\rho}$ and $\frac{b}{v}$ for $\frac{kd}{\lambda}$ in eqn. (8.20), we get

$$pv\left(1-\frac{b}{v}\right)=\frac{\overline{C}^2}{3}=RT$$

or $$p(v-b)=RT \qquad \text{...(8.21)}$$

Next consider the *forces of cohesion* which act between a molecule and those surrounding it. When the molecule is sufficiently far removed from the surface of the gas in all directions the resultant of these cohesives forces are equally probable, as the individual forces are varying continuously as the surrounding molecules change their positions. Hence if the resultant is averaged over a sufficient length of time the aggregate force will be nil. *This is not true, however, when the molecule is near the surface*. Let the force from each molecule be resolved into normal and tangential components. All directions for the resultant in the tangential plane are equally likely, but the resultant normal component is most often directed inwards. Averaged over a sufficient length of time the total resultant force will therefore be a normal force always directed inwards. Thus the average effect of the cohesive forces is the same as if there was a permanent field of force acting at and near the surface. This field of force can be regarded as exerting a pressure p_1 over the boundary of the gas. The pressure is proportional to the number of molecules per unit area near the boundary surface and to the normal component of the force. Both of these factors are proportional to the density, so p_1 will be proportional to the square of the density.

i.e., $$p_1 = a\rho^2 \qquad \text{...(8.22)}$$

where a is a constant.

Hence the molecules are not deflected by impact alone on reaching the boundary, but as the total result of their impact and of the action of the supposed field of force. That is, their change of momentum may be supposed to be produced by a total pressure $p + p_1$ instead of by the simple pressure p.

Hence eqn. (8.21) now becomes :

$$(p + p_1)(v - b) = RT,$$

or $$\left(p+\frac{a}{v^2}\right)(v-b)=RT$$

by substitution from p_1 from (8.22) and replacing ρ^2 by $\frac{1}{v^2}$.

Evaluation of constants *a* and *b* :

The general form of the isothermals for carbon dioxide given by Van der Waals' equation is shown in Fig. 8.7. These curves are obtained from the equation,

$$\left(p+\frac{0.00874}{v^2}\right)(v-0.0023)=\frac{1.00646}{273}\,T \qquad \text{...(8.23)}$$

where the unit of pressure is the atmosphere, and the unit of volume that of the gas at 0°C under one atmosphere pressure.

Van der Waals' equation being a cubic in v has three roots which may be either all real, or two imaginary and one real, as imaginary roots always occur in pairs. In Fig. 8.7, the 40°C isothermal corresponds to the first condition, and the other isothermals to the latter. There is one isothermal where there are three real coincident roots at a point of inflexion. All the isothermals for temperatures higher than that corresponding to the isothermal with three real coincident roots have no horizontal tangent, and all those lower have a maximum and minimum. Consequently this curve is identified with the critical isothermal. The temperature of the critical isothermal is obtained in the following manner. Equation (8.19) may be written

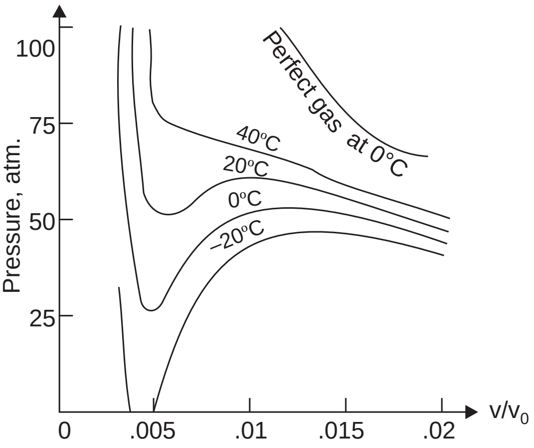


Fig. 8.7. Van der Waals' Isothermal for CO_2.

$$v^3 - \left(b + \frac{RT}{p}\right) v^2 + \frac{av}{p} - \frac{ab}{p} = 0 \quad ...(8.24)$$

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$= pv - pb + \frac{a}{v^2} \times v - \frac{a}{v^2} \times b - RT = 0$$

$$= pv - pb + \frac{a}{v} - \frac{ab}{v^2} - RT = 0$$

Multiplying both sides by $\frac{v^2}{p}$, we get

$$pv \times \frac{v^2}{p} - pb \times \frac{v^2}{p} + \frac{a}{v} \times \frac{v^2}{p} - \frac{ab}{v^2} \times \frac{v^2}{p} - \frac{RT}{p} v^2 = 0$$

$$v^3 - \left(b + \frac{RT}{p}\right) v^2 + \frac{av}{p} - \frac{ab}{p} = 0$$

Now at the critical point, as the three roots are equal, the equation must be of the form :

$$(v - v_c)^3 = 0 \quad ...(8.25)$$

where the suffix c denotes conditions at the critical point. For the critical point equation (8.24) becomes

$$v^3 - \left(b + \frac{RT_c}{p_c}\right) v^2 + \frac{av}{p_c} - \frac{ab}{p_c} = 0 \quad ...(8.26)$$

Equations (8.25) and (8.26) are identical, hence equating coefficients

$$3v_c = b + \frac{RT_c}{p_c},$$

$$3{v_c}^2 = \frac{a}{p_c},$$

$$v_c^3 = \frac{ab}{p_c},$$

and from these by a simple reduction, we have

$$\left.\begin{aligned} v_c &= 3b \\ p_c &= \frac{a}{27b^2} \\ T_c &= \frac{8}{27} \cdot \frac{a}{bR} \end{aligned}\right\} \quad \text{...(8.27)}$$

From these equations it follows that *the critical volume, pressure, and temperature are all completely determined by the constants of equation (8.19).*

The equation (8.27) indicates the critical constants for a particular gas and leads to the following results :

The values of a and b are also given by

$$a = 3p_c\, v_c^2 = \frac{9}{8}\, RT_c\, v_c = \frac{27}{64} \cdot \frac{R^2 T_c^2}{p_c} \quad \text{...}(i)$$

$$b = \frac{v_c}{3} = \frac{RT_c}{8p_c} \quad \text{...}(ii)$$

and

$$R = \frac{8}{3}\, \frac{p_c v_c}{T_c} \quad \text{...}(iii)$$

Using the values of a, b and R in equation (8.23), and substituting in (8.26), we have for carbon dioxide

$$p_c = 61.2 \text{ atmospheres},$$
$$T_c = 305.3 \text{ K or } 32.2°\text{C}.$$

It is frequently assumed that the approximate agreement between the calculated and experimental values of the critical temperature for carbon dioxide is a sufficient verification of Van der Waals' theory, but the constant b cannot be calculated with the required degree of accuracy from Regnault's experiments to make this an adequate test of the theory.

Also from equations (8.27), we have

$$\frac{p_c v_c}{RT_c} = \frac{3}{8} = 0.375$$

whereas experiment shows that about 0.27 as the average value of this ratio, varying considerably, however, from gas to gas.

The Reduced Equation :

When the pressure, volume and temperature of the fluid are expressed as fractions of the critical pressure, volume and temperature the reduced form of Van der Waals' equation is obtained. Thus, writing

$$p = ep_c = \frac{ea}{27b^2},$$
$$v = nv_c = 3nb,$$
$$T = mT_c = \frac{8}{27} \cdot \frac{ma}{bR}$$

and substituting these values in eqn. (8.19), this reduces to

$$\left(e + \frac{3}{n^2}\right)(3n - 1) = 8m$$

In this "*reduced*" equation the three constants which characterised a particular fluid have disappeared. The equation is accordingly true of any substance which satisfies an equation of the Van der Waals type, and the form of the curves connecting e, n and m is the same for all these substances. Thus we see that two substances, the behaviour of each of which is represented by Van der Waals' equation, will be in corresponding states when the pressure, volume and temperature are the same multiples of their critical values.

This *theorem of corresponding states*, enunciated by Van der Waals, was tested by Amagat and found to be approximately true for a large number of fluids. The theorem of corresponding states is not unique to the equation of Van der Waals. Any equation of state giving a critical point and having not more than three constants will serve equally well to give a reduced equation, in which the constants peculiar to any one fluid disappear, and therefore become the basis of the theorem of corresponding states.

It must be remembered in applying the theorem that the accuracy of results deduced by its aid cannot be greater than the accuracy with which the original equation represents the behaviour of the fluids under consideration.

Amagat's Experiments

As per Amagat's experiments Van der Waals' equation accounts for the variation of the product pv with increasing pressure as follows.

Writing equation (8.19) in the form

$$pv = \frac{RTv}{v - b} - \frac{a}{v},$$

and differentiating with respect to p, keeping T constant, we have

$$\left(\frac{d(pv)}{dp}\right)_T = \left\{\frac{a}{v^2} - \frac{RTb}{(v-b)^2}\right\}\left(\frac{dv}{dp}\right)_T \qquad ...(8.28)$$

Since the condition for a minimum on any isothermal is

$$\left(\frac{d(pv)}{dp}\right)_T = 0,$$

the right-hand side of equation (8.28) must vanish at this point. Now $\left(\frac{dv}{dp}\right)_T$ is *never zero,* so we have as the condition for a *minimum* :

$$\frac{RTb}{(v-b)^2} = \frac{a}{v^2} \quad \text{or} \quad RT \cdot \frac{b}{a} = \left(1 - \frac{b}{v}\right)^2 \qquad ...(8.29)$$

This equation shows that the *volume at which the minimum value of pv occurs on any isothermal gradually increases as the temperature is raised*.

To find the locus of minima the temperature T must be eliminated from equation (8.29) by substitution from the original equation. Thus from equation (8.19)

$$RT = \left(p + \frac{a}{v^2}\right)(v - b),$$

and substituting this in equation (8.28), we have

$$\frac{b}{a}\left(p+\frac{a}{v^2}\right)(v-b)=\left(1-\frac{b}{v}\right)^2,$$

which reduces to $v = b\left(\frac{pv^2}{a}+2\right)$

Multiply each side of this equation by p, and put $pv = y$ and $p = x$, and we obtain

$$y = b\left(\frac{y^2}{a}+2x\right) \quad \text{or} \quad y(a-by)=2abx$$

The above expression gives the locus of minima and is a parabola with axis parallel to the x-axis as shown in Fig. 8.8.

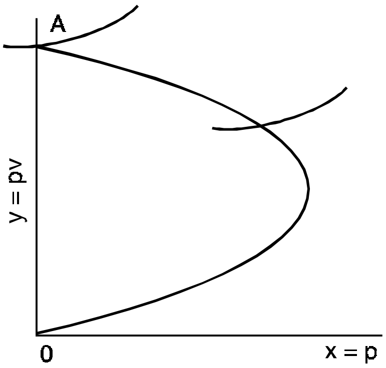


Fig. 8.8

Consider the isothermal which goes through the point A. Here $x = 0$ and $y = \frac{a}{b}$.

Writing Van der Waals' equation in terms of x and y, we have

$$\left(1+\frac{ax}{y^2}\right)(y-bx)=RT,$$

and substituting the coordinates of the point A

$$RT=\frac{a}{b}$$

or $$T=\frac{a}{bR} \qquad ...(8.30)$$

For temperatures above that given by equation (8.30) the minima lie in the region of negative pressure, so an Amagat isothermal for a temperature equal to or greater than $\frac{a}{bR}$ will slope upwards along its whole length for increasing values of p, but for a temperature less than $\frac{a}{bR}$ the isothermals first dip to a minimum and then rise.

Using the result from equation (8.27)

$$T_c=\frac{8}{27}\,\frac{a}{bR},$$

we see that the limiting temperature for an isothermal to show a *minimum* is

$$T=\frac{27}{8}\,.\,T_c$$

The reason for Amagat finding no dip in the isothermals for hydrogen is now apparent. The critical temperature is 35 K, and therefore the limiting temperature above which minima do not occur is $\frac{27}{8} \times 35 = 118.1$ K or – 155°C, and all Amagat's experiments were conducted between 0°C and 100°C.

The Cooling effect :

The most gases show an inversion of the cooling effect at a certain temperature. The equation of Van der Waals indicates at what temperature this occurs.

We have

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \qquad \text{[From eqn. (8.15)]}$$

Keeping p constant and differentiating with respect to T, we get

$$\left\{p - \frac{a}{v^2} + \frac{2ab}{v^3}\right\}\left(\frac{dv}{dT}\right)_p = R$$

or

$$\left(\frac{dv}{dT}\right)_p = \frac{R}{\left\{p - \frac{a}{v^2} + \frac{2ab}{v^3}\right\}}$$

Substituting this value of $\left(\frac{dv}{dT}\right)_p$ in the equation $c_p\mu = T\left(\frac{dv}{dT}\right)_p - v$ (where μ is a measure of cooling effect), we get

$$c_p\mu = \frac{RT}{\left\{p - \frac{a}{v^2} + \frac{2ab}{v^3}\right\}} - v$$

and substituting for RT from equation (8.19) this reduces to

$$c_p\mu = \frac{-bp + \frac{2a}{v} - \frac{3ab}{v^2}}{p - \frac{a}{v^2} + \frac{2ab}{v^3}}$$

The denominator of this expression is *always positive*, since it is $R\left(\frac{dT}{dv}\right)_p$. Hence the cooling effect, μ, is *positive* if

$$bp < \frac{2a}{v} - \frac{3ab}{v^2} \qquad \text{...(8.31)}$$

and *negative* if

$$bp > \frac{2a}{v} - \frac{3ab}{v^2} \qquad \text{...(8.32)}$$

and *inversion occurs* when

$$bp = \frac{2a}{v} - \frac{3ab}{v^2}$$

or

$$p = \frac{a}{b}\left(\frac{2}{v} - \frac{3b}{v^2}\right) \qquad \text{...(8.33)}$$

In order to get the temperature of inversion this equation must be combined with the original equation. Thus

$$\frac{2a}{b}\left(1 - \frac{b}{v}\right)^2 = RT \qquad \text{...(8.34)}$$

Since v is necessarily always greater than b, it will be seen that as v *increases so also does the temperature of inversion*.

The form of curve given by equation (8.33) is shown in Fig. 8.9. The pressure is zero when $v = \frac{3b}{2}$, or infinity. These values of v determine the limiting values of the temperature of inversion, as it is only between these limits that p is positive. Substituting these limits of v in equation (8.34) the limiting values of the temperature of inversion are $\frac{2}{9}\frac{a}{bR}$ and $\frac{2a}{bR}$, or from (8.27), $\frac{3}{4}T_c$ and $\frac{27}{4}T_c$.

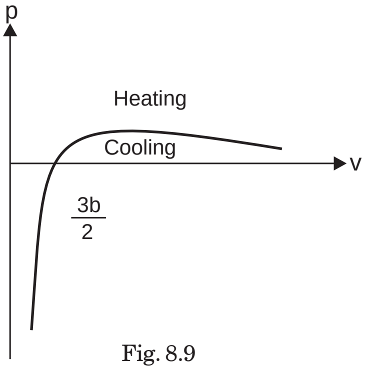


Fig. 8.9

The equation (8.33) being quadratic there are two values of v for a constant value of p at which inversion occurs, as may also be seen by reference to Fig. 8.9. Consequently by equation (8.34) there are two temperatures for a constant value of p at which inversion occurs. As the temperature increases through the lower of these values the change is from a heating to a cooling effect, and as it increases through the higher of these values the change is from a cooling to a heating effect.

The inversion will occur when the maximum value of p is $\frac{a}{3b^2}$, when $v = 3b$. For any value of p less than this there is a cooling effect provided the condition of the substance is represented by a point inside the area enclosed by the curve and the axis of volume, Fig. 8.9, and for any greater value of p there is a heating effect as indicated by equations (8.31) and (8.32) respectively.

Let us take the case of *hydrogen*. In the experiments of Joule and Thomson the pressure used was 4.7 atmospheres. The critical temperature and pressure are 35 K and 15 atmospheres. From equation (8.33) we can find the values of $\frac{b}{v}$ corresponding to the pressure used by Joule and Thomson, and by substitution in equation (8.34) find the two temperatures at which inversion occurs at this pressure. Equation (8.33) can be written as :

$$p = 27p_c\left[2\frac{b}{v} - 3\left(\frac{b}{v}\right)^2\right]$$

Hence
$$\frac{b}{v} = \frac{2 \pm \sqrt{4 - \frac{12p}{27\,p_c}}}{6} = 0.6608 \text{ or } 0.0058$$

by substitution of the above values for p and p_c.

Writing equation (8.34) in the form

$$T = \frac{27}{4}\,T_c\left(1 - \frac{b}{v}\right)^2,$$

we have by substitution for $\frac{b}{v}$: T = 233.5 K or 27.2 K that is, below – 245.9°C there would be a *heating effect*, between – 245.9°C and – 39.6°C *a cooling effect*, and above – 39.6°C a heating effect. Thus **Van der Waals' equation qualitatively accounts for the heating effect observed at ordinary temperatures.**

Limitations of Van der Waals' Equation

Van der Waals' equation under actual condition becomes *invalid* as discussed below :

— The values of a and b (which are assumed to be constant) are found to vary with temperature. Thus the results obtained from the equation are *incorrect when the variation of a and b is large with respect to temperature.*

— The equation is *not accurate enough in the critical region* and it is also obvious from its derivation.

8.8. VIRIAL EQUATION OF STATE

The virial (a Latin word used for force which refers to interaction forces between molecules) equation of state may be expressed as follows :

$$\frac{pv}{RT} = A_0 + A_1 p + A_2 p^2 + A_3 p^3 + \ldots\ldots. \qquad \ldots(8.35)$$

or

$$\frac{pv}{RT} = B_0 + \frac{B_1}{v} + \frac{B_2}{v^2} + \frac{B_3}{v^3} + \ldots\ldots \qquad \ldots(8.36)$$

where A_0, A_1, ... and B_0, B_1, ... are called the *virial co-efficients which are functions of temperature only*.

— The virial equation can be used only for gases at low and medium densities.

— The *advantage* of virial equation is that the *virial co-efficients can be determined from experimental p-v-T data*.

8.9. BEATTIE-BRIDGEMAN EQUATION

Beattie-Bridgeman equation is expressed as follows :

$$p = \frac{R_0 T(1-e)}{(\bar{v})^2}(\bar{v}+B) - \frac{A}{(\bar{v})^2} \qquad \ldots(8.37)$$

where p = pressure

$$A = A_0\left(1 - \frac{a}{\bar{v}}\right)$$

$$B = B_0\left(1 - \frac{b}{\bar{v}}\right)$$

and $e = \dfrac{c}{\bar{v}T^3}$

The factors A_0, a, B_0, b and c are constants whose values for different gases are given in Table 8.2.

— This equation is normally used for substances at pressures less than critical pressure.

— The equation is accurate enough when the volumes involved are greater than twice the critical volume.

— The equation fits the data of fourteen gases down to the critical point and over a wide range of pressure within ± 0.5% error. However, it is *inaccurate near critical point.*

Table 8.2. Constants of the Beattie-Bridgeman Equation of State

Gas	A_0	*a*	B_0	*b*	$c \times 10^{-4}$
Hydrogen (H_2)	20.0117	– 0.00506	0.02096	– 0.04359	0.0504
Oxygen (O_2)	151.0857	0.02562	0.04624	0.004208	4.80
Carbon dioxide (CO_2)	507.2836	0.07132	0.10476	0.07235	66.0
Helium (He)	2.1886	0.05984	0.01400	0.0	0.0040
Air	131.8441	0.01931	0.04611	– 0.001101	4.34
Nitrogen	136.2315	0.02617	0.05046	– 0.00691	4.20
Argon	130.7802	0.02328	0.03931	0.0	5.99

8.10. REDUCED PROPERTIES

The ratios of pressure, temperature and specific volume of a real gas to the corresponding critical values are called the **reduced properties**.

$$p_r = \frac{p}{p_c}\ ,\ T_r = \frac{T}{T_c}\ ,\ v_r = \frac{v}{v_c} \qquad \text{...(8.38)}$$

Table 8.3. Critical Constants

Substance	*Pressure (p_c) bar*	*Temperature (T_c) K*
Air	37.69	132.5
Argon	48.64	151.0
Carbon dioxide	73.86	304.2
Carbon monoxide	34.96	133.0
Helium	2.29	5.3
Hydrogen	12.97	33.3
Nitrogen	33.94	126.2
Oxygen	50.76	154.8
Water	228.59	647.15
Ethane	48.84	305.5
Ethylene	51.17	282.4
Methane	46.41	191.1
Propane	42.55	370.0

Table 8.4. Properties of Gases

Gas	*Molecular weight (M)*	c_p *(kJ/kg K)*	c_v *(kJ/kg K)*	$R = c_p - c_v$ *(kJ/kg K)*	$\gamma = \frac{c_p}{c_v}$	$Z_c = \frac{p_c \bar{v}}{R_0 T_c}$
Air	28.97	1.005	0.718	0.287	1.4	0.284
Oxygen	32	0.920	0.660	0.260	1.4	0.307
Nitrogen	28	1.046	0.754	0.292	1.39	0.291
Hydrogen	2	14.40	10.40	4.0	1.38	0.304
Carbon monoxide	28	1.046	0.754	0.292	1.39	—

Carbon dioxide	44	0.840	0.650	0.190	1.3	0.274
Water	18	—	—	0.462	—	0.230
Methane	16	2.22	1.70	0.520	1.3	—
Sulphur dioxide	64	0.796	0.67	0.126	1.19	0.268
Ammonia	17	—	—	0.488	—	—

8.11. LAW OF CORRESPONDING STATES

If any two gases have equal values of reduced pressure and reduced temperature, then they have same values of reduced volume ; i.e., $v_R = f(T_r, p_r)$ *for all gases and the function is the same*.

This law is most accurate in the *vicinity of the critical point*.

8.12. COMPRESSIBILITY CHART

The *compressibility factor* (Z) of any gas is a function of only two properties, usually temperature and pressure, so that $Z = f(T_r, p_r)$ except near the critical point. The value of Z for any real gas may be less or more than unity, depending on pressure and temperature conditions of the gas.

The general compressibility chart is plotted with Z versus p_r for various values of T_r. This is constructed by plotting the known data of one or more gases and can be used for any gas. Such a chart is shown in Fig. 8.10. This chart gives best results for the regions well removed from the critical state for all gases.

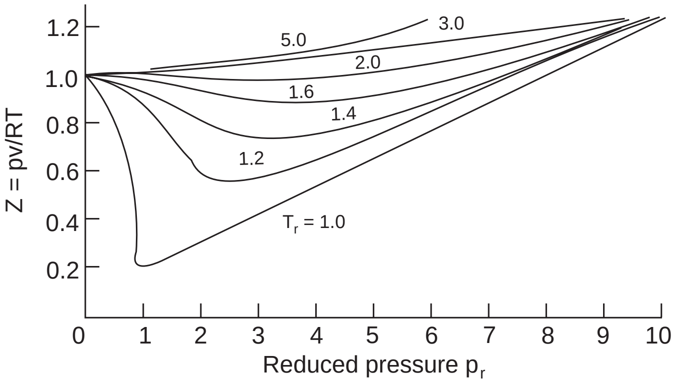


Fig. 8.10. Generalised compressibility chart.

IDEAL GASES

Example 8.1. *The volume of a high altitude chamber is 40 m³. It is put into operation by reducing pressure from 1 bar to 0.4 bar and temperature from 25°C to 5°C.*

How many kg of air must be removed from the chamber during the process ? Express this mass as a volume measured at 1 bar and 25°C.

Take R = 287 J/kg K for air.

Solution. $V_1 = 40 \text{ m}^3$ $V_2 = 40 \text{ m}^3$

$p_1 = 1 \text{ bar}$ $p_2 = 0.4 \text{ bar}$

$T_1 = 25 + 273 = 298 \text{ K}$ $T_2 = 5 + 273 = 278 \text{ K}$

kg of air to be removed :

Assuming nitrogen to be a perfect gas,

$$p_1V_1 = m_1RT_1 \qquad \left(\text{or } m_1 = \frac{p_1V_1}{RT_1}\right)$$

$$p_2V_2 = m_2RT_2 \qquad \left(\text{or } m_2 = \frac{p_2V_2}{RT_2}\right)$$

Mass of air removed during the process = $(m_1 – m_2)$ kg

$$(m_1 – m_2) = \frac{p_1V_1}{RT_1} - \frac{p_2V_2}{RT_2}$$

$$= \frac{1}{R}\left(\frac{p_1V_1}{T_1} - \frac{p_2V_2}{T_2}\right) = \frac{1}{287}\left[\frac{(1\times10^5)\times40}{298} - \frac{(0.4\times10^5)\times40}{278}\right]$$

$$= \mathbf{26.71\ kg.} \quad \textbf{(Ans.)}$$

Volume of this mass of gas at 1 bar and 25°C is given by

$$V = \frac{mRT}{p} = \frac{26.71\times287\times298}{1\times10^5} = \mathbf{22.84\ m^3.} \quad \textbf{(Ans.)}$$

Example 8.2. *A steel flask of 0.04 m^3 capacity is to be used to store nitrogen at 120 bar, 20°C. The flask is to be protected against excessive pressure by a fusible plug which will melt and allow the gas to escape if the temperature rises too high.*

(*i*) *How many kg of nitrogen will the flask hold at the designed conditions ?*

(*ii*) *At what temperature must the fusible plug melt in order to limit the pressure of a full flask to a maximum of 150 bar ?*

Solution. Capacity of the steel flask, $V = 0.04\ \text{m}^3$

Pressure, $p = 120$ bar

Temperature, $T = 20 + 273 = 293$ K

(*i*) **kg of nitrogen the flask can hold :**

Now, R for nitrogen (molecular weight, $M = 28$)

$$= \frac{R_0}{M} = \frac{8314}{28} = 296.9 \text{ J/kg K}$$

Assuming nitrogen to be a perfect gas, we get

Mass of nitrogen in the flask at designed condition

$$= m = \frac{pV}{RT} = \frac{120\times10^5\times0.04}{296.9\times293} = \mathbf{5.51\ kg.\ (Ans.)}$$

(*ii*) **Temperature at which fusible plug should melt, t :**

When the fusible plug is about to melt

$$p = 150 \text{ bar} ;\ V = 0.04\ \text{m}^3 ;\ m = 5.51 \text{ kg}$$

Therefore, temperature t at which fusible plug must melt is given by

$$T = \frac{pV}{mR} = \frac{150\times10^5\times0.04}{5.51\times296.9} = 366.7 \text{ K}$$

$$\therefore \qquad \mathbf{t} = 366.7 - 273 = \mathbf{93.7°C.} \quad \textbf{(Ans.)}$$

Example 8.3. *A balloon of spherical shape 6 m in diameter is filled with hydrogen gas at a pressure of 1 bar abs. and 20°C. At a later time, the pressure of gas is 94 per cent of its original pressure at the same temperature :*

(*i*) *What mass of original gas must have escaped if the dimensions of the balloon is not changed ?*

(*ii*) *Find the amount of heat to be removed to cause the same drop in pressure at constant volume.*

Solution. Diameter of the spherical balloon = 6 m

Pressure of hydrogen gas, p_1 = 1 bar abs.

Temperature of hydrogen gas, T_1 = 20°C or 293 K

At a later time pressure of the gas, $p_2 = 0.94p_1$ at 293 K.

(*i*) **Mass of original gas escaped :**

$$\Delta m = m_1 - m_2$$

[where m_1 and m_2 are the initial and final masses of the gas]

$$= \frac{p_1V_1}{RT_1} - \frac{p_2V_2}{RT_2} = \frac{V_1}{RT_1}(p_1 - p_2) \quad [\because \quad V_1 = V_2,\ T_1 = T_2 \text{ and } p_2 = 0.94p_1]$$

$$= \frac{V_1}{RT_1}(p_1 - 0.94p_1) = \frac{p_1V_1}{RT_1}(1 - 0.94)$$

$\therefore$ **%age mass escaped** $= \frac{\Delta m}{m_1} \times 100$

$$= \frac{\frac{p_1V_1}{RT_1}(1-0.94)}{\frac{p_1V_1}{RT_1}} = \textbf{6\%}. \quad \textbf{(Ans.)}$$

(*ii*) **Amount of heat to be removed :**

Using the gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

or
$$\frac{p_1}{T_1} = \frac{0.94p_1}{T_2} \qquad (\because \quad V_1 = V_2 \text{ and } p_2 = 0.94p_1)$$

$\therefore$ $T_2 = 0.94T_1 = 0.94 \times 293 = 275.4$ K or 2.42°C

The heat to be removed is given by

$$Q = mc_v(T_1 - T_2)$$

where
$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times \frac{4}{3}\pi \times 3^3}{\frac{8314}{2} \times 293} = 9.28 \text{ kg} \qquad \begin{bmatrix} \because & MR = 8314 \\ \therefore & R = \frac{8314}{2} \\ \text{as } M \text{ for } H_2 = 2 & \end{bmatrix}$$

c_v = 10400 J/kg K for H_2

$\therefore$ Q (heat to be removed) = 9.28 × 10400 (293 – 275.4) = **1.69 MJ. (Ans.)**

Example 8.4. *A vessel of capacity 3 m^3 contains 1 kg mole of N_2 at 90°C.*

(*i*) *Calculate pressure and the specific volume of the gas.*

(*ii*) *If the ratio of specific heats is 1.4, evaluate the values of c_p and c_v.*

(*iii*) *Subsequently, the gas cools to the atmospheric temperature of 20°C ; evaluate the final pressure of gas.*

(*iv*) *Evaluate the increase in specific internal energy, the increase in specific enthalpy, increase in specific entropy and magnitude and sign of heat transfer.*

Solution. Mass of N_2, m = 1 kg mole *i.e.,* 28 kg

Capacity of the vessel, $V_1 = 3 \text{ m}^3$

Temperature, $T_1 = 90 + 273 = 363 \text{ K}$

(*i*) **Pressure (p_1) and specific volume (v_1) of the gas :**

Using the relation

$$p_1V_1 = mRT_1$$

$$p_1 \times 3 = 28 \times \left(\frac{8314}{28}\right) \times 363 \qquad \left[\because \ R = \frac{R_0}{M} = \frac{8314}{28}\right]$$

$\therefore$ $\mathbf{p_1 = 1005994 \ J/m^2}$ **or 10.06 bar. (Ans.)**

Specific volume, $\mathbf{v_1} = \frac{V_1}{m} = \frac{3}{28} = \mathbf{0.107 \ m^3/kg.}$ **(Ans.)**

(*ii*) $\mathbf{c_p = ?, c_v = ?}$

$$\frac{c_p}{c_v} = 1.4 \text{ (given)} \qquad ...(i)$$

But $$c_p - c_v = R = \frac{8314}{28} \qquad ...(ii)$$

Solving for c_p and c_v between (*i*) and (*ii*)

$\mathbf{c_p = 1.039 \ kJ/kg \ K \ ; \ c_v = 0.742 \ kJ/kg \ K.}$ **(Ans.)**

(*iii*) **Final pressure of the gas after cooling to 20°C :**

Initially	*After cooling*
$p_1 = 10.06$ bar	p_2 = ?
$V_1 = 3 \text{ m}^3$	$V_2 = 3 \text{ m}^3$
$T_1 = 363$ K	$T_2 = 20 + 273 = 293$ K

Now, $$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

or $$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad (\text{as } V_1 = V_2)$$

$\therefore$ $$\mathbf{p_2} = \frac{p_1T_2}{T_1} = \frac{10.06 \times 293}{363} = \mathbf{8.12 \ bar.} \ \textbf{(Ans.)}$$

(*iv*) $\mathbf{\Delta u, \Delta h, \Delta s, Q}$ **:**

For a perfect gas,

Increase in specific internal energy

$$\Delta \mathbf{u} = c_v(T_2 - T_1) = 0.742(293 - 363) = \mathbf{-51.94 \ kJ/kg.} \ \textbf{(Ans.)}$$

Increase in specific enthalpy,

$$\Delta \mathbf{h} = c_p(T_2 - T_1) = 1.039(293 - 363) = \mathbf{-72.73 \ kJ/kg.} \ \textbf{(Ans.)}$$

Increase in specific entropy,

$$\Delta \mathbf{s} = c_v \log_e \left(\frac{T_2}{T_1}\right) + R \log_e \left(\frac{v_2}{v_1}\right)$$

But $v_1 = v_2$

$$\therefore \quad \Delta s = c_v \log_e \left(\frac{T_2}{T_1}\right) = 0.742 \log_e \left(\frac{293}{363}\right) = \mathbf{-0.1589\ kJ/kg\ K.} \quad \textbf{(Ans.)}$$

Now, $Q = \Delta u + W$

Here $W = 0$ as change in volume is zero

$\therefore$ $Q = \Delta u$

$\therefore$ **Heat transfer,** $\mathbf{Q} = -51.94$ kJ/kg $= -51.94 \times 28 =$ **– 1454.32 kJ. (Ans.)**

Example 8.5. (*a*) *1 kg of air at a pressure of 8 bar and a temperature of 100°C undergoes a reversible polytropic process following the law* $pv^{1.2}$ = *constant. If the final pressure is 1.8 bar determine :*

(*i*) *The final specific volume, temperature and increase in entropy ;*

(*ii*) *The work done and the heat transfer.*

Assume $R = 0.287$ *kJ/kg K and* $\gamma = 1.4$.

(*b*) *Repeat (a) assuming the process to be irreversible and adiabatic between end states.*

Solution. (*a*) Mass of air, $m = 1$ kg

Pressure, $p_1 = 8$ bar

Temperature, $T_1 = 100 + 273 = 373$ K

The law followed : $pv^{1.2}$ = constant

Final pressure, $p_2 = 1.8$ bar

Characteristic gas constant, $R = 0.287$ kJ/kg K

Ratio of specific heats, $\gamma = 1.4$

(*i*) $\mathbf{v_2}$, $\mathbf{T_2}$ **and** Δ**s :**

Assuming air to be a perfect gas,

$$p_1 v_1 = RT_1$$

$$\therefore \quad v_1 = \frac{RT_1}{p_1} = \frac{(0.287 \times 1000) \times 373}{8 \times 10^5} = 0.1338 \text{ m}^3/\text{kg}$$

Also, $p_1 v_1^{1.2} = p_2 v_2^{1.2}$

or
$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/1.2}$$

or
$$v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{1/1.2} = 0.1338 \left(\frac{8}{1.8}\right)^{1/1.2} = 0.4637 \text{ m}^3/\text{kg}$$

i.e., Final specific volume, $\mathbf{v_2 = 0.4637\ m^3/kg.}$ **(Ans.)**

Again, $p_2 v_2 = RT_2$

$$T_2 = \frac{p_2 v_2}{R} = \frac{1.8 \times 10^5 \times 0.4637}{(0.287 \times 1000)} = 290.8 \text{ K}$$

i.e., Final temperature, $\mathbf{t_2} = 290.8 - 273 =$ **17.8°C. (Ans.)**

Increase in entropy Δs is given by,

$$\Delta s = c_v \log_e \left(\frac{T_2}{T_1}\right) + R \log_e \left(\frac{v_2}{v_1}\right)$$

But $$\gamma = \frac{c_p}{c_v} = 1.4 \text{ (given)} \quad ...(i)$$

and $$c_p - c_v = R \text{ (= 0.287 kJ/kg K for air)} \quad ...(ii)$$

Solving for c_v between (*i*) and (*ii*),

$$c_v = 0.717 \text{ kJ/kg K}$$

$$\therefore \quad \Delta s = 0.717 \log_e \left(\frac{290.8}{373}\right) + 0.287 \log_e \left(\frac{0.4637}{0.1338}\right)$$

$$= -0.1785 + 0.3567 = 0.1782 \text{ kJ/kg K}$$

i.e., Increase in entropy, **Δs = 0.1782 kJ/kg K. (Ans.)**

(*ii*) **Work done and heat transfer :**

The work done in a polytropic process is given by,

$$W = \frac{p_1v_1 - p_2v_2}{n-1} = \frac{R(T_1 - T_2)}{n-1}$$

$$= \frac{0.287\,(373 - 290.8)}{(1.2 - 1)} = 117.96 \text{ kJ/kg}$$

i.e., **Work done = 117.96 kJ/kg. (Ans.)**

Heat transfer, $Q = \Delta u + W$

where $\Delta u = c_v(T_2 - T_1)$

$= 0.717\ (290.8 - 373) = -58.94$ kJ/kg

$\therefore$ $Q = -58.94 + 117.96 = 59.02$ kJ/kg

Hence **heat transfer = 59.02 kJ/kg. (Ans.)**

(*b*) (*i*) Though the process is assumed now to be irreversible and adiabatic, the end states are given to be the same as in (*a*). Therefore, **all the properties at the end of the process are the same as in (a). (Ans.)**

(*ii*) As the process is adiabatic, Q (heat transfer) = **0. (Ans.)**

$$\Delta u = \Delta u \text{ in } (a)$$

Applying first law for this process

$$Q = \Delta u + W$$

$$0 = \Delta u + W$$

or $$W = -\Delta u$$

$$= -(-58.94) = 58.94$$

$\therefore$ **Work done = 58.94 kJ/kg. (Ans.)**

Example 8.6. *Two spheres each 2.5 m in diameter are connected to each other by a pipe with a valve as shown in Fig. 8.11. One sphere contains 16 kg of air and other 8 kg of air when the valve is closed. The temperature of air in both sphere is 25°C. The valve is opened and the whole system is allowed to come to equilibrium conditions. Assuming there is no loss or gain of energy, determine the pressure in the spheres when the system attains equilibrium.*

Neglect the volume of the pipe.

Solution. Volume of each sphere $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times \left(\frac{2.5}{2}\right)^3 = 8.18 \text{ m}^3$

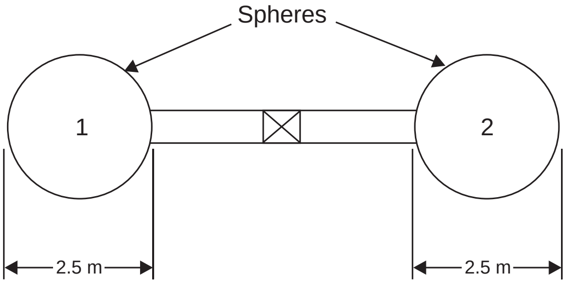


Fig. 8.11

The temperature in both spheres is same (25°C)

i.e., $$T_1 = T_2 = 25 + 273 = 298 \text{ K}$$

As no energy exchange occurs, the temperature reached after equilibrium is 298 K.

Mass of air in sphere 1, $m_1 = 16$ kg

Mass of air in sphere 2, $m_2 = 8$ kg

After opening the valve

Total volume, $V = 8.18 + 8.18 = 16.36 \text{ m}^3$

Total mass, $m = m_1 + m_2 = 16 + 8 = 24$ kg

Now using characteristic gas equation

$$pV = mRT$$

$$\therefore \quad p = \frac{mRT}{V} = \frac{24 \times 287 \times 298}{16.36} = 1.255 \times 10^5 \text{ N/m}^2 \text{ or } 1.255 \text{ bar}$$

Hence **pressure in the spheres when the system attains equilibrium = 1.255 bar. (Ans.)**

Example 8.7. *CO_2 flows at a pressure of 10 bar and 180°C into a turbine, located in a chemical plant, and there it expands reversibly and adibatically to a final pressure of 1.05 bar. Calculate the final specific volume, temperature and increase in entropy. Neglect changes in velocity and elevation.*

If the mass flow rate is 6.5 kg/min. evaluate the heat transfer rate from the gas and the power delivered by the turbine.

Assume CO_2 to be a perfect gas and c_v = 0.837 kJ/kg K.

Solution. *At entry to turbine* *At exit of turbine*

Pressure, $p_1 = 10$ bar Pressure, $p_2 = 1.05$ bar

Temperature, $T_1 = 180 + 273 = 453$ K

Since the expansion is reversible and adiabatic, therefore, the equation pv^γ = constant is applicable.

$$\therefore \quad p_1 v_1^\gamma = p_2 v_2^\gamma \quad \text{...}(i)$$

Eliminating v_1 and v_2 using the perfect gas equation

$$v = \frac{RT}{p}$$

We can write equation (*i*) as

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(\gamma - 1)/\gamma}$$

$$\therefore \qquad \frac{453}{T_2} = \left(\frac{10}{1.05}\right)^{(\gamma-1)/\gamma}$$

$$c_v = 0.837 \text{ kJ/kg K} \quad \text{(given)}$$

$$R = \frac{R_0}{M} = \frac{8.314}{44} \quad \text{(Molecular weight of } CO_2 = 44\text{)}$$

$$= 0.1889 \text{ kJ/kg K}$$

Also $\qquad c_p - c_v = R$

$\therefore \qquad c_p - 0.837 = 0.1889$

$$c_p = 1.0259 \text{ kJ/kg K}$$

$$\therefore \qquad \gamma = \frac{c_p}{c_v} = \frac{1.0259}{0.837} = 1.23$$

Substituting for γ in equation (*ii*)

$$\frac{453}{T_2} = \left(\frac{10}{1.05}\right)^{(1.23-1)/1.23}$$

$\therefore \qquad T_2 = 297$ K

Final temperature $= 297 - 273 =$ **24°C. (Ans.)**

$$p_2v_2 = RT_2$$

$$\therefore \qquad 1.05 \times 10^5 \times v_2 = (0.1889 \times 1000) \times 297$$

$$\therefore \qquad v_2 = \frac{(0.1889 \times 1000) \times 297}{1.05 \times 10^5} = 0.5343 \text{ m}^3\text{/kg}$$

i.e., **Final specific volume = 0.5343 m³/kg. (Ans.)**

As the process is reversible and adiabatic

$$\Delta s = 0$$

i.e., **Increase in entropy = 0. (Ans.)**

Since the process is adiabatic, therefore, **heat transfer rate from turbine = 0. (Ans.)**

Applying steady flow energy equation (S.F.E.E.) on unit time basis,

$$\dot{m}\left[h_1 + \frac{C_1^2}{2} + Z_1\right] + \dot{Q} = \dot{m}\left[h_2 + \frac{C_2^2}{2} + Z_2\right] + W$$

By data changes in velocity and elevation are negligible, and $Q = 0$.

$\therefore$ S.F.E.E. reduces to

i.e., $\qquad W = \dot{m}(h_1 - h_2)$

$$= \dot{m}\, c_p(T_1 - T_2) \qquad \left[as\ \frac{dh}{dT} = c_p, h_1 - h_2 = c_p(T_1 - T_2)\right]$$

$$= \frac{6.5}{60} \times 1.0259\,(453 - 297) = 17.34 \text{ kW}$$

Hence power delivered by the turbine = 17.34 kW. (Ans.)

Example 8.8. *A certain quantity of air initially at a pressure of 8 bar and 280°C has a volume of 0.035 m³. It undergoes the following processes in the following sequence in a cycle :*

(*a*) *Expands at constant pressure to 0.1 m³,* (*b*) *Follows polytropic process with* $n = 1.4$, *and*

(*c*) *A constant temperature process (which completes the cycle).*

Evaluate the following :

(*i*) *The heat received in the cycle ;*

(*ii*) *The heat rejected in the cycle ;*

(*iii*) *Efficiency of the cycle.*

Solution. Fig. 8.12 shows the cycle on *p-V* and *T-s* planes.

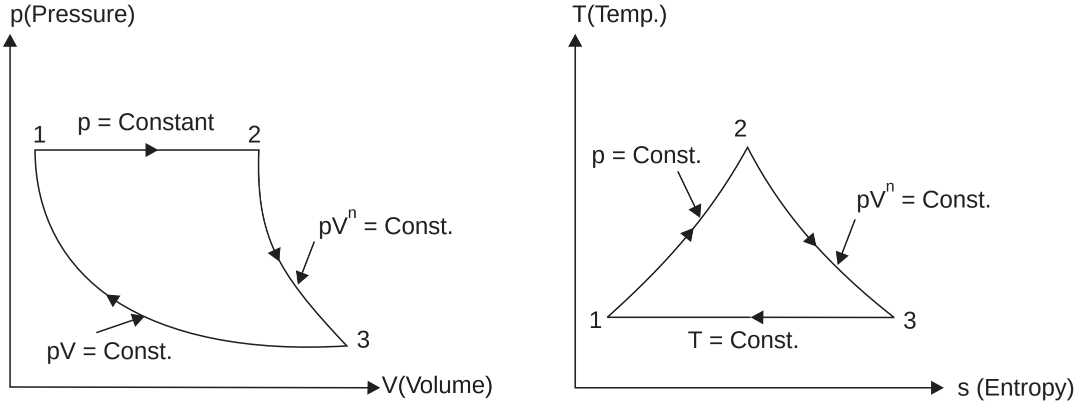


Fig. 8.12

Pressure, $p_1 = 8$ bar

Volume, $V_1 = 0.035$ m^3

Temperature, $T_1 = 280 + 273 = 553$ K

Pressure, $p_2 = 8$ bar $(= p_1)$

Volume, $V_2 = 0.1$ m^3

Index, $n = 1.4$

To find mass of air, use the relation

$$p_1V_1 = mRT_1$$

$$\therefore \quad m = \frac{p_1V_1}{RT_1} = \frac{8 \times 10^5 \times 0.035}{287 \times 553} = 0.1764 \text{ kg}$$

From $\quad p_2V_2 = mRT_2$

$$T_2 = \frac{p_2V_2}{mR} = \frac{8 \times 10^5 \times 0.1}{0.1764 \times 287} = 1580 \text{ K}$$

Also, $\quad p_2V_2^{1.4} = p_3V_3^{1.4}$

and $$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{(1.4-1)/1.4}$$

But $T_3 = T_1$ as 1 and 3 are on an isothermal line.

$$\therefore \quad \frac{1580}{553} = \left(\frac{8}{p_3}\right)^{0.4/1.4}$$

$$2.857 = \left(\frac{8}{p_3}\right)^{0.2857}$$

$$p_3 = \frac{8}{(2.857)^{1/0.2857}} = \frac{8}{(2.857)^{3.5}} = 0.2 \text{ bar}$$

Now, $p_3V_3 = mRT_3$

$\therefore$ $0.2 \times 10^5 \times V_3 = 0.1764 \times 287 \times 553$

$\therefore$ $$V_3 = \frac{0.1764 \times 287 \times 553}{0.2 \times 10^5} = 1.399 \text{ m}^3$$

(*i*) **The heat received in the cycle :**

Applying first law to the constant pressure *process 1-2*,

$$Q = \Delta U + W$$

$$W = \int_1^2 pdV \qquad \text{(as the process is reversible)}$$

$= p(V_2 - V_1)$

$= 8 \times 10^5 (0.1 - 0.035)$

$= 52000$ J or 52 kJ (work done by air)

$\therefore$ $Q = m \times c_v(T_2 - T_1) + 52$

$= 0.1764 \times 0.71(1580 - 553) + 52 = 180.6$ kJ

i.e., Heat received = 180.6 kJ

Applying first law to reversible polytropic *process 2-3*

$$Q = \Delta U + W$$

But $$W = \frac{p_2V_2 - p_3V_3}{n-1} = \frac{mR(T_2 - T_3)}{n-1}$$

$$= \frac{0.1764 \times 0.287\,(1580 - 553)}{1.4 - 1} = 129.98 \text{ kJ (work done } \textbf{by air}\text{)}$$

$\therefore$ $Q = mc_v(T_3 - T_2) + 129.98$

$= 0.1764 \times 0.71\,(553 - 1580) + 129.98$

$= -\,128.6 + 129.98 = 1.354$ kJ (heat received)

$\therefore$ **Total heat received in the cycle** = 180.6 + 1.354 = **181.954 kJ. (Ans.)**

(*ii*) **The heat rejected in the cycle :**

Applying first law to reversible isothermal *process 3-1*,

$$Q = \Delta U + W$$

$$W = p_3V_3 \log_e \left(\frac{V_1}{V_3}\right)$$

$$= 0.2 \times 10^5 \times 1.399 \times \log_e \left(\frac{0.035}{1.399}\right) \times 10^{-3}$$

$= -\,103.19$ kJ (work done **on** the air)

$\therefore$ $Q = mc_v(T_1 - T_3) + W$

$= 0 - 103.19 = -\,103.19$ kJ $\qquad (\because \; T_1 = T_3)$

Hence heat rejected in the cycle = 103.19 kJ. (Ans.)

(*ii*) **Efficiency of the cycle, η_{cycle} :**

$$\eta_{cycle} = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$$

$$= \frac{181.954 - 103.19}{181.954} = 0.433 \text{ or } 43.3\%$$

i.e., **Efficiency of the cycle = 43.3%. (Ans.)**

REAL GASES

Example 8.9. *One kg of CO_2 has a volume of 1 m^3 at 100°C. Compute the pressure by*

(*i*) *Van der Waals' equation*

(*ii*) *Perfect gas equation.*

Solution. (*i*) **Using Van der Waals' equation** :

Molar specific volume, $\bar{v} = 1 \times 44 = 44$ m^3/kg-mol ($\because$ M for $CO_2 = 44$)

Temperature, $T = 100 + 273 = 373$ K

The values of a and b for CO_2 (from Table 8.1)

$a = 362850$ $Nm^4/(kg\text{-}mol)^2$

and $b = 0.0423$ m^3/kg-mol

$R_0 = 8314$ Nm/kg-mol K

Van der Waals' equation is written as

$$\left(p + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = R_0 T$$

or

$$p = \left(\frac{R_0 T}{\bar{v} - b} - \frac{a}{\bar{v}^2}\right)$$

Substituting the values in the above equation, we get

$$\therefore \quad \mathbf{p} = \frac{8314 \times 373}{44 - 0.0423} - \frac{362850}{44^2}$$

$$= 70548 - 187 = \mathbf{70361 \ N/m^2 \ or \ 0.7036 \ bar.} \quad \textbf{(Ans.)}$$

(*ii*) **Using perfect gas equation :**

$$p\bar{v} = R_0 T$$

$$\therefore \quad \mathbf{p} = \frac{R_0 T}{\bar{v}} = \frac{8314 \times 373}{44} = \mathbf{70480 \ N/m^2 \ or \ 0.7048 \ bar.} \quad \textbf{(Ans.)}$$

Example 8.10. *A container of 3 m^3 capacity contains 10 kg of CO_2 at 27°C. Estimate the pressure exerted by CO_2 by using :*

(*i*) *Perfect gas equation*

(*ii*) *Van der Waals' equation*

(*iii*) *Beattie Bridgeman equation.*

Solution. Capacity of the container, $V = 3$ m^3

Mass of CO_2, $m = 10$ kg

Temperature of CO_2, $T = 27 + 273 = 300$ K

Pressure exerted by CO_2, p :

(*i*) **Using perfect gas equation :**

Characteristic gas constant, $R = \frac{R_0}{M} = \frac{8314}{44} = 188.95$ Nm/kg K (for CO_2)

Using perfect gas equation

$$pV = mRT$$

$$\therefore \quad \mathbf{p} = \frac{mRT}{V} = \frac{10 \times 188.95 \times 300}{3}$$

$$= \mathbf{188950\ N/m^2\ or\ 1.889\ bar.\ (Ans.)}$$

(*ii*) **Using Van der Waals' equation :**

$$\left(p + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = R_0T$$

$$p = \frac{R_0T}{\bar{v} - b} - \frac{a}{\bar{v}^2}$$

From Table 8.1

For CO_2 : $a = 362850$ Nm⁴/(kg-mol)²

$b = 0.0423$ m³/(kg-mol)

$\bar{v}$ = Molar specific volume = $\frac{3 \times 44}{10} = 13.2$ m³/kg-mol

Now substituting the values in the above equation, we get

$$p = \frac{8314 \times 300}{13.2 - 0.0423} - \frac{362850}{(13.2)^2}$$

$$= 189562 - 2082.5 = \mathbf{187479.5\ N/m^2\ or\ 1.875\ bar.\quad (Ans.)}$$

(*iii*) **Using Beattie Bridgeman equation :**

$$p = \frac{R_0T(1-e)}{(\bar{v})^2}(\bar{v} + B) - \frac{A}{\bar{v}^2}$$

where p = pressure, $A = A_0\left(1 - \frac{a}{\bar{v}}\right)$, $B = B_0\left(1 - \frac{b}{\bar{v}}\right)$ and $e = \frac{c}{\bar{v}T^3}$

From Table 8.2 $A_0 = 507.2836$, $a = 0.07132$

$B_0 = 0.10476$, $b = 0.07235$

$C = 66 \times 10^4$

$$\therefore \quad A = 507.2836\left(1 - \frac{0.07132}{13.2}\right) = 504.5$$

$$B = 0.10476\left(1 - \frac{0.07235}{13.2}\right) = 0.1042$$

$$C = \frac{66 \times 10^4}{13.2 \times (300)^3} = 0.001852$$

Now substituting the various values in the above equation, we get

$$\mathbf{p} = \frac{8314 \times 300(1 - 0.001852)}{(13.2)^2}(13.2 + 0.1042) - \frac{504.5}{(13.2)^2}$$

$$= 190093 - 2.89 \simeq 1.9 \times 10^5\ \text{N/m}^2 = \mathbf{1.9\ bar.\quad (Ans.)}$$

Example 8.11. *One kg-mol of oxygen undergoes a reversible non-flow isothermal compression and the volume decreases from 0.2 m^3/kg to 0.08 m^3/kg and the initial temperature is 60°C. If the gas obeys Van der Waals' equation find :*

(*i*) *The work done during the process* (*ii*) *The final pressure*.

Solution. The Van der Waals' equation is written as

$$\left(p + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = R_0 T$$

where p = pressure of the gas ; a, b = constants ; $\bar{v}$ = molar volume ; R_0 = universal gas constant

From Table 8.1

For O_2 : $a = 139250$ Nm^4/(kg-mol)^2

$b = 0.0314$ m^3/kg-mol

and $R_0 = 8314$ Nm/kg-mol K

$\bar{v}_1 = 0.2 \times 32 = 6.4$ m^3/kg-mol

$\bar{v}_2 = 0.08 \times 32 = 2.56$ m^3/kg-mol.

(*i*) **Work done during the process :**

The work done per kg mole of O_2 is given by

$$W = \int_1^2 p \, . d\bar{v} = \int_1^2 \left[\left(\frac{R_0 T}{\bar{v} - b}\right) - \frac{a}{\bar{v}^2}\right] d\bar{v}$$

$$= R_0 T \left[\log_e(\bar{v} - b)\right]_{\bar{v}_1}^{\bar{v}_2} + \left[\frac{a}{\bar{v}}\right]_{\bar{v}_1}^{\bar{v}_2}$$

$$= R_0 T \left[\log_e\left(\frac{\bar{v}_2 - b}{\bar{v}_1 - b}\right)\right] + \left[a\left(\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right)\right]$$

$$= 8314 \times (60 + 273) \left[\log_e\left(\frac{2.56 - 0.0314}{6.4 - 0.0314}\right)\right] + \left[139250\left(\frac{1}{2.56} - \frac{1}{6.4}\right)\right]$$

$= -\ 2557359 + 32636 =$ **– 2524723 Nm/kg-mol. (Ans.)**

(*ii*) **The final pressure, p_2 :**

$$p_2 = \frac{R_0 T}{\bar{v}_2 - b} - \frac{a}{\bar{v}_2^{\,2}}$$

$$= \frac{8314 \times 333}{2.56 - 0.0314} - \frac{139250}{(2.56)^2}$$ = **1073651 N/m^2 or 10.73 bar. (Ans.)**

Example 8.12. *If the values for reduced pressure and compressibility factor for ethylene are 20 and 1.25 respectively, compute the temperature.*

Solution. Reduced pressure, $p_r = 20$

Compressibility factor, $Z = 1.25$

Temperature, T = ?

From the generalised compressibility chart on $Z - p_r$ co-ordinates corresponding to $p_r = 20$ and $Z = 1.25$, $T_r = 8.0$.

Now, since $T = T_c \, T_r$

∴ **T** $= 282.4 \times 8.0$ [From Table 8.3, $T_c = 282.4$ K]

= **2259.2 K. (Ans.)**

Example 8.13. *Calculate the density of* N_2 *at 260 bar and 15°C by using the compressibility chart.*

Solution. Pressure, $p = 260$ bar

Temperature, $T = 15 + 273 = 288$ K

Density, $\rho = ?$

For N_2 (from Table 8.3) **:** $p_c = 33.94$ bar

$T_c = 126.2$ K

Now
$$p_r = \frac{p}{p_c} = \frac{260}{33.94} = 7.6$$

and
$$T_r = \frac{T}{T_c} = \frac{288}{126.2} = 2.28$$

From the compressibility chart for $p_r = 7.6$ and $T_r = 2.28$, $Z \simeq 1.08$

Also
$$Z = \frac{pv}{RT} = \frac{p}{\rho RT}, \text{ where } \rho \text{ stands for density}$$

or
$$\rho = \frac{p}{ZRT} = \frac{260 \times 10^5}{1.08 \times \frac{8314}{28} \times 288} = \mathbf{281.5\ kg/m^3}. \quad \textbf{(Ans.)}$$

Example 8.14. *What should be the temperature of 1.3 kg of* CO_2 *gas in a container at a pressure of 200 bar to behave as an ideal ?*

Solution. Pressure, $p = 200$ bar

Temperature, $\mathbf{T = ?}$

For CO_2 (from Table 8.3) $p_c = 73.86$ bar

$T_c = 304.2$ K

As the gas behaves like an ideal gas, $Z = 1$

$$p_r = \frac{p}{p_c} = \frac{200}{73.86} = 2.7$$

From compressibility chart for $Z = 1$, $p_r = 2.7$

$T_r = 2.48$

$\therefore$ $\mathbf{T} = T_r \times T_c = 2.48 \times 304.2 =$ **754.4 K. (Ans.)**

Example 8.15. *A spherical shaped balloon of 12 m diameter contains* H_2 *at 30°C and 1.21 bar. Find the mass of* H_2 *in the balloon using real gas equation.*

Solution. Diameter of spherical balloon = 12 m

$\therefore$ Volume, $V = 4/3\ \pi \times (6)^3 = 904.78\ \text{m}^3$

Temperature, $T = 30 + 273 = 303$ K

Pressure, $p = 1.21$ bar

Mass of H_2 in the balloon, m :

For H_2 (from Table 8.3) $p_c = 12.97$ bar

$T_c = 33.3$ K

Now,
$$p_r = \frac{p}{p_c} = \frac{1.21}{12.97} = 0.093$$

$$T_r = \frac{T}{T_c} = \frac{303}{33.3} = 9.1$$

From compressibility charge, corresponding to $p_r = 0.093$ and $T_r = 9.1$

$$Z \simeq 1$$

(This indicates that the gas having higher critical pressure and lower critical temperature behaves like an ideal gas at normal pressure and temperature conditions.)

Also, $$pV = ZmRT$$

or $$\mathbf{m} = \frac{pV}{ZRT} = \frac{1.21\times10^5\times904.78}{1\times\left(\frac{8314}{2}\right)\times303} = \mathbf{86.9\ kg.}\quad \textbf{(Ans.)}$$

Example 8.16. *Determine the value of compressibility factor at critical point* (Z_{cp}) *for the Van der Waals' gas.*

Solution. Refer Fig. 8.13.

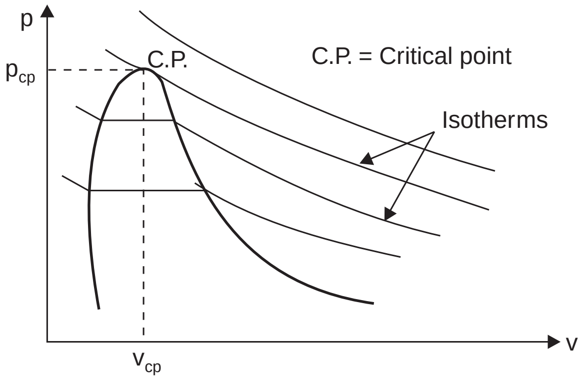


Fig. 8.13

From the isotherms plotted on *p-v* diagram in Fig. 8.13 it can be seen that the critical isotherms has an inflection point, whose *tangent is horizontal* at the *critical point*.

$$\left(\frac{\partial p_c}{\partial v}\right)_{cp} = 0 \text{ and } \left|\frac{\partial^2 p}{\partial v^2}\right|_{cp} = 0$$

The Van der Waal's equation at the critical point is

$$p_{cp} = \frac{R_0\,T_{cp}}{\bar{v}_{cp} - b} - \frac{a}{\bar{v}_{cp}{}^2} \qquad \text{...}(i)$$

As T_{cp} is constant $$\left(\frac{\partial p_{cp}}{\partial v_{cp}}\right) = -\frac{R_0 T_{cp}}{(\bar{v}_{cp} - b)^2} + \frac{2a}{\bar{v}_{cp}{}^3} = 0 \qquad \text{...}(ii)$$

$$\left(\frac{\partial^2 p_{cp}}{\partial v_{cp}{}^2}\right) = \frac{2R_0\,T_{cp}}{(\bar{v}_{cp} - b)^3} - \frac{6a}{(\bar{v}_{cp})^4} = 0 \qquad \text{...}(iii)$$

$\left(\frac{3}{\bar{v}_{cp}}\right) \times (ii) + (iii)$ gives

$$-\frac{3R_0\,T}{\bar{v}_{cp}(\bar{v}_{cp}-b)^2}+\frac{2R_0\,T}{(\bar{v}_{cp}-b)^3}=0$$

or
$$\frac{3}{\bar{v}_{cp}}=\frac{2}{(\bar{v}_{cp}-b)}$$

or
$$\bar{v}_{cp}=3b$$

Substituting for b in (ii), we get

$$-\frac{R_0T_{cp}}{[\bar{v}_{cp}-(1/3)\,\bar{v}_{cp}]^2}+\frac{2a}{(\bar{v}_{cp})^3}=0$$

$$\therefore \qquad a=\frac{9}{8}\,R_0T_{cp}\,\bar{v}_{cp}$$

Substituting for a and b in (i), we get

$$p_{cp}=\frac{R_0T_{cp}}{\bar{v}_{cp}-(1/3)\,\bar{v}_{cp}}-\frac{(9/8)\,R_0\,T_{cp}\,\bar{v}_{cp}}{\bar{v}_{cp}{}^2}$$

$$\therefore \qquad \frac{p_{cp}\,\bar{v}_{cp}}{R_0\,T_{cp}}=\frac{1}{(2/3)}-\frac{(9/8)}{1}$$

But
$$\frac{p_{cp}\,\bar{v}_{cp}}{R_0\,T_{cp}}=Z_{cp}$$

$$\therefore \qquad \mathbf{Z_{cp}}=\frac{3}{2}-\frac{9}{8}=\mathbf{\frac{3}{8}}. \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. An '*ideal gas*' is defined as a gas having no forces of intermolecular attraction. It obeys the law $pv = RT$. The specific heat capacities are *not constant* but are functions of temperature.

   A '*perfect gas*' obeys the law $pv = RT$ and has *constant* specific heat capacities.
2. The relation between the independent properties, such as pressure, specific volume and temperature for a pure substance is known as '*equation of state*'.
3. Each point on a *p-v-T* surface represents an equilibrium state and a line on the surface represents a process.
4. *Joule's law* states that the specific internal energy of a gas depends only on the temperature of the gas and is independent of both pressure and volume.
5. Van der Waals' equation may be written as

$$\left(p+\frac{a}{v^2}\right)(v-b)=RT$$

where a and b are constants for the particular fluid and R is the gas constant.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. (*a*) A perfect gas does not obey the law $pv = RT$
(*b*) A perfect gas obeys the law $pv = RT$ and has constant specific heat
(*c*) A perfect gas obeys the law $pv = RT$ but have variable specific heat capacities.

2. Boyle's law states that, when temperature is constant, the volume of a given mass of a perfect gas
(*a*) varies directly as the absolute pressure
(*b*) varies inversely as the absolute pressure
(*c*) varies as square of the absolute pressure
(*d*) does not vary with the absolute pressure.

3. Charle's law states that if any gas is heated at constant pressure, its volume
(*a*) changes directly as it absolute temperature
(*b*) changes inversely as its absolute temperature
(*c*) changes as square of the absolute temperature
(*d*) does not change with absolute temperature.

4. The equation of the state per kg of a perfect gas is given by
(*a*) $p^2v = RT$
(*b*) $pv = RT$
(*c*) $pv^2 = RT$
(*d*) $p^2v^2 = RT$.

where p, v, R and T are the pressure, volume, characteristic gas constant and temperature of the gas respectively.

5. The equation of state of an ideal gas is a relationship between the variables :
(*a*) pressure and volume
(*b*) pressure and temperature
(*c*) pressure, volume and temperature
(*d*) none of the above.

6. Joule's law states that the specific internal energy of a gas depends only on
(*a*) the pressure of the gas
(*b*) the volume of the gas
(*c*) the temperature of the gas
(*d*) none of the above.

7. Equation for specific heat at constant pressure of an ideal gas is given by
(*a*) $c_p = a + KT + K_1T^2 + K_2T^3$
(*b*) $c_p = a + KT^2 + K_1T^3 + K_2T^4$
(*c*) $c_p = a + KT^2 + K_1T^4 + K_2T$
(*d*) $c_p = a + KT^2 + K_1T^3 + K_2T^2$.

where a, K, K_1 and K_2 are constants.

8. Van der Waals' equation may be written as
(*a*) $\left(p + \frac{a}{v}\right)(v - b) = RT$
(*b*) $\left(p + \frac{a}{v^2}\right)(v - b) = RT$
(*c*) $\left(p + \frac{a}{v^2}\right)(v^2 - b) = RT$
(*d*) $\left(p + \frac{a}{v^2}\right)(v^2 - b) = RT^2$.

Answers

1. (*b*) **2.** (*b*) **3.** (*a*) **4.** (*b*) **5.** (*c*) **6.** (*c*) **7.** (*a*)
8. (*b*).

THEORETICAL QUESTIONS

1. What is an ideal gas ?
2. What is the difference between an ideal and a perfect gas ?
3. What are semi-perfect or permanent gases ?
4. Define 'Equation of state'.
5. State Boyle's and Charle's laws and derive an equation of the state for a perfect gas.
6. What is a p-v-T surface ? Draw a portion of a such a surface.
7. Derive the relationship between the two principal specific heats and characteristic gas constant for a perfect gas.
8. Write a short note on Van der Waals' equation.

UNSOLVED PROBLEMS

IDEAL GASES

1. A vessel of 0.03 m^3 capacity contains gas at 3.5 bar pressure and 35°C temperature. Determine the mass of the gas in the vessel. If the pressure of this gas is increased to 10.5 bar while the volume remains constant, what will be the temperature of the gas ?

For the gas take $R = 290$ J/kg K. [**Ans.** 0.118 kg, 650°C]

2. The tyre of an automobile contains a certain volume of air at a gauge pressure of 2 bar and 20°C. The barometer reads 75 cm of Hg. The temperature of air in the tyre rises to 80°C due to running of automobile for two hours. Find the new pressure in the tyre.

Assume that the air is an ideal gas and tyre does not stretch due to heating. [**Ans.** 2.62 bar]

3. A tank made of metal is designed to bear an internal gauge pressure of 7 bar. The tank is filled with a gas at a pressure of 5.5 bar abs., and 15°C. The temperature in the tank may reach to 50°C when the tank stands in the sun.

(*i*) If the tank does not expand with temperature, will the design pressure be exceeded on a day when atmospheric pressure is 1 bar and air in the tank reaches 50°C when exposed to hot sun ?

(*ii*) What temperature would have to be reached to raise the air pressure to the design limit ?

[**Ans.** (*i*) 6.16 bar, (*ii*) 147°C]

4. A vessel of spherical shape is 1.5 m in diameter and contains air at 40°C. It is evacuated till the vacuum inside the vessel is 735 mm of mercury. Determine :

(*i*) The mass of air pumped out ;

(*ii*) If the tank is then cooled to 10°C what is the pressure in the tank ?

The barometer reads 760 mm of mercury. Assume that during evacuation, there is no change in temperature of air. [**Ans.** (*i*) 1.91 kg, (*ii*) 3 kPa]

5. A balloon of spherical shape is 8 m in diameter and is filled with hydrogen at a pressure of 1 bar abs. and 15°C. At a later time, the pressure of gas is 95 per cent of its original pressure at the same temperature.

(*i*) What mass of original gas must have escaped if the dimensions of the balloon are not changed ?

(*ii*) Find the amount of heat to be removed to cause the same drop in pressure at constant volume.

[**Ans.** (*i*) 5 per cent, (*ii*) 3.26 MJ]

6. Find the molecular weight and gas constant for the gas whose specific heats are as follows :

$$c_p = 1.967 \text{ kJ/kg K}, \; c_v = 1.507 \text{ kJ/kg K}.$$

[**Ans.** 180.461 kJ/kg K]

7. A constant volume chamber of 0.3 m^3 capacity contains 1 kg of air at 20°C. Heat is transferred to the air until its temperature is 200°C. Find :

(*i*) Heat transferred ;

(*ii*) Change in entropy and enthalpy. [**Ans.** (*i*) 128.09 kJ, (*ii*) 0.339 kJ/kg K, 180.8 kJ]

8. 1 kg of air at 20°C occupying a volume of 0.3 m^3 undergoes a reversible constant pressure process. Heat is transferred to the air until its temperature is 200°C. Determine :

(*i*) The work and heat transferred.

(*ii*) The change in internal energy, enthalpy and entropy.

[**Ans.** (*i*) 51.5 kJ, 180.8 kJ ; (*ii*) 128.09 kJ, 180.8 kJ, 0.479 kJ/kg K]

9. A balloon of spherical shape, 10 m in diameter is filled with hydrogen at 20°C and atmospheric pressure. The surrounding air is at 15°C and barometer reads 75 mm of Hg. Determine the load lifting capacity of the balloon. [**Ans.** 587.2 kg]

10. Air expands in a cylinder in a reversible adiabatic process from 13.73 bar to 1.96 bar. If the final temperature is to be 27°C, what would be the initial temperature ?

Also calculate the change in specific enthalpy, heat and work transfers per kg of air.

[**Ans.** 524 K, 224.79 kJ/kg, zero, 160.88 kJ/kg]

11. 1 kg mole of N_2 is contained in a vessel of volume 2.5 m^3 at 100°C.

(*i*) Find the mass, the pressure and the specific volume of the gas.

(*ii*) If the ratio of the specific heats is 1.4, evaluate the values of c_p and c_v.

(*iii*) Subsequently, the gas cools to the atmospheric temperature of 30°C, evaluate the final pressure of the gas.

(*iv*) Evaluate the increase in specific internal energy, the increase in specific enthalpy, increase in specific entropy and magnitude and sign of heat transfer.

[**Ans.** (*i*) 28 kg, 12.37 bar, 0.089 m^3/kg ; (*ii*) c_p = 1.038 kJ/kg K, c_v = 0.745 kJ/kg K ; (*iii*) 10.22 bar ; (*iv*) – 52.16 kJ/kg, – 72.67 kJ/kg, – 0.1536 kJ/kg K, 1465.1 kJ]

12. The pressure and volume of a gas, during a process, change from 1 bar absolute and 2 m^3 respectively to 6 bar absolute and 0.4 m^3 respectively. During the process the increase in the enthalpy of the gas is 200 kJ. Taking c_v = 10.4 kJ/kg K, determine c_p, R and ΔU. [**Ans.** 13 kJ/kg K, 2.6 kJ/kg K, 160 kJ]

13. 1 kg of air at 27°C is heated reversibly at constant pressure until the volume is doubled and then heated reversibly at constant volume until the pressure is doubled. For the total path find :

(*i*) The work ; (*ii*) Heat transfer ;

(*iii*) Change of entropy. [**Ans.** (*i*) 86.14 kJ, (*ii*) 728.36 kJ, (*iii*) 1.186 kJ/kg K]

14. A mass of air initially at 260°C and a pressure of 6.86 bar has a volume of 0.03 m^3. The air is expanded at constant pressure to 0.09 m^3, a polytropic process with $n = 1.5$ is then carried out, followed by a constant temperature process which completes the cycle. All processes are reversible. Find (*i*) The heat received and rejected in the cycle, (*ii*) The efficiency of the cycle.

Show the cycle on p-v and T-s planes. [**Ans.** (*i*) 143.58 kJ, – 20.3 kJ ; (*ii*) 38.4%]

REAL GASES

15. One kg-mol of oxygen undergoes a reversible non-flow isothermal compression and the volume decreases from 0.15 m^3/kg to 0.06 m^3/kg and the initial temperature is 50°C. The gas obeys Van der Waals' equation during the compression. Find :

(*i*) The work done during the process ;

(*ii*) The final pressure. [**Ans.** (*i*) – 6706500 Nm/kg-mol, (*ii*) 13.825 bar]

16. Determine the compressibility factor for O_2 at (*i*) 100 bar – 70°C and (*ii*) at 5 bar and 30°C.

[**Ans.** (*i*) 0.71, (*ii*) 0.98]

17. Determine the pressure of air at 205°C having a specific volume of 0.00315 m^3/kg by means of :

(*i*) Ideal gas equation.

(*ii*) Van der Waals' equation ;

(*iii*) Beattie-Bridgeman equation. [**Ans.** (*i*) 435.7 bar, (*ii*) 557.3 bar, (*iii*) 525.8 bar]

9

Gases and Vapour Mixtures

9.1. INTRODUCTION

- A *pure substance* is defined as *a substance having a constant and uniform chemical composition*. A homogeneous mixture of gases which do not react with one another may, therefore, be considered a pure substance. For example, air is a homogeneous mixture of nitrogen, oxygen and traces of other substances like argon, helium, carbon dioxide, etc., and as they do not react with one another, air is regarded a pure substance. The properties of such a mixture can be determined and tabulated just like those of any other pure substance. The properties of air and some combustion products have been determined and tabulated in gas tables. But it is not possible to determine the properties of the unlimited number of mixtures possible, the properties of the mixtures are determined from the properties of the constituent gases.
- In this chapter the mixtures to be considered are those composed of perfect gases, and perfect gases and vapours. The properties of such mixtures are important in combustion calculations. Air and water vapour mixtures are considered later in the chapter with reference to surface condensers, but for moist atmospheric air there is a special nomenclature and this is considered in a separate chapter on *Psychrometrics*.

9.2. DALTON'S LAW AND GIBBS-DALTON LAW

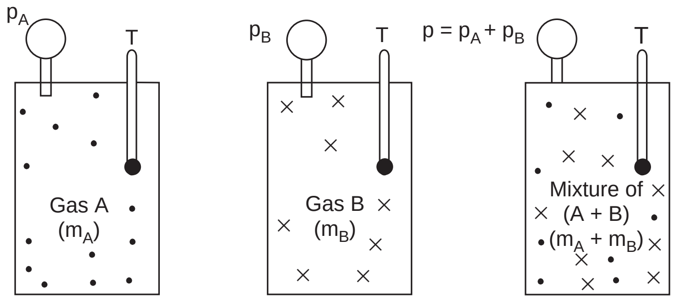
Dalton's law

Let us consider a closed vessel of volume *V* at temperature *T*, which contains a mixture of perfect gases at a known pressure. If some of the mixture were removed, then the pressure would be *less* than the initial value. If the gas removed were the full amount of one of the constituents then the reduction in pressure would be equal to the contribution of that constituent to the initial total pressure. Each constituent contributes to the total pressure by an amount which is known as the *partial pressure* of the constituent.

The *relationship between the partial pressures of the constituents* is expressed by Dalton's law, as follows :

- *The pressure of a mixture of gases is equal to the sum of the partial pressures of the constituents.*
- *The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixtures at the same temperature.*

This is expressed diagrammatically in Fig. 9.1. The gases A and B, originally occupying volume V at temperature T are mixed in the third vessel which is of the *same volume* and is *at the same temperature*.



(*i*) (*ii*) (*iii*)

Fig. 9.1

By the *consideration of mass*,

$$m = m_A + m_B \qquad \text{...(9.1)}$$

By *Dalton's law*,
$$p = p_A + p_B \qquad \text{...(9.2)}$$

Dalton's law is based on experiment and is found to be obeyed more accurately by gas mixtures at low pressures. As shown in Fig. 9.1 each occupant occupies the whole vessel. The example given in Fig. 9.1 and relationship in eqns. (9.1) and (9.2) refer to a mixture of two gases, but the law can be extended to any number of gases,

i.e.,
$$m = m_A + m_B + m_C + \text{......} \text{ or } m = \Sigma\, m_i \qquad \text{...(9.3)}$$

where m_i = Mass of a constituent.

Similarly
$$p = p_A + p_B + p_C + \text{......} \text{ or } p = \Sigma p_i \qquad \text{...(9.4)}$$

where p_i = The partial pressure of a constituent.

Gibbs-Dalton law

Dalton's law was re-formulated by Gibbs to include a second statement on the properties of mixtures. The combined statement is known as the Gibbs-Dalton law, and is as follows :

— *The internal energy, enthalpy, and entropy of a gaseous mixture are respectively equal to the sums of the internal energies, enthalpies, and entropies, of the constituents.*

— *Each constituent has that internal energy, enthalpy and entropy, which it could have if it occupied alone that volume occupied by the mixture at the temperature of the mixture.*

This statement leads to the following equations :

$$mu = m_A u_A + m_A u_B + \text{......} \text{ or } mu = \Sigma\, m_i u_i \qquad \text{...(9.5)}$$

and
$$mh = m_A h_A + m_B h_B + \text{......} \text{ or } mh = \Sigma\, m_i h_i \qquad \text{...(9.6)}$$

and
$$ms = m_A s_A + m_B s_B + \text{......} \text{ or } ms = \Sigma\, m_i s_i \qquad \text{...(9.7)}$$

— ***Properties of air.*** The properties of air which is the most common mixture are given below :

Constituent	*Molecular weight*	*Chemical symbol*	*Volumetric analysis %*	*Gravimetric analysis %*
Oxygen	31.999	O_2	20.95	23.14
Nitrogen	28.013	N_2	78.09	75.53
Argon	39.948	Ar	0.93	1.28
Carbon dioxide	44.01	CO_2	0.03	0.05

Mean molecular weight of air = 28.96

Gas constant $R = 0.287$ kJ/kg K

For *approximate* calculations the air is said to be composed of oxygen and 'atmospheric nitrogen'.

Constituent	*Molecular weight*	*Volumetric analysis %*	*Gravimetric analysis %*
Oxygen	32	21	23.3
Atmospheric nitrogen	28	79	76.7
Nitrogen/Oxygen	—	3.76 : 1	3.29 : 1

Note. Volumetric analysis is the analysis by **volume** ; gravimetric analysis is the analysis by **weight or mass**.

9.3. VOLUMETRIC ANALYSIS OF A GAS MIXTURE

It is usual practice to quote the analysis of a mixture by volume as this is the most convenient for practical determinations. In article 11.15, the analysis of exhaust or flue gases by means of the Orsat apparatus is described. The volume of the gas sample is measured at atmospheric pressure, and the temperature is held constant by means of a water jacket round the gas sample. The constituents are absorbed chemically one by one, and the remainder of the sample is measured after each absorption ; the difference in volume gives the partial volume occupied by the constituent in the mixture.

Let us consider a volume V of a gaseous mixture at a temperature T, consisting of three constituents A, B and C [Fig. 9.2 (*a*)]. Let us further assume that each of the constituents is compressed to a pressure p equal to the total pressure of the mixture, and let the *temperature remain constant*. The partial volumes then occupied by the constituents will be V_A, V_B and V_C.

$$m = m_A + m_B + m_C = \Sigma m_i$$

$$p = p_A + p_B + p_C = \Sigma p_i$$

$$n = n_A + n_B + n_C = \Sigma n_i$$

(*a*)

V_A	V_B	V_C
p	p	p
m_A	m_B	m_C
n_A	n_B	n_C

(*b*)

Fig. 9.2

Now using the eqn. $pV = mRT$, we get

$$m_A = \frac{p_A V}{R_A T} \qquad \text{... referring to Fig. 9.2 } (a)$$

and

$$m_A = \frac{p V_A}{R_A T} \qquad \text{... referring to Fig. 9.2 } (b)$$

Now equating the two values for m_A, we have

$$\frac{p_A V}{R_A T} = \frac{p V_A}{R_A T} \quad i.e., \quad p_A V = p V_A$$

or

$$V_A = \frac{p_A}{p} V$$

In general therefore,

$$V_i = \frac{p_i}{p} V \qquad ...(9.8)$$

i.e.,

$$\Sigma V_i = \sum \frac{p_i V}{p} = \frac{V}{p} \Sigma p_i$$

Now from eqn. (9.4), $p = \Sigma p_i$, therefore,

$$\Sigma V_i = V \qquad ...(9.9)$$

Thus, the *volume of a mixture of gases is equal to the sum of the volumes of the individual constituents when each exists alone at the pressure and temperature of the mixture.*

This is the statement of another empirical law, *the law of partial volumes*, sometimes called *Amagat's law* or *Leduc's law*.

— The analysis of mixtures, oftenly, is simplified if it is carried out in moles. The mole is given by the equation

$$n = \frac{m}{M}$$

where, n = Number of moles,

m = Mass of gas, and

M = Molecular weight.

According to *Avogadro's law*, the *number of moles of any gas is proportional to the volume of the gas at a given pressure and temperature.* Referring to Fig. 9.2 (*a*), the volume V contains n moles of the mixture at p and T. In Fig. 9.2 (*b*), the gas A occupies a volume V_A at p and T, and this volume contains n_A moles. Similarly there are n_B moles of gas B in volume V_B and n_C moles of gas C in volume V_C.

From eqn. (9.9), $\Sigma V_i = V$

or $V_A + V_B + V_C = V$

∴ The *total number of moles in the vessel must equal the sum of the moles of the individual constituents*,

$$n = n_A + n_B + n_C = \Sigma n_i \qquad ...(9.10)$$

9.4. THE APPARENT MOLECULAR WEIGHT AND GAS CONSTANT

The Apparent Molecular Weight

In a gas mixture if a gas occupies a total volume of V at a temperature T, then from the definition of partial pressure and equation $pV = nR_0T$, we have

$$p_iV = n_iR_0T \qquad ...(9.11)$$

(where R_0 is the *universal gas constant*)

∴ $\Sigma p_iV = \Sigma n_iR_0T$

i.e., $V \Sigma p_i = R_0T \Sigma n_i$

Also $p = \Sigma p_i$ [from eqn. (9.4)]

∴ $pV = R_0T \Sigma n_i$

Also $n = \Sigma n_i$ [from eqn. (9.10)]

∴ $pV = nR_0T$

The *mixture therefore acts as a perfect gas,* and *this is the characteristic equation for mixture.*

An apparent molecular weight is defined by the equation

$$M = \frac{m}{n}$$

where, m = Mass of the mixture, and

n = Number of moles of mixture.

The Gas Constant

The *apparent gas constant* (similarly as above) is defined by the equation

$$R = \frac{R_0}{M}$$

It can be assumed that a mixture of perfect gases obeys all the perfect gas laws.

In order to determine the gas constant for the mixture in terms of the gas constants of the constituents let us consider the equation $pV = mRT$ both for the mixture and for a constituent as follows

$$pV = mRT$$

and $$p_iV = m_iR_iT$$

Then $$\Sigma\, p_iV = \Sigma\, m_iR_iT$$

$\therefore$ $$V \Sigma\, p_i = T \Sigma\, m_iR_i$$

Also $$p = \Sigma\, p_i$$

$\therefore$ $$pV = T \Sigma\, m_iR_i$$

or $$pV = mRT = T \Sigma\, m_iR_i$$

i.e., $$mR = \Sigma\, m_iR_i$$

or $$R = \Sigma\, \frac{m_i}{m}\, R_i \qquad ...(9.12)$$

where $\frac{m_i}{m}$ = mass fraction of a constituent.

— From equation (9.11), $p_iV = n_iR_0T$, and combining this with eqn. (9.8) $\left(V_i = \frac{p_i}{p} V\right)$ applied to the mixture (*i.e.,* $pV = nR_0T$), we have

$$\frac{p_iV}{pV} = \frac{n_iR_0T}{nR_0T}$$

or $$\frac{p_i}{p} = \frac{n_i}{n} \qquad ...(9.13)$$

On combining this with eqn. (9.8), we get

$$\frac{p_i}{p} = \frac{n_i}{n} = \frac{V_i}{V} \qquad ...(9.14)$$

This means that the *molar analysis is identical with the volumetric analysis,* and *both are equal to the ratio of the partial pressure to the total pressure.*

— The *apparent molecular weight* can be also be determined by the following method.

Let us apply characteristic equation to each constituent and to mixture, we have

$$m_i = \frac{p_iV}{R_iT}$$

$$m = \frac{pV}{RT}$$

Also $m = \Sigma\, m_i$ [from eqn. (9.3)]

$$\therefore \quad \frac{pV}{RT} = \sum \frac{p_i V}{R_i T}$$

$$\therefore \quad \frac{p}{R} = \sum \frac{p_i}{R_i}$$

Using the relation $R = \dfrac{R_0}{M}$, and substituting, we have

$$\frac{pM}{R_0} = \sum \frac{p_i M_i}{R_0}$$

or
$$pM = \sum p_i M_i$$

i.e.,
$$M = \sum \frac{p_i}{p} M_i \quad \text{...(9.15)}$$

Now using eqn. (9.14), we have

$$M = \sum \frac{V_i}{V} M_i \quad \text{...(9.16)}$$

and
$$M = \sum \frac{n_i}{n} M_i \quad \text{...(9.17)}$$

Alternately

$$p = \Sigma\, p_i = p_A + p_B + \text{......}\ p_i$$

Also $pV = mRT$

Similarly $p_A V = m_A R_A T$

$$p_B V = m_B R_B T$$

$$\therefore \quad pV = p_A V + p_B V + \text{......}$$

or
$$mRT = m_A R_A T + m_B R_B T + \text{......}$$

$$\therefore \quad mR = m_A R_A + m_B R_B + \text{......}$$

But
$$R = \frac{R_0}{M},\ R_A = \frac{R_0}{M_A},\ R_2 = \frac{R_0}{M_B}$$

Substituting this in the above equation, we get

$$m\,\frac{R_0}{M} = m_A\,\frac{R_0}{M_A} + m_B\,\frac{R_0}{M_B} + \text{......}$$

$$\therefore \quad \frac{1}{M} = \frac{m_A}{m} \cdot \frac{1}{M_A} + \frac{m_B}{m} \cdot \frac{1}{M_B} + \text{......}$$

$$= \frac{m_{fA}}{M_A} + \frac{m_{fB}}{M_B} + \text{......}$$

where m_{fA}, m_{fB} etc. are the *mass fractions* of the constituents.

$$\therefore \quad \frac{1}{M} = \sum \frac{m_{fi}}{M_i}$$

$$\therefore \quad M = \frac{1}{\sum \dfrac{m_{fi}}{M_i}} \quad \text{...(9.18)}$$

9.5. SPECIFIC HEATS OF A GAS MIXTURE

— As per Gibbs-Dalton law, the internal energy of a mixture of gases is given by

$$mu = \Sigma\, m_i u_i \qquad \text{...}(i) \qquad \text{[from eqn. (9.5)]}$$

Also $$u = c_v T \qquad \text{...}(ii) \qquad \text{[from perfect gas equation]}$$

Now from (*i*) and (*ii*), we have

$$mc_v T = \Sigma\, m_i c_{vi} T$$

$\therefore$ $$mc_v = \Sigma\, m_i c_{vi}$$

or $$c_v = \sum \frac{m_i}{m}\, c_{vi} \qquad \text{...(9.19)}$$

Similarly from equations, $mh = \Sigma\, m_i h_i$

and $h = c_p T$, we get

$$mc_p T = \Sigma\, m_i c_{pi} T$$

$\therefore$ $$m_{cp} = \Sigma\, m_i c_{pi}$$

or $$c_p = \sum \frac{m_i}{m}\, c_{pi} \qquad \text{...(9.20)}$$

From eqns. (9.18) and (9.19),

$$c_p - c_v = \sum \frac{m_i}{m}\, c_{pi} - \sum \frac{m_i}{m}\, c_{vi} = \sum \frac{m_i}{m}\, (c_{pi} - c_{vi})$$

Also $c_{pi} - c_{vi} = R_i$, therefore,

$$c_p - c_v = \sum \frac{m_i}{m}\, R_i$$

Also from eqn. (9.12), $R = \sum \frac{m_i}{m}\, R_i$, therefore for the mixture

$$c_p - c_v = R$$

The following equations can be applied to a mixture of gases

$$\gamma = \frac{c_p}{c_v}\,; \quad c_v = \frac{R}{\gamma - 1}\,; \quad c_p = \frac{\gamma R}{\gamma - 1}$$

It should be borne in mind that γ *must be determined from the eqn.* $\gamma = \frac{c_p}{c_v}$; *there is no weighted mean expression as there is for R,* c_v *and* c_p.

— In problems on mixtures it is often convenient to work in moles and the specific heats can be expressed in terms of the mole. These are known as *molar heats*, and are denoted by C_p and C_v.

Molar heats are defined as follows :

$$C_p = Mc_p \quad \text{and} \quad C_v = Mc_v \qquad \text{...(9.21)}$$

But $$c_p - c_v = R$$

$\therefore$ $$C_p - C_v = Mc_p - Mc_v = M(c_p - c_v) = MR$$

But $$MR = R_0$$

$\therefore$ $$C_p - C_v = R_0 \qquad \text{...(9.22)}$$

Also $$U = mc_v T = \frac{mMc_v T}{M}$$

Again, $\frac{m}{M} = n$ and $Mc_v = C_v$

$\therefore$ $$U = nC_vT \qquad \text{...(9.23)}$$

Similarly, $$H = nC_pT \qquad \text{...(9.24)}$$

By the Gibbs-Dalton law,

$$U = \Sigma\, U_i \quad \text{and} \quad H = \Sigma\, H_i$$

$\therefore$ $$nC_vT = \Sigma\, n_iC_{vi}\, T \quad \text{and} \quad nC_pT = \Sigma\, n_iC_{pi}T$$

i.e., $$C_v = \sum \frac{n_i}{n} C_{vi} \qquad \text{...(9.25)}$$

and $$C_p = \sum \frac{n_i}{n} C_{pi}. \qquad \text{...(9.26)}$$

9.6. ADIABATIC MIXING OF PERFECT GASES

— Fig. 9.3 shows two gases A and B separated from each other in a closed vessel by a thin diaphragm. If the diaphragm is removed or punctured then the gases mix and each then occupies the total volume, behaving as if the other gas were not present. This process is equivalent to a *free expansion* of each gas, and is *irreversible*. The process can be simplified by the assumption that it is adiabatic ; this means that the vessel is perfectly thermally insulated and there will therefore be an *increase in entropy of the system.*

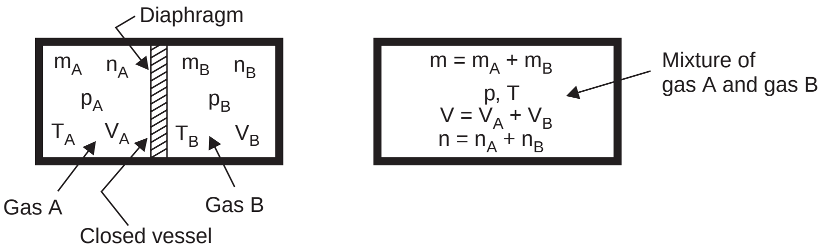


Fig. 9.3

In a free *expansion process, the internal energy initially is equal to the internal energy finally.* In this case, from eqn. (9.23),

$$U_1 = n_AC_{vA}\, T_A + n_BC_{vB}T_B$$

and $$U_2 = (n_AC_{vA} + n_BC_{vB})T$$

If this result is extended to any number of gases, we have

$$U_1 = \Sigma\, n_iC_{vi}T_i \quad \text{and} \quad U_2 = T\, \Sigma\, n_iC_{vi}$$

Then $$U_1 = U_2$$

i.e., $$\Sigma\, n_iC_{vi}T_i = T\, \Sigma\, n_iC_{vi}$$

i.e., $$T = \frac{\Sigma\, n_iC_{vi}T_i}{\Sigma\, n_iC_{vi}} \qquad \text{...(9.27)}$$

— When two streams of fluid meet to form a *common stream in steady flow*, they give *another form of mixing* (Fig. 9.4).

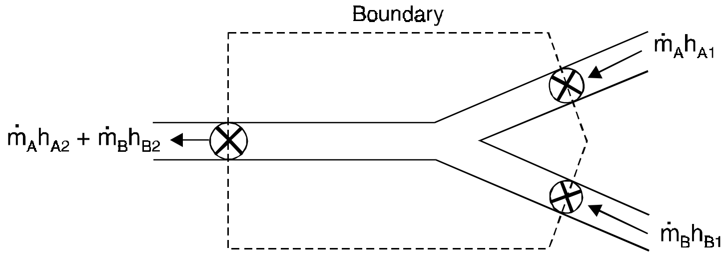


Fig. 9.4

Applying steady-flow energy equation to the *mixing section* (neglecting changes in kinetic and potential energy), we get

$$\dot{m}_A h_{A_1} + \dot{m}_B h_{B_1} + Q = \dot{m}_A h_{A_2} + \dot{m}_B h_{B_2} + W$$

In case of *adiabatic flow* : $Q = 0$, and also $W = 0$ in this case

$$\therefore \qquad \dot{m}_A h_{A_1} + \dot{m}_B h_{B_1} = \dot{m}_A h_{A_2} + \dot{m}_B h_{B_2}$$

Also $h = c_p T$, hence,

$$\dot{m}_A c_{pA} T_A + \dot{m}_B c_{pB} T_B = \dot{m}_A c_{pA} T + \dot{m}_B c_{pB} T$$

For any number of gases this becomes

$$\Sigma\, \dot{m}_i c_{pi} T_i = T\, \Sigma\, \dot{m}_i c_{pi}$$

i.e.,

$$T = \frac{\Sigma\, m_i c_{pi} T_i}{\Sigma\, m_i c_{pi}} \qquad \text{...(9.28)}$$

Also, $C_p = Mc_p$ and $M = m/n$

$$\therefore \qquad nC_p = mc_p$$

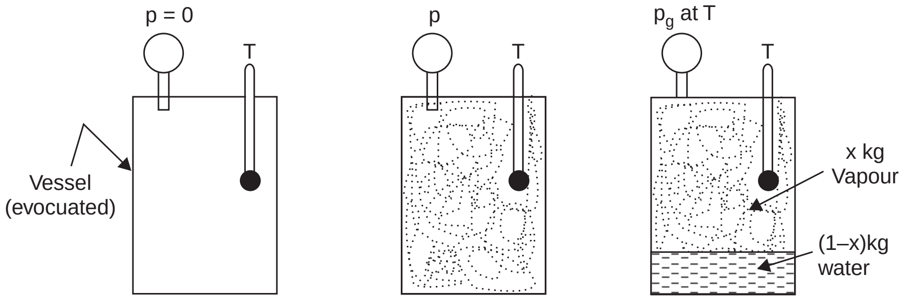
Hence,

$$T = \frac{\Sigma\, n_i C_{pi} T_i}{\Sigma\, n_i C_{pi}} \qquad \text{...(9.29)}$$

Eqns. (9.28) and (9.29) represent *one condition which must be satisfied in an adiabatic mixing process of perfect gas in steady flow*. In a particular problem some other information must be known (*e.g.,* specific volume or the final pressure) before a complete solution is possible.

9.7. GAS AND VAPOUR MIXTURES

Fig. 9.5 (*i*) shows a vessel of fixed volume which is maintained at a constant temperature. The vessel is *evacuated* and the absolute pressure is therefore *zero*.



(*i*) (*ii*) (*iii*)

Fig. 9.5

In Fig. 9.5 (*ii*) a small quantity of water is introduced into the vessel and it evaporated to occupy the whole volume. For a small quantity of water introduced, the pressure in the vessel will be *less* than the saturation pressure corresponding to the temperature of the vessel. At this condition of pressure and temperature the vessel will be occupied by *superheated vapour.* As more water is introduced the pressure increases and the water continues to evaporate until such a condition is reached that the *volume can hold no water.* Any additional water introduced into the vessel after this will not evaporate but will exist as water, the condition being as in Fig. 9.5 (*iii*), which shows the vapour in contact with its liquid per kg of water introduced, the vessel can be thought of as containing either $(1 - x)$ kg of water plug x kg of dry saturated vapour, or as containing 1 kg of wet steam of dryness fraction x.

The *temperature remains constant during the whole process of evaporation.* If the temperature is now raised by the addition of heat, then more vapour will evaporate and the pressure in the vessel will increase. Eventually the *vessel will contain a superheated vapour as before, but at a higher pressure and temperature.*

In Fig. 9.5 the vessel is considered to be initially evacuated, but the water would evaporate in exactly the same way, if the vessel contains a gas or a mixture of gases. As stated in the Gibbs-Dalton law, each constituent behaves as if it occupies the whole vessel at the temperature of the vessel. When a little water is sprayed into a vessel containing a gas mixture, then the vapour forced will exert the saturation pressure corresponding to the temperature of the vessel, and this is the partial pressure of the vapour in the mixture.

In case there is a *saturated vapour in a mixture*, then the partial pressure of the vapour can be found from tables at the temperature of the mixture. This means that a saturated vapour obeys the Gibbs-Dalton law ; *this is only a good approximation at low values of the total pressure.*

Mixtures of air and water vapour have been considered in this chapter ; in chapter 9 moist atmospheric air (*i.e.,* a mixture of dry air and water vapour) has been considered as a separate topic-*Psychrometrics.*

Example 9.1. *A vessel of 0.35 m^3 capacity contains 0.4 kg of carbon monoxide (molecular weight = 28) and 1 kg of air at 20°C. Calculate :*

(*i*) *The partial pressure of each constituent,*

(*ii*) *The total pressure in the vessel, and*

The gravimetric analysis of air is to be taken as 23.3% oxygen (molecular weight = 32) and 76.7% nitrogen (molecular weight = 28).

Solution. Capacity of the vessel, $V = 0.35$ m^3

Mass of carbon monoxide = 0.4 kg

Mass of air = 1 kg

Temperature, T = 20°C or 293 K

Mass of oxygen present in 1 kg of air $= \dfrac{23.3}{100} \times 1 = 0.233$ kg

Mass of nitrogen present in 1 kg of air $= \dfrac{76.7}{100} \times 1 = 0.767$ kg

But, characteristic gas constant,

$$R = \frac{R_0}{M} \qquad ...(i)$$

where, R_0 = Universal gas constant (= 8.314 kJ/kg K), and

M = Molecular weight.

Also, the characteristic gas equation is given by

$$pV = mRT \quad ...(ii)$$

$$\therefore \quad pV = \frac{mR_0T}{M} \quad ...(iii)$$

i.e.,

$$p = \frac{mR_0T}{MV} \quad ...(iv)$$

Hence, for a constituent,

$$p_i = \frac{m_iR_0T}{M_iV} \quad ...(v)$$

Substituting the values, we get the partial pressures as follows :

(*i*) **Partial pressures :**

For O_2, $$p_{O_2} = \frac{0.233 \times (8.314 \times 10^3) \times 293}{32 \times 0.35 \times 10^5} = \mathbf{0.5068\ bar.} \quad \textbf{(Ans.)}$$

For N_2, $$p_{N_2} = \frac{0.767 \times (8.314 \times 10^3) \times 293}{28 \times 0.35 \times 10^5} = \mathbf{1.9065\ bar.} \quad \textbf{(Ans.)}$$

For CO, $$p_{CO} = \frac{0.40 \times (8.314 \times 10^3) \times 293}{28 \times 0.35 \times 10^5} = \mathbf{0.9943\ bar.} \quad \textbf{(Ans.)}$$

(*ii*) **Total pressure in the vessel, p :**

$$p = \Sigma p_i = p_{O_2} + p_{N_2} + p_{CO}$$

$$= 0.5068 + 1.9065 + 0.9943 = \mathbf{3.4076\ bar.} \quad \textbf{(Ans.)}$$

Example 9.2. *The gravimetric analysis of air and other data are as follows :*

Constituent	*Percentage*	*Molecular weight*
Oxygen	*23.14*	*32*
Nitrogen	*75.53*	*28*
Argon	*1.28*	*40*
Carbon dioxide	*0.05*	*44*

Calculate : (i) Gas constant for air ;

(ii) Apparent molecular weight.

Solution. Using the relation, $R = \frac{R_0}{M}$ ...(*i*)

$$R_{O_2} = \frac{8.314}{32} = 0.2598 \text{ kJ/kg K}$$

$$R_{N_2} = \frac{8.314}{28} = 0.2969 \text{ kJ/kg K}$$

$$R_{Ar} = \frac{8.314}{40} = 0.2078 \text{ kJ/kg K}$$

$$R_{CO_2} = \frac{8.314}{44} = 0.1889 \text{ kJ/kg K}$$

(*i*) **Gas constant for air :**

Now using the equation,

$$R = \sum \frac{m_i}{m} R_i, \text{ we have} \qquad ...(ii)$$

$$R = 0.2314 \times 0.2598 + 0.7553 \times 0.2969 + 0.0128 \times 0.2078 + 0.0005 \times 0.1889$$
$$= 0.0601 + 0.2242 + 0.00266 + 0.00009 = 0.2871 \text{ kJ/kg K}$$

Hence gas constant for air = 0.2871 kJ/kg K. **(Ans.)**

(*ii*) **Apparent molecular weight :**

Now from eqn. (*i*), we have $M = \dfrac{8.314}{0.2871} = 28.96$

Hence apparent molecular weight = 28.96. **(Ans.)**

Example 9.3. *Following is the gravimetric analysis of air :*

Constituent	*Percentage*
Oxygen	*23.14*
Nitrogen	*75.53*
Argon	*1.28*
Carbon dioxide	*0.05*

Calculate the analysis by volume and the partial pressure of each constituent when the total pressure is 1 bar.

Solution. We know that the analysis by volume V_i/V, is the same as the mole fraction n_i/n. Also $n_i = m_i/M_i$; therefore considering 1 kg of mixture and using a tabular method, we have

Constituent	m_i	M_i	$n_i = \frac{m_i}{M_i}$	$\frac{n_i}{n} \times 100\% = \frac{V_i}{V} \times 100\%$. **(Ans.)**
Oxygen	0.2314	32	0.00723	$\frac{0.00723 \times 100}{0.03453}$ = **20.94%. (Ans.)**
Nitrogen	0.7553	28	0.02697	$\frac{0.02697 \times 100}{0.03453}$ = **78.10%. (Ans.)**
Argon	0.0128	40	0.00032	$\frac{0.00032 \times 100}{0.03453}$ = **0.93%. (Ans.)**
Carbon dioxide	0.0005	44	0.00001	$\frac{0.00001 \times 100}{0.03453}$ = **0.03%. (Ans.)**
			$n = \Sigma n_i = 0.03453$	

Also, $$\frac{p_i}{p} = \frac{V_i}{V} = \frac{n_i}{n}$$

∴ $$p_i = \frac{n_i}{n} p \text{ hence}$$

For O_2, $p_{O_2} = 0.2094 \times 1 =$ **0.2094 bar. (Ans.)**

For N_2, $p_{N_2} = 0.7810 \times 1 =$ **0.7810 bar. (Ans.)**

For Ar, $p_{Ar} = 0.0093 \times 1 =$ **0.0093 bar. (Ans.)**

For CO_2, $p_{CO_2} = 0.0003 \times 1 =$ **0.0003 bar. (Ans.)**

☞**Example 9.4.** *A vessel contains at 1 bar and 20°C a mixture of 1 mole of* CO_2 *and 4 moles of air. Calculate for the mixture :*

(*i*) *The masses of* CO_2*,* O_2 *and* N_2*, and the total mass ;*

(*ii*) *The percentage carbon content by mass ;*

(*iii*) *The apparent molecular weight and the gas constant for the mixture ;*

(*iv*) *The specific volume of the mixture.*

The volumetric analysis of air can be taken as 21% oxygen and 79% nitrogen.

Solution. The pressure in the vessel, p = 1 bar

Temperature in the vessel, T = 20 + 273 = 293 K

No. of moles of CO_2 = 1 mole

No. of moles of air = 4 mole

From equation, $n_i = \left(\frac{V_i}{V}\right) n$, we have

$$n_{O_2} = 0.21 \times 4 = 0.84$$

$$n_{N_2} = 0.79 \times 4 = 3.16$$

(*i*) From equation, $m_i = n_i M_i$, we have

$$m_{CO_2} = 1 \times 44 = \mathbf{44\ kg.}\quad \textbf{(Ans.)}$$

$$m_{O_2} = 0.84 \times 32 = \mathbf{26.88\ kg.}\quad \textbf{(Ans.)}$$

and
$$m_{N_2} = 3.16 \times 28 = \mathbf{88.48\ kg.}\quad \textbf{(Ans.)}$$

The *total mass,* $m = m_{CO_2} + m_{O_2} + m_{N_2}$

$$= 44 + 26.88 + 88.48 = \mathbf{159.36\ kg.}\quad \textbf{(Ans.)}$$

(*ii*) Since the molecular weight of carbon is 12, therefore, there are 12 kg of carbon present for every mole of CO_2,

i.e., **Percentage carbon in mixture** $= \frac{12 \times 100}{159.36}$ = **7.53% by mass. (Ans.)**

(*iii*) From equation $n = \Sigma\, n_i$, we have

$$n = n_{CO_2} + n_{O_2} + n_{N_2}$$
$$= 1 + 0.84 + 3.16 = 5.0$$

Now using the equation $M = \sum \frac{n_i}{n} M_i$, we have

$$M = \frac{1}{5} \times 44 + \frac{0.84}{5} \times 32 + \frac{3.16}{5} \times 28$$
$$= 8.8 + 5.376 + 17.696 = 31.872$$

i.e., **Apparent molecular weight = 31.872. (Ans.)**

From equation, $R = \frac{R_0}{M}$, we have

$$R = \frac{8.314}{31.872} = 0.2608 \text{ kJ/kg K}$$

i.e., **Gas constant for the mixture = 0.2608 kJ/kg K. (Ans.)**

(*iv*) To find specific volume of the mixture, v using the relation :

$$pv = RT$$

$$v = \frac{RT}{p} = \frac{0.2608 \times 10^3 \times 293}{1 \times 10^5} = 0.7641 \text{ m}^3/\text{kg}$$

i.e., **Specific volume of the mixture = 0.7641 m^3/kg. (Ans.)**

Example 9.5. *A mixture of hydrogen (H_2) and oxygen (O_2) is to be made so that the ratio of H_2 to O_2 is 2 : 1 by volume. If the pressure and temperature are 1 bar and 25°C respectively, calculate :*

(*i*) *The mass of O_2 required ;* (*ii*) *The volume of the container.*

Solution. Pressure, p = 1 bar

Temperature, $T = 25 + 273 = 298$ K

Ratio of H_2 to O_2 = 2 : 1 by volume.

(*i*) **The mass of O_2 required :**

Let the mass of O_2 per kg of H_2 = x kg

Now, $$n_i = \frac{m_i}{M_i}$$

$$\therefore \quad n_{H_2} = \frac{1}{2} = 0.5$$

and $$n_{O_2} = \frac{x}{32}$$

From equation, $\frac{V_i}{V} = \frac{n_i}{n}$, we have

$$\frac{V_{H_2}}{V_{O_2}} = \frac{n_{H_2}}{n_{O_2}} \quad \text{and} \quad \frac{V_{H_2}}{V_{O_2}} = 2 \text{ (given)}$$

$$\therefore \quad \frac{0.5}{x/32} = 2 \qquad \therefore \quad x = \frac{32 \times 0.5}{2} = 8$$

i.e., Mass of O_2 per kg of H_2 = 8 kg. **(Ans.)**

(*ii*) **The volume of the container, V :**

The total number of moles in the vessel per kg of H_2 is

$$n = n_{H_2} + n_{O_2} = 0.5 + \frac{x}{32} = 0.5 + \frac{8}{32} = 0.75$$

Now using the relation,

$$pV = nR_0T$$

$$\therefore \quad V = \frac{nR_0T}{p} = \frac{0.75 \times (8.314 \times 10^3)}{1 \times 10^5} \times 298 = 18.58 \text{ m}^3$$

i.e., **The volume of the container = 18.58 m^3. (Ans.)**

Example 9.6. *A gaseous mixture of composition by volume, 78% H_2 and 22% CO is contained in a vessel. It is desired that the mixture should be made in proportion 52% H_2 and 48% CO by removing some of the mixture and adding some CO. Calculate per mole of mixture the mass of mixture to be removed, and mass of CO to be added.*

Assume that the pressure and temperature in the vessel remain constant during the procedure.

Solution. Composition of mixture by volume : H_2 = 78%, CO = 22%

Final composition desired : H_2 = 52%, CO = 48%

Since the pressure and temperature remain constant, then the number of moles in the vessel remain the same throughout.

∴ Moles of mixture removed = Moles of CO added.

Let x kg of mixture be removed and y kg of CO be added.

For the mixture, $$M = \sum \frac{V_i}{V} M_i$$

∴ $$M = 0.78 \times 2 + 0.22 \times 28 = 7.72$$

Also from equation, $n = \frac{m}{M}$, we have

$$\text{Moles of mixture removed} = \frac{x}{7.72} = \text{moles of CO added} = \frac{y}{28}$$

From equation, $\frac{V_i}{V} = \frac{n_i}{n}$, we have

Moles of H_2 in the mixture removed

$$= 0.78 \times \frac{x}{7.72} = 0.101\, x$$

and Moles of H_2 initially = 0.78 × 1 = 0.78

Hence, Moles of H_2 remaining in vessel = $0.78 - 0.101\, x$

But 1 mole of the new mixture is 52% H_2 and 48% CO, therefore

$$0.78 - 0.101\, x = 0.52$$

∴ $$0.101\, x = 0.26 \quad \text{or} \quad x = 2.57$$

i.e., **Mass of mixture removed = 2.57 kg. (Ans.)**

Also since $$\frac{x}{7.72} = \frac{y}{28}$$

∴ $$y = \frac{28}{7.72} \times x = \frac{28}{7.72} \times 2.57 = 9.32 \text{ kg}$$

i.e., **Mass of CO added = 9.32 kg. (Ans).**

☞**Example 9.7.** *In an engine cylinder a gas has a volumetric analysis of 13% CO_2, 12.5% O_2, and 74.5% N_2. The temperature at the beginning of expansion is 950°C and the gas mixture expands reversibly through a volume ratio of 8 : 1, according to the law $pv^{1.2}$ = constant. Calculate per kg of gas :*

(*i*) *The workdone ;*

(*ii*) *The heat flow ;*

(*iii*) *Change of entropy per kg of mixture.*

The values of c_p for the constituents CO_2, O_2 and N_2 are 1.235 kJ/kg K, 1.088 kJ/kg K and 1.172 kJ/kg K respectively.

Solution. From equation $m_i = n_i M_i$, the conversion from volume fraction to mass fraction is as follows :

Consider 1 mole of the mixture.

Constituent	n_i	M_i	$m_i = n_i M_i$	$\frac{m_i}{m}$ = *Fraction by mass*
CO_2	0.13	44	5.72	$\frac{5.72}{30.58} = 0.187$
O_2	0.125	32	4.00	$\frac{4.0}{30.58} = 0.131$
N_2	0.745	28	20.86	$\frac{20.86}{30.58} = 0.682$
			$\Sigma m_i = 30.58$	

Now using the equation $c_p = \sum \frac{m_i}{m} c_{pi}$

$$\therefore \quad c_p = 0.187 \times 1.235 + 0.131 \times 1.088 + 0.682 \times 1.172$$
$$= 0.231 + 0.1425 + 0.799 = 1.1725 \text{ kJ/kg K}$$

From equations, $R = \sum \frac{m_i}{m} R_i$ and $R_i = \frac{R_0}{M_i}$, we have

$$R = 0.187 \times \frac{8.314}{44} + 0.131 \times \frac{8.314}{32} + 0.682 \times \frac{8.314}{28}$$
$$= 0.0353 + 0.0340 + 0.2025 = 0.2718 \text{ kJ/kg K}$$

From equation, $c_p - c_v = R$, we have

$$c_v = 1.1725 - 0.2718 = 0.9 \text{ kJ/kg K}$$

(*i*) **The workdone, W :**

$$W = \frac{R(T_1 - T_2)}{n - 1}.$$

T_2 can be found by using the equation

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{1}{8}\right)^{1.2-1} = 0.659$$

$$\therefore \quad T_2 = (950 + 273) \times 0.659 = 805.9 \text{ K}$$

$$\therefore \quad \mathbf{W} = \frac{0.2718(1223 - 805.9)}{1.2 - 1} = \mathbf{566.8 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

(*ii*) **The heat flow, Q :**

Also from equation,

$$u_2 - u_1 = c_v(T_2 - T_1) \text{ ... for 1 kg}$$
$$= 0.9(805.9 - 1223) = -375.39 \text{ kJ/kg}$$

Now from the non-flow energy equation,

$$Q = (u_2 - u_1) + W = -375.39 + 566.8 = 191.41 \text{ kJ/kg}$$

i.e., **Heat supplied = 191.41 kJ/kg. (Ans.)**

(*iii*) **Change of entropy per kg of mixture :**

Refer Fig. 9.6. The change of entropy between state 1 and state 2 can be found by imagining the process replaced by two other processes 1 to *A* and *A* to 2.

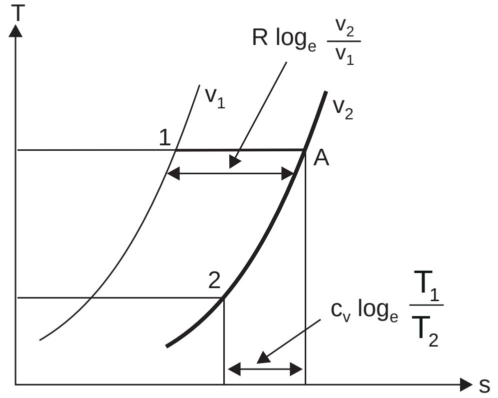


Fig. 9.6

For *isothermal process* 1-*A*

$$s_A - s_1 = R \log_e \frac{v_2}{v_1}$$

$$= 0.2718 \log_e 8 = 0.565 \text{ kJ/kg K}$$

For *constant volume process* *A*-2 :

$$s_A - s_2 = c_v \int_2^A \frac{dT}{T} = c_v \log_e \frac{T_1}{T_2}$$

$$= 0.9 \log_e \frac{1223}{805.9}$$

i.e., $s_A - s_2 = 0.375$ kJ/kg K

Then by subtraction,

$$s_2 - s_1 = 0.565 - 0.375 = 0.19 \text{ kJ/kg K}$$

i.e., **Change of entropy per kg of mixture**
= 0.19 kJ/kg K. (Ans.)

Example 9.8. *The following is the volumetric analysis of a producer gas : CO = 28%, H_2 = 13%, CH_4 = 4%, CO_2 = 4%, N_2 = 51%. The values of C_p for the constituents CO, H_2, CH_4, CO_2 and N_2 are 29.27 kJ/mole K, 28.89 kJ/mole K, 35.8 kg/mole K, 37.22 kJ/mole K, 29.14 kJ/mole K respectively. Calculate the values of C_p, C_v, c_p and c_v for the mixture.*

Solution. Using the relation

$$C_p = \sum \frac{n_i}{n} C_{pi}$$

$$\therefore \quad C_p = 0.28 \times 29.27 + 0.13 \times 28.89 + 0.04 \times 35.8 + 0.04 \times 37.22 + 0.51 \times 29.14$$

i.e., $\mathbf{C_p}$ **= 29.733 kJ/mole K. (Ans.)**

Also $C_p - C_v = R_0$

$\therefore$ $\mathbf{C_v} = C_p - R_0 = 29.733 - 8.314 =$ **21.419 kJ/mole K. (Ans.)**

To find the molecular weight, using the equation :

$$M = \sum \frac{n_i}{n} M_i$$

$$= 0.28 \times 28 + 0.13 \times 2 + 0.04 \times 16 + 0.04 \times 44 + 0.51 \times 28 = 24.78$$

Now $\mathbf{c_p} = \frac{C_p}{M} = \frac{29.733}{24.78} =$ **1.199 kJ/kg K. (Ans.)**

and $\mathbf{c_p} = \frac{C_v}{M} = \frac{21.419}{24.78} =$ **0.864 kJ/kg K. (Ans.)**

Example 9.9. *The analysis by weight of a perfect gas mixture at 20°C and 1.3 bar is 10% O_2, 70% N_2, 15% CO_2 and 5% CO. For a reference state of 0°C and 1 bar determine :*

(*i*) *Partial pressures of the constituents ;* (*ii*) *Gas constant of mixture.*

Solution. Using the relation, $M = \dfrac{1}{\sum \dfrac{m_{f_i}}{M_i}}$

where, M = Molecular weight of the mixture ;
m_{fi} = Mass fraction of a constituent, ;
M_i = Molecular weight of a constituent.

$$M = \frac{1}{\frac{(m_f)_{O_2}}{M_{O_2}} + \frac{(m_f)_{N_2}}{M_{N_2}} + \frac{(m_f)_{CO_2}}{M_{CO_2}} + \frac{(m_f)_{CO}}{M_{CO}}} = \frac{1}{\frac{0.1}{32} + \frac{0.7}{28} + \frac{0.15}{44} + \frac{0.05}{28}}$$

$$= \frac{1}{0.03332} = 30 \text{ kg/kg mole}$$

Considering 1 kg of the mixture

$$n = \frac{m}{M} = \frac{1}{30} = 0.033$$

Now mole fraction $x_i = \frac{n_i}{\Sigma n_i} = \frac{n_i}{n} = \frac{m_i / M_i}{n}$

$$\therefore \quad x_{O_2} = \frac{(0.1/32)}{0.033} = 0.094 \; ; \quad x_{N_2} = \frac{(0.7/28)}{0.033} = 0.757$$

$$x_{CO_2} = \frac{(0.15/44)}{0.033} = 0.103 \; ; \; x_{CO} = \frac{(0.05/28)}{0.033} = 0.054$$

(*i*) **Partial pressures of the constituents :**

$$\mathbf{P_{O_2}} = x_{O_2} \times p = 0.094 \times 1.3 = \mathbf{0.132\ bar.} \quad \textbf{(Ans.)}$$

$$\mathbf{P_{N_2}} = x_{N_2} \times p = 0.757 \times 1.3 = \mathbf{0.984\ bar.} \quad \textbf{(Ans.)}$$

$$\mathbf{P_{CO_2}} = x_{CO_2} \times p = 0.103 \times 1.3 = \mathbf{0.1334\ bar.} \quad \textbf{(Ans.)}$$

$$\mathbf{P_{CO}} = x_{CO} \times p = 0.054 \times 1.3 = \mathbf{0.0702\ bar.} \quad \textbf{(Ans.)}$$

(*ii*) **Gas constant of mixture, $\mathbf{R_{mix}}$:**

$$\mathbf{R_{mix}} = \frac{R_0}{M} = \frac{8.314}{30} = \mathbf{0.277\ kJ/kg\ K.} \quad \textbf{(Ans.)}$$

☞**Example 9.10.** *A mixture of ideal gases consists of 4 kg of nitrogen and 6 kg of carbon dioxide at a pressure of 4 bar and a temperature of 20°C. Find :*

(*i*) *The mole fraction of each constituent,*
(*ii*) *The equivalent molecular weight of the mixture,*
(*iii*) *The equivalent gas constant of the mixture,*
(*iv*) *The partial pressures and partial volumes,*
(*v*) *The volume and density of the mixture, and*
(*vi*) *The c_p and c_v of the mixture.*

If the mixture is heated at constant volume to 50°C, find the changes in internal energy, enthalpy and entropy of the mixture. Find the changes in internal energy, enthalpy and entropy of the mixture if the heating is done at constant pressure.

Take γ : for CO_2 = 1.286 and for N_2 = 1.4.

Solution. (*i*) **The mole fraction of each constituent :**

Since mole fraction, $x_i = \frac{n_i}{\Sigma n_i}$

$$\therefore \quad x_{N_2} = \frac{\frac{4}{28}}{\frac{4}{28} + \frac{6}{44}} = \frac{0.1428}{0.1428 + 0.1364} = \frac{0.1428}{0.2792} = \mathbf{0.511.} \quad \textbf{(Ans.)}$$

$$x_{CO_2} = \frac{\frac{6}{44}}{\frac{4}{28} + \frac{6}{44}} = \frac{0.1364}{0.1428 + 0.1364} = \mathbf{0.488.} \quad \textbf{(Ans.)}$$

(*ii*) **The equivalent molecular weight of the mixture, M :**

$$M = 0.511 \times 28 + 0.488 \times 44 = \mathbf{35.78\ kg/kg\ mole.} \quad \textbf{(Ans.)}$$

(*iii*) **The equivalent gas constant of the mixture, $\mathbf{R_{mix}}$:**

Total mass, $m = m_{N_2} + m_{CO_2} = 4 + 6 = 10$ kg

$$\therefore \quad R_{mix} = \frac{m_{N_2} R_{N_2} + m_{CO_2} R_{CO_2}}{m}$$

$$= \frac{4 \times \left(\frac{8.314}{28}\right) + 6 \times \left(\frac{8.314}{44}\right)}{10} \qquad \left[\because \ R = \frac{R_0}{M}\right]$$

$$= \mathbf{0.232\ kJ/kg\ K.} \quad \textbf{(Ans.)}$$

(*iv*) **The partial pressures and partial volumes :**

$$\mathbf{p_{N_2}} = x_{N_2} \times p = 0.511 \times 4 = \mathbf{2.044\ bar.} \quad \textbf{(Ans.)}$$

$$\mathbf{p_{CO_2}} = x_{CO_2} \times p = 0.488 \times 4 = \mathbf{1.952\ bar.} \quad \textbf{(Ans.)}$$

$$\mathbf{V_{N_2}} = \frac{m_{N_2} R_{N_2} T}{p} = \frac{4 \times \frac{8.314}{28} \times 293 \times 10^3}{4 \times 10^5} = \mathbf{0.87\ m^3.} \quad \textbf{(Ans.)}$$

$$\mathbf{V_{CO_2}} = \frac{m_{CO_2} R_{CO_2} T}{p} = \frac{6 \times \frac{8.314}{44} \times 293 \times 10^3}{4 \times 10^5} = \mathbf{0.83\ m^3.} \quad \textbf{(Ans.)}$$

(*v*) **The volume and density of the mixture :**

Total volume of the mixture,

$$V = \frac{mRT}{p} = \frac{m_{N_2} R_{N_2} T}{p_{N_2}} = \frac{m_{CO_2} R_{CO_2} T}{p_{CO_2}}$$

$$\therefore \quad V = \frac{10 \times 0.232 \times 293 \times 10^3}{4 \times 10^5} = \mathbf{1.699\ m^3.} \quad \textbf{(Ans.)}$$

Density of the mixture,

$$\rho_{mix} = \rho_{N_2} + \rho_{CO_2} = \frac{m}{V} = \frac{10}{1.699} = \mathbf{5.88\ kg/m^3.} \quad \textbf{(Ans.)}$$

(*vi*) $\mathbf{c_p}$ **and** $\mathbf{c_v}$ **of the mixture :**

$$c_{p_{N_2}} - c_{v_{N_2}} = R_{N_2}$$

$$\therefore \quad c_{v_{N_2}} = \frac{R_{N_2}}{\gamma - 1} = \frac{8.314}{28(1.4 - 1)} = 0.742 \text{ kJ/kg K.}$$

and $\quad c_{p_{N_2}} = 1.4 \times 0.742 = 1.039$ kJ/kg K

Now $$c_{v_{CO_2}} = \frac{R_{CO_2}}{\gamma - 1} = \frac{8.314}{44(1.286 - 1)} \quad [\because \gamma \text{ for } CO_2 = 1.286]$$

$$= 0.661 \text{ kJ/kg K}$$

and $$c_{p_{CO_2}} = 1.286 \times 0.661 = 0.85 \text{ kJ/kg K}$$

For the mixture :

$$c_p = \frac{m_{N_2} c_{p_{N_2}} + m_{CO_2} c_{p_{CO_2}}}{m_{N_2} + m_{CO_2}}$$

$$= \frac{4 \times 1.039 + 6 \times 0.85}{(4 + 6)} = \mathbf{0.925 \text{ kJ/kg K.}} \quad \textbf{(Ans.)}$$

$$c_v = \frac{m_{N_2} c_{v_{N_2}} + m_{CO_2} c_{v_{CO_2}}}{m_{N_2} + m_{CO_2}}$$

$$= \frac{4 \times 0.742 + 6 \times 0.661}{(4 + 6)} = \mathbf{0.693 \text{ kJ/kg K.}} \quad \textbf{(Ans.)}$$

When the mixture is heated at constant volume :

Change in internal energy,

$$U_2 - U_1 = mc_v(T_2 - T_1) = 10 \times 0.693(50 - 20) = \mathbf{207.9 \text{ kJ.}} \quad \textbf{(Ans.)}$$

Change in entropy,

$$H_2 - H_1 = mc_p(T_2 - T_1) = 10 \times 0.925(50 - 20) = \mathbf{277.5 \text{ kJ.}} \quad \textbf{(Ans.)}$$

Change in entropy,

$$S_2 - S_1 = mc_v \log_e \frac{T_2}{T_1} + mR \log_e \frac{V_2}{V_1}$$

$$= mc_v \log_e \frac{T_2}{T_1} \quad (\because V_1 = V_2)$$

$$= 10 \times 0.693 \times \log_e \left(\frac{273 + 50}{273 + 20}\right) = \mathbf{0.675 \text{ kJ/kg K.}} \quad \textbf{(Ans.)}$$

When the mixture is heated at constant pressure :

If the mixture is heated at constant pressure ΔU *and* ΔH *will remain the same.*

The **change in entropy** will be

$$S_2 - S_1 = mc_p \log_e \frac{T_2}{T_1} - mR \log_e \frac{p_2}{p_1}$$

$$= mc_p \log_e \frac{T_2}{T_1} \quad (\because p_1 = p_2)$$

$$= 10 \times 0.925 \log_e \left(\frac{323}{293}\right) = \mathbf{0.902 \text{ kJ/kg K.}} \quad \textbf{(Ans.)}$$

Example 9.11. *A vessel of 1.8 m*^{3} *capacity contains oxygen at 8 bar and 50°C. The vessel is connected to another vessel of 3.6 m*^{3} *capacity containing carbon monoxide at 1 bar and 20°C. A connecting valve is opened and the gases mix adiabatically. Calculate :*

(*i*) *The final temperature and pressure of the mixture ;*

(*ii*) *The change of entropy of the system.*

Take : For oxygen C_v *= 21.07 kJ/mole K.*

For carbon monoxide C_v *= 20.86 kJ/mole K.*

Solution. Using the relation, $n = \dfrac{pV}{R_0T}$

$$\therefore \quad n_{O_2} = \frac{8 \times 10^5 \times 1.8}{(8.314 \times 10^3) \times 323} = 0.536 \text{ (where } T_{O_2} = 50 + 273 = 323 \text{ K)}$$

and
$$n_{CO} = \frac{1 \times 10^5 \times 3.6}{(8.314 \times 10^3) \times 293} = 0.1478 \text{ (where } T_{CO} = 20 + 273 = 293 \text{ K).}$$

(*i*) **Final temperature (T) and pressure (p) of the mixture :**

Before mixing :

$$U_1 = \Sigma\, n_i C_{vi} T_i = 0.536 \times 21.07 \times 323 + 0.1478 \times 20.86 \times 293$$

i.e., $U_1 = 4551.15$ kJ

After mixing :

$$U_2 = T\, \Sigma\, n_i C_{vi} = T\,(0.536 \times 21.07 + 0.1478 \times 20.86)$$

i.e., $U_2 = 14.37\ T$

For **adiabatic mixing,**

$$U_1 = U_2$$

$$\therefore \quad 4551.15 = 14.37\ T$$

$$\therefore \quad T = \frac{4551.15}{14.37} = 316.7 \text{ K}$$

$\therefore$ Temperature of the mixture = 316.7 – 273 = **43.7°C. (Ans.)**

Now
$$p = \frac{nR_0T}{V}$$

$$\therefore \quad p = \frac{(0.536 + 0.1478) \times 8.314 \times 10^3 \times 316.7}{(1.8 + 3.6) \times 10^5} = 3.33 \text{ bar}$$

i.e., Pressure after mixing = 3.33 bar. **(Ans.)**

(*ii*) **Change of entropy of the system :**

Change of entropy of the system = change of entropy of the O_2 + change of entropy of CO
...... Gibbs-Dalton law

Referring to Fig. 9.7, the change of entropy of O_2 can be calculated by replacing the process undergone by the oxygen by the two processes 1 to *A* and *A* to 2.

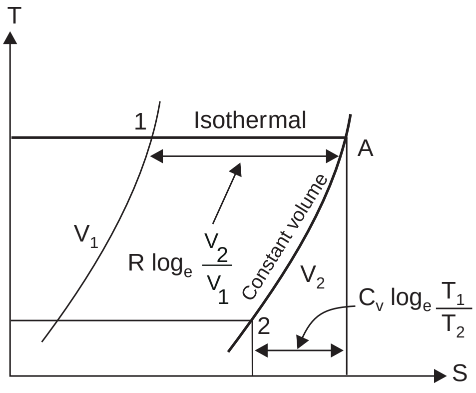


Fig. 9.7

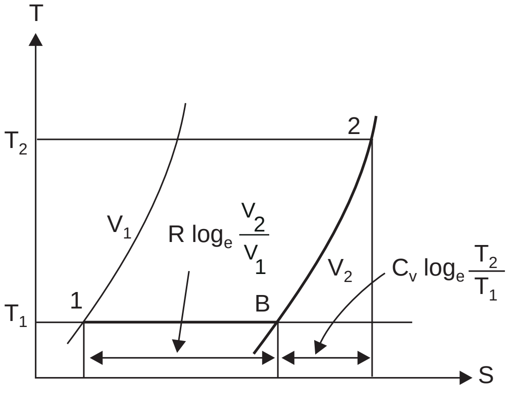


Fig. 9.8

For an *isothermal process* 1-A :

$$S_A - S_1 = mR \log_e \frac{V_A}{V_1}$$

or

$$S_A - S_1 = nR_0 \log_e \frac{V_A}{V_1}$$

$$= 0.536 \times 8.314 \times \log_e \frac{5.4}{1.8} = 4.896 \text{ kJ/K}$$

For *constant volume process* A-2 :

$$S_A - S_2 = mc_v \int_2^A \frac{dT}{T} = mc_v \log_e \frac{T_1}{T_2}$$

i.e.,

$$S_A - S_2 = nC_v \log_e \frac{T_1}{T_2} = 0.536 \times 21.07 \times \log_e \frac{323}{316.7} = 0.222 \text{ kJ/K}$$

$\therefore$

$$S_2 - S_1 = (S_A - S_1) - (S_A - S_2)$$
$$= 4.896 - 0.222 = 4.674 \text{ kJ/K}$$

Referring to Fig. 9.8, the change of entropy of CO can be found in a similar way to the above,

i.e.,

$$S_2 - S_1 = (S_B - S_1) + (S_2 - S_B)$$

$\therefore$

$$S_2 - S_1 = nR_0 \log_e \frac{V_B}{V_1} + nC_v \log_e \frac{T_2}{T_1}$$

$$= 0.1478 \times 8.314 \times \log_e \frac{5.4}{3.6} + 0.1478 \times 20.86 \log_e \frac{316.7}{293}$$
$$= 0.498 + 0.239 = 0.737 \text{ kJ/K}$$

Hence the change of entropy of the whole system is given by

$$(S_2 - S_1)_{system} = (S_2 - S_1)_{O_2} + (S_2 - S_1)_{CO}$$

i.e., **Change of entropy of system** = 4.674 + 0.737 = **5.411 kJ/K. (Ans.)**

☞**Example 9.12.** *Two vessels, A and B, both containing nitrogen, are connected by a valve which is opened to allow the contents to mix and achieve an equilibrium temperature of 30°C. Before mixing the following information is known about the gases in the two vessels.*

Vessel	*Pressure (p)*	*Tem. (t)*	*Contents*
A	*16 bar*	*55°C*	*0.6 kg mole*
B	*6.4 bar*	*25°C*	*3.6 kg*

(*a*) *Calculate : (i) The final equilibrium pressure ;*

(ii) The amount of heat transferred to the surroundings ;

(*b*) *If the vessel had been perfectly insulated, calculate :*

(*i*) *The final temperature ;*

(*ii*) *The final pressure.*

Assume γ = 1.4.

Solution. Refer Fig. 9.9.

Vessel A :

$$p_A V_A = n_A R_0 T_A \qquad \text{(where } V_A = \text{volume of vessel } A\text{)}$$

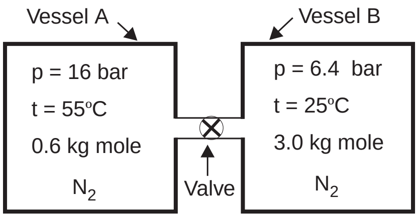


Fig. 9.9

$$16 \times 10^5 \times V_A = 0.6 \times 8.314 \times 10^3 \times (55 + 273)$$

$$\therefore \quad V_A = 1.022 \text{ m}^3$$

The mass of gas in vessel A

$$m_A = n_A M_A = 0.6 \times 28 = 16.8 \text{ kg}$$

Characteristic gas constant R of nitrogen

$$R = \frac{8.314}{28} = 0.297 \text{ kJ/kg K}$$

Vessel B :

$$p_B V_B = m_B R T_B$$

$$6.4 \times 10^5 \times V_B = 3.0 \times 0.297 \times 10^3 \times (25 + 273)$$

$$\therefore \quad V_B = 0.415 \text{ m}^3$$

Total volume of A and B

$$V = V_A + V_B = 1.022 + 0.415 = 1.437 \text{ m}^3$$

Total mass of gas

$$m = m_A + m_B = 16.8 + 3 = 19.8 \text{ kg}$$

Final temperature after mixing

$$T = 30 + 273 = 303 \text{ K}$$

(*a*) (*i*) **Final equilibrium pressure, p :**

$$pV = mRT$$

or $$p \times 1.437 = 19.8 \times 0.297 \times 303 \times 10^3 = 12.4 \times 10^5 \text{ N/m}^2$$

i.e., **p = 12.4 bar. (Ans.)**

Also $$c_v = \frac{R}{\gamma - 1} = \frac{0.297}{1.4 - 1} = 0.743 \text{ kJ/kg K.}$$

(*ii*) **Amount of heat transferred, Q :**

As there is no work transfer, the amount of heat transfer,

$$Q = \text{change of internal energy}$$
$$= U_2 - U_1$$

Measuring the internal energy above the datum of absolute zero (at $T = 0$ K, $u = 0$ kJ/kg).

Internal energy U_1 (before mixing)

$$= m_A c_v T_A + m_B c_v T_B$$
$$= 16.8 \times 0.743 \times 328 + 3.0 \times 0.743 \times 298 = 4758.5 \text{ kJ}$$

Final internal energy U_2 (after mixing)

$$= m c_v T = 19.8 \times 0.743 \times 303 = 4457.5 \text{ kJ}$$

$$\therefore \quad \mathbf{Q} = U_2 - U_1 = 4457.5 - 4758.5 = \mathbf{-\ 301\ kJ.\ (Ans.)}$$

(*b*) **If the vessel were insulated :**

(*i*) **Final temperature, t_2 :**

If the vessel were insulated

$$Q = U_2 - U_1 = 0$$

i.e., $$U_1 = U_2$$

$$m_A c_v T_A + m_B c_v T_B = m c_v T$$

$$\therefore \quad T = \frac{m_A c_v T_A + m_B c_v T_B}{m c_v} = \frac{m_A T_A + m_B T_B}{m}$$

$$= \frac{16.8 \times 328 + 3.0 \times 298}{19.8} = 323.5 \text{ K}$$

i.e., $$\mathbf{t} = 323.5 - 273 = \mathbf{50.5°C.} \quad \textbf{(Ans.)}$$

(*ii*) **Final pressure, p :**

$$pV = mRT$$

or $$p = \frac{mRT}{V} = \frac{19.8 \times 0.297 \times 10^3 \times 323.5}{1.437} = 13.24 \times 10^5 \text{ N/m}^2$$

i.e., $$\mathbf{p = 13.24 \text{ bar.}} \quad \textbf{(Ans.)}$$

Example 9.13. *Calculate the increase in entropy when 3 kg of O_2 at 50°C are mixed with 9 kg of N_2 at the same temperature. The initial pressure of each constituent is 11 bar and is the same as that of the mixture.*

Solution. $$x_{O_2} = \frac{p_{O_2}}{p} = \frac{3/32}{3/32 + 9/28} = 0.225$$

$$x_{N_2} = \frac{p_{N_2}}{p} = \frac{9/28}{3/32 + 9/28} = 0.774$$

Increase of entropy due to diffusion

$$\Delta S = -\, m_{O_2} R_{O_2} \log_e \frac{p_{O_2}}{p} - m_{N_2} R_{N_2} \log_e \frac{p_{N_2}}{p}$$

$$= -3 \times \left(\frac{8.314}{32}\right) \log_e 0.225 - 9 \times \left(\frac{8.314}{28}\right) \times \log_e 0.774$$

$$= 1.1626 + 0.6846 = 1.8472 \text{ kJ/kg K.}$$

Example 9.14. *2.5 kg of N_2 at 15 bar and 40°C is contained a rigid vessel. Adequate quantity of O_2 is added to increase the pressure to 20 bar while the temperature remains constant at 40°C.*

Calculate the mass of O_2 added.

Solution. m_{N_2} = 2.5 kg, M_{N_2} = 28

Initial pressure, p_{N_2} = 15 bar

Final pressure, $p_{\text{total}}\ (= p_{N_2} + p_{O_2})$ = 20 bar

Temperature = 40°C

In this process, the number of nitrogen moles remains *constant* at

$$n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{2.5}{28} = 0.0893$$

After the oxygen is added, p_{N_2} = 15 bar, since the volume remains unchanged

$$p_{O_2} = p_{\text{total}} - p_{N_2} = 20 - 15 = 5 \text{ bar}$$

Now $$p_{N_2} = \frac{n_{N_2} R_0 T}{V} \quad ...(i)$$

and $$p_{O_2} = \frac{n_{O_2} R_0 T}{V} \quad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{p_{O_2}}{p_{N_2}} = \frac{n_{O_2}}{n_{N_2}}$$

$$\therefore \quad n_{O_2} = \frac{p_{O_2} \times n_{N_2}}{p_{N_2}} = \frac{5 \times 10^5 \times 0.0893}{15 \times 10^5} = 0.0297$$

∴ **Mass of O_2 added,**

$$m_{O_2} = n_{O_2} \times M_{O_2}$$

$$= 0.0297 \times 32 = \mathbf{0.9504\ kg.} \quad \textbf{(Ans.)}$$

Example 9.15. *Given that air consists of 21% oxygen and 79% nitrogen by volume. Determine :*

(*i*) *The moles of nitrogen per mole of oxygen ;*
(*ii*) *The partial pressure of oxygen and nitrogen if the total pressure is atmosphere ;*
(*iii*) *The kg of nitrogen per kg of mixture.*

Solution. Let $n_{O_2} = 1.0$ and V = volume of air

so that $V_{O_2} = 0.21V$; $V_{N_2} = 0.79V$

Let V contain $n = n_{O_2} + n_{N_2}$ moles of air at p and T.

(*i*) **Moles of N_2 per mole of O_2 :**

Now $$pV_{O_2} = n_{O_2} R_0 T \quad ...(i)$$

and $$pV_{N_2} = n_{N_2} R_0 T \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{V_{O_2}}{V_{N_2}} = \frac{n_{O_2}}{n_{N_2}}$$

$$\therefore \quad \mathbf{n_{N_2}} = \frac{n_{O_2} \times V_{N_2}}{V_{O_2}} = \frac{1 \times 0.79V}{0.21V} = \mathbf{3.76\ moles.} \quad \textbf{(Ans.)}$$

(*ii*) $\mathbf{p_{O_2}}$ **and** $\mathbf{p_{N_2}}$ **:**

Now $$x_{O_2} = \frac{n_{O_2}}{4} = \frac{p_{O_2}}{p} \quad (n = n_{O_2} + n_{N_2})$$

(where p = total pressure = 1 atm.)

$$\therefore \quad \mathbf{p_{O_2}} = \frac{n_{O_2}}{n} \times p = \frac{1}{(1 + 3.76)} \times 1 = \mathbf{0.21\ atm.} \quad \textbf{(Ans.)}$$

Again, $$x_{N_2} = \frac{n_{N_2}}{n} = \frac{p_{N_2}}{p}$$

$\therefore$ $$\mathbf{p_{N_2}} = \frac{n_{N_2}}{n} \times p = \frac{3.76}{1+3.76} \times 1 = \mathbf{0.79\ atm.}\quad \textbf{(Ans.)}$$

(*iii*) **The kg of nitrogen per kg of mixture :**

$$\frac{m_{N_2}}{m_{N_2}+m_{O_2}} = \frac{n_{N_2} M_{N_2}}{n_{N_2} M_{N_2} + n_{O_2} M_{O_2}}$$

$$= \frac{3.76 \times 28}{3.76 \times 28 + 1 \times 32} = \mathbf{0.77\ kg\ N_2/kg\ mix.}\quad \textbf{(Ans.)}$$

Example 9.16. *Air (N_2 = 77%, O_2 = 23% by weight) at 25°C and 12 bar is contained in a vessel of capacity 0.6 m^3. Some quantity of CO_2 is forced into the vessel so that the temperature remains at 25°C but the pressure rises to 18 bar.*

Find the masses of O_2, N_2 and CO_2 in the cylinder.

Solution. Volume of the vessel, V = 0.6 m^3

Temperature (constant), T = 25 + 273 = 298 K

Initial pressure = 12 bar

Final pressure = 18 bar

Now $$pV = nR_0T$$

or $$n = \frac{pV}{R_0T} = \frac{12 \times 10^5 \times 0.6}{8.314 \times 10^3 \times 298} = 0.29$$

Also $$R_{air} = \left(\frac{m_{O_2}}{M_{O_2}} + \frac{m_{N_2}}{M_{N_2}}\right)\frac{R_0}{m} \qquad (m = n \times M)$$

Considering 100 kg of air

$$m_{O_2} = 23 \text{ kg},\ m_{N_2} = 77 \text{ kg}$$

$\therefore$ $$R_{air} = \left(\frac{23}{32} + \frac{77}{28}\right) \times \frac{8.314}{100} = 0.288 \text{ kJ/kg K}$$

$$M_{air} = \frac{R_0}{R_{air}} = \frac{8.314}{0.288} = 28.87$$

Now $$pV = mRT$$

$\therefore$ $$m = \frac{pV}{RT} = \frac{12 \times 10^5 \times 0.6}{0.288 \times 298 \times 10^3} = 8.39 \text{ kg of air}$$

Mass of O_2, m_{O_2} = 0.23 × 8.39 = **1.93 kg. (Ans.)**

Mass of N_2, m_{N_2} = 0.77 × 8.39 = **6.46 kg. (Ans.)**

After adding CO_2 in the vessel :

$$p_{N_2} + p_{O_2} = 12 \text{ bar} \qquad \text{... before adding } CO_2$$

$$p_{CO_2} + (p_{N_2} + p_{O_2}) = 18 \text{ bar} \qquad \text{... after adding } CO_2$$

or $$p_{CO_2} + 12 = 18$$

$$p_{CO_2} = 6 \text{ bar}$$

Now $$p_{CO_2} = \frac{m_{CO_2} R_{CO_2} T_{CO_2}}{V}$$

$\therefore$ $$m_{CO_2} = \frac{p_{CO2}}{R_{CO_2} T_{CO_2}} = \frac{6 \times 10^5 \times 0.6}{\left(\frac{8.314}{44}\right) \times 298 \times 10^3} = 6.39 \text{ kg}$$

i.e., **Mass of CO_2 = 6.39 kg. (Ans.)**

Example 9.17. *A vessel of 6 m^3 capacity contains two gases A and B in proportion of 45 per cent and 55 per cent respectively at 30°C. If the value of R for the gases is 0.288 kJ/kg K and 0.295 kJ/kg K and if the total weight of the mixture is 2 kg, calculate :*

(*i*) *The partial pressure ;* (*ii*) *The total pressure ;*

(*iii*) *The mean value of R for the mixture.*

Solution. Capacity of the vessel, $V = 6 \text{ m}^3$

%age content of gas A = 45%

%age content of gas B = 55%

R for gas A, $R_A = 0.288$ kJ/kg K

R for gas B, $R_B = 0.295$ kJ/kg K

Total weight of the mixture = 2 kg

Temperature, $T = 30 + 273 = 303$ K.

(*i*) **The partial pressures, p_A, p_B :**

$$m_A = 2 \times 0.45 = 0.9 \text{ kg}$$

$$m_B = 2 \times 0.55 = 1.1 \text{ kg}$$

Now, $$p_A V = m_A R_A T_A$$

$\therefore$ $$\mathbf{p_A} = \frac{m_A R_A T_A}{V} = \frac{0.9 \times 0.288 \times 303 \times 10^3}{6 \times 10^5} = \mathbf{0.13 \text{ bar.}} \quad \textbf{(Ans.)}$$

and $$\mathbf{p_B} = \frac{m_B R_B T_B}{V} = \frac{1.1 \times 0.295 \times 303 \times 10^3}{6 \times 10^5} = \mathbf{0.164 \text{ bar.}} \quad \textbf{(Ans.)}$$

(*ii*) **The total pressure, p :**

$$p = p_A + p_B = 0.13 + 0.164 = \mathbf{0.294 \text{ bar.}} \quad \textbf{(Ans.)}$$

(*iii*) **The mean value of R for the mixture, R_m :**

$$R_m = \frac{m_A R_A + m_B R_B}{m_A + m_B}$$

$$= \frac{0.9 \times 0.288 + 1.1 \times 0.295}{(0.9 + 1.1)} = 0.292 \text{ kJ/kg K}$$

i.e., **Mean value of R for the mixture = 0.292 kJ/kg K. (Ans.)**

Example 9.18. *The pressure and temperature of mixture of 4 kg of O_2 and 6 kg of N_2 are 4 bar and 27°C respectively. For the mixture determine the following :*

(*i*) *The mole fraction of each component ;* (*ii*) *The average molecular weight ;*

(*iii*) *The specific gas constant ;* (*iv*) *The volume and density ;*

(*v*) *The partial pressures and partial volumes.*

Solution. Mass of oxygen, m_{O_2} = 4 kg

Mass of nitrogen, m_{N_2} = 6 kg

Pressure, p = 4 bar

Temperature, T = 27 + 273 = 300 K.

(*i*) **The mole fraction of each component :**

$$n = \frac{m}{M}$$

∴ $$n_{O_2} = \frac{4}{32} = 0.125 \text{ and } n_{N_2} = \frac{6}{28} = 0.214$$

Now $$\mathbf{x_{O_2}} = \frac{0.125}{0.125 + 0.214} = \mathbf{0.3687.} \quad \textbf{(Ans.)}$$

and $$\mathbf{x_{N_2}} = \frac{0.214}{0.125 + 0.214} = \mathbf{0.6313.} \quad \textbf{(Ans.)}$$

(*ii*) **The average molecular weight, M :**

$$M = \frac{n_{O_2} M_{O_2} + n_{N_2} \times M_{N_2}}{n_{O_2} + n_{N_2}} = \frac{0.125 \times 32 + 0.214 \times 28}{0.125 + 0.214} = 29.475$$

i.e., **Average molecular weight = 29.475. (Ans.)**

(*iii*) **The specific gas constant, R :**

$$\mathbf{R} = \frac{R_0}{M} = \frac{8.314}{29.475} = \mathbf{0.282\ kJ/kg\ K.} \quad \textbf{(Ans.)}$$

(*iv*) **The volume and density :**

$pV = mRT$ for mixture

$$\mathbf{V} = \frac{mRT}{p} = \frac{(4+6) \times 0.282 \times 10^3 \times 300}{4 \times 10^5} = \mathbf{2.115\ m^3.} \quad \textbf{(Ans.)}$$

Density, $\rho = \rho_{O_2} + \rho_{N_2}$

$$\rho_{O_2} = \frac{m_{O_2}}{V} = \frac{4}{2.115} = 1.891 \text{ kg/m}^3$$

$$\rho_{N_2} = \frac{m_{N_2}}{V} = \frac{6}{2.115} = 2.837 \text{ kg/m}^3$$

∴ ρ = 1.891 + 2.837 = **4.728 kg/m³. (Ans.)**

(*v*) **The partial pressures and partial volumes :**

$$p_{O_2} V = n_{O_2} R_0 T$$

∴ $$\mathbf{p_{O_2}} = \frac{n_{O_2} R_0 T}{V} = \frac{0.125 \times 8.314 \times 10^3 \times 300}{2.115 \times 10^5} = \mathbf{1.474\ bar.} \quad \textbf{(Ans.)}$$

and $\mathbf{p_{N_2}}$ = 4 – 1.474 = **2.526 bar. (Ans.)**

$\mathbf{V_{O_2}} = x_{O_2} V$ = 0.3687 × 2.115 = **0.779 m³. (Ans.)**

$\mathbf{V_{N_2}} = x_{N_2} V$ = 0.6313 × 2.115 = **1.335 m³. (Ans.)**

☞**Example 9.19.** *4 kg of carbon dioxide at 40°C and 1.4 bar are mixed with 8 kg of nitrogen at 160°C and 1.0 bar to form a mixture at a final pressure of 0.7 bar. The process occurs adiabatically in a steady flow apparatus. Calculate :*

(*i*) *The final temperature of the mixture ;* (*ii*) *The change in entropy.*

Take value of c_p *: for* CO_2 *= 0.85 kJ/kg K and* N_2 *= 1.04 kJ/kg K.*

Solution. Refer Fig. 9.10.

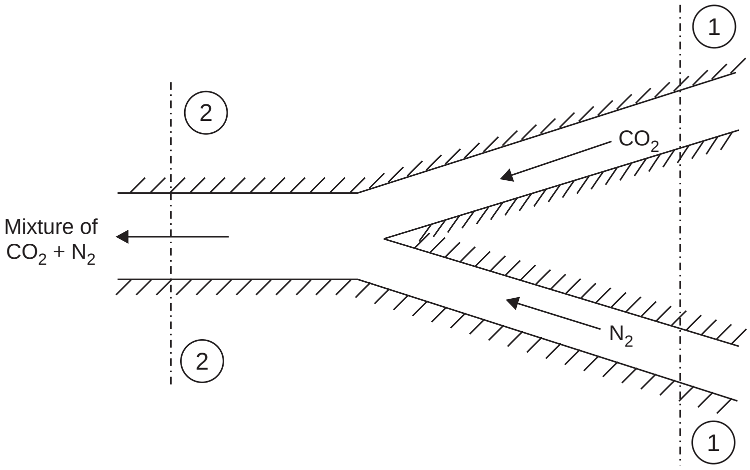


Fig. 9.10

(*i*) **Final temperature, T_2 :**

In this process, $W = 0,\ Q = 0$

$\therefore$ The steady flow equation may be written as

$$H_1 = H_2$$

i.e.,
$$(mh_1)_{CO_2} + (mh_1)_{N_2} = (mh_2)_{\text{mixture}} = (mh_2)_{CO_2} + (mh_2)_{N_2}$$

or
$$[m(h_1 - h_2)]_{CO_2} + [m(h_1 - h_2)]_{N_2} = 0$$

or
$$[mc_p(T_1 - T_2)]_{CO_2} + [mc_p(T_1 - T_2)]_{N_2} = 0$$

or
$$4 \times 0.85(40 - T_2) + 8 \times 1.04(160 - T_2) = 0$$

or
$$136 - 3.4T_2 + 1331.2 - 8.32T_2 = 0$$

or
$$1467.2 - 11.72T_2 = 0$$

$\therefore$ $T_2 = 125.2°\text{C} =$ **398.2 K. (Ans.)**

(*ii*) **Change in entropy :**

Now
$$n_{CO_2} = \frac{4}{44} = 0.0909$$

$$n_{N_2} = \frac{8}{28} = 0.2857$$

$\therefore$
$$n = n_{CO_2} + n_{N_2} = 0.0909 + 0.2857 = 0.3766$$

Again,
$$\frac{p_{(CO_2)_2}}{p_2} = x_{CO_2}$$ [p_2 = pressure of the mixture]

$$\therefore \qquad p_{(CO_2)_2} = 0.7 \times \frac{0.0909}{0.3766} = 0.17 \text{ bar}$$

Similarly,
$$p_{(N_2)_2} = 0.7 \times \frac{0.2857}{0.3766} = 0.53 \text{ bar}$$

$\therefore$ Change in entropy, ΔS

$$= \left[mc_p \log_e \frac{T_2}{T_1} - mR \log_e \frac{p_2}{p_1} \right]_{CO_2} + \left[mc_p \log_e \frac{T_2}{T_1} - mR \log_e \frac{p_2}{p_1} \right]_{N_2}$$

$$= 4 \left[0.85 \log_e \frac{398.2}{313} - \frac{8.314}{44} \log_e \frac{0.17}{1.4} \right] + 8 \left[1.04 \log_e \frac{398.2}{433} - \frac{8.314}{28} \log_e \frac{0.53}{1.0} \right]$$

$$= 4(0.2046 + 0.3984) + 8(-0.0871 + 0.1885) = 3.2232 \text{ kJ/K}$$

i.e., **Change in entropy = 3.2232 kJ/K. (Ans.)**

☞**Example 9.20.** *An insulated vessel containing 1 mole of oxygen at a pressure of 2.5 bar and a temperature of 293 K is connected through a valve to a second insulated rigid vessel containing 2 mole nitrogen at a pressure of 1.5 bar and a temperature of 301 K. The valve is opened and adiabatic mixing takes place. Assuming that oxygen and nitrogen are perfect gases calculate the entropy change in the mixing process.*

Assume the following specific heats at constant volume :

$$c_{v(O_2)} = 0.39 \text{ kJ/kg K}$$

$$c_{v(N_2)} = 0.446 \text{ kJ/kg K.}$$

Solution. Consider the system within the boundary of Fig. 9.11.

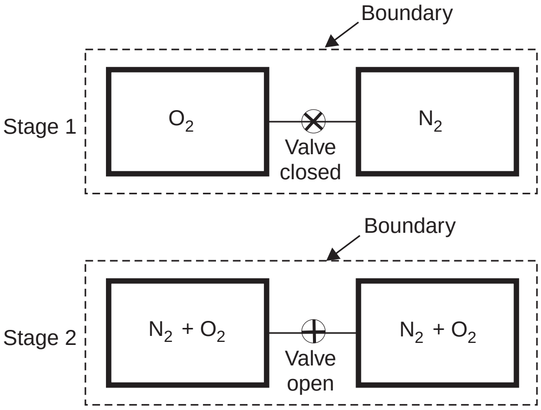


Fig. 9.11

In the process : $Q = 0$, $W = 0$, hence by the first law for a non-flow process $\Delta U = 0$. Let T_2 be the final temperature of the mixture.

$$\therefore \qquad \left[m_{O_2} c_{v_{(O_2)}} T_{O_2} + m_{N_2} c_{v_{(N_2)}} T_{N_2} \right] = \left[m_{O_2} c_{v_{(O_2)}} + m_{N_2} c_{v_{(N_2)}} \right] T_2$$

or $[(1 \times 32) \times 0.39 \times 293 + (2 \times 28) \times 0.446 \times 301] = (1 \times 32 \times 0.39 + (2 \times 28) \times 0.446]\ T_2$

or
$$T_2 = \frac{32 \times 0.39 \times 293 + 56 \times 0.446 \times 301}{32 \times 0.39 + 56 \times 0.446} = \frac{11174.4}{37.45} = 298.4 \text{ K}$$

Now entropy change in the mixing process is given by

$$\Delta S = m_{O_2}\, \Delta_{s_{(O_2)}} + m_{N_2}\, \Delta_{s_{(N_2)}}$$

where $\Delta_s = c_v \log_e \dfrac{T_2}{T_1} + R \log_e \dfrac{V_2}{V_1}$

Now to find initial and final volumes of O_2 and N_2 using the relation,

$$V = \frac{nR_0T}{p}$$

$\therefore$ The initial volume of $O_2 = \dfrac{1 \times 8.314 \times 293 \times 10^3}{2.5 \times 10^5} = 9.74\ m^3$

and Initial volume of $N_2 = \dfrac{2 \times 8.314 \times 301 \times 10^3}{1.5 \times 10^5} = 33.4\ m^3$

Final volume of the mixture = 9.74 + 33.4 = 43.14 m^3

Thus

$$\Delta S = \left[(1 \times 32)\left\{0.39 \times \log_e \frac{298.4}{293} + \frac{8.314}{32} \log_e \frac{43.14}{9.74}\right\}\right]$$

$$+ \left[(2 \times 28)\left\{0.446 \times \log_e \frac{298.4}{301} + \frac{8.314}{28} \log_e \frac{43.14}{33.4}\right\}\right]$$

$$= 12.60 + 4.04 = 16.64 \text{ kJ}$$

i.e., **Entropy change in the mixing process = 16.64 kJ. (Ans.)**

☞**Example 9.21.** *A tank of capacity 0.45 m^3 is insulated and is divided into two sections through a partition. One section initially contains H_2 at 3 bar and 130°C and has a volume of 0.3 m^3 and the other section initially holds N_2 at 6 bar and 30°C. The gases are then allowed to mix after removing the adiabatic partition. Determine :*

(*i*) *The temperature of the equilibrium mixture ;*

(*ii*) *The pressure of the mixture ;*

(*iii*) *The change in entropy for each component and total value.*

Assume : $c_{v_{(N_2)}} = 0.744$ *kJ/kg K,* $c_{v_{(H_2)}} = 10.352$ *kJ/kg K*

$c_{p_{(N_2)}} = 1.041$ *kJ/kg K,* $c_{p_{(H_2)}} = 14.476$ *kJ/kg K.*

Solution. Total capacity of the tank, $V = 0.45\ m^3$

$V_{H_2} = 0.3\ m^3$; $T_{H_2} = 130 + 273 = 403$ K

$p_{H_2} = 3$ bar ; $V_{N_2} = 0.15\ m^3$ (*i.e.,* 0.45 – 0.3 = 0.15 m^3)

$p_{N_2} = 6$ bar ; $T_{N_2} = 30 + 273 = 303$ K.

(*i*) **Temperature of equilibrium mixture, T_2 :**

Now $p_{H_2} V_{H_2} = m_{H_2} R_{H_2} T_{H_2}$

$$\therefore \quad m_{H_2} = \frac{3 \times 10^5 \times 0.3}{\left(\dfrac{8.314}{2}\right) \times 403 \times 10^3} = 0.0537 \text{ kg} \qquad \left[\because\ R = \frac{R_0}{M}\right]$$

and $$p_{N_2} V_{N_2} = m_{N_2} R_{N_2} T_{N_2}$$

$$\therefore \quad m_{N_2} = \frac{6 \times 10^5 \times 0.15}{\left(\frac{8.314}{28}\right) \times 303 \times 10^3} = 1.0 \text{ kg}$$

According to the first law for a closed system

$$\Delta U = 0$$

i.e., $$U_{\text{initial}} = U_{\text{final}}$$

or $$m_{H_2} c_{v(H_2)} T_{(H_2)_1} + m_{N_2} c_{v(N_2)} T_{(N_2)_1} = m_{H_2} c_{v(H_2)} T_{(H_2)_2} + m_{N_2} c_{v(N_2)} T_{(N_2)_2}$$

or $$m_{H_2} c_{v(H_2)} \left[T_{(H_2)_2} - T_{(H_2)_1}\right] + m_{N_2} c_{v(N_2)} \left[T_{(N_2)_2} - T_{(N_2)_1}\right] = 0$$

or $$0.0537 \times 10.352 \left[T_{(H_2)_2} - 403\right] + 1.0 \times 0.744 \left[T_{(N_2)_2} - 303\right] = 0$$

$$T_{(H_2)_2} = T_{(N_2)_2} = T_2$$

or $$0.556(T_2 - 403) + 0.744(T_2 - 303) = 0$$

or $$0.556T_2 - 224 + 0.744T_2 - 225.4 = 0$$

$$\therefore \quad T_2 = 345.7 \text{ K}$$

i.e., Temperature of the mixture **= 345.7 K. (Ans.)**

(*ii*) **Pressure of the mixture, p_2 :**

Now $$p_{(H_2)_2} V = m_{H_2} R_{H_2} T_2$$

$$\therefore \quad p_{(H_2)_2} = \frac{0.0537 \times \left(\frac{8.314}{2}\right) \times 345.7 \times 10^3}{0.45 \times 10^5} = 1.71 \text{ bar}$$

Similarly $$p_{(N_2)_2} = \frac{1.0 \times \left(\frac{8.314}{28}\right) \times 345.7 \times 10^3}{0.45 \times 10^5} = 2.28 \text{ bar}$$

$$\therefore \quad \mathbf{p_2} = p_{(H_2)_2} + p_{(N_2)_2} = 1.71 + 2.28 = \mathbf{3.99 \text{ bar. (Ans.)}}$$

(*iii*) **Change in entropy :**

Now $$(\Delta S)_{H_2} = m\left(c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}\right)_{H_2}$$

$$= 0.0537 \left[14.476 \log_e \frac{345.7}{403} - \frac{8.314}{2} \log_e \frac{1.71}{3}\right]$$

$$= \mathbf{0.00626 \text{ kJ/K. (Ans.)}}$$

$$(\Delta S)_{H_2} = m\left(c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}\right)$$

$$= 1.0 \left[1.041 \log_e \frac{345.7}{303} - \frac{8.314}{28} \log_e \frac{2.28}{6}\right]$$

$$= \mathbf{0.424 \text{ kJ/K. (Ans.)}}$$

$$\therefore \qquad \Delta S = (\Delta S)_{H_2} + (\Delta S)_{N_2} = 0.00626 + 0.424 = 0.43026 \text{ kJ/K}$$

i.e., **Total change in entropy = 0.43026 kJ/K. (Ans.)**

Example 9.22. *A perfect gas mixture consists of 4 kg of* N_2 *and 6 kg of* CO_2 *at a pressure of 4 bar and a temperature of 25°C. Calculate* c_v *and* c_p *of the mixture.*

If the mixture is heated at constant volume to 50°C, find the change in internal energy, enthalpy and entropy of the mixture.

Take : $c_{v(N_2)} = 0.745$ *kJ/kg K,* $c_{v(CO_2)} = 0.653$ *kJ/kg K*

$c_{p(N_2)} = 1.041$ *kJ/kg K,* $c_{p(CO_2)} = 0.842$ *kJ/kg K.*

Solution. $m_{N_2} = 4$ kg, $m_{CO_2} = 6$ kg, $p_{mix} = 4$ bar

$T_1 = 25 + 273 = 298$ K, $T_2 = 50 + 273 = 323$ K

$\mathbf{c_{v(mix)} = ?,\ c_{p(mix)} = ?}$

Using the relation,

$$(m_{N_2} + m_{CO_2}) c_{v(mix)} = m_{N_2} c_{v(N_2)} + m_{CO_2} c_{v(CO_2)}$$

$$(4 + 6)\ c_{v(mix)} = 4 \times 0.745 + 6 \times 0.653$$

$$\therefore \qquad \mathbf{c_{v(mix)}} = \frac{4 \times 0.745 + 6 \times 0.653}{4 + 6} = \mathbf{0.6898\ kJ/kg\ K.\quad (Ans.)}$$

Similarly,

$$\mathbf{c_{p(mix)}} = \frac{4 \times 1.041 + 6 \times 0.842}{4 + 6} = \mathbf{0.9216\ kJ/kg\ K.\quad (Ans.)}$$

Change in internal energy, ΔU :

$$\mathbf{\Delta U} = [mc_v(T_2 - T_1)]_{mix}$$
$$= (4 + 6) \times 0.6898(323 - 298) = \mathbf{172.45\ kJ.\quad (Ans.)}$$

Change in enthalpy, ΔH :

$$\mathbf{\Delta H} = [mc_p(T_2 - T_1)]_{mix}$$
$$= (4 + 6) \times 0.9216(323 - 298) = \mathbf{230.4\ kJ.\quad (Ans.)}$$

Change in entropy, ΔS :

$$(\Delta s) = \left(c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}\right)$$

$$= \left(c_v \log_e \frac{T_2}{T_1}\right) \qquad (\because v_2 = v_1)$$

$$\therefore \qquad (\Delta s)_{N_2} = \left(c_v \log_e \frac{T_2}{T_1}\right)_{N_2}$$

and

$$(\Delta s)_{CO_2} = \left(c_v \log_e \frac{T_2}{T_1}\right)_{CO_2}$$

Hence,

$$\mathbf{\Delta S} = \left(mc_v \log_e \frac{T_2}{T_1}\right)_{N_2} + \left(mc_v \log_e \frac{T_2}{T_1}\right)_{CO_2}$$

$$= 4 \times 0.745 \log_e \frac{323}{298} + 6 \times 0.653 \log_e \frac{323}{298}$$

$$= \mathbf{0.5557\ kJ/K.} \quad \textbf{(Ans.)}$$

Note. ΔS may also be found out as follows :

$$\Delta S = (m_{N_2} + m_{CO})\ c_{v(\text{mix})} \log_e \frac{T_2}{T_1}$$

$$= (4 + 6) \times 0.6898 \log_e \frac{323}{298} = 0.5557 \text{ kJ/K.}$$

HIGHLIGHTS

1. According to Dalton's law :

(*i*) The pressure of a mixture of gases is equal to the sum of the partial pressures of the constituents.

(*ii*) The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixture at the same temperature.

2. According to Gibbs-Dalton law :

(*i*) The internal energy, enthalpy and entropy of a gaseous mixture are respectively equal to the sum of the internal energies, enthalpies and entropies of the constituents.

(*ii*) Each constituent has that internal energy, enthalpy and entropy, which it would have if it occupied alone that volume occupied by the mixture at the temperature of the mixture.

3. The characteristic equation for mixture is given as :

$$pV = nR_0T$$

where n = Number of moles of mixture, and
R_0 = Universal gas constant.

4. Molecular weight (M) may be found out by using the following relations :

$$M = \sum \frac{n_i}{n} M_i \quad \text{and} \quad M = \frac{1}{\sum \frac{m_{fi}}{M_i}}$$

where $m_f = \frac{m_i}{m}$ = mass fraction of a constituent.

5. The following condition must be satisfied in an adiabatic mixing process of perfect gas in steady flow :

$$T = \frac{\Sigma m_i c_{pc} T_i}{\Sigma m_i c_{pi}} = \frac{\Sigma n_i C_{pi} T_i}{\Sigma n_i C_{pi}} .$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. In an ideal gas the partial pressure of a component is

(*a*) inversely proportional to the square of the mole fraction

(*b*) directly proportional to the mole fraction

(*c*) inversely proportional to the mole fraction

(*d*) equal to the mole fraction.

2. The value of the universal gas constant is

(*a*) 8.314 J/kg K (*b*) 83.14 kJ/kg K

(*c*) 848 kJ/kg K (*d*) 8.314 kJ/kg K.

3. In a mixture of gases, the partial pressure p_i of any constituent gas can be found by

(*a*) $p_i = n_i RT/V$ (*b*) $p_i = m_i RT/V_i$

(*c*) $p_i = n_i R_0 T/V_i$ (*d*) $p_i = n_i R_0 T/V$

where, R = Characteristic gas constant,
R_0 = Universal gas constant,
V = Volume of the mixture,
V_i = Volume of the ith constituent, and
T = Temperature of the mixture.

4. Mole fraction of a component of gas mixture is equal to

(*a*) $1/f$ (*b*) f^2

(*c*) f (*d*) f/p

where, f = Volume fraction, and
p = Pressure of the mixture.

5. In a gaseous mixture the specific volume of each component is given by

(*a*) V/m (*b*) V_i/m_i

(*c*) V/m_i (*d*) none of the above.

where, V = Volume of the mixture,
V_i = Volume of the ith component,
m = Mass of mixture, and
m_i = Mass of the ith component.

Answers

1. (*b*) **2.** (*d*) **3.** (*d*) **4.** (*c*) **5.** (*c*).

THEORETICAL QUESTIONS

1. Define the following terms :
(*i*) Partial pressure (*ii*) Mole fraction
(*iii*) Volume fraction of a gas constituent in a mixture.

2. Explain briefly Dalton's law and Gibbs-Dalton law.

3. State and explain Amagat's law or Leduc's law.

4. Prove that the molar analysis is identical with the volumetric analysis, and both are equal to the ratio of the partial pressure to the total pressure.

5. Prove the following relation

$$M = \frac{\Sigma n_i M_i}{\Sigma n_i} \quad \frac{1}{\sum \frac{m_{fi}}{M_i}}$$

where, M = Molecular weight of the mixture,
n_i = Number of moles of an any constituent,
m_{fi} = Mass fraction of the constituent, and
M_i = Molecular weight of the constituent.

UNSOLVED EXAMPLES

1. 0.45 kg of carbon monoxide (28) and 1 kg of air at 15°C are contained in a vessel of volume 0.4 m^3. Calculate the partial pressure of each constituent and the total pressure in the vessel. The gravimetric analysis of air is to be taken as 23.3% oxygen (32) and 76.7% nitrogen (28).

[**Ans.** p_{O_2} = 0.4359 bar ; p_{N_2} = 1.64 bar, p_{CO} = 0.962 bar]

2. A mixture of CO and O_2 is to be prepared in the proportion of 7 kg to 4 kg in a vessel of 0.3 m^3 capacity. If the temperature of the mixture is 15°C, determine the pressure to which the vessel is subjected. If the temperature is raised to 40°C, what will then be the pressure in the vessel ? [**Ans.** 29.9 bar, 32.5 bar]

3. Assuming that air may be treated as a mixture of ideal gases which has a mass composition 23.2% oxygen and 76.8% nitrogen, find the gas constant and apparent molecular weight of air.

[**Ans.** 0.288 kJ/kg K, 28.86]

4. A mixture of 1 mole CO_2 and 3.5 moles of air is contained in a vessel at 1 bar and 15°C. The volumetric analysis of air can be taken as 21% oxygen and 79% nitrogen. Calculate for the mixture : (*i*) The masses of CO_2, O_2 and N_2, and the total mass. (*ii*) The percentage carbon content by mass. (*iii*) The apparent molecular weight and the gas constant for the mixture. (*iv*) The specific volume of the mixture.

[**Ans.** (*i*) 44 kg, 23.55 kg, 77.5 kg ; (*ii*) 8.27% ; (*iii*) 32.2, 0.2581 kJ/kg K ; (*iv*) 0.7435 m^3/kg]

5. A mixture of H_2 and O_2 is to be made so that the ratio of H_2 to O_2 is 2 to 1 by volume. Calculate the mass of O_2 required and the volume of the container, per kg of H_2, if the pressure and temperature are 1 bar and 15°C respectively. [**Ans.** 8 kg, 17.96 m^3]

6. A vessel contains a gaseous mixture of composition by volume, 80% H_2 and 20% CO. It is desired that the mixture should be made in proportion 50% H_2 and 50% CO by removing some of the mixture and adding some CO. Calculate per mole of mixture the mass of mixture to be removed, and the mass of CO to be added. The pressure and temperature in the vessel remain constant during the procedure.

[**Ans.** 2.7 kg, 10.5 kg]

7. The gas in an engine cylinder has a volumetric analysis of 12% of CO_2, 11.5% O_2 and 76.5% N_2. The temperature at the beginning of expansion is 1000°C and the gas mixture expands reversibly through a volume ratio of 7 : 1, according to a law $pv^{1.25}$ = constant. Calculate the work done and the heat flow per kg of gas. The values of c_p for the constituents are as follows : c_p for CO_2 = 1.235 kJ/kg K ; c_p for O_2 = 1.088 kJ/kg K ; c_p for N_2 = 1.172 kJ/kg K. [**Ans.** 536.3 kJ/kg ; 96 kJ/kg (heat supplied)]

8. A producer gas has the following volumetric analysis : 29% CO, 12% H_2, 3% CH_4, 4% CO_2, 52% N_2. Calculate the values of C_p, C_v, c_p and c_v for the mixture. The values of C_p for the constituents are as follows : for CO C_p = 29.27 kJ/mole K ; for H_2 C_p = 28.89 kJ/mole K ; for CH_4 C_p = 35.8 kJ/mole K ; for CO_2 C_p = 37.22 kJ/mole K ; for N_2 C_p = 29.14 kJ/mole K.

[**Ans.** 29.676 kJ/mole K, 21.362 kJ/mole K ; 1.178 kJ/kg K ; 0.8476 kJ/kg K]

9. An exhaust gas is analysed and is found to contain, by volume, 78% N_2, 12% CO_2 and 10% O_2. What is the corresponding gravimetric analysis ? Calculate the mass of mixture per mole, and the density if the temperature is 550°C and the total pressure is 1 bar.

[**Ans.** 72% N_2, 17.3% CO_2, 10.6% O_2 ; 30.28 kg/mole ; 0.442 kg/m^3]

10. A vessel of 3 m^3 capacity contains a mixture of nitrogen and carbon dioxide, the analysis by volume showing equal quantities of each. The temperature is 15°C and the total pressure is 3.5 bar. Determine the mass of each constituent. [**Ans.** 6.14 kg N_2 ; 9.65 kg CO_2]

11. A mixture is made up of 25% N_2 35% O_2, 20% CO_2 and 20% CO by volume. Calculate : (*i*) The molecular weight of the mixture. (*ii*) C_p and C_v for the mixture. (*iii*) γ for the mixture. (*iv*) The partial pressure of each constituent when the total pressure is 1.5 bar. (*v*) The density of the mixture at 1.5 bar and 15°C.

[**Ans.** (*i*) 32.6 ; (*ii*) 30.9, 22.53 kJ/mole K ; (*iii*) 1.37 ; (*iv*) 0.375, 0.525, 0.3, 0.3 bar ; (*v*) 2.04 kg/m^3]

12. A mixture of ideal gases consists of 3 kg of nitrogen and 5 kg of carbon dioxide at a pressure of 3 bar and a temperature of 20°C. Find (*i*) the mole fraction of each constituent, (*ii*) the equivalent molecular weight of the mixture, (*iii*) the equivalent gas constant of the mixture, (*iv*) the partial pressures and partial volumes, (*v*) the volume and density of the mixture, and (*vi*) the c_p and c_v of the mixture.

If the mixture is heated at constant volume to 40°C, find the changes in internal energy, enthalpy and entropy of the mixture. Find the changes in internal energy, enthalpy and entropy of the mixture if the heating is done at constant pressure.

Take γ : for CO_2 = 1.286 and N_2 = 1.4.

[**Ans.** (*i*) 0.485, 0.515 ; (*ii*) 36.25 kg/kg mole ; (*iii*) 0.229 kJ/kg K ;
(*iv*) 1.455 bar, 1.545 bar ; 0.87 m^3, 0.923 m^3 ; (*v*) 1.79 m^3, 4.46 kg/m^3 ;
(*vi*) 0.92 kJ/kg K, 0.69 kJ/kg K ; 110.4 kJ, 147.2 kJ, 0.368 kJ/kg K ; 0.49 kJ/kg K]

13. A vessel of 1.5 m^3 capacity contains oxygen at 7 bar and 40°C. The vessel is connected to another vessel of 3 m^3 capacity containing carbon monoxide at 1 bar and 15°C. A connecting valve is opened and the gases mix adiabatically. Calculate : (*i*) The final temperature and pressure of the mixture. (*ii*) The change in entropy of the system.

Assume : For oxygen C_v = 21.07 kJ/mole K

For carbon monoxide C_v = 20.86 kJ/mole K. [**Ans.** (*i*) 34°C, 3 bar ; (*ii*) 4.108 kJ/K]

14. Two vessels both containing nitrogen, are connected by a valve which is opened to allow the contents to mix and achieve an equilibrium temperature of 27°C. Before mixing the following information is known about the gases in the two vessels.

Vessel A	*Vessel B*
p = 15 bar	p = 6 bar
t = 50°C	t = 20°C
Contents = 0.5 kg mole	Contents = 2.5 kg

Calculate the final equilibrium pressure and amount of heat transferred to the surroundings. If the vessel had been perfectly insulated, calculate the final temperature and pressure which would have been reached.

Take γ = 1.4. [**Ans.** 11.68 bar, – 226.2 kJ, 45.5°C, 12.4 bar]

15. Find the increase in entropy when 2 kg of oxygen at 60°C are mixed with 6 kg of nitrogen at the same temperature. The initial pressure of each constituent is 10.3 bar and is the same as that of the mixture. [**Ans.** 1.2314 kJ/kg K]

16. A gas mixture contains hydrogen, nitrogen and carbon monoxide in equal molar proportions. Treating the components of the mixture as perfect gases, determine the gas constant and the ratio of specific heats for the mixture. Calculate the work required to compress 1 kg of the mixture in reversible adiabatic flow through a pressure ratio of 2 from an initial temperature of 15°C when kinetic and potential energy changes are negligible. [**Ans.** 0.43 kJ/kg K, 1.4, – 82.76 kJ]

17. 2 kg of nitrogen at 20 bar and 38°C is contained in a rigid vessel. A sufficient quantity of oxygen is added to increase the pressure to 26.66 bar while the temperature remains constant at 38°C.

Calculate the mass of oxygen added. [**Ans.** 0.761 kg]

18. Air (N_2 = 77%, O_2 = 23% by weight) at 20°C and 10 bar is contained in a vessel of capacity of 0.5 m^3. Some quantity of carbon dioxide is forced into the vessel so that the temperature remains at 20°C but the pressure rises to 15 bar. Find the masses of oxygen, nitrogen and carbon dioxide in the cylinder. The universal gas constant is 8.3143 kJ/kg K. [**Ans.** 1.35 kg, 4.54 kg, 4.51 kg]

19. A vessel of 5 m^3 capacity contains a mixture of two gases in proportion of 40 per cent and 60 per cent at 20°C. If the value of R for the gases is 0.287 kJ/kg K and 0.294 kJ/kg K and if the total weight of the mixture is 1.5 kg, calculate :

(*i*) The partial pressure ; (*ii*) Total pressure ;

(*iii*) Mean value of R for the mixture. [**Ans.** 0.1 bar ; 0.155 bar ; 0.291 kJ/kg K]

20. A mixture consisting of 6 kg of O_2 and 9 kg of N_2 has a pressure of 3 bar and temperature of 20°C. For the mixture determine the following :

(*i*) The mole fraction of each component ; (*ii*) The average molecular weight ;

(*iii*) The specific gas constant ; (*iv*) The volume and density ;

(*v*) The partial pressures and partial volumes.

[**Ans.** (*i*) 0.3684, 0.6315 ; (*ii*) 29.475 ; (*iii*) 0.282 kJ/kg K ;

(*iv*) 4.13 m^3, 3.629 kg/m^3 ; (*v*) 1.1 bar, 1.894 bar ; 1.52 m^3, 2.61 m^3]

21. A closed vessel of 0.1 m^3 capacity contained air at 1 bar pressure and 27°C. Hydrogen was added and the total pressure in the vessel was raised to 1.2 bar at the same temperature. Find the weight of oxygen, nitrogen and hydrogen finally in the vessel and their respective partial pressures.

Air contains 77% N_2 by weight. Take the value of universal gas constant as 8.3143 kJ/kg K and molecular weight of air as 29. [**Ans.** m_{O_2} = 0.0267 kg, m_{N_2} = 0.0893 kg, m_{H_2} = 0.001603 kg]

22. A vessel of capacity 0.3 m^3 is insulated and divided into two sections by a partition. One section is 0.2 m^3 in volume and initially contains H_2 at 2 bar and 127°C. The remaining section initially holds N_2 at 4 bar and 27°C. The adiabatic partition is then removed, and the gases are allowed to mix. Determine :

(*i*) The temperature of the equilibrium mixture,
(*ii*) The pressure of the mixture, and
(*iii*) The change in entropy for each component and total value.

$$c_{v_{(N_2)}} = 0.744 \text{ kJ/kg K}, \qquad c_{v_{(H_2)}} = 10.352 \text{ kJ/kg K}$$

$$c_{p_{(N_2)}} = 1.041 \text{ kJ/kg K}, \qquad c_{p_{(N_2)}} = 14.476 \text{ kJ/kg K}.$$

[**Ans.** (*i*) 342.6 K ; (*ii*) 2.661 bar ; (*iii*) $(\Delta S)_{H_2} = 0.00235$ kJ/K ; $(\Delta S)_{N_2} = 0.1908$ kJ/K ; $\Delta S = 0.19315$ kJ/K]

23. 3 kg of N_2 and 5 kg of CO_2 at a pressure of 3 bar and a temperature of 20°C comprise a perfect gas mixture. Calculate c_v and c_p of the mixture.
If the mixture is heated at constant volume to 40°C, find the change in internal energy, enthalpy and entropy of the mixture.

Take : $c_{v_{(N_2)}} = 0.7448$ and $c_{p_{(N_2)}} = 1.0416$ kJ/kg K

$c_{v_{(CO_2)}} = 0.6529$ and $c_{p_{(CO_2)}} = 0.8418$ kJ/kg K.

[**Ans.** 0.6873 kJ/kg K, 0.9167 kJ/kg K ; 109.96 kJ, 146.67 kJ, 0.363 kJ/K]

10

Psychrometrics

10.1. CONCEPT OF PSYCHROMETRY AND PSYCHROMETRICS

Air comprises of fixed gases principally, nitrogen and oxygen with an admixture of water vapour in varying amounts. In atmospheric air water is always present and its relative weight averages less than 1% of the weight of atmospheric air in temperate climates and less than 3% by weight under the most extreme natural climatic conditions, it is nevertheless one of most important factors in human comfort and has significant effects on many materials. Its effect on human activities is in fact altogether disproportionate to its relative weights. *The art of measuring* the *moisture content of air is termed* "**psychrometry**". *The science which investigates the thermal properties of moist air, considers the measurement and control of the moisture content of air, and studies the effect of atmospheric moisture on material and human comfort may properly be termed* "**psychrometrics**".

10.2. DEFINITIONS

Some of the more important definitions are given below :

1. **Dry air.** The international joint committee on Psychrometric Data has adopted the following exact composition of air expressed in mole fractions (Volumetric) Oxygen 0.2095, Nitrogen 0.7809, Argon 0.0093, Carbon dioxide 0.0003. Traces of rare gases are neglected. Molecular weight of air for all air conditioning calculations will be taken as 28.97. Hence the gas constant,

$$R_{air} = \frac{8.3143}{28.97} = 0.287 \text{ kJ/kg K}$$

Dry air is never found in practice. Air always contains some moisture. Hence the common designation "air" *usually means moist* air. The term '*dry air*' is used to indicate the *water free contents* of air having any degree of moisture.

2. **Saturated air.** Moist air is said to be saturated when its condition is such that it can co-exist in natural equilibrium with an associated condensed moisture phase presenting a flat surface to it. For *a given temperature, a given quantity of air can be saturated with a fixed quantity of moisture.* At higher temperatures, it requires a larger quantity of moisture to saturate it. At saturation, vapour pressure of moisture in air corresponds to the saturation pressure given in steam tables corresponding to the given temperature of air.

3. **Dry-bulb temperature (DBT).** It *is the temperature of air as registered by an ordinary thermometer* (t_{db}).

4. **Wet-bulb temperature (WBT).** *It is the temperature registered by a thermometer when the bulb is covered by a wetted wick and is exposed to a current of rapidly moving air* (t_{wb}).

5. **Adiabatic saturation temperature.** *It is the temperature at which the water or ice can saturate air by evaporating adiabatically into it*. It is numerically equivalent to the measured wet bulb temperature (as corrected, if necessary for radiation and conduction) (t_{wb}).

6. **Wet bulb depression.** It *is the difference between dry-bulb and wet bulb temperatures* ($t_{db} - t_{wb}$).

7. **Dew point temperature (DPT).** *It is the temperature to which air must be cooled at constant pressure in order to cause condensation of any of its water vapour*. It is equal to steam table saturation temperature corresponding to the actual partial pressure of water vapour in the air (t_{dp}).

8. **Dew point depression.** *It* is *the difference between the dry bulb and dew point temperatures* ($t_{db} - t_{dp}$).

9. **Specific humidity (Humidity ratio).** *It is the ratio of the mass of water vapour per unit mass of dry air in the mixture of vapour and air*, it is generally expressed as grams of water per kg of dry air. For a given barometric pressure it is a function of dew point temperature alone.

10. **Relative humidity (RH), (ϕ).** *It is the ratio of the partial pressure of water vapour in the mixture to the saturated partial pressure at the dry bulb temperature, expressed as percentage.*

11. **Sensible heat.** *It is the heat that changes the temperature of a substance when added to or abstracted from it.*

12. **Latent heat.** *It is the heat that does not affect the temperature but changes the state of substance when added to or abstracted from it.*

13. **Enthalpy.** *It is the combination energy which represents the sum of internal and flow energy in a steady flow process*. It is determined from an arbitrary datum point for the air mixture and is expressed as kJ per kg of dry air (h).

Note. When air is saturated DBT, WBT, DPT are *equal*.

10.3. PSYCHROMETRIC RELATIONS

Pressure

Dalton's law of partial pressure is employed to determine the pressure of a mixture of gases. This law *states that the total pressure of a mixture of gases is equal to the sum of partial pressures which the component gases would exert if each existed alone in the mixture volume at the mixture temperature.*

Precise measurements made during the last few years indicate that this law as well as Boyle's and Charle's laws are only approximately correct. Modern tables of atmospheric air properties are based on the correct versions.

For calculating partial pressure of water vapour in the air many equations have been proposed, probably Dr. Carrier's equation is most widely used.

$$p_v = (p_{vs})_{wb} - \frac{[pt - (p_{vs})_{wb}](t_{db} - t_{wb})}{1527.4 - 1.3\,t_{wb}} \qquad \text{...(10.1)}$$

where p_v = Partial pressure of water vapour,

p_{vs} = Partial pressure of water vapour when air is fully saturated,

p_t = Total pressure of moist air,

t_{db} = Dry bulb temperature (°C), and

t_{wb} = Wet bulb temperature (°C).

Specific humidity W :

$$\text{Specific humidity} = \frac{\text{Mass of water vapour}}{\text{Mass of dry air}}$$

or

$$W = \frac{m_v}{m_a}$$

Also,

$$m_a = \frac{p_a V}{R_a T} \quad \text{...(10.2)}$$

and

$$m_v = \frac{p_v \times V}{R_v \times T} \quad \text{...(10.3)}$$

where p_a = Partial pressure of dry air,
p_v = Partial pressure of water vapour,
V = Volume of mixture,
R_a = Characteristic gas constant for dry air, and
R_v = Characteristic gas constant for water vapour.

From equations (10.2) and (10.3)

$$W = \frac{p_v \times V}{R_v \times T} \times \frac{R_a T}{p_a V} = \frac{R_a}{R_v} \times \frac{p_v}{p_a}$$

But

$$R_a = \frac{R_0}{M_a} \qquad \left(= \frac{8.3143}{28.97} = 0.287 \text{ kJ/kg K in SI units} \right)$$

$$R_v = \frac{R_0}{M_v} \qquad \left(= \frac{8.3143}{18} = 0.462 \text{ kJ/kg K in SI units} \right)$$

where R_0 = Universal gas constant,
M_a = Molecular weight of air, and
M_v = Molecular weight of water vapour.

$$\therefore \quad W = \frac{0.287}{0.462} \cdot \frac{p_v}{p_a} = 0.622 \frac{p_v}{p_t - p_v}$$

i.e.,

$$W = 0.622 \frac{p_v}{p_t - p_v} \quad \text{...(10.4)}$$

The masses of air and water vapour in terms of specific volumes are given by expression as

$$m_a = \frac{V}{v_a} \quad \text{and} \quad m_v = \frac{V}{v_v}$$

where v_a = Specific volume of dry air, and
v_v = Specific volume of water vapour.

$$W = \frac{v_a}{v_v} \quad \text{...(10.5)}$$

Degree of saturation (μ) :

$$\text{Degree of saturation} = \frac{\text{Mass of water vapour associated with unit mass of dry air}}{\text{Mass of water vapour associated with saturated unit mass of dry saturated air}}$$

i.e.,
$$\mu = \frac{W}{W_s} \qquad \text{...(10.6)}$$

where, W_s = Specific humidity of air when air is fully saturated

$$\therefore \quad \mu = \frac{0.622\left(\frac{p_v}{p_t - p_v}\right)}{0.622\left(\frac{p_{vs}}{p_t - p_{vs}}\right)} = \frac{p_v(p_t - p_{vs})}{p_{vs}(p_t - p_v)}$$

$$= \frac{p_v}{p_s}\left[\frac{\left(1 - \frac{p_{vs}}{p_t}\right)}{\left(1 - \frac{p_v}{p_t}\right)}\right] \qquad \text{...(10.7)}$$

where p_{vs} = Partial pressure of water vapour when air is fully saturated (p_{vs} can be calculated from steam tables corresponding to the dry bulb temperature of the air).

Relative humidity (RH), ϕ :

$$\text{Relative humidity, } \phi = \frac{\text{Mass of water vapour in a given volume}}{\text{Mass of water vapour in the same volume if saturated at the same temp.}}$$

$$= \frac{m}{m_{vs}} = \frac{\frac{p_v\,T}{R_v\,T}}{\frac{p_{vs}\,T}{R_v\,T}} = \frac{p_v}{p_{vs}} \qquad \text{...(10.8)}$$

Inserting the value of equation (10.8) into equation (10.7), we get

$$\mu = \phi\left[\frac{1 - \frac{p_{vs}}{p_v}}{1 - \frac{p_v}{p_t}}\right] = \phi\left[\frac{1 - \frac{p_{vs}}{p_t}}{1 - \frac{\phi p_{vs}}{p_t}}\right] = \phi\left(\frac{p_t - p_{vs}}{p_t - \phi\, p_{vs}}\right)$$

$$\therefore \quad \phi(p_t - p_{vs}) = \mu(p_t - \phi\, p_{vs})$$

or
$$\phi(p_t - p_{vs} + \mu p_{vs}) = \mu p_t$$

$$\therefore \quad \phi = \frac{\mu p_t}{p_t - p_{vs} + \mu p_{vs}} = \frac{\mu}{1 - (1 - \mu)\frac{p_{vs}}{p_t}} \qquad \text{...(10.9)}$$

Since $p_{vs} << p_t$

$\therefore \quad \phi \simeq \mu$

Inserting the value of the p_v from equation (10.4) into equation (10.8), we get

$$\phi = \frac{p_a\,W}{0.622} \times \frac{1}{p_{vs}} = 1.6\ W\,\frac{p_a}{p_{vs}} \qquad \text{...(10.10)}$$

Note 1. Relative humidity as compared to specific humidity plays a vital role in comfort air-conditioning and industrial air-conditioning. *Relative humidity signifies the absorption capacity of air*. If initial relative humidity of air is less it will absorb more moisture.

2. W, μ and ϕ cannot be conveniently measured as they require measurement of p_v and p_{vs}. The value of p_v can be obtained from the measurement of the wet bulb temperature and the value of p_{vs} can be calculated from steam tables corresponding to given air temperature.

Enthalpy of moist air

It is the sum of enthalpy of dry air and enthalpy of water vapour associated with dry air. It is expressed in kJ/kg of dry air

$$h = h_{\text{air}} + W \,.\, h_{\text{vapour}}$$
$$= c_p t_{db} + W \,.\, h_{\text{vapour}}$$

where h = Enthalpy of mixture/kg of dry air,

h_{air} = Enthalpy of 1 kg of dry air,

h_{vapour} = Enthalpy of 1 kg of vapour obtained from steam tables,

W = Specific humidity in kg/kg of dry air, and

c_p = Specific heat of dry air normally assumed as 1.005 kJ/kg K.

Also $$h_{\text{vapour}} = h_g + c_{ps}\,(t_{db} - t_{dp})$$

where h_g = Enthalpy of saturated steam at dew point temperature,

and c_{ps} = 1.88 kJ/kg K.

$$\therefore \quad h = c_p t_{db} + W[h_g + c_{ps}(t_{db} - t_{dp})] \qquad \text{...(10.11)}$$
$$= (c_p + c_{ps}\, W)\, t_{db} + W(h_g - c_{ps} t_{dp})$$
$$= c_{pm} t_{db} + W(h_g - c_{ps} t_{dp}) \qquad \text{...[10.11(}a\text{)]}$$

where $c_{pm} = (c_p + c_{ps}\, W)$ is the *specific heat of humid air or humid specific heat.*

The value of c_{pm} is taken as 1.021 kJ/kg dry air per K. It is the heat capacity of $(1 + W)$ kg of moisture per kg of dry air.

$$h_{\text{vapour}} \simeq h_g \text{ at dry bulb temperature. So,}$$
$$h = c_p t_{db} + W\, h_g. \qquad \text{...(10.12)}$$

However, a better approximation is given by the following relationship :

$$h_{\text{vapour}} = 2500 + 1.88 t_{db} \text{ kJ/kg of water vapour} \qquad \text{...[10.12 (}a\text{)]}$$

where t_{db} is dry bulb temperature in °C, and the datum state is liquid water at 0°C.

$$\therefore \quad h = 1.005\, t_{db} + W(2500 + 1.88\, t_{db}) \text{ kJ/kg dry air.} \qquad \text{...[10.12 (}b\text{)]}$$

Adiabatic saturation process

In an insulated chamber when unsaturated air flows over a long sheet of water (Fig. 10.1), the water evaporates, and the specific humidity of the air increases. As the evaporation takes place both the air and water are cooled. *The process continues until the energy transferred from the air to the water is equal to the energy required to vapourise the water.* When this point is reached,

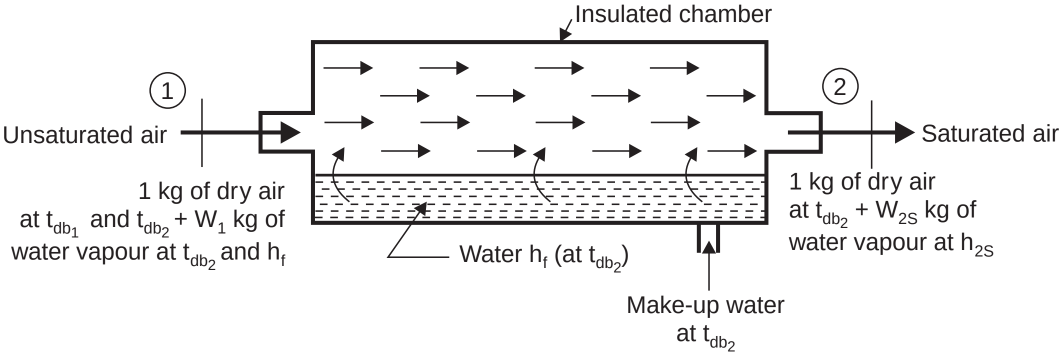


Fig. 10.1. Adiabatic saturation process.

thermal equilibrium exists with respect to water, air and water vapour, and consequently the air is saturated. The *equilibrium temperature is called the* ***adiabatic saturation temperature or the thermodynamic wet bulb temperature***. The make-up water is introduced at *this temperature* to make the water level constant. The '*adiabatic*' cooling process is shown in Fig. 10.2 for the vapour in the air-vapour mixture. Although the total pressure of the mixture is constant, the partial pressure of the vapour increases, and in the saturated state corresponds to the adiabatic saturation temperature. The vapour is initially at *DBT* t_{db_1} and is cooled adiabatically to *DBT* t_{db_2} which is equal to the adiabatic saturation t_{wb_2}. The adiabatic saturation temperature and wet bulb temperatures are taken to be equal for all practical purposes. The wet bulb temperature lies between the dry bulb temperature and dew point temperature.

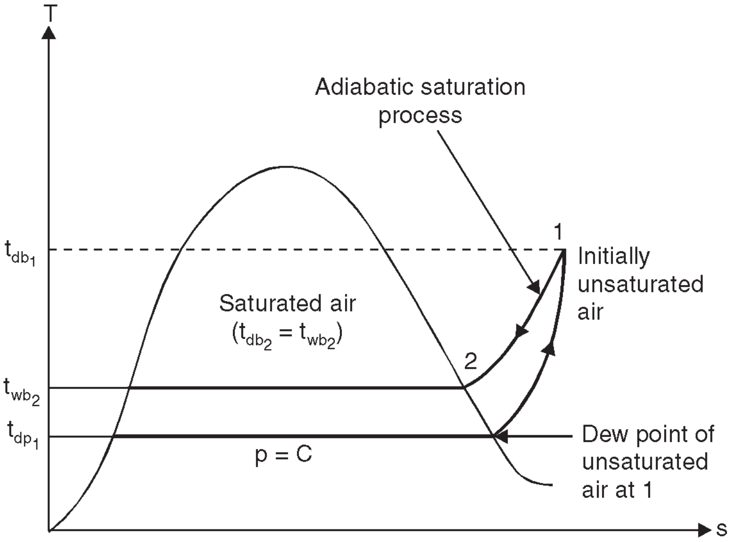


Fig. 10.2. Adiabatic cooling process.

Let us now apply the first law to the entire process. Considering the process to be steady state steady flow, neglecting changes in kinetic and potential energies, we have

$$h_1 + (W_{2s} - W_1)h_{f_2} = h_{2s} \quad ...(10.13)$$

The quantities W_{2s}, h_{2s} and h_{f_2} are the functions of temperature t_{db_2}. The term $(W_{2s} - W_1)h_{f_2}$ is quite small and if this term is neglected, it can be seen that the enthalpy remains constant in adiabatic saturation.

Equation (10.13) may be rewritten as

$$h_1 - W_1 h_{f_2} = h_{2s} - W_{2s} h_{f_2}$$

The inlet term can be generalized and the expression can be written as follows :

$$\Sigma = h_{2s} - W_{2s} h_{f_2} = h_1 - W_1 h_{f_2} = h_x - W_x h_{f_2} \quad ...(10.14)$$

This means that sigma function (Σ) as defined by the equation, is constant for any wet bulb temperature.

Also $$h_1 = h_{2s} - (W_{2s} - W_1)\, h_{f_2} \quad ...(10.15)$$

Equation (10.15) indicates that the enthalpy of an air-water vapour mixture is equal to the enthalpy of saturated air at the same wet bulb temperature, less small correction term $(W_{2s} - W_1) h_{f_2}$. This correction term is called ***enthalpy deviation.***

$$h_1 = h_{\text{air (1)}} + W_1 \, h_{\text{vapour (1)}} \quad \text{...(10.16)}$$

$$h_{2s} = h_{\text{air (2)}} + W_{2s} \, h_{\text{vapour (2)}} \quad \text{...(10.17)}$$

or

$$h_{\text{air (1)}} + W_1 h_{g_1} + (W_{2s} - W_1) h_{f_2} = h_{\text{air (2)}} + W_{2s} h_{g_2}$$

$$c_p t_{db_1} + W_1 h_{g_1} + (W_{2s} - W_1) h_{f_2} = c_p t_{db_2} + W_{2s} h_{g_2} \quad \text{...(10.18)}$$

After arranging, we get

$$W_1 = \frac{c_p(t_{db_2} - t_{db_1}) + W_{2s}(h_{g_2} - h_{f_2})}{h_{g_1} - h_{f_2}}$$

or

$$W_1 = \frac{c_p(t_{db_2} - t_{db_1}) + W_{2s} h_{g_2}}{h_{g_1} - h_{f_2}} \quad \text{...[10.18(a)]}$$

Note. The wet bulb temperature is not a property of moist air as it is influenced by heat and mass transfer rates. Thus in psychrometric equations and psychrometric charts where the wet bulb temperature appears, *it is always the thermodynamic wet bulb temperature that is considered.*

10.4. PSYCHROMETERS

A psychrometer is a device which is used for measuring dry bulb and wet bulb temperatures simultaneously.

The psychrometers may be classified as follows :

1. Laboratory psychrometer
2. Sling psychrometer
3. Aspirating psychrometer
4. Continuous recording psychrometer.

The description of a sling psychrometer is given below :

Refer Fig. 10.3. The sling psychrometer consists of two thermometers mounted on a base plate. The one with the 'sock' is wet-bulb thermometer ; the other is dry-bulb. The wet bulb exists below the dry-bulb. This is done purposely so that 'sock' can be dipped in water without wetting the dry-bulb. The handle of the frame helps for rotating the psychrometer to produce necessary air motion. As the psychrometer is rotated it provides necessary air velocity over the thermometer. Fast movement of air past the 'sock' is necessary to bring the air at temperature t_{db} always in immediate contact with the wet 'sock'. The temperature spread between dry bulb and wet bulb readings depends upon the *amount of moisture in the air*. *Dry air*, or air that has low moisture

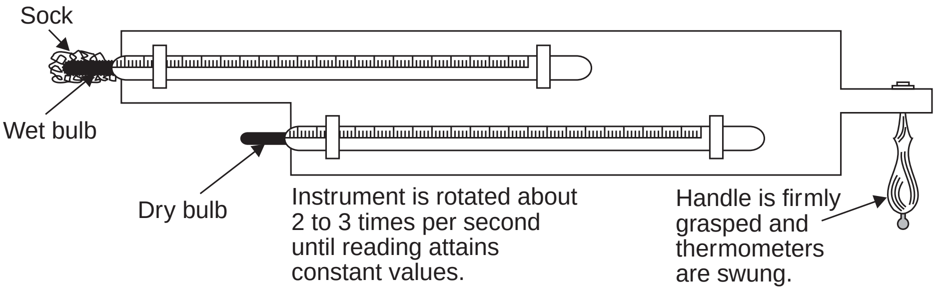


Fig. 10.3. Sling psychrometer.

content has a low wet bulb temperature ; humid air that has a high moisture content, has a high wet-bulb temperature.

When dry and wet bulb temperatures are known the other psychrometric properties like relative humidity, dew point temperature, degree of saturation, humidity ratio, and volume of air-vapour mixture per kg of dry air are determined by calculations.

10.5. PSYCHROMETRIC CHARTS

The psychrometric charts are prepared to represent graphically all the necessary moist air properties used for air conditioning calculations. The values are based on actual measurements verified for thermodynamic consistency.

For psychrometric charts the most convenient co-ordinates are dry bulb temperature of air vapour mixture as the abcissa and moisture content (kg/kg of dry air) or water vapour pressure as the ordinate. Depending upon whether the humidity contents is abcissa or ordinate with temperature co-ordinate, the charts are generally classified as Mollier chart and Carrier chart. Carrier chart having t_{db} as the abcissa and W as the ordinate finds a wide application. The chart is constructed as under :

1. *The dry bulb temperature* (°C) of unit mass of dry air for different humidity contents or humidity ratios are indicated by *vertical lines drawn parallel to the ordinate.*

2. *The mass of water vapour* in kg (or grams) per kg of dry air is drawn parallel to the abcissa for different values of dry bulb temperature. It is the *major vertical scale of the chart.*

3. *Pressure of water vapour in mm of mercury* is shown in the scale at left and is the absolute pressure of steam.

4. *Dew point temperatures* are temperatures corresponding to the boiling points of water at low pressures of water vapour and are shown in the scale on the *upper curved line.* The dew points for different low pressures are read on *diagonal co-ordinates.*

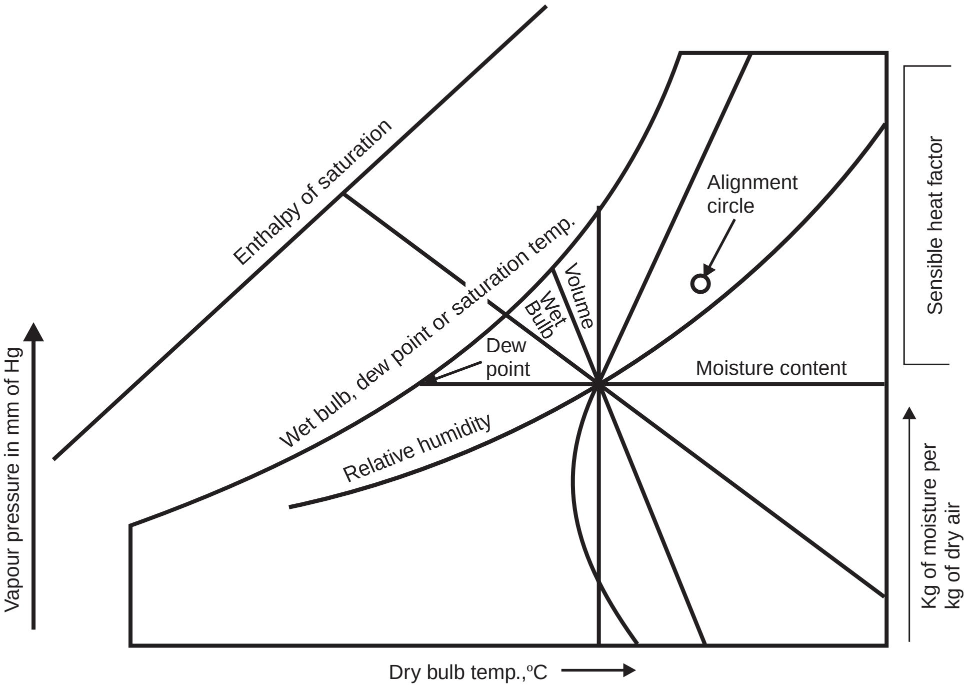


Fig. 10.4. Skeleton psychrometric chart.

5. Constant relative humidity lines in per cent are indicated by marking off vertical distances between the saturation line or the upper curved line and the base of the chart. The relative humidity curve depicts quantity (kg) of moisture actually present in the air as a percentage of the total amount possible at various dry bulb temperatures and masses of vapour.

6. *Enthalpy or total heat* at saturation temperature in kJ/kg of dry air is shown by a diagonal system of co-ordinates. The scale on the diagonal line is separate from the body of the chart and is indicated above the saturation line.

7. *Wet bulb temperatures* are shown on the diagonal co-ordinates coinciding with heat co-ordinates. The scale of wet bulb temperatures is shown on the saturation curve. The diagonals run downwards to the right at an angle of 30° to the horizontal.

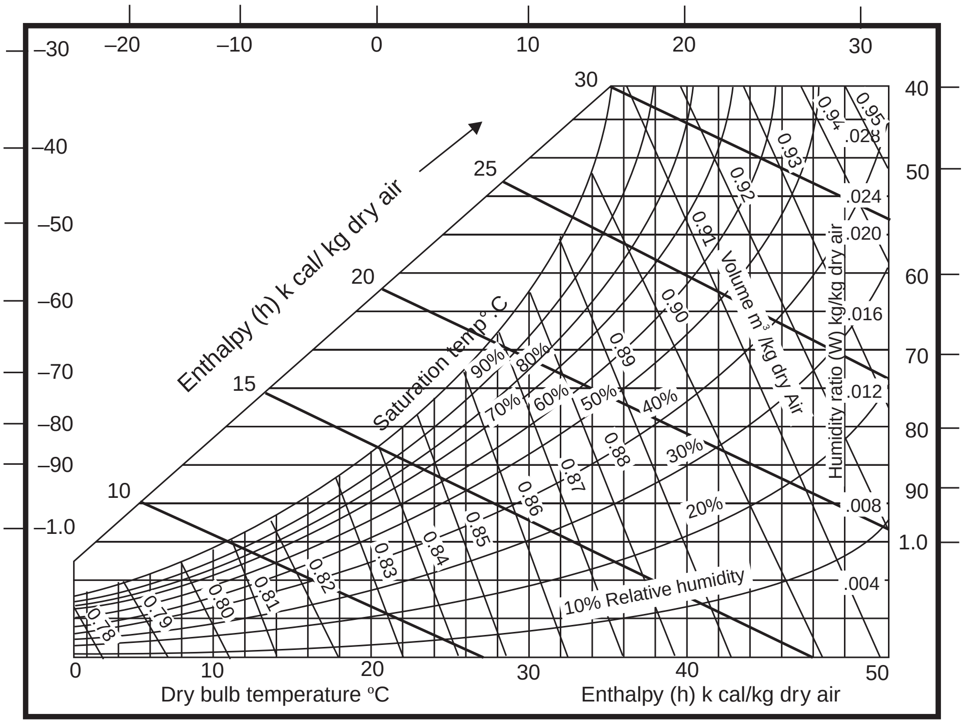


Fig. 10.5. Carrier chart.

8. The volume of air vapour mixture per kg of dry air (specific volume) is also indicated by a set of diagonal co-ordinates but at an angle of 60° with the horizontal.

The other properties of air vapour mixtures can be determined by using formulae (already discussed).

In relation to the psychrometric chart, these terms can quickly indicate many things about the condition of air, for example :

1. If dry bulb and wet bulb temperatures are known, the relative humidity can be read from the chart.

2. If the dry bulb and relative humidity are known, the wet bulb temperature can be determined.

3. If wet bulb temperature and relative humidity are known, the dry bulb temperature can be found.

4. If wet bulb and dry bulb temperatures are known, the dew point can be found.
5. If wet bulb and relative humidity are known, dew point can be read from the chart.
6. If dry-bulb and relative humidity are known, dew point can be found.
7. The quantity (kg) of moisture in air can be determined from any of the following combinations :

(*i*) Dry bulb temperature and relative humidity ;
(*ii*) Dry bulb temperature and dew point ;
(*iii*) Wet bulb temperature and relative humidity ;
(*iv*) Wet bulb temperature and dew point temperature ;
(*v*) Dry bulb temperature and wet bulb temperature ; and
(*vi*) Dew point temperature alone.

Figs. 10.4 and 10.5 show the skeleton psychrometric chart and lines on carrier chart respectively.

10.6. PSYCHROMETRIC PROCESSES

In order to condition air to the conditions of human comfort or of the optimum control of an industrial process required, certain processes are to be carried out on the outside air available. The processes affecting the *psychrometric properties of air are called* ***psychrometric processes.*** These processes involve mixing of air streams, heating, cooling, humidifying, dehumidifying, adiabatic saturation and mostly the combinations of these.

The important psychrometric processes are enumerated and explained in the following text :

1. Mixing of air streams
2. Sensible heating
3. Sensible cooling
4. Cooling and dehumidification
5. Cooling and humidification
6. Heating and dehumidification
7. Heating and humidification.

10.6.1. Mixing of Air Streams

Refer Figs. 10.6 and 10.7. Mixing of several air streams is the process which is very frequently used in air conditioning. This mixing normally takes place without the addition or rejection of either heat or moisture, *i.e., adiabatically and at constant total moisture content. Thus we can write the following equations* :

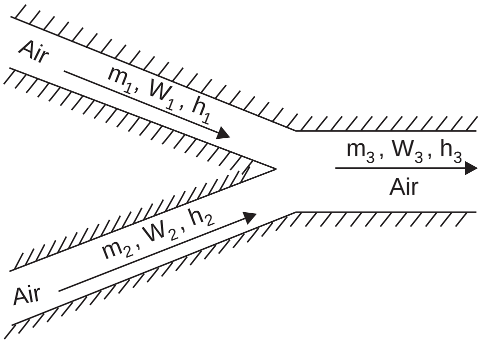


Fig. 10.6. Mixing of air streams.

$$m_1 + m_2 = m_3 \quad ...(10.19)$$

$$m_1W_1 + m_2W_2 = m_3W_3 \quad ...(10.20)$$

$$m_1h_1 + m_2h_2 = m_3h_3 \quad ...(10.21)$$

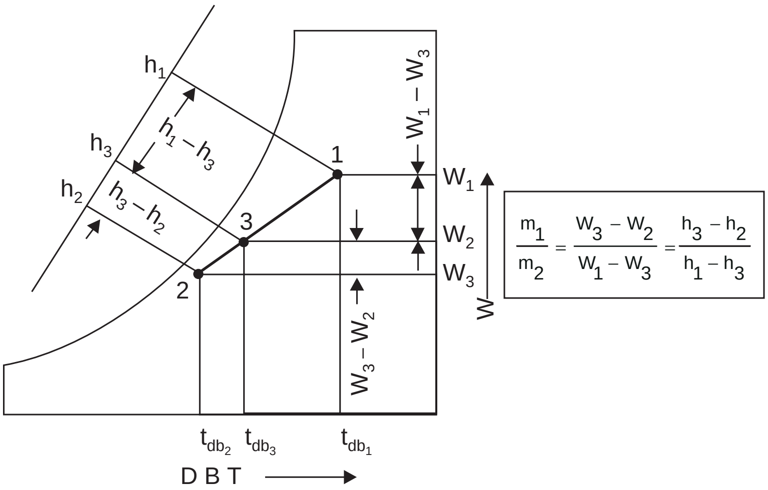


Fig. 10.7

Rearranging of last two equations gives the following :

$$m_1(W_1 - W_3) = m_2(W_3 - W_2)$$

$$m_1(h_1 - h_3) = m_2(h_3 - h_2)$$

or

$$\frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3} = \frac{h_3 - h_2}{h_1 - h_3}$$

where $\left.\begin{aligned} m &= \text{Mass of dry air} \\ W &= \text{Specific humidity} \\ h &= \text{Enthalpy} \end{aligned}\right]$ at particular state points.

On the psychrometric chart, the specific humidity and enthalpy scales are linear, ignoring enthalpy deviations. Therefore, the final state 3 lies on a straight line connecting the initial states of the two streams before mixing, and the final state 3 divides this line into two parts that are in the same ratio as were the two masses of air before mixing.

If the air quantities are known in volume instead of mass units, it is generally sufficiently accurate to units of m^3 or m^3/min. in the mixing equations. The inaccuracy introduced is due to the difference in specific volume at two initial states. This difference in densities is small for most of the comfort air conditioning problems.

10.6.2. Sensible Heating

When air passes over a dry surface which is at a temperature *greater* than its (air) dry bulb temperature, it undergoes *sensible heating*. Thus the heating can be achieved by passing the air over heating coil like electric resistance heating coils or steam coils. During such a process, the specific *humidity remains constant but the dry bulb temperature rises and approaches that of the surface.* The extent to which it approaches the mean effective surface temperature of the coil is conveniently expressed in terms of the equivalent **by-pass factor.**

The by-pass factor (BF) for the process is defined as the ratio of the difference between the mean surface temperature of the coil and leaving air temperature to the difference between the mean surface temperature and the entering air temperature. Thus on Fig. 10.8, air at temperature t_{db_1}, passes over a heating coil with an average surface temperature t_{db_3} and leaves at temperature t_{db_2} .

The by-pass factor is expressed as follows :

$$BF = \frac{t_{db_3} - t_{db_2}}{t_{db_3} - t_{db_1}} \qquad ...(10.22)$$

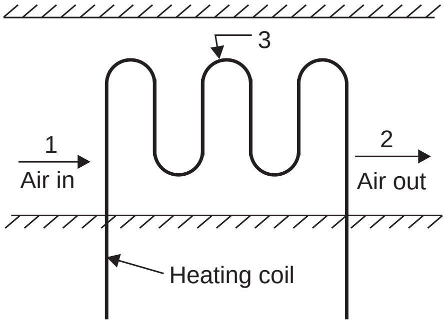


Fig. 10.8. Sensible heating.

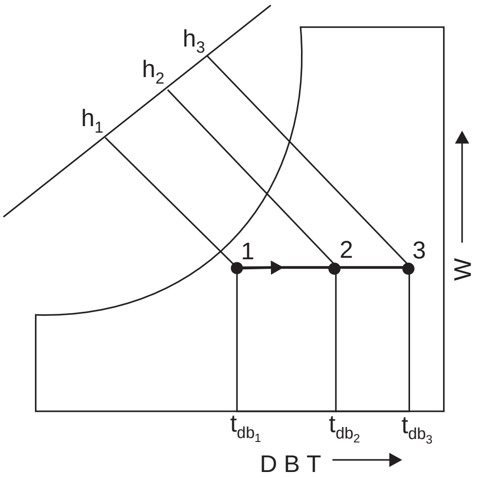


Fig. 10.9

Or in terms of lengths on the chart (Fig. 10.9) it is $\frac{\text{length } 2\text{-}3}{\text{length } 1\text{-}3}$. The *value of the by-pass factor is a function of coil design and velocity.* The heat added to the air can be obtained directly from the entering and leaving enthalpies ($h_2 - h_1$) or it can be obtained from the humid specific heat multiplied by the temperature difference ($t_{db_2} - t_{db_1}$).

In a complete air conditioning system the *preheating* and *reheating* of air are among the familar examples of sensible heating.

Note. 'By-pass factor' can be considered to represent the fraction of air which *does not* come into contact with coil surface.

10.6.3. Sensible Cooling

Refer Fig. 10.10. Air undergoes *sensible cooling* whenever it passes over a surface that is at a temperature less than the *dry bulb* temperature of the *air but greater than the dew point temperature.* Thus sensible cooling can be achieved by passing the air over cooling coil like *evaporating coil of the refrigeration cycle or secondary brine coil.* During the process, *the specific humidity remains constant* and *dry bulb temperature decreases*, approaching the mean effective surface temperature. On a psychrometric chart the process will appear as a horizonal line 1–2 (Fig. 10.11), where point 3 represents the effective surface temperature. For this process :

$$\text{By-pass factor BF} = \frac{t_{db_2} - t_{db_3}}{t_{db_1} - t_{db_3}} \qquad ...(10.23)$$

The heat removed from air can be obtained from the enthalpy difference ($h_1 - h_2$) or from humid specific heat multiplied by the temperature difference ($t_{db_1} - t_{db_2}$).

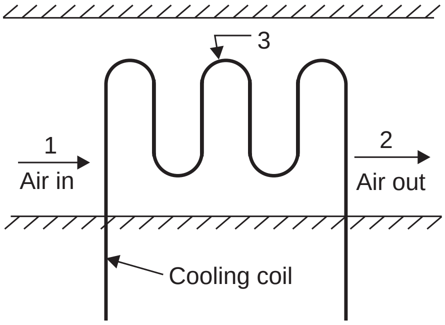


Fig. 10.10. Sensible cooling.

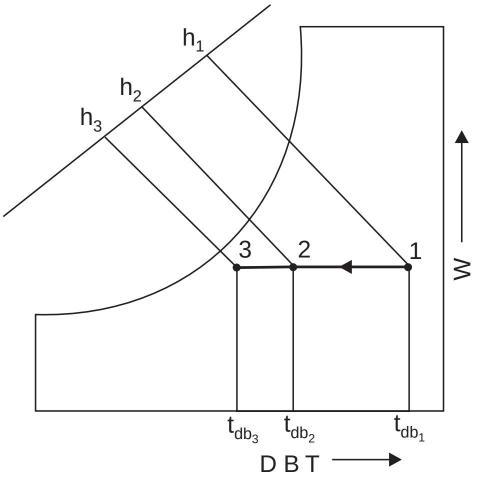


Fig. 10.11

10.6.4. Cooling and Dehumidification

Refer Fig. 10.12. Whenever air is made to pass over a surface or through a spray of water that is at a *temperature less than the dew point temperature of the air, condensation of some of the water vapour in air will occur simultaneously with the sensible cooling process.* Any air that comes into sufficient contact with the cooling surface will be reduced in temperature to the mean surface temperature along a path such as 1-2-3 in Fig. 10.12, with condensation and therefore dehumidification occurring between points 2 and 3. The air that does not contact the surface will be finally cooled by mixing with the portion that did, and the final state point will somewhere on the straight line connecting points 1 and 3. The actual path of air during the path will not be straight line shown but will be something similarly to the curved dashed line 1–4. It will result

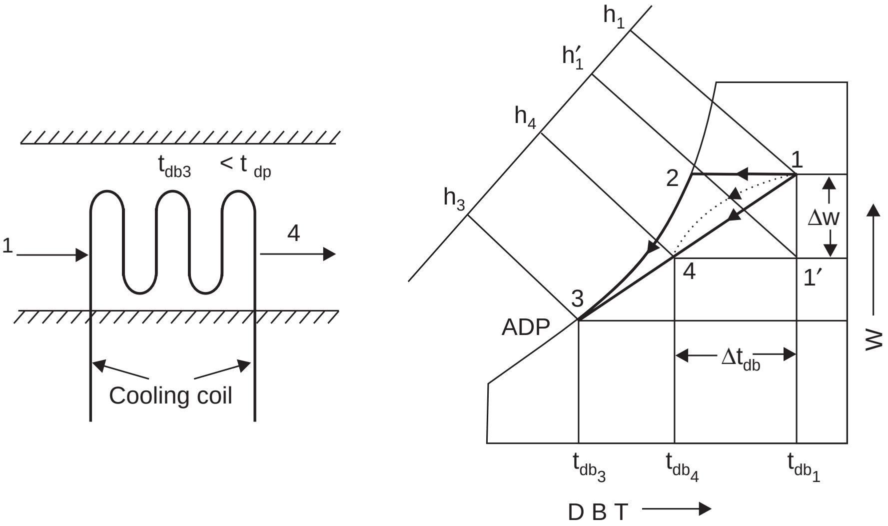


Fig. 10.12. Cooling and dehumidification.

from a continuous mixing of air which is connecting a particular part of the coil and air which is by passing it. It is convenient, however to analyse the problem with the straight line shown, and to assume that the final air state results from the mixing of air that has completely by passed the coil with air that has been cooled to the mean effective surface temperature. If there is enough contact between air and surface for all the air to come to the mean surface temperature, the process is one of zero by pass. In any practical system, complete saturation is not obtained and final state will be a point such as 4 in Fig. 10.12 with an equivalent by pass factor equal to $\frac{\text{length } 3\text{-}4}{\text{length } 3\text{-}1}$. For processes involving condensation, the effective surface temperature, *e.g.* t_{db_3} in Fig. 10.12 is called '**apparatus dew point**' (ADP). The final state point of air passing through a cooling and dehumidifying apparatus is in effect a mixture condition that results from mixing the fraction of the air, which is equal to the equivalent by-pass factor (BF) and is at initial state point and the remaining fraction which is equal to one minus by pass factor (1–BF) and is saturated at the apparatus dew point (ADP).

Total heat removed from the air is given by

$$Q_t = h_1 - h_4 = (h_1 - h_1') + (h_1' - h_4)$$
$$= Q_L + Q_S$$

where, W_L = Latent heat removed $(h_1 - h_1')$, and

Q_S = Sensible heat removed $(h_1' - h_4)$

The ratio $\frac{Q_S}{Q_L}$ *is called sensible heat factor* (SHF) Or

sensible heat ratio (SHR)

$$\therefore \qquad \text{SHF} = \frac{Q_S}{Q_L + Q_S} \qquad \text{...(10.24)}$$

The ratio fixes the slope of the line 1—4 on the psychrometric chart. Sensible heat factor slope lines are given on the psychrometric chart. If the initial condition and SHF are known for the given process, then the process line can be drawn through the given initial condition at a slope given by SHF on the psychrometric chart.

The capacity of the cooling coil in *tonnes* of refrigeration is given by,

$$\text{Capacity in TR} = \frac{m_a(h_1 - h_4) \times 60}{14000}, \qquad \text{...(10.25)}$$

where m_a = mass of air, kg/min and h = enthalpy in kJ/kg of air.

10.6.5. Cooling and Humidification

If unsaturated air is passed through a spray of continuously recirculated water, the specific humidity will increase while the dry bulb temperature decreases. This is the process of ***adiabatic saturation* or *evaporative cooling*.** This process is one of constant adiabatic-saturation temperature and for all practical purposes, one of constant wet bulb temperature. The process is illustrated as path 1-2 on Fig. 10.13, with wet bulb temperature of air being that of point 3, which is also equilibrium temperature of the recirculated water if there is sufficient contact between air and spray, the air will leave at a condition very close to that of point 3. The concept of equivalent by pass can be applied to this process but another term is more used to describe the performance of a humidifying apparatus. It is the ***'saturating'*** or ***'humidifying efficiency'*** which is *defined as the*

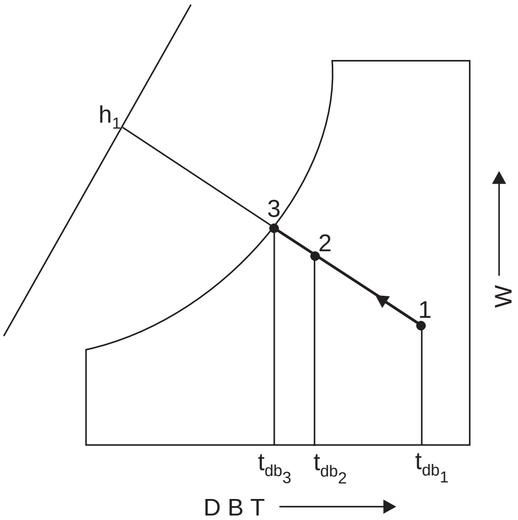


Fig. 10.13. Cooling and humidification.

ratio of dry-bulb temperature decrease to the entering wet bulb depression usually expressed as percentage. Thus, from Fig. 10.13, the saturating efficiency is :

$$\% \ \eta_{sat} = \left(\frac{t_{db_1} - t_{db_2}}{t_{db_1} - t_{db_3}} \right) \times 100 \qquad ...(10.26)$$

As a fraction, it is equal to one minus the by pass factor for the process.

This adiabatic process, for all practical purposes, is line of constant *enthalpy*. The moisture added can be obtained from the increase in specific humidity.

10.6.6. Heating and Dehumidification

If air is passed over a solid absorbent surface or through a liquid absorbent spray simultaneous heating and dehumidification is accompanied. In either case the dehumidification results from adsorbent or absorbent having a lower water vapour pressure than air. Moisture is condensed out of the air, and consequently the latent heat of condensation is liberated, causing sensible heating of air. If these were the only energies involved, the process would be the inverse of the adiabatic saturation process. There is, however, an additional energy absorbed or liberated by the active material, termed the *heat of adsorption* or *absorption.* For the solid adsorbents used commercially, such as silica gel or activated alumina, and for the more common liquid absorbents, such as solutions of organic salts or inorganic compounds like ethylene, glycol, heat is involved and results in additional sensible heating. Thus the path lies above a constant wet bulb line on the psychrometric chart such as path 1-2 in Fig. 10.14.

10.6.7. Heating and Humidification

If air is passed through a humidifier which has heated water sprays instead of simply recirculated spray, the air is humidified and may be heated, cooled or unchanged in temperature. In such a process the *air increases in specific humidity and the* enthalpy, and the *dry bulb temperature will increase or decrease according to the initial temperature of the air and that of the spray*. If sufficient water is supplied relative to the mass flow of air, the air will approach saturation at water temperature. Examples of such processes are shown on Fig. 10.15.

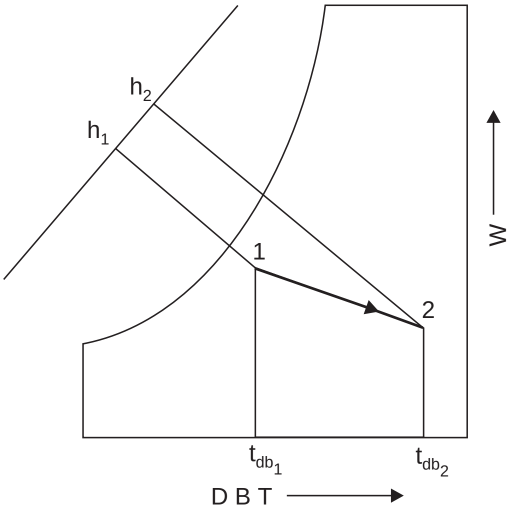


Fig. 10.14. Heating and dehumidification.

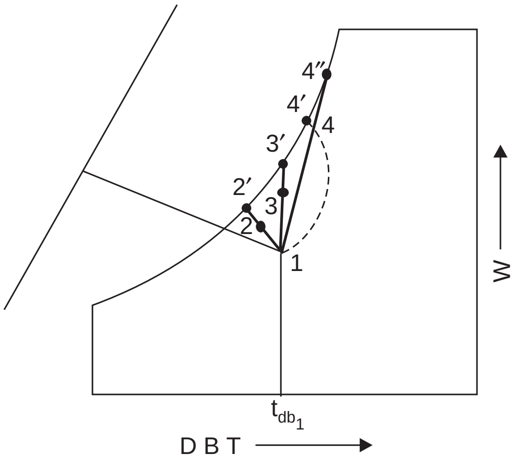


Fig. 10.15. Heating and humidification.

Process 1-2 : It denotes the cases in which the temperature of the heated spray water is less than the air DBT.

Process 1-3 : It denotes the cases in which the temperature is *equal to the air DBT.*

Process 1-4 : It denotes the cases in which a spray temperature is *greater than air DBT.*

As in the case of adiabatic saturation, the degree to which the process approaches saturation can be expressed in terms of the by-pass factor or a saturating efficiency.

If the water rate relative to the air quantity is smaller, the water temperature will drop significantly during the process. The resultant process will be a curved line such as the dashed 1-4 where 4 represents the leaving water temperature.

Note. It is possible to accomplish heating and humidification by evaporation from an open pan of heated water, or by direct injection of heated water or steam. The latter is more common. The process line for it is of little value because the process is essentially an instantaneous mixing of steam and the air. The final state point of the air can be found, however by making a humidity and enthalpy balance for the process. The solution of such a problem usually involves cut-and-try procedure.

Example 10.1. *The atmospheric conditions are ; 20°C and specific humidity of 0.0095 kg/kg of dry air. Calculate the following :*

(*i*) *Partial pressure of vapour* (*ii*) *Relative humidity*

(*iii*) *Dew point temperature.*

Solution. Dry bulb temperature, t_{db} = 20°C

Specific humidity, W = 0.0095 kg/kg of dry air

(*i*) **Partial pressure of vapour, $\mathbf{p_v}$:**

The specific humidity is given by

$$W = \frac{0.622\, p_v}{p_t - p_v}$$

$$0.0095 = \frac{0.622\, p_v}{1.0132 - p_v}$$

$$0.0095(1.0132 - p_v) = 0.622\, p_v$$

$$0.009625 - 0.0095\ p_v = 0.622\ p_v$$

$$p_v = \mathbf{0.01524\ bar.}\quad \textbf{(Ans.)}$$

(*ii*) **Relative humidity ϕ :**

Corresponding to 20ºC, from steam tables,

$$p_{vs} = 0.0234 \text{ bar}$$

∴ **Relative humidity,** $\phi = \dfrac{p_v}{p_{vs}} = \dfrac{0.01524}{0.0234} = \mathbf{0.65\ or\ 65\%.}\quad \textbf{(Ans.)}$

(*iii*) **Dew point temperature, t_{dp} :**

The dew point temperature is the saturation temperature of water vapour at a pressure of 0.01524 bar,

t_{dp} [from steam tables by interpolation]

$$= 13 + \frac{(14 - 13)}{(0.01598 - 0.0150)} \times [0.01524 - 0.0150]$$

$$= 13 + \frac{0.00024}{0.00098} = \mathbf{13.24°C.}\quad \textbf{(Ans.)}$$

Example 10.2. *The air supplied to a room of a building in winter is to be at 17°C and have a relative humidity of 60%. If the barometric pressure is 1.01325 bar, find : (i) The specific humidity ; (ii) The dew point under these conditions.*

Solution. Dry bulb temperature, t_{db} = 17ºC

Relative humidity, ϕ = 60%

Barometric or total pressure, p_t = 1.01325 bar

Specific humidity, W :

Corresponding to 17ºC, from steam tables,

$$p_{vs} = 0.0194 \text{ bar}$$

Also, $\phi = \dfrac{p_v}{p_{vs}}$

i.e., $0.6 = \dfrac{p_v}{0.0194}$

∴ $p_v = 0.6 \times 0.0194 = 0.01164$ bar.

Specific humidity, $W = \dfrac{0.622\ p_v}{p_t - p_v} = \dfrac{0.622 \times 0.01164}{1.01325 - 0.01164}$

$= \mathbf{0.00723\ kg/kg\ of\ dry\ air.}\quad \textbf{(Ans.)}$

Dew point temperature, t_{dp} :

If the air is cooled at constant pressure the vapour will begin to condense at the saturation temperature corresponding to 0.01164 bar. By interpolation from steam tables, the dew point temperature, t_{dp} is then

$$\mathbf{t_{dp}} = 9 + (10 - 9) \times \frac{0.01164 - 0.01150}{0.01230 - 0.01150} = \mathbf{9.18°C.}\quad \textbf{(Ans.)}$$

Example 10.3. *0.004 kg of water vapour per kg of atmospheric air is removed and temperature of air after removing the water vapour becomes 20°C. Determine :*

(*i*) *Relative humidity* (*ii*) *Dew point temperature.*

Assume that condition of atmospheric air is 30°C and 55% R.H. and pressure is 1.0132 bar.

Solution. Corresponding to 30ºC, from steam tables, p_{vs} = 0.0425 bar

∴ Relative humidity (R.H.), $\phi = \frac{p_v}{p_{vs}}$

i.e., $$0.55 = \frac{p_v}{0.0425}$$

∴ $$p_v = 0.02337 \text{ bar.}$$

Also the specific humidity,

$$W = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.02337}{1.0132 - 0.02337} = 0.01468 \text{ kg/kg of dry air.}$$

The specific humidity after removing 0.004 kg of water vapour becomes,

$$0.01468 - 0.004 = 0.01068 \text{ kg/kg of dry air}$$

and the temperature t_{db} is given as 20ºC.

The partial pressure of water vapour, p_v, at this condition can be calculated as follows :

$$W = \frac{0.622\, p_v}{p_t - p_v}$$

$$0.01068 = \frac{0.622\, p_v}{1.0132 - p_v}$$

or, $$0.01068\,(1.0132 - p_v) = 0.622\, p_v$$

or, $$0.01082 - 0.01068\, p_v = 0.622\, p_v$$

$$0.6327\, p_v = 0.01082$$

∴ $$p_v = 0.0171 \text{ bar}$$

Corresponding to 20ºC, from steam tables, p_{vs} = 0.0234 bar.

(*i*) **Relative humidity,** $\phi = \frac{p_v}{p_{vs}} = \frac{0.0171}{0.0234}$ = **0.73 or 73%. (Ans.)**

(*ii*) **Dew point temperature, t_{dp} :**

Corresponding to 0.0171 bar, from steam tables, t_{dp} = **15°C. (Ans.)**

Example 10.4. *The sling psychrometer in a laboratory test recorded the following readings :*

Dry bulb temperature = 35°C

Wet bulb temperature = 25°C.

Calculate the following :

(*i*) *Specific humidity* (*ii*) *Relative humidity*

(*iii*) *Vapour density in air* (*iv*) *Dew point temperature*

(*v*) *Enthalpy of mixture per kg of dry air*

Take atmospheric pressure = 1.0132 bar.

Solution. For finding the partial pressure of vapour, using the equation :

$$p_v = (p_{vs})_{wb} - \frac{[p_t - (p_{vs})_{wb}]\,(t_{db} - t_{wb})}{1527.4 - 1.3\, t_{wb}}$$

Corresponding to 25ºC (from steam tables),

$$(p_{vs})_{wb} = 0.0317 \text{ bar}$$

Substituting the values in the above equation, we get

$$p_v = 0.0317 - \frac{[1.0132 - 0.0317]\,(35 - 25)}{1527.4 - 1.3 \times 25} = 0.0317 - 0.0065 = 0.0252 \text{ bar.}$$

(*i*) **Specific humidity,**

$$W = \frac{0.622\ p_v}{p_t - p_v} = \frac{0.622 \times 0.0252}{(1.0312 - 0.0252)} = \mathbf{0.01586\ kg/kg\ of\ dry\ air.\ (Ans.)}$$

(*ii*) **Relative humidity,** $\phi = \dfrac{p_v}{p_{vs}} = \dfrac{0.0252}{0.0563}$

[p_{vs} = 0.0563 bar corresponding to 35°C, from steam tables]

$= \mathbf{0.447\ or\ 44.7\%.}$ **(Ans.)**

(*iii*) **Vapour density :**

From characteristic gas equation

$$p_v V_v = m_v R_v T_v$$

$$p_v = \frac{m_v}{V_v} R_v T_v = \rho_v R_v T_v$$

$$\left[\text{where } \rho_v = \text{vapour density, } R_v = \frac{\text{Universal gas constant}}{\text{Molecular weight of H}_2\text{O}} = \frac{8314.3}{18}\right]$$

$$\therefore \qquad 0.0252 \times 10^5 = \rho_v \times \frac{8314.3}{18} \times (273 + 35)$$

$$\therefore \qquad \rho_v = \frac{0.0252 \times 10^5 \times 18}{8314.3 \times 308} = \mathbf{0.0177\ kg/m^3.}\ \ \textbf{(Ans.)}$$

(*iv*) **Dew point temperature, $\mathbf{t_{dp}}$:**

Corresponding to 0.0252 bar, from steam tables (by interpolation),

$$\mathbf{t_{dp}} = 21 + (22 - 21) \times \frac{(0.0252 - 0.0249)}{(0.0264 - 0.0249)} = \mathbf{21.2°C.}\ \ \textbf{(Ans.)}$$

(*v*) **Enthalpy of mixture per kg of dry air, h :**

$$h = c_p t_{db} + W h_{\text{vapour}}$$

$$= 1.005 \times 35 + 0.01586\ [h_g + 1.88\ (t_{db} - t_{dp})]$$

$$= 35.175 + 0.01586\ [2565.3 + 1.88\ (35 - 21.2)]$$

(where h_g = 2565.3 kJ/kg corresponding to 35°C t_{db})

$= \mathbf{76.27\ kJ/kg\ of\ dry\ air.}$ **(Ans.)**

Example 10.5. Adiabatic mixing : *One kg of air at 35°C DBT and 60% R.H. is mixed with 2 kg of air at 20°C DBT and 13°C dew point temperature. Calculate the specific humidity of the mixture.*

Solution. For the air at 35°C DBT and 60% R.H. :

Corresponding to 35°C, from steam tables,

$$p_{vs} = 0.0563 \text{ bar}$$

Relative humidity, $\phi = \dfrac{p_v}{p_{vs}}$

$$\therefore \qquad p_v = \phi\ p_{vs} = 0.6 \times 0.0563 = 0.0338 \text{ bar}$$

$$W = \frac{0.622\ p_v}{p_t - p_v} = \frac{0.622 \times 0.0338}{1.0132 - 0.0338} = 0.0214 \text{ kg/kg of dry air}$$

Corresponding to 0.0338 bar, from steam tables,

$$t_{dp} = 26 + (27 - 26)\ \frac{(0.0338 - 0.0336)}{(0.0356 - 0.0336)} = 26.1°\text{C}$$

Enthalpy, $h = c_p t_{db} + Wh_{\text{vapour}}$
$= 1.005\ t_{db} + W\ [h_g + 1.88\ (t_{db} - t_{dp})]$
$= 1.005 \times 35 + 0.0214\ [2565.3 + 1.88\ (35 - 26.1)]$
$= 90.43$ kJ/kg of dry air.

For the air at 20°C DBT and 13°C dew point temperature :

p_v is the vapour pressure corresponding to the saturation pressure of steam at 13ºC.

$\therefore \quad p_v = 0.0150$ bar

$$W = \frac{0.622\ p_v}{p_t - p_v} = \frac{0.622 \times 0.015}{1.0132 - 0.015} = 0.00935 \text{ kg/kg of dry air}$$

Enthalpy, $h = c_p t_{db} + Wh_{\text{vapour}}$
$= 1.005 \times 20 + 0.00935\ [h_g + 1.88\ (t_{db} - t_{dp})]$
$= 20.1 + 0.00935\ [2538.1 + 1.88\ (20 - 13)]$
$= 43.95$ kJ/kg of dry air

Now enthalpy per kg of moist air

$$= \frac{1}{3}\left[\frac{90.43}{1.0214} + \frac{43.95 \times 2}{1.00935}\right] = 58.54 \text{ kJ/kg of moist air}$$

Mass of vapour/kg of moist air

$$= \frac{1}{3}\left[\frac{0.0214}{1.0214} + \frac{0.00935 \times 2}{1.00935}\right] = 0.01316 \text{ kg/kg of moist air}$$

Specific humidity of mixture

$$= \frac{0.01316}{1 - 0.01316} = \mathbf{0.01333 \text{ kg/kg of dry air.}} \quad \textbf{(Ans.)}$$

Example 10.6. Sensible heating : *90 m^3 of air per minute at 20°C and 75% R.H. is heated until its temperature becomes 30°C. Calculate :*

(*i*) *R.H. of the heated air.* (*ii*) *Heat added to air per minute.*

Solution. (*i*) ***For air at 20°C and 75% R.H. :***

$p_{vs} = 0.0234$ bar (from steam tables, at 20ºC)
$p_v = \phi \times p_{vs} = 0.75 \times 0.0234 = 0.01755$ bar

$$t_{dp} = 15 + (16 - 15)\ \frac{(0.01755 - 0.017)}{(0.0182 - 0.017)} \simeq 15.5\text{ºC}$$

$$W_1 = \frac{0.622\ p_v}{p_t - p_v} = \frac{0.622 \times 0.01755}{1.0132 - 0.01755} = 0.0109 \text{ kg/kg of dry air}$$

Enthalpy, $h_1 = c_p t_{db} + Wh_{\text{vapour}}$
$= 1.005 \times 20 + 0.0109\ [h_g + 1.88\ (t_{db} - t_{dp})]$
$= 1.005 \times 20 + 0.0109\ [2538.1 + 1.88(20 - 15.5)] = 47.85$ kJ/kg of dry air

(*i*) **Relative humidity of heated air :**

For air at 30°C DBT :

Since the saturation pressure of water vapour at 30ºC is higher than the saturation pressure of water vapour at 20ºC so it is *sensible heating,* where p_v is same after heating.

$$\therefore \quad \text{Relative humidity, } \phi = \frac{p_v}{p_{vs}} = \frac{0.01755}{0.0425} = 0.412 \text{ or } 41.2\%$$

($p_{vs} = 0.0425$ bar, corresponding to 30ºC)

i.e., Relative humidity of heated air **= 41.2%. (Ans.)**

(*ii*) **Heat added to air per minute :**

Enthalpy, $h_2 = c_p t_{db} + W h_{\text{vapour}}$

$$= 1.005 \times 30 + 0.0109 \times [h_g + 1.88\,(t_{db} - t_{dp})]$$

$$= 1.005 \times 30 + 0.0109\,[2556.3 + 1.88\,(30 - 15.5)]$$

$$= 58.31 \text{ kJ/kg of dry air}$$

Mass of dry air in 90 m^3 of air supplied

$$m_a = \frac{pV}{RT} = \frac{(p_t - p_v)V}{RT}$$

$$= \frac{(1.0132 - 0.01755) \times 10^5 \times 90}{287 \times (273 + 20)} = 106.5 \text{ kg/min.}$$

Amount of heat added per minute

$$= 106.5\,(h_2 - h_1) = 106.5\,(58.31 - 47.85) \simeq \mathbf{1114\ kJ.}\quad \mathbf{(Ans.)}$$

Example 10.7. Sensible cooling : *40 m^3 of air at 35°C DBT and 50% R.H. is cooled to 25°C DBT maintaining its specific humidity constant. Determine :*

(*i*) *Relative humidity (R.H.) of cooled air ;*

(*ii*) *Heat removed from air.*

Solution. ***For air at 35°C DBT and 50% R.H. :***

$$p_{vs} = 0.0563 \text{ bar (At 35°C, from steam tables)}$$

$$\phi = \frac{p_v}{p_{vs}}$$

$$\therefore \quad p_v = \phi \times p_{vs} = 0.5 \times 0.0563 = 0.02815 \text{ bar}$$

$$W_1 = \frac{0.622\,p_v}{p_t - p_v} = \frac{0.622 \times 0.02815}{1.0132 - 0.02815} = 0.0177 \text{ kg/kg of dry air}$$

$$h_1 = c_p\,t_{db_1} + W_1\,[h_{g_1} + 1.88\,(t_{db_1} - t_{dp_1})]$$

$$t_{dp_1} \simeq 23°\text{C (corresponding to 0.02815 bar)}$$

$$\therefore \quad h_1 = 1.005 \times 35 + 0.0177\,[2565.3 + 1.88\,(35 - 23)] = 80.98 \text{ kJ/kg of dry air}$$

For air at 25°C DBT :

(*i*) ***R.H. of cooled air :***

Since the specific humidity remains constant the vapour pressure in the air remains constant.

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.02815}{0.0317} = 0.888 \text{ or } 88.8\%$$

i.e., Relative humidity of the cooled air **= 88.8%. (Ans.)**

(*ii*) ***Heat removed from air :***

$$h_2 = c_p\,t_{db_2} + W_2\,[h_{g_2} + 1.88\,(t_{db_2} - t_{dp_2})]$$

$$= 1.005 \times 25 + 0.0177\,[2547.2 + 1.88\,(25 - 23)]$$

$$= 70.27 \text{ kJ/kg of dry air.}$$

$$\left[\begin{array}{l}\because\ W_1 = W_2 = 0.0177 \text{ kg/kg of dry air} \\ t_{dp_2} = t_{dp_1} = 23°\text{C since } p_v \text{ does not change}\end{array}\right]$$

To find mass of dry air (m_a), using the relation :

$$p_a v_a = m_a R_a T_a$$

$$\therefore \quad m_a = \frac{p_a v_a}{R_a T_a} = \frac{(1.0132 - 0.02815) \times 10^5 \times 40}{287 \times (273 + 35)} = 44.57 \text{ kg}$$

$\therefore$ Heat removed from 40 m^3 of air

$$= m_a\,(h_1 - h_2) = 44.57\,(80.98 - 70.27) = \mathbf{477.3\ kJ.}\quad \mathbf{(Ans.)}$$

Example 10.8. Cooling and dehumidification : *120 m^3 of air per minute at 35°C DBT and 50% relative humidity is cooled to 20°C DBT by passing through a cooling coil.*

Determine the following :

(*i*) *Relative humidity of out coming air and its wet bulb temperature.*

(*ii*) *Capacity of cooling coil in tonnes of refrigeration.*

(*iii*) *Amount of water vapour removed per hour.*

Solution. ***For the air at 35°C DBT and 50% R.H. :***

$$p_{vs} = 0.0563 \text{ bar (At 35°C, from steam tables)}$$

$$p_v = \phi \times p_{vs} = 0.5 \times 0.0563 = 0.02815 \text{ bar.}$$

$$W_1 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.02815}{1.0132 - 0.02815} = 0.0177 \text{ kg/kg of dry air.}$$

$$h_1 = c_p\, t_{db_1} + W_1\, [h_{g_1} + 1.88\, (t_{db_1} - t_{dp_1})]$$

$$t_{dp_1} \approx 23°\text{C (Corresponding to 0.02815 bar).}$$

$$\therefore \quad h_1 = 1.005 \times 35 + 0.0177\, [2565.3 + 1.88\, (35 - 23)]$$

$$= 80.98 \text{ kJ/kg of dry dir.}$$

For the air at 20°C. As the saturation vapour pressure at 20°C is 0.0234 bar, less than the vapour pressure 0.02815 bar at 35°C, so that *condensation takes place* and air *will be saturated at 20°C.*

(*i*) ∴ **Relative humidity of exit air is 100 per cent. (Ans.)**

Since the air is saturated, wet bulb temperature is equal to dry bulb temperature = 20°C. (Ans.)

$$\therefore \quad p_v = p_{vs} = 0.0234 \text{ bar.}$$

$$W_2 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.0234}{(1.0132 - 0.0234)} = 0.0147 \text{ kg/kg of dry air}$$

$$h_2 = c_p\, t_{db_2} + W_2\, [h_{g_2} + 1.88\, (t_{db_2} - t_{dp_2})]$$

$$= 1.005 \times 20 + 0.0147\, [2538.1 + 1.88\, (20 - 20)]$$

[∵ When air is saturated $t_{db} = t_{dp}$]

$$= 57.41 \text{ kJ/kg of dry air}$$

The weight of water vapour removed per kg of dry air

$$= 0.0177 - 0.0147 = 0.003 \text{ kg/kg of dry air}$$

Heat removed per kg of dry air

$$= h_1 - h_2 = 80.98 - 57.41 = 23.57 \text{ kJ/kg of dry air}$$

Mass of dry air passing per minute

$$m_a = \frac{p_a V_a}{R_a T_a} = \frac{(1.0132 - 0.02815) \times 10^5 \times 120}{287 \times (35 + 273)} = 133.7 \text{ kg/min}$$

(*ii*) **Capacity of the cooling coil in tonnes of refrigeration**

$$= \frac{m_a\, (h_1 - h_2)}{14000} = \frac{133.7 \times 23.57 \times 60}{14000} = \mathbf{13.5\ TR.}\quad \textbf{(Ans.)}$$

(*iii*) **Amount of water removed per hour**

$$= m_a\, (W_1 - W_2) \times 60$$

$$= 133.7\, (0.0177 - 0.0147) \times 60 = \mathbf{24.066\ kg/h.}\quad \textbf{(Ans.)}$$

Example 10.9. Adiabatic humidification : *150 m^3 of air per minute is passed through the adiabatic humidifier. The condition of air at inlet is 35°C DBT and 20 per cent relative humidity and the outlet condition is 20°C DBT and 15°C WBT.*

Determine the following :

(*i*) *Dew point temperature* (*ii*) *Relative humidity of the exit air*

(*iii*) *Amount of water vapour added to the air per minute.*

Solution. ***For air at 35°C DBT and 20% relative humidity.***

$$p_{vs} = 0.0563 \text{ bar (At 35°C from steam tables)}$$

$$p_v = \phi \times p_{vs} = 0.2 \times 0.0563 = 0.01126 \text{ bar}$$

$$W_1 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.01126}{1.0132 - 0.01126} = 0.00699 \text{ kg/kg of dry air}$$

(*i*) The dew point temperature of air which is the saturation temperature of steam corresponding to the pressure 0.01126 bar is

$$8 + (9 - 8) \times \frac{(0.01126 - 0.01072)}{(0.01150 - 0.01072)} = 8.7°\text{C}$$

i.e., **Dew point temperature = 8.7°C. (Ans.)**

(*ii*) **Relative humidity of the exit air** :

For air at 20°C *DBT* and 15°C *WBT*.

$$p_v = (p_{vs})_{wb} - \frac{[p_t - (p_{vs})_{wb}]\,(t_{db} - t_{wb})}{1527.4 - 1.3\, t_{wb}}$$

$$= 0.0170 - \frac{[1.0132 - 0.0170]\,(20 - 15)}{1527.4 - 1.3 \times 15} = 0.0137 \text{ bar}$$

$$W_2 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.0137}{(1.0132 - 0.0137)} = 0.00852 \text{ kg/kg of dry air}$$

Relative humidity $= \dfrac{p_v}{p_{vs}} = \dfrac{0.0137}{0.0234} =$ **0.585 or 58.5%. (Ans.)**

($\because$ p_{vs} = 0.0234 bar, corresponding to 20°C, from steam tables)

The *dew point temperature of air* which is the saturation temperature of steam corresponding to 0.0137 bar is **11°C** (from steam tables). **(Ans.)**

The amount of water vapour per kg of dry air

$$= W_2 - W_1 = 0.00852 - 0.00699 = 0.00153 \text{ kg}$$

The mass of dry air in 150 m^3 of mixture

$$m_a = \frac{p_a V_a}{R_a T_a} = \frac{(1.0132 - 0.01126) \times 10^5 \times 150}{287 \times (35 + 273)} = 170 \text{ kg}$$

(*iii*) **The amount of water vapour added to air per minute**

$$= m_a\,(W_2 - W_1) = 170 \times 0.00153 = \mathbf{0.26 \text{ kg/min.}} \quad \textbf{(Ans.)}$$

Example 10.10. Adiabatic saturation process : *An air-water vapour mixture enters an adiabatic saturation chamber at 28°C and leaves at 18°C, which is the adiabatic saturation temperature. The pressure remains constant at 1.0 bar.*

Determine the relative humidity and humidity ratio of the inlet mixture.

Solution. The specific humidity at the *exit*

$$W_{2s} = \frac{0.622\, p_s}{p_t - p_s} = \frac{0.622 \times 0.0206}{(1.00 - 0.0206)} = 0.01308 \text{ kg/kg of dry air}$$

The specific humidity at the inlet (equation 10.18)

$$W_1 = \frac{c_p\,(t_{db_2} - t_{db_1}) + W_{2s}\,(h_{g_2} - h_{f_2})}{h_{g_1} - h_{f_2}}$$

$$= \frac{1.005\,(18 - 28) + 0.01308\,(2534.4 - 75.6)}{(2552.6 - 75.6)}$$

$$= \frac{42.211}{2477} = 0.01704 \text{ kg/kg of dry air}$$

$$W_1 = \frac{0.622\,p_{v_1}}{p_t - p_{v_1}}$$

$\therefore$ $$0.01704 = \frac{0.622\,p_{v_1}}{1.00 - p_{v_1}}$$

or $$0.01704\,(1.00 - p_{v_1}) = 0.622\,p_{v_1}$$

or $$0.01704 - 0.01704\,p_{v_1} = 0.622\,p_{v_1}$$

or $$0.0639\,p_{v_1} = 0.01704$$

$\therefore$ $$p_{v_1} = 0.02666 \text{ bar}$$

$\therefore$ **Relative humidity** $= \dfrac{p_{v_1}}{p_{s_1}} = \dfrac{0.02666}{0.03782} =$ **0.7 or 70%. (Ans.)**

Example 10.11. *An air-water vapour mixture enters an air-conditioning unit at a pressure of 1.0 bar. 38°C DBT, and a relative humidity of 75%. The mass of dry air entering is 1 kg/s. The air-vapour mixture leaves the air-conditioning unit at 1.0 bar, 18°C, 85% relative humidity. The moisture condensed leaves at 18°C. Determine the heat transfer rate for the process.*

Solution. $t_{db_1} = 38°\text{C}$, R.H., $\phi_1 = 75\%$

$t_{tb_2} = 18°\text{C}$, R.H., $\phi_2 = 85\%$

The flow diagram and the process are shown in Figs. 10.16 (*a*) and (*b*) respectively.

At 38°C

From steam tables : $p_{vs} = 0.0663$ bar, $h_{g_1} = 2570.7$ kJ/kg

$\therefore$ $$p_v = \phi \times p_{vs} = 0.75 \times 0.0663 = 0.0497 \text{ bar}$$

$$W_1 = \frac{0.622 \times 0.0497}{1.0 - 0.0497} = 0.0325 \text{ kg/kg of dry air}$$

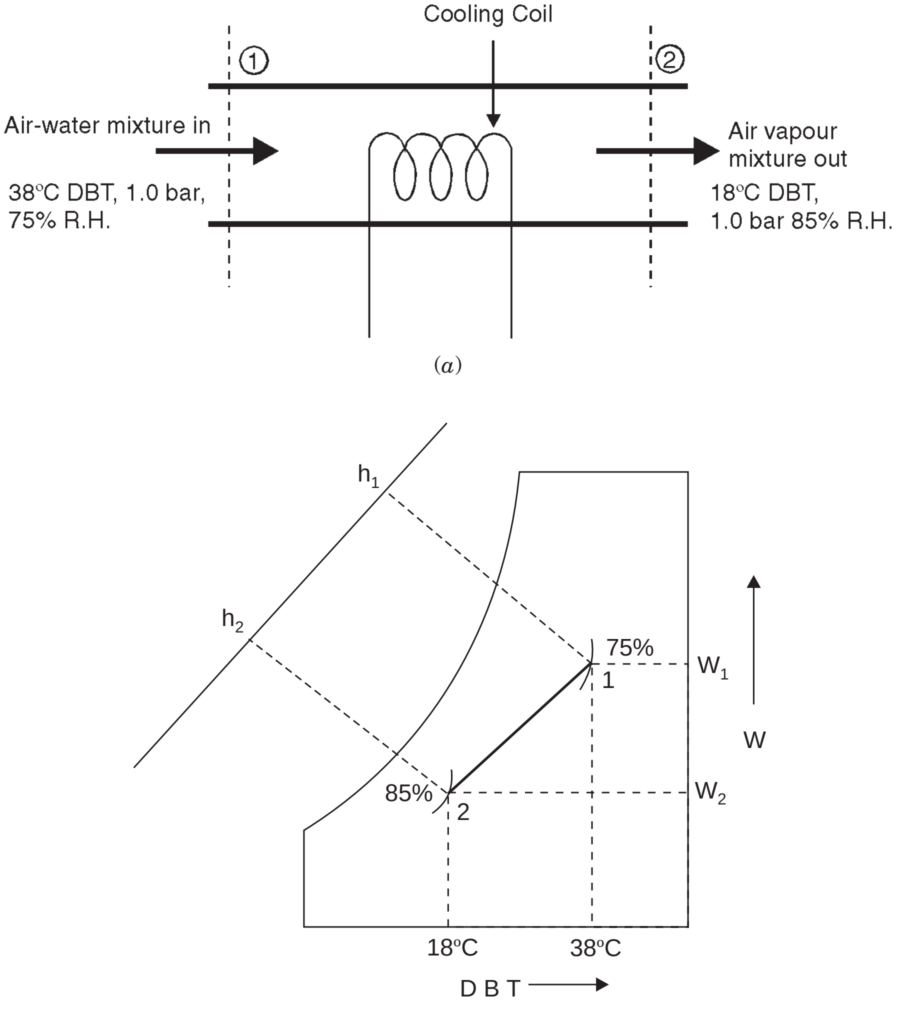
At 18°C

From steam tables : $p_{vs} = 0.0206$ bar, $h_{g_2} = 2534.4$ kJ/kg

$h_{f_2} = 75.6$ kJ/kg

$p_v = 0.85 \times 0.0206 = 0.01751$ bar

$$W_2 = \frac{0.622 \times 0.01751}{1 - 0.01751} = 0.01108 \text{ kg/kg of dry air}$$



(*a*)

(*b*)

Fig. 10.16

Heat transfer rate,

$$q = (W_2 h_{g_2} - W_1 h_{g_1}) + c_p\, (t_{db_2} - t_{db_1}) + (W_1 - W_2)\, h_{f_2}$$

$$= (0.01108 \times 2534.4 - 0.0325 \times 2570.7) + 1.005\ (18 - 38) + (0.0325 - 0.01108) \times 75.6$$

$$= -\ 55.46 - 20.1 + 1.62 = -\ \mathbf{73.94\ kJ/kg\ of\ dry\ air.}\quad \textbf{(Ans.)}$$

Example 10.12. Evaporative Cooler : *Atmospheric air at 38°C and 25% relative humidity passes through an evaporator cooler. If the final temperature of air is 18°C, how much water is added per kg of dry air and what is the final relative humidity ?*

Solution. At 38°C :

$$p_{vs} = 0.0663 \text{ bar},\ h_{g_1} = 2570.7 \text{ kJ/kg}$$

and $$p_v = \phi \times p_{vs} = 0.25 \times 0.0663 = 0.01657 \text{ bar}$$

At 18°C :

$$h_{g_2} = 2534.4 \text{ kJ/kg}, \; p_{vs} = 0.0206 \text{ bar}$$

$$\therefore \quad W_1 = \frac{0.622 \times 0.01657}{1.0132 - 0.01657} = 0.01037 \text{ kg/kg of dry air}$$

Since enthalpy remains constant during the process

$$\therefore \quad c_p t_{db_1} + W_1 h_{g_1} = c_p t_{db_2} + W_2 h_{g_2}$$

$$1.005 \times 38 + 0.01034 \times 2570.7 = 1.005 \times 18 + W_2 \times 2534.4$$

($\because$ At 18°C, $h_{g_2} = 2534.4$ kJ/kg)

i.e.,
$$W_2 = \frac{1.005 \times 38 + 0.01034 \times 2570.7 - 1.005 \times 18}{2534.4}$$

$$= 0.01842 \text{ kg/kg of dry air}$$

$\therefore$ **Amount of water added** $= W_2 - W_1 = 0.01842 - 0.01034$

$=$ **0.00808 kg/kg of dry air. (Ans.)**

Also,
$$0.00808 = \frac{0.622 \, p_{v_2}}{1.0132 - p_{v_2}}$$

or
$$0.00808\,(1.0132 - p_{v_2}) = 0.622 \; p_{v_2}$$

$$0.00818 - 0.00808 \; p_{v_2} = 0.622 \; p_{v_2}$$

$$\therefore \quad p_{v_2} = 0.01298 \text{ bar}$$

$\therefore$ **Final relative humidity** $= \dfrac{0.01298}{0.0206} =$ **0.63 or 63%. (Ans.)**

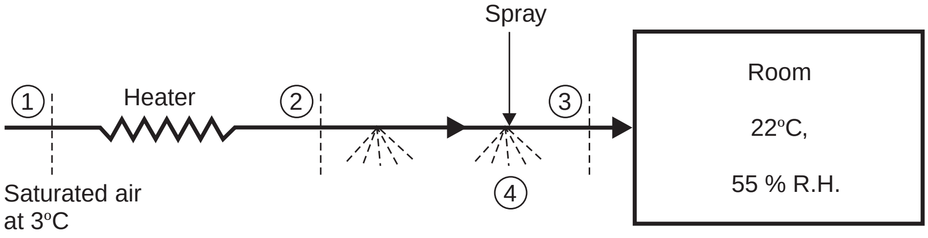
Example 10.13. *Saturated air at 3°C is required to be supplied to a room where the temperature must be held at 22°C with a relative humidity of 55%. The air is heated and then water at 10°C is sprayed to give the required humidity. Determine :*

(*i*) *The mass of spray water required per m*^{3} *of air at room conditions.*

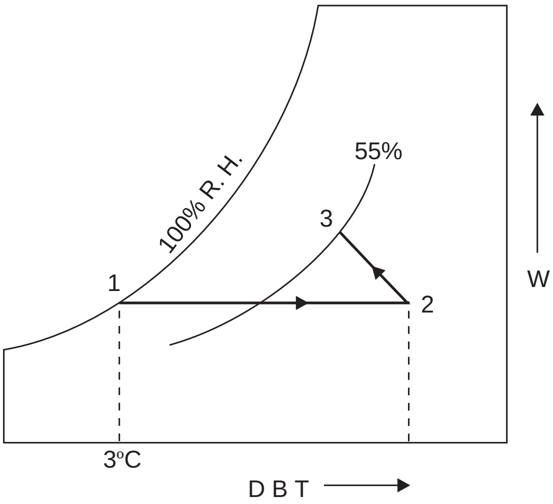
(*ii*) *The temperature to which the air must be heated.*

Neglect the fan power. Assume that the total pressure is constant at 1.0132 bar.

Solution. (*i*) The flow diagram is shown in Fig. 10.17 (*a*) and the processes are shown in Fig. 10.17 (*b*).



(*a*)



(*b*)

Fig. 10.17

(*i*) **Mass of spray water required**

At 22°C

From steam tables : $p_{vs} = 0.0264$ bar

$$\phi_3 = \frac{p_{v_3}}{P_{vs_3}} = \frac{p_{vs_3}}{0.0264} = 0.55$$

$\therefore$ $p_{v_3} = 0.55 \times 0.0264 = 0.01452$ bar

$\therefore$ $W_3 = \dfrac{0.622 \times 0.01452}{(1.0132 - 0.01452)} = 0.00904$ kg/kg of dry air

At 3°C

From steam tables : $p_{vs} = 0.0076$ bar

$$\phi_1 = \frac{p_{v_1}}{p_{vs_1}} = 1.00$$

$\therefore$ $p_{v_1} = p_{vs_1} = 0.0076$ bar

$$W_1 = \frac{0.622 \times 0.0076}{1.0132 - 0.0076} = 0.0047 \text{ kg/kg of dry air}$$

$$W_3 - W_1 = 0.00904 - 0.0047 = 0.00434 \text{ kg/kg of dry air}$$

$$v_{a_3} = \frac{R_a T_3}{p_{a_3}} = \frac{287 \times (273 + 22)}{(1.0132 - p_{v_3}) \times 10^5} = \frac{287 \times 295}{0.9987 \times 10^5}$$

$$= 0.847 \text{ m}^3\text{/kg of dry air}$$

Spray water = $\dfrac{0.00434}{0.847}$ = **0.005124 kg moisture/m³. (Ans.)**

(*ii*) **Temperature to which the air must be heated $\mathbf{t_{db_2}}$:**

Now $h_2 + (W_3 - W_2)\, h_4 = h_3$

$$[c_p t_{db_2} + W_2 h_{\text{vapour (2)}}] + (W_3 - W_2) h_2 = c_p t_{db_3} + W_3 h_{\text{vapour (3)}}$$

∴ $$c_p(t_{db_3} - t_{db_2}) + W_3 h_{\text{vapour (3)}} - W_2 h_{\text{vapour (2)}} - (W_3 - W_2) h_4 = 0$$

From the steam tables at $p_v = 0.01452$ bar : $h_g = 2524$ kJ/kg

and $$t_{dp} = t_{sat} = 12.5°\text{C}$$

∴ $$1.005(22 - t_{db_2}) + 0.00904[2524 + 1.88(22 - 12.5)]$$

$$- 0.0047\ [2524 + 1.88(t_{db_2} - 12.5)] - (0.00434 \times 4.187 \times 10) = 0$$

$$22.11 - 1.005\ t_{db_2} + 22.97 - 11.86 - 0.0088\ t_{db_2} + 0.11 - 0.1817 = 0$$

$$1.014\, t_{db_2} = 33.148$$

∴ $$\mathbf{t_{db_2} = 32.7°C.}$$ **(Ans.)**

Example 10.14. Cooling tower : *A small-size cooling tower is designed to cool 5.5 litres of water per second, the inlet temperature of which is 44°C. The motor-driven fan induces 9 m^3/s of air through the tower and the power absorbed is 4.75 kW. The air entering the tower is at 18°C, and has a relative humidity of 60%. The air leaving the tower can be assumed to be saturated and its temperature is 26°C. Calculate :*

(*i*) *The amount of cooling water (make-up) required per second.*

(*ii*) *The final temperature of the water.*

Assume that the pressure remains constant throughout the tower at 1.013 bar.

Solution. The cooling tower is shown diagrammatically in Fig. 10.18.

(*i*) **Make-up water required :**

At 18°C $$p_{vs} = 0.0206 \text{ bar},$$

∴ $$p_v = \phi \times p_{vs} = 0.6 \times 0.0206 = 0.01236 \text{ bar}$$

∴ $$p_{a_1} = 1.013 - 0.01236 = 1.00064 \text{ bar}$$

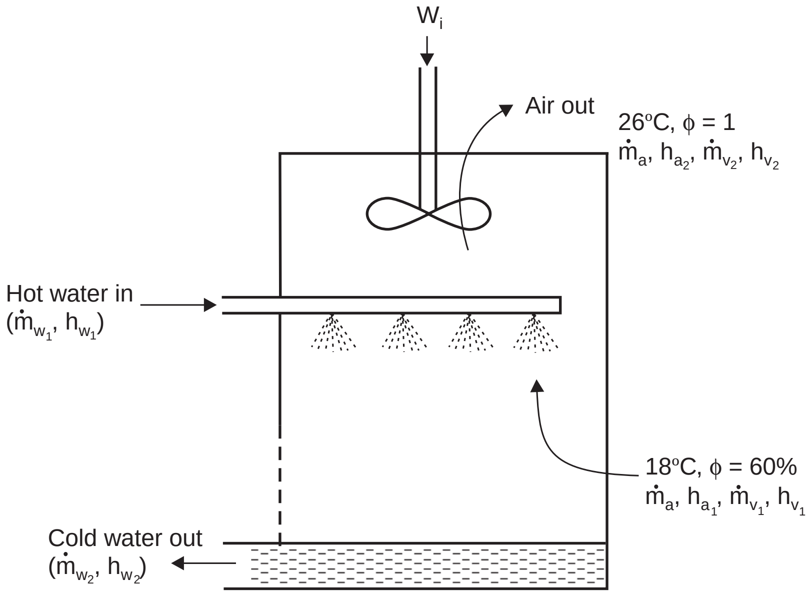


Fig. 10.18

Then, $$\dot{m}_a = \frac{10^5 \times 1.00064 \times 9}{(0.287 \times 10^3) \times (18 + 273)} = 10.78 \text{ kg/s}$$

and $$\dot{m}_{v_1} = \frac{10^5 \times 0.01236 \times 9}{(0.4618 \times 10^3) \times (18 + 273)} = 0.0828 \text{ kg/s}$$

(Script '*v*' denotes vapour and the script '*a*' denotes the air).

At exit at 26°C, $p_{vs} = 0.0336$ bar and $\phi = 1$

$\therefore$ $p_v = p_{vs} = 0.0336$ bar

$$W_2 = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.0336}{1.013 - 0.0336} = 0.02133 \text{ kg.}$$

But $$W = \frac{m_v}{m_a}$$

$\therefore$ $\dot{m}_{v_2} = 0.02133 \times 10.78 = 0.23$ kg/s

Hence, **make-up water required**

$= 0.23 - 0.0828 =$ **0.1472 kg/s. (Ans.)**

(*ii*) **Final temperature of the water :**

Also, $\dot{m}_{w_1} = 5.5 \times 1 = 5.5$ kg/s

and $\dot{m}_{w_2} = \dot{m}_{w_1}$ – (make-up water)

$= 5.5 - 0.1472 = 5.353$ kg/s

Applying the steady flow energy equation and neglecting changes in kinetic energy and potential energy, we have

$$W_i + \dot{m}_{w_1} h_{w_1} + \dot{m}_{a_1} h_{a_1} + \dot{m}_{v_1} h_{v_1} = \dot{m}_{a_2} h_{a_2} + \dot{m}_{v_2} h_{v_2} + \dot{m}_{w_2} h_{w_2}$$

Now, W_i (*i.e.*, work input) = 4.75 kW = 4.75 kJ/s.

Evaluating the enthalpies from a datum of 0°C, we have :

$h_{w_1} = h_f$ at 44°C = 184.3 kJ/kg,

$h_{a_1} = 1.005\ (18 - 0) = 18.09$ kJ/kg,

$h_{v_1} = 2519.7 + 1.88\ (18 - 10) = 2534.74$ kJ/kg.

[Corresponding to $p_v = 0.01236$ bar, $t_s = t_{dp} \simeq 10$°C *i.e.*, the vapour is superheated]

$h_{v_1} = h_g$ at 26°C = 2549 kJ/kg

$h_{a_2} = 1.005\ (26 - 0) = 26.13$ kJ/kg.

Then, substituting, we get

$4.75 + 5.5 \times 184.3 + 10.78 \times 18.09 + 0.0828 \times 2534.74$

$= 10.78 \times 26.13 + 0.23 \times 2549 + 5.353 \times h_{w_2}$

or $5.353\ h_{w_2} = 1423.28 - 867.95 = 555.33$

or $h_{w_2} = 103.74$ kJ/kg.

By interpolation, $h_f = 103.74$ kJ/kg at 26.7°C.

Hence, **final temperature of water = 26.7°C. (Ans.)**

Example 10.15. *A cooling tower used in power plant consists of 10 big fans, $\dot{m}_{water}$ = 1000 kg/min. It is cooled from 35°C to 30°C. Atmospheric conditions are 35°C DBT, 25°C WBT. Air leaves the tower at 30°C, 90% RH. Find out the quantity of air handled per fan hour and the quantity of make-up water required per hour.* **(AMIE Winter, 1999)**

Solution. Refer Fig. 10.19.

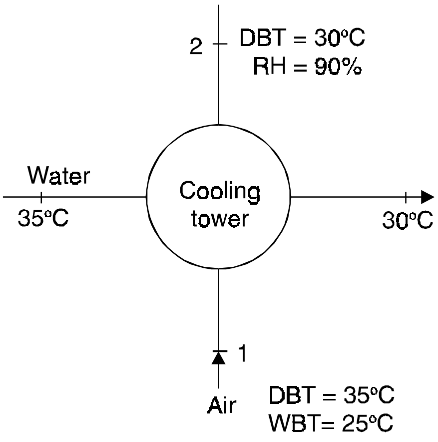


Fig. 10.19

Heat absorbed from the cooling tower

$$= \dot{m}_{\text{water}} \times c \times \Delta T$$
$$= (1000 \times 60) \times 4.186 \times (35 - 30)$$
$$= 1.256 \times 10^6 \text{ kJ/h}$$

From psychrometric chart, we have

At 35°C DBT and 25°C WBT :

h_1 = 76.5 kJ/kg ; W_1 = 0.016 kg/kg of air

At 30°C and 90% RH :

h_2 = 92.5 kJ/kg ; W_2 = 0.0246 kg/kg of air

Heat gained by air = Heat lost by water

$$\dot{m}_{\text{air}}\ (h_2 - h_1) = 1.256 \times 10^6$$

$$\therefore \quad \text{Mass of air, } \dot{m}_{\text{air}} = \frac{1.256 \times 10^6}{(h_2 - h_1)}$$

$$= \frac{1.256 \times 10^6}{(92.5 - 76.5)} = 78.5 \times 10^3 \text{ kg/h}$$

∴ **Quantity of air handled per fan** = $\frac{78.5 \times 10^3}{10}$ = **7850 kg/h. (Ans.)**

Quantity of make-up water = $\dot{m}_{air}\ (W_2 - W_1)$

= 78.5 × 10^3 (0.0246 – 0.016) = **675.1 kg/h. (Ans.)**

SOLUTIONS USING PSYCHROMETRIC CHARTS

Example 10.16. *The following data pertain to an air-conditioning system :*

Unconditioned space DBT = 30°C

Unconditioned space WBT = 22°C

Cold air duct supply surface temperature = 14°C.

Determine : (i) Dew point temperature.

(ii) Whether or not condensation will form on the duct.

Solution. Refer Fig. 10.20.

(*i*) To determine the dew point temperature for the given conditions, find the intersection of 30°C *DBT* and 22°C *WBT* and move horizontally (as shown by the arrow) to the dew point temperature scale. The **dew point (t_{dp}) is 18.6°C. (Ans.)**

(*ii*) Since the duct temperature (14°C) is less than t_{dp} (18.6°C) therefore *moisture will condense on the duct surface.* **(Ans.)**

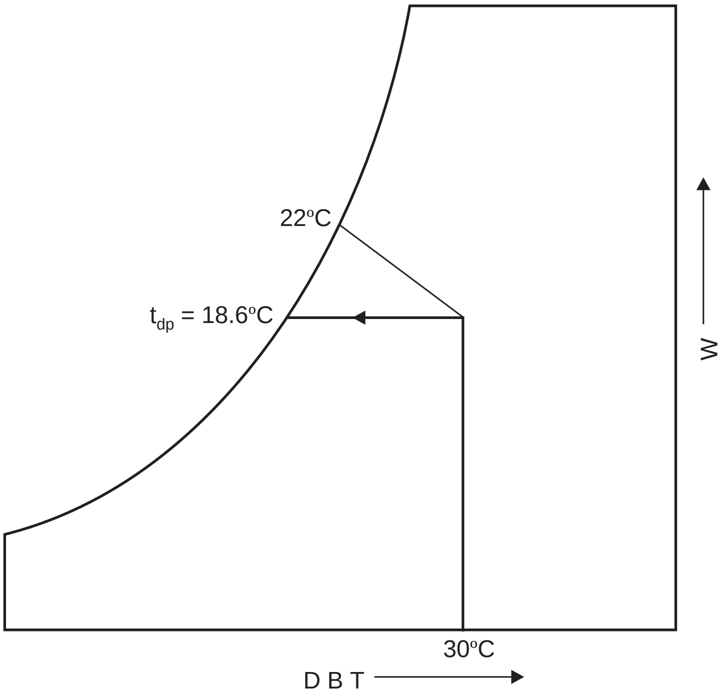


Fig. 10.20

Example 10.17. *200 m^3 of air per minute at 15°C DBT and 75% R.H. is heated until its temperature is 25°C.*

Find : (*i*) *R.H. of heated air.*

(*ii*) *Wet bulb temperature of heated air.*

(*iii*) *Heat added to air per minute.*

Solution. Refer Fig. 10.21.

- Locate point 1 on the psychrometric chart on intersection of 15°C *DBT* and 75% R.H. lines.
- Through point 1 draw a horizontal line to cut 25°C *DBT* line and get point 2.
- Read the following values from the psychrometric chart :

$$h_1 = 35.4 \text{ kJ/kg}$$

$$h_2 = 45.2 \text{ kJ/kg}$$

$$v_{s_1} = 0.8267 \text{ m}^3\text{/kg.}$$

(*i*) **R.H. of heated air** (read from chart) = **41%. (Ans.)**

(*ii*) **WBT of heated air** (read from chart) = **16.1°C. (Ans.)**

(*iii*) Mass of air circulated per min., $m_a = \dfrac{200}{0.8267} = 241.9$ kg.

∴ **Heat added to air/min.**

$$= m_a (h_2 - h_1) = 241.9\ (45.2 - 35.4) = \mathbf{2370.6\ kJ.}\quad \textbf{(Ans.)}$$

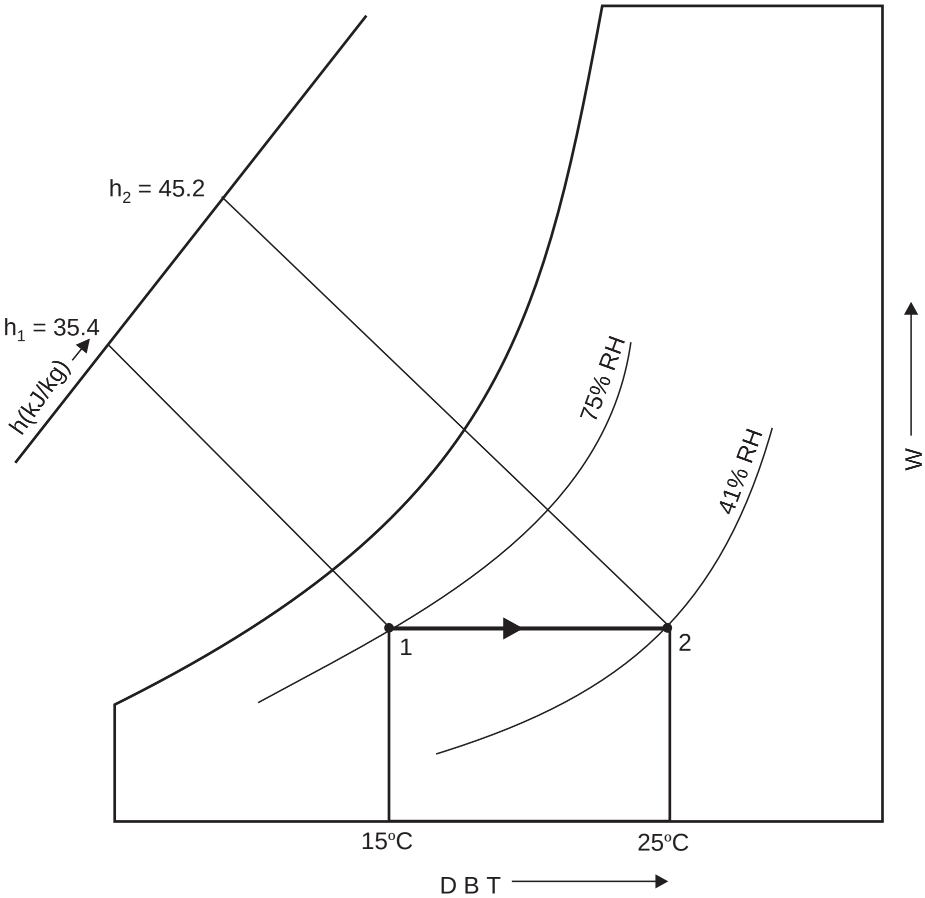


Fig. 10.21

Example 10.18. *It is required to design an air-conditioning plant for a small office room for following winter conditions :*

Outdoor conditions ...... 14°C DBT and 10°C WBT

Required conditions ...... 20°C DBT and 60% R.H.

Amount of air circulation ...... 0.30 m^3*/min./person.*

Seating capacity of office ...... 60.

The required condition is achieved first by heating and then by adiabatic humidifying.

Determine the following :

(*i*) *Heating capacity of the coil in kW and the surface temperature required if the by pass factor of coil is 0.4.*

(*ii*) *The capacity of the humidifier.*

Solve the problem by using psychrometric chart.

Solution. Refer Fig. 10.22.

- Locate the points '1' and '3' on the psychrometric chart.
- Draw a constant enthalpy line through '3' and constant specific humidity line through '1'.

- Locate the point '2' where the above two lines intersect.

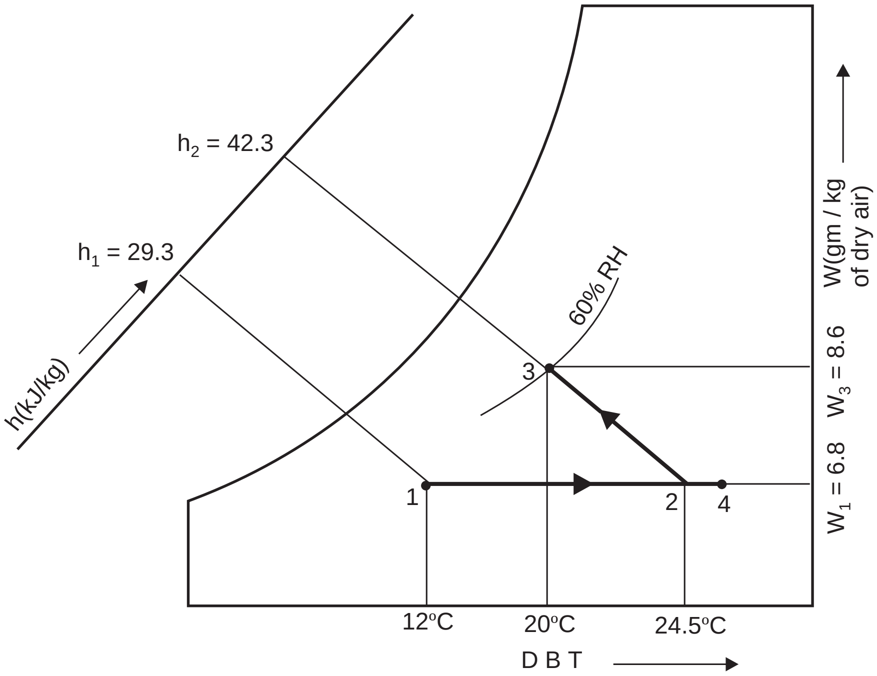


Fig. 10.22

From the psychrometric chart :

$$h_1 = 29.3 \text{ kJ/kg}, \; h_2 = h_3 = 42.3 \text{ kJ/kg}$$

$$t_{db_2} = 24.5°\text{C}, \; v_{s_1} = 0.817 \text{ m}^3\text{/kg}$$

The mass of air circulated per minute,

$$m_a = \frac{0.30 \times 60}{0.817} = 22.03 \text{ kg/min.}$$

(*i*) **Heating capacity of the heating coil**

$$= m_a(h_2 - h_1) = 22.03\,(42.3 - 29.3) = 286.4 \text{ kJ/min.}$$

$$= \mathbf{4.77\ kJ/s\ or\ 4.77\ kW.} \quad \textbf{(Ans.)}$$

The by-pass factor (*BF*) of heating coil is given by :

$$BF = \frac{t_{db_4} - t_{db_2}}{t_{db_4} - t_{db_1}}$$

$$0.4 = \frac{t_{db_4} - 24.5}{t_{db_4} - 12}$$

$$\therefore \quad 0.4\,(t_{db_4} - 12) = t_{db_4} - 24.5$$

i.e., t_{db_4} *(coil surface temperature)* **= 32.8°C. (Ans.)**

(*ii*) **The capacity of the humidifier**

$$= \frac{m_a\,(W_3 - W_1)}{1000} \times 60 \text{ kg/h} = \frac{22.03\,(8.6 - 6.8)}{1000} \times 60 = \mathbf{2.379\ kg/h.} \quad \textbf{(Ans.)}$$

Example 10.19. *It is required to design an air-conditioning system for an industrial process for the following hot and wet summer conditions :*

Outdoor conditions ...... 32°C DBT and 65% R.H.

Required air inlet conditions ...... 25°C DBT and 60% R.H.

Amount of free air circulated ...... 250 m^3*/min.*

Coil dew temperature ...... 13°C.

The required condition is achieved by first cooling and dehumidifying and then by heating.

Calculate the following :

(*i*) *The cooling capacity of the cooling coil and its by-pass factor.*

(*ii*) *Heating capacity of the heating coil in kW and surface temperature of the heating coil if the by-pass factor is 0.3.*

(*iii*) *The mass of water vapour removed per hour.*

Solve this problem with the use of psychrometric chart.

Solution. Refer Fig. 10.23.

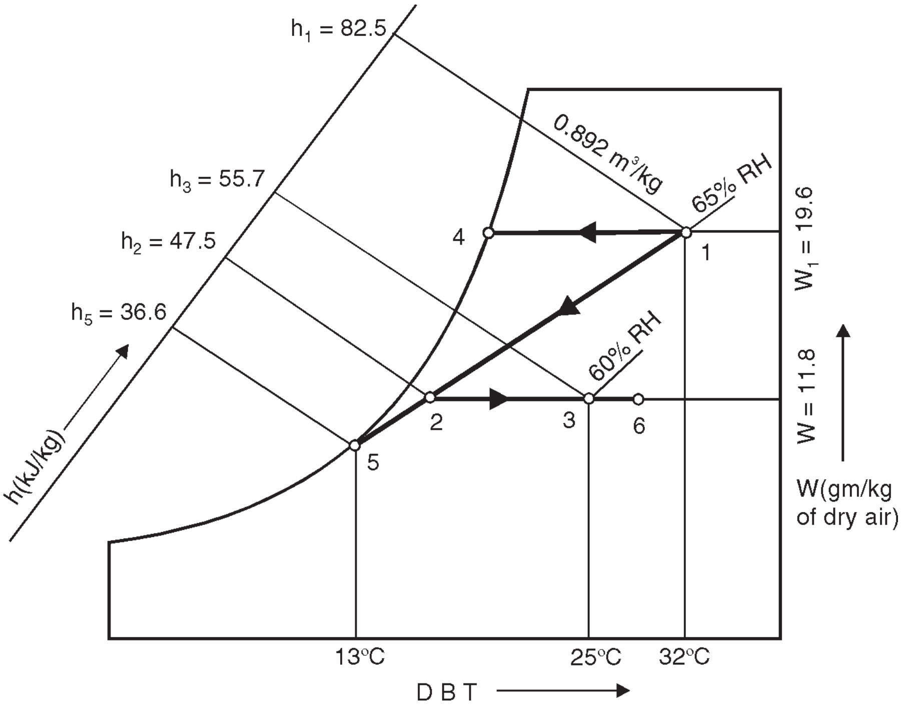


Fig. 10.23

- Locate the points '1', '5' and '3' as shown on psychrometric chart.
- Join the line 1-5.
- Draw constant specific humidity line through '3' which cuts the line 1-5 at point '2'. The point '2' is located in this way.

From psychrometric chart :

$$h_1 = 82.5 \text{ kJ/kg}, \qquad h_2 = 47.5 \text{ kJ/kg}$$

$$h_3 = 55.7 \text{ kJ/kg}, \qquad h_5 = 36.6 \text{ kJ/kg}$$

W_1 = 19.6 gm/kg, W_3 = 11.8 gm/kg

t_{db_2} = 17.6°C, v_{s_1} = 0.892 m^3/kg.

The mass of air supplied per minute,

$$m_a = \frac{250}{0.892} = 280.26 \text{ kg/min.}$$

(*i*) **The capacity of the cooling coil**

$$= \frac{m_a(h_1 - h_2) \times 60}{14000} = \frac{280.26\,(82.5 - 47.5) \times 60}{14000} = \mathbf{42.04\ TR.} \quad \textbf{(Ans.)}$$

The *by-pass factor of the cooling coil* is given by :

$$BF = \frac{h_2 - h_5}{h_1 - h_5} = \frac{47.5 - 36.6}{82.5 - 36.6} = \mathbf{0.237.} \quad \textbf{(Ans.)}$$

(*ii*) **The heating capacity of the heating coil**

$$= m_a\,(h_3 - h_2) = 280.26\,(55.7 - 47.5) = 2298.13 \text{ kJ/min} = \frac{2298.13}{60} \text{ kJ/s}$$

$$= \mathbf{38.3\ kW.} \quad \textbf{(Ans.)}$$

The by-pass factor of the *heating coil* is given by

$$BF = \frac{t_{db_6} - t_{db_3}}{t_{db_6} - t_{db_2}}$$

$$0.3 = \frac{t_{db_6} - 25}{t_{db_6} - 17.6}$$

$$\therefore \quad t_{db_6} = 28.2°\text{C}.$$

Hence surface temperature of heating coil = 28.2°C. (Ans.)

(*iii*) **The mass of water vapour removed per hour**

$$= \frac{280.26\,(W_1 - W_3) \times 60}{1000} = \frac{280.26\,(19.6 - 11.8)}{1000} \times 60 = \mathbf{131.16\ kg/h.} \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. 'Air-conditioning' is the simultaneous control of temperature, humidity, motion and purity of the atmosphere in a confined space.
2. 'Psychrometry' is an art of measuring moisture content of air.

   The science which investigates the thermal properties of moist air, considers the measurement and control of the moisture content of air, and studies the effects of atmospheric moisture on material and human comfort may properly be termed 'Psychrometrics'.
3. When air is saturated *DBT, WBT, DPT* are equal.
4. *Dalton's law of partial pressure* states, that the total pressure of a mixture of gases is equal to the sum of partial pressure which the component gases would exert if each existed alone in the mixture volume at the mixture temperature.
5. Specific humidity, $W = \dfrac{0.622\, p_v}{p_t - p_v}$.
6. Degree of saturation (μ) = $\dfrac{\text{Mass of water vapour associated with unit mass of dry air } (W)}{\text{Mass of water vapour associated with saturated unit mass of dry air } (W_s)}$.

7. Relative humidity, $\phi = \dfrac{p_v}{p_{vs}}$.

8. A 'Psychrometer' is a device which is used for measuring dry bulb and wet-bulb temperatures simultaneously.

9. The processes which affect the psychrometric properties of air are called '*psychrometric processes*'.

Important psychrometric processes are :

(*i*) Mixing of air streams
(*ii*) Sensible heating
(*iii*) Sensible cooling
(*iv*) Cooling and dehumidification
(*v*) Cooling and humidification
(*vi*) Heating and dehumidification
(*vii*) Heating and humidification.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. In an unsaturated air the state of a vapour is
(*a*) wet
(*b*) superheated
(*c*) saturated
(*d*) unsaturated.

2. For saturated air
(*a*) Wet bulb depression is zero
(*b*) Wet bulb depression is positive
(*c*) Wet bulb depression is negative
(*d*) Wet bulb depression can be either positive or negative.

3. Which one of the following statements is *correct* ?
(*a*) Dew point temperature can be measured with the help of thermometer
(*b*) Dew point temperature is the saturation temperature corresponding to the partial pressure of the water vapour in moist air.
(*c*) Dew point temperature is the same as the thermodynamic wet bulb temperature.
(*d*) For saturated air, dew point temperature is less than the wet bulb temperature.

4. During sensible heating of moist air, enthalpy
(*a*) increases
(*b*) decreases
(*c*) remains constant
(*d*) none of the above.

5. During sensible cooling, wet bulb temperature
(*a*) decreases
(*b*) increases
(*c*) remains constant
(*d*) can decrease or increase.

6. Which one of the following statements is *correct* ?
(*a*) Evaporative cooling and sensible cooling is the same
(*b*) Evaporative cooling is a cooling and humidification process
(*c*) Evaporative cooling is a cooling and dehumidification process
(*d*) Evaporative cooling is not effective for hot and dry climates.

7. An air washer can work as a
(*a*) filter only
(*b*) humidifier only
(*c*) dehumidifier only
(*d*) all of the above.

8. The relative humidity, during sensible heating,
(*a*) can increase or decrease
(*b*) increases
(*c*) decreases
(*d*) remains constant.

9. The vapour pressure, during sensible heating of moist air,
(*a*) increases
(*b*) decreases
(*c*) can increase or decrease
(*d*) remains constant.

10. The relative humidity, during heating and humidification,
(*a*) increases (*b*) decreases
(*c*) may increase or decrease (*d*) remains constant.

11. The relative humidity, during cooling and dehumidification of moist air
(*a*) increases (*b*) decreases
(*c*) can increase or decrease (*d*) remains constant.

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*a*) **5.** (*a*) **6.** (*b*) **7.** (*d*)
8. (*b*) **9.** (*d*) **10.** (*a*) **11.** (*c*).

THEORETICAL QUESTIONS

1. Define the following terms :
(*i*) Saturated air (*ii*) Dry bulb temperature
(*iii*) Dew point temperature (*iv*) Relative humidity
(*v*) Specific humidity.

2. State 'Dalton's law of partial pressure'.

3. Derive the following relations :

(*i*) Specific humidity, $W = 0.622 \dfrac{p_v}{p_t - p_v}$. (*ii*) Degree of saturation, $\mu = \dfrac{p_v}{p_s}\left[\dfrac{\left(1 - \dfrac{p_s}{p_t}\right)}{\left(1 - \dfrac{p_v}{p_t}\right)}\right]$.

4. Explain briefly with a neat sketch a 'sling psychrometer'.

5. Describe briefly any two of the following processes :
(*i*) Sensible heating (*ii*) Cooling and dehumidification
(*iii*) Heating and humidification (*iv*) Heating and dehumidification.

6. Write a short note on 'by-pass factor'.

UNSOLVED EXAMPLES

1. The atmospheric conditions are 30°C and specific humidity of 0.0215 kg/kg of air. Determine :
(*i*) Partial pressure of air (*ii*) Relative humidity
(*iii*) Dew point temperature.
Atmospheric pressure = 756 mm Hg. [**Ans.** (*i*) 14.89 mm of Hg, (*ii*) 46.8%, (*iii*) 17°C]

2. A mixture of air and water vapour at 1 bar and 25°C has a dew point temperature of 15°C. Determine the relative humidity and specific humidity. [**Ans.** 53.8%, 0.01078 kg/kg of dry air]

3. An air-water vapour mixture at 1.24 bar has a temperature of 38°C and relative humidity of 60%. Calculate the kg-mass of water vapour per kg of air and per kg of mixture. Also find the dew point.
[**Ans.** 0.0203 kg/kg of dry air, 0.0199 kg/kg of mixture, 28.5°C]

4. In a house, the temperature of the windows on a day in winter is 5°C. When the temperature in the room is 23°C, and the barometric pressure is 748.8 mm Hg, what would be maximum relative humidity that could be maintained in the room without condensation on the window panes ? Under these conditions, find the partial pressure of the water vapour and air, the specific humidity, and the density of the mixture.
[**Ans.** 30.7%, 0.00872 bar, 0.989 bar, 0.00548 kg/kg of dry air, 1.164 kg/m^3]

5. Atmospheric air enters a heater at 4.5°C and 60% relative humidity and leaves at a temperature of 21°C. Calculate :

(*i*) The heat supplied to the air ;

(*ii*) The final relative humidity. [**Ans.** 16.8 kJ/kg, 29.2%]

6. The air supplied to a room of building in winter is to be at 17°C and have a relative humidity of 60%. If the barometric pressure is 1.01325 bar, calculate the specific humidity. What would be the dew point under these conditions ? [**Ans.** 0.00723 kg/kg of dry air, 9.18°C]

7. If air at the condition of example 6, is passed at the rate of 0.5 m^3/s over a cooling coil which is at a temperature of 6°C, calculate the amount of vapour which will be condensed. Assume that the barometric pressure is the same as in example 6, and that the air leaving the coil is saturated. [**Ans.** 3.082 kg/h]

8. An air and water vapour mixture at 1 bar and 26.7°C has a specific humidity of 0.0085. Determine the percentage saturation. [**Ans.** 37.7%]

9. A mixture of air and water vapour at 1.013 bar and 16°C has a dew point of 5°C. Determine the relative and specific humidities. [**Ans.** 48%, 0.0054 kg/kg of dry air]

10. Atmospheric air at a pressure of 760 mm Hg has a temperature of 32°C and a percentage saturation as determined from a psychrometric chart of 52%. Calculate

(*i*) The partial pressure of the vapour and the dry air

(*ii*) The specific humidity

(*iii*) The dew point

(*iv*) The density of the mixture.

[**Ans.** (*i*) 0.0247 bar, 0.988 bar, (*ii*) 0.01556, (*iii*) 20.9°C, (*iv*) 1.147 kg/m^3]

11. In a laboratory test, a psychrometer recorded 36°C DBT and 30°C WBT. Calculate :

(*i*) Vapour pressure (*ii*) Relative humidity (*iii*) Specific humidity

(*iv*) Degree of saturation (*v*) Dew point temperature (*vi*) Enthalpy of the mixture.

[**Ans.** (*i*) 0.0385 bar, (*ii*) 64.5%, (*iii*) 0.025 kg/kg dry air, (*iv*) 0.63, (*v*) 28°C, (*vi*) 99.2 kJ]

12. The pressure and temperature of the air in a room is 1 bar and 28°C. If the relative humidity is found to be 30 per cent, determine :

(*i*) The partial pressure of the water vapour and dew point,

(*ii*) The specific volume of each constituent, and

(*iii*) The specific humidity.

[**Ans.** (*i*) 0.0378 bar, 8.8°C ; (*ii*) v_{air} = 0.874 m^3/kg, v_{vap} = 122.7 m^3/kg ; (*iii*) 0.00712 kg/kg dry air]

13. 100 m^3 of air per minute at 35°C DBT and 60% relative humidity is cooled to 20°C DBT by passing through a cooling coil. Find the following :

(*i*) Capacity of cooling coil in kJ/h

(*ii*) Amount of water vapour removed per hour, and

(*iii*) Relative humidity of air coming out and its wet-bulb temperature.

[**Ans.** (*i*) 1037088 kJ/h, (*ii*) 465.36 kg/h, (*iii*) 100%, 20°C]

14. Atmospheric air at 38°C and 40 per cent relative humidity is to be cooled and dehumidified to a state of saturated air at 10°C. The mass rate of flow of atmospheric air entering the dehumidifier is 45.4 kg/h. Neglecting any pressure drop, determine :

(*i*) The mass of water removed ; (*ii*) The quantity of heat removed.

[**Ans.** (*i*) 0.397 kg/h, (*ii*) 2287 kJ/h]

15. 1 kg of air at 24°C and a relative humidity of 70% is to be mixed adiabatically in a steady state, steady flow device with 1 kg of air at 16°C and a relative humidity of 10%. Assuming that the mixing is to be carried out at a constant pressure of 1.0 atm, determine the temperature and relative humidity of the stream leaving the device. [**Ans.** 19.5°C, 50%]

16. An air-water vapour mixture enters an adiabatic saturator at 30°C and leaves at 20°C, which is the adiabatic saturation temperature. The pressure remains constant at 1 bar. Determine the relative humidity and the humidity ratio of the inlet mixture. [**Ans.** 39.8%, 0.0107 kg/kg dry air]

11

Fuels and Combustion

(Including Chemical Thermodynamics)

11.1. INTRODUCTION

- Fuel may be *chemical* or *nuclear*. Here we shall consider briefly *chemical fuels* only.

A chemical fuel is a substance which releases heat energy on combustion. The principal combustible elements of each fuel are *carbon* and *hydrogen*. Though *sulphur* is a combustible element too but its presence in the fuel is considered to be *undesirable*.

- In **chemical thermodynamics** the study of systems involving chemical reactions is an important topic. *A chemical reaction may be defined as the rearrangement of atoms due to redistribution of electrons*. In a chemical reaction the terms, **reactants** and the **products** are frequently used. '*Reactants*' comprise of initial constituents which start the reaction while '*products*' comprise of *final constituents* which are formed by the chemical reaction. Although the basic principles which will be discussed in this chapter apply to any chemical reaction, here main attention will be focused on an important type of chemical reaction—**combustion**.

11.2. CLASSIFICATION OF FUELS

Fuels can be classified according to whether :

1. They occur in nature called **primary fuels** or are prepared called **secondary fuels** ;

2. They are in solid, liquid or gaseous state. The detailed classification of fuels can be given in a summary form as follows :

Type of fuel	*Natural (Primary)*	*Prepared (Secondary)*
Solid	Wood	Coke
	Peat	Charcoal
	Lignite coal	Briquettes

Liquid	Petroleum	Gasoline Kerosene Fuel oil Alcohol Benzol Shale oil
Gaseous	Natural gas	Petroleum gas Producer gas Coal gas Coke-oven gas Blast furnace gas Carburetted gas Sewer gas

11.3. SOLID FUELS

Coal. Its main constituents are carbon, hydrogen, oxygen, nitrogen, sulphur, moisture and ash. Coal passes through different stages during its formation from vegetation. These stages are enumerated and discussed below :

Plant debris—Peat—Lignite—Brown coal—sub-bituminous coal—Bituminous coal—Semi-bituminous coal—Semi-anthracite coal—Anthracite coal—Graphite.

Peat. It is the first stage in the formation of coal from wood. It contains huge amount of moisture and therefore it is dried for about 1 to 2 months before it is put to use. It is used as a domestic fuel in Europe and for power generation in Russia. In India it does not come in the categories of good fuels.

Lignites and brown coals. These are intermediate stages between peat and coal. They have a woody or often a clay like appearance associated with high moisture, high ash and low heat contents. Lignites are usually amorphous in character and impose transport difficulties as they break easily. They burn with a smoky flame. Some of this type are suitable for local use only.

Bituminous coal. It burns with long yellow and smoky flames and has high percentages of volatile matter. The average calorific value of bituminous coal is about 31350 kJ/kg. It may be of two types, namely *caking* or *noncaking*.

Semi-bituminous coal. It is softer than the anthracite. It burns with a very small amount of smoke. It contains 15 to 20 per cent volatile matter and has a tendency to break into small sizes during storage or transportation.

Semi-anthracite. It has less fixed carbon and less lustre as compared to true anthracite and gives out longer and more luminous flames when burnt.

Anthracite. It is very hard coal and has a shining black lustre. It ignites slowly unless the furnace temperature is high. It is non-caking and has high percentage of fixed carbon. It burns either with very short blue flames or without flames. The calorific value of this fuel is high to the tune of 35500 kJ/kg and as such is *very suitable for steam generation*.

Wood charcoal. It is obtained by destructive distillation of wood. During the process the volatile matter and water are expelled. The physical properties of the residue (charcoal), however depends upon the rate of heating and temperature.

Coke. It consists of carbon, mineral matter with about 2% sulphur and small quantities of hydrogen, nitrogen and phosphorus. It is solid residue left after the destructive distillation of certain kinds of coals. It is smokeless and clear fuel and can be produced by several processes. It is *mainly used in blast furnace* to produce heat and at the same time to reduce the iron ore.

Briquettes. These are prepared from fine coal or coke by compressing the material under high pressure.

11.4. LIQUID FUELS

The chief source of liquid fuels is *petroleum* which is obtained from wells under the earth's crust. These fuels have proved *more advantageous in comparison to sold fuels* in the following respects.

Advantages :

1. Require less space for storage.
2. Higher calorific value.
3. Easy control of consumption.
4. Staff economy.
5. Absence of danger from spontaneous combustion.
6. Easy handling and transportation.
7. Cleanliness.
8. No ash problem.
9. Non-deterioration of the oil in storage.

Petroleum. There are different opinions regarding the origin of petroleum. However, now it is accepted that petroleum has originated probably from organic matter like fish and plant life etc., by bacterial action or by their distillation under pressure and heat. It consists of a mixture of gases, liquids and solid hydrocarbons with small amounts of nitrogen and sulphur compounds. In India, the main sources of Petroleum are Assam and Gujarat.

Heavy fuel oil or crude oil is imported and then refined at different refineries. The refining of crude oil supplies the most important product called *petrol*. Petrol can also be made by polymerization of refinery gases.

Other liquid fuels are kerosene, fuels oils, colloidal fuels and alcohol.

11.5. GASEOUS FUELS

Natural gas. The main constituents of natural gas are *methane* (CH_4) and *ethane* (C_2H_6). It has calorific value nearly 21000 kJ/m^3. Natural gas is used alternately or simultaneously with oil for internal combustion engines.

Coal gas. Mainly consists of *hydrogen, carbon monoxide* and *hydrocarbons*. It is prepared by carbonisation of coal. It finds its use in boilers and sometimes used for commercial purposes.

Coke-oven gas. It is obtained during the production of coke by heating the bituminous coal. The volatile content of coal is driven off by heating and major portion of this gas is utilised in heating the ovens. This gas *must be thoroughly filtered before using in gas engines.*

Blast furnance gas. It is obtained from smelting operation in which air is forced through layers of coke and iron ore, the example being that of pig iron manufacture where this gas is produced as by product and contains about 20% carbon monoxide (CO). After filtering it may be blended with richer gas or used in gas engines directly. The heating value of this gas is very low.

Producer gas. It results from the partial oxidation of coal, coke or peat when they are burnt with an insufficient quantity of air. It is produced in specially designed retorts. It has low heating value and in general is suitable for large installations. It is also used in steel industry for firing open hearth furnaces.

Water or illuminating gas. It is produced by blowing steam into white hot coke or coal. The decomposition of steam takes place liberating free hydrogen, and oxygen in the steam combines with carbon to form carbon monoxide according to the reaction :

$$C + H_2O \longrightarrow CO + H_2$$

The gas composition varies as the hydrogen content if the coal is used.

Sewer gas. It is obtained from sewage disposal vats in which fermentation and decay occur. It consists of mainly marsh gas (CH_4) and is collected at large disposal plants. It works as a fuel for gas engines which in turn drive the plant pumps and agitators.

Gaseous fuels are becoming popular because of following *advantages* they possess.

Advantages :

1. Better control of combustion.
2. Much less excess air is needed for complete combustion.
3. Economy in fuel and more efficiency of furnace operation.
4. Easy maintenance of oxidizing or reducing atmosphere.
5. Cleanliness.
6. No problem of storage if the supply is available from public supply line.
7. The distribution of gaseous fuels even over a wide area is easy through the pipe lines and as such handling of the fuel is altogether eliminated.
8. Gaseous fuels give economy of heat and produce higher temperatures (as they can be preheated in regenerative furnances and thus heat from hot flue gases can be recovered).

11.6. BASIC CHEMISTRY

Before considering combustion problems it is necessary to understand the construction and use of chemical formulae. This involves elementary concepts which are discussed below briefly.

Atoms. It is not possible to divide the chemical elements *indefinitely*, and the *smallest particle which can take part in a chemical change* is called an **'atom'**. If an atom is split as in nuclear reaction, the *divided atom does not retain the original chemical properties*.

Molecules. It is rare to find elements to exist naturally as single atom. Some elements have atoms which exist in pairs, each pair forming a molecule (*e.g.* oxygen), and the atoms of each molecule are held together by stronger *inter-atomic forces*. The isolation of a molecule of oxygen would be tedious, but possible ; the isolation of an atom of oxygen would be a different prospect. The molecules of some substances are formed by the mating up of atoms of different elements. For example, water has a molecule which consists of two atoms of hydrogen and one atom of oxygen. The atoms of different elements have different masses and these values are important when a quantitative analysis is required. The actual masses are infinitesimally small, and the ratios of the masses of atoms are used. These ratios are indicated by **atomic weight** quoted on a scale which defines the atomic weight of oxygen as 16.

The symbols and molecular weights of some important elements, compounds and gases are given in Table 11.1.

Table 11.1. Symbols and Molecular weights

Elements / Compounds / Gases	*Molecule*		*Atom*	
	Symbol	*Molecular weight*	*Symbol*	*Molecular weight*
Hydrogen	H_2	2	H	1
Oxygen	O_2	32	O	16
Nitrogen	N_2	28	N	14
Carbon	C	12	C	12
Sulphur	S	32	S	32
Water	H_2O	18	—	—
Carbon monoxide	CO	28	—	—
Carbon dioxide	CO_2	44	—	—
Sulphur dioxide	SO_2	64	—	—
Marsh gas (Methane)	CH_4	16	—	—
Ethylene	C_2H_4	28	—	—
Ethane	C_2H_6	30	—	—

11.7. COMBUSTION EQUATIONS

- In a combustion chamber proportionate masses of air and fuel enter where the chemical reaction takes place, and then the combustion products pass to the exhaust. By the conservation of mass the mass flow remains constant (*i.e.,* total mass of *products* = total mass of *reactants*), but the reactants are chemically different from the products, and the products leave at a higher temperature. The *total number of atoms of each element concerned in the combustion remains constant, but the atoms are rearranged into groups having different chemical properties.* This information is expressed in the chemical equation which shows (*i*) the reactants and the products of combustion, (*ii*) the relative quantities of the reactants and products. The two sides of the equation must be **consistent**, each having the same number of atoms of each element involved.
- The oxygen supplied for combustion is *usually* provided by *atmospheric air*, and it is necessary to use accurate and consistent analysis of air by *mass* and by *volume*. It is usual in combustion calculations to take air as 23.3% O_2, 76.7% N_2 by mass, and 21% O_2, 79% N_2 by volume. The small traces of other gases in dry air are included in nitrogen, which is sometimes called '*atmospheric nitrogen*'.

Some important *combustion equations* are given below :

1. **Combustion of hydrogen**

$$2H_2 + O_2 = 2H_2O \qquad ...(11.1)$$

The above equation of combustion of hydrogen tell us that :

(*i*) Hydrogen reacts with water to form steam or water.

(*ii*) Two molecules of hydrogen react with one molecule of oxygen to give two molecules of steam or water,

i.e., $$2 \text{ volumes } H_2 + 1 \text{ volume } O_2 \longrightarrow 2 \text{ volumes } H_2O$$

The H_2O may be liquid or a vapour depending on whether the product has been cooled sufficiently to cause condensation.

The *proportions by mass* are obtained by using *atomic weights* as follows :

$$2H_2 + O_2 \longrightarrow 2H_2O$$

$$\therefore \qquad 2(2 \times 1) + 2 \times 16 \longrightarrow 2(2 \times 1 + 16)$$

i.e.,
$$4 \text{ kg } H_2 + 32 \text{ kg } O_2 \longrightarrow 36 \text{ kg } H_2O$$

or
$$\mathbf{1 \text{ kg } H_2 + 8 \text{ kg } O_2 \longrightarrow 9 \text{ kg } H_2O}$$

$$\left[\begin{array}{c}\text{The same proportions are obtained by writing the equation (11.1) as :} \\ H_2 + \tfrac{1}{2} O_2 \longrightarrow H_2O\text{, and this is sometimes done.}\end{array}\right]$$

It will be noted from equation (11.1) that the total volume of the *reactants* is

$$2 \text{ volumes } H_2 + 1 \text{ volume } O_2 = 3 \text{ volumes.}$$

The total volume of the *product* is only 2 volumes. There is therefore a *volumetric contraction on combustion*.

Since the oxygen is accompanied by nitrogen if air is supplied for the combustion, then this nitrogen should be included in the equation. As nitrogen is *inert* as far as chemical reaction is concerned, it will appear on both sides of the equation.

With one mole of oxygen there are 79/21 moles of nitrogen, hence equation (11.1) becomes,

$$2H_2 + O_2 + \tfrac{79}{21} N_2 \longrightarrow 2H_2O + \tfrac{79}{21} N_2 \qquad \text{...(11.2)}$$

2. **Combustion of carbon**

(*i*) ***Complete combustion of carbon to carbon dioxide***

$$C + O_2 \longrightarrow CO_2 \qquad \text{...(11.3)}$$

and including the nitrogen,

$$C + O_2 + \tfrac{79}{21} N_2 \longrightarrow CO_2 + \tfrac{79}{21} N_2 \qquad \text{...(11.4)}$$

By volume :

$$0 \text{ volume C} + 1 \text{ volume } O_2 + \tfrac{79}{21} \text{ volumes } N_2 \longrightarrow 1 \text{ volume } CO_2 + \tfrac{79}{21} \text{ volumes } N_2 \qquad \text{...[11.4 (a)]}$$

The volume of carbon is written as zero since the volume of solid is negligible in comparison with that of a gas.

By mass :

$$12 \text{ kg C} + (2 \times 16) \text{ kg } O_2 + \tfrac{79}{21} (2 \times 14) \text{ kg } N_2 \longrightarrow (12 + 2 \times 16) \text{ kg } CO_2 + \tfrac{79}{21} (2 \times 14) N_2$$

i.e.,
$$12 \text{ kg C} + 32 \text{ kg } O_2 + 105.3 \text{ kg } N_2 \longrightarrow 44 \text{ kg } CO_2 + 105.3 \text{ kg } N_2$$

or
$$1 \text{ kg C} + \frac{8}{3} \text{ kg } O_2 + \frac{105.3}{12} \text{ kg } N_2 \longrightarrow \frac{11}{3} \text{ kg } CO_2 + \frac{105.3}{12} \text{ kg } N_2. \qquad \text{...[11.4 (b)]}$$

(*ii*) ***The incomplete combustion of carbon.*** The incomplete combustion of carbon occurs when there is an insufficient supply of oxygen to burn the carbon completely to carbon dioxide.

$$2C + O_2 \longrightarrow 2CO \qquad \text{...(11.5)}$$

and including the nitrogen,

$$2C + O_2 + \frac{79}{21} N_2 \longrightarrow 2CO + \frac{79}{21} N_2 \qquad \text{...(11.6)}$$

By mass :

$$(2 \times 12) \text{ kg C} + (2 \times 16) \text{ kg } O_2 + \frac{79}{21} (2 \times 14) \text{ kg } N_2 \longrightarrow 2(12 + 16) \text{ kg CO} + \frac{79}{21} (2 \times 14) \text{ kg } N_2$$

i.e.,
$$24 \text{ kg C} + 32 \text{ kg } O_2 + 105.3 \text{ kg } N_2 \longrightarrow 56 \text{ kg CO} + 105.3 \text{ kg } N_2$$

or
$$1 \text{ kg C} + \frac{4}{3} \text{ kg } O_2 + \frac{105.3}{24} \text{ kg } N_2 \longrightarrow \frac{7}{3} \text{ kg CO} + \frac{105.3}{24} \text{ kg } N_2 \qquad \text{...[11.6 (a)]}$$

If a further supply of oxygen is available then the combustion can continue to completion,

$$2CO + O_2 + \tfrac{79}{21}\ N_2 \longrightarrow 2CO_2 + \tfrac{79}{21}\ N_2 \qquad \text{...(11.7)}$$

By mass :

$$56\ \text{kg CO} + 32\ \text{kg } O_2 + \frac{79 \times 28}{21}\ \text{kg } N_2 \longrightarrow 88\ \text{kg } CO_2 + \frac{79 \times 28}{21}\ \text{kg } N_2$$

or

$$1\ \text{kg CO} + \frac{4}{7}\ \text{kg } O_2 + \frac{105.3}{56}\ \text{kg } N_2 \longrightarrow \frac{11}{7}\ \text{kg } CO_2 + \frac{105.3}{56}\ \text{kg } N_2. \qquad \text{...[11.7 (a)]}$$

11.8. THEORETICAL AIR AND EXCESS AIR

The *minimum amount of air* that supplies sufficient oxygen for the complete combustion of all the carbon, hydrogen, and any other elements in the fuel that may oxidise is called the **"theoretical air"**. When complete combustion is achieved with theoretical air, the products contain no oxygen.

In practice, it is found that complete combustion is not likely to be achieved unless the amount of air supplied is somewhat greater than the theoretical amount. Thus 150 per cent theoretical air means that air actually supplied is 1.5 times the theoretical air.

The complete combustion of methane with minimum amount of theoretical air and 150 per cent theoretical air respectively is written as :

$$CH_4 + 2O_2 + 2\left(\frac{79}{21}\right) N_2 \longrightarrow CO_2 + 2H_2O + 2\left(\frac{79}{21}\right) N_2 \qquad \text{...(11.8)}$$

$$CH_4 + 2(1.5)\ O_2 + 2\left(\frac{79}{21}\right)(1.5)\ N_2 \longrightarrow CO_2 + 2H_2O + O_2 + 3\left(\frac{29}{21}\right) N_2 \qquad \text{...(11.9)}$$

(with 150 per cent theoretical air)

The amount of air actually supplied may also be expressed in terms of per cent excess air. The excess air is the amount of air supplied over and above the theoretical air. Thus 150 per cent theoretical air is equivalent to 50 per cent excess air.

Note. For complete combustion of fuel we need air. As per theoretical basis there is a minimum amount of air which is required by the fuel to burn completely, *but always, air in excess is used because whole of air supplied for combustion purposes does not come in contact with the fuel completely and as such portion of fuel may be left unburnt*. But if a large quantity of excess air is used it exercises a *cooling effect on combustion process which however can be avoided by preheating the air. The weight of excess air supplied can be determined from the weight of oxygen which is left unused*. The amount of excess air supplied varies with the type of fuel and the firing conditions. It may approach a value of 100% but modern practice is to use 25% to 50% excess air.

11.9. STOICHIOMETRIC AIR FUEL (A/F) RATIO

Stoichiometric (or chemically correct) *mixture* of air and fuel is one that contains *just sufficient oxygen for complete combustion of the fuel*.

A *weak mixture* is one which has an *excess of air*.

A *rich mixture* is one which has a *deficiency* of air.

The percentage of excess air is given as under :

$$\%\text{age excess air} = \frac{\text{Actural A/F ratio} - \text{Stoichiometric A/F ratio}}{\text{Stoichiometric A/F ratio}} \qquad \text{...(11.10)}$$

(where A and F denote *air* and *fuel* respectively)

The ratios are expressed as follows :

For *gaseous fuels*	*By volume*
For solid and liquid fuels	*By mass*

For *boiler* plant the mixture is usually greater than 20% weak ; for *gas turbines* it can be as much as 300% weak. *Petrol engines* have to meet various conditions of load and speed, and operate over a wide range of mixture strength. The following definition is used :

$$\text{Mixture strength} = \frac{\text{Stoichiometric A/F ratio}}{\text{Actual A/F ratio}} \quad \text{...(11.11)}$$

The working value range between 80% (weak) and 120% (rich).

Note. The reciprocal of the air fuel ratio is called the *fuel-air (F/A) ratio*.

11.10. AIR-FUEL RATIO FROM ANALYSIS OF PRODUCTS

When analysis of combustion products is known air-fuel ratio can be calculated by the following methods :

1. **Fuel composition known**

(*i*) Carbon balance method (*ii*) Hydrogen balance method

(*iii*) Carbon-hydrogen balance method.

2. **Fuel composition unknown**

(*i*) Carbon-hydrogen balance method.

1. **Fuel composition known**

(*i*) *Carbon balance method*. When the fuel composition is known, the carbon balance method is quite accurate if *combustion takes place with excess air and when free (solid) carbon is not present in the products*. It may be noted that the Orsat analysis will not determine the quantity of solid carbon in the products.

(*ii*) *Hydrogen balance method*. This method is used when solid carbon is suspected to be present.

(*iii*) *Carbon-hydrogen balance method*. This method may be employed *when there is some uncertainty about the nitrogen percentage reported by the Orsat analysis.*

2. **Fuel composition unknown**

When the fuel composition is not known the carbon-hydrogen balance method has to be employed.

11.11. HOW TO CONVERT VOLUMETRIC ANALYSIS TO WEIGHT ANALYSIS ?

The conversion of volumetric analysis to weight analysis involves the following steps :

1. Multiply the volume of each constituent by its molecular weight.
2. Add all these weights and then divide each weight by the total of all and express it as percentage.

11.12. HOW TO CONVERT WEIGHT ANALYSIS TO VOLUMETRIC ANALYSIS ?

1. Divide the weight of each constituent by its molecular weight.
2. Add up these volumes and divide each volume by the total of all and express it as a percentage.

11.13. WEIGHT OF CARBON IN FLUE GASES

The weight of carbon contained in one kg of flue or exhaust gas can be calculated from the amounts of CO_2 and CO contained in it.

In eqn. [11.4 (*b*)], it was shown that 1 kg of carbon produces 11/3 kg of CO_2 when completely burnt. Hence 1 kg of CO_2 will contain 3/11 kg of carbon.

In eqn. [11.6 (*a*)], it can be seen that 1 kg of carbon produces 7/3 kg of CO, hence 1 kg CO contains 3/7 kg of carbon.

Therefore, weight of carbon per kg of fuel

$$= \left(\frac{3}{11} CO_2 + \frac{3}{7} CO\right)$$

where CO_2 and CO are the quantities of carbon dioxide and carbon monoxide present in 1 kg of flue or exhaust gas.

11.14. WEIGHT OF FLUE GASES PER KG OF FUEL BURNT

Due to supply of air, the weight of flue gas or exhaust gas is always more than that of fuel burnt. The actual weight of dry flue gases can be obtained by comparing the weight of carbon present in the flue gases with the weight of carbon in the fuel, since there is no loss of carbon during the combustion process. As the analysis of the exhaust gases is volumetric, so this *must first be reduced to weight analysis.*

Also, *total weight of carbon in one kg of flue gas is*

$$= \left(\frac{3}{11} CO_2 + \frac{3}{7} CO\right)$$

∴ The *weight of flue gas/kg of fuel burnt*

$$= \frac{\text{Weight of carbon in one kg of fuel}}{\text{Weight of carbon in one kg of flue gas}}.$$

11.15. ANALYSIS OF EXHAUST AND FLUE GAS

The combustion products are mainly gaseous. When a sample is taken for analysis it is usually cooled down to a temperature which is below the saturation temperature of the steam present. The *steam content* is therefore *not included* in the analysis, which is then quoted as the *analysis of the dry products*. Since the products are gaseous, it is usual to quote the analysis by *volume*. *An analysis which includes the steam in the exhaust is called a wet analysis*.

Practical analysis of combustion products :

The most common means of analysis of the combustion products is the ***Orsat apparatus*** which is described below :

Construction. An Orsat's apparatus consists of the following :

(*i*) A burette

(*ii*) A gas cleaner

(*iii*) Four absorption pipettes 1, 2, 3, 4.

The pipettes are interconnected by means of a manifold fitted with cocks S_1, S_2, S_3 and S_4 and contain different chemicals to absorb carbon dioxide (CO_2), carbonmonoxide (CO) and oxygen (O_2). Each pipette is also fitted with a number of small glass tubes which provide a greater amount of surface. These tubes are wetted by the absorbing agents and are exposed to the gas under analysis. The measuring burrette is surrounded by a *water jacket* to prevent, changes in temperature and density of the gas. The pipettes 1, 2, 3, 4 contain the following chemicals :

Pipette 1 **:** Contains '*KOH*' (caustic soda) *to absorb* CO_2 (carbon dioxide)

Pipette 2 **:** Contains *an alkaline solution of 'pyrogallic acid' to absorb* O_2 (oxygen)

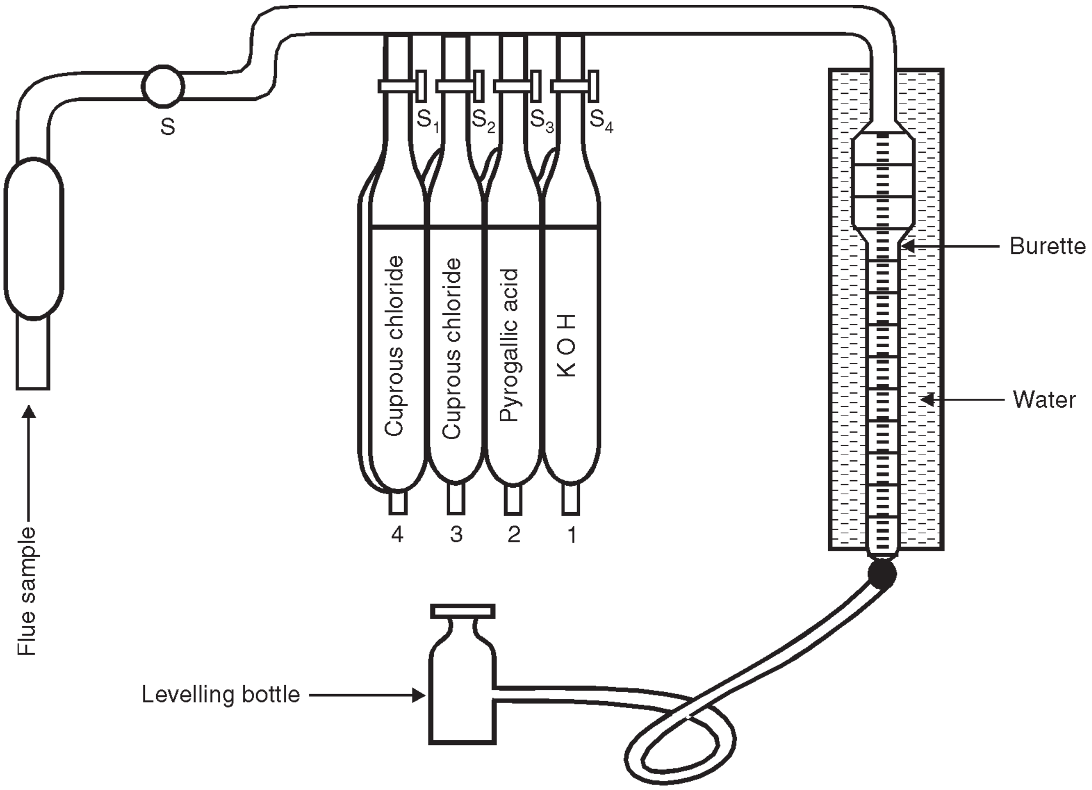


Fig. 11.1. Orsat's apparatus.

Pipette 3, 4 : Contain an *acid solution of 'cuprous chloride' to absorb CO* (carbonmonoxide)

Furthermore the apparatus has a *levelling bottle* and a three way cock to connect the apparatus either to gases or to atmosphere.

Procedure. 100 cm^3 of gas whose analysis is to be made is drawn into the bottle by lowering the levelling bottle. The stop cock S_4 is then opened and the whole flue gas is forced to pipette 1. The gas remains in this pipette for sometime and most of the carbondioxide is absorbed. The levelling bottle is then lowered to allow the chemical to come to its original level. The volume of gas thus absorbed is read on the scale of the measuring bottle. The flue gas is then forced through the pipette 1 for a number of times to ensure that the whole of the CO_2 is absorbed. Further, the remaining flue gas is then forced to the pipette 2 which contains pyrogallic acid to absorb whole of O_2. The reading on the measuring burette will be the sum of volume of CO_2 and O_2. The oxygen content can then be found out by subtraction. Finally, as before, the sample of gas is forced through the pipettes 3 and 4 to absorb carbonmonoxide completely.

The amount of nitrogen in the sample can be determined by subtracting from total volume of gas the sum of CO_2, CO and O_2 contents.

Orsat apparatus *gives an analysis of the dry products of combustion*. Steps may have been taken to remove the steam from the sample by condensing, but as the sample is collected over water it becomes saturated with water. The resulting analysis is nevertheless a true analysis of the dry products. This is because the volume readings are taken at a constant temperature and pressure, and the partial pressure of the vapour is constant. This means that the sum of the partial pressures of the remaining constituents is constant. The vapour then occupies the same proportion of the total volume at each measurement. Hence the vapour does not affect the result of the analysis.

Note. Quantitatively the dry product analysis can be used to calculate A/F ratio. This method of obtaining the A/F ratio is not so reliable as direct measurement of air consumption and fuel consumption of the engine. More caution is required when analysing the products of consumption of a *solid fuel* since some of the products do not appear in the flue gases (*e.g.* ash and unburnt carbon). The residual solid must be analysed as well in order to determine the carbon content, if any. With an engine using *petrol or diesel fuel* the exhaust may include unburnt particles of carbon and this quantity will not appear in the analysis. The exhaust from internal combustion engines may contain also some CH_4 and H_2 due to incomplete combustion. Another piece of equipment called the **Heldane apparatus** measures the CH_4 content as well as CO_2, O_2 and CO.

11.16. INTERNAL ENERGY AND ENTHALPY OF FORMATION

The first law of thermodynamics can be applied to any system. Non-flow and steady-flow energy equations deducted from this law must be applicable to systems undergoing combustion processes.

It has been proved experimentally that the energy released, when a unit mass of a fuel undergoes complete combustion, depends on the *temperature at which the process is carried out*. Thus such quantities quoted are related to temperature. Now it will be shown that if the energy released by a fuel at one temperature is known then it can be calculated at other temperatures.

The process of combustion is defined as taking place from reactants at a state identified by the reference temperature T_0 and another property, either pressure or volume, to products at the same state.

Let U_{R_0} = Internal energy of the *reactants* (which is a mixture of fuel and air) at T_0,

U_{P_0} = Internal energy of *products* of combustion at T_0,

U_{R_1} = Internal energy of *reactants* at temperature T_1,

U_{P_1} = Internal energy of *products* at temperature T_1,

U_{R_2} = Internal energy of *reactants* at temperature T_2,

U_{P_2} = Internal energy of *products* at temperature T_2,

ΔU_0 = Constant volume heat of combustion,

Q = Heat transferred to the surroundings during the process, and

W = Work obtained during combustion process.

Analysis for a non-flow process involving combustion at 'constant volume' :

When the combustion process is carried out at *constant volume* then the non-flow energy equation, $Q = (U_2 - U_1) + W$, can be applied to give

$$Q = (U_{P_0} - U_{R_0}) \qquad \text{...(11.12)}$$

where, $W = 0$ for constant volume combustion,

$$U_1 = U_{R_0}\text{, and}$$

$$U_2 = U_{P_0}.$$

The internal energy change is independent of the path between the two states and depends only on the initial and final values and is given by the quantity Q. This is illustrated in Fig. 11.2 and *property diagram* of Fig. 11.3. The heat so transferred is called the *internal energy of combustion* at T_0 (or *constant volume heat of combustion*), and is denoted by ΔU_0. Thus,

$$\Delta U_0 = U_{P_0} - U_{R_0} \qquad \text{...(11.13)}$$

ΔU_0 is a *negative quantity* since the internal energy of the reactants includes the potential chemical energy and heat is transferred *from* the system.

It may be noted that in case of real constant volume combustion processes the initial and final temperatures will not be same as T_0 (reference temperature). The change in internal energy, for analytical purposes, between reactants at state 1 to products at state 2 can be considered in the following *three steps* (stages) :

(*i*) The change for the reactants from state 1 to T_0.

(*ii*) The constant volume combustion process from reactants to products at T_0.

(*iii*) The change for the products from T_0 to state 2.

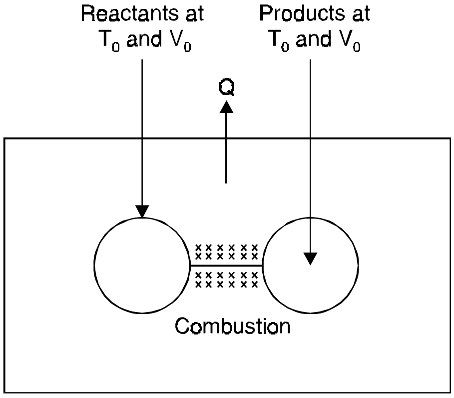


Fig. 11.2

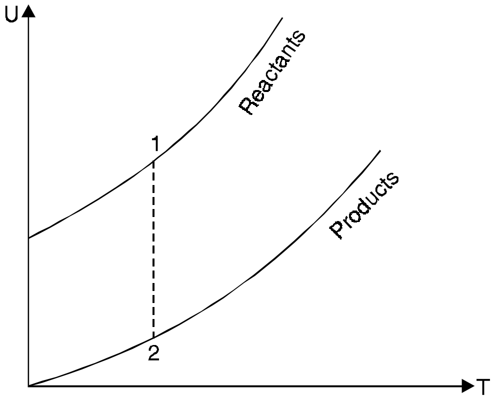


Fig. 11.3

The entire process can be thought of as taking place in piston-cylinder device as shown in Fig. 11.4.

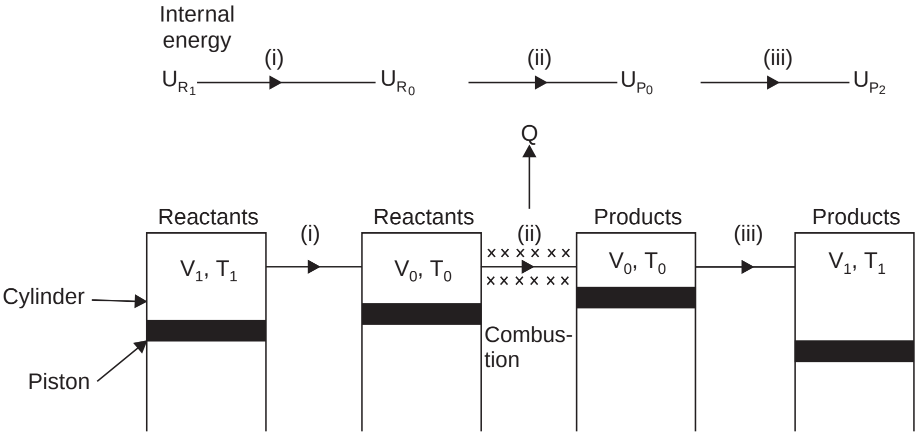


Fig. 11.4

Thus $U_2 - U_1$, the change in internal energy between states 1 and 2, can be written as $(U_{P_2} - U_{R_1})$ to show the chemical changes involved and this can be further expanded for analytical purposes as follows :

$$U_{P_2} - U_{R_1} = (U_{P_2} - U_{P_0}) + (U_{P_0} - U_{R_0}) + (U_{R_0} - U_{R_1})$$

i.e.,

$$U_{P_2} - U_{R_1} = \underbrace{(U_{P_2} - U_{P_0})}_{\substack{\text{Products}\\(iii)}} + \underbrace{\Delta U_0}_{(ii)} + \underbrace{(U_{R_0} - U_{R_1})}_{\substack{\text{Reactants}\\(i)}} \quad ...(11.14)$$

The values of $(U_{R_0} - U_{R_1})$ and $(U_{P_2} - U_{P_0})$ can be calculated from the following relations :

$$U_{R_0} - U_{R_1} = \sum_R n_i(u_{i_0} - u_{i_1}) \quad ...(11.15)$$

where, u_i = Tabulated value of the internal energy for any constituent at the required temperature T_0 or T_1 in heat unit per mole

n_i = Number of moles of the constituent, and

$\sum_R$ = Summation for all the constituents of the reactants denoted by i.

If *mass* base is used for tabulated values or calculation, then

$$U_{R_0} - U_{R_1} = \sum_R m_i(u_{i_0} - u_{i_1}) \quad ...(11.16)$$

where, u_i = Internal energy per unit mass.

The above expression in terms of the *specific heats* (average values for the required temperature range) may be written as

$$U_{R_0} - U_{R_1} = \sum_R m_i c_{vi}(T_0 - T_1) = (T_0 - T_1)\sum_R m_i c_{vi} \quad ...(11.17)$$

For *products*, similar expressions may be written as :

$$U_{P_2} - U_{P_0} = \sum_P n_i(u_{i_2} - u_{i_0}) \quad \text{... on mole basis}$$

$$U_{P_2} - U_{P_0} = \sum_P m_i(u_{i_2} - u_{i_0}) \quad \text{... on mass basis}$$

$$U_{P_2} - U_{P_0} = \sum_P m_i c_{vi}(T_2 - T_0)$$

$$= (T_2 - T_0)\sum_P m_i c_{vi} \quad \text{... in terms of mean specific heats}$$

It may be noted that $n_i C_{vi} = m_i c_{vi}$

Analysis for a steady flow or 'constant pressure' combustion process :

In such an analysis the *changes in enthalpy (H)* are important. An analysis carried out as above will give the following expressions :

$$H_{P_2} - H_{R_1} = \underbrace{(H_{P_2} - H_{P_0})}_{\text{Products}} + \Delta H_0 + \underbrace{(H_{R_0} - H_{R_1})}_{\text{Reactants}} \quad ...(11.18)$$

where, $\Delta H_0 = H_{P_0} - H_{R_0}$, and is always *negative* ...(11.19)

[ΔH_0 = *Enthalpy of combustion* at T_0 or the *constant pressure heat of combustion* at T_0]

Expressions for change of enthalpy of reactants and products :

Reactants :

$$H_{R_0} - H_{R_1} = \sum_R n_i(h_{i_0} - h_{i_1}) \quad \text{... on mole basis} \quad ...(11.20)$$

$$H_{R_0} - H_{R_1} = \sum_R m_i(h_{i_0} - h_{i_1}) \quad \text{... on mass basis} \quad \text{...(11.21)}$$

$$H_{R_0} - H_{R_1} = \sum_R m_i c_{pi}(T_0 - T_1)$$

$$= (T_0 - T_1) \sum_R m_i c_{pi} \quad \text{... in terms of } \textit{mean specific heats}$$

Products : $$H_{P_2} - H_{P_0} = \sum_P n_i(h_{i_2} - h_{i_0}) \quad \text{... on mole basis} \quad \text{...(11.22)}$$

$$H_{P_2} - H_{P_0} = \sum_P m_i(h_{i_2} - h_{i_0}) \quad \text{... on mass basis} \quad \text{...(11.23)}$$

$$H_{P_2} - H_{P_0} = \sum_P m_i c_{pi}(T_2 - T_0)$$

$$= (T_2 - T_0) \sum_P m_i c_{pi} \quad \text{... in terms of } \textit{mean specific heats} \quad \text{...[11.23 (a)]}$$

It may be noted that $n_i C_{pi} = m_i c_{pi}$

From the definition of the enthalpy of a perfect gas

$$H = U + pV = U + nR_0T$$

So if we are concerned only with gaseous mixtures in the reaction then for products and reactants

$$H_{P_0} = U_{P_0} + n_P R_0 T_0$$

and

$$H_{R_0} = U_{R_0} + n_R R_0 T_0$$

where n_P and n_R are the moles of products and reactants respectively and the temperature is the reference temperature T_0.

Thus, using eqns. (11.13) and (11.19), we have

$$\Delta H_0 = \Delta U_0 + (n_P - n_R)R_0T_0 \quad \text{...(11.24)}$$

If there is no change in the number of moles during the reaction or if the reference temperature is absolute zero, then ΔH_0 *and* ΔU_0 *will be equal.*

11.17. ENTHALPY OF FORMATION (ΔH_f)

A combustion reaction is a particular kind of chemical reaction in which products are formed from reactants with the release or absorption of energy as heat is transferred to as from the surroundings. In some substances like hydrocarbon fuels which are many in number and complex in structure the *heat of reaction* or *combustion* may be calculated on the basis of known values of the enthalpy of formation, ΔH_f of the constituent of the reactants and products at the temperature T_0 (reference temperature). *The enthalpy of formation* (ΔH_f) *is the increase in enthalpy when a compound is formed from its constituent elements in their natural form and in a standard state.* The standard state is 25°C, and 1 atm. pressure, but it must be borne in mind that not all substances can exist in natural form, *e.g.* H_2O cannot be a vapour at 1 atm. and 25°C.

The expression of a particular reaction, for calculation purposes, may be given as :

$$\Delta H_0 = \sum_P n_i \Delta H_{f_i} - \sum_R n_i \Delta H_{f_i} \quad \text{...(11.25)}$$

Typical values of ΔH_f for different substances at 25°C (298 K) in **kJ/mole** are given below :

S. No.	*Substance*	*Formula*	*State*	ΔH_f
1.	Oxygen	O	gas	249143
		O_2	gas	zero
2.	Water	H_2O	liquid	– 285765
			vapour	– 241783
3.	Carbon	C	gas	714852
			diamond	1900
			graphite	zero
4.	Carbon monoxide	CO	gas	– 111508
5.	Carbon dioxide	CO_2	gas	– 393443
6.	Methane	CH_4	gas	– 74855
7.	Methyl alcohol	CH_3OH	vapour	– 240532
8.	Ethyl alcohol	C_2H_5OH	vapour	– 281102
9.	Ethane	C_2H_6	gas	– 83870
10.	Ethene	C_2H_4	gas	51780
11.	Propane	C_3H_8	gas	– 102900
12.	Butane	C_4H_{10}	gas	– 125000
13.	Octane	C_8H_{18}	liquid	– 247600

11.18. CALORIFIC OR HEATING VALUES OF FUELS

The "calorific value or heating value" of the fuel is defined as the energy liberated by the complete oxidation of a unit mass or volume of a fuel. It is expressed in kJ/kg for solid and liquid fuels and kJ/m^3 for gases.

If a fuel contains hydrogen water will be formed as one of the products of combustion. If this *water is condensed, a large amount of heat will be released* than if the water exists in the vapour phase. For this reason *two heating values* are defined ; the *higher or gross heating value* and the *lower* or *net heating value*.

The *higher heating value*, HHV, is obtained when the *water formed by combustion is completely condensed*.

The *lower heating value*, LHV, is obtained when the water formed by combustion exists *completely in the vapour phase*.

Thus :

$$(\text{HHV})_p = (\text{LHV})_p + m\, h_{fg} \quad \text{...(11.26)}$$

$$(\text{HHV})_v = (\text{LHV})_v + m(u_g - u_f) \quad \text{...(11.27)}$$

where m = Mass of water formed by combustion,
h_{fg} = Enthalpy of vaporisation of water, kJ/kg,
u_g = Specific internal energy of vapour, kJ/kg, and
u_f = Specific internal energy of liquid, kJ/kg.

In almost all practical cases, the water vapour in the products is vapour, the lower value is the one *which usually applies*.

11.19. DETERMINATION OF CALORIFIC OR HEATING VALUES

The calorific value of fuels can be determined either from chemical analysis or in the laboratory.

11.19.1. Solid and Liquid Fuels

Dulong's formula. Dulong suggested a formula for the calculation of the calorific value of the solid or liquid fuels from their chemical composition which is as given below.

Gross calorific value

or $$\text{H.H.V.} = \frac{1}{100}\left[33800\,\text{C} + 144000\left(\text{H} - \frac{0}{8}\right) + 9270\,\text{S}\right] \text{ kJ/kg} \qquad ...(11.28)$$

where C, H, O and S are carbon, hydrogen, oxygen and sulphur in percentages respectively in 100 kg of fuel. In the above formula the oxygen is assumed to be in combination with hydrogen and only extra surplus hydrogen supplies the necessary heat.

Laboratory method (Bomb calorimeter)

The calorific value of solid and liquid fuels is determined in the laboratory by '*Bomb calorimeter*'. It is so named because its shape resembles that of a bomb. Fig. 11.5 shows the schematic sketch of a bomb calorimeter.

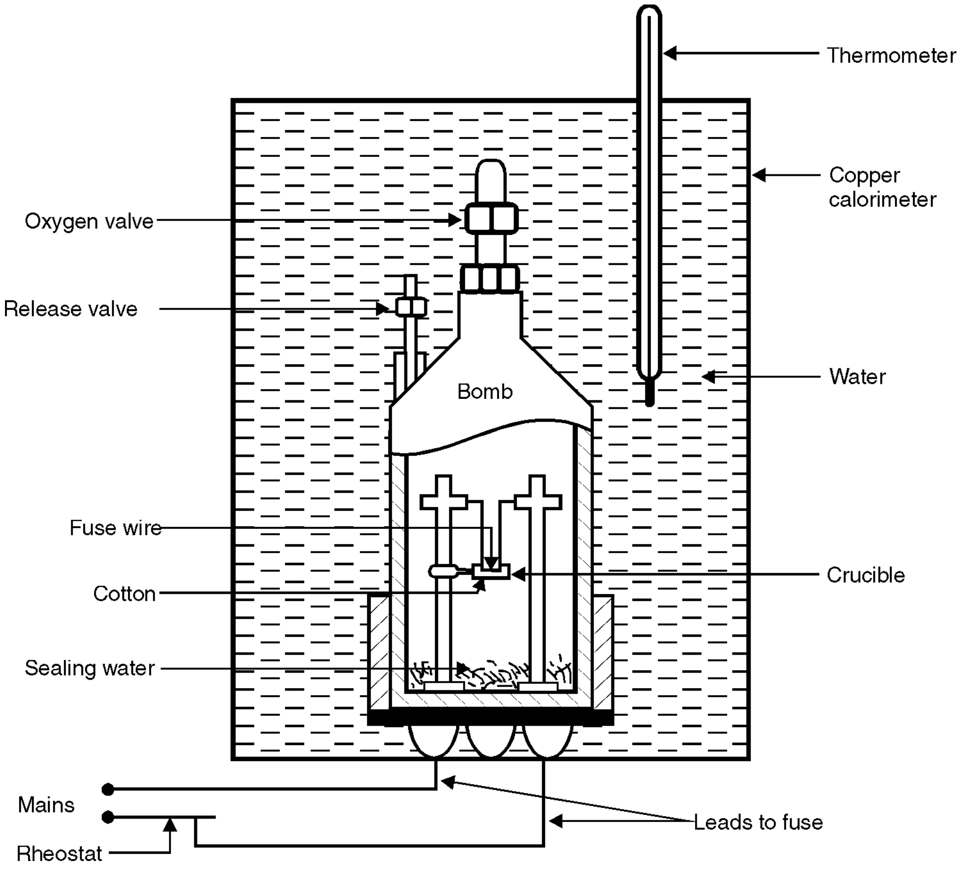


Fig. 11.5. Bomb calorimeter.

The *calorimeter* is made of austenitic steel which provides considerable resistance to corrosion and enables it to withstand high pressure. In the calorimeter is a *strong cylindrical bomb* in which combustion occurs. The bomb has *two valves* at the top. One supplies oxygen to the bomb and other releases the exhaust gases. A *crucible* in which a weighted quantity of fuel sample is burnt is arranged between the *two electrodes* as shown in Fig. 11.5. The calorimeter is fitted with *water jacket* which surrounds the bomb. To reduce the losses due to radiation, calorimeter is further provided with a jacket of water and air. A stirrer for keeping the temperature of water uniform and a thermometer to measure the temperature up to an accuracy of 0.001°C are fitted through the lid of the calorimeter.

Procedure. To start with, about 1 gm of fuel sample is accurately weighed into the crucible and a fuse wire (whose weight is known) is stretched between the electrodes. It should be ensured that *wire is in close contact with the fuel.* To absorb the combustion products of sulphur and nitrogen 2 ml of water is poured in the bomb. Bomb is then supplied with pure oxygen through the valve to an amount of 25 atmosphere. The bomb is then placed in the weighed quantity of water, in the calorimeter. The stirring is started after making necessary electrical connections, and when the thermometer indicates a steady temperature fuel is fired and temperature readings are recorded after 1/2 minute intervals until maximum temperature is attained. The bomb is then removed ; the pressure slowly released through the exhaust valve and the contents of the bomb are carefully weighed for further analysis.

The heat released by the fuel on combustion is absorbed by the surrounding water and the calorimeter.

From the above data the calorific value of the fuel can be found in the following way :

Let w_f = Weight of fuel sample (kg),
w = Weight of water (kg),
C = Calorific value (higher) of the fuel (kJ/kg),
w_e = Water equivalent of calorimeter (kg),
t_1 = Initial temperature of water and calorimeter,
t_2 = Final temperature of water and calorimeter,
t_c = Radiation corrections, and
c = Specific heat of water.

Heat released by the fuel sample = $w_f \times C$

Heat received by water and calorimeter

$$= (w_w + w_e) \times c \times [(t_2 - t_1) + t_c].$$

Heat lost = Heat gained

$$\therefore \quad w_f \times C = (w + w_e) \times c \times [(t_2 - t_1) + t_c]$$

i.e.,
$$C = \frac{(w + w_e) \times c \times [(t_2 - t_1) + t_c]}{w_f} \quad \text{...(11.29)}$$

[Value of c is 4.18 in SI units and unity in MKS units.]

Note 1. Corrections pertain to the heat of oxidation of fuse wire, heat liberated as a result of formation of sulphuric and nitric acids in the bomb itself.

2. It should be noted that *bomb calorimeter measures the higher or gross calorific value because the fuel sample is burnt at a constant volume in the bomb.* Further the bomb calorimeter will measure the H.C.V. directly if the bomb contains adequate amount of water before firing to saturate the oxygen. Any water formed from combustion of hydrogen will, therefore, be condensed.

The procedure of determining calorific values of *liquid fuels* is similar to that described above. However, if the liquid fuel sample is volatile, it is weighed in a glass bulb and broken in a tray just before the bomb is closed. In this way the loss of volatile constituents of fuels during weighing operation is prevented.

Example 11.1. *The following particulars refer to an experimental determination of the calorific value of a sample of coal containing 88% C and 4.2%* H_2*. Weight of coal = 0.848 gm,* *weight of fuse wire 0.027 gm, of calorific value 6700 J/gm, weight of water in the calorimeter = 1950 gm, water equivalent of calorimeter = 380 gm, observed temperature rise = 3.06°C, cooling correction = + 0.017°C.*

Find the higher and lower calorific values of the coal.

Solution. Percentage of *carbon* in coal $= 88\%$

Percentage of *hydrogen* in coal $= 4.2\%$

Weight of coal, $w_f = 0.848$ gm

Weight of fuse wire, $w_{fw} = 0.027$ gm

Weight of water in the calorimeter, $w = 1950$ gm

Water equivalent of calorimeter, $w_e = 380$ gm

Observed temperature rise $(t_2 - t_1) = 3.06°\text{C}$

Cooling correction, $t_c = + 0.017°\text{C}$

$\therefore$ Corrected temperature rise $= (t_2 - t_1) + t_c$

$= 3.06 + 0.017 = 3.077°\text{C}$

Calorific value of fuse wire $= 6700$ J/gm

Heat received by water $= (w + w_e) \times 4.18 \times [(t_2 - t_1) + t_c]$

$= (1950 + 380) \times 4.18 \times 3.077 = 29968$ J

Heat given out by fuse wire $= w_{fw} \times$ calorific value $= 0.027 \times 6700 = 180.9$ J

Heat produced due to combustion of fuel $= 29968 - 180.9 = 29787$ J

$\therefore$ *Higher calorific value of fuel, H.C.V.* $= \dfrac{29787}{0.848} = 35126$ J/gm $=$ **35126 kJ/kg. (Ans.)**

Steam produced per kg of coal $= 9 \times 0.042 = 0.378$ kg

Lower calorific value of coal, L.C.V. $=$ H.C.V. $- 2465 \times 0.378 = 35126 - 931.7$

= 34194.3 kJ/kg. (Ans.)

11.19.2. Gaseous Fuels

The calorific value of gaseous fuels can be determined by ***Junker's gas calorimeter.***

Fig. 11.6 illustrates *Junker's gas calorimeter*. Its principle is some what similar to Bomb calorimeter ; in respect that heat evolved by burning the gas is taken away by the water. In its simplest construction it consists of a combustion chamber in which the gas is burnt (in a gas burner). A water jacket through which a set of tubes called flues pass surrounds this chamber. Thermometers are incorporated at different places (as shown in Fig. 11.6) to measure the temperatures.

Procedure. A metered quantity of gas whose calorific value is to be determined is supplied to the gas burner via a gas meter which records its volume and a gas pressure regulator which measures the pressure of the gas by means of a manometer. When the gas burns the hot products of combustion travel upwards in the chamber and then downwards through the flues and finally escape to the atmosphere through the outlet. The temperature of the escaping gas is recorded by the thermometer fitted at the exit and this temperature should be as close to room temperature as possible so that entire heat of combustion is absorbed by water. The cold water enters the calorimeter near the bottom and leaves near the top. Water which is formed by condensation of steam is collected in a pot.

The quantity of gas used during the experiment is accurately measured by the meter and temperature of ingoing and outgoing water are indicated by the thermometers. From the above data the calorific value of the gas can be calculated.

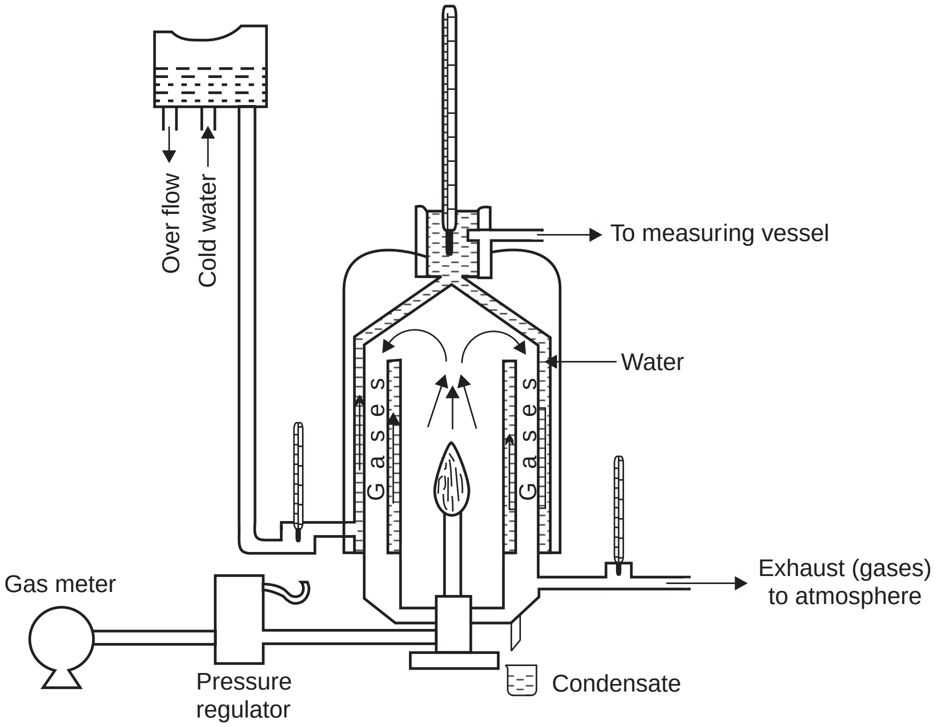


Fig. 11.6. Junker's gas calorimeter.

Note 1. The volume of gas used during the experiment should be converted to volume at S.T.P. (standard temperature and pressure *i.e.*, 15°C, 760 mm respectively).

2. The gross calorific value is obtained by dividing the heat given out by corrected volume of gas. The net or lower calorific value of the gas is obtained by subtracting from total heat the heat associated with condensed water (which is obtained by multiplying the weighed condensate by latent heat of vapourisation of water.).

Example 11.2. *Following results were obtained when a sample of gas was tested by Junker's gas calorimeter :*

Gas burnt in the calorimeter = 0.08 m³, Pressure of gas supply = 5.2 cm of water, Barometer = 75.5 cm of Hg. Temperature of gas = 13°C, Weight of water heated by gas = 28 kg, Temperature of water at inlet = 10°C, Temperature of water at outlet = 23.5°C, Steam condensed = 0.06 kg.

Determine the higher and lower calorific values per m³ of the gas at a temperature of 15°C and barometric pressure of 76 cm of Hg.

Solution. The volume of the gas is measured at a temperature of 13°C and pressure of 5.2 cm of water. Let us reduce this volume to S.T.P. by using the general gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$p_1 = 75.5 + \left(\frac{5.2}{13.6}\right) = 75.882 \text{ cm of Hg} \; ; \; T_1 = 273 + 13 = 286 \text{ K}$$

$$V_1 = 0.08 \text{ m}^3 \; ; \; p_2 = 76 \text{ cm of Hg}$$

$$V_2 = ? \; ; \; T_2 = 273 + 15 = 288 \text{ K}$$

$$\therefore \quad \frac{75.882 \times 0.08}{286} = \frac{76 \times V_2}{288}$$

i.e., $$V_2 = \frac{75.882 \times 0.08 \times 288}{286 \times 76} = 0.0804 \text{ m}^3$$

Heat received by water $= 28 \times 4.18 \times (23.5 - 10) = 1580$ kJ

Higher calorific value of fuel $= \frac{1580}{0.08}$ = **19750 kJ/m^3. (Ans.)**

Amount of water vapour formed (*i.e.*, steam condensed) *per m^3 of gas burnt* $= \frac{0.06}{0.08} = 0.75$ kg

Lower calorific value, L.C.V. = H.C.V. – 2465 × 0.75

= 19750 – 1848.7 = **17901.3 kJ/kg. (Ans.)**

11.20. ADIABATIC FLAME TEMPERATURE

In a given combustion process, that takes place *adiabatically* and with no work or changes in kinetic or potential energy involved, *the temperature of the products is referred to as the 'adiabatic flame temperature'*. With the assumptions of no work and no changes in kinetic or potential energy, this is the *maximum temperature* that can be achieved for the given reactants because any heat transfer from the reacting substances and any incomplete combustion would tend to lower the temperature of the products.

The following points are worthnoting :

(*i*) *The maximum temperature achieved through adiabatic complete combustion varies with the type of reaction and per cent of theoretical air supplied.*

An *increase in the air-fuel* ratio will effect a *decrease in the maximum temperature*.

(*ii*) For a given fuel and given pressure and temperature of the reactants, the *maximum adiabatic flame temperature that can be achieved is with a 'stoichiometric' mixture.*

(*iii*) The adiabatic flame temperature can be *controlled* by the *amount of excess air* that is used. This is important, for example, in *gas turbines*, where the maximum permissible temperature is determined by *metallurgical considerations* in the turbine, and *close control of the temperature of the products is essential*.

11.21. CHEMICAL EQUILIBRIUM

The calculation of the *adiabatic flame temperature* is based, in part, on the assumption that the reaction goes to *completion*. Owing to *dissociation*, complete conversion of the reactants to the products is not accomplished. As a consequence of the failure to achieve complete conversion of the reactants, the maximum reaction temperature cannot attain the level of the theoretical adiabatic flame temperature.

The combination of CO and O_2 produces CO_2 together with a release of energy. In an adiabatic system no heat is transferred to the surroundings, hence the temperature of the mixture of the products and reacting substances rises rapidly. As the mixture temperature increases to higher levels the rate of dissociation of the CO_2 becomes increasingly more pronounced. Since the dissociation of CO_2 requires absorption of energy, *a condition is reached where the rate of evolution and the rate of absorption of energy are in balance. At this point no further increase in temperature can be observed* and the reaction is in *chemical equilibrium*. For this condition

$$C + O_2 \rightleftharpoons CO_2$$

At each temperature of the equilibrium mixture the substances participating in the reaction exist in unique proportions. For the combustion of CO the right-hand side of the equation

$$CO + \tfrac{1}{2} O_2 = (1 - x)\, CO_2 + x\, CO + \tfrac{x}{2} O_2 \qquad \text{...(11.30)}$$

represents the distribution of the equilibrium products resulting from a reaction between CO and O_2. In this equation x denotes the fraction of dissociated CO_2. At low temperatures the fraction $(1 - x)$ approaches unity while at high temperatures $(1 - x)$ shows a substantial reduction in magnitude.

For the combustion of H_2 with O_2

$$H_2 + \tfrac{1}{2} O_2 \rightleftharpoons H_2O$$

and
$$H_2 + \tfrac{1}{2} O_2 = (1 - x)\, H_2O + x\, H_2 + \tfrac{x}{2} O_2 \qquad ...(11.31)$$

It is essential to distinguish between the effects of dissociation and the losses resulting from incomplete combustion of fuel. Incomplete combustion, which may be attributed to a number of factors, results in a discharge from the system of combustible substances. *Dissociation*, on the other hand, is of *transient nature*. Usually any appreciable degree of dissociation extends over a very short time interval at the highest level of temperature attained in the reaction. The gaseous products are likely to be discharged from the system at a temperature that is indicative of a low degree of dissociation. *For example, dissociation does not influence the heating value determined in a fuel calorimeter*. Although the maximum temperature attained in the calorimeter is limited by chemical equilibrium, the combustion process moves to completion with the decrease in the temperature of the products. The reduction in temperature is a result of heat transfer to the jacket water. Dissociation of the products is negligible at room temperature, which is essentially the calorimeter reaction temperature.

The temperature of the products discharged from the combustion chamber of the gas turbine power plant is limited to approximately 870°C by introduction of a *large quantity of excess air*. Absorption of energy in the water walls of a boiler furnace limits the outlet gas temperature to approximately 1100°C. The quantity of dissociated products at temperatures ranging upward to 1100°C is not appreciable. In the cylinder of I.C. engine, considerably higher maximum temperatures—that is, in excess of 1100°C are attained, hence in the analysis of this thermal power system consideration must be given to the effects of dissociation. Of particular significance is the effect of reduced maximum temperature on the system availability. As a result of heat transfer and work performed by the gaseous medium the products are discharged from the system at a temperature below the level at which an appreciable degree of dissociation is observed.

The proportions of the dissociated products in chemical equilibrium at temperature T are established from the *equilibrium constant*. The evaluation of the equilibrium constant is achieved in accordance with the analysis presented by Van't Hoff.

11.22. ACTUAL COMBUSTION ANALYSIS

In evaluating the performance of an actual combustion process a number of different parameters can be defined depending on the nature of the process and the system considered. The combustion efficiency in a gas turbine for instance can be defined as

$$\eta_{\text{combustion}} = \frac{(F/A)_{\text{ideal}}}{(F/A)_{\text{actual}}} \qquad ...(11.32)$$

where $(F/A)_{\text{ideal}}$ = Fuel-air ratio required for adiabatic and complete combustion and in which the products would attain the adiabatic flame temperature.

In case of a *steam generator (boiler)*

$$\eta_{\text{steam generator}} = \frac{\text{Heat transferred to steam / kg fuel}}{\text{Higher heating value of the fuel}} \qquad ...(11.33)$$

In case of an *internal combustion engine,*

$$\eta_{\text{thermal}} = \frac{W_{\text{actual}}}{\text{Heating value}} \qquad ...(11.34)$$

Example 11.3. *A coal sample gave the following analysis by weight, Carbon 85 per cent, Hydrogen 6 per cent, Oxygen 6 per cent, the remainder being incombustible. Determine minimum weight of air required per kg of coal for chemically correct composition.*

Solution.

Element, wt. (kg)	*O_2 required (kg)*
$C = 0.85$	$0.85 \times \frac{8}{3} = 2.27$
$H_2 = 0.06$	$0.06 \times 8 = 0.48$
$O_2 = 0.06$	— — — — — —
	Total $O_2 = 2.75$

Weight of O_2 to be supplied = Wt. of O_2 needed – Wt. of O_2 already present in fuel

= 2.75 – 0.06 = 2.69 kg

Weight of air needed $= 2.69 \times \frac{100}{23}$ = **11.70 kg. (Ans.)**

☞**Example 11.4.** *The percentage composition of sample of liquid fuel by weight is, C = 84.8 per cent, and H_2 = 15.2 per cent. Calculate (i) the weight of air needed for the combustion of 1 kg of fuel ; (ii) the volumetric composition of the products of combustion if 15 per cent excess air is supplied.*

Solution.

Element, wt. (kg)	*O_2 used (kg)*	*Dry products (kg)*
$C = 0.848$	$0.848 \times \frac{8}{3} = 2.261$	$\frac{0.848 \times 11}{3} = 3.109\ (CO_2)$
$H_2 = 0.152$	$0.152 \times 8 = 1.216$	
	Total $O_2 = 3.477$	

(*i*) **Minimum weight of air needed for combustion**

$= \frac{3.477 \times 100}{23}$ = **15.11 kg. (Ans.)**

Excess air supplied $= \frac{15.11 \times 15}{100} = 2.266$ kg

Wt. of oxygen in excess air $= \frac{2.266 \times 23}{100} = 0.521$ kg

Total air supplied for combustion = Minimum air + Excess air

= 15.11 + 2.266 = 17.376 kg

∴ Wt. of nitrogen (N_2) in flue gases $= \frac{17.376 \times 77}{100} = 13.38$ kg.

(*ii*) To get **volumetric composition of the product of combustion** let us use tabular method.

Name of gas	*Weight (x)*	*Molecular weight (y)*	*Proportional volume (z)* $= \frac{(x)}{(y)}$	*Percentage volume* $= \frac{(z)}{\Sigma(z)} \times 100$
CO_2	3.109	44	0.0707	12.51 per cent. **(Ans.)**
O_2	0.521	32	0.0163	2.89 per cent. **(Ans.)**
N_2	13.38	28	0.4780	84.60 per cent. **(Ans.)**
			$\Sigma z = 0.5650$	

☞**Example 11.5.** *Percentage volumetric analysis of a sample of flue gases of a coal fired boiler gave* CO_2 *= 10.4 ; CO = 0.2 ;* O_2 *= 7.8 and* N_2 *= 81.6 (by difference). Gravemetric percentage analysis of coal was C = 78,* H_2 *= 6,* O_2 *= 3 and incombustible = 13. Estimate :*

(*i*) *Weight of dry flue gases per kg of fuel.*

(*ii*) *Weight of excess air per kg of fuel.*

Solution.

Element, wt. (kg)	O_2 *reqd. (kg)*	*Dry products (kg)*
C = 0.78	$0.78 \times \frac{8}{3} = 2.08$	$0.78 \times \frac{11}{3} = 2.86\ (CO_2)$
$H_2 = 0.06$	$0.06 \times 8 = 0.48$	
$O_2 = 0.03$		
	Total $O_2 = 2.56$	

$$\text{Minimum wt. of air needed for combustion} = (2.56 - .03) \times \frac{100}{23} = 11 \text{ kg.}$$

(*i*) **Weight of dry flue gases per kg of fuel :**

To determine the wt. of flue gases per kg of fuel let us use tabular method to convert volumetric analysis to analysis by weight.

Name of gas	*Volume per* m^3 *of flue gas (x)*	*Molecular weight (y)*	*Relative volume* $z = x \times y$	*Weight per kg of flue gas* $= \frac{z}{\Sigma z}$
CO_2	0.104	44	4.576	0.1525
CO	0.002	28	0.056	0.0019
N_2	0.816	28	22.848	0.7616
O_2	0.078	32	2.496	0.0832
			$\Sigma z = 29.976$ (say 30)	

Amount of carbon present per kg of gases

= Amount of carbon in 0.1525 kg of CO_2 + Amount of carbon present in 0.0019 kg of CO

$$= \frac{3}{11} \times 0.1525 + \frac{3}{7} \times 0.0019 = 0.0416 + 0.0008 = 0.0424 \text{ kg.}$$

Also carbon in the fuel = 0.78 kg.

∴ *Weight of dry flue gas per kg of fuel*

$$= \frac{\text{Weight of carbon in 1 kg of fuel}}{\text{Weight of carbon in 1 kg of flue gas}} = \frac{0.78}{0.0424} = \mathbf{18.4\ kg.} \quad \textbf{(Ans.)}$$

(*ii*) **Weight of excess air per kg of fuel :**

Weight of excess oxygen per kg of flue gas $= 0.0832 - \frac{4}{7} \times .0019$

$$\left[\begin{array}{c} 2CO + O_2 = 2CO_2 \\ 56 \quad 32 \quad 88 \\ 1\text{ kg} + \frac{4}{7}\text{ kg} = \frac{11}{7}\text{ kg} \end{array}\right]$$

= 0.0832 – 0.0011 (allowing for unburnt carbon monoxide)

= 0.0821 kg.

Weight of excess O_2 per kg of fuel = 18.4 × 0.0821 = 1.51 kg

∴ *Weight of excess air per kg of fuel* $= \frac{1.51 \times 100}{23}$ = **6.56 kg. (Ans.)**

Example 11.6. *A single cylinder was supplied with a gas having the following percentage volumetric analysis ; $CO = 5$, $CO_2 = 10$, $H_2 = 50$, $CH_4 = 25$, $N_2 = 10$. The percentage volumetric analysis of dry gases was $CO_2 = 8$, $O_2 = 6$ and $N_2 = 86$. Determine the air-fuel ratio by volume.*

Solution. Combustion equations are :

$$2H_2 + O_2 = 2H_2O$$
1 vol + 1/2 vol = 1 vol
$$2CO + O_2 = 2CO_2$$
1 vol + 1/2 vol = 1 vol
$$CH_4 + 2O_2 = CO_2 + 2H_2O$$
1 vol + 2 vol = 1 vol + 2 vol.

Gas	*Vol (m^3)*	*O_2 needed (m^3)*	*Products (m^3)*	
			CO_2	*N_2*
CO	0.05	0.025	0.05	—
CO_2	0.10	—	0.10	—
H_2	0.50	0.25	—	—
CH_4	0.25	0.50	0.25	—
N_2	0.10	—	—	0.10
Total	1.0	0.775	0.4	0.10

Volume of air required = 0.775 × 100/21 = 3.69 m^3

Volume of nitrogen in the air = 3.69 × 79/100 = 2.92 m^3

Dry combustion products of 1 m^3 of gases (V) contain 0.4 m^3 of CO_2 + 0.1 m^3 of N_2 (as given in the table) + 2.92 m^3 of N_2 (from air supplied for complete combustion) = 3.42 m^3.

Excess air supplied $= \dfrac{O_2 \times V}{(21 - O_2)} = \dfrac{6.0 \times 3.42}{(21 - 6)} = \dfrac{20.52}{15} = 1.37 \text{ m}^3$

Total quantity of air supplied $= 3.69 + 1.37 = 5.06 \text{ m}^3$.

Air fuel ratio $= \dfrac{\text{Volume of air}}{\text{Volume of fuel}} = \dfrac{5.06}{1} =$ **5 (say). (Ans.)**

☞**Example 11.7.** *The following is the ultimate analysis of a sample of petrol by weight :*

Carbon = 85 per cent ; Hydrogen = 15 per cent.

Calculate the ratio of air to petrol consumption by weight if the volumetric analysis of the dry exhaust gas is :

CO_2 = 11.5 per cent ; CO = 1.2 per cent ; O_2 = 0.9 per cent ; N_2 = 86 per cent.

Also find percentage excess air.

Solution.

Name of gas	*Volume per m^3 of flue gas* (x)	*Molecular weight* (y)	*Relative weight* $z = x \times y$	*Weight per kg of flue gas* $\dfrac{z}{\Sigma z}$
CO_2	0.115	44	5.06	0.1700
CO	0.012	28	0.336	0.0113
O_2	0.009	32	0.288	0.0096
N_2	0.86	28	24.08	0.8091
			$\Sigma z = 29.76$	

∴ Weight of carbon per kg of flue gas

= Weight of carbon in 0.17 kg of CO_2 + Weight of carbon in 0.0113 kg of CO

$= \dfrac{3}{11} \times 0.17 + \dfrac{3}{7} \times 0.0113 = 0.0512$ kg

∴ Weight of dry flue gas per kg of fuel $= \dfrac{0.85}{0.0512} = 16.6$ kg

Vapour of combustion $= 9 \times 0.15 = 1.35$ kg

Total weight of gas $= 16.6 + 1.35 = 17.95$ kg per kg of fuel

∴ Air supplied $= (17.95 - 1) = 16.95$ kg/kg of fuel

∴ *Ratio of air to petrol* = **16.95 : 1. (Ans.)**

Stoichiometric air $= \left[\left(0.85 \times \dfrac{8}{3}\right) + (0.15 \times 8)\right] \times \dfrac{100}{23}$

= 15.07 kg per kg of fuel

∴ Excess air $= 16.95 - 15.07 = 1.88$ kg

∴ *Percentage excess air* $= \dfrac{1.88}{15.07} \times 100 =$ **12.47%. (Ans.)**

☞**Example 11.8.** *A sample of fuel has the following percentage composition : Carbon = 86 per cent ; Hydrogen = 8 per cent ; Sulphur = 3 per cent ; Oxygen = 2 per cent ; Ash = 1 per cent.*

For an air-fuel ratio of 12 : 1, calculate :

(*i*) *Mixture strength as a percentage rich or weak.*

(*ii*) *Volumetric analysis of the dry products of combustion.*

Solution.

Element, wt. (kg)	*O_2 reqd. (kg)*
C = 0.86	$0.86 \times \frac{8}{3} = 2.29$
$H_2 = 0.08$	$0.08 \times 8 = 0.64$
S = 0.03	$0.03 \times \frac{1}{1} = 0.03$
$O_2 = 0.02$	
	Total $O_2 = 2.96$

Weight of oxygen to be supplied per kg of fuel = 2.96 – 0.02 = 294 kg.

Weight of minimum air required for complete combustion = $\frac{2.94 \times 100}{23}$ = 12.78 kg

Hence "*correct*" fuel-air ratio $= \frac{1}{12.78} : 1$

But actual ratio is $\frac{1}{12} : 1$.

(*i*) **Mixture strength** $= \frac{12.78}{12} \times 100 = 106.5\%$

This show that mixture is **6.5% rich. (Ans.)**

Deficient amount of air = 12.78 – 12 = 0.78 kg

Amount of air saved by burning 1 kg of C to CO instead of CO_2

$$= \text{Oxygen saved} \times \frac{100}{23}$$

$$= \left[\frac{8}{3}(CO_2) - \frac{4}{3}(CO)\right] \times \frac{100}{23} = 5.8 \text{ kg}$$

Hence $\frac{0.78}{5.8}$ = 0.134 kg of carbon burns to CO and as such 0.86 – 0.134 = 0.726 kg of carbon burns to CO_2.

∴ CO formed $= 0.134 \times \frac{7}{3} = 0.313$ kg

CO_2 formed $= 0.726 \times \frac{11}{3} = 2.662$ kg

N_2 supplied = 12 × 0.77 = 9.24 kg

SO_2 formed = 0.03 × 2 = 0.06 kg.

(*ii*) **The percentage composition of dry flue gases** is given as below :

Dry products	*Weight (kg)* x	*Molecular weight* y	*Proportional volume* $z = \frac{x}{y}$	*Percentage volume* $\frac{z}{\Sigma z} \times 100$
CO	0.313	28	0.0112	**2.78 per cent (Ans.)**
CO_2	2.662	44	0.0605	**15.03 per cent (Ans.)**
N_2	9.24	28	0.3300	**81.97 per cent (Ans.)**
SO_2	0.06	64	0.0009	**0.22 per cent (Ans.)**
			$\Sigma z = 0.4026$	

Example 11.9. *A fuel ($C_{10}H_{22}$) is burnt using an air-fuel ratio of 13 : 1 by weight. Determine the complete volumetric analysis of the products of combustion, assuming that the whole amount of hydrogen burns to form water vapour and there is neither any free oxygen nor any free carbon. The carbon burns to CO_2 and CO.*

Air contains 77% of nitrogen and 23% of oxygen by weight.

Solution. Combustion equation is :

$$2C_{10}H_{22} + 31O_2 = 20CO_2 + 22H_2O$$

$$2 \times 142 + 31 \times 32 = 20 \times 44 + 22 \times 18$$

or

$$284 + 992 = 880 + 396$$

∴ Air required for complete combustion

$$= \frac{992 \times 100}{284 \times 23} = 15.2 \text{ kg/kg of fuel}$$

Air actually supplied = 13 kg/kg of fuel

∴ Deficiency of air = 15.2 – 13 = 2.2 kg/kg of fuel

Also 1 kg of C requires $\frac{4}{3} \times \frac{100}{23}$ = 5.8 kg of less air to burn to CO instead of CO_2.

Hence $\frac{2.2}{5.8}$ = 0.379 kg C is burnt to CO ;

and $\left(\frac{12 \times 10}{142}\right) - 0.379$ = 0.466 kg of C is burnt to CO_2.

Weight of CO_2 formed $= 0.466 \times \frac{11}{3} = 1.708$ kg

Weight of CO formed $= 0.379 \times \frac{7}{3} = 0.884$ kg

Weight of H_2O formed $= \left(\frac{22}{142}\right) \times 9 = 1.394$ kg

Weight of N_2 from air $= 13 \times 0.77 = 10.01$ kg.

The percentage composition of products of combustion

Products of combustion	*Weight* x	*Molecular weight* y	*Proportional volume* $z = \frac{x}{y}$	*Percentage* $= \frac{z}{\Sigma z} \times 100$
CO_2	1.708	44	0.0388	**7.678 per cent (Ans.)**
CO	0.884	28	0.0316	**6.254 per cent (Ans.)**
H_2O	1.394	18	0.0774	**15.317 per cent (Ans.)**
N_2	10.01	28	0.3575	**70.750 per cent (Ans.)**
			$\Sigma z = 0.5053$	

Example 11.10. *Find an expression for the weight of air supplied per kg of fuel when carbon content of the fuel and the volumetric analysis of the fuel gas is known.*

Show that carbon burnt to CO per kg of fuel is $\frac{(CO)(C)}{CO + CO_2}$, *where CO and* CO_2 *represent percentage monoxide and carbon dioxide in dry flue gas by volume and C represents percentage by weight of carbon that is actually burnt.*

Solution. Let C = Percentage of carbon, by weight, in the fuel burnt,

CO_2 = Percentage by volume of carbon dioxide in the dry flue gas,

CO = Percentage by volume of carbon monoxide in the dry flue gas, and

N_2 = Percentage by volume of nitrogen in the dry flue gas.

Consider 100 m^3 of the gas.

The actual weight of CO_2 $= CO_2 \times a \times b \times 44$ in 100 m^3 of gas

The actual weight of CO $= CO \times a \times b \times 28$ in 100 m^3 of gas

The actual weight of N_2 $= N_2 \times a \times b \times 28$ in 100 m^3 of gas

The molecular weight of CO = 28

The molecular weight of N_2 = 28

Total mass of carbon per 100 m^3 of flue gas

$$= [a \times b \times 44 \times CO_2] \times \frac{12}{44} + [\, a \times b \times 28 \times CO] \times \frac{12}{28}$$

The reaction producing CO_2 and CO are

$$\begin{matrix} C + O_2 = CO_2 \\ 12 \quad 32 \quad 44 \end{matrix}$$

$$\therefore \qquad \text{C per kg of } CO_2 = \frac{12}{44}$$

Also

$$\begin{matrix} 2C + O_2 = 2CO \\ 24 \quad 32 \quad 56 \end{matrix}$$

$$\therefore \qquad \text{C per kg of CO} = \frac{12}{28}$$

All the carbon available in carbon dioxide and carbon monoxide comes from carbon present in the fuel. Also it can be safely assumed that all nitrogen comes from air (*this analysis being not applicable to the fuels containing nitrogen*).

Total carbon in the flue gas $= a \times b \times 12\ (CO_2 + CO)$ per kg of carbon in the flue gas

$$= \frac{a \times b \times 28 \times N_2}{a \times b \times 12(CO + CO_2)} \qquad ...(i)$$

For every kg of fuel supplied for combustion $\frac{C}{100}$ kg of carbon pass out of the flue, whence the weight of N_2 per kg of fuel is

$$\frac{28 N_2}{12(CO + CO_2)} \times \frac{C}{100} \text{ kg} \qquad ...(ii)$$

If all this nitrogen has come from the atmosphere, then air supplied per kg of fuel

$$= \frac{28 \times N_2 \times C \times 100}{12(CO + CO_2) \times 100 \times 77} \qquad ...(iii)$$

(since air contains 77% nitrogen by weight)

∴ *Air per kg of fuel* $= \dfrac{N_2 \times C}{\dfrac{100 \times 77 \times 12}{28 \times 100}(CO + CO_2)} = \boldsymbol{\dfrac{N_2 \times C}{33(CO + CO_2)}}$. **(Ans.)**

It has been shown above that he total carbon of the flue gas is given by $a \times b \times 12\ (CO_2 + CO)$ and that in CO by $a \times b \times 12$ CO, whence the proportion of carbon burnt to CO is given by

$$\frac{a \times b \times 12 \times CO}{a \times b \times 12(CO + CO_2)} = \frac{CO}{CO + CO_2}$$

and if C is the proportion of carbon actually burnt per kg of fuel, then

Carbon burnt to CO per kg of fuel $= \dfrac{(CO)(C)}{CO + CO_2}$. **Proved.**

☞ **Example 11.11.** *The following results were obtained in a trial on a boiler fitted with economiser :*

	CO_2	CO	O_2	N_2
Analysis of gas entering the economiser	*8.3*	*0*	*11.4*	*80.3*
Analysis of gas leaving the economiser	*7.9*	*0*	*11.5*	*80.6*

(*i*) *Determine the air leakage into the economiser if the carbon content of the fuel is 80 per cent.*

(*ii*) *Determine the reduction in temperature of the gas due to air leakage if atmospheric temperature is 20°C and flue gas temperature is 410°C. Ash collected from ash pan is 15 per cent by weight of the fuel fired.*

Take : c_p *for air = 1.005 kJ/kg K and* c_p *for flue gas = 1.05 kJ/kg K.*

Solution. (*i*) Air supplied $= \dfrac{N_2 \times C}{33(CO_2 + CO)}$

Air supplied on the basis of conditions at entry to the economiser

$$= \frac{80.3 \times 80}{33(8.3 + 0)} = 23.45 \text{ kg}$$

Air applied on the basis of conditions at exit

$$= \frac{80.6 \times 80}{33(7.9 + 0)} = 24.73 \text{ kg}$$

∴ *Air leakage* = 24.73 – 23.45 = **1.28 kg of air per kg of fuel. (Ans.)**

For each kg of fuel burnt, the ash collected is 15% *i.e.*, 0.15 kg.

∴ Weight of fuel passing up the chimney = 1 – 0.15 = 0.85 kg

∴ Total weight of products

= Weight of air supplied per kg of fuel
+ Weight of fuel passing through chimney per kg of fuel

= 23.45 + 0.85 = 24.3 kg

Heat in flue gases per kg of coal

= Weight of flue gases × Specific heat × Temperature rise above 0°C

= 24.3 × 1.05 × (410 – 0) = 10461 kJ

Heat in leakage air = Weight of leakage air × Specific heat
× Temperature rise of air above 0°C

= 1.28 × 1.005 × (20 – 0) = 25.73 kJ.

We can still consider, in the mixture, the gas and the air as separate and having their own specific heats, but sharing a common temperature t.

For heat balance :

$$(1.005 \times 1.28 + 24.3 \times 1.05)\, t = 10461 + 25.73$$

$$26.8\, t = 10486.73$$

$$\therefore \quad t = 391.3°\text{C}$$

∴ *Fall in temperature as a result of the air leakage into the economiser*

= 410 – 391.3 = **18.7°C. (Ans.)**

AIR-FUEL RATIO AND ANALYSIS OF PRODUCTS OF COMBUSTION

Example 11.12. *The chemical formula for alcohol is C_2H_6O. Calculate the stoichiometric air/fuel ratio by mass and the percentage composition of the products of combustion per kg of C_2H_6O.* **(GATE 1998)**

Solution. Chemical equation for complete combustion of given fuel can be written as follows :

$$C_2H_6O + 3O_2 \longrightarrow 2CO_2 + 3H_2O$$

$$(1 \times 46) + (3 \times 32) = (2 \times 44) + (3 \times 18)$$

For complete combustion of 1 kg of C_2H_6O, oxygen required

$$= \frac{3 \times 32}{46} = 2.087 \text{ kg of oxygen}$$

$$= 2.087 \times \frac{100}{23} = 9.074 \text{ kg of air}$$

A : F ratio for complete combustion = **9.074 : 1. Ans.**

Also 46 kg of fuel produces products of combustion = 88 + 54 = 142 kg

$$\left[\therefore \quad 1 \text{ kg of fuel produces } \frac{142}{46} = 3.087 \text{ kg of products of combustion } (i.e.,\ CO_2 \text{ and } H_2O)\right]$$

Hence $\mathbf{CO_2}$ produced by fuel = $\frac{88}{46} \times 100 = 1.913$ or **191.3%. (Ans.)**

$\mathbf{H_2O}$ produced by fuel = $\frac{54}{46} \times 100 = 1.174$ or **117.4%. (Ans.)**

Example 11.13. *Calculate the amount of theoretical air required for the combustion of 1 kg of acetylene* (C_2H_2) *to* CO_2 *and* H_2O.

Solution. For combustion of acetylene (C_2H_2) the stoichiometric equation is written as

$$C_2H_2 + x\,O_2 \longrightarrow a\,CO_2 + b\,H_2O \qquad \ldots(i)$$

Balancing the *carbon atoms* on both sides of the combustion eqn. (*i*), we get

$$2C = a\,C \quad i.e., \quad a = 2$$

Now balancing *hydrogen atoms* on both sides, we get

$$2H = 2bH$$
$$b = 1$$

Thus, eqn. (*i*) becomes

$$C_2H_2 + x\,O_2 \longrightarrow 2CO_2 + H_2O$$

Now, balancing *oxygen atoms* in the above equation

$$2x = 2 \times 2 + 1 = 5 \quad i.e., \quad x = 2.5$$

Hence, the final combustion eqn. (*i*) is

$$C_2H_2 + 2.5O_2 \longrightarrow 2CO_2 + H_2O \qquad \ldots(ii)$$

Thus, for combustion of C_2H_2 in air, we get

$$C_2H_2 + 2.5O_2 + 2.5\left(\frac{79}{21}\right)N_2 \longrightarrow 2CO_2 + H_2O + 2.5\left(\frac{79}{21}\right)N_2$$

On a mass basis, this becomes

$$(2 \times 12 + 2 \times 1)\,C_2H_2 + 2.5\,(2 \times 16)\,O_2 + 2.5\left(\frac{79}{21}\right)(2 \times 14)\,N_2$$

$$\longrightarrow 2\,(12 + 2 \times 16)\,CO_2 + (1 \times 2 + 1 \times 16)\,H_2O + 2.5\left(\frac{79}{21}\right)(2 \times 14)\,N_2$$

$$26 \text{ kg } C_2H_2 + 80 \text{ kg } O_2 + 263.3\,N_2 \longrightarrow 88 \text{ kg } CO_2 + 18 \text{ kg } H_2O + 263.3 \text{ kg } N_2$$

or $\quad 1 \text{ kg } C_2H_2 + 3.076 \text{ kg } O_2 + 10.12 \text{ kg } N_2 \longrightarrow 3.38 \text{ kg } CO_2 + 0.69 \text{ kg } H_2O + 10.12 \text{ kg } N_2$

i.e., Amount of air = 3.076 + 10.12 = 13.196 kg of air per kg of C_2H_2

Hence amount of theoretical air required for combustion of 1 kg acetylene = 13.196 kg. (Ans.)

Example 11.14. *Determine the gravimetric analysis of the products of complete combustion of acetylene with 200 per cent stoichiometric air.*

Solution. The stoichiometric air equation (Example 11.3) is written as :

$$C_2H_2 + 2.5\,O_2 + 2.5\left(\frac{79}{21}\right)N_2 \longrightarrow 2CO_2 + H_2O + 2.5\left(\frac{79}{21}\right)N_2$$

If *200 per cent* stoichiometric air is used, the combustion equation for C_2H_2 becomes

$$C_2H_2 + (2)(2.5)\,O_2 + (2)(2.5)\left(\frac{79}{21}\right)N_2 \longrightarrow 2CO_2 + H_2O + (2)(2.5)\left(\frac{79}{21}\right)N_2 + (1)(2.5)\,O_2$$

$$(2 \times 12 + 2 \times 1) \text{ kg } C_2H_2 + (2)(2.5)(2 \times 16) \text{ kg } O_2 + (2)(2.5)\left(\frac{79}{21}\right)(2 \times 14) \text{ kg } N_2$$

$$\longrightarrow 2\,(12 + 2 \times 16)\text{ kg } CO_2 + (2 \times 1 + 16)\text{ kg } H_2O + (2)\,(2.5)\left(\frac{79}{21}\right)(2 \times 14)\text{ kg } N_2 + (1)\,(2.5)\,(2 \times 16)\text{ kg } O_2$$

or $$26\text{ kg } C_2H_2 + 160\text{ kg } O_2 + 526.6\text{ kg } N_2 \longrightarrow 88\text{ kg } CO_2 + 18\text{ kg } H_2O + 526.6\text{ kg } N_2 + 80\text{ kg } O_2$$

or $$1\text{ kg } C_2H_2 + 6.15\text{ kg } O_2 + 20.25\text{ kg } N_2 \longrightarrow 3.38\text{ kg } CO_2 + 0.69\text{ kg } H_2O + 20.25\text{ kg } N_2 + 3.07\text{ kg } O_2$$

Thus for 1 kg of fuel, the products will consist of

$$CO_2 = 3.38\text{ kg}$$
$$H_2O = 0.69\text{ kg}$$
$$O_2 = 3.07\text{ kg}$$
$$N_2 = 20.25\text{ kg}$$

Total mass of products $= 27.39$ kg

∴ Mass fractions are :

$$CO_2 = \frac{3.38}{27.39} = 0.123$$

$$H_2O = \frac{0.69}{27.39} = 0.025$$

$$O_2 = \frac{3.07}{27.39} = 0.112$$

$$N_2 = \frac{20.25}{27.39} = 0.739.$$

Hence the gravimetric analysis of the complete combustion is :

$CO_2 = 12.3\%$, $H_2O = 2.5\%$, $O_2 = 11.2\%$, $N_2 = 73.9\%$. (Ans.)

Example 11.15. *Calculate the theoretical air-fuel ratio for the combustion of octane, C_8H_{18}. The combustion equation is :*

$$C_8H_{18} + 12.5\,O_2 + 12.5\left(\frac{79}{21}\right)N_2 \longrightarrow 8CO_2 + 9H_2O + 12.5\left(\frac{79}{21}\right)N_2.$$

Solution. The air-fuel ratio on a mole basis is

$$A/F = \frac{12.5 + 12.5\left(\frac{79}{21}\right)}{1} = 59.5\text{ mol air/mol fuel}$$

The theoretical air-fuel ratio on a mass basis is found by introducing the molecular weight of the air and fuel

$$A/F = \frac{59.5(28.97)}{(8 \times 12 + 1 \times 18)} = \textbf{15.08 kg air/kg fuel.} \quad \textbf{(Ans.)}$$

Example 11.16. *One kg of octane (C_8H_{18}) is burned with 200% theoretical air. Assuming complete combustion determine :*

(*i*) *Air-fuel ratio*

(*ii*) *Dew point of the products at a total pressure 100 kPa.*

Solution. The equation for the combustion of C_8H_{18} with theoretical air is

$$C_8H_{18} + 12.5\,O_2 + 12.5\left(\frac{79}{21}\right)N_2 \longrightarrow 8CO_2 + 9H_2O + 12.5\left(\frac{79}{21}\right)N_2$$

For *200% theoretical air* the combustion equation would be

$$C_8H_{18} + (2)\ (12.5)\ O_2 + (2)\ (12.5)\ \left(\frac{79}{21}\right) N_2$$

$$\longrightarrow 8CO_2 + 9H_2O + (1)\ (12.5)\ O_2 + (2)\ (12.5)\ \left(\frac{79}{21}\right) N_2$$

Mass of fuel = (1) (8 × 12 + 1 × 18) = 114 kg/mole

Mass of air = $(2)\ (12.5)\ \left(1 + \frac{79}{21}\right)\ 28.97$ = 3448.8 kg/mole of fuel

(*i*) **Air-fuel ratio :**

Air-fuel ratio, $$A/F = \frac{\text{Mass of air}}{\text{Mass of fuel}} = \frac{3448.8}{114} = 30.25$$

i.e., **A/F = 30.25. (Ans.)**

(*ii*) **Dew point of the products, t_{dp} :**

Total number of moles of products

$$= 8 + 9 + 12.5 + (2)\ (12.5)\ \left(\frac{79}{21}\right) = 123.5 \text{ moles/mole fuel}$$

Mole fraction of $H_2O = \frac{9}{123.5} = 0.0728$

Partial pressure of H_2O = 100 × 0.0728 = 7.28 kPa

The saturation temperature corresponding to this pressure is 39.7°C which is also the dew-point temperature.

Hence t_{dp} = 39.7°C. (Ans.)

Note. The water condensed from the products of combustion usually contains some dissolved gases and therefore may be quite *corrosive*. For this reason the products of combustion are often *kept above the dew point until discharged to the atmosphere*.

Example 11.17. *One kg of ethane (C_2H_6) is burned with 90% of theoretical air. Assuming complete combustion of hydrogen in the fuel determine the volumetric analysis of the dry products of combustion.*

Solution. The complete combustion equation for C_2H_6 is written as :

$$C_2H_6 + 3.5O_2 \longrightarrow 2CO_2 + 3H_2O$$

The combustion equation for C_2H_6 for 90% theoretical air is written as :

$$C_2H_6 + (0.9)\ (3.5)\ O_2 + (0.9)\ (3.5)\ \left(\frac{79}{21}\right) N_2 \longrightarrow a\ CO_2 + b\ CO + 3H_2O + (0.9)\ (3.5)\ \left(\frac{79}{21}\right) N_2$$

By balancing *carbon atoms* on both the sides, we get

$$2 = a + b \quad ...(i)$$

By balancing *oxygen atoms* on both the sides, we get

$$(0.9)\ (3.5)\ (2) = 2a + b + 3 \quad ...(ii)$$

Substituting the value of b (= 2 – a) from eqn. (*i*) in eqn. (*ii*), we get

$$(0.9)\ (3.5)\ (2) = 2a + 2 - a + 3$$

$$6.3 = a + 5$$

$$\therefore \quad a = 1.3$$

and $$b = 2 - a = 2 - 1.3 = 0.7$$

Thus the combustion equation becomes :

$$C_2H_6 + (0.9)(3.5)\,O_2 + (0.9)(3.5)\left(\frac{79}{21}\right)N_2$$

$$\longrightarrow 1.3CO_2 + 0.7CO + 3H_2O + (0.9)(3.5)\left(\frac{79}{21}\right)N_2$$

Total number of moles of *dry products* of combustion

$$= 1.3 + 0.7 + (0.9)(3.5)\left(\frac{79}{21}\right)$$

$$= 1.3 + 0.7 + 11.85 = 13.85 \text{ moles/mole of fuel}$$

Volumetric analysis of dry products of combustion is as follows :

$$\mathbf{CO_2} = \frac{1.3}{13.85} \times 100 = \mathbf{9.38\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{CO} = \frac{0.7}{13.85} \times 100 = \mathbf{5.05\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{11.85}{13.85} \times 100 = \mathbf{85.56\%.} \quad \textbf{(Ans.)}$$

☞ **Example 11.18.** *Methane (CH_4) is burned with atmospheric air. The analysis of the products on a 'dry' basis is as follows :*

$$CO_2 = 10.00\%,\ O_2 = 2.37\%,\ CO = 0.53\%,\ N_2 = 87.10\%.$$

(*i*) *Determine the combustion equation ;* (*ii*) *Calculate the air-fuel ratio ;*

(*iii*) *Percent theoretical air.*

Solution. (*i*) **Combustion equation :**

From the analysis of the products, the following equation can be written, keeping in mind that this analysis is on a *dry basis*.

$$x\,CH_4 + y\,O_2 + z\,N_2 \longrightarrow 10.0\,CO_2 + 0.53\,CO + 2.37\,O_2 + a\,H_2O + 87.1\,N_2$$

To determine all the unknown co-efficients let us find balance for each of the elements.

Nitrogen balance **:** $z = 87.1$

Since all the nitrogen comes from the air,

$$\frac{z}{y} = \frac{79}{21}\ ;\ y = \frac{87.1}{(79/21)} = 23.16$$

Carbon balance **:** $x = 10.00 + 0.53 = 10.53$

Hydrogen balance **:** $a = 2x = 2 \times 10.53 = 21.06$

Oxygen balance. All the unknown co-efficients have been solved for, and in this case the oxygen balance provides a check on the accuracy. Thus, y can also be determined by an oxygen balance

$$y = 10.00 + \frac{0.53}{2} + 2.37 + \frac{21.06}{2} = 23.16$$

Substituting these values for x, y, z and a, we have,

$$10.53\,CH_4 + 23.16\,O_2 + 87.1\,N_2 \longrightarrow 10.0\,CO_2 + 0.53\,CO + 2.37\,O_2 + 21.06\,H_2O + 87.1\,N_2$$

Dividing both sides by 10.53, we get the **combustion equation** per mole of fuel,

$$\mathbf{CH_4 + 2.2\,O_2 + 8.27\,N_2 \longrightarrow 0.95\,CO_2 + 0.05\,CO + 2H_2O + 0.225\,O_2 + 8.27\,N_2.} \quad \textbf{(Ans.)}$$

(*ii*) **Air-fuel ratio A/F :**

The air-fuel ratio on a *mole basis* is

$$2.2 + 8.27 = \mathbf{10.47\ moles\ air/mole\ fuel.}\quad \textbf{(Ans.)}$$

The air-fuel ratio on a *mass basis* is found by introducing the molecular weights

$$\text{A/F} = \frac{10.47 \times 28.97}{(12 + 1 \times 4)} = \mathbf{18.96\ kg\ air/kg\ fuel.}\quad \textbf{(Ans.)}$$

The theoretical air-fuel ratio is found by writing the combustion equation for theoretical air,

$$CH_4 + 2O_2 + 2\left(\frac{79}{21}\right) N_2 \longrightarrow CO_2 + 2H_2O + (2)\left(\frac{79}{21}\right) N_2$$

$$\text{A/F}_{\text{theor.}} = \frac{\left[2 + (2)\left(\frac{79}{21}\right)\right] 28.97}{(12 + 1 \times 4)} = \mathbf{17.24\ kg\ air/kg\ fuel.}\quad \textbf{(Ans.)}$$

(*iii*) **Percent theoretical air :**

$$\text{Per cent theoretical air} = \frac{18.96}{17.24} \times 100 = \mathbf{110\%.}\quad \textbf{(Ans.)}$$

Example 11.19. *The gravimetric analysis of a sample of coal is given as 82% C, 10% H_2 and 8% ash. Calculate :*

(*i*) *The stoichiometric A/F ratio ;* (*ii*) *The analysis of the products by volume ;*

Solution. (*i*) **The stoichiometric A/F ratio :**

1 kg of coal contains 0.82 kg C and 0.10 kg H_2.

$$\therefore\quad \text{1 kg of coal contains } \frac{0.82}{12} \text{ moles C and } \frac{0.10}{2} \text{ moles } H_2$$

Let the oxygen required for complete combustion = x moles

Then the nitrogen supplied with the oxygen = $x \times \frac{79}{21} = 3.76x$ moles

For 1 kg of coal the combustion equation is therefore as follows :

$$\frac{0.82}{12} C + \frac{0.10}{2} H_2 + x\ CO_2 + 3.76x\ N_2 \longrightarrow a\ CO_2 + b\ H_2O + 3.76\ x\ N_2$$

Then, **Carbon balance :** $\frac{0.82}{12} = a$ $\quad\therefore\quad a = 0.068$ moles

Hydrogen balance : $2 \times \frac{0.10}{2} = 2b$ $\quad\therefore\quad b = 0.05$ moles

Oxygen balance : $2x = 2a + b$ $\quad\therefore\quad x = \left(\frac{2 \times 0.068 + 0.05}{2}\right) = 0.093$ moles

The mass of 1 mole of oxygen is 32 kg, therefore, the mass of O_2 supplied per kg of coal = 32 × 0.093 = 2.976 kg

i.e., $\quad$ *Stoichiometric A/F ratio* $= \frac{2.976}{0.233} = \mathbf{12.77}$ **(Ans.)**

(where air is assumed to contain 23.3% O_2 and 76.7% N_2 by mass)

Total moles of products $= a + b + 3.76x = 0.068 + 0.05 + 3.76 \times 0.093 = 0.467$ moles

Hence *the analysis of the products* is

$$\mathbf{CO_2} = \frac{0.068}{0.467} \times 100 = \mathbf{14.56\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{H_2} = \frac{0.05}{0.467} \times 100 = \mathbf{10.7\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{(3.76 \times 0.093)}{0.467} \times 100 = \mathbf{74.88\%.} \quad \textbf{(Ans.)}$$

☞**Example 11.20.** *Calculate the stoichiometric air-fuel ratio for the combustion of a sample of dry anthracite of the following composition by mass :*

Carbon (C) = 88 per cent *Hydrogen (H_2) = 4 per cent*
Oxygen (O_2) = 3.5 per cent *Nitrogen (N_2) = 1 per cent*
Sulphur (S) = 0.5 per cent *Ash = 3 per cent*

If 30 per cent excess air is supplied determine :

(*i*) *Air-fuel ratio ;*

(*ii*) *Wet dry analysis of the products of combustion by volume.*

Solution. Stoichiometric air-fuel (A/F) ratio :

In case of a fuel with several constituents a *tubular method* is advisable, as shown below. Each constituent is taken separately and the amount of oxygen required for complete combustion is found from the chemical equation. The oxygen in the fuel is included in the column headed '*oxygen required*' as a *negative quantity*.

	Mass per kg coal	*Combustion equation*	*Oxygen required per kg of coal*	*Products per kg of coal*
C	0.88	$C + O_2 \longrightarrow CO_2$ $12 \text{ kg} + 32 \text{ kg} \longrightarrow 44 \text{ kg}$	$0.88 \times \frac{32}{12} = 2.346$ kg	$0.88 \times \frac{44}{12} = 3.23$ kg CO_2
H_2	0.04	$2H_2 + O_2 \longrightarrow 2H_2O$ $1 \text{ kg} + 8 \text{ kg} \longrightarrow 9 \text{ kg}$	$0.04 \times 8 = 0.32$ kg	$0.04 \times 9 = 0.36$ kg H_2O
O_2	0.035	—	— 0.035 kg	—
N_2	0.01	—	—	0.01 kg N_2
S	0.005	$S + O_2 \longrightarrow SO_2$ $32 \text{ kg} + 32 \text{ kg} \longrightarrow 64 \text{ kg}$	$0.005 \times \frac{32}{32} = 0.005$ kg	$0.005 \times \frac{64}{32} = 0.01$ kg SO_2
Ash	0.03	—	—	—
			Total $O_2 = 2.636$ kg	

From table :

O_2 required per kg of coal = 2.636 kg

$\therefore$ Air required per kg of coal $= \frac{2.636}{0.233} = 11.31$ kg

(where air is assumed to contain 23.3% O_2 *by mass*)

N_2 associated with this air = 0.767 × 11.31 = 8.67 kg

$\therefore$ Total N_2 in products = 8.67 + 0.01 = 8.68 kg

The stoichiometric A/F ratio = 11.31/1. (Ans.)

When 30 per cent excess air is used :

(*i*) **Actual A/F ratio :**

Actual A/F ratio $= 11.31 + 11.31 \times \frac{30}{100} =$ **14.7/1. Ans.**

(*ii*) **Wet and dry analyses of products of combustion by volume :**

As per actual A/F ratio, N_2 supplied = 0.767 × 14.7 = 11.27 kg

Also O_2 supplied = 0.233 × 14.7 = 3.42 kg

(where air is assumed to contain N_2 = 76.7% and O_2 = 23.3)

In the products then, we have

$$N_2 = 11.27 + 0.01 = 11.28 \text{ kg}$$

and

$$\text{excess } O_2 = 3.42 - 2.636 = 0.78 \text{ kg}$$

The products are entered in the following table and the analysis by volume is obtained :

— In column 3 the percentage by mass is given by the mass of each product divided by the total mass of 15.66 kg.

— In column 5 the moles per kg of coal are given by equation $n = \frac{m}{M}$. The total of column 5 gives the total moles of wet products per kg of coal, and by subtracting the moles of H_2O from this total, the total moles of dry products is obtained as 0.5008.

— Column 6 gives the proportion of each constituent of column 5 expressed as a percentage of the total moles of the wet products.

— Similarly column 7 gives the percentage by volume of the dry products.

Product	*Mass / kg coal*	*% by mass*	*M*	*Moles / kg coal*	*% by vol. wet*	*% by vol. dry*
1	*2*	*3*	*4*	*5*	*6*	*7*
CO_2	3.23	20.62	44	0.0734	14.10	14.66
H_2O	0.36	2.29	18	0.0200	3.84	—
SO_2	0.01	0.06	64	0.0002 (say)	0.04	0.04
O_2	0.78	4.98	32	0.0244	4.68	4.87
N_2	11.28	72.03	28	0.4028	77.34	80.43
	15.66 kg			Total wet = 0.5208 $-H_2O$ = 0.0200 Total dry = 0.5008	100.00	100.00 **(Ans.)**

Example 11.21. *The following analysis relate to coal gas :*

H_2 = *50.4 per cent*	*CO = 17 per cent*
CH_4 = *20 per cent*	C_4H_8 = *2 per cent*
O_2 = *0.4 per cent*	N_2 = *6.2 per cent*
CO_2 = *4 per cent.*	

(*i*) *Calculate the stoichiometric A/F ratio.*

(*ii*) *Find also the wet and dry analyses of the products of combustion if the actual mixture is 30 per cent weak.*

Solution. The example is solved by a tabular method ; a specimen calculation is given below :

For CH_4 :

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

i.e., $$1 \text{ mole } CH_4 + 2 \text{ moles } O_2 \longrightarrow 1 \text{ mole } CO_2 + 2 \text{ moles } H_2O$$

There are 0.2 moles of CH_4 per mole of the coal gas, hence

$$0.2 \text{ moles } CH_4 + 0.2 \times 2 \text{ moles } O_2 \longrightarrow 0.2 \text{ moles } CO_2 + 0.2 \times 2 \text{ moles } H_2O$$

∴ O_2 required for the CH_4 in the coal gas = 0.4 moles per mole of coal gas.

The oxygen in the fuel (0.004 moles) is included in column 4 as a *negative quantity*.

Product	*Moles/mole fuel*	*Combustion equation*	*O_2 moles/ mole fuel*	*Products CO_2*	*H_2O*
1	*2*	*3*	*4*	*5*	*6*
H_2O	0.504	$2H_2 + O_2 \longrightarrow 2H_2O$	0.252	—	0.504
CO	0.17	$2CO + O_2 \longrightarrow 2CO_2$	0.085	0.17	—
CH_4	0.20	$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$	0.400	0.20	0.40
C_4H_8	0.02	$C_4H_8 + 6O_2 \longrightarrow 4CO_2 + 4H_2O$	0.120	0.08	0.08
O_2	0.004	—	– 0.004	—	—
N_2	0.062	—	—	—	—
CO_2	0.04	—	—	0.04	—
			Total = 0.853	0.49	0.984

(*i*) **Stoichiometric A/F ratio :**

$$\text{Air required} = \frac{0.853}{0.21} = 4.06 \text{ moles/mole of fuel}$$

(where air is assumed to contain 21% O_2 by volume)

∴ **Stoichiometric A/F ratio = 4.06/1 by volume. (Ans.)**

(*ii*) **Wet and dry analyses of the products of combustion if the actual mixture is 30% weak :**

Actual A/F ratio with 30% weak mixture

$$= 4.06 + \frac{30}{100} \times 4.06 = 1.3 \times 4.06 = 5.278/1$$

Associated N_2 = 0.79 × 5.278 = 4.17 moles/mole fuel

Excess oxygen = 0.21 × 5.278 – 0.853 = 0.255 moles

Total moles of N_2 in products = 4.17 + 0.062 = 4.232 moles/mole fuel.

Analysis by volume of wet and dry products :

Product	*Moles/mole fuel*	*% by vol. (dry)*	*% by vol. (wet)*
CO_2	0.490	9.97	8.31
H_2O	0.984	—	16.68
O_2	0.255	5.19	4.32
N_2	4.170	84.84	70.69
	Total wet = 5.899 – H_2O = 0.984 Total dry = 4.915	100.00	100.00 **(Ans.)**

Example 11.22. *Find the stoichiometric air-fuel ratio for the combustion of ethyl alcohol (C_2H_6O), in a petrol engine. Calculate the air-fuel ratios for the extreme mixture strengths of 80 per cent and 130 per cent. Determine also the wet and dry analyses by volume of the exhaust gas for each mixture strength.*

Solution. The equation for combustion of ethyl alcohol is as follows :

$$C_2H_6O + 3O_2 \longrightarrow 2CO_2 + 3H_2O$$

— Since there are two atoms of carbon in each mole of C_2H_6O then there must be two moles of CO_2 in the products, giving two atoms of carbon on each side of the equation.

— Similarly, since there are six atoms of hydrogen in each mole of ethyl alcohol then there must be three moles of H_2O in the products, giving six atoms of hydrogen on each side of the equation.

— Then balancing the atoms of oxygen, it is seen that there are (2 × 2 + 3) = 7 atoms on the right hand side of the equation, hence seven atoms must appear on the left hand side of the equation. There is one atom of oxygen in ethyl alcohol, therefore a further six atoms of oxygen must be supplied, and hence three moles of oxygen are required as shown.

Since the O_2 is supplied as air, the associated N_2 must appear in the equation,

i.e.,
$$C_2H_6O + 3O_2 + 3 \times \frac{79}{21}\, N_2 \longrightarrow 2CO_2 + 3H_2O + 3 \times \frac{79}{21}\, N_2$$

One mole of fuel has a mass of (2 × 12 + 1 × 6 + 16) = 46 kg. Three moles of oxygen have a mass of (3 × 32) = 96 kg.

$$\therefore \quad O_2 \text{ required per kg of fuel} = \frac{96}{46} = 2.09 \text{ kg}$$

$$\therefore \quad \textbf{Stoichiometric A/F ratio} = \frac{2.09}{0.233} = \textbf{8.96/1.} \quad \textbf{(Ans.)}$$

Considering a mixture strength of 80% :

$$\text{Now, mixture strength} = \frac{\text{Stoichiometric A/F ratio}}{\text{Actual A/F ratio}}$$

i.e.,
$$0.8 = \frac{8.96/1}{\text{Actual A/F ratio}}$$

$$\therefore \quad \textbf{Actual A/F ratio} = \frac{8.96}{0.8} = \textbf{11.2/1.} \quad \textbf{(Ans.)}$$

This means that 1/0.8 or 1.25 times as much air is supplied as is necessary for complete combustion. The exhaust will therefore contain 0.25 stoichiometric oxygen.

i.e.,
$$C_2H_6O + 1.25\left(3O_2 + 3 \times \frac{79}{21} N_2\right) \longrightarrow 2CO_2 + 3H_2O + 0.25 \times 3O_2 + 1.25 \times 3 \times \frac{79}{21}\, N_2$$

i.e., The products are :

$$2 \text{ moles } CO_2 + 3 \text{ moles } H_2O + 0.75 \text{ moles } O_2 + 14.1 \text{ moles } N_2$$

The total moles = 2 + 3 + 0.75 + 14.1 = 19.85

Hence wet analysis is :

$$\mathbf{CO_2} = \frac{2}{19.85} \times 100 = \textbf{10.08\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{H_2O} = \frac{3}{19.85} \times 100 = \textbf{15.11\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{O_2} = \frac{0.75}{19.85} \times 100 = \mathbf{3.78\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{14.1}{19.85} \times 100 = \mathbf{71.03\%.} \quad \textbf{(Ans.)}$$

The total dry moles = 2 + 0.75 + 14.1 = 16.85

Hence dry analysis is :

$$\mathbf{CO_2} = \frac{2}{16.85} \times 100 = \mathbf{11.87\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{O_2} = \frac{0.75}{16.85} \times 100 = \mathbf{4.45\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{14.1}{16.85} \times 100 = \mathbf{83.68\%.} \quad \textbf{(Ans.)}$$

Considering a mixture strength of 130% :

Now, $$1.3 = \frac{\text{Stoichiometric ratio}}{\text{Actual A/F ratio}}$$

$\therefore$ **Actual A/F ratio** $= \dfrac{8.96}{1.3} = \mathbf{6.89/1.}$ **(Ans.)**

This means that $\dfrac{1}{1.3}$ or 0.769 of the stoichiometric air is supplied. *The combustion cannot be complete, as the necessary oxygen is not available. It is usual to assume that all hydrogen is burned to* H_2O*, since hydrogen atoms have a greater affinity for oxygen than carbon atoms. The carbon in the fuel will burn to CO and* CO_2*, but the relative proportions have to be determined.*

Let, a = Number of moles of CO_2 in the products, and

b = Number of moles of CO in the products.

Then the combustion equation is as follows :

$$C_2H_6O + 0.769\left(3O_2 + 3 \times \frac{79}{21}N_2\right) \longrightarrow a\ CO_2 + b\ CO + 3H_2O + 0.769 \times 3 \times \frac{79}{21}\ N_2$$

To find a and b a balance of carbon and oxygen atoms can be made,

i.e., ***Carbon balance* :**

$$2 = a + b \qquad ...(i)$$

and ***Oxygen balance* :**

$$1 + 2 \times 0.769 \times 3 = 2a + b + 3$$

or $$2.614 = 2a + b \qquad ...(ii)$$

From eqns. (*i*) and (*ii*), we get

$$a = 0.614,\ b = 1.386$$

i.e., The products are : 0.614 moles CO_2 + 1.386 moles CO + 3 moles H_2O + 8.678 moles N_2

The total moles = 0.614 + 1.386 + 3 + 8.678 = 13.678.

Hence wet analysis is :

$$\mathbf{CO_2} = \frac{0.614}{13.678} \times 100 = \mathbf{4.49\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{CO} = \frac{1.386}{13.678} \times 100 = \mathbf{10.13\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{H_2O} = \frac{3}{13.678} \times 100 = \mathbf{21.93\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{8.678}{13.678} \times 100 = \mathbf{63.45\%.} \quad \textbf{(Ans.)}$$

The total dry moles = 0.614 + 1.386 + 8.678 = 10.678

Hence dry analysis is :

$$\mathbf{CO_2} = \frac{0.614}{10.678} \times 100 = \mathbf{5.75\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{CO} = \frac{1.386}{10.678} \times 100 = \mathbf{12.98\%.} \quad \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{8.678}{10.678} \times 100 = \mathbf{81.27\%.} \quad \textbf{(Ans.)}$$

Example 11.23. *For the stoichiometric mixture of example 11.22 calculate :*

(*i*) *The volume of the mixture per kg of fuel at a temperature of 50°C and a pressure of 1.013 bar.*

(*ii*) *The volume of the products of combustion per kg of fuel after cooling to a temperature of 130°C at a pressure of 1 bar.*

Solution. As before,

$$C_2H_6O + 3O_2 + 3 \times \frac{79}{21} N_2 \longrightarrow 2CO_2 + 3H_2O + 3 \times \frac{79}{21} N_2$$

$\therefore$ Total moles reactants $= 1 + 3 + 3 \times \frac{79}{21} = 15.3$

From equation, $pV = nR_0T$

$$V = \frac{nR_0T}{p} = \frac{15.3 \times 8.314 \times 10^3 \times (50 + 273)}{1.013 \times 10^5} = 405.6 \text{ m}^3\text{/mole of fuel}$$

In 1 mole of fuel there are (2 × 12 + 6 + 16) = 46 kg

(*i*) $\therefore$ **Volume of reactants per kg of fuel** $= \frac{405.6}{46} = \mathbf{8.817\ m^3.}$ **(Ans.)**

When the products are cooled to 130°C the H_2O exists as steam, since the temperature is well above the saturation temperature corresponding to the partial pressure of the H_2O. This must be so since the saturation temperature corresponding to the *total* pressure is 99.6°C, and the *saturation temperature decreases with pressure*. The total moles of the products is

$$= \left(2 + 3 + 3 \times \frac{79}{21}\right) = 16.3$$

From equation, $pV = nR_0T$

$$V = \frac{nR_0T}{p} = \frac{16.3 \times 8.314 \times 10^3 \times (130 + 273)}{1 \times 10^5} = 546.14 \text{ m}^3\text{/mole of fuel.}$$

(*ii*) $\therefore$ **Volume of products per kg of fuel** $= \frac{546.14}{46} = \mathbf{11.87\ m^3.}$ **(Ans.)**

☞**Example 11.24.** *The following is the composition of coal gas supplied to a gas engine :*

H_2 = *50.6 per cent ;* CO = *10 per cent ;* CH_4 = *26 per cent ;* C_4H_8 = *4 per cent ;* O_2 = *0.4 per cent ;* CO_2 = *3 per cent ;* N_2 = *6 per cent.*

If the air-fuel ratio is 7/1 by volume, calculate the analysis of the dry products of combustion. It can be assumed that the stoichiometric A/F ratio is less than 7/1.

Solution. Since it is given that the actual *A/F ratio is greater than the stoichiometric,* therefore it follows that *excess air has been supplied*. The products will therefore consist of CO_2, H_2O, O_2 and N_2.

The combustion equation can be written as follows :

$$0.506H_2 + 0.1CO + 0.26CH_4 + 0.04C_4H_8 + 0.004O_2 + 0.03CO_2 + 0.06N_2 + 0.21 \times 7O_2 + 0.79 \times 7N_2 \longrightarrow a\ CO_2 + b\ H_2O + c\ O_2 + d\ N_2$$

Then,

Carbon balance **:** $0.1 + 0.26 + 4 \times 0.04 + 0.03 = a$ $\quad\therefore\ a = 0.55$

Hydrogen balance **:** $2 \times 0.506 + 4 \times 0.26 + 8 \times 0.04 = 2b$ $\quad\therefore\ b = 1.186$

Oxygen balance **:** $0.1 + 2 \times 0.004 + 2 \times 0.03 + 0.21 \times 7 \times 2 = 2a + b + 2c$ $\quad\therefore\ c = 0.411$

Nitrogen balance **:** $2 \times 0.06 + 2 \times 0.79 \times 7 = 2d$ $\quad\therefore\ d = 5.59$

$\therefore$ Total moles of dry products = 0.55 + 0.411 + 5.59 = 6.65

Then analysis by volume is :

$$\mathbf{CO_2} = \frac{0.55}{6.55} \times 100 = \mathbf{8.39\%.}\quad \textbf{(Ans.)}$$

$$\mathbf{O_2} = \frac{0.411}{6.55} \times 100 = \mathbf{6.27\%.}\quad \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{5.59}{6.55} \times 100 = \mathbf{85.34\%.}\quad \textbf{(Ans.)}$$

Example 11.25. *The following is the analysis (by weight) of a chemical fuel :*

Carbon = 60 per cent ; Hydrogen = 20 per cent ; Oxygen = 5 per cent ; Sulphur = 5 per cent and Nitrogen = 10 per cent.

Find the stoichiometric amount of air required for complete combustion of this fuel.

Solution. On the basis of 100 kg fuel let us assume an equivalent formula of the form :

$$C_aH_bO_cN_dS_e$$

From the given analysis by weight, we can write

$12a = 60$ or $a = 5$

$1b = 20$ or $b = 20$

$16c = 5$ or $c = 0.3125$

$14d = 10$ or $d = 0.7143$

$32e = 5$ or $e = 0.1562$

Then the formula of the fuel can be written as

$$C_5H_{20}O_{0.3125}N_{0.7143}S_{0.1562}$$

The combustion equation is

$$C_5H_{20}O_{0.3125}N_{0.7143}S_{0.1562} + x\ O_2 + x\left(\frac{79}{21}\right)N_2 \longrightarrow p\ CO_2 + q\ H_2O + r\ SO_2 + s\ N_2$$

Then,

Carbon balance **:** $5 = p$ $\quad\therefore\ p = 5$

Hydrogen balance **:** $20 = 2q$ $\quad\therefore\ q = 10$

Sulphur balance **:** $0.1562 = r$ $\quad\therefore\ r = 0.1562$

Oxygen balance **:** $0.3125 + 2x = (2p + q + 2r)$

i.e., $$x = p + \frac{q}{2} + r - \frac{0.3125}{2} = 5 + \frac{10}{2} + 0.1562 - \frac{0.3125}{2} = 9.99$$

Nitrogen balance **:** $0.7143 + 2x \times \frac{79}{21} = 2s$

$$\therefore \qquad s = \frac{0.7143}{2} + x \times \frac{79}{21} = \frac{0.7143}{2} + 9.99 \times \frac{79}{21} = 37.94$$

Hence the combustion equation is written as follows :

$$C_5H_{20}O_{0.3125}N_{0.7143}S_{0.1562} + 9.99O_2 + 9.99\left(\frac{79}{21}\right)N_2 \longrightarrow 5CO_2 + 10H_2O + 0.1562SO_2 + 37.94N_2$$

$\therefore$ **Stoichiometric air required** $= \dfrac{9.92 \times 32 + 9.99 \times \left(\frac{79}{21}\right) \times 28}{100}$ **= 13.7 kg/kg of fuel. (Ans.)**

(**Note.** This example can also be solved by tabular method as explained in example 11.20.).

Example 11.26. *A sample of fuel has the following percentage composition by weight :*

Carbon = 84 per cent *Hydrogen = 10 per cent*
Oxygen = 3.5 per cent *Nitrogen = 1.5 per cent*
Ash = 1 per cent

(*i*) *Determine the stoichiometric air-fuel ratio by mass.*

(*ii*) *If 20 per cent excess air is supplied, find the percentage composition of dry flue gases by volume.*

Solution. (*i*) **Stoichiometric air fuel ratio :**

On the basis of 100 kg of fuel let us assume an equivalent formula of the form :

$$C_aH_bO_cN_d$$

From the given analysis by weight, we can write

$12a = 84$ *i.e.,* $a = 7$
$1b = 10$ *i.e.,* $b = 10$
$16c = 3.5$ *i.e.,* $c = 0.218$
$14d = 1.5$ *i.e.,* $d = 0.107$

The formula of fuel is

$$C_7H_{10}O_{0.218}N_{0.107}$$

The combustion equation is written as

$$C_7H_{10}O_{0.218}N_{0.107} + x\,O_2 + x\left(\frac{79}{21}\right)N_2 \longrightarrow p\,CO_2 + q\,H_2O + r\,N_2$$

Then,

Carbon balance **:** $7 = p$ *i.e.,* $p = 7$

Hydrogen balance **:** $10 = 2q$ *i.e.,* $q = 5$

Oxygen balance **:** $0.218 + 2x = (2p + q)$

or $0.218 + 2x = 2 \times 7 + 5$ *i.e.,* $x = 9.39$

Nitrogen balance **:** $0.107 + 2x\left(\frac{79}{21}\right) = 2r$

or $0.107 + 2 \times 9.39 \times \frac{79}{21} = 2r$ *i.e.,* $r = 35.4$

Hence the combustion equation becomes

$$C_7H_{10}O_{0.218}N_{0.107} + 9.39O_2 + 9.39\left(\frac{79}{21}\right)N_2 \longrightarrow 7CO_2 + 5H_2O + 35.4N_2$$

∴ **Stoichiometric A/F ratio** $= \dfrac{9.39 \times 32 + 9.39 \times \frac{79}{21} \times 28}{100}$ = **12.89. (Ans.)**

(*ii*) **Percentage composition of dry flue gases by volume with 20 per cent excess air :**

If 20 per cent excess air is used, the combustion equation becomes

$$C_7H_{10}O_{0.218}N_{0.107} + (1.2)(9.39)\ O_2 + (1.2)(9.39)\left(\frac{79}{21}\right)N_2$$
$$\longrightarrow 7CO_2 + 5H_2O + (0.2)(9.39)\ O_2 + (1.2)(35.4)\ N_2$$

Total number of moles of dry products of combustion

$$n = 7 + (0.2)(9.39) + (1.2)(35.4)$$
$$= 7 + 1.878 + 42.48 = 51.358$$

∴ **Percentage composition of dry flue gases by volume is as follows :**

$$\mathbf{CO_2} = \frac{7}{51.358} \times 100 = \mathbf{13.63\%.}\ \ \textbf{(Ans.)}$$

$$\mathbf{O_2} = \frac{1.878}{51.358} \times 100 = \mathbf{3.66\%.}\ \ \textbf{(Ans.)}$$

$$\mathbf{N_2} = \frac{42.48}{51.358} \times 100 = \mathbf{82.71\%.}\ \ \textbf{(Ans.)}$$

Example 11.27. *Orsat analysis of the products of combustion of a hydrocarbon fuel of unknown composition is as follows :*

Carbon dioxide (CO_2) *= 8%*	*Carbon monoxide (CO) = 0.5%*
Oxygen (O_2) *= 6.3%*	*Nitrogen* (N_2) *= 85.2%*

Determine the following :

(*i*) *Air-fuel ratio ;*

(*ii*) *Percent theoretical air required for combustion.*

Solution. From the given Orsat analysis the combustion equation is written as follows :

$$a\ C + b\ H + c\ O_2 + \left(\frac{79}{21}\right) c\ N_2 = 8CO_2 + 0.5CO + 6.3O_2 + x\ H_2O + 85.2N_2$$

Then,

Carbon balance : $a = 8 + 0.5 = 8.5$ *i.e.,* $a = 8.5$

Nitrogen balance : $\frac{79}{21}$ C = 85.2 *i.e.,* $c = 22.65$

Oxygen balance : $c = 8 + \frac{0.5}{2} + 6.3 + \frac{x}{2}$

or $22.65 = 8 + 0.25 + 6.3 + \frac{x}{2}$ *i.e.,* $x = 16.2$

Hydrogen balance : $b = 2x = 2 \times 16.2 = 32.4$ *i.e.,* $b = 32.4$

(*i*) **Air-fuel ratio :**

The air supplied per 100 moles of dry products is

$$= 22.65 \times 32 + \left(\frac{79}{21}\right) \times 22.65 \times 28 = 3110.6 \text{ kg}$$

∴ **Air-fuel ratio** $= \dfrac{3110.6}{8.5 \times 12 + 32.4 \times 1}$ = **23.1 kg of air/kg of fuel. (Ans.)**

(*ii*) **Per cent theoretical air required for combustion :**

Mass fraction of carbon $= \dfrac{12 \times 8.5}{12 \times 8.5 + 32.4 \times 1} = 0.759$

Mass fraction of hydrogen $= \dfrac{32.4 \times 1}{12 \times 8.5 + 32.4} = 0.241$

Considering 1 kg of fuel, the *air required for complete combustion* is

$$= \left[0.759 \times \left(\frac{8}{3}\right) \times \frac{100}{23.3}\right] + \left[0.241 \times 8 \times \frac{100}{23.3}\right] = 16.96 \text{ kg}$$

∴ **Per cent theoretical air required for combustion** $= \dfrac{23.1}{16.96} \times 100$ = **136.2%. (Ans.)**

Example 11.28. *The following is the volumetric analysis of the dry exhaust from an internal combustion engine :*

CO_2 = 8.9% ; CO = 8.2% ; H_2 = 4.3% ; CH_4 = 0.5% and N_2 = 78.1%.

If the fuel used is octane (C_8H_{18}) determine air-fuel ratio on mass basis :

(*i*) *By a carbon balance.* (*ii*) *By a hydrogen-oxygen balance.*

Solution. (*i*) As per analysis of dry products, the combustion equation is written as

$$a\, C_8H_{18} + 78.1N_2 + 78.1 \left(\frac{21}{79}\right) O_2$$
$$\longrightarrow 8.9CO_2 + 8.2CO + 4.3H_2 + 0.5CH_4 + 78.1N_2 + x\, H_2O$$

***Carbon balance* :** $8a = 8.9 + 8.2 + 0.5 = 17.6$ *i.e.,* $a = 2.2$

∴ **Air-fuel (A/F) ratio** $= \dfrac{78.1 \times 28 + 78.1 \times \frac{21}{79} \times 32}{2.2(8 \times 12 + 1 \times 18)}$

$$= \frac{2186.8 + 664.3}{250.8} = \frac{2851.1}{250.8} = \mathbf{11.37.} \quad \textbf{(Ans.)}$$

(*ii*) In this case the combustion equation is written as

$$a\, C_8H_{18} + b\, O_2 + b \left(\frac{79}{21}\right) N_2 \longrightarrow 8.9CO_2 + 8.2CO + 4.3H_2 + 0.5CH_4 + b \left(\frac{79}{21}\right) N_2 + x\, H_2O$$

***Carbon balance* :** $8a = 8.9 + 8.2 + 0.5 = 17.6$ *i.e.,* $a = 2.2$

***Hydrogen balance* :** $18a = 4.3 \times 2 + 0.5 \times 4 + 2x$

or $18 \times 2.2 = 8.6 + 2 + 2x$ *i.e.,* $x = 14.5$

***Oxygen balance* :** $2b = 8.9 \times 2 + 8.2 + x$

or $2b = 17.8 + 8.2 + 14.5$ *i.e.,* $b = 20.25$

∴ **Air-fuel (A/F) ratio** $= \dfrac{(20.25 \times 32) + (20.25)\left(\frac{79}{21}\right) \times 28}{2.2(8 \times 12 + 1 \times 18)} = \dfrac{2781}{250.8}$ = **11.09. (Ans.)**

☞ **Example 11.29.** *The exhaust from an engine running on benzole was measured with the help of Orsat apparatus. Orsat analysis showed a CO_2 content of 12%, but no CO. Assuming that the remainder of the exhaust contains only oxygen and nitrogen, calculate the air-fuel ratio of the engine.*

The ultimate analysis of benzole is C = 88% and H_2 = 12%.

Solution. 1 kg of fuel, consisting of 0.88 kg C and 0.12 kg H_2, can be written as 0.88/2 moles C and 0.12/2 moles H_2. Therefore, considering 1 mole of *dry exhaust gas* (D.E.G.) we can write the combustion equation as follows :

$$X\left(\frac{0.88}{12}\text{C}+\frac{0.12}{2}\text{H}_2\right)+Y\,\text{O}_2+\frac{79}{21}\,Y\,\text{N}_2 \longrightarrow 0.12\text{CO}_2+a\,\text{O}_2+(0.88-a)\,\text{N}_2+b\,\text{H}_2\text{O}$$

$$\begin{bmatrix}\text{Let the D.E.G. contain } a \text{ moles of O}_2\text{. The moles of CO}_2 \text{ in 1 mole of D.E.G. are 0.12.}\\ \text{Therefore the D.E.G. contains } (1-a-0.12)=(0.88-a) \text{ moles of N}_2.\end{bmatrix}$$

where, X = Mass of fuel per mole D.E.G.,

Y = Moles of O_2 per mole D.E.G.,

a = Moles of *excess* O_2 per mole D.E.G., and

b = Moles of H_2O per mole D.E.G.

Now,

Carbon balance : $\frac{0.88}{12}X = 0.12$ $\therefore$ $X = 1.636$

Hydrogen balance : $0.06X = b$ $\therefore$ $b = 0.06 \times 1.636 = 0.098$

Oxygen balance : $2Y = 2 \times 0.12 + 2a + b$

or $2Y = 0.24 + 2a + 0.098$ $\therefore$ $Y = 0.169 + a$

Nitrogen balance : $\frac{79}{21}Y = (0.88 - a)$ $\therefore$ $Y = 0.234 - 0.266a$

Equating the expressions for Y gives

$$0.234 - 0.266a = 0.169 + a \qquad \therefore \quad a = 0.0513$$

i.e., $Y = 0.169 + 0.0513 = 0.2203$

$\therefore$ O_2 supplied = 0.2203 × 32 kg/mole D.E.G.

i.e., Air supplied = $\frac{0.2203 \times 32}{0.233}$ = 30.26 kg/mole D.E.G.

Since X = 1.636, then, the fuel supplied per mole D.E.G. is 1.636 kg

$\therefore$ **A/F ratio** = $\frac{30.26}{1.636}$ = **18.5/1. (Ans.)**

Example 11.30. *The analysis of the dry exhaust from an internal combustion engine is as follows :*

Carbon dioxide (CO_2) = 15 per cent *Carbon monoxide (CO) = 3 per cent*

Methane (CH_4) = 3 per cent *Hydrogen (H_2) = 1 per cent*

Oxygen (O_2) = 2 per cent *Nitrogen (N_2) = 76 per cent*

Calculate the proportions by mass of carbon to hydrogen in the fuel, assuming it to be a pure hydrocarbon.

Solution. Let 1 kg of fuel contain x kg of carbon (C) and y kg hydrogen (H_2). Then considering 1 mole of D.E.G. and introducing X and Y, we can write

$$X\left(\frac{x\,\mathrm{C}}{12}+\frac{y}{2}\mathrm{H_2}\right)+Y\,O_2+\frac{79}{21}\,Y\,N_2 \longrightarrow 0.15CO_2 + 0.03CO + 0.03CH_4$$
$$+ 0.01H_2 + 0.02O_2 + a\ H_2O + 0.76N_2$$

Then,

Nitrogen balance : $\frac{79}{21}\ Y = 0.76 \qquad \therefore\ Y = 0.202$

Oxygen balance : $Y = 0.15 + \frac{0.03}{2} + 0.02 + \frac{a}{2}$

or $0.202 = 0.15 + 0.015 + 0.02 + \frac{a}{2} \qquad \therefore\ a = 0.034$

Carbon balance : $\frac{Xx}{12} = 0.15 + 0.03 + 0.03 \qquad \therefore\ Xx = 2.52$ ...(*i*)

Hydrogen balance : $\frac{Xy}{2} = 2 \times 0.03 + 0.01 + a = 0.06 + 0.01 + 0.034$

$\therefore \qquad Xy = 0.208$ ...(*ii*)

Dividing equations (*i*) and (*ii*), we get

$$\frac{Xx}{Xy} = \frac{2.52}{0.208} \quad \text{or} \quad \frac{x}{y} = 12.1$$

i.e., **Ratio of C to H_2 in fuel** $= \frac{x}{y} = \mathbf{\frac{12.1}{1}}$. **(Ans.)**

INTERNAL ENERGY AND ENTHALPY OF COMBUSTION

Example 11.31. *ΔH_0 (enthalpy of combustion at reference temperature T_0) for benzene vapour (C_6H_6) at 25°C is – 3301000 kJ/mole with the H_2O in the liquid phase. Calculate ΔH_0 for the H_2O in the vapour phase.*

Solution. If H_2O remains as a vapour the heat transferred to the surroundings will be less than when the vapour condenses by the amount due to the change in enthalpy of the vapour during condensation at the reference temperature.

$$\Delta H_0 \text{ (vapour)} = \Delta H_0 \text{ (liquid)} + m_s\, h_{fgo}$$

where, m_s = Mass of H_2O formed, and

h_{fgo} = Change in enthalpy of steam between saturated liquid and saturated vapour at the reference temperature T_0

= 2441.8 kJ at 25°C

For the reaction :

$$C_6H_6 + 7.5O_2 \longrightarrow 6CO_2 + 3H_2O$$

3 moles of H_2O are formed on combustion of 1 mole of C_6H_6 ; 3 moles of H_2O

$$= 3 \times 18 = 54 \text{ kg } H_2O$$

$\therefore$ **ΔH_0 (vapour)** = – 3301000 + 54 × 2441.8 = **– 3169143 kJ/mole. (Ans.)**

Example 11.32. *Calculate ΔU_0 in kJ/kg for the combustion of benzene (C_6H_6) vapour at 25°C given that ΔH_0 = – 3169100 kJ/mole and the H_2O is in the vapour phase.*

Solution. Given : ΔH_0 = – 3169100 kJ

The combustion equation is written as

$$C_6H_6 + 7.5O_2 \longrightarrow 6CO_2 + 3H_2O \text{ (vapour)}$$

$$n_R = 1 + 7.5 = 8.5,\ n_P = 6 + 3 = 9$$

Using the relation,

$$\begin{aligned}\Delta U_0 &= \Delta H_0 - (n_P - n_R)R_0T_0\\ &= -\ 3169100 - (9 - 8.5) \times 8.314 \times (25 + 273)\\ &= -\ 3169100 - 1239 = -\ 3170339 \text{ kJ/mole}\end{aligned}$$

(It may be noted that ΔU_0 is negligibly different from ΔH_0)

1 mole of $C_6H_6 = 6 \times 12 + 1 \times 6 = 78$ kg

$$\therefore \qquad \mathbf{\Delta U_0} = \frac{-\ 3170339}{78} = \mathbf{-\ 40645\ kJ/kg.} \quad \textbf{(Ans.)}$$

Example 11.33. *ΔH_0 for CO at 60°C is given as – 285200 kJ/mole. Calculate ΔH_0 at 2500°C given that the enthalpies of gases concerned in kJ/mole are as follows :*

Gas	*60°C*	*2500°C*
CO	*9705*	*94080*
O	*9696*	*99790*
CO_2	*10760*	*149100*

Solution. The reaction equation is given by

$$CO + \tfrac{1}{2}O_2 \longrightarrow CO_2$$

Refer Fig. 11.7.

It can be seen from the property diagram of Fig. 11.7 that the enthalpy of combustion at temperature T, ΔH_T can be obtained from ΔH_0 and T_0 by the relationship

$$-\ \Delta H_T = -\ \Delta H_0 + (H_{R_T} - H_{R_0}) - (H_{P_T} - H_{P_0}) \qquad ...(i)$$

where $H_{R_T} - H_{R_0}$ = increase in enthalpy of the *reactants* from T_0 to T

and $H_{P_T} - H_{P_0}$ = increase in enthalpy of the *products* from T_0 to T.

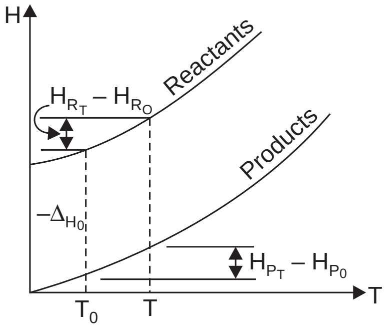


Fig. 11.7

Now, from the given data, we have

$$H_{R_0} = 1 \times 9705 + \tfrac{1}{2} \times 9696 = 14553 \text{ kJ}$$

$$H_{R_T} = 1 \times 94080 + \tfrac{1}{2} \times 99790 = 143975, \text{ kJ}$$

$$H_{P_0} = 1 \times 10760 = 10760 \text{ kJ}$$

$$H_{P_T} = 1 \times 149100 \text{ kJ} = 149100 \text{ kJ}$$

Using equation (i), we get

$$\begin{aligned}-\ \Delta H_T &= +\ 285200 + (143975 - 14553) - (149100 - 10760)\\ &= 285200 + 129422 - 138340 = 276282\end{aligned}$$

$$\therefore \qquad \mathbf{\Delta H_T = -\ 276282\ kJ/mole.} \quad \textbf{(Ans.)}$$

HEATING VALUES OF FUELS

Example 11.34. *The lower heating value of propane at constant pressure and 25°C is 2044009 kJ per kg mole. Find the higher heating value at constant pressure and at constant volume.*

Solution. (*i*) **Higher heating value at constant pressure, $(\mathbf{HHV})_\mathbf{P}$:**

The combustion reaction for propane is written as

$$C_3H_8 + 5O_2 = 3CO_2 + 4H_2O$$

Now $$(\text{HHV})_p = (\text{LHV})_p + mh_{fg}$$

where,
HHV = Higher heating value at constant pressure,
LHV = Lower heating value,
m = Mass of water formed by combustion
= 4 × 18 = 72 kg per kg mole, and
h_{fg} = Latent heat of vaporisation at given temperature per unit mass of water
= 2443 kJ/kg at 25°C.

∴ $(\mathbf{HHV})_\mathbf{p}$ = 2044009 + 72(2442) = **2219833 kJ/kg. (Ans.)**

(*ii*) **Higher heating value at constant volume, $(\mathbf{HHV})_\mathbf{v}$:**

Now $$(\Delta U) = \Delta H - \Delta n R_0 T$$

or $$-(\text{HHV})_v = -(\text{HHV})_p - \Delta n R_0 T$$

or $$(\text{HHV})_v = (\text{HHV})_p + \Delta n R_0 T$$

where R_0 = universal gas constant = 8.3143 kJ/kg mol K

$\Delta n = n_P - n_R$

$$\begin{bmatrix} n_P = \text{number of moles of gaseous products} \\ n_R = \text{number of moles of gaseous reactants} \end{bmatrix}$$

Now, the reaction for higher heating value is

$$C_3H_8 + 5O_2 = 3CO_2 + 4H_2O \text{ (liquid)}$$

$$\Delta n = 3 - (1 + 5) = -3$$

∴ $(\mathbf{HHV})_\mathbf{v}$ = 2219905 – 3(8.3143)(25 + 273) = **2212472 kJ/kg. (Ans.)**

Example 11.35. *Calculate the lower heating value of gaseous octane at constant volume if $(\Delta U)_{25°C} = -5494977$ kJ for the reaction :*

$$[C_8H_{18} \text{ (gas)} + 12.5O_2 \text{ (gas)}] = [8CO_2 \text{ (gas)} + 9H_2O \text{ (liquid)}].$$

Solution. The given value of ΔU corresponds to the *higher heating value* at constant volume because the water in the products is in *liquid phase*.

$$\text{HHV} = 5494977 \text{ kJ/kg}$$

$$(\text{LHV})_v = (\text{HHV})_v - m(u_g - u_f)$$

$$m = 9 \times 18 = 162 \text{ kg/kg mole } C_8H_{18}$$

$$(u_g - u_f) = 2305 \text{ kJ/kg at 25°C}$$

∴ $(\mathbf{LHV})_\mathbf{v}$ = 5494977 – 162(2305) = **5121567 kJ/kg. (Ans.)**

Example 11.36. *Calculate the lower and higher heating values at constant pressure per kg of mixture at 25°C, for the stoichiometric mixtures of :*

(*i*) *Air and benzene vapour (C_6H_6), and*

(*ii*) *Air and octane vapour (C_8H_{18}).*

Given that the enthalpies of combustion at 25°C are :

$$C_6H_6 = -\ 3169500 \text{ kJ/mole}$$
$$C_8H_{18} = -\ 5116200 \text{ kJ/mole}$$

Both the above figures are for the case where the water in the products is in the vapour phase.

Solution. (*i*) **Air and benzene vapour :**

For benzene, the combustion equation is as follows :

$$C_6H_6(g) + 7.5O_2(g) + 7.5\left(\frac{79}{21}\right)N_2(g) = 6CO_2(g) + 3H_2O(g) + 7.5\left(\frac{79}{21}\right)N_2(g)$$

Since the water in the products is in *vapour phase*, therefore, the given value of enthalpy of combustion corresponds to the *lower heating* value at constant pressure.

i.e., $(\text{LHV})_p = 3169500$ kJ/mole

$$\textbf{(LHV)}_\textbf{v} \textbf{ per kg of mixture} = \frac{3169500}{(12\times6+6\times1)+(7.5\times32)+7.5\left(\frac{79}{21}\right)(28)}$$

$$= \frac{3169500}{78+240+790} = \textbf{2861 kJ/kg.} \quad \textbf{(Ans.)}$$

Now, $(\text{HHV})_p = (\text{LHV})_p + mh_{fg}$

where $(\text{HHV})_p$ = Higher heating value at constant pressure,
$(\text{LHV})_p$ = Lower heating value at constant pressure,
m = Mass of water formed by combustion.
= 3 × 18 = 54 kg/kg mole of fuel, and
h_{fg} = Latent heat of vapourisation at given temperature per unit mass of water
= 2442 kJ/kg at 25°C.

$\therefore$ $(\text{HHV})_p = 3169500 + 54 \times 2442 = 3301368$ kJ/mole

Thus, $\textbf{(HHV)}_\textbf{p} \textbf{ per kg of mixture} = \dfrac{3301368}{78+240+790} = \textbf{2980 kJ/kg.}$ **(Ans.)**

(*ii*) **Air and octane vapour :**

$(\text{LHV})_p = 5116200$ kJ/mole of C_8H_{18}

For octane, the combustion equation is written as follows :

$$C_8H_{18}(g) + 12.5O_2(g) \longrightarrow 8CO_2(g) + 9H_2O(g) + 12.5\left(\frac{79}{21}\right)N_2(g)$$

$$\textbf{(LHV)}_\textbf{p} \textbf{ per kg of mixture} = \frac{5116200}{(12\times8+18\times1)+12.5\times32+12.5\times\frac{79}{21}\times28}$$

$$= \frac{5116200}{114+400+1317} = \textbf{2794 kJ/kg.} \quad \textbf{(Ans.)}$$

$$(\text{HHV})_p = (\text{LHV})_p + mh_{fg}$$
$$m = 9 \times 18 = 162 \text{ kJ/kg mole of fuel}$$

$\therefore$ $(\text{HHV})_p = 5116200 + 162 \times 2442 = 5511804$

Hence, $\textbf{(HHV)}_\textbf{p} \textbf{ per kg of mixture} = \dfrac{5511804}{114+400+1317} = \textbf{3010 kJ/kg.}$ **(Ans.)**

☞**Example 11.37.** *The higher heating value of kerosene at constant volume whose ultimate analysis is 88% and 12% hydrogen, was found to be 45670 kJ/kg. Calculate the other three heating values.*

Solution. Combustion of 1 kg of fuel produces the following products :

$$CO_2 = \frac{44}{12} \times 0.88 = 3.23 \text{ kg}$$

$$H_2O = \frac{18}{2} \times 0.12 = 1.08 \text{ kg}$$

At 25°C : $(u_g - u_f)$ *i.e.,* $u_{fg} = 2304$ kJ/kg

$$h_{fg} = 2442 \text{ kJ/kg}$$

(*i*) **(LHV)$_v$:**

$$(\text{LHV})_v = (\text{HHV})_v - m(u_g - u_f)$$
$$= 45670 - 1.08 \times 2304 = 43182 \text{ kJ/kg}$$

Hence **(LHV)$_v$ = 43182 kJ/kg. (Ans.)**

(*ii*) **(HHV)$_p$, (LHV)$_p$:**

The combustion equation is written as follows :

$$1 \text{ mole fuel} + \frac{x}{32} O_2 \longrightarrow \frac{3.23}{44} CO_2 + \frac{1.08}{18} H_2O$$

i.e.,
$$\frac{x}{32} = \frac{3.23}{44} + \frac{1.08}{18 \times 2}$$

or
$$x = 3.31 \text{ kg}$$

i.e., $1 \text{ kg fuel} + 3.31 \text{ kg } O_2 = 3.23CO_2 + 1.08H_2O$

Also, $\Delta H = \Delta U + \Delta n R_0 T$

i.e., $-(\text{HHV})_p = -(\text{HHV})_v + \Delta n R_0 T$

or $(\text{HHV})_p = (\text{HHV})_v - \Delta n R_0 T$

where $\Delta n = n_p - n_R$

$$= \left(\frac{3.23}{44} - \frac{3.31}{32}\right) \quad \begin{bmatrix} n_p = \text{number of moles of gaseous products} \\ n_R = \text{number of moles of gaseous reactants} \end{bmatrix}$$

Since in case of higher heating value, H_2O will appear in liquid phase

$$\textbf{(HHV)}_\mathbf{p} = 45670 - \left(\frac{3.23}{44} - \frac{3.31}{32}\right) \times 8.3143 \times (25 + 273)$$

$$= \textbf{45744 kJ/kg.} \quad \textbf{(Ans.)}$$

$$\textbf{(LHV)}_\mathbf{p} = (\text{HHV})_p - 1.08 \times 2442 = 45774 - 1.08 \times 2442$$

$$= \textbf{43107 kJ/kg.} \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. A *chemical reaction* may be defined as the rearrangement of atoms due to redistribution of electrons. '*Reactants*' comprise of initial constituents which start the reaction while '*products*' comprise of final constituents which are formed by the chemical reaction.
2. A chemical fuel is a substance which releases heat energy on combustion.
3. The total number of atoms of each element concerned in the combustion remains constant, but the atoms are rearranged into groups having different chemical properties.

4. The amount of excess air supplied varies with the type of the fuel and the firing conditions. It may approach a value of 100 per cent but modern practice is to use 25% to 50% excess air.

5. *Stoichiometric* (or chemically correct) *mixture* of air and fuel is one that contains just sufficient oxygen for complete combustion of the fuel.

6. Mixture strength $= \dfrac{\text{Stoichiometric } A/F \text{ ratio}}{\text{Actual } A/F \text{ ratio}}$.

7. When analysis of combustion products is known air fuel ratio can be calculated by the following methods :

(*a*) Fuel composition known

(*i*) Carbon balance method (*ii*) Hydrogen balance method

(*iii*) Carbon hydrogen balance method

(*b*) Fuel composition unknown

(*i*) Carbon hydrogen balance method.

8. The most common means of analysis of the combustion products is Orsat apparatus.

9. The *enthalpy of formation* (ΔH_f) is the increase in enthalpy when a compound is formed from its constituent elements in their natural form and in a standard state. The standard state is 25°C and 1 atm. pressure (but it must be borne in mind that not all substances can exist in natural form, *e.g.* H_2O cannot be a vapour at 1 atm. and 25°C).

10.
$$(\text{HHV})_p = (\text{LHV})_p + m\,h_{fg}$$
$$(\text{HHV})_v = (\text{LHV})_v + m(u_g - u_f)$$

where HHV = Higher heating value,
LHV = Lower heating value,
m = Mass of water formed by combustion,
h_{fg} = Enthalpy of vapourisation of water, kJ/kg,
u_g = Specific internal energy of vapour, kJ/kg, and
u_f = Specific internal energy of liquid, kJ/kg.

11. In a given combustion process, that takes place adiabatically and with no work or changes in kinetic or potential energy involved, the temperature of the products is referred to as the '*adiabatic flame temperature*'.

12. For a given fuel and given pressure and temperature of the reactants, the maximum adiabatic flame temperature that can be achieved is with a 'stoichiometric' mixture.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The smallest particle which can take part in a chemical change is called
(*a*) atom (*b*) molecule
(*c*) electron (*d*) compound.

2. A chemical fuel is a substance which releases ...... on combustion.
(*a*) chemical energy (*b*) heat energy
(*c*) sound energy (*d*) magnetic energy.

3. The most important solid fuel is
(*a*) wood (*b*) charcoal
(*c*) coal (*d*) all of the above.

4. For each mole of oxygen, number of moles of nitrogen required for complete combustion of carbon are
(*a*) 20/21 (*b*) 2/21
(*c*) 77/21 (*d*) 79/21.

5. Modern practice is to use ...... excess air.
(*a*) 5 to 10 per cent (*b*) 15 to 20 per cent
(*c*) 20 to 25 per cent (*d*) 25 to 50 per cent.

6. Stoichiometric air-fuel ratio by mass for combustion of petrol is
(*a*) 5 (*b*) 10
(*c*) 12 (*d*) 15.05.

7. An analysis which includes the steam in the exhaust is called
(*a*) dry analysis (*b*) wet analysis
(*c*) dry and wet analysis (*d*) none of the above.

8. The Orsat apparatus gives
(*a*) volumetric analysis of the dry products of combustion
(*b*) gravimetric analysis of the dry products of combustion
(*c*) gravimetric analysis of products of combustion including H_2O
(*d*) volumetric analysis of products of combustion including H_2O.

9. In the Orsat apparatus KOH solution is used to absorb
(*a*) carbon monoxide (*b*) carbon dioxide
(*c*) oxygen (*d*) none of the above.

10. Enthalpy of formation is defined as enthalpy of compounds at
(*a*) 25°C and 10 atmospheres (*b*) 25°C and 1 atmosphere
(*c*) 0°C and 1 atmosphere (*d*) 100°C and 1 atmosphere.

11. Bomb calorimeter is used to find the calorific value of ...... fuels.
(*a*) solid (*b*) gaseous
(*c*) solid and gaseous (*d*) none of the above.

12. When the fuel is burned and the water appears in the vapour phase, the heating value of fuel is called
(*a*) enthalpy of formation (*b*) lower heating value
(*c*) higher heating value (*d*) none of the above.

13. Heat released in a reaction at constant pressure is called
(*a*) entropy change (*b*) enthalpy of reaction
(*c*) internal energy of reaction (*d*) none of the above
(*e*) all of the above.

14. When the fuel is burned and water is released in the liquid phase, the heating value of fuel is called
(*a*) higher heating value (*b*) lower heating value
(*c*) enthalpy of formation (*d*) none of the above.

15. Choose the *correct* statement :
(*a*) Number of atoms of each constituent are not conserved in a chemical reaction.
(*b*) The mass of all the substances on one side of the equation may not be equal to the mass of all the substances on the other side.
(*c*) The number of atoms of each constituent are conserved in a chemical reaction.
(*d*) The number of moles of the reactants in a chemical equation are equal to the number of moles of the products.

ANSWERS

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*d*) **5.** (*d*) **6.** (*d*) **7.** (*b*)
8. (*a*) **9.** (*b*) **10.** (*b*) **11.** (*a*) **12.** (*b*) **13.** (*b*) **14.** (*a*)
15. (*c*).

THEORETICAL QUESTIONS

1. What is chemical thermodynamics ?
2. What is a chemical fuel ?
3. What are primary fuels ? List some important primary fuels.

4. What are secondary fuels ? List some important secondary fuels.
5. Write a short note on 'excess air'.
6. What do you mean by stoichiometric air-fuel (A/F) ratio ?
7. Enumerate the methods by which air fuel ratio can be calculated when analysis of combustion products is known.
8. How is analysis of exhaust and flue gas carried out ?
9. Derive relations for internal energy and enthalpy of reaction.
10. What is enthalpy of formation (ΔH_f) = ?
11. Define heating value of fuel.
12. What is the difference between higher heating value (HHV) and lower heating value (LHV) of the fuel ?
13. Describe with the help of neat sketches the following calorimeters used for the determination of heating values :
    (*i*) Bomb calorimeter (*ii*) Junkers gas calorimeter.
14. What is 'adiabatic flame temperature' ?
15. Write a short note on chemical equilibrium.

UNSOLVED EXAMPLES

1. A fuel has the following composition by weight : Carbon = 86% ; hydrogen = 11.75% and oxygen = 2.25%. Calculate the theoretical air supply per kg of fuel, and the weight of products of combustion per kg of fuel.
   [(**Ans.**) 13.98 kg ; 4.21 kg]
2. The volumetric analysis of a fuel gas is : CO_2 = 14% ; CO = 1% ; O_2 = 5% and N_2 = 80%. Calculate the fuel gas composition by weight. [(**Ans.**) 20.24% ; 0.93% ; 5.25% ; 73.58%]
3. The ultimate analysis of a dry coal burnt in a boiler gauge C = 84% ; H_2 = 9% and incombustibles 7% by weight. Determine the weight of dry flue gases per kg of coal burnt, if volumetric combustion of the flue gas is :
   CO_2 = 8.75%, CO = 2.25% ; O_2 = 80% and N_2 = 81%. [(**Ans.**) 18.92 kg]
4. During a trial in a boiler, the dry flue gas analysis by volume was obtained as CO_2 = 13%, CO = 0.3%, O_2 = 6%, N_2 = 80.7%. The coal analysis by weight was reported as C = 62.4%, H_2 = 4.2%, O_2 = 4.5%, moisture = 15% and ash 13.9%. Estimate :
   (*a*) Theoretical air required to burn 1 kg of coal.
   (*b*) Weight of air actually supplied per kg of coal.
   (*c*) The amount of excess air supplied per kg of coal burnt. [**Ans.** 8.5 kg ; 11.5 kg ; 3 kg]
5. A steam boiler uses pulverised coal in the furnace. The ultimate analysis of coal (by weight) as received is : C = 78% ; H_2 = 3% ; O_2 = 3% ; ash 10% and moisture 5%. Excess air supplied is 30%. Calculate the weight of air to be supplied and weight of gaseous product formed per kg of coal burnt.
   [**Ans.** 13 kg ; CO_2 = 2.86 kg ; H_2 = 0.27 kg ; excess O_2 = 0.69 kg and N_2 = 9.81 kg per kg of coal]
6. The percentage composition by mass of a crude oil is given as follows : C = 90% ; H_2 = 3.3% ; O_2 = 3%, N_2 = 0.8% ; S = 0.9% and remaining incombustible. If 50% excess air is supplied find the percentage of dry exhaust gases found by volume. [**Ans.** CO_2 = 12.7% ; SO_2 = 0.05% ; O_2 = 7%, N_2 = 80.25%]
7. In a boiler trial, the analysis of the coal used is as follows : C = 20%, H_2 = 4.5% , O_2 = 7.5%, remainder—incombustible matter.
   The dry flue gas has the following composition by volume :
   CO_2 = 8.5%, CO = 1.2%, N_2 = 80.3%, O_2 = 10%. Determine :
   (*i*) Minimum weight of air required per kg of coal.
   (*ii*) Percentage excess air. [**Ans.** (*i*) 3.56 kg, (*ii*) 63.2%]
8. The ultimate analysis of a sample of petrol by weight is : Carbon 0.835 ; hydrogen 0.165. Calculate the ratio of air to petrol consumption by weight, if the volumetric analysis of the dry exhaust gas is : Carbon dioxide 12.1 ; Carbon monoxide 1.1 ; Oxygen 0.8 ; Nitrogen 85.4 per cent. Also find the percentage excess air.
   [**Ans.** 16.265 : 1 ; 5.6%]

9. The percentage composition of a sample of fuel was found to be C = 85%. H_2 = 9%, S = 3%, O_2 = 1.5%, Ash = 1.5%. For an air-fuel ratio of 12 : 1. Calculate : (*i*) The mixture strength as a percentage rich or weak. (*ii*) The volumetric analysis of the dry products of combustion.

[**Ans.** 8.83% rich ; CO_2 = 12.16%, CO = 2.54%, N_2 = 84.83%, SO_2 = 0.47%]

10. Determine the gravimetric analysis of the products of complete combustion of acetylene (C_2H_2) with 125 per cent stoichiometric air. [**Ans.** CO_2 = 19.5%, H_2O = 3.9%, O_2 = 4.4%, N_2 = 72.2%]

11. One kg of ethane (C_2H_6) is burned with 80% of theoretical air. Assuming complete combustion of the hydrogen in the fuel determine the volumetric analysis of the dry products of combustion.

[**Ans.** CO_2 = 4.8%, CO = 11.2%, N_2 = 84%]

12. The gravimetric analysis of a sample of coal is given as 80% C, 12% H_2 and 8% ash. Calculate the stoichiometric A/F ratio and the analysis of the products by volume. [**Ans.** CO_2 = 13.6%, H_2 = 12.2%, N_2 = 74.2%]

13. Calculate the stoichiometric air-fuel ratio for the combustion of a sample of dry anthracite of the following composition by mass :

C = 90 per cent ; H_2 = 3 per cent ; N_2 = 1 per cent ; Sulphur = 0.5 per cent ; ash = 3 per cent.

If 20 per cent excess air is supplied determine :

(*i*) Air-fuel ratio

(*ii*) Wet analysis of the products of combustion by volume.

[**Ans.** 11.25/1 (*i*) 13.5/1 ; (*ii*) CO_2 = 16.3%, H_2O = 0.03%, SO_2 = 3.51%, N_2 = 80.3%]

14. The following is the analysis of a supply of coal gas :

H_2 = 49.4 per cent ; CO = 18 per cent ; CH_4 = 20 per cent ; C_4H_8 = 2 per cent ; O_2 = 0.4 per cent ; N_2 = 6.2 per cent ; CO_2 = 4 per cent.

(*i*) Calculate the stoichiometric A/F ratio.

(*ii*) Find also the wet and dry analyses of the products of combustion if the actual mixture is 20 per cent weak.

[**Ans.** (*i*) 4.06/1 by volume ; (*ii*) Wet analysis : CO_2 = 9.0%, H_2O = 17.5%, O_2 = 3.08%, N_2 = 70.4%. Dry analysis : CO_2 = 10.9%, O_2 = 3.72%, N_2 = 85.4%]

15. Find the stoichiometric air-fuel ratio for the combustion of ethyl alcohol (C_2H_6O), in a petrol engine. Calculate the air-fuel ratios for the extreme mixture strengths of 90% and 120%. Determine also the wet and dry analyses by volume of the exhaust gas for each mixture strength.

[**Ans.** 8.96/1 ; 9.95/1 ; 7.47/1, Wet analysis : CO_2 = 11.2%, H_2O = 16.8%, O_2 = 1.85%, N_2 = 70.2%
Dry analysis : CO_2 = 13.45%, O_2 = 2.22%, N_2 = 84.4%
Wet analysis : CO_2 = 6.94%, CO = 6.94%, H_2 = 20.8%, N_2 = 65.3%
Dry analysis : CO_2 = 8.7%, CO = 8.7%, N_2 = 82.5%]

16. The chemical analysis of a fuel by weight is as follows :

Carbon = 50 per cent ; Hydrogen = 25 per cent ; Oxygen = 10 per cent ; Sulphur = 5 per cent and Nitrogen = 10 per cent.

Find the stoichiometric amount of air required for complete combustion of this fuel. [**Ans.** 14.26 kg]

17. The percentage composition of a fuel by weight is as follows :

Carbon = 89.3 per cent ; Hydrogen = 5 per cent ; Oxygen = 4.2 per cent ; Nitrogen = 1.5 per cent and the remainder ash. Determine the stoichiometric air-fuel ratio by mass.

If 30 per cent excess air is supplied, find the percentage composition of dry flue gases by volume.

[**Ans.** 11.74 ; CO_2, = 14.3%, O_2 = 4.9%, N_2 = 80.8%]

18. Orsat analysis of the products of combustion of hydrocarbon fuel of unknown composition is as follows :

Carbon dioxide (CO_2) = 9% Carbon monoxide (CO) = 0.6%
Oxygen (O_2) = 7.3% Nitrogen (N_2) = 83.1%

Determine the following :

(*i*) Air-fuel ratio (*ii*) Per cent theoretical air required for combustion.

[**Ans.** (*i*) 22.1, (*ii*) 146.2%]

19. An Orsat analysis of the exhaust from an engine running on benzole showed a CO_2 content of 15 per cent, but no CO. Assuming that the remainder of the exhaust contains only oxygen and nitrogen, calculate the air-fuel ratio of the engine.

The ultimate analysis of benzole is C = 90 per cent and H_2 = 10%. [**Ans.** 15.2/1]

20. The analysis of the dry exhaust from and internal-combustion engine gave :
CO_2 = 12 per cent ; CO = 2 per cent ; CH_4 = 4 per cent ; H_2 = 1 per cent ; O_2 = 4.5 per cent and the remainder nitrogen.
Calculate the proportions by mass of carbon to hydrogen in the fuel, assuming it to be a pure hydrocarbon.
[**Ans.** 7.35/1]

21. The following is the percentage analysis by mass of a fuel :
Hydrogen (H_2) = 10 per cent   Oxygen (O_2) = 2 per cent
Sulphur (S) = 1 per cent   Nitrogen (N_2) = 3 per cent
Determine the following :
(*i*) The amount of air required to completely burn 1 kg of this fuel.
(*ii*) The products of combustion as a percentage by mass.
[**Ans.** 13.17 kg ; CO_2 = 21.7% ; H_2O = 6.35% ; SO_2 = 0.141% ; N_2 = 71.75%]

22. An Orsat analysis of the products of combustion resulting from the burning in air of a hydrocarbon fuel yielded the following :
Carbon dioxide (CO_2) = 12.2%   Oxygen (O_2) = 1.1%
Carbon monoxide (CO) = 0.5%   Nitrogen (N_2) = 86.2%
Determine : (*i*) The mass fraction of carbon in the fuel.
(*ii*) Air-fuel ratio.
(*iii*) Per cent of air theoretically needed for complete combustion.
[**Ans.** (*i*) 80.4%, (*ii*) 16.57/1, (*iii*) 103.9%]

23. If the higher heating value at constant pressure (C_6H_6) at 25°C is 3298354 kJ/kg mole, determine its lower calorific value at constant pressure. [**Ans.** 316647 kJ/mole]

24. The lower heating value of propane at constant pressure and 25°C is 2042055 kJ/kg mole. Find the higher heating value at constant pressure and at constant volume.
[**Ans.** 2217816 kJ/kg mole, 2210333 kJ/kg mole]

25. The higher heating value of kerosene at constant volume whose ultimate analysis is 86% carbon and 14% hydrogen, was found to be 46890 kJ/kg. Calculate the other three heating values.
[**Ans.** $(LHV)_v$ = 43987 kJ/kg ; $(HHV)_p$ = 46977 kJ/kg ; $(LHV)_p$ = 43900 kJ/kg]

12

Vapour Power Cycles

12.1. Carnot cycle. 12.2. Rankine cycle. 12.3. Modified Rankine cycle. 12.4. Regenerative cycle. 12.5. Reheat cycle. 12.6. Binary vapour cycle. Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

12.1. CARNOT CYCLE

Figure 12.1 shows a Carnot cycle on *T-s* and *p-V* diagrams. It consists of (*i*) two constant pressure operations (4-1) and (2-3) and (*ii*) two frictionless adiabatics (1-2) and (3-4). These operations are discussed below :

1. **Operation (4-1).** 1 kg of boiling water at temperature T_1 is heated to form wet steam of dryness fraction x_1. Thus heat is absorbed at constant temperature T_1 and pressure p_1 during this operation.

2. **Operation (1-2).** During this operation steam is expanded isentropically to temperature T_2 and pressure p_2. The point '2' represents the condition of steam after expansion.

3. **Operation (2-3).** During this operation heat is rejected at constant pressure p_2 and temperature T_2. As the steam is exhausted it becomes wetter and cooled from 2 to 3.

4. **Operation (3-4).** In this operation the wet steam at '3' is compressed isentropically till the steam regains its original state of temperature T_1 and pressure p_1. Thus cycle is completed.

Refer *T-s* diagram :

Heat supplied at constant temperature T_1 [operation (4-1)] = area 4-1-*b*-*a* = $T_1 (s_1 - s_4)$ or $T_1 (s_2 - s_3)$.

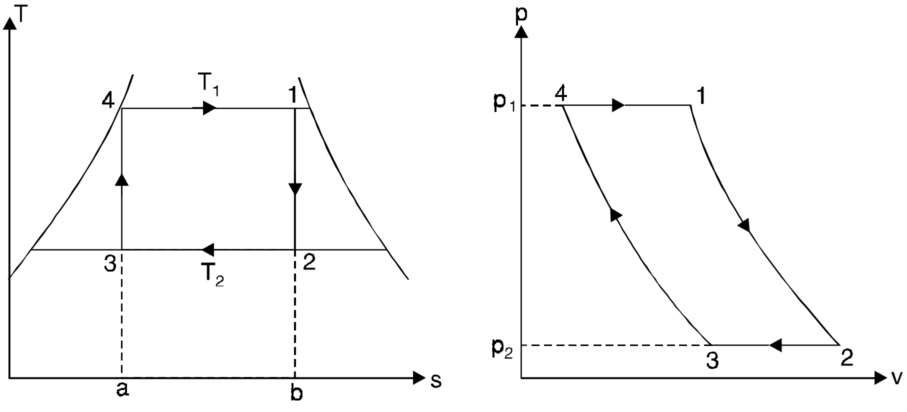


Fig. 12.1. Carnot cycle on *T-s* and *p-V* diagrams.

Heat rejected at constant temperature T_2 (operation 2-3) = area 2-3-*a*-*b* = $T_2 (s_2 - s_3)$.

Since there is no exchange of heat during isentropic operations (1-2) and (3-4)

Net work done = Heat supplied – heat rejected

$$= T_1 (s_2 - s_3) - T_2 (s_2 - s_3)$$

$$= (T_1 - T_2)(s_2 - s_3).$$

$$\text{Carnot cycle } \eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{(T_1 - T_2)(s_2 - s_3)}{T_1 (s_2 - s_3)} = \frac{T_1 - T_2}{T_1} \quad \ldots(12.1)$$

Limitations of Carnot Cycle

Though Carnot cycle is simple (thermodynamically) and has the *highest thermal efficiency* for given values of T_1 and T_2, yet it is *extremely difficult to operate in practice* because of the following *reasons* :

1. It is difficult to compress a wet vapour isentropically to the saturated state as required by the process 3-4.

2. It is difficult to control the quality of the condensate coming out of the condenser so that the state '3' is exactly obtained.

3. The efficiency of the Carnot cycle is greatly affected by the temperature T_1 at which heat is transferred to the working fluid. Since the critical temperature for steam is only 374°C, therefore, if the cycle is to be operated in the *wet region*, the maximum possible temperature is severely limited.

4. The cycle is still more difficult to operate in practice with superheated steam due to the necessity of supplying the superheat at constant temperature instead of constant pressure (as it is customary).

- *In a practical cycle, limits of pressure and volume are far more easily realised than limits of temperature so that at present no practical engine operates on the Carnot cycle, although all modern cycles aspire to achieve it.*

12.2. RANKINE CYCLE

Rankine cycle *is the theoretical cycle on which the steam turbine (or engine) works.*

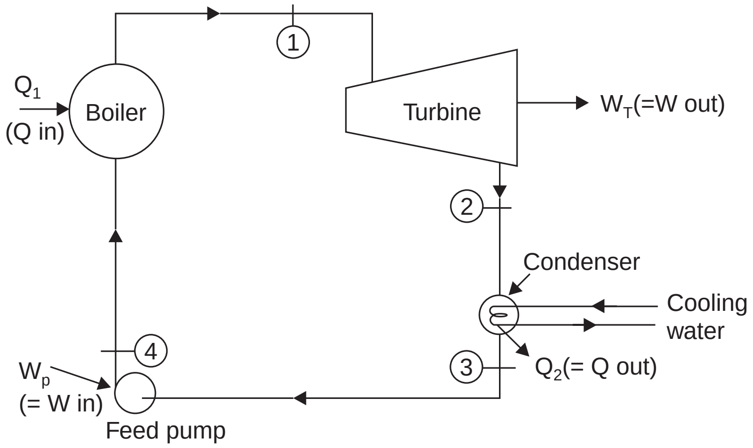
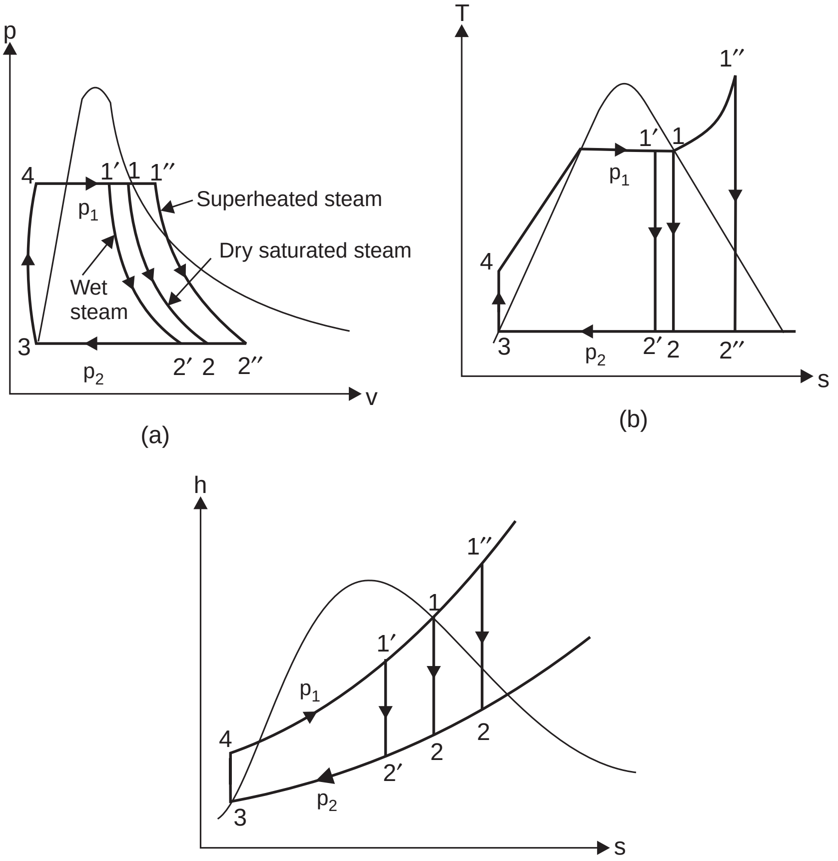


Fig. 12.2. Rankine cycle.



(a)

(b)

(c)

Fig. 12.3. (*a*) *p-v* diagram ; (*b*) *T-s* diagram ; (*c*) *h-s* diagram for Rankine cycle.

The Rankine cycle is shown in Fig. 12.2. It comprises of the following *processes* :

Process 1-2 : Reversible adiabatic expansion in the turbine (or steam engine).

Process 2-3 : Constant-pressure transfer of heat in the condenser.

Process 3-4 : Reversible adiabatic pumping process in the feed pump.

Process 4-1 : Constant-pressure transfer of heat in the boiler.

Fig. 12.3 shows the Rankine cycle on *p-v*, *T-s* and *h-s* diagrams (when the saturated steam enters the turbine, the steam can be wet or superheated also).

Considering 1 kg of fluid :

Applying *steady flow energy equation* (S.F.E.E.) to boiler, turbine, condenser and pump :

(*i*) **For boiler** (as control volume), we get

$$h_{f_4} + Q_1 = h_1$$

$$\therefore \qquad Q_1 = h_1 - h_{f_4} \qquad ...(12.2)$$

(*ii*) **For turbine** (as control volume), we get

$$h_1 = W_T + h_2, \text{ where } W_T = \text{turbine work}$$

$$\therefore \qquad W_T = h_1 - h_2 \qquad ...(12.3)$$

(*iii*) **For condenser**, we get

$$h_2 = Q_2 + h_{f_3}$$

$$\therefore \quad Q_2 = h_2 - h_{f_3} \qquad \text{...(12.4)}$$

(*iv*) **For the feed pump**, we get

$$h_{f_3} + W_P = h_{f_4}, \qquad \text{where, } W_P = \text{Pump work}$$

$$\therefore \quad W_P = h_{f_4} - h_{f_3}$$

Now, efficiency of Rankine cycle is given by

$$\eta_{\text{Rankine}} = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$= \frac{(h_1 - h_2) - (h_{f_4} - h_{f_3})}{(h_1 - h_{f_4})} \qquad \text{...(12.5)}$$

The feed pump handles liquid water which is incompressible which means with the increase in pressure its density or specific volume undergoes a little change. Using general property relation for reversible adiabatic compression, we get

$$Tds = dh - vdp$$

$$\because \quad ds = 0$$

$$\therefore \quad dh = vdp$$

or $$\Delta h = v \, \Delta p \qquad \text{...... (since change in specific volume is negligible)}$$

or $$h_{f_4} - h_{f_3} = v_3 \, (p_1 - p_2)$$

When p is in bar and v is in m^3/kg, we have

$$h_{f_4} - h_{f_3} = v_3 \, (p_1 - p_2) \times 10^5 \text{ J/kg}$$

The feed pump term $(h_{f_4} - h_{f_3})$ being a small quantity in comparison with turbine work, W_T, is usually neglected, *especially when the boiler pressures are low*.

Then, $$\eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_4}} \qquad \text{...[12.5 (a)]}$$

Comparison between Rankine Cycle and Carnot Cycle

The following points are worth noting :

(*i*) Between the same temperature limits Rankine cycle provides a higher specific work output than a Carnot cycle, consequently Rankine cycle *requires a smaller steam flow rate resulting in smaller size plant for a given power output*. However, Rankine cycle calls for higher *rates of heat transfer in boiler and condenser*.

(*ii*) Since in Rankine cycle only part of the heat is supplied isothermally at constant higher temperature T_1, therefore, its *efficiency is lower* than that of Carnot cycle. The efficiency of the Rankine cycle will approach that of the Carnot cycle more nearly if the *superheat temperature rise is reduced*.

(*iii*) The advantage of using pump to feed liquid to the boiler instead to compressing a wet vapour is obvious that the *work for compression is very large compared to the pump*.

Fig. 12.4 shows the plots between efficiency and specific steam consumption against boiler pressure for Carnot and ideal Rankine cycles.

Effect of Operating Conditions on Rankine Cycle Efficiency

The Rankine cycle efficiency can be *improved* by :

(*i*) *Increasing the average temperature at which heat is supplied.*

(*ii*) *Decreasing/reducing the temperature at which heat is rejected.*

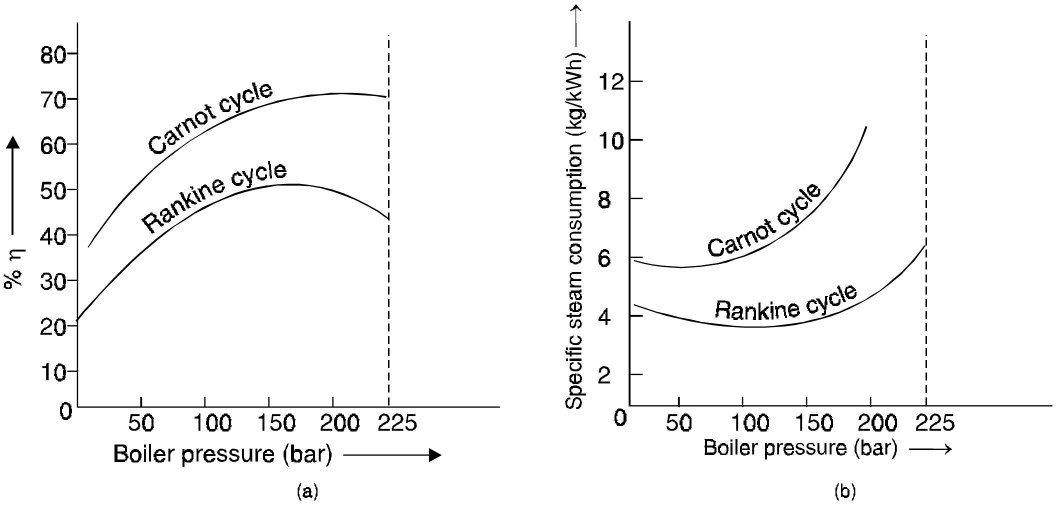


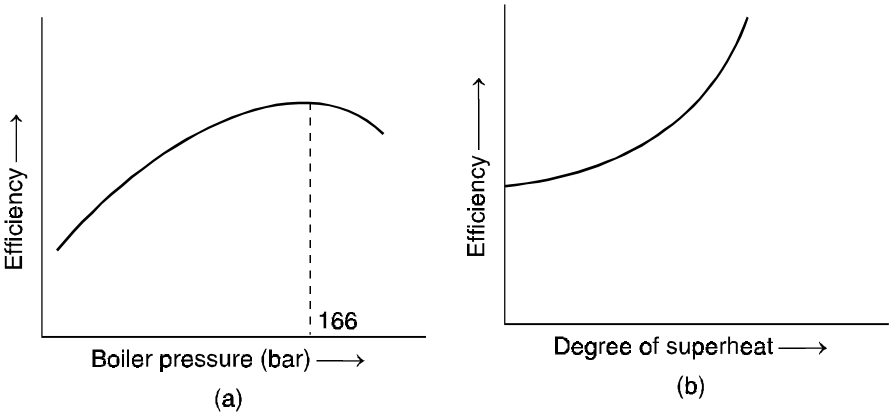
Fig. 12.4

This can be achieved by making suitable changes in the conditions of steam generation or condensation, as discussed below :

1. ***Increasing boiler pressure.*** It has been observed that by increasing the boiler pressure (other factors remaining the same) the cycle tends to rise and reaches a maximum value at a boiler pressure of about 166 bar [Fig. 12.5 (*a*)].

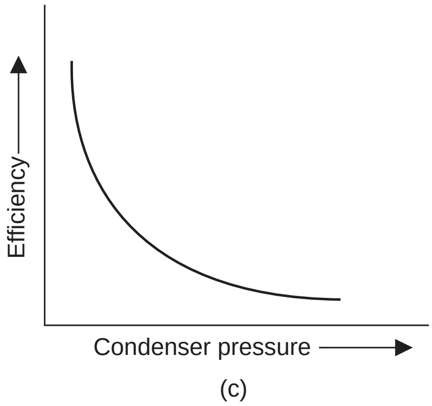
2. ***Superheating.*** All other factors remaining the same, if the steam is superheated before allowing it to expand the Rankine cycle efficiency may be increased [Fig. 12.5 (*b*)]. The use of superheated steam also ensures longer turbine blade life because of the absence of erosion from high velocity water particles that are suspended in wet vapour.

3. ***Reducing condenser pressure.*** The thermal efficiency of the cycle can be amply improved by reducing the condenser pressure [Fig. 12.5 (*c*)] (hence by reducing the temperature at which heat is rejected), especially in high vacuums. But the increase in efficiency is obtained at the *increased cost of condensation apparatus.*



(a)

(b)



(c)

Fig. 12.5. Effect of operating conditions on the thermal efficiency of the Rankine cycle.

The thermal efficiency of the Rankine cycle is also *improved* by the following methods :

(*i*) *By regenerative feed heating.*
(*ii*) *By reheating of steam.*
(*iii*) *By water extraction.*
(*iv*) *By using binary-vapour.*

☞**Example 12.1.** *The following data refer to a simple steam power plant :*

S. No.	*Location*	*Pressure*	*Quality/temp.*	*Velocity*
1.	*Inlet to turbine*	*6 MPa (= 60 bar)*	*380°C*	—
2.	*Exit from turbine inlet to condenser*	*10 kPa (= 0.1 bar)*	*0.9*	*200 m/s*
3.	*Exit from condenser and inlet to pump*	*9 kPa (= 0.09 bar)*	*Saturated liquid*	—
4.	*Exit from pump and inlet to boiler*	*7 MPa (= 70 bar)*	—	—
5.	*Exit from boiler*	*6.5 MPa (= 65 bar)*	*400°C*	—
	Rate of steam flow = 10000 kg/h.			

Calculate :

(*i*) *Power output of the turbine.*
(*ii*) *Heat transfer per hour in the boiler and condenser separately.*
(*iii*) *Mass of cooling water circulated per hour in the condenser. Choose the inlet temperature of cooling water 20°C and 30°C at exit from the condenser.*
(*iv*) *Diameter of the pipe connecting turbine with condenser.*

Solution. Refer Fig. 12.6.

(*i*) **Power output of the turbine, P :**

At 60 bar, 380°C : From steam tables,

$$h_1 = 3043.0 \text{ (at 350°C)} + \frac{3177.2 - 3043.0}{(400 - 350)} \times 30 \text{ ... By interpolation}$$

$$= 3123.5 \text{ kJ/kg}$$

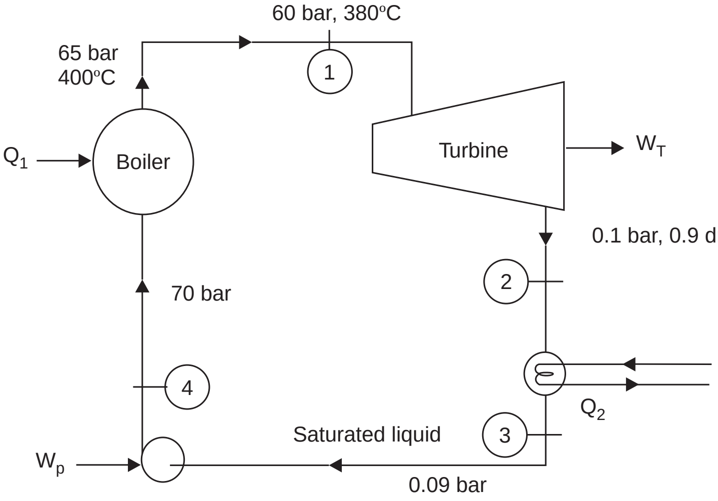


Fig. 12.6

At 0.1 bar :

$$h_{f_2} = 191.8 \text{ kJ/kg}, \; h_{fg_2} = 2392.8 \text{ kJ/kg (from steam tables)}$$

and $$x_2 = 0.9 \text{ (given)}$$

$$\therefore \quad h_2 = h_{f_2} + x_2 \, h_{fg_2} = 191.8 + 0.9 \times 2392.8 = 2345.3 \text{ kJ/kg}$$

Power output of the turbine = m_s ($h_1 - h_2$) kW,

[where m_s = Rate of steam flow in kg/s and h_1, h_2 = Enthalpy of steam in kJ/kg]

$$= \frac{10000}{3600} (3123.5 - 2345.3) = 2162 \text{ kW}$$

Hence power output of the turbine **= 2162 kW. (Ans.)**

(*ii*) **Heat transfer per hour in the boiler and condenser :**

At 70 bar : h_{f_4} = 1267.4 kJ/kg

At 65 bar, 400°C : $$h_a = \frac{3177.2\,(60 \text{ bar}) + 3158.1\,(70 \text{ bar})}{2} = 3167.6 \text{ kJ/kg}$$

......(By interpolation)

∴ *Heat transfer per hour in the boiler,*

$$Q_1 = 10000\,(h_a - h_{f_4}) \text{ kJ/h}$$

$$= 10000\,(3167.6 - 1267.4) = \mathbf{1.9 \times 10^7 \text{ kJ/h.}} \quad \textbf{(Ans.)}$$

At 0.09 bar : h_{f_3} = 183.3 kJ/kg

Heat transfer per hour in the condenser,

$$Q_1 = 10000\,(h_2 - h_{f_3})$$

$$= 10000\,(2345.3 - 183.3) = \mathbf{2.16 \times 10^7 \text{ kJ/h.}} \quad \textbf{(Ans.)}$$

(*iii*) **Mass of cooling water circulated per hour in the condenser, $\mathbf{m_w}$:**

Heat lost by steam = Heat gained by the cooling water

$$Q_2 = m_w \times c_{pw}\,(t_2 - t_1)$$

$$2.16 \times 10^7 = m_w \times 4.18\,(30 - 20)$$

$$\therefore \quad \mathbf{m_w} = \frac{2.16 \times 10^7}{4.18\,(30 - 20)} = \mathbf{1.116 \times 10^7 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

(*iv*) **Diameter of the pipe connecting turbine with condenser, d :**

$$\frac{\pi}{4} d^2 \times C = m_s\, x_2\, v_{g_2} \qquad \text{...}(i)$$

Here, d = Diameter of the pipe (m),

C = Velocity of steam = 200 m/s (given),

m_s = Mass of steam in kg/s,

x_2 = Dryness fraction at '2', and

v_{g_2} = Specific volume at pressure 0.1 bar (= 14.67 m^3/kg).

Substituting the various values in eqn. (*i*), we get

$$\frac{\pi}{4} d^2 \times 200 = \frac{10000}{3600} \times 0.9 \times 14.67$$

$$d = \left(\frac{10000 \times 0.9 \times 14.67 \times 4}{3600 \times \pi \times 200}\right)^{1/2} = 0.483 \text{ m or } \mathbf{483\ mm.} \quad \textbf{(Ans.)}$$

Example 12.2. *In a steam power cycle, the steam supply is at 15 bar and dry and saturated. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle. Neglect pump work.*

Solution. Steam supply pressure, p_1 = 15 bar, x_1 = 1

Condenser pressure, p_2 = 0.4 bar

Carnot and Rankine efficiencies :

From steam tables :

At 15 bar : t_s = 198.3°C, h_g = 2789.9 kJ/kg, s_g = 6.4406 kJ/kg K

At 0.4 bar : t_s = 75.9°C, h_f = 317.7 kJ/kg, h_{fg} = 2319.2 kJ/kg,

s_f = 1.0261 kJ/kg K, s_{fg} = 6.6448 kJ/kg K

T_1 = 198.3 + 273 = 471.3 K

T_2 = 75.9 + 273 = 348.9 K

$$\therefore \quad \eta_{\textbf{carnot}} = \frac{T_1 - T_2}{T_1} = \frac{471.3 - 348.9}{471.3}$$

= **0.259 or 25.9%. (Ans.)**

$$\eta_{\textbf{Rankine}} = \frac{\text{Adiabatic or isentropic heat drop}}{\text{Heat supplied}} = \frac{h_1 - h_2}{h_1 - h_{f_2}}$$

where $h_2 = h_{f_2} + x_2\, h_{fg_2} = 317.7 + x_2 \times 2319.2$ ...(*i*)

Value of x_2 :

As the steam expands isentropically,

$$\therefore \quad s_1 = s_2$$

$$6.4406 = s_{f_2} + x_2\, s_{fg_2} = 1.0261 + x_2 \times 6.6448$$

$$\therefore \quad x_2 = \frac{6.4406 - 1.0261}{6.6448} = 0.815$$

$$\therefore \quad h_2 = 317.7 + 0.815 \times 2319.2 = 2207.8 \text{ kJ/kg} \qquad \text{[From eqn. }(i)]$$

Hence, $$\eta_{\textbf{Rankine}} = \frac{2789.9 - 2207.8}{2789.9 - 317.7} = \mathbf{0.2354 \text{ or } 23.54\%.} \quad \textbf{(Ans.)}$$

Example 12.3. *In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam the net work and the cycle efficiency.*

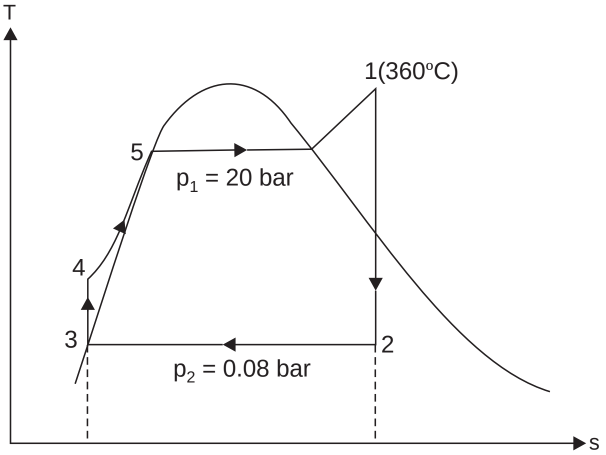


Fig. 12.7

Solution. Boiler pressure, p_1 = 20 bar (360°C)

Condenser pressure, p_2 = 0.08 bar

From steam tables :

At 20 bar (p_1), 360°C : h_1 = 3159.3 kJ/kg

s_1 = 6.9917 kJ/kg-K

At 0.08 bar (p_2) : $h_3 = h_{f(p_2)}$ = 173.88 kJ/kg,

$s_3 = s_{f(p_2)}$ = 0.5926 kJ/kg-K

$h_{fg(p_2)}$ = 2403.1 kJ/kg, $s_{g(p_2)}$ = 8.2287 kJ/kg-K

$v_{f(p_2)}$ = 0.001008 m³/kg $\therefore$ $s_{fg(p_2)}$ = 7.6361 kJ/kg-K

Now $s_1 = s_2$

$$6.9917 = s_{f(p_2)} + x_2\, s_{fg(p_2)} = 0.5926 + x_2 \times 7.6361$$

$$\therefore \quad x_2 = \frac{0.69917 - 0.5926}{7.6361} = 0.838$$

$$\therefore \quad h_2 = h_{f(p_2)} + x_2\, h_{fg(p_2)}$$

$$= 173.88 + 0.838 \times 2403.1 = 2187.68 \text{ kJ/kg.}$$

Net work, W_{net} :

$$W_{net} = W_{turbine} - W_{pump}$$

$$W_{pump} = h_{f_4} - h_{f(p_2)}\ (= h_{f_3}) = v_{f(p_2)}\ (p_1 - p_2)$$

$$= 0.00108 \text{ (m}^3\text{/kg)} \times (20 - 0.08) \times 100 \text{ kN/m}^2$$

$$= 2.008 \text{ kJ/kg}$$

$$[\text{and} \quad h_{f_4} = 2.008 + h_{f(p_2)} = 2.008 + 173.88 = 175.89 \text{ kJ/kg}]$$

$$W_{\text{turbine}} = h_1 - h_2 = 3159.3 - 2187.68 = 971.62 \text{ kJ/kg}$$

$$\therefore \quad W_{\text{net}} = 971.62 - 2.008 = \mathbf{969.61 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

Cycle efficiency, η_{cycle} **:**

$$Q_1 = h_1 - h_{f_4} = 3159.3 - 175.89 = 2983.41 \text{ kJ/kg}$$

$$\therefore \quad \eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_1} = \frac{969.61}{2983.41} = \mathbf{0.325 \text{ or } 32.5\%.} \quad \textbf{(Ans.)}$$

Example 12.4. *A Rankine cycle operates between pressures of 80 bar and 0.1 bar. The maximum cycle temperature is 600°C. If the steam turbine and condensate pump efficiencies are 0.9 and 0.8 respectively, calculate the specific work and thermal efficiency. Relevant steam table extract is given below.*

p(bar)	*t(°C)*	*Specific volume (m³/kg)*		*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*		
		v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0.1	*45.84*	*0.0010103*	*14.68*	*191.9*	*2392.3*	*2584.2*	*0.6488*	*7.5006*	*8.1494*
80	*295.1*	*0.001385*	*0.0235*	*1317*	*1440.5*	*2757.5*	*3.2073*	*2.5351*	*5.7424*

80 bar, 600°C	*v*	*0.486 m³/kg*
Superheat	*h*	*3642 kJ/kg*
table	*s*	*7.0206 kJ/kgK*

(GATE, 1998)

Solution. Refer Fig. 12.8

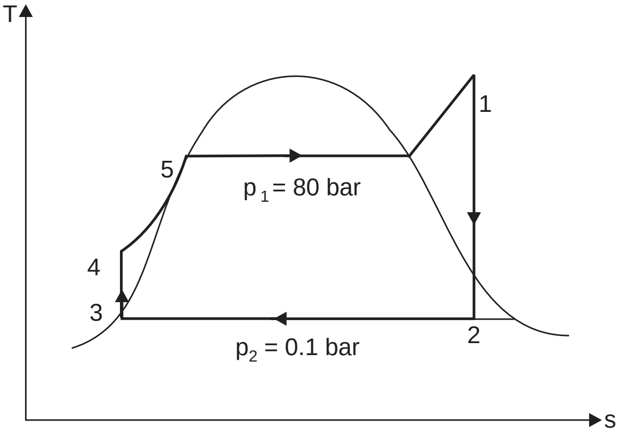


Fig. 12.8

At 80 bar, 600°C :

h_1 = 3642 kJ / kg ;

s_1 = 7.0206 kJ / kg K.

Since $s_1 = s_2$,

$$\therefore \quad 7.0206 = s_{f_2} + x_2 \, s_{fg_2}$$

$$= 0.6488 + x_2 \times 7.5006$$

or

$$x_2 = \frac{7.0206 - 0.6488}{7.5006} = 0.85$$

Now, $h_2 = h_{f_2} + x_2 \, h_{fg_2}$

$$= 191.9 + 0.85 \times 2392.3$$

$$= 2225.36 \text{ kJ/kg}$$

Actual turbine work

$$= \eta_{\text{turbine}} \times (h_1 - h_2)$$

$$= 0.9\ (3642 - 2225.36) = 1275 \text{ kJ/kg}$$

Pump work $= v_{f(p_2)}\,(p_1 - p_2)$

$$= 0.0010103\ (80 - 0.1) \times \frac{10^5}{10^3} \text{ kN/m}^2 = 8.072 \text{ kJ/kg}$$

Actual pump work $= \frac{8.072}{\eta_{\text{pump}}} = \frac{8.072}{0.8} = 10.09$ kJ/kg

Specific work $(W_{\text{net}}) = 1275 - 10.09 =$ **1264.91 kJ / kg. (Ans.)**

Thermal efficiency $= \frac{W_{\text{net}}}{Q_1}$

where, $Q_1 = h_1 - h_{f_4}$

But $h_{f_4} = h_{f_3}$ + pump work = 191.9 + 10.09 = 202 kJ/kg

∴ Thermal efficiency, $\eta_{\text{th}} = \frac{1264.91}{3642 - 202}$ = **0.368 or 36.8 %. (Ans.)**

Example 12.5. *A simple Rankine cycle works between pressures 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cycle efficiency, work ratio and specific steam consumption.*

Solution.

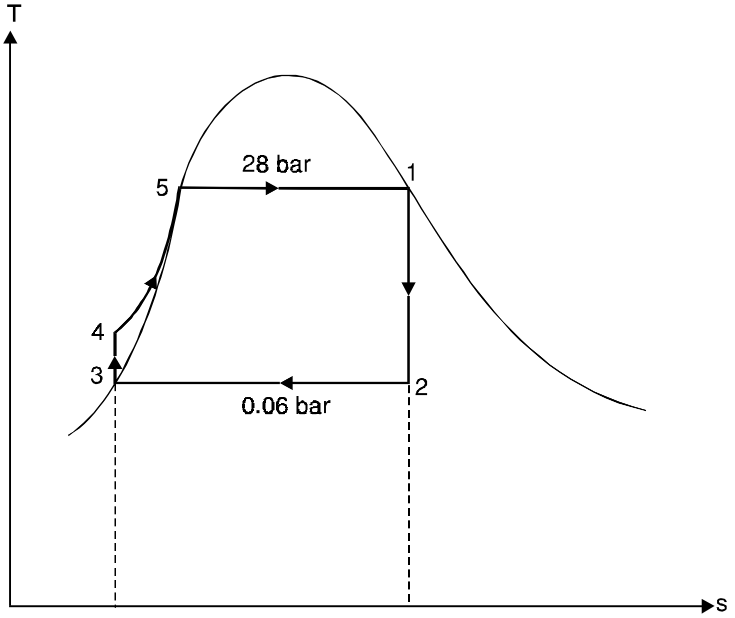


Fig. 12.9

From steam tables,

At 28 bar : $h_1 = 2802$ kJ/kg, $s_1 = 6.2104$ kJ/kg K

At 0.06 bar : $h_{f_2} = h_{f_3} = 151.5$ kJ/kg, $h_{fg_2} = 2415.9$ kJ/kg,

$s_{f_2} = 0.521$ kJ/kg K, $s_{fg_2} = 7.809$ kJ/kg K

$v_f = 0.001$ m³/kg

Considering turbine *process 1-2*, we have :

$$s_1 = s_2$$

$$6.2104 = s_{f_2} + x_2\, s_{fg_2} = 0.521 + x_2 \times 7.809$$

$\therefore \quad x_2 = \dfrac{6.2104 - 0.521}{7.809} = 0.728$

$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2}$

$= 151.5 + 0.728 \times 2415.9 = 1910.27$ kJ/kg

$\therefore$ Turbine work, $W_{\text{turbine}} = h_1 - h_2 = 2802 - 1910.27 = 891.73$ kJ/kg

Pump work, $W_{\text{pump}} = h_{f_4} - h_{f_3} = v_f\,(p_1 - p_2)$

$= \dfrac{0.001\,(28 - 0.06) \times 10^5}{1000} = 2.79$ kJ/kg

$[\because \quad h_{f_4} = h_{f_3} + 2.79 = 151.5 + 2.79 = 154.29$ kJ/kg$]$

$\therefore$ Net work, $W_{\text{net}} = W_{\text{turbine}} - W_{\text{pump}}$

$= 891.73 - 2.79 = 888.94$ kJ/kg

Cycle efficiency $= \dfrac{W_{\text{net}}}{Q_1} = \dfrac{888.94}{h_1 - h_{f_4}}$

$= \dfrac{888.94}{2802 - 154.29}$ = **0.3357 or 33.57%. (Ans.)**

Work ratio $= \dfrac{W_{\text{net}}}{W_{\text{turbine}}} = \dfrac{888.94}{891.73}$ = **0.997. (Ans.)**

Specific steam consumption $= \dfrac{3600}{W_{\text{net}}} = \dfrac{3600}{888.94}$ = **4.049 kg/kWh. (Ans.)**

☞**Example 12.6.** *In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 35 bar and the exhaust pressure is 0.2 bar. Determine :*

(*i*) *The pump work,* (*ii*) *The turbine work,*
(*iii*) *The Rankine efficiency,* (*iv*) *The condenser heat flow,*
(*v*) *The dryness at the end of expansion.*

Assume flow rate of 9.5 kg/s.

Solution. Pressure and condition of steam, at inlet to the turbine,

$p_1 = 35$ bar, $x = 1$

Exhaust pressure, $p_2 = 0.2$ bar

Flow rate, $\dot{m} = 9.5$ kg/s

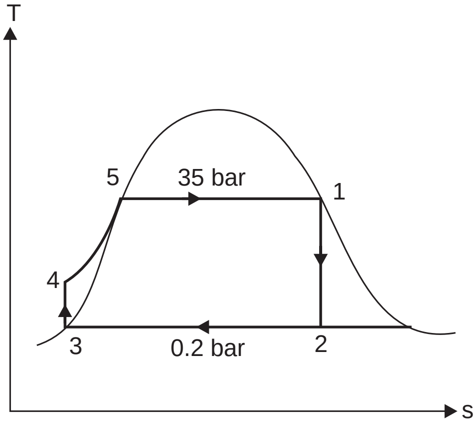


Fig. 12.10

From steam tables :

At 35 bar : $h_1 = h_{g_1}$ = 2802 kJ/kg, s_{g_1} = 6.1228 kJ/kg K

At 0.26 bar : h_f = 251.5 kJ/kg, h_{fg} = 2358.4 kJ/kg,

v_f = 0.001017 m^3/kg, s_f = 0.8321 kJ/kg K, s_{fg} = 7.0773 kJ/kg K.

(*i*) **The pump work :**

Pump work $= (p_4 - p_3)\, v_f = (35 - 0.2) \times 10^5 \times 0.001017$ J or 3.54 kJ/kg

$$\begin{bmatrix} \text{Also } h_{f_4} - h_{f_3} = \text{Pump work} = 3.54 \\ \therefore \quad h_{f_4} = 251.5 + 3.54 = 255.04 \text{ kJ / kg} \end{bmatrix}$$

Now power required to drive the pump

= 9.5 × 3.54 kJ/s or **33.63 kW. (Ans.)**

(*ii*) **The turbine work :**

$$s_1 = s_2 = s_{f_2} + x_2 \times s_{fg_2}$$

$$6.1228 = 0.8321 + x_2 \times 7.0773$$

$$\therefore \quad x_2 = \frac{6.1228 - 0.8321}{7.0773} = 0.747$$

$$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2} = 251.5 + 0.747 \times 2358.4 = 2013 \text{ kJ/kg}$$

∴ *Turbine work* $= \dot{m}\,(h_1 - h_2) = 9.5\,(2802 - 2013) =$ **7495.5 kW. (Ans.)**

It may be noted that pump work (33.63 kW) is very small as compared to the turbine work (7495.5 kW).

(*iii*) **The Rankine efficiency :**

$$\eta_{\text{rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_2}} = \frac{2802 - 2013}{2802 - 251.5} = \frac{789}{2550.5} = \mathbf{0.3093 \text{ or } 30.93\%.} \quad \textbf{(Ans.)}$$

(*iv*) **The condenser heat flow :**

The condenser heat flow $= \dot{m}\,(h_2 - h_{f_3}) = 9.5\,(2013 - 251.5) =$ **16734.25 kW. (Ans.)**

(*v*) **The dryness at the end of expansion, x_2 :**

The dryness at the end of expansion,

x_2 = **0.747 or 74.7%. (Ans.)**

Example 12.7. *The adiabatic enthalpy drop across the primemover of the Rankine cycle is 840 kJ/kg. The enthalpy of steam supplied is 2940 kJ/kg. If the back pressure is 0.1 bar, find the specific steam consumption and thermal efficiency.*

Solution. Adiabatic enthalpy drop, $h_1 - h_2$ = 840 kJ/kg

Enthalpy of steam supplied, h_1 = 2940 kJ/kg

Back pressure, p_2 = 0.1 bar

From steam tables, corresponding to 0.1 bar : h_f = 191.8 kJ/kg

Now, $$\eta_{\text{rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_2}} = \frac{840}{2940 - 191.8} = 0.3056 = \mathbf{30.56\%.} \quad \textbf{(Ans.)}$$

Useful work done per kg of steam = 840 kJ/kg

∴ *Specific steam consumption* $= \frac{1}{840}$ kg/s $= \frac{1}{840} \times 3600 =$ **4.286 kg/kWh. (Ans.)**

☞**Example 12.8.** *A 35 kW (I.P.) system engines consumes 284 kg/h at 15 bar and 250°C. If condenser pressure is 0.14 bar, determine :*

(*i*) *Final condition of steam ;* (*ii*) *Rankine efficiency ;*
(*iii*) *Relative efficiency*.

Solution. Power developed by the engine = 35 kW (I.P.)
Steam consumption = 284 kg/h
Condenser pressure = 0.14 bar
Steam inlet pressure = 15 bar, 250°C.
From steam tables :

At 15 bar, 250°C : h = 2923.3 kJ/kg, s = 6.709 kJ/kg K

At 0.14 bar : h_f = 220 kJ/kg, h_{fg} = 2376.6 kJ/kg,
s_f = 0.737 kJ/kg K, s_{fg} = 7.296 kJ/kg K

(*i*) **Final condition of steam :**

Since steam expands isentropically.

$$\therefore \quad s_1 = s_2 = s_{f_2} + x_2\, s_{fg_2}$$

$$6.709 = 0.737 + x_2 \times 7.296$$

$$\therefore \quad x_2 = \frac{6.709 - 0.737}{7.296} = \mathbf{0.818 \simeq 0.82.} \quad \textbf{(Ans.)}$$

$$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2} = 220 + 0.82 \times 2376.6 = 2168.8 \text{ kJ/kg.}$$

(*ii*) **Rankine efficiency :**

$$\eta_{\text{rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_2}} = \frac{2923.3 - 2168.8}{2923.3 - 220} = \mathbf{0.279 \text{ or } 27.9\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Relative efficiency :**

$$\eta_{\text{thermal}} = \frac{\text{I.P.}}{\dot{m}\left(h_1 - h_{f_2}\right)} = \frac{35}{\frac{284}{3600}(2923.3 - 220)} = 0.1641 \text{ or } 16.41\%$$

$$\eta_{\text{relative}} = \frac{\eta_{\text{thermal}}}{\eta_{\text{rankine}}} = \frac{0.1641}{0.279}$$

$$= \mathbf{0.588 \text{ or } 58.8\%.} \quad \textbf{(Ans.)}$$

Example 12.9. *Calculate the fuel oil consumption required in a industrial steam plant to generate 5000 kW at the turbine shaft. The calorific value of the fuel is 40000 kJ/kg and the Rankine cycle efficiency is 50%. Assume appropriate values for isentropic turbine efficiency, boiler heat transfer efficiency and combustion efficiency.* **(AMIE Summer, 2000)**

Solution. Power to be generated at the turbine shaft, P = 5000 kW

The calorific value of the fuel, C = 40000 kJ/kg

Rankine cycle efficiency, η_{rankine} = 50%

Fuel oil combustion, $\mathbf{m_f}$:

Assume : η_{turbine} = 90% ; $\eta_{\text{heat transfer}}$ = 85% ; $\eta_{\text{combustion}}$ = 98%

$$\eta_{\text{rankine}} = \frac{\text{Shaft power} / \eta_{\text{turbine}}}{m_f \times C \times \eta_{\text{heat transfer}} \times \eta_{\text{combustion}}}$$

or

$$0.5 = \frac{(5000/0.9)}{m_f \times 40000 \times 0.85 \times 0.98}$$

$$\therefore \qquad m_f = \frac{(5000/0.9)}{0.5 \times 40000 \times 0.85 \times 0.98} = \textbf{0.3335 kg/s or 1200.6 kg/h.} \quad \textbf{(Ans.)}$$

12.3. MODIFIED RANKINE CYCLE

Figures 12.11 and 12.12 show the modified Rankine cycle on *p-V* and *T-s* diagrams (neglecting pump work) respectively. It will be noted that *p-V* diagram is very narrow at the toe *i.e.,* point '2″ and the work obtained near to *e* is very small. In fact this work is too inadequate to overcome friction (due to reciprocating parts) even. Therefore, the adiabatic is terminated at '2' ; the pressure drop decreases suddenly whilst the volume remains constant. This operation is represented by the line 2-3. By this doing the stroke length is reduced ; in other words the cylinder dimensions reduce but at the expense of small loss of work (area 2-3-2′) which, however, is negligibly small.

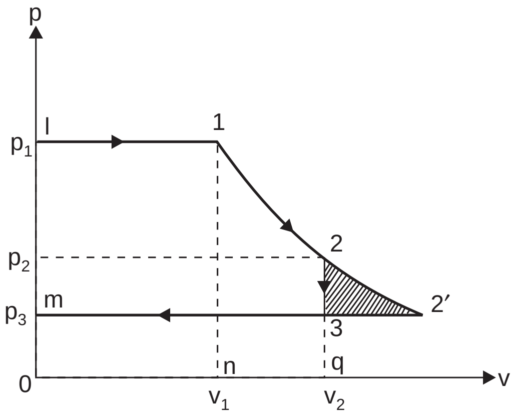


Fig. 12.11. *p-V* diagram of Modified Rankine Cycle.

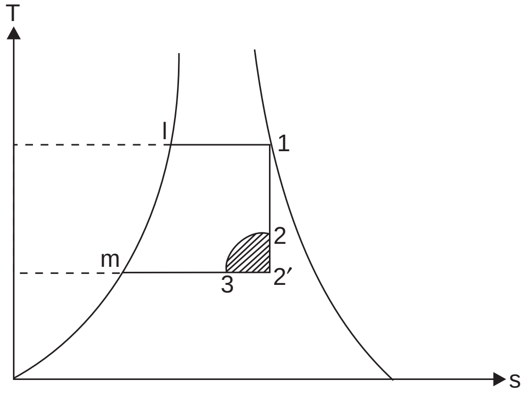


Fig. 12.12. *T-s* diagram of Modified Rankine cycle.

The work done during the modified Rankine cycle can be calculated in the following way :

Let p_1, v_1, u_1 and h_1 correspond to initial condition of steam at '1'.

p_2, v_2, u_2 and h_2 correspond to condition of steam at '2'.

p_3, h_3 correspond to condition of steam at '3″.

Work done during the cycle/kg of steam

$= \text{area } l\text{-1-2-3-}m$
$= \text{area '}o\text{-}l\text{-1-}n\text{' + area '1-2-}q\text{-}n\text{' – area '}o\text{-}m\text{-3-}q\text{'}$
$= p_1v_1 + (u_1 - u_2) - p_3v_2$

Heat supplied $= h_1 - h_{f_3}$

∴ The modified Rankine efficiency

$$= \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{p_1v_1 + (u_1 - u_2) - p_3v_2}{h_1 - h_{f_3}} \quad \text{...(12.6)}$$

Alternative method for finding modified Rankine efficiency :

Work done during the cycle/kg of steam

$= \text{area '}l\text{-1-2-3-}m\text{'}$
$= \text{area '}l\text{-1-2-}s\text{' + area '}s\text{-2-3-}m\text{'}$
$= (h_1 - h_2) + (p_2 - p_3)\, v_2$

Heat supplied $= h_1 - h_{f_3}$

Modified Rankine efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}}$

$$= \frac{(h_1 - h_2) + (p_2 - p_3)\, v_2}{h_1 - h_{f_3}} \quad \text{...(12.7)}$$

Note. Modified Rankine cycle is used for 'reciprocating steam engines' because stroke length and hence cylinder size is reduced with the sacrifice of practically a quite negligible amount of work done.

☞**Example 12.10. (Modified Rankine Cycle).** *Steam at a pressure of 15 bar and 300°C is delivered to the throttle of an engine. The steam expands to 2 bar when release occurs. The steam exhaust takes place at 1.1 bar. A performance test gave the result of the specific steam consumption of 12.8 kg/kWh and a mechanical efficiency of 80 per cent. Determine :*

(*i*) *Ideal work or the modified Rankine engine work per kg.*
(*ii*) *Efficiency of the modified Rankine engine or ideal thermal efficiency.*
(*iii*) *The indicated and brake work per kg.*
(*iv*) *The brake thermal efficiency.*
(*v*) *The relative efficiency on the basis of indicated work and brake work.*

Solution. Fig. 12.13 shows the *p-v* and *T-s* diagrams for modified Rankine cycle.

From steam tables :

1. ***At 15 bar, 300°C :*** $h_1 = 3037.6$ kJ/kg, $v_1 = 0.169$ m^3/kg,
$s_1 = 6.918$ kJ/kg K.

2. ***At 2 bar :*** $t_{s_2} = 120.2$°C, $h_{f_2} = 504.7$ kJ/kg, $h_{fg_2} = 2201.6$ kJ/kg,
$s_{f_2} = 1.5301$ kJ/kg K, $s_{fg_2} = 5.5967$ kJ/kg K,
$v_{f_2} = 0.00106$ m^3/kg, $v_{g_2} = 0.885$ m^3/kg.

3. ***At 1.1 bar :*** $t_{s_3} = 102.3$°C, $h_{f_3} = 428.8$ kJ/kg, $h_{fg_3} = 2250.8$ kJ/kg,
$s_{f_3} = 1.333$ kJ/kg K, $s_{fg_3} = 5.9947$ kJ/kg K,
$v_{f_3} = 0.001$ m^3/kg, $v_{g_3} = 1.549$ m^3/kg.

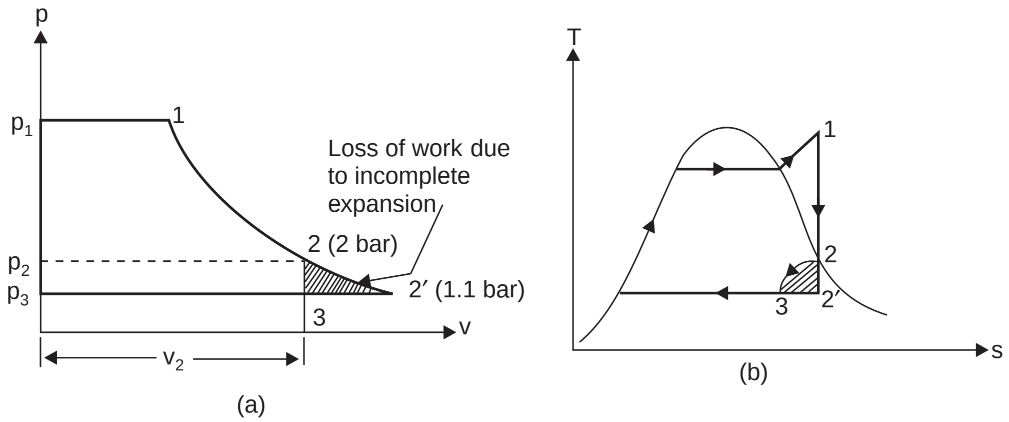


Fig. 12.13. p-V and T-s diagrams.

During isentropic expansion 1-2, we have

$$s_1 = s_2$$

$$6.918 = s_{f_2} + x_2\, s_{fg_2} = 1.5301 + x_2 \times 5.5967$$

$$\therefore \qquad x_2 = \frac{6.918 - 1.5301}{5.5967} = 0.96.$$

Then

$$h_2 = h_{f_2} + x_2 h_{fg_2} = 504.7 + 0.96 \times 2201.6 = 2618.2 \text{ kJ/kg}$$

$$v_2 = x_2\, v_{g_2} + (1 - x_2)\, v_{f_2}$$

$$= 0.96 \times 0.885 + (1 - 0.96) \times 0.00106 = 0.849 \text{ m}^3\text{/kg.}$$

(*i*) **Ideal work :**

Ideal work or modified Rankine engine work/kg,

$$W = (h_1 - h_2) + (p_2 - p_3)\, v_2$$

$$= (3037.6 - 2618.2) + (2 - 1.1) \times 10^5 \times 0.849/1000$$

$$= 419.4 + 76.41 = \mathbf{495.8 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

(*ii*) **Rankine engine efficiency :**

$$\eta_{\text{rankine}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{495.8}{(h_1 - h_{f_3})}$$

$$= \frac{495.8}{3037.6 - 428.8} = \mathbf{0.19} \text{ or } \mathbf{19\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Indicated and brake work per kg :**

Indicated work/kg,

$$W_{\text{indicated}} = \frac{\text{I.P.}}{\dot{m}}$$

$$= \frac{1 \times 3600}{12.8} = \mathbf{281.25 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

Brake work/kg,

$$W_{\text{brake}} = \frac{\text{B.P.}}{\dot{m}} = \frac{\eta_{\text{mech.}} \times \text{I.P.}}{\dot{m}}$$

$$= \frac{0.8 \times 1 \times 3600}{12.8} = \mathbf{225 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

(*iv*) **Brake thermal efficiency :**

$$\text{Brake thermal efficiency} = \frac{W_{\text{brake}}}{h_1 - h_{f_3}} = \frac{225}{3037.6 - 428.8} = \mathbf{0.086 \text{ or } 8.6\%.} \quad \textbf{(Ans.)}$$

(*v*) **Relative efficiency :**

Relative efficiency *on the basis of indicated work*

$$= \frac{\dfrac{W_{\text{indicated}}}{h_1 - h_{f_3}}}{\dfrac{W}{h_1 - h_{f_3}}} = \frac{W_{\text{indicated}}}{W} = \frac{281.25}{495.8} = \mathbf{0.567 \text{ or } 56.7\%.} \quad \textbf{(Ans.)}$$

Relative efficiency *on the basis of brake work*

$$= \frac{\dfrac{W_{\text{indicated}}}{(h_1 - h_{f_3})}}{\dfrac{W}{(h_1 - h_{f_3})}} = \frac{W_{\text{brake}}}{W} = \frac{225}{495.8} = \mathbf{0.4538 \text{ or } 45.38\%.} \quad \textbf{(Ans.)}$$

Example 12.11. *Superheated steam at a pressure of 10 bar and 400°C is supplied to a steam engine. Adiabatic expansion takes place to release point at 0.9 bar and it exhausts into a condenser at 0.3 bar. Neglecting clearance determine for a steam flow rate of 1.5 kg/s :*

(*i*) *Quality of steam at the end of expansion and the end of constant volume operation.*

(*ii*) *Power developed.*

(*iii*) *Specific steam consumption.*

(*iv*) *Modified Rankine cycle efficiency.*

Solution. Fig. 12.14 shows the *p-V* and *T-s* diagrams for modified Rankine cycle.

From steam tables :

1. At 10 bar, 400°C : $h_1 = 3263.9$ kJ/kg, $v_1 = 0.307$ m^3/kg, $s_1 = 7.465$ kJ/kg K

2. At 0.9 bar : $t_{s_2} = 96.7°\text{C}$, $h_{g_2} = 2670.9$ kJ/kg, $s_{g_2} = 7.3954$ kJ/kg K,

$v_{g_2} = 1.869$ m^3/kg

3. At 0.3 bar : $h_{f_3} = 289.3$ kJ/kg, $v_{g_3} = 5.229$ m^3/kg

(*i*) **Quality of steam at the end of expansion,** T$_{sup2}$ **:**

For isentropic expansion 1-2, we have

$$s_1 = s_2$$

$$= s_{g_2} + c_p \log_e \frac{T_{sup\,2}}{T_{s_2}}$$

$$7.465 = 7.3954 + 2.1 \log_e \frac{T_{sup2}}{(96.7 + 273)}$$

$$\left(\frac{7.465 - 7.3954}{2.1}\right) = \log_e \frac{T_{sup2}}{369.7} \quad \text{or} \quad \log_e \frac{T_{sup2}}{369.7} = 0.0033$$

$$\frac{T_{sup2}}{369.7} = 1.0337 \quad \text{or} \quad T_{sup2} = 382 \text{ K}$$

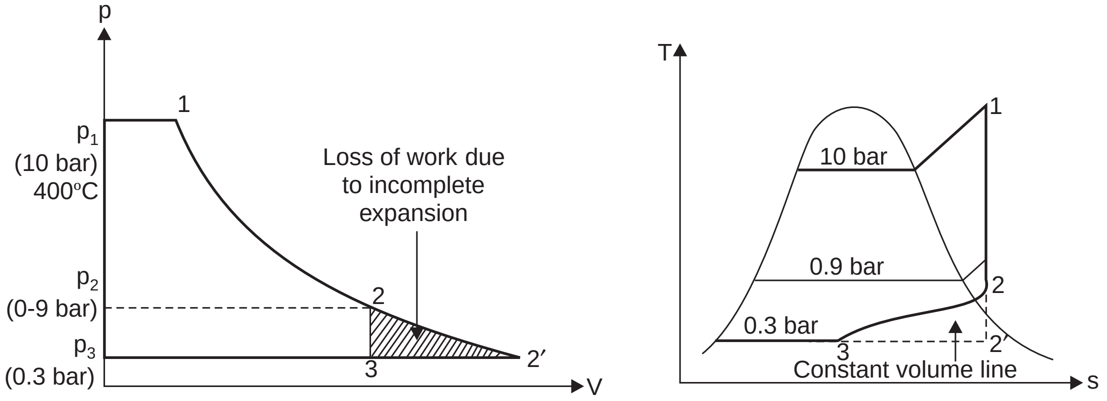


Fig. 12.14. *p-V* and *T-s* diagrams.

or $$t_{sup2} = 382 - 273 = \mathbf{109°C.} \quad \textbf{(Ans.)}$$

∴ $$h_2 = h_{g_2} + c_{ps}\,(T_{sup2} - T_{s_2})$$

$$= 2670.9 + 2.1\,(382 - 366.5) = 2703.4 \text{ kJ/kg.}$$

(*ii*) **Quality of steam at the end of constant volume operation, $\mathbf{x_3}$:**

For calculating v_2 using the relation

$$\frac{v_{g_2}}{T_{s_2}} = \frac{v_2}{T_{sup2}} \text{ (Approximately)}$$

$$\frac{1.869}{369.7} = \frac{v_2}{382}$$

or $$v_2 = \frac{1.869 \times 382}{369.7} = 1.931 \text{ m}^3\text{/kg}$$

Also $$v_2 = v_3 = x_3\, v_{g_3}$$

$$1.931 = x_3 \times 5.229$$

or $$\mathbf{x_3} = \frac{1.931}{5.229} = \mathbf{0.37.} \quad \textbf{(Ans.)}$$

(*iii*) **Power developed, P :**

Work done $$= (h_1 - h_2) + (p_2 - p_3)\, v_2$$

$$= (3263.9 - 2703.4) + \frac{(0.75 - 0.3) \times 10^5 \times 1.931}{1000}$$

$$= 560.5 + 86.9 = 647.4 \text{ kJ/kg}$$

∴ **Power developed** = Steam flow rate × work done (per kg)

$$= 1 \times 647.4 = \mathbf{647.4\ kW.} \quad \textbf{(Ans.)}$$

(*iv*) **Specific steam consumption, ssc :**

$$\textbf{ssc} = \frac{3600}{\text{Power}} = \frac{1 \times 3600}{647.4} = \mathbf{5.56\ kg/kWh.} \quad \textbf{(Ans.)}$$

(*v*) **Modified Rankine cycle efficiency, η_{mR} :**

$$\eta_{mR} = \frac{(h_1 - h_2) + (p_2 - p_3)\, v_2}{h_1 - h_{f_3}}$$

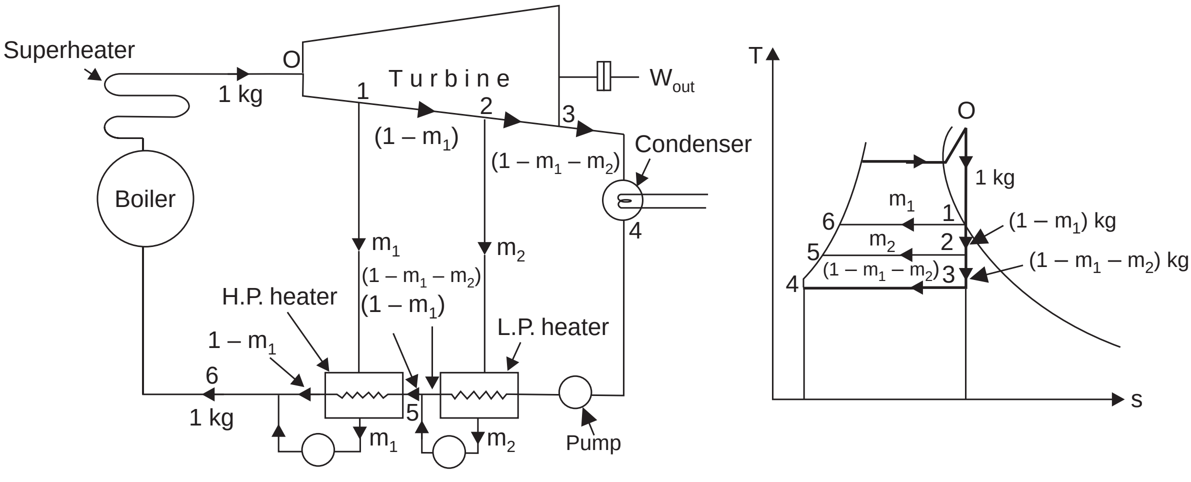
$$= \frac{647.4}{3263.9 - 289.3} = \mathbf{0.217 \text{ or } 21.7\%.} \quad \textbf{(Ans.)}$$

12.4. REGENERATIVE CYCLE

In the Rankine cycle it is observed that the condensate which is fairly at low temperature has an irreversible mixing with hot boiler water and this results in decrease of cycle efficiency. Methods are, therefore, adopted to heat the feed water from the hot well of condenser irreversibly by interchange of heat within the system and thus improving the cycle efficiency. This heating method is called regenerative feed heat and the cycle is called *regenerative cycle*.

The principle of regeneration can be practically utilised by extracting steam from the turbine at several locations and supplying it to the regenerative heaters. The resulting cycle is known as *regenerative or bleeding cycle.* The heating arrangement comprises of : (*i*) For medium capacity turbines—not more than 3 heaters ; (*ii*) For high pressure high capacity turbines—not more than 5 to 7 heaters ; and (*iii*) For turbines of super critical parameters 8 to 9 heaters. The most advantageous condensate heating temperature is selected depending on the turbine throttle conditions and this determines the number of heaters to be used. The final condensate heating temperature is kept 50 to 60°C below the boiler saturated steam temperature so as to prevent evaporation of water in the feed mains following a drop in the boiler drum pressure. The conditions of steam bled for each heater are so selected that the temperature of saturated steam will be 4 to 10°C higher than the final condensate temperature.

Fig. 12.15 (*a*) shows a diagrammatic layout of a condensing steam power plant in which a surface condenser is used to condense all the steam that is not extracted for feed water heating. The turbine is double extracting and the boiler is equipped with a superheater. The cycle diagram (*T-s*) would appear as shown in Fig. 12.15 (*b*). This arrangement constitutes a *regenerative cycle*.



(a) (b)

Fig. 12.15. Regenerative cycle.

Let, m_1 = kg of high pressure (H.P.) steam per kg of steam flow,

m_2 = kg of low pressure (L.P.) steam extracted per kg of steam flow, and

$(1 - m_2 - m_2)$ = kg of steam entering condenser per kg of steam flow.

Energy/Heat balance equation for H.P. heater :

$$m_1 (h_1 - h_{f_6}) = (1 - m_1)(h_{f_6} - h_{f_5})$$

or $$m_1[(h_1 - h_{f_6}) + (h_{f_6} - h_{f_5})] = (h_{f_6} - h_{f_5})$$

or $$m_1 = \frac{h_{f_6} - h_{f_5}}{h_1 - h_{f_5}} \quad ...(12.8)$$

Energy/Heat balance equation for L.P. heater :

$$m_2 (h_2 - h_{f_5}) = (1 - m_1 - m_2)(h_{f_5} - h_{f_3})$$

or $$m_2[(h_2 - h_{f_5}) + (h_{f_5} - h_{f_3})] = (1 - m_1)(h_{f_5} - h_{f_3})$$

or $$m_2 = \frac{(1 - m_1)(h_{f_5} - h_{f_3})}{(h_2 - h_{f_3})} \quad ...(12.9)$$

All enthalpies may be determined ; therefore m_1 and m_2 may be found. The maximum temperature to which the water can be heated is dictated by that of bled steam. The condensate from the bled steam is added to feed water.

Neglecting pump work :

The heat supplied externally in the cycle

$$= (h_0 - h_{f_6})$$

Isentropic work done $= m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)$

The thermal efficiency of regenerative cycle is

$$\eta_{\text{thermal}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)}{(h_0 - h_{f_6})} \quad ...(12.10)$$

[The work done by the turbine may also be calculated by summing up the products of the steam flow and the corresponding heat drop in the turbine stages.

i.e., Work done $= (h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)$]

Advantages of Regenerative cycle over Simple Rankine cycle :

1. The heating process in the boiler tends to become reversible.

2. The thermal stresses set up in the boiler are minimised. This is due to the fact that temperature ranges in the boiler are reduced.

3. The thermal efficiency is improved because the average temperature of heat addition to the cycle is increased.

4. Heat rate is reduced.

5. The blade height is less due to the reduced amount of steam passed through the low pressure stages.

6. Due to many extractions there is an improvement in the turbine drainage and it reduces erosion due to moisture.

7. A small size condenser is required.

Disadvantages :

1. The plant becomes more complicated.
2. Because of addition of heaters greater maintenance is required.
3. For given power a large capacity boiler is required.
4. The heaters are costly and the gain in thermal efficiency is not much in comparison to the heavier costs.

Note. In the absence of precise information (regarding actual temperature of the feed water entering and leaving the heaters and of the condensate temperatures) the following assumption should always be made while doing calculations :

1. Each heater is ideal and bled steam just condenses.
2. The feed water is heated to saturation temperature at the pressure of bled steam.
3. Unless otherwise stated the work done by the pumps in the system is considered negligible.
4. There is equal temperature rise in all the heaters (usually 10°C to 15°C).

Example 12.12. *A steam turbine is fed with steam having an enthalpy of 3100 kJ/kg. It moves out of the turbine with an enthalpy of 2100 kJ/kg. Feed heating is done at a pressure of 3.2 bar with steam enthalpy of 2500 kJ/kg. The condensate from a condenser with an enthalpy of 125 kJ/kg enters into the feed heater. The quantity of bled steam is 11200 kg/h. Find the power developed by the turbine. Assume that the water leaving the feed heater is saturated liquid at 3.2 bar and the heater is direct mixing type. Neglect pump work.*

Solution. Arrangement of the components is shown in Fig. 12.16.

At 3.2 bar, $h_{f_2} = 570.9$ kJ/kg.

Consider m kg out of 1 kg is taken to the feed heater (Fig. 12.16).

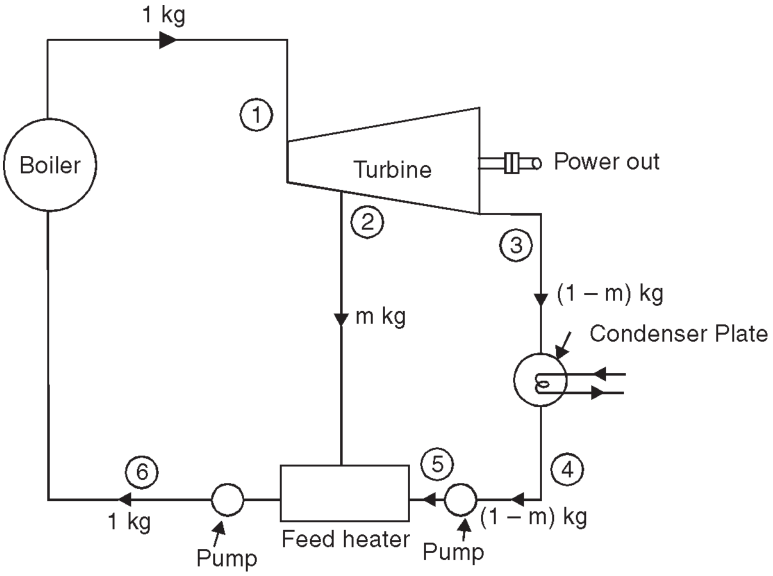


Fig. 12.16

Energy balance for the feed heater is written as :

$$mh_2 + (1 - m)\, h_{f_5} = 1 \times h_{f_2}$$

$$m \times 2100 + (1 - m) \times 125 = 1 \times 570.9$$

$$2100\, m + 125 - 125\, m = 570.9$$

$$1975\, m = 570.9 - 125$$

$\therefore$ $m = 0.226$ kg per kg of steam supplied to the turbine

$\therefore$ Steam supplied to the turbine per hour

$$= \frac{11200}{0.226} = 49557.5 \text{ kg/h}$$

Net work developed per kg of steam

$$= (h_1 - h_2) + (1 - m)(h_2 - h_3)$$

$$= (3100 - 2500) + (1 - 0.226)(2500 - 2100)$$

$$= 600 + 309.6 = 909.6 \text{ kJ/kg}$$

$\therefore$ **Power developed by the turbine**

$$= 909.6 \times \frac{49557.5}{3600} \text{ kJ/s}$$

$$= \textbf{12521.5 kW.} \quad \textbf{(Ans.)} \qquad (\because\ 1 \text{ kJ/s} = 1 \text{ kW})$$

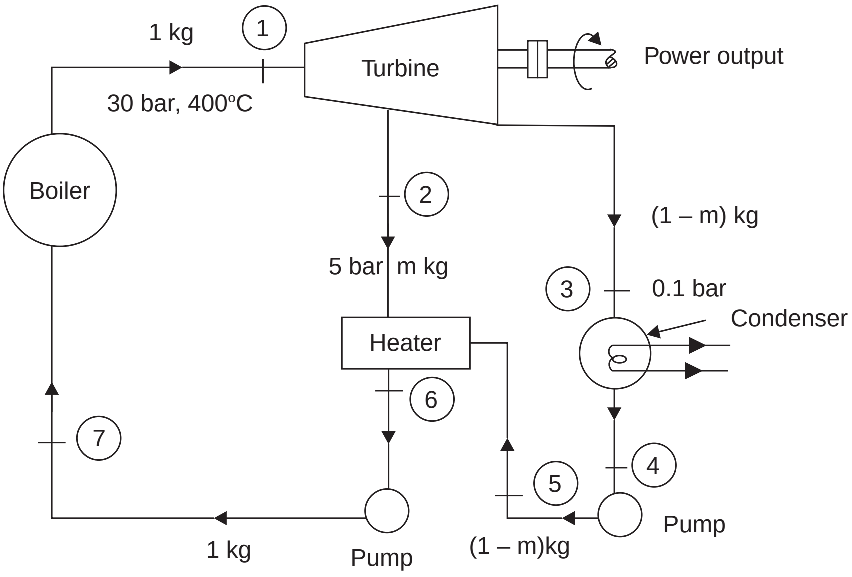
☞**Example 12.13.** *In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Find :*

(*i*) *The efficiency and the steam rate of the cycle.*

(*ii*) *The increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration).*

Pump work may be neglected.

Solution. Fig. 12.17 shows the flow, *T-s* and *h-s* diagrams.



(a)

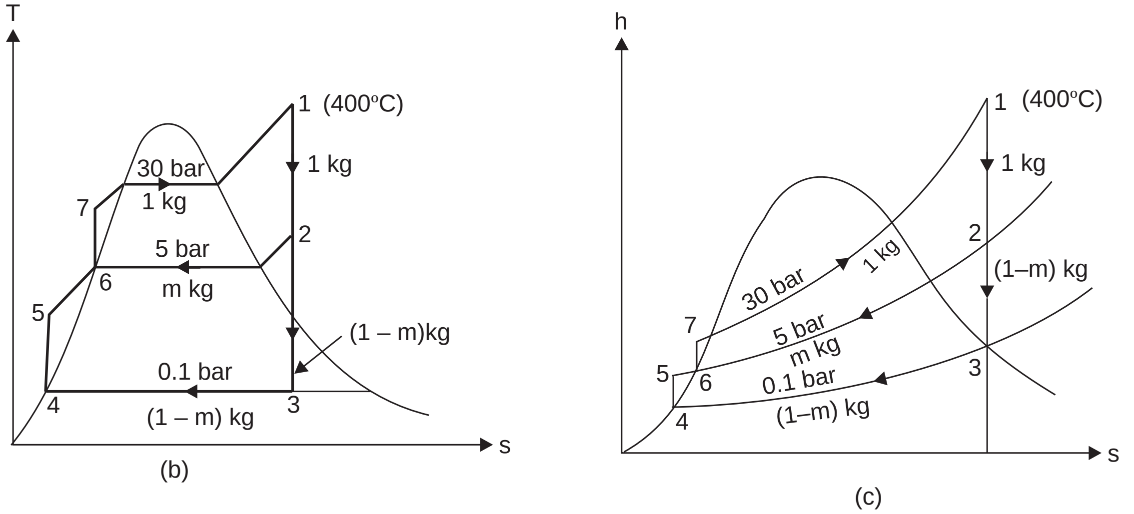


Fig. 12.17

From steam tables :

At 30 bar, 400°C : $h_1 = 3230.9$ kJ/kg, $s_1 = 6.921$ kJ/kg K $= s_2 = s_3$,

At 5 bar : $s_f = 1.8604$, $s_g = 6.8192$ kJ/kg K, $h_f = 640.1$ kJ/kg

Since $s_2 > s_g$, the state 2 must lie in the superheated region. From the table for superheated steam $t_2 = 172$°C, $h_2 = 2796$ kJ/kg.

At 0.1 bar : $s_f = 0.649$, $s_{f_g} = 7.501$, $h_f = 191.8$, $h_{f_g} = 2392.8$

Now, $s_2 = s_3$

i.e., $6.921 = s_{f_3} + x_3\, s_{fg_3} = 0.649 + x_3 \times 7.501$

$$\therefore \quad x_3 = \frac{6.921 - 0.649}{7.501} = 0.836$$

$$\therefore \quad h_3 = h_{f_3} + x_3\, h_{fg_3} = 191.8 + 0.836 \times 2392.8 = 2192.2 \text{ kJ/kg}$$

Since pump work is neglected

$$h_{f_4} = 191.8 \text{ kJ/kg} = h_{f_5}$$

$$h_{f_6} = 640.1 \text{ kJ/kg (at 5 bar)} = h_{f_7}$$

Energy balance for heater gives

$$m\,(h_2 - h_{f_6}) = (1 - m)\,(h_{f_6} - h_{f_5})$$

$$m\,(2796 - 640.1) = (1 - m)\,(640.1 - 191.8) = 448.3\,(1 - m)$$

$$2155.9\, m = 448.3 - 448.3\, m$$

$$\therefore \quad m = 0.172 \text{ kg}$$

$\therefore$ Turbine work, $W_T = (h_1 - h_2) + (1 - m)\,(h_2 - h_3)$

$$= (3230.9 - 2796) + (1 - 0.172)\,(2796 - 2192.2)$$

$$= 434.9 + 499.9 = 934.8 \text{ kJ/kg}$$

Heat supplied, $Q_1 = h_1 - h_{f_6} = 3230.9 - 640.1 = 2590.8$ kJ/kg.

(*i*) **Efficiency of cycle, η_{cycle} :**

$$\eta_{\text{cycle}} = \frac{W_T}{Q_1} = \frac{934.8}{2590.8} = \mathbf{0.3608 \text{ or } 36.08\%.} \quad \textbf{(Ans.)}$$

$$\textbf{Steam rate} = \frac{3600}{934.8} = \mathbf{3.85 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

(*ii*)
$$T_{m_1} = \frac{h_1 - h_{f_7}}{s_1 - s_7} = \frac{2590.8}{6.921 - 1.8604} = 511.9 \text{ K} = 238.9°\text{C}.$$

T_{m_1} (without regeneration)

$$= \frac{h_1 - h_{f_4}}{s_1 - s_4} = \frac{3230.9 - 191.8}{6.921 - 0.649} = \frac{3039.1}{6.272} = 484.5 \text{ K} = 211.5°\text{C}.$$

Increase in $\mathbf{T_{m_1}}$ due to regeneration

$$= 238.9 - 211.5 = \mathbf{27.4°C.} \quad \textbf{(Ans.)}$$

W_T (without regeneration)

$$= h_1 - h_3 = 3230.9 - 2192.2 = 1038.7 \text{ kJ/kg}$$

Steam rate without regeneration

$$= \frac{3600}{1038.7} = 3.46 \text{ kg/kWh}$$

∴ **Increase in steam rate due to regeneration**

$$= 3.85 - 3.46 = \mathbf{0.39 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

$$\eta_{\text{cycle}} \text{ (without regeneration)} = \frac{h_1 - h_3}{h_1 - h_{f_4}} = \frac{1038.7}{3230.9 - 191.8} = \mathbf{0.3418 \text{ or } 34.18\%.} \quad \textbf{(Ans.)}$$

Increase in cycle efficiency due to regeneration

$$= 36.08 - 34.18 = \mathbf{1.9\%.} \quad \textbf{(Ans.)}$$

Example 12.14. *Steam is supplied to a turbine at a pressure of 30 bar and a temperature of 400°C and is expanded adiabatically to a pressure of 0.04 bar. At a stage of turbine where the pressure is 3 bar a connection is made to a surface heater in which the feed water is heated by bled steam to a temperature of 130°C. The condensed steam from the feed heater is cooled in a drain cooler to 27°C. The feed water passes through the drain cooler before entering the feed heater. The cooled drain water combines with the condensate in the well of the condenser.*

Assuming no heat losses in the steam, calculate the following :

(*i*) *Mass of steam used for feed heating per kg of steam entering the turbine ;*

(*ii*) *Thermal efficiency of the cycle.*

Solution. Refer Fig. 12.18.

From steam tables :

At 3 bar : $t_s = 133.5°\text{C}$, $h_f = 561.4$ kJ/kg.

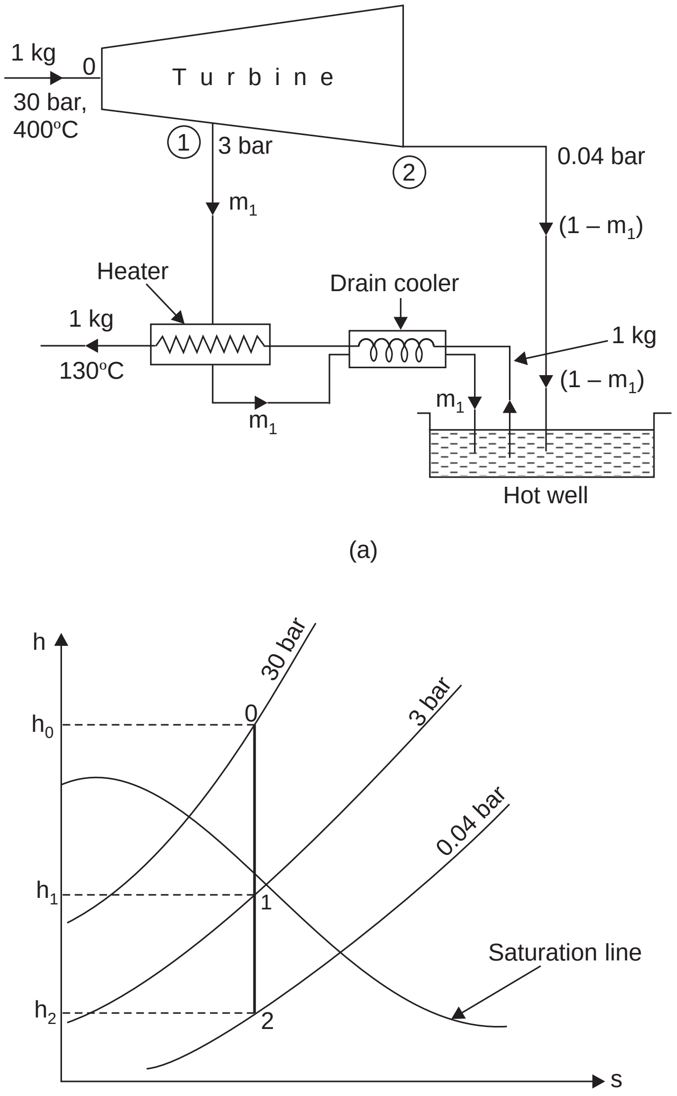
At 0.04 bar : $t_s = 29°\text{C}$, $h_f = 121.5$ kJ/kg.

From Mollier chart :

$h_0 = 3231$ kJ/kg (at 30 bar, 400°C)

$h_1 = 2700$ kJ/kg (at 3 bar)

$h_2 = 2085$ kJ/kg (at 0.04 bar).



(a)

(*b*)

Fig. 12.18

(*i*) **Mass of steam used, $\mathbf{m_1}$:**

Heat lost by the steam = Heat gained by water.

Taking the feed-heater and drain-cooler combined, we have :

$$m_1 (h_1 - h_{f_2}) = 1 \times 4.186 \ (130 - 27)$$

or

$$m_1 \ (2700 - 121.5) = 4.186 \ (130 - 27)$$

$$\therefore \quad \mathbf{m_1} = \frac{4.186 \ (130 - 27)}{(2700 - 121.5)} = \mathbf{0.1672 \ kg.} \quad \textbf{(Ans.)}$$

(*ii*) **Thermal efficiency of the cycle :**

Work done per kg of steam

$$= 1(h_0 - h_1) + (1 - m_1) \ (h_1 - h_2)$$

$$= 1(3231 - 2700) + (1 - 0.1672)\,(2700 - 2085)$$

$$= 1043.17 \text{ kJ/kg}$$

Heat supplied per kg of steam $= h_0 - 1 \times 4.186 \times 130$

$$= 3231 - 544.18 = 2686.82 \text{ kJ/kg.}$$

$$\eta_{\text{Thermal}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{1043.17}{2686.82} = \mathbf{0.3882 \text{ or } 38.82\%.} \quad \textbf{(Ans.)}$$

Example 12.15. *Steam is supplied to a turbine at 30 bar and 350°C. The turbine exhaust pressure is 0.08 bar. The main condensate is heated regeneratively in two stages by steam bled from the turbine at 5 bar and 1.0 bar respectively. Calculate masses of steam bled off at each pressure per kg of steam entering the turbine and the theoretical thermal efficiency of the cycle.*

Solution. Refer Fig. 12.19.

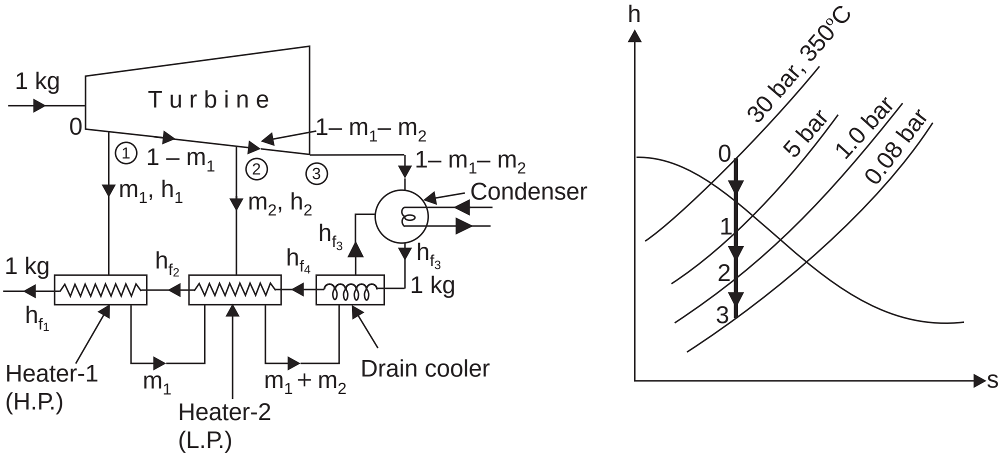


Fig. 12.19

The following *assumptions* are made :

1. The condensate is heated to the saturation temperature in each heater.

2. The drain water from H.P. (high pressure) heater passes into the steam space of the L.P. (low pressure) heater without loss of heat.

3. The combined drains from the L.P. heater are cooled in a drain cooler to the condenser temperature.

4. The expansion of the steam in the turbine is adiabatic and frictionless.

Enthalpy at 30 bar, 350°C, $h_0 = 3115.3$ kJ/kg.

After adiabatic expansion (from Mollier chart)

Enthalpy at 5 bar, $h_1 = 2720$ kJ/kg

Enthalpy at 1.0 bar, $h_2 = 2450$ kJ/kg

Enthalpy at 0.08 bar, $h_3 = 2120$ kJ/kg

From steam tables : $h_{f_1} = 640.1$ kJ/kg (at 5.0 bar)

$h_{f_2} = 417.5$ kJ/kg (at 1.0 bar)

$h_{f_3} = 173.9$ kJ/kg (at 0.08 bar)

At heater No. 1 :

$$m_1 h_1 + h_{f_2} = m_1 h_{f_1} + h_{f_1}$$

$$\mathbf{m_1} = \frac{h_{f_1} - h_{f_2}}{h_1 - h_{f_1}} = \frac{640.1 - 417.5}{2720 - 640.1} = \textbf{0.107 kJ/kg of entering steam.}$$

(Ans.)

At heater No. 2 :

$$m_2 h_2 + m_1 h_{f_1} + h_{f_4} = (m_1 + m_2)\, h_{f_2} + h_{f_2} \quad ...(i)$$

At drain cooler :

$$(m_1 + m_2)\, h_{f_2} + h_{f_3} = h_{f_4} + (m_1 + m_2)\, h_{f_3}$$

$$\therefore \quad h_{f_4} = (m_1 + m_2)\,(h_{f_2} - h_{f_3}) + h_{f_3} \quad ...(ii)$$

Inserting the value of h_{f_4} in eqn. (*i*), we get

$$m_2 h_2 + m_1 h_{f_1} + (m_1 + m_2)\,(h_{f_2} - h_{f_3}) + h_{f_3} = (m_1 + m_2)\, h_{f_2} + h_{f_2}$$

$$m_2 h_2 + m_1 h_{f_1} + (m_1 + m_2) h_{f_2} - (m_1 + m_2) h_{f_3} + h_{f_3} = (m_1 + m_2) h_{f_2} + h_{f_2}$$

$$m_2 h_2 + m_1 h_{f_1} - m_1 h_{f_3} - m_2 h_{f_3} + h_{f_3} = h_{f_2}$$

$$m_2\,(h_2 - h_{f_3}) = (h_{f_2} - h_{f_3}) - m_1 (h_{f_1} - h_{f_3})$$

$$\mathbf{m_2} = \frac{(h_{f_2} - h_{f_3}) - m_1\,(h_{f_1} - h_{f_3})}{(h_2 - h_{f_3})}$$

$$= \frac{(417.5 - 173.9) - 0.107\,(640.1 - 173.9)}{(2450 - 173.9)}$$

$$= \frac{193.7}{2276.1} = \textbf{0.085 kJ/kg. (Ans.)}$$

Work done $= 1\,(h_0 - h_1) + (1 - m_1)\,(h_1 - h_2) + (1 - m_1 - m_2)\,(h_2 - h_3)$

$= 1\,(3115.3 - 2720) + (1 - 0.107)\,(2720 - 2450)$
$+ (1 - 0.107 - 0.085)\,(2450 - 2120)$

$= 395.3 + 241.11 + 266.64 = 903.05$ kJ/kg

Heat supplied/kg $= h_0 - h_{f_1}$

$= 3115.3 - 640.1 = 2475.2$ kJ/kg

∴ **Thermal efficiency of the cycle**

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{903.05}{2475.2} = \textbf{0.3648 or 36.48\%. (Ans.)}$$

☞**Example 12.16.** *Steam at a pressure of 20 bar and 250°C enters a turbine and leaves it finally at a pressure of 0.05 bar. Steam is bled off at pressures of 5.0, 1.5 and 0.3 bar. Assuming (i) that the condensate is heated in each heater upto the saturation temperature of the steam in that heater, (ii) that the drain water from each heater is cascaded through a trap into the next heater on the low pressure side of it, (iii) that the combined drains from the heater operating at 0.3 bar are cooled in a drain cooler to condenser temperature, calculate the following :*

(*i*) *Mass of bled steam for each heater per kg of steam entering the turbine*

(*ii*) *Thermal efficiency of the cycle,* (*iii*) *Thermal efficiency of the Rankine cycle*

(*iv*) *Theoretical gain due to regenerative feed heating,*

(*v*) *Steam consumption in kg/kWh with or without regenerative feed heating, and*

(*vi*) *Quantity of steam passing through the last stage nozzle of a 50000 kW turbine with and without regenerative feed heating.*

Solution. Refer Fig. 12.20 (*a*), (*b*).

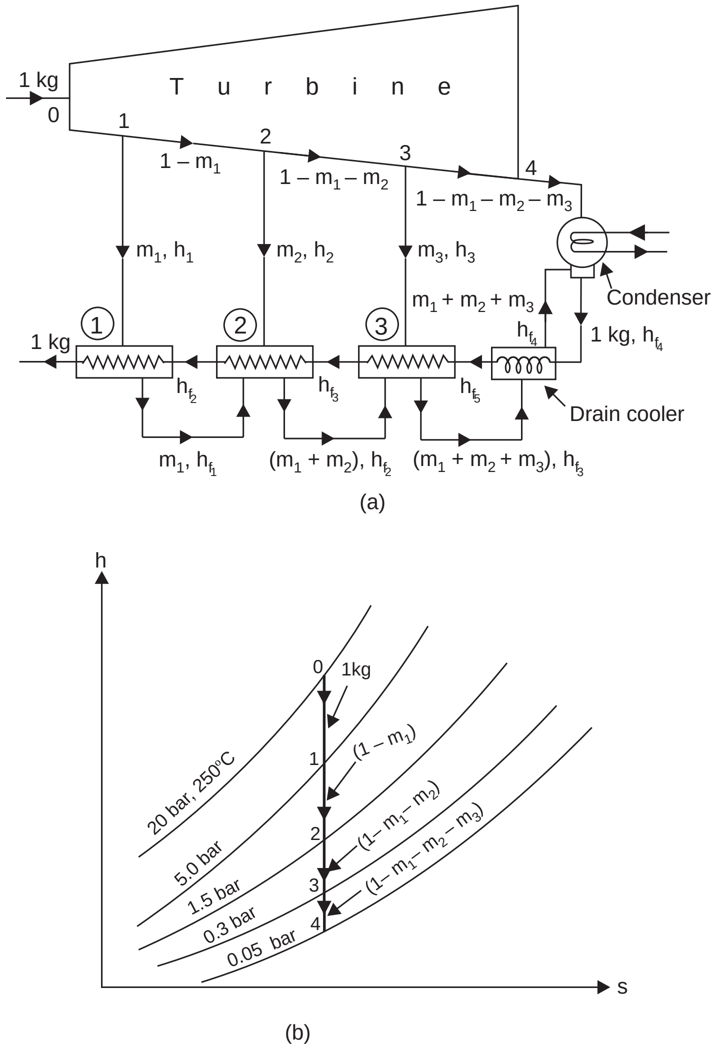


Fig. 12.20

From Mallier Chart : h_0 = 2905 kJ/kg, h_1 = 2600 kJ/kg, h_2 = 2430 kJ/kg
h_3 = 2210 kJ/kg, h_4 = 2000 kJ/kg

From steam tables :

At 5 bar : h_{f_1} = 640.1 kJ/kg

At 1.5 bar : h_{f_2} = 467.1 kJ/kg

At 0.3 bar : $h_{f_3} = 289.3$ kJ/kg

At 0.05 bar : $h_{f_4} = 137.8$ kJ/kg.

(*i*) **Mass of bled steam for each heater per kg of steam :**

Using heat balance equation :

At heater No. 1 :

$$m_1h_1 + h_{f_2} = m_1h_{f_1} + h_{f_1}$$

$$\therefore \qquad \mathbf{m_1} = \frac{h_{f_1} - h_{f_2}}{h_1 - h_{f_1}} = \frac{640.1 - 467.1}{2600 - 640.1}$$

= **0.088 kJ/kg of entering steam. (Ans.)**

At heater No. 2 :

$$m_2h_2 + h_{f_3} + m_1h_{f_1} = h_{f_2} + (m_1 + m_2)h_{f_2}$$

$$\mathbf{m_2} = \frac{(h_{f_2} + h_{f_3}) - m_1(h_{f_1} - h_{f_2})}{(h_2 - h_{f_2})}$$

$$= \frac{(467.1 - 289.3) - 0.088\,(640.1 - 467.1)}{(2430 - 467.1)} = \frac{162.57}{1962.9}$$

= **0.0828 kJ/kg of entering steam. (Ans.)**

At heater No. 3 :

$$m_3h_3 + h_{f_5} + (m_1 + m_2)h_{f_2} = h_{f_3} + (m_1 + m_2 + m_3)h_{f_3} \qquad ...(i)$$

At drain cooler :

$$(m_1 + m_2 + m_3)h_{f_3} + h_{f_4} = h_{f_5} + (m_1 + m_2 + m_3)h_{f_4}$$

$$\therefore \qquad h_{f_5} = (m_1 + m_2 + m_3)\,(h_{f_3} - h_{f_4}) + h_{f_4} \qquad ...(ii)$$

Inserting the value of h_{f_5} in eqn. (*i*), we get

$$m_3h_3 + (m_1 + m_2 + m_3)\,(h_{f_3} - h_{f_4}) + h_{f_4} + (m_1 + m_2)\,h_{f_2} = h_{f_3} + (m_1 + m_2 + m_3)h_{f_3}$$

$$\therefore \qquad \mathbf{m_3} = \frac{(h_{f_3} - h_{f_4}) - (m_1 + m_2)(h_{f_2} - h_{f_4})}{h_3 - h_{f_4}}$$

$$= \frac{(289.3 - 137.8) - (0.088 + 0.0828)(467.1 - 137.8)}{(2210 - 137.8)}$$

$$= \frac{151.5 - 56.24}{2072.2}$$ = **0.046 kJ/kg of entering steam. (Ans.)**

Work done/kg (*neglecting pump work*)

$= (h_0 - h_1) + (1 - m_1)\,(h_1 - h_2) + (1 - m_1 - m_2)\,(h_2 - h_3) + (1 - m_1 - m_2 - m_3)\,(h_3 - h_4)$

$= (2905 - 2600) + (1 - 0.088)\,(2600 - 2430) + (1 - 0.088 - 0.0828)\,(2430 - 2210)$

$+ (1 - 0.088 - 0.0828 - 0.046)\,(2210 - 2000)$

$= 305 + 155.04 + 182.42 + 164.47 = 806.93$ kJ/kg

Heat supplied/kg = $h_0 - h_{f_1} = 2905 - 640.1 = 2264.9$ kJ/kg.

(*ii*) **Thermal efficiency of the cycle, η_{Thermal} :**

$$\eta_{\text{Thermal}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{806.93}{2264.9} = \mathbf{0.3563 \text{ or } 35.63\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Thermal efficiency of Rankine cycle, η_{Rankine} :**

$$\eta_{\text{Rankine}} = \frac{h_0 - h_4}{h_0 - h_{f_4}} = \frac{2905 - 2000}{2905 - 137.8} = \mathbf{0.327 \text{ or } 32.7\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Theoretical gain due to regenerative feed heating**

$$= \frac{35.63 - 32.7}{35.63} = \mathbf{0.0822 \text{ or } 8.22\%.} \quad \textbf{(Ans.)}$$

(*v*) **Steam consumption with regenerative feed heating**

$$= \frac{1 \times 3600}{\text{Work done / kg}} = \frac{1 \times 3600}{806.93} = \mathbf{4.46 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

Steam consumption without regenerative feed heating

$$= \frac{1 \times 3600}{\text{Work done / kg without regeneration}} = \frac{1 \times 3600}{h_0 - h_4}$$

$$= \frac{1 \times 3600}{2905 - 2000} = \mathbf{3.97 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

(*vi*) **Quantity of steam passing through the last stage of a 50000 kW turbine with regenerative feed-heating**

$$= 4.46\,(1 - m_1 - m_2 - m_3) \times 50000$$
$$= 4.46\,(1 - 0.088 - 0.0828 - 0.046) \times 50000 = \mathbf{174653.6 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

Same without regenerative arrangement

$$= 3.97 \times 50000 = \mathbf{198500 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

Example 12.17. *A steam turbine plant developing 120 MW of electrical output is equipped with reheating and regenerative feed heating arrangement consisting of two feed heaters—one surface type on H.P. side and other direct contact type on L.P. side. The steam conditions before the steam stop valve are 100 bar and 530°C. A pressure drop of 5 bar takes place due to throttling in valves.*

Steam exhausts from the H.P. turbine at 25 bar. A small quantity of steam is bled off at 25 bar for H.P. surface heater for feed heating and the remaining is reheated in a reheater to 550°C and the steam enters at 22 bar in L.P. turbine for further expansion. Another small quantity of steam is bled off at pressure 6 bar for the L.P. heater and the rest of steam expands up to the back pressure of 0.05 bar. The drain from the H.P. heater is led to the L.P. heater and the combined feed from the L.P. heater is pumped to the high-pressure feed heater and finally to the boiler with the help of boiler feed pump.

The component efficiencies are : Turbine efficiency 85%, pump efficiency 90%, generator efficiency 96%, boiler efficiency 90% and mechanical efficiency 95%. It may be assumed that the feed-water is heated up to the saturation temperature at the prevailing pressure in feed heater. Work out the following :

(*i*) *Sketch the feed heating system and show the process on T-s and h-s diagrams.*
(*ii*) *Amounts of steam bled off.*
(*iii*) *Overall thermal efficiency of turbo-alternator considering pump work.*
(*iv*) *Specific steam consumption in kg/kWh.* **(AMIE Summer, 1998)**

Solution. (*i*) The *schematic arrangement* including feed heating system, and *T-s* and *h-s* diagrams of the process are shown in Figs. 12.21 and 12.22 respectively.

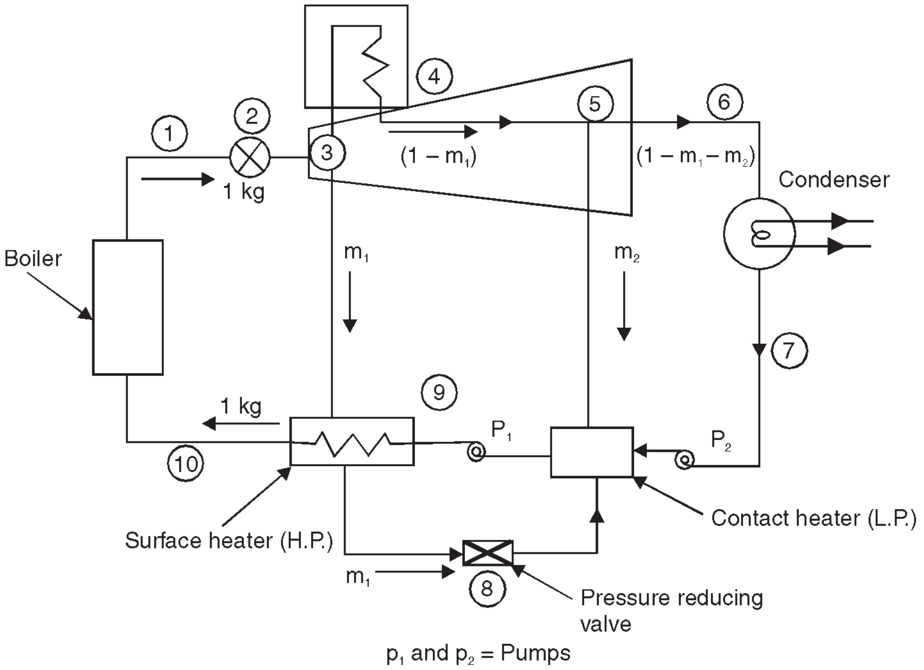


Fig. 12.21

(*ii*) **Amounts of bled off.** The enthalpies at various state points as read from *h-s* diagram/steam tables, in kJ/kg, are :

$$h_1 = h_2 = 3460$$

$$h_3' = 3050, \text{ and } \therefore\ h_3 = 3460 - 0.85(3460 - 3050) = 3111.5$$

$$h_4 = 3585$$

$$h_5' = 3140, \text{ and } \therefore\ h_5 = 3585 - 0.85(3585 - 3140) = 3207$$

$$h_6' = 2335, \text{ and } \therefore\ h_6 = 3207 - 0.85\ (3207 - 2335) = 2466$$

$$h_7 = 137.8 \text{ kJ/kg } (h_f \text{ at } 0.05 \text{ bar})$$

$$h_8 = h_{10} = 962 \text{ kJ/kg } (h_f \text{ at } 25 \text{ bar})$$

and $$h_9 = 670.4\ (h_f \text{ at } 6 \text{ bar}).$$

Enthalpy balance for ***surface heater :***

$$m_1h_3 + h_9 = m_1h_8 + h_{10}, \text{ neglecting pump work}$$

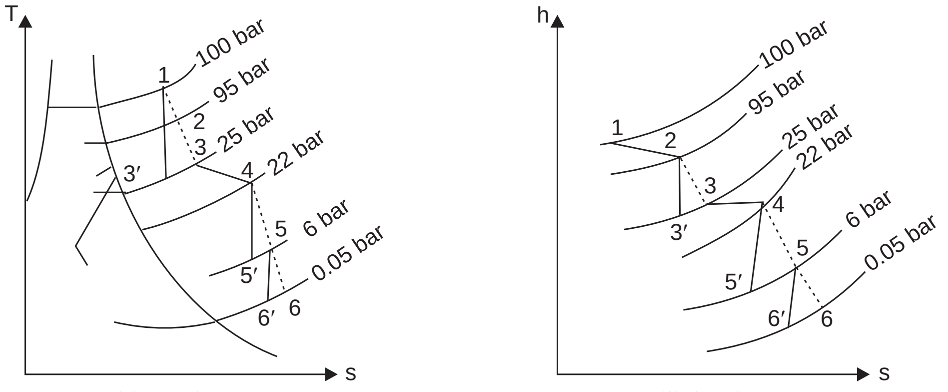
or $$m_1 = \frac{h_{10} - h_9}{h_3 - h_8} = \frac{962 - 670.4}{3111.5 - 962} = 0.13566 \text{ kg}$$

Enthalpy balance for ***contact heater :***

$$m_2h_5 + (1 - m_1 - m_2)h_7 + m_1h_8 = h_9, \text{ neglecting pump work}$$

or $$m_2 \times 3207 + (1 - 0.13566 - m_2) \times 137.8 + 0.13566 \times 962 = 670.4$$

or $$m_2 = 0.1371 \text{ kg.}$$



(*a*) *T-s* diagram (*b*) *h-s* diagram

Fig. 12.22

Pump Work. Take specific volume of water as 0.001 m^3/kg.

$$(W_{\text{pump}})_{\text{L.P.}} = (1 - m_1 - m_2)(6 - 0.05) \times 0.001 \times 10^2$$
$$= (1 - 0.13566 - 0.1371) \times 5.95 \times 0.1 = 0.4327 \text{ kJ/kg.}$$
$$(W_{\text{pump}})_{\text{H.P.}} = 1 \times (100 - 6) \times 0.001 \times 10^2 = 9.4 \text{ kJ/kg}$$

Total pump work (actual) $= \dfrac{0.4327 + 9.4}{0.9} = 10.925$ kJ/kg

Turbine output (indicated) $= (h_2 - h_3) + (1 - m_1)(h_4 - h_5) + (1 - m_1 - m_2)(h_5 - h_6)$

$= (3460 - 3111.5) + (1 - 0.13566)(3585 - 3207)$
$+ (1 - 0.13566 - 0.1371)(3207 - 2466)$

$= 1214.105$ kJ/kg

Net electrical output $=$ (Indicated work – Pump work) $\times\ \eta_{mech.} \times \eta_{gen}$.

$= (1214.105 - 10.925) \times 0.9 \times 0.96 = 1039.55$ kJ/kg

[**Note.** All the above calculations are for 1 kg of main (boiler) flow.]

$\therefore$ Main steam flow rate $= \dfrac{120 \times 10^3 \times 3600}{1039.55} = 4.155 \times 10^5$ kJ/h.

Amounts of bled off are :

(*a*) Surface (high pressure) heater,

$= 0.13566$ kg/kg of boiler flow

or $= 0.13566 \times 4.155 \times 10^5$

i.e., **$= 5.6367 \times 10^4$ kg/h. (Ans.)**

(*b*) Direct contact (low pressure) heater

$= 0.1371$ kg/kg of boiler flow

or $= 0.1371 \times 4.155 \times 10^5$

i.e., **$= 5.697 \times 10^4$ kg/h. (Ans.)**

(*iii*) **Overall thermal efficiency, η_{overall} :**

Heat input in boiler $= \dfrac{h_1 - h_{10}}{\eta_{\text{boiler}}} = \dfrac{3460 - 962}{0.9}$

$= 2775.6$ kJ/kg of boiler flow.

Heat input in reheater $= \dfrac{h_4 - h_3}{\eta_{\text{boiler}}} = \dfrac{3585 - 3111.5}{0.9} = 526.1$ kJ/kg of boiler flow

$$\therefore \qquad \eta_{\text{overall}} = \frac{1039.55}{2775.6 + 526.1} \times 100 = \mathbf{31.48\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Specific steam consumption :**

$$\text{Specific steam consumption} = \frac{4.155 \times 10^5}{120 \times 10^3} = \mathbf{3.4625\ kg/kWh.} \quad \textbf{(Ans.)}$$

12.5. REHEAT CYCLE

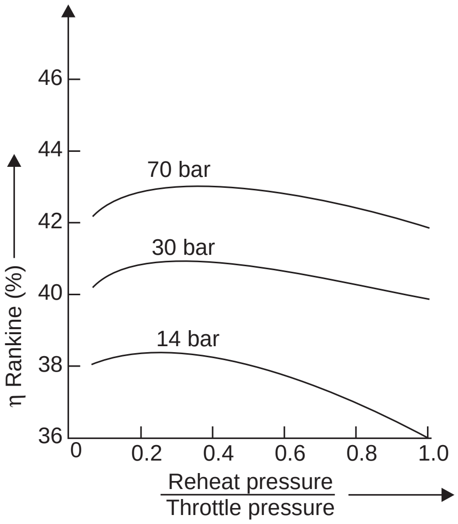
For attaining greater thermal efficiencies when the initial pressure of steam was raised beyond 42 bar it was found that resulting condition of steam after, expansion was increasingly wetter and exceeded in the safe limit of 12 per cent condensation. It, therefore, became necessary to *reheat* the steam after part of expansion was over so that the resulting condition after complete expansion fell within the region of permissible wetness.

The reheating or resuperheating of steam is now universally used when high pressure and temperature steam conditions such as 100 to 250 bar and 500°C to 600°C are employed for throttle. For plants of *still higher pressures and temperatures, a double reheating may be used*.

In actual practice reheat *improves* the cycle efficiency by about 5% for a 85/15 bar cycle. A *second reheat* will give a *much less gain* while the initial cost involved would be so high as to prohibit use of two stage reheat except in case of very high initial throttle conditions. The cost of reheat equipment consisting of boiler, piping and controls may be 5% to 10% more than that of the conventional boilers and this additional expenditure is justified only if gain in thermal efficiency is sufficient to promise a return of this investment. *Usually a plant with a base load capacity of 50000 kW and initial steam pressure of 42 bar would economically justify the extra cost of reheating*.

The improvement in thermal efficiency due to reheat is greatly dependent upon the *reheat pressure* with respect to the original pressure of steam.

Fig. 12.23 shows the reheat pressure selection on cycle efficiency.



Condenser pressure : 12.7 mm Hg
Temperature of throttle and heat : 427ºC
Fig. 12.23. Effect of reheat pressure selection on cycle efficiency.

Fig. 12.24 shows a schematic diagram of a theoretical single-stage reheat cycle. The corresponding representation of ideal reheating process on *T-s* and *h-s* chart is shown in Figs. 12.25 (*a* and *b*).

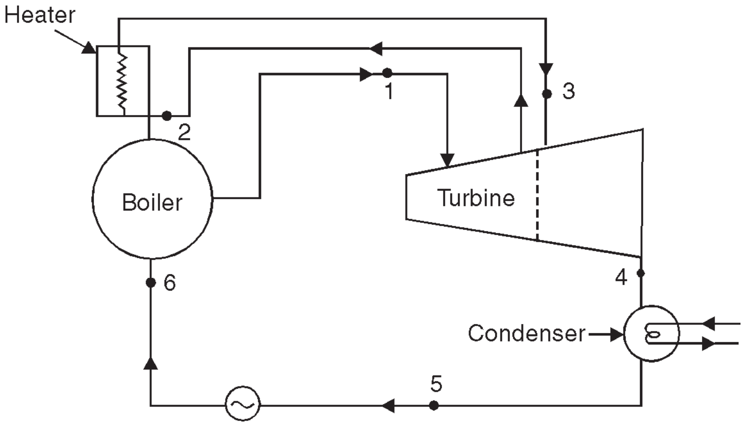


Fig. 12.24. Reheat cycle.

Refer to Fig. 12.25. 5-1 shows the formation of steam in the boiler. The steam as at state point 1 (*i.e.,* pressure p_1 and temperature T_1) enters the turbine and expands isentropically to a certain pressure p_2 and temperature T_2. From this state point 2 the whole of steam is drawn out of the turbine and is reheated in a reheater to a temperature T_3. (Although there is an *optimum pressure* at which the steam should be removed for reheating, if the highest return is to be obtained, yet, for simplicity, the whole steam is removed from the high pressure exhaust, where the pressure is about *one-fifth* of boiler pressure, and after undergoing a 10% pressure drop, in circulating through the heater, it is returned to intermediate pressure or low pressure turbine). This reheated steam is then readmitted to the turbine where it is expanded to condenser pressure isentropically.

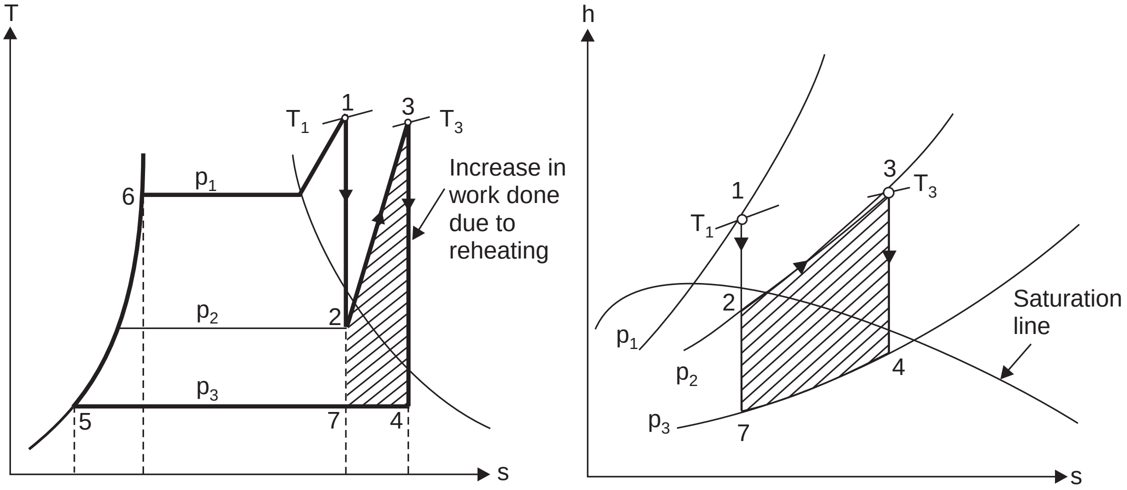


Fig. 12.25. Ideal reheating process on *T-s* and *h-s* chart.

Note. Superheating of steam. The primary object of superheating steam and supplying it to the primemovers is *to avoid too much wetness at the end of expansion*. Use of inadequate degree of superheat in *steam engines would cause greater condensation* in the engine cylinder ; while in case of *turbines* the moisture content of steam would result in *undue blade erosion*. The *maximum wetness in the final condition of steam that may be tolerated without any appreciable harm to the turbine blades is about 12 per cent*. Broadly each *1 per cent* of moisture in steam reduces the efficiency of that part of the turbine in which wet steam passes by 1 per cent to 1.5 per cent and in engines about 2 per cent.

Advantages of superheated steam :

(*i*) Superheating reduces the initial condensation losses in steam engines.

(*ii*) Use of superheated steam results in *improving the plant efficiency* by effecting a *saving in cost of fuel*. This saving may be of the order of 6% to 7% due to first 38°C of superheat and 4% to 5% for next 38°C and so on. This saving results due to the fact that the heat content and consequently the capacity to do work in superheated steam is increased and the quantity of steam required for a given output of power is reduced. Although additional heat has to be added in the boiler there is reduction in the work to be done by the feed pump, the condenser pump and other accessories due to reduction in quantity of steam used. It is estimated that the *quantity of steam may be reduced by 10% to 15% for first 38°C of superheat and somewhat less for the next 38°C of superheat in the case of condensing turbines.*

(*iii*) When a superheater is used in a boiler it helps in *reducing the stack temperatures* by extracting heat from the flue gases before these are passed out of chimney.

Thermal efficiency with '*Reheating*' (neglecting pump work) :

Heat supplied $= (h_1 - h_{f_4}) + (h_3 - h_2)$

Heat rejected $= h_4 - h_{f_4}$

Work done by the turbine = Heat supplied – heat rejected

$$= (h_1 - h_{f_4}) + (h_3 - h_2) - (h_4 - h_{f_4})$$

$$= (h_1 - h_2) + (h_3 - h_4)$$

Thus, theoretical thermal efficiency of reheat cycle is

$$\eta_{\text{thermal}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)} \quad \text{...(12.11)}$$

If pump work, $W_p = \dfrac{v_f\,(p_1 - p_b)}{1000}$ kJ/kg is considered, the thermal efficiency is given by :

$$\eta_{\text{thermal}} = \frac{[(h_1 - h_4) + (h_3 - h_4)] - W_p}{[(h_1 - h_{f_4}) + (h_3 - h_2)] - W_p} \quad \text{...(12.12)}$$

W_p is usually small and *neglected*.

Thermal efficiency *without reheating* is

$$\eta_{\text{thermal}} = \frac{h_1 - h_7}{h_1 - h_{f_4}} \quad (\because \quad h_{f_4} = h_{f_7}) \quad \text{...(12.13)}$$

Note 1. The reheater may be incorporated in the walls of the main boiler ; it may be a separately fired superheater or it may be heated by a coil carrying high-pressure superheated steam, this system being analogous to a steam jacket.

2. Reheating should be done at '*optimum pressure*' because if the steam is reheated early in its expansion then the additional *quantity of heat supplied will be small* and thus thermal efficiency gain will be small ; and if the reheating is done at a fairly low pressure, then, although a large amount of additional heat is supplied, the steam will have a high degree of superheat (as is clear from Mollier diagram), thus a large proportion of the heat supplied in the reheating process *will be thrown to waste in the condenser*.

Advantages of 'Reheating' :

1. There is an increased output of the turbine.
2. Erosion and corrosion problems in the steam turbine are eliminated/avoided.
3. There is an improvement in the thermal efficiency of the turbines.
4. Final dryness fraction of steam is improved.
5. There is an increase in the nozzle and blade efficiencies.

Disadvantages :

1. Reheating requires more maintenance.
2. The increase in thermal efficiency is not appreciable in comparison to the expenditure incurred in reheating.

☞**Example 12.18.** *Steam at a pressure of 15 bar and 250°C is expanded through a turbine at first to a pressure of 4 bar. It is then reheated at constant pressure to the initial temperature of 250°C and is finally expanded to 0.1 bar. Using Mollier chart, estimate the work done per kg of steam flowing through the turbine and amount of heat supplied during the process of reheat. Compare the work output when the expansion is direct from 15 bar to 0.1 bar without any reheat. Assume all expansion processes to be isentropic.*

Solution. Refer Fig. 12.26.

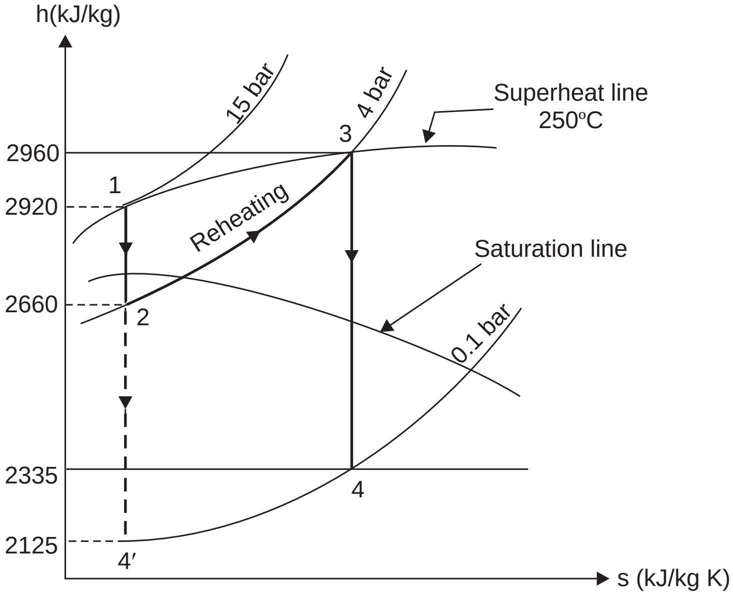


Fig. 12.26

Pressure, $p_1 = 15$ bar ;

$p_2 = 4$ bar ;

$p_4 = 0.1$ bar.

Work done per kg of steam,

$$W = \text{Total heat drop}$$

$$= [(h_1 - h_2) + (h_3 - h_4)] \text{ kJ/kg} \quad ...(i)$$

Amount of heat supplied during process of reheat,

$$h_{\text{reheat}} = (h_3 - h_2) \text{ kJ/kg} \qquad ...(ii)$$

From Mollier diagram or *h-s* chart,

$$h_1 = 2920 \text{ kJ/kg},\ h_2 = 2660 \text{ kJ/kg}$$

$$h_3 = 2960 \text{ kJ/kg},\ h_2 = 2335 \text{ kJ/kg}$$

Now, by putting the values in eqns. (*i*) and (*ii*), we get

$$W = (2920 - 2660) + (2960 - 2335)$$

$$= \mathbf{885\ kJ/kg.} \quad \textbf{(Ans.)}$$

Hence *work done per kg of steam* = **885 kJ/kg. (Ans.)**

Amount of heat supplied during reheat,

$$h_{\text{reheat}} = (2960 - 2660) = \mathbf{300\ kJ/kg.} \quad \textbf{(Ans.)}$$

If the expansion would have been continuous without reheating *i.e.,* 1 to 4′, the work output is given by

$$W_1 = h_1 - h_4'$$

From Mollier diagram,

$$h_{4'} = 2125 \text{ kJ/kg}$$

$$\therefore \qquad W_1 = 2920 - 2125 = \mathbf{795\ kJ/kg.} \quad \textbf{(Ans.)}$$

Example 12.19. *A steam power plant operates on a theoretical reheat cycle. Steam at boiler at 150 bar, 550°C expands through the high pressure turbine. It is reheated at a constant pressure of 40 bar to 550°C and expands through the low pressure turbine to a condenser at 0.1 bar. Draw T-s and h-s diagrams. Find :*

(*i*) *Quality of steam at turbine exhaust ;* (*ii*) *Cycle efficiency ;*

(*iii*) *Steam rate in kg/kWh.* **(AMIE Summer, 1999)**

Solution. Refer Figs. 12.27 and 12.28.

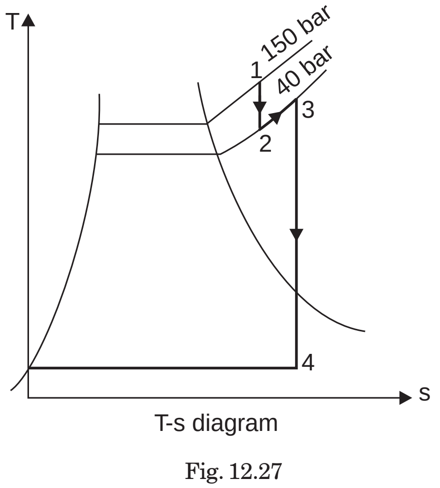


Fig. 12.27

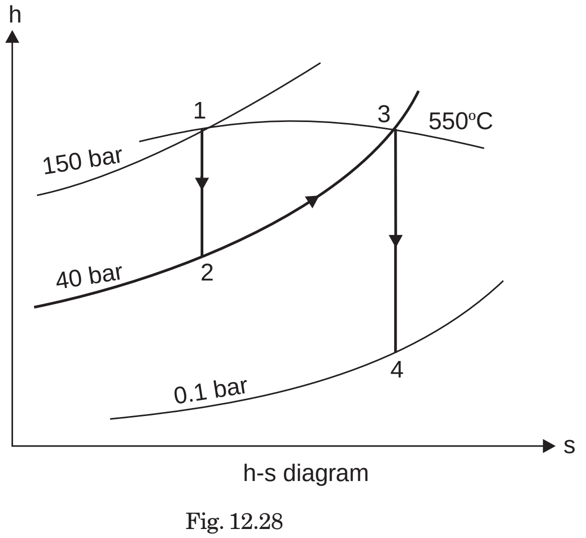


Fig. 12.28

From Mollier diagram (*h-s* diagram) :

$$h_1 = 3450 \text{ kJ/kg} \ ;\ h_2 = 3050 \text{ kJ/kg} \ ;\ h_3 = 3560 \text{ kJ/kg} \ ;\ h_4 = 2300 \text{ kJ/kg}$$

$$h_{f_4} \text{ (from steam tables, at 0.1 bar)} = 191.8 \text{ kJ/kg}$$

(*i*) **Quality of steam at turbine exhaust, x_4 :**

$$x_4 = 0.88 \text{ (From Mollier diagram)}$$

(*ii*) **Cycle efficiency, η_{cycle} :**

$$\eta_{\text{cycle}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)}$$

$$= \frac{(3450 - 3050) + (3560 - 2300)}{(3450 - 191.8) + (3560 - 3050)} = \frac{1660}{3768.2} = \mathbf{0.4405 \text{ or } 44.05\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Steam rate in kg/kWh :**

$$\text{Steam rate} = \frac{3600}{(h_1 - h_2) + (h_3 - h_4)} = \frac{3600}{(3450 - 3050) + (3560 - 2300)}$$

$$= \frac{3600}{1660} = \mathbf{2.17 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

Example 12.20. *A turbine is supplied with steam at a pressure of 32 bar and a temperature of 410°C. The steam then expands isentropically to a pressure of 0.08 bar. Find the dryness fraction at the end of expansion and thermal efficiency of the cycle.*

If the steam is reheated at 5.5 bar to a temperature of 395°C and then expanded isentropically to a pressure of 0.08 bar, what will be the dryness fraction and thermal efficiency of the cycle ?

Solution. First case. Refer Fig. 12.29.

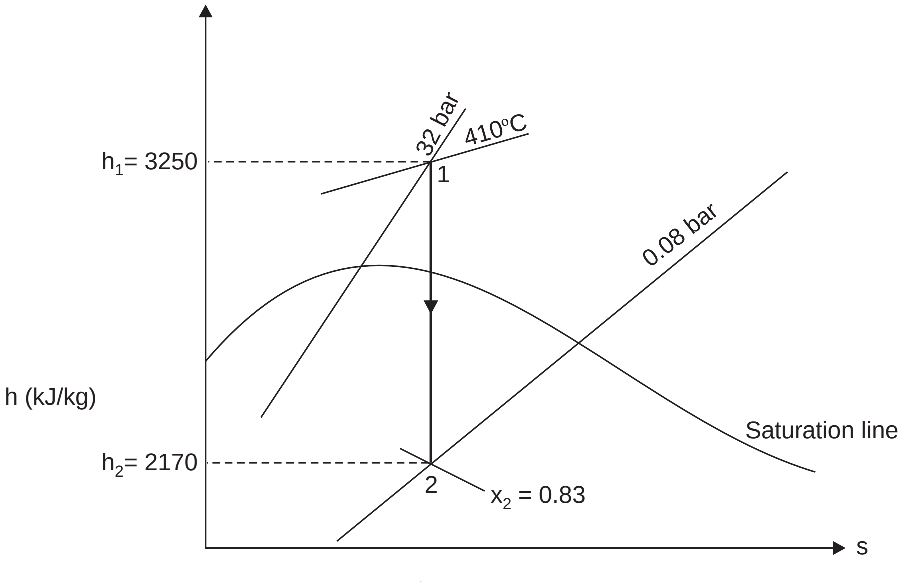


Fig. 12.29

From Mollier chart :

$$h_1 = 3250 \text{ kJ/kg}$$
$$h_2 = 2170 \text{ kJ/kg}$$

Heat drop (or work done) = $h_1 - h_2$

= 3250 – 2170 = 1080 kJ/kg

Heat supplied $= h_1 - h_{f_2}$

$= 3250 - 173.9$ $[h_{f_2} = 173.9 \text{ kJ/kg at } 0.08 \text{ bar}]$

$= 3076.1$ kJ/kg

Thermal efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{1080}{3076.1}$ = **0.351 or 35.1%. (Ans.)**

Exhaust steam condition, x_2

= **0.83 (From Mollier chart). (Ans.)**

Second case. Refer Fig. 12.30 (*b*).

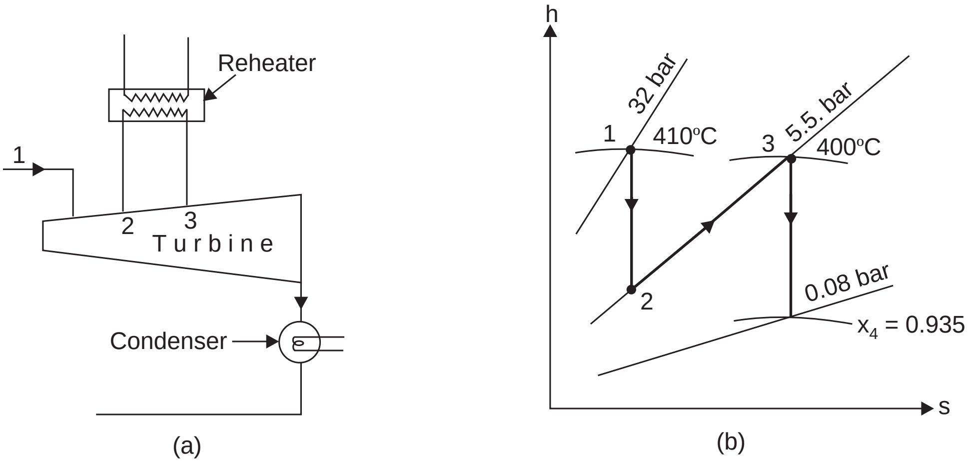


Fig. 12.30

From Mollier chart :

$h_1 = 3250$ kJ/kg ;
$h_2 = 2807$ kJ/kg ;
$h_3 = 3263$ kJ/kg ;
$h_4 = 2426$ kJ/kg.

Work done $= (h_1 - h_2) + (h_3 - h_4) = (3250 - 2807) + (3263 - 2426) = 1280$ kJ/kg

Heat supplied $= (h_1 - h_{f_4}) + (h_3 - h_2)$

$= (3250 - 173.9) + (3263 - 2807) = 3532$ kJ/kg

Thermal efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{1280}{3532}$ = **0.362 or 36.2%. (Ans.)**

Condition of steam at the exhaust,

x_4 = 0.935 [*From Mollier chart*]. **(Ans.)**

Example 12.21. (*a*) *How does erosion of turbine blades occur ? State the methods of preventing erosion of turbine blades.*

(*b*) *What do you mean by TTD of a feed water heater ? Draw temperature-path-line diagram of a closed feed water heater used in regenerative feed heating cycle.*

(*c*) *In a 15 MW steam power plant operating on ideal reheat cycle, steam enters the H.P. turbine at 150 bar and 600°C. The condenser is maintained at a pressure of 0.1 bar. If the moisture content at the exit of the L.P. turbine is 10.4%, determine :*

(*i*) *Reheat pressure ;* (*ii*) *Thermal efficiency ;* (*iii*) *Specific steam consumption* ; *and* (*iv*) *Rate of pump work in kW. Assume steam to be reheated to the initial temperature.*

(AMIE Summer, 1998)

Solution. (*a*) The erosion of the moving blades is *caused by the presence of water particles in (wet) steam in the L.P. stages*. The water particles strike the leading surface of the blades. Such impact, if sufficiently heavy, produces *severe local stresses in the blade material causing the surface metal to fail and flake off*.

The erosion, if any, is more likely to occur in the region where the steam is wettest, *i.e.*, in the last one or two stages of the turbine. Moreover, the water droplets are concentrated in the outer parts of the flow annuals where the velocity of impact is highest.

Erosion difficulties due to moisture in the steam may be avoided by reheating (see Fig. 12.31). The whole of steam is taken from the turbine at a suitable point 2, and a further supply of heat is given to it along 2-3 after which the steam is readmitted to the turbine and expanded along 3-4 to condenser pressure.

Erosion *may also be reduced by using steam traps in between the stages to separate moisture from the steam*.

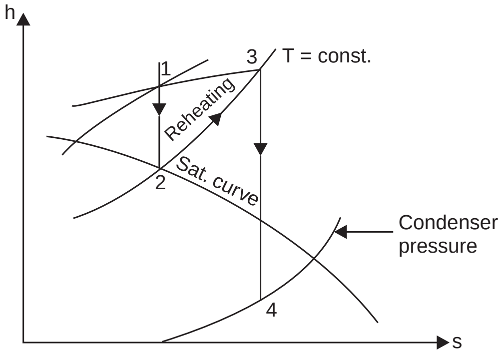


Fig. 12.31

(*b*) TTD means *"Terminal temperature difference"*. It is the *difference between temperatures of bled steam/condensate and the feed water at the two ends of the feed water heater.*

The required temperature-path-line diagram of a closed feed water heater is shown in Fig. 12.32.

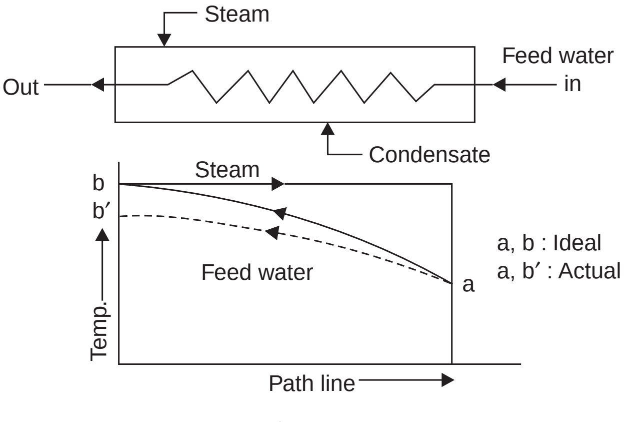


Fig. 12.32

(*c*) The cycle is shown on *T-s* and *h-s* diagrams in Figs. 12.33 and 12.34 respectively. The following values are read from the Mollier diagram :

$$h_1 = 3580 \text{ kJ/kg}, \; h_2 = 3140 \text{ kJ/kg}, \; h_3 = 3675 \text{ kJ/kg, and } h_4 = 2335 \text{ kJ/kg}$$

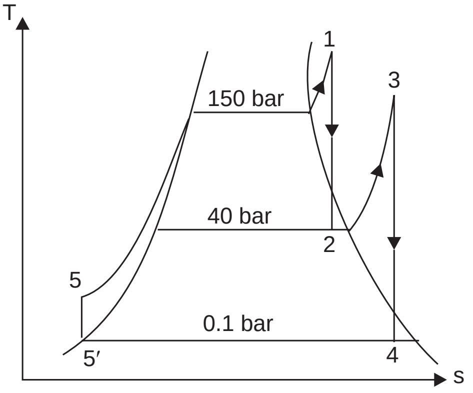


Fig. 12.33

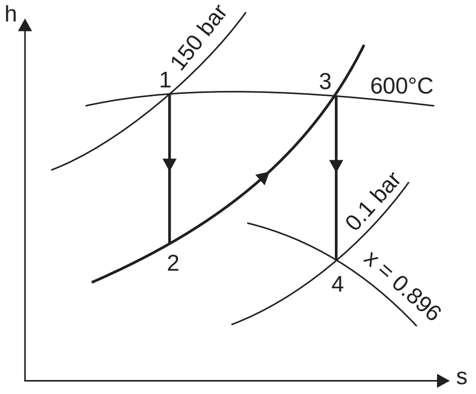


Fig. 12.34

Moisture contents in exit from L.P. turbine = 10.4%

$$x_4 = 1 - 0.104 = 0.896$$

(*i*) **Reheat pressure :** From the Mollier diagram, the **reheat pressure is 40 bar. (Ans.)**

(*ii*) **Thermal efficiency, η_{th} :**

Turbine work $= (h_1 - h_2) + (h_3 - h_4)$

$= (3580 - 3140) + (3675 - 2335) = 1780$ kJ/kg.

Assuming specific volume of water = 10^{-3} m^3/kg, the pump work = 10^{-3} $(150 - 0.1)$ = 0.15 kJ/kg, *i.e.*, may be neglected in computing of η_{th}, $h_5 = h_4 = 191.8$ kJ/kg, (h_f at 0.1 bar) from steam tables,

$$Q_{\text{input}} = (h_1 - h_5) + (h_3 - h_2)$$

$$= (3580 - 191.8) + (3675 - 3140) = 3923.2 \text{ kJ/kg}$$

$$\%\eta_{th} = \frac{1780}{3923.2} \times 100 = \mathbf{45.37\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Specific steam consumption :**

Steam consumption $= \dfrac{15 \times 10^3}{1780} = 8.427$ kg/s

Specific steam consumption $= \dfrac{8.427 \times 3600}{15 \times 10^3} =$ **2.0225 kg/kWh. (Ans.)**

(*iv*) **Rate of pump work :**

Rate of pump work = 8.427 × 0.15 = **1.26 kW. (Ans.)**

12.6. BINARY VAPOUR CYCLE

Carnot cycle gives the highest thermal efficiency which is given by $\dfrac{T_1 - T_2}{T_1}$. To approach this cycle in an actual engine it is necessary that whole of heat must be supplied at constant temperature T_1 and rejected at T_2. This can be achieved only by using a vapour in the wet field but not in the superheated. The efficiency depends on temperature T_1 since T_2 is fixed by the natural sink to which heat is rejected. This means that T_1 should be *as large as possible*, *consistent with the vapour being saturated*.

If we use steam as the working medium the temperature rise is accompanied by rise in pressure and at critical temperature of 374.15°C the pressure is as high as 225 bar which will create many difficulties in design, operation and control. It would be desirable to use some fluid other than steam which has more desirable thermodynamic properties than water. An ideal fluid for this purpose should have a *very high critical temperature combined with low pressure*. Mercury, diphenyl oxide and similar compounds, aluminium bromide and zinc ammonium chloride are fluids which possess the required properties in varying degrees. Mercury is the only working fluid which has been successfully used in practice. It has high critical temperature (588.4°C) and correspondingly low critical pressure (21 bar abs.). *The mercury alone cannot be used as its saturation temperature at atmospheric pressure is high (357°C). Hence* ***binary vapour cycle*** *is generally used to increase the overall efficiency of the plant*. Two fluids (mercury and water) are used in cascade in the binary cycle for production of power.

The *few more properties* required for an ideal binary fluid used in high temperature limit are listed below :

1. It should have high critical temperature at reasonably low pressure.
2. It should have high heat of vaporisation to keep the weight of fluid in the cycle to minimum.
3. Freezing temperature should be below room temperature.
4. It should have chemical stability through the working cycle.
5. It must be non-corrosive to the metals normally used in power plants.
6. It must have an ability to wet the metal surfaces to promote the heat transfer.
7. The vapour pressure at a desirable condensation temperature should be nearly atmospheric which will eliminate requirement of power for maintenance of vacuum in the condenser.
8. After expansion through the primemover the vapour should be nearly saturated so that a desirable heat transfer co-efficient can be obtained which will reduce the size of the condenser required.
9. It must be available in large quantities at reasonable cost.
10. It should not be toxic and, therefore, dangerous to human life.

Although mercury does not have all the required properties, it is more favourable than any other fluid investigated. It is most stable under all operating conditions.

Although, mercury does not cause any corrosion to metals, but it is extremely dangerous to human life, therefore, elaborate precautions must be taken to prevent the escape of vapour. The *major disadvantage* associated with mercury is that it *does not wet surface of the metal and forms a serious resistance to heat flow*. This difficulty can be considerably reduced by adding magnesium and titanium (2 parts in 100000 parts) in mercury.

Thermal properties of mercury :

Mercury fufills practically all the *desirable thermodynamic properties* stated above.

1. Its freezing point is – 3.3°C and boiling point is – 354.4°C at atmospheric pressure.
2. The pressure required when the temperature of vapour is 540°C is only 12.5 bar (app.) and, therefore, heavy construction is not required to get high initial temperature.
3. Its liquid saturation curve is very steep, approaching the isentropic of the Carnot cycle.
4. It has no corrosive or erosive effects upon metals commonly used in practice.
5. Its critical temperature is so far removed from any possible upper temperature limit with existing metals as to cause no trouble.

Some *undesirable properties* of mercury are listed below :

1. Since the latent heat of mercury is quite low over a wide range of desirable condensation temperatures, therefore, several kg of mercury must be circulated per kg of water evaporated in binary cycle.
2. The cost is a considerable item as the quantity required is 8 to 10 times the quantity of water circulated in binary system.
3. Mercury vapour in larger quantities is poisonous, therefore, the system must be perfect and tight.

Fig. 12.35 shows the schematic line diagram of binary vapour cycle using mercury and water as working fluids. The processes are represented on *T-s* diagram as shown in Fig. 12.36.

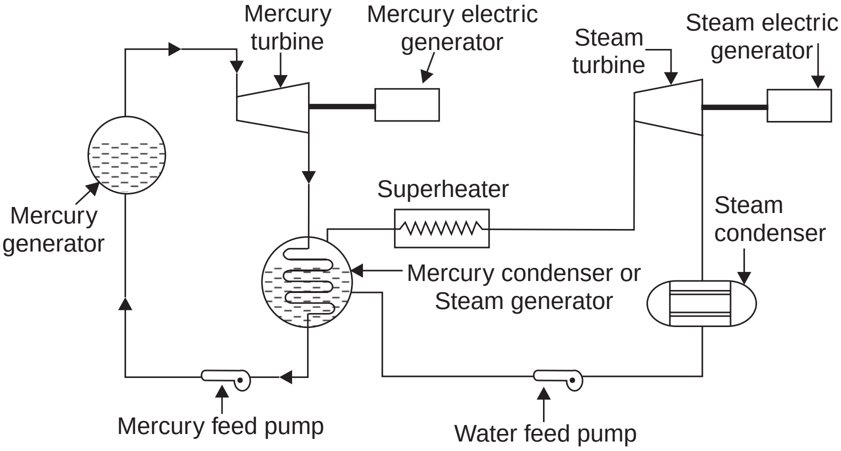


Fig. 12.35. Line diagram of binary vapour cycle.

Analysis of Binary vapour cycle :

h_{hg_1} = Heat supplied per kg of Hg (mercury) vapour formed in the mercury boiler.

h_{hg_2} = Heat lost by one kg of Hg vapour in the mercury condenser.

h_s = Heat given per kg of steam generated in the mercury condenser or steam boiler.

W_{hg} = Work done per kg of Hg in the cycle.

W_s = Work done per kg of steam in the steam cycle.

η_s = Thermal efficiency of the steam cycle.

η_{hg} = Thermal efficiency of the Hg cycle.

m = Mass of Hg in the Hg cycle per kg of steam circulated in the steam cycle.

The heat losses to the surroundings, in the following analysis, are neglected and steam generated is considered one kg and Hg in the circuit is *m* kg per kg of water in the steam cycle.

Heat supplied in the Hg boiler

$$h_t = m \times h_{hg_1} \qquad \text{...(12.14)}$$

Work done in the mercury cycle

$$= m \, . \, W_{hg} \qquad \text{...(12.15)}$$

Work done in the steam cycle

$$= 1 \times W_s \qquad \text{...(12.16)}$$

Total work done in the binary cycle is given by

$$W_t = m\ W_{hg} + W_s \qquad ...(12.17)$$

$\therefore$ Overall efficiency of the binary cycle is given by

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_t}{h_t} = \frac{mW_{hg} + W_s}{mh_{hg_1}} \qquad ...(12.18)$$

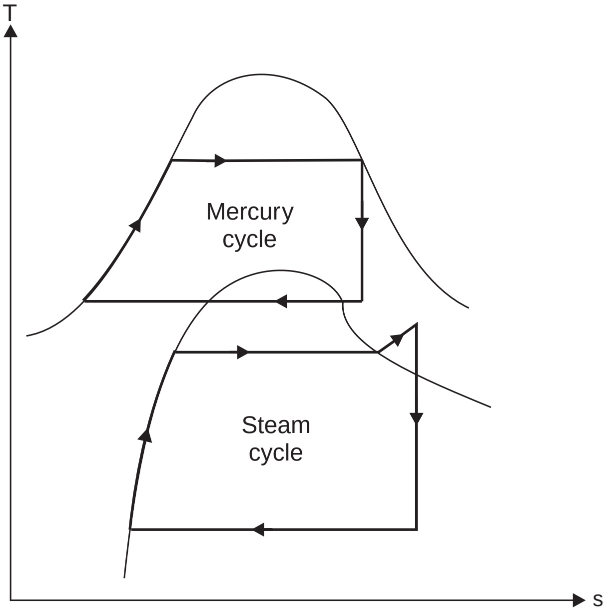


Fig. 12.36. Binary vapour cycle on *T-s* diagram.

Thermal efficiency of the mercury cycle is given by

$$\eta_{hg} = \frac{mW_{hg}}{mh_{hg_1}}$$

$$= \frac{W_{hg}}{h_{hg_1}} = \frac{h_{hg_1} - h_{hg_2}}{h_{hg_1}} = 1 - \frac{h_{hg_2}}{h_{hg_1}} \qquad ...(12.19)$$

$$= \frac{mh_{hg_1} - h_s}{mh_{hg_1}} = 1 - \frac{1}{m} \cdot \frac{h_s}{h_{hg_1}} \qquad ...(12.20)$$

Heat lost by mercury vapour = Heat gained by steam

$$\therefore \qquad m\,h_{hg_2} = 1 \times h_s \qquad ...(12.21)$$

Substituting the value of m from eqn. (12.21) into eqn. (12.20), we get

$$\eta_{hg} = 1 - \frac{h_{hg_2}}{h_{hg_1}} \qquad ...(12.22)$$

The thermal efficiency of the steam cycle is given by

$$\eta_s = \frac{W_s}{h_s} = \frac{h_{s_1} - h_{s_2}}{h_{s_1}} = \frac{h_{s_1} - h_{s_2}}{mh_{hg_2}} \qquad ...(12.23)$$

From the eqns. (12.18), (12.20), (12.21), (12.22) and (12.23), we get

$$\eta = \eta_{hg}(1 - \eta_s) + \eta_s \qquad ...(12.24)$$

To solve the problems eqns. (12.19), (12.23), (12.24) are used.

In the design of binary cycle, another important problem is the limit of exhaust pressure of the mercury (location of optimum exhaust pressure) which will provide maximum work per kg of Hg circulated in the system and high thermal efficiency of the cycle. It is not easy to decide as number of controlling factors are many.

☞**Example 12.22.** *A binary vapour cycle operates on mercury and steam. Standard mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhausts at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.*

(*i*) *Determine the overall efficiency of the cycle.*

(*ii*) *If 48000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?*

(*iii*) *Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?*

(*iv*) *If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler and if the internal efficiencies of the mercury and steam turbines are 0.84 and 0.88 respectively, calculate the overall efficiency of the cycle. The properties of standard mercury are given below :*

p (bar)	*t (°C)*	h_f *(kJ/kg)*	h_g *(kJ/kg)*	s_f *(kJ/kg K)*	s_g *(kJ/kg K)*	v_f *(m³/kg)*	v_g *(m³/kg)*
4.5	*450*	*62.93*	*355.98*	*0.1352*	*0.5397*	79.9×10^{-6}	*0.068*
0.04	*216.9*	*29.98*	*329.85*	*0.0808*	*0.6925*	76.5×10^{-6}	*5.178.*

Solution. The binary vapour cycle is shown in Fig. 12.37.

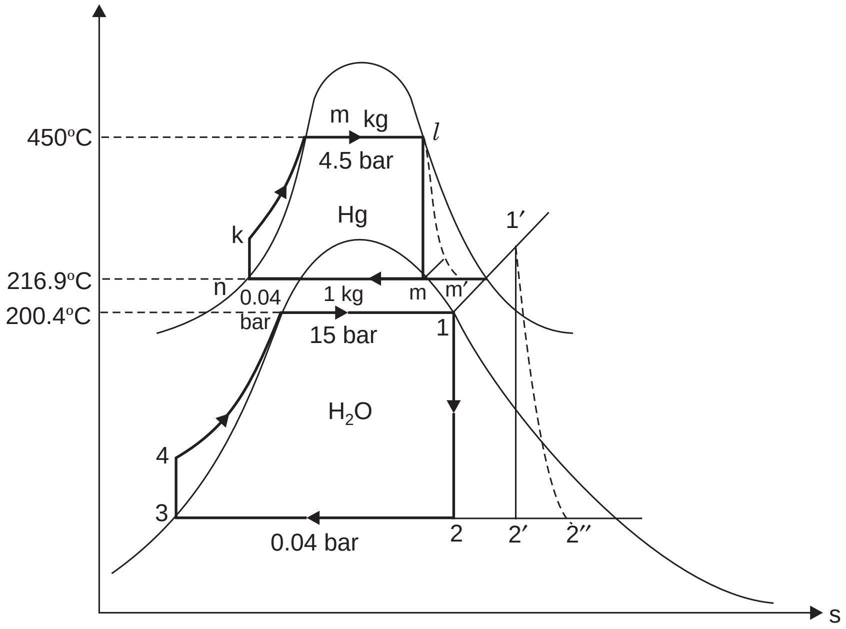


Fig. 12.37

Mercury cycle :

$$h_l = 355.98 \text{ kJ/kg}$$

$$s_l = 0.5397 = s_m = s_f + x_m\, s_{fg}$$

or $$0.5397 = 0.0808 + x_m = (0.6925 - 0.0808)$$

$\therefore$ $$x_m = \frac{(0.5397 - 0.0808)}{(0.6925 - 0.0808)} = 0.75$$

$$h_m = h_f + x_m\, h_{fg} = 29.98 + 0.75 \times (329.85 - 29.98)$$
$$= 254.88 \text{ kJ/kg}$$

Work obtained from mercury turbine

$$(W_T)_{Hg} = h_l - h_m = 355.98 - 254.88 = 101.1 \text{ kJ/kg}$$

Pump work in mercury cycle,

$$(W_P)_{Hg} = h_{f_k} - h_{f_n} = 76.5 \times 10^{-6} \times (4.5 - 0.04) \times 100 = 0.0341 \text{ kJ/kg}$$

$\therefore$ $$W_{net} = 101.1 - 0.0341 \simeq 101.1 \text{ kJ/kg}$$

$$Q_1 = h_l - h_{f_k} = 355.98 - 29.98 = 326 \text{ kJ/kg} \qquad (\because\ h_{f_n} \simeq h_{f_k})$$

$\therefore$ $$\eta_{\text{Hg cycle}} = \frac{W_{net}}{Q_1} = \frac{101.1}{326} = 0.31 \quad \text{or} \quad 31\%.$$

Steam cycle :

At 15 bar : $h_1 = 2789.9$ kJ/kg, $s_1 = 6.4406$ kJ/kg

At 0.04 bar : $h_f = 121.5$ kJ/kg, $h_{f_g} = 2432.9$ kJ/kg,

$s_f = 0.432$ kJ/kg K, $s_{fg_2} = 8.052$ kJ/kg K, $v_f = 0.0001$ kJ/kg K

Now, $$s_1 = s_2$$

$$6.4406 = s_f + x_2\, s_{fg} = 0.423 + x_2 \times 8.052$$

$\therefore$ $$x_2 = \frac{6.4406 - 0.423}{8.052} = 0.747$$

$$h_2 = h_{f_2} + x_2\, h_{f_g} = 121.5 + 0.747 \times 2432.9 = 1938.8 \text{ kJ/kg}$$

Work obtained from steam turbine,

$$(W_T)_{steam} = h_1 - h_2 = 2789.9 - 1938.8 = 851.1 \text{ kJ/kg}$$

Pump work in steam cycle,

$$(W_P)_{steam} = h_{f_4} - h_{f_3} = 0.001\ (15 - 0.04) \times 100 = 1.496 \text{ kJ/kg} \simeq 1.5 \text{ kJ/kg}$$

or $$h_{f_4} = h_{f_3} + 1.5 = 121.5 + 1.5 = 123 \text{ kJ/kg}$$

$$Q_1 = h_1 - h_{f_4} = 2789.9 - 123 = 2666.9 \text{ kJ/kg}$$

$$(W_{net})_{steam} = 851.1 - 1.5 = 849.6 \text{ kJ/kg}$$

$\therefore$ $$\eta_{\text{steam cycle}} = \frac{W_{net}}{Q_1} = \frac{849.6}{2666.6} = 0.318 \text{ or } 31.8\%.$$

(*i*) **Overall efficiency of the binary cycle :**

Overall efficiency of the binary cycle

$$= \eta_{Hg_{\text{cycle}}} + \eta_{\text{steam cycle}} - \eta_{Hg_{\text{cycle}}} \times \eta_{\text{steam cycle}}$$

$$= 0.31 + 0.318 - 0.31 \times 0.318 = 0.5294 \text{ or } 52.94\%$$

Hence *overall efficiency of the binary cycle* = **52.94%. (Ans.)**

η_{overall} can also be found out as follows :

Energy balance for a mercury condenser-steam boiler gives :

$$m\,(h_m - h_{f_n}) = 1(h_1 - h_{f_4})$$

where m is the amount of mercury circulating for 1 kg of steam in the bottom cycle

$$\therefore \quad m = \frac{h_1 - h_{f_4}}{h_m - h_{f_n}} = \frac{2666.9}{254.88 - 29.98} = 11.86 \text{ kg}$$

$$(Q_1)_{\text{total}} = m\,(h_l - h_{f_k}) = 11.86 \times 326 = 3866.36 \text{ kJ/kg}$$

$$(W_{\text{T}})_{\text{total}} = m\,(h_l - h_m) + (h_1 - h_2)$$

$$= 11.86 \times 101.1 + 851.1 = 2050.1 \text{ kJ/kg}$$

$(W_{\text{P}})_{\text{total}}$ may be *neglected*

$$\eta_{\text{overall}} = \frac{W_T}{Q_1} = \frac{2050.1}{3866.36} = 0.53 \text{ or } 53\%.$$

(*ii*) **Flow through mercury turbine :**

If 48000 kg/h of steam flows through the steam turbine, the flow rate of mercury,

m_{Hg} = 48000 × 11.86 = **569280 kg/h. (Ans.)**

(*iii*) **Useful work in binary vapour cycle :**

Useful work, $(W_T)_{\text{total}} = 2050.1 \times 48000 = 9840.5 \times 10^4$ kJ/h

$$= \frac{9840.5 \times 10^4}{3600} = 27334.7 \text{ kW} = \mathbf{27.33\ MW.}\ \ \textbf{(Ans.)}$$

(*iv*) **Overall efficiency under new conditions :**

Considering the efficiencies of turbines, we have :

$$(W_T)_{\text{Hg}} = h_l - h_m' = 0.84 \times 101.1 = 84.92 \text{ kJ/kg}$$

$$\therefore \quad h_{m'} = h_l - 84.92 = 355.98 - 84.92 = 271.06 \text{ kJ/kg}$$

$$\therefore \quad m'\,(h_{m'} - h_{n'}) = (h_1 - h_{f_4})$$

or

$$m' = \frac{h_1 - h_{f_4}}{h_{m'} - h_{n'}} = \frac{2666.9}{271.06 - 29.98} = 11.06 \text{ kg}$$

$$(Q_1)_{\text{total}} = m'\,(h_l - h_{f_k}) + 1\,(h_1' - h_1)$$

[***At 15 bar, 300°C :*** h_g = 3037.6 kJ/kg, s_g = 6.918 kJ/kg K]

$$= 11.06 \times 326 + (3037.6 - 2789.9) = 3853.26 \text{ kJ/kg}$$

$$s_1' = 6.918 = s_2' = 0.423 + x_2' \times 8.052$$

$$\therefore \quad x_2' = \frac{6.918 - 0.423}{8.052} = 0.80.$$

$$h_2' = 121.5 + 0.807 \times 2432.9 = 2084.8 \text{ kJ/kg}$$

$$(W_T)_{\text{steam}} = h_1' - h_2' = 0.88\,(3037.6 - 2084.8)$$

$$= 838.46 \text{ kJ/kg}$$

$$(W_T)_{\text{total}} = 11.06 \times 84.92 + 838.46 = 1777.67 \text{ kJ/kg}$$

Neglecting pump work,

$$\eta_{\text{overall}} = \frac{1777.67}{3853.26} = \mathbf{0.4613 \text{ or } 46.13\%.} \quad \textbf{(Ans.)}$$

ADDITIONAL / TYPICAL EXAMPLES

Example 12.23. *The following data relate to a regenerative steam power plant generating 22500 kW energy, the alternator directly coupled to steam turbine :*

Condition of steam supplied to the steam turbine	*... 60 bar, 450°C*
Condenser vacuum	*... 707.5 mm*
Pressure at which steam is bled from the steam turbine	*... 3 bar*
Turbine efficiency of each portion of expansion	*... 87 per cent*
Boiler efficiency	*... 86 per cent*
Alternator efficiency	*... 94 per cent*
Mechanical efficiency from turbine to generator	*... 97 per cent*

Neglecting the pump work in calculating the input to the boiler, determine :

(*i*) *The steam bled per kg of steam supplied to the turbine.*

(*ii*) *The steam generated per hour if the 9 percent of the generator output is used to run the pumps.*

(*iii*) *The overall efficiency of the plant.*

Solution. The schematic arrangement of the steam power plant is shown in Fig. 12.38 (*a*), while the conditions of the fluid passing through the components are represented on *T-s* and *h-s* diagrams as shown in Figs. 12.38 (*b*) and (*c*). The conditions of the fluid entering and leaving the pump are shown by the same point as the rise in temperature due to pump work is neglected.

Given : Power generated = 22500 kW ;

$$p_1 = 60 \text{ bar} ; t_1 = 450°\text{C} ; p_2 (= p_2') = 3 \text{ bar} ;$$

$$p_3 (= p_3') = \frac{760 - 707.5}{760} \times 1.013 = 0.07 \text{ bar} ; \eta_{\text{turbine (each portion)}} = 87\% ;$$

$$\eta_{\text{boiler}} = 86\% ; \eta_{\text{alt.}} = 94\%, \eta_{\text{mech.}} = 97\%$$

- Locate point 1 corresponding to the conditions : $p_1 = 60$ bar ; $t_1 = 450°$C on the *h-s* chart *(Mollier chart)*.

  From *h-s* chart ; we find : $h_1 = 3300$ kJ/kg.

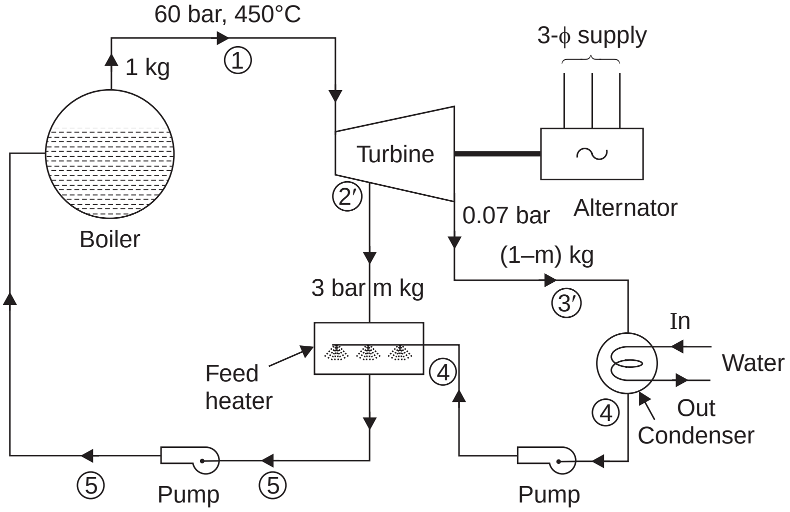
- Draw vertical line through point 1 till it cuts the 3 bar pressure line, then locate point 2.

  $\therefore \quad h_2 = 2607$ kJ/kg

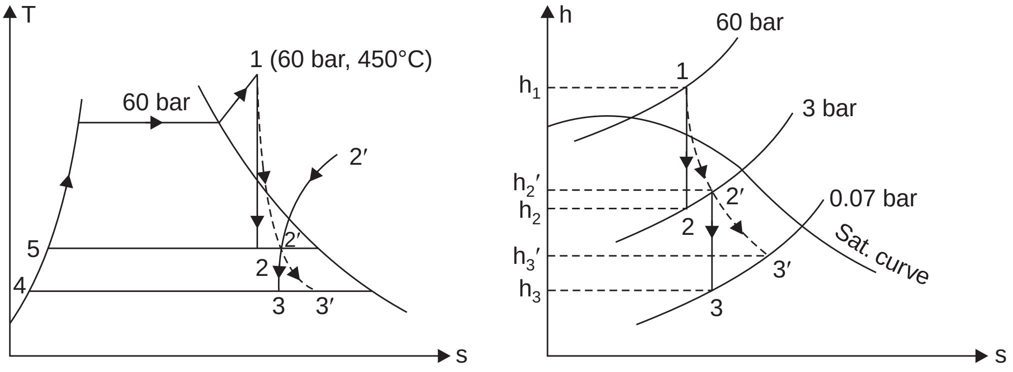
  Now, $\quad \eta_{\text{turbine}} = 0.87 = \dfrac{h_1 - h_2'}{h_1 - h_2} \quad$ or $\quad 0.87 = \dfrac{3300 - h_2'}{3300 - 2607}$

  $\therefore \quad h_2' = 2697$ kJ/kg

- Locate the point 2 on the *h-s* chart as enthalpy and pressure are known and then draw a vertical line through the point 2 till it cuts the 0.07 bar pressure line and then locate the point 3.



(*a*) Schematic arrangement of the steam power plant



(*b*) *T-s* diagram (Neglecting pump work)

(*c*) *h-s* diagram

Fig. 12.38

∴ $h_3 = 2165$ kJ/kg

Again, $\eta_{\text{turbine}} = 0.87 = \dfrac{h_2' - h_3'}{h_2' - h_3}$ or $0.87 = \dfrac{2697 - h_3'}{2697 - 2165}$

∴ $h_3' = 2234$ kJ/kg

From steam tables, corresponding to pressures 3 bar and 0.02 bar, the saturated liquid heats at points 4 and 5 are :

$$h_{f4} = 163.4 \text{ kJ/kg} ; h_{f5} = 561.4 \text{ kJ/kg}.$$

(*i*) **The steam bled per kg of steam supplied to the turbine, m :**

Considering the *energy balance for feed heater* we have ;

$$m(h_2' - h_{f5}) = (1 - m)(h_{f5} - h_{f4})$$

or $$m(2697 - 561.4) = (1 - m)(561.4 - 163.4)$$

or $$2135.6\, m = 398\,(1 - m)$$

∴ m = **0.157 kJ/kg of steam generated. (Ans.)**

(*ii*) **Steam generated per hour :**

Work developed per kg of steam in the turbine

$$= 1(h_1 - h_2') + (1 - m)(h_2' - h_3')$$

$$= (3300 - 2697) + (1 - 0.157)(2697 - 2234) = 993.3 \text{ kJ/kg}$$

Actual work developed by the turbine

$$= \frac{22500}{\eta_{\text{alt.}} \times \eta_{\text{mech.}}} = \frac{22500}{0.94 \times 0.97} = 24676.5 \text{ kW}$$

$$\therefore \quad \text{Steam generated per hour} = \frac{24676.5}{993.3} \times \frac{3600}{1000} \text{ tonnes/h} = \textbf{89.43 tonnes/h.} \quad \textbf{(Ans.)}$$

(*iii*) **The overall efficiency of the plant, η_{overall} :**

Net power available deducting pump power

$$= 22500(1 - 0.09) = 20475 \text{ kW}$$

$$\text{Heat supplied in the boiler} = \frac{89.43 \times 1000 (h_1 - h_{f5})}{0.86} \text{ kJ/h}$$

$$= \frac{89.43 \times 1000 (3300 - 561.4)}{0.86 \times 3600} \text{ kW} = 79106.3 \text{ kW}$$

$$\therefore \quad \eta_{\text{overall}} = \frac{\text{Net power available}}{\text{Heat supplied by the boiler}}$$

$$= \frac{20475}{79106.3} = 0.2588 \quad \text{or} \quad \textbf{25.88\%.} \quad \textbf{(Ans.)}$$

Example 12.24. *A steam power plant of 110 MW capacity is equipped with regenerative as well as reheat arrangement. The steam is supplied at 80 bar and 55°C of superheat. The steam is extracted at 7 bar for feed heating and remaining steam is reheated to 350°C, and then expanded to 0.4 bar in the L.P. stage. Assume indirect type of feed heaters. Determine :*

(*i*) *The ratio of steam bled to steam generated,*

(*ii*) *The boiler generating capacity in tonnes of steam/hour, and*

(*iii*) *Thermal efficiency of the cycle.*

Assume no losses and ideal processes of expansion.

Solution. The schematic arrangement of the plant is shown in Fig. 12.39 (*a*) and the processes are represented on *h-s* chart in Fig. 12.39 (*b*).

Given : Capacity of plant = 110 MW ;

$$t_1 = 350°\text{C } \textit{i.e.,}\ t_s \text{ at 80 bar} \simeq 295°\text{C} + 55°\text{C} = 350°\text{C})$$

$$p_2 = p_3 = 7 \text{ bar} ;\ t_3 = 350°\text{C} ;\ p_4 = 0.4 \text{ bar}$$

- Locate point 1 corresponding to the condition p_1 = 80 bar and t_1 = 350°C, on the *h-s* chart.
- Locate point 2 by drawing vertical line through point 1 till it cuts the 7 bar pressure line.
- Locate point 3 as the cross point of 7 bar and 350°C temperature line.
- Locate point 4 by drawing vertical line through the point 3 till it cuts the 0.4 bar pressure line.

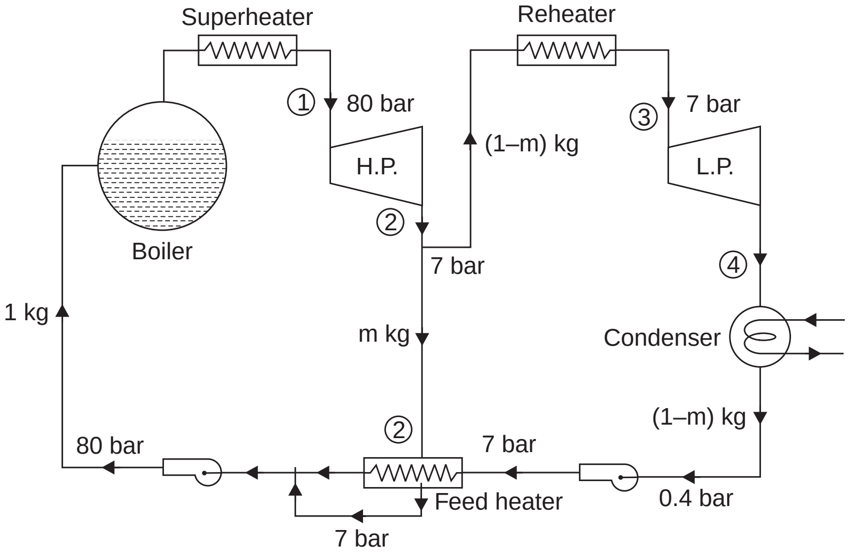
From *h-s* chart, we find :

$$h_1 = 2985 \text{ kJ/kg} ;\ h_2 = 2520 \text{ kJ/kg} ;$$

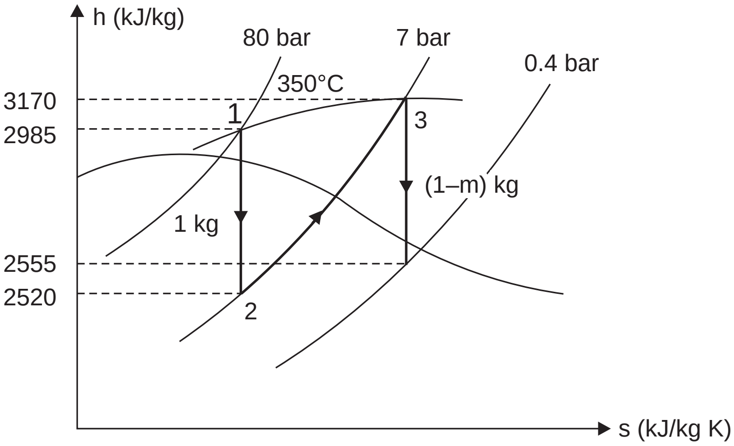
$$h_3 = 3170 \text{ kJ/kg} ;\ h_4 = 2555 \text{ kJ/kg.}$$

Also, from steam tables, we have :

$$h_{f2} \text{ (at 7 bar)} = 697.1 \text{ kJ/kg} ;\ h_{f4} \text{ (at 0.4 bar)} = 317.7 \text{ kJ/kg.}$$



(*a*) Schematic arrangement of the plant



(*b*) *h-s* diagram

Fig. 12.39

(*i*) **The ratio of steam bled to steam generated :**

Consider energy/heat balance of feed heater :

Heat lost by m kg of steam = Heat gained by $(1 - m)$ kg of condensed steam

$$m(h_2 - h_{f2}) = (1 - m)\,(h_{f2} - h_{f4})$$

$$m(2520 - 697.1) = (1 - m)\,(697.1 - 317.7)$$

$$1822.9\,m = (1 - m) \times 379.4$$

$$\therefore \quad m = 0.172 \text{ kg}$$

i.e. Amount of steam bled per kg of steam supplied to the turbine = 0.172 kg

$$\therefore \quad \frac{\text{Steam generated}}{\text{Steam bled}} = \frac{1}{0.172} = \mathbf{5.814.} \quad \textbf{(Ans.)}$$

(*ii*) **The boiler generating capacity :**

If m_s is the mass of steam supplied to the power plant per second, then the work developed is given by :

$$m_s(h_1 - h_2) + m_s(1 - m)(h_3 - h_4) = 110 \times 10^3$$

or, $$m_s(2985 - 2520) + m_s(1 - 0.172)(3170 - 2555) = 110 \times 10^3$$

or, $$m_s(465 + 509.22) = 110 \times 10^3$$

$\therefore$ $m_s = 112.91$ kg/s or **406.48 tonnes/hour (Ans.)**

(*iii*) **Thermal efficiency of the cycle, η_{thermal} :**

$$\eta_{\text{thermal}} = \frac{\text{Output / kg of steam}}{\text{Input / kg of steam}} = \frac{(h_1 - h_2) + (1 - m)(h_3 - h_4)}{(h_1 - h_{f_2}) + (1 - m)(h_3 - h_2)}$$

$$= \frac{(2985 - 2520) + (1 - 0.172)(3170 - 2555)}{(2985 - 697.1) + (1 - 0.172)(3170 - 2520)}$$

$$= \frac{974.22}{2826.1} = 0.3447$$ or **34.47%. (Ans.)**

Example 12.25. *A steam power plant equipped with regenerative as well as reheat arrangement is supplied with steam to the H.P. turbine at 80 bar 470°C. For feed heating, a part of steam is extracted at 7 bar and remainder of the steam is reheated to 350°C in a reheater and then expanded in L.P. turbine down to 0.035 bar. Determine :*

(*i*) *Amount of steam bled-off for feed heating,*

(*ii*) *Amount of steam supplied to L.P. turbine,*

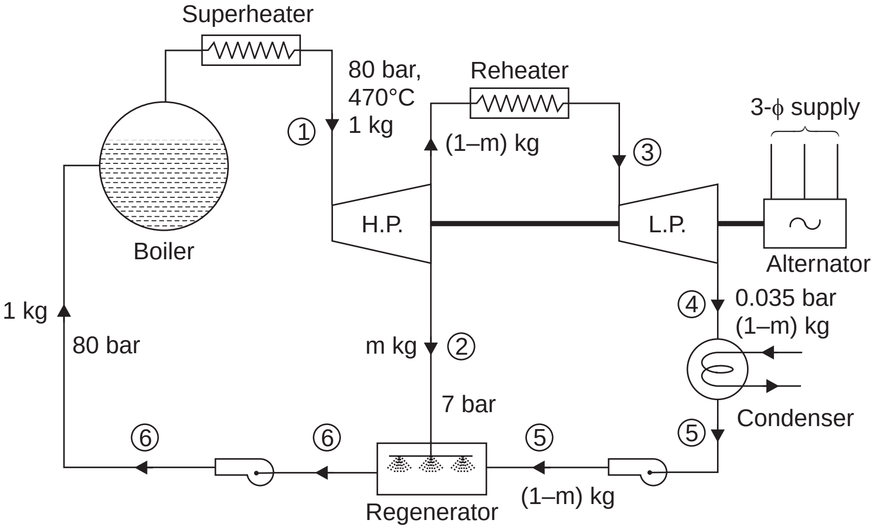
(*iii*) *Heat supplied in the boiler and reheater*

(*iv*) *Cycle efficiency, and*

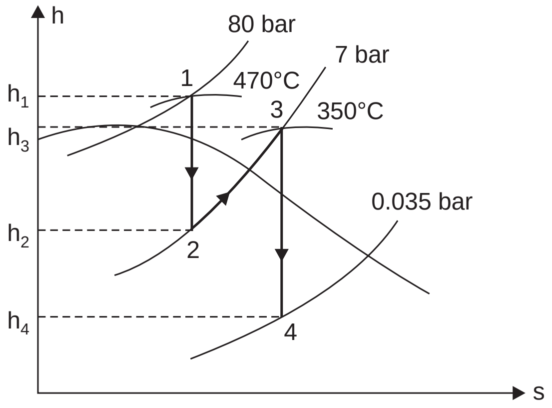
(*v*) *Power developed by the system.*

The steam supplied by the boiler is 50 kg/s. **(B.U. Dec., 2000)**

Solution. The schematic arrangement is the steam power plant of shown in Fig. 12.40 (*a*) and the processes are represented on *h-s* diagram as shown in Fig. 12.40 (*b*).



(*a*) Schematic arrangement of the steam power plant



(*b*) *h-s* diagram

Fig. 12.40

From *h-s* chart and steam tables, we have enthalpies at different points as follows :

$$\left.\begin{array}{ll} h_1 = 3315 \text{ kJ/kg ;} & h_2 = 2716 \text{ kJ/kg} \\ h_3 = 3165 \text{ kJ/kg ;} & h_4 = 2236 \text{ kJ/kg} \end{array}\right\} \text{ From } h\text{-}s \text{ chart}$$

$$h_{f6} = h_{f2} = 697.1 \text{ kJ/kg ;} \qquad h_{f5} = h_{f4} = 101.9 \text{ kJ/kg } \} \text{ From steam table.}$$

(*i*) **Amount of steam bled off for feed heating :**

Considering *energy balance at regenerator, we have :*

Heat lost by steam = Heat gained by water

$$m(h_2 - h_{f6}) = (1 - m)\,(h_{f6} - h_{f5})$$

or $$m(h_2 - h_{f2}) = (1 - m)\,(h_{f2} - h_{f4}) \qquad [\because \; h_{f6} = h_{f2}\,;\, h_{f5} = h_{f4}]$$

or $$m(2716 - 697.1) = (1 - m)\,(697.1 - 111.9)$$

or $$2018.9\, m = 585.2\,(1 - m)$$

$$\therefore \qquad m = 0.225 \text{ g of steam supplied}$$

Hence amount of **steam bled off is 22.5% of steam generated by the boiler. (Ans.)**

(*ii*) **Amount of steam supplied to L.P. turbine :**

Amount of steam supplied to L.P. turbine

= 100 – 22.5

= **77.5% of the steam generated by the boiler. (Ans.)**

(*iii*) **Heat supplied in the boiler and reheater**

Heat supplied in the *boiler* per kg of steam generated

= $h_1 - h_{f6}$ = 3315 – 697.1 = **2617.9 kJ/kg. (Ans.)**

$(\because \; h_{f6} = h_{f2})$

Heat supplied in the *reheater* per kg of steam generated

= $(1 - m)\,(h_3 - h_2)$

= (1 – 0.225) (3165 – 2716) = **347.97 kJ/kg. (Ans.)**

Total amount of heat supplied by the boiler and reheater per kg of steam generated,

$$Q_s = 2617.9 + 347.97 = 2965.87 \text{ kJ/kg}$$

(*iv*) **Cycle efficiency, η_{cycle} :**

Amount of work done by per kg of steam generated by the boiler,

$$W = 1(h_1 - h_2) + (1 - m)(h_3 - h_4), \text{ Neglecting pump work}$$

$$= (3315 - 2716) + (1 - 0.225)(3165 - 2236) \simeq 1319 \text{ kJ/kg}$$

$$\therefore \quad \eta_{\text{cycle}} = \frac{W}{Q_s} = \frac{1319}{2965.87} = 0.4447 \quad \text{or} \quad \mathbf{44.47\%} \quad \textbf{(Ans.)}$$

(*v*) **Power developed by the system :**

Power developed by the system

$$= m_s \times W = 50 \times 1319 \text{ kJ/s} = \frac{50 \times 1319}{1000}$$

$$= \mathbf{65.95\ MW} \quad \textbf{(Ans.)}$$

Example 12.26. *A steam power plant operates on ideal Rankine cycle using reheater and regenerative feed water heaters. It has one open feed heater. Steam is supplied at 150 bar and 600°C. The condenser pressure is 0.1 bar. Some steam is extracted from the turbine at 40 bar for closed feed water heater and remaining steam is reduced at 40 bar to 600°C. Extracted steam is completely condensed in this closed feed water heater and is pumped to 150 bar before mixing with the feed water heater. Steam for the open feed water heater is bled from L.P. turbine at 5 bar. Determine :*

(*i*) *Fraction of steam extracted from the turbines at each bled heater, and*

(*ii*) *Thermal efficiency of the system.*

Draw the line diagram of the components and represent the cycle on T-s diagram.

(P.U. Dec., 2001)

Solution. The arrangement of the components is shown in Fig. 12.41 (*a*) and the processes are represented on *T-s* diagram as shown in Fig. 12.41 (*b*).

From *h-s* chart and steam tables we have enthalpies at different points as follows :

$$\left.\begin{aligned} &h_1 = 3578 \text{ kJ/kg} ; \quad h_2 = 3140 \text{ kJ/kg} ; \\ &h_3 = 3678 \text{ kJ/kg} ; \quad h_4 = 3000 \text{ kJ/kg} ; \\ &h_5 = 2330 \text{ kJ/kg} ; \end{aligned}\right\} \text{From } h\text{-}s \text{ chart}$$

$$\left.\begin{aligned} &h_{f1} \text{ (at 150 bar)} = 1611 \text{ kJ/kg} \\ &h_{f2} \text{ (at 40 bar)} = 1087.4 \text{ kJ/kg} ; \ h_{f4} \text{ (at 5 bar)} = 640.1 \text{ kJ/kg} ; \\ &h_{f5} = h_{f6} \text{ (at 0.1 bar)} = 191.8 \text{ kJ/kg} \end{aligned}\right\} \text{Steam tables}$$

(*i*) **Fraction of steam extracted from the turbines at each bled heater $\mathbf{m_1, m_2}$:**

Considering *energy balance for closed feed heater,* we have :

$$m_1(h_2 - h_{f2}) = (1 - m_1)(h_{f2} - h_{f4})$$

$$m_1(3140 - 1087.4) = (1 - m_1)(1087.4 - 640.1)$$

or
$$2052.6\, m_1 = (1 - m_1) \times 447.3$$

$$\therefore \quad \mathbf{m_1 = 0.179\ kg/kg\ of\ steam\ supplied\ by\ the\ boiler.} \quad \textbf{(Ans.)}$$

Considering *energy balance for open feed heater,* we have :

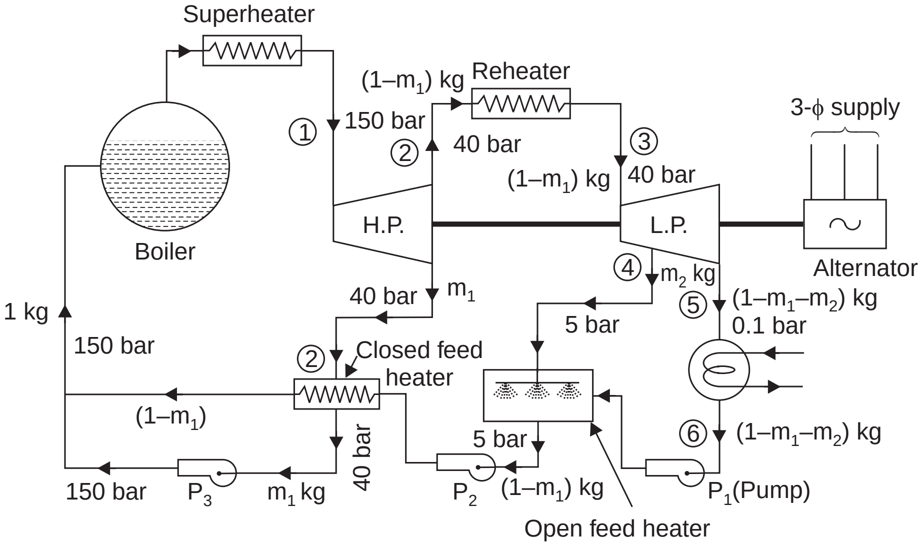
$$m_2(h_4 - h_{f4}) = (1 - m_1 - m_2)(h_{f4} - h_{f6})$$

or
$$m_2(h_4 - h_{f4}) = (1 - m_1 - m_2)(h_{f4} - h_{f5}) \qquad (\because \ h_{f6} = h_{f5})$$

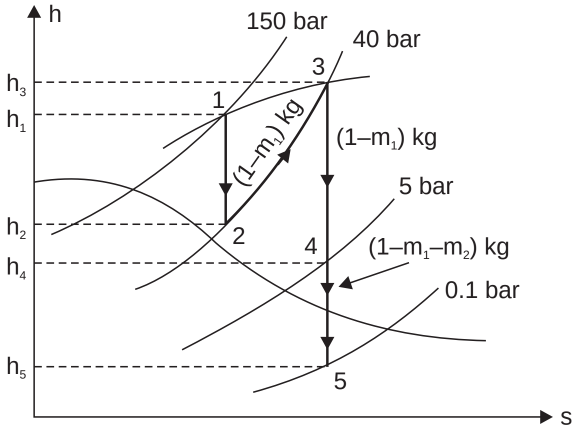
or
$$m_2(3000 - 640.1) = (1 - 0.179 - m_2)(640.1 - 191.8)$$

or
$$2359.9\, m_2 = (0.821 - m_2) \times 448.3 = 368.05 - 448.3\, m_2$$

$$\therefore \quad \mathbf{m_2 = 0.131\ kg/kg\ of\ steam\ supplied\ by\ boiler.} \quad \textbf{(Ans.)}$$



(*a*) Schematic arrangement of the steam power plant



(*b*) *h-s* diagram

Fig. 12.41

(*ii*) **Thermal efficiency of the system, η_{thermal} :**

Total work done per kg of steam supplied by the boiler

$$= 1 \times (h_1 - h_2) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$$

$$= (3578 - 3140) + (1 - 0.179)(3678 - 3000) + (1 - 0.179 - 0.131)(3000 - 2330)$$

$$= 438 + 556.64 + 462.3 = 1456.94 \text{ kJ/kg}$$

Work done by the pump P_1

$$W_{P1} = v_{w1}\,(1 - m_1 - m_2)(5 - 0.1) \times 10^5 \times 10^{-3} \text{ kJ/kg}$$

$$= \frac{1}{1000}\,(1 - 0.179 - 0.131)(5 - 0.1) \times 10^5 \times 10^{-3} = 0.338 \text{ kJ/kg}$$

$$\left[\text{Taking } v_{w1} = v_{w2} = v_{w3} = \frac{1}{1000}\text{ m}^3\text{/kg}\right]$$

Work done by the pump P_2,

$$W_{P2} = v_{w2}\,(1 - m_1)(150 - 5) \times 10^5 \times 10^{-3}\ \text{kJ/kg}$$

$$= \frac{1}{1000}\,(1 - 0.179)(150 - 5) \times 10^5 \times 10^{-3} = 11.9\ \text{kJ/kg}$$

Work done by pump P_3,

$$W_{P3} = v_{w3} \times m_1 \times (150 - 40) \times 10^5 \times 10^{-3}$$

$$= \frac{1}{1000} \times 0.179\,(150 - 40) \times 10^5 \times 10^{-3} = 1.97\ \text{kJ/kg}$$

Total pump work $= W_{P1} + W_{P2} + W_{P3}$

$= 0.338 + 11.9 + 1.97 = 14.21$ kJ/kg of steam supplied by boiler

∴ Net work done by the turbine per kg of steam supplied by the boiler,

$$W_{\text{net}} = 1456.94 - 14.21 = 1442.73\ \text{kJ/kg}$$

Heat of feed water extering the boiler

$$= (1 - m_1) \times 1611 + m_1 \times 1611 = 1611\ \text{kJ/kg}$$

Heat supplied by the boiler per kg of steam,

$$Q_{s1} = h_1 - 1610 = 3578 - 1610 = 1968\ \text{kJ/kg}$$

Q_{s2} = Heat supplied in the reheater

$= (1 - m_1)(h_3 - h_2) = (1 - 0.179)(3678 - 3140)$

$= 441.7$ kJ/kg of steam supplied by the boiler

Q_{st} (Total heat supplied) $= Q_{s1} + Q_{s2} = 1968 + 441.7 = 2409.7$ kJ/kg

$$\therefore \quad \eta_{\text{thermal}} = \frac{W_{\text{net}}}{Q_{st}} = \frac{1442.73}{2409.7} = 0.5987 \quad \text{or} \quad \mathbf{59.87\%.}\ \textbf{(Ans.)}$$

Example 12.27. *Steam at 70 bar and 450°C is supplied to a steam turbine. After expanding to 25 bar in high pressure stages, it is reheated to 420°C at the constant pressure. Next ; it is expanded in intermediate pressure stages to an appropriate minimum pressure such that part of the steam bled at this pressure heats the feed water to a temperature of 180°C. The remaining steam expands from this pressure to a condenser pressure of 0.07 bar in the low pressure stage. The isentropic efficiency of H.P. stage is 78.5%, while that of the intermediate and L.P. stages is 83% each. From the above data, determine :*

(*i*) *The minimum pressure at which bleeding is necessary.*

(*ii*) *The quantity of steam bled per kg of flow at the turbine inlet.*

(*iii*) *The cycle efficiency.*

Neglect pump work. **(Roorkee University)**

Solution. The schematic arrangement of the plant is shown in Fig. 12.42 (*a*) and the processes are represented on *T-s* and *h-s* diagrams as shown in Figs. 12.42 (*b*) and (*c*) respectively.

(*i*) **The minimum pressure at which bleeding is necessary :**

It would be assumed that the feed water heater is an open heater. Feed water is heated to 180°C. So p_{sat} at 180°C $\simeq$ 10 bar is the pressure at which the heater operates.

Thus, *the pressure at which bleeding is necessary is* **10 bar. (Ans.)**

From the *h-s* chart (Mollier chart), we have :

$$h_1 = 3285\ \text{kJ/kg}\ ;\ h_2 = 2980\ \text{kJ/kg}\ ;\ h_3 = 3280\ \text{kJ/kg}\ ;\ h_4 = 3030\ \text{kJ/kg}$$

$$h_3 - h_4' = 0.83(h_3 - h_4) = 0.83(3280 - 3030) = 207.5\ \text{kJ/kg}$$

$$\therefore \quad h_4' = h_3 - 207.5 = 3280 - 207.5 = 3072.5\ \text{kJ/kg}$$

$$h_5 = 2210 \text{ kJ/kg}$$

$$h_4' - h_5' = 0.83(h_4' - h_5) = 0.83(3072.5 - 2210) \simeq 715.9 \text{ kJ/kg}$$

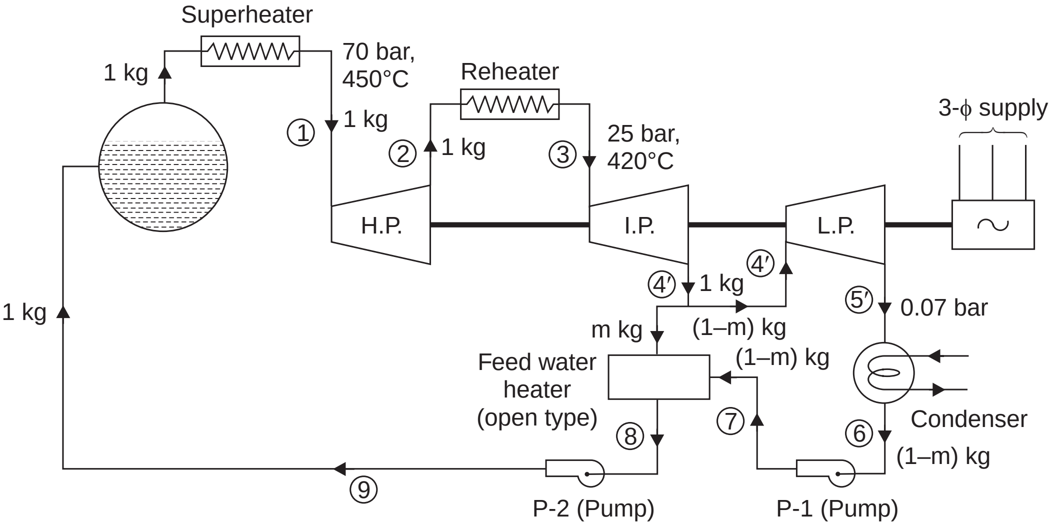
$$\therefore \quad h_5' = h_4' - 715.9 = 3072.5 - 715.9 = 2356.6 \text{ kJ/kg}$$

From steam tables, we have :

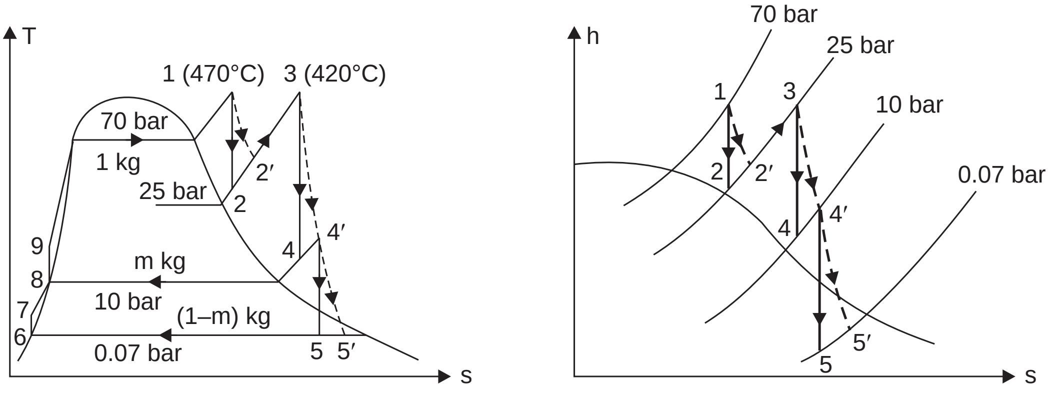
$$h_{f6} = 163.4 \text{ kJ/kg} \ ; \quad h_{f8} = 762.6 \text{ kJ/kg}$$

$$h_1 - h_2' = 0.785(h_1 - h_2) = 0.785(3285 - 2980) = 239.4 \text{ kJ/kg}$$

$$\therefore \quad h_2' = h_1 - 239.4 = 3285 - 239.4 = 3045.6 \text{ kJ/kg}$$



(*a*) Schematic arrangement of the plant



(*b*) *T-s* diagram (*c*) *h-s* diagram (Pump work not shown)

Fig. 12.42

(*ii*) **The quantity of steam bled per kg of flow at the turbine inlet, m :**

Considering *energy balance for the feed water heater, we have :*

$$m \times h_4' + (1 - m)\, h_{f7} = 1 \times h_{f8}$$

$$m \times 3072.5 + (1 - m) \times 163.4 = 1 \times 762.6 \qquad (\because \ h_{f7} = h_{f6})$$

$$3072.5\, m + 163.4 - 163.4\, m = 762.6$$

$$\therefore \quad m = \frac{(762.6 - 163.4)}{(3072.5 - 163.4)}$$

= **0.206 kg of steam flow at turbine inlet. (Ans.)**

(*iii*) **Cycle efficiency, η_{cycle} :**

$$\eta_{\text{cycle}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{1(h_1 - h_2) + 1(h_3 - h_4) + (1 - m)(h_4' - h_5')}{(h_1 - h_{f8}) + (h_3 - h_2')}$$

$$= \frac{(3285 - 3045.6) + 207.5 + (1 - 0.206)(715.9)}{(3285 - 762.6) + (3280 - 3045.6)} = \frac{1015.3}{2756.8}$$

= 0.3683 or **36.83%. (Ans.)**

HIGHLIGHTS

1. Carnot cycle efficiency = $\dfrac{T_1 - T_2}{T_1}$.

2. Rankine cycle is the theoretical cycle on which steam primemovers work.

Rankine efficiency = $\dfrac{h_1 - h_2}{h_1 - h_{f_2}}$.

3. The thermal efficiency of Rankine cycle is increased by

(*i*) *Increasing the average temperature at which heat is added to the cycle.*

(*ii*) *Decreasing the average temperature at which heat is rejected to the cycle.*

4. Thermal efficiency of regenerative cycle

$$= \frac{(h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)}{(h_0 - h_{f_6})} .$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. Rankine cycle efficiency of a good steam power plant may be in the range of

(*a*) 15 to 20% (*b*) 35 to 45%

(*c*) 70 to 80% (*d*) 90 to 95% .

2. Rankine cycle operating on low pressure limit of p_1 and high pressure limit of p_2

(*a*) has higher thermal efficiency than the Carnot cycle operating between same pressure limits

(*b*) has lower thermal efficiency than Carnot cycle operating between same pressure limits

(*c*) has same thermal efficiency as Carnot cycle operating between same pressure limits

(*d*) may be more or less depending upon the magnitudes of p_1 and p_2.

3. Rankine efficiency of a steam power plant

(*a*) improves in summer as compared to that in winter

(*b*) improves in winter as compared to that in summer

(*c*) is unaffected by climatic conditions

(*d*) none of the above.

4. Rankine cycle comprises of

(*a*) two isentropic processes and two constant volume processes

(*b*) two isentropic processes and two constant pressure processes

(*c*) two isothermal processes and two constant pressure processes
(*d*) none of the above.

5. In Rankine cycle the work output from the turbine is given by
(*a*) change of internal energy between inlet and outlet
(*b*) change of enthalpy between inlet and outlet
(*c*) change of entropy between inlet and outlet
(*d*) change of temperature between inlet and outlet.

6. Regenerative heating *i.e.,* bleeding steam to reheat feed water to boiler
(*a*) decreases thermal efficiency of the cycle
(*b*) increases thermal efficiency of the cycle
(*c*) does not affect thermal efficiency of the cycle
(*d*) may increase or decrease thermal efficiency of the cycle depending upon the point of extraction of steam.

7. Regenerative cycle thermal efficiency
(*a*) is always greater than simple Rankine thermal efficiency
(*b*) is greater than simple Rankine cycle thermal efficiency only when steam is bled at particular pressure
(*c*) is same as simple Rankine cycle thermal efficiency
(*d*) is always less than simple Rankine cycle thermal efficiency.

8. In a regenerative feed heating cycle, the optimum value of the fraction of steam extracted for feed heating
(*a*) decreases with increase in Rankine cycle efficiency
(*b*) increases with increase in Rankine cycle efficiency
(*c*) is unaffected by increase in Rankine cycle efficiency
(*d*) none of the above.

9. In a regenerative feed heating cycle, the greatest economy is affected
(*a*) when steam is extracted from only one suitable point of steam turbine
(*b*) when steam is extracted from several places in different stages of steam turbine
(*c*) when steam is extracted only from the last stage of steam turbine
(*d*) when steam is extracted only from the first stage of steam turbine.

10. The maximum percentage gain in Regenerative feed heating cycle thermal efficiency
(*a*) increases with number of feed heaters increasing
(*b*) decreases with number of feed heaters increasing
(*c*) remains same unaffected by number of feed heaters
(*d*) none of the above.

Answers

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*b*) **5.** (*b*) **6.** (*b*) **7.** (*a*)
8. (*b*) **9.** (*b*) **10.** (*a*).

THEORETICAL QUESTIONS

1. Explain the various operation of a Carnot cycle. Also represent it on a T-s and p-V diagrams.
2. Describe the different operations of Rankine cycle. Derive also the expression for its efficiency.
3. State the methods of increasing the thermal efficiency of a Rankine cycle.
4. Explain with the help of neat diagram a 'Regenerative Cycle'. Derive also an expression for its thermal efficiency.
5. State the advantages of regenerative cycle/simple Rankine cycle.
6. Explain with a neat diagram the working of a Binary vapour cycle.

UNSOLVED EXAMPLES

1. A simple Rankine cycle works between pressure of 30 bar and 0.04 bar, the initial condition of steam being dry saturated, calculate the cycle efficiency, work ratio and specific steam consumption.

[**Ans.** 35%, 0.997, 3.84 kg/kWh]

2. A steam power plant works between 40 bar and 0.05 bar. If the steam supplied is dry saturated and the cycle of operation is Rankine, find :

(*i*) Cycle efficiency (*ii*) Specific steam consumption.

[**Ans.** (*i*) 35.5%, (*ii*) 3.8 kg/kWh]

3. Compare the Rankine efficiency of a high pressure plant operating from 80 bar and 400°C and a low pressure plant operating from 40 bar 400°C, if the condenser pressure in both cases is 0.07 bar.

[**Ans.** 0.391 and 0.357]

4. A steam power plant working on Rankine cycle has the range of operation from 40 bar dry saturated to 0.05 bar. Determine :

(*i*) The cycle efficiency (*ii*) Work ratio

(*iii*) Specific fuel consumption. [**Ans.** (*i*) 34.64%, (*ii*) 0.9957, (*iii*) 3.8 kg/kWh]

5. In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 30 bar and the exhaust pressure is 0.25 bar. Determine :

(*i*) The pump work (*ii*) Turbine work

(*iii*) Rankine efficiency (*iv*) Condenser heat flow

(*v*) Dryness at the end of expansion.

Assume flow rate of 10 kg/s. [**Ans.** (*i*) 30 kW, (*ii*) 7410 kW, (*iii*) 29.2%, (*iv*) 17900 kW, (*v*) 0.763]

6. In a regenerative cycle the inlet conditions are 40 bar and 400°C. Steam is bled at 10 bar in regenerative heating. The exit pressure is 0.8 bar. Neglecting pump work determine the efficiency of the cycle.

[**Ans.** 0.296]

7. A turbine with one bleeding for regenerative heating of feed water is admitted with steam having enthalpy of 3200 kJ/kg and the exhausted steam has an enthalpy of 2200 kJ/kg. The ideal regenerative feed water heater is fed with 11350 kg/h of bled steam at 3.5 bar (whose enthalpy is 2600 kJ/h). The feed water (condensate from the condenser) with an enthalpy of 134 kJ/kg is pumped to the heater. It leaves the heater dry saturated at 3.5 bar. Determine the power developed by the turbine. [**Ans.** 16015 kW]

8. A binary-vapour cycle operates on mercury and steam. Saturated mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhaust at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.

(*i*) Find the overall efficiency of the cycle.

(*ii*) If 50000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?

(*iii*) Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?

(*iv*) If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler, and if the internal efficiencies of the mercury and steam turbines are 0.85 and 0.87 respectively, calculate the overall efficiency of the cycle. The properties of saturated mercury are given below :

p (bar)	*t (°C)*	h_f	h_g	s_f	s_g	v_f	v_g
		(kJ/kg)		*(kJ/kg K)*		*(m^3/kg)*	
4.5	450	63.93	355.98	0.1352	0.5397	79.9×10^{-6}	0.068
0.04	216.9	29.98	329.85	0.0808	0.6925	76.5×10^{-3}	5.178

[**Ans.** (*i*) 52.94%, (*ii*) 59.35×10^4 kg/h, (*iii*) 28.49 MW, (*iv*) 46.2%]

13

Gas Power Cycles

13.1. DEFINITION OF A CYCLE

A **cycle** is defined as a *repeated series of operations occurring in a certain order*. It may be repeated by repeating the processes in the same order. The cycle may be of *imaginary perfect engine or actual engine*. The former is called ***ideal cycle*** and the latter ***actual cycle.*** *In ideal cycle all accidental heat losses are prevented and the working substance is assumed to behave like a perfect working substance.*

13.2. AIR STANDARD EFFICIENCY

To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed to behave as a perfect gas) as the working substance in the engine cylinder. *The efficiency of engine using air as the working medium is known as an "**Air standard efficiency**"*. This efficiency is oftenly called ***ideal efficiency.***

The actual efficiency of a cycle is always *less* than the air-standard efficiency of that cycle under ideal conditions. This is taken into account by introducing a new term "***Relative efficiency***" which is defined as :

$$\eta_{\text{relative}} = \frac{\text{Actual thermal efficiency}}{\text{Air standard efficiency}} \qquad ...(13.1)$$

The analysis of all air standard cycles is based upon the following *assumptions* :

Assumptions :

1. The gas in the engine cylinder is a *perfect gas i.e.*, it obeys the gas laws and has constant specific heats.
2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures *i.e.*, the molecular weight of cylinder gas is 29.

$$c_p = 1.005 \text{ kJ/kg-K}, \; c_v = 0.718 \text{ kJ/kg-K}.$$

3. The compression and expansion processes are adiabatic and they take place without internal friction, *i.e.*, these processes are *isentropic*.
4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.

5. The cycle is considered closed with the same 'air' always remaining in the cylinder to repeat the cycle.

13.3. THE CARNOT CYCLE

This cycle has the *highest possible efficiency* and consists of four simple operations namely,

(*a*) Isothermal expansion

(*b*) Adiabatic expansion

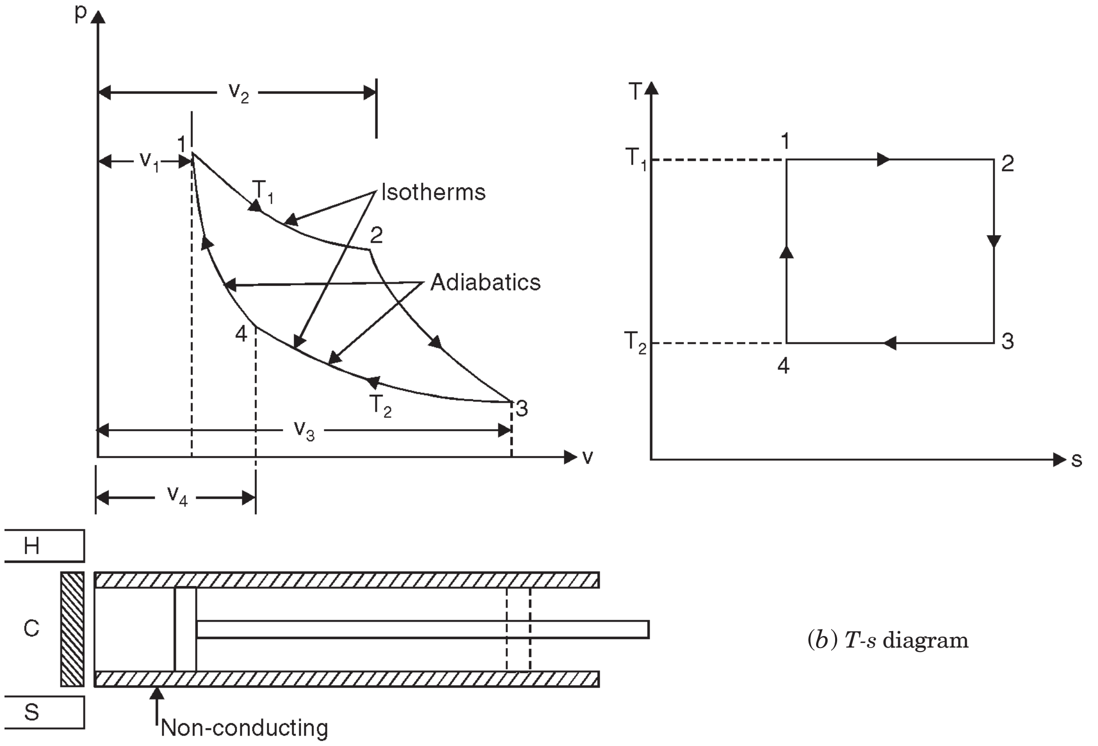
(*c*) Isothermal compression

(*d*) Adiabatic compression.

The condition of the Carnot cycle may be imagined to occur in the following way :

One kg of a air is enclosed in the cylinder which (except at the end) is made of perfect non-conducting material. A source of heat '*H*' is supposed to provide unlimited quantity of heat, non-conducting cover '*C*' and a sump '*S*' which is of infinite capacity so that its temperature remains unchanged irrespective of the fact how much heat is supplied to it. The temperature of source *H* is T_1 and the same is of the working substance. The working substance while rejecting heat to sump '*S*' has the temperature. T_2 *i.e.*, the same as that of sump *S*.

Following are the *four stages* of the Carnot cycle. Refer Fig. 13.1 (*a*).



(*a*) Four stages of the carnot cycle

(*b*) *T-s* diagram

Fig. 13.1. Carnot cycle.

Stage (1). Line 1-2 [Fig. 13.1 (*a*)] represents the isothermal expansion which takes place at temperature T_1 when source of heat *H* is applied to the end of cylinder. Heat supplied in this case is given by $RT_1 \log_e r$ and where *r* is the ratio of expansion.

Stage (2). Line 2-3 represents the application of non-conducting cover to the end of the cylinder. This is followed by the adiabatic expansion and the temperature falls from T_1 to T_2.

Stage (3). Line 3-4 represents the isothermal compression which takes place when sump '*S*' is applied to the end of cylinder. Heat is rejected during this operation whose value is given by $RT_2 \log_e r$ where *r* is the ratio of compression.

Stage (4). Line 4-1 represents repeated application of non-conducting cover and adiabatic compression due to which temperature increases from T_2 to T_1.

It may be noted that ratio of expansion during isotherm 1-2 and ratio of compression during isotherm 3-4 must be equal to get a closed cycle.

Fig. 13.1 (*b*) represents the Carnot cycle on *T-s* coordinates.

Now according to law of conservation of energy,

Heat supplied = Work done + Heat rejected

Work done = Heat supplied – Heat rejected

$$= RT_1 \,.\, \log_e r - RT_2 \log_e r$$

$$\text{Efficiency of cycle} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{R \log_e r\,(T_1 - T_2)}{RT_1 \,.\, \log_e r}$$

$$= \frac{T_1 - T_2}{T_1} \qquad ...(13.2)$$

From this equation, it is quite obvious that if temperature T_2 decreases efficiency increases and it becomes 100% if T_2 becomes absolute zero which, of course is impossible to attain. Further more *it is not possible to produce an engine that should work on Carnot's cycle as it would necessitate the piston to travel very slowly during first portion of the forward stroke (isothermal expansion) and to travel more quickly during the remainder of the stroke (adiabatic expansion) which however is not practicable.*

Example 13.1. *A Carnot engine working between 400°C and 40°C produces 130 kJ of work. Determine :*

(*i*) *The engine thermal efficiency.*

(*ii*) *The heat added.*

(*iii*) *The entropy changes during heat rejection process.*

Solution. Temperature, $T_1 = T_2 = 400 + 273 = 673$ K

Temperature, $T_3 = T_4 = 40 + 273 = 313$ K

Work produced, $W = 130$ kJ.

(*i*) **Engine thermal efficiency, η_{th} :**

$$\eta_{th.} = \frac{673 - 313}{673} = \mathbf{0.535 \text{ or } 53.5\%.} \quad \textbf{(Ans.)}$$

(*ii*) **Heat added :**

$$\eta_{th.} = \frac{\text{Work done}}{\text{Heat added}}$$

i.e.,

$$0.535 = \frac{130}{\text{Heat added}}$$

$$\therefore \quad \text{Heat added} = \frac{130}{0.535} = \mathbf{243 \text{ kJ.}} \quad \textbf{(Ans.)}$$

(*iii*) **Entropy change during the heat rejection process, $(S_3 - S_4)$:**

Heat rejected = Heat added – Work done

= 243 – 130 = 113 kJ

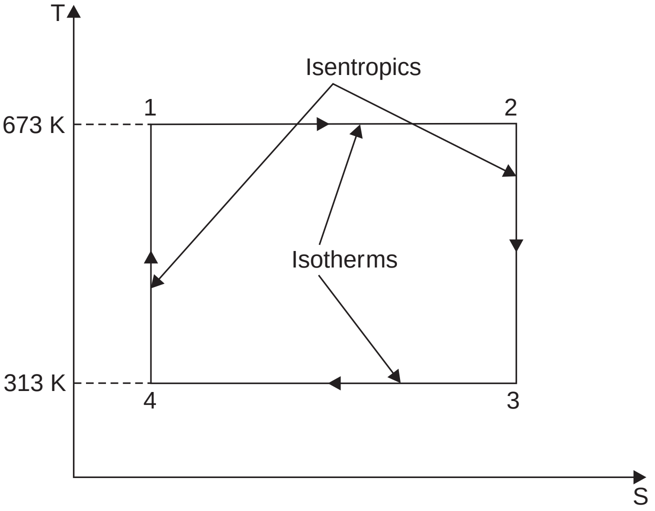


Fig. 13.2

Heat rejected $= T_3 (S_3 - S_4) = 113$

$$\therefore \quad (S_3 - S_4) = \frac{113}{T_3} = \frac{113}{313} = \mathbf{0.361\ kJ/K.} \quad \textbf{(Ans.)}$$

Example 13.2. *0.5 kg of air (ideal gas) executes a Carnot power cycle having a thermal efficiency of 50 per cent. The heat transfer to the air during the isothermal expansion is 40 kJ. At the beginning of the isothermal expansion the pressure is 7 bar and the volume is 0.12 m³. Determine :*

(i) The maximum and minimum temperatures for the cycle in K ;

(ii) The volume at the end of isothermal expansion in m³ ;

(iii) The heat transfer for each of the four processes in kJ.

For air c_v = 0.721 kJ/kg K, and c_p = 1.008 kJ/kg K. **(U.P.S.C. 1993)**

Solution. Refer Fig. 13.3. *Given* : m = 0.5 kg ; η_{th} = 50% ; Heat transferred during isothermal expansion = 40 kJ ; p_1 = 7 bar, V_1 = 0.12 m³ ; c_v = 0.721 kJ/kg K ; c_p = 1.008 kJ/kg K.

(*i*) **The maximum and minimum temperatures, T_1, T_2 :**

$$p_1V_1 = mRT_1$$

$$7 \times 10^5 \times 0.12 = 0.5 \times 287 \times T_1$$

$$\therefore \quad \text{Maximum temperature, } \mathbf{T_1} = \frac{7 \times 10^5 \times 0.12}{0.5 \times 287} = \mathbf{585.4\ K.} \quad \textbf{(Ans.)}$$

$$\eta_{cycle} = \frac{T_1 - T_2}{T_1} \Rightarrow 0.5 = \frac{585.4 - T_2}{585.4}$$

$$\therefore \quad \text{Minimum temperature, } \mathbf{T_2} = 585.4 - 0.5 \times 585.4 = \mathbf{292.7\ K.} \quad \textbf{(Ans.)}$$

(*ii*) **The volume at the end of isothermal expansion, $\mathbf{V_2}$:**

Heat transferred during isothermal expansion

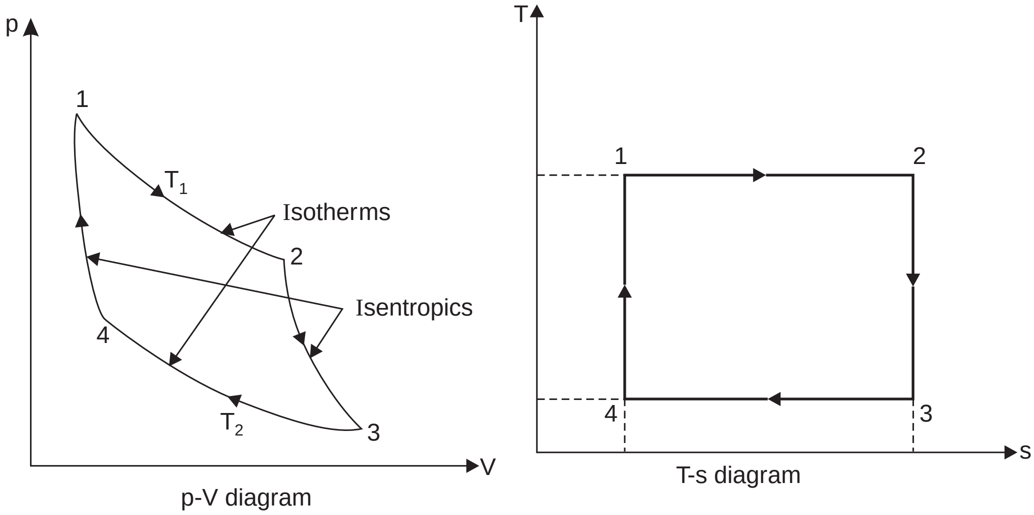


Fig. 13.3. Carnot cycle.

$$= p_1V_1 \ln(r) = mRT_1 \ln\left(\frac{V_2}{V_1}\right) = 40 \times 10^3 \qquad \text{...... (Given)}$$

or
$$0.5 \times 287 \times 585.4 \ln\left(\frac{V_2}{0.12}\right) = 40 \times 10^3$$

or
$$\ln\left(\frac{V_2}{0.12}\right) = \frac{40 \times 10^3}{0.5 \times 287 \times 585.4} = 0.476$$

or
$$\mathbf{V_2} = 0.12 \times (e)^{0.476} = \mathbf{0.193\ m^3}. \quad \textbf{(Ans.)}$$

(*iii*) **The heat transfer for each of the four processes :**

Process	*Classification*	*Heat transfer*
1—2	Isothermal expansion	40 kJ
2—3	Adiabatic reversible expansion	zero
3—4	Isothermal compression	– 40 kJ
4—1	Adiabatic reversible compression	zero. **(Ans.)**

☞**Example 13.3.** *In a Carnot cycle, the maximum pressure and temperature are limited to 18 bar and 410°C. The ratio of isentropic compression is 6 and isothermal expansion is 1.5. Assuming the volume of the air at the beginning of isothermal expansion as 0.18 m*^{3}*, determine :*

(*i*) *The temperature and pressures at main points in the cycle.*

(*ii*) *Change in entropy during isothermal expansion.*

(*iii*) *Mean thermal efficiency of the cycle.*

(*iv*) *Mean effective pressure of the cycle.*

(*v*) *The theoretical power if there are 210 working cycles per minute.*

Solution. Refer Fig. 13.4.

Maximum pressure, $p_1 = 18$ bar

Maximum temperature, $T_1 = (T_2) = 410 + 273 = 683$ K

Ratio of isentropic (or adiabatic) compression, $\dfrac{V_4}{V_1} = 6$

Ratio of isothermal expansion, $\dfrac{V_2}{V_1} = 1.5.$

Volume of the air at the beginning of isothermal expansion, $V_1 = 0.18$ m^3.

(*i*) **Temperatures and pressures at the main points in the cycle :**

For the *isentropic process* 4-1 :

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma - 1} = (6)^{1.4 - 1} = (6)^{0.4} = 2.05$$

$$\therefore \qquad T_4 = \frac{T_1}{2.05} = \frac{683}{2.05} = 333.2 \text{ K} = T_3$$

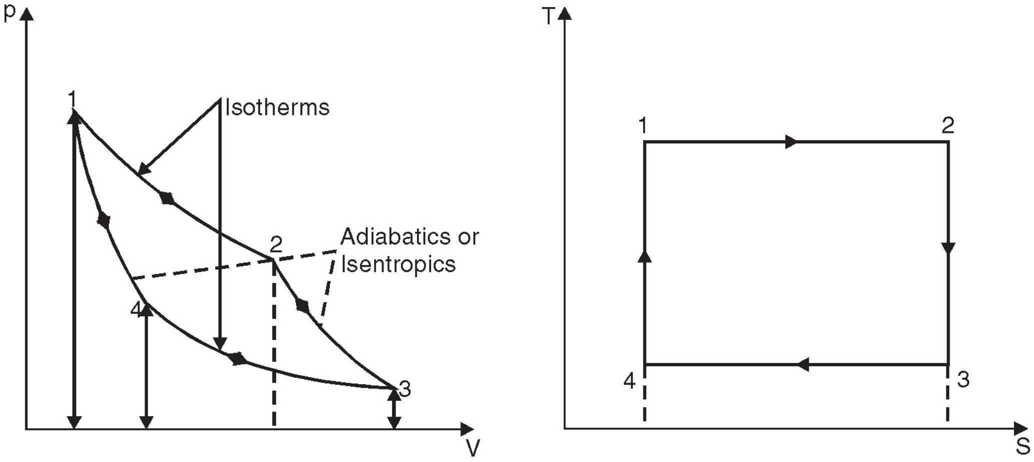


Fig. 13.4

$$\text{Also,} \qquad \frac{p_1}{p_4} = \left(\frac{V_4}{V_1}\right)^{\gamma} = (6)^{1.4} = 12.29$$

$$\therefore \qquad p_4 = \frac{p_1}{12.29} = \frac{18}{12.29} = 1.46 \text{ bar}$$

For the *isothermal process* 1-2 :

$$p_1V_1 = p_2V_2$$

$$p_2 = \frac{p_1V_1}{V_2} = \frac{18}{1.5} = 12 \text{ bar}$$

For *isentropic process* 2-3, we have :

$$p_2V_2^{\gamma} = p_3V_3^{\gamma}$$

$$p_3 = p_2 \times \left(\frac{V_2}{V_3}\right)^{\gamma} = 12 \times \left(\frac{V_1}{V_4}\right)^{\gamma} \qquad \left[\because \frac{V_4}{V_1} = \frac{V_3}{V_2}\right]$$

$$= 12 \times \left(\frac{1}{6}\right)^{1.4} = \mathbf{0.97\ bar.} \quad \textbf{(Ans.)}$$

Hence

$$\left.\begin{array}{ll} p_1 = \mathbf{18\ bar} & T_1 = T_2 = 683 \text{ K} \\ p_2 = \mathbf{12\ bar} & \\ p_3 = \mathbf{0.97\ bar} & T_3 = T_4 = 333.2 \text{ K} \\ p_4 = \mathbf{1.46\ bar} & \end{array}\right\} \textbf{(Ans.)}$$

(*ii*) **Change in entropy :**

Change in entropy during isothermal expansion,

$$S_2 - S_1 = mR \log_e \left(\frac{V_2}{V_1}\right) = \frac{p_1V_1}{T_1} \log_e \left(\frac{V_2}{V_1}\right) \qquad \left[\begin{array}{ll} \because & pV = mRT \\ \text{or} & mR = \dfrac{pV}{T} \end{array}\right]$$

$$= \frac{18 \times 10^5 \times 0.18}{10^3 \times 683} \log_e (1.5) = \mathbf{0.192\ kJ/K.} \quad \textbf{(Ans.)}$$

(*iii*) **Mean thermal efficiency of the cycle :**

Heat supplied, $Q_s = p_1V_1 \log_e \left(\frac{V_2}{V_1}\right)$

$= T_1 (S_2 - S_1)$

$= 683 \times 0.192 = 131.1$ kJ

Heat rejected, $Q_r = p_4V_4 \log_e \left(\frac{V_3}{V_4}\right)$

$= T_4 (S_3 - S_4)$ because increase in entropy during heat addition is equal to decrease in entropy during heat rejection.

∴ $Q_r = 333.2 \times 0.192 = 63.97$ kJ

∴ *Efficiency,* $\eta = \frac{Q_s - Q_r}{Q_s} = 1 - \frac{Q_r}{Q_s}$

$$= 1 - \frac{63.97}{131.1} = \mathbf{0.512} \quad \textbf{or} \quad \mathbf{51.2\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Mean effective pressure of the cycle, $\mathbf{p_m}$:**

The mean effective pressure of the cycle is given by

$$p_m = \frac{\text{Work done per cycle}}{\text{Stroke volume}}$$

$$\frac{V_3}{V_1} = 6 \times 1.5 = 9$$

Stroke volume, $V_s = V_3 - V_1 = 9V_1 - V_1 = 8V_1 = 8 \times 0.18 = 1.44$ m^3

∴ $p_m = \frac{(Q_s - Q_r) \times J}{V_s} = \frac{(Q_s - Q_r) \times 1}{V_s}$ $(\because J = 1)$

$$= \frac{(131.1 - 63.97) \times 10^3}{1.44 \times 10^5} = \mathbf{0.466\ bar.} \quad \textbf{(Ans.)}$$

(*v*) **Power of the engine, P :**

Power of the engine working on this cycle is given by

$$P = (131.1 - 63.97) \times (210/60) = \mathbf{234.9\ kW.}\quad \textbf{(Ans.)}$$

Example 13.4. *A reversible engine converts one-sixth of the heat input into work. When the temperature of the sink is reduced by 70°C, its efficiency is doubled. Find the temperature of the source and the sink.*

Solution. Let, T_1 = temperature of the source (*K*), and

T_2 = temperature of the sink (*K*)

First case :

$$\frac{T_1 - T_2}{T_1} = \frac{1}{6}$$

i.e., $6T_1 - 6T_2 = T_1$

or $5T_1 = 6T_2$ or $T_1 = 1.2T_2$ ...(*i*)

Second case :

$$\frac{T_1 - [T_2 - (70 + 273)]}{T_1} = \frac{1}{3}$$

$$\frac{T_1 - T_2 + 343}{T_1} = \frac{1}{3}$$

$$3T_1 - 3T_2 + 1029 = T_1$$

$$2T_1 = 3T_2 - 1029$$

$$2 \times (1.2T_2) = 3T_2 - 1029 \qquad (\because\ T_1 = 1.2T_2)$$

$$2.4T_2 = 3T_2 - 1029$$

or $0.6T_2 = 1029$

$\therefore$ $T_2 = \dfrac{1029}{0.6} = \mathbf{1715\ K\ \ or\ \ 1442°C.}$ **(Ans.)**

and $T_1 = 1.2 \times 1715 = \mathbf{2058\ K\ \ or\ \ 1785°C.}$ **(Ans.)**

Example 13.5. *An inventor claims that a new heat cycle will develop 0.4 kW for a heat addition of 32.5 kJ/min. The temperature of heat source is 1990 K and that of sink is 850 K. Is his claim possible ?*

Solution. Temperature of heat source, T_1 = 1990 K

Temperature of sink, T_2 = 850 K

Heat supplied, = 32.5 kJ/min

Power developed by the engine, P = 0.4 kW

The *most efficient engine is one that works on Carnot cycle*

$$\eta_{\text{carnot}} = \frac{T_1 - T_2}{T_1} = \frac{1990 - 850}{1990} = 0.573 \text{ or } 57.3\%$$

Also, thermal efficiency of the engine,

$$\eta_{\text{th}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{0.4}{(32.5/60)} = \frac{0.4 \times 60}{32.5}$$

$$= 0.738 \text{ or } 73.8\%$$

which is not feasible as no engine can be more efficient than that working on Carnot cycle.

Hence claims of the inventor is **not true. (Ans.)**

Example 13.6. *An ideal engine operates on the Carnot cycle using a perfects gas as the working fluid. The ratio of the greatest to the least volume is fixed and is x : 1, the lower temperature of the cycle is also fixed, but the volume compression ratio 'r' of the reversible adiabatic compression is variable. The ratio of the specific heats is γ.*

Show that if the work done in the cycle is a maximum then,

$$(\gamma - 1) \log_e \frac{x}{r} + \frac{1}{r^{\gamma - 1}} - 1 = 0.$$

Solution. Refer Fig. 13.1.

$$\frac{V_3}{V_1} = x \; ; \; \frac{V_4}{V_1} = r$$

During isotherms, since compression ratio = expansion ratio

$$\therefore \qquad \frac{V_3}{V_4} = \frac{V_2}{V_1}$$

Also $\frac{V_3}{V_4} = \frac{V_3}{V_1} \times \frac{V_1}{V_4} = x \times \frac{1}{r} = \frac{x}{r}$

Work done per kg of the gas

$$= \text{Heat supplied} - \text{Heat rejected} = RT_1 \log_e \frac{x}{r} - RT_2 \log_e \frac{x}{r}$$

$$= R(T_1 - T_2) \log_e \frac{x}{r} = RT_2 \left(\frac{T_1}{T_2} - 1 \right) \log_e \frac{x}{r}$$

But $\frac{T_1}{T_2} = \left(\frac{V_4}{V_1} \right)^{\gamma - 1} = (r)^{\gamma - 1}$

∴ Work done per kg of the gas,

$$W = RT_2 \, (r^{\gamma - 1} - 1) \log_e \frac{x}{r}$$

Differentiating W w.r.t. 'r' and equating to zero

$$\frac{dW}{dr} = RT_2 \left[(r^{\gamma - 1} - 1) \left\{ \frac{r}{x} \times (-xr^{-2}) \right\} + \log_e \frac{x}{r} \{ (\gamma - 1) r^{\gamma - 2} \} \right] = 0$$

or $$(r^{\gamma - 1} - 1)\left(-\frac{1}{r} \right) + (\gamma - 1) \times r^{\gamma - 2} \log_e \frac{x}{r} = 0$$

or $$- r^{\gamma - 2} + \frac{1}{r} + r^{\gamma - 2} (\gamma - 1) \log_e \frac{x}{r} = 0$$

or $$r^{\gamma - 2} \left\{ -1 + \frac{1}{r \, . \, r^{\gamma - 2}} + (\gamma - 1) \log_e \frac{x}{r} \right\} = 0$$

or $$-1 + \frac{1}{r \, . \, r^{\gamma - 2}} + (\gamma - 1) \log_e \frac{x}{r} = 0$$

$$(\gamma - 1) \log_e \frac{x}{r} + \frac{1}{r^{\gamma - 1}} - 1 = 0. \quad \text{...... \textbf{Proved.}}$$

13.4. CONSTANT VOLUME OR OTTO CYCLE

This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines.

Figs. 13.5 (*a*) and (*b*) shows the theoretical *p-V* diagram and *T-s* diagrams of this cycle respectively.

- The point 1 represents that cylinder is full of air with volume V_1, pressure p_1 and absolute temperature T_1.
- Line 1-2 represents the *adiabatic compression* of air due to which p_1, V_1 and T_1 change to p_2, V_2 and T_2, respectively.
- Line 2-3 shows the *supply of heat* to the air *at constant volume* so that p_2 and T_2 change to p_3 and T_3 (V_3 being the same as V_2).
- Line 3-4 represents the *adiabatic expansion* of the air. During expansion p_3, V_3 and T_3 change to a final value of p_4, V_4 or V_1 and T_4, respectively.
- Line 4-1 shows the *rejection of heat* by air at *constant volume* till original state (point 1) reaches.

Consider **1 kg of air** (working substance) :

Heat supplied at constant volume $= c_v(T_3 - T_2)$.

Heat rejected at constant volume $= c_v (T_4 - T_1)$.

But, work done = Heat supplied – Heat rejected

$$= c_v (T_3 - T_2) - c_v (T_4 - T_1)$$

$$\therefore \quad \text{Efficiency} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v (T_3 - T_2) - c_v (T_4 - T_1)}{c_v (T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \qquad \text{...}(i)$$

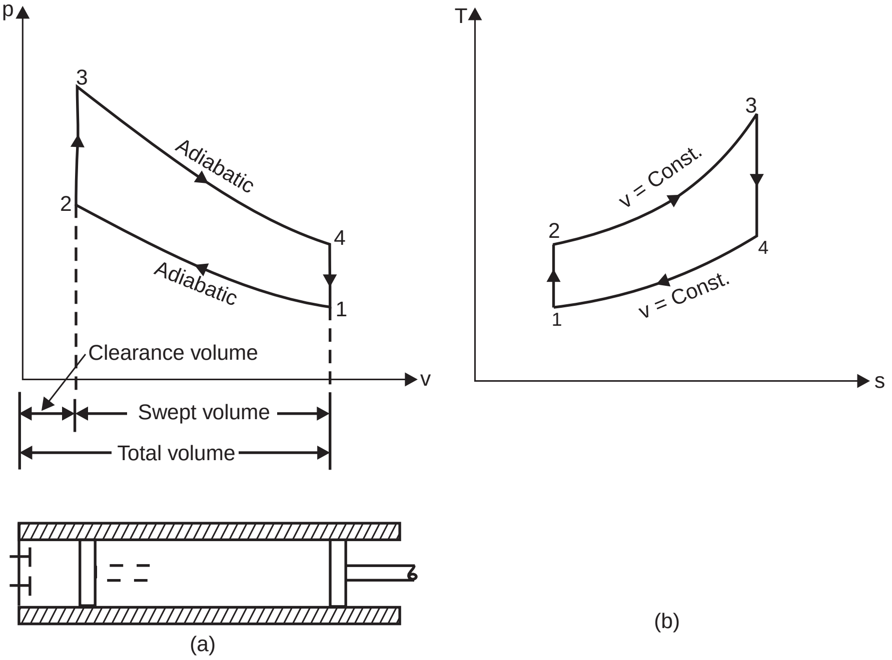


Fig. 13.5

Let compression ratio, $r_c\ (= r) = \dfrac{v_1}{v_2}$

and expansion ratio, $r_e\ (= r) = \dfrac{v_4}{v_3}$

(These *two ratios are same* in this cycle)

As $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

Then, $$T_2 = T_1 \cdot (r)^{\gamma - 1}$$

Similarly, $$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1}$$

or $$T_3 = T_4 \cdot (r)^{\gamma - 1}$$

Inserting the values of T_2 and T_3 in equation (*i*), we get

$$\eta_{otto} = 1 - \frac{T_4 - T_1}{T_4 \cdot (r)^{\gamma - 1} - T_1 \cdot (r)^{\gamma - 1}} = 1 - \frac{T_4 - T_1}{r^{\gamma - 1}(T_4 - T_1)}$$

$$= 1 - \frac{1}{(r)^{\gamma - 1}} \qquad \text{...(13.3)}$$

This expression is known as the ***air standard efficiency of the Otto cycle.***

It is clear from the above expression that efficiency increases with the increase in the value of r, which means we can have maximum efficiency by increasing r to a considerable extent, but *due to practical difficulties its value is limited to about 8.*

The *net work done per kg* in the Otto cycle can also be expressed in terms of p, v. If p is expressed in bar *i.e.,* 10^5 N/m^2, then work done

$$W = \left(\frac{p_3v_3 - p_4v_4}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1}\right) \times 10^2 \text{ kJ} \qquad \text{...(13.4)}$$

Also $$\frac{p_3}{p_4} = r^{\gamma} = \frac{p_2}{p_1}$$

$\therefore$ $$\frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p$$

where r_p stands for *pressure ratio*.

and $$v_1 = rv_2 = v_4 = rv_3 \qquad \left[\because\ \frac{v_1}{v_2} = \frac{v_4}{v_3} = r\right]$$

$\therefore$ $$W = \frac{1}{\gamma - 1}\left[p_4v_4\left(\frac{p_3v_3}{p_4v_4} - 1\right) - p_1v_1\left(\frac{p_2v_2}{p_1v_1} - 1\right)\right]$$

$$= \frac{1}{\gamma - 1}\left[p_4v_4\left(\frac{p_3}{p_4r} - 1\right) - p_1v_1\left(\frac{p_2}{p_1r} - 1\right)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[p_4\,(r^{\gamma - 1} - 1) - p_1\,(r^{\gamma - 1} - 1)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[(r^{\gamma - 1} - 1)(p_4 - p_1)\right]$$

$$= \frac{p_1 v_1}{\gamma - 1}\left[(r^{\gamma - 1} - 1)(r_p - 1)\right] \qquad \text{...[13.4 }(a)\text{]}$$

Mean effective pressure (p_m) is given by :

$$p_m = \left[\left(\frac{p_3v_3 - p_4v_4}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1}\right) \div (v_1 - v_2)\right] \text{ bar} \qquad \text{...(13.5)}$$

Also

$$p_m = \frac{\left[\dfrac{p_1v_1}{\gamma - 1}(r^{\gamma - 1} - 1)(r_p - 1)\right]}{(v_1 - v_2)}$$

$$= \frac{\dfrac{p_1v_1}{\gamma - 1}[(r^{\gamma - 1} - 1)(r_p - 1)}{v_1 - \dfrac{v_1}{r}}$$

$$= \frac{\dfrac{p_1v_1}{\gamma - 1}[(r^{\gamma - 1} - 1)(r_p - 1)]}{v_1\left(\dfrac{r - 1}{r}\right)}$$

i.e.,

$$\mathbf{p_m} = \frac{p_1 r[(r^{\gamma - 1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)} \qquad \text{...(13.6)}$$

Example 13.7. *The efficiency of an Otto cycle is 60% and $\gamma = 1.5$. What is the compression ratio ?*

Solution. Efficiency of Otto cycle, $\eta = 60\%$

Ratio of specific heats, $\gamma = 1.5$

Compression ratio, $r = ?$

Efficiency of Otto cycle is given by,

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma - 1}}$$

$$0.6 = 1 - \frac{1}{(r)^{1.5 - 1}}$$

or

$$\frac{1}{(r)^{0.5}} = 0.4 \quad \text{or} \quad (r)^{0.5} = \frac{1}{0.4} = 2.5 \quad \text{or} \quad r = 6.25$$

Hence, *compression ratio* = **6.25. (Ans.)**

Example 13.8. *An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearance volume is 0.00263 m^3. The initial pressure and temperature are 1 bar and 50°C. If the maximum pressure is limited to 25 bar, find the following :*

(*i*) *The air standard efficiency of the cycle.*

(*ii*) *The mean effective pressure for the cycle.*

Assume the ideal conditions.

Solution. Bore of the engine, $D = 250 \text{ mm} = 0.25 \text{ m}$

Stroke of the engine, $L = 375 \text{ mm} = 0.375 \text{ m}$

Clearance volume, $V_c = 0.00263 \text{ m}^3$

Initial pressure, $p_1 = 1 \text{ bar}$

Initial temperature, $T_1 = 50 + 273 = 323 \text{ K}$

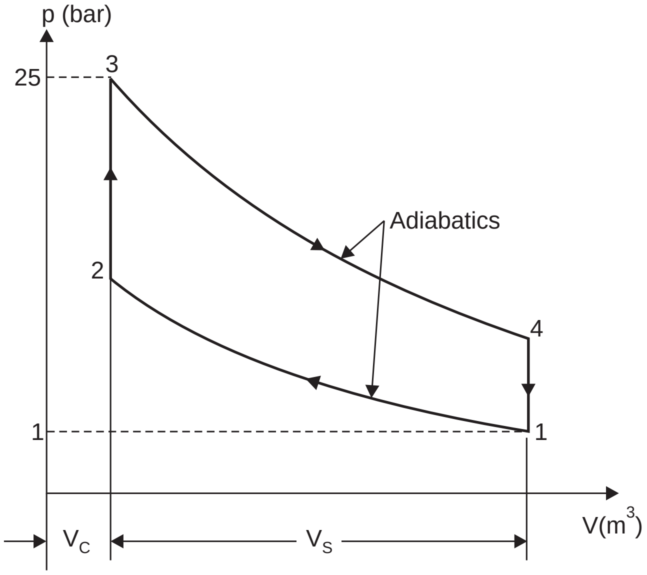


Fig. 13.6

Maximum pressure, $p_3 = 25 \text{ bar}$

Swept volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.25^2 \times 0.375 = 0.0184 \text{ m}^3$

Compression ratio, $r = \dfrac{V_s + V_c}{V_c} = \dfrac{0.0184 + 0.00263}{0.00263} = 8.$

(*i*) **Air standard efficiency :**

The air standard efficiency of Otto cycle is given by

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(8)^{1.4-1}} = 1 - \frac{1}{(8)^{0.4}}$$

$$= 1 - 0.435 = \mathbf{0.565 \quad or \quad 56.5\%.} \quad \textbf{(Ans.)}$$

(*ii*) **Mean effective pressure, p_m :**

For adiabatic (or isentropic) process 1-2

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or $$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (r)^{1.4} = 1 \times (8)^{1.4} = 18.38 \text{ bar}$$

$\therefore$ Pressure ratio, $r_p = \dfrac{p_3}{p_2} = \dfrac{25}{18.38} = 1.36$

The mean effective pressure is given by

$$p_m = \frac{p_1 r[(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)} = \frac{1 \times 8\,[\{(8)^{1.4-1} - 1\}\,(1.36 - 1)]}{(1.4 - 1)\,(8 - 1)}$$

... [Eqn. (13.6)]

$$= \frac{8\,(2.297 - 1)(0.36)}{0.4 \times 7} = 1.334 \text{ bar}$$

Hence *mean effective pressure* = **1.334 bar. (Ans.)**

Example 13.9. *The minimum pressure and temperature in an Otto cycle are 100 kPa and 27°C. The amount of heat added to the air per cycle is 1500 kJ/kg.*

(i) Determine the pressures and temperatures at all points of the air standard Otto cycle.

(ii) Also calculate the specific work and thermal efficiency of the cycle for a compression ratio of 8 : 1.

Take for air : $c_v = 0.72$ *kJ/kg K, and* $\gamma = 1.4$. **(GATE, 1998)**

Solution. Refer Fig. 13.7. *Given* : $p_1 = 100$ kPa $= 10^5$ N/m^2 or 1 bar ;

$T_1 = 27 + 273 = 300$ K ; Heat added = 1500 kJ/kg ;

$r = 8 : 1$; $c_v = 0.72$ kJ/kg ; $\gamma = 1.4$.

Consider 1 kg of air.

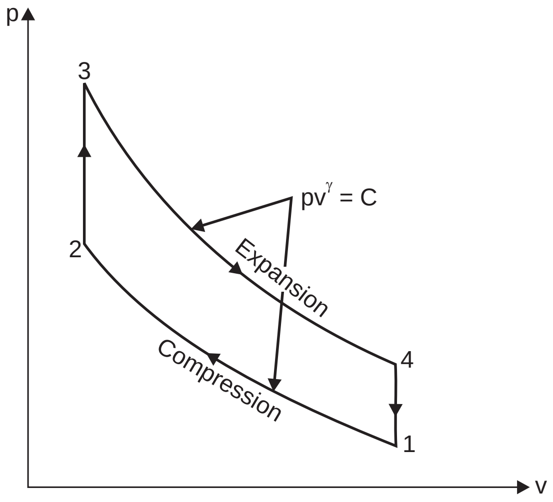


Fig. 13.7

(*i*) **Pressures and temperatures at all points :**

Adiabatic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (8)^{1.4 - 1} = 2.297$$

$\therefore$ $\mathbf{T_2} = 300 \times 2.297 =$ **689.1 K. (Ans.)**

Also $p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$

or
$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (8)^{1.4} = 18.379$$

$\therefore$ $\mathbf{p_2} = 1 \times 18.379 =$ **18.379 bar. (Ans.)**

Constant volume process 2-3 :

Heat added during the process,

$$c_v\,(T_3 - T_2) = 1500$$

or $$0.72\,(T_3 - 689.1) = 1500$$

or $$\mathbf{T_3} = \frac{1500}{0.72} + 689.1 = \mathbf{2772.4\ K.}\quad \textbf{(Ans.)}$$

Also, $$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow \mathbf{p_3} = \frac{p_2 T_3}{T_2} = \frac{18.379 \times 2772.4}{689.1} = \mathbf{73.94\ bar.}\quad \textbf{(Ans.)}$$

Adiabatic Expansion process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (8)^{1.4 - 1} = 2.297$$

$$\therefore \quad \mathbf{T_4} = \frac{T_3}{2.297} = \frac{2772.4}{2.297} = \mathbf{1206.9\ K.}\quad \textbf{(Ans.)}$$

Also, $$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \Rightarrow \mathbf{p_4} = p_3 \times \left(\frac{v_3}{v_4}\right)^{\gamma} = 73.94 \times \left(\frac{1}{8}\right)^{1.4} = \mathbf{4.023\ bar.}\quad \textbf{(Ans.)}$$

(*ii*) **Specific work and thermal efficiency :**

Specific work = Heat added – heat rejected

$$= c_v\,(T_3 - T_2) - c_v(T_4 - T_1) = c_v\,[(T_3 - T_2) - (T_4 - T_1)]$$
$$= 0.72\,[(2772.4 - 689.1) - (1206.9 - 300)] = \mathbf{847\ kJ/kg.}\quad \textbf{(Ans.)}$$

Thermal efficiency, $$\eta_{\mathbf{th}} = 1 - \frac{1}{(r)^{\gamma - 1}}$$

$$= 1 - \frac{1}{(8)^{1.4 - 1}} = \mathbf{0.5647 \quad or \quad 56.47\%.}\quad \textbf{(Ans.)}$$

Example 13.10. *An air standard Otto cycle has a volumetric compression ratio of 6, the lowest cycle pressure of 0.1 MPa and operates between temperature limits of 27°C and 1569°C.*

(i) Calculate the temperature and pressure after the isentropic expansion (ratio of specific heats = 1.4).

(ii) Since it is observed that values in (i) are well above the lowest cycle operating conditions, the expansion process was allowed to continue down to a pressure of 0.1 MPa. Which process is required to complete the cycle ? Name the cycle so obtained.

(iii) Determine by what percentage the cycle efficiency has been improved. **(GATE, 1994)**

Solution. Refer Fig. 13.8. *Given :* $\frac{v_1}{v_2} = \frac{v_4}{v_3} = r = 6$; $p_1 = 0.1$ MPa = 1 bar ; $T_1 = 27 + 273 = 300$ K ; $T_3 = 1569 + 273 = 1842$ K ; $\gamma = 1.4$.

(*i*) **Temperature and pressure after the isentropic expansion, $\mathbf{T_4}$, $\mathbf{p_4}$:**

Consider 1 kg of air :

For *the compression process 1-2 :*

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \Rightarrow p_2 = p_1 \times \left(\frac{v_1}{v_2}\right)^{\gamma} = 1 \times (6)^{1.4} = 12.3 \text{ bar}$$

Also $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (6)^{1.4 - 1} = 2.048$$

$$\therefore \quad T_2 = 300 \times 2.048 = 614.4 \text{ K}$$

For *the constant volume process 2-3 :*

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow p_3 = \frac{p_2 T_3}{T_2} = 12.3 \times \frac{1842}{614.4} = 36.9 \text{ bar}$$

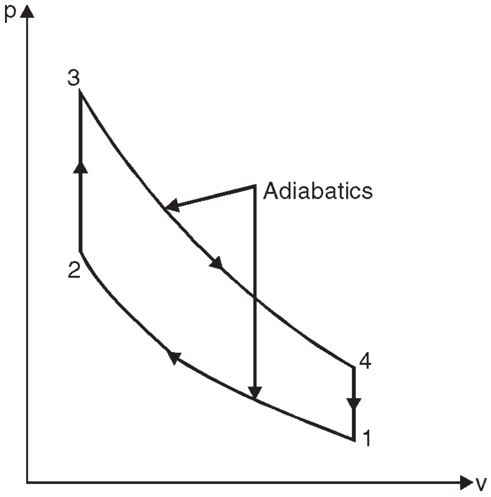


Fig. 13.8

For *the expansion process 3-4 :*

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1} = (6)^{1.4-1} = 2.048$$

$\therefore$ $$\mathbf{T_4} = \frac{T_3}{2.048} = \frac{1842}{2.048} = \mathbf{900\ K.}\quad \textbf{(Ans.)}$$

Also $$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \Rightarrow p_4 = p_3 \times \left(\frac{v_3}{v_4}\right)^{\gamma}$$

or $$\mathbf{p_4} = 36.9 \times \left(\frac{1}{6}\right)^{1.4} = \mathbf{3\ bar.}\quad \textbf{(Ans.)}$$

(*ii*) **Process required to complete the cycle :**

Process required to complete the cycle is the *constant pressure scavenging.*

The cycle is called **Atkinson cycle** (Refer Fig. 13.9).

(*iii*) **Percentage improvement/increase in efficiency :**

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(6)^{1.4-1}} = 0.5116 \text{ or } \mathbf{51.16\%.}\quad \textbf{(Ans.)}$$

$$\eta_{\text{Atkinson}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= \frac{c_v(T_3 - T_2) - c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{\gamma(T_5 - T_1)}{(T_3 - T_2)}$$

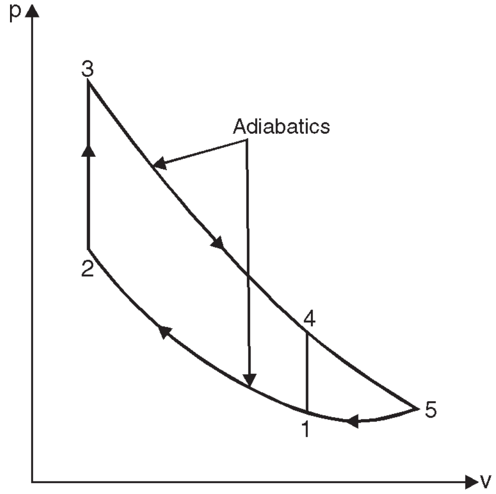


Fig. 13.9. Atkinson cycle.

Now, $$\frac{T_5}{T_3} = \left(\frac{p_5}{p_3}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad T_5 = 1842 \times \left(\frac{1.0}{36.9}\right)^{\frac{1.4-1}{1.4}} = 657 \text{ K}$$

$\therefore$ $$\eta_{\text{Atkinson}} = 1 - \frac{1.4(657-300)}{(1842-614.4)} = \mathbf{0.5929} \quad \textbf{or} \quad \mathbf{59.29\%.}$$

$\therefore$ **Improvement in efficiency** = 59.29 – 51.16 = **8.13%. (Ans.)**

Example 13.11. *A certain quantity of air at a pressure of 1 bar and temperature of 70°C is compressed adiabatically until the pressure is 7 bar in Otto cycle engine. 465 kJ of heat per kg of air is now added at constant volume. Determine :*

(*i*) *Compression ratio of the engine.*

(*ii*) *Temperature at the end of compression.*

(*iii*) *Temperature at the end of heat addition.*

Take for air $c_p = 1.0$ *kJ/kg K,* $c_v = 0.706$ *kJ/kg K.*

Show each operation on p-V and T-s diagrams.

Solution. Refer Fig. 13.10.

Initial pressure, $p_1 = 1$ bar

Initial temperature, $T_1 = 70 + 273 = 343$ K

Pressure after adiabatic compression, $p_2 = 7$ bar

Heat addition at constant volume, $Q_s = 465$ kJ/kg of air

Specific heat at constant pressure, $c_p = 1.0$ kJ/kg K

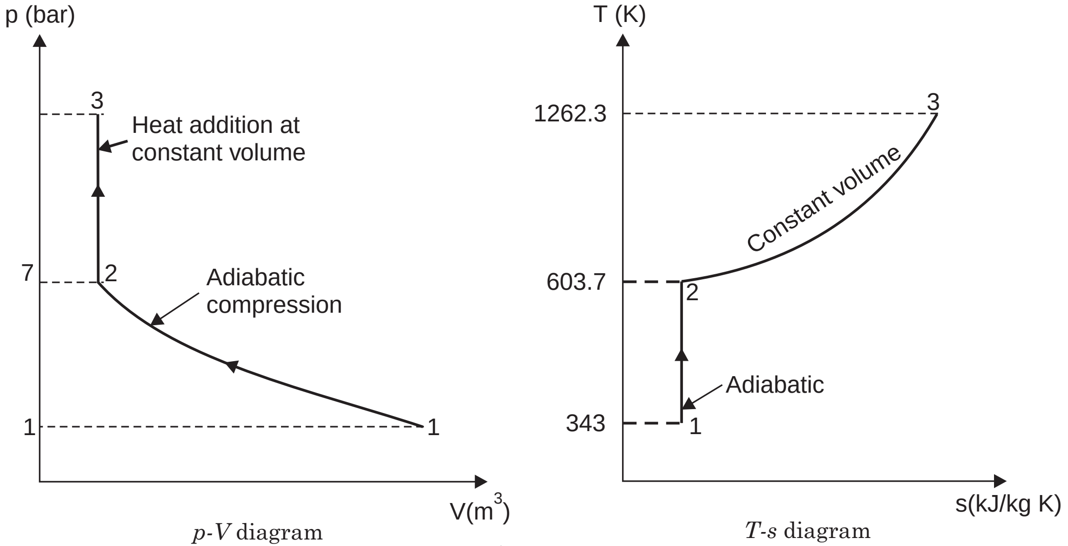
Specific heat at constant volume, $c_v = 0.706$ kJ/kg K

$\therefore$ $$\gamma = \frac{c_p}{c_v} = \frac{1.0}{0.706} = 1.41$$

(*i*) **Compression ratio of engine, r :**

According to *adiabatic compression 1-2*

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$



p-V diagram *T-s* diagram

Fig. 13.10

or $$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

or $$(r)^{\gamma} = \frac{p_2}{p_1} \qquad \left(\because \frac{V_1}{V_2} = r\right)$$

or $$r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{7}{1}\right)^{\frac{1}{1.41}} = (7)^{0.709} = 3.97$$

Hence *compression ratio of the engine* = **3.97. (Ans.)**

(*ii*) **Temperature at the end of compression, T_2 :**

In case of *adiabatic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (3.97)^{1.41 - 1} = 1.76$$

$$\therefore \quad T_2 = 1.76\ T_1 = 1.76 \times 343 = 603.7 \text{ K or } 330.7°\text{C}$$

Hence *temperature at the end of compression* = **330.7°C. (Ans.)**

(*iii*) **Temperature at the end of heat addition, T_3 :**

According to *constant volume heating operation 2-3*

$$Q_s = c_v\,(T_3 - T_2) = 465$$

$$0.706\,(T_3 - 603.7) = 465$$

or $$T_3 - 603.7 = \frac{465}{0.706}$$

or $$T_3 = \frac{465}{0.706} + 603.7 = 1262.3 \text{ K or } 989.3°\text{C}$$

Hence *temperature at the end of heat addition* = **989.3°C. (Ans.)**

Example 13.12. *In a constant volume 'Otto cycle', the pressure at the end of compression is 15 times that at the start, the temperature of air at the beginning of compression is 38°C and maximum temperature attained in the cycle is 1950°C. Determine :*

(*i*) *Compression ratio.*

(*ii*) *Thermal efficiency of the cycle.*

(*iii*) *Work done.*

Take γ *for air = 1.4.*

Solution. Refer Fig. 13.11.

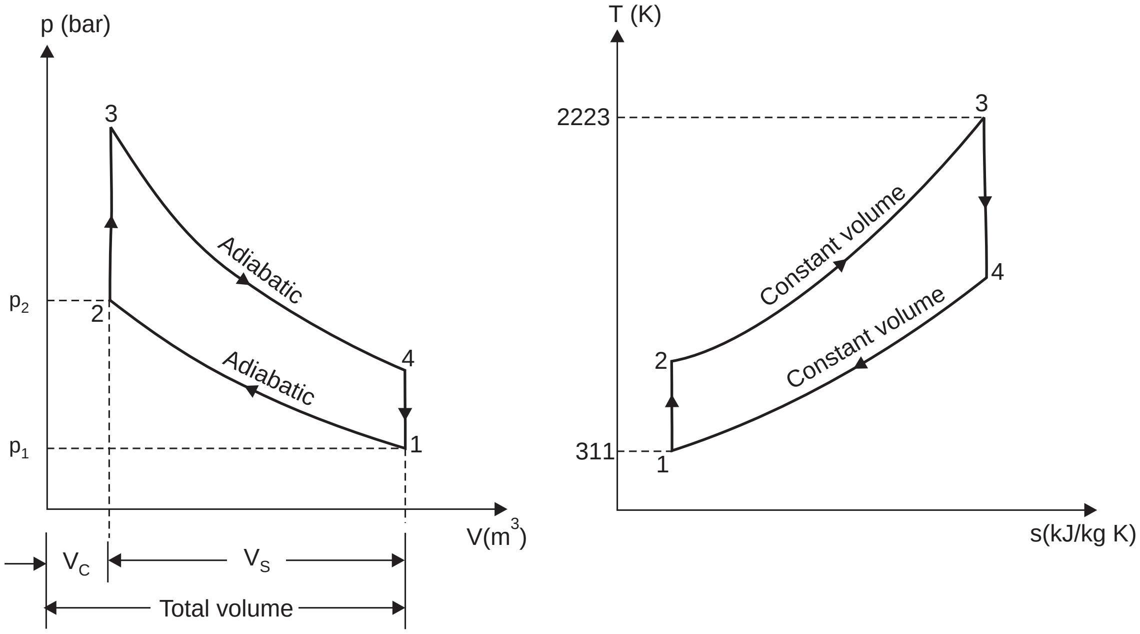


Fig. 13.11

Initial temperature, $T_1 = 38 + 273 = 311$ K

Maximum temperature, $T_3 = 1950 + 273 = 2223$ K.

(*i*) **Compression ratio, r :**

For *adiabatic compression 1-2*,

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or $$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

But $$\frac{p_2}{p_1} = 15 \qquad \text{...(given)}$$

$\therefore$ $$(r)^{\gamma} = 15 \qquad \left[\because \; r = \frac{V_1}{V_2}\right]$$

or $$(r)^{1.4} = 15$$

or $$r = (15)^{\frac{1}{1.4}} = (15)^{0.714} = 6.9$$

Hence *compression ratio* = **6.9. (Ans.)**

(*ii*) **Thermal efficiency :**

Thermal efficiency, $$\eta_{th} = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(6.9)^{1.4 - 1}} = \mathbf{0.538 \text{ or } 53.8\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Work done :**

Again, for *adiabatic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (6.9)^{1.4 - 1} = (6.9)^{0.4} = 2.16$$

or $$T_2 = T_1 \times 2.16 = 311 \times 2.16 = 671.7 \text{ K or } 398.7°\text{C}$$

For *adiabatic expansion process 3-4*

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (6.9)^{0.4} = 2.16$$

or $$T_4 = \frac{T_3}{2.16} = \frac{2223}{2.16} = 1029 \text{ K or } 756°\text{C}$$

Heat supplied *per kg of air*

$$= c_v(T_3 - T_2) = 0.717(2223 - 671.7)$$

$$= 1112.3 \text{ kJ/kg or air}$$

$$\left[c_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.717 \text{ kJ/ kg K} \right]$$

Heat rejected per kg of air

$$= c_v(T_4 - T_1) = 0.717(1029 - 311)$$

$$= 514.8 \text{ kJ/kg of air}$$

∴ *Work done per kg of air* = Heat supplied – heat rejected

= 1112.3 – 514.8

= **597.5 kJ or 597500 N-m. (Ans.)**

☞**Example 13.13.** *An engine working on Otto cycle has a volume of 0.45 m*^{3}*, pressure 1 bar and temperature 30°C at the beginning of compression stroke. At the end of compression stroke, the pressure is 11 bar. 210 kJ of heat is added at constant volume. Determine :*

(*i*) *Pressures, temperatures and volumes at salient points in the cycle.*

(*ii*) *Percentage clearance.*

(*iii*) *Efficiency.*

(*iv*) *Net work per cycle.*

(*v*) *Mean effective pressure.*

(*vi*) *Ideal power developed by the engine if the number of working cycles per minute is 210. Assume the cycle is reversible.*

Solution. Refer Fig. 13.12

Volume,	$V_1 = 0.45$ m^3
Initial pressure,	$p_1 = 1$ bar
Initial temperature,	$T_1 = 30 + 273 = 303$ K
Pressure at the end of compression stroke,	$p_2 = 11$ bar
Heat added at constant volume	= 210 kJ
Number of working cycles/min.	= 210.

(*i*) **Pressures, temperatures and volumes at salient points :**

For *adiabatic compression 1-2*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

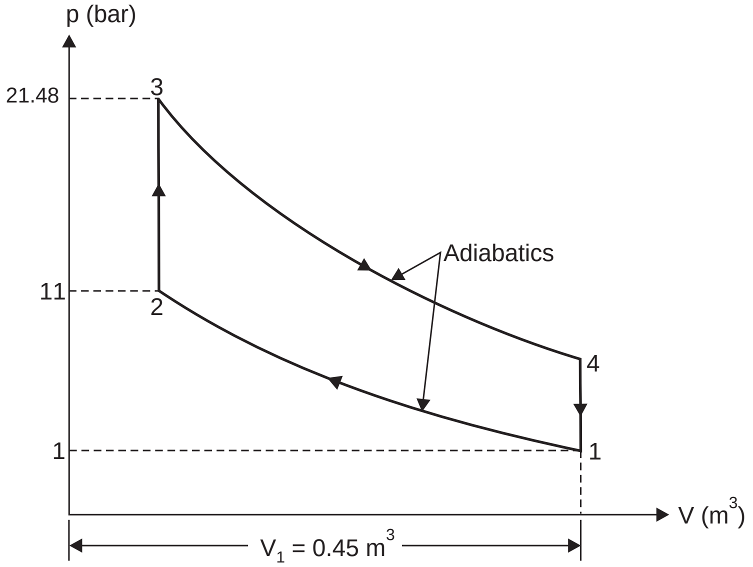


Fig. 13.12

or
$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma} \quad \text{or} \quad r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1}{1.4}} = (11)^{0.714} = 5.5$$

Also
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (5.5)^{1.4-1} = 1.977 \simeq 1.98$$

$\therefore$ $T_2 = T_1 \times 1.98 = 303 \times 1.98 =$ **600 K. (Ans.)**

Applying gas laws to points 1 and 2
$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$\therefore$
$$V_2 = \frac{T_2}{T_1} \times \frac{p_1}{p_2} \times V_1 = \frac{600 \times 1 \times 0.45}{303 \times 11} = \mathbf{0.081\ m^3.}\quad \textbf{(Ans.)}$$

The heat supplied during the *process 2-3* is given by :
$$Q_s = m\ c_v\ (T_3 - T_2)$$

where
$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.45}{287 \times 303} = 0.517 \text{ kg}$$

$\therefore$
$$210 = 0.517 \times 0.71\ (T_3 - 600)$$

or
$$T_3 = \frac{210}{0.517 \times 0.71} + 600 = \mathbf{1172\ K.}\quad \textbf{(Ans.)}$$

For the *constant volume process 2-3*
$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$\therefore$
$$p_3 = \frac{T_3}{T_2} \times p_2 = \frac{1172}{600} \times 11 = \mathbf{21.48\ bar.}\quad \textbf{(Ans.)}$$
$$V_3 = V_2 = \mathbf{0.081\ m^3.}\quad \textbf{(Ans.)}$$

For the *adiabatic (or isentropic) process 3-4*

$$p_3V_3^{\gamma} = p_4V_4^{\gamma}$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma} = p_3 \times \left(\frac{1}{r}\right)^{\gamma}$$

$$= 21.48 \times \left(\frac{1}{5.5}\right)^{1.4} = \mathbf{1.97\ bar.}\quad \textbf{(Ans.)}$$

Also $$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} = \left(\frac{1}{5.5}\right)^{1.4-1} = 0.505$$

$\therefore$ $$T_4 = 0.505\ T_3 = 0.505 \times 1172 = \mathbf{591.8\ K.}\quad \textbf{(Ans.)}$$

$$V_4 = V_1 = \mathbf{0.45\ m^3.}\quad \textbf{(Ans.)}$$

(*ii*) **Percentage clearance :**

Percentage clearance

$$= \frac{V_c}{V_s} = \frac{V_2}{V_1 - V_2} \times 100 = \frac{0.081}{0.45 - 0.081} \times 100$$

$$= \mathbf{21.95\%.}\quad \textbf{(Ans.)}$$

(*iii*) **Efficiency :**

The heat rejected per cycle is given by

$$Q_r = mc_v(T_4 - T_1)$$

$$= 0.517 \times 0.71\ (591.8 - 303) = 106\ \text{kJ}$$

The air-standard efficiency of the cycle is given by

$$\eta_{\text{otto}} = \frac{Q_s - Q_r}{Q_s} = \frac{210 - 106}{210} = \mathbf{0.495\ or\ 49.5\%.}\quad \textbf{(Ans.)}$$

[Alternatively :

$$\eta_{\text{otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5.5)^{1.4-1}} = \mathbf{0.495\ or\ 49.5\%.}\quad \textbf{(Ans.)}$$]

(*iv*) **Mean effective pressure, $\mathbf{p_m}$:**

The mean effective pressure is given by

$$p_m = \frac{W\ (\text{work done})}{V_s(\text{swept volume})} = \frac{Q_s - Q_r}{(V_1 - V_2)}$$

$$= \frac{(210 - 106) \times 10^3}{(0.45 - 0.081) \times 10^5} = \mathbf{2.818\ bar.}\quad \textbf{(Ans.)}$$

(*v*) **Power developed, P :**

Power developed, P = Work done per second

= Work done per cycle × number of cycles per second

= (210 – 106) × (210/60) = **364 kW.** **(Ans.)**

Example 13.14. (*a*) *Show that the compression ratio for the maximum work to be done per kg of air in an Otto cycle between upper and lower limits of absolute temperatures T_3 and T_1 is given by*

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}$$

(*b*) *Determine the air-standard efficiency of the cycle when the cycle develops maximum work with the temperature limits of 310 K and 1220 K and working fluid is air. What will be the percentage change in efficiency if helium is used as working fluid instead of air ? The cycle operates between the same temperature limits for maximum work development.*

Consider that all conditions are ideal.

Solution. Refer Fig. 13.13.

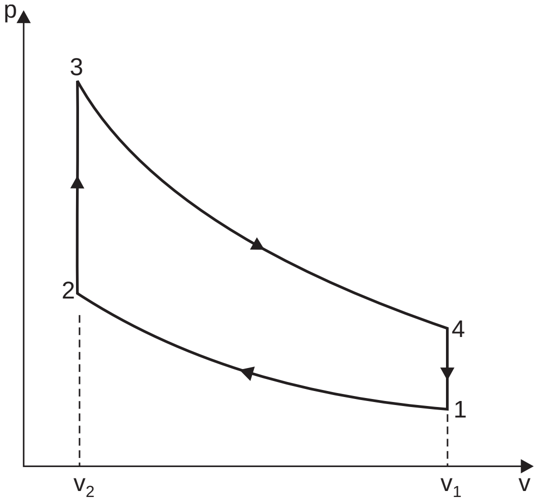


Fig. 13.13

(*a*) The work done per kg of fluid in the cycle is given by

$$W = Q_s - Q_r = c_v (T_3 - T_2) - c_v (T_4 - T_1)$$

But $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1}$$

∴ $$T_2 = T_1 \, . \, (r)^{\gamma - 1} \qquad ...(i)$$

Similarly, $$T_3 = T_4 \, . \, (r)^{\gamma - 1} \qquad ...(ii)$$

∴ $$W = c_v \left[T_3 - T_1 \, . \, (r)^{\gamma - 1} - \frac{T_3}{(r)^{\gamma - 1}} + T_1 \right] \qquad ...(iii)$$

This expression is a function of r when T_3 and T_1 are fixed. The value of W will be maximum when,

$$\frac{dW}{dr} = 0.$$

∴ $$\frac{dW}{dr} = - T_1 \, . \, (\gamma - 1) \, (r)^{\gamma - 2} - T_3 (1 - \gamma) \, (r)^{-\gamma} = 0$$

or $$T_3 \, (r)^{-\gamma} = T_1 \, (r)^{\gamma - 2}$$

or $$\frac{T_3}{T_1} = (r)^{2(\gamma - 1)}$$

∴ $$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma - 1)}. \qquad \textbf{......Proved.}$$

(*b*) **Change in efficiency :**

For *air* $\gamma = 1.4$

$$\therefore \qquad r = \left(\frac{T_3}{T_1}\right)^{1/2(1.4-1)} = \left(\frac{1220}{310}\right)^{1/0.8} = 5.54$$

The air-standard efficiency is given by

$$\eta_{\text{otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5.54)^{1.4-1}} = \mathbf{0.495 \text{ or } 49.5\%.} \quad \textbf{(Ans.)}$$

If *helium* is used, then the values of

$$c_p = 5.22 \text{ kJ/kg K and } c_v = 3.13 \text{ kJ/kg K}$$

$$\therefore \qquad \gamma = \frac{c_p}{c_v} = \frac{5.22}{3.13} = 1.67$$

The compression ratio for maximum work for the temperature limits T_1 and T_3 is given by

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)} = \left(\frac{1220}{310}\right)^{1/2(1.67-1)} = 2.77$$

The air-standard efficiency is given by

$$\eta_{\text{otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(2.77)^{1.67-1}} = \mathbf{0.495 \quad or \quad 49.5\%.}$$

Hence change in efficiency is nil. (Ans.)

Example 13.15. (*a*) *An engine working on Otto cycle, in which the salient points are 1, 2, 3 and 4, has upper and lower temperature limits* T_3 *and* T_1. *If the maximum work per kg of air is to be done, show that the intermediate temperature is given by*

$$T_2 = T_4 = \sqrt{T_1 T_3}\,.$$

(*b*) *If an engine works on Otto cycle between temperature limits 1450 K and 310 K, find the maximum power developed by the engine assuming the circulation of air per minute as 0.38 kg.*

Solution. (*a*) Refer Fig. 13.13 (Example 13.14).

Using the equation (*iii*) of example 13.14.

$$W = c_v \left[T_3 - T_1 \,.\, (r)^{\gamma-1} - \frac{T_3}{(r)^{\gamma-1}} + T_1\right]$$

and differentiating W w.r.t. r and equating to zero

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}$$

$$T_2 = T_1 (r)^{\gamma-1} \text{ and } T_4 = T_3/(r)^{\gamma-1}$$

Substituting the value of r in the above equation, we have

$$T_2 = T_1 \left[\left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}\right]^{\gamma-1} = T_1 \left(\frac{T_3}{T_1}\right)^{1/2} = \sqrt{T_1 T_3}$$

Similarly,
$$T_4 = \frac{T_3}{\left[\left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}\right]^{\gamma-1}} = \frac{T_3}{\left(\frac{T_3}{T_1}\right)^{1/2}} = \sqrt{T_3 T_1}$$

$\therefore \qquad T_2 = T_4 = \sqrt{T_1 T_3}$. **Proved.**

(*b*) **Power developed, P :**

$$\left.\begin{aligned} T_1 &= 310 \text{ K} \\ T_3 &= 1450 \text{ K} \\ m &= 0.38 \text{ kg} \end{aligned}\right\} \quad \text{....(given)}$$

Work done $\qquad W = c_v\,[(T_3 - T_2) - (T_4 - T_1)]$

$$T_2 = T_4 = \sqrt{T_1 T_3} = \sqrt{310 \times 1450} = 670.4 \text{ K}$$

$\therefore \qquad W = 0.71\ [(1450 - 670.4) - (670.4 - 310)]$

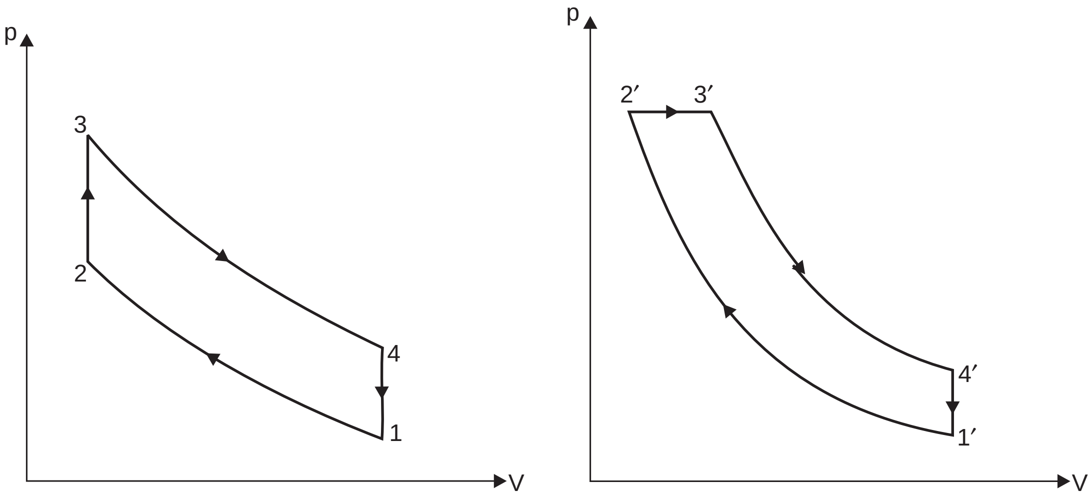
$\qquad = 0.71\ (779.6 - 360.4) = 297.6$ kJ/kg

Work done per second $\qquad = 297.6 \times (0.38/60) = 1.88$ kJ/s

Hence **power developed, P = 1.88 kW. (Ans.)**

Example 13.16. *For the same compression ratio, show that the efficiency of Otto cycle is greater than that of Diesel cycle.*

Solution. Refer Fig. 13.14.



Otto cycle Diesel cycle

Fig. 13.14

We know that

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma - 1}}$$

and

$$\eta_{\text{Diesel}} = 1 - \frac{1}{(r)^{\gamma - 1}} \times \frac{1}{\gamma}\left\{\frac{\rho^{\gamma} - 1}{\rho - 1}\right\}$$

As the compression ratio is same,

$$\frac{V_1}{V_2} = \frac{V_1'}{V_2'} = r$$

If $\dfrac{V_4'}{V_3'} = r_1$, then cut-off ratio, $\rho = \dfrac{V_3'}{V_2'} = \dfrac{r}{r_1}$

Putting the value of ρ in η_{Diesel}, we get

$$\eta_{\text{Diesel}} = 1 - \frac{1}{(r)^{\gamma-1}} \times \frac{1}{\gamma}\left[\frac{\left(\frac{r}{r_1}\right)^{\gamma} - 1}{\frac{r}{r_1} - 1}\right]$$

From above equation, we observe

$$\frac{r}{r_1} > 1$$

Let $r_1 = r - \delta$, where δ is a small quantity.

Then $$\frac{r}{r_1} = \frac{r}{r-\delta} = \frac{r}{r\left(1-\frac{\delta}{r}\right)} = \left(1-\frac{\delta}{r}\right)^{-1} = 1 + \frac{\delta}{r} + \frac{\delta^2}{r^2} + \frac{\delta^3}{r^3} + \ldots\ldots$$

and $$\left(\frac{r}{r_1}\right)^{\gamma} = \frac{r^{\gamma}}{r^{\gamma}\left(1-\frac{\delta}{r}\right)^{\gamma}} = \left(1-\frac{\delta}{r}\right)^{-\gamma} = 1 + \frac{\gamma\delta}{r} + \frac{\gamma(\gamma+1)}{2!}\cdot\frac{\delta^2}{r^2} + \ldots\ldots$$

$$\therefore \quad \eta_{\text{Diesel}} = 1 - \frac{1}{(r)^{\gamma-1}} \times \frac{1}{\gamma}\left[\frac{\frac{\gamma\,.\,\delta}{r} + \frac{\gamma\,(\gamma+1)}{2!}\cdot\frac{\delta^2}{r^2} + \ldots\ldots}{\frac{\delta}{r} + \frac{\delta^2}{r^2} + \ldots\ldots}\right]$$

$$= 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\frac{\delta}{r} + \frac{\gamma+1}{2}\cdot\frac{\delta^2}{r^2} + \ldots\ldots}{\frac{\delta}{r} + \frac{\delta^2}{r^2} + \ldots\ldots}\right]$$

The ratio inside the bracket is greater than 1 since the co-efficients of terms δ^2/r^2 is greater than 1 in the numerator. Its means that something more is subtracted in case of diesel cycle than in Otto cycle.

Hence, *for same compression ratio* $\eta_{otto} > \eta_{diesel}$.

13.5. CONSTANT PRESSURE OR DIESEL CYCLE

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle in that *heat is supplied at constant pressure instead of at constant volume*. Fig. 13.15 (*a* and *b*) shows the *p-v* and *T-s* diagrams of this cycle respectively.

This cycle comprises of the following ***operations*** :

(*i*) 1-2......*Adiabatic compression.*

(*ii*) 2-3......*Addition of heat at constant pressure.*

(*iii*) 3-4......*Adiabatic expansion.*

(*iv*) 4-1......*Rejection of heat at constant volume.*

Point 1 represents that the cylinder is full of air. Let p_1, V_1 and T_1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (*i.e.,* pV^{γ} = constant) till the values become p_2, V_2 and T_2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let

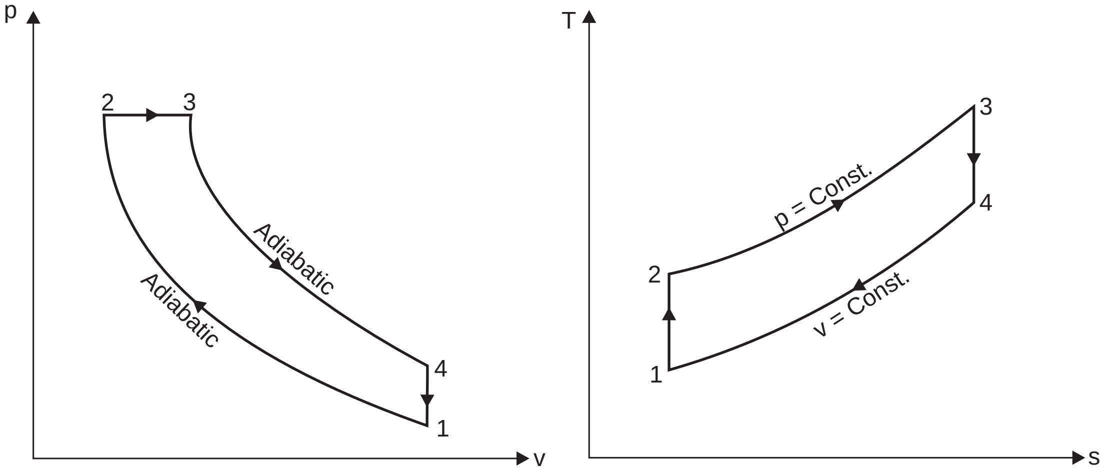


Fig. 13.15

volume increases from V_2 to V_3 and temperature T_2 to T_3, corresponding to point 3. This point (3) is called the ***point of cut-off***. The air then expands adiabatically to the conditions p_4, V_4 and T_4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.

Consider 1 kg of air.

Heat supplied at constant pressure = $c_p(T_3 - T_2)$

Heat rejected at constant volume = $c_v(T_4 - T_1)$

Work done = Heat supplied – heat rejected

$$= c_p(T_3 - T_2) - c_v(T_4 - T_1)$$

$$\therefore \qquad \eta_{\text{diesel}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \qquad \text{...}(i) \quad \left[\because \frac{c_p}{c_v} = \gamma\right]$$

Let compression ratio, $r = \dfrac{v_1}{v_2}$, and cut-off ratio, $\rho = \dfrac{v_3}{v_2}$ *i.e.*, $\dfrac{\text{Volume at cut-off}}{\text{Clearance volume}}$

Now, during *adiabatic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad \text{or} \quad T_2 = T_1 \,.\, (r)^{\gamma - 1}$$

During *constant pressure process 2-3,*

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho \quad \text{or} \quad T_3 = \rho \,.\, T_2 = \rho \,.\, T_1 \,.\, (r)^{\gamma - 1}$$

During *adiabatic expansion 3-4*

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma - 1} \qquad \left(\because \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = \frac{r}{\rho}\right)$$

$$\therefore \qquad T_4 = \frac{T_3}{\left(\frac{r}{\rho}\right)^{\gamma - 1}} = \frac{\rho \,.\, T_1 (r)^{\gamma - 1}}{\left(\frac{r}{\rho}\right)^{\gamma - 1}} = T_1 \,.\, \rho^{\gamma}$$

By inserting values of T_2, T_3 and T_4 in eqn. (*i*), we get

$$\eta_{\text{diesel}} = 1 - \frac{(T_1 \,.\, \rho^{\gamma} - T_1)}{\gamma\,(\rho \,.\, T_1 \,.\, (r)^{\gamma - 1} - T_1 \,.\, (r)^{\gamma - 1})} = 1 - \frac{(\rho^{\gamma} - 1)}{\gamma (r)^{\gamma - 1} (\rho - 1)}$$

or

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right] \qquad \text{...(13.7)}$$

It may be observed that eqn. (13.7) for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $\rho > 1$. Hence *for a given compression ratio, the Otto cycle is more efficient*.

The *net work* for diesel cycle can be expressed in terms of *pv* as follows :

$$W = p_2(v_3 - v_2) + \frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_2\,(\rho v_2 - v_2) + \frac{p_3 \rho v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1}$$

$$\left[\begin{array}{l} \because \ \dfrac{v_3}{v_2} = \rho \quad \therefore \ v_3 = \rho v_2 \ \text{ and } \dfrac{v_1}{v_2} = r \quad \therefore \ v_1 = r v_2 \\ \text{But } v_4 = v_1 \quad \therefore \quad v_4 = r v_2 \end{array}\right]$$

$$= p_2 v_2\,(\rho - 1) + \frac{p_3\,\rho\, v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1}$$

$$= \frac{v_2\,[p_2\,(\rho - 1)(\gamma - 1) + p_3 \rho - p_4 r - (p_2 - p_1 r)]}{\gamma - 1}$$

$$= \frac{v_2 \left[p_2\,(\rho - 1)(\gamma - 1) + p_3\left(\rho - \dfrac{p_4 r}{p_3}\right) - p_2\left(1 - \dfrac{p_1 r}{p_2}\right)\right]}{\gamma - 1}$$

$$= \frac{p_2 v_2\,[(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} \,.\, r^{1 - \gamma} - (1 - r^{1 - \gamma})}{\gamma - 1}$$

$$\left[\because \ \frac{p_4}{p_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} = \rho^{\gamma}\, r^{-\gamma}\right]$$

$$= \frac{p_1 v_1 r^{\gamma - 1}\,[(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} r^{1 - \gamma} - (1 - r^{1 - \gamma})}{\gamma - 1}$$

$$\left[\because \ \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \ \text{ or } \ p_2 = p_1 \,.\, r^{\gamma} \text{ and } \frac{v_1}{v_2} = r \ \text{ or } \ v_2 = v_1\, r^{-1}\right]$$

$$= \frac{p_1 v_1 r^{\gamma - 1}\,[\gamma(\rho - 1) - r^{1 - \gamma}\,(\rho^{\gamma} - 1)]}{(\gamma - 1)} \qquad \text{...(13.8)}$$

Mean effective pressure p_m is given by :

$$p_m = \frac{p_1 v_1 r^{\gamma-1}\,[\gamma(\rho-1) - r^{1-\gamma}\,(\rho^\gamma - 1)]}{(\gamma-1)v_1\left(\frac{r-1}{r}\right)}$$

or
$$\mathbf{p_m} = \frac{p_1 r^\gamma\,[\gamma(\rho-1) - r^{1-\gamma}\,(\rho^\gamma-1)]}{(\gamma-1)(r-1)}\ . \qquad ...(13.9)$$

Example 13.17. *A diesel engine has a compression ratio of 15 and heat addition at constant pressure takes place at 6% of stroke. Find the air standard efficiency of the engine.*

Take γ for air as 1.4.

Solution. Refer Fig. 13.16.

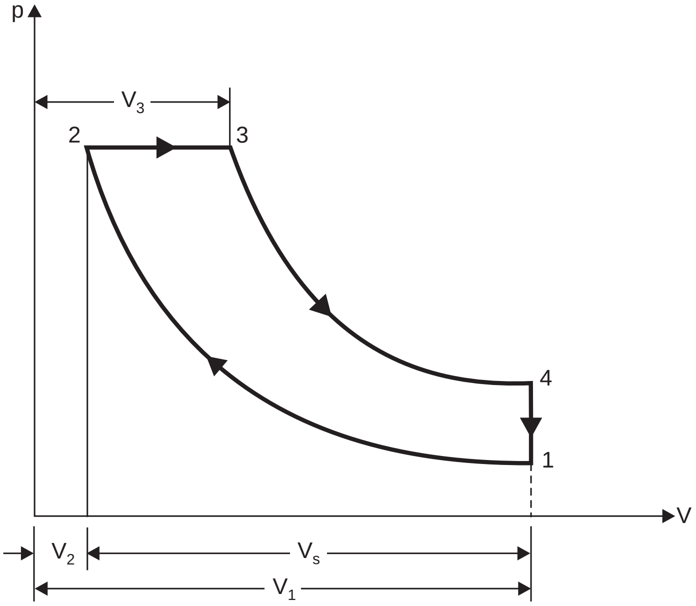


Fig. 13.16

Compression ratio, $r\left(=\frac{V_1}{V_2}\right) = 15$

γ for air = 1.4

Air standard efficiency of diesel cycle is given by

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] \qquad ...(i)$$

where ρ = cut-off ratio = $\frac{V_3}{V_2}$

But $$V_3 - V_2 = \frac{6}{100}\ V_s \qquad (V_s = \text{stroke volume})$$

$$= 0.06\,(V_1 - V_2) = 0.06\,(15\,V_2 - V_2)$$

$$= 0.84\,V_2 \quad \text{or} \quad V_3 = 1.84\,V_2$$

$$\therefore \qquad \rho = \frac{V_3}{V_2} = \frac{1.84\,V_2}{V_2} = 1.84$$

Putting the value in eqn. (*i*), we get

$$\eta_{diesel} = 1 - \frac{1}{1.4\,(15)^{1.4-1}}\left[\frac{(1.84)^{1.4} - 1}{1.84 - 1}\right]$$

$$= 1 - 0.2417 \times 1.605 = \mathbf{0.612 \text{ or } 61.2\%.} \quad \textbf{(Ans.)}$$

Example 13.18. *The stroke and cylinder diameter of a compression ignition engine are 250 mm and 150 mm respectively. If the clearance volume is 0.0004 m*^{3} *and fuel injection takes place at constant pressure for 5 per cent of the stroke determine the efficiency of the engine. Assume the engine working on the diesel cycle.*

Solution. Refer Fig. 13.16.

Length of stroke, $L = 250 \text{ mm} = 0.25 \text{ m}$

Diameter of cylinder, $D = 150 \text{ mm} = 0.15 \text{ m}$

Clearance volume, $V_2 = 0.0004 \text{ m}^3$

Swept volume, $V_s = \pi/4\, D^2 L = \pi/4 \times 0.15^2 \times 0.25 = 0.004418 \text{ m}^3$

Total cylinder volume

$$= \text{Swept volume} + \text{clearance volume}$$

$$= 0.004418 + 0.0004 = 0.004818 \text{ m}^3$$

Volume at point of cut-off, $V_3 = V_2 + \dfrac{5}{100}\,V_s$

$$= 0.0004 + \frac{5}{100} \times 0.004418 = 0.000621 \text{ m}^3$$

$\therefore$ Cut-off ratio, $\rho = \dfrac{V_3}{V_2} = \dfrac{0.000621}{0.0004} = 1.55$

Compression ratio, $r = \dfrac{V_1}{V_2} = \dfrac{V_s + V_2}{V_2} = \dfrac{0.004418 + 0.0004}{0.0004} = 12.04$

Hence,

$$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4 \times (12.04)^{1.4-1}}\left[\frac{(1.55)^{1.4} - 1}{1.55 - 1}\right]$$

$$= 1 - 0.264 \times 1.54 = \mathbf{0.593 \text{ or } 59.3\%.} \quad \textbf{(Ans.)}$$

Example 13.19. *Calculate the percentage loss in the ideal efficiency of a diesel engine with compression ratio 14 if the fuel cut-off is delayed from 5% to 8%.*

Solution. Let the clearance volume (V_2) be unity.

Then, compression ratio, $r = 14$

Now, when the fuel is cut off at 5%, we have

$$\frac{\rho - 1}{r - 1} = \frac{5}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = 0.05 \quad \text{or} \quad \rho - 1 = 13 \times 0.05 = 0.65$$

$$\therefore \qquad \rho = 1.65$$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4 \times (14)^{1.4-1}}\left[\frac{(1.65)^{1.4} - 1}{1.65 - 1}\right]$$

$$= 1 - 0.248 \times 1.563 = 0.612 \quad \text{or} \quad 61.2\%$$

When the fuel is cut-off at 8%, we have

$$\frac{\rho - 1}{r - 1} = \frac{8}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = \frac{8}{100} = 0.08$$

$$\therefore \qquad \rho = 1 + 1.04 = 2.04$$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4 \times (14)^{1.4 - 1}} \left[\frac{(2.04)^{1.4} - 1}{2.04 - 1} \right]$$

$$= 1 - 0.248 \times 1.647 = 0.591 \quad \text{or} \quad 59.1\%.$$

Hence percentage loss in efficiency due to delay in fuel cut off

$$= 61.2 - 59.1 = \mathbf{2.1\%}. \quad \textbf{(Ans.)}$$

Example 13.20. *The mean effective pressure of a Diesel cycle is 7.5 bar and compression ratio is 12.5. Find the percentage cut-off of the cycle if its initial pressure is 1 bar.*

Solution. Mean effective pressure, $p_m = 7.5$ bar

Compression ratio, $r = 12.5$

Initial pressure, $p_1 = 1$ bar

Refer Fig. 13.15.

The mean effective pressure is given by

$$p_m = \frac{p_1 r^{\gamma} [\gamma (\rho - 1) - r^{1-\gamma} (\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)}$$

$$7.5 = \frac{1 \times (12.5)^{1.4} [1.4 (\rho - 1) - (12.5)^{1 - 1.4} (\rho^{1.4} - 1)]}{(1.4 - 1)(12.5 - 1)}$$

$$7.5 = \frac{34.33 [1.4 \rho - 1.4 - 0.364 \rho^{1.4} + 0.364]}{4.6}$$

$$7.5 = 7.46 \, (1.4 \, \rho - 1.036 - 0.364 \, \rho^{1.4})$$

$$1.005 = 1.4 \, \rho - 1.036 - 0.364 \, \rho^{1.4}$$

or $$2.04 = 1.4 \, \rho - 0.364 \, \rho^{1.4} \quad \text{or} \quad 0.346 \, \rho^{1.4} - 1.4 \, \rho + 2.04 = 0$$

Solving by trial and error method, we get

$$\rho = 2.24$$

$$\therefore \qquad \% \; cut\text{-}off = \frac{\rho - 1}{r - 1} \times 100 = \frac{2.24 - 1}{12.5 - 1} \times 100 = \mathbf{10.78\%}. \quad \textbf{(Ans.)}$$

☞**Example 13.21.** *An engine with 200 mm cylinder diameter and 300 mm stroke works on theoretical Diesel cycle. The initial pressure and temperature of air used are 1 bar and 27°C. The cut-off is 8% of the stroke. Determine :*

(*i*) *Pressures and temperatures at all salient points.*

(*ii*) *Theoretical air standard efficiency.*

(*iii*) *Mean effective pressure.*

(*iv*) *Power of the engine if the working cycles per minute are 380.*

Assume that compression ratio is 15 and working fluid is air.

Consider all conditions to be ideal.

Solution. Refer Fig. 13.17.

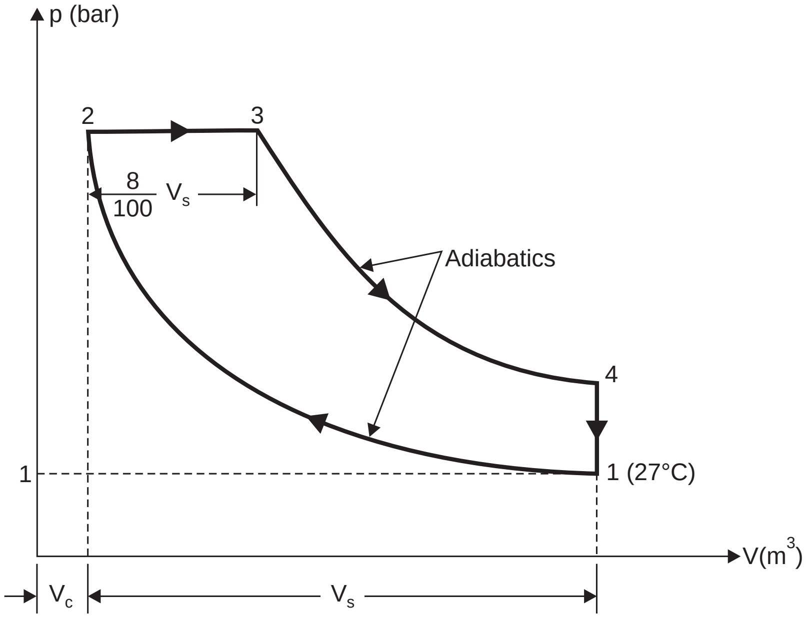


Fig. 13.17

Cylinder diameter, $D = 200$ mm or 0.2 m

Stroke length, $L = 300$ mm or 0.3 m

Initial pressure, $p_1 = 1.0$ bar

Initial temperature, $T_1 = 27 + 273 = 300$ K

Cut-off $= \frac{8}{100} V_s = 0.08\ V_s$

(*i*) **Pressures and temperatures at salient points :**

Now, stroke volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.2^2 \times 0.3 = 0.00942$ m³

$$V_1 = V_s + V_c = V_s + \frac{V_s}{r-1} \qquad \left[\because\ V_c = \frac{V_s}{r-1}\right]$$

$$= V_s\left(1 + \frac{1}{r-1}\right) = \frac{r}{r-1} \times V_s$$

i.e., $\mathbf{V_1} = \frac{15}{15-1} \times V_s = \frac{15}{14} \times 0.00942 =$ **0.0101 m³. (Ans.)**

Mass of the air in the cylinder can be calculated by using the gas equation,

$$p_1V_1 = mRT_1$$

$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.0101}{287 \times 300} = 0.0117 \text{ kg/cycle}$$

For the *adiabatic (or isentropic) process 1-2*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma}$$

$\therefore$ $\boldsymbol{p_2} = p_1 \cdot (r)^{\gamma} = 1 \times (15)^{1.4} = \mathbf{44.31\ bar.}$ **(Ans.)**

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.4-1} = 2.954$$

$\therefore$ $\boldsymbol{T_2} = T_1 \times 2.954 = 300 \times 2.954 = \mathbf{886.2\ K.}$ **(Ans.)**

$$\boldsymbol{V_2} = V_c = \frac{V_s}{r-1} = \frac{0.00942}{15-1} = \mathbf{0.0006728\ m^3.}\ \textbf{(Ans.)}$$

$\boldsymbol{p_2} = p_3 = \mathbf{44.31\ bar.}$ **(Ans.)**

% cut-off ratio $$= \frac{\rho - 1}{r - 1}$$

$$\frac{8}{100} = \frac{\rho-1}{15-1}$$

i.e., $\rho = 0.08 \times 14 + 1 = 2.12$

$\therefore$ $V_3 = \rho\, V_2 = 2.12 \times 0.0006728 = \mathbf{0.001426\ m^3.}$ **(Ans.)**

$$\left[\begin{array}{l} V_3 \text{ can also be calculated as follows:} \\ V_3 = 0.08V_s + V_c = 0.08 \times 0.00942 + 0.0006728 = 0.001426\ \text{m}^3 \end{array}\right]$$

For the *constant pressure process 2-3,*

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

$\therefore$ $$\boldsymbol{T_3} = T_2 \times \frac{V_3}{V_2} = 886.2 \times \frac{0.001426}{0.0006728} = \mathbf{1878.3\ K.}\ \textbf{(Ans.)}$$

For the *isentropic process 3-4,*

$$p_3V_3^{\gamma} = p_4V_4^{\gamma}$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma} = p_3 \times \frac{1}{(7.07)^{1.4}}$$

$$= \frac{44.31}{(7.07)^{1.4}} = \mathbf{2.866\ bar.}\ \textbf{(Ans.)}$$

$$\left[\begin{array}{l} \because\ \dfrac{V_4}{V_3} = \dfrac{V_4}{V_2} \times \dfrac{V_2}{V_3} = \dfrac{V_1}{V_2} \times \dfrac{V_2}{V_3} \\ \qquad = \dfrac{r}{\rho},\ \because\ V_4 = V_1 = \dfrac{15}{2.12} = 7.07 \end{array}\right]$$

Also, $$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{7.07}\right)^{1.4-1} = 0.457$$

$\therefore$ $\boldsymbol{T_4} = T_3 \times 0.457 = 1878.3 \times 0.457 = \mathbf{858.38\ K.}$ **(Ans.)**

$\boldsymbol{V_4} = V_1 = \mathbf{0.0101\ m^3.}$ **(Ans.)**

(*ii*) **Theoretical air standard efficiency :**

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right] = 1 - \frac{1}{1.4\,(15)^{1.4-1}}\left[\frac{(2.12)^{1.4}-1}{2.12-1}\right]$$

$$= 1 - 0.2418 \times 1.663 = \mathbf{0.598\ or\ 59.8\%.}\ \textbf{(Ans.)}$$

(*iii*) **Mean effective pressure, p_m :**

Mean effective pressure of Diesel cycle is given by

$$\boldsymbol{p_m} = \frac{p_1(r)^{\gamma}[\gamma(\rho-1)-r^{1-\gamma}(\rho^{\gamma}-1)}{(\gamma-1)(r-1)}$$

$$= \frac{1\times(15)^{1.4}[1.4(2.12-1)-(15)^{1-1.4}(2.12^{1.4}-1)]}{(1.4-1)(15-1)}$$

$$= \frac{44.31[1.568-0.338\times1.863]}{0.4\times14} = \textbf{7.424 bar.} \quad \textbf{(Ans.)}$$

(*iv*) **Power of the engine, P :**

Work done per cycle $= p_m V_s = \dfrac{7.424\times10^5\times0.00942}{10^3} = 6.99$ kJ/cycle

Work done per second = Work done per cycle × no. of cycles per second

= 6.99 × 380/60 = 44.27 kJ/s = 44.27 kW

Hence *power of the engine* = **44.27 kW. (Ans.)**

Example 13.22. *The volume ratios of compression and expansion for a diesel engine as measured from an indicator diagram are 15.3 and 7.5 respectively. The pressure and temperature at the beginning of the compression are 1 bar and 27°C.*

Assuming an ideal engine, determine the mean effective pressure, the ratio of maximum pressure to mean effective pressure and cycle efficiency.

Also find the fuel consumption per kWh if the indicated thermal efficiency is 0.5 of ideal efficiency, mechanical efficiency is 0.8 and the calorific value of oil 42000 kJ/kg.

Assume for air : c_p = 1.005 kJ/kg K ; c_v = 0.718 kJ/kg K, γ = 1.4. **(U.P.S.C., 1996)**

Solution. Refer Fig. 13.18. *Given :* $\dfrac{V_1}{V_2} = 15.3$; $\dfrac{V_4}{V_3} = 7.5$

p_1 = 1 bar ; T_1 = 27 + 273 = 300 K ; $\eta_{th(I)} = 0.5 \times \eta_{air\text{-}standard}$; $\eta_{mech.} = 0.8$; C = 42000 kJ/kg.

The cycle is shwon in Fig. 13.18, the subscripts denote the respective points in the cycle.

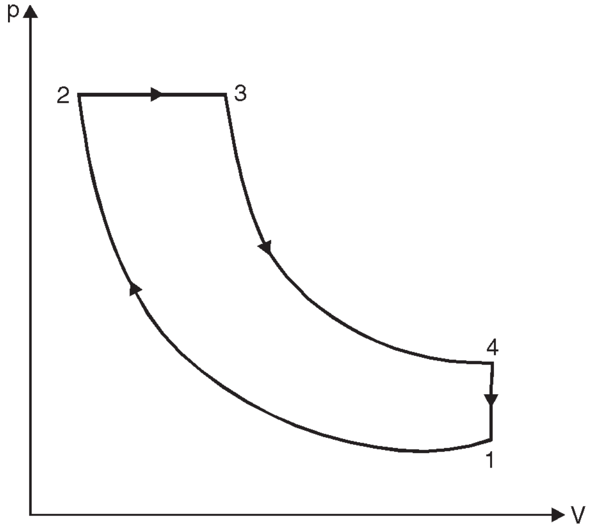


Fig. 13.18. Diesel cycle.

Mean effective pressure, $\mathbf{p_m}$:

$$p_m = \frac{\text{Work done by the cycle}}{\text{Swept volume}}$$

Work done = Heat added – heat rejected

Heat added $= mc_p\,(T_3 - T_2)$, and

Heat rejected $= mc_v\,(T_4 - T_1)$

Now assume air as a perfect gas and mass of oil in the air-fuel mixture is negligible and is not taken into account.

Process 1-2 is an *adiabatic compression process*, thus

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \text{or} \quad T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{1.4-1} \quad (\text{since } \gamma = 1.4)$$

or $$T_2 = 300 \times (15.3)^{0.4} = 893.3 \text{ K}$$

Also, $$p_1V_1^{\gamma} = p_2V_2^{\gamma} \;\Rightarrow\; p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (15.3)^{1.4} = 45.56 \text{ bar}$$

Process 2-3 is a *constant pressure process*, hence

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \;\Rightarrow\; T_3 = \frac{V_3T_2}{V_2} = 2.04 \times 893.3 = 1822.3 \text{ K}$$

Assume that the volume at point 2 (V_2) is 1 m^3. Thus the mass of air involved in the process,

$$m = \frac{p_2V_2}{RT_2} = \frac{45.56 \times 10^5 \times 1}{287 \times 893.3} = 17.77 \text{ kg} \qquad \left[\begin{array}{l} \because \; \dfrac{V_4}{V_3} = \dfrac{V_1}{V_3} = \dfrac{V_1}{V_2} \times \dfrac{V_2}{V_3} \\ \text{or} \; \dfrac{V_3}{V_2} = \dfrac{V_1}{V_2} \times \dfrac{V_3}{V_4} = \dfrac{15.3}{7.5} = 2.04 \end{array}\right]$$

Process 3-4 is an *adiabatic expansion process*, thus

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{7.5}\right)^{1.4-1} = 0.4466$$

or $$T_4 = 1822.3 \times 0.4466 = 813.8 \text{ K}$$

∴ Work done $= mc_p\,(T_3 - T_2) - mc_v\,(T_4 - T_1)$

$= 17.77\,[1.005\,(1822.3 - 893.3) - 0.718\,(813.8 - 300)] = 10035$ kJ

∴ $$\mathbf{p_m} = \frac{\text{Work done}}{\text{Swept volume}} = \frac{10035}{(V_1 - V_2)} = \frac{10035}{(15.3V_2 - V_2)} = \frac{10035}{14.3}$$

$= 701.7$ kN/m^2 = **7.017 bar. (Ans.)**

($\because \; V_2 = 1 \text{ m}^3$ assumed)

Ratio of maximum pressure to mean effective pressure

$$= \frac{p_2}{p_m} = \frac{45.56}{7.017} = \mathbf{6.49.} \quad \textbf{(Ans.)}$$

Cycle efficiency, η_{cycle} :

$$\eta_{\text{cycle}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{10035}{mc_p\,(T_3 - T_2)} = \frac{10035}{17.77 \times 1.005\,(1822.3 - 897.3)} = \mathbf{0.6048 \quad or \quad 60.48\%.\quad (Ans.)}$$

Fuel consumption per kWh ; $\mathbf{m_f}$:

$\eta_{th(I)} = 0.5\ \eta_{cycle} = 0.5 \times 0.6048 = 0.3024$ or 30.24%

$$\eta_{th(B)} = 0.3024 \times 0.8 = 0.242$$

Also,

$$\eta_{th(B)} = \frac{\text{B.P.}}{m_f \times C} = \frac{1}{\frac{m_f}{3600} \times 42000} = \frac{3600}{m_f \times 42000}$$

or

$$0.242 = \frac{3600}{m_f \times 42000}$$

or

$$\mathbf{m_f} = \frac{3600}{0.242 \times 42000} = \mathbf{0.354\ kg/kWh.\quad (Ans.)}$$

13.6. DUAL COMBUSTION CYCLE

This cycle (also called the *limited pressure cycle or mixed cycle*) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; *the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.*

The dual combustion cycle (Fig. 13.19) consists of the following **operations** :

(*i*) 1-2—*Adiabatic compression*

(*ii*) 2-3—*Addition of heat at constant volume*

(*iii*) 3-4—*Addition of heat at constant pressure*

(*iv*) 4-5—*Adiabatic expansion*

(*v*) 5-1—*Rejection of heat at constant volume.*

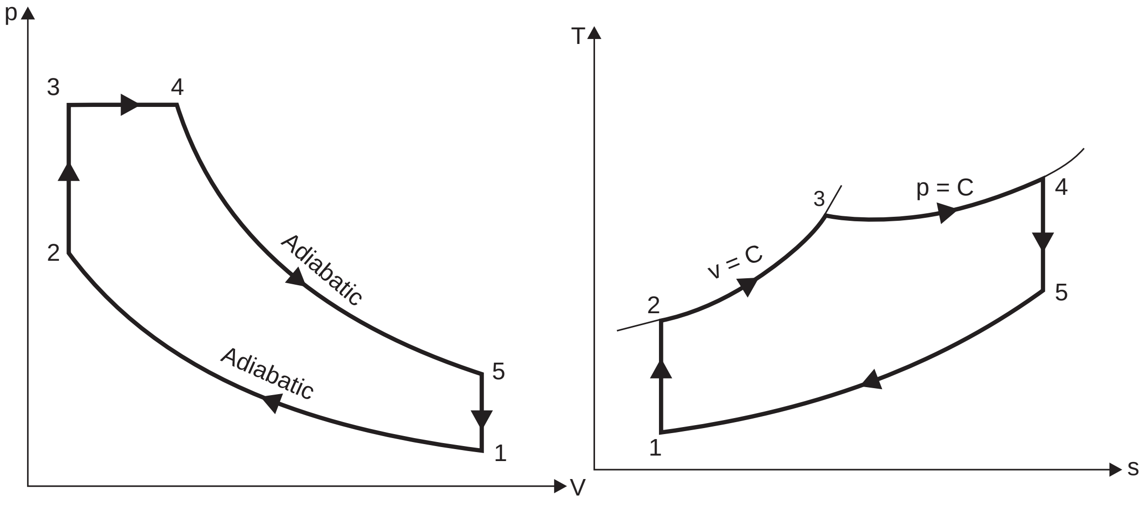


Fig. 13.19.

Consider 1 kg of air.

Total heat supplied = Heat supplied during the operation 2-3 + heat supplied during the operation 3-4

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

Heat rejected during operation 5-1 $= c_v(T_5 - T_1)$

Work done = Heat supplied – heat rejected

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)$$

$$\eta_{\text{dual}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \quad ...(i) \quad \left(\because \ \gamma = \frac{c_p}{c_v}\right)$$

Compression ratio, $r = \dfrac{v_1}{v_2}$

During *adiabatic compression process 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad ...(ii)$$

During *constant volume heating process,*

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

or $\dfrac{T_3}{T_2} = \dfrac{p_3}{p_2} = \beta$, where β is known as **pressure or explosion ratio.**

or $$T_2 = \frac{T_3}{\beta} \quad ...(iii)$$

During *adiabatic expansion process,*

$$\frac{T_4}{T_5} = \left(\frac{v_5}{v_4}\right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma - 1} \quad ...(iv)$$

$$\left(\because \ \frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{v_1}{v_2} \times \frac{v_2}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{r}{\rho}, \ \rho \text{ being the cut-off ratio}\right)$$

During *constant pressure heating process,*

$$\frac{v_3}{T_3} = \frac{v_4}{T_4}$$

$$T_4 = T_3 \frac{v_4}{v_3} = \rho \, T_3 \quad ...(v)$$

Putting the value of T_4 in the eqn. (*iv*), we get

$$\frac{\rho T_3}{T_5} = \left(\frac{r}{\rho}\right)^{\gamma - 1} \quad \text{or} \quad T_5 = \rho \, . \, T_3 \, . \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

Putting the value of T_2 in eqn. (*ii*), we get

$$\frac{\dfrac{T_3}{\beta}}{T_1} = (r)^{\gamma - 1}$$

$$T_1 = \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma - 1}}$$

Now inserting the values of T_1, T_2, T_4 and T_5 in eqn. (*i*), we get

$$\eta_{\text{dual}} = 1 - \frac{\left[\rho \, . \, T_3 \left(\dfrac{\rho}{r}\right)^{\gamma - 1} - \dfrac{T_3}{\beta} \cdot \dfrac{1}{(r)^{\gamma - 1}}\right]}{\left[\left(T_3 - \dfrac{T_3}{\beta}\right) + \gamma(\rho T_3 - T_3)\right]} = 1 - \frac{\dfrac{1}{(r)^{\gamma - 1}}\left(\rho^{\gamma} - \dfrac{1}{\beta}\right)}{\left(1 - \dfrac{1}{\beta}\right) + \gamma(\rho - 1)}$$

i.e.,
$$\eta_{\text{dual}} = 1 - \frac{1}{(r)^{\gamma - 1}} \cdot \frac{(\beta \, . \, \rho^{\gamma} - 1)}{[(\beta - 1) + \beta\gamma(\rho - 1)]} \qquad \text{...(13.10)}$$

Work done is given by,

$$W = p_3(v_4 - v_3) + \frac{p_4 v_4 - p_5 v_5}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_3 v_3(\rho - 1) + \frac{(p_4 \rho v_3 - p_5 r v_3) - (p_2 v_3 - p_1 r v_3)}{\gamma - 1}$$

$$= \frac{p_3 v_3 (\rho - 1)(\gamma - 1) + p_4 v_3 \left(\rho - \dfrac{p_5}{p_4} r\right) - p_2 v_3 \left(1 - \dfrac{p_1}{p_2} r\right)}{\gamma - 1}$$

Also
$$\frac{p_5}{p_4} = \left(\frac{v_4}{v_5}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} \quad \text{and} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma}$$

also,
$$p_3 = p_4, \; v_2 = v_3, \; v_5 = v_1$$

∴
$$W = \frac{v_3[p_3 (\rho - 1)(\gamma - 1) + p_3 (\rho - \rho^{\gamma} r^{1 - \gamma}) - p_2 (1 - r^{1 - \gamma})]}{(\gamma - 1)}$$

$$= \frac{p_2 v_2[\beta(\rho - 1)(\gamma - 1) + \beta(\rho - \rho^{\gamma} r^{1 - \gamma}) - (1 - r^{1 - \gamma})]}{(\gamma - 1)}$$

$$= \frac{p_1 (r)^{\gamma} v_1 / r[\beta\gamma(\rho - 1) + (\beta - 1) - r^{1 - \gamma} (\beta\rho^{\gamma} - 1)]}{\gamma - 1}$$

$$= \frac{p_1 v_1 r^{\gamma - 1}[\beta\gamma(\rho - 1) + (\beta - 1) - r^{\gamma - 1}(\beta\rho^{\gamma} - 1)]}{\gamma - 1} \qquad \text{...(13.11)}$$

Mean effective pressure ($\mathbf{p_m}$) is given by,

$$p_m = \frac{W}{v_1 - v_2} = \frac{W}{v_1\left(\dfrac{r - 1}{r}\right)} = \frac{p_1 v_1 [r^{1 - \gamma} \beta\gamma (\rho - 1) + (\beta - 1) - r^{1 - \gamma} (\beta\rho^{\gamma} - 1)]}{(\gamma - 1) v_1 \left(\dfrac{r - 1}{r}\right)}$$

$$\mathbf{p_m} = \frac{p_1 (r)^{\gamma}[\beta (\rho - 1) + (\beta - 1) - r^{1 - \gamma} (\beta\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)} \qquad \text{...(13.12)}$$

Example 13.23. *The swept volume of a diesel engine working on dual cycle is 0.0053 m*^{3} *and clearance volume is 0.00035 m*^{3}*. The maximum pressure is 65 bar. Fuel injection ends at 5 per cent of the stroke. The temperature and pressure at the start of the compression are 80°C and 0.9 bar. Determine the air standard efficiency of the cycle. Take* γ *for air = 1.4.*

Solution. Refer Fig. 13.20.

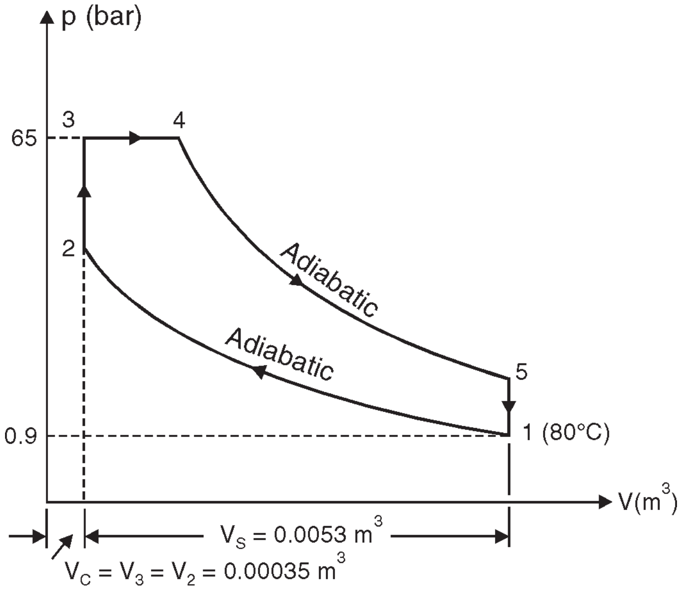


Fig. 13.20

Swept volume, $V_s = 0.0053\ \text{m}^3$

Clearance volume, $V_c = V_3 = V_2 = 0.00035\ \text{m}^3$

Maximum pressure, $p_3 = p_4 = 65$ bar

Initial temperature, $T_1 = 80 + 273 = 353$ K

Initial pressure, $p_1 = 0.9$ bar

η_{dual} **= ?**

The efficiency of a dual combustion cycle is given by

$$\eta_{\text{dual}} = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\beta \,.\, \rho^{\gamma} - 1}{(\beta - 1) + \beta\gamma(\rho - 1)}\right] \quad ...(i)$$

Compression ratio, $r = \dfrac{V_1}{V_2} = \dfrac{V_s + V_c}{V_c} = \dfrac{0.0053 + 0.00035}{0.00035} = 16.14$

$[\because\ V_2 = V_c = \text{Clearance volume}]$

Cut-off ratio, $\rho = \dfrac{V_4}{V_3} = \dfrac{\frac{5}{100}V_s + V_3}{V_3} = \dfrac{0.05V_s + V_c}{V_c} \quad (\because\ V_2 = V_3 = V_c)$

$$= \frac{0.05 \times 0.0053 + 0.00035}{0.00035} = 1.757 \text{ say } 1.76$$

Also during the *compression operation 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or
$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (16.14)^{1.4} = 49.14$$

or
$$p_2 = p_1 \times 49.14 = 0.9 \times 49.14 = 44.22 \text{ bar}$$

Pressure or explosion ratio, $\beta = \frac{p_3}{p_2} = \frac{65}{44.22} = 1.47$

Putting the value of r, ρ and β in eqn. (i), we get

$$\eta_{\textbf{dual}} = 1 - \frac{1}{(16.14)^{1.4-1}}\left[\frac{1.47 \times (1.76)^{1.4} - 1}{(1.47 - 1) + 1.47 \times 1.4\,(1.76 - 1)}\right]$$

$$= 7 - 0.328\left[\frac{3.243 - 1}{0.47 + 1.564}\right] = \mathbf{0.6383 \text{ or } 63.83\%.} \quad \textbf{(Ans.)}$$

Example 13.24. *An oil engine working on the dual combustion cycle has a compression ratio 14 and the explosion ratio obtained from an indicator card is 1.4. If the cut-off occurs at 6 per cent of stroke, find the ideal efficiency. Take γ for air = 1.4.*

Solution. Refer Fig. 13.19.

Compression ratio, $r = 14$

Explosion ratio, $\beta = 1.4$

If ρ is the cut-off ratio, then $\frac{\rho - 1}{r - 1} = \frac{6}{100}$ or $\frac{\rho - 1}{14 - 1} = 0.06$

$\therefore \quad \rho = 1.78$

Ideal efficiency is given by

$$\eta_{\text{ideal or dual}} = 1 - \frac{1}{(r)^{\gamma - 1}}\left[\frac{(\beta\rho^{\gamma} - 1)}{(\beta - 1) + \beta\gamma(\rho - 1)}\right]$$

$$= 1 - \frac{1}{(14)^{1.4-1}}\left[\frac{1.4 \times (1.78)^{1.4} - 1}{(1.4 - 1) + 1.4 \times 1.4(1.78 - 1)}\right]$$

$$= 1 - 0.348\left[\frac{3.138 - 1}{0.4 + 1.528}\right] = \mathbf{0.614 \text{ or } 61.4\%.} \quad \textbf{(Ans.)}$$

Example 13.25. *The compression ratio for a single-cylinder engine operating on dual cycle is 9. The maximum pressure in the cylinder is limited to 60 bar. The pressure and temperature of the air at the beginning of the cycle are 1 bar and 30°C. Heat is added during constant pressure process upto 4 per cent of the stroke. Assuming the cylinder diameter and stroke length as 250 mm and 300 mm respectively, determine :*

(*i*) *The air standard efficiency of the cycle.*

(*ii*) *The power developed if the number of working cycles are 3 per second.*

Take for air c_v = 0.71 kJ/kg K and : c_p = 1.0. kJ/kg K

Solution. Refer Fig. 13.21.

Cylinder diameter, $D = 250 \text{ mm} = 0.25 \text{ m}$

Compression ratio, $r = 9$

Stroke length, $L = 300 \text{ mm} = 0.3 \text{ m}$

Initial pressure, $p_1 = 1$ bar

Initial temperature, $T_1 = 30 + 273 = 303$ K

Maximum pressure, $p_3 = p_4 = 60$ bar

Cut-off = 4% of stroke volume

Number of working cycles/sec. = 3.

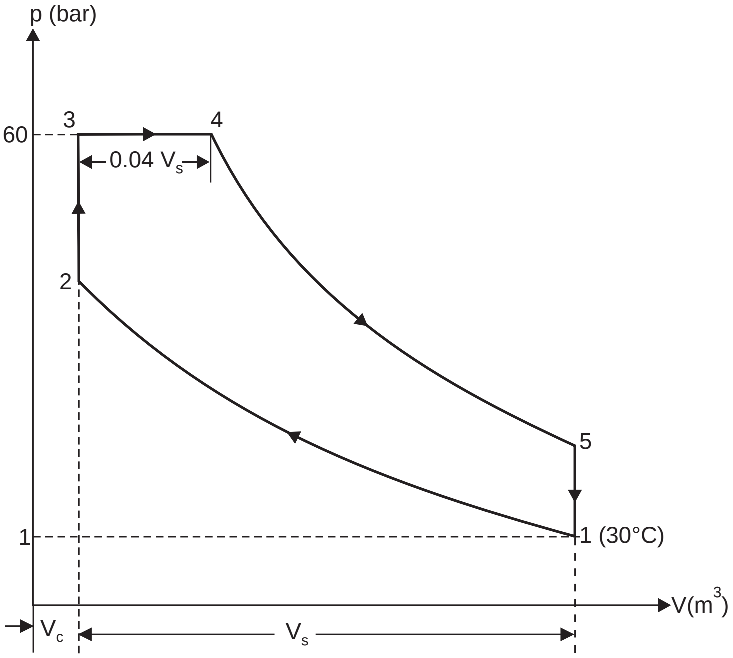


Fig. 13.21

(*i*) **Air standard efficiency :**

Now, swept volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.25^2 \times 0.3$
$= 0.0147\ \text{m}^3$

Also, compression ratio, $r = \dfrac{V_s + V_c}{V_c}$

i.e., $$9 = \frac{0.0147 + V_c}{V_c}$$

$\therefore$ $$V_c = \frac{0.0147}{8} = 0.0018\ \text{m}^3$$

$\therefore$ $$V_1 = V_s + V_c = 0.0147 + 0.0018 = 0.0165\ \text{m}^3$$

For the *adiabatic (or isentropic) process 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

$$p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (r)^{\gamma} = 1 \times (9)^{1.4} = 21.67 \text{ bar}$$

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (9)^{1.4 - 1} = (9)^{0.4} = 2.408$$

∴ $T_2 = T_1 \times 2.408 = 303 \times 2.408 = 729.6$ K

For the *constant volume process 2-3,*

$$\frac{T_3}{p_3} = \frac{T_2}{p_2}$$

∴ $T_3 = T_2 \cdot \frac{p_3}{p_2} = 729.6 \times \frac{60}{21.67} = 2020$ K

Also, $\frac{\rho - 1}{r - 1} = \frac{4}{100}$ or 0.04

∴ $\frac{\rho - 1}{9 - 1} = 0.04$ or $\rho = 1.32$

For the *constant pressure process 3-4,*

$$\frac{V_4}{T_4} = \frac{V_3}{T_3} \quad \text{or} \quad \frac{T_4}{T_3} = \frac{V_4}{V_3} = \rho$$

∴ $T_4 = T_3 \times \rho = 2020 \times 1.32 = 2666.4$ K

Also expansion ratio, $\frac{V_5}{V_4} = \frac{V_5}{V_2} \times \frac{V_2}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{r}{\rho}$ $[\because V_5 = V_1 \text{ and } V_2 = V_3]$

For *adiabatic process 4-5,*

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma - 1} = \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

∴ $T_5 = T_4 \times \left(\frac{\rho}{r}\right)^{\gamma - 1} = 2666.4 \times \left(\frac{1.32}{9}\right)^{1.4 - 1} = 1237$ K

Also $p_4V_4^{\gamma} = p_5V_5^{\gamma}$

$$p_5 = p_4 \cdot \left(\frac{V_4}{V_5}\right)^{\gamma} = 60 \times \left(\frac{r}{\rho}\right)^{\gamma} = 60 \times \left(\frac{1.32}{9}\right)^{1.4} = 4.08 \text{ bar}$$

Heat supplied, $Q_s = c_v(T_3 - T_2) + c_p(T_4 - T_3)$

$= 0.71\ (2020 - 729.6) + 1.0\ (2666.4 - 2020) = 1562.58$ kJ/kg

Heat rejected, $Q_r = c_v\ (T_5 - T_1)$

$= 0.71\ (1237 - 303) = 663.14$ kJ/kg

$$\eta_{\text{air-standard}} = \frac{Q_s - Q_r}{Q_s} = \frac{1562.85 - 663.14}{1562.58} = \mathbf{0.5756 \text{ or } 57.56\%.} \quad \textbf{(Ans.)}$$

(*ii*) **Power developed by the engine, P :**

Mass of air in the cycle is given by

$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.0165}{287 \times 303} = 0.0189 \text{ kg}$$

∴ Work done per cycle $= m(Q_s - Q_r)$

$= 0.0189\ (1562.58 - 663.14) = 16.999$ kJ

Power developed = Work done per cycle × no. of cycles per second

= 16.999 × 3 = **50.99 say 51 kW. (Ans.)**

☞**Example 13.26.** *In an engine working on Dual cycle, the temperature and pressure at the beginning of the cycle are 90°C and 1 bar respectively. The compression ratio is 9. The maximum pressure is limited to 68 bar and total heat supplied per kg of air is 1750 kJ. Determine :*

(*i*) *Pressure and temperatures at all salient points*

(*ii*) *Air standard efficiency*

(*iii*) *Mean effective pressure.*

Solution. Refer Fig. 13.22.

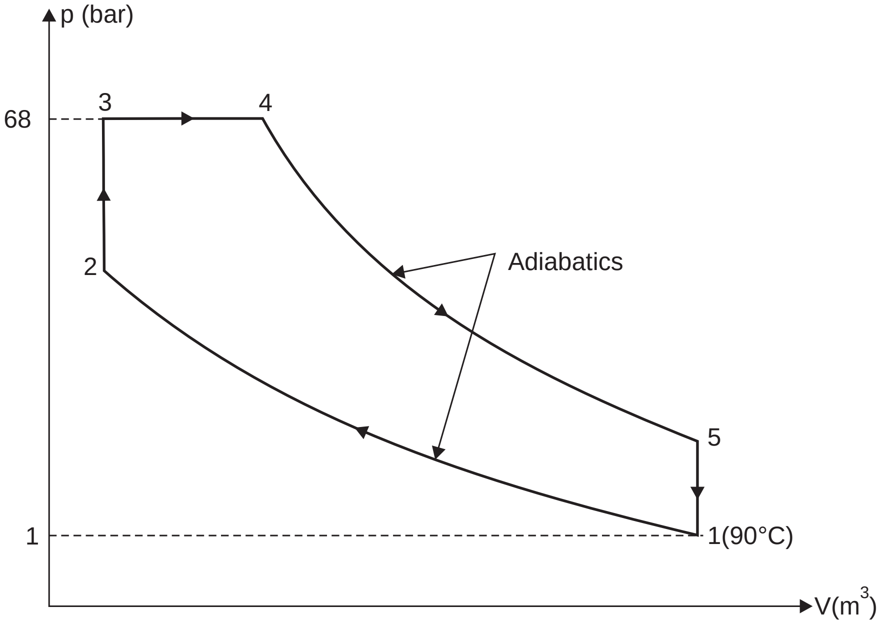


Fig. 13.22

Initial pressure, $p_1 = 1$ bar

Initial temperature, $T_1 = 90 + 273 = 363$ K

Compression ratio, $r = 9$

Maximum pressure, $p_3 = p_4 = 68$ bar

Total heat supplied = 1750 kJ/kg

(*i*) **Pressures and temperatures at salient points :**

For the *isentropic process 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

$$\boldsymbol{p_2} = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (r)^{\gamma} = 1 \times (9)^{1.4} = \textbf{21.67 bar.} \quad \textbf{(Ans.)}$$

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (9)^{1.4 - 1} = 2.408$$

∴ $$T_2 = T_1 \times 2.408 = 363 \times 2.408 = \textbf{874.1 K.} \quad \textbf{(Ans.)}$$

$$\boldsymbol{p_3 = p_4 = } \textbf{68 bar.} \quad \textbf{(Ans.)}$$

For the *constant volume process* 2-3,

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

$$\therefore \quad \boldsymbol{T_3} = T_2 \times \frac{p_3}{p_2} = 874.1 \times \frac{68}{21.67} = \mathbf{2742.9\ K.} \quad \textbf{(Ans.)}$$

Heat added at constant volume

$$= c_v\,(T_3 - T_2) = 0.71\,(2742.9 - 874.1) = 1326.8 \text{ kJ/kg}$$

∴ Heat added at constant pressure

= Total heat added – heat added at constant volume

= 1750 – 1326.8 = 423.2 kJ/kg

$$\therefore \quad c_p(T_4 - T_3) = 423.2$$

or $$1.0(T_4 - 2742.9) = 423.2$$

$$\therefore \quad \boldsymbol{T_4} = \mathbf{3166\ K.} \quad \textbf{(Ans.)}$$

For *constant pressure process* 3-4,

$$\rho = \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{3166}{2742.9} = 1.15$$

For *adiabatic (or isentropic) process* 4-5,

$$\frac{V_5}{V_4} = \frac{V_5}{V_2} \times \frac{V_2}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{r}{\rho} \qquad \left(\because \ \rho = \frac{V_4}{V_3} \right)$$

Also $$p_4 V_4^{\gamma} = p_5 V_5^{\gamma}$$

$$\therefore \quad p_5 = p_4 \times \left(\frac{V_4}{V_5}\right)^{\gamma} = 68 \times \left(\frac{\rho}{r}\right)^{\gamma} = 68 \times \left(\frac{1.15}{9}\right)^{1.4} = \mathbf{3.81\ bar.} \quad \textbf{(Ans.)}$$

Again, $$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma - 1} = \left(\frac{\rho}{r}\right)^{\gamma - 1} = \left(\frac{1.15}{9}\right)^{1.4 - 1} = 0.439$$

$$\therefore \quad T_5 = T_4 \times 0.439 = 3166 \times 0.439 = \mathbf{1389.8\ K.} \quad \textbf{(Ans.)}$$

(*ii*) **Air standard efficiency :**

Heat rejected during constant volume process 5-1,

$$Q_r = c_v(T_5 - T_1) = 0.71(1389.8 - 363) = 729 \text{ kJ/kg}$$

$$\therefore \quad \eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s}$$

$$= \frac{1750 - 729}{1750} = \mathbf{0.5834\ or\ 58.34\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Mean effective pressure, $\mathbf{p_m}$:**

Mean effective pressure is given by

$$p_m = \frac{\text{Work done per cycle}}{\text{Stroke volume}}$$

or $$p_m = \frac{1}{V_s}\left[p_3\,(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1}\right]$$

$$V_1 = V_5 = r\,V_c,\ V_2 = V_3 = V_c,\ V_4 = \rho V_c, \qquad \left[\begin{array}{l} \because \quad r = \dfrac{V_s + V_c}{V_c} = 1 + \dfrac{V_s}{V_c} \\ \therefore \quad V_s = (r - 1)\,V_c \end{array}\right]$$

$$V_s = (r - 1)\,V_c$$

$$\therefore \qquad p_m = \frac{1}{(r-1)V_c}\left[p_3\,(\rho\, V_c - V_c) + \frac{p_4\rho V_c - p_5 \times rV_c}{\gamma - 1} - \frac{p_2V_c - p_1rV_c}{\gamma - 1}\right]$$

$$r = 9,\ \rho = 1.15,\ \gamma = 1.4$$

$$p_1 = 1 \text{ bar},\ p_2 = 21.67 \text{ bar},\ p_3 = p_4 = 68 \text{ bar},\ p_5 = 3.81 \text{ bar}$$

Substituting the above values in the above equation, we get

$$p_m = \frac{1}{(9-1)}\left[68\,(1.15-1) + \frac{68 \times 1.15 - 3.81 \times 9}{1.4-1} - \frac{21.67 - 9}{1.4-1}\right]$$

$$= \tfrac{1}{8}\ (10.2 + 109.77 - 31.67) = 11.04 \text{ bar}$$

Hence, *mean effective pressure* = **11.04 bar. (Ans.)**

Example 13.27. *An I.C. engine operating on the dual cycle (limited pressure cycle) the temperature of the working fluid (air) at the beginning of compression is 27°C. The ratio of the maximum and minimum pressures of the cycle is 70 and compression ratio is 15. The amounts of heat added at constant volume and at constant pressure are equal. Compute the air standard thermal efficiency of the cycle. State three main reasons why the actual thermal efficiency is different from the theoretical value.* **(U.P.S.C. 1997)**

Take γ *for air = 1.4.*

Solution. Refer Fig. 13.23. *Given :* $T_1 = 27 + 273 = 300$ K ; $\frac{p_3}{p_1} = 70$, $\frac{v_1}{v_2} = \frac{v_1}{v_3} = 15$

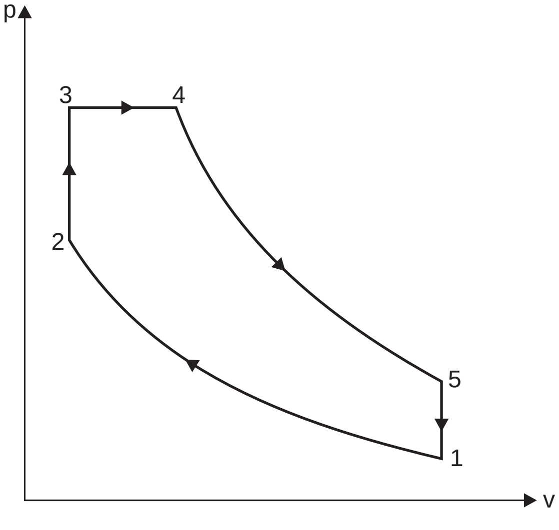


Fig. 13.23. Dual cycle.

Air standard efficiency, $\eta_{\text{air-standard}}$ **:**

Consider 1 kg of air.

Adiabatic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (15)^{1.4-1} = 2.954$$

$\therefore \quad T_2 = 300 \times 2.954 = 886.2 \text{ K}$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (15)^{1.4} \Rightarrow p_2 = 44.3\, p_1$$

Constant pressure process 2-3 :

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or $$T_3 = T_2 \times \frac{p_3}{p_2} = 886.2 \times \frac{70 p_1}{44.3 p_1} = 1400 \text{ K}$$

Also, Heat added at constant volume = Heat added at constant pressure ...(Given)

or $c_v\,(T_3 - T_2) = c_p\,(T_4 - T_3)$

or $T_3 - T_2 = \gamma\,(T_4 - T_3)$

or $$T_4 = T_3 + \frac{T_3 - T_2}{\gamma} = 1400 + \frac{1400 - 886.2}{1.4} = 1767 \text{ K.}$$

Constant volume process 3-4 :

$$\frac{v_3}{T_3} = \frac{v_4}{T_4} \Rightarrow \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1767}{1400} = 1.26$$

Also, $$\frac{v_4}{v_3} = \frac{v_4}{(v_1/15)} = 1.26 \quad \text{or} \quad v_4 = 0.084\, v_1$$

Also, $v_5 = v_1$

Adiabatic expansion process 4-5 :

$$\frac{T_4}{T_5} = \left(\frac{v_5}{v_4}\right)^{\gamma - 1} = \left(\frac{v_1}{0.084 v_1}\right)^{1.4 - 1} = 2.69$$

$$\therefore \quad T_5 = \frac{T_4}{2.69} = \frac{1767}{2.69} = 656.9 \text{ K}$$

$$\therefore \quad \eta_{\textbf{air-standard}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= 1 - \frac{\text{Heat rejected}}{\text{Heat supplied}}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

$$= 1 - \frac{(656.9 - 300)}{(1400 - 886.2) + 1.4(1767 - 1400)} = \mathbf{0.653 \text{ or } 65.3\%.} \quad \textbf{(Ans.)}$$

Reasons for actual thermal efficiency being different from the theoretical value :

1. In theoretical cycle working substance is taken *air* whereas in actual cycle *air with fuel acts as working substance.*

2. The fuel combustion phenomenon and associated problems like dissociation of gases, dilution of charge during suction stroke, etc. have *not* been taken into account.

3. Effect of variable specific heat, heat loss through cylinder walls, inlet and exhaust velocities of air/gas etc. have *not* been taken into account.

☞**Example 13.28.** *A Diesel engine working on a dual combustion cycle has a stroke volume of 0.0085* m^3 *and a compression ratio 15 : 1. The fuel has a calorific value of 43890 kJ/kg. At the end of suction, the air is at 1 bar and 100°C. The maximum pressure in the cycle is 65 bar and air fuel ratio is 21 : 1. Find for ideal cycle the thermal efficiency. Assume* $c_p = 1.0$ *and* $c_v = 0.71$.

Solution. Refer Fig. 13.24.

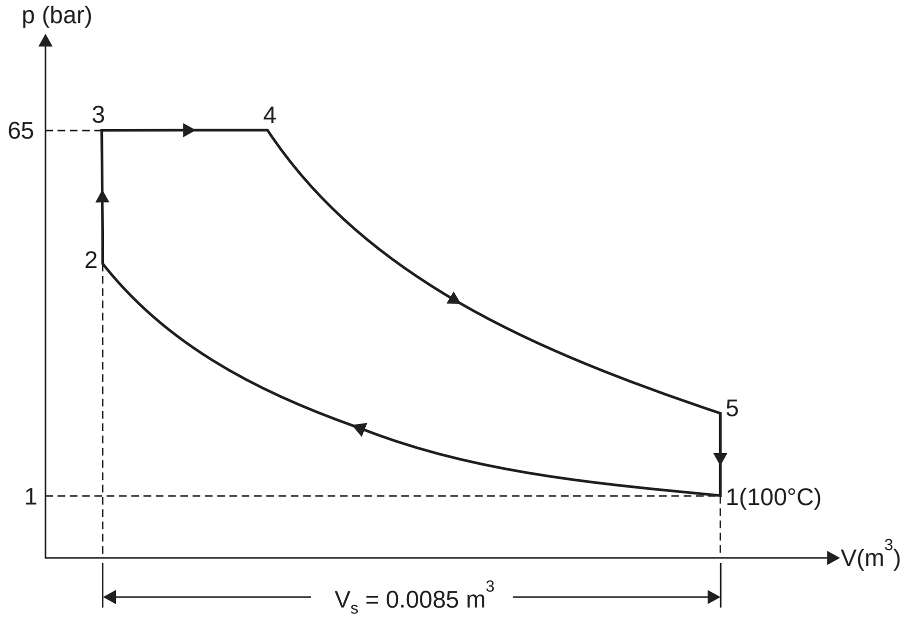


Fig. 13.24

Initial temperature, $T_1 = 100 + 273 = 373$ K

Initial pressure, $p_1 = 1$ bar

Maximum pressure in the cycle, $p_3 = p_4 = 65$ bar

Stroke volume, $V_s = 0.0085$ m^3

Air-fuel ratio $= 21 : 1$

Compression ratio, $r = 15 : 1$

Calorific value of fuel, $C = 43890$ kJ/kg

$c_p = 1.0, \; c_v = 0.71$

Thermal efficiency :

$$V_s = V_1 - V_2 = 0.0085$$

and as $$r = \frac{V_1}{V_2} = 15, \text{ then } V_1 = 15V_2$$

$\therefore$ $$15V_2 - V_2 = 0.0085$$

or $$14V_2 = 0.0085$$

or $$V_2 = V_3 = V_c = \frac{0.0085}{14} = 0.0006 \text{ m}^3$$

or $$V_1 = 15V_2 = 15 \times 0.0006 = 0.009 \text{ m}^3$$

For *adiabatic compression process 1-2,*

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or $$p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (15)^{1.41} \qquad \left[\gamma = \frac{c_p}{c_v} = \frac{1.0}{0.71} = 1.41\right]$$

$$= 45.5 \text{ bar}$$

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (15)^{1.41 - 1} = 3.04$$

∴ $$T_2 = T_1 \times 3.04 = 373 \times 3.04 = 1134 \text{ K or } 861°\text{C}$$

For *constant volume process 2-3,*

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or $$T_3 = T_2 \times \frac{p_3}{p_2} = 1134 \times \frac{65}{45.5} = 1620 \text{ K or } 1347°\text{C}$$

According to characteristic equation of gas,

$$p_1V_1 = mRT_1$$

∴ $$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.009}{287 \times 373} = 0.0084 \text{ kg (air)}$$

Heat added during constant volume process 2-3,

$$= m \times c_v\,(T_3 - T_2)$$

$$= 0.0084 \times 0.71\,(1620 - 1134)$$

$$= 2.898 \text{ kJ}$$

Amount of fuel added during the constant volume process 2-3,

$$= \frac{2.898}{43890} = 0.000066 \text{ kg}$$

Also as air-fuel ratio is 21 : 1.

∴ Total amount of fuel added $$= \frac{0.0084}{21} = 0.0004 \text{ kg}$$

Quantity of fuel added during the process 3-4,

$$= 0.0004 - 0.000066 = 0.000334 \text{ kg}$$

∴ *Heat added during the constant pressure operation 3-4*

$$= 0.000334 \times 43890 = 14.66 \text{ kJ}$$

But $$(0.0084 + 0.0004)\, c_p\,(T_4 - T_3) = 14.66$$

or $$0.0088 \times 1.0\,(T_4 - 1620) = 14.66$$

∴ $$T_4 = \frac{14.66}{0.0088} + 1620 = 3286 \text{ K or } 3013°\text{C}$$

Again for *process 3-4*,

$$\frac{V_3}{T_3} = \frac{V_4}{T_4} \quad \text{or} \quad V_4 = \frac{V_3T_4}{T_3} = \frac{0.0006 \times 3286}{1620} = 0.001217 \text{ m}^3$$

For *adiabatic expansion operation 4-5*,

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma - 1} = \left(\frac{0.009}{0.001217}\right)^{1.41 - 1} = 2.27$$

or $$T_5 = \frac{T_4}{2.27} = \frac{3286}{2.27} = 1447.5 \text{ K or } 1174.5°\text{C}$$

Heat rejected during constant volume process 5-1,

$$= m\, c_v\, (T_5 - T_1)$$
$$= (0.00854 + 0.0004) \times 0.71\, (1447.5 - 373) = 6.713 \text{ kJ}$$

Work done $= \text{Heat supplied} - \text{Heat rejected}$

$$= (2.898 + 14.66) - 6.713 = 10.845 \text{ kJ}$$

∴ *Thermal efficiency,*

$$\eta_{th.} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{10.845}{(2.898 + 14.66)} = \mathbf{0.6176 \text{ or } 61.76\%.} \quad \textbf{(Ans.)}$$

☞**Example 13.29.** *The compression ratio and expansion ratio of an oil engine working on the dual cycle are 9 and 5 respectively. The initial pressure and temperature of the air are 1 bar and 30°C. The heat liberated at constant pressure is twice the heat liberated at constant volume. The expansion and compression follow the law* $pV^{1.25}$ *= constant. Determine :*

(*i*) *Pressures and temperatures at all salient points.*

(*ii*) *Mean effective pressure of the cycle.*

(*iii*) *Efficiency of the cycle.*

(*iv*) *Power of the engine if working cycles per second are 8.*

Assume : Cylinder bore = 250 mm and stroke length = 400 mm.

Solution. Refer Fig. 13.25.

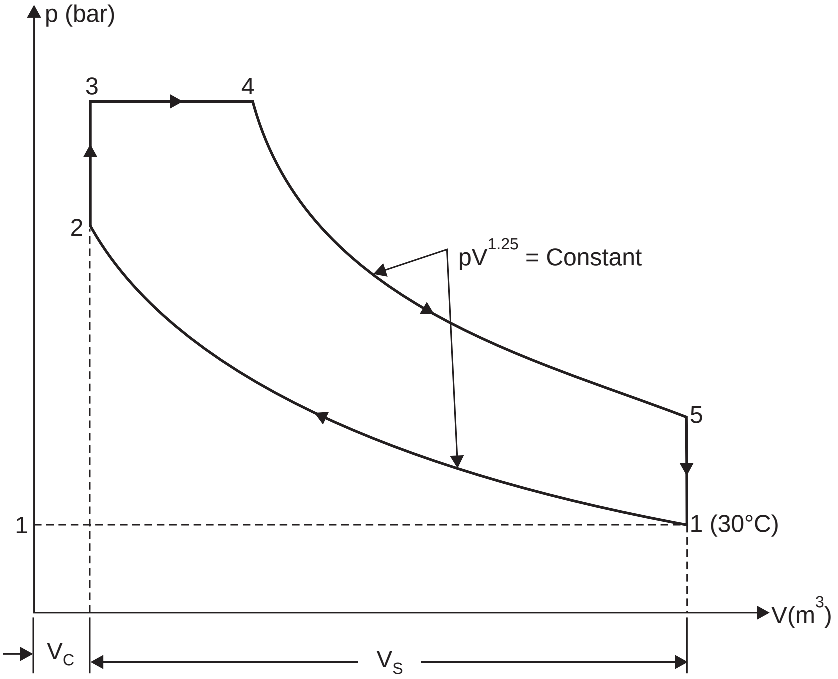


Fig. 13.25

Initial temperature, $T_1 = 30 + 273 = 303$ K

Initial pressure, $p_1 = 1$ bar

Compression and expansion law,

$$pV^{1.25} = \text{Constant}$$

Compression ratio, $r_c = 9$

Expansion ratio, $r_e = 5$

Number of cycles/sec. $= 8$

Cylinder diameter, $D = 250 \text{ mm} = 0.25 \text{ m}$

Stroke length, $L = 400 \text{ mm} = 0.4 \text{ m}$

Heat liberated at constant pressure

= 2 × heat liberated at constant volume

(*i*) **Pressure and temperatures at all salient points :**

For *compression process 1-2,*

$$p_1 V_1^n = p_2 V_2^n$$

$$\therefore \quad \boldsymbol{p_2} = p_1 \times \left(\frac{V_1}{V_2}\right)^n = 1 \times (9)^{1.25} = \textbf{15.59 bar.} \quad \textbf{(Ans.)}$$

A1so,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = (9)^{1.25-1} = 1.732$$

$$\therefore \quad \boldsymbol{T_2} = T_1 \times 1.732 = 303 \times 1.732$$

$$= \textbf{524.8 K or 251.8°C.} \quad \textbf{(Ans.)}$$

Also, $c_p(T_4 - T_3) = 2 \times c_v(T_3 - T_2)$ ...... (given) ...(*i*)

For *constant pressure process 3-4,*

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} = \rho = \frac{\text{Compression ratio } (r_c)}{\text{Expansion ratio } (r_e)}$$

$$= \frac{9}{5} = 1.8$$

$$T_4 = 1.8 T_3$$

$$\left[\begin{aligned} \frac{V_5}{V_4} (i.e., r_e) &= \frac{V_5}{V_3} \times \frac{V_3}{V_4} \\ &= \frac{V_1}{V_3} \times \frac{1}{\rho} \\ &= \frac{V_1}{V_2} \times \frac{1}{\rho} = \frac{r_c}{\rho} \\ \therefore \ \rho &= \frac{r_c}{\dfrac{V_5}{V_4}} = \frac{r_c}{r_e} \end{aligned}\right]$$

Substituting the values of T_2 and T_4 in the eqn. (*i*), we get

$$1.0(1.8T_3 - T_3) = 2 \times 0.71(T_3 - 524.8)$$

$$0.8T_3 = 1.42(T_3 - 524.8)$$

$$0.8T_3 = 1.42T_3 - 745.2$$

$$\therefore \quad 0.62T_3 = 745.2$$

$$T_3 = \textbf{1201.9 K or 928.9°C.} \quad \textbf{(Ans.)}$$

Also,

$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \quad \text{...... for } \textit{process 2-3}$$

$$\therefore \quad \boldsymbol{p_3} = p_2 \times \frac{T_3}{T_2} = 15.59 \times \frac{1201.9}{524.8} = \textbf{35.7 bar.} \quad \textbf{(Ans.)}$$

$$\boldsymbol{p_4} = \boldsymbol{p_3} = \textbf{35.7 bar.} \quad \textbf{(Ans.)}$$

$$\boldsymbol{T_4} = 1.8T_3 = 1.8 \times 1201.9 = \textbf{2163.4 K or 1890.4°C.} \quad \textbf{(Ans.)}$$

For expansion process 4-5,

$$p_4 V_4^n = p_5 V_5^n$$

$$p_5 = p_4 \times \left(\frac{V_4}{V_5}\right)^n = p_4 \times \frac{1}{(r_e)^n} = \frac{35.7}{(5)^{1.25}} = \mathbf{4.77\ bar.}\quad \textbf{(Ans.)}$$

Also $$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{n-1} = \frac{1}{(r_e)^{n-1}} = \frac{1}{(5)^{1.25-1}} = 0.668$$

$\therefore$ $$T_5 = T_4 \times 0.668 = 2163.4 \times 0.668 = \mathbf{1445\ K\ or\ 1172°C.}\quad \textbf{(Ans.)}$$

(*ii*) **Mean effective pressure, $\mathbf{p_m}$:**

Mean effective pressure is given by

$$p_m = \frac{1}{V_s}\left[p_3(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{n-1} - \frac{p_2V_2 - p_1V_1}{n-1}\right]$$

$$= \frac{1}{(r_c - 1)}\left[p_3\,(\rho - 1) + \frac{p_4\rho - p_5r_c}{n-1} - \frac{p_2 - p_1r_c}{n-1}\right]$$

Now, $r_c = \rho$, $\rho = 1.8$, $n = 1.25$, $p_1 = 1$ bar, $p_2 = 15.59$ bar, $p_3 = 35.7$ bar, $p_4 = 35.7$ bar, $p_5 = 4.77$ bar

$\therefore$ $$p_m = \frac{1}{(9-1)}\left[35.7\,(1.8 - 1) + \frac{35.7 \times 1.8 - 4.77 \times 9}{1.25 - 1} - \frac{15.59 - 1 \times 9}{1.25 - 1}\right]$$

$$= \frac{1}{8}\ [28.56 + 85.32 - 26.36] = 10.94 \text{ bar}$$

Hence *mean effective pressure* = **10.94 bar. (Ans.)**

(*iii*) **Efficiency of the cycle :**

Work done per cycle is given by $W = p_mV_s$

Here, $V_s = \pi/4D^2L = \pi/4 \times 0.25^2 \times 0.4 = 0.0196 \text{ m}^3$

$\therefore$ $$W = \frac{10.94 \times 10^5 \times 0.0196}{1000} \text{ kJ/cycle} = 21.44 \text{ kJ/cycle}$$

Heat supplied per cycle $= mQ_s$,

where m is the mass of air per cycle which is given by

$$m = \frac{p_1V_1}{RT_1} \quad \text{where } V_1 = V_s + V_c = \frac{r_c}{r_c - 1}\ V_s$$

$$\left[\begin{array}{l} r_c = \dfrac{V_s + V_c}{V_c} = 1 + \dfrac{V_s}{V_c} \quad \text{or} \quad V_c = \dfrac{V_s}{r_c - 1} \\ \therefore\ V_1 = V_s + \dfrac{V_s}{r_c - 1} = V_s\left(1 + \dfrac{1}{r_c - 1}\right) = \dfrac{r_c}{r_c - 1}\,V_s \end{array}\right]$$

$$= \frac{9}{9-1} \times 0.0196 = 0.02205 \text{ m}^3$$

$\therefore$ $$m = \frac{1 \times 10^5 \times 0.02205}{287 \times 303} = 0.02535 \text{ kg/cycle}$$

$\therefore$ Heat supplied per cycle

$$= mQ_s = 0.02535[c_v(T_3 - T_2) + c_p(T_4 - T_3)]$$

$$= 0.02535[0.71(1201.9 - 524.8) + 1.0(2163.4 - 1201.9)]$$

$$= 36.56 \text{ kJ/cycle}$$

$$\textit{Efficiency} = \frac{\text{Work done per cycle}}{\text{Heat supplied per cycle}} = \frac{21.44}{36.56}$$

$$= \mathbf{0.5864\quad or\quad 58.64\%.}\quad \textbf{(Ans.)}$$

(*iv*) **Power of the engine, P :**

Power of the engine, P = Work done per second

= Work done per cycle × no. of cycles/sec.

= 21.44 × 8 = **171.52 kW. (Ans.)**

13.7. COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

Following are the *important variable factors which are used as a basis for comparison of the cycles :*

- Compression ratio.
- Maximum pressure
- Heat supplied
- Heat rejected
- Net work

Some of the above mentioned variables are fixed when the performance of Otto, Diesel and dual combustion cycles is to be compared.

13.7.1. Efficiency Versus Compression Ratio

Fig. 13.26 shows the comparison for the air standard efficiencies of the Otto, Diesel and Dual combustion cycles at various compression ratios and with given cut-off ratio for the Diesel and Dual combustion cycles. It is evident from the Fig. 13.26 that the air standard efficiencies *increase with the increase in the compression ratio. For a given compression ratio Otto cycle is the most efficient while the Diesel cycle is the least efficient.* ($\eta_{otto} > \eta_{dual} > \eta_{diesel}$).

Note. The maximum compression ratio for the petrol engine is limited by detonation. In their respective ratio ranges, the Diesel cycle is more efficient than the Otto cycle.

13.7.2. For the Same Compression Ratio and the Same Heat Input

A comparison of the cycles (Otto, Diesel and Dual) on the *p-v* and *T-s* diagrams for the *same compression ratio and heat supplied* is shown in the Fig. 13.27.

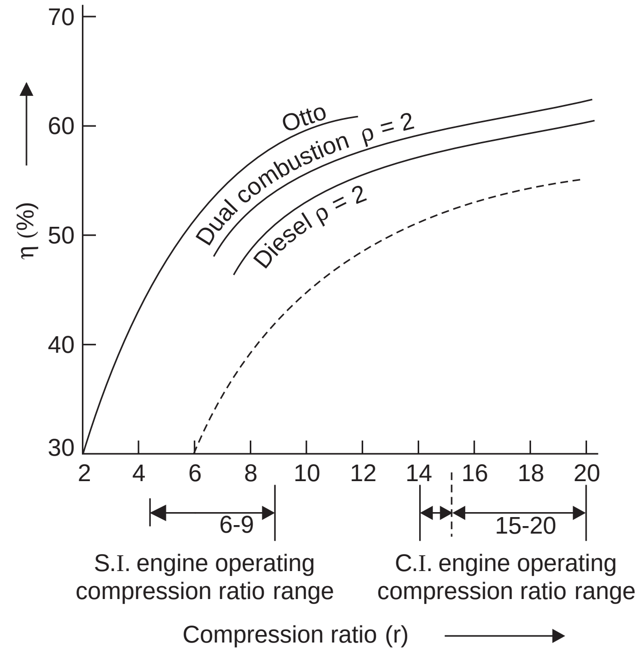


Fig. 13.26. Comparison of efficiency at various compression ratios.

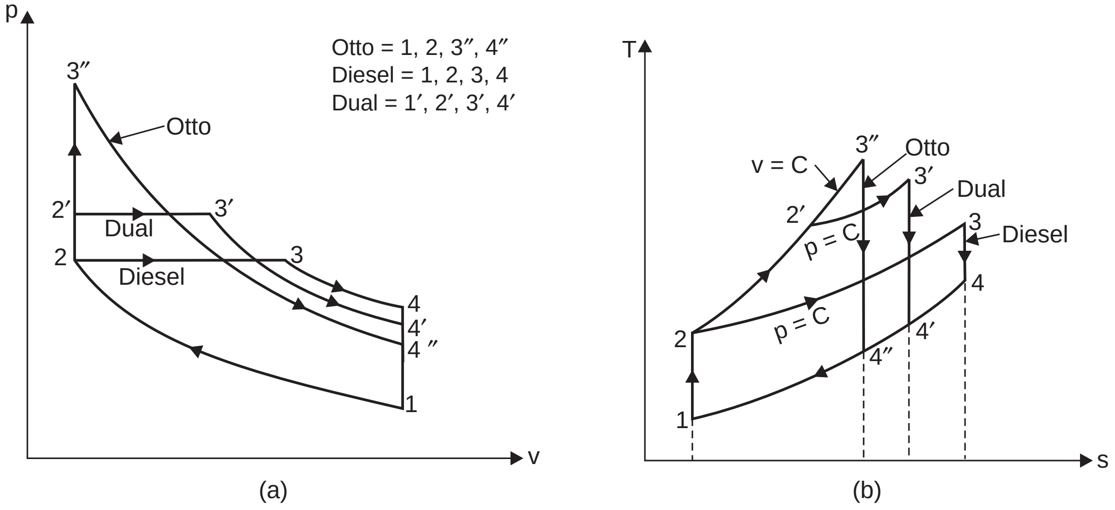


Fig. 13.27. (*a*) *p-v* diagram, (*b*) *T-s* diagram.

We know that,
$$\eta = 1 - \frac{\text{Heat rejected}}{\text{Heat supplied}} \qquad \text{...(13.13)}$$

Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of *heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram*. As is evident from the eqn. (13.13) the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.

i.e.,
$$\eta_{\text{otto}} > \eta_{\text{dual}} > \eta_{\text{diesel}}\ .$$

13.7.3. For Constant Maximum Pressure and Heat Supplied

Fig. 13.28 shows the Otto and Diesel cycles on *p-v* and *T-s* diagrams for constant maximum pressure and heat input respectively.

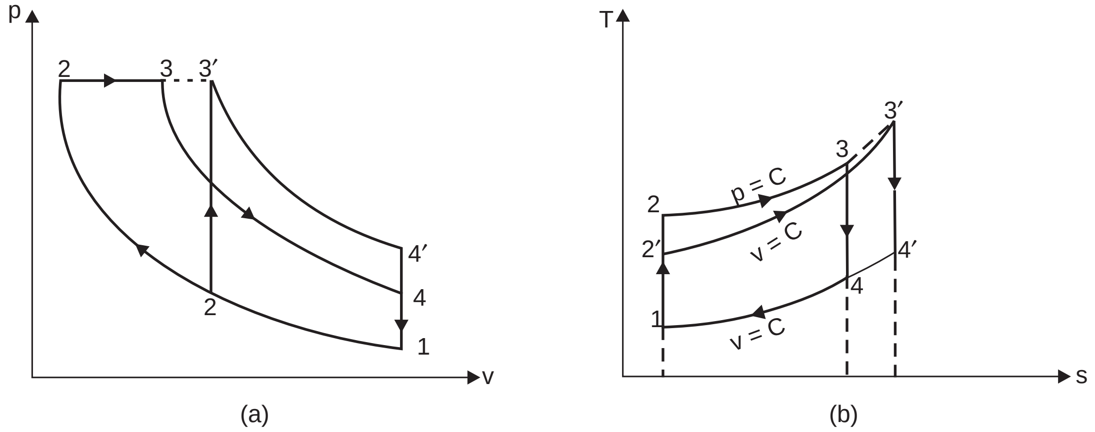


Fig. 13.28. (*a*) *p-v* diagram, (*b*) *T-s* diagram.

— For the maximum pressure the points 3 and 3′ must lie on a constant pressure line.

— On *T-s* diagram the heat rejected from the Diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve 4′ to 1 ; hence the *Diesel cycle is more efficient than the Otto cycle for the condition of maximum pressure and heat supplied.*

Example 13.30. *With the help of p-v and T-s diagram compare the cold air standard otto, diesel and dual combustion cycles for same maximum pressure and maximum temperature.*

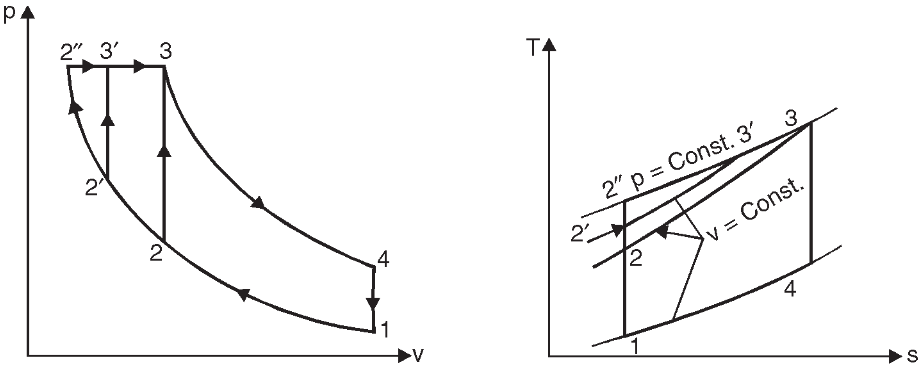
(AMIE Summer, 1998)

Solution. Refer Figs. 13.29 (*a*) and (*b*).

The air-standard Otto, Dual and Diesel cycles are drawn on common *p-v* and *T-s* diagrams for the same maximum pressure and maximum temperature, for the purpose of comparison.

Otto 1-2-3-4-1, Dual 1-2′-3′-3-4-1, Diesel 1-2″-3-4-1 (Fig 13.29 (*a*)).

Slope of constant volume lines on *T-s* diagram is higher than that of constant pressure lines. (Fig. 13.29 (*b*)).



(a) (b)

Fig. 13.29

Here the otto cycle must be limited to a low compression ratio (*r*) to fulfill the condition that point 3 (same maximum pressure and temperature) is to be a common state for all the three cycles.

The construction of cycles on *T-s* diagram proves that for the given conditions the heat rejected is same for all the three cycles (area under process line 4-1). Since, by definition,

$$\eta = 1 - \frac{\text{Heat rejected, } Q_r}{\text{Heat supplied, } Q_s} = 1 - \frac{\text{Const.}}{Q_s}$$

the cycle, with greater heat addition will be more efficient. From the *T-s* diagram,

$$Q_{s(\text{diesel})} = \text{Area under } 2''\text{-}3$$
$$Q_{s(\text{dual})} = \text{Area under } 2'\text{-}3'\text{-}3$$
$$Q_{s(\text{otto})} = \text{Area under } 2\text{-}3.$$

It can be seen that, $Q_{s(\text{diesel})} > Q_{s(\text{dual})} > Q_{s(\text{otto})}$

and thus, $\eta_{\text{diesel}} > \eta_{\text{dual}} > \eta_{\text{otto}}$.

13.8. ATKINSON CYCLE

This cycle consists of two adiabatics, a constant volume and a constant pressure process. *p-V* diagram of this cycle is shown in Fig. 13.30. It consists of the following *four operations* :

(*i*) 1-2—*Heat rejection at constant pressure*

(*ii*) 2-3—*Adiabatic compression*

(*iii*) 3-4—*Addition of heat at constant volume*

(*iv*) 4-1—*Adiabatic expansion.*

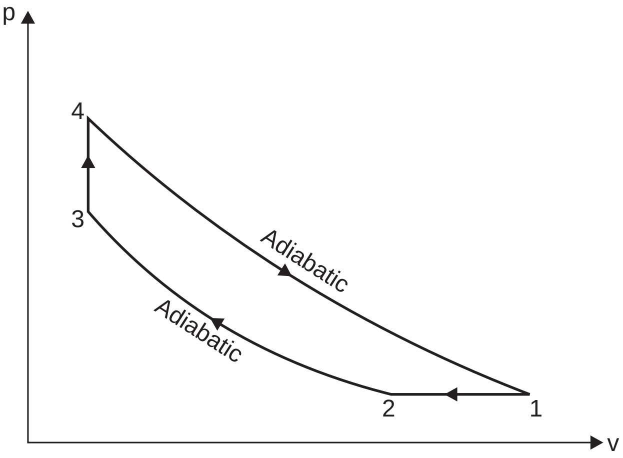


Fig. 13.30

Considering 1 kg of air.

Compression ratio $= \dfrac{v_2}{v_3} = \alpha$

Expansion ratio $= \dfrac{v_1}{v_4} = r$

Heat supplied at constant volume $= c_v(T_4 - T_3)$

Heat rejected $= c_v(T_1 - T_2)$

Work done = Heat supplied – heat rejected

$= c_v(T_4 - T_3) - c_v(T_1 - T_2)$

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v\,(T_4 - T_3) - c_p(T_1 - T_2)}{c_v\,(T_4 - T_3)}$$

$$= 1 - \gamma\,.\,\frac{(T_1 - T_2)}{(T_4 - T_3)} \qquad ...(i)$$

During *adiabatic compression 2-3,*

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} = (\alpha)^{\gamma - 1}$$

or

$$T_3 = T_2\ (\alpha)^{\gamma - 1} \qquad ...(ii)$$

During *constant pressure operation 1-2,*

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}$$

or
$$\frac{T_2}{T_1} = \frac{v_2}{v_1} = \frac{\alpha}{r} \quad ...(iii) \quad \left(\frac{v_2}{v_1} = \frac{v_2}{v_3} \times \frac{v_3}{v_1} = \frac{v_2}{v_3} \times \frac{v_4}{v_1} = \frac{\alpha}{r}\right)$$

During *adiabatic expansion 4-1,*

$$\frac{T_4}{T_1} = \left(\frac{v_1}{v_4}\right)^{\gamma - 1} = (r)^{\gamma - 1}$$

$$T_1 = \frac{T_4}{(r)^{\gamma - 1}} \quad ...(iv)$$

Putting the value of T_1 in eqn. (*iii*), we get

$$T_2 = \frac{T_4}{(r)^{\gamma - 1}} \cdot \frac{\alpha}{r}$$

$$= \frac{\alpha T_4}{r^{\gamma}} \quad ...(v)$$

Substituting the value of T_2 in eqn. (*ii*), we get

$$T_3 = \frac{\alpha T_4}{r^{\gamma}} (\alpha)^{\gamma - 1} = \left(\frac{\alpha}{r}\right)^{\gamma} . T_4$$

Finally putting the values of T_1, T_2 and T_3 in eqn. (*i*), we get

$$\eta = 1 - \gamma \left(\frac{\frac{T_4}{r^{\gamma - 1}} - \frac{\alpha T_4}{(r)^{\gamma}}}{T_4 - \left(\frac{\alpha}{r}\right)^{\gamma} . T_4}\right) = 1 - \gamma \left(\frac{r - \alpha}{r^{\gamma} - \alpha^{\gamma}}\right)$$

Hence, air standard efficiency $= 1 - \gamma . \left(\frac{r - \alpha}{r^{\gamma} - \alpha^{\gamma}}\right)$ ...(13.14)

Example 13.31. *A perfect gas undergoes a cycle which consists of the following processes taken in order :*

(*a*) *Heat rejection at constant pressure.*

(*b*) *Adiabatic compression from 1 bar and 27°C to 4 bar.*

(*c*) *Heat addition at constant volume to a final pressure of 16 bar.*

(*d*) *Adiabatic expansion to 1 bar.*

Calculate : (*i*) *Work done/kg of gas.*

(*ii*) *Efficiency of the cycle.*

Take : $c_p = 0.92$, $c_v = 0.75$.

Solution. Refer Fig. 13.31.

Pressure, $p_2 = p_1 = 1$ bar

Temperature, $T_2 = 27 + 273 = 300$ K

Pressure after adiabatic compression, $p_3 = 4$ bar

Final pressure after heat addition, $p_4 = 16$ bar

For *adiabatic compression 2-3,*

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.22 - 1}{1.22}} = 1.284 \quad \left[\gamma = \frac{c_p}{c_v} = \frac{0.92}{0.75} = 1.22\right]$$

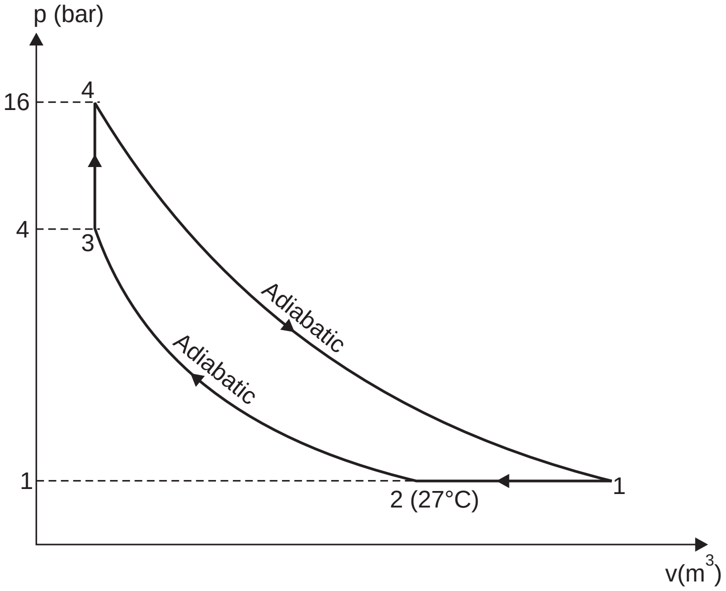


Fig. 13.31

$$\therefore \qquad T_3 = T_2 \times 1.284 = 300 \times 1.284 = 385.2 \text{ K or } 112.2°\text{C}$$

For *constant volume process 3-4*,

$$\frac{p_4}{T_4} = \frac{p_3}{T_3}$$

$$T_4 = \frac{p_4 T_3}{p_3} = \frac{16 \times 385.2}{4} = 1540.8 \text{ K or } 1267.8°\text{C}$$

For *adiabatic expansion process 4-1*,

$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{1}\right)^{\frac{1.22-1}{1.22}} = 1.648$$

or

$$T_1 = \frac{T_4}{1.648} = \frac{1540.8}{1.648} = 934.9 \text{ K or } 661.9°\text{C}.$$

(*i*) **Work done per kg of gas, W** :

Heat supplied $= c_v (T_4 - T_3)$

$= 0.75\ (1540.8 - 385.2) = 866.7$ kJ/kg

Heat rejected $= c_p (T_1 - T_2) = 0.92(934.9 - 300) = 584.1$ kJ/kg

Work done/kg of gas, W = Heat supplied – heat rejected

$= 866.7 - 584.1 = 282.6$ kJ/kg = **282600 N-m/kg.** **(Ans.)**

(*ii*) **Efficiency of the cycle :**

Efficiency, $\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{282.6}{866.7}$ = **0.326 or 32.6%.** **(Ans.)**

13.9. ERICSSON CYCLE

It is so named as it was invented by Ericsson. Fig. 13.32 shows *p-v* diagram of this cycle.

It comprises of the following *operations* :

(*i*) 1-2—*Rejection of heat at constant pressure*

(*ii*) 2-3—*Isothermal compression*

(*iii*) 3-4—*Addition of heat at constant pressure*

(*iv*) 4-1—*Isothermal expansion.*

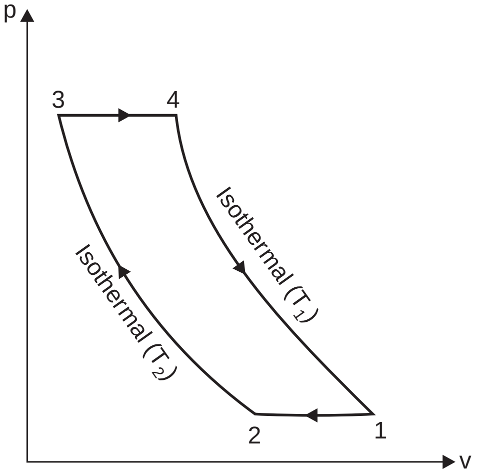


Fig. 13.32

Considering 1 kg of air.

Volume ratio, $r = \frac{v_2}{v_3} = \frac{v_1}{v_4}$

Heat supplied to air from an external source

$= $ Heat supplied during the isothermal expansion 4-1

$= RT_1 \log_e r$

Heated rejected by air to an external source $= RT_2 \,.\, \log_e r$

Work done $= $ Heat supplied – heat rejected

$= RT_1 \,.\, \log_e r - RT_2 \,.\, \log_e r = R \log_e r \, (T_1 - T_2)$

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{R \log_e r \, (T_1 - T_2)}{RT_1 \,.\, \log_e r}$$

$$= \frac{T_1 - T_2}{T_1} \qquad ...(13.15)$$

which is the same as Carnot cycle.

Note. For **'Stirling cycle', Miller cycle and Lenoir cycle** please refer to the Author's popular book on "I.C. Engines".

13.10. GAS TURBINE CYCLE—BRAYTON CYCLE

13.10.1. Ideal Brayton Cycle

Brayton cycle is *a constant pressure cycle for a perfect gas*. It is also called **Joule cycle.** The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gas turbine plant would perform the processes that make up a Brayton cycle. The cycle is shown in the Fig. 13.33 (*a*) and it is represented on *p-v* and *T-s* diagrams as shown in Figs. 13.33 (*b*) and (*c*).

The various operations are as follows :

Operation 1-2. The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

Operation 2-3. Heat flows into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2. Heat received = $mc_p\ (T_3 - T_2)$.

Operation 3-4. The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

Operation 4-1. Heat is rejected from the system as the volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1. Heat rejected = mc_p $(T_4 - T_1)$.

$$\eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat received}}$$

$$= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}}$$

$$= \frac{mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)}{mc_p\,(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

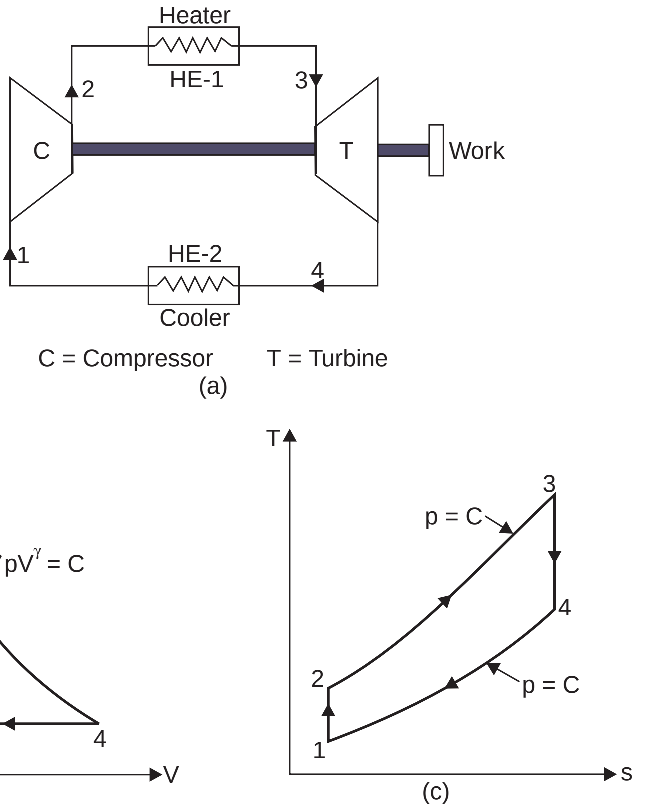


Fig. 13.33. Brayton cycle : (*a*) Basic components of a gas turbine power plant (*b*) *p-V* diagram (*c*) *T-s* diagram.

Now, from isentropic expansion,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1\,(r_p)^{\frac{\gamma-1}{\gamma}}\text{ , where } r_p = \text{pressure ratio.}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad\text{or}\quad T_3 = T_4\,(r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \qquad \eta_{\text{air-standard}} = 1 - \frac{T_4 - T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}} - T_1(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \qquad ...(13.16)$$

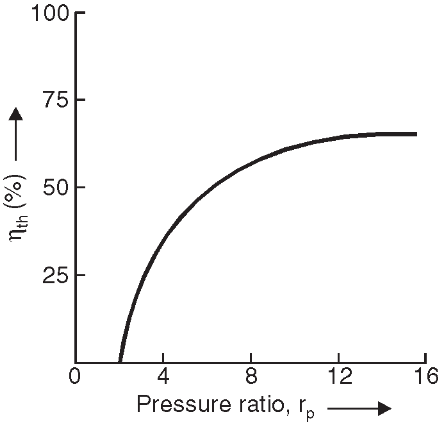


Fig. 13.34. Effect of pressure ratio on the efficiency of Brayton cycle.

The eqn. (13.16) shows that the *efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.*

13.10.2. Pressure Ratio for Maximum Work

Now we shall prove that the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

Work output during the cycle

$$= \text{Heat received/cycle} - \text{heat rejected/cycle}$$

$$= mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)$$

$$= mc_p\,(T_3 - T_4) - mc_p\,(T_2 - T_1)$$

$$= mc_p\,T_3\left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)$$

In case of a given turbine the minimum temperature T_1 and the maximum temperature T_3 are prescribed, T_1 being the temperature of the atmosphere and T_3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure c_p to be constant. Then,

Since, $$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

Using the constant $'z' = \dfrac{\gamma - 1}{\gamma}$,

we have, work output/cycle

$$W = K\left[T_3\left(1 - \frac{1}{{r_p}^z}\right) - T_1\,({r_p}^z - 1)\right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K\left[T_3 \times \frac{z}{r_p(z+1)} - T_1\, z{r_p}^{(z-1)}\right] = 0 \text{ for a maximum}$$

$$\therefore \qquad \frac{zT_3}{{r_p}^{(z+1)}} = T_1 z\,(r_p)^{(z-1)}$$

$$\therefore \qquad {r_p}^{2z} = \frac{T_3}{T_1}$$

$$\therefore \qquad r_p = (T_3/T_1)^{1/2z} \quad \textit{i.e.,} \quad r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}} \qquad \text{...(13.17)}$$

Thus, the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

13.10.3. Work Ratio

Work ratio is defined as the *ratio of net work output to the work done by the turbine.*

$$\therefore \qquad \text{Work ratio} = \frac{W_T - W_C}{W_T}$$

$$\left[\begin{array}{ll}\text{where,} & W_T = \text{Work obtained from this turbine,} \\ \text{and} & W_C = \text{Work supplied to the compressor.}\end{array}\right]$$

$$= \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p\,(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$= 1 - \frac{T_1}{T_3}\left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \dfrac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}\right] = 1 - \frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}} \qquad \text{...(13.18)}$$

13.10.4. Open Cycle Gas Turbine—Actual Brayton Cycle

Refer Fig. 13.35. The fundamental gas turbine unit is one operating on the open cycle in which a rotary compressor and a turbine are mounted on a common shaft. Air is drawn into the compressor and after compression passes to a combustion chamber. Energy is supplied in the combustion chamber by spraying fuel into the air stream, and the resulting hot gases expand through the turbine to the atmosphere. In order to achieve net work output from the unit, the turbine must develop more gross work output than is required to drive the compressor and to overcome mechanical losses in the drive. The products of combustion coming out from the turbine are exhausted to the atmosphere as they cannot be used any more. The working fluids (air and fuel) must be replaced continuously as they are exhausted into the atmosphere.

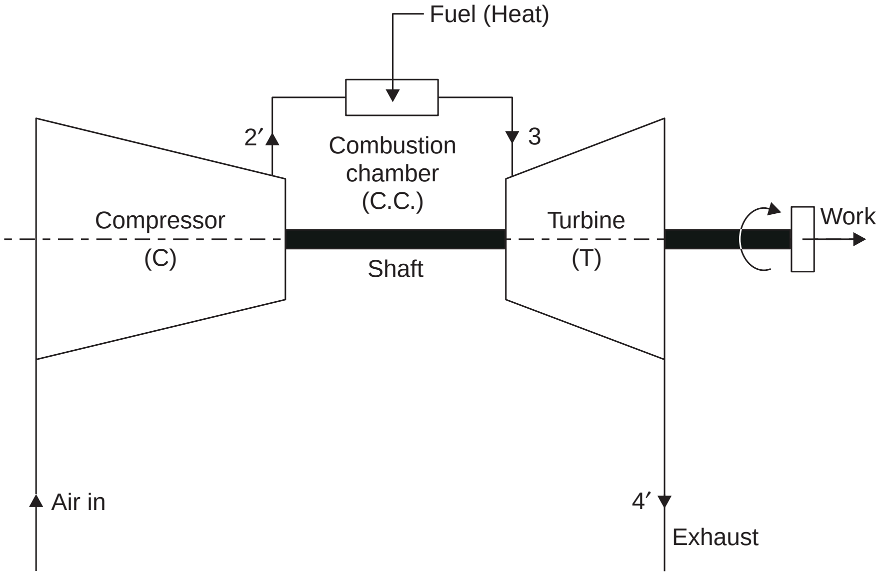


Fig. 13.35. Open cycle gas turbine.

If pressure loss in the combustion chamber is neglected, this cycle may be drawn on a *T-s* diagram as shown in Fig. 13.36.

- 1-2′ represents : *irreversible adiabatic compression.*
- 2′-3 represents : *constant pressure heat supply in the combustion chamber.*
- 3-4′ represents : *irreversible adiabatic expansion.*
- 1-2 represents : *ideal isentropic compression.*
- 3-4 represents : *ideal isentropic expansion.*

Assuming change in kinetic energy between the various points in the cycle to be negligibly small compared with enthalpy changes and then applying the flow equation to each part of cycle, for unit mass, we have

Work input (compressor) $= c_p\,(T_2' - T_1)$

Heat supplied (combustion chamber) $= c_p\,(T_3 - T_2')$

Work output (turbine) $= c_p\,(T_3 - T_4')$

$\therefore$ Net work output = Work output – Work input

$= c_p\,(T_3 - T_4') - c_p(T_2' - T_1)$

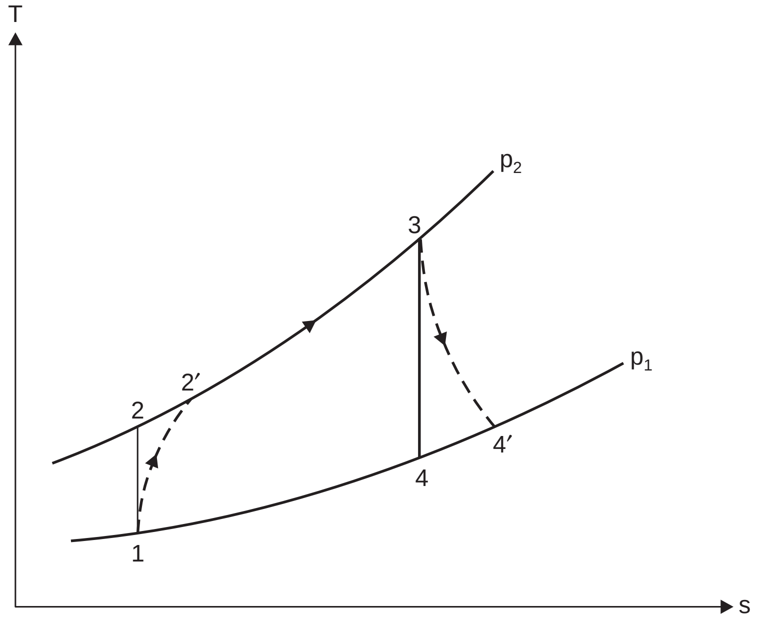


Fig. 13.36

and
$$\eta_{thermal} = \frac{\text{Net work output}}{\text{Heat supplied}}$$

$$= \frac{c_p(T_3 - T_4') - c_p(T_2' - T_1)}{c_p(T_3 - T_2')}$$

Compressor isentropic efficiency, η_{comp}

$$= \frac{\text{Work input required in isentropic compression}}{\text{Actual work required}}$$

$$= \frac{c_p(T_2 - T_1)}{c_p(T_2' - T_1)} = \frac{T_2 - T_1}{T_2' - T_1} \quad ...(13.19)$$

Turbine isentropic efficiency, $\eta_{turbine}$

$$= \frac{\text{Actual work output}}{\text{Isentropic work output}}$$

$$= \frac{c_p(T_3 - T_4')}{c_p(T_3 - T_4)} = \frac{T_3 - T_4'}{T_3 - T_4} \quad ...(13.20)$$

Note. With the variation in temperature, the value of the specific heat of a real gas varies, and also in the open cycle, the specific heat of the gases in the combustion chamber and in turbine is different from that in the compressor because fuel has been added and a chemical change has taken place. Curves showing the variation of c_p with temperature and air/fuel ratio can be used, and a suitable mean value of c_p and hence γ can be found out. It is usual in gas turbine practice to assume fixed mean value of c_p and γ for the expansion process, and fixed mean values of c_p and γ for the compression process. In an open cycle gas turbine unit the mass flow of gases in turbine is greater than that in compressor due to mass of fuel burned, but it is possible to neglect mass of fuel, since the air/fuel ratios used are large. Also, in many cases, air is bled from the compressor for cooling purposes, or in the case of air-craft at high altitudes, bled air is used for de-icing and cabin air-conditioning. This amount of air bled is approximately the same as the mass of fuel injected therein.

13.10.5. Methods for Improvement of Thermal Efficiency of Open Cycle Gas Turbine Plant

The following methods are employed *to increase the specific output and thermal efficiency of the plant :*

1. Intercooling 2. Reheating 3. Regeneration.

1. **Intercooling.** A compressor in a gas turbine cycle utilises the major percentage of power developed by the gas turbine. The work required by the compressor can be reduced by compressing the air in two stages and *incorporating an intercooler* between the two as shown in Fig. 13.37. The corresponding *T-s* diagram for the unit is shown in Fig. 13.38. The actual processes take place as follows :

1-2′	...	L.P. (Low pressure) compression
2′-3	...	Intercooling
3-4′	...	H.P. (High pressure) compression
4′-5	...	C.C. (Combustion chamber)-heating
5-6′	...	T (Turbine)-expansion

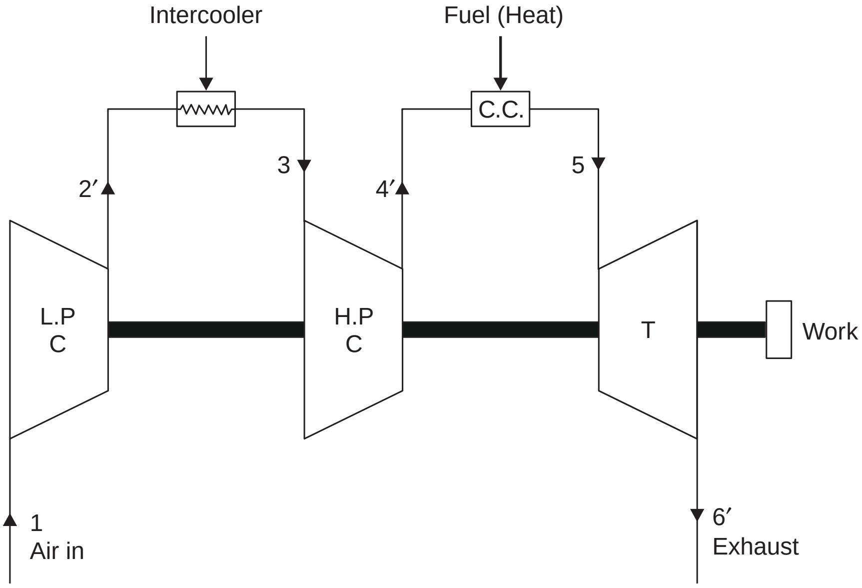


Fig. 13.37. Turbine plant with intercooler.

The ideal cycle for this arrangement is 1-2-3-4-5-6 ; the compression process without intercooling is shown as 1-*L*′ in the actual case, and 1-*L* in the ideal isentropic case.

Now,

Work input (*with intercooling*)

$$= c_p(T_2' - T_1) + c_p(T_4' - T_3) \quad ...(13.21)$$

Work input (*without intercooling*)

$$= c_p(T_L' - T_1) = c_p(T_2' - T_1) + c_p(T_L' - T_2') \quad ...(13.22)$$

By comparing equation (13.22) with equation (13.21) it can be observed that the *work input with intercooling is less than the work input with no intercooling*, when $c_p\,(T_4' - T_3)$ is *less* than $c_p(T_L' - T_2')$. This is so if it is assumed that isentropic efficiencies of the two compressors, operating separately, are each equal to the isentropic efficiency of the single compressor which would be required if no intercooling were used. Then $(T_4' - T_3) < (T_L' - T_2')$ since the pressure lines diverge on the T-s diagram from left to the right.

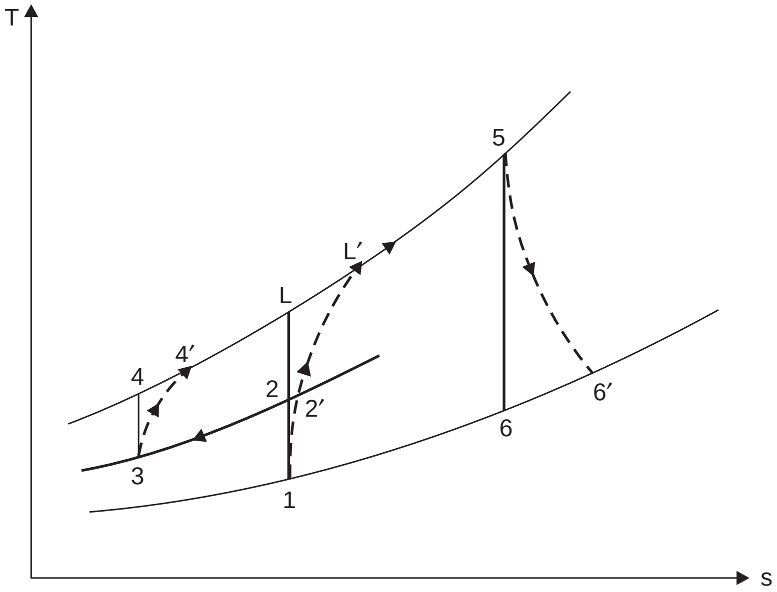


Fig. 13.38. T-s diagram for the unit.

$$\text{Again, work ratio} = \frac{\text{Net work output}}{\text{Gross work output}}$$

$$= \frac{\text{Work of expansion} - \text{Work of compression}}{\text{Work of expansion}}$$

From this we may conclude that *when the compressor work input is reduced then the work ratio is increased*.

However the heat supplied in the combustion chamber when intercooling is used in the cycle, is given by,

Heat supplied *with intercooling* $= c_p(T_5 - T_4')$

Also the heat supplied when intercooling is not used, with the same maximum cycle temperature T_5, is given by

Heat supplied *without intercooling* $= c_p\,(T_5 - T_L')$

Thus, the *heat supplied when intercooling is used is greater than with no intercooling. Although the net work output is increased by intercooling it is found in general that the increase in heat to be supplied causes the thermal efficiency to decrease*. When intercooling is used a supply of cooling water must be readily available. The additional bulk of the unit may offset the advantage to be gained by increasing the work ratio.

2. **Reheating.** The output of a gas turbine can be amply improved by expanding the gases in two stages with a *reheater* between the two as shown in Fig. 13.39. The H.P. turbine drives the compressor and the L.P. turbine provides the useful power output. The corresponding T-s diagram is shown in Fig. 13.40. The line $4'$-L' represents the expansion in the L.P. turbine if reheating is *not* employed.

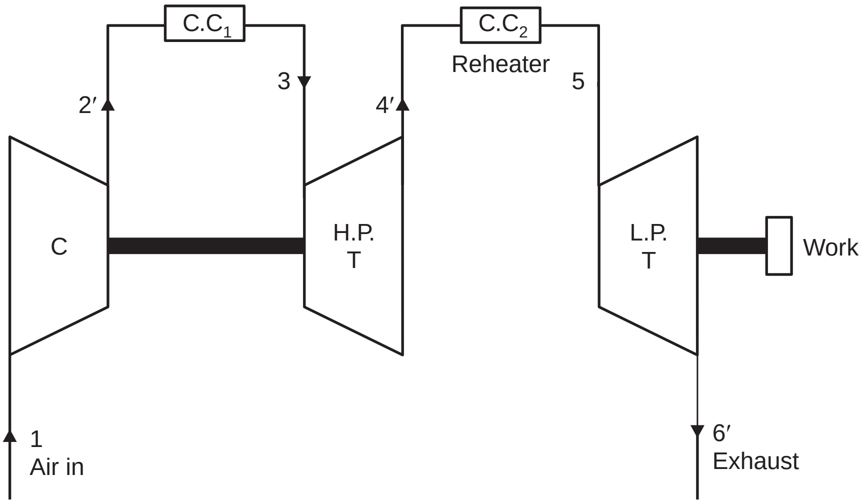


Fig. 13.39. Gas turbine with reheater.

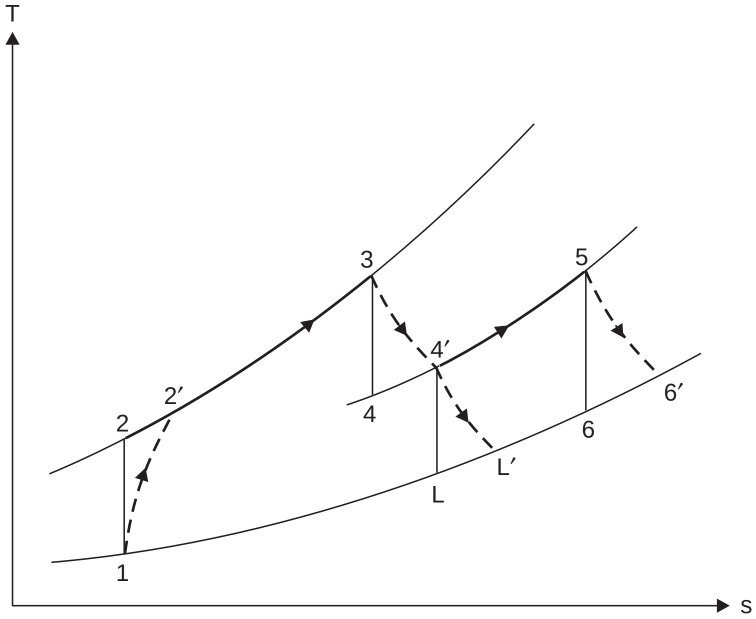


Fig. 13.40. T-s diagram for the unit.

Neglecting mechanical losses the work output of the H.P. turbine must be exactly equal to the work input required for the compressor i.e., $c_{pa}\ (T_2' - T_1) = c_{pg}\ (T_3 - T_4')$

The work output (net output) of L.P. turbine is given by,

Net work output (with reheating) $= c_{pg}\ (T_5 - T_6')$

and Net work output (without reheating) $= c_{pg}\ (T_4' - T_L')$

Since the pressure lines diverge to the right on T-s diagram it can be seen that the temperature difference $(T_5 - T_6')$ is always *greater* than $(T_4' - T_L')$, so that *reheating increases the net work output*.

Although net work is increased by reheating the heat to be supplied is also increased, and the net effect *can be to reduce the thermal efficiency*

Heat supplied $= c_{pg}\,(T_3 - T_2') + c_{pg}\,(T_5 - T_4')$.

Note. c_{pa} and c_{pg} stand for specific heats of air and gas respectively at constant pressure.

3. **Regeneration.** The exhaust gases from a gas turbine carry a large quantity of heat with them since their temperature is far above the ambient temperature. They can be used to heat the air coming from the compressor thereby reducing the mass of fuel supplied in the combustion chamber. Fig. 13.41 shows a gas turbine plant with a regenerator. The corresponding *T-s* diagram is shown in Fig. 13.42. 2′-3 represents the heat flow into the compressed air during its passage through the heat exchanger and 3-4 represents the heat taken in from the combustion of fuel. Point 6 represents the temperature of exhaust gases at discharge from the heat exchanger. The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice because a temperature gradient must exist for an unassisted transfer of energy. The *effectiveness* of the heat exchanger is given by :

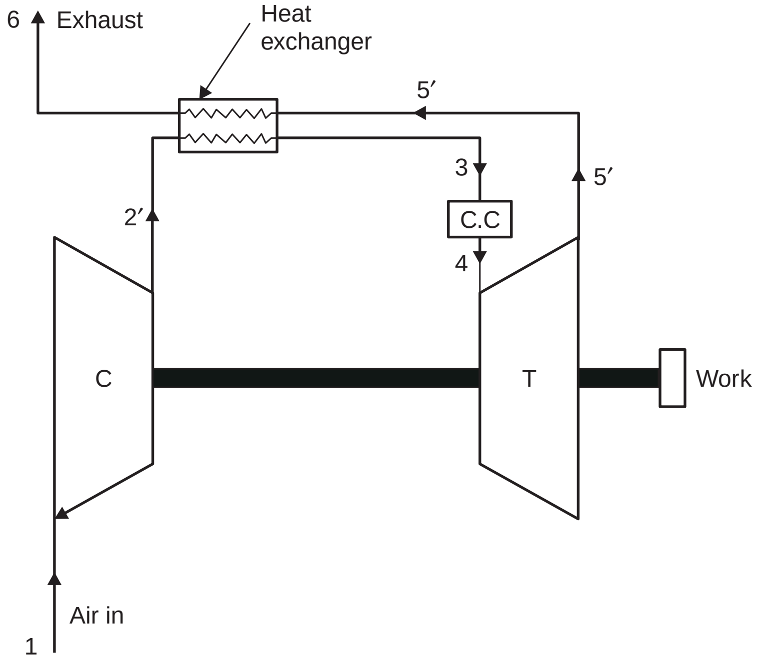


Fig. 13.41. Gas turbine with regenerator.

Effectiveness,
$$\varepsilon = \frac{\text{Increase in enthalpy per kg of air}}{\text{Available increase in enthalpy per kg of air}}$$

$$= \frac{(T_3 - T_2')}{(T_5' - T_2')} \qquad \text{...(13.23)}$$

(assuming c_{pa} and c_{pg} to be equal)

A heat exchanger is usually used in large gas turbine units for marine propulsion or industrial power.

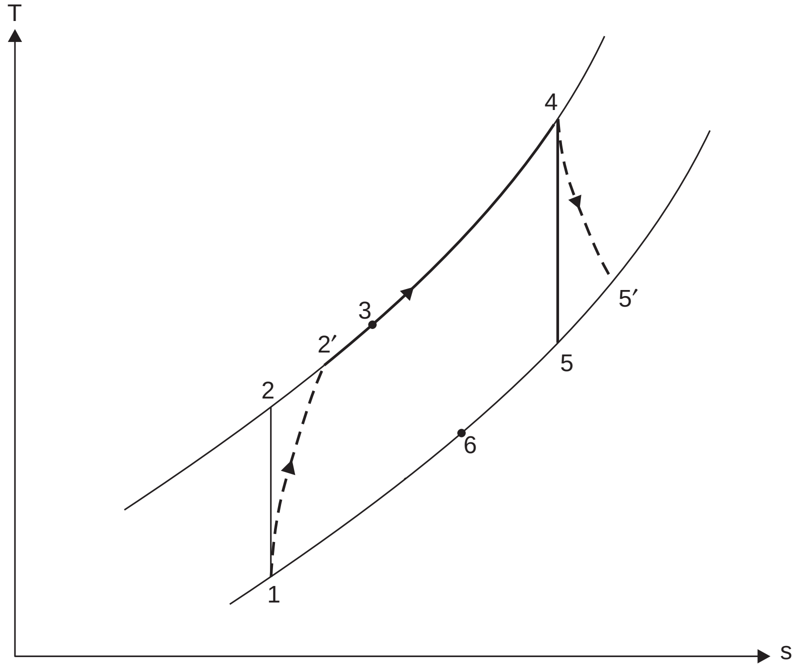


Fig. 13.42. *T-s* diagram for the unit.

13.10.6. Effect of Operating Variables on Thermal Efficiency

The thermal efficiency of *actual open cycle* depends on the following thermodynamic variables :

(*i*) Pressure ratio

(*ii*) Turbine inlet temperature (T_3)

(*iii*) Compressor inlet temperature (T_1)

(*iv*) Efficiency of the turbine ($\eta_{turbine}$)

(*v*) Efficiency of the compressor (η_{comp}).

Effect of turbine inlet temperature and pressure ratio :

If the permissible turbine inlet-temperature (with the other variables being constant) of an *open cycle gas turbine power plant* is increased its *thermal efficiency* is *amply improved*. A practical limitation to increasing the turbine inlet temperature, however, is the ability of the material available for the turbine blading to *withstand the high rotative and thermal stresses.*

Refer Fig. 13.43. For *a given turbine inlet temperature, as the pressure ratio increases, the heat supplied as well as the heat rejected are reduced. But the ratio of change of heat supplied is not the same as the ratio of change heat rejected.* As a consequence, there *exists an optimum pressure ratio producing maximum thermal efficiency for a given turbine inlet temperature.*

As the pressure ratio increases, the thermal efficiency also increases until it becomes maximum and then it drops off with a further increase in pressure ratio (Fig. 13.44). Further, as the turbine inlet temperature increases, the peaks of the curves flatten out giving a greater range of ratios of pressure optimum efficiency.

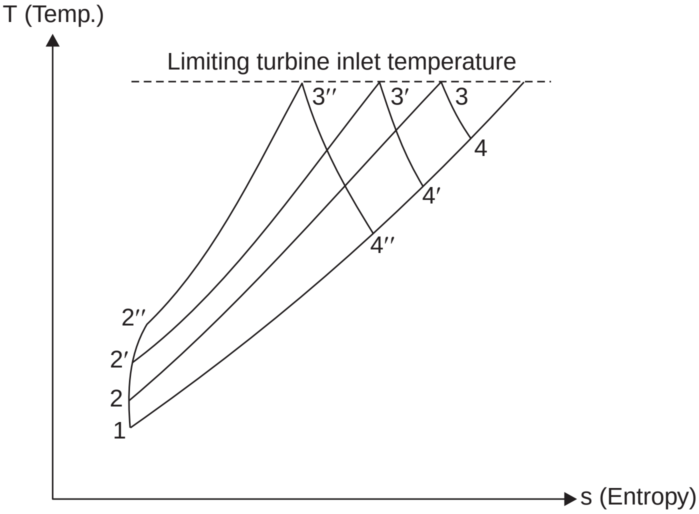


Fig. 13.43

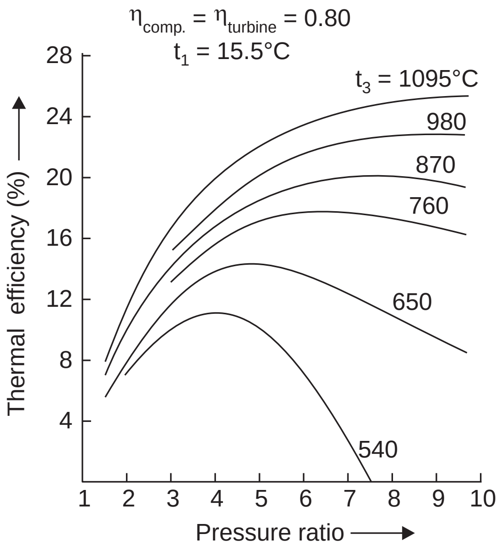


Fig. 13.44. Effect of pressure ratio and turbine inlet temperature.

Following particulars are worthnoting :

Gas temperatures	*Efficiency (gas turbine)*
550 to 600°C	20 to 22%
900 to 1000°C	32 to 35%
Above 1300°C	more than 50%

Effect of turbine and compressor efficiencies :

Refer Fig. 13.45. The thermal efficiency of the actual gas turbine cycle is very sensitive to variations in the efficiencies of the compressor and turbine. There is a particular pressure ratio at which maximum efficiencies occur. For lower efficiencies, the peak of the thermal efficiency occurs at lower pressure ratios and vice versa.

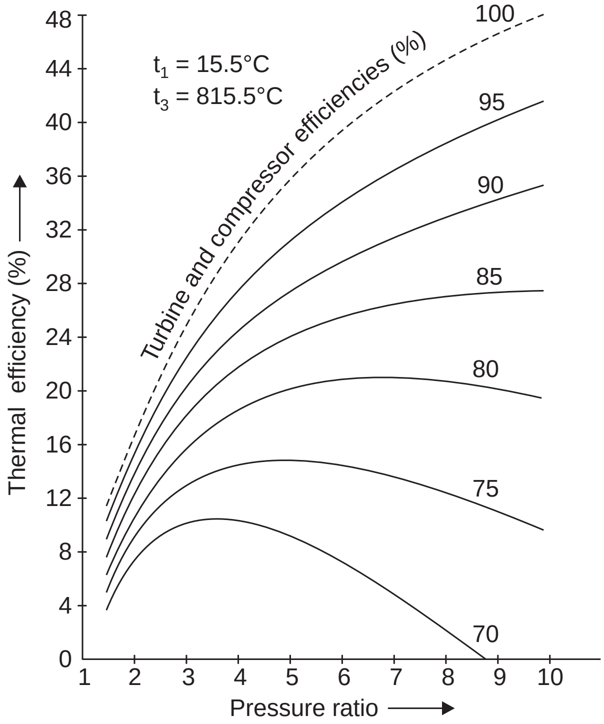


Fig. 13.45. Effect of component efficiency.

Effect of compressor inlet temperature :

Refer Fig. 13.46 (on next page). *With the decrease in the compressor inlet temperature there is increase in thermal efficiency of the plant.* Also the peaks of thermal efficiency occur at high pressure ratios and the curves become flatter giving thermal efficiency over a wider pressure ratio range.

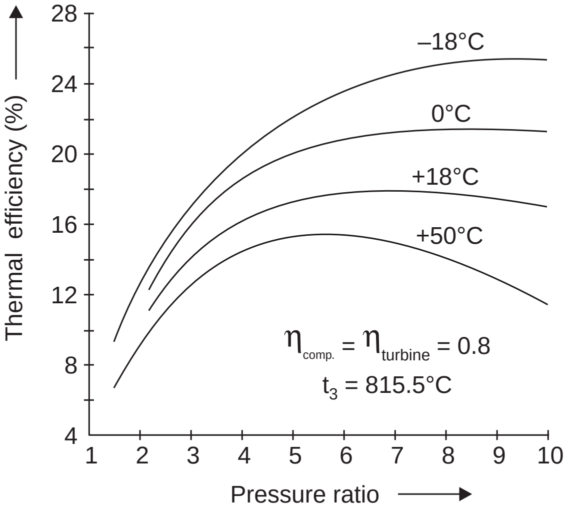


Fig. 13.46

13.10.7. Closed Cycle Gas Turbine (Constant pressure or joule cycle).

Fig. 13.47 shows a gas turbine operating on a constant pressure cycle in which the closed system consists of air behaving as an ideal gas. The various operations are as follows : Refer Figs. 13.48 and 13.49.

Operation 1-2 : The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

Operation 2-3 : Heat flow into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2. Heat received = $mc_p\,(T_3 - T_2)$.

Operation 3-4 : The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

Operation 4-1 : Heat is rejected from the system as the volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1. Heat rejected = $mc_p\,(T_4 - T_1)$

$$\eta_{air\text{-}standard} = \frac{\text{Work done}}{\text{Heat received}}$$

$$= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}}$$

$$= \frac{mc_p(T_3 - T_2) - mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now, from isentropic expansion

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

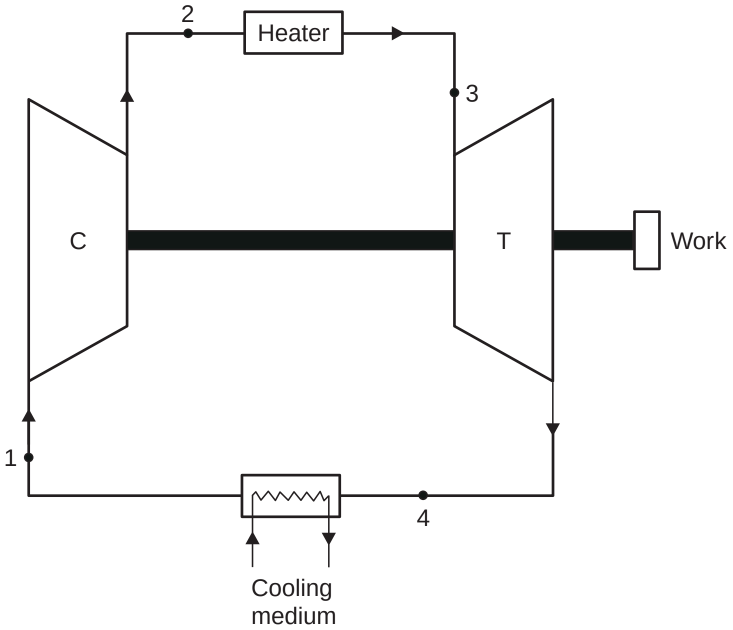


Fig. 13.47. Closed cycle gas turbine.

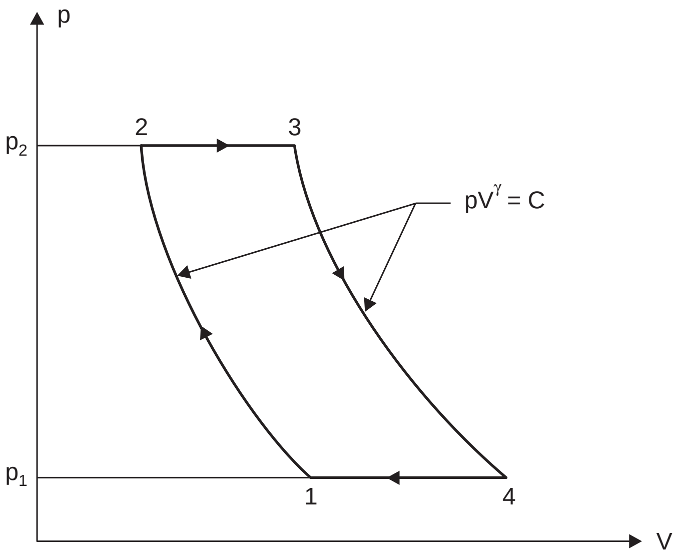


Fig. 13.48. *p-V* diagram.

$$T_2 = T_1\,(r_p)^{\frac{\gamma-1}{\gamma}}\text{, where } r_p = \text{Pressure ratio}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ or } T_3 = T_4\,(r_p)^{\frac{\gamma-1}{\gamma}}$$

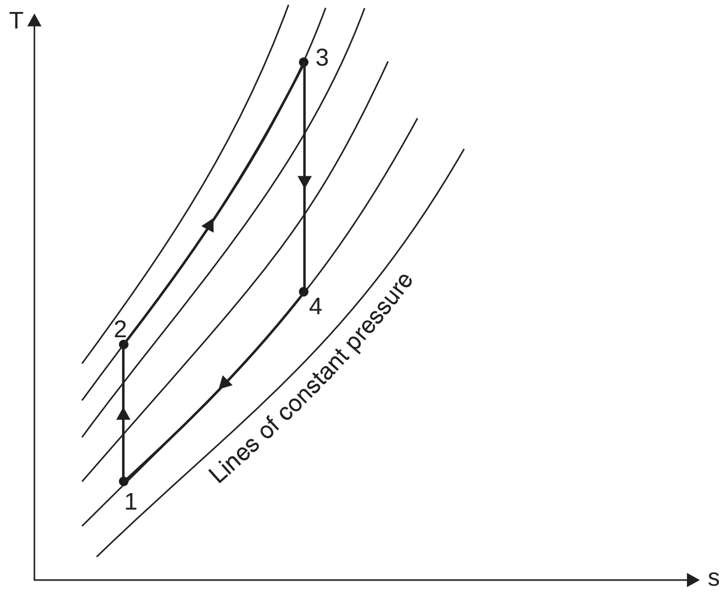


Fig. 13.49. *T-s* diagram.

$$\therefore \qquad \eta_{air\text{-}standard} = 1 - \frac{T_4 - T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}} - T_1(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \qquad ...(13.24)$$

The expression shows that the *efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use*.

Now we shall prove that the *pressure ratio for maximum work is a function of the limiting temperature ratio*.

Work output during the cycle

$$= \text{Heat received/cycle} - \text{Heat rejected/cycle}$$

$$= mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1) = mc_p\,(T_3 - T_4) - mc_p\,(T_2 - T_1)$$

$$= mc_pT_3\left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)$$

In case of a given turbine the minimum temperature T_1 and the maximum temperature T_3 are prescribed, T_1 being the temperature of the atmosphere and T_3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure c_p to be constant. Then,

Since, $$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

Using the constant $$'z' = \frac{\gamma - 1}{\gamma},$$

we have, work output/cycle $$W = K\left[T_3\left(1 - \frac{1}{r_p{}^z}\right) - T_1(r_p{}^z - 1)\right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K\left[T_3 \times \frac{z}{r_p(z+1)} - T_1 z r_p{}^{(z-1)}\right] = 0 \text{ for a maximum}$$

$$\therefore \qquad \frac{zT_3}{r_p{}^{(z+1)}} = T_1 z (r_p)^{(z-1)}$$

$$\therefore \qquad r_p{}^{2z} = \frac{T_3}{T_1}$$

$$\therefore \qquad r_p = (T_3/T_1)^{1/2z} \text{ i.e., } r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}}$$

Thus the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

Fig. 13.50 shows an arrangement of closed cycle stationary gas turbine plant in which air is continuously circulated. This ensures that the air is not polluted by the addition of combustion waste product, since the heating of air is carried out in the form of heat exchanger shown in the diagram as air heater. The air exhausted from the power turbine is cooled before readmission to L.P. compressor. The various operations as indicated on *T-s* diagram (Fig. 13.51) are as follows :

Operation 1-2′ : Air is compressed from p_1 to p_x in the L.P. compressor.

Operation 2′-3 : Air is cooled in the intercooler at constant pressure p_x.

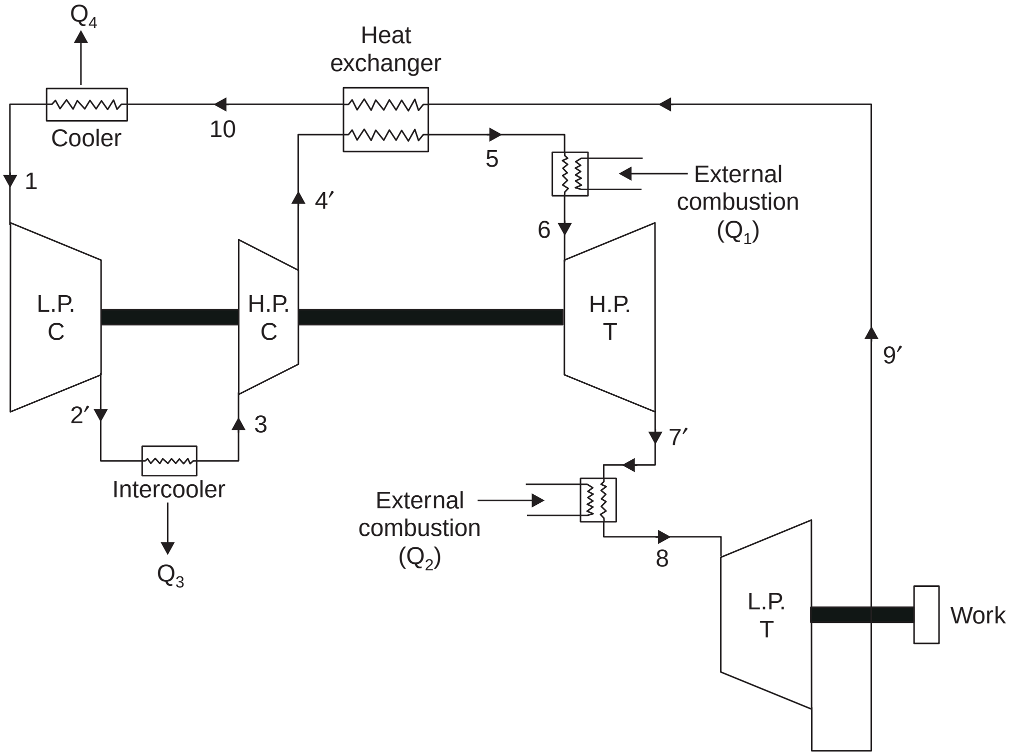


Fig. 13.50. Closed cycle gas turbine plant.

Operation 3-4′ : Air is compressed in the H.P. compressor from p_x to p_2.

Operation 4′-5 : High pressure air is heated at constant pressure by exhaust gases from power turbine in the heat exchanger to T_5.

Operation 5-6 : High pressure air further heated at constant pressure to the maximum temperature T_6 by an air heater (through external combustion).

Operation 6-7′ : The air is expanded in the H.P. turbine from p_2 to p_x producing work to drive the compressor.

Operation 7′-8 : Exhaust air from the H.P. turbine is heated at constant pressure in the air heater (through external combustion) to the maximum temperature $T_8 (= T_6)$.

Operation 8-9′ : The air is expanded in the L.P. turbine from p_x to p_1, producing energy for a flow of work externally.

Operation 9′-10 : Air from L.P. turbine is passed to the heat exchanger where energy is transferred to the air delivered from the H.P. compressor. The temperature of air leaving the heat exchanger and entering the cooler is T_{10}.

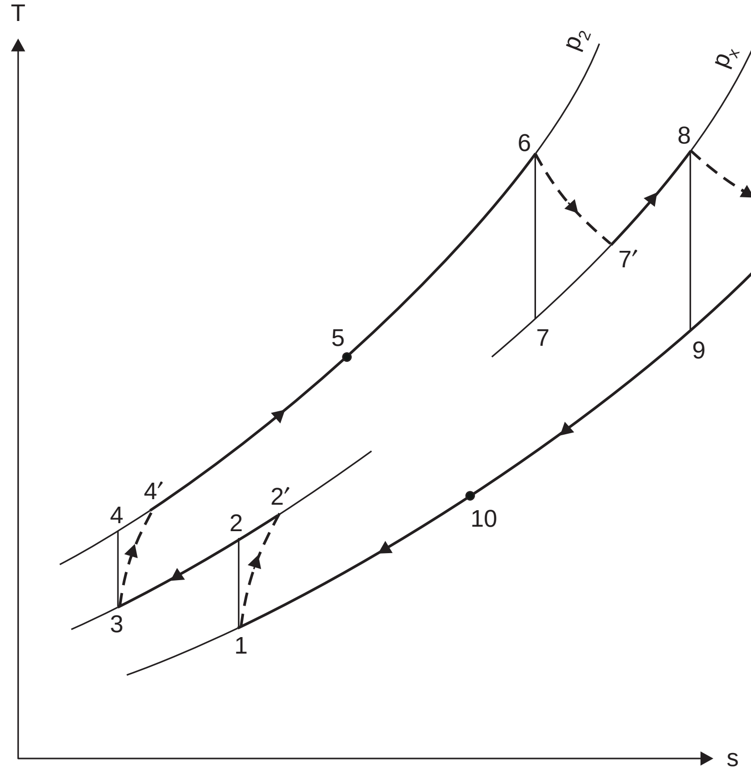


Fig. 13.51. *T-s* diagram for the plant.

Operation 10-11 : Air cooled to T_1 by the cooler before entering the L.P. compressor.

The energy balance for the whole plant is as follows :

$$Q_1 + Q_2 - Q_3 - Q_4 = W$$

In a closed cycle plant, in practice, the control of power output is achieved by varying the mass flow by the use of a reservoir in the circuit. The *reservoir maintains the design pressure and temperature and therefore achieves an approximately constant level of efficiency for varying* loads. In this cycle since it is closed, *gases other than air with favourable properties can be used* ; furthermore it is possible to burn solid fuels in the combustion heaters. *The major factor responsible for inefficiency in this cycle is the large irreversible temperature drop which occurs in the air heaters between the furnace and circulating gas*.

Note 1. In a closed cycle gas turbines, although air has been extensively used, the use of 'helium' which though of a lower density, has been inviting the attention of manufacturers for its use, for large output gas turbine units. *The specific heat of helium at constant pressure is about 'five times' that of air, therefore for each kg mass flow the heat drop and hence energy dealt with in helium machines is nearly five times of those in case of air*. The *surface area of the heat exchanger for helium can be kept as low as 1/3* of that *required for gas turbine plant using air as working medium*. For the same temperature ratio and for the plants of the same output the *cross-sectional area required for helium is much less than that for air*. It may therefore be concluded that the *size of helium unit is considerably small comparatively*.

2. Some gas turbine plants work on a combination of two cycles the open cycle and the closed cycle. Such a combination is called the *semi-closed cycle. Here a part of the working fluid is confined within the plant and another part flows from and to atmosphere.*

13.10.8. Gas Turbine Fuels

The various fuels used in gas turbines are enumerated and discussed below :

1. Gaseous fuels
2. Liquid fuels
3. Solid fuels

1. **Gaseous fuels.** *Natural gas is the ideal fuel for gas turbines*, but this is not available everywhere.

Blast furnace and *producer gases* may also be used for gas turbine power plants.

2. **Liquid fuels.** Liquid fuels of petroleum origin such as distillate oils or residual oils are most commonly used for gas turbine plant. The essential qualities of these fuels include *proper volatility, viscosity and calorific value*. At the same time it *should be* free *from any contents of moisture* and *suspended impurities that would log the small passages of the nozzles and damage valves and plungers of the fuel pumps.*

Minerals like *sodium, vanadium* and *calcium* prove *very harmful* for the *turbine blading* as these build deposits or corrode the blades. The sodium in ash should be less than 30% of the vanadium content as otherwise the ratio tends to be critical. The actual sodium content may be between 5 ppm to 10 ppm (part per million). If the vanadium is over 2 ppm, the magnesium in ash tends to become critical. *It is necessary that the magnesium in ash is at least three times the quantity of vanadium.* The content of calcium and lead should not be over 10 ppm and 5 ppm respectively.

Sodium is removed from residual oils by mixing with 5% of water and then double centrifuging when sodium leaves with water. Magnesium is added to the washed oil in the form of epsom salts, before the oil is sent into the combustor. This checks the corrosive action of vanadium. Residual oils burn with less ease than distillate oils and the latter are often used to start the unit from cold, after which the residual oils are fed in the combustor. In cold conditions residual oils need to be preheated.

3. **Solid fuels.** The use of solid fuels such as coal in pulverised form in gas turbines presents several difficulties most of which have been only partially overcome yet. The pulverising plant for coal in gas turbines applications is much lighter and small than its counterpart in steam generators. *Introduction of fuel in the combustion chamber of a gas turbine is required to be done against a high pressure whereas the pressure in the furnace of a steam plant is atmospheric.* Furthermore, the *degree of completeness of combustion in gas turbine applications has to be very high as otherwise soot and dust in gas would deposit on the turbine blading.*

Some practical applications of solid fuel burning in turbine combustors have been commercially, made available in recent years. In one such design finely crushed coal is used instead of pulverised fuel. This fuel is carried in stream of air tangentially into one end of a cylindrical furnace while gas comes out at the centre of opposite end. As the fuel particles roll around the circumference of the furnace they are burnt and a high temperature of about 1650°C is maintained which causes the mineral matter of fuel to be converted into a liquid slag. The slag covers the walls of the furnace and runs out through a top hole in the bottom. The result is that fly-ash is reduced to a very small content in the gases. In *another design* a regenerator is used to transfer the heat to air, the combustion chamber being located on the outlet of the turbine, and the combustion is carried out in the turbine exhaust stream. The advantage is that only clean air is handled by the turbine.

Example 13.32. *Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kPa, 27°C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume* $W_T = 2.5\ W_C$, *where* W_T *and* W_C *are the turbine and the compressor work respectively. Take* $\gamma = 1.4$. **(P.U.)**

Solution. Pressure of intake air, $p_1 = 101.325$ kPa

Temperature of intake air, $T_1 = 27 + 273 = 300$ K

The pressure ratio in the cycle, $r_p = 6$

(*i*) **Maximum temperature in the cycle, T_3 :**

Refer Fig. 13.52.

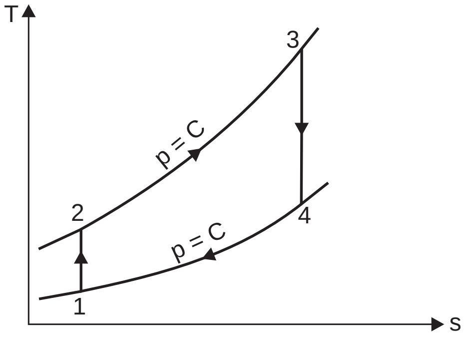


Fig. 13.52

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.668$$

$\therefore$ $T_2 = 1.668\ T_1 = 1.668 \times 300 = 500.4$ K

Also, $$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.668$$

$\therefore$ $$T_4 = \frac{T_3}{1.668}$$

But, $W_T = 2.5\ W_C$ (given)

$\therefore$ $mc_p\ (T_3 - T_4) = 2.5\ mc_p\ (T_2 - T_1)$

or, $$T_3 - \frac{T_3}{1.668} = 2.5\ (500.4 - 300) = 501 \quad \text{or} \quad T_3\left(1 - \frac{1}{1.668}\right) = 501$$

$\therefore$ $$T_3 = \frac{501}{\left(1 - \frac{1}{1.668}\right)} = \textbf{1251 K or 978°C.} \quad \textbf{(Ans.)}$$

(*ii*) **Cycle efficiency, η_{cycle} :**

Now, $$T_4 = \frac{T_3}{1.668} = \frac{1251}{1.668} = 750 \text{ K}$$

$$\therefore \qquad \eta_{cycle} = \frac{\text{Net work}}{\text{Heat added}} = \frac{mc_p\,(T_3 - T_4) - mc_p\,(T_2 - T_1)}{mc_p(T_3 - T_2)}$$

$$= \frac{(1251 - 750) - (500.4 - 300)}{(1251 - 500.4)} = \mathbf{0.4 \quad or \quad 40\%. \quad (Ans.)}$$

$$\left[\textbf{\textit{Check}}\,;\ \eta_{cycle} = 1 - \frac{1}{(r_p)^{\left(\frac{\gamma-1}{\gamma}\right)}} = 1 - \frac{1}{(6)^{\frac{1.4-1}{1.4}}} = \mathbf{0.4 \quad or \quad 40\%. \quad (Ans.)} \right]$$

Example 13.33. *A gas turbine is supplied with gas at 5 bar and 1000 K and expands it adiabatically to 1 bar. The mean specific heat at constant pressure and constant volume are 1.0425 kJ/kg K and 0.7662 kJ/kg K respectively.*

(i) Draw the temperature-entropy diagram to represent the processes of the simple gas turbine system.

(ii) Calculate the power developed in kW per kg of gas per second and the exhaust gas temperature. **(GATE, 1995)**

Solution. *Given* : $p_1 = 1$ bar ; $p_2 = 5$ bar ; $T_3 = 1000$ K ; $c_p = 1.0425$ kJ/kg K ;
$c_v = 0.7662$ kJ/kg K

$$\gamma = \frac{c_p}{c_v} = \frac{1.0425}{0.7662} = 1.36$$

(*i*) **Temperature-entropy (T-s) diagram :**

Temperature-entropy diagram representing the processes of the simple gas turbine system is shown in Fig. 13.53.

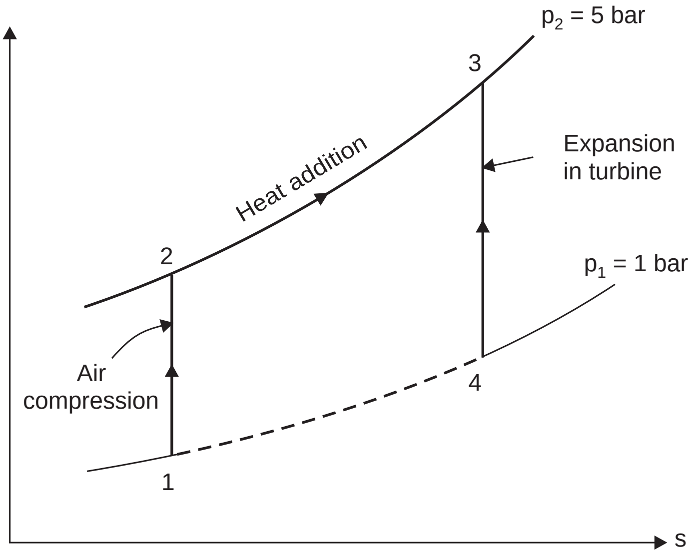


Fig. 13.53

(*ii*) **Power required :**

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.36-1}{1.36}} = 0.653$$

$\therefore$ $T_4 = 1000 \times 0.653 = 653$ K

Power developed per kg of gas per second

$= c_p (T_3 - T_4)$

$= 1.0425 (1000 - 653) =$ **361.7 kW. (Ans.)**

Example 13.34. *An isentropic air turbine is used to supply 0.1 kg/s of air at 0.1 MN/m^2 and at 285 K to a cabin. The pressure at inlet to the turbine is 0.4 MN/m^2. Determine the temperature at turbine inlet and the power developed by the turbine. Assume c_p= 1.0 kJ/kg K.* **(GATE, 1999)**

Solution. *Given* : $\dot{m}_a = 0.1$ kg/s ; $p_1 = 0.1$ MN/m^2 = 1 bar, $T_4 = 285$ K ; $p_2 = 0.4$ MN/m^2 = 4 bar ; $c_p = 1.0$ kJ/kg K.

Temperature at turbine inlet, T_3 :

$$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore$ $T_3 = 285 \times 1.486 =$ **423.5 K. (Ans.)**

Power developed, P :

$$P = \dot{m}_a c_p (T_3 - T_4)$$

$= 0.1 \times 1.0 (423.5 - 285)$

$=$ **13.85 kW. (Ans.)**

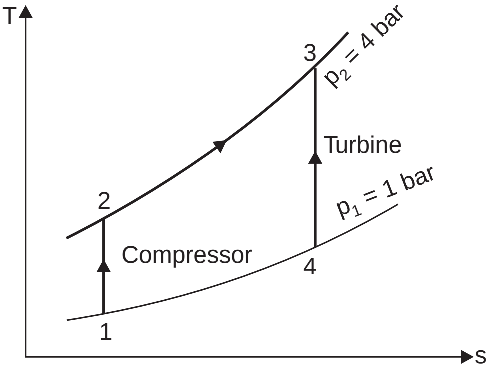


Fig. 13.54

Example 13.35. *Consider an air standard cycle in which the air enters the compressor at 1.0 bar and 20°C. The pressure of air leaving the compressor is 3.5 bar and the temperature at turbine inlet is 600°C. Determine per kg of air :*

(*i*) *Efficiency of the cycle,* (*ii*) *Heat supplied to air,*

(*iii*) *Work available at the shaft,* (*iv*) *Heat rejected in the cooler, and*

(*v*) *Temperature of air leaving the turbine.*

For air $\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K.

Solution. Refer Fig. 13.52.

Pressure of air entering the compressor, $p_1 = 1.0$ bar

Temperature at the inlet of compressor, $T_1 = 20 + 273 = 293$ K

Pressure of air leaving the compressor, $p_2 = 3.5$ bar

Temperature of air at turbine inlet, $T_3 = 600 + 273 = 873$ K

(*i*) **Efficiency of the cycle, η_{cycle} :**

$$\eta_{cycle} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(3.5)^{\frac{1.4-1}{1.4}}} = \mathbf{0.30 \text{ or } 30\%.\ (Ans.)} \quad \left(\because\ r_p = \frac{p_2}{p_1} = \frac{3.5}{1.0} = 3.5\right)$$

(*ii*) **Heat supplied to air :**

For compression process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.5}{1}\right)^{\frac{1.4-1}{1.4}} = 1.43$$

$\therefore$ $T_2 = T_1 \times 1.43 = 293 \times 1.43 \simeq 419$ K

$\therefore$ Heat supplied to air, $Q_1 = c_p\,(T_3 - T_2) = 1.005\ (873 - 419) =$ **456.27 kJ/kg. (Ans.)**

(*iii*) **Work available at the shaft, W :**

We know that, $\eta_{\text{cycle}} = \dfrac{\text{Work output }(W)}{\text{Heat input }(Q_1)}$

or $0.30 = \dfrac{W}{456.27}$ or $W = 0.3 \times 456.27 = 136.88$ kJ/kg

(*iv*) **Heat rejected in the cooler, Q_2 :**

Work output (W) = Heat supplied (Q_1) – heat rejected (Q_2)

$\therefore$ $Q_2 = Q_1 - W = 456.27 - 136.88 =$ **319.39 kJ/kg. (Ans.)**

(*v*) **Temperature of air leaving the turbine, T_4 :**

For *expansion (isentropic) process 3-4,* we have

$$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = (3.5)^{\frac{1.4-1}{1.4}} = 1.43$$

$\therefore$ $T_4 = \dfrac{T_3}{1.43} = \dfrac{873}{1.43} =$ **610.5 K. (Ans.)**

[**Check** : Heat rejected in the air cooler at constant pressure during the process 4-1 can also be calculated as : Heat rejected $= m \times c_p\,(T_4 - T_1) = 1 \times 1.005 \times (610.5 - 293) = 319.1$ kJ/kg]

Example 13.36. *A closed cycle ideal gas turbine plant operates between temperature limits of 800°C and 30°C and produces a power of 100 kW. The plant is designed such that there is no need for a regenerator. A fuel of calorific 45000 kJ/kg is used. Calculate the mass flow rate of air through the plant and rate of fuel consumption.*

Assume c_p = 1 kJ/kg K and γ = 1.4. **(GATE, 2000)**

Solution. *Given* : $T_1 = 30 + 273 = 303$ K ; $T_3 = 800 + 273 = 1073$ K ; $C = 45000$ kJ/kg ; $c_p = 1$ kJ/kg K ; $\gamma = 1.4$; $W_{\text{turbine}} - W_{\text{compressor}} = 100$ kW.

$\dot{m}_a$, $\dot{m}_f$:

Since no regenerator is used we can assume the turbine expands the gases upto T_4 in such a way that the exhaust gas temperature from the turbine is equal to the temperature of air coming out of the compressor *i.e.,* $T_2 = T_4$

$$\frac{p_2}{p_1} = \frac{p_3}{p_4},\quad \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \text{ and } \frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_3}{T_2}$$

$(\because\ T_2 = T_4$ ......assumed)

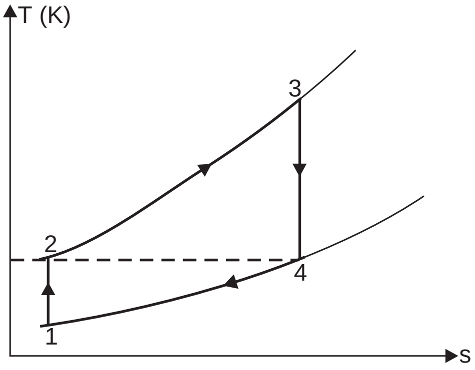


Fig. 13.55

or, $$T_2^2 = T_1T_3 \quad \text{or} \quad T_2 = \sqrt{T_1T_3}$$

or, $$T_2 = \sqrt{303 \times 1073} = 570.2 \text{ K}$$

Now, $$W_{\text{turbine}} - W_{\text{compressor}} = \dot{m}_f \times C \times \eta$$

or, $$100 = \dot{m}_f \times 45000 \times \left[1 - \frac{T_4 - T_1}{T_3 - T_2}\right]$$

$$= \dot{m}_f \times 45000 \left[1 - \frac{570.2 - 303}{1073 - 570.2}\right]$$

$$= \dot{m}_f \times 21085.9$$

or, $$\mathbf{\dot{m}_f = \frac{100}{21085.9} = 4.74 \times 10^{-3}\ kg/s.} \quad \textbf{(Ans.)}$$

Again, $$W_{\text{turbine}} - W_{\text{compressor}} = 100 \text{ kW}$$

$$(\dot{m}_a + \dot{m}_f)(T_3 - T_4) - \dot{m}_a \times 1 \times (T_2 - T_1) = 100$$

or, $$(\dot{m}_a + 0.00474)(1073 - 570.2) - \dot{m}_a (570.2 - 303) = 100$$

or, $$(\dot{m}_a + 0.00474) \times 502.8 - 267.2\ \dot{m}_a = 100$$

or, $$502.8\ \dot{m}_a + 2.383 - 267.2\ \dot{m}_a = 100$$

or, $$235.6\ \dot{m}_a = 97.617$$

$\therefore$ $$\mathbf{\dot{m}_a = 0.414\ kg/s.} \quad \textbf{(Ans.)}$$

Example 13.37. *In a gas turbine plant working on Brayton cycle, the air at inlet is 27°C, 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C. The turbine and compressor efficiencies are each 80%. Find compressor work, turbine work, heat supplied, cycle efficiency and turbine exhaust temperature. Mass of air may be considered as 1 kg. Draw T-s diagram.* **(N.U.)**

Solution. Refer to Fig. 13.56.

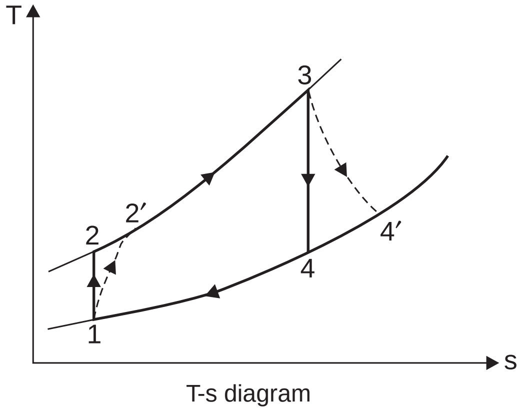


Fig. 13.56

Given : $T_1 = 27 + 273 = 300$ K ; $p_1 = 0.1$ MPa ; $r_p = 6.25$, $T_3 = 800 + 273 = 1073$ K ; $\eta_{\text{comp.}} = \eta_{\text{turbine}} = 0.8$.

For the *compression process 1-2*, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6.25)^{\frac{1.4-1}{1.4}} = 1.688$$

or $$T_2 = 300 \times 1.688 = 506.4 \text{ K}$$

Also, $$\eta_{\text{comp.}} = \frac{T_2 - T_1}{T_2' - T_1} \quad \text{or} \quad 0.8 = \frac{506.4 - 300}{T_2' - 300}$$

or $$T_2' = \frac{506.4 - 300}{0.8} + 300 = 558 \text{ K}$$

∴ **Compressor work,** $W_{\text{comp.}} = 1 \times c_p \times (T_2' - T_1)$

$= 1 \times 1.005 \, (558 - 300) =$ **259.29 kJ/kg. (Ans.)**

For *expansion process 3-4,* we have

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6.25)^{\frac{1.4-1}{1.4}} = 1.688$$

or $$T_4 = \frac{T_3}{1.688} = \frac{1073}{1.688} = 635.66 \text{ K}$$

Also, $$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4} \quad \text{or} \quad 0.8 = \frac{1073 - T_4'}{1073 - 635.66}$$

or $$T_4' = 1073 - 0.8 \, (1073 - 635.66) = 723.13 \text{ K}$$

∴ **Turbine work,** $W_{\text{turbine}} = 1 \times c_p \times (T_3 - T_4')$ (neglecting fuel mass)

$= 1 \times 1.005 \, (1073 - 723.13) =$ **351.6 kJ/kg. (Ans.)**

Net work output, $W_{\text{net}} = W_{\text{turbine}} - W_{\text{comp.}} = 351.6 - 259.29 = 92.31$ kJ/kg

Heat supplied, $Q_s = 1 \times c_p \times (T_3 - T_2')$

$= 1 \times 1.005 \times (1073 - 558) =$ **517.57 kJ/kg. (Ans.)**

Cycle efficiency, $$\eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_s} = \frac{92.31}{517.57} = 0.1783 \text{ or } \mathbf{17.83\%}. \quad \textbf{(Ans.)}$$

Turbine exhaust temperature, T_4' = 723.13 K or **450.13°C. (Ans.)**

The *T-s* diagram is shown in Fig. 13.56.

Example 13.38. *Find the required air-fuel ratio in a gas turbine whose turbine and compressor efficiencies are 85% and 80%, respectively. Maximum cycle temperature is 875°C. The working fluid can be taken as air (c_p = 1.0 kJ/kg K, γ = 1.4) which enters the compressor at 1 bar and 27°C. The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg. There is a loss of 10% of calorific value in the combustion chamber.* **(GATE, 1998)**

Solution. *Given :* η_{turbine} = 85% ; $\eta_{\text{compressor}}$ = 80% ; T_3 = 273 + 875 = 1148 K ; T_1 = 27 + 273 = 300 K ; c_p = 1.0 kJ/kg K ; γ = 1.4 ; p_1 = 1 bar; p_2 = 4 bar (since pressure ratio is 4) ; C = 42000 kJ/kg K, η_{cc} = 90% (since loss in the combustion chamber is 10%)

For isentropic compression 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

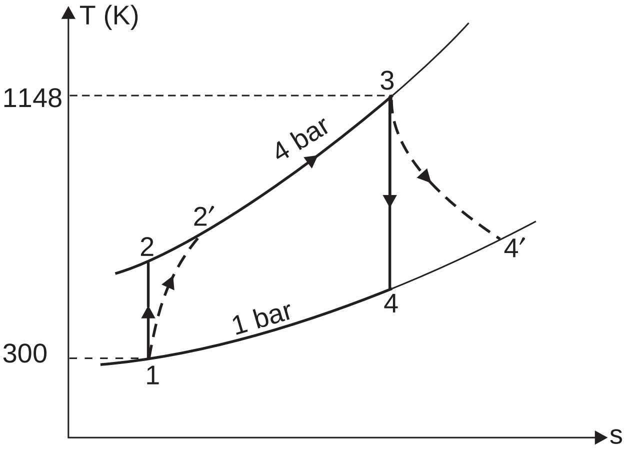


Fig. 13.57

∴ $$T_2 = 300 \times 1.486 = 445.8 \text{ K}$$

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

or $$0.8 = \frac{445.8 - 300}{T_2' - 300}$$

or $$T_2' = \frac{445.8 - 300}{0.8} + 300 = 482.2 \text{ K}$$

Now, heat supplied by the fuel = heat taken by the burning gases

$$0.9 \times m_f \times C = (m_a + m_f) \times c_p \times (T_3 - T_2')$$

∴ $$C = \left(\frac{m_a + m_f}{m_f}\right) \times \frac{c_p (T_3 - T_2')}{0.9} = \left(\frac{m_a}{m_f} + 1\right) \times \frac{c_p (T_3 - T_2')}{0.9}$$

or $$42000 = \left(\frac{m_a}{m_f} + 1\right) \times \frac{1.00(1148 - 482.27)}{0.9} = 739.78 \left(\frac{m_a}{m_f} + 1\right)$$

∴ $$\frac{m_a}{m_f} = \frac{42000}{739.78} - 1 = 55.77 \text{ say } 56$$

∴ **A/F ratio = 56 : 1. (Ans.)**

Example 13.39. *A gas turbine unit receives air at 1 bar and 300 K and compresses it adiabatically to 6.2 bar. The compressor efficiency is 88%. The fuel has a heating valve of 44186 kJ/kg and the fuel-air ratio is 0.017 kJ/kg of air. The turbine internal efficiency is 90%. Calculate the work of turbine and compressor per kg of air compressed and thermal efficiency.*

For products of combustion, $c_p = 1.147$ *kJ/kg K and* $\gamma = 1.333$. **(UPSC, 1992)**

Solution. *Given* : $p_1 (= p_4) = 1$ bar, $T_1 = 300$ K ; $p_2 (= p_3) = 6.2$ bar ; $\eta_{\text{compressor}} = 88\%$;

$C = 44186$ kJ/kg ; Fuel-air ratio = 0.017 kJ/kg of air,

$\eta_{\text{turbine}} = 90\%$; $c_p = 1.147$ kJ/kg K ; $\gamma = 1.333$.

For isentropic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6.2}{1}\right)^{\frac{1.4-1}{1.4}} = 1.684$$

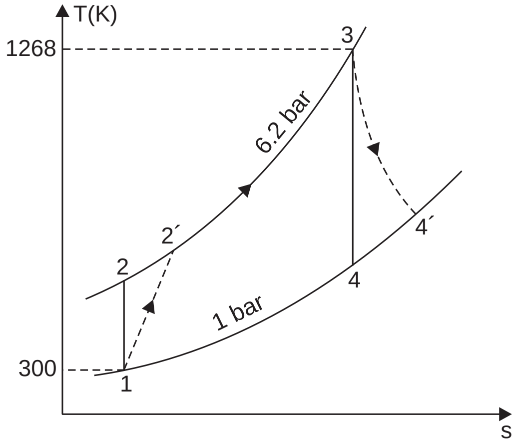


Fig. 13.58

$\therefore \quad T_2 = 300 \times 1.684 = 505.2$ K

Now, $\eta_{\text{compressor}} = \dfrac{T_2 - T_1}{T_2' - T_1}$

$$0.88 = \frac{505.2 - 300}{T_2' - 300}$$

$$T_2' = \left(\frac{505.2 - 300}{0.88} + 300\right)$$

$$= 533.2 \text{ K}$$

Heat supplied $= (m_a + m_f) \times c_p(T_3 - T_2') = m_f \times C$

or $$\left(1 + \frac{m_f}{m_a}\right) \times c_p(T_3 - T_2') = \frac{m_f}{m_a} \times C$$

or $$(1 + 0.017) \times 1.005(T_3 - 533.2) = 0.017 \times 44186$$

$$\therefore \quad T_3 = \frac{0.017 \times 44186}{(1 + 0.017) \times 1.005} + 533.2 = 1268 \text{ K}$$

For isentropic expression process 3-4 :

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6.2}\right)^{\frac{1.333-1}{1.333}} = 0.634$$

$\therefore \quad T_4 = 1268 \times 0.634 = 803.9$ K $\quad$ ($\because$ $\gamma_g = 1.333$ ......*Given*)

Now, $$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.9 = \frac{1268 - T_4'}{1268 - 803.9}$$

$$\therefore \quad T_4' = 1268 - 0.9(1268 - 803.9) = 850.3 \text{ K}$$

$$W_{\text{compresssor}} = c_p(T_2' - T_1) = 1.005(533.2 - 300) = 234.4 \text{ kJ/kg}$$

$$W_{\text{turbine}} = c_{pg}(T_3 - T_4') = 1.147(1268 - 850.3) = 479.1 \text{ kJ/kg}$$

$$\text{Net work} = W_{\text{turbine}} - W_{\text{compressor}}$$

$$= 479.1 - 234.4 = 244.7 \text{ kJ/kg}$$

Heat supplied per kg of air $= 0.017 \times 44186 = 751.2$ kJ/kg

$\therefore$ Thermal efficiency, $\eta_{\text{th.}} = \dfrac{\text{Net work}}{\text{Heat supplied}}$

$$= \frac{244.7}{751.2} = 0.3257 \quad \text{or} \quad \mathbf{32.57\%.}$$ **(Ans.)**

Example 13.40. *The air enters the compressor of an open cycle constant pressure gas turbine at a pressure of 1 bar and temperature of 20°C. The pressure of the air after compression is 4 bar. The isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The air-fuel ratio used is 90 : 1. If flow rate of air is 3.0 kg/s, find :*

(*i*) *Power developed.*

(*ii*) *Thermal efficiency of the cycle.*

Assume $c_p = 1.0$ *kJ/kg K and* $\gamma = 1.4$ *for air and gases.*

Calorific value of fuel = 41800 kJ/kg.

Solution. *Given* : $p_1 = 1$ bar ; $T_1 = 20 + 273 = 293$ K

$p_2 = 4$ bar ; $\eta_{\text{compressor}} = 80\%$; $\eta_{\text{turbine}} = 85\%$

Air-fuel ratio = 90 : 1 ; Air flow rate, $m_a = 3.0$ kg/s

(*i*) **Power developed, P :**

Refer to Fig. 13.59 (*b*)

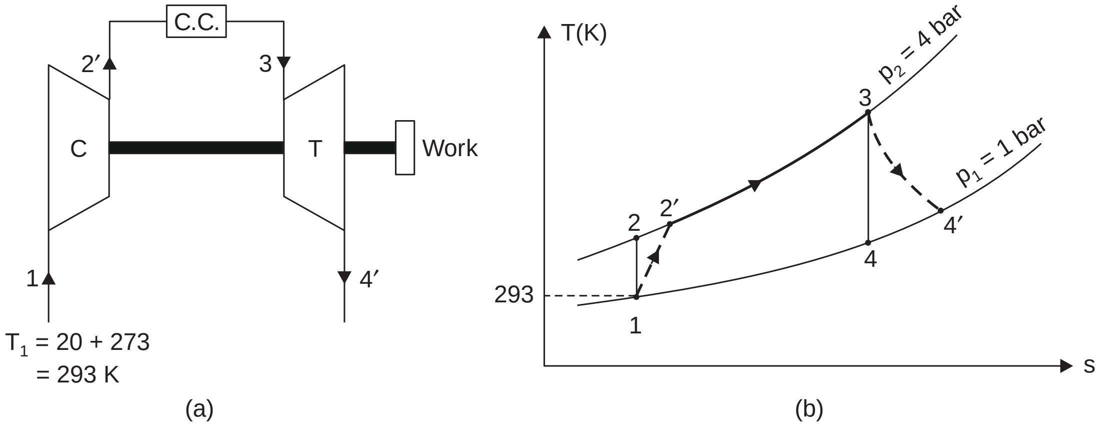
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$$\therefore \quad T_2 = (20 + 273) \times 1.486 = 435.4 \text{ K}$$

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2' - 293}$$

$$\therefore \quad T_2' = \frac{435.4 - 293}{0.8} + 293 = 471 \text{ K}$$



(a) (b)

Fig. 13.59

Heat supplied by fuel = Heat taken by burning gases

$$m_f \times C = (m_a + m_f)\, c_p (T_3 - T_2')$$

(where m_a = mass of air, m_f = mass of fuel)

$$\therefore \qquad C = \left(\frac{m_a}{m_f} + 1\right) c_p \, (T_3 - T_2')$$

$$\therefore \qquad 41800 = (90 + 1) \times 1.0 \times (T_3 - 471)$$

i.e.,
$$T_3 = \frac{41800}{91} + 471 = 930 \text{ K}$$

Again,
$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{0.4}{0.4}} = 0.672$$

$$\therefore \qquad T_4 = 930 \times 0.672 = 624.9 \text{ K}$$

$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.85 = \frac{930 - T_4'}{930 - 624.9}$$

$$\therefore \qquad T_4' = 930 - 0.85 \, (930 - 624.9) = 670.6 \text{ K}$$

$$W_{\text{turbine}} = m_g \times c_p \times (T_3 - T_4')$$

(where m_g is the mass of hot gases formed per kg of air)

$$\therefore \qquad W_{\text{turbine}} = \left(\frac{90+1}{90}\right) \times 1.0 \times (930 - 670.6)$$

$$= 262.28 \text{ kJ/kg of air.}$$

$$W_{\text{compressor}} = m_a \times c_p \times (T_2' - T_1) = 1 \times 1.0 \times (471 - 293)$$

$$= 178 \text{ kJ/kg of air}$$

$$W_{\text{net}} = W_{\text{turbine}} - W_{\text{compressor}}$$

$$= 262.28 - 178 = 84.28 \text{ kJ/kg of air.}$$

Hence *power developed*, $P = 84.28 \times 3 =$ **252.84 kW/kg of air. (Ans.)**

(*ii*) **Thermal efficiency of cycle, η_{thermal} :**

Heat supplied per kg of air passing through combustion chamber

$$= \frac{1}{90} \times 41800 = 464.44 \text{ kJ/kg of air}$$

$$\therefore \qquad \eta_{\text{thermal}} = \frac{\text{Work output}}{\text{Heat supplied}} = \frac{84.28}{464.44} = 0.1814 \text{ or } \mathbf{18.14\%.} \quad \textbf{(Ans.)}$$

Example 13.41. *A gas turbine unit has a pressure ratio of 6 : 1 and maximum cycle temperature of 610°C. The isentropic efficiencies of the compressor and turbine are 0.80 and 0.82 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 16 kg/s.*

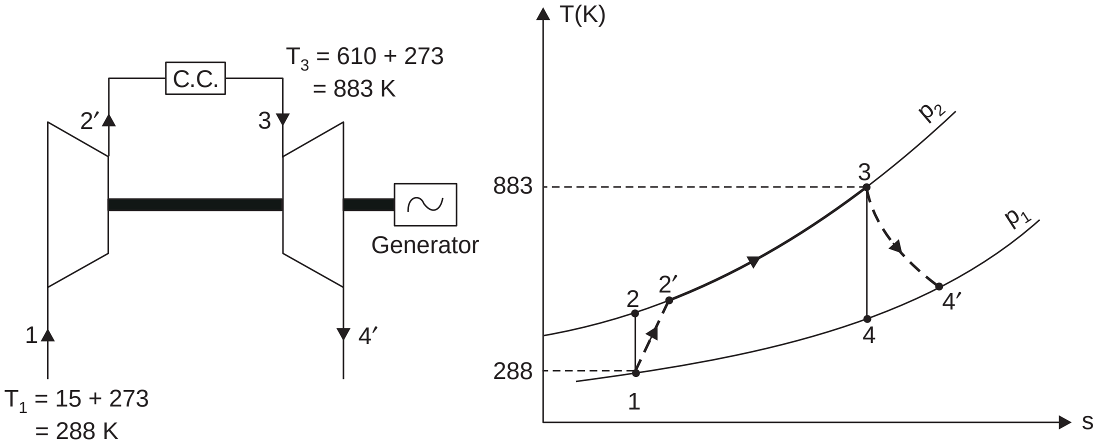
Take c_p = 1.005 kJ/kg K and γ = 1.4 for the compression process, and take c_p = 1.11 kJ/kg K and γ = 1.333 for the expansion process.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K ; $T_3 = 610 + 273 = 883$ K ; $\frac{p_2}{p_1} = 6$,

$\eta_{\text{compressor}} = 0.80$; $\eta_{\text{turbine}} = 0.82$; Air flow rate = 16 kg/s

For compression process : c_p = 1.005 kJ/kg K, γ = 1.4

For expansion process : c_p = 1.11 kJ/kg K, γ = 1.333



(a) (b)

Fig. 13.60

In order to evaluate the net work output it is necessary to calculate temperatures T_2' and T_4'. To calculate these temperatures we must first calculate T_2 and then use the isentropic efficiency.

For an isentropic process, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.67$$

$\therefore$ $$T_2 = 288 \times 1.67 = 481 \text{ K}$$

Also, $$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{481 - 288}{T_2' - T_1}$$

$\therefore$ $$T_2' = \frac{481 - 288}{0.8} + 288 = 529 \text{ K}$$

Similarly for the turbine, $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.333-1}{1.333}} = 1.565$$

$\therefore$ $$T_4 = \frac{T_3}{1.565} = \frac{883}{1.565} = 564 \text{ K}$$

Also, $$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{883 - T_4'}{883 - 564}$$

$\therefore$ $$0.82 = \frac{883 - T_4'}{883 - 564}$$

$\therefore$ $$T_4' = 883 - 0.82\,(883 - 564) = 621.4 \text{ K}$$

Hence,

Compressor work input, $W_{\text{compressor}} = c_p\,(T_2' - T_1)$

$= 1.005\,(529 - 288) = 242.2$ kJ/kg

Turbine work output, $W_{\text{turbine}} = c_p\,(T_3 - T_4')$

$= 1.11\,(883 - 621.4) = 290.4$ kJ/kg.

∴ Net work output, $W_{\text{net}} = W_{\text{turbine}} - W_{\text{compressor}}$

$= 290.4 - 242.2 = 48.2$ kJ/kg

Power in kilowatts $= 48.2 \times 16 =$ **771.2 kW. (Ans.)**

Example 13.42. *Calculate the thermal efficiency and work ratio of the plant is example 5.2, assuming that* c_p *for the combustion process is 1.11 kJ/kg K.*

Solution. Heat supplied $= c_p(T_3 - T_2')$

$= 1.11\,(883 - 529) = 392.9$ kJ/kg

$$\eta_{\text{thermal}} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{48.2}{392.9} = 0.1226 \text{ or } \mathbf{12.26\%}. \quad \textbf{(Ans.)}$$

Now,
$$\textit{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{48.2}{W_{\text{turbine}}} = \frac{48.2}{290.4} = \mathbf{0.166}. \quad \textbf{(Ans.)}$$

Example 13.43. *In a constant pressure open cycle gas turbine air enters at 1 bar and 20°C and leaves the compressor at 5 bar. Using the following data : Temperature of gases entering the turbine = 680°C, pressure loss in the combustion chamber = 0.1 bar,* $\eta_{compressor} = 85\%$, $\eta_{turbine} = 80\%$, $\eta_{combustion} = 85\%$, $\gamma = 1.4$ *and* $c_p = 1.024$ *kJ/kg K for air and gas, find :*

(*i*) *The quantity of air circulation if the plant develops 1065 kW.*

(*ii*) *Heat supplied per kg of air circulation.*

(*iii*) *The thermal efficiency of the cycle.*

Mass of the fuel may be neglected.

Solution. *Given* : $p_1 = 1$ bar, $p_2 = 5$ bar, $p_3 = 5 - 0.1 = 4.9$ bar, $p_4 = 1$ bar,

$T_1 = 20 + 273 = 293$ K, $T_3 = 680 + 273 = 953$ K,

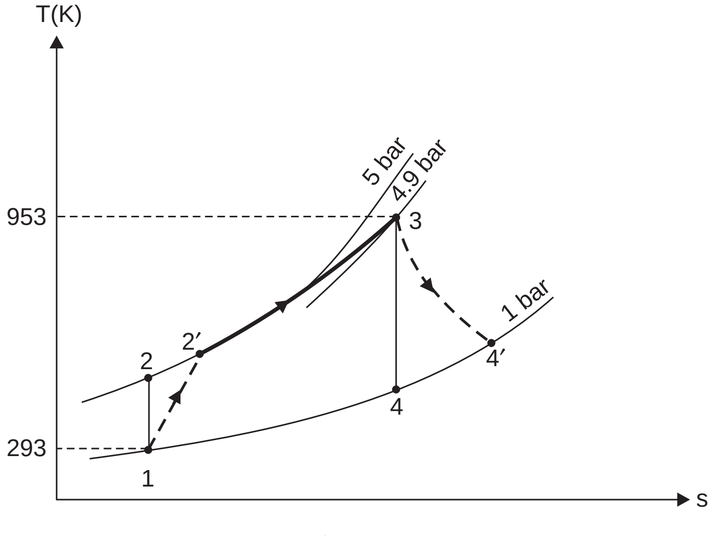


Fig. 13.61

$$\eta_{\text{compressor}} = 85\%,\ \eta_{\text{turbine}} = 80\%,\ \eta_{\text{combustion}} = 85\%.$$

For air and gases : $c_p = 1.024$ kJ/kg K, $\gamma = 1.4$

Power developed by the plant, $P = 1065$ kW.

(*i*) **The quantity of air circulation, m_a :**

For *isentropic compression 1-2*,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 1.584$$

$$\therefore \qquad T_2 = 293 \times 1.584 = 464 \text{ K}$$

Now, $$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1} \quad i.e. \quad 0.85 = \frac{464 - 293}{T_2' - 293}$$

$$\therefore \qquad T_2' = \frac{464 - 293}{0.85} + 293 = 494 \text{ K}$$

For *isentropic expansion process 3-4,*

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4.9}\right)^{\frac{1.4-1}{1.4}} = 0.635$$

$$\therefore \qquad T_4 = 953 \times 0.635 = 605 \text{ K}$$

Now, $$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.8 = \frac{953 - T_4'}{953 - 605}$$

$$\therefore \qquad T_4' = 953 - 0.8(953 - 605) = 674.6 \text{ K}$$

$$W_{\text{compressor}} = c_p\,(T_2' - T_1) = 1.024\,(494 - 293) = 205.8 \text{ kJ/kg}$$

$$W_{\text{turbine}} = c_p\,(T_3 - T_4') = 1.024\,(953 - 674.6) = 285.1 \text{ kJ/kg.}$$

$$\therefore \qquad W_{\text{net}} = W_{\text{turbine}} - W_{\text{compressor}} = 285.1 - 205.8 = 79.3 \text{ kJ/kg of air}$$

If the mass of air flowing is m_a kg/s, the power developed by the plant is given by

$$P = m_a \times W_{net} \text{ kW}$$

$$1065 = m_a \times 79.3$$

$$\therefore \qquad m_a = \frac{1065}{79.3} = 13.43 \text{ kg.}$$

i.e., **Quantity of air circulation = 13.43 kg. (Ans.)**

(*ii*) **Heat supplied per kg of air circulation :**

Actual heat supplied per kg of air circulation

$$= \frac{c_p\,(T_3 - T_2')}{\eta_{\text{combustion}}} = \frac{1.024\,(953 - 494)}{0.85} = 552.9 \text{ kJ/kg.}$$

(*iii*) **Thermal efficiency of the cycle, η_{thermal} :**

$$\eta_{\text{thermal}} = \frac{\text{Work output}}{\text{Heat supplied}} = \frac{79.3}{552.9} = 0.1434 \quad \text{or} \quad \mathbf{14.34\%.\ (Ans.)}$$

Example 13.44. *In a gas turbine the compressor is driven by the high pressure turbine. The exhaust from the high pressure turbine goes to a free low pressure turbine which runs the load. The air flow rate is 20 kg/s and the minimum and maximum temperatures are respectively 300 K and 1000 K. The compressor pressure ratio is 4. Calculate the pressure ratio of the low pressure turbine and the temperature of exhaust gases from the unit. The compressor and turbine are isentropic.* C_p *of air and exhaust gases = 1 kJ/kg K and* $\gamma = 1.4$. **(GATE, 1995)**

Solution. *Given :* $\dot{m}_a = 20$ kg/s ; $T_1 = 300$ K ; $T_3 = 1000$ K, $\frac{p_2}{p_1} = 4$; $c_p = 1$ kJ/kg K ; $\gamma = 1.4$.

Pressure ratio of low pressure turbine, $\mathbf{\frac{p_4}{p_5}}$ **:**

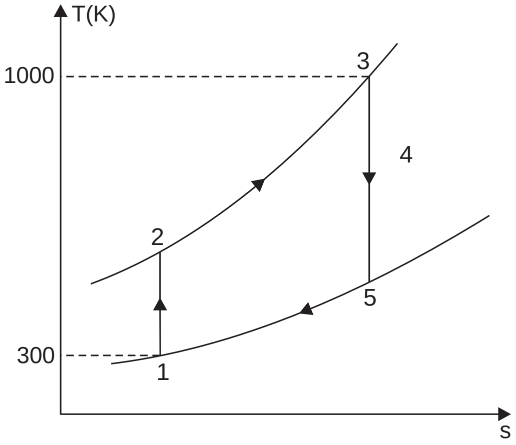


Fig. 13.62

Since the compressor is driven by high pressure turbine,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{0.4}{1.4}} = 1.486$$

or $T_2 = 300 \times 1.486 = 445.8$ K

Also, $\dot{m}_a c_p (T_2 - T_1) = \dot{m}_a c_p (T_3 - T_4)$

(neglecting mass of fuel)

or $T_2 - T_1 = T_3 - T_4$

$445.8 - 300 = 1000 - T_4$, or $T_4 = 854.2$ K

For process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad \frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{1.4}{0.4}}$$

or
$$\frac{p_3}{p_4} = \left(\frac{1000}{854.2}\right)^{3.5} = 1.736$$

Now,
$$\frac{p_3}{p_4} = \frac{p_3}{p_5} \times \frac{p_5}{p_4} = 4 \times \frac{p_5}{p_4} \qquad \left(\because \frac{p_3}{p_5} = \frac{p_2}{p_1} = 4\right)$$

$$\therefore \quad \frac{p_5}{p_4} = \frac{1}{4}\left(\frac{p_3}{p_4}\right) = \frac{1}{4} \times 1.736 = 0.434$$

Hence pressure ratio of low pressure turbine = $\frac{p_4}{p_5} = \frac{1}{0.434}$ = **2.3. (Ans.)**

Temperature of the exhaust from the unit, $\mathbf{T_5}$ **:**

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = (2.3)^{\frac{1.4-1}{1.4}} = 1.269$$

$$\therefore \quad \mathbf{T_5} = \frac{T_4}{1.269} = \frac{854.2}{1.269} = \mathbf{673\ K.} \quad \textbf{(Ans.)}$$

☞**Example 13.45.** *Air is drawn in a gas turbine unit at 15°C and 1.01 bar and pressure ratio is 7 : 1. The compressor is driven by the H.P. turbine and L.P. turbine drives a separate* *power shaft. The isentropic efficiencies of compressor, and the H.P. and L.P. turbines are 0.82, 0.85 and 0.85 respectively. If the maximum cycle temperature is 610°C, calculate :*

(*i*) *The pressure and temperature of the gases entering the power turbine.*

(*ii*) *The net power developed by the unit per kg/s mass flow.*

(*iii*) *The work ratio.*

(*iv*) *The thermal efficiency of the unit.*

Neglect the mass of fuel and assume the following :

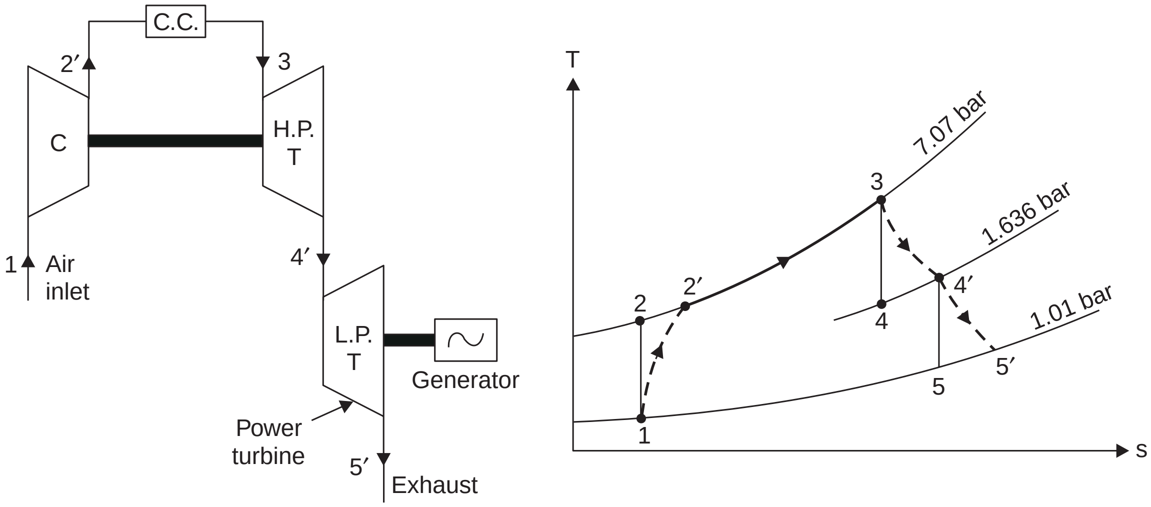
For compression process $c_{pa} = 1.005$ *kJ/kg K and* $\gamma = 1.4$

For combustion and expansion processes : $c_{pg} = 1.15$ *kJ/kg K and* $\gamma = 1.333$.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K, $p_1 = 1.01$ bar, Pressure ratio $= \dfrac{p_2}{p_1} = 7$,

$\eta_{\text{compressor}} = 0.82$, η_{turbine} (H.P.) = 0.85, η_{turbine} (L.P.) = 0.85,

Maximum cycle temperature, $T_3 = 610 + 273 = 883$ K



(a) (b)

Fig. 13.63

(*i*) **Pressure and temperature of the gases entering the power turbine, p_4' and T_4' :**

Considering isentropic compression 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = (7)^{\frac{1.4 - 1}{1.4}} = 1.745$$

$$\therefore \qquad T_2 = 288 \times 1.745 = 502.5 \text{ K}$$

Also
$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{502.5 - 288}{T_2' - 288}$$

$$\therefore \qquad T_2' = \frac{502.5 - 288}{0.82} + 288 = 549.6 \text{ K}$$

$$W_{\text{compressor}} = c_{pa}(T_2' - T_1) = 1.005 \times (549.6 - 288) = 262.9 \text{ kJ/kg}$$

Now, the *work output of H.P. turbine = Work input to compressor*

$$\therefore \qquad c_{pg}(T_3 - T_4') = 262.9$$

i.e.,
$$1.15(883 - T_4') = 262.9$$

$$\therefore \qquad T_4' = 883 - \frac{262.9}{1.15} = 654.4 \text{ K}$$

i.e., Temperature of gases entering the power turbine = **654.4 K. (Ans.)**

Again, for H.P. turbine :

$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

i.e.,
$$0.85 = \frac{883 - 654.4}{883 - T_4}$$

$$\therefore \qquad T_4 = 883 - \left(\frac{883 - 654.4}{0.85}\right) = 614 \text{ K}$$

Now, considering isentropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

or
$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{883}{614}\right)^{\frac{1.33}{0.33}} = 4.32$$

i.e.,
$$p_4 = \frac{p_3}{4.32} = \frac{7.07}{4.32} = 1.636 \text{ bar}$$

i.e., Pressure of gases entering the power turbine = **1.636 bar. (Ans.)**

(*ii*) **Net power developed per kg/s mass flow, P :**

To find the power output it is now necessary to calculate T_5'.

The pressure ratio, $\frac{p_4}{p_5}$, is given by $\frac{p_4}{p_3} \times \frac{p_3}{p_5}$

i.e.,
$$\frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_2}{p_1} \qquad (\because \quad p_2 = p_3 \text{ and } p_5 = p_1)$$

$$= \frac{7}{4.32} = 1.62$$

Then,
$$\frac{T_4'}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma - 1}{\gamma}} = (1.62)^{\frac{0.33}{1.33}} = 1.127$$

$$\therefore \qquad T_5 = \frac{T_4'}{1.127} = \frac{654.4}{1.127} = 580.6 \text{ K.}$$

Again, for L.P. turbine :

$$\eta_{\text{turbine}} = \frac{T_4' - T_5'}{T_4' - T_5}$$

i.e.,
$$0.85 = \frac{654.4 - T_5'}{654.4 - 580.6}$$

$\therefore$
$$T_5' = 654.4 - 0.85\,(654.4 - 580.6) = 591.7 \text{ K}$$

$$W_{\text{L.P. turbine}} = c_{pg}\,(T_4' - T_5') = 1.15\,(654.4 - 591.7) = 72.1 \text{ kJ/kg}$$

Hence *net power output* (per kg/s mass flow)

$$= \mathbf{72.1\ kW.}\quad \textbf{(Ans.)}$$

(*iii*) **Work ratio :**

$$\textit{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{72.1}{72.1 + 262.9} = \mathbf{0.215.}\quad \textbf{(Ans.)}$$

(*iv*) **Thermal efficiency of the unit, η_{thermal} = ?**

Heat supplied $= c_{pg}(T_3 - T_2') = 1.15(883 - 549.6) = 383.4$ kJ/kg

$\therefore$
$$\eta_{\text{thermal}} = \frac{\text{Net work output}}{\text{Heat supplied}}$$

$$= \frac{72.1}{383.4} = 0.188 \text{ or } \mathbf{18.8\%.}\quad \textbf{(Ans.)}$$

Example 13.46. *In a gas turbine the compressor takes in air at a temperature of 15°C and compresses it to four times the initial pressure with an isentropic efficiency of 82%. The air is then passes through a heat exchanger heated by the turbine exhaust before reaching the combustion chamber. In the heat exchanger 78% of the available heat is given to the air. The maximum temperature after constant pressure combustion is 600°C, and the efficiency of the turbine is 70%. Neglecting all losses except those mentioned, and assuming the working fluid throughout the cycle to have the characteristic of air find the efficiency of the cycle.*

Assume R = 0.287 kJ/kg K and γ = 1.4 for air and constant specific heats throughout.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K, Pressure ratio, $\frac{p_2}{p_1} = \frac{p_3}{p_4} = 4$, $\eta_{\text{compressor}} = 82\%$.

Effectiveness of the heat exchanger, $\varepsilon = 0.78$,

$\eta_{\text{turbine}} = 70\%$, Maximum temperature, $T_3 = 600 + 273 = 873$ K.

Efficiency of the cycle, η_{cycle} :

Considering the isentropic compression 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = (4)^{\frac{1.4 - 1}{1.4}} = 1.486$$

$\therefore$
$$T_2 = 288 \times 1.486 = 428 \text{ K}$$

Now,
$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

i.e.,
$$0.82 = \frac{428 - 288}{T_2' - 288}$$

$\therefore$
$$T_2' = \frac{428 - 288}{0.82} + 288 = 459 \text{ K}$$

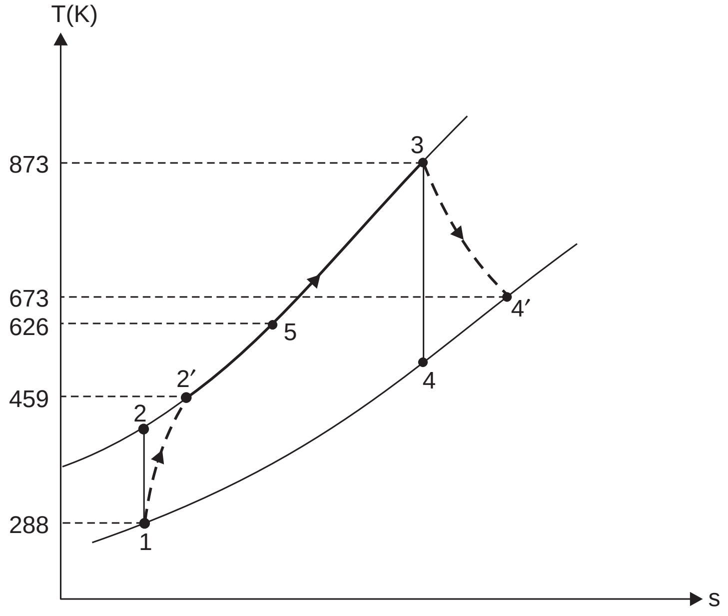


Fig. 13.64

Considering the isentropic expansion process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

$$\therefore \qquad T_4 = \frac{T_3}{1.486} = \frac{873}{1.486} = 587.5 \text{ K.}$$

Again, $$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{873 - T_4'}{873 - 587.5}$$

i.e., $$0.70 = \frac{873 - T_4'}{873 - 587.5}$$

$$\therefore \qquad T_4' = 873 - 0.7(873 - 587.5) = 673 \text{ K}$$

$$W_{\text{compressor}} = c_p(T_2' - T_1)$$

But $$c_p = R \times \frac{\gamma}{\gamma - 1} = 0.287 \times \frac{1.4}{1.4 - 1} = 1.0045 \text{ kJ/kg K}$$

$$\therefore \qquad W_{\text{compressor}} = 1.0045(459 - 288) = 171.7 \text{ kJ/kg}$$

$$W_{\text{turbine}} = c_p(T_3 - T_4') = 1.0045(873 - 673) = 200.9 \text{ kJ/kg}$$

$$\therefore \qquad \text{Net work} = W_{\text{turbine}} - W_{\text{compressor}} = 200.9 - 171.7 = 29.2 \text{ kJ/kg.}$$

Effectiveness for heat exchanger, $$\varepsilon = \frac{T_5 - T_2'}{T_4' - T_2'}$$

i.e., $$0.78 = \frac{T_5 - 459}{673 - 459}$$

$$\therefore \qquad T_5 = (673 - 459) \times 0.78 + 459 = 626 \text{ K}$$

∴ Heat supplied by fuel per kg

$$= c_p(T_3 - T_5) = 1.0045(873 - 626) = 248.1 \text{ kJ/kg}$$

∴ $$\eta_{\text{cycle}} = \frac{\text{Net work done}}{\text{Heat supplied by the fuel}} = \frac{29.2}{248.1}$$

$$= 0.117 \text{ or } \mathbf{11.7\%.} \quad \textbf{(Ans.)}$$

Example 13.47. *A gas turbine employs a heat exchanger with a thermal ratio of 72%. The turbine operates between the pressures of 1.01 bar and 4.04 bar and ambient temperature is 20°C. Isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The pressure drop on each side of the heat exchanger is 0.05 bar and in the combustion chamber 0.14 bar. Assume combustion efficiency to be unity and calorific value of the fuel to be 41800 kJ/kg.*

Calculate the increase in efficiency due to heat exchanger over that for simple cycle.

Assume c_p is constant throughout and is equal to 1.024 kJ/kg K, and assume $\gamma = 1.4$.

For simple cycle the air-fuel ratio is 90 : 1, and for the heat exchange cycle the turbine entry temperature is the same as for a simple cycle.

Solution. Simple Cycle. Refer Fig. 13.65.

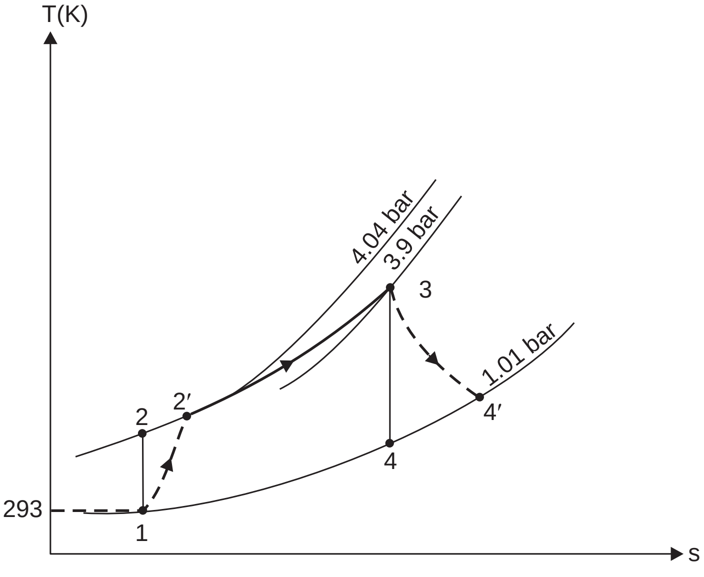


Fig. 13.65

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ $$T_2 = 293 \times 1.486 = 435.4$$

Also, $$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2' - 293}$$

∴ $$T_2' = \frac{435.4 - 293}{0.8} + 293 = 471 \text{ K}$$

Now $$m_f \times C = (m_a + m_f) \times c_p \times (T_3 - T_2')$$

[m_a = mass of air, m_f = mass of fuel]

∴ $$T_3 = \frac{m_f \times C}{c_p\,(m_a + m_f)} + T_2' = \frac{1 \times 41800}{1.024\,(90+1)} + 471 = 919.5 \text{ K}$$

Also, $$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$T_4 = T_3 \times \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = 919.5 \times \left(\frac{1.01}{3.9}\right)^{\frac{1.4-1}{1.4}} = 625 \text{ K}$$

Again, $$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

∴ $$0.85 = \frac{919.5 - T_4'}{919.5 - 625}$$

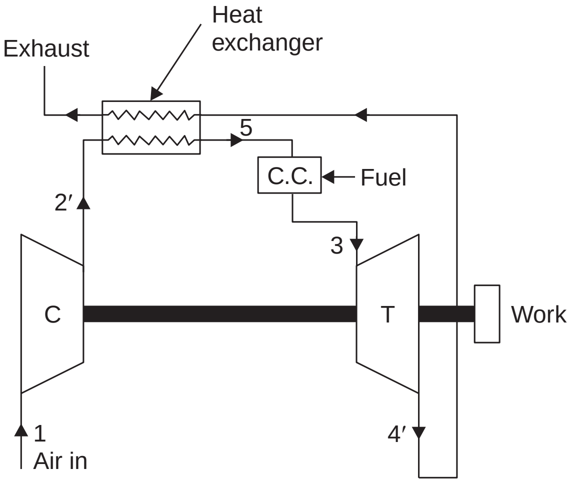
∴ $$T_4' = 919.5 - 0.85(919.5 - 625) = 669 \text{ K}$$

$$\eta_{\text{thermal}} = \frac{(T_3 - T_4') - (T_2' - T_1)}{(T_3 - T_2')}$$

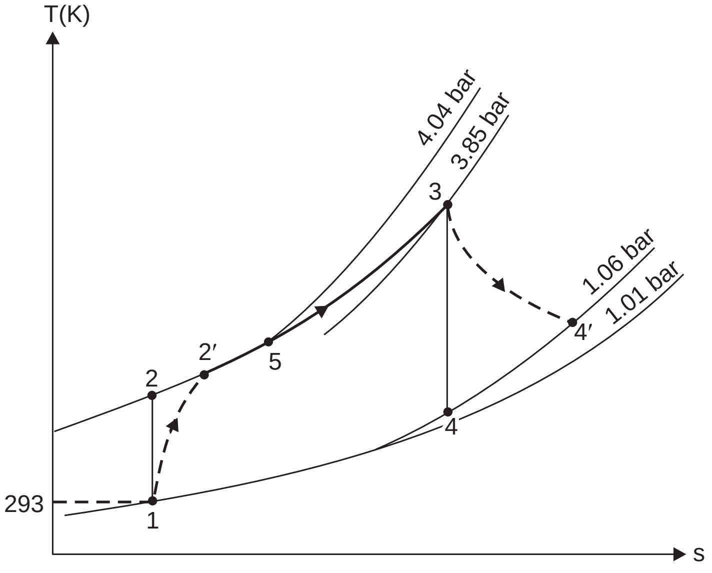
$$= \frac{(919.5 - 669) - (471 - 293)}{(919.5 - 471)} = \frac{72.5}{448.5} = 0.1616 \text{ or } \mathbf{16.16\%.}$$ **(Ans.)**

Heat Exchanger Cycle. Refer Figs. 13.66 (*a*) and (*b*)

$$T_2' = 471 \text{ K (as for simple cycle)}$$
$$T_3 = 919.5 \text{ K (as for simple cycle)}$$



(a)



(b)

Fig. 13.66

To find T_4' :

$$p_3 = 4.04 - 0.14 - 0.05 = 3.85 \text{ bar}$$

$$p_4 = 1.01 + 0.05 = 1.06 \text{ bar}$$

$$\therefore \quad \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.06}{3.85}\right)^{\frac{1.4-1}{1.4}} = 0.69$$

i.e., $$T_4 = 919.5 \times 0.69 = 634 \text{ K}$$

$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4} \; ; \quad 0.85 = \frac{919.5 - T_4'}{919.5 - 634}$$

$$\therefore \quad T_4' = 919.5 - 0.85\,(919.5 - 634) = 677 \text{ K}$$

To find T_5 :

Thermal ratio (or effectiveness),

$$\varepsilon = \frac{T_5 - T_2'}{T_4' - T_2'} \qquad \therefore \quad 0.72 = \frac{T_5 - 471}{677 - 471}$$

$$\therefore \quad T_5 = 0.72(677 - 471) + 471 = 619 \text{ K}$$

$$\eta_{\text{thermal}} = \frac{(T_3 - T_4') - (T_2' - T_1)}{(T_3 - T_5)}$$

$$= \frac{(919.5 - 677) - (471 - 293)}{(919.5 - 619)} = \frac{64.5}{300.5} = 0.2146 \text{ or } \mathbf{21.46\%}$$

∴ **Increase in thermal efficiency** = 21.46 – 16.16 = **5.3%. (Ans.)**

☞**Example 13.48.** *A 5400 kW gas turbine generating set operates with two compressor stages, the overall pressure ratio is 9 : 1. A high pressure turbine is used to drive the compressors,* *and a low-pressure turbine drives the generator. The temperature of the gases at entry to the high pressure turbine is 625°C and the gases are reheated to 625°C after expansion in the first turbine. The exhaust gases leaving the low-pressure turbine are passed through a heat exchanger to heat the air leaving the high pressure stage compressor. The compressors have equal pressure ratios and intercooling is complete between the stages. The air inlet temperature to the unit is 20°C. The isentropic efficiency of each compressor stage is 0.8, and the isentropic efficiency of each turbine stage is 0.85, the heat exchanger thermal ratio is 0.8. A mechanical efficiency of 95% can be assumed for both the power shaft and compressor turbine shaft. Neglecting all pressure losses and changes in kinetic energy calculate :*

(*i*) *The thermal efficiency* (*ii*) *Work ratio of the plant*

(*iii*) *The mass flow in kg/s.*

Neglect the mass of the fuel and assume the following :

For air : c_{pa} *= 1.005 kJ/kg K and* $\gamma = 1.4$.

For gases in the combustion chamber and in turbines and heat exchanger, c_{pg} *= 1.15 kJ/kg K and* $\gamma = 1.333$.

Solution. Refer Fig. 13.67.

Given : $T_1 = 20 + 273 = 293$ K, $T_6 = T_8 = 625 + 273 = 898$ K

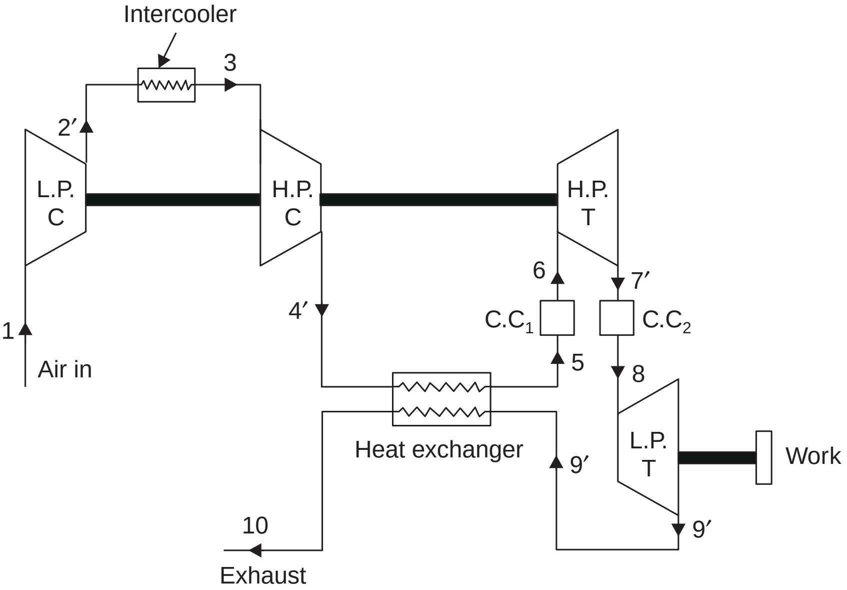
Efficiency of each compressor stage = 0.8

Efficiency of each turbine stage = 0.85

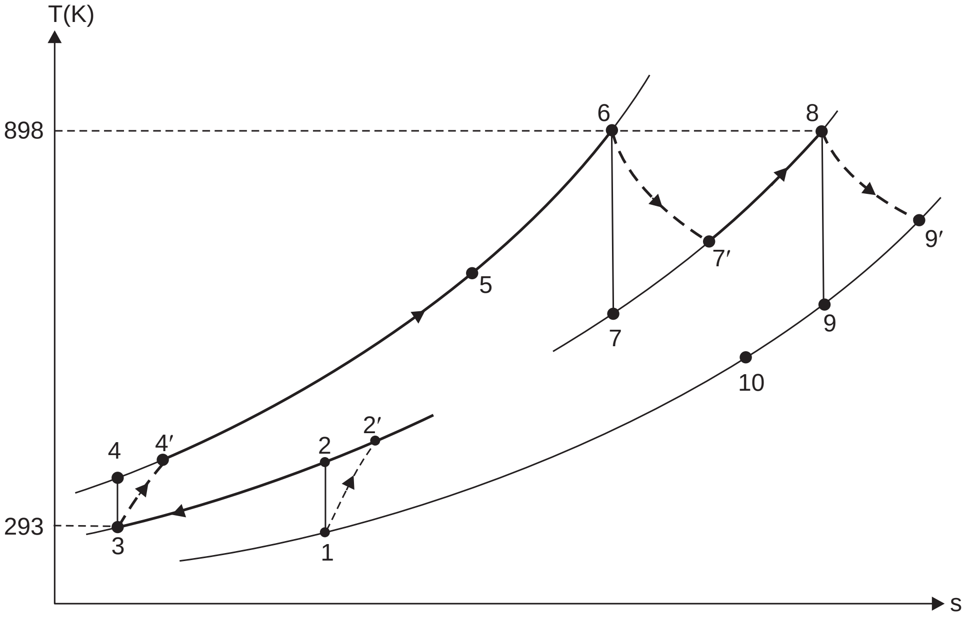
$$\eta_{\text{mech.}} = 0.95, \ \varepsilon = 0.8$$

(*i*) **Thermal efficiency,** $\eta_{\textbf{thermal}}$ **:**

Since the pressure ratio and the isentropic efficiency of each compressor is the same then the work input required for each compressor is the same since both compressor have the same air inlet temperature *i.e.*, $T_1 = T_3$ and $T_2' = T_4'$.



(a)



(b)

Fig. 13.67

Also, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$ and $\frac{p_2}{p_1} = \sqrt{9} = 3$

∴ $T_2 = (20 + 273) \times (3)^{\frac{1.4 - 1}{1.4}} = 401 \text{ K}$

Now, $\eta_{\text{compressor}} \text{ (L.P.)} = \frac{T_2 - T_1}{T_2' - T_1}$

$$0.8 = \frac{401 - 293}{T_2' - 293}$$

i.e., $T_2' = \frac{401 - 298}{0.8} + 293 = 428 \text{ K}$

Work input per compressor stage

$$= c_{pa}(T_2' - T_1) = 1.005\ (428 - 293) = 135.6 \text{ kJ/kg}$$

The H.P. turbine is required to drive both compressors and to overcome mechanical friction.

i.e., Work output of H.P. turbine $= \frac{2 \times 135.6}{0.95} = 285.5$ kJ/kg

∴ $c_{pg}\ (T_6 - T_7') = 285.5$

i.e., $1.15\ (898 - T_7') = 285.5$

∴ $T_7' = 898 - \frac{285.5}{1.15} = 650 \text{ K}$

Now, $\eta_{\text{turbine (H.P.)}} = \frac{T_6 - T_7'}{T_6 - T_7}$; $0.85 = \frac{898 - 650}{898 - T_7}$

∴ $$T_7 = 898 - \left(\frac{898 - 650}{0.85}\right) = 606 \text{ K}$$

Also, $$\frac{T_6}{T_7} = \left(\frac{p_6}{p_7}\right)^{\frac{\gamma - 1}{\gamma}}$$

or $$\frac{p_6}{p_7} = \left(\frac{T_6}{T_7}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{898}{606}\right)^{\frac{1.333}{0.333}} = 4.82$$

Then, $$\frac{p_8}{p_9} = \frac{9}{4.82} = 1.86$$

Again, $$\frac{T_8}{T_9} = \left(\frac{p_8}{p_9}\right)^{\frac{\gamma - 1}{\gamma}} = (1.86)^{\frac{1.333 - 1}{1.333}} = 1.16$$

∴ $$T_9 = \frac{T_8}{1.16} = \frac{898}{1.16} = 774 \text{ K}$$

Also, $$\eta_{\text{turbine (L.P.)}} = \frac{T_8 - T_9'}{T_8 - T_9} \; ; \quad 0.85 = \frac{898 - T_9'}{898 - 774}$$

∴ $$T_9' = 898 - 0.85\,(898 - 774) = 792.6 \text{ K}$$

∴ Net work output $= c_{pg}(T_8 - T_9') \times 0.95$

$= 1.15\,(898 - 792.6) \times 0.95 = 115.15$ kJ/kg

Thermal ratio or effectiveness of heat exchanger,

$$\varepsilon = \frac{T_5 - T_4'}{T_9' - T_4'} = \frac{T_5 - 428}{792.6 - 428}$$

i.e., $$0.8 = \frac{T_5 - 428}{792.6 - 428}$$

∴ $$T_5 = 0.8\,(792.6 - 428) + 428 = 719.7 \text{ K}$$

Now, Heat supplied $= c_{pg}(T_6 - T_5) + c_{pg}(T_8 - T_7')$

$= 1.15\,(898 - 719.7) + 1.15\,(898 - 650) = 490.2$ kJ/kg

∴ $$\eta_{\text{thermal}} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{115.15}{490.2}$$

$= 0.235$ or **23.5%. (Ans.)**

(*ii*) **Work ratio :**

Gross work of the plant $= W_{\text{turbine (H.P.)}} + W_{\text{turbine (L.P.)}}$

$$= 285.5 + \frac{115.15}{0.95} = 406.7 \text{ kJ/kg}$$

∴ **Work ratio** $$= \frac{\text{Net work output}}{\text{Gross work output}} = \frac{115.15}{406.7} = \mathbf{0.283.}$$ **(Ans.)**

(*iii*) **Mass flow rate, $\dot{m}$:**

Let the mass flow be $\dot{m}$, then

$$\dot{m} \times 115.15 = 4500$$

$$\therefore \qquad \dot{m} = \frac{4500}{115.15} = 39.08 \text{ kg/s}$$

i.e., **Mass flow** **= 39.08 kg/s. (Ans.)**

Example 13.49. *In a closed cycle gas turbine there is a two stage compressor and a two stage turbine. All the components are mounted on the same shaft. The pressure and temperature at the inlet of the first stage compressor are 1.5 bar and 20°C. The maximum cycle temperature and pressure are limited to 750°C and 6 bar. A perfect intercooler is used between the two stage compressors and a reheater is used between the two turbines. Gases are heated in the reheater to 750°C before entering into the L.P. turbine. Assuming the compressor and turbine efficiencies as 0.82, calculate :*

(*i*) *The efficiency of the cycle without regenerator.*

(*ii*) *The efficiency of the cycle with a regenerator whose effectiveness is 0.70.*

(*iii*) *The mass of the fluid circulated if the power developed by the plant is 350 kW.*

The working fluid used in the cycle is air. For air :

$$\gamma = 1.4 \text{ and } c_p = 1.005 \text{ kJ/kg K.}$$

Solution. *Given* : $T_1 = 20 + 273 = 293$ K, $T_5 = T_7 = 750 + 273 = 1023$ K, $p_1 = 1.5$ bar,

$p_2 = 6$ bar, $\eta_{\text{compressor}} = \eta_{\text{turbine}} = 0.82$,

Effectiveness of regenerator, $\varepsilon = 0.70$, Power developed, $P = 350$ kW.

For air : $c_p = 1.005$ kJ/kgK, $\gamma = 1.4$

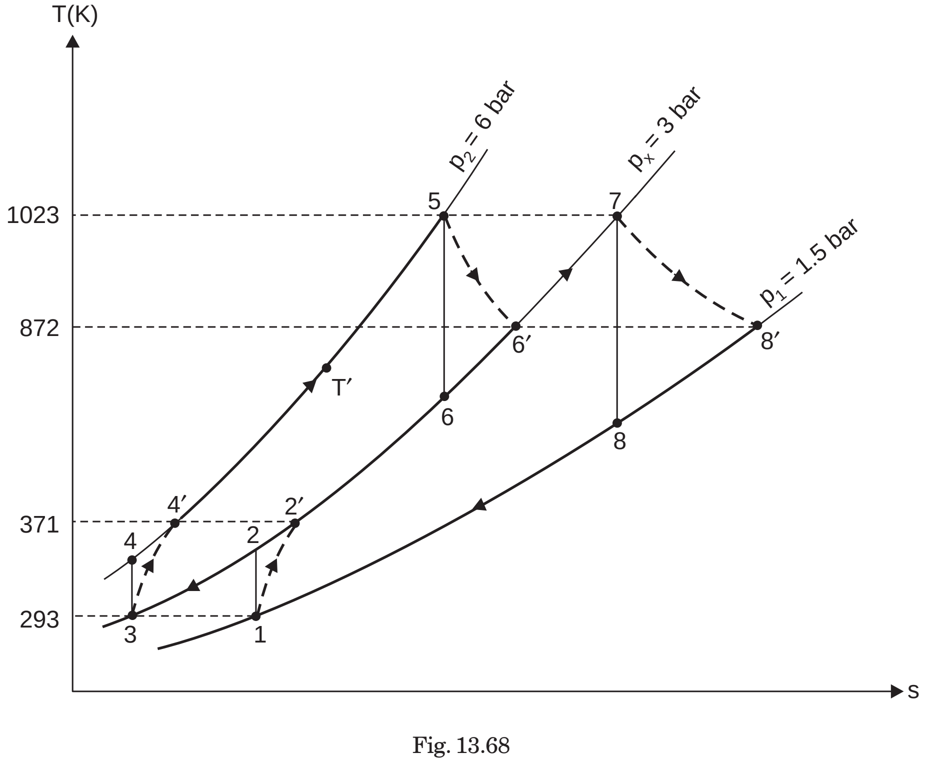


Fig. 13.68

As per given conditions : $T_1 = T_3$, $T_2' = T_4'$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad p_x = \sqrt{p_1 p_2} = \sqrt{1.5 \times 6} = 3 \text{ bar}$$

Now,
$$T_2 = T_1 \times \left(\frac{p_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}} = 293 \times \left(\frac{3}{1.5}\right)^{\frac{1.4-1}{1.4}} = 357 \text{ K}$$

$$\eta_{\text{compressor (L.P.)}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{357 - 293}{T_2' - 293}$$

$\therefore$
$$T_2' = \frac{357 - 293}{0.82} + 293 = 371 \text{ K}$$

i.e.,
$$T_2' = T_4' = 371 \text{ K}$$

Now,
$$\frac{T_5}{T_6} = \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_x}\right)^{\frac{1.4-1}{1.4}} \qquad \left[\because \begin{array}{l} p_5 = p_2 \\ p_6 = p_x \end{array}\right]$$

$$\frac{1023}{T_6} = \left(\frac{6}{3}\right)^{0.286} = 1.219$$

$\therefore$
$$T_6 = \frac{1023}{1.219} = 839 \text{ K}$$

$$\eta_{\text{turbine (H.P.)}} = \frac{T_5 - T_6'}{T_5 - T_6}$$

$$0.82 = \frac{1023 - T_6'}{1023 - 839}$$

$\therefore$
$$T_6' = 1023 - 0.82\,(1023 - 839) = 872 \text{ K}$$

$$T_8' = T_6' = 872 \text{ K} \quad \text{as} \quad \eta_{\text{turbine (H.P.)}} = \eta_{\text{turbine (L.P.)}}$$

and
$$T_7 = T_5 = 1023 \text{ K}$$

Effectiveness of regenerator, $\varepsilon = \dfrac{T' - T_4'}{T_8' - T_4'}$

where T' is the temperature of air coming out of regenerator

$\therefore$
$$0.70 = \frac{T' - 371}{872 - 371} \quad \textit{i.e.,} \quad T' = 0.70\,(872 - 371) + 371 = 722 \text{ K}$$

Net work available, $W_{\text{net}} = [W_{T(\text{L.P.})} + W_{T(\text{L.P.})}] - [W_{C(\text{H.P.})} + W_{C(\text{L.P.})}]$

$= 2\,[W_{T(\text{L.P.})} - W_{C(\text{L.P.})}]$ as the *work developed by each turbine is same and work absorbed by each compressor is same.*

$\therefore$
$$W_{\text{net}} = 2c_p\,[(T_5 - T_6') - (T_2' - T_1)]$$
$$= 2 \times 1.005\,[(1023 - 872) - (371 - 293)] = 146.73 \text{ kJ/kg of air}$$

Heat supplied per kg of air *without regenerator*

$$= c_p(T_5 - T_4') + c_p(T_7 - T_6')$$
$$= 1.005\,[(1023 - 371) + (1023 - 872)] = 807 \text{ kJ/kg of air}$$

Heat supplied per kg of *air with regenerator*

$$= c_p(T_5 - T') + c_p(T_7 - T_6')$$

$$= 1.005\ [(1023 - 722) + (1023 - 872)]$$

$$= 454.3 \text{ kJ/kg}$$

(*i*) $\eta_{\textbf{thermal (without regenerator)}} = \dfrac{146.73}{807} = 0.182$ or **18.2%. (Ans.)**

(*ii*) $\eta_{\textbf{thermal (with regenerator)}} = \dfrac{146.73}{454.3} = 0.323$ or **32.3%. (Ans.)**

(*iii*) **Mass of fluid circulated, $\dot{m}$:**

Power developed, $P = 146.73 \times \dot{m}$ kW

$\therefore$ $350 = 146.73 \times \dot{m}$

i.e., $\dot{m} = \dfrac{350}{146.73} = 2.38$ kg/s

i.e., **Mass of fluid circulated = 2.38 kg/s. (Ans.)**

HIGHLIGHTS

1. A cycle is defined as a repeated series of operations occurring in a certain order.
2. The efficiency of an engine using air as the working medium is known as an '**Air standard efficiency**'.
3. Relative efficiency, $\eta_{\text{relative}} = \dfrac{\text{Actual thermal efficiency}}{\text{Air standard efficiency}}$.
4. **Carnot cycle** efficiency, $\eta_{\text{Carnot}} = \dfrac{T_1 - T_2}{T_1}$.
5. **Otto cycle** efficiency, $\eta_{\text{Otto}} = 1 - \dfrac{1}{(r)^{\gamma - 1}}$.

   Mean effective pressure, $p_{m(\text{Otto})} = \dfrac{p_1\, r[(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)}$.
6. **Diesel cycle** efficiency, $\eta_{\text{Diesel}} = 1 - \dfrac{1}{\gamma(r)^{\gamma-1}}\left[\dfrac{\rho^\gamma - 1}{\rho - 1}\right]$

   Mean effective pressure, $p_{m(\text{Diesel})} = \dfrac{p_1 r^\gamma [\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)}$.
7. **Dual cycle** efficiency, $\eta_{\text{Dual}} = 1 - \dfrac{1}{(r)^{\gamma-1}}\left[\dfrac{(\beta.\rho^\gamma - 1)}{(\beta - 1) + \beta\gamma(\rho - 1)}\right]$

   Mean effective pressure, $p_{m(\text{Dual})} = \dfrac{p_1 r^\gamma [\beta(\rho - 1) + (\beta - 1) - r^{1-\gamma}(\beta\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)}$.
8. Atkinson cycle efficiency, $\eta_{\text{Atkinson}} = 1 - \gamma \,.\, \dfrac{(r - \alpha)}{r^\gamma - \alpha^\gamma}$

   where α = Compression ratio, r = Expansion ratio.

THEORETICAL QUESTIONS

1. What is a cycle ? What is the difference between an ideal and actual cycle ?
2. What is an air-standard efficiency ?
3. What is relative efficiency ?
4. Derive expressions of efficiency in the following cases :
   (*i*) Carnot cycle (*ii*) Diesel cycle (*iii*) Dual combustion cycle.
5. Explain "Air standard analysis" which has been adopted for I.C. engine cycles. State the assumptions made for air standard cycles.
6. Derive an expression for 'Atkinson cycle'.
7. Explain briefly Brayton cycle. Derive expression for optimum pressure ratio.
8. Describe with neat sketches the working of a simple constant pressure open cycle gas turbine.
9. Discuss briefly the methods employed for improvement of thermal efficiency of open cycle gas turbine plant.
10. Describe with neat diagram a closed cycle gas turbine. State also its merits and demerits.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The air standard Otto cycle comprises
   (*a*) two constant pressure processes and two constant volume processes
   (*b*) two constant pressure and two constant entropy processes
   (*c*) two constant volume processes and two constant entropy processes
   (*d*) none of the above.
2. The air standard efficiency of Otto cycle is given by

   (*a*) $\eta = 1 + \dfrac{1}{(r)^{\gamma+1}}$ (*b*) $\eta = 1 - \dfrac{1}{(r)^{\gamma-1}}$

   (*c*) $\eta = 1 - \dfrac{1}{(r)^{\gamma+1}}$ (*d*) $\eta = 2 - \dfrac{1}{(r)^{\gamma-1}}$.
3. The thermal efficiency of theoretical Otto cycle
   (*a*) increases with increase in compression ratio (*b*) increases with increase in isentropic index γ
   (*c*) does not depend upon the pressure ratio (*d*) follows all the above.
4. The work output of theoretical Otto cycle
   (*a*) increases with increase in compression ratio (*b*) increases with increase in pressure ratio
   (*c*) increases with increase in adiabatic index γ (*d*) follows all the above.
5. For same compression ratio
   (*a*) thermal efficiency of Otto cycle is greater than that of Diesel cycle
   (*b*) thermal efficiency of Otto cycle is less than that of Diesel cycle
   (*c*) thermal efficiency of Otto cycle is same as that for Diesel cycle
   (*d*) thermal efficiency of Otto cycle cannot be predicted.
6. In air standard Diesel cycle, at fixed compression ratio and fixed value of adiabatic index (γ)
   (*a*) thermal efficiency increases with increase in heat addition cut-off ratio
   (*b*) thermal efficiency decreases with increase in heat addition cut-off ratio
   (*c*) thermal efficiency remains same with increase in heat addition cut-off ratio
   (*d*) none of the above.

7. Thermal efficiency of a gas turbine plant as compared to Diesel engine plant is

(*a*) higher
(*b*) lower
(*c*) same
(*d*) may be higher or lower.

8. Mechanical efficiency of a gas turbine as compared to internal combustion reciprocating engine is

(*a*) higher
(*b*) lower
(*c*) same
(*d*) un-predictable.

9. For a gas turbine the pressure ratio may be in the range

(*a*) 2 to 3
(*b*) 3 to 5
(*c*) 16 to 18
(*d*) 18 to 22.

10. The air standard efficiency of closed gas turbine cycle is given by (r_p = pressure ratio for the compressor and turbine)

(*a*) $\eta = 1 - \dfrac{1}{(r_p)^{\gamma - 1}}$
(*b*) $\eta = 1 - (r_p)^{\gamma - 1}$
(*c*) $\eta = 1 - \left(\dfrac{1}{r_p}\right)^{\frac{\gamma - 1}{\gamma}}$
(*d*) $\eta = (r_p)^{\frac{\gamma - 1}{\gamma}} - 1$.

11. The work ratio of closed cycle gas turbine plant depends upon

(*a*) pressure ratio of the cycle and specific heat ratio
(*b*) temperature ratio of the cycle and specific heat ratio
(*c*) pressure ratio, temperature ratio and specific heat ratio
(*d*) only on pressure ratio.

12. Thermal efficiency of closed cycle gas turbine plant increases by

(*a*) reheating
(*b*) intercooling
(*c*) regenerator
(*d*) all of the above.

13. With the increase in pressure ratio thermal efficiency of a simple gas turbine plant with fixed turbine inlet temperature

(*a*) decreases
(*b*) increases
(*c*) first increases and then decreases
(*d*) first decreases and then increases.

14. The thermal efficiency of a gas turbine cycle with ideal regenerative heat exchanger is

(*a*) equal to work ratio
(*b*) is less than work ratio
(*c*) is more than work ratio
(*d*) unpredictable.

15. In a two stage gas turbine plant reheating after first stage

(*a*) decreases thermal efficiency
(*b*) increases thermal efficiency
(*c*) does not effect thermal efficiency
(*d*) none of the above.

16. In a two stage gas turbine plant, reheating after first stage

(*a*) increases work ratio
(*b*) decreases work ratio
(*c*) does not affect work ratio
(*d*) none of the above.

17. In a two stage gas turbine plant, with intercooling and reheating

(*a*) both work ratio and thermal efficiency improve
(*b*) work ratio improves but thermal efficiency decreases
(*c*) thermal efficiency improves but work ratio decreases
(*d*) both work ratio and thermal efficiency decrease.

18. For a jet propulsion unit, ideally the compressor work and turbine work are

(*a*) equal
(*b*) unequal
(*c*) not related to each other
(*d*) unpredictable.

19. Greater the difference between jet velocity and aeroplane velocity

(*a*) greater the propulsive efficiency (*b*) less the propulsive efficiency

(*c*) unaffected is the propulsive efficiency (*d*) none of the above.

ANSWERS

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*d*) **5.** (*a*) **6.** (*b*) **7.** (*b*)
8. (*b*) **9.** (*d*) **10.** (*d*) **11.** (*a*) **12.** (*b*) **13.** (*b*) **14.** (*b*)
15. (*d*) **16.** (*d*) **17.** (*a*) **18.** (*a*) **19.** (*c*).

UNSOLVED EXAMPLES

1. A Carnot engine working between 377°C and 37°C produces 120 kJ of work. Determine :

(*i*) The heat added in kJ. (*ii*) The entropy change during heat rejection process.

(*iii*) The engine thermal efficiency. [**Ans.** (*i*) 229.5 kJ ; (*ii*) 0.353 kJ/K (*iii*) 52.3%]

2. Find the thermal efficiency of a Carnot engine whose hot and cold bodies have temperatures of 154°C and 15°C respectively. [**Ans.** 32.55%]

3. Derive an expression for change in efficiency for a change in compression ratio. If the compression ratio is increased from 6 to 8, what will be the percentage increase in efficiency ? [**Ans.** 8%]

4. The efficiency of an Otto cycle is 50% and γ is 1.5. What is the compression ratio ? [**Ans.** 4]

5. An engine working on Otto cycle has a volume of 0.5 m^3, pressure 1 bar and temperature 27°C at the commencement of compression stroke. At the end of compression stroke, the pressure is 10 bar. Heat added during the constant volume process is 200 kJ. Determine :

(*i*) Percentage clearance (*ii*) Air standard efficiency

(*iii*) Mean effective pressure

(*iv*) Ideal power developed by the engine if the engine runs at 400 r.p.m. so that there are 200 complete cycles per minutes. [**Ans.** (*i*) 23.76% ; (*ii*) 47.2% ; (*iii*) 2.37 bar (*iv*) 321 kW]

6. The compression ratio in an air-standard Otto cycle is 8. At the beginning of compression process, the pressure is 1 bar and the temperature is 300 K. The heat transfer to the air per cycle is 1900 kJ/kg of air. Calculate :

(*i*) Thermal efficiency (*ii*) The mean effective pressure.

[**Ans.** (*i*) 56.47% ; (*ii*) 14.24 bar]

7. An engine 200 mm bore and 300 mm stroke works on Otto cycle. The clearance volume is 0.0016 m^3. The initial pressure and temperature are 1 bar and 60°C. If the maximum pressure is limited to 24 bar, find :

(*i*) The air-standard efficiency of the cycle (*ii*) The mean effective pressure for the cycle.

Assume ideal conditions. [**Ans.** (*i*) 54.08% ; (*ii*) 1.972 bar]

8. Calculate the air standard efficiency of a four stroke Otto cycle engine with the following data :

Piston diameter (bore) = 137 mm ; Length of stroke = 130 mm ;

Clearance volume 0.00028 m^3.

Express clearance as a percentage of swept volume. [**Ans.** 56.1% ; 14.6%]

9. In an ideal Diesel cycle, the temperatures at the beginning of compression, at the end of compression and at the end of the heat addition are 97°C, 789°C and 1839°C. Find the efficiency of the cycle.

[**Ans.** 59.6%]

10. An air-standard Diesel cycle has a compression ratio of 18, and the heat transferred to the working fluid per cycle is 1800 kJ/kg. At the beginning of the compression stroke, the pressure is 1 bar and the temperature is 300 K. Calculate : (*i*) Thermal efficiency, (*ii*) The mean effective pressure.

[**Ans.** (*i*) 61% ; (*ii*) 13.58 bar]

11. 1 kg of air is taken through a Diesel cycle. Initially the air is at 15°C and 1 ata. The compression ratio is 15 and the heat added is 1850 kJ. Calculate : (*i*) The ideal cycle efficiency, (*ii*) The mean effective pressure. [**Ans.** (*i*) 55.1% ; (*ii*) 13.4 bar]

12. What will be loss in the ideal efficiency of a Diesel engine with compression ratio 14 if the fuel cut-off is delayed from 6% to 9% ? [**Ans.** 2.1%]

13. The pressures on the compression curve of a diesel engine are at $\frac{1}{8}$ th stroke 1.4 bar and at $\frac{7}{8}$ th stroke 14 bar. Estimate the compression ratio. Calculate the air standard efficiency of the engine if the cut-off occurs at $\frac{1}{15}$th of the stroke. [**Ans.** 18.54 ; 63.7%]

14. A compression ignition engine has a stroke 270 mm, and a cylinder diameter of 165 mm. The clearance volume is 0.000434 m^3 and the fuel ignition takes place at constant pressure for 4.5 per cent of the stroke. Find the efficiency of the engine assuming it works on the Diesel cycle. [**Ans.** 61.7%]

15. The following data belong to a Diesel cycle :

Compression ratio = 16 : 1 ; Heat added = 2500 kJ/kg ; Lowest pressure in the cycle = 1 bar ; Lowest temperature in the cycle = 27°C.

Determine :

(*i*) Thermal efficiency of the cycle. (*ii*) Mean effective pressure.

[**Ans.** (*i*) 45% ; (*ii*) 16.8 bar]

16. The compression ratio of an air-standard Dual cycle is 12 and the maximum pressure in the cycle is limited to 70 bar. The pressure and temperature of cycle at the beginning of compression process are 1 bar and 300 K. Calculate : (*i*) Thermal efficiency, (*ii*) Mean effective pressure.

Assume : cylinder bore = 250 mm, stroke length = 300 mm, c_p = 1.005, c_v = 0.718 and γ = 1.4.

[**Ans.** (*i*) 61.92% ; (*ii*) 9.847 bar]

17. The compression ratio of a Dual cycle is 10. The temperature and pressure at the beginning of the cycle are 1 bar and 27°C. The maximum pressure of the cycle is limited to 70 bar and heat supplied is limited to 675 kJ/kg of air. Find the thermal efficiency of the cycle. [**Ans.** 59.5%]

18. An air standard Dual cycle has a compression ratio of 16, and compression begins at 1 bar, 50°C. The maximum pressure is 70 bar. The heat transferred to air at constant pressure is equal to that at constant volume. Determine :

(*i*) The cycle efficiency. (*ii*) The mean effective pressure of the cycle.

Take : c_p = 1.005 kJ/kg-K, c_v = 0.718 kJ/kg-K. [**Ans.** (*i*) 66.5% ; (*ii*) 4.76 bar]

19. In an air standard gas turbine engine, air at a temperature of 15°C and a pressure of 1.01 bar enters the compressor, where it is compressed through a pressure ratio of 5. Air enters the turbine at a temperature of 815°C and expands to original pressure of 1.01 bar. Determine the ratio of turbine work to compressor work and the thermal efficiency when the engine operates on ideal Brayton cycle.

Take : γ = 1.4, c_p = 1.005 kJ/kg K. [**Ans.** 2.393 ; 37.03%]

20. In an open cycle constant pressure gas turbine air enters the compressor at 1 bar and 300 K. The pressure of air after the compression is 4 bar. The isentropic efficiencies of compressor and turbine are 78% and 85% respectively. The air-fuel ratio is 80 : 1. Calculate the power developed and thermal efficiency of the cycle if the flow rate of air is 2.5 kg/s.

Take c_p = 1.005 kJ/kg K and γ = 1.4 for air and c_{pg} = 1.147 kJ/kg K and γ = 1.33 for gases. R = 0.287 kJ/kg K. Calorific value of fuel = 42000 kJ/kg. [**Ans.** 204.03 kW/kg of air ; 15.54%]

21. A gas turbine has a pressure ratio of 6/1 and a maximum cycle temperature of 600°C. The isentropic efficiencies of the compressor and turbine are 0.82 and 0.85 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 15 kg/s.

Take : c_p = 1.005 kJ/kg K and γ = 1.4 for the compression process, and take c_p = 1.11 kJ/kg K and γ = 1.333 for the expansion process. [**Ans.** 920 kW]

22. Calculate the thermal efficiency and the work ratio of the plant in example 3 (above), assuming that c_p for the combustion process is 1.11 kJ/kg K. [**Ans.** 15.8% ; 0.206]

23. The gas turbine has an overall pressure ratio of 5 : 1 and a maximum cycle temperature of 550°C. The turbine drives the compressor and an electric generator, the mechanical efficiency of the drive being 97%. The ambient temperature is 20°C and the isentropic efficiencies for the compressor and turbine are 0.8 and

0.83 respectively. Calculate the power output in kilowatts for an air flow of 15 kg/s. Calculate also the thermal efficiency and the work ratio.

Neglect changes is kinetic energy, and the loss of pressure in combustion chamber.

[**Ans.** 655 kW ; 12% ; 0.168]

24. Air is drawn in a gas turbine unit at 17°C and 1.01 bar and the pressure ratio is 8 : 1. The compressor is driven by the H.P. turbine and the L.P. turbine drives a separate power shaft. The isentropic efficiencies of the compressor, and the H.P. and L.P. turbines are 0.8, 0.85 and 0.83, respectively. Calculate the pressure and temperature of the gases entering the power turbine, the net power developed by the unit per kg/s of mass flow, the work ratio and the thermal efficiency of the unit. The maximum cycle temperature is 650°C.

For the compression process take $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$

For the combustion process and expansion process, take

$$c_p = 1.15 \text{ kJ/kg K and } \gamma = 1.333$$

Neglect the mass of fuel. [**Ans.** 1.65 bar, 393°C ; 74.5 kW ; 0.201 ; 19.1%]

25. In a gas turbine plant, air is compressed through a pressure ratio of 6 : 1 from 15°C. It is then heated to the maximum permissible temperature of 750°C and expanded in two stages each of expansion ratio $\sqrt{6}$, the air being reheated between the stages to 750°C. A heat exchanger allows the heating of the compressed gases through 75 per cent of the maximum range possible. Calculate : (*i*) The cycle efficiency (*ii*) The work ratio (*iii*) The work per kg of air.

The isentropic efficiencies of the compressor and turbine are 0.8 and 0.85 respectively.

[**Ans.** (*i*) 32.75% (*ii*) 0.3852 (*iii*) 152 kJ/kg]

26. At the design speed the following data apply to a gas turbine set employing the heat exchanger : Isentropic efficiency of compressor = 75%, isentropic efficiency of the turbine = 85%, mechanical transmission efficiency = 99%, combustion efficiency = 98%, mass flow = 22.7 kg/s, pressure ratio = 6 : 1, heat exchanger effectiveness = 75%, maximum cycle temperature = 1000 K.

The ambient air temperature and pressure are 15°C and 1.013 bar respectively. Calculate :

(*i*) The net power output (*ii*) Specific fuel consumption

(*iii*) Thermal efficiency of the cycle.

Take the lower calorific value of fuel as 43125 kJ/kg and assume no pressure-loss in heat exchanger and combustion chamber. [**Ans.** (*i*) 2019 kW (*ii*) 0.4799 kg/kWh (*iii*) 16.7%]

27. In a gas turbine plant air at 10°C and 1.01 bar is compressed through a pressure ratio of 4 : 1. In a heat exchanger and combustion chamber the air is heated to 700°C while its pressure drops 0.14 bar. After expansion through the turbine the air passes through a heat exchanger which cools the air through 75% of maximum range possible, while the pressure drops 0.14 bar, and the air is finally exhausted to atmosphere. The isentropic efficiency of the compressor is 0.80 and that of turbine 0.85.

Calculate the efficiency of the plant. [**Ans.** 22.76%]

28. In a marine gas turbine unit a high-pressure stage turbine drives the compressor, and a low-pressure stage turbine drives the propeller through suitable gearing. The overall pressure ratio is 4 : 1, and the maximum temperature is 650°C. The isentropic efficiencies of the compressor, H.P. turbine, and L.P. turbine are 0.8, 0.83, and 0.85 respectively, and the mechanical efficiency of both shafts is 98%. Calculate the pressure between turbine stages when the air intake conditions are 1.01 bar and 25°C. Calculate also the thermal efficiency and the shaft power when the mass flow is 60 kg/s. Neglect kinetic energy changes, and pressure loss in combustion. [**Ans.** 1.57 bar ; 14.9% ; 4560 kW]

29. In a gas turbine unit comprising L.P. and H.P. compressors, air is taken at 1.01 bar 27°C. Compression in L.P. stage is upto 3.03 bar followed by intercooling to 30°C. The pressure of air after H.P. compressor is 58.7 bar. Loss in pressure during intercooling is 0.13 bar. Air from H.P. compressor is transferred to heat exchanger of effectiveness 0.60 where it is heated by gases from L.P. turbine. The temperature of gases supplied to H.P. turbine is 750°C. The gases expand in H.P. turbine to 3.25 bar and are then reheated to 700°C before expanding in L.P. turbine. The loss of pressure in reheater is 0.1 bar. If isentropic efficiency of compression in both stages is 0.80 and isentropic efficiency of expansion in turbine is 0.85, calculate : (*i*) Overall efficiency (*ii*) Work ratio (*iii*) Mass flow rate when the gas power generated is 6500 kW. Neglect the mass of fuel.

Take, for air : c_p = 1.005 kJ/kg K, γ = 1.4

for gases : c_{pg} = 1.15 kJ/kg K, γ = 1.3. [**Ans.** (*i*) 16.17% (*ii*) 0.2215 (*iii*) 69.33 kg of air/sec.]

30. In a gas turbine installation, air is taken in L.P. compressor at 15°C 1.1 bar and after compression it is passed through intercooler where its temperature is reduced to 22°C. The cooled air is further compressed in H.P. unit and then passed in the combustion chamber where its temperature is increased to 677°C by burning the fuel. The combustion products expand in H.P. turbine which runs the compressor and further expansion is continued in the L.P. turbine which runs the alternator. The gases coming out from L.P. turbine are used for heating the incoming air from H.P. compressor and then exhausted to atmosphere.

Taking the following data determine : (*i*) power output (*ii*) specific fuel consumption (*iii*) Thermal efficiency :

Pressure ratio of each compressor = 2, isentropic efficiency of each compressor stage = 85%, isentropic efficiency of each turbine stage = 85%, effectiveness of heat exchanger = 0.75, air flow = 15 kg/sec., calorific value of fuel = 45000 kJ/kg, c_p(for gas) = 1 kJ/kg K, c_p (for gas) = 1.15 kJ/kg K, γ (for air) = 1.4, γ (for gas) = 1.33.

Neglect the mechanical, pressure and heat losses of the system and fuel mass also.

[**Ans.** (*i*) 1849.2 kW (*ii*) 0.241 kg/kWh (*iii*) 33.17%]

14

Refrigeration Cycles

14.1. Fundamentals of refrigeration : Introduction—Elements of refrigeration systems—Refrigeration systems—Co-efficient of performance (C.O.P.)—Standard rating of refrigeration machine. 14.2. Air refrigeration system : Introduction—Reversed Carnot cycle—Reversed Brayton cycle—Merits and demerits of air refrigeration system. 14.3. Simple vapour compression system : Introduction—Simple vapour compression cycle—Functions of parts of a simple vapour compression system—Vapour compression cycle on temperature-entropy (*T-s*) diagram—Pressure enthalpy (*p-h*) chart—Simple vapour compression cycle on *p-h* chart—Factors affecting the performance of a vapour compression system—Actual vapour compression cycle—Volumetric efficiency—Mathematical analysis of vapour compression refrigeration. 14.4. Vapour absorption system : Introduction—Simple vapour absorption system—Practical vapour absorption system—Comparison between vapour compression and vapour absorption systems. 14.5. Refrigerants—Classification of refrigerants—Desirable properties of an ideal refrigerant—Properties and uses of commonly used refrigerants—Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

14.1. FUNDAMENTALS OF REFRIGERATION

14.1.1. Introduction

Refrigeration *is the science of producing and maintaining temperatures below that of the surrounding atmosphere.* This means the removing of heat from a substance to be cooled. Heat always passes downhill, from a warm body to a cooler one, until both bodies are at the same temperature. Maintaining perishables at their required temperatures is done by refrigeration. Not only perishables but to-day many human work spaces in offices and factory buildings are air-conditioned and a refrigeration unit is the heart of the system.

Before the advent of mechanical refrigeration water was kept cool by storing it in semi-porous jugs so that the water could seep through and evaporate. The evaporation carried away heat and cooled the water. This system was used by the Egyptians and by Indians in the South-west. Natural ice from lakes and rivers was often cut during winter and stored in caves, straw-lined pits, and later in sawdust-insulated buildings to be used as required. The Romans carried pack trains of snow from Alps to Rome for cooling the Emperor's drinks. Though these methods of cooling all make use of natural phenomena, they were used to maintain a lower temperature in a space or product and may properly be called refrigeration.

In simple, *refrigeration means the cooling of or removal of heat from a system.* The equipment employed to maintain the system at a low temperature is termed as *refrigerating system* and the system which is kept at lower temperature is called *refrigerated system.* Refrigeration is generally produced in one of the following *three ways* :

(*i*) By *melting of a solid.*

(*ii*) By *sublimation of a solid.*

(*iii*) By *evaporation of a liquid.*

Most of the commercial refrigeration is produced by the evaporation of a liquid called *refrigerant. Mechanical refrigeration* depends upon the evaporation of liquid refrigerant and its circuit includes the equipments naming ***evaporator, compressor, condenser*** and ***expansion valve.*** It is used for preservation of food, manufacture of ice, solid carbon dioxide and control of air temperature and humidity in the air-conditioning system.

Important refrigeration applications :

1. Ice making
2. Transportation of foods above and below freezing
3. Industrial air-conditioning
4. Comfort air-conditioning
5. Chemical and related industries
6. Medical and surgical aids
7. Processing food products and beverages
8. Oil refining and synthetic rubber manufacturing
9. Manufacturing and treatment of metals
10. Freezing food products
11. Miscellaneous applications :
    (*i*) Extremely low temperatures
    (*ii*) Plumbing
    (*iii*) Building construction etc.

14.1.2. Elements of Refrigeration Systems

All refrigeration systems must include atleast *four basic units* as given below :

(*i*) *A low temperature thermal "sink" to which heat will flow from the space to be cooled.*

(*ii*) *Means of extracting energy from the sink, raising the temperature level of this energy, and delivering it to a heat receiver.*

(*iii*) *A receiver to which heat will be transferred from the high temperature high-pressure refrigerant.*

(*iv*) *Means of reducing of pressure and temperature of the refrigerant as it returns from the receiver to the "sink".*

14.1.3. Refrigeration Systems

The various refrigeration systems may be enumerated as below :

1. Ice refrigeration
2. Air refrigeration system
3. Vapour compression refrigeration system
4. Vapour absorption refrigeration system
5. Special refrigeration systems
    (*i*) Adsorption refrigeration system
    (*ii*) Cascade refrigeration system
    (*iii*) Mixed refrigeration system
    (*iv*) Vortex tube refrigeration system
    (*v*) Thermoelectric refrigeration
    (*vi*) Steam jet refrigeration system.

14.1.4. Co-efficient of Performance (C.O.P.)

The performance of a refrigeration system is expressed by a term known as the *"co-efficient of performance"*, which is defined as the *ratio of heat absorbed by the refrigerant while passing through the evaporator to the work input required to compress the refrigerant in the compressor* ; in short it is the *ratio between heat extracted and work done* (in heat units).

If, R_n = Net refrigerating effect,

W = Work expanded in by the machine during the same interval of time, Then, $\text{C.O.P.} = \dfrac{R_n}{W}$

and $\text{Relative C.O.P.} = \dfrac{\text{Actual C.O.P.}}{\text{Theoretical C.O.P.}}$

where, Actual C.O.P. = Ratio of R_n and W *actually measured during a test*

and, Theoretical C.O.P. = Ratio of *theoretical values* of R_n and W *obtained by applying laws of thermodynamics to the refrigeration cycle.*

14.1.5. Standard Rating of a Refrigeration Machine

The rating of a refrigeration machine is obtained by refrigerating effect or amount of heat extracted in a given time from a body. The rating of the refrigeration machine is given by a unit of refrigeration known as ***"standard commercial tonne of refrigeration"*** which is defined *as the refrigerating effect produced by the melting of 1 tonne of ice from and at 0°C in 24 hours*. Since the latent heat of fusion of ice is 336 kJ/kg, the refrigerating effect of 336 × 1000 kJ in 24 hours is rated as one *tonne*, *i.e.,*

$$\text{1 tonne of refrigeration (TR)} = \frac{336 \times 1000}{24} = 14000 \text{ kJ/h.}$$

Note : ***Ton of refrigeration (TR).*** A ton of refrigeration is basically an American unit of refrigerating effect (R.E.). It originated from the rate at which heat is required to be removed to freeze *one ton of water from and at 0°C.* Using American units this is equal to removal of 200 BTU of heat per minute, and MKS unit it is adopted as 50 kcal/min or 3000 kcal/hour. *In S.I. units its conversion is rounded of to 3.5 kJ/s (kW) or 210 kJ/min.*

In this book we shall be adopting,

1 tonne of refrigeration = 14000 kJ/h (1 ton = 0.9 tonne).

14.2. AIR REFRIGERATION SYSTEM

14.2.1. Introduction

Air cycle refrigeration is one of the earliest methods of cooling developed. It became obsolete for several years because of its low co-efficient of performance (C.O.P.) and high operating costs. It has, however, been applied to aircraft refrigeration systems, where with low equipment weight, it can utilise a portion of the cabin air according to the supercharger capacity. The main characteristic feature of *air refrigeration system, is that throughout the cycle the refrigerant remains in gaseous state.*

The air refrigeration system can be divided in two systems :

(*i*) Closed system (*ii*) Open system.

In ***closed*** (or *dense air*) ***system*** the *air refrigerant is contained within the piping or components parts of the system at all times and refrigerator with usually pressures above atmospheric pressure.*

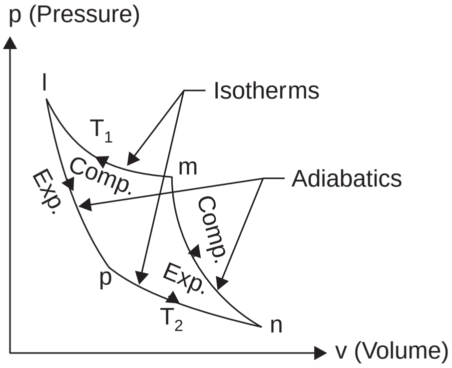
In the ***open system*** the refrigerator is replaced by the *actual space to be cooled with the air expanded to atmospheric pressure, circulated through the cold room and then compressed to the cooler pressure.* The pressure of operation in this system is inherently limited to operation *at atmospheric pressure* in the refrigerator.

A closed system claims the following *advantages* over open system : (*i*) In a closed system the suction to compressor may be at high pressure. The sizes of expander and compressor can be kept within reasonable limits by using dense air ; (*ii*) In open air system, the air picks up moisture from the products kept in the refrigerated chamber ; the moisture may freeze during expansion and is likely to choke the valves whereas it does not happen in closed system and (*iii*) In open system, the expansion of the refrigerant can be carried only upto atmospheric pressure prevailing in the cold chamber but for a closed system there is no such restriction.

14.2.2. Reversed Carnot Cycle

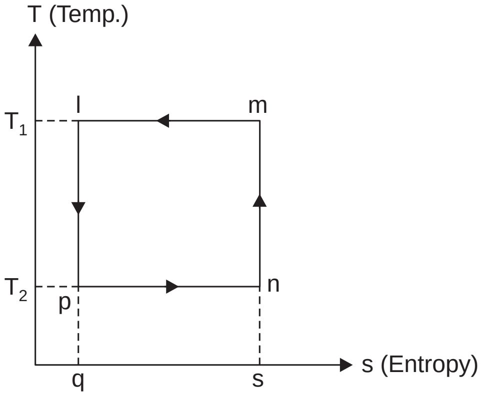
If a machine working on reversed Carnot cycle is driven from an external source, it will work or function as a refrigerator. The *production of such a machine has not been possible practically because the adiabatic portion of the stroke would need a high speed while during isothermal portion of stroke a very low speed will be necessary. This variation of speed during the stroke, however is not practicable.*

p-V and *T-s* diagrams of reversed Carnot cycle are shown in Figs. 14.1 (*a*) and (*b*). Starting from point *l*, the clearance space of the cylinder is full of air, the air is then expanded adiabatically to point *p* during which its temperature falls from T_1 to T_2, the cylinder is put in contact with a cold body at temperature T_2. The air is then expanded isothermally to the point *n*, as a result of which heat is extracted from the cold body at temperature T_2. Now the cold body is removed ; from *n* to *m* air undergoes adiabatic compression with the assistance of some external power and temperature rises to T_1. A hot body at temperature T_1 is put in contact with the cylinder. Finally the air is compressed isothermally during which process heat is rejected to the hot body.



(a)

Fig. 14.1 (*a*) *p-V* diagram for reversed Carnot cycle.



(b)

Fig. 14.1 (*b*) *T-s* diagram for a reversed Carnot cycle.

Refer Fig. 14.1 (*b*)

Heat abstracted from the cold body = Area '*npqs*' = $T_2 \times pn$

Work done per cycle = Area '*lpnm*'

$= (T_1 - T_2) \times pn$

Co-efficient of performance,

$$\text{C.O.P.} = \frac{\text{Heat extracted from the cold body}}{\text{Work done per cycle}}$$

$$= \frac{\text{Area '}npqs\text{'}}{\text{Area '}lpnm\text{'}} = \frac{T_2 \times pn}{(T_1 - T_2) \times pn} = \frac{T_2}{T_1 - T_2} \quad ...(14.1)$$

Since the co-efficient of performance (C.O.P.) means the ratio of the desired effect in kJ/kg to the energy supplied in kJ/kg, therefore C.O.P. in case of Carnot cycle run either as a refrigerating machine or a heat pump or as a heat engine will be as given below :

(*i*) **For a reversed Carnot cycle 'refrigerating machine' :**

$$\text{C.O.P.}_{\text{(ref.)}} = \frac{\text{Heat extracted from the cold body /cycle}}{\text{Work done per cycle}}$$

$$= \frac{T_2 \times pn}{(T_1 - T_2) \times pn} = \frac{T_2}{T_1 - T_2} \quad ...(14.2)$$

(*ii*) **For a Carnot cycle 'heat pump' :**

$$\text{C.O.P.}_{\text{(heat pump)}} = \frac{\text{Heat rejected to the hot body/cycle}}{\text{Work done per cycle}} = \frac{T_1 \times lm}{(T_1 - T_2) \times pn}$$

$$= \frac{T_1 \times pn}{(T_1 - T_2) \times pn} \qquad (\because \ lm = pn)$$

$$= \frac{T_1}{T_1 - T_2} \qquad ...(14.3)$$

$$= 1 + \frac{T_2}{T_1 - T_2} \qquad ...(14.4)$$

This indicates that *C.O.P. of heat pump is greater than that of a refrigerator working on reversed Carnot cycle between* the same *temperature* limits T_1 and T_2 by *unity*.

(*iii*) **For a Carnot cycle 'heat engine' :**

$$\text{C.O.P.}_{\text{(heat engine)}} = \frac{\text{Work obtained/cycle}}{\text{Heat supplied/cycle}} = \frac{(T_1 - T_2) \times pn}{T_1 \times lm} = \frac{(T_1 - T_2) \times pn}{T_1 \times pn}$$

$$(\because \ lm = pn)$$

$$= \frac{T_1 - T_2}{T_1} \qquad ...(14.5)$$

Example 14.1. *A Carnot refrigerator requires 1.3 kW per tonne of refrigeration to maintain a region at low temperature of – 38°C. Determine :*

(*i*) *C.O.P. of Carnot refrigerator*

(*ii*) *Higher temperature of the cycle*

(*iii*) *The heat delivered and C.O.P. when this device is used as heat pump.*

Solution. $T_2 = 273 - 38 = 235$ K

Power required per tonne of refrigeration = 1.3 kW

(*i*) **C.O.P. of Carnot refrigerator :**

$$\text{C.O.P.}_{\text{(Carnot ref.)}} = \frac{\text{Heat absorbed}}{\text{Work done}}$$

$$= \frac{1 \text{ tonne}}{1.3} = \frac{14000 \text{ kJ/h}}{1.3 \times 60 \times 60 \text{ kJ/h}} = \mathbf{2.99.} \quad \textbf{(Ans.)}$$

(*ii*) **Higher temperature of the cycle, T_1 :**

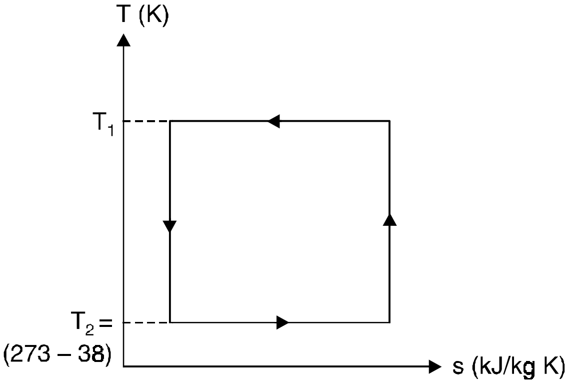


Fig. 14.2

$$\text{C.O.P.}_{\text{(Carnot ref.)}} = \frac{T_2}{T_1 - T_2}$$

i.e.,
$$2.99 = \frac{235}{T_1 - 235}$$

∴
$$T_1 = \frac{235}{2.99} + 235 = 313.6 \text{ K}$$

$$= 313.6 - 273 = \mathbf{40.6°C.} \quad \textbf{(Ans.)}$$

(*iii*) **Heat delivered as heat pump**

$$= \text{Heat absorbed} + \text{Work done}$$

$$= \frac{14000}{60} + 1.3 \times 60 = \mathbf{311.3\ kJ/min.} \quad \textbf{(Ans.)}$$

$$\mathbf{C.O.P.}_{\text{(heat pump)}} = \frac{\text{Heat delivered}}{\text{Work done}} = \frac{311.3}{1.3 \times 60} = \mathbf{3.99.} \quad \textbf{(Ans.)}$$

Example 14.2. *A refrigerating system operates on the reversed Carnot cycle. The higher temperature of the refrigerant in the system is 35°C and the lower temperature is – 15°C. The capacity is to be 12 tonnes. Neglect all losses. Determine* :

(*i*) *Co-efficient of performance.*
(*ii*) *Heat rejected from the system per hour.*
(*iii*) *Power required.*

Solution. (*i*)
$$T_1 = 273 + 35 = 308 \text{ K}$$
$$T_2 = 273 - 15 = 258 \text{ K}$$
$$\text{Capacity} = 12 \text{ tonne}$$

$$\mathbf{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{258}{308 - 258} = \mathbf{5.16.} \quad \textbf{(Ans.)}$$

(*ii*) **Heat rejected from the system per hour :**

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work input}}$$

$$5.16 = \frac{12 \times 14000 \text{ kJ/h}}{\text{Work input}}$$

∴
$$\text{Work input} = \frac{12 \times 14000}{5.16} = 32558 \text{ kJ/h.}$$

Thus, *heat rejected/hour* = Refrigerating effect/hour + Work input/hour

$$= 12 \times 14000 + 32558 = \mathbf{200558\ kJ/h.} \quad \textbf{(Ans.)}$$

(*iii*) **Power required :**

$$\textit{Power required} = \frac{\text{Work input/hour}}{60 \times 60} = \frac{32558}{60 \times 60} = \mathbf{9.04\ kW.} \quad \textbf{(Ans.)}$$

Example 14.3. *A cold storage is to be maintained at – 5°C while the surroundings are at 35°C. The heat leakage from the surroundings into the cold storage is estimated to be 29 kW. The actual C.O.P. of the refrigeration plant used is one third that of an ideal plant working between the same temperatures. Find the power required to drive the plant.* **(AMIE)**

Solution. $T_2 = -5 + 273 = 268$ K ; $T_1 = 35 + 273 = 308$ K

Heat leakage from the surroundings into the cold storage = 29 kW

$$\text{Ideal C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{268}{308 - 268} = 6.7$$

$$\text{Actual C.O.P.} = \frac{1}{3} \times 6.7 = 2.233 = \frac{R_n}{W}$$

(where R_n = net refrigerating effect, and W = work done)

or $$2.233 = \frac{29}{W} \quad \text{or} \quad W = \frac{29}{2.233} = 12.98 \text{ kJ/s}$$

Hence power required to drive the plant = **12.98 kW. (Ans.)**

Example 14.4. *Ice is formed at 0°C from water at 20°C. The temperature of the brine is – 8°C. Find out the kg of ice formed per kWh. Assume that the refrigeration cycle used is perfect reversed Carnot cycle. Take latent heat of ice as 335 kJ/kg.*

Solution. Latent heat of ice = 335 kJ/kg

$$T_1 = 20 + 273 = 293 \text{ K}$$

$$T_2 = -8 + 273 = 265 \text{ K}$$

$$\text{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{265}{293 - 265} = 9.46$$

Heat to be extracted per kg of water (to form ice at 0°C *i.e.*, 273 K), R_n

$$= 1 \times c_{pw} \times (293 - 273) + \text{latent heat of ice}$$

$$= 1 \times 4.18 \times 20 + 335 = 418.6 \text{ kJ/kg}$$

Also, $$1 \text{ kWh} = 1 \times 3600 = 3600 \text{ kJ}$$

Also, $$\text{C.O.P.} = \frac{R_n}{W} = \frac{\text{Refrigerating effect in kJ/kg}}{\text{Work done in kJ}}$$

∴ $$9.46 = \frac{m_{\text{ice}} \times 418.6}{3600}$$

i.e., $$m_{\text{ice}} = \frac{9.46 \times 3600}{418.6} = 81.35 \text{ kg}$$

Hence ice formed per kWh = 81.35 kg. (Ans.)

Example 14.5. *Find the least power of a perfect reversed heat engine that makes 400 kg of ice per hour at – 8°C from feed water at 18°C. Assume specific heat of ice as 2.09 kJ/kg K and latent heat 334 kJ/kg.*

Solution. $$T_1 = 18 + 273 = 291 \text{ K}$$

$$T_2 = -8 + 273 = 265 \text{ K}$$

$$\text{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{265}{291 - 265} = 10.19$$

Heat absorbed per kg of water (to form ice at – 8°C *i.e.*, 265 K), R_n

$$= 1 \times 4.18\ (291 - 273) + 334 + 1 \times 2.09 \times (273 - 265) = 425.96 \text{ kJ/kg}$$

Also, $$\text{C.O.P.} = \frac{\text{Net refrigerating effect}}{\text{Work done}} = \frac{R_n}{W}$$

i.e., $$10.19 = \frac{425.96 \times 400}{W}$$

$$\therefore \qquad W = \frac{425.96 \times 400}{10.19} = 16720.7 \text{ kJ/h}$$

$$= 4.64 \text{ kJ/s or } 4.64 \text{ kW}$$

Hence least power = 4.64 kW. (Ans.)

☞**Example 14.6.** *The capacity of the refrigerator (working on reversed Carnot cycle) is 280 tonnes when operating between – 10°C and 25°C. Determine :*

(*i*) *Quantity of ice produced within 24 hours when water is supplied at 20°C.*

(*ii*) *Minimum power (in kW) required.*

Solution. (*i*) **Quantity of ice produced :**

Heat to be extracted per kg of water (to form ice at 0°C)

$$= 4.18 \times 20 + 335 = 418.6 \text{ kJ/kg}$$

Heat extraction capacity of the refrigerator

$$= 280 \text{ tonnes}$$

$$= 280 \times 14000 = 3920000 \text{ kJ/h}$$

∴ Quantity of ice produced in 24 hours,

$$m_{\text{ice}} = \frac{3920000 \times 24}{418.6 \times 1000} = \mathbf{224.75 \text{ tonnes.}} \quad \textbf{(Ans.)}$$

(*ii*) **Minimum power required :**

$$T_1 = 25 + 273 = 298 \text{ K}$$

$$T_2 = -10 + 273 = 263 \text{ K}$$

$$\text{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{263}{298 - 263} = 7.51$$

Also, $$\text{C.O.P.} = \frac{\text{Net refrigerating effect}}{\text{Work done /min}} = \frac{R_n}{W}$$

i.e., $$7.51 = \frac{3920000}{W}$$

$$\therefore \qquad W = \frac{3920000}{7.51} \text{ kJ/h} = 145 \text{ kJ/s}$$

∴ **Power required = 145 kW. (Ans.)**

Example 14.7. *A cold storage plant is required to store 20 tonnes of fish. The temperature of the fish when supplied = 25°C ; storage temperature of fish required = – 8°C ; specific heat of fish above freezing point = 2.93 kJ/kg-°C ; specific heat of fish below freezing point = 1.25 kJ/kg-°C ; freezing point of fish = – 3°C. Latent heat of fish = 232 kJ/kg.*

If the cooling is achieved within 8 hours ; find out :

(*i*) *Capacity of the refrigerating plant.*

(*ii*) *Carnot cycle C.O.P. between this temperature range.*

(*iii*) *If the actual C.O.P. is* $\frac{1}{3}$*rd of the Carnot C.O.P. find out the power required to run the plant.*

Solution. Heat removed in 8 hours from each kg of fish

$$= 1 \times 2.93 \times [25 - (-3)] + 232 + 1 \times 1.25\,[-3 - (-8)]$$

$$= 82.04 + 232 + 6.25 = 320.29 \text{ kJ/kg}$$

Heat removed by the plant /min

$$= \frac{320.29 \times 20 \times 1000}{8} = 800725 \text{ kJ/h}$$

(*i*) **Capacity of the refrigerating plant** $= \frac{800725}{14000}$ = **57.19 tonnes. (Ans.)**

(*ii*) $T_1 = 25 + 273 = 298$ K

$T_2 = -8 + 273 = 265$ K

∴ **C.O.P. of reversed Carnot cycle**

$$= \frac{T_2}{T_1 - T_2} = \frac{265}{298 - 265} = \textbf{8.03. (Ans.)}$$

(*iii*) **Power required :**

Actual C.O.P. $= \frac{1}{3} \times$ Carnot C.O.P. $= \frac{1}{3} \times 8.03 = 2.67$

But actual C.O.P. $= \frac{\text{Net refrigerating effect/min}}{\text{Work done /min}} = \frac{R_n}{W}$

$$2.67 = \frac{800725}{W} \text{ kJ/h}$$

∴ $W = \frac{800725}{2.67} = 299897$ kJ/h = 83.3 kJ/s

∴ *Power required to run the plant* **= 83.3 kW. (Ans.)**

Example 14.8. *A heat pump is used for heating the interior of a house in cold climate. The ambient temperature is – 5°C and the desired interior temperature is 25°C. The compressor of heat pump is to be driven by a heat engine working between 1000°C and 25°C. Treating both cycles as reversible, calculate the ratio in which the heat pump and heat engine share the heating load.* **(P.U.)**

Solution. Refer Fig. 14.3. *Given :* $T_1 = 1000 + 273 = 1273$ K ; $T_2 = 25 + 273 = 298$ K ;
$T_3 = -5 + 273 = 268$ K ; $T_4 = 25 + 273 = 298$ K

The ratio in which the heat pump and heat engine share the heating load, $\mathbf{\frac{Q_4}{Q_1}}$ **:**

Since both the cycles are *reversible*, therefore,

$$\frac{Q_3}{Q_4} = \frac{T_3}{T_4} \text{ and } \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

or $$\frac{Q_3}{Q_4} = \frac{268}{298} \text{ or } Q_3 = \frac{268}{298} Q_4 \text{ and } \frac{Q_2}{Q_1} = \frac{298}{1273}$$

Heat engine drives the heat pump,

∴ $W = (Q_1 - Q_2) = Q_4 - Q_3$

Dividing both sides by Q_1, we have

$$1 - \frac{Q_2}{Q_1} = \frac{Q_4 - Q_3}{Q_1}$$

$$1 - \frac{298}{1273} = \frac{Q_4 - \frac{268}{298} Q_4}{Q_1}$$

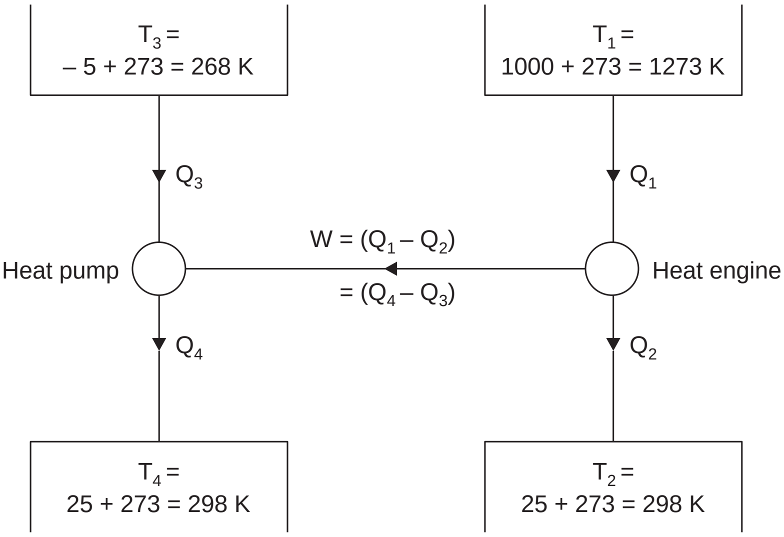


Fig. 14.3

$$\frac{975}{1273} = \frac{30}{298} \times \frac{Q_4}{Q_1}$$

$$\therefore \qquad \frac{Q_4}{Q_1} = \frac{975}{1273} \times \frac{298}{30} = \mathbf{7.608.} \quad \textbf{(Ans.)}$$

14.2.3. Reversed Brayton Cycle

Fig. 14.4 shows a schematic diagram of an air refrigeration system working on reversed Brayton cycle. *Elements* of this systems are :

1. Compressor
2. Cooler (Heat exchanger)
3. Expander
4. Refrigerator.

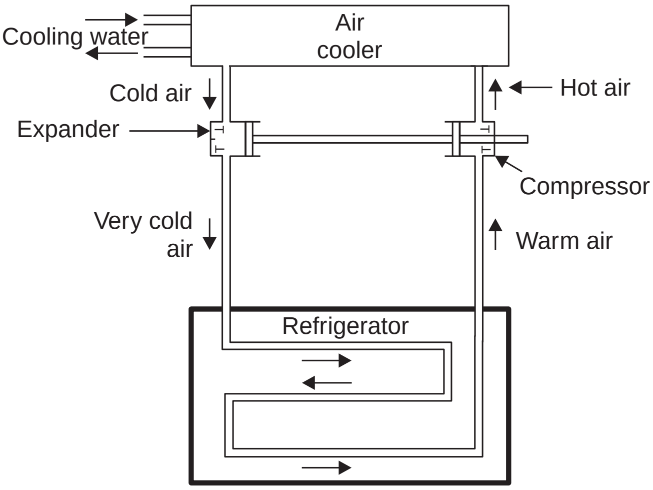


Fig. 14.4. Air refrigeration system.

In this system, work gained from expander is employed for compression of air, consequently less external work is needed for operation of the system. In practice it may or may not be done *e.g.*, in some aircraft refrigeration systems which employ air refrigeration cycle the expansion work may be used for driving other devices.

This system uses reversed *Brayton cycle* which is described below :

Figs. 14.5 (*a*) and (*b*) shows *p-V* and *T-s* diagrams for a reversed Brayton cycle. Here it is *assumed* that (*i*) *absorption and rejection of heat are constant pressure processes and* (*ii*) Compression and expansion are *isentropic processes*.

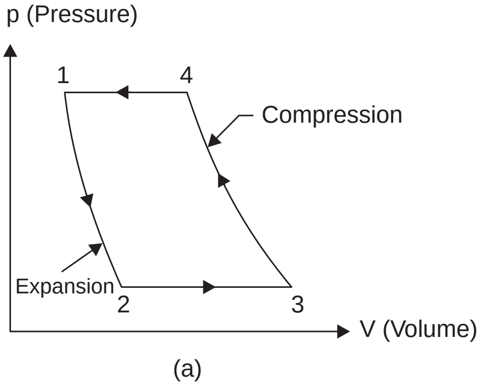


Fig. 14.5. (*a*) *p-V* diagram.

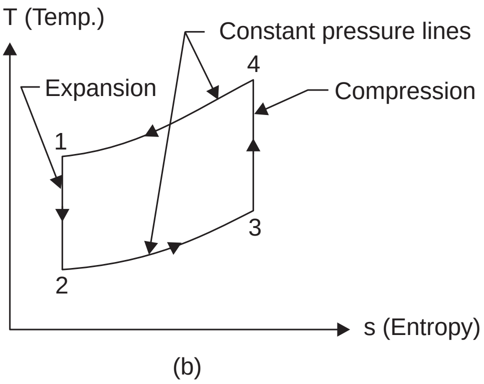


Fig. 14.5. (*b*) *T-s* diagram.

Considering m kg of air :

Heat absorbed in refrigerator, $Q_{\text{added}} = m \times c_p \times (T_3 - T_2)$

Heat rejected is cooler, $Q_{\text{rejected}} = m \times c_p \times (T_4 - T_1)$

If the process is considered to be polytropic, the *steady flow work of compression is* given by,

$$W_{\text{comp}} = \frac{n}{n-1}\,(p_4V_4 - p_3V_3) \qquad \text{...(14.6)}$$

Similarly work of expansion is given by,

$$W_{\text{exp.}} = \frac{n}{n-1}\,(p_1V_1 - p_2V_2) \qquad \text{...(14.7)}$$

Equations (14.6) and (14.7) may easily be reduced to the theoretical isentropic process shown in Fig. 14.5 (*b*) by substituting $\gamma = n$ and the known relationship.

$$R = c_p\left(\frac{\gamma-1}{\gamma}\right)\text{J}$$

The net external work required for operation of the cycle

$$\begin{aligned}
&= \text{Steady flow work of compression} - \text{Steady flow work of expansion}\\
&= W_{\text{comp.}} - W_{\text{exp.}}\\
&= \left(\frac{n}{n-1}\right)(p_4V_4 - p_3V_3 - p_1V_1 + p_2V_2)\\
&= \left(\frac{n}{n-1}\right)mR(T_4 - T_3 - T_1 + T_2)\\
&= \left(\frac{n}{n-1}\right)\frac{mR}{J}(T_4 - T_3 - T_1 + T_2)
\end{aligned}
\qquad
\left[\begin{array}{ll} \because & p_1V_1 = mRT_1\\ & p_2V_2 = mRT_2\\ & p_3V_3 = mRT_3\\ & p_4V_4 = mRT_4 \end{array}\right]$$

in heat units.

But
$$R = c_p \left(\frac{\gamma - 1}{\gamma}\right) J$$
$$(J = 1 \text{ in S.I. units})$$

$$\therefore \quad W_{\text{comp.}} - W_{\text{exp.}} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right) mc_p(T_4 - T_3 + T_2 - T_1) \qquad \text{...(14.8)}$$

For *isentropic compression* and *expansion*,
$$W_{\text{net}} = mc_p(T_4 - T_3 + T_2 - T_1)$$

Now according to law of conservation of energy the net work on the gas must be equivalent to the net heat rejected.

Now,
$$\text{C.O.P.} = \frac{W_{\text{added}}}{Q_{\text{rejected}} - Q_{\text{added}}} = \frac{Q_{\text{added}}}{W_{\text{net}}}$$

For the air cycle assuming polytropic compression and expansion, co-efficient of performance is :

$$\text{C.O.P.} = \frac{m \times c_p \times (T_3 - T_2)}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right) m \times c_p \times (T_4 - T_3 + T_2 - T_1)}$$
$$= \frac{(T_3 - T_2)}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)(T_4 - T_3 + T_2 - T_1)} \qquad \text{...(14.9)}$$

Note. The reversed Brayton cycle is same as the Bell-Coleman cycle. Conventionally Bell-Coleman cycle refers to a closed cycle with expansion and compression taking place in reciprocating expander and compressor respectively, and heat rejection and heat absorption taking place in condenser and evaporator respectively.

With the development of efficient centrifugal compressors and gas turbines, the processes of compression and expansion can be carried out in centrifugal compressors and gas turbines respectively. Thus the shortcoming encountered with conventional reciprocating expander and compressor is overcome. Reversed Brayton cycle finds its application for air-conditioning of aeroplanes where air is used as refrigerant.

14.2.4. Merits and Demerits of Air refrigeration System

Merits

1. Since air is non-flammable, therefore there is no risk of fire as in the machine using NH_3 as the refrigerant.
2. It is cheaper as air is easily available as compared to the other refrigerants.
3. As compared to the other refrigeration systems the weight *of air refrigeration system per tonne of refrigeration is quite low, because of this reason this system is employed in aircrafts*.

Demerits

1. The C.O.P. of this system is very low in comparison to other systems.
2. The weight of air required to be circulated is more compared with refrigerants used in other systems. This is due to the fact that heat is carried by air in the form of *sensible heat*.

Example 14.9. *A Bell-Coleman refrigerator operates between pressure limits of 1 bar and 8 bar. Air is drawn from the cold chamber at 9°C, compressed and then it is cooled to 29°C before entering the expansion cylinder. Expansion and compression follow the law* $pv^{1.35}$ = *constant. Calculate the theoretical C.O.P. of the system.*

For air take $\gamma = 1.4$, $c_p = 1.003$ *kJ/kg K.*

Solution. Fig. 14.6 shows the working cycle of the refrigerator.

Given :
$p_2 = 1.0$ bar ;
$p_1 = 8.0$ bar ;
$T_3 = 9 + 273 = 282$ K ;
$T_4 = 29 + 273 = 302$ K.

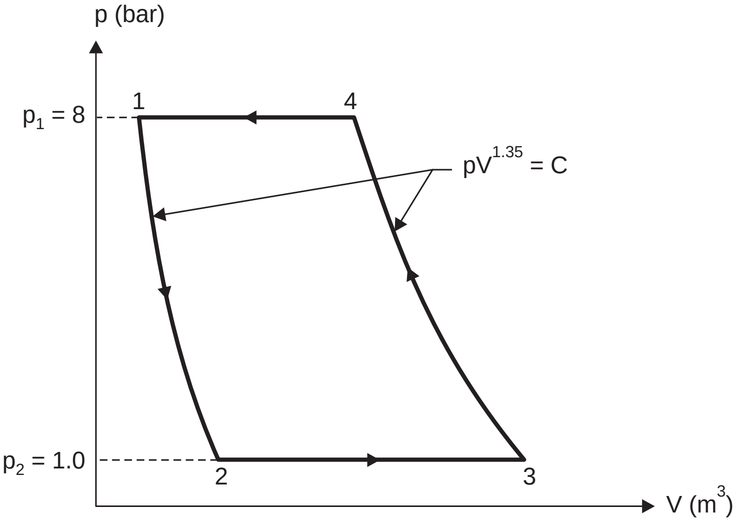


Fig. 14.6

Considering *polytropic compression 3-4*, we have

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{8}{1}\right)^{\frac{1.35-1}{1.35}} = (8)^{0.259} = 1.71$$

or $$T_4 = T_3 \times 1.71 = 282 \times 1.71 = 482.2 \text{ K}$$

Again, considering *polytropic expansion 1-2*, we have

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{8}{1}\right)^{\frac{1.35-1}{1.35}} = 1.71$$

$$T_2 = \frac{T_1}{1.71} = \frac{302}{1.71} = 176.6 \text{ K}$$

Heat extracted from cold chamber per kg of air

$$= c_p\,(T_3 - T_2) = 1.003\,(282 - 176.6) = 105.7 \text{ kJ/kg.}$$

Heat rejected in the cooling chamber per kg of air

$$= c_p\,(T_4 - T_1) = 1.003\,(482.2 - 302) = 180.7 \text{ kJ/kg.}$$

Since the compression and expansion are not isentropic, difference between heat rejected and heat absorbed is not equal to the work done because there are heat transfers to the surroundings and from the surroundings during compression and expansion.

To find the work done, the area of the diagram '1-2-3-4' is to be considered :

Work done $$= \frac{n}{n-1}\,(p_4V_4 - p_3V_3) - \frac{n}{n-1}\,(p_1V_1 - p_2V_2)$$

$$= \frac{n}{n-1} R[(T_4 - T_3) - (T_1 - T_2)]$$

The value of R can be calculated as follows

$$\frac{c_p}{c_v} = \gamma$$

$$\therefore \quad c_v = \frac{c_p}{\gamma} = \frac{1.003}{1.4} = 0.716$$

$$R = (c_p - c_v) = 1.003 - 0.716 = 0.287 \text{ kJ/kg K.}$$

$$\therefore \quad \text{Work done} = \frac{1.35}{0.35} \times 0.287\ [(482.2 - 282) - (302 - 176.6)] = 82.8 \text{ kJ/kg.}$$

$$\therefore \quad \textbf{C.O.P.} = \frac{\text{Heat abstracted}}{\text{Work done}} = \frac{105.7}{82.4} = \textbf{1.27.} \quad \textbf{(Ans.)}$$

Example 14.10. *An air refrigeration open system operating between 1 MPa and 100 kPa is required to produce a cooling effect of 2000 kJ/min. Temperature of the air leaving the cold chamber is – 5°C and at leaving the cooler is 30°C. Neglect losses and clearance in the compressor and expander. Determine :*

(*i*) *Mass of air circulated per min.* ;

(*ii*) *Compressor work, expander work, cycle work* ;

(*iii*) *COP and power in kW required.* **(AMIE)**

Solution. Refer Fig. 14.7.

Pressure, p_1 = 1 MPa = 1000 kPa ; p_2 = 100 kPa

Refrigerating effect produced = 2000 kJ/min

Temperature of air leaving the cold chamber, T_3 = – 5 + 273 = 268 K

Temperature of air leaving the cooler, T_1 = 30 + 273 = 303 K

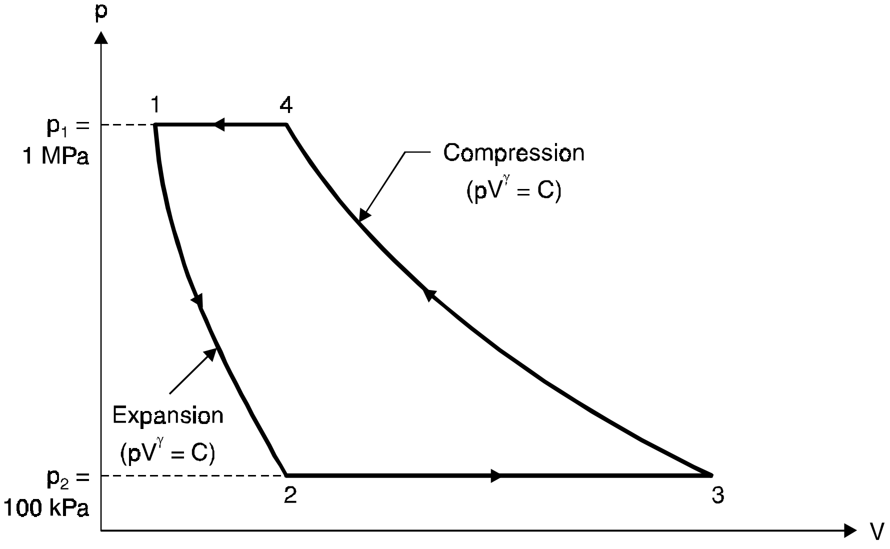


Fig. 14.7

(*i*) **Mass of air circulated per minute, m :**

For the *expansion process 1-2*, we have

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1000}{100}\right)^{\frac{1.4-1}{1.4}} = 1.9306$$

or $$T_2 = \frac{T_1}{1.9306} = \frac{303}{1.9306} = 156.9 \text{ K}$$

Refrigerating effect per kg = $1 \times c_p (T_3 - T_2) = 1.005 (268 - 156.9) = 111.66$ kJ/kg

$$\therefore \quad \text{Mass of air circulated per minute} = \frac{\text{Refrigerating effect}}{\text{Refrigerating effect per kg}}$$

$$= \frac{2000}{111.66} = \textbf{17.91 kg/min.} \quad \textbf{(Ans.)}$$

(*ii*) **Compressor work ($W_{comp.}$), expander work ($W_{exp.}$) and cycle work (W_{cycle}) :**

For *compression process 3-4*, we have

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1000}{10}\right)^{\frac{1.4-1}{1.4}} = 1.9306$$

or $$T_4 = 268 \times 1.9306 = \textbf{517.4 K.} \quad \textbf{(Ans.)}$$

Compressor work, $W_{comp.}$: $\frac{\gamma}{\gamma-1} mR (T_4 - T_3)$

$$= \frac{1.4}{1.4-1} \times 17.91 \times 0.287 (517.4 - 268)$$

$$= \textbf{4486.85 kJ/min.} \quad \textbf{(Ans.)}$$

Expander work, $W_{exp.}$: $\frac{\gamma}{\gamma-1} mR (T_1 - T_2)$

$$= \frac{1.4}{1.4-1} \times 17.91 \times 0.287 (303 - 156.9)$$

$$= \textbf{2628.42 kJ/min.} \quad \textbf{(Ans.)}$$

Cycle work, W_{cycle} : $W_{comp.} - W_{exp.}$

$= 4486.85 - 2628.42 =$ **1858.43 kJ/min. (Ans.)**

(*iii*) **C.O.P. and power required (P) :**

$$\text{COP} = \frac{\text{Refrigerating effect}}{\text{Work required}} = \frac{2000}{1858.43} = \textbf{1.076} \quad \textbf{(Ans.)}$$

Power required, P = Work per second = $\frac{1858.43}{60}$ kJ/s or kW = **30.97 kW. (Ans.)**

☞**Example 14.11.** *A refrigerating machine of 6 tonnes capacity working on Bell-Coleman cycle has an upper limit of pressure of 5.2 bar. The pressure and temperature at the start of the compression are 1.0 bar and 16°C respectively. The compressed air cooled at constant pressure to a temperature of 41°C enters the expansion cylinder. Assuming both expansion and compression processes to be adiabatic with γ = 1.4, calculate :*

(*i*) *Co-efficient of performance.*

(*ii*) *Quantity of air in circulation per minute.*

(*iii*) *Piston displacement of compressor and expander.*

(*iv*) *Bore of compressor and expansion cylinders. The unit runs at 240 r.p.m. and is double-acting. Stroke length = 200 mm.*

(*v*) *Power required to drive the unit*

For air take $\gamma = 1.4$ *and* $c_p = 1.003$ *kJ/kg K.*

Solution. Refer Fig. 14.8.

$$T_3 = 16 + 273 = 289 \text{ K} ; \qquad T_1 = 41 + 273 = 314 \text{ K}$$

$$p_1 = 5.2 \text{ bar} ; \qquad p_2 = 1.0 \text{ bar.}$$

Considering the *adiabatic compression 3-4*, we have

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5.2}{1}\right)^{\frac{1.4-1}{1.4}} = (5.2)^{0.286} = 1.6$$

$$\therefore \qquad T_4 = 1.6 ; T_3 = 1.6 \times 289 = 462.4 \text{ K}$$

Considering the *adiabatic expansion 1-2*, we have

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{314}{T_2} = \left(\frac{5.2}{1}\right)^{\frac{0.4}{1.4}} = 1.6 \quad \text{or} \quad T_2 = \frac{314}{1.6} = 196.25 \text{ K.}$$

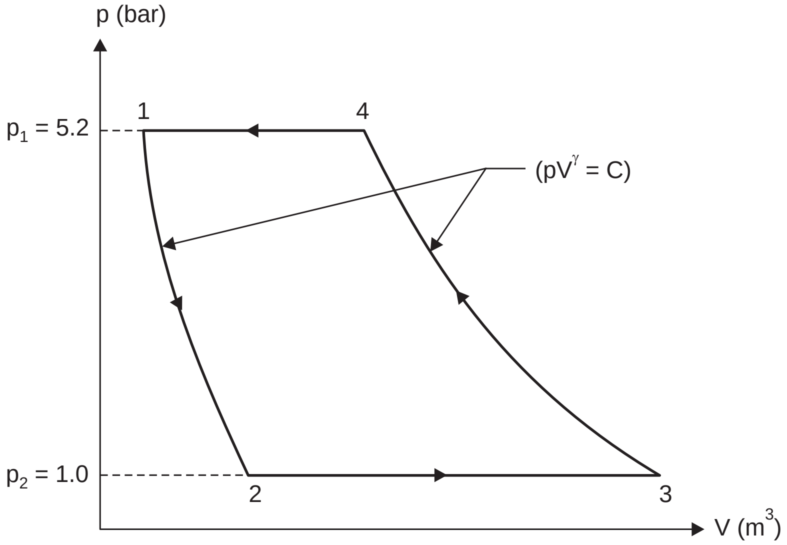


Fig. 14.8

(*i*) **C.O.P. :**

Since both the compression and expansion processes are isentropic/adiabatic reversible,

$$\therefore \quad \textit{C.O.P. of the machine} = \frac{T_2}{T_1 - T_2} = \frac{196.25}{314 - 196.25} = \mathbf{1.67.} \quad \textbf{(Ans.)}$$

(*ii*) **Mass of air in circulation :**

Refrigerating effect per kg of air

$$= c_p\,(T_3 - T_2) = 1.003\,(289 - 196.25) = 93.03 \text{ kJ/kg.}$$

Refrigerating effect produced by the refrigerating machine

$$= 6 \times 14000 = 84000 \text{ kJ/h.}$$

Hence mass of air in circulation

$$= \frac{84000}{93.03 \times 60} = \mathbf{15.05 \text{ kg/min.}} \quad \textbf{(Ans.)}$$

(*iii*), (*iv*) **Piston displacement of compressor**

= Volume corresponding to point 3 *i.e.,* V_3

$$\therefore \quad V_3 = \frac{mRT_3}{p_2} = \frac{15.05 \times 0.287 \times 1000 \times 289}{1.0 \times 10^5} = \mathbf{12.48 \text{ m}^3\text{/min.}} \quad \textbf{(Ans.)}$$

$\therefore$ Swept volume per stroke

$$= \frac{12.48}{2 \times 240} = 0.026 \text{ m}^3$$

If, d_c = Dia. of compressor cylinder, and
l = Length of stroke,

then $$\frac{\pi}{4} d_c^2 \times l = 0.026$$

or $$\frac{\pi}{4} d_c^2 \times \left(\frac{200}{1000}\right) = 0.026$$

$$\therefore \quad d_c = \left(\frac{0.026 \times 1000 \times 4}{\pi \times 200}\right)^{1/2} = 0.407 \text{ m or } 407 \text{ mm}$$

i.e., Diameter or bore of the compressor cylinder **= 407 mm. (Ans.)**

Piston displacement of expander

= Volume corresponding to point 2 *i.e.,* V_2

$$\therefore \quad V_2 = \frac{mRT_2}{p_2} = \frac{15.05 \times 0.287 \times 1000 \times 196.25}{1 \times 10^5} = \mathbf{8.476 \text{ m}^3\text{/min.}} \quad \textbf{(Ans.)}$$

$\therefore$ Swept volume per stroke

$$= \frac{8.476}{2 \times 240} = 0.0176 \text{ m}^3.$$

If d_e = dia. of the expander, and
l = length of stroke,

then $$\frac{\pi}{4} d_e^2 \times l = 0.0176$$

or $$\frac{\pi}{4} d_e^2 \times \left(\frac{200}{1000}\right) = 0.0176$$

$$\therefore \quad d_e = \left(\frac{0.0176 \times 1000 \times 4}{\pi \times 200}\right)^{1/2} = 0.335 \text{ m or } 335 \text{ mm}$$

i.e., Diameter or bore of the expander cylinder **= 335 mm. (Ans.)**

(*v*) **Power required to drive the unit :**

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{R_n}{W}$$

$$1.67 = \frac{6 \times 14000}{W}$$

$$W = \frac{6 \times 14000}{1.67} = 50299.4 \text{ kJ/h} = 13.97 \text{ kJ/s.}$$

Hence power required **= 13.97 kW. (Ans.)**

14.3. SIMPLE VAPOUR COMPRESSION SYSTEM

14.3.1. Introduction

Out of all refrigeration systems, the vapour compression system is the most important system from the view point of *commercial* and *domestic utility*. It is the most practical form of refrigeration. In this system the *working fluid is a vapour*. It readily evaporates and condenses or changes alternately between the vapour and liquid phases without leaving the refrigerating plant. During evaporation, it absorbs heat from the cold body. This heat is used as its latent heat for converting it from the liquid to vapour. In condensing or cooling or liquifying, it rejects heat to external body, thus creating a cooling effect in the working fluid. This refrigeration system thus acts as a latent heat pump since it pumps its latent heat from the cold body or brine and rejects it or delivers it to the external hot body or cooling medium. The principle upon which the vapour compression system works apply to all the vapours for which tables of Thermodynamic properties are available.

14.3.2. Simple Vapour Compression Cycle

In a simple vapour compression system fundamental processes are completed in one cycle. These are :

1. Compression 2. Condensation 3. Expansion 4. Vapourisation.

The flow diagram of such a cycle is shown in Fig. 14.9.

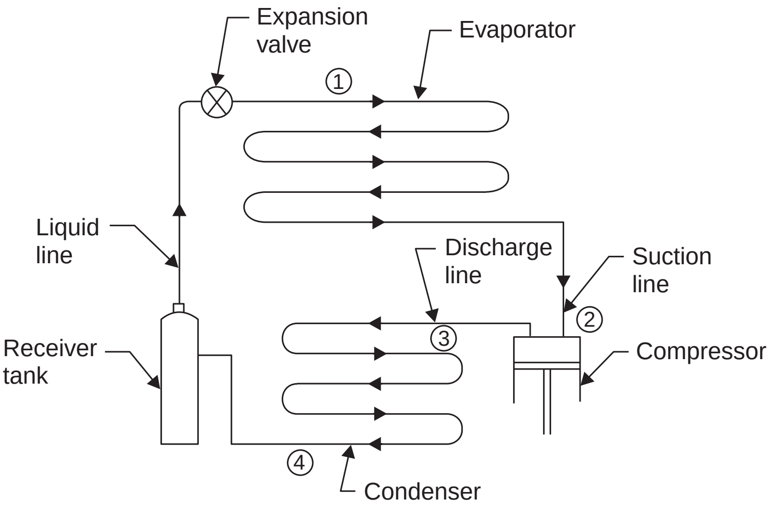


Fig. 14.9. Vapour compression system.

The *vapour at low temperature and pressure* (state '2') enters the "*compressor*" where it is compressed isentropically and subsequently its temperature and pressure increase considerably (state '3'). This vapour after leaving the compressor enters the "*condenser*" where it is condensed into *high pressure liquid* (state '4') and is collected in a "*receiver tank*". From receiver tank it passes through the "*expansion valve*", here it is *throttled down to a lower pressure* and has a low temperature (state '1'). After finding its way through expansion "valve" it finally passes on to "*evaporator*" where it *extracts heat from the surroundings* or *circulating fluid being refrigerated and vapourises to low pressure vapour (state '2').*

Merits and demerits of vapour compression system over Air refrigeration system :

Merits :

1. C.O.P. is quite high as the working of the cycle is very near to that of reversed Carnot cycle.

2. When used on ground level the running cost of vapour-compression refrigeration system is only 1/5th of air refrigeration system.

3. For the same refrigerating effect the size of the evaporator is smaller.

4. The required temperature of the evaporator can be achieved simply by adjusting the throttle valve of the same unit.

Demerits :

1. Initial cost is high.

2. The major disadvantages are *inflammability*, *leakage of vapours* and *toxity*. These have been overcome to a great extent by improvement in design.

14.3.3. Functions of Parts of a Simple Vapour Compression System

Here follows the brief description of various parts of a simple vapour compression system shown in Fig. 14.9.

1. Compressor. The function of a compressor is to remove the *vapour* from the evaporator, and to *raise its temperature and pressure to a point such that it* (vapour) *can be condensed with available condensing media.*

2. Discharge line (or **hot gas line**). A hot gas or discharge line *delivers the high-pressure, high-temperature vapour from the discharge of the compressor to the condenser.*

3. Condenser. The function of a condenser is to *provide a heat transfer surface through which heat passes from the hot refrigerant vapour to the condensing medium.*

4. Receiver tank. A receiver tank is used to provide *storage for a condensed liquid* so that a constant supply of liquid is available to the evaporator as required.

5. Liquid line. A liquid line carries the liquid refrigerant from the receiver tank to the refrigerant flow control.

6. Expansion valve (refrigerant flow control). Its function is to *meter the proper amount of refrigerant to the evaporator and to reduce the pressure of liquid entering the evaporator so that liquid will vapourize in the evaporator at the desired low temperature and take out sufficient amount of heat.*

7. Evaporator. An evaporator *provides a heat transfer surface through which heat can pass from the refrigerated space into the vapourizing refrigerant.*

8. Suction line. The suction line *conveys the low pressure vapour from the evaporator to the suction inlet of the compressor.*

14.3.4. Vapour Compression Cycle on Temperature-Entropy (T-s) Diagram

We shall consider the following three cases :

1. When the vapour is dry and saturated at the end of compression. Fig. 14.10 represents the vapour compression cycle, on *T-s* diagram the points 1, 2, 3 and 4 correspond to the state points 1, 2, 3 and 4 in Fig. 14.9.

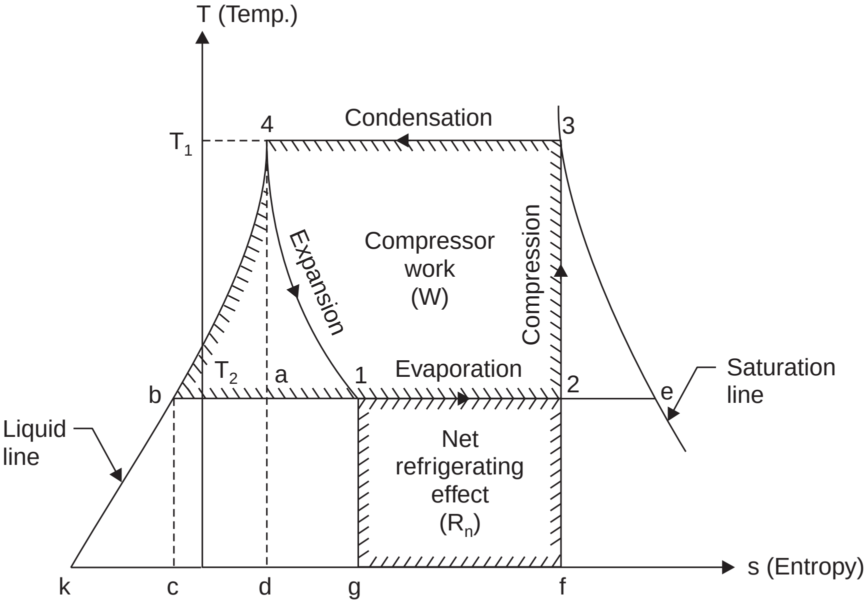


Fig. 14.10. *T-s* diagram.

At point '2' the vapour which is at low temperature (T_2) and low pressure enters the compressor's cylinder and is compressed adiabatically to '3' when its temperature increases to the temperature T_1. It is then condensed in the condenser (line 3-4) where it gives up its latent heat to the condensing medium. It then undergoes throttling expansion while passing through the expansion valve and its again reduces to T_2, it is represented by the line 4-1. From the *T-s* diagram it may be noted that due to this expansion the liquid partially evaporates, as its dryness fraction is represented by the ratio $\frac{b_1}{b_2}$. At '1' it enters the evaporator where it is further evaporated at constant pressure and constant temperature to the point '2' and the cycle is completed.

Work done by the compressor = W = Area '2-3-4-*b*-2'

Heat absorbed = Area '2-1-*g*-*f*-2'

$$\therefore \quad \text{C.O.P.} = \frac{\text{Heat extracted or refrigerating effect}}{\text{Work done}} = \frac{\text{Area '2-1-}g\text{-}f\text{-2'}}{\text{Area '2-3-4-}b\text{-2'}}$$

or
$$\text{C.O.P} = \frac{h_2 - h_1}{h_3 - h_2} \quad \text{...[14.10 (a)]}$$

$$= \frac{h_2 - h_4}{h_3 - h_2} \quad \text{...[14.10 (b)]}$$

($\because$ $h_1 = h_4$, *since during the throttling expansion 4-1 the total heat content remains unchanged*)

2. When the vapour is superheated after compression. If the compression of the vapour is continued after it has become dry, the vapour will be superheated, its effect on T-s diagram is shown in Fig. 14.11. The vapour enters the compressor at condition ‘2’ and is compressed to ‘3’ where it is superheated to temperature T_{sup}. Then it enters the condenser. Here firstly superheated vapour cools to temperature T_1 (represented by line 3-3′) and then it condenses at constant temperature along the line 3′-4 ; the remaining of the cycle ; however is the same as before.

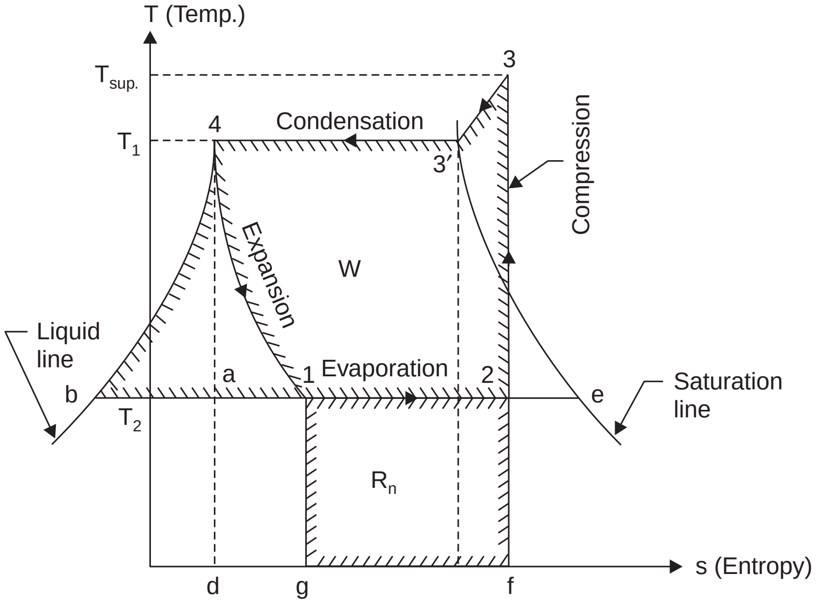


Fig. 14.11. T-s diagram.

Now, Work done = Area ‘2-3-3′-4-b-2’

and Heat extracted/absorbed = Area ‘2-1-g-f-2’

$$\therefore \quad \text{C.O.P.} = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{\text{Area ‘2-1-}g\text{-}f\text{-2’}}{\text{Area ‘2-3-3′-4-}b\text{-2’}} = \frac{h_2 - h_1}{h_3 - h_2} \qquad \text{...[14.10 }(c)]$$

In this case $h_3 = h_3' + c_p\,(T_{sup.} - T_{sat.})$ and h_3' = total heat of dry and saturated vapour at the point ‘3″.

3. When the vapour is wet after compression. Refer Fig. 14.12.

Work done by the compressor = Area ‘2-3-4-b-2’

Heat extracted = Area ‘2-1-g-f-2’

$$\therefore \quad \text{C.O.P.} = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{\text{Area ‘2-1-}g\text{-}f\text{-2’}}{\text{Area ‘2-3-4-}b\text{-2’}} = \frac{h_2 - h_1}{h_3 - h_2} \qquad \text{...[14.10}(d)]$$

Note. If the vapour is not superheated after compression, the operation is called ‘WET COMPRESSION’ and if the vapour is superheated at the end of compression, it is known as ‘DRY COMPRESSION’. Dry compression, in actual practice is always preferred as it gives *higher volumetric efficiency* and *mechanical efficiency* and there are *less chances of compressor damage*.

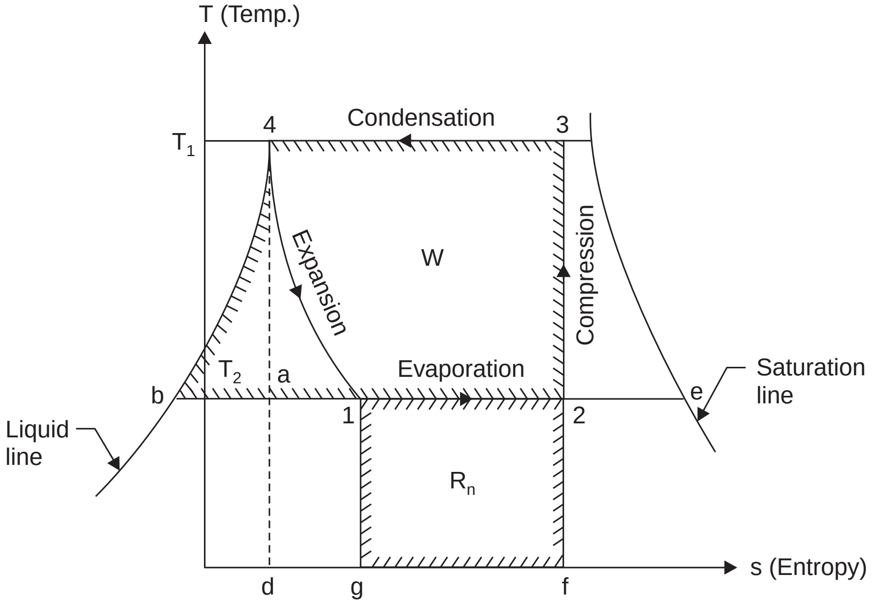


Fig. 14.12. *T-s* diagram.

14.3.5. Pressure-Enthalpy (p-h) Chart

The diagram commonly used in the analysis of the refrigeration cycle are :

(*i*) Pressure-enthalpy (*p-h*) chart (*ii*) Temperature-entropy (*T-s*) chart.

Of the two, the pressure-enthalpy diagram seems to be the more useful.

The condition of the refrigerant in any thermodynamic state can be represented as a point on the *p-h* chart. The point on the *p-h* chart that represents the condition of the refrigerant in any one particular thermodynamic state may be located if any two properties of the refrigerant for that state are known, the other properties of the refrigerant for that state can be determined directly from the chart for studying the performance of the machines.

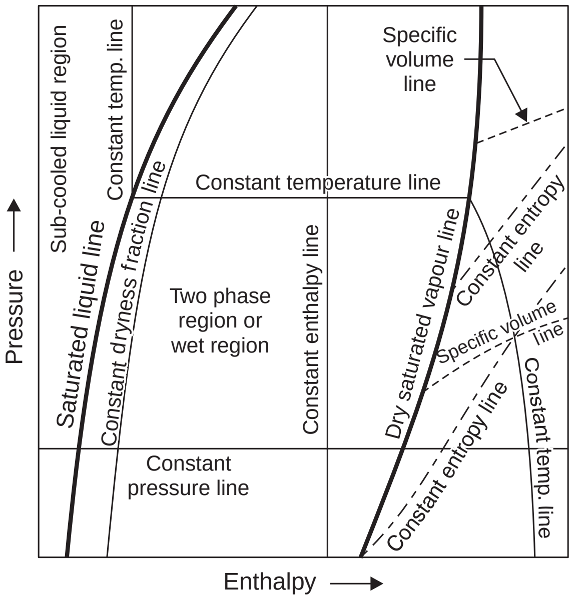


Fig. 14.13. Pressure enthalpy (*p-h*) chart.

Refer Fig. 14.13. The chart is dividing into three areas that are separated from each other by the saturated liquid and saturated vapour lines. The region on the chart to the *left* of the saturated liquid line is called the *sub-cooled region.* At any point in the sub-cooled region the refrigerant is in the liquid phase and its temperature is below the saturation temperature corresponding to its pressure. The area to the *right* of the saturated vapour line is superheated region and the refrigerant is in the form of a *superheated vapour.* The section of the chart between the saturated liquid and saturated vapour lines is the two phase region and represents the change in phase of the refrigerant between liquid and vapour phases. At any point between two saturation lines the refrigerant is in the form of a liquid vapour mixture. *The distance between the two lines along any constant pressure line, as read on the enthalpy scale at the bottom of the chart, is the latent heat of vapourisation of the refrigerant at that pressure.*

The horizontal lines extending across the chart are lines of 'constant pressure' and the vertical lines are lines of constant enthalpy. The lines of 'constant temperature' in the sub-cooled region are almost vertical on the chart and parallel to the lines of constant enthalpy. In the centre section, since the refrigerant changes state at a constant temperature and pressure, the lines of constant temperature are parallel to and coincide with the lines of constant pressure. At the saturated vapour line the lines of constant temperature change direction again and, in the superheated vapour region, fall of sharply toward the bottom of the chart.

The straight lines which extend diagonally and almost vertically across the superheated vapour region are lines of constant entropy. The curved, nearly horizontal lines crossing the superheated vapour region are lines of constant volume.

p-h chart gives directly the changes in enthalpy and pressure during a process for thermodynamic analysis.

14.3.6. Simple Vapour Compression Cycle on p-h Chart

Fig. 14.14 shows a simple vapour compression cycle on a *p-h* chart. The points 1, 2, 3 and 4 correspond to the points marked in Fig. 14.9.

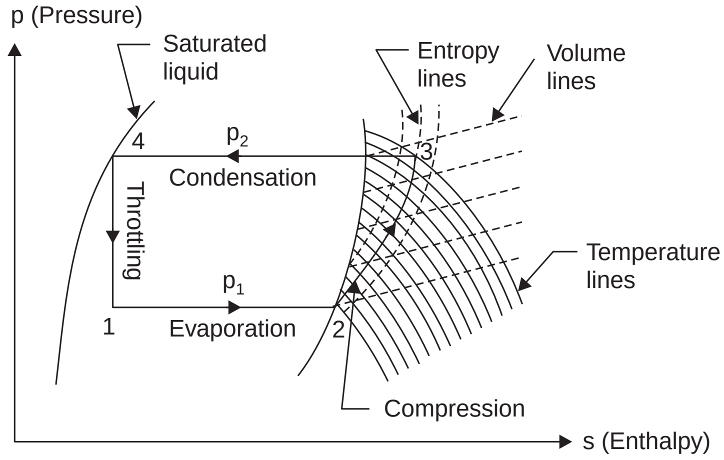


Fig. 14.14. Simple vapour compression cycle on *p-h* chart.

The dry saturated vapour (at state 2) is drawn by the compressor from evaporator at lower pressure p_1 and then it (vapour) is compressed isentropically to the upper pressure p_2. The isentropic compression is shown by the line 2-3. Since the vapour is dry and saturated at the start of compression it becomes superheated at the end of compression as given by point 3. The process of *condensation which takes place at constant pressure* is given by the line 3-4. The vapour now reduced to saturated liquid is throttled through the expansion valve and the process is shown by the line 4-1. At the point 1 a mixture of vapour and liquid enters the evaporator where it gets dry saturated as shown by the point 2. The cycle is thus completed.

Heat extracted (or refrigerating effect produced),

$$R_n = h_2 - h_1$$

Work done, $W = h_3 - h_2$

$$\therefore \qquad \text{C.O.P.} = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2}$$

The values of h_1, h_2 and h_3 can be directly read from *p-h* chart.

14.3.7. Factors Affecting the Performance of a Vapour Compression System

The factors which affect the performance of a vapour compression system are given below :

1. **Effect of suction pressure.** The effect of *decrease* in suction pressure is shown in Fig. 14.15.

The C.O.P. of the original cycle,

$$\text{C.O.P.} = \frac{h_2 - h_1}{h_3 - h_2}$$

The C.O.P. of the cycle when suction pressure is decreased,

$$\text{C.O.P.} = \frac{h_2' - h_1'}{h_3' - h_2'}$$

$$= \frac{(h_2 - h_1) - (h_2 - h_2')}{(h_3 - h_2) + (h_2 - h_2') + (h_3' - h_3)} \qquad (\because \;\; h_1 = h_1')$$

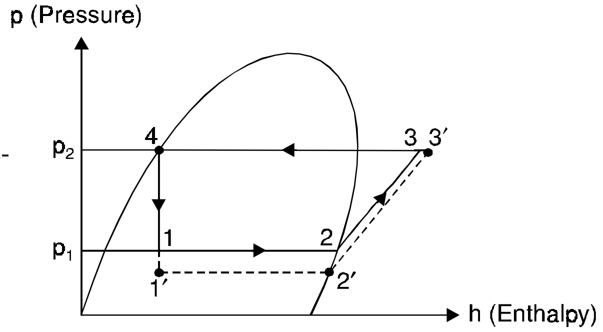


Fig. 14.15. Effect of decrease in suction pressure.

This shows that the *refrigerating effect is decreased and work required is increased. The net effect is to reduce the refrigerating capacity of the system (with the same amount of refrigerant flow) and the C.O.P.*

2. **Effect of delivery pressure.** Fig. 14.16 shows the effect of *increase in delivery pressure*.

C.O.P. of the original cycle,

$$\text{C.O.P.} = \frac{h_2 - h_1}{h_3 - h_2}$$

C.O.P. of the cycle when delivery pressure is increased,

$$\text{C.O.P.} = \frac{h_2 - h_1'}{h_3' - h_2} = \frac{(h_2 - h_1) - (h_1' - h_1)}{(h_3 - h_2) + (h_3' - h_3)}$$

The effect of increasing the delivery/discharge pressure is just similar to the effect of decreasing the suction pressure. *The only difference is that the effect of decreasing the suction pressure is more predominant than the effect of increasing the discharge pressure.*

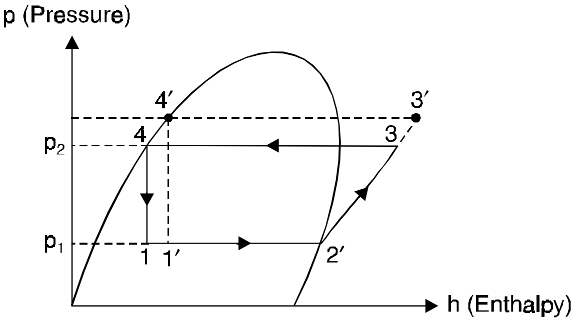


Fig. 14.16. Effect of increase in delivery pressure

The following points may be noted :

(*i*) *As the discharge temperature required in the summer is more as compared with winter, the same machine will give less refrigerating effect (load capacity decreased) at a higher cost.*

(*ii*) *The increase in discharge pressure is necessary for high condensing temperatures and decrease in suction pressure is necessary to maintain low temperature in the evaporator.*

3. **Effect of superheating.** As may be seen from the Fig. 14.17 the effect of superheating is to increase the refrigerating effect but this increase in refrigerating effect is at the cost of increase in amount of work spent to attain the upper pressure limit. *Since the increase in work is more as compared to increase in refrigerating effect, therefore overall effect of superheating is to give a low value of C.O.P.*

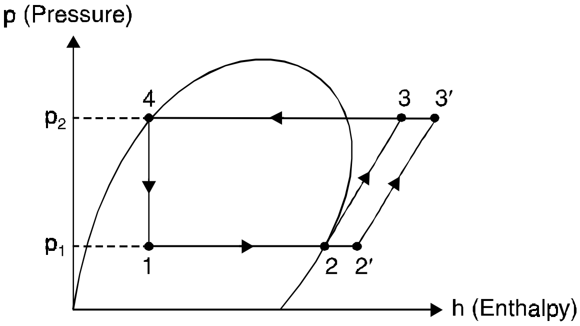


Fig. 14.17. Effect of superheating.

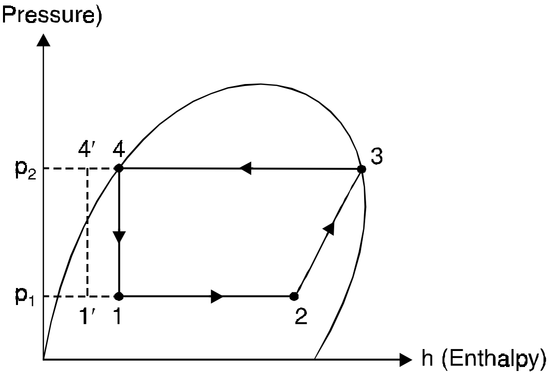


Fig. 14.18. Effect of sub-cooling of liquid.

4. **Effect of sub-cooling of liquid.** *'Sub-cooling' is the process of cooling the liquid refrigerant below the condensing temperature for a given pressure*. In Fig. 14.18 the process of sub-cooling is shown by 4-4′. As is evident from the figure the *effect of subcooling is to increase the refrigerating effect. Thus sub-cooling results in increase of C.O.P. provided that no further energy has to be spent to obtain the extra cold coolant required*.

The *sub-cooling* or *undercooling* may be done by any of the following methods :

(*i*) Inserting a special coil between the condenser and the expansion valve.

(*ii*) Circulating greater quantity of cooling water through the condenser.

(*iii*) Using water cooler than main circulating water.

5. **Effect of suction temperature and condenser temperature.** The performance of the vapour compression refrigerating cycle varies considerably with both vapourising and condensing temperatures. Of the two, the *vapourising temperature has far the greater effect*. It is seen that *the capacity and performance of the refrigerating system improve as the vapourising temperature increases and the condensing temperature decreases*. Thus refrigerating system *should always be designed to operate at the highest possible vapourising temperature and lowest possible condensing temperature*, of course, keeping in view the requirements of the application.

14.3.8. Actual Vapour Compression Cycle

The actual vapour compression cycle differs from the theoretical cycle in several ways because of the following *reasons* :

(*i*) Frequently the liquid refrigerant is sub-cooled before it is allowed to enter the expansion valve, and usually the gas leaving the evaporator is superheated a few degrees before it enters the compressor. This superheating may occur as a result of the type of expansion control used or through a pick up of heat in the suction line between the evaporator and compressor.

(*ii*) Compression, although usually assumed to be isentropic, may actually prove to be neither isentropic nor polytropic.

(*iii*) Both the compressor suction and discharge valves are actuated by pressure difference and this process requires the actual suction pressure inside the compressor to be slightly below that of the evaporator and the discharge pressure to be above that of condenser.

(*iv*) Although isentropic compression assumes no transfer of heat between the refrigerant and the cylinder walls, actually the cylinder walls are hotter than the incoming gases from the evaporator and colder than the compressed gases discharged to the condenser.

(*v*) Pressure drop in long suction and liquid line piping and any vertical differences in head created by locating the evaporator and condenser at different elevations.

Fig. 14.19 shows the actual vapour compression cycle on *T-s* diagram. The various processes are discussed as follows :

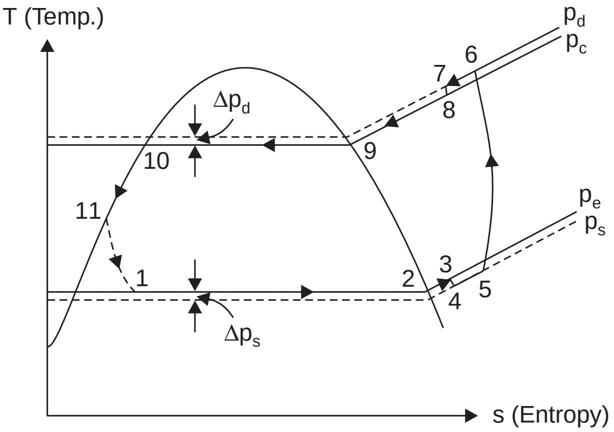


Fig. 14.19. Actual vapour compression cycle (*T-s* diagram).

Process 1-2-3. This process represents passage of refrigerant through the evaporator, with *1-2* indicating *gain of latent heat of vapourisation, and 2-3, the gain of superheat before entrance to compressor*. Both of these processes approach very closely to the constant pressure conditions (assumed in theory).

Process 3-4-5-6-7-8. This path/process represents the passage of the vapour refrigerant from entrance to the discharge of the compressor. Path 3-4 represents the throttling action that occurs during passage through the suction valves, and path 7-8 represents the throttling during passage through exhaust valves. Both of these actions are accompanied by an entropy increase and a slight drop in temperature.

Compression of the refrigerant occurs along path 5-6, which is actually neither isentropic nor polytropic. The heat transfers indicated by path 4-5 and 6-7 occur essentially at constant pressure.

Process 8-9-10-11. This process represents the passage of refrigerant through the condenser with 8-9 indicating removal of superheat, 9-10 the removal of latent heat, and 10-11 removal of heat of liquid or sub-cooling.

Process 11-1. This process represents passage of the refrigerant through the expansion valve, both theoretically and practically an irreversible adiabatic path.

14.3.9. Volumetric Efficiency

A compressor which is theoretically perfect would have neither clearance nor losses of any type and would pump on each stroke a quantity of refrigerant equal to piston displacement. No actual compressor is able to do this, since it is impossible to construct a compressor without clearance or one that will have no wire drawing through the suction and discharge valves, no superheating of the suction gases upon contact with the cylinder walls, or no leakage of gas past the piston or the valves. All these factors effect the volume of gas pumped or the capacity of the compressor, some of them affect the H.P. requirements per tonne of refrigeration developed.

'Volumetric efficiency' is defined as the *ratio of actual volume of gas drawn into the compressor* (at evaporator temperature and pressure) on *each stroke to the piston displacement*. If the effect of *clearance alone* is considered, the resulting expression may be termed *clearance volumetric efficiency*. The expression used for grouping into one constant all the factors affecting efficiency may be termed *total volumetric efficiency*.

Clearance volumetric efficiency. *'Clearance volume' is the volume of space between the end of the cylinder and the piston when the latter is in dead centre position.* The clearance volume is expressed as a percentage of piston displacement. In Fig. 14.20 the piston displacement is shown as 4′-1.

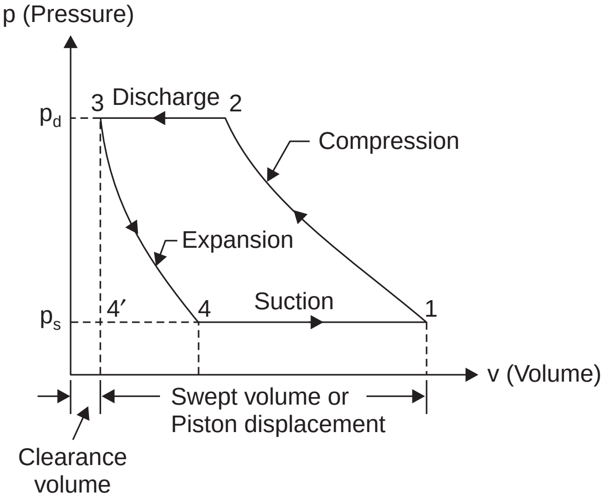


Fig. 14.20

During the suction stroke 4′-1, the vapour filled in clearance space at a discharge pressure p_d expands along 3-4 and the suction valve opens only when pressure has dropped to suction pressure p_s, therefore actual volume sucked will be $(v_1 - v_4)$ while the swept volume is $(v_1 - v_4')$. The *ratio of actual volume of vapour sucked to the piston displacement is defined as clearance volumetric efficiency*.

Thus,

$$\text{Clearance volumetric efficiency, } \eta_{cv} = \frac{v_1 - v_4}{v_1 - v_4'} = \frac{v_1 - v_4}{v_1 - v_3} \qquad (\because \quad v_4' = v_3)$$

Considering *polytropic expansion process 3-4*, we have

$$p_s v_4^{\,n} = p_d v_3^{\,n}$$

or
$$\frac{p_d}{p_s} = \left(\frac{v_4}{v_3}\right)^n \quad \text{or} \quad v_4 = v_3 \cdot \left(\frac{p_d}{p_s}\right)^{1/n}$$

If the clearance ratio,

$$C = \frac{v_3}{v_1 - v_3} = \frac{\text{Clearance volume}}{\text{Swept volume}}$$

Thus
$$\eta_{cv} = \frac{v_1 - v_4}{v_1 - v_3} = \frac{(v_1 - v_4') - (v_4 - v_4')}{(v_1 - v_3)}$$

$$= \frac{(v_1 - v_3) - (v_4 - v_3)}{(v_1 - v_3)} \qquad (\because \quad v_4' = v_3)$$

$$= 1 - \frac{v_4 - v_3}{v_1 - v_3}$$

$$= 1 - \frac{v_3\left(\frac{p_d}{p_s}\right)^{1/n} - v_3}{v_1 - v_3} = 1 + \frac{v_3}{v_1 - v_3}\left[1 - \left(\frac{p_d}{p_s}\right)^{1/n}\right]$$

$$= 1 + C - C\left(\frac{p_d}{p_s}\right)^{1/n}$$

Hence clearance volumetric efficiency,

$$\eta_{cv} = 1 + C - C\left(\frac{p_d}{p_s}\right)^{1/n} \qquad \text{...(14.11)}$$

Total volumetric efficiency. The total volumetric efficiency (η_{tv}) of a compressor is best obtained by actual *laboratory measurements of the amount of refrigerant compressed and delivered to the condenser*. It is very difficult to predict the effects of wire-drawing, cylinder wall heating, and piston leakage to allow any degree of accuracy in most cases. The total volumetric efficiency can be approximately calculated if the pressure drop through the suction valves and the temperature of the gases at the end of the suction stroke are known and if it is assumed that there is no leakage past the piston during compression, it can be calculated (by modifying the eqn. 14.11) by using the following equation :

$$\eta_{tv} = \left[1 + C - C\left(\frac{p_d}{p_s}\right)^{1/n}\right] \times \frac{p_c}{p_s} \times \frac{T_s}{T_c} \qquad \text{...(14.12)}$$

where the subscript '*c*' refers to compressor cylinder and '*s*' refers to the evaporator or the suction line just adjacent to the compressor.

14.3.10. Mathematical Analysis of Vapour Compression Refrigeration

(*i*) **Refrigerating effect.** *Refrigerating effect is the amount of heat absorbed by the refrigerant in its travel through the evaporator*. In Fig. 14.10 this effect is represented by the expression.

$$Q_{evap.} = (h_2 - h_1) \text{ kJ/kg} \qquad \text{...(14.13)}$$

In addition to the latent heat of vaporization it may include any heat of superheat absorbed in the evaporator.

(*ii*) **Mass of refrigerant.** Mass of refrigerant circulated (per second per tonne of refrigeration) may be calculated by *dividing the amount of heat by the refrigerating effect.*

∴ Mass of refrigerant circulated,

$$m = \frac{14000}{3600\,(h_2 - h_1)} \text{ kg/s-tonne} \qquad ...(14.14)$$

because one tonne of refrigeration means cooling effect of 14000 kJ/h.

(*iii*) **Theoretical piston displacement.** Theoretical piston displacement (per tonne of refrigeration per minute) may be found by *multiplying the mass of refrigerant to be circulated (per tonne of refrigeration per sec.) by the specific volume of the refrigerant gas,* $(v_g)_2$, at its entrance of compressor. Thus,

$$\text{Piston displacement}_{(Theoretical)} = \frac{14000}{3600\,(h_2 - h_1)}\,(v_g)_2 \text{ m}^3\text{/s-tonne} \qquad ...(14.15)$$

(*iv*) **Power (Theoretical) required.** Theoretical power per tonne of refrigeration is the power, *theoretically required to compress the refrigerant.* Here volumetric and mechanical efficiencies are not taken into consideration. Power required may be calculated as follows :

(*a*) ***When compression is isentropic :***

Work of compression $= h_3 - h_2$ ...(14.16)

Power required $= m(h_3 - h_2)$ kW

where, m = Mass of refrigerant circulated in kg/s.

(*b*) ***When compression follows the general law pV^n = constant :***

$$\text{Work of compression} = \frac{n}{n-1}\,(p_3v_3 - p_2v_2) \text{ Nm/kg}$$

$$\text{Power required} = m \times \frac{n}{n-1}\,(p_3v_3 - p_2v_2) \times \frac{1}{10^3} \text{ kW } (p \text{ is in N/m}^2) \qquad ...(14.17)$$

(*v*) **Heat rejected to compressor cooling water.** If the compressor cylinders are jacketed, an appreciable amount of heat may be rejected to the cooling water during compression. If the suction and discharge compression conditions are known, this heat can be determined as follows :

Heat rejected to compressor cooling water

$$= \left[\frac{n}{(n-1)}\left(\frac{p_3v_3 - p_2v_2}{1000}\right) - (h_3 - h_2)\right] \text{ kJ/kg} \qquad (p \text{ is in N/m}^2) \qquad ...(14.18)$$

(*vi*) **Heat removed through condenser.** Heat removed through condenser includes all heat removed through the condenser, either as latent heat, heat of superheat, or heat of liquid. This is *equivalent to the heat absorbed in the evaporator plus the work of compression.*

∴ Heat removed through condenser

$$= m(h_3 - h_4) \text{ kJ/s} \qquad (m = \text{mass of refrigerant circulated in kg/s}) \qquad ...(14.19)$$

14.4. VAPOUR ABSORPTION SYSTEM

14.4.1. Introduction

In a *vapour absorption system the refrigerant is absorbed on leaving the evaporator, the absorbing medium being a solid or liquid.* In order that the sequence of events should be continuous it is necessary for the refrigerant to be separated from the absorbent and subsequently condensed before being returned to the evaporator. The separation is accomplished by the application of direct heat in a '*generator*'. The solubility of the refrigerant and absorbent must be suitable and the plant which uses *ammonia as the refrigerant and water as absorbent* will be described.

14.4.2. Simple Vapour Absorption System

Refer Fig. 14.21 for a simple absorption system. The solubility of ammonia in water at low temperatures and pressures is higher than it is at higher temperatures and pressures. The ammonia vapour leaving the evaporator at point 2 is readily absorbed in the low temperature hot solution in the absorber. This process is accompanied by the rejection of heat. The ammonia in water solution is pumped to the higher pressure and is heated in the generator. *Due to reduced solubility of ammonia in water at the higher pressure and temperature, the vapour is removed from the solution*. The vapour then passes to the condenser and the weakened ammonia in water solution is returned to the absorber.

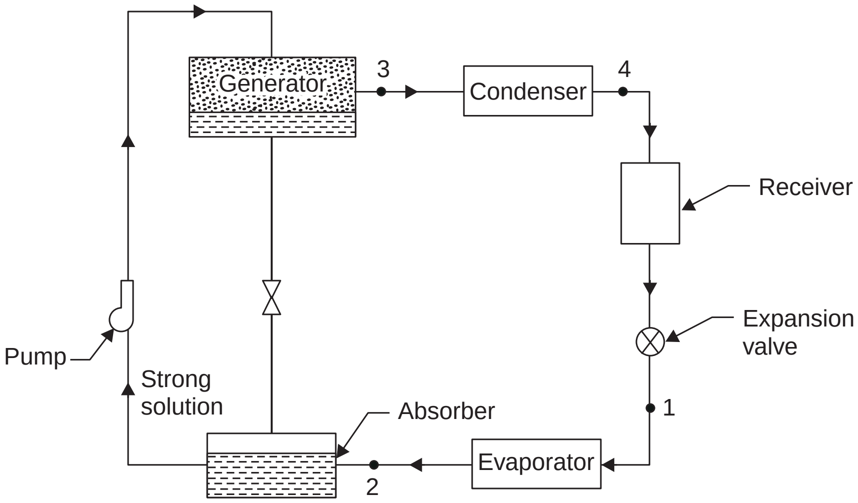


Fig. 14.21. (*a*) Simple vapour absorption system.

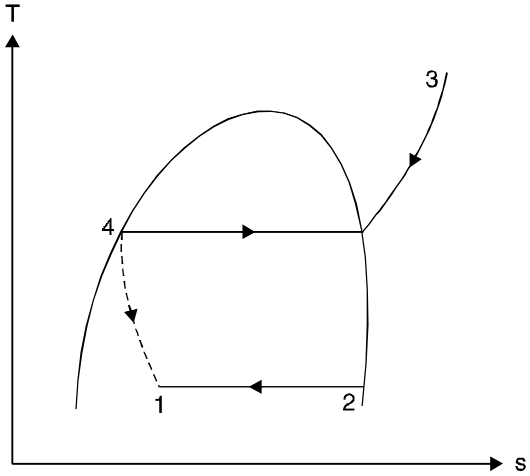


Fig. 14.21. (*b*) Simple vapour absorption system—*T-s* diagram.

In this system the *work done on compression is less than in vapour compression cycle* (since pumping a liquid requires much less work than compressing a vapour between the same pressures) but a heat input to the generator is required. The heat may be supplied by any convenient form *e.g.* steam or gas heating.

14.4.3. Practical Vapour Absorption System

Refer Fig. 14.22. Although a simple vapour absorption system can provide refrigeration *yet its operating efficiency is low*. The following *accessories* are fitted to make the system more practical and improve the performance and working of the plant.

1. Heat exchanger. 2. Analyser. 3. Rectifier.

1. **Heat exchanger.** A heat exchanger is located between the generator and the absorber. The strong solution which is pumped from the absorber to the generator must be heated ; and the weak solution from the generator to the absorber must be cooled. This is accomplished by a heat exchanger and consequently *cost of heating the generator and cost of cooling the absorber are reduced*.

2. **Analyser.** An analyser consists of a series of trays mounted above the generator. Its *main function is to remove partly some of the unwanted water particles associated with ammonia vapour going to condenser. If these water vapours are permitted to enter condenser they may enter the expansion valve and freeze ; as a result the pipe line may get choked.*

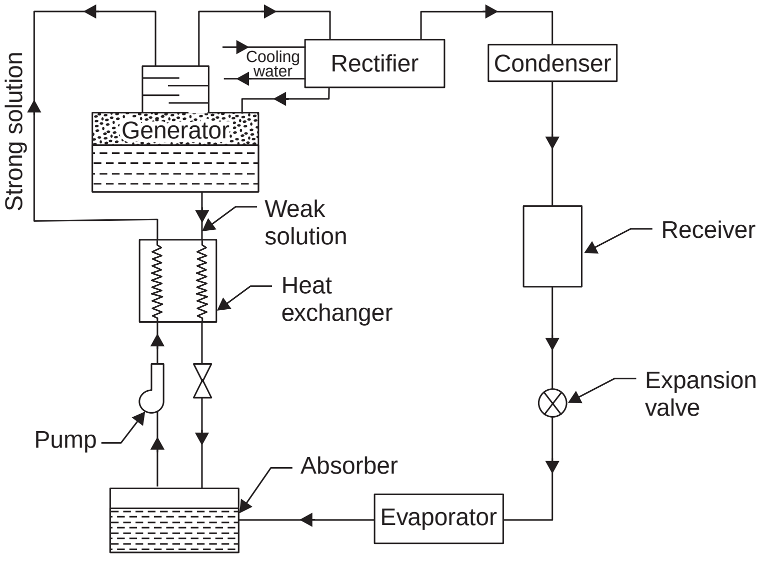


Fig. 14.22

3. **Rectifier.** A rectifier is a water-cooled heat exchanger *which condenses water vapour and some ammonia and sends back to the generator*. Thus *final reduction or elemination of the percentage of water vapour takes place in a rectifier*.

The co-efficient of performance (C.O.P.) of this system is given by :

$$\text{C.O.P.} = \frac{\text{Heat extracted from the evaporator}}{\text{Heat supplied in the generator + Work done by the liquid pump}} \, .$$

14.4.4. Comparison between Vapour Compression and Vapour Absorption Systems

S. No.	*Particulars*	*Vapour compression system*	*Vapour absorption system*
1.	*Type of energy supplied*	Mechanical—a high grade energy	Mainly heat—a low grade energy
2.	*Energy supply*	Low	High
3.	*Wear and tear*	More	Less
4.	*Performance at part loads*	Poor	System not affected by variations of loads.
5.	*Suitability*	Used where high grade mechanical energy is available	Can also be used at remote places as it can work even with a simple kerosene lamp (of course in small capacities)
6.	*Charging of refrigerant*	Simple	Difficult
7.	*Leakage of refrigerant*	More chances	No chance as there is no compressor or any reciprocating component to cause leakage.
8.	*Damage*	Liquid traces in suction line may damage the compressor	Liquid traces of refregerant present in piping at the exit of evaporator constitute no danger.

WORKED EXAMPLES

Example 14.12. *A refrigeration machine is required to produce i.e., at 0°C from water at 20°C. The machine has a condenser temperature of 298 K while the evaporator temperature is 268 K. The relative efficiency of the machine is 50% and 6 kg of Freon-12 refrigerant is circulated through the system per minute. The refrigerant enters the compressor with a dryness fraction of 0.6. Specific heat of water is 4.187 kJ/kg K and the latent heat of ice is 335 kJ/kg. Calculate the amount of ice produced on 24 hours. The table of properties of Freon-12 is given below :*

Temperature K	*Liquid heat kJ/kg*	*Latent heat kJ/g*	*Entropy of liquid kJ/kg*
298	*59.7*	*138.0*	*0.2232*
268	*31.4*	*154.0*	*0.1251*

(U.P.S.C. 1992)

Solution. *Given* : m = 6 kg/min. ; η_{relative} = 50% ; x_2 = 0.6 ; c_{pw} = 4.187 kJ/kg K ; Latent heat of ice = 335 kJ/kg.

Refer Fig. 14.23

h_{f_2} = 31.4 kJ/kg ; h_{fg_2} = 154.0 kJ/kg ; h_{f_3} = 59.7 kJ/kg ;

h_{fg_3} = 138 kJ/kg ; h_{f_4} = 59.7 kJ/kg ...From the table given above

$$h_2 = h_{f_2} + x_2 h_{fg_2}$$
$$= 31.4 + 0.6 \times 154 = 123.8 \text{ kJ/kg}$$

For *isentropic compression 2-3*, we have

$$s_3 = s_2$$

$$s_{f_3} + x_3 \frac{h_{fg_3}}{T_3} = s_{f_2} + x_2 \frac{h_{fg_2}}{T_2}$$

$$0.2232 + x_3 \times \frac{138}{298} = 0.1251 + 0.6 \times \frac{154}{268}$$

$$= 0.4698$$

$$\therefore \qquad x_3 = (0.4698 - 0.2232) \times \frac{298}{138} = 0.5325$$

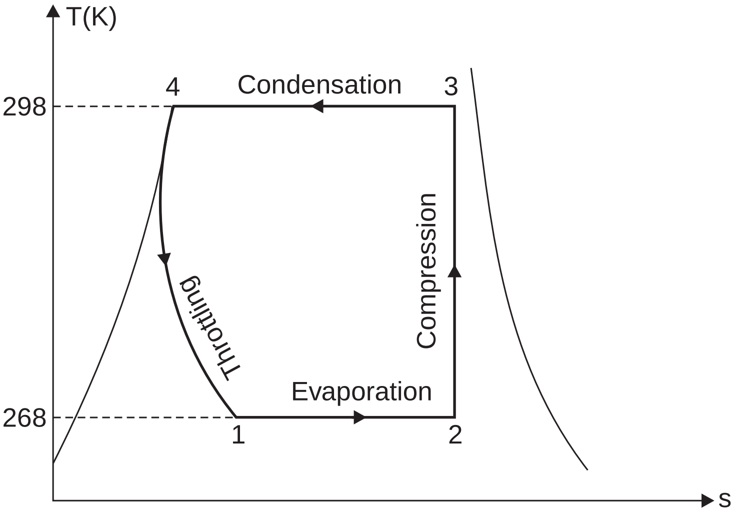


Fig. 14.23

Now, $h_3 = h_{f_3} + x_3\, h_{fg_3} = 59.7 + 0.5325 \times 138 = 133.2$ kJ/kg

Also, $h_1 = h_{f_4} = 59.7$ kJ/kg

Theoretical $\text{C.O.P.} = \dfrac{R_n}{W} = \dfrac{h_2 - h_1}{h_3 - h_2} = \dfrac{123.8 - 59.7}{133.2 - 123.8} = 6.82$

Actual $\text{C.O.P.} = \eta_{\text{relative}} \times (\text{C.O.P.})_{\text{theoretical}} = 0.5 \times 6.82 = 3.41$

Heat extracted from 1 kg of water at 20°C for the formation of 1 kg of ice at 0°C

$$= 1 \times 4.187 \times (20 - 0) + 335 = 418.74 \text{ kJ/kg}$$

Let m_{ice} = Mass of ice formed in kg/min.

$$(\text{C.O.P.})_{\text{actual}} = 3.41 = \frac{R_n(\text{actual})}{W} = \frac{m_{\text{ice}} \times 418.74}{m(h_3 - h_2)} = \frac{m_{\text{ice}} \times 418.74\ (\text{kJ/min})}{6(133.2 - 123.8)\ (\text{kJ/min})}$$

$$\therefore \qquad m_{\text{ice}} = \frac{6(133.2 - 123.8) \times 3.41}{418.74} = 0.459 \text{ kg/min}$$

$$= \frac{0.459 \times 60 \times 24}{1000} \text{ tonnes (in 24 hours)} = \mathbf{0.661\ tonne.} \quad \textbf{(Ans.)}$$

Example 14.13. *28 tonnes of ice from and at 0°C is produced per day in an ammonia refrigerator. The temperature range in the compressor is from 25°C to – 15°C. The vapour is dry and saturated at the end of compression and an expansion valve is used. Assuming a co-efficient of performance of 62% of the theoretical, calculate the power required to drive the compressor.*

Temp. °*C*	*Enthalpy (kJ/kg)*		*Entropy of liquid (kJ/kg K)*	*Entropy of vapour kJ/kg K*
	Liquid	*Vapour*		
25	*100.04*	*1319.22*	*0.3473*	*4.4852*
– 15	*– 54.56*	*1304.99*	*– 2.1338*	*5.0585*

Take latent heat of ice = 335 kJ/kg.

Solution. Theoretical C.O.P. $= \dfrac{h_2 - h_1}{h_3 - h_2}$

Here, $h_3 = 1319.22$ kJ/kg ;

$h_1 = h_4$ (*i.e.,* h_{f4}) $= 100.04$ kJ/kg ...From the table above.

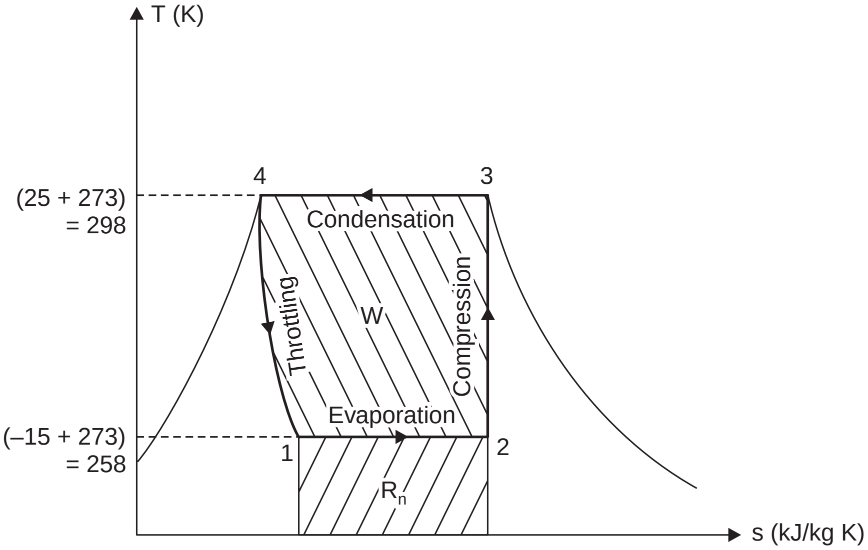


Fig. 14.24

To find h_2, let us first find dryness at point 2.

Entropy at '2' = Entropy at '3' (Process 2-3 being isentropic)

$$s_{f_2} + x_2\, s_{fg_2} = s_{g_3}$$

$$-\,2.1338 + x_2 \times [5.0585 - (-\,2.1338)] = 4.4852$$

$$\therefore \quad x_2 = \frac{4.4852 + 2.1338}{5.0585 + 2.1338} = 0.92$$

$$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2} = -\,54.56 + 0.92 \times [1304.99 - (-\,54.56)]$$

$$= 1196.23 \text{ kJ/kg.}$$

$$\therefore \quad \text{C.O.P.}_{\text{(theoretical)}} = \frac{1196.23 - 100.04}{1319.22 - 1196.23} = 8.91.$$

$\therefore$ C.O.P.$_{\text{(actual)}} = 0.62 \times$ C.O.P.$_{\text{(theoretical)}}$ ... Given

i.e., C.O.P.$_{\text{(actual)}} = 0.62 \times 8.91 = 5.52$

Actual refrigerating effect per kg

$$= \text{C.O.P.}_{\text{(actual)}} \times \text{work done}$$
$$= 5.52 \times (h_3 - h_2) = 5.52 \times (1319.22 - 1196.23)$$
$$= 678.9 \text{ kJ/kg}$$

Heat to be extracted per hour

$$= \frac{28 \times 1000 \times 335}{24} = 390833.33 \text{ kJ}$$

Heat to be extracted per second $= \dfrac{390833.33}{3600} = 108.56$ kJ/s.

∴ Mass of refrigerant circulated per second $= \dfrac{108.56}{678.9} = 0.1599$ kg

Total work done by the compressor per second

$$= 0.1599 \times (h_3 - h_2) = 0.1599\ (1319.22 - 1196.23)$$
$$= 19.67 \text{ kJ/s}$$

i.e., **Power required to drive the compressor = 19.67 kW. (Ans.)**

Example 14.14. *A refrigerating plant works between temperature limits of – 5°C and 25°C. The working fluid ammonia has a dryness fraction of 0.62 at entry to compressor. If the machine has a relative efficiency of 55%, calculate the amount of ice formed during a period of 24 hours. The ice is to be formed at 0°C from water at 15°C and 6.4 kg of ammonia is circulated per minute. Specific heat of water is 4.187 kJ/kg and latent heat of ice is 335 kJ/kg.*

Properties of NH_3 *(datum – 40°C).*

Temp. °C	*Liquid heat* kJ/kg	*Latent heat* kJ/kg	*Entropy of liquid* kJ/kg K
25	298.9	1167.1	1.124
– 5	158.2	1280.8	0.630

Solution. Fig. 14.25 shows the *T-s* diagram of the cycle.

Enthalpy at point '2', $h_2 = h_{f_2} + x_2\, h_{fg_2} = 158.2 + 0.62 \times 1280.8 = 952.3$ kJ/kg

Enthalpy at point '1', $h_1 = h_{f_4} = 298.9$ kJ/kg

Also, entropy at point '2' = entropy at point '3'

i.e.,
$$s_2 = s_3$$
$$s_{f_2} + x_2\, s_{fg_2} = s_{f_3} + x_3\, s_{fg_3}$$
$$0.630 + 0.62 \times \frac{1280.8}{(-5+273)} = 1.124 + x_2 \times \frac{1167.1}{(25+273)}$$

i.e.,
$$x_3 = 0.63$$

∴ Enthalpy at point '3', $h_3 = h_{f_3} + x_3\, h_{fg_3}$

$$= 298.9 + 0.63 \times 1167.1 = 1034.17 \text{ kJ/kg}$$

$$\text{C.O.P.}_{\text{(theoretical)}} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{952.3 - 298.9}{1034.17 - 952.3} = \frac{653.4}{81.87} = 7.98.$$

$$\text{C.O.P.}_{\text{(actual)}} = 0.55 \times 7.98 = 4.39$$

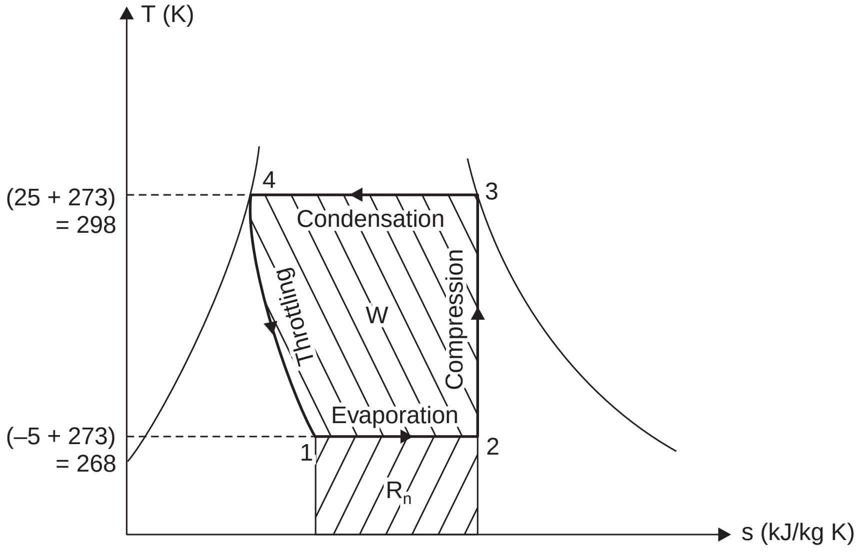


Fig. 14.25

Work done per kg of refrigerant $= h_3 - h_2 = 1034.17 - 952.3 = 81.87$ kJ/kg

Refrigerant in circulation, $m = 6.4$ kg/min.

$$\therefore \quad \text{Work done per second} = 81.87 \times \frac{6.4}{60} = 8.73 \text{ kJ/s}$$

Heat extracted per kg of ice formed = 15 × 4.187 + 335 = 397.8 kJ.

Amount of ice formed in 24 hours,

$$m_{\text{ice}} = \frac{8.73 \times 3600 \times 24}{397.8} = \mathbf{1896.1\ kg.} \quad \textbf{(Ans.)}$$

☞**Example 14.15.** *A simple vapour compression plant produces 5 tonnes of refrigeration. The enthalpy values at inlet to compressor, at exit from the compressor, and at exit from the condenser are 183.19, 209.41 and 74.59 kJ/kg respectively. Estimate :*

(*i*) *The refrigerant flow rate,* (*ii*) *The C.O.P.,*

(*iii*) *The power required to drive the compressor, and*

(*iv*) *The rate of heat rejection to the condenser.* **(AMIE)**

Solution. Total refrigeration effect produced = 5 *TR* (tonnes of refrigeration)

= 5 × 14000 = 70000 kJ/h or 19.44 kJ/s (∵ 1 *TR* = 14000 kJ/h)

Refer Fig. 14.26.

Given : $h_2 = 183.19$ kJ/kg ; $h_3 = 209.41$ kJ/kg ;

$h_4 (= h_1) = 74.59$ kJ/kg (Throttling process)

(*i*) **The refrigerant flow rate, $\dot{m}$:**

Net refrigerating effect produced per kg $= h_2 - h_1$

$= 183.19 - 74.59 = 108.6$ kJ/kg

$$\therefore \text{ Refrigerant flow rate, } \dot{m} = \frac{19.44}{108.6} = \mathbf{0.179\ kg/s.} \quad \textbf{(Ans.)}$$

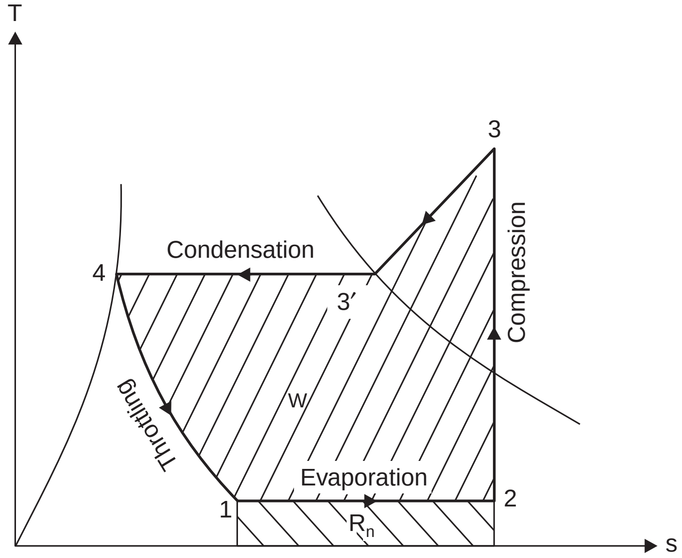


Fig. 14.26

(*ii*) **The C.O.P. :**

$$\text{C.O.P.} = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{183.19 - 74.59}{209.41 - 183.19} = \mathbf{4.142.} \quad \textbf{(Ans.)}$$

(*iii*) **The power required to drive the compressor, P :**

$$P = \dot{m}\ (h_3 - h_2) = 0.179\ (209.41 - 183.19) = \mathbf{4.69\ kW.} \quad \textbf{(Ans.)}$$

(*iv*) **The rate of heat rejection to the condenser :**

The rate of heat rejection to the condenser

$$= \dot{m}\ (h_3 - h_4) = 0.179\ (209.41 - 74.59) = \mathbf{24.13\ kW.} \quad \textbf{(Ans.)}$$

Example 14.16. (*i*) *What are the advantages of using an expansion valve instead of an expander in a vapour compression refrigeration cycle ?*

(*ii*) *Give a comparison between centrifugal and reciprocating compressors.*

(*iii*) *An ice-making machine operates on ideal vapour compression refrigeration cycle using refrigerant R-12. The refrigerant enters the compressor as dry saturated vapour at – 15°C and leaves the condenser as saturated liquid at 30°C. Water enters the machine at 15°C and leaves as ice at – 5°C. For an ice production rate of 2400 kg in a day, determine the power required to run the unit. Find also the C.O.P. of the machine. Use refrigerant table only to solve the problem. Take the latent heat of fusion for water as 335 kJ/kg.* **(AMIE Summer, 1998)**

Solution. (*i*) If an expansion cylinder is used in a vapour compression system, *the work recovered would be extremely small, in fact not even sufficient to overcome the mechanical friction*. It will not be possible to gain any work. Further, the *expansion cylinder is bulky*. On the other hand the *expansion valve is a very simple and handy device, much cheaper than the expansion cylinder*. It *does not need installation, lubrication or maintenance*.

The expansion valve *also controls the refrigerant flow rate according to the requirement, in addition to serving the function of reducting the pressure of the refrigerant.*

(*ii*) **The comparison between centrifugal and reciprocating compressors :**

The comparison between centrifugal and reciprocating compressors is given in the table below :

S. No.	*Particulars*	*Centrifugal compressor*	*Reciprocating compressor*
1.	*Suitability*	Suitable for handling large volumes of air at low pressures	Suitable for low discharges of air at high pressure.
2.	*Operational speeds*	Usually high	Low
3.	*Air supply*	Continuous	Pulsating
4.	*Balancing*	Less vibrations	Cyclic vibrations occur
5.	*Lubrication system*	Generally simple lubrication systems are required.	Generally complicated
6.	*Quality of air delivered*	Air delivered is relatively more clean	Generally contaminated with oil.
7.	*Air compressor size*	Small for given discharge	Large for same discharge
8.	*Free air handled*	2000-3000 m^3/min	250-300 m^3/min
9.	*Delivery pressure*	Normally below 10 bar	500 to 800 bar
10.	*Usual standard of compression*	Isentropic compression	Isothermal compression
11.	*Action of compressor*	Dynamic action	Positive displacement.

(*iii*) Using property table of R-12 :

$$h_2 = 344.927 \text{ kJ/kg}$$

$$h_4 = h_1 = 228.538 \text{ kJ/kg}$$

$$(c_p)_v = 0.611 \text{ kJ/kg°C}$$

$$s_2 = s_3$$

or
$$1.56323 = 1.5434 + 0.611 \log_e \left[\frac{t_3 + 273}{30 + 273}\right]$$

or
$$t_3 = 39.995°\text{C}$$

$$h_3 = 363.575 + 0.611(39.995 - 30)$$
$$= 369.68 \text{ kJ/kg.}$$

$$R_n/\text{kg} = h_2 - h_1 = 344.927 - 228.538$$
$$= 116.389 \text{ kJ/kg}$$

$$W/\text{kg} = h_3 - h_2 = 369.68 - 344.927 = 24.753$$

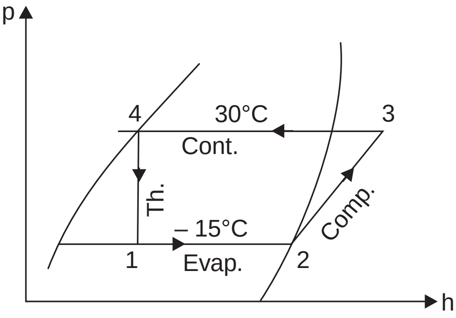


Fig. 14.27

$$\textbf{C.O.P.} = \frac{R_n}{W} = \frac{116.389}{24.753} = \textbf{4.702.} \quad \textbf{(Ans.)}$$

Assuming c_p for ice = 2.0935 kJ/kg°C

Heat to be removed to produce ice

$$= \frac{2400}{24 \times 3600} [4.187(15 - 0) + 335 + 2.0935(0 - (- 5))]$$

$$= 11.3409 \text{ kJ/s} = \text{Work required, kJ/s (kW)} \times \text{C.O.P.}$$

$$\therefore \quad \text{Work required (Power)} = \frac{11.3409}{4.702} = \textbf{2.4 kW.} \quad \textbf{(Ans.)}$$

Example 14.17. *A R-12 refrigerator works between the temperature limits of – 10°C and + 30°C. The compressor employed is of 20 cm × 15 cm, twin cylinder, single-acting compressor having a volumetric efficiency of 85%. The compressor runs at 500 r.p.m. The refrigerant is* *sub-cooled and it enters at 22°C in the expansion valve. The vapour is superheated and enters the compressor at – 2°C. Work out the following :*

(*i*) *Show the process on T-s and p-h diagrams ;* (*ii*) *The amount of refrigerant circulated per minute ;* (*iii*) *The tonnes of refrigeration ;* (*iv*) *The C.O.P. of the system.* **(M.U.)**

Solution. (*i*) **Process on T-s and p-h diagrams :**

The processes on *T-s* and *p-h* diagrams are shown in Fig. 14.28.

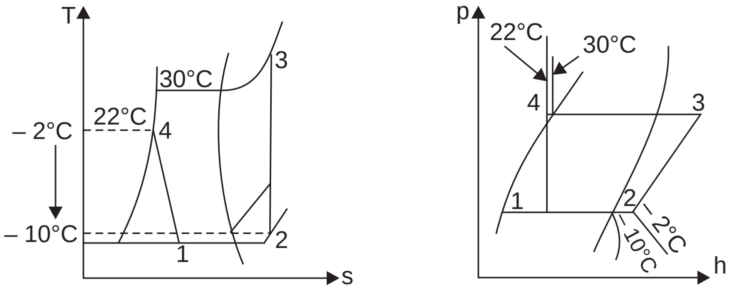


Fig. 14.28

(*ii*) **Mass of refrigerant circulated per minute :**

The value of enthalpies and specific volume read from *p-h* diagram are as under :

$$h_2 = 352 \text{ kJ/kg} \; ; \; h_3 = 374 \text{ kJ/kg}$$

$$h_4 = h_1 = 221 \text{ kJ/kg} \; ; \; v_2 = 0.08 \text{ m}^3\text{/kg}$$

Refrigerants effect per kg $= h_2 - h_1 = 352 - 221 = 131$ kJ/kg

Volume of refrigerant admitted per min.

$$= \frac{\pi}{4} D^2L \times \text{r.p.m.} \times 2 \times \eta_{vol}, \text{ for twin cylinder, single acting}$$

$$= \frac{\pi}{4} (0.2)^2 \times 0.15 \times 500 \times 2 \times 0.85 = 4 \text{ m}^3\text{/min}$$

Mass of refrigerant per min $= \dfrac{4}{0.08} =$ **50 kg/min. (Ans.)**

(*iii*) **Cooling capacity in tonnes of refrigeration :**

Cooling capacity $= 50(h_2 - h_1) = 50 \times 131$

$= 6550$ kJ/min or 393000 kJ/h

or $= \dfrac{393000}{14000} =$ **28.07 TR. (Ans.)**

($\because$ 1 tonne of refrigeration TR = 14000 kJ/h)

(*iv*) Work per kg $= (h_2 - h_1) = 374 - 352 = 22$ kJ/kg

$$\text{C.O.P.} = \frac{131}{22} = \mathbf{5.95.} \quad \textbf{(Ans.)}$$

Example 14.18. *In a standard vapour compression refrigeration cycle, operating between an evaporator temperature of – 10°C and a condenser temperature of 40°C, the enthalpy of the refrigerant, Freon-12, at the end of compression is 220 kJ/kg. Show the cycle diagram on T-s plane. Calculate :*

(i) The C.O.P. of the cycle.

(ii) The refrigerating capacity and the compressor power assuming a refrigerant flow rate of 1 kg/min. You may use the extract of Freon-12 property table given below :

t(°C)	*p(MPa)*	h_f*(kJ/kg)*	h_g*(kJ/kg)*
– 10	*0.2191*	*26.85*	*183.1*
40	*0.9607*	*74.53*	*203.1*

(GATE 1997)

Solution. The cycle is shown on *T-s* diagram in Fig. 14.29.

Given : Evaporator temperature = – 10°C

Condenser temperature = 40°C

Enthalpy at the end of compression, h_3 = 220 kJ/kg

From the table given, we have

$$h_2 = 183.1 \text{ kJ/kg} \; ; \; h_1 = h_{f_4} = 26.85 \text{ kJ/kg}$$

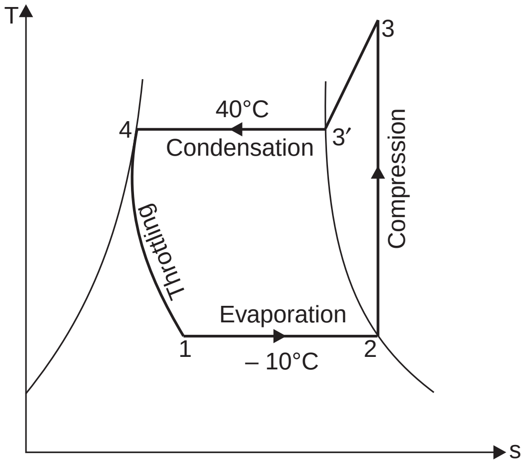


Fig. 14.29

(*i*) **The C.O.P. the cycle :**

$$\text{C.O.P.} = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2}$$

$$= \frac{183.1 - 74.53}{220 - 183.1} = \mathbf{2.94.} \quad \textbf{(Ans.)}$$

(*ii*) **Refrigerating capacity :**

Refrigerating capacity $= m(h_2 - h_1)$

[where m = mass flow rate of refrigerant = 1 kg/min ...(Given)]

= 1 × (183.1 – 74.53) = **108.57 kJ/min. (Ans.)**

Compressor power :

Compressor power $= m(h_3 - h_2)$

= 1 × (220 – 183.1) = 36.9 kJ/min or 0.615 kJ/s

= **0.615 kW. (Ans.)**

Example 14.19. *A Freon-12 refrigerator producing a cooling effect of 20 kJ/s operates on a simple cycle with pressure limits of 1.509 bar and 9.607 bar. The vapour leaves the evaporator dry saturated and there is no undercooling. Determine the power required by the machine.*

If the compressor operaters at 300 r.p.m. and has a clearance volume of 3% of stroke volume, determine the piston displacement of the compressor. For compressor assume that the expansion following the law $pv^{1.13}$ = *constant.*

Given :

Temperature °*C*	p_s *bar*	v_g m^3/kg	*Enthalpy* h_f	*kJ/kg* h_g	*Entropy* s_f	*kJ/kg K* s_g	*Specific heat kJ/kg K*
– 20	1.509	0.1088	17.8	178.61	0.073	0.7082	—
40	9.607	—	74.53	203.05	0.2716	0.682	0.747

(U.P.S.C. 1996)

Solution. *Given* : (From the table above) :

h_2 = 178.61 kJ/kg ; h_3' = 203.05 kJ/kg ; h_{f_4} = 74.53 kJ/kg = h_1

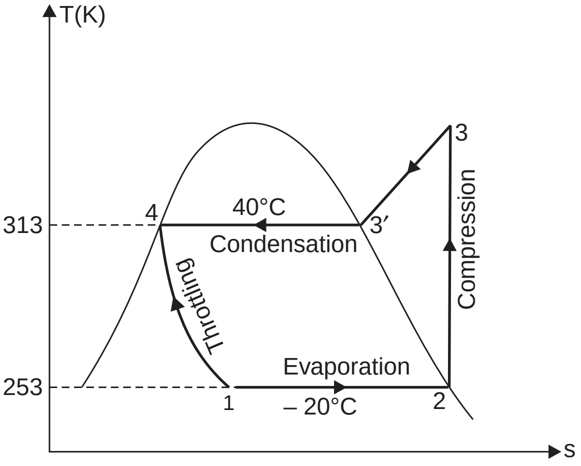


Fig. 14.30

Now, cooling effect $= \dot{m}(h_2 - h_1)$

$$20 = \dot{m}(178.61 - 74.53)$$

$$\therefore \quad \dot{m} = \frac{20}{(178.61 - 74.53)} = 0.192 \text{ kg/s}$$

Also, $s_3 = s_2$

$$s_3' + c_p \ln\left(\frac{T_3}{T_3'}\right) = 0.7082$$

$$0.682 + 0.747 \ln\left(\frac{T_3}{313}\right) = 0.7082$$

or $$\ln\left(\frac{T_3}{313}\right) = \frac{0.7082 - 0.682}{0.747} = 0.03507$$

or $$\frac{T_3}{313} = e^{0.03507} = 1.0357$$

∴ $$T_3 = 313 \times 1.0357 = 324.2 \text{ K}$$

Now, $$h_3 = h_3' + c_p(324.2 - 303)$$
$$= 203.05 + 0.747(324.2 - 313) = 211.4 \text{ kJ/kg}$$

Power required :

Power required by the machine $= \dot{m}(h_3 - h_2)$

$$= 0.192(211.4 - 178.61) = \mathbf{6.29\ kW.}\quad \textbf{(Ans.)}$$

Piston displacement, V :

Volumetric efficiency, $$\eta_{\text{vol.}} = 1 + C - C\left(\frac{p_d}{p_s}\right)^{1/n}$$

$$= 1 + 0.03 - 0.03\left(\frac{9.607}{1.509}\right)^{\frac{1}{1.13}} = 0.876 \quad \text{or} \quad 87.6\%$$

The volume of refrigerant at the intake conditions is

$$\dot{m} \times v_g = 0.192 \times 0.1088 = 0.02089 \text{ m}^3/\text{s}$$

Hence the swept volume $$= \frac{0.02089}{\eta_{\text{vol.}}} = \frac{0.02089}{0.876} = 0.02385 \text{ m}^3/\text{s}$$

∴ $$\mathbf{V} = \frac{0.02385 \times 60}{300} = \mathbf{0.00477\ m^3.}\quad \textbf{(Ans.)}$$

Example 14.20. *A food storage locker requires a refrigeration capacity of 50 kW. It works between a condenser temperature of 35°C and an evaporator temperature of – 10°C. The refrigerant is ammonia. It is sub-cooled by 5°C before entering the expansion valve by the dry saturated vapour leaving the evaporator. Assuming a single cylinder, single-acting compressor operating at 1000 r.p.m. with stroke equal to 1.2 times the bore.*

Determine : (*i*) *The power required, and*
(*ii*) *The cylinder dimensions.*

Properties of ammonia are :

Saturation temperature, °C	*Pressure bar*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*		*Specific volume, m³/kg*		*Specific heat kJ/kg K*	
		Liquid	*Vapour*	*Liquid*	*Vapour*	*Liquid*	*Vapour*	*Liquid*	*Vapour*
– 10	*2.9157*	*154.056*	*1450.22*	*0.82965*	*5.7550*	—	*0.417477*	—	*2.492*
35	*13.522*	*366.072*	*1488.57*	*1.56605*	*5.2086*	*1.7023*	*0.095629*	*4.556*	*2.903*

(U.P.S.C. 1997)

Solution. *Given* : (From the table above)

h_2 = 1450.22 kJ/kg ; h_3' = 1488.57 kJ/kg ; h_{f_4} = 366.072 kJ/kg ;

$$h_{f_4'} = h_1 = h_{f_4} - 4.556(308 - 303)$$
$$= 366.07 - 4.556(308 - 303) = 343.29 \text{ kJ/kg}$$

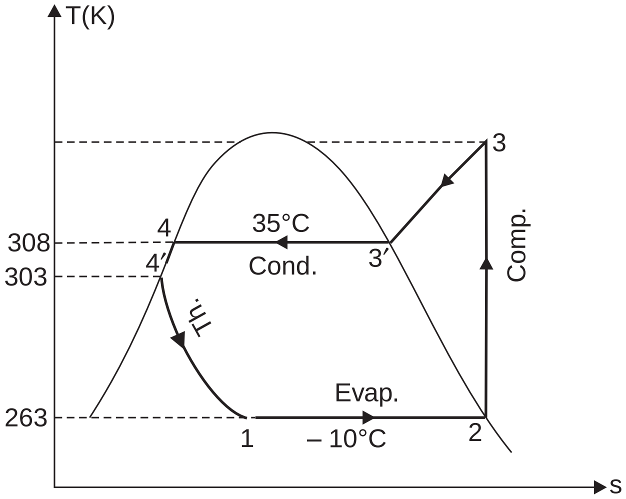


Fig. 14.31

Also $$s_3 = s_2$$

or $$s_3' + c_p \ln\left(\frac{T_3}{T_3'}\right) = 5.755$$

$$5.2086 + 2.903 \ln\left(\frac{T_3}{308}\right) = 5.755$$

or $$\ln\left(\frac{T_3}{308}\right) = \frac{5.755 - 5.2086}{2.903} = 0.1882$$

$$\frac{T_3}{308} = e^{0.1882} = 371.8 \text{ K}$$

Now, $$h_3 = h_3' + c_p(T_3 - T_3')$$
$$= 1488.57 + 2.903\ (371.8 - 308) = 1673.8 \text{ kJ/kg}$$

Mass of refrigerant, $$\dot{m} = \frac{50}{h_2 - h_1} = \frac{50}{1450.22 - 343.29}$$
$$= 0.04517 \text{ kJ/s}$$

(*i*) **Power required :**

Power required $$= \dot{m}\ (h_3 - h_2)$$
$= 0.04517\ (1673.8 - 1450.22) =$ **10.1 kW. (Ans.)**

(*ii*) **Cylinder dimensions :**

$$\dot{m} = \frac{\pi}{4} D^2 \times L \times \frac{N}{60} \times 0.417477 = 0.04517 \text{ (calculated above)}$$

or $$\frac{\pi}{4} D^2 \times 1.2D \times \frac{1000}{60} \times 0.417477 = 0.04517$$

or $$D^3 = \frac{0.04517 \times 4 \times 60}{\pi \times 1.2 \times 1000 \times 0.417477} = 0.006888$$

∴ Diameter of cylinder, **D** $= (0.006888)^{1/3} =$ **0.19 m. (Ans.)**

and, Length of the cylinder, $L = 1.2D = 1.2 \times 0.19 =$ **0.228 m. (Ans.)**

Example 14.21. *A refrigeration cylce uses Freon-12 as the working fluid. The temperature of the refrigerant in the evaporator is – 10°C. The condensing temperature is 40°C. The cooling load is 150 W and the volumetric efficiency of the compressor is 80%. The speed of the compressor is 720 rpm. Calculate the mass flow rate of the refrigerant and the displacement volume of the compressor.*

Properties of Freon-12

Temperature (°C)	*Saturation pressure (MPa)*	*Enthalpy (kJ/kg)*		*Specific volume (m^3/kg)* *Saturated vapour*
		Liquid	*Vapour*	
– 10	*0.22*	*26.8*	*183.0*	*0.08*
40	*0.96*	*74.5*	*203.1*	*0.02*

(GATE, 1995)

Solution. *Given* : Cooling load = 150 W ; $\eta_{vol.}$ = 0.8 ; N = 720 r.p.m.

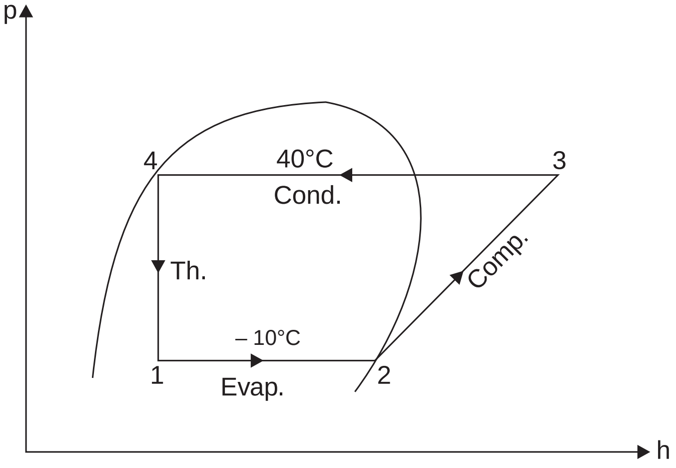


Fig. 14.32

Mass flow rate of the refrigerant $\dot{m}$:

Refrigerating effect = $h_2 - h_1$

= 183 – 74.5 = 108.5 kJ/kg

Cooling load = $\dot{m}$ × (108.5 × 1000) = 150

or $$\dot{m} = \frac{150}{108.5 \times 1000} = \mathbf{0.001382\ kJ/s.}\quad \textbf{(Ans.)}$$

Displacement volume of the compressor :

Specific volume at entry to compressor,

v_2 = 0.08 m^3/kg (From table)

∴ Displacement volume of compressor = $\dfrac{\dot{m} v_2}{\eta_{vol.}} = \dfrac{0.001382 \times 0.08}{0.8}$

= **0.0001382 m^3/s. (Ans.)**

Example 14.22. *In a simple vapour compression cycle, following are the properties of the refrigerant R-12 at various points :*

Compressor inlet : $h_2 = 183.2$ *kJ/kg* $v_2 = 0.0767$ *m³/kg*

Compressor discharge : $h_3 = 222.6$ *kJ/kg* $v_3 = 0.0164$ *m³/kg*

Compressor exit : $h_4 = 84.9$ *kJ/kg* $v_4 = 0.00083$ *m³/kg*

The piston displacement volume for compressor is 1.5 litres per stroke and its volumetric efficiency is 80%. The speed of the compressor is 1600 r.p.m.

Find : (*i*) *Power rating of the compressor (kW) ;*

(*ii*) *Refrigerating effect (kW).* **(GATE 1996)**

Solution. Piston displacement volume = $\frac{\pi}{4}d^2 \times l = 1.5$ litres

$= 1.5 \times 1000 \times 10^{-6}$ m³/stoke = 0.0015 m³/revolution.

(*i*) **Power rating of the compressor (kW) :**

Compressor discharge

$= 0.0015 \times 1600 \times 0.8\ (\eta_{vol.}) = 1.92$ m³/min.

Mass flow rate of compressor,

$$m = \frac{\text{Compressor discharge}}{v_2}$$

$$= \frac{1.92}{0.0767} = 25.03 \text{ kg/min.}$$

Power rating of the compressor

$$= \dot{m}(h_3 - h_2)$$

$$= \frac{25.03}{60}(222.6 - 183.2) = \mathbf{16.44\ kW.}$$ **(Ans.)**

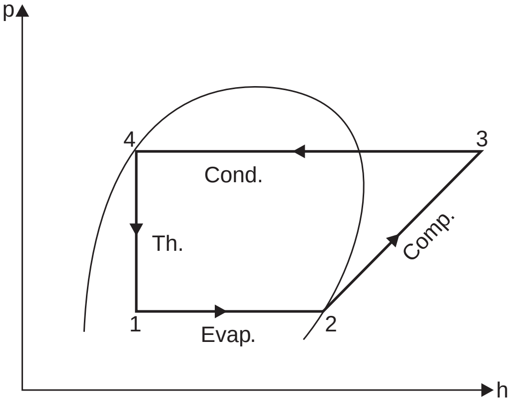


Fig. 14.33

(*ii*) **Refrigerating effect (kW) :**

Refrigerating effect $= \dot{m}(h_2 - h_1) = \dot{m}(h_2 - h_4)$ $(\because\ h_1 = h_4)$

$$= \frac{25.03}{60}(183.2 - 84.9)$$

= **41 kW. (Ans.)**

Example 14.23. *A refrigerator operating on standard vapour compression cycle has a co-efficiency performance of 6.5 and is driven by a 50 kW compressor. The enthalpies of saturated liquid and saturated vapour refrigerant at the operating condensing temperature of 35°C are 62.55 kJ/kg and 201.45 kJ/kg respectively. The saturated refrigerant vapour leaving evaporator has an enthalpy of 187.53 kJ/kg. Find the refrigerant temperature at compressor discharge. The* c_p *of refrigerant vapour may be taken to be 0.6155 kJ/kg°C.* **(GATE 1992)**

Solution. *Given* : C.O.P. = 6.5 ; $W = 50$ kW, $h_3' = 201.45$ kJ/kg,

$$h_{f_4} = h_1 = 69.55 \text{ kJ/kg} ;\ h_2 = 187.53 \text{ kJ/kg}$$

$$c_p = 0.6155 \text{ kJ/kg K}$$

Temperature, t_3 :

Refrigerating capacity = 50 × C.O.P.

= 50 × 6.5 = 325 kW

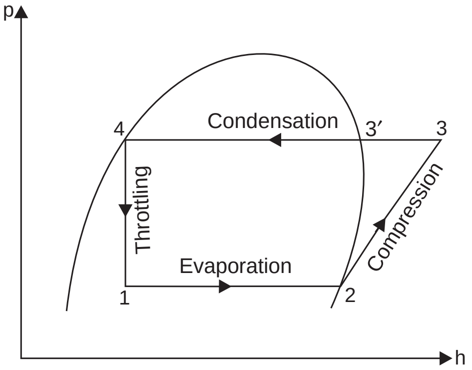


Fig. 14.34

Heat extracted per kg of refrigerant

$$= 187.53 - 69.55 = 117.98 \text{ kJ/kg}$$

$$\text{Refrigerant flow rate} = \frac{325}{117.98} = 2.755 \text{ kg/s}$$

Compressor power = 50 kW

$$\therefore \quad \text{Heat input per kg} = \frac{50}{2.755} = 18.15 \text{ kJ/kg}$$

Enthalpy of vapour after compression

$$= h_2 + 18.15 = 187.53 + 18.15$$

$$= 205.68 \text{ kJ/kg}$$

$$\text{Superheat} = 205.68 - h_3' = 205.68 - 201.45$$

$$= 4.23 \text{ kJ/kg}$$

$$\text{But} \quad 4.23 = 1 \times c_p\,(t_3 - t_3') = 1 \times 0.6155 \times (t_3 - 35)$$

$$\therefore \quad \mathbf{t_3} = \frac{4.23}{0.6155} + 35 = \mathbf{41.87°C.} \quad \textbf{(Ans.)}$$

Note. The compressor rating of 50 kW is assumed to be the enthalpy of compression, in the absence of any data on the efficiency of compressor.

Example 14.24. *A vapour compression heat pump is driven by a power cycle having a thermal efficiency of 25%. For the heat pump, refrigerant-12 is compressed from saturated vapor at 2.0 bar to the condenser pressure of 12 bar. The isentropic efficiency of the compressor is 80%. Saturated liquid enters the expansion valve at 12 bar. For the power cycle 80% of the heat rejected by it is transferred to the heated space which has a total heating requirement of 500 kJ/min. Determine the power input to the heat pump compressor. The following data for refrigerant-12 may be used :*

Pressure, bar	*Temperature, °C*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
		Liquid	*Vapour*	*Liquid*	*Vapour*
2.0	*– 12.53*	*24.57*	*182.07*	*0.0992*	*0.7035*
12.0	*49.31*	*84.21*	*206.24*	*0.3015*	*0.6799*

Vapour specific heat at constant pressure = 0.7 kJ/kg K. **(U.P.S.C. 1995)**

Solution. Heat rejected by the cycle $= \frac{500}{0.8} = 625$ kJ/min.

Assuming isentropic compression of refrigerant, we have

Entropy of dry saturated vapour at 2 bar

= Entropy of superheated vapour at 12 bar

$$0.7035 = 0.6799 + c_p \ln \frac{T}{(49.31 + 273)} = 0.6799 + 0.7 \times \ln\left(\frac{T}{322.31}\right)$$

$$\text{or} \qquad \ln\left(\frac{T}{322.31}\right) = \frac{0.7035 - 0.6799}{0.7} = 0.03371$$

$$\text{or} \qquad T = 322.31\,(e)^{0.03371} = 333.4 \text{ K}$$

∴ Enthalpy of superheated vapour at 12 bar

$$= 206.24 + 0.7(333.4 - 322.31) = 214 \text{ kJ/kg}$$

Heat rejected per cycle = 214 – 84.21 = 129.88 kJ/kg

$$\text{Mass flow rate of refrigerant} = \frac{625}{129.88} = 4.812 \text{ kg/min}$$

Work done on compressor $= 4.812\ (214 - 182.07)$

$= 153.65$ kJ/min $= 2.56$ kW

$$\text{Actual work of compresson} = \frac{2.56}{\eta_{\text{compressor}}} = \frac{2.56}{0.8} = 3.2 \text{ kW}$$

Hence power input to the heat pump compressor = **3.2 kW. (Ans.)**

☞**Example 14.25.** *A food storage locker requires a refrigeration system of 2400 kJ/min. capacity at an evaporator temperature of 263 K and a condenser temperature of 303 K. The refrigerant used is freon-12 and is subcooled by 6°C before entering the expansion valve and vapour is superheated by 7°C before leaving the evaporator coil. The compression of refrigerant is reversible adiabatic. The refrigeration compressor is two-cylinder single-acting with stroke equal to 1.25 times the bore and operates at 1000 r.p.m.*

Properties of freon-12

Saturation temp, K	*Absolute pressure, bar*	*Specific volume of vapour, m^3/kg*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
			Liquid	*Vapour*	*Liquid*	*Vapour*
263	*2.19*	*0.0767*	*26.9*	*183.2*	*0.1080*	*0.7020*
303	*7.45*	*0.0235*	*64.6*	*199.6*	*0.2399*	*0.6854*

Take : Liquid specific heat = 1.235 kJ/kg K ; Vapour specific heat = 0.733 kJ/kg K. Determine :

(*i*) *Refrigerating effect per kg.*
(*ii*) *Mass of refrigerant to be circulated per minute.*
(*iii*) *Theoretical piston displacement per minute.*
(*iv*) *Theoretical power required to run the compressor, in kW.*
(*v*) *Heat removed through condenser per min.*
(*vi*) *Theoretical bore and stroke of compressor.*

Solution. The cycle of refrigeration is represented on *T-s* diagram on Fig. 14.35.

Enthalpy at '2', $h_2 = h_2' + c_p\ (T_2 - T_2')$

From the given table :

$$h_2' = 183.2 \text{ kJ/kg}$$

$(T_2 - T_2')$ = Degree of superheat as the vapour enters the compressor = 7°C

$$\therefore \quad h_2 = 183.2 + 0.733 \times 7 = 188.33 \text{ kJ/kg}$$

Also, entropy at '2', $s_2 = s_2' + c_p \log_e \dfrac{T_2}{T_2'}$

$$= 0.7020 + 0.733 \log_e \left(\frac{270}{263}\right) = 0.7212 \text{ kJ/kg K}$$

For isentropic process 2-3

Entropy at '2' = Entropy at '3'

$$0.7212 = s_3' + c_p \log_e \left(\frac{T_3}{T_3'}\right)$$

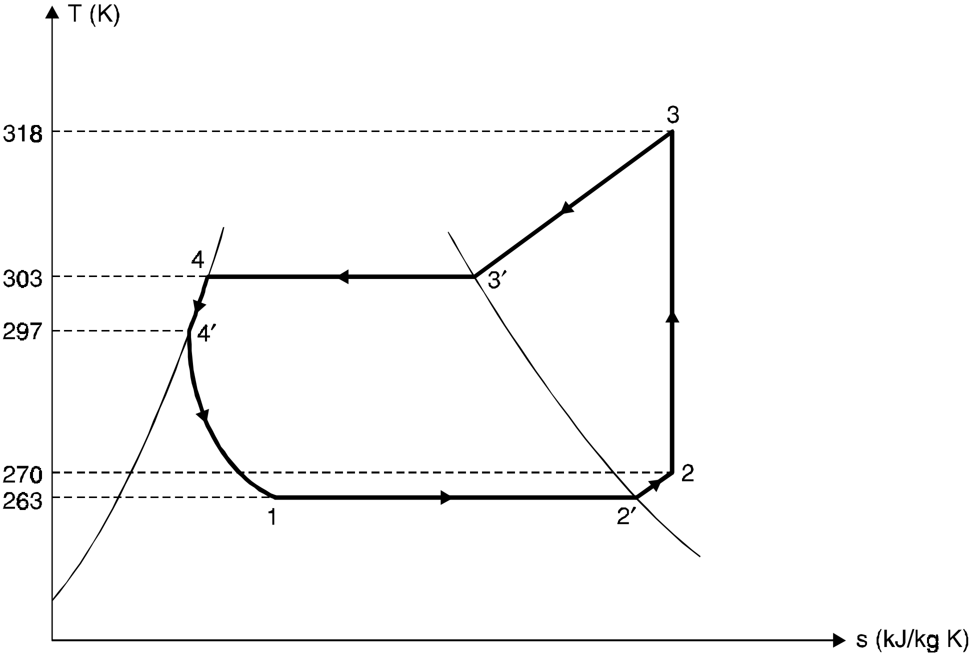


Fig. 14.35

$$= 0.6854 + 0.733 \log_e \left(\frac{T_3}{303}\right)$$

$$\therefore \qquad \log_e \left(\frac{T_3}{303}\right) = 0.0488$$

i.e., $T_3 = 318$ K

Now, enthalpy at '3', $h_3 = h_3' + c_p (T_3 - T_3')$

$$= 199.6 + 0.733 (318 - 303) = 210.6 \text{ kJ/kg.}$$

Also, enthalpy at 4′, $h_{f_4'} = h_{f_4} - (c_p)_{\text{liquid}} (T_4 - T_4') = 64.6 - 1.235 \times 6 = 57.19$ kJ/kg

For the *process 4′-1,*

Enthalpy at 4′ = enthalpy at 1 = 57.19 kJ/kg

For specific volume at 2,

$$\frac{v_2'}{T_2'} = \frac{v_2}{T_2}$$

$$\therefore \qquad v_2 = \frac{v_2'}{T_2'} \times T_2 = 0.0767 \times \frac{270}{263} = 0.07874 \text{ m}^3\text{/kg}$$

(*i*) **Refrigerating effect per kg**

$= h_2 - h_1 = 188.33 - 57.19 =$ **131.14 kJ/kg. (Ans.)**

(*ii*) **Mass of refrigerant to be circulated per minute** for producing effect of 2400 kJ/min.

$= \dfrac{2400}{131.14} =$ **18.3 kg/min. (Ans.)**

(*iii*) **Theoretical piston displacement per minute**

= Mass flow/min. × specific volume at suction

= 18.3 × 0.07874 = 1.441 m^3/min.

(*iv*) **Theoretical power required to run the compressor**

= Mass flow of refrigerant per sec. × compressor work/kg

$$= \frac{18.3}{60} \times (h_3 - h_2) = \frac{18.3}{60} (210.6 - 188.33) \text{ kJ/s} = 6.79 \text{ kJ/s}$$

or **6.79 kW. (Ans.)**

(*v*) **Heat removed through the condenser per min.**

= Mass flow of refrigerant × heat removed per kg of refrigerant

= 18.3 ($h_3 - h_{f_4'}$) = 18.3 (210.6 – 57.19) = **2807.4 kJ/min. (Ans.)**

(*vi*) **Theoretical bore (d) and stroke (l) :**

Theoretical piston displacement per cylinder

$$= \frac{\text{Total displacement per minute}}{\text{Number of cylinder}} = \frac{1.441}{2} = 0.7205 \text{ m}^3\text{/min.}$$

Also, length of stroke = 1.25 × diameter of piston

Hence, $0.7205 = \pi/4\ d^2 \times (1.25\ d) \times 1000$

i.e., $d = 0.09$ m or **90 mm. (Ans.)**

and $l = 1.25\ d = 1.25 \times 90 =$ **112.5 mm. (Ans.)**

☞**Example 14.26.** *A refrigeration system of 10.5 tonnes capacity at an evaporator temperature of – 12°C and a condenser temperature of 27°C is needed in a food storage locker. The refrigerant ammonia is sub-cooled by 6°C before entering the expansion valve. The vapour is 0.95 dry as it leaves the evaporator coil. The compression in the compressor is of adiabatic type.*

Using p-h chart find :

(*i*) *Condition of volume at outlet of the compressor*

(*ii*) *Condition of vapour at entrance to evaporator*

(*iii*) *C.O.P.*

(*iv*) *Power required, in kW.*

Neglect valve throttling and clearance effect.

Solution. Refer Fig. 14.36.

Using *p-h* chart for ammonia,

- Locate point '2' where – 12°C cuts 0.95 dryness fraction line.
- From point '2' move along constant entropy line and locate point '3' where it cuts constant pressure line corresponding to + 27°C temperature.
- From point '3' follow constant pressure line till it cuts + 21°C temperature line to get point '4'.
- From point '4' drop a vertical line to cut constant pressure line corresponding to – 12°C and get the point '5'.

The values as read from the chart are :

h_2 = 1597 kJ/kg

h_3 = 1790 kJ/kg

$h_4 = h_1$ = 513 kJ/kg

t_3 = 58°C

x_1 = 0.13.

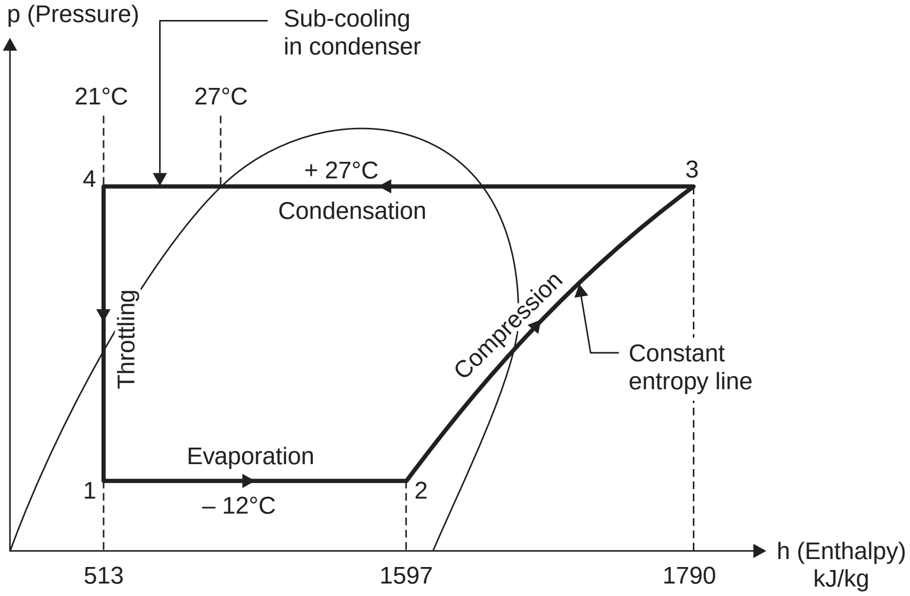


Fig. 14.36

(*i*) **Condition of the vapour at the outlet of the compressor**

$$= 58 - 27 = \mathbf{31°C\ superheat.}\quad \textbf{(Ans.)}$$

(*ii*) **Condition of vapour at entrance to evaporator,**

$$x_1 = \mathbf{0.13.}\quad \textbf{(Ans.)}$$

(*iii*) $$\textbf{C.O.P.} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{1597 - 513}{1790 - 1597} = \mathbf{5.6.}\quad \textbf{(Ans.)}$$

(*iv*) **Power required :**

$$\text{C.O.P.} = \frac{\text{Net refrigerating effect}}{\text{Work done}} = \frac{R_n}{W}$$

$$5.6 = \frac{10.5 \times 14000}{W \times 60}$$

$$\therefore \quad W = \frac{10.5 \times 14000}{5.6 \times 60} \text{ kJ/min} = 437.5 \text{ kJ/min.}$$

$$= 7.29 \text{ kJ/s.}$$

i.e., **Power required = 7.29 kW. (Ans.)**

☞**Example 14.27.** *The evaporator and condenser temperatures of 20 tonnes capacity freezer are – 28°C and 23°C respectively. The refrigerant – 22 is subcooled by 3°C before it enters the expansion valve and is superheated to 8°C before leaving the evaporator. The compression is isentropic. A six-cylinder single-acting compressor with stroke equal to bore running at 250 r.p.m. is used. Determine :*

(*i*) *Refrigerating effect/kg.*

(*ii*) *Mass of refrigerant to be circulated per minute.*

(*iii*) *Theoretical piston displacement per minute.*

(*iv*) *Theoretical power.*

(*v*) *C.O.P.*

(*vi*) *Heat removed through condenser.*

(*vii*) *Theoretical bore and stroke of the compressor.*

Neglect valve throttling and clearance effect.

Solution. Refer Fig. 14.37. Following the procedure as given in the previous example plot the points 1, 2, 3 and 4 on *p-h* chart for freon-22. The following values are obtained :

$$h_2 = 615 \text{ kJ/kg}$$

$$h_3 = 664 \text{ kJ/kg}$$

$$h_4 = h_1 = 446 \text{ kJ/kg}$$

$$v_2 = 0.14 \text{ m}^3\text{/kg.}$$

(*i*) **Refrigerating effect per kg** = $h_2 - h_1$ = 615 – 446 = **169 kJ/kg. (Ans.)**

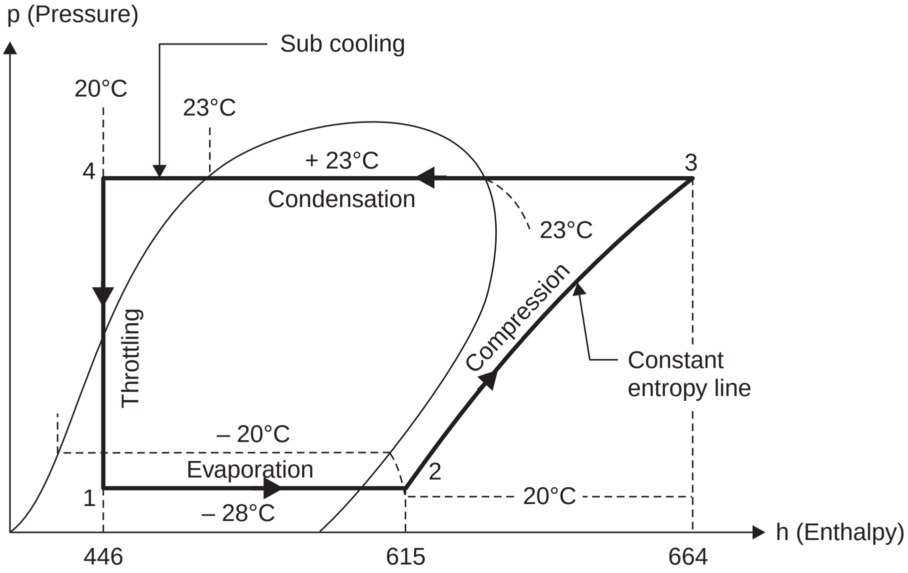


Fig. 14.37

(*ii*) **Mass of refrigerant to be circulated per minute,**

$$m = \frac{20 \times 14000}{169 \times 60} = \textbf{27.6 kg/min. (Ans.)}$$

(*iii*) **Theoretical piston displacement**

= Specific volume at suction × Mass of refrigerant used/min

= 0.14 × 27.6 = 3.864 m^3/min

(*iv*) **Theoretical power**

$$= m \times (h_3 - h_2) = \frac{27.6}{60} (664 - 615) = 22.54 \text{ kJ/s}$$

$$= \textbf{22.54 kW. (Ans.)}$$

(*v*) $$\textbf{C.O.P.} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{615 - 446}{664 - 615} = \textbf{3.45. (Ans.)}$$

(*vi*) ***Heat removed through the condenser***

$$= m\,(h_3 - h_4) = 27.6\,(664 - 446) = \mathbf{6016.8\ kJ/min.}\quad \textbf{(Ans.)}$$

(*vii*) Theoretical displacement per minute per cylinder

$$= \frac{\text{Total displacement/ min.}}{\text{Number of cylinders}} = \frac{3.864}{6} = 0.644\ \text{m}^3/\text{min}$$

Let diameter of the cylinder = d

Then, stroke length, $l = d$

Now, $$\frac{\pi}{4}\,d^2 \times l = \frac{0.644}{950}$$

or $$\frac{\pi}{4}\,d^2 \times d = \frac{0.644}{950}$$

i.e., $\mathbf{d} = 0.0952$ m or **95.2 mm. (Ans.)**

and $\mathbf{l} =$ **95.2 mm. (Ans.)**

14.5. REFRIGERANTS

A ***'refrigerant'*** *is defined as any substance that absorbs heat through expansion or vaporisation and loses it through condensation in a refrigeration system*. The term 'refrigerant' in the broadest sense is also applied to such *secondary cooling mediums* as cold water or brine, solutions. Usually refrigerants include only those working mediums which pass through the cycle of *evaporation, recovery, compression, condensation* and *liquification. These substances absorb heat at one place at low temperature level and reject the same at some other place having higher temperature and pressure. The rejection of heat takes place at the cost of some mechanical work.* Thus circulating cold mediums and cooling mediums (such as ice and solid carbondioxide) are not primary refrigerants. In the early days only four refrigerants, *Air, ammonia* (NH_3), *Carbon dioxide* (CO_2), *Sulphur dioxide* (SO_2), possessing chemical, physical and thermodynamic properties permitting their efficient application and service in the practical design of refrigeration equipment were used. All the refrigerants change from liquid state to vapour state during the process.

14.5.1. Classification of Refrigerants

The refrigerants are classified as follows :

1. Primary refrigerants.
2. Secondary refrigerants.

1. **Primary refrigerants** *are those working mediums or heat carriers which directly take part in the refrigeration system and cool the substance by the absorption of latent heat e.g. Ammonia, Carbon dioxide, Sulphur dioxide, Methyl chloride, Methylene chloride, Ethyl chloride and Freon group etc.*

2. **Secondary refrigerants** *are those circulating substances which are first cooled with the help of the primary refrigerants and are then employed for cooling purposes*, *e.g. ice, solid carbon dioxide* etc. These refrigerants cool substances by absorption of their sensible heat.

The primary refrigerants are grouped as follows :

(*i*) **Halocarbon compounds.** In 1928, Charles Kettening and Dr. Thomas Mighey invented and developed this group of refrigerant. In this group are included refrigerants which contain one or more of three halogens, chlorine and bromine and they are sold in the market under the names as *Freon, Genetron, Isotron*, and *Areton*. Since the refrigerants belonging to this group have outstanding merits over the other group's refrigerants, therefore they find wide field of application in domestic, commercial and industrial purposes.

The list of the halocarbon-refrigerants commonly used is given below :

R-10 — Carbon tetrachloride (CCl_4)
R-11 — Trichloro-monofluoro methane (CCl_3F)
R-12 — Dichloro-difluoro methane (CCl_2F_2)
R-13 — Mono-bromotrifluoro methane ($CBrF_3$)
R-21 — Dichloro monofluoro methane ($CHCl_2F$)
R-22 — Mono chloro difluoro methane ($CHClF_2$)
R-30 — Methylene-chloride (CH_2Cl_2)
R-40 — Methyle chloride (CH_3Cl)
R-41 — Methyle fluoride (CH_3F)
R-100— Ethyl chloride (C_2H_5Cl)
R-113— Trichloro trifluoroethane ($C_2F_3Cl_3$)
R-114— Tetra-fluoro dichloroethane ($Cl_2F_4Cl_2$)
R-152— Difluoro-ethane ($C_2H_6F_2$)

(*ii*) **Azeotropes.** The refrigerants belonging to this group consists of mixtures of different substances. These substances cannot be separated into components by distillations. They possess fixed thermodynamic properties and do not undergo any separation with changes in temperature and pressure. An azeotrope behaves like a simple substance.

Example. R-500. It contains 73.8% of (R-12) and 26.2% of (R-152).

(*iii*) **Hydrocarbons.** Most of the refrigerants of this group are organic compounds. Several hydrocarbons are used successfully in commercial and industrial installations. Most of them possess satisfactory thermodynamic properties but are highly inflammable. Some of the important refrigerants of this group are :

R-50 — Methane (CH_4)
R-170— Ethane (C_2H_6)
R-290— Propane (C_2H_8)
R-600— Butane (C_4H_{10})
R-601— Isobentane [$CH(CH_3)_3$]

(*iv*) **Inorganic compounds.** Before the introduction of hydrocarbon group these refrigerants were most commonly used for all purposes.

The important refrigerants of this group are :

R-717— Ammonia (NH_3)
R-718— Water (H_2O)
R-729— Air (mixture of O_2, N_2, CO_2 etc.)
R-744— Carbon dioxide (CO_2)
R-764— Sulphur dioxide (SO_2)

(*v*) **Unsaturated organic compound.** The refrigerants belonging to this group possess ethylene or propylene as their constituents. They are :

R-1120 — Trichloroethylene ($C_3H_4Cl_3$)

R-1130 — Dichloroethylene ($C_2H_4Cl_2$)
R-1150 — Ethylene (C_3H_6)
R-1270 — Propylene.

14.5.2. Desirable properties of an ideal refrigerant

An ideal refrigerant should possess the following properties :

1. **Thermodynamic properties :**

(*i*) Low boiling point
(*ii*) Low freezing point
(*iii*) Positive pressures (but not very high) in condenser and evaporator.
(*iv*) High saturation temperature
(*v*) High latent heat of vapourisation.

2. **Chemical Properties :**

(*i*) Non-toxicity
(*ii*) Non-flammable and non-explosive
(*iii*) Non-corrosiveness
(*iv*) Chemical stability in reacting
(*v*) No effect on the quality of stored (food and other) products like flowers, with other materials *i.e.*, furs and fabrics.
(*vi*) Non-irritating and odourless.

3. **Physical Properties :**

(*i*) Low specific volume of vapour
(*ii*) Low specific heat
(*iii*) High thermal conductivity
(*iv*) Low viscosity
(*v*) High electrical insulation.

4. **Other Properties :**

(*i*) Ease of leakage location
(*ii*) Availability and low cost
(*iii*) Ease of handling
(*iv*) High C.O.P.
(*v*) Low power consumption per tonne of refrigeration.
(*vi*) Low pressure ratio and pressure difference.

Some important properties (mentioned above) are discussed below :

Freezing point. As the refrigerant must operate in the cycle above its freezing point, it is evident that the same for the refrigerant *must be lower than system temperatures.* It is found that except in the case of water for which the freezing point is 0°C, other refrigerants have reasonably low values. Water, therefore, can be used only in air-conditioning applications which are above 0°C.

Condenser and evaporator pressures. The evaporating pressure should be as near atmospheric as possible. If it is *too low,* it would result in *a large volume of the suction vapour.* If it is *too high, overall high pressures including condenser pressure would result necessitating stronger equipment and consequently higher cost. A positive pressure is required in order to eliminate the possibility of the entry of air and moisture into the system. The normal boiling point of the refrigerant should, therefore, be lower than the refrigerant temperature.*

Critical temperature and pressure. Generally, for high C.O.P. the *critical temperature should be very high so that the condenser temperature line on p-h diagram is far removed from the critical point.* This ensures reasonable refrigerating effect as it is very small with the state of liquid before expansion near the critical point.

The *critical pressure should be low so as to give low condensing pressure.*

Latent heat of vapourisation. It should be *as large as possible to reduce the weight of the refrigerant to be circulated in the system. This reduces initial cost of the refrigerant. The size of the system will also be small and hence low initial cost.*

Toxicity. Taking into consideration comparative hazard to life due to gases and vapours underwriters Laboratories have divided the compounds into *six groups.* Group six contains compounds with a very low degree of toxicity. It includes R_{12}, R_{114}, R_{13}, etc. Group one, at the other end of the scale, includes the most toxic substances such as SO_2.

Ammonia is not used in comfort air-conditioning and in domestic refrigeration because of inflammability and toxicity.

Inflammability. Hydrocarbons (*e.g.* methane, ethane etc.) are highly explosive and inflammable. Fluorocarbons are neither explosive nor inflammable. *Ammonia is explosive* in a mixture with air in concentration of 16 to 25% by volume of ammonia.

Volume of suction vapour. The *size of the compressor depends on the volume of suction vapour per unit* (say per tonne) *of refrigeration. Reciprocating compressors are used with refrigerants with high pressures and small volumes of the suction vapour. Centrifugal or turbo-compressors are used with refrigerants with low pressures and large volumes of the suction vapour.* A high volume flow rate for a given capacity is required for centrifugal compressors to permit flow passages of sufficient width to minimise drag and obtain high efficiency.

Thermal conductivity. *For a high heat transfer co-efficient a high thermal conductivity is desirable.* R_{22} has better heat transfer characteristics than R_{12} ; R_{21} is still better, R_{13} has poor heat transfer characteristics.

Viscosity. For *a high heat transfer co-efficient a low viscosity is desirable.*

Leak tendency. The *refrigerants should have low leak tendency.* The greatest drawback of *fluorocarbons* is the fact that they are *odourless.* This, at times, results in a complete loss of costly gas from leaks without being detected. An ammonia leak can be very easily detected by pungent odour.

Refrigerant cost. The cost factor is only relevant to the extent of the price of the initial charge of the refrigerant which is very small compared to the total cost of the plant and its installation. The cost of losses due to leakage is also important. In small-capacity units requiring only a small charge of the refrigerant, the cost of refrigerant is immaterial.

The *cheapest refrigerant is Ammonia.* R_{12} *is slightly cheaper than* R_{22}. R_{12} and R_{22} have replaced ammonia in the dairy and frozen food industry (and even in cold storages) *because of the tendency of ammonia to attack some food products.*

Co-efficient of performance and power per tonne. Practically all common refrigerants have approximately same C.O.P. and power requirement.

Table 14.1 gives the values of C.O.P. for some important refrigerants.

Table 14.1. C.O.P. of some important refrigerants

S. No.	*Refrigerant*	*C.O.P.*
1.	Carnot value	5.74
2.	R_{11}	5.09
3.	R_{113}	4.92
4.	Ammonia	4.76
5.	R_{12}	4.70
6.	R_{22}	4.66
7.	R_{144}	4.49
	CO_2	2.56

Action with oil. No chemical reaction between refrigerant and lubricating oil of the compressor should take place. Miscibility of the oil is quite important as some oil should be carried out of the compressor crankcase with the hot refrigerant vapour to lubricate the pistons and discharge valves properly.

Reaction with materials of construction. While selecting a material to contain the refrigerant this material should be given a due consideration. Some metals are attacked by the refrigerants ; *e.g. ammonia reacts with copper, brass or other cuprous alloys in the presence of water*, therefore in *ammonia systems the common metals used are iron and steel. Freon group* does not react with steel, copper, brass, zinc, tin and aluminium but is corrosive to magnesium and aluminium having magnesium more than 2%. Freon group refrigerants tend to dissolve natural rubber in packing and gaskets but synthetic rubber such as neoprene are entirely suitable. The hydrogenerated hydrocarbons may react with zinc but not with copper, aluminium, iron and steel.

14.5.3. Properties and Uses of Commonly Used Refrigerants

1. **Air**

Properties :

(*i*) No cost involved ; easily available.

(*ii*) Completely non-toxic.

(*iii*) Completely safe.

(*iv*) The C.O.P. of air cycle operating between temperatures of 80°C and – 15°C is 1.67.

Uses :

(*i*) Air is one of the earliest refrigerants and was widely used even as late as World War I wherever a completely non-toxic medium was needed.

(*ii*) Because of low C.O.P., it is used only where *operating efficiency is secondary* as in *aircraft refrigeration.*

2. **Ammonia (NH_3)**

Properties :

(*i*) It is highly toxic and flammable.

(*ii*) It has the excellent thermal properties.

(*iii*) It has the *highest refrigerating effect per kg of refrigerant.*

(*iv*) Low volumetric displacement.

(*v*) Low cost.

(*vi*) Low weight of liquid circulated per tonne of refrigeration.

(*vii*) High efficiency.

(*viii*) The evaporator and condenser pressures are 3.5 bar abs. and 13 bar abs. (app.) respectively at standard conditions of – 15°C and 30°C.

Uses :

(*i*) It is widely used in large industrial and commercial reciprocating compression systems where high toxicity is secondary.

It is extensively used in *ice plants, packing plants, large cold storages and skating rinks* etc.

(*ii*) It is widely used as the refrigerant in *absorption systems.*

The following points are worth noting :

- Ammonia *should never be used with copper, brass and other copper alloys ; iron and steel should be* used in *ammonia systems instead*.
- In ammonia systems, to *detect the leakage a sulphur candle is used which gives off a dense white smoke when ammonia vapour is present.*

3. **Sulphur dioxide (SO_2)**

Properties :

(*i*) It is a colourless gas or liquid.

(*ii*) It is extremely toxic and has a pungent irritating odour.

(*iii*) It is non-explosive and non-flammable.

(*iv*) It has a liquid specific gravity of 1.36.

(*v*) Works at low pressures.

(*vi*) Possesses small latent heat of vapourisation.

Uses :

It finds little use these days. However its use was made in small machines in early days.

• *The leakage of sulphur dioxide may be detected by bringing aqueous ammonia near the leak, this gives off a white smoke.*

4. **Carbon dioxide (CO_2)**

Properties :

(*i*) It is a colourless and odourless gas, and is heavier than air.

(*ii*) It has liquid specific gravity of 1.56.

(*iii*) It is non-toxic and non-flammable.

(*iv*) It is non-explosive and non-corrosive.

(*v*) It has *extremely high operating pressures.*

(*vi*) It gives *very low refrigerating effect.*

Uses :

This refrigerant has received only limited use because of the high power requirements per tonne of refrigeration and the high operating pressures. In *former years* it was selected for *marine refrigeration,* for *theater air-conditioning systems,* and for *hotel and institutional refrigeration* instead of ammonia because it is non-toxic.

At the present-time its use is limited primarily to the *manufacture of dry ice* (solid carbon dioxide).

• The *leak detection of CO_2* is done by *soap solution.*

5. Methyl Chloride (CH_3Cl)

Properties :

(*i*) It is a colourless liquid with a faint, sweet, non-irritating odour.

(*ii*) It has liquid specific gravity of 1.002 at atmospheric pressure.

(*iii*) It is neither flammable nor toxic.

Uses :

It has been used in the past in both domestic and commercial applications. It should never be used with aluminium.

6. R-11 (Trichloro monofluoro methane)

Properties :

(*i*) It is composed of one carbon, three chlorine and one fluorine atoms (or parts by weight) and is *non-corrosive, non-toxic* and *non-flammable.*

(*ii*) It dissolves natural rubber.

(*iii*) It has a boiling point of – 24°C.

(*iv*) It mixes completely with mineral lubricating oil under all conditions.

Uses :

It is employed for 50 tonnes capacity and over in small office buildings and factories. A centrifugal compressor is used in the plants employing this refrigerant.

- *Its leakage is detected by a halide torch.*

7. R-12 (Dichloro-difluoro methane) or Freon-12

Properties :

(*i*) It is *non-toxic, non-flammable*, and *non-explosive*, therefore it is *most suitable refrigerant*.

(*ii*) It is fully oil miscible therefore it simplifies the problem of oil return.

(*iii*) The operating pressures of R-12 in evaporator and condenser under *standard tonne of refrigeration* are 1.9 bar abs. and 7.6 bar abs. (app.).

(*iv*) Its latent heat at – 15°C is 161.6 kJ/kg.

(*v*) C.O.P. = 4.61.

(*vi*) It does not break even under the extreme operating conditions.

(*vii*) It condenses at moderate pressure and under atmospheric conditions.

Uses :

1. It is suitable for high, medium and low temperature applications.
2. It is used for domestic applications.
3. It is *excellent electric insulator therefore it is universally used in sealed type compressors.*

8. R-22 (Monochloro-difluoro methane) or Freon-22

R-22 refrigerant is superior to R-12 in many respects. It has the following properties and uses :

Properties :

(*i*) The compressor displacement per tonne of refrigeration with R-22 is 60% less than the compressor displacement with R-12 as refrigerant.

(*ii*) R-22 is miscible with oil at condenser temperature but tries to separate at evaporator temperature when the system is used for very low temperature applications (– 90°C). Oil separators must be incorporated to return the oil from the evaporator when the system is used for such low temperature applications.

(*iii*) The pressures in the evaporator and condenser at standard tonne of refrigeration are 2.9 bar abs. and 11.9 bar abs. (app.).

(*iv*) The latent heat at – 15°C is low and is 89 kJ/kg.

The *major disadvantage of R-22 compared with R-12 is the high discharge temperature which requires water cooling of the compressor head and cylinder.*

***Uses* :**

R-22 is universally used in commercial and industrial low temperature systems.

HIGHLIGHTS

1. Refrigeration is the science of producing and maintaining temperatures below that of the surrounding atmosphere.

2. Refrigeration is generally produced in one of the following three ways :

(*i*) By melting a solid ; (*ii*) By sublimation of a solid ;

(*iii*) By evaporation of a liquid.

3. Co-efficient of performance (C.O.P.) is defined as the ratio of heat absorbed by the refrigerant while passing through the evaporator to the work input required to compress the refrigerant in the compressor ; in short it is the ratio between heat extracted and work done (in heat units).

4. Relative C.O.P. = $\dfrac{\text{Actual C.O.P}}{\text{Theoretical C.O.P}}$.

5. 1 tonne of refrigeration = 14000 kJ/h.

6. The main characteristic feature of air refrigeration system is that throughout the cycle the refrigerant remains in *gaseous state*.

The air refrigeration system may be of two types :

(*i*) Closed system and (*ii*) Open system.

7. Co-efficient of performance of a 'refrigerator' working on a reversal Carnot cycle

$$= \frac{T_2}{T_1 - T_2}$$

For a Carnot cycle 'heat pump' C.O.P. $= \dfrac{T_1}{T_1 - T_2}$

For a Carnot cycle 'heat engine' C.O.P. $= \dfrac{T_1 - T_2}{T_1}$.

8. For air refrigeration system working on reversed Brayton cycle.

$$\text{C.O.P.} = \frac{(T_3 - T_2)}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma - 1}{\gamma}\right)(T_4 - T_3 + T_2 - 1)} .$$

9. The following air refrigeration systems are used in aeroplanes :

(*i*) Simple cooling system (*ii*) Boot strap system

(*iii*) Regenerative cooling system.

10. In a simple vapour compression cycle the following processes are completed :

(*i*) Compression (*ii*) Condensation

(*iii*) Expansion (*iv*) Vaporisation.

11. The various parts of a simple vapour compression cycle are : Compressor, Discharge line (or hot gas line), Condenser, Receiver tank, Liquid line, Expansion valve, Evaporator and Suction line.

12. If the vapour is not superheated after compression, the operation is called 'Wet compression' and if the vapour is superheated at the end of compression, it is known as 'Dry compression'. Dry compression, in actual practice is always preferred as it gives higher volumetric efficiency and mechanical efficiency and there are less chances of compressor damage.

13. *p-h* chart gives directly the changes in enthalpy and pressure during a process for thermodynamic analysis.

14. When suction pressure is decreased, the refrigerating effect is decreased and work required is increased. The net effect is to reduce the refrigerating capacity of the system and the C.O.P.

15. The overall effect of superheating is to give a low value of C.O.P.

16. 'Subcooling' results in increase of C.O.P. provided that no further energy has to be spent to obtain the extra cold coolant required.

17. The refrigerating system should always be designed to operate at the highest possible vaporising temperature and lowest possible condensing temperature, of course, keeping in view the requirements of the application.

18. 'Volumetric efficiency' is defined as the ratio of the actual volume of gas drawn into the compressor (at evaporator temperature and pressure) on each stroke to the piston displacement. If the effect of clearance alone is considered, the resulting expression may be termed 'clearance volumetric efficiency'. The expression used for grouping into one constant all the factors affecting efficiency may be termed 'total volumetric efficiency'.

19. Clearance volumetric efficiency, $\eta_{cv} = 1 + C - C\left(\frac{p_d}{p_s}\right)^{1/n}$

where, $C = \frac{\text{Clearance volume}}{\text{Swept volume}}$

p_d = Displacement pressure

p_s = Suction pressure.

20. Total volumetric efficiency,

$$\eta_{tv} = \left[1 + C - C\left(\frac{p_d}{p_s}\right)^{1/n}\right] \times \frac{p_c}{p_s} \times \frac{T_s}{T_c}$$

where subscript '*c*' refers to compressor cylinder and '*s*' refers to the evaporator on the suction line just adjacent to the compressor.

OBJECTIVE TYPE QUESTIONS

Fill in the blanks :

1. ...... means the cooling of or removal of heat from a system.

2. Most of the commercial refrigeration is produced by the evaporation of a liquid ......

3. ...... is the ratio between the heat extracted and the work done.

4. ...... = $\frac{\text{Actual C.O.P}}{\text{Theoretical C.O.P.}}$.

5. The C.O.P. for Carnot refrigerator is equal to ......

6. The C.O.P. for a Carnot heat pump is equal to .....

7. The C.O.P. for a Carnot refrigerator is ...... than that of Carnot heat pump.

8. The C.O.P. of an air refrigeration system is ...... than a vapour compression system.

9. In a refrigeration system the heat rejected at higher temperature = ... + ...

10. Out of all the refrigeration systems, the ...... system is the most important system from the stand point of commercial and domestic utility.

11. The function of a ...... is to remove the vapour from the evaporator and to raise its temperature and pressure to a point such that it (vapour) can be condensed with normally available condensing media.

12. The function of a ...... is to provide a heat transfer surface through which a heat passes from the hot refrigerant vapour to the condensing medium.

13. The function of ...... is to meter the proper amount of refrigerant to the evaporator and to reduce the pressure of liquid entering the evaporator so that liquid will vaporise in the evaporator at the desired low temperature.

14. ...... provides a heat transfer surface through which heat can pass from the refrigerated space or product into the vaporising refrigerant.

15. If the vapour is not superheated after compression, the operation is called .....

16. If the vapour is superheated at the end of compression, the operation is called ......

17. When the suction pressure decreases the refrigerating effect and C.O.P. are ......

18. ...... results in increase of C.O.P. provided that no further energy has to be spent to obtain the extra cold coolant required.

19. ...... efficiency is defined as the ratio of actual volume of gas drawn into the compressor (at evaporator temperature and pressure) on each stroke to the piston displacement.

ANSWERS

1. Refrigeration
2. Refrigerant
3. C.O.P.
4. Relative C.O.P.
5. $\dfrac{T_2}{T_1 - T_2}$
6. $\dfrac{T_1}{T_1 - T_2}$
7. Less
8. Less
9. Refrigeration effect + work done
10. Vapour compression
11. Compressor
12. Condenser
13. Expansion valve
14. Evaporator
15. Wet compression
16. Dry compression
17. Reduced
18. Sub-cooling
19. Volumetric.

THEORETICAL QUESTIONS

1. Define the following :
(*i*) Refrigeration
(*ii*) Refrigerating system
(*iii*) Refrigerated system.

2. Enumerate different ways of producing refrigeration.

3. Enumerate important refrigeration applications.

4. State elements of refrigeration systems.

5. Enumerate systems of refrigeration.

6. Define the following :
(*i*) Actual C.O.P.
(*ii*) Theoretical C.O.P.
(*iii*) Relative C.O.P.

7. What is a standard rating of a refrigeration machine ?

8. What is main characteristic feature of an air refrigeration system ?

9. Differentiate clearly between open and closed air refrigeration systems.

10. Explain briefly an air refrigerator working on a reversed Carnot cycle. Derive expression for its C.O.P.

11. Derive an expression for C.O.P. for an air refrigeration system working on reversed Brayton cycle.

12. State merits and demerits of an air refrigeration system.

13. Describe a simple vapour compression cycle giving clearly its flow diagram.

14. State merits and demerits of 'Vapour compression system' over 'Air refrigeration system'.

15. State the functions of the following parts of a simple vapour compression system :

(*i*) Compressor, (*ii*) Condenser,
(*iii*) Expansion valve, and (*iv*) Evaporator.

16. Show the vapour compression cycle on 'Temperature-Entropy' (T-s) diagram for the following cases :

(*i*) When the vapour is dry and saturated at the end of compression.
(*ii*) When the vapour is superheated after compression.
(*iii*) When the vapour is wet after compression.

17. What is the difference between 'Wet compression' and 'Dry compression' ?

18. Write a short note on 'Pressure Enthalpy (p-h) chart'.

19. Show the simple vapour compression cycle on a p-h chart.

20. Discuss the effect of the following on the performance of a vapour compression system :

(*i*) Effect of suction pressure (*ii*) Effect of delivery pressure
(*iii*) Effect of superheating (*iv*) Effect of sub-cooling of liquid
(*v*) Effect of suction temperature and condenser temperature.

21. Show with the help of diagrams, the difference between theoretical and actual vapour compression cycles.

22. Define the terms 'Volumetric efficiency' and 'Clearance volumetric efficiency'.

23. Derive an expression for 'Clearance volumetric efficiency'.

24. Explain briefly the term 'Total volumetric efficiency'.

25. Explain briefly simple vapour absorption system.

26. Give the comparison between a vapour compression system and a vapour absorption system.

UNSOLVED EXAMPLES

1. The co-efficient of performance of a Carnot refrigerator, when it extracts 8350 kJ/min from a heat source, is 5. Find power required to run the compressor. [**Ans.** 27.83 kW]

2. A reversed cycle has refrigerating C.O.P. of 4,

(*i*) Determine the ratio T_1/T_2 ; and
(*ii*) If this cycle is used as heat pump, determine the C.O.P. and heat delivered.

[**Ans.** (*i*) 1.25 (*ii*) 50 kW, 5]

3. An ice plant produces 10 tonnes of ice per day at 0°C, using water at room temperature of 20°C. Estimate the power rating of the compressor motor if the C.O.P. of the plant is 2.5 and overall electromechanical efficiency is 0.9.

Take latent heat of freezing for water = 335 kJ/kg
Specific heat of water = 4.18 kJ/kg. [**Ans.** 21.44 kW]

4. An air refrigeration system operating on Bell Coleman cycle, takes in air from cold room at 268 K and compresses it from 1.0 bar to 5.5 bar. The index of compression being 1.25. The compressed air is cooled to 300 K. The ambient temperature is 20°C. Air expands in an expander where the index of expansion is 1.35.

Calculate : (*i*) C.O.P. of the system (*ii*) Quantity of air circulated per minute for production of 1500 kg of ice per day at 0°C from water at 20°C. (*iii*) Capacity of the plant in terms of kJ/s.

Take c_p = 4.18 kJ/kg K for water, c_p = 1.005 kJ/kg K for air
Latent heat of ice = 335 kJ/kg. [**Ans.** 1.974 ; 5.814 kg/min ; 7.27 kJ/s]

5. The temperature in a refrigerator coil is 267 K and that in the condenser coil is 295 K. Assuming that the machine operates on the reversed Carnot cycle, calculate :

(*i*) C.O.P.$_{\text{(ref.)}}$
(*ii*) The refrigerating effect per kW of input work.
(*iii*) The heat rejected to the condenser. [**Ans.** (*i*) 9.54 (*ii*) 9.54 kW (*iii*) 10.54 kW]

6. An ammonia vapour-compression refrigerator operates between an evaporator pressure of 2.077 bar and a condenser pressure of 12.37 bar. The following cycles are to be compared ; in each case there is no undercooling in the condenser, and isentropic compression may be assumed :

(*i*) The vapour has a dryness fraction of 0.9 at entry to the compressor.

(*ii*) The vapour is dry saturated at entry to the compressor.

(*iii*) The vapour has 5 K of superheat at entry to the compressor.

In each case calculate the C.O.P.$_{\text{(ref.)}}$ and the refrigerating effect per kg.

What would be the C.O.P.$_{\text{(ref.)}}$ of a reversed Carnot cycle operating between the same saturation temperatures ? [**Ans.** 4.5 ; 957.5 kJ/kg ; 4.13 ; 1089.9 kJ/kg ; 4.1 ; 1101.4 kJ/kg]

7. A refrigerator using Freon-12 operates between saturation temperatures of – 10°C and 60°C, at which temperatures the latent heats are 156.32 kJ/kg and 113.52 kJ/kg respectively. The refrigerant is dry saturated at entry to the compressor and the liquid is not undercooled in the condenser. The specific heat of liquid freon is 0.970 kJ/kg K and that of the superheated freon vapour is 0.865 kJ/kg K. The vapour is compressed isentropically in the compressor. Using no other information than that given, calculate the temperature at the compressor delivery, and the refrigerating effect per kg of Freon.

[**Ans.** 69.6°C ; 88.42 kJ/kg]

8. A heat pump using ammonia as the refrigerant operates between saturation temperatures of 6°C and 38°C. The refrigerant is compressed isentropically from dry saturation and there is 6 K of undercooling in the condenser. Calculate :

(*i*) C.O.P.$_{\text{(heat pump)}}$ (*ii*) The mass flow of refrigerant

(*iii*) The heat available per kilowatt input. [**Ans.** 8.8 ; 25.06 kg/h ; 8.8 kW]

9. An ammonia vapour-compression refrigerator has a single-stage, single-acting reciprocating compressor which has a bore of 127 mm, a stroke of 152 mm and a speed of 240 r.p.m. The pressure in the evaporator is 1.588 bar and that in the condenser is 13.89 bar. The volumetric efficiency of the compressor is 80% and its mechanical efficiency is 90%. The vapour is dry saturated on leaving the evaporator and the liquid leaves the condenser at 32°C. Calculate the mass flow of refrigerant, the refrigerating effect, and the power ideally required to drive the compressor. [**Ans.** 0.502 kg/min ; 9.04 kW ; 2.73 kW]

10. An ammonia refrigerator operates between evaporating and condensing temperatures of – 16°C and 50°C respectively. The vapour is dry saturated at the compressor inlet, the compression process is isentropic and there is no undercooling of the condensate.

Calculate :

(*i*) The refrigerating effect per kg,

(*ii*) The mass flow and power input per kW of refrigeration, and

(*iii*) The C.O.P.$_{\text{(ref.)}}$. [**Ans.** 1003.4 kJ/kg ; 3.59 kg/h ; 0.338 kW ; 2.96]

11. 30 tonnes of ice from and at 0°C is produced in a day of 24 hours by an ammonia refrigerator. The temperature range in the compressor is from 298 K to 258 K. The vapour is dry saturated at the end of compression and expansion valve is used. Assume a co-efficient of performance of 60% of the theoretical and calculate the power in kW required to drive the compressor. Latent heat of ice is 334.72 kJ/kg.

Temp. *K*	*Enthalpy* *kJ/kg*		*Entropy of liquid* *kJ/kg K*	*Entropy of vapour* *kJ/kg*
	Liquid	*Vapour*		
298	100.04	1319.22	0.3473	4.4852
258	– 54.56	1304.99	– 2.1338	5.0585

[**Ans.** 21.59 kW]

12. A refrigerant plant works between temperature limits of – 5°C (in the evaporator) and 25°C (in the condenser). The working fluid ammonia has a dryness fraction of 0.6 at entry to the compressor. If the machine has a relative efficiency of 50%, calculate the amount of ice formed during a period of 24 hours.

The ice is to be formed at 0°C from water at 20°C and 6 kg of ammonia is circulated per minute. Specific heat of water is 4.187 kJ/kg and latent heat of ice is 335 kJ/kg.

Properties of a ammonia (datum – 40°C) :

Temp. *K*	*Liquid heat* *kJ/kg*	*Latent heat* *kJ/kg*	*Entropy of liquid* *kJ/kg°C*
298	298.9	1167.1	1.124
268	158.2	1280.8	0.630

[**Ans.** 1640.5 kg]

13. A food storage locker requires a refrigeration system of 2500 kJ/min capacity at an evaporator temperature of – 10°C and a condenser temperature of 30°C. The refrigerant used is Freon-12 and sub-cooled by 5°C before entering the expansion valve and vapour is superheated by 6°C before leaving the evaporator coil. The compression of refrigerant is reversible adiabatic. The refrigeration compressor is two-cylinder single-acting with stroke equal to 1.3 times the bore and operates at 975 r.p.m. Determine (using thermodynamic tables of properties for Freon-12) :

(*i*) Refrigerating effect per kg.
(*ii*) Mass of refrigerant to be circulated per minute.
(*iii*) Theoretical piston displacement per minute.
(*iv*) Theoretical power required to run the compressor, in kW.
(*v*) Heat removed through the condenser per minute.
(*vi*) Theoretical bore and stroke of compressor.

Properties of Freon-12

Saturation temp. *°C*	*Absolute pressure*	*Specific volume of vapour* *m^3/kg*	*Enthalpy*		*Entropy*	
			Liquid *kJ/kg*	*Vapour* *kJ/kg*	*Liquid* *kJ/kg K*	*Vapour* *kJ/kg K*
– 10°C	2.19	0.0767	26.9	183.2	0.1080	0.7020
30°C	7.45	0.0235	64.6	199.6	0.2399	0.6854

Take : Liquid specific heat = 1.235 kJ/kg K
Vapour specific heat = 0.735 kJ/kg K.

[**Ans.** (*i*) 129.17 kJ/kg (*ii*) 19.355 kg/min (*iii*) 1.518 m^3/min (*iv*) 7.2 kW
(*v*) 2931 kJ/min (*vi*) 91 mm, 118 mm]

14. A vapour compression refrigerator uses methyl chloride and works in the pressure range of 11.9 bar and 5.67 bar. At the beginning of the compression, the refrigerant is 0.96 dry and at the end of isentropic compression, it has a temperature of 55°C. The refrigerant liquid leaving the condenser is saturated. If the mass flow of refrigerant is 1.8 kg/min. Determine :

(*i*) Co-efficient of performance.
(*ii*) The rise in temperature of condenser cooling water if the water flow rate is 16 kg/min.
(*iii*) The ice produced in the evaporator in kg/hour from water at 15°C and ice at 0°C.

Properties of methyl chloride :

Saturation temp. *(°C)*	*Pressure* *(bar)*	*Enthalpy (kJ/kg)*		*Entropy (kJ/kg K)*	
		h_f	h_g	s_f	s_g
– 20	11.9	30.1	455.2	0.124	1.803
25	5.67	100.5	476.8	0.379	1.642

Take : Specific enthalpy of fusion of ice = 336 kJ/kg

Specific heat of water = 4.187 kJ/kg. [**Ans.** 4.97, 10.9°C, 91.3 kg]

15. A vapour compression refrigerator circulates 4.5 kg of NH_3 per hour. Condensation take place at 30°C and evaporation at – 15°C. There is no under-cooling of the refrigerant. The temperature after isentropic compression is 75°C and specific heat of superheated vapour is 2.82 kJ/kg K. Determine :

(*i*) Co-efficient of performance.

(*ii*) Ice produced in kg per hour in the evaporator from water at 20°C and ice at 0°C. Take : Enthalpy of fusion of ice = 336 kJ/kg, specific heat of water = 4.187 kJ/kg.

(*iii*) The effective swept volume of the compressor in m^3/min.

Properties of ammonia :

Sat. temp. (K)	*Enthalpy (kJ/kg)*		*Entropy (kJ/kg K)*		*Volume (m^3/kg)*	
	h_f	h_g	s_f	s_g	v_f	v_g
303	323.1	1469	1.204	4.984	0.00168	0.111
258	112.3	1426	0.457	5.549	0.00152	0.509

[**Ans**. 4.95, 682 kg/h, 2.2 m^3/min]

15

Heat Transfer

15.1. MODES OF HEAT TRANSFER

"*Heat transfer*" which is defined as the *transmission of energy from one region to another as a result of temperature gradient* takes place by the following *three modes :*

(*i*) Conduction ; (*ii*) Convection ; (*iii*) Radiation.

Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. *Example :* The water in a boiler shell receives its heat from the fire-bed by conducted, convected and radiated heat from the fire to the shell, conducted heat through the shell and conducted and convected heat from the inner shell wall, to the water. *Heat always flows in the direction of lower temperature.*

The above three modes are similar in that a temperature differential must exist and the heat exchange is in the direction of decreasing temperature ; each method, however, has different controlling laws.

(*i*) **Conduction.** '*Conduction*' is the *transfer of heat from one part of a substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance*.

In *solids*, the heat is conducted by the following *two mechanisms :*

(*i*) ***By lattice vibration*** (The faster moving molecules or atoms in the hottest part of a body transfer heat by impacts some of their energy to adjacent molecules).

(*ii*) ***By transport of free electrons*** (Free electrons provide an energy flux in the direction of decreasing temperature—For metals, especially good electrical conductors, the electronic mechanism is responsible for the major portion of the heat flux except at low temperature).

In case of *gases*, the mechanisam of heat conduction is simple. The kinetic energy of a molecule is a function of temperature. These molecules are in a continuous random motion exchanging energy and momentum. When a molecule from the high temperature region collides with a molecule from the low temperature region, it loses energy by collisions.

In liquids, the mechanism of heat is nearer to that of gases. However, the molecules are more closely spaced and intermolecular forces come into play.

(*ii*) **Convection.** *'Convection' is the transfer of heat within a fluid by mixing of one portion of the fluid with another.*

- Convection is possible only in a fluid medium and is *directly linked* with the *transport of medium itself.*
- Convection constitutes the *macroform* of the heat transfer since macroscopic particles of a fluid moving in space cause the heat exchange.
- The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid.

This mode of heat transfer is met with in situations where energy is transferred as heat to a flowing fluid at any surface over which flow occurs. This mode is *basically conduction in a very thin fluid layer at the surface and then mixing caused by the flow.* The heat flow depends on the properties of fluid and is independent of the properties of the material of the surface. However, the shape of the surface will influence the flow and hence the heat transfer.

Free or natural convection. *Free or natural convection occurs where the fluid circulates by virtue of the natural differences in densities of hot and cold fluids ; the denser portions of the fluid move downward because of the greater force of gravity, as compared with the force on the less dense.*

Forced convection. *When the work is done to blow or pump the fluid, it is said to be forced convection.*

(*iii*) **Radiation.** *'Radiation' is the transfer of heat through space or matter by means other than conduction or convection.*

Radiation heat is thought of as *electromagnetic waves or quanta* (as convenient) an emanation of the same nature as light and radio waves. *All bodies radiate heat ; so a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits*. Radiant *energy* (being electromagnetic radiation) *requires no medium for propagation and will pass through a vacuum*.

Note. The rapidly oscillating molecules of the hot body produce electromagnetic waves in hypothetical medium called *ether*. These waves are identical with light waves, radio waves and X-rays, differ from them only in *wavelength* and travel with an approximate velocity of 3×10^8 m/s. These waves carry energy with them and *transfer it to the relatively slow-moving molecules of the cold body* on which they happen to fall. The molecular energy of the later increases and results in a rise of its temperature. Heat travelling by radiation is known as *radiant heat.*

The properties of radiant heat in general, are similar to those of light. Some of the properties are :

(*i*) It does not require the presence of a material medium for its transmission.

(*ii*) Radiant heat can be reflected from the surfaces and obeys the ordinary laws of reflection.

(*iii*) It travels with velocity of light.

(*iv*) Like light, it shows interference, diffraction and polarisation etc.

(*v*) It follows the law of inverse square.

The wavelength of heat radiations is longer than that of light waves, hence they are invisible to the eye.

15.2. HEAT TRANSFER BY CONDUCTION

15.2.1. Fourier's Law of Heat Conduction

Fourier's law of heat conduction is an empirical law based on observation and states as follows :

"The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of the heat flow".

Mathematically, it can be represented by the equation :

$$Q \propto A \,.\, \frac{dt}{dx}$$

where, Q = Heat flow through a body per unit time (in watts), W,

A = Surface area of heat flow (*perpendicular to the direction of flow*), m^2,

dt = Temperature difference of the faces of block (homogeneous solid) of thickness 'dx' through which heat flows,°C or K, and

dx = Thickness of body in the direction of flow, m.

Thus, $$Q = -\,k \,.\, A \,\frac{dt}{dx} \qquad ...(15.1)$$

where, k = Constant of proportionality and is known as *thermal conductivity of the body.*

The –ve sign of k [eqn. (15.1)] is to take care of the decreasing temperature alongwith the direction of increasing thickness or the direction of heat flow. The temperature gradient $\frac{dt}{dx}$ is *always negative along positive x* direction and therefore the value of Q *becomes* +ve.

Assumptions :

The following are the *assumptions* on which Fourier's law is based :

1. Conduction of heat takes place under *steady state conditions.*
2. The heat flow is unidirectional.
3. The temperatures gradient is *constant* and the temperature profile is *linear.*
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (*i.e.*, the value of thermal conductivity is *constant in all directions*).

Some essential features of Fourier's Law :

Following are some *essential features of Fourier's law* :

1. It is applicable to all matter (may be solid, liquid or gas).
2. It is based on experimental evidence and cannot be derived from first principle.
3. It is a vector expression indicating that heat flow rate is in the direction of decreasing temperature and is normal to an isotherm.
4. It helps to define thermal conductivity 'k' (transport property) of the medium through which heat is conducted.

15.2.2. Thermal Conductivity of Materials

From eqn. (15.1), we have $k = \frac{Q}{A} \cdot \frac{dx}{dt}$

The value of k = 1 when Q = 1, A = 1 and $\frac{dt}{dx}$ = 1

Now $k = \frac{Q}{1} \,.\, \frac{dx}{dt}$ (unit of k : $\text{W} \times \frac{1}{\text{m}^2} \times \frac{\text{m}}{\text{K (or °C)}}$ = W/mK. or W/m°C)

Thus, the *thermal conductivity of a material is defined* as follows :

"The amount of energy conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference".

It follows from eqn. (15.1) that materials with high thermal conductivities are good conductors of heat, whereas materials with low thermal conductives are good thermal insulator. *Conduction of heat occurs most readily in pure metals, less so in alloys, and much less readily in non-metals.* The very low thermal conductivities of certain thermal insulators *e.g.,* cork is due to their porosity, the air trapped within the material acting as an insulator.

Thermal conductivity (a property of material) depends essentially upon the following factors :

(*i*) Material structure (*ii*) Moisture content

(*iii*) Density of the material

(*iv*) Pressure and temperature (operating conditions)

Thermal conductivities (average values at normal pressure and temperature) of some common materials are as under :

Material	*Thermal conductivity (k) (W/mK)*	*Material*	*Thermal conductivity (k) (W/mK)*
1. Silver	410	8. Asbestos sheet	0.17
2. Copper	385	9. Ash	0.12
3. Aluminum	225	10. Cork, felt	0.05–0.10
4. Cast-iron	55–65	11. Saw dust	0.07
5. Steel	20–45	12. Glass wool	0.03
6. Concrete	1.20	13. Water	0.55–0.7
7. Glass (window)	0.75	14. Freon	0.0083

Following points *regarding thermal conductivity—its variation for different materials and under different conditions* are worth noting :

1. Thermal conductivity of a material is due to flow of free electrons (in case of *metals*) and lattice vibrational waves (in case of *fluids*).
2. Thermal conductivity in case of *pure metals* is the highest (k = 10 to 400 W/m°C). It *decreases with increase in impurity*.

   The range of k for other materials is as follows :

   Alloys : = k = 12 to 120 W/m°C

   Heat insulating and building materials : k = 0.023 to 2.9 W/m°C

   Liquids : k = 0.2 to 0.5 W/m°C

   Gases and vapours : k = 0.006 to 0.05 W/m°C.
3. Thermal conductivity of a metal varies considerably when it (metal) is heat treated or mechanically processed/formed.
4. Thermal conductivity of *most metals decreases with the increase in temperature* (*aluminium* and *uranium* being the *exceptions*).

   — In most of *liquids* the value of thermal conductivity tends to decrease with temperature (water being an exception) due to decrease in density with increase in temperature.

   — In case of gases the value of thermal conductivity *increases with temperature.* Gases with higher molecular weights have smaller thermal conductivities than with lower molecular weights. This is because the mean molecular path of gas molecules decreases with increase in density and k is directly proportional to the mean free path of the molecule.
5. The dependence of thermal conductivity (k) on temperature, for most materials is almost linear ;

$$k = k_0 (1 + \beta t) \qquad ...(15.2)$$

where, k_0 = Thermal conductivity at 0°C, and

β = Temperature coefficient of thermal conductivity, 1/°C (It is usually *positive for non-metals* and *insulating materials* (magnesite bricks being the exception) and *negative for metallic conductors* (aluminium and certain non-ferrous alloys are the exceptions).

6. In case of solids and liquids, thermal conductivity (k) is only very weakly dependent on pressure ; in case of gases the value of k is independent of pressure (near standard atmospheric).
7. In case of non-metallic solids :

   — Thermal conductivity of porous materials depends upon the type of gas or liquid present in the voids.

   — Thermal conductivity of a *damp material is considerably higher than that of the dry material and water taken individually*.

   — Thermal conductivity *increases with increase in density*.
8. The Wiedemann and Franz law (based on experiment results), regarding thermal and electrical conductivities of a material, states as follows :

   "*The ratio of the thermal and electrical conductivities is the same for all metals at the same temperature ; and that the ratio is directly proportional to the absolute temperature of the metal.*"

   Mathematically, $$\frac{k}{\sigma} \propto T$$

   or $$\frac{k}{\sigma T} = C \qquad \text{...(15.3)}$$

   where, k = Thermal conductivity of metal at temperature T(K),

   σ = Electrical conductivity of metal at temperature T(K), and

   C = Constant (for all metals) is referred to as Lorenz number

   (= 2.45×10^{-8} WΩ/K^2 ; Ω stands for ohms).

   This law conveys that the materials which are *good conductors of electricity are also good conductors of heat.*

15.2.3. Thermal Resistance (R_{th})

When two physical systems are described by similar equations and have similar boundary conditions, these are said to be *analogous.* The heat transfer processes may be compared by *analogy* with the flow of electricity in an electrical resistance. As the flow of electric current in the electrical resistance is directly proportional to potential difference (dV) ; similarly heat flow rate, Q, is directly proportional to temperature difference (dt), the driving force for heat conduction through a medium.

As per Ohm's law (in electric-circuit theory), we have

$$\text{Current } (I) = \frac{\text{Potential difference } (dV)}{\text{Electrical resistance } (R)} \qquad \text{...(15.4)}$$

By analogy, the heat flow equation (Fourier's equation) may be written as

$$\text{Heat flow rate } (Q) = \frac{\text{Temperature difference } (dt)}{\left(\dfrac{dx}{kA}\right)} \qquad \text{...(15.5)}$$

By comparing eqns. (15.4) and (15.5), we find that I is analogus to, Q, dV is analogous to dt and R is analogous to the quantity $\left(\dfrac{dx}{kA}\right)$. The quantity $\dfrac{dx}{kA}$ is called *thermal conduction resistance* $(\mathbf{R_{th}})_{\text{cond.}}$ *i.e.,*

$$(R_{\text{th}})_{\text{cond.}} = \frac{dx}{kA}$$

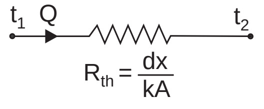


Fig. 15.1

The reciprocal of the thermal resistance is called *thermal conductance.*

It may be noted that *rules for combining electrical resistances in series and parallel apply equally well to thermal resistances.*

The concept of thermal resistance is quite helpful white making calculations for flow of heat.

15.2.4. General Heat Conduction Equation in Cartesian Coordinates

Consider an infinitesimal rectangular parallelopiped (volume element) of sides dx, dy and dz parallel, respectively, to the three axes (X, Y, Z) in a medium in which temperature is varying with location and time as shown in Fig. 15.2.

Let, t = Temperature at the left face *ABCD* ; this temperature may be assumed uniform over the entire surface, since the area of this face can be made arbitrarily *small*.

$\dfrac{dt}{dx}$ = Temperature changes and rate of change along X-direction.

Then, $\left(\dfrac{\partial t}{\partial x}\right) dx$ = Change of temperature through distance dx, and

$t + \left(\dfrac{\partial t}{\partial x}\right) dx$ = Temperature on the right face *EFGH* (at distance dx from the left face *ABCD*).

Further, let, k_x, k_y, k_z = Thermal conductivities (direction characteristics of the material) along X, Y and Z axes.

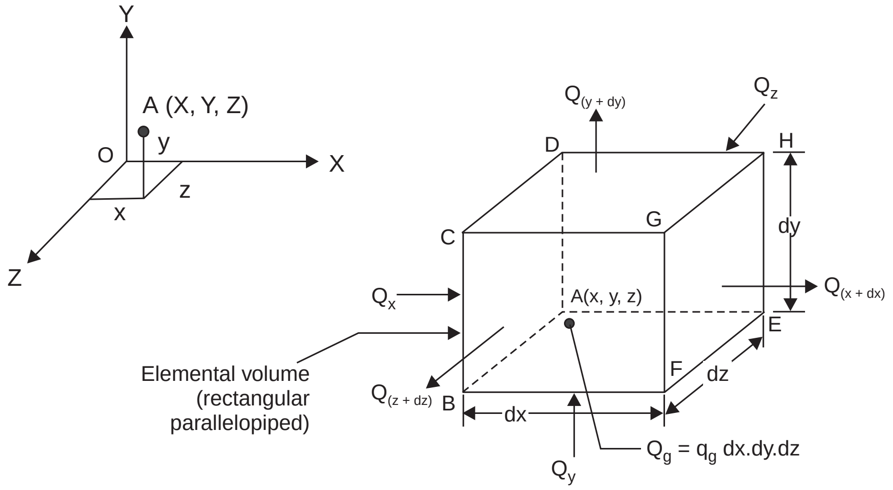


Fig. 15.2. Elemental volume for three-dimensional heat conduction analysis—Cartesian co-ordinates.

If the directional characteristics of a material are equal/same, it is called an "*Isotropic material*" and if unequal/different '*Anisotropic material*'.

q_g = Heat generated per unit volume per unit time.

Inside the control volume there may be heat sources due to flow of electric current in electric motors and generators, nuclear fission etc.

[**Note.** q_g may be function of position or time, or both].

ρ = Mass density of material.

c = Specific heat of the material.

Energy balance/equation for volume element :

Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered (*A*) + heat generated within the element (*B*) = Energy stored in the element (*C*) ...(1)

Let, Q = Rate of *heat flow* in a direction, and

$Q' = (Q.d\tau)$ = *Total* heat flow (flux) in that direction (in time $d\tau$).

A. *Net heat accumulated in the element due to conduction of heat from all the directions considered :*

Quantity of heat flowing into the element from the left face *ABCD* during the time interval $d\tau$ in *X*-direction is given by :

$$\text{Heat influx. } Q_x' = -\,k_x(dy.dz)\frac{\partial t}{\partial x}\,.\,d\tau \qquad ...(i)$$

During the same time interval $d\tau$ the heat flowing out of the right face of control volume (*EFGH*) will be :

$$\text{Heat efflux. } Q'_{(x+dx)} = Q_x' + \frac{\partial}{\partial x}(Q_x')\,dx \qquad ...(ii)$$

∴ Heat accumulation in the element due to heat flow in *X*-direction,

$$dQ_x' = Q_x' - \left[Q_x' + \frac{\partial}{\partial x}(Q_x')\,dx\right] \qquad \text{[Subtracting } (ii) \text{ from } (i)]$$

$$= -\frac{\partial}{\partial x}(Q_x')\,dx$$

$$= -\frac{\partial}{\partial x}\left[-\,k_x(dy.dz)\frac{\partial t}{\partial x}\,.\,d\tau\right]dx$$

$$= \frac{\partial}{\partial x}\left[k_x\,\frac{\partial t}{\partial x}\right]dx.dy.dz.d\tau \qquad ...(15.6)$$

Similarly the heat accumulated due to heat flow by conduction along *Y* and *Z* directions in time $d\tau$ will be :

$$dQ_y' = \frac{\partial}{\partial x}\left[k_y\,\frac{\partial t}{\partial y}\right]dx.dy.dz.d\tau \qquad ...(15.7)$$

$$dQ_z' = \frac{\partial}{\partial z}\left[k_z\,\frac{\partial t}{\partial z}\right]dx.dy.dz.d\tau \qquad ...(15.8)$$

∴ *Net heat accumulated* in the element due to conduction of heat from all the co-ordinate directions considered

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dx.dy.dz.d\tau$$

$$= \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau \qquad ...(15.9)$$

B. *Total heat generated within the element* (Q_g') :

The total heat generated in the element is given by :

$$Q_g' = q_g(dx.dy.dz)\, d\tau \qquad ...(15.10)$$

C. *Energy stored in the element :*

The total heat accumulated in the element due to heat flow along coordinate axes (eqn. 15.9) and the haet generated within the element (eqn. 15.10) together serve to increase the thermal energy of the element/lattice. This increase in thermal energy is given by :

$$\rho(dx.dy.dz)c.\frac{\partial t}{\partial \tau}.d\tau \qquad ...(15.11)$$

[$\because$ Heat stored in the body = Mass of the body × specific heat of the body material × rise in the temperature of body].

Now, substituting eqns. (15.9), (15.10), (15.11), in the eqn. (1), we have

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau + q_g(dx.dy.dz)d\tau = \rho(dx.dy.dz)c.\frac{\partial t}{\partial \tau}.d\tau$$

Dividing both sides by $dx.dy.dz.d\tau$, we have

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right) + q_g = \rho.c.\frac{\partial t}{\partial \tau} \qquad ...(15.12)$$

or, using the vector operator ∇, we get

$$\nabla . (k\nabla t) + q_g = \rho.c.\frac{\partial t}{\partial \tau}$$

This is known as the **general heat conduction equation** for **'non-homogeneous material', self heat generating'** and **'unsteady three-dimensional flow'**. This equation establishes in *differential form the relationship between the time and space variation of temperature at any point of solid through which heat flow by conduction takes place.*

General heat conduction equation for constant thermal conductivity :

In case of *homogeneous* (in which properties *e.g.,* specific heat, density, thermal conductivity etc. are same everywhere in the material) and *isotropic* (in which properties are independent of surface orientation) material, $k_x = k_y = k_z = k$ and diffusion equation eqn. (15.12) becomes

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho.c}{k}.\frac{\partial t}{\partial \tau} = \frac{1}{\alpha}.\frac{\partial t}{\partial \tau} \qquad ...(15.13)$$

where $\alpha = \dfrac{k}{\rho.c} = \dfrac{\text{Thermal conductivity}}{\text{Thermal capacity}}$

The quantity, $\alpha = \dfrac{k}{\rho.c}$ is known as **thermal diffusivity.**

— The larger the value of α, the faster will the heat diffuse through the material and its temperature will change with time. This will result either due to a high value of thermal conductivity k or a low value of heat capacity $\rho.c$. A low value of heat capacity means the less amount of heat entering the element would be absorbed and used to raise its temperature and more would be available for onward transmission. Metals and gases have relatively high value of α and their response to temperature changes is quite rapid. The non-metallic solids and liquids respond slowly to temperature changes because of their relatively small value of thermal diffusivity.

— Thermal diffusivity is an important characteristic quantity for *unsteady conduction situations.*

Eqn. (15.13) by using Laplacian ∇^2, may be written as :

$$\nabla^2 t + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...[15.13 } (a)]$$

Eqn. (15.13), governs the temperature distribution under unsteady heat flow through a material which is homogeneous and isotropic.

Other simplified forms of heat conduction equation in cartesian co-ordinates :

(*i*) For the case when *no internal source of heat generation is present.* Eqn. (15.13) reduces to $\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$ [Unsteady state $\left(\frac{\partial t}{\partial \tau} \neq 0\right)$ heat flow with no internal heat generation]

or

$$\nabla^2 t = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...(Fourier's equation)} \quad \text{...(15.14)}$$

(*ii*) Under the situations when temperature does not depend on time, the conduction then takes place in steady state $\left(i.e., \frac{\partial t}{\partial \tau} = 0\right)$ and the eqn. (15.13) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

or

$$\nabla^2 t + \frac{q_g}{k} = 0 \quad \text{...(Poisson's equation)} \quad \text{...(15.15)}$$

In the absence of internal heat generation, eqn. (15.15) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

or

$$\nabla^2 t = 0 \quad \text{...(Laplace equation)} \quad \text{...(15.16)}$$

(*iii*) *Steady state and one-dimensional heat transfer*

$$\frac{\partial^2 t}{\partial x^2} + \frac{q_g}{k} = 0 \quad \text{...(15.17)}$$

(*iv*) *Steady state, one-dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = 0 \quad \text{...(15.18)}$$

(*v*) *Steady state, two dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \quad \text{...(15.19)}$$

(*vi*) *Unsteady state, one dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...(15.20)}$$

15.2.5. Heat Conduction Through Plane and Composite Walls

15.2.5.1. Heat conduction through a plane wall

Refer Fig. 15.3 (*a*). Consider a plane wall of homogeneous material through which heat is flowing *only in x-direction.*

Let, L = Thickness of the plane wall,

A = Cross-sectional area of the wall,

k = Thermal conductivity of the wall material, and

t_1, t_2 = Temperatures maintained at the two faces 1 and 2 of the wall, respectively.

The general heat conduction equation in cartesian coordinates is given by :

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \qquad \text{...[Eqn. 15.13]}$$

If the heat conduction takes place under the conditions, steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, one-dimensional $\left[\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0\right]$ and with no internal heat generation $\left(\frac{q_g}{k} = 0\right)$ then the above equation is reduced to :

$$\frac{\partial^2 t}{\partial x^2} = 0, \quad \text{or} \quad \frac{d^2 t}{dx^2} = 0 \qquad \text{...(15.21)}$$

By integrating the above differential twice, we have

$$\frac{\partial t}{\partial x} = C_1 \text{ and } t = C_1 x + C_2 \qquad \text{...(15.22)}$$

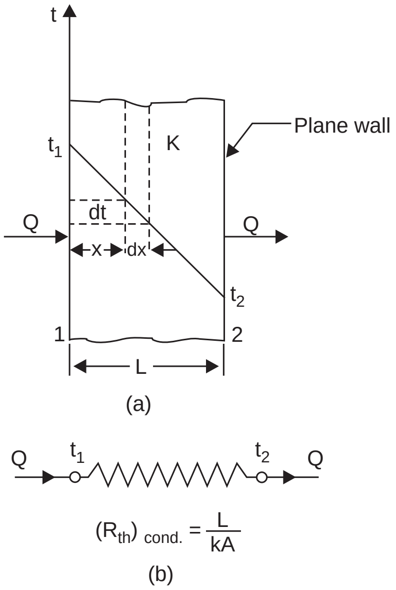
where C_1 and C_2 are the arbitrary constants. The values of these constants may be calculated from the known boundary conditions as follows :

At $x = 0$ $\quad t = t_1$

At $x = L$ $\quad t = t_2$

Substituting the values in the eqn. (15.22), we get

$$t_1 = O + C_2 \text{ and } t_2 = C_1 L + C_2$$



(a)

(b)

Fig. 15.3. Heat conduction through a plane wall.

After simplification, we have, $C_2 = t_1$ and $C_1 = \frac{t_2 - t_1}{L}$

Thus, the eqn. (15.22) reduces to :

$$t = \left(\frac{t_2 - t_1}{L}\right) x + t_1 \qquad \text{...(15.23)}$$

The eqn. (15.23) indicates that *temperature distribution across a wall is linear* and is *independent of thermal conductivity.* Now heat through the plane wall can be found by using Fourier's equation as follows :

$$Q = -\, kA \frac{dt}{dx}, \left(\text{where } \frac{dt}{dx} = \text{temperature gradient}\right) \qquad \text{...[Eqn. (1.1)]}$$

But, $$\frac{dt}{dx} = \frac{d}{dx}\left[\left(\frac{t_2 - t_1}{L}\right)x + t_1\right] = \frac{t_2 - t_1}{L}$$

$\therefore$ $$Q = -\,kA\,\frac{(t_2 - t_1)}{L} = \frac{kA\,(t_1 - t_2)}{L} \qquad ...(15.24)$$

Eqn. (15.24) can be written as :

$$Q = \frac{(t_1 - t_2)}{(L/kA)} = \frac{(t_1 - t_2)}{(R_{th})_{\text{cond.}}} \qquad ...(15.25)$$

where, $(R_{th})_{\text{cond.}}$ = Thermal resistance to heat conduction. Fig. 15.3 (*b*) shows the *equivalent thermal circuit* for heat flow through the plane wall.

Let us now find out the condition when instead of space, weight is the main criterion for selection of the insulation of a plane wall.

Thermal resistance (conduction) of the wall, $(R_{th})_{\text{cond.}} = \dfrac{L}{kA}$ ...(*i*)

Weight of the wall, $W = \rho\, A\, L$ ...(*ii*)

Eliminating L from (*i*) and (*ii*), we get

$$W = \rho A.(R_{th})_{\text{cond.}}\ kA = (\rho.k)A^2.(R_{th})_{\text{cond.}} \qquad ...(15.26)$$

The eqn. (15.26) stipulates the condition that, for a specified thermal resistance, the *lightest insulation will be one which has the smallest product of density* (ρ) *and thermal conductivity* (*k*).

15.2.5.2. Heat conduction through a composite wall

Refer Fig. 15.4 (*a*). Consider the transmission of heat through a composite wall consisting of a number of slabs.

Let L_A, L_B, L_C = Thicknesses of slabs *A, B* and *C* respectively (also called path lengths),

k_A, k_B, k_C = Thermal conductivities of the slabs *A, B* and *C* respectively,

$t_1, t_4 (t_1 > t_4)$ = Temperatures at the wall surfaces 1 and 4 respectively, and

t_2, t_3 = Temperatures at the interfaces 2 and 3 respectively.

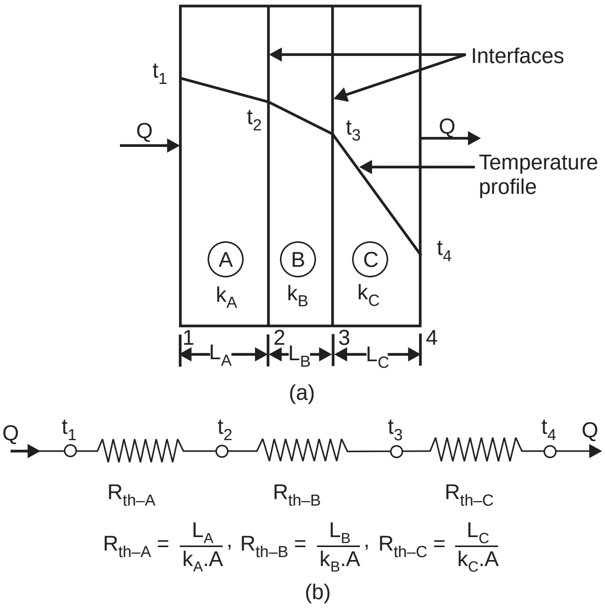


Fig. 15.4. Steady state conduction through a composite wall.

Since the quantity of heat transmitted per unit time through each slab/layer is same, we have

$$Q = \frac{k_A \,.A\,(t_1 - t_2)}{L_A} = \frac{k_B \,.\, A\,(t_2 - t_3)}{L_B} = \frac{k_C \,.\, A\,(t_3 - t_4)}{L_C}$$

(Assuming that there is a perfect contact between the layers and no temperature drop occurs across the interface between the materials).

Rearranging the above expression, we get

$$t_1 - t_2 = \frac{Q \,.\, L_A}{k_A \,.\, A} \qquad ...(i)$$

$$t_2 - t_3 = \frac{Q \,.\, L_B}{k_B \,.\, A} \qquad ...(ii)$$

$$t_3 - t_4 = \frac{Q \,.\, L_C}{k_C \,.\, A} \qquad ...(iii)$$

Adding (*i*), (*ii*) and (*iii*), we have

$$(t_1 - t_4) = Q\left[\frac{L_A}{k_A \,.\, A} + \frac{L_B}{k_B \,.\, A} + \frac{L_C}{k_C \,.\, A}\right]$$

or

$$Q = \frac{A\,(t_1 - t_4)}{\left[\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C}\right]} \qquad ...(15.27)$$

or

$$Q = \frac{(t_1 - t_4)}{\left[\dfrac{L_A}{k_A \,.\, A} + \dfrac{L_B}{k_B \,.\, A} + \dfrac{L_C}{k_C \,.\, A}\right]} = \frac{(t_1 - t_4)}{[R_{th-A} + R_{th-B} + R_{th-C}]} \qquad ...(15.28)$$

If the composite wall consists of n slabs/layers, then

$$Q = \frac{[t_1 - t_{(n+1)}]}{\displaystyle\sum_1^n \frac{L}{kA}} \qquad ...(15.29)$$

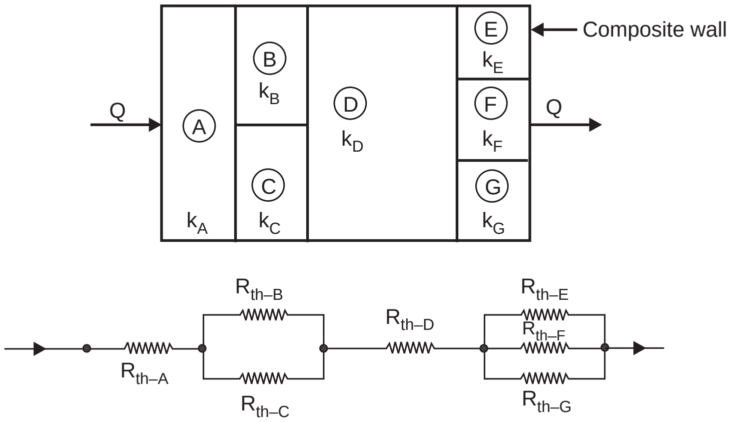


Fig. 15.5. Series and parallel one-dimensional heat transfer through a composite wall and electrical analog.

In order to solve more complex problems involving both series and parallel thermal resistances, the electrical analogy may be used. A typical problem and its analogous electric circuit are shown in Fig. 15.5.

$$Q = \frac{\Delta t_{\text{overall}}}{\Sigma R_{th}} \quad ...(15.30)$$

Thermal contact resistance. In a composite (multi-layer) wall, the calculations of heat flow are made on the assumptions : (*i*) The contact between the adjacent layers is perfect, (*ii*) At the interface there is no fall of temperature, and (*iii*) At the interface the temperature is continuous, although there is discontinuity in temperature gradient. In real systems, however, due to surface roughness and void spaces (usually filled with air) the contact surfaces *touch only at discrete locations.* Thus there is not a single plane of contact, which means that the area available for the flow of heat at the interface will be small compared to geometric face area. Due to this reduced area and presence of air voids, *a large resistance to heat flow at the interface occurs.* This resistance is known as *thermal contact resistance* and it *causes temperature drop between two materials at the interface* as shown in Fig. 15.6.

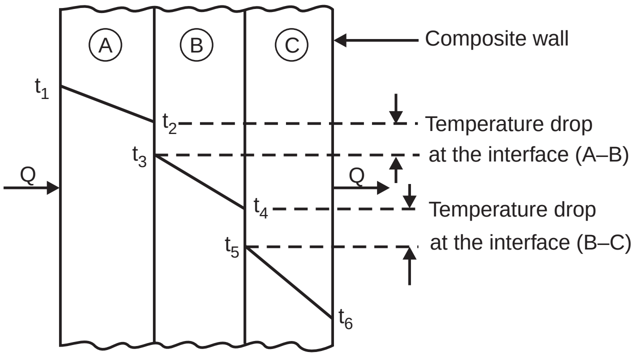


Fig. 15.6. Temperature drops at the interfaces.

Refer Fig. 15.6. The contact resistances are given by

$$(R_{th\text{–}AB})_{\text{cond.}} = \frac{(t_2 - t_3)}{Q/A} \text{ and } (R_{th\text{–}BC})_{\text{cont.}} = \frac{(t_4 - t_5)}{Q/A}.$$

15.2.6. The Overall Heat-transfer Coefficient

While dealing with the problems of fluid to fluid heat transfer across a metal boundary, it is usual to adopt an overall heat transfer coefficient U which *gives the heat transmitted per unit area per unit time per degree temperature difference between the bulk fluids on each side of the metal.*

Refer Fig. 15.7.

Let, L = Thickness of the metal wall,
k = Thermal conductivity of the wall material,
t_1 = Temperature of the surface-1,
t_2 = Temperature of the surface-2,
t_{hf} = Temperature of the hot fluid,
t_{cf} = Temperature of the cold fluid,
h_{hf} = Heat transfer coefficient from hot fluid to metal surface, and
h_{cf} = Heat transfer coefficient from metal surface to cold fluid.

(The suffices *hf* and *cf* stand for hot fluid and cold fluid respectivley.)

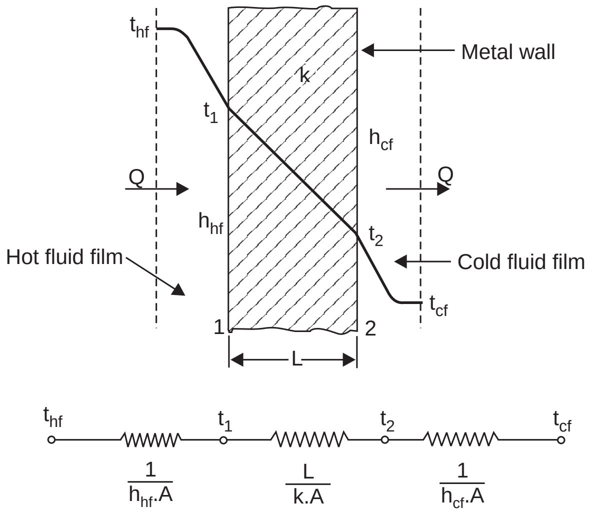


Fig. 15.7. The overall heat transfer through a plane wall.

The equations of heat flow through the fluid and the metal surface are given by

$$Q = h_{hf} \, . \, A(t_{hf} - t_1) \quad \text{...}(i)$$

$$Q = \frac{k_A \, (t_1 - t_2)}{L} \quad \text{...}(ii)$$

$$Q = h_{cf} \, . \, A(t_2 - t_{cf}) \quad \text{...}(iii)$$

By rearranging (*i*), (*ii*) and (*iii*), we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} . A} \quad \text{...}(iv)$$

$$t_1 - t_2 = \frac{QL}{k \, . \, A} \quad \text{...}(v)$$

$$t_2 - t_{cf} = \frac{Q}{h_{cf} \, . \, A} \quad \text{...}(vi)$$

Adding (*iv*), (*v*) and (*vi*), we get

$$t_{hf} - t_{cf} = Q \left[\frac{1}{h_{hf} \, . \, A} + \frac{L}{k \, . \, A} + \frac{1}{h_{cf} \, . \, A} \right]$$

or

$$Q = \frac{A \, (t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}} \quad \text{...(15.31)}$$

If U is the overall coefficient of heat transfer, then

$$Q = U.A \; (t_{hf} - t_{cf}) = \frac{A \, (t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

or

$$U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}} \quad \text{...(15.32)}$$

It may be noticed from the above equation that if the individual coefficients differ greatly in magnitude only a change in the *least* will have significant effect on the rate of heat transfer.

Example 15.1. *The inner surface of a plane brick wall is at 60°C and the outer surface is at 35°C. Calculate the rate of heat transfer per* m^2 *of surface area of the wall, which is 220 mm thick. The thermal conductivity of the brick is 0.51 W/m°C.*

Solution. Temperature of the inner surface of the wall $t_1 = 60°C$.

Temperature of the outer surface of the wall,

$$t_2 = 35°C$$

The thickness of the wall, $L = 220 \text{ mm} = 0.22 \text{ m}$

Thermal conductivity of the brick,

$$k = 0.51 \text{ W/m°C}$$

Rate of heat transfer per m², q :

Rate of heat transfer per unit area,

$$q = \frac{Q}{A} = \frac{k(t_1 - t_2)}{L}$$

or $$q = \frac{0.51 \times (60 - 35)}{0.22} = \mathbf{57.95 \ W/m^2}. \quad \textbf{(Ans.)}$$

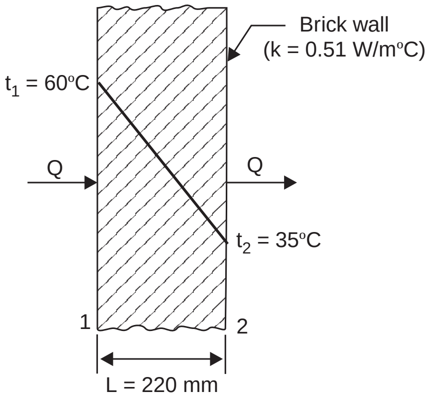


Fig. 15.8

Example 15.2. *A reactor's wall 320 mm thick, is made up of an inner layer of fire brick (k = 0.84 W/m°C) covered with a layer of insulation (k = 0.16 W/m°C). The reactor operates at a temperature of 1325°C and the ambient temperature is 25°C.*

(*i*) *Determine the thickness of fire brick and insulation which gives minimum heat loss.*

(*ii*) *Calculate the heat loss presuming that the insulating material has a maximum temperature of 1200°C.*

If the calculated heat loss is not acceptable, then state whether addition of another layer of insulation would provide a satisfactory solution.

Solution. Refer Fig. 15.9.

Given : $t_1 = 1325°C$; $t_2 = 1200°C$, $t_3 = 25°C$;

$L_A + L_B = L = 320$ mm or 0.32 m

$\therefore \quad L_B = (0.32 - L_A)$; ...(*i*)

$k_A = 0.84$ W/m°C ;

$k_B = 0.16$ W/m°C.

(*i*) **L_A : ; L_B :**

The heat flux, under steady state conditions, is constant throughout the wall and is same for each layer. Then for *unit area* of wall,

$$q = \frac{t_1 - t_3}{L_A/k_A + L_B/k_B} = \frac{t_1 - t_2}{L_A/k_A} = \frac{t_2 - t_3}{L_B/k_B}$$

Considering first two quantities, we have

$$\frac{(1325 - 25)}{L_A/0.84 + L_B/0.16} = \frac{(1325 - 1200)}{L_A/0.84}$$

or $$\frac{1300}{1.190\, L_A + 6.25(0.32 - L_A)} = \frac{105}{L_A}$$

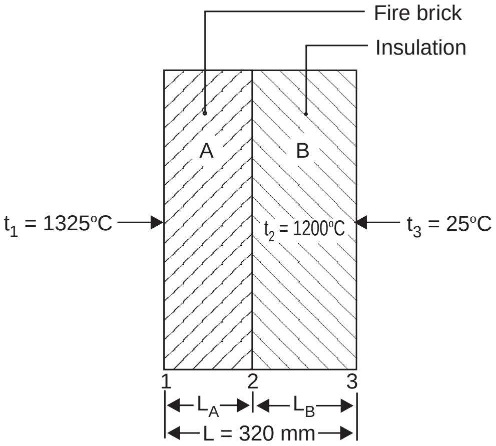


Fig. 15.9

or $$\frac{1300}{1.190\,L_A + 2 - 6.25\,L_A} = \frac{105}{L_A}$$

or $$\frac{1300}{2 - 5.06\,L_A} = \frac{105}{L_A}$$

or $$1300L_A = 105\,(2 - 5.06\,L_A)$$

or $$1300\,L_A = 210 - 531.3\,L_A$$

or $$L_A = \frac{210}{(1300 + 531.3)} = 0.1146 \text{ m or } \mathbf{114.6\ mm.}\ \textbf{(Ans.)}$$

∴ Thickness of insulation $L_B = 320 - 114.6 =$ **205.4 mm.** **(Ans.)**

(*ii*) **Heat loss per unit area,** q :

Heat loss per unit area, $$q = \frac{t_1 - t_2}{L_A/k_A} = \frac{1325 - 1200}{0.1146/0.84} = \mathbf{916.23\ W/m^2.}\ \textbf{(Ans.)}$$

If another layer of insulating material is added, the heat loss from the wall will reduce ; consequently the temperature drop across the fire brick lining will drop and the interface temperature t_2 will *rise*. As the interface temperature is *already fixed*. Therefore, a *satisfactory solution will not be available* by *adding layer of insulation.*

Example 15.3. *An exterior wall of a house may be approximated by a 0.1 m layer of common brick (k = 0.7 W/m°C) followed by a 0.04 m layer of gypsum plaster (k = 0.48 W/m°C). What thickness of loosely packed rock wool insulation (k = 0.065 W/m°C) should be added to reduce the heat loss or (gain) through the wall by 80 per cent ?* **(AMIE Summer, 1997)**

Solution. Refer Fig. 15.10.

Thickness of common brick, $L_A = 0.1$ m

Thickness of gypsum plaster, $L_B = 0.04$ m

Thickness of rock wool, $L_C = x$ (in m) = ?

Thermal conductivities :

Common brick, $k_A = 0.7$ W/m°C

Gypsum plaster, $k_B = 0.48$ W/m°C

Rock wool, $k_C = 0.065$ W/m°C

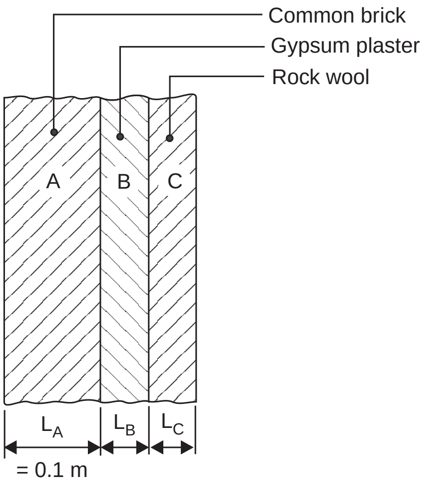


Fig. 15.10

Case I. *Rock wool insulation* **not** *used :*

$$Q_1 = \frac{A\,(\Delta t)}{\frac{L_A}{k_A} + \frac{L_B}{k_B}} = \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48}} \quad ...(i)$$

Case II. *Rock wool insulation used :*

$$Q_2 = \frac{A\,(\Delta t)}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} \quad ...(ii)$$

But $Q_2 = (1 - 0.8)Q_1 = 0.2\,Q_1$ ...(Given)

$$\therefore \quad \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} = 0.2 \times \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48}}$$

or $$\frac{0.1}{0.7} + \frac{0.04}{0.48} = 0.2\left[\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}\right]$$

or $$0.1428 + 0.0833 = 0.2\ [0.1428 + 0.0833 + 15.385\ x]$$

or $$0.2261 = 0.2\ (0.2261 + 15.385\ x)$$

or $$x = 0.0588 \text{ m or } 58.8 \text{ mm}$$

Thus, the *thickness of rock wool insulation should be* **58.8 mm. (Ans.)**

Example 15.4. *A furnace wall consists of 200 mm layer of refractory bricks, 6 mm layer of steel plate and a 100 mm layer of insulation bricks. The maximum temperature of the wall is 1150°C on the furnace side and the minimum temperature is 40°C on the outermost side of the wall. An accurate energy balance over the furnace shows that the heat loss from the wall is 400 W/m². It is known that there is a thin layer of air between the layers of refractory bricks and steel plate. Thermal conductivities for the three layers are 1.52, 45 and 0.138 W/m°C respectively. Find :*

(*i*) *To how many millimetres of insulation brick is the air layer equivalent ?*

(*ii*) *What is the temperature of the outer surface of the steel plate ?*

Solution. Refer Fig. 15.11.

Thickness of refractory bricks, $L_A = 200 \text{ mm} = 0.2 \text{ m}$

Thickness of steel plate, $L_C = 6 \text{ mm} = 0.006 \text{ m}$

Thickness of insulation bricks, $L_D = 100 \text{ mm} = 0.1 \text{ m}$

Difference of temperature between the innermost and outermost side of the wall,

$$\Delta t = 1150 - 40 = 1110°\text{C}$$

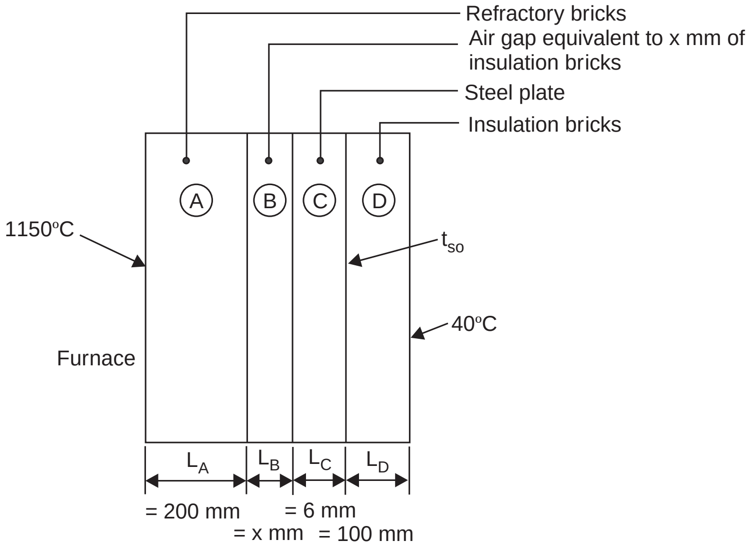


Fig. 15.11

Thermal conductivities :

$$k_A = 1.52 \text{ W/m°C} ;\ k_B = k_D = 0.138 \text{ W/m°C} ;\ k_C = 45 \text{ W/m°C}$$

Heat loss from the wall, $q = 400 \text{ W/m}^2$

(*i*) **The value of x = ($\mathbf{L_C}$) :**

We know, $$Q = \frac{A \cdot \Delta t}{\Sigma \frac{L}{k}} \quad \text{or} \quad \frac{Q}{A} = q = \frac{\Delta t}{\Sigma \frac{L}{k}}$$

or
$$400 = \frac{1110}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C} + \dfrac{L_D}{k_D}}$$

or
$$400 = \frac{1110}{\dfrac{0.2}{1.52} + \dfrac{(x/1000)}{0.138} + \dfrac{0.006}{45} + \dfrac{0.1}{0.138}}$$

$$= \frac{1110}{0.1316 + 0.0072x + 0.00013 + 0.7246} = \frac{1110}{0.8563 + 0.0072x}$$

or
$$0.8563 + 0.0072\,x = \frac{1110}{400} = 2.775$$

or
$$x = \frac{2.775 - 0.8563}{0.0072} = \textbf{266.5 mm.} \quad \textbf{(Ans.)}$$

(*ii*) **Temperature of the outer surface of the steel plate t_{so} :**

$$q = 400 = \frac{(t_{so} - 40)}{L_D/k_D}$$

or
$$400 = \frac{(t_{so} - 40)}{(0.1/0.138)} = 1.38(t_{so} - 40)$$

or
$$t_{so} = \frac{400}{1.38} + 40 = \textbf{329.8°C.} \quad \textbf{(Ans.)}$$

Example 15.5. *Find the heat flow rate through the composite wall as shown in Fig. 15.12. Assume one dimensional flow.*

k_A = 150 W/m°C,
k_B = 30 W/m°C,
k_C = 65 W/m°C and
k_D = 50 W/m°C.

(M.U. Winter, 1997)

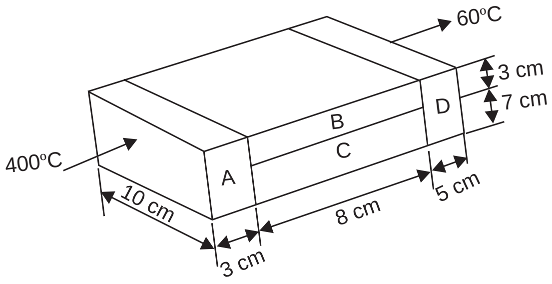


Fig. 15.12

Solution. The thermal circuit for heat flow in the given composite system (shown in Fig. 15.12) has been illustrated in Fig. 15.13.

Thickness :

L_A = 3 cm = 0.03 m ; $L_B = L_C$ = 8 cm = 0.08 m ; L_D = 5 cm = 0.05 m

Areas :

$A_A = 0.1 \times 0.1 = 0.01 \text{ m}^2$; $A_B = 0.1 \times 0.03 = 0.003 \text{ m}^2$
$A_C = 0.1 \times 0.07 = 0.007 \text{ m}^2$; $A_D = 0.1 \times 0.1 = 0.01 \text{ m}^2$

Heat flow rate, Q :

The thermal resistances are given by

$$R_{th-A} = \frac{L_A}{k_A A_A} = \frac{0.03}{150 \times 0.01} = 0.02$$

$$R_{th-B} = \frac{L_B}{k_B A_B} = \frac{0.08}{30 \times 0.003} = 0.89$$

$$R_{th-C} = \frac{L_C}{k_C A_C} = \frac{0.08}{65 \times 0.007} = 0.176$$

$$R_{th-D} = \frac{L_D}{k_D A_D} = \frac{0.05}{50 \times 0.01} = 0.1$$

The equivalent thermal resistance for the parallel thermal resistances R_{th-B} and R_{th-C} is given by :

$$\frac{1}{(R_{th})_{eq.}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176}$$

$$= 6.805$$

$$\therefore \quad (R_{th})_{eq.} = \frac{1}{6.805} = 0.147$$

Now, the total thermal resistance is given by

$$(R_{th})_{total} = R_{th-A} + (R_{th})_{eq.} + R_{th-D}$$

$$= 0.02 + 0.147 + 0.1 = 0.267$$

$$\therefore \quad Q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(400 - 60)}{0.267} = \textbf{1273.4 W. (Ans.)}$$

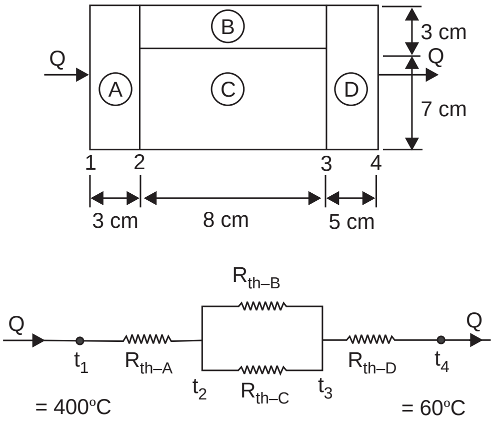


Fig. 15.13. Thermal circuit for heat flow in the composite system.

Example 15.6. *A mild steel tank of wall thickness 12 mm contains water at 95°C. The thermal conductivity of mild steel is 50 W/m°C, and the heat transfer coefficients for the inside and outside the tank are 2850 and 10 W/m²°C, respectively. If the atmospheric temperature is 15°C, calculate :*

(*i*) *The rate of heat loss per m² of the tank surface area ;*

(*ii*) *The temperature of the outside surface of the tank.*

Solution. Refer Fig. 15.14.

Thickness of mild steel tank wall

$$L = 12 \text{ mm} = 0.012 \text{ m}$$

Temperature of water, $t_{hf} = 95°\text{C}$

Temperature of air, $t_{cf} = 15°\text{C}$

Thermal conductivity of mild steel,

$$k = 50 \text{ W/m°C}$$

Heat transfer coefficients :

Hot fluid (water), $h_{hf} = 2850 \text{ W/m}^2\text{°C}$

Cold fluid (air), $h_{cf} = 10 \text{ W/m}^2\text{°C}$

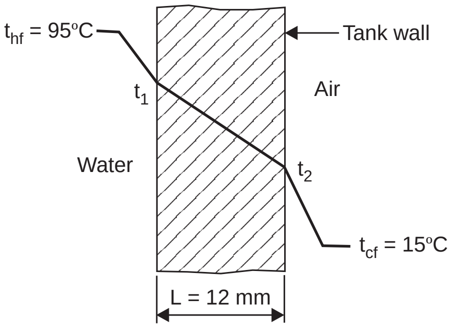


Fig. 15.14

(*i*) **Rate of heat loss per m² of the tank surface area, q :**

Rate of heat loss per m² of tank surface,

$$q = UA(t_{hf} - t_{cf})$$

The overall heat transfer coefficient, U is found from the relation ;

$$\frac{1}{U} = \frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}} = \frac{1}{2850} + \frac{0.012}{50} + \frac{1}{10}$$

$$= 0.0003508 + 0.00024 + 0.1 = 0.1006$$

$$\therefore \qquad U = \frac{1}{0.1006} = 9.94 \text{ W/m}^2\text{°C}$$

$$\therefore \qquad q = 9.94 \times 1 \times (95 - 15) = \mathbf{795.2\ W/m^2.} \quad \textbf{(Ans.)}$$

(*ii*) **Temperature of the outside surface of the tank, t_2 :**

We know that, $\quad q = h_{cf} \times 1 \times (t_2 - t_{cf})$

or $\quad 795.2 = 10(t_2 - 15)$

or $$t_2 = \frac{795.2}{10} + 15 = \mathbf{94.52°C.} \quad \textbf{(Ans.)}$$

Example 15.7. *The interior of a refrigerator having inside dimensions of 0.5 m × 0.5 m base area and 1 m height, is to be maintained at 6°C. The walls of the refrigerator are constructed of two mild steel sheets 3 mm thick (k = 46.5 W/m°C) with 50 mm of glass wool insulation (k = 0.046 W/m°C) between them. If the average heat transfer coefficients at the inner and outer surfaces are 11.6 W/m²°C and 14.5 W/m²°C respectively, calculate :*

(*i*) *The rate at which heat must be removed from the interior to maintain the specified temperature in the kitchen at 25°C, and*

(*ii*) *The temperature on the outer surface of the metal sheet.*

Solution. Refer Fig. 15.15

Given : $\quad L_A = L_C = 3 \text{ mm} = 0.003 \text{ m}$;

$L_B = 50 \text{ mm} = 0.05 \text{ m}$;

$k_A = k_C = 46.5 \text{ W/m°C}$; $k_B = 0.046 \text{ W/m°C}$;

$h_0 = 11.6 \text{ W/m}^2\text{°C}$; $h_i = 14.5 \text{ W/m}^2\text{°C}$;

$t_0 = 25°\text{C}$; $t_i = 6°\text{C}$.

The total area through which heat is coming into the refrigerator

$$A = 0.5 \times 0.5 \times 2 + 0.5 \times 1 \times 4 = 2.5 \text{ m}^2$$

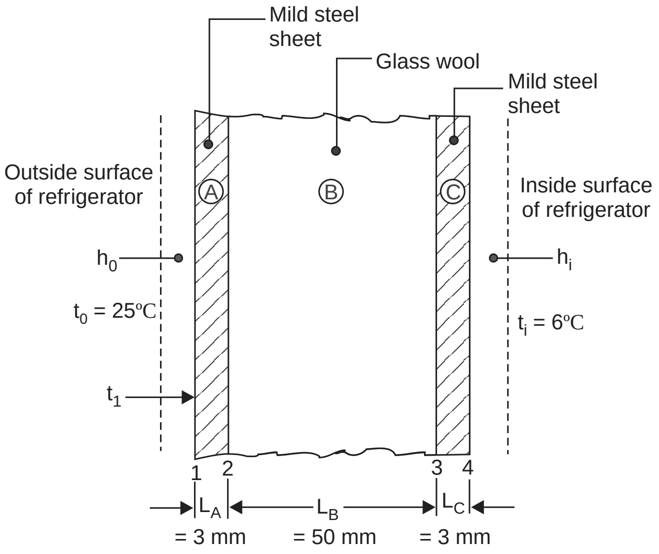


Fig. 15.15

(*i*) **The rate of removal of heat, Q :**

$$Q = \frac{A(t_0 - t_i)}{\frac{1}{h_o} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_i}}$$

$$= \frac{2.5\,(25 - 6)}{\frac{1}{11.6} + \frac{0.003}{46.5} + \frac{0.05}{0.046} + \frac{0.003}{46.5} + \frac{1}{14.5}} = \mathbf{38.2\ W.}\quad \textbf{(Ans.)}$$

(*ii*) **The temperature at the outer surface of the metal sheet, t_1 :**

$$Q = h_o\,A(25 - t_1)$$

or $$38.2 = 11.6 \times 2.5\,(25 - t_1)$$

or $$t_1 = 25 - \frac{38.2}{11.6 \times 2.5} = \mathbf{23.68°C.}\quad \textbf{(Ans.)}$$

Example 15.8. *A furnace wall is made up of three layers of thicknesses 250 mm, 100 mm and 150 mm with thermal conductivities of 1.65, k and 9.2 W/m°C respectively. The inside is exposed to gases at 1250°C with a convection coefficient of 25 W/m²°C and the inside surface is at 1100°C, the outside surface is exposed air at 25°C with convection coefficient of 12 W/m²°C. Determine :*

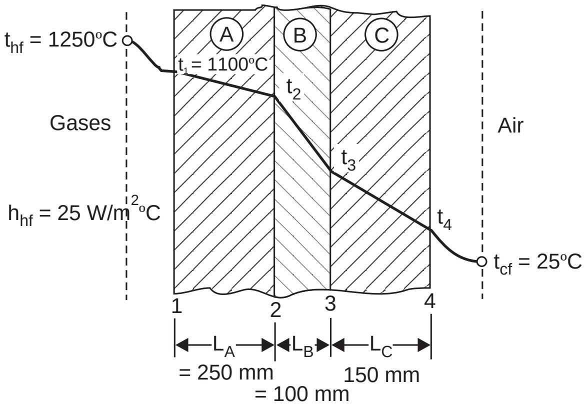
(*i*) *The unknown thermal conductivity 'k' ;*

(*ii*) *The overall heat transfer coefficient ;*

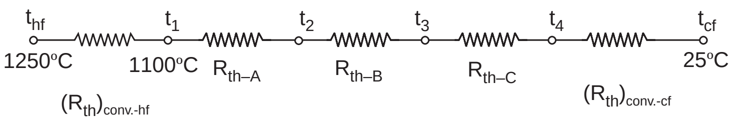
(*iii*) *All surface temperatures.*

Solution. L_A = 250 mm 0.25 m ; L_B = 100 mm = 0.1 m ;
L_C = 150 mm = 0.15 m ; k_A = 1.65 W/m°C ;
k_C = 9.2 W/m°C ; t_{hf} = 1250°C ; t_1 = 1100°C
h_{hf} = 25 W/m²°C ; h_{cf} = 12 W/m²°C

(*i*) **Thermal conductivity, k (= k_B) :**



(*a*) Composite system.



(*b*) Thermal circuit.

Fig. 15.16

The rate of heat transfer per unit area of the furnace wall,

$$q = h_{hf}\,(t_{hf} - t_1)$$
$$= 25(1250 - 1100) = 3750 \text{ W/m}^2$$

Also, $$q = \frac{(\Delta t)_{\text{overall}}}{(R_{th})_{\text{total}}}$$

or $$q = \frac{(t_{hf} - t_{cf})}{(R_{th})_{\text{conv}-hf} - R_{th-A} + R_{th-B} + R_{th-C} + (R_{th})_{\text{conv}-cf}}$$

or $$3750 = \frac{(1250 - 25)}{\dfrac{1}{h_{hf}} + \dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C} + \dfrac{1}{h_{cf}}} \text{ or } 3750 = \frac{1225}{\dfrac{1}{25} + \dfrac{0.25}{1.65} + \dfrac{0.1}{k_B} + \dfrac{0.15}{9.2} + \dfrac{1}{12}}$$

$$= \frac{1225}{0.04 + 0.1515 + \dfrac{0.1}{k_B} + 0.0163 + 0.0833} = \frac{1225}{0.2911 + \dfrac{0.1}{k_B}}$$

or $$3750\left(0.289 + \frac{0.1}{k_B}\right) = 1225 \text{ or } \frac{0.1}{k_B} = \frac{1225}{3750} - 0.2911 = 0.0355$$

$\therefore$ $$k_B = k = \frac{0.1}{0.0355} = \mathbf{2.817\ W/m^2{}^\circ C.}\quad \textbf{(Ans.)}$$

(*ii*) **The overall transfer coefficient, U :**

The overall heat transfer coefficient, $U = \dfrac{1}{(R_{\text{th}})_{\text{total}}}$

Now, $$(R_{\text{th}})_{\text{total}} = \frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{2.817} + \frac{0.15}{9.2} + \frac{1}{12}$$
$$= 0.04 + 0.1515 + 0.0355 + 0.0163 + 0.0833 = 0.3266^\circ\text{C m}^2\text{/W}$$

$\therefore$ $$U = \frac{1}{(R_{\text{th}})_{\text{total}}} = \frac{1}{0.3266} = \mathbf{3.06\ W/m^2{}^\circ C.}\quad \textbf{(Ans.)}$$

(*iii*) **All surface temperature ; t_1, t_2, t_3, t_4 :**

$$q = q_A = q_B = q_C$$

or $$3750 = \frac{(t_1 - t_2)}{L_A/k_A} = \frac{(t_2 - t_3)}{L_B/k_B} = \frac{(t_3 - t_4)}{L_C/k_C}$$

or $$3750 = \frac{(1110 - t_2)}{0.25/1.65} \text{ or } t_2 = 1100 - 3750 \times \frac{0.25}{1.65} = \mathbf{531.8^\circ C}$$

Similarly, $$3750 = \frac{(531.8 - t_3)}{0.1/2.817} \text{ or } t_3 = 531.8 - 3750 \times \frac{0.1}{2.817} = \mathbf{398.6^\circ C}$$

and $$3750 = \frac{(398.6 - t_4)}{(0.15/9.2)} \text{ or } t_4 = 398.6 - 3750 \times \frac{0.5}{9.2} = \mathbf{337.5^\circ C}$$

[Check using outside convection, $q = \dfrac{(337.5 - 25)}{1/h_{cf}} = \dfrac{(337.5 - 25)}{1/12} = 3750 \text{ W/m}^2$]

15.2.7. Heat Conduction Through Hollow and Composite Cylinders

15.2.7.1. Heat conduction through a hollow cylinder

Refer Fig. 15.17. Consider a hollow cylinder made of material having constant thermal conductivity and insulated at both ends.

Let r_1, r_2 = Inner and outer radii ;
t_1, t_2 = Temperature of inner and outer surfaces, and
k = Constant thermal conductivity within the given temperature range.

Consider an element at radius '*r*' and thickness '*dr*' for a length of the hollow cylinder through which heat is transmitted. Let *dt* be the temperature drop over the element.

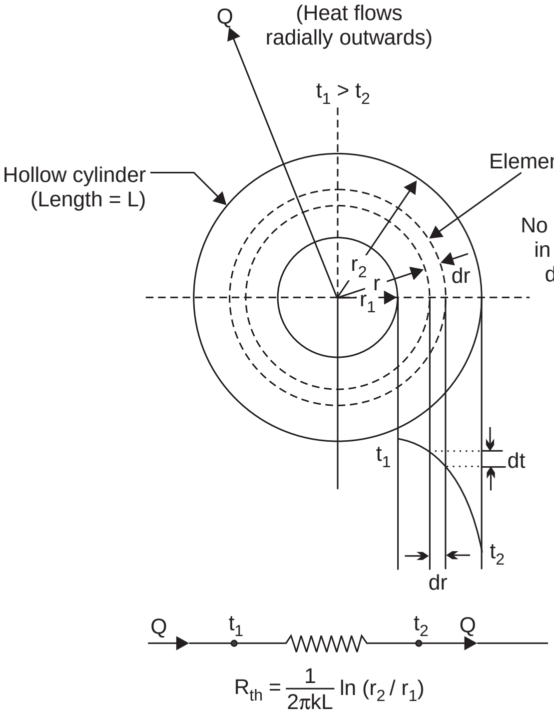


Fig. 15.17

Area through which heat is transmitted. $A = 2\pi\ r.\ L.$
Path length = dr (over which the temperature fall is dt)

$$\therefore \qquad Q = -\,kA\ .\left(\frac{dt}{dr}\right) = -\,k\ .\ 2\pi r\ .\ L\ \frac{dt}{dr} \text{ per unit time} \quad \text{or} \quad Q\ .\ \frac{dr}{r} = -\,k\ .\ 2\pi L.dt$$

Integrating both sides, we get

$$Q\int_{r_1}^{t_2}\frac{dr}{r} = -\,k.2\pi L\int_{t_1}^{t_2} dt \quad \text{or} \quad Q\left[\ln(r)\right]_{r_1}^{r_2} = k.2\pi L\left[t\right]_{t_1}^{t_2}$$

or $$Q.\ln(r_2/r_1) = k.2\pi L(t_2 - t_1) = k.2\pi L(t_1 - t_2)$$

$$\therefore \qquad Q = \frac{k.2\pi L(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{(t_1 - t_2)}{\left[\dfrac{\ln(r_2/r_1)}{2\pi k L}\right]} \qquad ...(15.33)$$

15.2.7.2. Heat conduction through a composite cylinder

Consider flow of heat through a composite cylinder as shown in Fig. 15.18.

Let t_{hf} = The temperature of the hot fluid flowing inside the cylinder,
t_{cf} = The temperature of the cold fluid (atmospheric air),

k_A = Thermal conductivity of the inside layer A,
k_B = Thermal conductivity of the outside layer B,
t_1, t_2, t_3 = Temperature at the points 1, 2 and 3 (see Fig. 15.18),
L = Length of the composite cylinder, and
h_{hf}, h_{cf} = Inside and outside heat transfer coefficients.

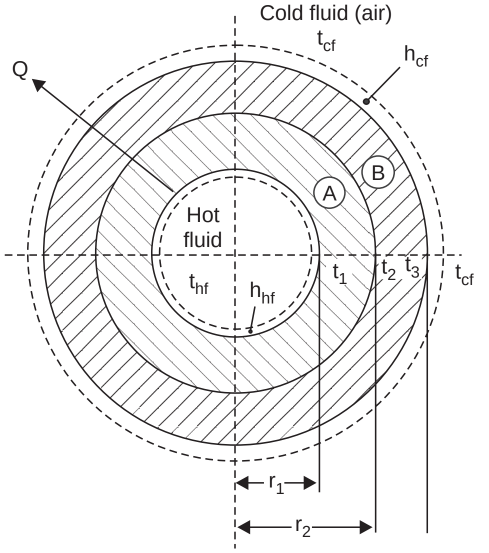


Fig. 15.18. Cross-section of a composite cylinder.

The rate of heat transfer is given by,

$$Q = h_{hf} \,.\, 2\pi r_1 \,.\, L(t_{hf} - t_1) = \frac{k_A \,.\, 2\pi L(t_1 - t_2)}{\ln (r_2/r_1)}$$

$$= \frac{k_B \,.\, 2\pi L\,(t_2 - t_3)}{\ln (r_3/r_2)} = h_{cf} \,.\, 2\pi r_3 \,.\, L(t_3 - t_{cf})$$

Rearranging the above expression, we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} \,.\, r_1 \,.\, 2\pi L} \qquad ...(i)$$

$$t_1 - t_2 = \frac{Q}{\dfrac{k_A \,.\, 2\pi L}{\ln (r_2/r_1)}} \qquad ...(ii)$$

$$t_2 - t_3 = \frac{Q}{\dfrac{k_B \,.\, 2\pi L}{\ln (r_3/r_2)}} \qquad ...(iii)$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} \,.\, r_3 \,.\, 2\pi L} \qquad ...(iv)$$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we have

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{hf} \,.\, r_1} + \frac{1}{\dfrac{k_A}{\ln (r_2/r_1)}} + \frac{1}{\dfrac{k_B}{\ln (r_3/r_2)}} + \frac{1}{h_{cf} \,.\, r_3}\right] = t_{hf} - t_{cf}$$

$$\therefore \qquad Q = \frac{2\pi L\,(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}\,.r_1} + \dfrac{1}{\dfrac{k_A}{\ln(r_2/r_1)}} + \dfrac{1}{\dfrac{k_B}{\ln(r_3/r_2)}} + \dfrac{1}{h_{cf}\,.r_3}\right]}$$

$$\therefore \qquad Q = \frac{2\pi L\,(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}\,.r_1} + \dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{1}{h_{cf}/r_3}\right]} \qquad \text{...(15.34)}$$

If there are '*n*' concentric cylinders, then

$$Q = \frac{2\pi L\,(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}\,.r_1} + \sum_{n=1}^{n=n} \dfrac{1}{k_n} \ln\{r_{(n+1)}/r_n\} + \dfrac{1}{h_{cf}\,.r_{(n+1)}}\right]} \qquad \text{...(15.35)}$$

If inside the outside heat transfer coefficients are *not* considered then the above equation can be written as

$$Q = \frac{2\pi L\,[t_1 - t_{(n+1)}]}{\sum_{n=1}^{n=n} \dfrac{1}{k_n} \ln[r_{(n+1)}/r_n]} \qquad \text{...(15.36)}$$

Example 15.9. *A thick walled tube of stainless steel with 20 mm inner diameter and 40 mm outer diameter is covered with a 30 mm layer of asbestos insulation (k = 0.2 W/m°C). If the inside wall temperature of the pipe is maintained at 600°C and the outside insulation at 1000°C, calculate the heat loss per metre of length.* **(AMIE Summer, 2000)**

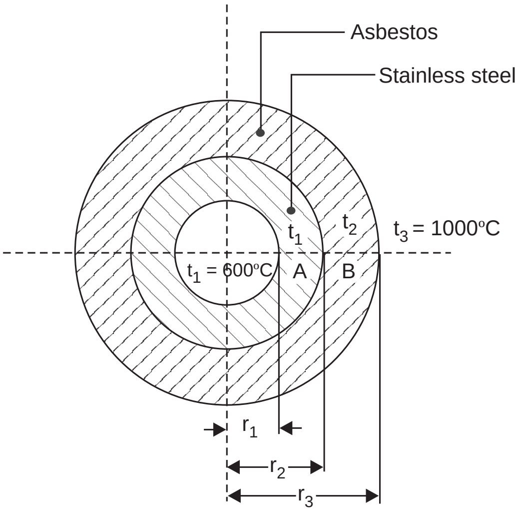


Fig. 15.19

Solution. Refer Fig. 15.19,

Given, $r_1 = \dfrac{20}{2} = 10 \text{ mm} = 0.01 \text{ m}$

$r_2 = \dfrac{40}{2} = 20 \text{ mm} = 0.02 \text{ m}$

$r_3 = 20 + 30 = 50 \text{ mm} = 0.05 \text{ m}$

$t_1 = 600°\text{C}, \; t_3 = 1000°\text{C}, \; k_B = 0.2 \text{ W/m°C}$

Heat transfer per metre of length, Q/L :

$$Q = \frac{2\pi L\,(t_1 - t_3)}{\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B}}$$

Since the thermal conductivity of satinless steel is not given therefore, neglecting the resistance offered by stainless steel to heat transfer across the tube, we have

$$\frac{Q}{L} = \frac{2\pi(t_1 - t_3)}{\dfrac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi(600 - 1000)}{\dfrac{\ln(0.05/0.02)}{0.2}} = -\ \mathbf{548.57\ W/m.} \quad \textbf{(Ans.)}$$

Negative sign indicates that the heat transfer takes place *radially inward.*

Example 15.10. *Hot air at a temperature of 65°C is flowing through a steel pipe of 120 mm diameter. The pipe is covered with two layers of different insulating materials of thickness 60 mm and 40 mm, and their corresponding thermal conductivities are 0.24 and 0.4 W/m°C. The inside and outside heat transfer coefficients are 60 and 12 W/m°C. The atmosphere is at 20°C. Find the rate of heat loss from 60 m length of pipe.*

Solution. Refer Fig. 15.20.

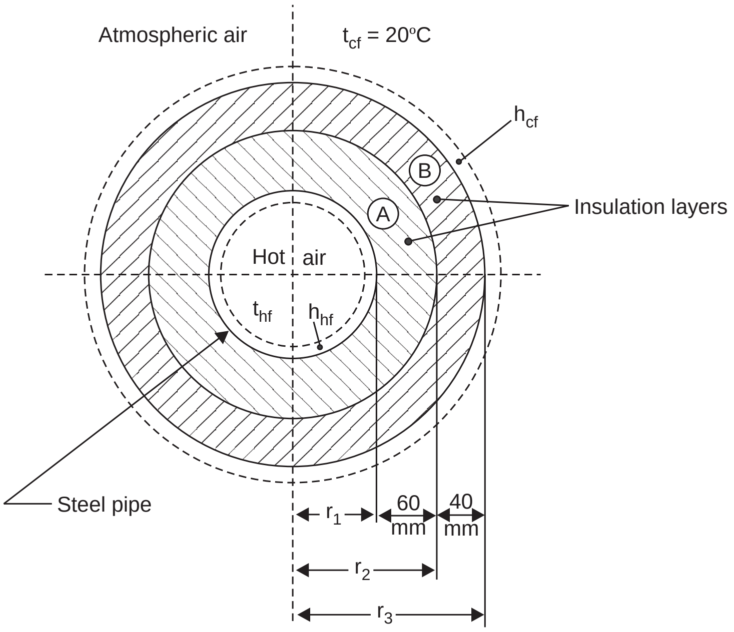


Fig. 15.20

Given : $r_1 = \dfrac{120}{2}$ = 60 mm = 0.06 m

r_2 = 60 + 60 = 120 mm = 0.12 m

r_3 = 60 + 60 + 40 = 160 mm = 0.16 m

k_A = 0.24 W/m°C ; k_B = 0.4 W/m°C

h_{hf} = 60 W/m^2°C ; h_{cf} = 12 W/m^2°C

t_{hf} = 65°C ; t_{cf} = 20°C

Length of pipe, L = 60 m

Rate of heat loss, Q :

Rate of heat loss is given by

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} \cdot r_1} + \dfrac{\ln (r_2/r_1)}{k_A} + \dfrac{\ln (r_3/r_2)}{k_B} + \dfrac{1}{h_{cf} \cdot r_3}\right]} \quad \text{[Eqn. (15.34)]}$$

$$= \frac{2\pi \times 60(65 - 20)}{\left[\dfrac{1}{60 \times 0.06} + \dfrac{\ln (0.12/0.06)}{0.24} + \dfrac{\ln (0.16/0.12)}{0.4} + \dfrac{1}{12 \times 0.16}\right]}$$

$$= \frac{16964.6}{0.2777 + 2.8881 + 0.7192 + 0.5208} = 3850.5 \text{ W}$$

i.e., Rate of heat loss = **3850.5 W (Ans.)**

Example 15.11. *A 150 mm steam pipe has inside dimater of 120 mm and outside diameter of 160 mm. It is insulated at the outside with asbestos. The steam temperature is 150°C and the air temperature is 20°C. h (steam side) = 100 W/m²°C, h (air side) = 30 W/m²°C, k (asbestos) = 0.8 W/m°C and k (steel) = 42 W/m°C. How thick should the asbestos be provided in order to limit the heat losses to 2.1 kW/m² ?* **(N.U.)**

Solution. Refer Fig. 15.21.

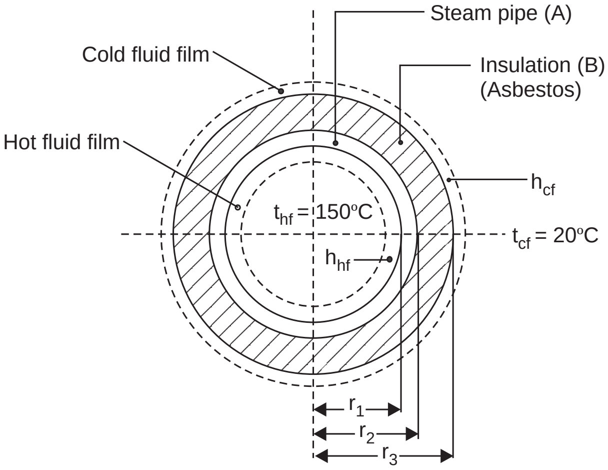


Fig. 15.21

Given : $r_1 = \frac{120}{2} = 60 \text{ mm} = 0.06 \text{ m}$

$r_2 = \frac{160}{2} = 80 \text{ mm} = 0.08 \text{ m}$

$k_A = 42$ W/m°C ; $k_B = 0.8$ W/m°C

$t_{hf} = 150$°C ; $t_{cf} = 20$°C

$h_{hf} = 100$ W/m²°C ; $h_{cf} = 30$ W/m²°C

Heat loss = 2.1 kW/m²

Thickness of insulation (asbestos), ($r_3 - r_2$) :

Area for heat transfer = $2\pi\, r\, L$ (where L = length of the pipe)

∴ Heat loss $= 2.1 \times 2\pi\, r\, L$ kW

$= 2.1 \times 2\pi \times 0.075 \times L = 0.989\, L$ kW

$= 0.989 \text{ L} \times 10^3$ watts

$\left(\text{where } r, \text{ mean radius} = \frac{150}{2} = 75 \text{ mm or } 0.075 \text{ m} \ldots \text{Given}\right)$

Heat transfer rate in such a case is given by

$$Q = \frac{2\pi L\,(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf}\,.\,r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf}\,.\,r_3}\right]} \quad \text{...[Eqn. (15.34)]}$$

$$0.989\ L \times 10^3 = \frac{2\pi L\,(150 - 20)}{\left[\dfrac{1}{100 \times 0.06} + \dfrac{\ln(0.08/0.06)}{42} + \dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30 \times r_3}\right]}$$

$$0.989 \times 10^3 = \frac{816.81}{\left[0.16666 + 0.00685 + \dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30 r_3}\right]}$$

or $$\frac{\ln(r_3/0.08)}{0.8} + \frac{1}{30\,r_3} = \frac{816.81}{0.989 \times 10^3} - (0.16666 + 0.00685) = 0.6524$$

or $$1.25 \ln(r_3/0.08) + \frac{1}{30 r_3} - 0.6524 = 0$$

Solving by hit and trial, we get

$$r_3 \simeq 0.105 \text{ m or } 105 \text{ mm}$$

∴ Thickness of insulation = $r_3 - r_2$ = 105 – 80 = **25 mm. (Ans.)**

15.2.8. Heat Conduction Through Hollow and Composite Spheres

15.2.8.1. Heat conduction through hollow sphere

Refer Fig. 15.22. Consider a hollow sphere made of material having constant thermal conductivity.

Let r_1, r_2 = Inner and outer radii,

t_1, t_2 = Temperature of inner and outer surfaces, and

k = Constant thermal conductivity of the material with the given temperature range.

Consider a small element of thickness dr at any radius r.

Area through which the heat is transmitted, $A = 4\pi r^2$

$$\therefore \quad Q = -k \,.\, 4\pi r^2 \,.\, \frac{dt}{dr}$$

Rearranging and integrating the above equation, we obtain

$$Q \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{t_1}^{t_2} dt$$

or $$Q \left[\frac{r^{-2+1}}{-2+1}\right]_{r_1}^{r_2} = -4\pi k \left[t\right]_{t_1}^{t_2}$$

or $$-Q\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = -4\pi k (t_2 - t_1)$$

or $$\frac{Q(r_2 - r_1)}{r_1 r_2} = 4\pi k\,(t_1 - t_2)$$

or $$Q = \frac{4\pi k r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)} = \frac{t_1 - t_2}{\left[\dfrac{(r_2 - r_1)}{(4\pi k r_1 r_2)}\right]} \qquad ...(15.37)$$

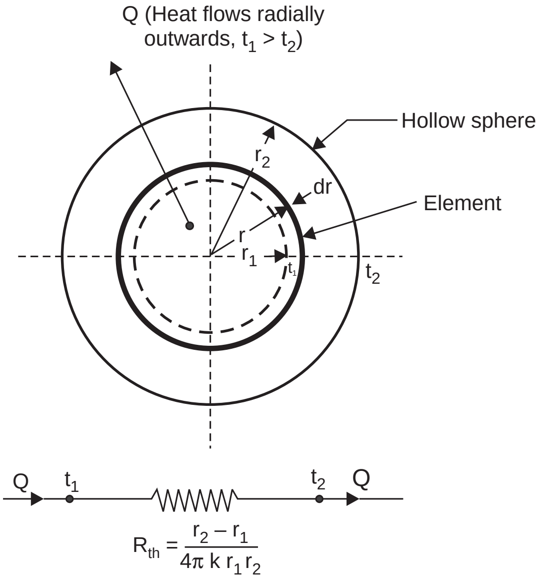


Fig. 15.22. Steady state conduction through a hollow sphere.

15.2.8.2. Heat conduction through a composite sphere

Considering Fig. 15.23 as cross-section of a composite sphere, the heat flow equation can be written as follows :

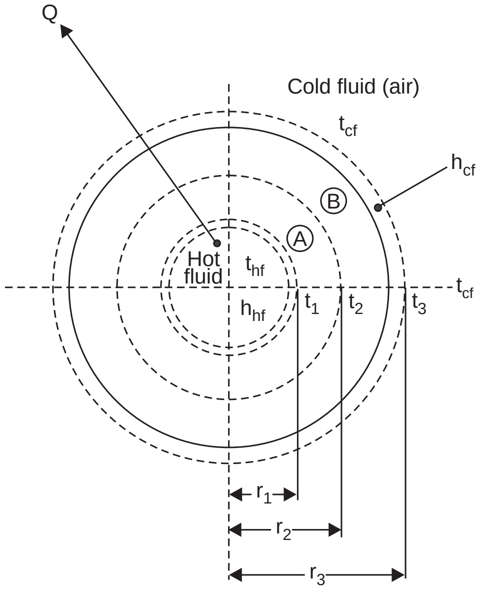


Fig. 15.23. Steady state conduction through a composite sphere.

$$Q = h_{hf} \,.\, 4\pi r_1^2 \;(t_{hf} - t_1) = \frac{4\pi k_A r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)} = \frac{4\pi k_B r_2 r_3 (t_1 - t_3)}{(r_3 - r_2)}$$

$$= h_{cf} \,.\, 4\pi r_3^2 \;(t_3 - t_{cf})$$

By rearranging the above equation, we have

$$t_{hf} - t_1 = \frac{Q}{h_{hf} \,.4\pi r_1^2} \qquad ...(i)$$

$$t_1 - t_2 = \frac{Q\,(r_2 - r_1)}{4\pi k_A \,.\, r_1 r_2} \qquad ...(ii)$$

$$t_2 - t_3 = \frac{Q(r_3 - r_2)}{4\pi k_B \,.\, r_2 r_3} \qquad ...(iii)$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} \,.\, 4\pi r_3^2} \qquad ...(iv)$$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we get

$$\frac{Q}{4\pi}\left[\frac{1}{h_{hf} \,.\, r_1^2} + \frac{(r_2 - r_1)}{k_A \,.\, r_1 r_2} + \frac{(r_3 - r_2)}{k_B \,.\, r_2 r_3} + \frac{1}{h_{cf} \,.\, r_3^2}\right] = t_{hf} - t_{cf}$$

$$\therefore \qquad Q = \frac{4\pi (t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} \,.\, r_1^2} + \dfrac{(r_2 - r_1)}{k_A \,.\, r_1 r_2} + \dfrac{(r_3 - r_2)}{k_B \,.\, r_2 r_3} + \dfrac{1}{h_{cf} \,.\, r_3^2}\right]} \qquad ...(15.38)$$

If there are n concentric spheres then the above equation can be written as follows :

$$Q = \frac{4\pi(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} \cdot r_1^2} + \displaystyle\sum_{n=1}^{n=n} \left\{\dfrac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right\} + \dfrac{1}{h_{cf} \cdot r^2_{(n+1)}}\right]} \quad \text{...(15.39)}$$

If inside and outside heat transfer coefficients are considered, then the above equation can be written as follows :

$$Q = \frac{4\pi(t_1 - t_{(n+1)})}{\displaystyle\sum_{n=1}^{n=n} \left[\dfrac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right]} \quad \text{...(15.40)}$$

Example 15.12. *A spherical shaped vessel of 1.4 m diameter is 90 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 220°C. Thermal conductivity of the material of the sphere is 0.083 W/m°C.*

Solution. Refer Fig. 15.24.

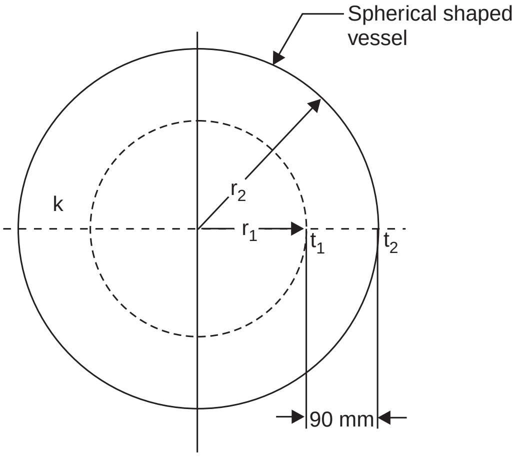


Fig. 15.24

Given : $r_2 = \frac{1.4}{2} = 0.7$ m ;

$r_1 = 0.7 - \frac{90}{1000} = 0.61$ m ;

$t_1 - t_2 = 220$°C ; $k = 0.083$ W/m°C

The rate of heat transfer/leakage is given by

$$Q = \frac{(t_1 - t_2)}{\left[\dfrac{(r_2 - r_1)}{4\pi k r_1 r_2}\right]} \quad \text{...[Eqn. (15.37)]}$$

$$= \frac{220}{\left[\dfrac{(0.7 - 0.61)}{4\pi \times 0.083 \times 0.61 \times 0.7}\right]} = 1088.67 \text{ W}$$

i.e., Rate of heat leakage = **1088.67 W. (Ans.)**

15.2.9. Critical Thickness of Insulation

15.2.9.1. Insulation-General aspects

Definition. *A material which retards the flow of heat with reasonable effectiveness* is known as '*Insulation*'. Insulation serves the following *two purposes :*

(*i*) It prevents the heat flow from the system to the surroundings ;

(*ii*) It prevents the heat flow from the surroundings to the system.

Applications :

The *fields of application* of insulations are :

(*i*) Boilers and steam pipes

(*ii*) Air-conditioning systems

(*iii*) Food preserving stores and refrigerators

(*iv*) Insulating bricks (employed in various types of furnaces)

(*v*) Preservation of liquid gases etc.

Factors affecting thermal conductivity

Some of the important factors which affect thermal conductivity (k) of the insulators (the value of k should be always low to reduce the rate of heat flow) are as follows :

1. *Temperature.* For most of the insulating materials, the value of *k increases with increase in temperature.*
2. *Density.* There is no mathematical relationship between k and ρ (density). The common understanding that high density insulating materials will have higher values of k in not always true.
3. *Direction of heat flow.* For most of the insulating materials (except few like wood) the effect of direction of heat flow on the values of k is negligible.
4. *Moisture.* It is always considered necessary to prevent ingress of moisture in the insulating materials during service, it is however difficult to find the effect of moisture on the values of k of different insulating materials.
5. *Air pressure.* It has been found that the value of *k decreases with decrease in pressure.*
6. *Convection in insulators.* The value of k increases due to the phenomenon of convection in insulators.

15.2.9.2. Critical Thickness of Insulation

The addition of insulation always increases the conductive thermal resistance. But when the total thermal resistance is made of conductive thermal resistance [$(R_{th})_{cond.}$] and convective thermal resistance [$(R_{th})_{conv.}$], the addition of insulation in *some cases* may *reduce the convective thermal resistance due to increase in surface area,* as in the case of a cylinder and a sphere, and the total thermal resistance may actually decrease resulating in increased heat flow. It may be shown that the thermal resistance actually decreases and then increases in some cases.

"*The thickness upto which heat flow increases and after which heat flow decreases is termed as* ***Critical thickness.*** In case of cylinders and spheres it is called ***'Critical radius'.***

A. Critical thickness of insulation for cylinder :

Consider a solid cylinder of radius r_1 insulated with an insulation of thickness $(r_2 - r_1)$ as shown in Fig. 15.25.

Let, L = Length of the cylinder,

t_1 = Surface temperature of the cylinder,

t_{air} = Temperature of air,

h_o = Heat transfer coefficient at the outer surface of the insulation, and

k = Thermal conductivity of insulating material.

Then the rate of heat transfer from the surface of the solid cylinder to the surroundings is given by

$$Q = \frac{2\pi L\,(t_1 - t_{air})}{\dfrac{\ln (r_2/r_1)}{k} + \dfrac{1}{h_o \cdot r_2}} \quad ...(15.41)$$

From eqn. (15.41) it is evident that as r_2 increases, the factor $\dfrac{\ln (r_2/r_1)}{k}$ increases but the factor $\dfrac{1}{h_o \cdot r_2}$ decreases. Thus Q becomes *maximum* when the *denominator* $\left[\dfrac{\ln (r_2/r_1)}{k} + \dfrac{1}{h_o \cdot r_2}\right]$ becomes *minimum.* The required condition is

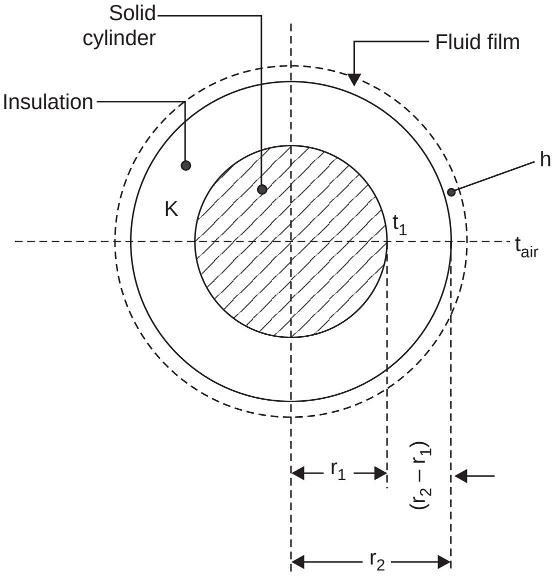


Fig. 15.25. Critical thickness of insulation for cylinder.

$$\frac{d}{dr_2}\left[\frac{\ln (r_2/r_1)}{k} + \frac{1}{h_o \cdot r_2}\right] = 0 \qquad (r_2 \text{ being the only variable})$$

$$\therefore \qquad \frac{1}{k}\cdot\frac{1}{r_2} + \frac{1}{h_o}\left(-\frac{1}{{r_2}^2}\right) = 0$$

or $$\frac{1}{k} - \frac{1}{h_o \cdot r_2} = 0 \qquad \text{or} \qquad h_o \cdot r_2 = k$$

or $$r_2\ (= r_c) = \frac{k}{h_o} \quad ...(15.42)$$

The above relation represents the *condition for minimum resistance and consequently *maximum heat flow rate.* The *insulation radius at which resistance to heat flow is minimum* is called the '*critical radius*' (r_c). The critical radius r_c is dependent of the thermal quantities k and h_o and is independent of r_1 (*i.e.,* cylinder radius).

*It may be noted that if the second derivative of the denominator is evaluated, it will come out to be *positive.* This would verify that heat flow rate will be maximum, when $r_2 = r_c$.

In eqn. (15.41) ln $(r_2/r_1)/k$ is the conduction (insulation) thermal resistance which *increases* with increasing r_2 and $1/h_o.r_2$ is convective thermal resistance which *decreases* with increasing r_2. At $r_2 = r_c$ *the rate of increase of conductive resistance of insulation is equal to the rate of decrease of convective resistance thus giving a minimum value for the sum of thermal resistances.*

In the physical sense we may arrive at the following conclusions :

(*i*) For cylindrical bodies with $r_1 < r_c$, the heat transfer increases by adding insulation till $r_2 = r_c$ as shown in Fig. [15.26 (*a*)]. If insulation thickness is further increased, the rate of heat loss will decrease from this peak value, but until a certain amount of insulation denoted by r_2' at b is added, the heat loss rate is still greater for the solid cylinder. This happens when r_1 is small and r_c is large, *viz.,* the thermal conductivity of the insulation k is high (poor insulating material) and h_o is low. A *practical application would be the insulation of electric cables which should be good insulator for current but poor for heat.*

(*ii*) For cylindrical bodies with $r_1 > r_c$, the heat transfer decreases by adding insulation [Fig. 15.26 (*b*)]. This happens when r_1 is large and r_c is small, *viz.,* a good insulating material is used with low k and h_o is high. In *steam and refrigeration pipes* heat insulation is the main objective. For insulation to be properly effective in restricting heat transmission, the *outer radius must be greater than or equal to the critical radius.*

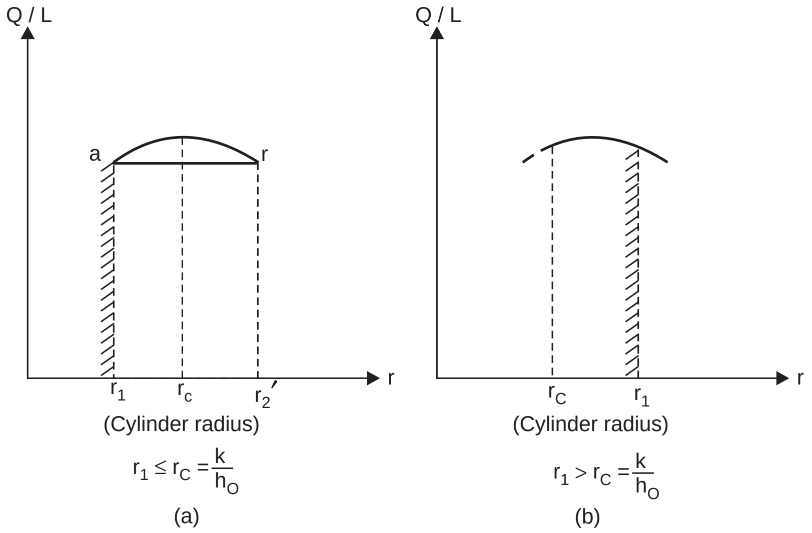


Fig. 15.26. Dependence of heat loss on insulation thickness.

B. Critical thickness of insulation for sphere :

Refer Fig. 15.27. The equation of heat flow through a sphere with insulation is given as

$$Q = \frac{(t_1 - t_{air})}{\left[\frac{r_2 - r_1}{4\pi k\, r_1 r_2}\right] + \frac{1}{4\pi\, r_2^2\, h_o}}$$

Adopting the same procedure as that of a cylinder, we have

$$\frac{d}{dr_2}\left[\frac{r_2 - r_1}{4\pi k\, r_1 r_2} + \frac{1}{4\pi r_2^2 \,.\, h_o}\right] = 0$$

or $$\frac{d}{dr_2}\left[\frac{1}{kr_1} - \frac{1}{kr_2} + \frac{1}{r_2^2\, h_o}\right] = 0$$

or $$\frac{1}{kr_2^2} - \frac{2}{r_2^3\, h_o} = 0$$

or $$r_2^3\, h_o = 2kr_2^2$$

or $$r_2 (= r_c) = \frac{2k}{h_o} \qquad ...(15.43)$$

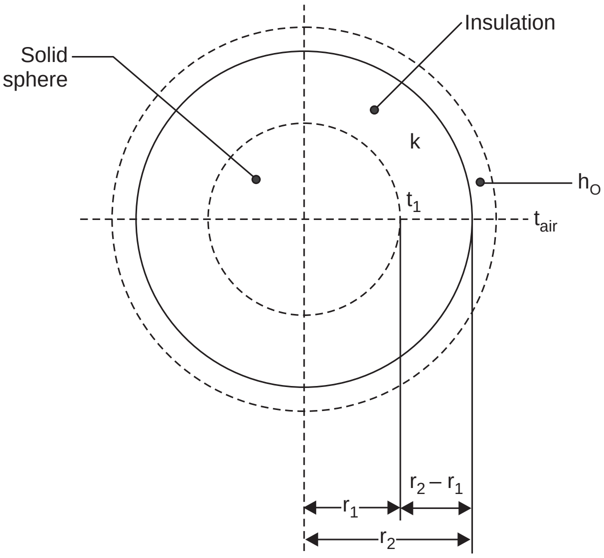


Fig. 15.27

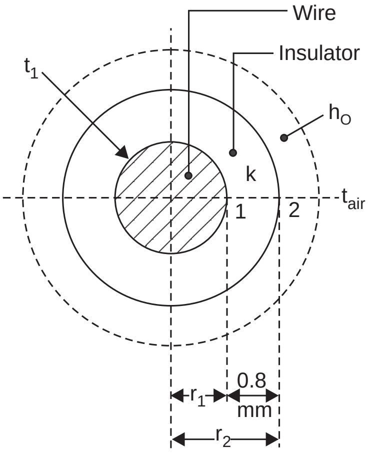


Fig. 15.28

Example 15.13. *A small electric heating application uses wire of 2 mm diameter with 0.8 mm thick insulation (k = 0.12 W/m°C). The heat transfer coefficient (h_o) on the insulated surface is 35 W/m²°C. Determine the critical thickness of insulation in this case and the percentage change in the heat transfer rate if the critical thickness is used, assuming the temperature difference between the surface of the wire and surrounding air remains unchanged.*

Solution. Refer Fig. 15.28.

Given : $r_1 = \frac{2}{2} = 1$ mm = 0.001 m

$r_2 = 1 + 0.8 = 1.8$ mm = 0.0018 m

$k = 0.12$ W/m°C, $h_o = 35$ W/m²°C

Critical thickness of insulation :

The critical radius of insulation is given by

$$r_c = \frac{k}{h_o} = \frac{0.12}{35} = 3.43 \times 10^{-3} \text{ m or } 3.43 \text{ mm.}$$

∴ Critical thickness of insulation

$= r_c - r_1 = 3.43 - 1 =$ **2.43 mm. (Ans.)**

Percentage change in heat transfer rate :

Case I : The heat flow through an insulated wire is given by

$$Q_1 = \frac{2\pi L\,(t_1 - t_{air})}{\dfrac{\ln(r_2/r_1)}{k} + \dfrac{1}{h_o \, . \, r_2}} = \frac{2\pi L\,(t_1 - t_{air})}{\dfrac{\ln(0.0018/0.001)}{0.12} + \dfrac{1}{35 \times 0.0018}} = \frac{2\pi L\,(t_1 - t_{air})}{20.77} \quad ...(i)$$

Case II : The heat flow through an insulated wire when critical thickness is used is given by

$$Q_2 = \frac{2\pi L\,(t_1 - t_{air})}{\dfrac{\ln(r_c/r_1)}{k} + \dfrac{1}{h_o\,.\,r_c}} = \frac{2\pi L\,(t_1 - t_{air})}{\dfrac{\ln(0.00343/0.001)}{0.12} + \dfrac{1}{35 \times 0.00343}}$$

$$= \frac{2\pi L\,(t_1 - t_{air})}{18.6} \qquad ...(ii)$$

∴ Percentage increases in heat flow by using critical thickness of insulation

$$= \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{\dfrac{1}{18.6} - \dfrac{1}{20.77}}{\dfrac{1}{20.77}} \times 100 = \mathbf{11.6\%}. \quad \textbf{(Ans.)}$$

15.3. HEAT TRANSFER BY CONVECTION

- The rate equation for the convective heat transfer (regardless of particular nature) between a surface and an adjacent fluid is prescribed by *Newton's law of cooling* (Refer Fig. 15.29)

$$Q = hA(t_s - t_f) \qquad ...(15.44)$$

where, Q = Rate of conductive heat transfer,

A = Area exposed to heat transfer,

t_s = Surface temperature,

t_f = Fluid temperature, and

h = Co-efficient of conductive heat transfer.

The units of h are, $h = \dfrac{Q}{A\,(t_s - t_f)} = \dfrac{\text{W}}{\text{m}^2\,°\text{C}}$ or W/m^2°C or W/m^{2}K

The coefficient of convective heat transfer 'h' (also known as film heat transfer coefficient) may be defined as "*the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time.*"

The value of 'h' depends on the following factors :

(*i*) Thermodynamic and transport properties (*e.g.,* viscosity, density, specific heat etc.) ;

(*ii*) Nature of fluid flow ;

(*iii*) Geometry of the surface ;

(*iv*) Prevailing thermal conditions.

Since 'h' depends upon several factors, it is difficult to frame a single equation to satisfy all the variations, however a dimensional analysis gives an equation for the purpose which is given as under :

$$\frac{h_D}{k} = Z\left(\frac{\rho C D}{\pi}\right)^a \left(\frac{c_p \mu}{k}\right)^b \left(\frac{D}{L}\right)^c \qquad ...(15.45)$$

or

$$Nu = Z\,(Re)^a\,(Pr)^b \left(\frac{D}{L}\right)^c$$

where, Nu = Nusselt number $\left(\dfrac{h_D}{k}\right)$,

Re = Reynolds number $\left(\dfrac{\rho \bar{u} D}{\mu}\right)$,

Pr = Prandtl number $\left(\frac{c_p \mu}{k}\right)$,

$\frac{D}{L}$ = Diameter to length ratio,

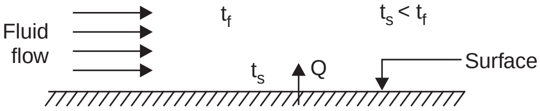
Z = A constant to be determined experimentally,

ρ = Density,

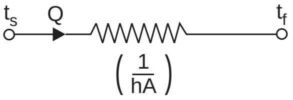
μ = Dynamic viscosity, and

C = Velocity.

The *mechanisms of convection* in which phase changes are involved lead to the important *fields of boiling and condensation.*



(*a*) Physical configuration



(*b*) Equivalent circuit

Fig. 15.29. Convective heat-transfer.

Refer Fig. 15.29 (*b*). The quantity $\frac{1}{hA}\left[Q = \frac{t_s - t_f}{(1/hA)} \ldots \text{Eqn}\,(28.44)\right]$ is called *convection thermal resistance* [$(R_{th})_{conv.}$] *to heat flow.*

- **Dimensionless numbers :**

Reynolds numbers, $Re = \frac{VL}{\nu}$

Prandtl number, $Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}$

Nusselt number, $Nu = \frac{hL}{k}$

Stanton number, $St = \frac{h}{\rho V c_p} = \frac{Nu}{Re \times Pr}$

Peclet number, $Pe = \frac{LV}{\alpha}\ (= Re . Pr)$

Graetz number, $G = Pe\left(\frac{\pi D}{4}\right)$

Grashoff number, $Gr = \frac{\rho^2 \beta g \Delta t L^3}{\mu^2}$.

Example 15.14. *A hot plate 1 m × 1.5 m is maintained at 300°C. Air at 25°C blows over the plate. If the convective heat transfer coefficient is 20 W/m²°C, calculate the rate of heat transfer.*

Solution. Area of the plate exposed to heat transfer, $A = 1 \times 1.5 = 1.5\ \text{m}^2$

Plate surface temperature, $t_s = 300°\text{C}$

Temperature of air (fluid), $t_f = 20°\text{C}$

Convective heat-transfer coeffficient, $h = 20\ \text{W/m}^2°\text{C}$

Rate of heat transfer, Q :

From Netwon's law of cooling,

$$Q = hA\,(t_s - t_f)$$

$$= 20 \times 1.5(300 - 20) = 8400\ \text{W or } \mathbf{8.4\ kW}. \quad \textbf{(Ans.)}$$

Example 15.15. *A wire 1.5 mm in diameter and 150 mm long is submerged in water at atmospheric pressure. An electric current is passed through the wire and is increased until the water boils at 100°C. Under the condition if convective heat transfer coefficient is 4500 W/m²°C find how much electric power must be supplied to the wire to maintain the wire surface at 120°C ?*

Solution. Diameter of the wire, d = 1.5 mm = 0.0015 m

Length of the wire, l = 150 mm = 0.15 m

∴ Surface area of the wire (exposed to heat transfer),

$$A = \pi\, d\, l = \pi \times 0.0015 \times 0.15 = 7.068 \times 10^{-4} \text{ m}^2$$

Wire surface temperature, t_s = 120°C

Water temperature, t_f = 100°C

Convective heat transfer coefficient, h = 4500 W/m^2°C

Electric power to be supplied :

Electric power which must be supplied = Total convection loss (Q)

∴ $Q = hA(t_s - t_f) = 4500 \times 7.068 \times 10^{-4}\ (120 - 100)$ = **63.6 W. (Ans.)**

Example 15.16. *Water flows inside a tube 45 mm in diameter and 3.2 m long at a velocity of 0.78 m/s. Determine the heat transfer co-efficient and the rate of heat transfer if the mean water temperature is 50°C and the wall is isothermal at 70°C. For water at 50°C take k = 0.66 W/mK,* $\nu = 0.478 \times 10^{-6}$ *m²/s and Prandtl number = 2.98.*

Solution. Diameter of the tube, D = 45 mm = 0.045 m

Length of the tube, l = 3.2 m

Velocity of water, $\bar{u}$ = 0.78 m/s

For water at 60°C, k = 0.66 W/mK

Kinematic viscosity, $\nu = 0.478 \times 10^{-6}$ m^2/s

Pr = 2.98

Reynolds number is given by

$$Re = \frac{D\bar{u}}{\nu} = \frac{0.045 \times 0.78}{0.478 \times 10^{-6}} = 73431$$

From Dittus and Boelter equation, Nusselt number,

$$Nu = 0.023\ (Re)^{0.8}\ (Pr)^{0.4}$$

$$\frac{hD}{k} = 0.023\ (73431)^{0.8}\ (2.98)^{0.4}$$

$$\frac{h \times 0.045}{0.66} = 0.023 \times 7810.9 \times 1.547$$

$$\therefore \quad h = 4076 \text{ W/m}^2 \text{ K}$$

i.e., Heat transfer co-efficient = **4076 W/m² K (Ans).**

$$Q = hA\ (t_w - t_f)$$
$$= 4076 \times \pi DL\ (70 - 50)$$
$$= 4076 \times \pi \times 0.045 \times 3.2 \times 20 = 36878 \text{ or } 36.878 \text{ kW}$$

i.e., Rate of heat transfer = **36.878 kW. (Ans).**

Example 15.17. *When 0.5 kg of water per minute is passed through a tube of 20 mm diameter, it is found to be heated from 20°C to 50°C. The heating is accomplished by condensing steam on the surface of the tube and subsequently the surface temperature of the tube is maintained at 85°C. Determine the length of the tube required for developed flow.*

Take the thermo-physical properties of water at 60°C as :

$\rho = 983.2$ *kg/m²,* $c_p = 4.178$ *kJ/kg K,* $k = 0.659$ *W/m°C,* $\nu = 0.478 \times 10^{-6}$ *m²/s.*

Solution. *Given :* m = 0.5 kg/min, D = 20 mm = 0.02 m, t_i = 20°C, t_o = 50°C

Length of the tube required for fully developed flow, L :

The mean film temperature, $t_f = \frac{1}{2}\left(85 + \frac{20 + 50}{2}\right) = 60°\text{C}$

Let us first determine the type of the flow

$$m = \rho A\bar{u} = 983.2 \times \frac{\pi}{4} \times (0.02)^2 \times \bar{u} = \frac{0.5}{60} \text{ (kg/s)}$$

or
$$\bar{u} = \frac{0.5}{60} \times \frac{4}{\pi} \times \frac{1}{983.2 \times (0.02)^2} = 0.0269 \text{ m/s}$$

Reynolds number, $Re = \frac{D\,.\,\bar{u}}{\nu} = \frac{0.02 \times 0.0269}{0.478 \times 10^{-6}} = 1125.5$

Since $Re < 2000$, hence the flow is *laminar.*

With *constant wall temperature* having *fully developed flow,*

$$Nu = \frac{hD}{k} = 3.65$$

or
$$h = \frac{3.65\, k}{D} = \frac{3.65 \times 0.659}{0.02} = 120.26 \text{ W/m}^2\text{°C}$$

The rate of heat transfer, $Q = A_s\, h\, (t_s - t_\infty) = m\, c_p\, (t_o - t_i)$

Here, $t_\infty = \frac{20 + 50}{2} = 35\text{°C} = t_b$

$$\therefore \quad (\pi \times 0.02 \times L) \times 120.26 \times (85 - 35) = \frac{0.5}{60} \times (4.178 \times 10^3)(50 - 20)$$

or
$$377.8\, L = 1044.5$$

or
$$L = \frac{1044.5}{377.8} = \mathbf{2.76\ m.} \quad \textbf{(Ans.)}$$

15.4. HEAT EXCHANGERS

15.4.1. Introduction

A **'heat exchanger'** may be defined *as an equipment which transfers the energy from a hot fluid to a cold fluid, with maximum rate and minimum investment and running costs.*

In heat exchangers the temperature of each fluid changes as it passes through the exchangers, and hence the temperature of the dividing wall between the fluids also changes along the length of the exchanger.

Examples of heat exchangers :

(*i*) Intercoolers and preheaters ;
(*ii*) Condensers and boilers in steam plant ;
(*iii*) Condensers and evaporators in refrigeration units ;
(*iv*) Regenerators ;
(*v*) Automobile radiators ;
(*vi*) Oil coolers of heat engine ;
(*vii*) Milk chiller of a pasteurising plant ;
(*viii*) Several other industrial processes.

15.4.2. Types of Heat Exchangers

In order to meet the widely varying applications, several types of heat exchangers have been developed which are classified on the basis of *nature of heat exchange process, relative direction of fluid motion, design and constructional features,* and *physical state of fluids.*

1. Nature of heat exchange process

Heat exchangers, on the basis of nature of heat exchange process, are classified as follows :

(*i*) Direct contact (or open) heat exchangers.
(*ii*) Indirect contact heat exchangers.
(*a*) Regenerators.
(*b*) Recuperators.

(*i*) ***Direct contact heat exchangers.*** In a direct contact or open heat exchanger the exchange of heat takes place by direct mixing of hot and cold fluids and transfer of heat and mass takes place simultaneously. The use of such units is made under conditions where mixing of two fluids is either harmless or desirable. *Examples* : (*i*) Cooling towers ; (*ii*) Jet condensers ; (*iii*) Direct contact feed heaters.

Fig. 15.30 shows a direct contact heat exchanger in which steam mixes with cold water, gives its latent heat to water and gets condensed. Hot water and non-condensable gases leave the container as shown in the figure.

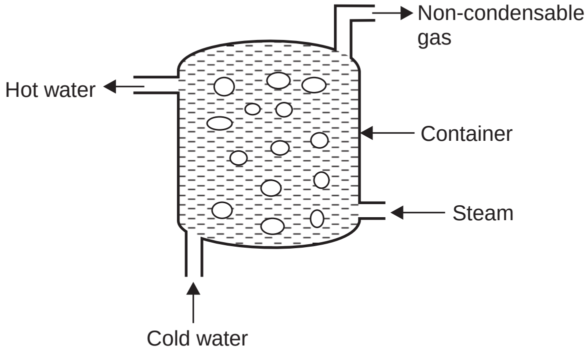


Fig. 15.30. Direct contact or open heat exchanger.

(*ii*) ***Indirect contact heat exchangers.*** In this type of heat exchanger, the heat transfer between two fluids could be carried out by transmission through wall which separates the two fluids. This type includes the following :

(*a*) Regenerators.

(*b*) Recuperators or surface exchangers.

(*a*) *Regenerators* : In a *regenerator* type of heat exchanger the *hot and cold fluids pass alternately through a space containing solid particles (matrix), these particles providing alternately a sink and a source for heat flow.*

Examples : (*i*) I.C. engines and gas turbines ; (*ii*) Open hearth and glass melting furnaces ; (*iii*) Air heaters of blast furnaces.

A *regenerator* generally operates periodically (the solid matrix alternately stores heat extracted from the hot fluid and then delivers it to the cold fluid). However, in some regenerators the matrix is made to rotate through the fluid passages arranged side by side which makes the heat exchange process *continuous.*

The performance of these regenerators is affected by the following *parameters :*

(*i*) Heat capacity of regenerating material,

(*ii*) The rate of absorption, and (*iii*) The release of heat.

Advantages :

1. Higher heat transfer coefficient ;
2. Less weight per kW of the plant ;
3. Minimum pressure loss ;
4. Quick response to load variation ;
5. Small bulk weight ;
6. Efficiency quite high.

Disadvantages :

1. Costlier compared to recuperative heat exchangers.
2. Leakage is the main trouble, therefore, perfect sealing is required.

(*b*) *Recuperators : 'Recuperator'* is the most important type of heat exchanger in which the flowing fluids exchanging heat are on either side of *dividing wall* (in the form of pipes or tubes generally). These heat exchangers are used when two fluids cannot be allowed to mix *i.e.,* when the mixing is undesirable.

Examples : (*i*) Automobile radiators, (*ii*) Oil coolers, intercoolers, *air preheaters,* economisers, superheaters, condensers and surface feed heaters of a steam power plant, (*iii*) Milk chiller of pasteurising plant, (*iv*) Evaporator of an ice plant :

Advantages :

1. Easy construction ;
2. More economical ;
3. More surface area for heat transfer ;
4. Much suitable for stationary plants.

Disadvantages :

1. Less heat transfer coefficient ;
2. Less generating capacity ;
3. Heavy and sooting problems.

The flow through *direct heat exchangers and recuperators* may be treated as *steady state* while through regenerators the *flow is essentially transient.*

2. Relative direction of fluid motion

According to the relative directions of two fluid streams the heat exchangers are classified into the following *three* categories :

(*i*) Parallel-flow or unidirection flow (*ii*) Counte-flow (*iii*) Cross-flow.

(*i*) ***Parallel-flow heat exchangers.*** In a *parallel-flow exchanger,* as the name suggests, the two fluid streams (hot and cold) travel in the *same direction.* The two streams enter at one end and leave at the other end. The flow arrangement and variation of temperatures of the fluid streams in case of parallel flow heat exchangers, are shown in Fig. 15.31. It is evident from the Fig. 15.31. (*b*) that the *temperature difference* between the hot and cold fluids goes on *decreasing* from inlet to outlet. Since this type of heat exchanger needs a large area of heat transfer, therefore, it is *rarely used in practice.*

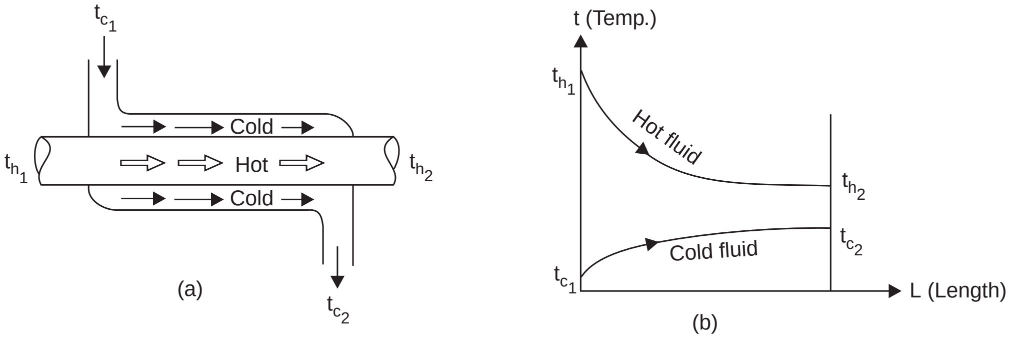


Fig. 15.31. Parallel-flow heat exchanger.

Examples : Oil coolers, oil heaters, water heaters etc.

As the two fluids are separated by a wall, this type of heat exchanger may be called *parallel-flow recuperator or surface heat exchanger*.

(*ii*) ***Counter-flow heat exchangers.*** In a counter-flow heat exchanger, the two fluids flow in *opposite directions*. The hot and cold fluids enter at the opposite ends. The flow arrangement and temperature distribution for such a heat exchanger are shown schematically in Fig. 15.32. The *temperature difference* between the two fluids remains more or less *nearly constant*. This type of heat exchanger, due to counter flow, gives *maximum rate of heat transfer for a given surface area.* Hence such heat exchangers are *most favoured* for heating and cooling of fluids.

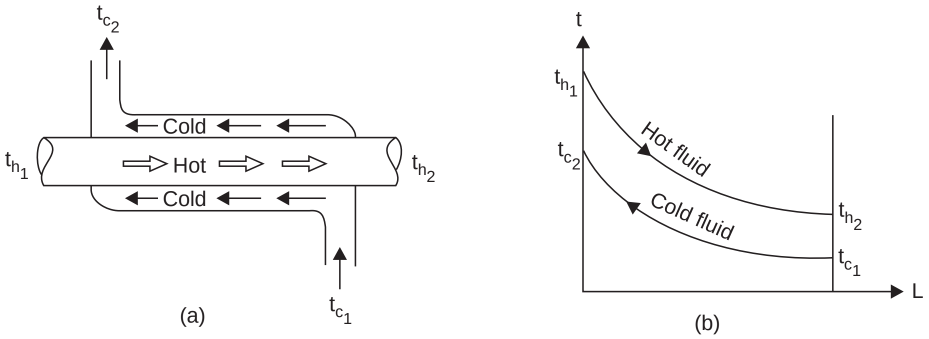


Fig. 15.32. Counter-flow heat exchanger.

(*iii*) ***Cross-flow heat exchangers.*** In cross-flow heat exchangers, the *two fluids (hot and cold) cross one another in space, usually at right angles.* Fig. 15.32 shows a schematic diagram of common arrangements of cross-flow heat exchangers.

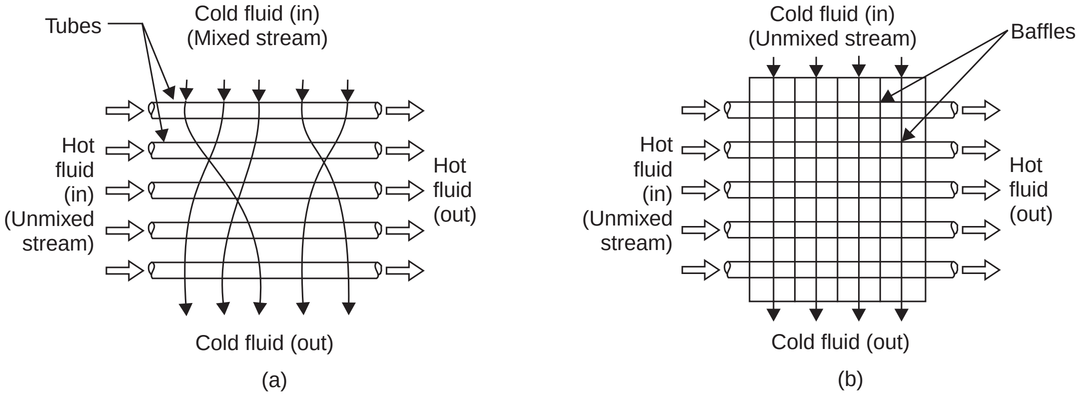


Fig. 15.33. Cross-flow heat exchangers.

- Refer Fig. 15.33 (*a*) : *Hot fluid* flows in the separate tubes and there is no mixing of the fluid streams. The *cold fluid* is perfectly mixed as it flows through the exchanger. The temperature of this mixed fluid will be uniform across any section and will vary only in the direction of flow.

Examples : The cooling unit of refrigeration system etc.

- Refer Fig. 15.33 (*b*) : In this case each of the fluids follows a prescribed path and is unmixed as it flows through heat exchanger. Hence the temperature of the fluid leaving the heater section is not uniform.

Examples : Automobile radiator etc.

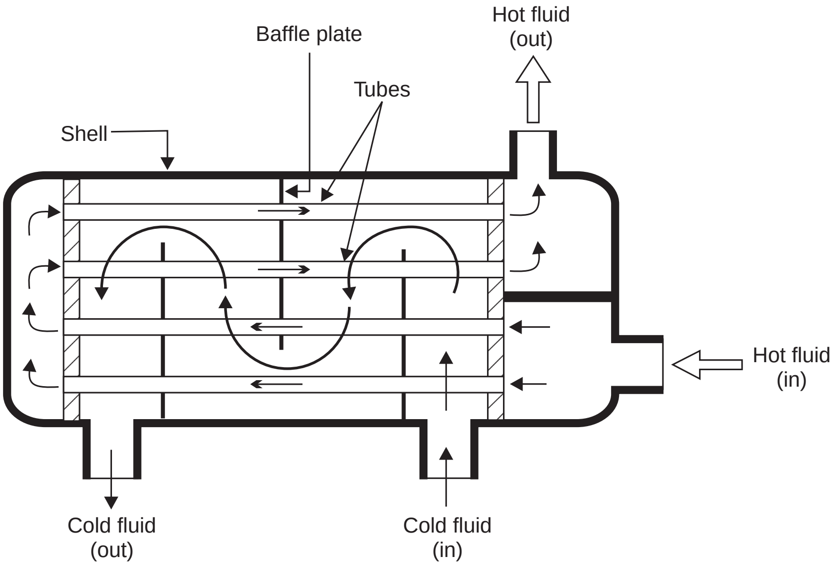
- In yet another arrangement, *both the fluids are mixed* while they travel through the exchanger ; consequently the temperature of both the fluids is uniform across the section and varies only in the direction in which flow takes place.

3. Design and constructional features

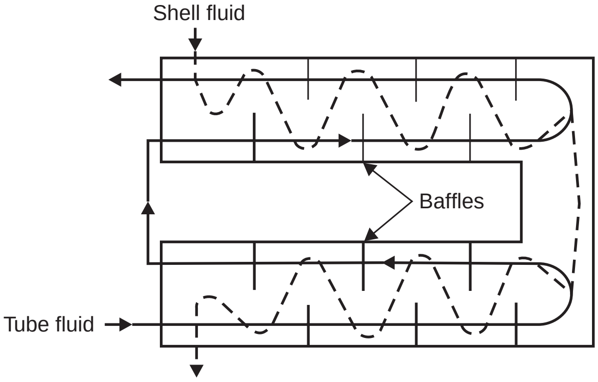
On the basis of design and constructional features, the heat exchangers are classified as under :

(*i*) ***Concentric tubes.*** In this type, two concentric tubes are used, each carrying one of the fluids. This direction of flow may be parallel or counter as depicted in Figs. 15.31 (*a*) and Fig. 15.32 (*a*). The effectiveness of the heat exchanger is increased by using swirling flow.

(*ii*) ***Shell and tube.*** In this type of heat exchanger one of the fluids flows through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it flows over the outside surface of the tubes. Such an arrangement is employed where *reliability* and *heat transfer effectiveness are important*. With the use of multiple tubes heat transfer rate is amply improved due to increased surface area.



(*a*) One-shell pass and two-tube pass heat exchanger.



(*b*) Two-shell pass and Four-tube pass heat exchanger

Fig. 15.34. Shell and tube heat exchangers.

(*iii*) ***Multiple shell and tube passes.*** Multiple shell and tube passes are used for *enhancing the overall heat transfer*. *Multiple shell pass* is possible where the fluid flowing through the shell is re-routed. The shell side fluid is forced to flow back and forth across the tubes by baffles. *Multiple tube pass* exchangers are those which re-route the fluid through tubes in the opposite direction.

(*iv*) ***Compact heat exchangers.*** These are special purpose heat exchangers and have a very large transfer surface area per unit volume of the exchanger. They are generally employed when convective heat transfer sufficient associated with one of the fluids is much smaller than that associated with the other fluid.

Example : Plate-fin, flattened fin tube exchangers etc.

4. Physical state of fluids

Depending upon the physical state of fluids the heat exchangers are classified as follows :

(*i*) Condensers (*ii*) Evaporators

(*i*) ***Condensers.*** In a condenser, the condensing fluid remains at constant temperature throughout the exchanger while the temperature of the colder fluid gradually increases from inlet to outlet. The hot fluid loses latent part of heat which is accepted by the cold fluid (Refer Fig. 15.35).

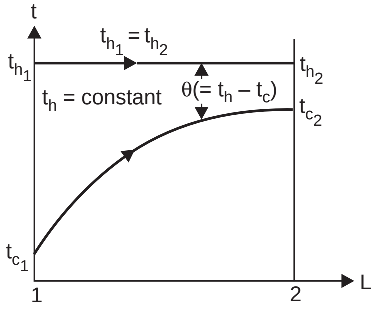


Fig. 15.35. Temperature distribution in a condenser.

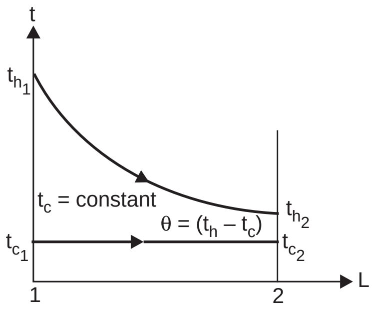


Fig. 15.36. Temperature distribution in an evaporator.

(*ii*) ***Evaporators.*** In this case, the boiling fluid (cold fluid) remains at constant temperature while the temperature of hot fluid gradually decreases from inlet to outlet. (Refer Fig. 15.36).

15.4.3. Heat Exchanger Analysis

For designing or predicting the performance of a heat exchanger it is necessary that the total heat transfer may be related with its governing parameters : (*i*) U (overall heat transfer coefficient due to various modes of heat transfer), (*ii*) A total surface area of the heat transfer, and (*iii*) t_1, t_2 (the inlet and outlet fluid temperatures). Fig. 15.37 shows the overall energy balance in a heat exchanger.

Let, $\dot{m}$ = Mass flow rate, kg/s,

c_p = Specific heat of fluid at constant pressure J/kg°C,

t = Temperature of fluid, °C, and

Δt = Temperature drop or rise of a fluid across the heat exchanger.

Subscripts *h* and *c* refer to the *hot* and *cold* fluids respectively ; subscripts 1 and 2 correspond to the *inlet* and *outlet* conditions respectively.

Assuming that there is no heat loss to the surroundings and potential and kinetic energy changes are negligible, from the energy balance in a heat exchanger, we have :

Heat given up by the hot fluid, $Q = m_h\, c_{ph}\, (t_{h_1} - t_{h_2})$ ...(15.46)

Heat picked up by the cold fluid, $Q = m_c\, c_{pc}\, (t_{c_2} - t_{c_1})$ ...(15.47)

Total heat transfer rate in the heat exchanger, $Q = UA\,\theta_m$ ...(15.48)

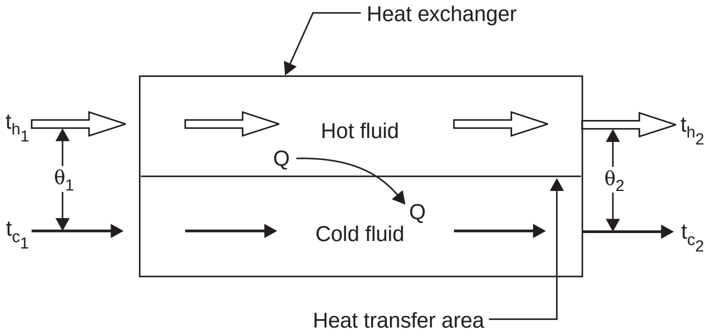


Fig. 15.37. Overall energy balance in a heat exchanger.

where, U = Overall heat transfer coefficient between the two fluids,

A = Effective heat transfer area, and

θ_m = *Appropriate mean value of temperature difference or logarithmic mean temperature difference (LMTD).*

15.4.4. Logarithmic Mean Temperature Difference (LMTD)

Logarithmic mean temperature difference (LMTD) is defined as *that temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference*.

In order to derive expression for *LMTD* for various types of heat exchangers, the following ***assumptions*** are made :

1. The overall heat transfer coefficient U is constant.
2. The flow conditions are steady.
3. The specific heats and mass flow rates of both fluids are constant.
4. There is no loss of heat to the surroundings, due to the heat exchanger being perfectly insulated.
5. There is no change of phase either of the fluids during the heat transfer.
6. The changes in potential and kinetic energies are negligible.
7. Axial conduction along the tubes of the heat exchanger is negligible.

15.4.4.1. Logarithmic Mean Temperature Difference for "Parallel-flow"

Refer Fig. 15.38, which shows the flow arrangement and distribution of temperature in a single-pass parallel-flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U\,dA\,(t_h - t_c) = U\,.\,dA\,.\,\Delta t$$

As a result of heat transfer dQ through the area dA, the hot fluid is cooled by dh whereas the cold fluid is heated up by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\,\dot{m}_h\,.\,c_{ph}\,.\,dt_h = \dot{m}_c\,.\,c_{pc}\,.\,dt_c = U\,.\,dA\,.\,(t_h - t_c) \quad ...(15.49)$$

(Here dt_h is – ve and dt_c is + ve)

or

$$dt_h = -\,\frac{dQ}{\dot{m}_h c_{ph}} = -\,\frac{dQ}{C_h}$$

and $$dt_c = \frac{dQ}{\dot{m}_c c_{pc}} = \frac{dQ}{C_c}$$

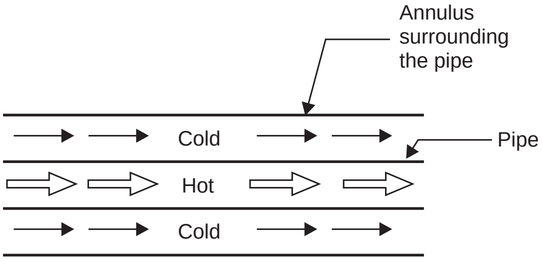
where, $C_h = \dot{m}_h\ c_{ph}$ = Heat capacity or water equivalent of hot fluid, and

$C_c = \dot{m}_c\ c_{pc}$ = Heat capacity or water equivalent of cold fluid.

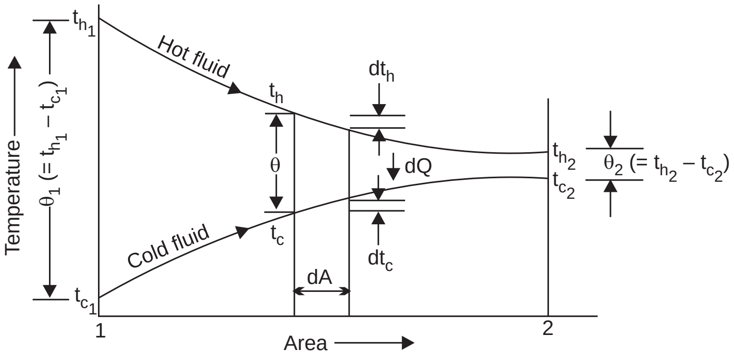
$\dot{m}_h$ and $\dot{m}_c$ are the mass flow rates of fluids and c_{ph} and c_{pc} are the respective specific heats.

$$\therefore \qquad dt_h - dt_c = -\,dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d\theta = -\,dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \qquad ...(15.50)$$



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Subscripts *h, c* refer to *hot* and *cold* fluids
Subscripts 1, 2 refer to *inlet* and *outlet* conditions.
Fig. 15.38. Calculation of LMTD for a parallel-flow heat exchanger.

Substituting the value of *dQ* from eqn. (15.49) the above equation becomes

$$d\theta = -\,U\,.\,dA\,(t_h - t_c)\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or $$d\theta = -\,U\,.\,dA\,.\,\theta\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or $$\frac{d\theta}{\theta} = -\,U.\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Integrating between inlet and outlet conditions (*i.e.,* from $A = 0$ to $A = A$), we get

$$\int_1^2 \frac{d\theta}{\theta} = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \int_{A=0}^{A=A} U \,.\, dA$$

or

$$\ln (\theta_2/\theta_1) = -\, UA \left[\frac{1}{C_h} + \frac{1}{C_c}\right] \qquad \text{...(15.51)}$$

Now, the total heat transfer rate between the two fluids is given by

$$Q = C_h \; (t_{h_1} - t_{h_2}) = C_c \; (t_{c_2} - t_{c_1}) \qquad \text{...(15.52)}$$

or

$$\frac{1}{C_h} = \frac{t_{h_1} - t_{h_2}}{Q} \qquad \text{...[15.52 (a)]}$$

$$\frac{1}{C_c} = \frac{t_{c_2} - t_{c_1}}{Q} \qquad \text{...[15.52 (b)]}$$

Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ into eqn. (15.51), we get

$$\ln (\theta_2/\theta_1) = -\, UA \left[\frac{t_{h_1} - t_{h_2}}{Q} + \frac{t_{c_2} - t_{c_1}}{Q}\right]$$

$$= \frac{UA}{Q} \; [(t_{h_2} - t_{c_2}) - (t_{h_1} - t_{c_1})] \; = \; \frac{UA}{Q} \; (\theta_2 - \theta_1)$$

$$Q = \frac{UA\,(\theta_1 - \theta_1)}{\ln\,(\theta_2 / \theta_1)}$$

The above equation may be written as

$$Q = U\,A\;\theta_m \qquad \text{...(15.53)}$$

where

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln\,(\theta_2 / \theta_1)} = \frac{\theta_1 - \theta_2}{\ln\,(\theta_1 / \theta_2)} \qquad \text{...(15.54)}$$

θ_m is called the *logarithmic mean temperature difference (LMTD).*

15.4.4.2. Logarithmic Mean Temperature Difference for "Counter-flow"

Ref Fig. 15.39, which shows the flow arrangement and temperature distribution in a single-pass counter-flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U \,.\, dA \; (t_h - t_c) = U \,.\, dA \,.\, \Delta t \qquad \text{...(15.55)}$$

In this case also, due to heat transfer dQ through the area dA, the hot fluid is cooled down by dt_h whereas the cold fluid is heated by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\, \dot{m}_h \,.\, c_{ph} \,.\, dt_h = -\, \dot{m}_c \,.\, c_{pc} \,.\, dt_c \qquad \text{...(15.56)}$$

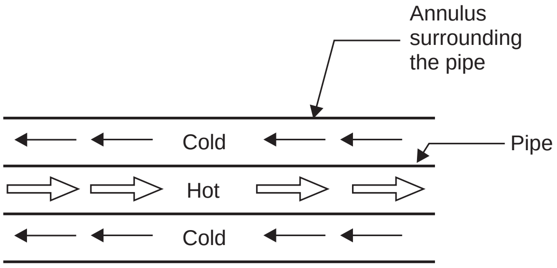
In a counter-flow system, the temperatures of both the fluids *decrease* in the direction of heat exchanger length, hence the –ve signs.

$$\therefore \qquad dt_h = -\,\frac{dQ}{\dot{m}_h c_{ph}} = -\,\frac{dQ}{C_h}$$

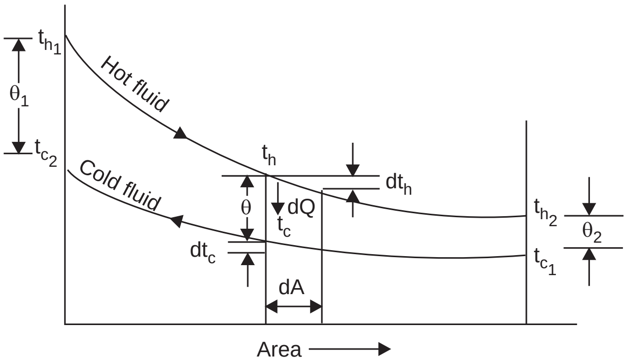
and

$$dt_c = -\,\frac{dQ}{\dot{m}_c c_{pc}} = -\,\frac{dQ}{C_c}$$

$$\therefore \qquad dt_h - dt_c = -\, dQ \left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Fig. 15.39. Calculation of LMTD for a counter-flow heat exchanger.

or $$d\theta = -\,dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \qquad ...(15.57)$$

Inserting the value of dQ from eqn. (15.55), we get

$$d\theta = -\,U\,dA\,(t_h - t_c)\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$= -\,U\,dA\,.\,\theta\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or $$\frac{d\theta}{\theta} = -\,U\,dA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

Integrating the above equation from $A = 0$ to $A = A$, we get

$$\ln\,(\theta_2/\theta_1) = -\,U\,.\,A\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \qquad ...(15.58)$$

Now, the total heat transfer rate between the two fluids is given by

$$\theta = C_h\,(t_{h_1} - t_{h_2}) = C_c\,(t_{c_2} - t_{c_1}) \qquad ...(15.59)$$

or $$\frac{1}{C_h} = \frac{t_{h_1} - t_{h_2}}{Q} \qquad ...[15.60\ (a)]$$

or $$\frac{1}{C_c} = \frac{t_{c_2} - t_{c_1}}{Q} \qquad ...[15.60\ (b)]$$

Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ into eqn. (15.58), we get

$$\ln(\theta_2/\theta_1) = -UA\left[\frac{t_{h_1}-t_{h_2}}{Q} - \frac{t_{c_2}-t_{c_1}}{Q}\right]$$

$$= -\frac{UA}{Q}\,[(t_{h_1}-t_{c_2})-(t_{h_2}-t_{c_1})] = -\frac{UA}{Q}(\theta_1-\theta_2) = \frac{UA}{Q}(\theta_2-\theta_1)$$

or

$$Q = \frac{UA(\theta_2-\theta_1)}{\ln(\theta_2/\theta_1)} \quad \text{...(15.61)}$$

Since $Q = UA\,\theta_m$

$$\therefore \quad \theta_m = \frac{\theta_2-\theta_1}{\ln(\theta_2/\theta_1)} = \frac{\theta_1-\theta_2}{\ln(\theta_1/\theta_2)} \quad \text{...(15.61(a))}$$

A special case arises when $\theta_1 = \theta_2 = \theta$ in case of a *counter-flow* heat exchanger. In such a case, we have

$$\theta_m = \frac{\theta-\theta}{\ln(\theta/\theta)} = \frac{0}{0}$$

This value is indeterminate. The value of θ_m for such can be found by applying L′ Hospital's rule :

$$\lim_{\theta_2\to\theta_1}\frac{\theta_2-\theta_1}{\ln(\theta_2/\theta_1)} = \lim_{(\theta_2/\theta_1)\to 1}\frac{\theta_1\left[\frac{\theta_2}{\theta_1}-1\right]}{\ln(\theta_2/\theta_1)}$$

Let $(\theta_2/\theta_1) = R$. Therefore, the above expression can be written as

$$\lim_{R\to 1}\frac{\theta(R-1)}{\ln(R)}$$

Differentiating the numerator and denominator with respect to R and taking limits, we get

$$\lim_{(R\to 1)}\frac{\theta}{(1/R)} = \theta$$

Hence when $\theta_1 = \theta_2$ eqn. (15.61) becomes

$$Q = UA\,\theta$$

θ_m *(LMTD) for a counter-flow unit is always greater than that for a parallel-flow unit ;* hence counter-flow heat exchanger can transfer *more* heat than parallel-flow one ; in other words a counter-flow heat exchanger needs a *smaller heating surface for the same rate of heat transfer.* For this reason, the counter-flow arrangement is usually used.

When the temperature variations of the fluids are relatively small, then temperature variation curves are approximately straight lines and adequately accurate results are obtained by taking the *arithmetic mean temperature difference (AMTD).*

$$AMTD = \frac{t_{h_1}+t_{h_2}}{2} - \frac{t_{c_1}+t_{c_2}}{2} = \frac{(t_{h_1}-t_{c_1})+(t_{h_2}-t_{c_2})}{2} = \frac{\theta_1+\theta_2}{2} \quad \text{...(15.62)}$$

However, practical considerations suggest that the logarithmic mean temperature difference (θ_m) should be invariably used when $\frac{\theta_1}{\theta_2} > 1.7$.

Example 15.18. *The flow rates of hot and cold water streams running through a parallel-flow heat exchanger are 0.2 kg/s and 0.5 kg/s respectively. The inlet temperatures on the hot and cold sides are 75°C and 20°C respectively. The exit temperature of hot water is 45°C. If the individual heat transfer coefficients on both sides are 650 W/m^2°C, calculate the area of the heat exchanger.*

Solution. *Given :* $\dot{m}_h = 0.2$ kg/s ; $\dot{m}_c = 0.5$ kg/s ; $t_{h_1} = 75°C$;

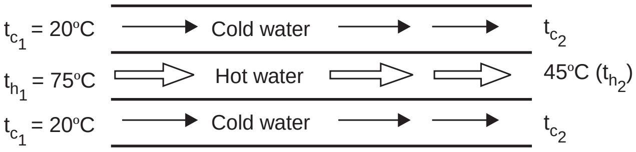
$t_{h_2} = 45°C$; $t_{c_1} = 20°C$; $h_i = h_0 = 650$ W/m²°C.

The area of heat exchanger, A :

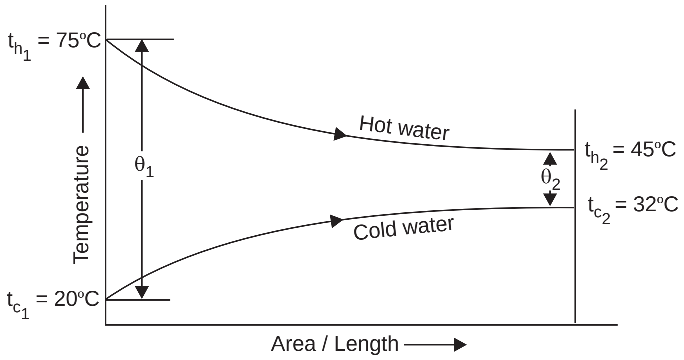
The heat exchanger is shown diagrammatically in Fig. 15.40.

The heat transfer rate, $Q = m_h \times c_{ph} \times (t_{h_1} - t_{h_2})$

$$= 0.2 \times 4.187 \times (75 - 45) = 25.122 \text{ kJ/s}$$



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Fig. 15.40. Parallel-flow heat exchanger.

Heat lost by hot water = Heat gained by cold water

$$\dot{m}_h \times c_{ph} \times (t_{h_1} - t_{h_2}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$0.2 \times 4.187 \times (75 - 45) = 0.5 \times 4.187 \times (t_{c_2} - 20)$$

$\therefore$ $$t_{c_2} = 32°C$$

Logarithmic mean temperature difference (*LMTD*) is given by,

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} \qquad \text{...[Eqn. (15.54)]}$$

or $$\theta_m = \frac{(t_{h_1} - t_{c_1}) - (t_{h_2} - t_{c_2})}{\ln[(t_{h_1} - t_{c_1})/(t_{h_2} - t_{c_2})]}$$

$$= \frac{(75 - 20) - (45 - 32)}{\ln[(75 - 20)/(45 - 32)]} = \frac{55 - 33}{\ln(55/13)} = 29.12°C$$

Overall heat transfer coefficient U is calculated from the relation

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{650} + \frac{1}{650} = \frac{1}{325}$$

$\therefore$ $$U = 325 \text{ W/m}^2°C$$

Also, $$Q = U A \theta_m$$

or $$A = \frac{Q}{U\theta_m} = \frac{25.122 \times 1000}{325 \times 29.12} = \mathbf{2.65\ m^2}. \quad \textbf{(Ans.)}$$

Example 15.19. *In a counter-flow double pipe heat exchanger, water is heated from 25°C to 65°C by an oil with a specific heat of 1.45 kJ/kg K and mass flow rate of 0.9 kg/s. The oil is cooled from 230°C to 160°C. If the overall heat transfer coefficient is 420 W/m²°C, calculate the following :*

(*i*) *The rate of heat transfer,*

(*ii*) *The mass flow rate of water, and*

(*iii*) *The surface area of the heat exchanger.*

Solution. *Given :* $t_{c_1} = 25°\text{C}$; $t_{c_2} = 65°\text{C}$, $c_{ph} = 1.45$ kJ/kg K ; $\dot{m}_h = 0.9$ kg/s ;

$$t_{h_1} = 230°\text{C} \ ; \ t_{h_2} = 160°\text{C}, \ U = 420 \text{ W/m}^2°\text{C}.$$

(*i*) **The rate of heat transfer, *Q* :**

$$Q = \dot{m}_h \times c_{ph} \times (t_{h_1} - t_{h_2})$$

or $$Q = 0.9 \times (1.45) \times (230 - 160) = \mathbf{91.35\ kJ/s}$$

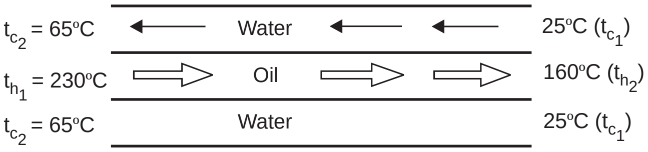
(*ii*) **The mass flow rate of water, $\dot{\mathbf{m}}_\mathbf{c}$:**

Heat lost by oil (hot fluid) = Heat gained by water (cold fluid)

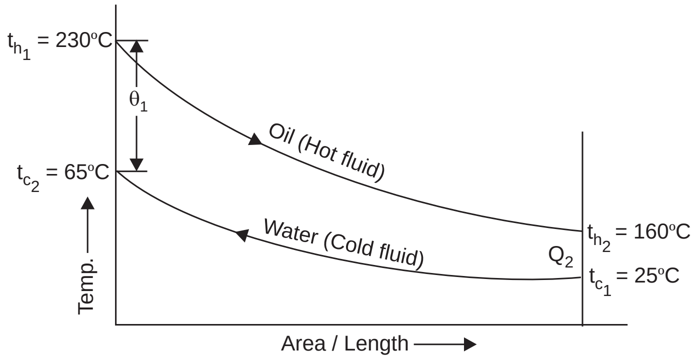
$$\dot{m}_h \times c_{hp} \times (t_{h_1} - t_{h_2}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$91.35 = \dot{m}_c \times 4.187 \ (65 - 25)$$

∴ $$\dot{m}_c = \frac{91.35}{4.187 \times (65 - 25)} = \mathbf{0.545\ kg/s}$$



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Fig. 15.41. Counter-flow heat exchanger.

(*iii*) **The surface area of heat exchanger, *A* :**

Logarithmic mean temperature difference (LMTD) is given by

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln (\theta_1 / \theta_2)}$$

$$= \frac{(t_{h_1} - t_{c_2}) - (t_{h_2} - t_{c_1})}{\ln [(t_{h_1} - t_{c_2})/(t_{h_2} - t_{c_1})]} = \frac{(230 - 65) - (160 - 25)}{\ln [(230 - 65)/(160 - 25)]}$$

or $$\theta_m = \frac{165 - 135}{\ln [(165/135)]} = 149.5°\text{C}$$

Also, $$Q = U A \theta_m$$

or $$A = \frac{Q}{U\theta_m} = \frac{91.35 \times 10^3}{420 \times 149.5} = \mathbf{1.45\ m^2}. \quad \textbf{(Ans.)}$$

Example 15.20. *Steam enters a counter-flow heat exchanger, dry saturated at 10 bar and leaves at 35°C. The mass flow of steam is 800 kg/min. The gas enters the heat exchanger at 650°C and mass flow rate is 1350 kg/min. If the tubes are 30 mm diameter and 3 m long, determine the number of tubes required. Neglect the resistance offered by metallic tubes. Use the following data :*

For steam : t_{sat} *= 180°C (at 10 bar) ;* c_{ps} *= 2.71 kJ/kg°C ;* h_s *= 600 W/m²°C*

For gas : c_{pg} *= 1 kJ/kg°C ;* h_g *= 250 W/m²°C* **(P.U.)**

Solution. *Given :* $\dot{m}_s = \dot{m}_c = \frac{800}{60} = 13.33$ kg/s ; $\dot{m}_g = \dot{m}_h = \frac{1350}{60} = 22.5$ kg/s ;

$t_{h_1} = 650°\text{C}$; $t_{c_1} (= t_{sat}) = 180°\text{C}$; $t_{c_2} = 350°\text{C}$; $d = 30$ mm $= 0.03$ m ; $L = 3$ m.

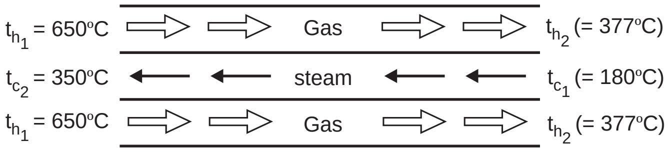
Number of tubes required, *N* :

Heat lost by gases = Heat gained by steam

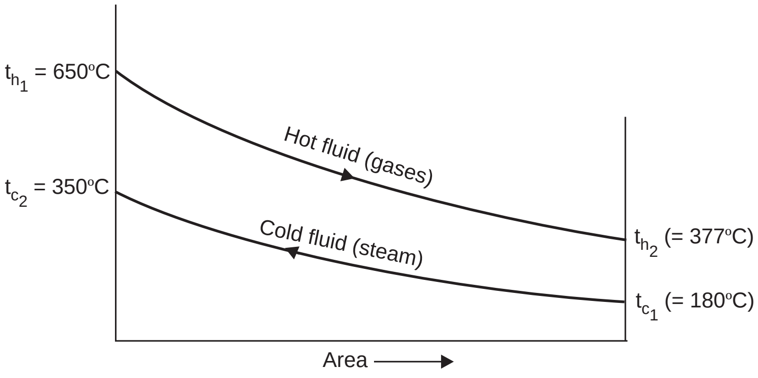
$$\dot{m}_h \times c_{ph} \times (t_{h_1} - t_{h_2}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$22.5 \times 1 \times (650 - t_{h_2}) = 13.33 \times 2.71 \times (350 - 180)$$

$$\therefore \quad t_{h_2} = 377°\text{C}$$



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Fig. 15.42. Counter-flow heat exchanger.

Overall heat transfer coefficient is given by

$$\frac{1}{U} = \frac{1}{h_g} + \frac{d_o}{d_i}\frac{1}{h_s} = \frac{1}{h_g} + \frac{1}{h_s} \text{ as } d_i \approx d_o \qquad \text{...(given)}$$

or
$$U = \frac{h_g \times h_s}{h_g + h_s} = \frac{250 \times 600}{250 + 600} = 176.5 \text{ W/m}^2\text{°C}$$

Total heat transfer rate is given by

$$Q = U A \, \theta_m \qquad \text{...}(i)$$

where $A = N \times (\pi \, d \, L) = N \times \pi \times 0.03 \times 3 = 0.2827 \text{ N m}^2$

$Q = 22.5 \times (1 \times 10^3) \times (650 - 377) = 6142.5 \times 10^3 \text{ W}$

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\ln[(t_{h1} - t_{c2})/(t_{h2} - t_{c1})]}$$

$$= \frac{(650 - 350) - (377 - 180)}{\ln[(650 - 300)/(377 - 180)]} = \frac{300 - 197}{\ln(300/197)} = 244.9\text{°C}$$

Substituting the values is eqn. (*i*), we get

$$6142.5 \times 10^3 = 176.5 \times 0.2827 \text{ N} \times 244.9$$

or
$$N = \frac{6142.5 \times 10^3}{176.5 \times 0.2827 \times 244.9} = \mathbf{503 \text{ tubes.}} \quad \textbf{(Ans.)}$$

Example 15.21. *A two-pass surface condenser is required to handle the exhaust from a turbine developing 15 MW with specific steam consumption of 5 kg/kWh. The condenser vacuum is 660 mm of Hg when the barometer reads 760 mm of Hg. The mean velocity of water is 3 m/s, water inlet temperature is 24°C. The condensate is saturated water and outlet temperature of cooling water in 4°C less than the condensate temperature. The quality of exhaust steam is 0.9 dry. The overall heat transfer coefficient based on outer area of tubes is 4000 W/m*^{2}°C. *The water tubes are 38.4 mm in outer diameter and 29.6 mm in inner diameter. Calculate the following :*

(*i*) *Mass of cooling water circulated in kg/min,*

(*ii*) *Condenser surface area,*

(*iii*) *Number of tubes required per pass, and*

(*iv*) *Tube length.* **(P.U.)**

Solution. *Given* : d_i = 29.6 mm = 0.0296 m ; d_o = 38.4 mm = 0.0384 m ;

U = 4000 W/m^2°C ; V = 3 m/s ; t_{c_1} = 24°C ; x (dryness fraction) = 0.9.

The pressure of the steam in the condenser,

$$p_s = \frac{760 - 660}{760} \times 1.0133 = 0.133 \text{ bar}$$

The properties of steam at p_s = 0.133 bar, from steam table, are :

$$t_{sat} = 51\text{°C} \; ; \; h_{fg} = 2592 \text{ kJ/kg}$$

$$\therefore \qquad t_{c_1} = 51 - 4 = 47\text{°C}$$

The steam condensed per minute,

$$\dot{m}_s \, (= \dot{m}_h) = \frac{(15 \times 1000) \times 5}{60} = 1250 \text{ kg/min}$$

(*i*) **Mass of cooling water circulated per minute, $\dot{\mathbf{m}}_\mathbf{w}$ (= $\dot{\mathbf{m}}_\mathbf{c}$) :**

Heat lost by steam = Heat gained by water

$$\dot{m}_h \times (x \, . \, h_{fg}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$1250 \times (0.9 \times 2592) = \dot{m}_c \times 4.187\ (47 - 24)$$

$$\therefore \qquad \dot{m}_c\ (= \dot{m}_w) = \mathbf{30280\ kg/min}$$

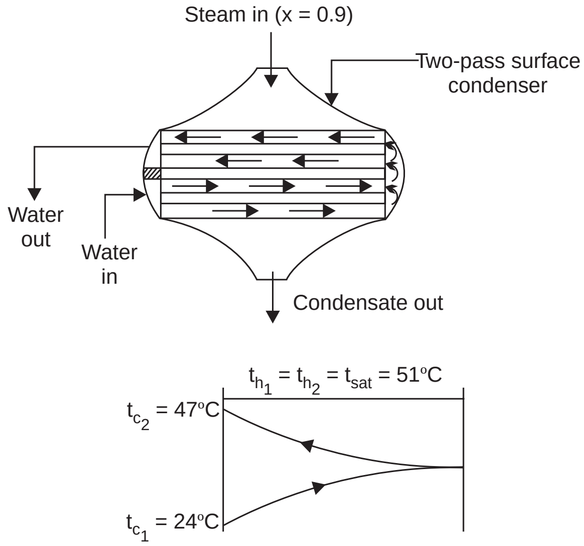


Fig. 15.43. A two-pass surface condenser.

(*ii*) **Condenser surface area, *A* :**

$$Q = \frac{\dot{m}_s \times (x \,.\, h_{fg})}{60} = U A\, \theta_m \qquad ...(i)$$

where,
$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\, (\theta_1 / \theta_2)} = \frac{(t_{h_1} - t_{c_1}) - (t_{h_2} - t_{c_2})}{\ln\, [(t_{h_1} - t_{c_1})/(t_{h_2} - t_{c_2})]}$$

$$= \frac{(51 - 24) - (51 - 47)}{\ln\, [(51 - 24)/(51 - 47)]} = \frac{(27 - 4)}{\ln\, (27/4)} = 12.04°C$$

Substituting the values in eqn. (*i*), we get

$$\frac{1250}{60} \times (0.9 \times 2592 \times 10^3) = 4000 \times A \times 12.04$$

or
$$A = \mathbf{1009.1\ m^2}$$

(*iii*) **Number of tubes required per pass, N_p :**

$$\dot{m}_w = \left(\frac{\pi}{4}\, d_i^{\,2} \times V \times \rho\right) \times N_p$$

$$\frac{30280}{60} = \frac{\pi}{4} \times (0.0296)^2 \times 3 \times 1000 \times N_p$$

or
$$N_p = \frac{30280 \times 4}{60 \times \pi \times (0.0296)^2 \times 3 \times 1000} = 244.46 \text{ say } \mathbf{245}$$

(Total number of tubes required, $N = 2N_p = 2 \times 245 = 490$)

(*iv*) **Tube length, L :**

$$A = (\pi d_o L) \times (2N_p)$$
$$1009.1 = (\pi \times 0.0384 \times L) \times (2 \times 245)$$

or
$$L = \frac{1009.1}{\pi \times 0.0384 \times 2 \times 245} \simeq \mathbf{17.1\ m.} \quad \textbf{(Ans.)}$$

Example 15.22. *A feed water heater which supplies hot water to a boiler comprises a shell and tube heat exchanger with one-shell pass and two-tube passes. One hundred thin-walled tubes each of 20 mm diameter and length of 2 m per pass are used. Under normal operating conditions, water enters the tubes at 10 kg/s and 17°C and is heated by condensing saturated steam at 1 atm. on the outer surface of the tubes. The convection coefficient of the saturated steam is 10 kW/m²°C. Determine the water exit temperature.*

Use the following properties of water :

$c_p = 4.18$ *kJ/kg°C ;* $\mu = 0.596 \times 10^{-3}$ *Ns/m² ;* $k = 0.635$ *W/m°C and* $Pr = 3.93$. **(M.U.)**

Solution. *Given* : p (number of tube passes) = 2, N (total number of tubes) = 200,

d = 20 mm = 0.02 m ; (length per pass) = 2 m, $\dot{m}_w = \dot{m}_c$ = 10 kg/s, t_{c_1} = 17°C.

Water exit temperature, $\mathbf{t_{c_2}}$:

$$\dot{m}_c = \frac{\pi}{4} d^2 \times V \times \rho \times N_p$$

$$\left[\text{where } V = \text{velocity of water ; } N_p = \text{number of tubes per pass} = \frac{N}{p} = \frac{200}{2} = 100\right]$$

or
$$10 = \frac{\pi}{4} \times 0.02^2 \times V \times 1000 \times 100$$

$\therefore$
$$V = \frac{10 \times 4}{\pi \times 0.02^2 \times 1000 \times 100} = 0.318 \text{ m/s}$$

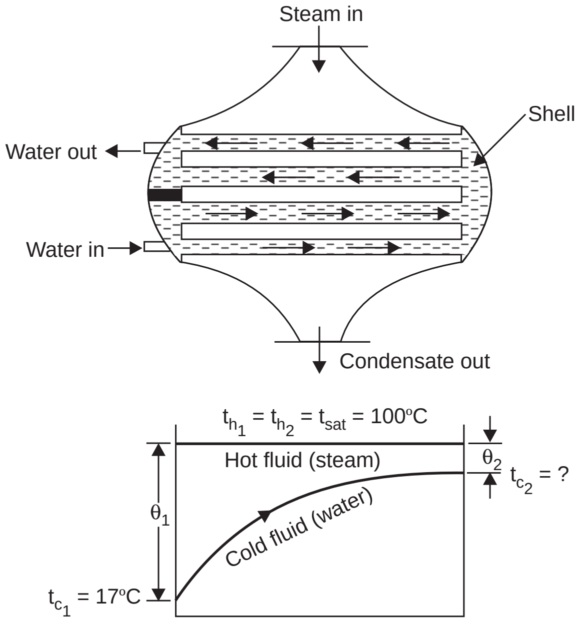


Fig. 15.44. One-shell pass and two-tube passes condenser.

Using non-dimensional heat transfer equation to water side, we get

$$Nu = \frac{h_i d}{k} = 0.023\ (Re)^{0.8}\ (Pr)^{0.33}$$

or $$h_i = \frac{k}{d} \times 0.023\ (Re)^{0.8}\ (Pr)^{0.33} \qquad \ldots(i)$$

$$Re = \frac{\rho V d}{\mu} = \frac{1000 \times 0.318 \times 0.02}{0.596 \times 10^{-3}} = 10671$$

Substituting the values in eqn. (*i*), we get

$$h_i = \frac{0.635}{0.02} \times 0.023\ (10671)^{0.8}\ (3.93)^{0.33} = 1915 \text{ W/m}^2\text{°C}$$

The overall heat transfer coefficient is given by the relation,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{U} = \frac{1}{1915} + \frac{1}{10 \times 10^3} = 0.000622$$

$$\therefore \quad U = \frac{1}{0.000622} = 1607.7 \text{ W/m}^2\text{°C}$$

Further, $$\theta_1 = t_{h_1} - t_{c_1} = 100 - 17 = 83\text{°C}$$

$$\theta_2 = t_{h_2} - t_{c_2} = 100 - t_{c_2}$$

∴ Arithmetic mean temperature difference,

$$AMTD = \frac{\theta_1 + \theta_2}{2} = \frac{83 + (100 - t_{c_2})}{2} = 91.5 - 0.5\ t_{c_2}$$

The heat transfer rate is given by,

$$Q = \dot{m}_c\ c_{pc}\ (t_{c_2} - t_{c_1}) = U A_s\ (AMTD) = U \times (\pi\ d\ L \times N)\ (AMTD)$$

(where A_s = Surface area of *all the tubes in both passes*)

or $$10 \times (4.18 \times 10^3)\ (t_{c_2} - 17) = 1607.7 \times (\pi \times 0.02 \times 2 \times 200) \times (91.5 - 0.5\ t_{c_2})$$

$$41800\ (t_{c_2} - 17) = 40406\ (91.5 - 5\ t_{c_2})$$

or $$t_{c_2} - 17 = \frac{40406}{41800}\ (91.5 - 0.5\ t_{c_2}) = 0.966\ (91.5 - 0.5\ t_{c_2})$$

$$= 88.39 - 0.483\ t_{c_2}$$

or $$t_{c_2} = \mathbf{71°C.} \quad \textbf{(Ans.)}$$

15.5. HEAT TRANSFER BY RADIATION

15.5.1. Introduction

'Radiation' heat transfer is defined as *"the transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused solely by a temperature difference."* Whereas the heat transfer by conduction and convection takes place only in the *presence of medium, radiation heat transfer does not require a medium.* Radiation exchange, in fact, occurs most effectively in *vacuum.* Further, the rate of heat transfer by *conduction and convection* varies as the temperature difference to the *first power,* whereas the *radiant heat exchange* between two bodies depends on the difference between their temperature to the *'fourth power'.* Both the amount of radiation and the quality of radiation depend upon temperature. The *dissipation from the filament of a vacuum tube* or the *heat leakage through the evacuated walls of a thermos flask* are some familiar *examples* of heat transfer by radiation.

The contribution of *radiation* to heat transfer is very significant at high absolute temperature levels such as those prevailing in *furnaces, combustion chambers, nuclear explosions* and in *space applications.* The *solar energy* incident upon the earth is also governed by the *laws of radiation.*

The energy which a radiating surface releases is *not continuous* but is in the form of successive and separate (discrete) packet or quanta of energy called *photons.* The photons are propagated through space as rays ; the movement of swarm of photons is described as *electromagnetic waves.* The photons travel (with speed equal to that of light) in straight paths with unchanged frequency ; when they approach the receiving surface, there occurs reconversion of wave motion into *thermal energy* which is partly absorbed, reflected or transmitted through the receiving surface (the magnitude of each fraction depends, upon the nature of the surface that receives the *thermal radiation*).

All types of electromagnetic waves are classified in terms of *wavelength* and are propagated at the speed of light (*c*) *i.e.,* 3×10^8 m/s. The electromagnetic spectrum is shown in Fig. 15.45. The distinction between one form of radiation and another lies only in its frequency (*f*) and wavelength (λ) which are related by

$$c = \lambda \times f \quad \text{...(15.63)}$$

The emission of thermal radiation (range lies between wavelength of 10^{-7} m and 10^{-4} m) *depends upon the nature, temperature and state of the emitting surface.* However, with *gases the dependence is also upon the thickness of the emitting layer and the gas pressure.*

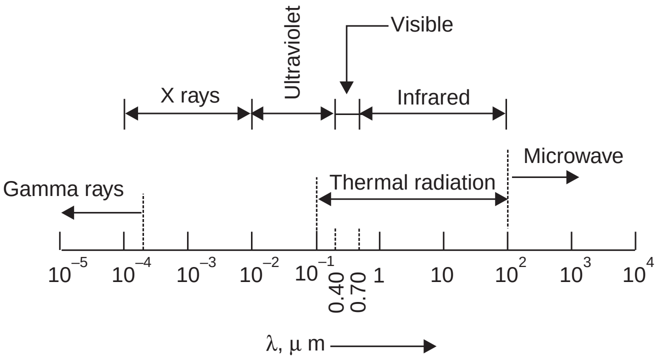


Fig. 15.45. Spectrum of electromagnetic radiation.

Thermal radiations exhibit characteristics similar to those of *visible light,* and *follow optical laws.* These can be *reflected, refracted* and are *subject to scattering and absorption when they pass through a media.* They get *polarised* and *weakened* in strength with inverse square of radial distance from the radiating surface.

15.5.2. Surface Emission Properties

The rate of emission of radiation by a body depends upon the following *factors* :

(*i*) The temperature of the surface,

(*ii*) The nature of the surface, and

(*iii*) The wavelength or frequency of radiation.

The parameters which deal with the surface emission properties are given below :

(*i*) **Total emissive power (E).** The *emissive power* is defined as the *total amount of radiation emitted by a body per unit area and time.* It is expressed in W/m^2. The *emissive power of a black body,* according to Stefan- Boltzmann, is *proportional to absolute temperature to the fourth power.*

$$E_b = \sigma T^4 \text{ W/m}^2 \quad \text{...(15.64)}$$

$$E_b = \sigma A T^4 \text{ W} \quad \text{...[15.64 (a)]}$$

where σ = Stefan-Boltzmann constant = 5.67×10^{-8} W/m^2 K^4.

(*ii*) **Monochromatic (spectral) emissive power (E_1).** It is often necessary to determine the spectral distribution of the energy radiated by a surface. At any given temperature the amount of radiation emitted per unit wavelength varies at different wavelengths. For this purpose the *monochromatic emissive power* E_λ of the surface is used. It is defined as *the rate of energy radiated per unit area of the surface per unit wavelength*.

The total emissive power is given by

$$E = \int_o^\infty E_\lambda \, d\lambda \text{ W/m}^2 \qquad ...(15.65)$$

(*iii*) **Emission from real surface-emissivity.** The emissive power from a real surface is given by

$$E = \varepsilon \, \sigma \, AT^4 \text{ W} \qquad ...(15.66)$$

where ε = emissivity of the material.

Emissivity(ε). It is defined as the *ability of the surface of a body to radiate heat.* It is also defined as the *ratio of the emissive power of any body to the emissive power of a black body of equal temperature* $\left(i.e., \varepsilon = \dfrac{E}{E_b}\right)$. Its values varies for different substances ranging from 0 to 1. For a black body $\varepsilon = 1$, for a white body surface $\varepsilon = 0$ and for gray bodies it lies between 0 and 1. It may vary with temperature or wavelength.

(*iv*) *Intensity of radiation.*

(*v*) *Radiation density and pressure.*

(*vi*) *Radiosity* (*J*). It refers to *all of the radiant energy leaving a surface.*

(*vii*) *Interrelationship between surface emission and irradiation properties.*

15.5.3. Absorptivity, Reflectivity and Transmissivity

When incident radiation also called **irradiation** (defined as the *total incident radiation on a surface from all directions per unit time and per unit area of surface),* expressed in W/m^2 and denoted by (G) impinges on a surface, three things happens ; a part is *reflected* back (G_r), a part is *transmitted* through (G_t), and the remainder is *absorbed* (G_a) depending upon the characteristics of the body, as shown in Fig. 15.46.

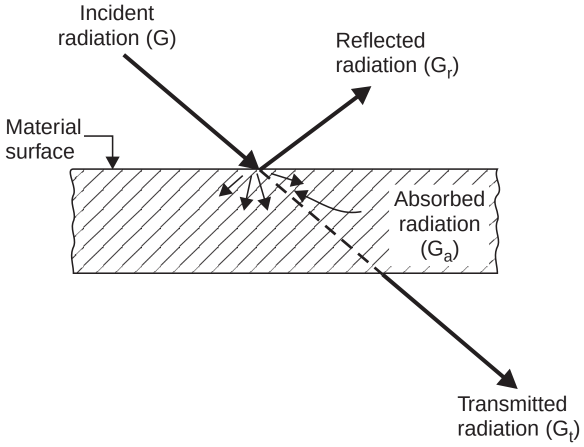


Fig. 15.46. Absorption, reflection and transmission of radiation.

By the conservation of energy principle,

$$G_a + G_r + G_t = G$$

Dividing both sides by G, we get

$$\frac{G_a}{G}+\frac{G_r}{G}+\frac{G_t}{G}=\frac{G}{G}$$

$$a+\rho+\tau=1 \quad ...(15.67)$$

where, α = *Absorptivity* (or fraction of incident radiation absorbed),

ρ = *Reflectivity* (or fraction of incident radiation reflected), and

τ = *Transmittivity* (or fraction of incident radiation transmitted).

When the incident radiation is absorbed, it is converted into internal energy.

Black body. For perfectly absorbing body, $\alpha = 1$, $\rho = 0$, $\tau = 0$. Such a body is called a '*black body*' (*i.e., a black body is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it*). In practice, a perfect black body ($\alpha = 1$) does not exist. However its concept is very important.

Opaque body. When no incident radiation is transmitted through the body, it is called an '*opaque body*'.

For the opaque body $\tau = 0$, and eqn. (15.67) reduces to

$$\alpha+\rho=1 \quad ...(15.68)$$

Solids generally do not transmit unless the material is of very thin section. Metals absorb radiation within a fraction of a micrometre, and insulators within a fraction of millimetre. Glasses and liquids are, therefore, generally considered as opaque.

White body. If all the incident radiation falling on the body are reflected, it is called a '*white body*'.

For a white body, $\rho = 1$, $\alpha = 0$ and $\tau = 0$.

Gases such as hydrogen, oxygen and nitrogen (and their mixture such as air) have a transmissivity of practically *unity*.

Reflections are of two types : Refer Fig. 15.47.

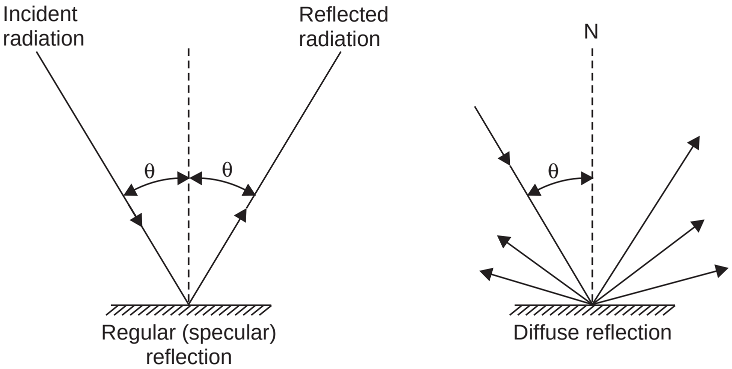


Fig. 15.47. Regular and diffuse reflections.

1. Regular (specular) reflection 2. Diffuse reflection.

Regular reflection implies that angle between the reflected beam and the normal to the surface *equals* the angle made by the incident radiation with the same normal. Reflection from highly polished and smooth surfaces approaches specular characteristics.

In a *diffused reflection,* the incident beam is reflected in *all directions*. Most of the engineering materials have rough surfaces, and these rough surfaces give diffused reflections.

Gray body. If the radiative properties, α, ρ τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called *gray body*. A *gray body* is also defined as one *whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation* [$\alpha = (\alpha)_\lambda$ = constant.]

A *coloured body* is one whose absorptivity of a surface *varies* with the wavelength of radiation [$\alpha \neq (\alpha)_\lambda$].

15.5.4. Concept of a Black Body

A black body is an object *that absorbs all the radiant energy reaching its surface* (for a black body $\alpha = 1$, $\rho = 0$, $\tau = 0$). No actual body is perfectly black ; the concept of a black body is an idealization with which the radiation characteristics of real bodies can be conveniently compared.

A *black body* has the following *properties* :

(*i*) It absorbs all the incident radiation falling on it and does not transmit or reflect regardless of wavelength and direction.

(*ii*) It emits maximum amount of thermal radiations at all wavelengths at any specified temperature.

(*iii*) It is a *diffuse emitter* (*i.e.,* the radiation emitted by a black body is independent of direction).

Consider a hollow enclosure with a very small hole for the passage of incident radiation as shown in Fig. 15.48. Incident radiant energy passes through the small opening ; some of this energy is absorbed by the inside surface and some is reflected. However, most of this energy is absorbed on a second incidence. Again, a small fraction is reflected. After a number of such reflections the amount unabsorbed is exceedingly small and very little of the original incident energy is reflected back out of the opening. A small hole leading into a cavity (Hohlraum) thus acts very nearly as a black body because all the radiant energy entering through it gets absorbed.

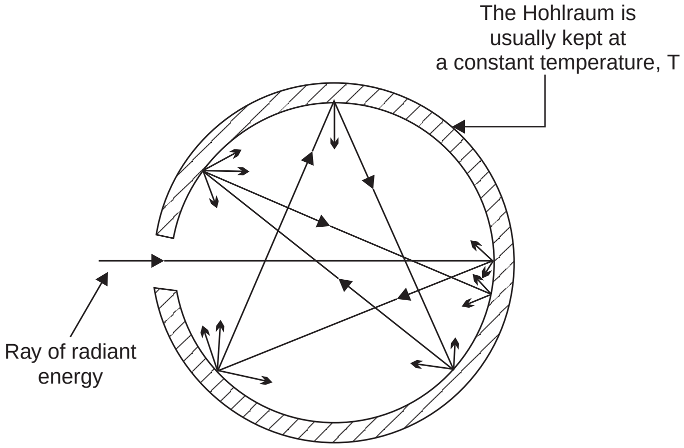


Fig. 15.48. Concept of a black body.

Isothermal furnaces, with small apertures, approximate a black body and are frequently used to *calibrate* heat flux gauges, thermometers and other radiometric devices.

15.5.5. The Stefan-Boltzmann Law

The law states that *the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.*

i.e., $$E_b = \sigma T^4$$

where, E_b = Emissive power of a black body, and ...(15.69)

σ = Stefan-Boltzmann constant

= 5.67×10^{-8} W/m^2 K^4.

Equation (15.69) can be rewritten as :

$$E_b = 5.67 \left(\frac{T}{100}\right)^4 \qquad \text{...(15.70)}$$

15.5.6. Kirchhoff's Law

The law states that *at any temperature the ratio of total emissive power E to the total absorptivity* α *is a constant for all substances which are in thermal equilibrium with their environment.*

Let us consider a large radiating body of surface area A which encloses a small body (1) of surface area A_1 (as shown in Fig. 15.49). Let the energy fall on the unit surface of the body at the rate E_b. Of this energy, generally, a fraction α, will be absorbed by the small body. Thus this energy absorbed by the small body (1) is $\alpha_1 A_1 E_b$, in which α_1 is the absorptivity of the body. When thermal equilibrium is attained, the *energy absorbed* by the body (1) must be equal to the *energy emitted*, say, E_1 per unit surface. Thus, at equilibrium, we may write

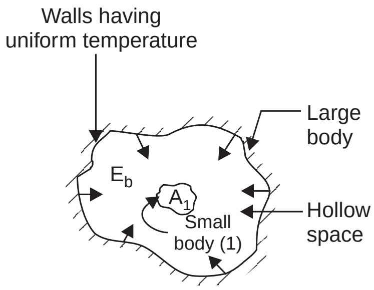


Fig. 15.49. Derivation of Kirchhoff's law.

$$A_1E_1 = \alpha_1 A_1 E_b \qquad \text{...(15.71)}$$

Now we remove body (1) and replace it by body (2) having absorptivity α_2. The radiative energy impinging on the surface of this body is again E_b. In this case, we may write

$$A_2E_2 = \alpha_2 A_2 E_b \qquad \text{...(15.72)}$$

By considering generality of bodies, we obtain

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha} \qquad \text{...(15.73)}$$

Also, as per definition of emissivity ε, we have

$$\varepsilon = \frac{E}{E_b}$$

or

$$E_b = \frac{E}{\varepsilon} \qquad \text{...(15.74)}$$

By comparing eqns. (15.73) and (15.74), we obtain

$$\varepsilon = \alpha \qquad \text{...(15.75)}$$

(α is always *smaller* than 1. Therefore, the emissive power E is always *smaller* than the emissive power of a black body at equal temperature).

Thus, kirchhoff's law also states that *the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings*.

15.5.7. Planck's Law

In 1900 Max Planck showed by quantum arguments that the spectral distribution of the radiation intensity of a black body is given by

$$(E_\lambda)_b = \frac{2\pi c^2 h \lambda^{-5}}{\exp\left(\frac{ch}{\lambda k T}\right) - 1} \qquad \text{...(Planck's law)} \qquad \text{...(15.76)}$$

where, $(E_\lambda)_b$ = Monochromatic (single wavelength) emissive power of a black body,

c = Velocity of light in vacuum, $2.998 \times 10^8 \simeq 3 \times 10^8$ m/s,

h = Planck's constant = 6.625×10^{-34} js,

λ = Wavelength, μm,

k = Boltzmann constant = 1.3805×10^{-23} J/K, and

T = Absolute temperature, K.

Hence the unit of $(E_\lambda)_b$ is W/m^2. μm

Quite often the Planck's law is written as

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left[\frac{C_2}{\lambda T}\right] - 1} \qquad \text{...(15.77)}$$

where, $C_1 = 2\pi c^2 h = 3.742 \times 10^8$ W.μm^4/m^2 ;

$$C_2 = \frac{ch}{k} = 1.4388 \times 10^4 \text{ μmK}$$

Equation (15.76) is of great importance as it provides quantitative results for the radiation from a black body.

The quantity $(E_\lambda)_b$, *monochromatic emissive power*, is defined as the *energy emitted by the black surface in all directions at a given wavelength* λ *per unit wavelength interval around* λ ; that is, the rate of energy emission in the interval $d\lambda$ is equal to $(E_\lambda)_b \; d\lambda$. The total emissive power and monochromatic emissive power are related by the equation

$$E_b = \int_0^\infty (E_\lambda)_b \, d\lambda \qquad \text{...(15.78)}$$

A plot of $(E_\lambda)_b$ as a function of temperature and wavelength is given in Fig. 15.50.

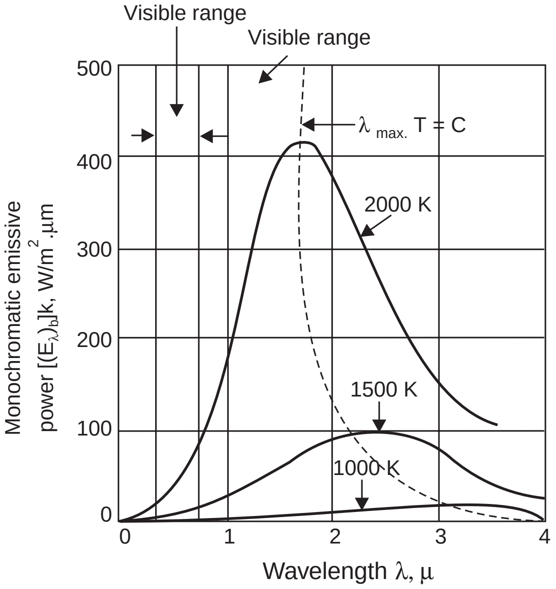


Fig. 15.50. Variation of emissive power with wavelength.

The plot shows that the following distinct characteristics of black body radiations :

1. *The energy emitted at all wavelengths increases with rise in temperature*.
2. *The peak spectral emissive power shifts towards a smaller wavelength at higher temperatures.* This shift signifies that at elevated temperature, much of the energy is emitted in a narrow band ranging on both sides of wavelength at which the monochromatic power is maximum.
3. The *area* under the monochromatic emissive power versus wavelength, at any temperature, gives the *rate of radiant energy emitted* within the wavelength interval $d\lambda$. Thus,

$$dE_b = (E_\lambda)_b \; d\lambda$$

or

$$E_b = \int_{\lambda = 0}^{\lambda = \infty} (E_\lambda)_b \; d\lambda \qquad \text{... over the entire range of length.}$$

The integral represents the total emissive power per unit area radiated from a black body.

15.5.8. Wien's Displacement Law

In 1893 Wien established a relationship between the temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs. A peak monochromatic emissive power occurs at a particular wavelength. Wien's displacement law states that the *product of* λ_{max} *and T is constant, i.e.*,

$$\lambda_{\text{max}} \, T = \text{constant} \qquad \text{...(15.79)}$$

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$$

$(E_\lambda)_b$ becomes maximum (if T remains constant) when

$$\frac{d(E_\lambda)_b}{d\lambda} = 0$$

i.e.,

$$\frac{d(E_\lambda)_b}{d\lambda} = \frac{d}{d\lambda}\left[\frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}\right] = 0$$

or

$$\frac{\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right](-5\, C_1\, \lambda^{-6}) - C_1\, \lambda^{-5}\left\{\exp\left(\frac{C_2}{\lambda T}\right)\frac{C_2}{T}\left(-\frac{1}{\lambda^2}\right)\right\}}{\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]^2} = 0$$

or

$$-5\, C_1\, \lambda^{-6} \exp\left(\frac{C_2}{\lambda T}\right) + 5\, C_1\, \lambda^{-6} + C_1\, C_2\, \lambda^{-5}\; \frac{1}{\lambda^2 T}\; \exp\left(\frac{C_2}{\lambda T}\right) = 0$$

Dividing both side by $5C_1\, \lambda^{-6}$, we get

$$-\exp\left(\frac{C_2}{\lambda T}\right) + 1 + \frac{1}{5}\, C_2\, \frac{1}{\lambda T}\, \exp\left(\frac{C_2}{\lambda T}\right) = 0$$

Solving this equation by trial and error method, we get

$$\frac{C_2}{\lambda T} = \frac{C_2}{\lambda_{\text{max}} T} = 4.965$$

$$\therefore \qquad \lambda_{\text{max}}\; T = \frac{C_2}{4.965} = \frac{1.439 \times 10^4}{4.965}\; \mu\text{mK} = 2898\; \mu\text{mK}\; (\simeq 2900\; \mu\text{mK})$$

i.e.,

$$\lambda_{\text{max}}\; T = 2898\; \mu\text{mK} \qquad \text{...(15.80)}$$

This law holds true for more *real substances* ; there is however some deviation in the case of a metallic conductor where the product ($\lambda_{\max}.T$) is found to vary with absolute temperature. It is used in *predicting a very high temperature through measurement of wavelength*.

A *combination of Planck's law and Wien's displacement law yields the condition for the maximum monochromatic emissive power for a black body*.

$$(E_{\lambda b})_{\max} = \frac{C_1\,(\lambda_{\max})^{-5}}{\exp\left[\dfrac{C_2}{\lambda_{\max}\,T}\right]-1} = \frac{0.374\times 10^{-15}\left(\dfrac{2.898\times 10^{-3}}{T}\right)^{-5}}{\exp\left[\dfrac{1.4388\times 10^{-2}}{2.898\times 10^{-3}}\right]-1}$$

or $$(E_{\lambda b})_{\max} = 1.285\times 10^{-5}\,T^5 \text{ W/m}^2 \text{ per metre wavelength} \quad ...(15.81)$$

15.5.9. Intensity of Radiation and Lambert's Consine Law

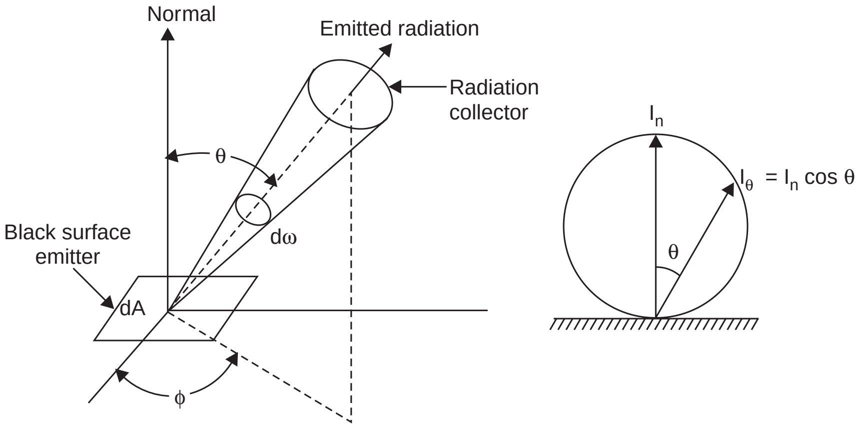
15.5.9.1. Intensity of Radiation

When a surface element emits radiation, all of it will be intercepted by a hemispherical surface placed over the element. The **intensity of radiation** (I) is defined as the *rate of energy leaving a surface in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space*. A **solid angle** is defined as a *portion of the space inside a sphere enclosed by a conical surface with the vertex of the cone at the centre of the sphere.* It is measured by the ratio of the spherical surface enclosed by the cone to the square of the radius of the sphere ; it unit is steradian (sr). The solid angle subtended by the complete hemisphere is given by : $\frac{2\pi r^2}{r^2} = 2\pi$.

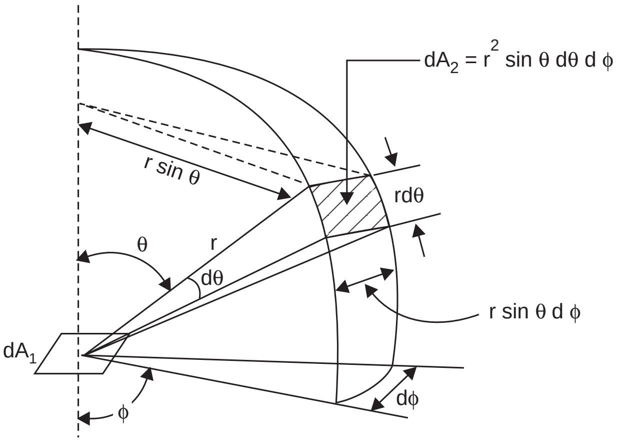
Fig. 15.51 (a) shows a small black surface of area dA (emitter) emitting radiation in different directions. A black body radiation collector through which the radiation pass is located at an angular position characterised by *zenith angle* θ towards the surface normal and angle ϕ of a spherical coordinate system. Further the collector subtends a solid angle $d\omega$ when viewed from a point on the emitter.

Let us now consider radiation from the elementary area dA_1 at the centre of a sphere as shown in Fig. 15.51. Suppose this radiation is absorbed by a second elemental area dA_2, a portion of the hemispherical surface.

The projected area of dA_1 on a plane perpendicular to the line joining dA_1 and $dA_2 = dA_1 \cos\theta$.



(a) Special distribution of radiations emitted from a surface.



(*b*) Illustration for evaluating area dA_2

Fig. 15.51. Radiation from an elementary surface.

The solid angle subtended by $dA_2 = \dfrac{dA_2}{r^2}$

$\therefore$ The intensity of radiation, $$I = \frac{dQ_{1-2}}{dA_1 \cos\theta \times \dfrac{dA_2}{r^2}} \qquad \text{...(15.82)}$$

where dQ_{1-2} is the rate of radiation heat transfer from dA_1 to dA_2.

It is evident from Fig. 15.51 (*b*) that,

$$dA_2 = r\, d\theta\, (r \sin\theta\, d\phi)$$

or $$dA_2 = r^2 \sin\theta . d\theta . d\phi \qquad \text{...(15.83)}$$

From eqns. (15.82) and (15.83), we obtain

$$dQ_{1-2} = I\, dA_1 \,.\, \sin\theta \,.\, \cos\theta \,.\, d\theta \,.\, d\phi$$

The total radiation through the hemisphere is given by

$$Q = I\, dA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{\phi=0}^{\phi=2\pi} \sin\theta \cos\theta\, d\theta\, d\phi$$

$$= 2\pi\, I\, dA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta \cos\theta\, d\theta$$

$$= \pi\, I\, dA_1 \int_{\theta=0}^{\frac{\pi}{2}} 2\sin\theta \cos\theta\, d\theta$$

$$= \pi\, I\, dA_1 \int_{\theta=0}^{\frac{\pi}{2}} \sin 2\theta\, d\theta$$

or $$Q = \pi\, I\, dA_1 \qquad \text{...(15.84)}$$

Also $$Q = E . dA_1$$

$\therefore$ $$E\, dA_1 = \pi\, I\, dA_1$$

or $$E = \pi I$$

i.e., The *total emissive power of a diffuse surface is equal to* π *times its intensity of radiation.*

15.5.9.2. Lambert's Cosine Law

The law states that the *total emissive power* E_θ *from a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission.* The angle of emission θ is the angle subtended by the normal to the radiating surface and the direction vector of emission of the receiving surface. If E_n be the total emissive power of the radiating surface in the direction of its normal, then

$$E_\theta = E_n \cos\theta \qquad \text{...(15.85)}$$

The above equation is true only for diffuse radiation surface. The radiation emanating from a point on a surface is termed diffused if the intensity, I is constant. This law is also known as *Lambert's law of diffuse radiation*.

Example 15.23. *The effective temperature of a body having an area of 0.12 m² is 527°C. Calculate the following* :

(*i*) *The total rate of energy emission.*

(*ii*) *The intensity of normal radiation, and*

(*iii*) *The wavelength of maximum monochromatic emissive power.*

Solution. *Given* : $A = 0.12\ \text{m}^2$; $T = 527 + 273 = 800$ K

(*i*) **The total rate of energy emission, E_b :**

$$E_b = \sigma A T^4 \text{ W (watts)} \qquad \text{...[Eqn. 15.64 }(a)]$$

$$= 5.67 \times 10^{-8} \times 0.12 \times (800)^4 = 5.67 \times 0.12 \times \left(\frac{800}{100}\right)^4 = \mathbf{2786.9\ W.} \quad \textbf{(Ans.)}$$

(*ii*) **The intensity of normal radiation, I_{bn} :**

$$I_{bn} = \frac{E_b}{\pi}, \text{ where } E_b \text{ is in W/m}^2\text{ K}^4$$

$$= \frac{\sigma T^4}{\pi} = \frac{5.67 \times \left(\frac{800}{100}\right)^4}{\pi} = \mathbf{7392.5\ W/m^2.sr.} \quad \textbf{(Ans.)}$$

(*iii*) **The wavelength of maximum monochromatic emissive power, λ_{max}** :

From Wien's displacement law,

$$\lambda_{max}.T = 2898\ \mu\text{m K} \qquad \text{...[Eqn. 15.80]}$$

or

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{800} = \mathbf{3.622\ \mu m.} \quad \textbf{(Ans.)}$$

Example 15.24. *Assuming the sun to be a black body emitting radiation with maximum intensity at λ = 0.49 μm, calculate the following* :

(*i*) *The surface temperature of the sun, and*

(*ii*) *The heat flux at surface of the sun.*

Solution. Given : $\lambda_{max} = 0.49\ \mu$m

(*i*) **The surface temperature of the sun, T :**

According to Wien's displacement law,

$$\lambda_{max} \,.\, T = 2898\ \mu\text{mK}$$

$$\therefore \qquad T = \frac{2898}{\lambda_{max}} = \frac{2898}{0.48} = \mathbf{5914\ K.} \quad \textbf{(Ans.)}$$

(*ii*) **The heat flux at the surface of the sun, $(E)_{sun}$:**

$$(E)_{sun} = \sigma T^4 = 5.67 \times 10^{-8}\, T^4 = 5.67 \left(\frac{T}{100}\right)^4$$

$$= 5.67 \times \left(\frac{5914}{100}\right)^4 = \mathbf{6.936 \times 10^7\ W/m^2}$$

Example 15.25. *Calculate the following for an industrial furnance in the form of a black body and emitting radiation of 2500°C :*

(*i*) *Monochromatic emissive power at 1.2 μm length,*

(*ii*) *Wavelength at which the emission is maximum,*

(*iii*) *Maximum emissive power,*

(*iv*) *Total emissive power, and*

(*v*) *Total emissive power of the furnance if it is assumed as a real surface with emissivity equal to 0.9.*

Solution. *Given :* $T = 2500 + 273 = 2773$ K ; $\lambda = 12$ μm, $\varepsilon = 0.9$

(*i*) **Monochromatic emissive power at 1.2 μm length, $(\mathbf{E}_\lambda)_\mathbf{b}$:**

According to Planck's law, $$(E_\lambda)_b = \frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1} \qquad \text{...[Eqn. (15.77)]}$$

where, $C_1 = 3.742 \times 10^8$ W. μm^4/m^2 $= 0.3742 \times 10^{-15}$ W.m^4/m^2 and

$C_2 = 1.4388 \times 10^{-2}$ mK

Substituting the values, we get

$$(E_\lambda)_b = \frac{0.3742 \times 10^{-15} \times (1.2 \times 10^{-6})^{-5}}{\exp\left(\frac{1.4388 \times 10^{-4}}{1.2 \times 10^{-6} \times 2773}\right) - 1} = \frac{1.5 \times 10^{14}}{74.48}$$

$$= \mathbf{2.014 \times 10^{12}\ W/m^2.} \quad \textbf{(Ans.)}$$

(*ii*) **Wavelength at which the emission is maximum, $\lambda_{\mathbf{max}}$:**

According to Wien's displacement law,

$$\lambda_{\max} = \frac{2898}{T} = \frac{2898}{2773} = \mathbf{1.045\ \mu m.} \quad \textbf{(Ans.)}$$

(*iii*) **Maximum emissive power, $(\mathbf{E}_{\lambda \mathbf{b}})_{\mathbf{max}}$:**

$$(E_{\lambda b})_{\max} = 1.285 \times 10^{-5}\, T^5 \text{ W/m}^2 \text{ per metre length} \qquad \text{...[Eqn. (15.81)]}$$

$$= 1.285 \times 10^{-5} \times (2773)^5 = \mathbf{2.1 \times 10^{12}\ W/m^2\ per\ metre\ length.} \quad \textbf{(Ans.)}$$

[**Note.** At high temperature the difference between $(E_\lambda)_b$ and $(E_{\lambda b})_{\max}$ is very small].

(*iv*) **Total emissive power, $\mathbf{E_b}$:**

$$E_b = \sigma T^4 = 5.67 \times 10^{-8}\,(2273)^4 = 5.67\left(\frac{2773}{100}\right)^4 = \mathbf{3.352 \times 10^6\ W/m^2}$$

(*v*) **Total emissive power, E with emisivity** (ε) = **0.9**

$$E = \varepsilon\,\sigma T^4 = 0.9 \times 5.67\left(\frac{2773}{100}\right)^4 = \mathbf{3.017 \times 10^6\ W/m^2.} \quad \textbf{(Ans.)}$$

15.5.10. Radiation Exchange Between Black Bodies Separated by a Non-absorbing Medium

Refer Fig. 15.52. Let us consider heat exchange between elementary areas dA_1 and dA_2 of two black radiating bodies, separated by a non-absorbing medium, and having areas A_1 and A_2 and temperatures T_1 and T_2 respectively. The elemenary areas are at a distance r apart and the normals to these areas make angles θ_1 and θ_2 with the line joining them. Each elemental area subtends a solid angle at the centre of the other. Let $d\omega_1$ be subtended at dA_1 by dA_2 and $d\omega_2$ subtended at dA_2 by dA_1. Then

$$d\omega_1 = \frac{dA_2 \cos\theta_2}{r^2}, \text{ and } d\omega_2 = \frac{dA_1 \cos\theta_1}{r^2} \qquad ...(15.86)$$

The energy leaving dA_1 in the direction given by the angle per unit solid angle = $I_{b_1}\ dA_1 \cos\theta_1$.

where, I_b = Black body intensity, and

$dA_1 \cos\ \theta_1$ = Projection of dA_1 on the line between the centres.

The rate of radiant energy leaving $dA_1$and striking on dA_2 is given by

$$dQ_{1-2} = I_{b_1}\ dA_1 \cos\theta_1 .\ d\omega_1$$

$$= \frac{I_{b_1} \cos\theta_1 \cos\theta_2\ dA_1\ dA_2}{r^2} \qquad ...(15.87)$$

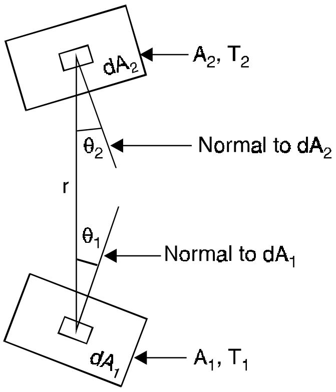


Fig. 15.52. Radiation heat exchange between two black surfaces.

This energy is absorbed by the elementary area dA_2, since both the surfaces are black. The quantity of energy radiated by dA_2 and absorbed by dA_1 is given by

$$dQ_{2-1} = \frac{I_{b_2} \cos\theta_2 \cos\theta_1\ dA_2\ dA_1}{r^2} \qquad ...(15.88)$$

The net rate of transfer of energy between dA_1 and dA_2 is

$$dQ_{12} = dQ_{1-2} - dQ_{2-1}$$

$$= \frac{dA_1\ dA_2 \cos\theta_1 \cos\theta_2}{r^2} \left(I_{b_1} - I_{b_2}\right)$$

But $$I_{b_1} = \frac{E_{b_1}}{\pi} \text{ and } I_{b_2} = \frac{E_{b_2}}{\pi} \qquad ...[\text{Eqn. } (15.88)]$$

$\therefore$ $$dQ_{12} = \frac{dA_1\ dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2} (E_{b_1} - E_{b_2}) \qquad ...(15.89)$$

or $$dQ_{12} = \frac{\sigma dA_1\ dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2} (T_1^2 - T_2^4) \qquad ...(15.90)$$

The rate of total net heat transfer for the total areas A_1 and A_2 is given by

$$Q_{12} = \int dQ_{12} = \sigma (T_1^2 - T_2^4) \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\ dA_1\ dA_2}{r^2} \qquad ...(15.91)$$

The rate of radiant energy emitted by A_1 that falls on A_2, from eqn. (15.87), is given by

$$Q_{1-2} = I_{b_1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\ dA_1\ dA_2}{r^2}$$

$$Q_{1-2} = \sigma T_1^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\ dA_1\ dA_2}{\pi r^2} \qquad ...(15.92)$$

The rate of total energy radiated by A_1 is given by,

$$Q_1 = A_1\ \sigma\ T_1^4$$

Hence the fraction of the rate of energy leaving area A_1 and impinging on area A_2 is given by

$$\frac{Q_{1-2}}{Q_1} = \frac{1}{A_1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\ dA_1\ dA_2}{\pi r^2} \qquad ...(15.93)$$

or $$\frac{Q_{1-2}}{Q_1} = F_{1-2} \qquad ...[15.93\ (a)]$$

F_{1-2} is known as '**configuration factor**' or **'surface factor'** or **'view factor'** between the two radiating surfaces and is a function of geometry only.

Thus, the ***shape factor*** may be defined as "*The fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections*."

Further, $$Q_{1-2} = F_{1-2}\, A_1\, \sigma\, T_1^4 \quad \text{...(15.94)}$$

Similarly, the rate of radiant energy by A_2 that falls on A_1, from eqn. (15.88), is given by

$$Q_{2-1} = \sigma\, T_2^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

The rate of total energy radiated by A_2 is given by

$$Q_2 = A_2\, \sigma\, T_2^4$$

Hence the fraction of the rate of energy leaving area A_2 and impinging on area A_1 is given by

$$\frac{Q_{2-1}}{Q_2} = \frac{1}{A_2} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2} \quad \text{...(15.95)}$$

or $$\frac{Q_{2-1}}{Q_2} = F_{2-1}$$

F_{2-1} is the shape factor of A_2 with respect to A_1.

$$Q_{2-1} = F_{2-1}\, A_2\, \sigma\, T_2^4 \quad \text{...(15.96)}$$

From eqns. (15.93) and (15.95), we get

$$A_1\, F_{1-2} = A_2\, F_{2-1} \quad \text{...(15.97)}$$

The above result is known as **reciprocity theorem.** It indicates that the net radiant interchange may be evaluated by computing one way configuration factor from either surface to the other. Thus the net rate of heat transfer between two surfaces A_1 and A_2 is given by

$$\begin{aligned} Q_{12} &= A_1\, F_{1-2}\, \sigma\, (T_1^4 - T_2^4) \\ &= A_2\, F_{2-1}\, \sigma\, (T_1^4 - T_2^4) \quad \text{...(15.98)} \end{aligned}$$

It may be noted that eqn. (15.98) is applicable to *black surfaces only and must not be used* for surfaces having emissivities very different from unity.

Example 15.26. *A body at 1000°C in black surroundings at 500°C has an emissivity of 0.42 at 1000°C and an emissivity of 0.72 at 500°C. Calculate the rate of heat loss by radiation per m*^{2}.

(*i*) *When the body is assumed to be grey with* $\varepsilon = 0.42$.

(*ii*) *When the body is not grey.*

Assume that the absorptivity is independent of the surface temperature.

Solution. (*i*) **When the body is grey with** ε **= 0.42** :

$$T_1 = 1000 + 273 = 1273 \text{ K}$$
$$T_2 = 500 + 273 = 773 \text{ K}$$
$$\varepsilon \text{ at } 1000°\text{C} = 0.42$$
$$\varepsilon \text{ at } 500°\text{C} = 0.72$$
$$\sigma = 5.67 \times 10^{-8}$$

Heat loss per m^2 by radiation,

$$\begin{aligned} q &= \varepsilon\sigma\, (T_1^4 - T_2^4) \\ &= 0.42 \times 5.67 \times 10^{-8}\, [(1273)^4 - 773)^4] = 54893 \text{ W} \end{aligned}$$

i.e., *Heat loss per m*^{2} *by radiation* = **54.893 kW. (Ans.)**

(*ii*) **When the body is not grey :**

Absorptivity when source is at 500°C = Emissivity when body is at 500°C

i.e., absorptivity, $\alpha = 0.72$

Then, energy emitted $= \varepsilon\sigma\, T_1^4 = 0.42 \times 5.67 \times 10^{-8} \times (1273)^4$

and, Energy absorbed $= \alpha\sigma T_2^4 = 0.72 \times 5.67 \times 10^{-8} \times (773)^4$

i.e., q = Energy emitted – Energy absorbed

$= 0.42 \times 5.67 \times 10^{-8} \times (1238)^4 - 0.72 \times 5.67 \times 10^{-8} \times (773)^4$

$= 62538 - 14576 = 47962$ W

i.e., Heat loss per m^2 by radiation = **47.962 kW. (Ans.)**

Example 15.27. *A long steel rod, 22 mm in diameter, is to be heated from 420°C to 540°C. It is placed concentrically in a long cylindrical furnace which has an inside diameter of 180 mm. The inner surface of the furnace is at a temperature of 1100°C, and has an emissivity of 0.82. If the surface of the rod has an emissivity of 0.62, find the time required for the heating operation.*

Take for steel : c = 0.67 kJ/kg K, ρ= 7845 kg/m^3.

Solution. Refer Fig. 15.53.

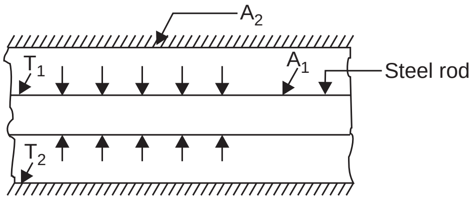


Fig. 15.53

Diameter of the steel rod

= 22 mm = 0.022 m

Inside diameter of the furnace

= 180 mm = 0.18 m

Emissivity $\varepsilon_1 = 0.62$

Emissivity $\varepsilon_2 = 0.82$

Specific heat of steel, $c = 0.67$ kJ/kg K

Density of steel, $\rho = 7845$ kg/m^3

$T_1 = 420 + 273 = 693$ K ..... 1st case

and $= 540 + 273 = 813$ K ..... 2nd case

$T_2 = 1100 + 273 = 1373$ K

The surface area of the rod, $A_1 = \pi \times 0.022 \times l$ m^2

The surface area of the furnace, $A_2 = \pi \times 0.18 \times l$ m^2

Time required for the heating operation, t_h :

Initial rate of heat absorption by radiation, when the rod is at 420°C or 693 K

$$Q_i = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{\pi \times 0.022 \times 1 \times 5.67 \times 10^{-8}\,(693^4 - 1373^4)}{\dfrac{1}{0.62} + \left(\dfrac{\pi \times 0.022 \times l}{\pi \times 0.18 \times l}\right)\left[\dfrac{1}{0.82} - 1\right]}$$

$$= \frac{-13022.5}{1.64} = -7940.5 \text{ W/m} \qquad [\because\ l = 1\,m \ldots\text{assumed}]$$

Rate of heat absorption at the end of the heating process, when the rod is at 540°C or 813 K

$$Q_e = \frac{A_2\,\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{\pi \times 0.022 \times 1 \times 5.67 \times 10^{-8}\,(813^4 - 1373^4)}{\frac{1}{0.62} + \frac{\pi \times 0.022 \times l}{\pi \times 0.18 \times l}\left[\frac{1}{0.82} - 1\right]}$$

$$= \frac{-12214.3}{1.64} = -7447.7 \text{ W/m}$$

∴ Average rate of heat absorption during the heating process

$$Q_{av.} = \frac{7940.5 + 7447.7}{2} = 7694.1 \text{ W/m}$$

Time required for heating, t_h is obtained from the equation

$$mc_p\,\Delta T = Q_{av.} \times t_h$$

$$\therefore \quad t_h = \frac{[\pi/4 \times (0.022)^2 \times 1 \times 7845] \times 0.67 \times (540 - 420) \times 1000}{7694.1}$$

$$= \mathbf{31.16\ s.} \quad \textbf{(Ans.)}$$

Example 15.28. *Calculate the heat transfer rate per m² area by radiation between the surfaces of two long cylinders having radii 100 mm and 50 mm respectively. The smaller cylinder being in the larger cylinder. The axes of the cylinders are parallel to each other and separated by a distance of 20 mm. The surfaces of inner and outer cylinders are maintained at 127°C and 27°C respectively. The emissivity of both the surfaces is 0.5.*

Assume the medium between the two cylinders is non-absorbing. **(P.U.)**

Solution. *Given* : $r_1 = 50$ mm $= 0.05$ m ; $r_2 = 100$ mm $= 0.1$ m, $T_1 = 127 + 273 = 400$ K,
$T_2 = 27 + 273 = 300$ K, $\varepsilon_1 = \varepsilon_2 = 0.5$

The heat transfer between two concentric or eccentric cylinders is given by

$$(Q_{12})_{net} = \frac{A_1\,\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

Here $F_{1-2} = 1$ and $\frac{A_1}{A_2} = \frac{2\pi r_1 L}{2\,\pi r_2 L} = \frac{r_1}{r_2}$

Substituting the values, we have

$$(Q_{12})_{net} = \frac{1 \times 5.67\left[\left(\frac{400}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right]}{\left(\frac{1-0.5}{0.5}\right) + 1\left(\frac{1-0.5}{0.5}\right) \times \frac{0.05}{0.1}} = \frac{992.25}{2.5} = \mathbf{396.9\ W/m^2}. \quad \textbf{(Ans.)}$$

Example 15.29. *Three thin walled infinitely long hollow cylinders of radii 5 cm, 10 cm and 15 cm are arranged concentrically as shown in Fig. 15.54. $T_1 = 1000$ K and $T_3 = 300$ K.*

Assuming $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.05$ and vacuum in the spaces between the cylinders, calculate the steady state temperature of cylinder surface 2 and heat flow per m² area of cylinder 1. **(P.U.)**

Solution. *Given* : $r_1 = 5$ cm ; $r_2 = 10$ cm ; $r_3 = 15$ cm ; $T_1 = 1000$ K ; $T_3 = 300$ K
$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.05.$

For steady state heat flow,

$$Q_{12} = Q_{23}$$

or

$$\frac{A_1\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}} = \frac{A_2\,\sigma(T_2^4 - T_3^4)}{\left(\frac{1-\varepsilon_2}{\varepsilon_2}\right) + \frac{1}{F_{2-3}} + \left(\frac{1-\varepsilon_3}{\varepsilon_3}\right)\frac{A_2}{A_3}}$$

In this case $F_{1-2} = F_{2-3} = 1$; and

$$\frac{A_1}{A_2} = \frac{r_1}{r_2} = \frac{5}{10} = 0.5$$

$$\frac{A_2}{A_3} = \frac{r_2}{r_3} = \frac{10}{15} = 0.67$$

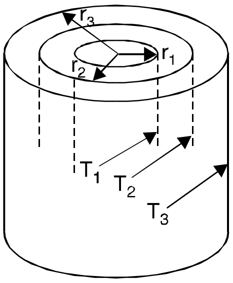


Fig. 15.54

$$\therefore \quad \frac{2\pi r_1\, L\left[\left(\frac{1000}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{\left(\frac{1-0.05}{0.05}\right) + 1 + \left(\frac{1-0.05}{0.05}\right) \times 0.5}$$

$$\frac{2\pi r_2\, L\left[\left(\frac{T_2}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right]}{\left(\frac{1-0.05}{0.05}\right) + 1 + \left(\frac{1-0.05}{0.05}\right) \times 0.67}$$

or $$\frac{0.05(10000 - x^4)}{29.4} = \frac{0.1(x^4 - 81)}{32.73}$$

or $$(1000 - x^4) = \frac{29.5 \times 0.1}{32.73 \times 0.05}\,(x^4 - 81) = 1.8(x^4 - 81)$$

or $$2.8x^4 = 10000 - 145.8 = 9854.2$$

or $$x = \left(\frac{9854.2}{2.8}\right)^{1/4} = 7.7$$

or $$\frac{T_2}{100} = 7.7 \text{ or } T_2 = 770 \text{ K}$$

∴ Heat flow per m^2 area of cylinder 1,

$$Q_{12} = \frac{A_1\, \sigma(T_1^{\,4} - T_2^{\,4})}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + 1 + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

$$= \frac{1 \times 5.67\left[\left(\frac{1000}{100}\right)^4 - \left(\frac{770}{100}\right)^4\right]}{\left(\frac{1-0.05}{0.05}\right) + 1 + \left(\frac{1-0.05}{0.05}\right) \times 0.5}$$

$$= \frac{5.67 \times (10000 - 3515.3)}{29.5} = \textbf{1246.4 W.} \quad \textbf{(Ans.)}$$

Example 15.30. *Two concentric spheres 210 mm and 300 mm diameters with the space between them evacuated are to be used to store liquid air (– 153°C) in a room at 27°C. The surfaces of the spheres are flushed with aluminium (ε = 0.03) and latent heat of vaporization of liquid air is 209.35 kJ/kg. Calculate the rate of evaporation of liquid air.* **(M.U.)**

Solution. *Given* : $r_1 = \frac{210}{2} = 105$ mm $= 0.105$ m ; $r_2 = \frac{300}{2} = 150$ mm $= 0.15$ m ;

$T_1 = -153 + 273 = 120$ K ; $T_2 = 27 + 273 = 300$ K ; $\varepsilon_1 = \varepsilon_2 = 0.03$, $h_{fg} = 209.35$ kJ/kg.

Rate of evaporation of liquid air :

The heat flow from the inner sphere surface to outer sphere surface is given by,

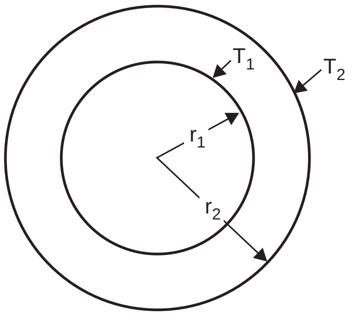


Fig. 15.55

$$Q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

$$= \frac{4\pi r_1^2\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + 1 + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{r_1^2}{r_2^2}}$$

$$= \frac{4\pi \times (0.105)^2 \times 5.67\left[\left(\frac{120}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right]}{\left(\frac{1-0.03}{0.03}\right) + 1 + \left(\frac{1-0.03}{0.03}\right) \times \left(\frac{0.105}{0.15}\right)^2}$$

$$= \frac{0.7855\,(2.07 - 81)}{32.33 + 1 + 15.84} = \frac{-61.99}{49.17} = -1.26 \text{ W}$$

– ve sign indicates that heat is gained by the surface 1, means, heat is flowing from outside surface to inside surface.

$$\therefore \quad \text{The rate of evaporation} = \frac{1.26 \times 3600}{209.35 \times 1000} = \mathbf{0.0217\ kg/h.\ (Ans.)}$$

Example 15.31. *Liquid oxygen (boiling temperature = – 182°C) is to be stored in spherical container of 30 cm diameter. The system is insulated by an evacuated space between inner sphere and surrounding 45 cm inner diameter concentric sphere. For both spheres ε = 0.03 and temperature of the outer sphere is 30°C. Estimate the rate of heat flow by radiation to the oxygen in the container.*

Solution. *Given* : $T_1 = -182 + 273 = 91$ K, $T_2 = 30 + 273 = 303$ K, $\varepsilon_1 = \varepsilon_2 = 0.03$
$d_1 = 30$ cm $= 0.3$ m, $d_2 = 45$ cm $= 0.45$ m.

Rate of heat flow, $\mathbf{Q_{12}}$:

The heat flow between the two concentric spheres by radiation is given by

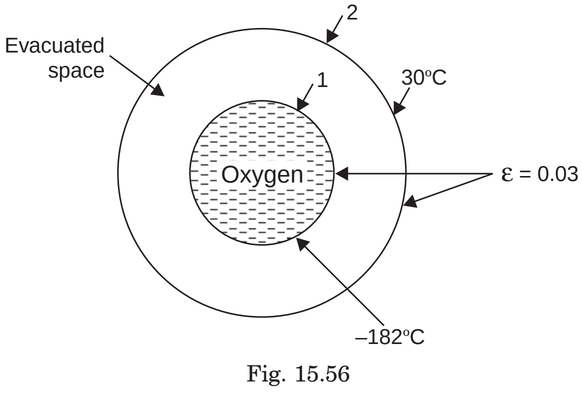


Fig. 15.56

$$Q_{12} = \frac{A_1\,\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{1-2}}\left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

For concentric spheres

$F_{1-2} = 1$

and $$\frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{0.3}{0.45}\right)^2 = 0.4444$$

$$A_1 = 4\pi\, r_1^2 = 4\pi \times \left(\frac{0.3}{2}\right)^2 = 0.283 \text{ m}^2$$

Now substituting the values in the above equation, we get

$$Q_{12} = \frac{0.283 \times 5.67 \left[\left(\frac{91}{100} \right)^4 - \left(\frac{303}{100} \right)^4 \right]}{\left(\frac{1 - 0.03}{0.03} \right) + 1 + \left(\frac{1 - 0.03}{0.03} \right) \times 0.4444}$$

$$= \frac{0.283 \times 5.67 (0.686 - 84.289)}{32.33 + 1 + 14.37} = \mathbf{-\,2.81\ W}$$

– ve sign shows *heat flows from outside to inside.* **(Ans.)**

Example 15.32 (Radiation shield). *The large parallel planes with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction when a polished aluminium shield of emissivity 0.04 is placed between them. Use the method of electrical analogy.*

Solution. *Given :* $\varepsilon_1 = 0.3$; $\varepsilon_2 = 0.8$; $\varepsilon_3 = 0.04$

Consider all resistances (surface resistances and space resistances) per unit surface area.

For steady state heat flow,

$$\frac{E_{b_1} - E_{b_3}}{\left(\frac{1 - \varepsilon_1}{\varepsilon_1} \right) + 1 + \left(\frac{1 - \varepsilon_3}{\varepsilon_3} \right)} = \frac{E_{b_3} - E_{b_2}}{\left(\frac{1 - \varepsilon_3}{\varepsilon_3} \right) + 1 + \left(\frac{1 - \varepsilon_2}{\varepsilon_2} \right)}$$

$$[\because\ A_1 = A_2 = A_3 = 1\,\text{m}^2 \text{ and } F_{1-3}, F_{3-2} = 1]$$

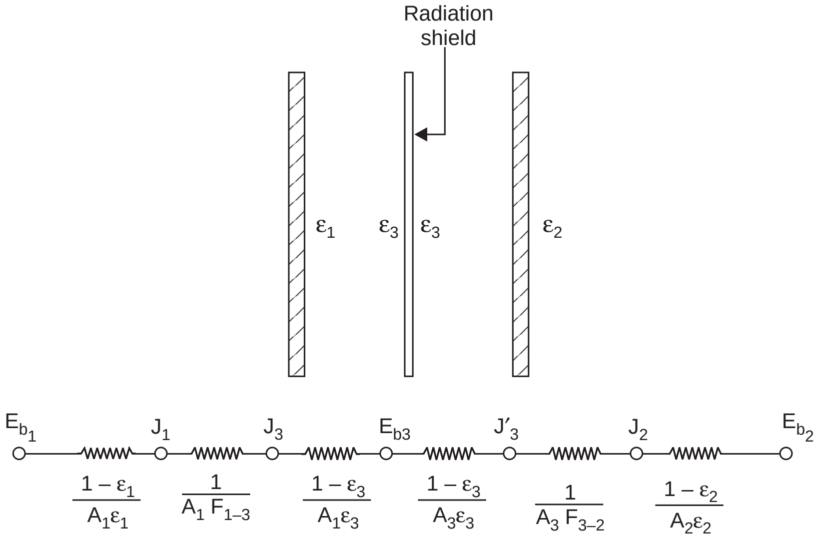


Fig. 15.57

or $$\frac{\sigma (T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma (T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

or $$\frac{T_1^4 - T_3^4}{\frac{1}{0.3} + \frac{1}{0.04} - 1} = \frac{T_3^4 - T_2^4}{\frac{1}{0.04} + \frac{1}{0.8} - 1}$$

or $$\frac{T_1^4 - T_3^4}{27.33} = \frac{T_3^4 - T_2^4}{25.25}$$

or $$T_1^4 - T_3^4 = \frac{27.33}{25.25}(T_3^4 - T_2^4)$$

$$= 1.08\ (T_3^4 - T_2^4) = 1.08\ T_3^4 - 1.08\ T_2^4$$

or $$2.08\ T_3^4 = T_1^4 + 1.08\ T_2^4$$

or $$T_3^4 = \frac{1}{2.08}\ (T_1^4 + 1.08\ T_2^4) = 0.48\ (T_1^4 + 1.08\ T_2^4) \quad ...(i)$$

Q_{12} (heat flow **without shield**)

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{0.3} + \frac{1}{0.8} - 1} = \frac{\sigma(T_1^4 - T_2^4)}{3.58} \quad ...(ii)$$

Q_{13} (heat flow **with shield**)

$$= \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{0.3} + \frac{1}{0.4} - 1} = \frac{\sigma(T_1^4 - T_3^4)}{27.33} \quad ...(iii)$$

∴ **Percentage reduction** in heat flow due to shield

$$= \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= 1 - \frac{Q_{13}}{Q_{12}} = 1 - \frac{\sigma(T_1^4 - T_3^4)/27.33}{\sigma(T_1^4 - T_2^4)/3.58}$$

$$= 1 - \frac{3.58}{27.33}\left[\frac{T_1^4 - T_3^4}{T_1^4 - T_2^4}\right]$$

$$= 1 - 0.131\left[\frac{T_1^4 - 0.48\,(T_1^4 + 1.08\,T_2^4)}{T_1^4 - T_2^4}\right]$$

$$= 1 - 0.131\left[\frac{T_1^4 - 0.48\,T_1^4 + 0.52\,T_2^4}{T_1^4 - T_2^4}\right]$$

$$= 1 - 0.131\left[\frac{0.52\,(T_1^4 - T_2^4}{(T_1^4 - T_2^4)}\right]$$

$= 1 - 0.131 \times 0.52 =$ **0.932 or 93.2%. (Ans.)**

HIGHLIGHTS

1. Heat transfer may be defined as the transmission of energy from one region to another as a result of temperature gradient and it takes place by three modes : conduction, convection and radiation.
2. Fourier's law of conduction : $Q = -\,kA\,\dfrac{dt}{dx}$

   where, Q = Heat flow through a body per unit time,

   A = Surface area of heat flow (perpendicular to the direction of flow),

   dt = Temperature difference,

dx = Thickness of body in the direction of flow, and

k = Thermal conductivity of the body.

3. Conduction of heat flow through slabs :

$$Q = \frac{A(t_1 - t_2)}{\left(\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3}\right)} \text{ ......3 slabs.}$$

4. Conduction of heat flow through pipe walls and lagging :

$$Q = \frac{(t_1 - t_2)}{\frac{1}{2\pi l}\left[\frac{1}{k}\log_e \frac{r_2}{r_1}\right]}.$$

5. Conduction of heat through a hollow sphere :

$$Q = \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{4\pi k r_1 r_2}\right)}.$$

6. Heat transfer by convection :

$$Q = hA\,(t_1 - t_2).$$

7. Overall heat transfer co-efficient :

$$U = \frac{1}{\frac{1}{h_{hf}} + \sum \frac{L}{k} + \frac{1}{h_{cf}}}.$$

8. Heat transfer between two fluids separated by the walls of a composite tube of solid material :

$$Q = \frac{(t_{hf} - t_{cf})}{\frac{1}{2\pi L}\left[\frac{1}{r_1 h_{hf}} + \frac{1}{k_1}\log_e \frac{r_2}{r_1} + \frac{1}{k_2}\log_e \frac{r_3}{r_2} + \frac{1}{r_4 h_{cf}}\right]}.$$

9. A heat exchanger may be defined as an equipment which transfers the energy from a hot fluid to a cold fluid, with maximum rate and minimum investment and running cost.

10. The net heat transfer in case of grey bodies with emissivities ε_1 and ε_2 is given by :

$$Q = \frac{\sigma A(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}\right) - 1}$$

In case of concentric or long co-axial cylinder,

$$Q = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}.$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The Fourier's law of heat transfer by conduction is expressed as

(*a*) $Q = kA^2 \frac{dt}{dx}$ (*b*) $Q = kA \frac{dt}{dx}$

(*c*) $Q = k^2 A \frac{dx}{dt}$ (*d*) $Q = k^3 A \frac{dx}{dt}$.

2. The heat transfer is constant when

(*a*) temperature remains constant with time (*b*) temperature decreases with time

(*c*) temperature increases with time (*d*) any of these.

3. The co-efficient of thermal conductivity is defined as

(*a*) Quantity of heat transfer per unit area per one degree drop in temperature

(*b*) Quantity of heat transfer per one degree temperature drop per unit area

(*c*) Quantity of heat transfer per unit time per unit area

(*d*) Quantity of heat transfer per unit time per unit area per one degree temperature drop per unit length.

4. The thermal conductivity is expressed as

(*a*) W/mK (*b*) W/m^2K

(*c*) W/hmK (*d*) W/h^2m^2K.

5. Heat transfer from higher temperature to low temperature takes place according to

(*a*) Fourier law (*b*) First law of thermodynamics

(*c*) Second law of thermodynamics (*d*) Zeroth law of thermodynamics.

6. Conduction through flat composite wall is given by :

(*a*) $Q = \dfrac{t_1 - t_4}{\dfrac{x_1}{k_1A} + \dfrac{x_2}{k_2A} + \dfrac{x_3}{k_3A}}$ (*b*) $Q = \dfrac{t_1 - t_4}{\dfrac{k_1A}{x_1} + \dfrac{k_2A}{x_2} + \dfrac{k_3A}{x_3}}$

(*c*) $Q = \dfrac{(t_1 - t_4)A}{\dfrac{k_1}{x_1} + \dfrac{k_2}{x_2} + \dfrac{k_3}{x_3}}$ (*d*) $Q = \dfrac{\dfrac{k_1A}{x_1} + \dfrac{k_2A}{x_2} + \dfrac{k_3A}{x_3}}{(t_1 - t_4)}$

where Q = heat transfer, t_1, t_2, t_3 and t_4 temperatures on surfaces of composite wall, x_1, x_2, x_3, x_4 thicknesses of different composite wall layers.

7. Conduction through hollow, radial one dimensional heat transfer is expressed as

(*a*) $Q = \dfrac{2\pi L(t_1 - t_2)k}{\log_e r_2/r_1}$ (*b*) $Q = \dfrac{2\pi L(t_1 - t_2)}{k(r_2 - r_1)}$

(*c*) $Q = \dfrac{2\pi L \log_e (t_1/t_2)}{(r_2 - r_1)k}$ (*d*) $Q = \dfrac{2\pi L(t_1 - t_2)k}{\log_e r_2/r_1}$.

8. The radial heat transfer rate through hollow cylinder increases as the ratio of outer radius to inner radius

(*a*) decreases (*b*) increases

(*c*) constant (*d*) none of the above.

9. Stefan-Boltzmann law is expressed as

(*a*) $Q = \sigma AT^4$ (*b*) $Q = \sigma A^2T^4$

(*c*) $Q = \sigma AT^2$ (*d*) $Q = AT^4$.

10. The quantity of heat radiation is dependent on

(*a*) area of the body only (*b*) shape of the body only

(*c*) temperature of the body only (*d*) on all (*a*), (*b*) and (*c*).

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*d*) **4.** (*a*) **5.** (*c*) **6.** (*a*) **7.** (*a*)

8. (*a*) **9.** (*a*) **10.** (*c*).

THEORETICAL QUESTIONS

1. Enumerate the three modes by which heat can be transferred from one place to another. Which is the slowest of all ?
2. How do you define the thermal conductivity of a material ?
3. What do you understand by the terms 'convective heat transfer co-efficient' and 'overall heat transfer co-efficient'.
4. Derive an expression for heat loss in kJ/m^2-hr through a composite wall of layers (*i*) without considering convective heat transfer co-efficients and (*ii*) considering the convective heat transfer co-efficients.
5. Classify the heat exchangers according to the flow directions of fluid and give few examples of each in actual field of application.
6. Prove that the mean temperature difference in a parallel-flow heat exchanger is given by

$$LMTD\ (t_m) = \frac{t_1 - t_2}{\log_e \dfrac{t_1}{t_2}} .$$

UNSOLVED EXAMPLES

1. The inner surface of a plane brick wall is at 40°C and the outer surface is at 20°C. Calculate the rate of heat transfer per m^2 of surface area of the wall, which is 250 mm thick. The thermal conductivity of the brick is 0.52 W/mK. [**Ans.** 41.6 W/m^2]
2. Determine the rate of heat flow through the boiler wall made of 2 cm thick steel and covered with an insulating material of 0.5 cm thick. The temperatures at the inner and outer surfaces of the wall are 300°C and 50°C respectively.

   k (steel) = 58 W/mK

   k (insulation) = 0.116 W/mK. [**Ans.** 5.8 kW/m^2]
3. A mild steel tank of wall thickness 10 mm contains water at 90°C. Calculate the rate of heat loss per m^2 of tank surface area when the atmospheric temperature is 15°C. The thermal conductivity of mild steel is 50 W/mK, and the heat transfer co-efficients for inside and outside the tank are 2800 and 11 W/m^2 K, respectively. Calculate also the temperature of the outside surface of the tank.

   [**Ans.** 820 W/m^2, 89.6°C]
4. A cold storage room has walls made of 0.23 m of brick on the outside, 0.08 m of plastic foam, and finally 15 mm of wood on the inside. The outside and inside air temperatures are 22°C and – 2°C respectively. If the inside and outside heat transfer co-efficients are respectively 29 and 12 W/m^2 K and the thermal conductivities of brick, foam and wood are 0.98, 0.02 and 0.17 W/mK respectively determine (*i*) the rate of heat removal by refrigeration if the total wall area is 90 m^2, and (*ii*) the temperature of the inside surface of the brick. [**Ans.** (*i*) 486.4 W, (*ii*) 20.28°C]
5. The wall of a refrigerated van is of 1.5 mm of steel sheet at outer surface, 10 mm plywood at the inner surface and 2 cm of glasswool in between. Calculate the rate of heat flow, if the temperatures of the inside and outside surfaces are – 15°C and 24°C.

   Take : k (steel) = 23.2 W/mK, k (glass-wool) = 0.014 W/mK

   and k (plywood) = 0.052 W/mK. [**Ans.** 6 kW/m^2]
6. Sheets of brass and steel, each 10 mm thick, are placed in contact. The outer surface of brass is kept at 100°C and outer surface of steel is kept at 0°C. What is the temperature of the common interface ? The thermal conductivities of brass and steel are in the ratio of 2 : 1. [**Ans.** 66.7°C]
7. The wall of a furnace is made up of 250 mm of fire brick, k = 1.05 W/mK ; 120 mm of insulation brick, k = 0.85 W/mK, and 200 mm of red brick, k = 0.85 W/mK. The inner and outer surface temperatures of the walls are 850°C and 65°C respectively. Calculate the temperatures at the contact surfaces.

   Neglect the resistance of mortar joints. [**Ans.** 703°C, 210°C]

8. Calculate the heat flowing through a furnace wall 0.23 m thick, the inside and outside surface temperatures of which are 1000°C and 200°C respectively. Assume that the mean thermal conductivity of the wall material is 1.1 W/mK. Assuming that 7 mm of insulation (k = 0.075 W/mK) is added to the outside surface of the wall and reduces the heat loss 20% ; calculate the outside surface temperature of the wall. If the cost of the insulation is Rs. 70 per sq m what time will be required to pay for the insulation ? Base the calculations on the 24 hours operation per day and 199 days per year. Heat energy may be valued at Rs. 10 per 1000 kWh. [**Ans.** 3826 W/h-m^2 ; 74.3°C ; 1.916 years]

9. A flat wall of a furnace is composed of two layers of different materials having thicknesses of 0.115 m and 0.6 m with thermal conductivities of 0.16 W/m K and 10.6 W/m K respectively. If 1 kW/h of heat passes through every sq m area, estimate the drop in temperature at the contact between the two walls. The temperature inside the furnace is 1000°C and that at outside layer is 150°C. [**Ans.** 74°C]

10. A furnace wall consists of 250 mm fire brick, 125 mm insulating brick, and 250 mm building brick. The inside wall is at temperature of 600°C and the atmospheric temperature is 20°C. Calculate the heat loss per m^2 of wall area and the temperature of the outside wall surface of the furnace. The heat transfer co-efficient for the outside surface is 10 W/m^2 K, and the thermal conductivities of the fire brick, insulating brick and building brick are 1.4, 0.2 and 0.7 W/m K respectively.

Neglect radiation. [**Ans**. 0.46 kW/m^2 ; 66°C]

11. Hot air at a temperature of 60°C is flowing through a steel pipe of 100 mm diameter. The pipe is covered with two layers of different insulating materials of thicknesses 50 mm and 30 mm, and their corresponding thermal conductivities are 0.23 and 0.37 W/m K. The inside and outside heat transfer co-efficients are 58 and 12 W/m^2 K. The atmosphere is at 25°C. Find the rate of heat loss from a 50 m length of pipe. Neglect the resistance of the steel pipe. [**Ans.** 2.334 kW]

12. A steel pipe of 100 mm bore and 7 mm wall thickness, carrying steam at 260°C, is insulated with 40 mm of a high temperature diatomaceous earth covering. This covering is in turn insulated with 60 mm of asbestos felt. If the atmospheric temperature is 15°C, calculate the rate at which heat is lost by the steam per m length of the pipe. The heat transfer co-efficients for the inside and outside surfaces are 550 and 15 W/m^2 K, respectively and the thermal conductivities of steel, diatomaceous earth and asbestos felt are 50, 10.09 and 0.07 W/m K respectively. Calculate also the temperature of the outside surface.

[**Ans.** 116 W ; 22.8°C]

13. A 250 mm steam main, 225 metres long is covered with 50 mm of high temperature insulation (k = 0.095 W/m K) and 40 mm of low temperature insulation (k = 0.065 W/m K). The inner and outer surface temperatures as measured are 400°C and 50°C respectively. Calculate :

(*i*) The total heat loss per hour.

(*ii*) The total heat loss per m^2 of outer surface.

(*iii*) The heat loss per m^2 of pipe surface.

(*iv*) The temperature between the two layers of insulation.

Neglect heat conduction through pipe material.

[**Ans.** (*i*) 265514 kJ/h, (*ii*) 873.5 kJ/h, (*iii*) 1502.5 kJ/h, (*iv*) 215°C]

14. A steam pipe of 160 mm inside diameter and 170 mm outside diameter (k = 58 W/m K) is covered with first layer of insulating material of 30 mm thickness (k = 0.17 W/m K) and second layer of insulating material of 50 mm thickness (k = 0.093 W/m K). The temperature of steam passing through the pipe is 300°C and ambient air temperature surrounding the pipe is 30°C. Taking inner and outer heat transfer co-efficients 30 and 5.8 W/m^2 K respectively, find the heat lost per metre length of pipe.

[**Ans.** 216 W/m]

15. A small hemispherical oven is built of an inner layer of insulating fire brick 125 mm thick, and an outer covering of 85% magnesia 40 mm thick. The inner surface of the oven is at 800°C and the heat transfer co-efficient for the outside surface is 10 W/m^2 K, the room temperature is 20°C. Calculate the heat loss through the hemisphere if the inside radius is 0.6 m. Take the thermal conductivities of fire brick and 85% magnesia as 0.31 and 0.05 W/mK, respectively. [**Ans.** 1.93 kW]

16. A spherical shaped vessel of 1.2 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 200°C. Thermal conductivity of the material is 0.3 kJ/m-h-°C. [**Ans.** 2262 kJ/h]

17. Exhaust gases flowing through a tubular heat exchanger at the rate of 0.3 kg/s are cooled from 400°C to 120°C by water initially at 10°C. The specific heat of exhaust gases and water may be taken as 1.13 and 4.19 kJ/kg K respectively, and overall heat transfer co-efficient from gases to water is 140 W/m^2 K. Calculate the surface area required when the cooling water flow is 0.4 kg/s.

(*i*) For parallel-flow ; (*ii*) For counter-flow. [**Ans.** (*i*) 4.0 m^2, (*ii*) 3.37 m^2]

18. Water flows inside a tube 50 mm in diameter and 3 m long at a velocity of 0.8 m/s. Determine the heat transfer co-efficient and the rate of heat transfer if the mean water temperature is 50°C and the wall is isothermal at 70°C. For water at 60°C, take $k = 0.66$ W/m K, ν (kinematic viscosity) $= 0.478 \times 10^{-6}$ m^2/s, and Prandtl number = 2.98. [**Ans.** 4075 W/m^2K ; 38.39 kW]

19. Liquid air at – 153°C is stored in the space of two concentric spheres of 21 cm and 30 cm diameters. The surface emissivities are 0.03. Assume the outer surface temperature is 27°C. Considering only radiation heat transfer and taking the latent heat of liquid air of 209 kJ/kg, find the rate of evaporation. Take $\sigma = 2.04 \times 10^{-4}$ kJ/h-m^2 K^4. [**Ans.** 21.7 kg/h]

20. A body at 1100°C in black surroundings at 550°C has an emissivity of 0.4 at 1100°C and an emissivity of 0.7 at 550°C. Calculate the ratio of heat loss by radiation per m^2,

(*i*) when the body is assumed to be grey with $\varepsilon = 0.4$

(*ii*) when the body is not grey. [**Ans.** (*i*) 70.22 kW, (*ii*) 62.42 kW]

21. A long steel rod, 20 mm in diameter, is to be heated from 427°C to 538°C. It is placed concentrically in a long cylindrical furnace which has an inside diameter of 160 mm. The inner surface of the furnace is at a temperature of 1093°C, and has an emissivity of 0.85. If the surface of the rod has an emissivity of 0.6, find the time required for the heating operation.

Take for steel : $\rho = 7845$ kg/m^3 and $c = 0.67$ kJ/kg K. [**Ans.** 29.88 s]

16

Compressible Flow

16.1. INTRODUCTION

A compressible flow is that flow in which the *density of the fluid changes during flow.* All real fluids are compressible to some extent and therefore their density will change with change in pressure or temperature. If the relative change in density $\Delta\rho/\rho$ is small, the fluid can be treated as incompressible. A compressible fluid, such as air, can be considered as incompressible with constant ρ if *changes in elevation are small, acceleration is small*, and/or *temperature changes are negligible.* In other words, if Mach's number U/C, where C is the sonic velocity, is small, compressible fluid can be treated as incompressible.

- The gases are treated as compressible fluids and study of this type of flow is often referred to as *'Gas dynamics'*.
- Some important problems where *compressibility effect* has to be considered are :

(*i*) Flow of gases through nozzles, orifices ;

(*ii*) Compressors ;

(*iii*) Flight of aeroplanes and projectiles moving at higher altitudes ;

(*iv*) Water hammer and accoustics.

- *'Compressibility' affects the drag coefficients of bodies by formation of shock waves, discharge coefficients of measuring devices such as orificemeters, venturimeters and pitot tubes, stagnation pressure and flows in converging-diverging sections.*

16.2. BASIC EQUATIONS OF COMPRESSIBLE FLUID FLOW

The basic equations of compressible fluid flow are : (*i*) *Continuity equation,* (*ii*) *Momentum equation,* (*iii*) *Energy equation*, and (*iv*) *Equation of state.*

16.2.1. Continuity Equation

In case of *one-dimensional flow*, mass per second = ρAV

(where ρ = mass density, A = area of cross-section, V = velocity)

Since the mass or mass per second is constant according to law of conservation of mass, therefore,

$$\rho AV = \text{constant} \qquad ...(16.1)$$

Differentiating the above equation, we get

$$d(\rho AV) = 0 \text{ or } \rho d(AV) + AVd\rho = 0$$

or $$\rho(AdV + VdA) + AVd\rho = 0 \text{ or } \rho AdV + \rho VdA + AVd\rho = 0$$

Dividing both sides by ρAV and rearranging, we get

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad ...(16.2)$$

Eqn. (16.2) is also known as equation of continuity in *differential form.*

16.2.2. Momentum Equation

The momentum equation for compressible fluids is similar to the one for incompressible fluids. This is because in momentum equation the *change in momentum flux is equated to force required to cause this change*.

Momentum flux = mass flux × velocity = $\rho AV \times V$

But the mass flux *i.e.*, ρAV = constant ...By continuity equation

Thus the momentum equation is completely independent of the compressibility effects and for compressible fluids the momentum equation, say in X-direction, may be expressed as :

$$\Sigma F_x = (\rho AVV_x)_2 - (\rho AVV_x)_1 \qquad ...(16.3)$$

16.2.3. Bernoulli's or Energy Equation

As the flow of compressible fluid is steady, the Euler equation is given as :

$$\frac{dp}{\rho} + VdV + gdz = 0 \qquad ...(16.4)$$

Integrating both sides, we get

$$\int \frac{dp}{\rho} + \int VdV + \int gdz = \text{constant}$$

or $$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant} \qquad ...(16.5)$$

In compressible flow since ρ is not constant it cannot be taken outside the integration sign. In compressible fluids the pressure (p) changes with change of density (ρ), depending on the type of process. Let us find out the Bernoulli's equation for isothermal and adiabatic processes.

(*a*) **Bernoulli's or energy equation for isothermal process :**

In case of an isothermal process,

$$pv = \text{constant or } \frac{p}{\rho} = \text{constant} = c_1 \ (say)$$

(where v = specific volume = $1/\rho$)

$$\therefore \qquad \rho = \frac{p}{c}$$

Hence $$\int \frac{dp}{\rho} = \int \frac{dp}{p/c_1} = \int \frac{c_1 dp}{p} = c_1 \int \frac{dp}{p} = c_1 \log_e p = \frac{p}{\rho} \log_e p \left(\because \ c_1 = \frac{p}{\rho}\right)$$

Substituting the value of $\int \frac{dp}{p}$ in eqn. (16.5), we get

$$\frac{p}{\rho} \log_e p + \frac{V^2}{2} + gz = \text{constant}$$

Dividing both sides by g, we get

$$\frac{p}{\rho g}\log_e p + \frac{V^2}{2g} + z = \text{constant} \qquad \text{...(16.6)}$$

Eqn. (16.6) is the Bernoulli's equation for compressible flow undergoing isothermal process.

(*b*) **Bernoulli's equation for adiabatic process :**

In case of an adiabatic process,

$$pv^{\gamma} = \text{constant or } \frac{p}{\rho^{\gamma}} = \text{constant} = c_2 \text{ (say)}$$

$$\therefore \qquad \rho^{\gamma} = \frac{p}{c_2} \text{ or } \rho = \left(\frac{p}{c_2}\right)^{1/\gamma}$$

Hence $\displaystyle\int \frac{dp}{\rho} = \int \frac{dp}{(p/c_2)^{1/\gamma}} = \, = (c_2)^{1/\gamma} \int \frac{1}{p^{1/\gamma}}\, dp = (c_2)^{1/\gamma} \int p^{-1/\gamma}\, dp$

$$= (c_2)^{1/\gamma}\left[\frac{p^{-\frac{1}{\gamma}+1}}{\left(-\frac{1}{\gamma}+1\right)}\right] = \frac{(c_2)^{1/\gamma}\,(p)^{\left(\frac{\gamma-1}{\gamma}\right)}}{\left(\frac{\gamma-1}{\gamma}\right)} = \frac{\gamma}{\gamma-1}\,(c_2)^{1/\gamma}\,(p)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{\rho^{\gamma}}\right)^{1/\gamma}(p)^{\left(\frac{\gamma-1}{\gamma}\right)} \qquad \left(\because\ c_2 = \frac{p}{\rho^{\gamma}}\right)$$

$$= \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p^{1/\gamma}}{\rho^{\gamma\times\frac{1}{\gamma}}}\right)(p)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= \left(\frac{\gamma}{\gamma-1}\right)\frac{(p)^{\left(\frac{1}{\gamma}+\frac{\gamma-1}{\gamma}\right)}}{\rho} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho}$$

Substituting the value of $\displaystyle\int \frac{dp}{p}$ in eqn. (16.6), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Dividing both sides by g, we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \qquad \text{...(16.7)}$$

Eqn. (16.7) is the Bernoulli's equation for compressible flow undergoing adiabatic process.

Example 16.1. *A gas with a velocity of 300 m/s is flowing through a horizontal pipe at a section where pressure is 78 kN/m*^{2} *absolute and temperature 40°C. The pipe changes in diameter and at this section, the pressure is 117 kN/m*^{2} *absolute. Find the velocity of the gas at this section if the flow of the gas is adiabatic. Take R = 287 J/kg K and* $\gamma = 1.4$.

(Punjab University)

Sol. Section 1 : Velocity of the gas, $V = 300$ m/s

Pressure, $p_1 = 78$ kN/m^2

Temperature, $T_1 = 40 + 273 = 313$ K

Section 2 : Pressure, $p_2 = 117$ kN/m^2

$R = 287$ J/kg K, $\gamma = 1.4$

Velocity of gas at section 2, V$_2$:

Applying Bernoulli's equations at sections 1 and 2 for *adiabatic process*, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} = z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 \qquad \text{[Eqn. (16.7)]}$$

But $z_1 = z_2$, since the pipe is horizontal.

$$\therefore \qquad \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g}$$

Cancelling 'g' on both sides, we get

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right) = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left(1 - \frac{p_2}{\rho_2} \times \frac{\rho_1}{p_1}\right) = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\therefore \qquad \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left(1 - \frac{p_2}{p_1} \times \frac{\rho_1}{\rho_2}\right) = \frac{V_2^2}{2} - \frac{V_1^2}{2} \qquad \ldots(i)$$

For an adiabatic flow : $\dfrac{p_1}{\rho_1^{\gamma}} = \dfrac{p_2}{\rho_2^{\gamma}}$ or $\dfrac{p_1}{p_2} = \left(\dfrac{\rho_1}{\rho_2}\right)^{\gamma}$ or $\dfrac{\rho_1}{\rho_2} = \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$

Substituting the value of $\dfrac{\rho_1}{\rho_2}$ in eqn (i), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1 - \frac{p_2}{p_1} \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}\right\} = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right\} = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} = \frac{V_2^2 - V_1^2}{2} \qquad \ldots(ii)$$

At section 1 : $\dfrac{p_1}{\rho_1} = RT_1 = 287 \times 313 = 89831$

$\dfrac{p_2}{p_1} = \dfrac{117}{78} = 1.5$, and $V_1 = 300$ m/s

Substituting the values in eqn. (*ii*), we get

$$\left(\frac{1.4}{1.4-1}\right) \times 89831 \left\{1-(1.5)^{\frac{1.4-1}{1.4}}\right\} = \frac{V_2^2}{2} - \frac{300^2}{2}$$

$$314408.5\ (1 - 1.1228) = \frac{V_2^2}{2} - 45000 \text{ or } - 38609.4 = \frac{V_2^2}{2} - 45000$$

or, $V_2^2 = 12781.2$ or $\mathbf{V_2 = 113.05}$ **m/s. (Ans.)**

Example 16.2. *In the case of air flow in a conduit transition, the pressure, velocity and temperature at the upstream section are 35 kN/m², 30 m/s and 150°C respectively. If at the downstream section the velocity is 150 m/s, determine the pressure and the temperature if the process followed is isentropic. Take γ = 1.4, R = 290 J/kg K.*

Sol. *Section 1 (upstream) :* Pressure, $p_1 = 35$ kN/m²,

Velocity, $V_1 = 30$ m/s,

Temperature, $T = 150 + 273 = 423$ K

Velocity, $V_2 = 150$ m/s

$R = 290$ J/kg K, $\gamma = 1.4$

Section 2 (downstream) :

Pressure, $\mathbf{p_2}$:

Applying Bernoulli's equation at sections 1 and 2 for isentropic (reversible adiabatic) process, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2$$

Assuming $z_1 = z_2$, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g}$$

Cancelling '*g*' on both the sides, and rearranging, we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left(1 - \frac{p_2}{p_1} \times \frac{\rho_1}{\rho_2}\right) = \frac{V_2^2}{2} - \frac{V_1^2}{2} \qquad ...(i)$$

For an isentropic flow : $\dfrac{p_1}{\rho_1^{\gamma}} = \dfrac{p_2}{\rho_2^{\gamma}}$ or $\dfrac{p_1}{p_2} = \left(\dfrac{\rho_1}{\rho_2}\right)^{\gamma}$ or $\dfrac{\rho_1}{\rho_2} = \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$

Substituting the value of $\dfrac{\rho_1}{\rho_2}$ in eqn. (*i*), we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1 - \frac{p_2}{p_1} \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}\right\} = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right\} = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} = \frac{V_2^2}{2}-\frac{V_1^2}{2}$$

Substituting the values, we get

$$\frac{1.4}{1.4-1} \times 122670 \left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{1.4-1}{1.4}}\right\} = \frac{150^2}{2}-\frac{30^2}{2} = 10800$$

$$\left(\because \ \frac{p_1}{\rho_1} = RT_1 = 290 \times 423 = 122670\right)$$

$$429345 \left\{1-\left(\frac{p_2}{p_1}\right)^{0.2857}\right\} = 10800$$

or $$\left(\frac{p_2}{p_1}\right)^{0.2857} = 1-\frac{10800}{429345} = 0.9748$$

or $$\frac{p_2}{p_1} = (0.9748)^{1/0.2857} = (0.9748)^{3.5} = 0.9145$$

or $p_2 = 35 \times 0.9145 =$ **32 kN/m² (Ans.)**

Temperature, T_2 :

For an isentropic process, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (0.9145)^{\frac{1.4-1}{1.4}} = (0.9145)^{0.2857} = 0.9748$$

$\therefore$ $T_2 = 423 \times 0.9748 = 412.3$ K

or $t_2 = 412.3 - 273 =$ **139.3°C (Ans.)**

16.3. PROPAGATION OF DISTURBANCES IN FLUID AND VELOCITY OF SOUND

The solids as well as fluids consist of molecules. Whereas the molecules in solids are close together, these are relatively apart in fluids. Consequently whenever there is a minor disturbance, it travels instantaneously in case of solids ; but in case of fluid the molecules change its position before the transmission or propagation of the disturbance. Thus the velocity of disturbance in case of fluids will be less than the velocity of the disturbance in solids. In case of fluid, the propagation of disturbance depends upon its *elastic properties. The velocity of disturbance depends upon the changes in pressure and density of the fluid.*

The propagation of disturbance is similar to the propagation of sound through a media. The *speed of propagation of sound in a media* is known as *acoustic* or *sonic velocity* and depends upon the difference of pressure. In compressible flow, velocity of sound (sonic velocity) is of paramount importance.

16.3.1. Derivation of Sonic Velocity (velocity of sound)

Consider a one-dimensional flow through long straight rigid pipe of uniform cross-sectional area filled with a frictionless piston at one end as shown in Fig. 16.1. The tube is filled with a compressible fluid initially at rest. If the piston is moved suddenly to the right with a velocity, a *pressure wave* would be propagated through the fluid with the velocity of sound wave.

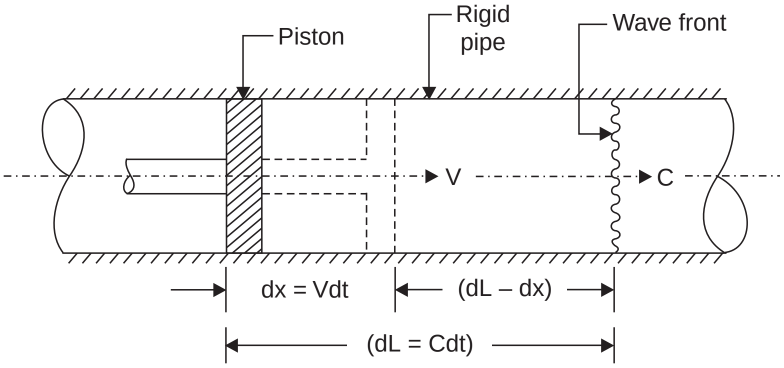


Fig. 16.1. One dimensional pressure wave propagation.

Let A = cross-sectional area of the pipe,

V = piston velocity,

p = fluid pressure in the pipe before the piston movement,

ρ = fluid density before the piston movement,

dt = a small interval of time during which piston moves, and

C = velocity of pressure wave or sound wave (travelling in the fluid).

Before the movement of the piston the length dL has an initial density ρ, and its total mass $= \rho \times dL \times A$.

When the piston moves through a distance dx, the fluid density within the compressed region of length $(dL - dx)$ will be increased and becomes $(\rho + d\rho)$ and subsequently the total mass of fluid in the compressed region $= (\rho + d\rho)\,(dL - dx) \times A$

$\therefore$ $\rho \times dL \times A = (\rho + d\rho)\,(dL - dx) \times A$ ...by principle of continuity.

But $dL = C\,dt$ and $dx = Vdt$; therefore, the above equation becomes

$$\rho C dt = (\rho + d\rho)\,(C - V)\,dt$$

or, $\rho C = (\rho + d\rho)\,(C - V)$ or $\rho C = \rho C - \rho V + d\rho \,.\, C - d\rho \,.\, V$

or, $0 = -\,\rho V + d\rho \,.\, C - d\rho \,.\, V$

Neglecting the term $d\rho.V$ (V being much smaller than C), we get

$$d\rho \,.\, C = \rho V \text{ or } C = \frac{\rho V}{d\rho} \qquad ...(16.8)$$

Further in the region of compressed fluid, the fluid particles have attained a velocity which is apparently equal to V (velocity of the piston), accompanied by an increase in pressure dp due to sudden motion of the piston. Applying inpulse-momentum equation for the fluid in the compressed region during dt, we get

$$\underset{\text{(force on the fluid)}}{dp \times A \times dt} = \underset{\text{(rate of change of momentum)}}{\rho \times dL \times A\,(V - 0)}$$

or, $$dp = \rho\,\frac{dL}{dt}\,V = \rho \times \frac{Cdt}{dt} \times V = \rho C V \qquad (\because\ dL = Cdt)$$

or, $$C = \frac{dp}{\rho V} \qquad ...(16.9)$$

Multiplying eqns. (16.8) and (16.9), we get

$$C^2 = \frac{\rho V}{d\rho} \times \frac{dp}{\rho V} = \frac{dp}{d\rho}$$

$$\therefore \qquad C = \sqrt{\frac{dp}{d\rho}} \qquad ...(16.10)$$

16.3.2. Sonic Velocity in Terms of Bulk Modulus

The bulk modulus of elasticity of fluid (K) is defined as

$$K = \frac{dp}{\left(\frac{dv}{v}\right)} \qquad ...(i)$$

where, dv = decrease in volume, and v = original volume
(– ve sign indicates that volume decreases with increase in pressure)

Also, volume $v \propto \frac{1}{\rho}$, or $v\rho$ = constant

Differentiating both sides, we get

$$vd\rho + \rho dv = 0 \quad \text{or} \quad -\frac{dv}{v} = \frac{d\rho}{\rho}$$

Substituting the value of $-\frac{dv}{v}\left(=\frac{dp}{K}\right)$ from eqn. (i), we get

$$\frac{dp}{K} = \frac{d\rho}{\rho} \quad \text{or} \quad \frac{dp}{d\rho} = \frac{K}{\rho}$$

Substituting this value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\frac{K}{\rho}} \qquad ...(16.11)$$

Eqn. (16.11) is applicable for liquids and gases.

16.3.3. Sonic Velocity for Isothermal Process

For isothermal process : $\frac{p}{\rho}$ = constant

Differentiating both sides, we get

$$\frac{\rho \,.\, dp - p \,.\, d\rho}{\rho^2} = 0 \qquad \text{or} \quad \frac{dp}{\rho} - \frac{p \,.\, d\rho}{\rho^2} = 0$$

or,

$$\frac{dp}{\rho} = \frac{p \,.\, d\rho}{\rho^2} \quad \text{or} \quad \frac{dp}{d\rho} = \frac{p}{\rho} = RT \qquad ...(16.12)$$

$$\left(\frac{p}{\rho} = RT \quad \text{...equation of state}\right)$$

Substituting the value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\frac{p}{\rho}} = \sqrt{RT} \qquad ...(16.13)$$

16.3.4. Sonic Velocity for Adiabatic Process

For isentropic (reversible adiabatic) process : $\frac{p}{\rho^{\gamma}}$ = constant

or $p \,.\, \rho^{-\gamma}$ = constant

Differentiating both sides, we have $p(-\gamma) \,.\, \rho^{-\gamma-1}\, d\rho + \rho^{-\gamma}\, dp = 0$

Dividing both sides by $\rho^{-\gamma}$, we get

$$-p\,\gamma\,\rho^{-1}\, d\rho + dp = 0 \text{ or } dp = p\,\gamma\,\rho^{-1}\, d\rho$$

or,
$$\frac{dp}{d\rho} = \frac{p}{\rho}\,\gamma = \gamma RT \qquad \left(\because \frac{p}{\rho} = RT\right)$$

Substituting the value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\gamma RT} \qquad ...(16.14)$$

The following points are worth noting :

(*i*) The process is assumed to be *adiabatic* when minor disturbances are to be propagated through air ; due to *very high velocity* of disturbances/pressure waves appreciable heat transfer does not take place.

(*ii*) For calculation of velocity of the sound/pressure waves, *isothermal process* is considered only when it is mentioned in the numerical problem (that the process is isothermal). When no process is mentioned in the problem, calculation are made assuming the process to be *adiabatic*.

16.4. MACH NUMBER

The mach number is an important parameter in dealing with the flow of compressible fluids, when elastic forces become important and predominant.

Mach number is defined as the *square root of the ratio of the inertia force of a fluid to the elastic force.*

$\therefore$ Mach number,
$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho A V^2}{KA}}$$

$$= \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \qquad \left[\because \sqrt{K/\rho} = C \text{ ...Eqn. (16.11)}\right]$$

i.e.
$$M = \frac{V}{C} \qquad ...(16.15)$$

Thus,
$$M = \frac{\text{Velocity at a point in a fluid}}{\text{Velocity of sound at that point at a given instant of time}}$$

Depending on the value of Mach number, the flow can be classified as follows :

1. *Subsonic flow :* Mach number is *less* than 1.0 (or $M < 1$) ; in this case $V < C$.
2. *Sonic flow :* Mach number is *equal* to 1.0 (or $M = 1$) ; in this case $V = C$.
3. *Supersonic flow :* Mach number is *greater* than 1.0 (or $M > 1$) ; in this case $V > C$.

When the Mach number in flow region is slightly less to slightly greater than 1.0, the flow is termed as *transonic flow*.

The following points are worth noting :

(*i*) Mach number is important in those problems in which the flow velocity is comparable with the sonic velocity (velocity of sound). It may happen in case of airplanes travelling at very high speed, projectiles, bullets etc.

(*ii*) If for any flow system the Mach number is less than about 0.4, the effects of compressibility may be neglected (for that flow system).

Example 16.3. *Find the sonic velocity for the following fluids :*

(*i*) *Crude oil of specific gravity 0.8 and bulk modulus 1.5 GN/m*^{2} *;*

(*ii*) *Mercury having a bulk modulus of 27 GN/m*^{2}.

Sol. Crude oil : Specific gravity = 0.8 **(Delhi University)**

∴ Density of oil, $\rho = 0.8 \times 1000 = 800$ kg/m^3

Bulk modulus, $K = 1.5$ GN/m^2

Mercury : Bulk modulus, $K = 27$ GN/m^2

Density of mercury, $\rho = 13.6 \times 1000 = 13600$ kg/m^3

Sonic velocity, C_{oil}, C_{Hg} :

Sonic velocity is given by the relation :

$$C = \sqrt{\frac{K}{\rho}} \qquad \text{[Eqn. (16.11)]}$$

$$\therefore \qquad C_{oil} = \sqrt{\frac{1.5 \times 10^9}{800}} = \mathbf{1369.3\ m/s} \quad \textbf{(Ans.)}$$

$$C_{Hg} = \sqrt{\frac{27 \times 10^9}{13600}} = \mathbf{1409\ m/s} \quad \textbf{(Ans.)}$$

Example 16.4. *An aeroplane is flying at a height of 14 km where temperature is – 45°C. The speed of the plane is corresponding to M = 2. Find the speed of the plane if R = 287 J/kg K and* $\gamma = 1.4$.

Sol. Temperature (at a height of 14 km), $t = -45$°C.

$T = -45 + 273 = 228$ K

Mach number, $M = 2$

Gas constant, $R = 287$ J/kg K

$\gamma = 1.4$

Speed of the plane, V :

Sonic velocity, (C) is given by,

$$C = \sqrt{\gamma RT} \text{ (assuming the process to be } \textit{adiabatic}) \qquad \text{...[Eqn. (16.14)]}$$

$$= \sqrt{1.4 \times 287 \times 228} = 302.67 \text{ m/s}$$

Also $$M = \frac{V}{C} \qquad \text{...[Eqn. (16.15)]}$$

or, $$2 = \frac{V}{302.67}$$

or, $$V = 2 \times 302.67 = 605.34 \text{ m/s} = \frac{605.34 \times 3600}{1000} = \mathbf{2179.2\ km/h} \quad \textbf{(Ans.)}$$

16.5. PROPAGATION OF DISTURBANCE IN COMPRESSIBLE FLUID

When some disturbance is created in a compressible fluid (elastic or pressure waves are also generated), it is propagated in all directions with sonic velocity (= C) and its nature of propagation depends upon the Mach number (M). Such disturbance may be created when an object moves in a relatively stationary compressible fluid or when a compressible fluid flows past a stationary object.

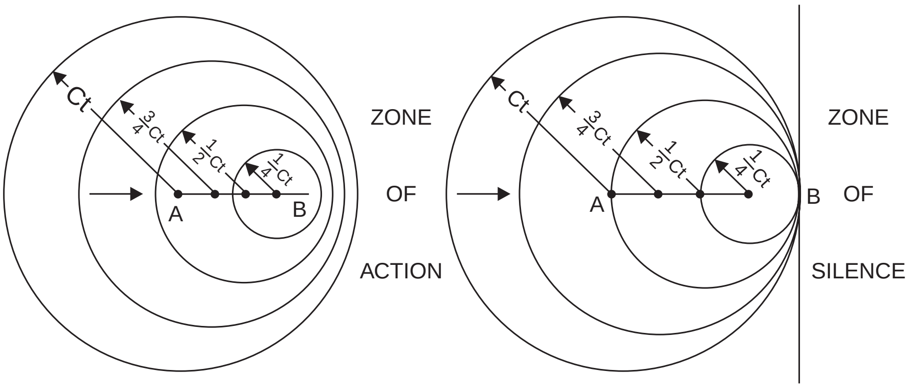
Consider a tiny projectile moving in a straight line with velocity V through a compressible fluid which is stationary. Let the projectile is at A when time $t = 0$, then in time t it will move through a distance $AB = Vt$. During this time the disturbance which originated from the projectile when it was at A will grow into the surface of sphere of radius Ct as shown in Fig. 16.2, which also shows the growth of the other disturbances which will originate from the projectile at every $t/4$ interval of time as the projectile moves from A to B.

Let us find nature of propagation of the disturbance for different Mach numbers.

Case I. When $M < 1$ (*i.e.,* $V < C$). In this case since $V < C$ the projectile lags behind the disturbance/pressure wave and hence as shown in Fig. 16.2 (*a*) the projectile at point B lies inside the sphere of radius Ct and also inside other spheres formed by the disturbances/waves started at intermediate points.

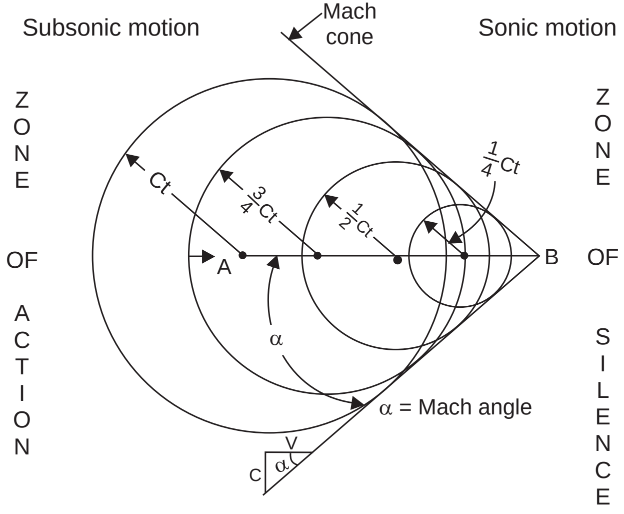
Case II. When $M = 1$ (*i.e.,* $V = C$). In this case, the disturbance always travels with the projectile as shown in Fig. 16.2 (*b*). The circle drawn with centre A will pass through B.

Case III. When $M > 1$ (*i.e.,* $V > C$). In this case the projectile travels faster than the disturbance. Thus the distance AB (which the projectile has travelled) is more than Ct, and hence



(*a*) M < 1 (V < C)
Subsonic motion

(*b*) M = 1 (V = C)
Sonic motion



(*c*) M > 1 (V > C)
Supersonic motion

Fig. 16.2. Nature of propagation of disturbances in compressible flow.

the projectile at point 'B' is outside the spheres formed due to formation and growth of disturbance at $t = 0$ and at the intermediate points (Fig. 16.2 (*c*)). If the tangents are drawn (from the point B) to the circles, the spherical pressure waves form a cone with its vertex at B. It is known as **Mach cone.** The semi-vertex angle α of the cone is known as **Mach angle** which is given by,

$$\sin \alpha = \frac{Ct}{Vt} = \frac{C}{V} = \frac{1}{M} \qquad \text{...(16.16)}$$

In such a case ($M > 1$), the effect of the disturbance is felt only in region inside the Mach cone, this region is called *zone of action*. The region outside the Mach cone is called *zone of silence*.

It has been observed that when an aeroplane is moving with supersonic speed, its noise is heard only after the plane has already passed over us.

Example 16.5. *Find the velocity of a bullet fired in standard air if its Mach angle is 40°.*

Sol. Mach angle, $\alpha = 40°$

$\gamma = 1.4$

For standard air : $R = 287$ J/kg K, $t = 15°$C or $T = 15 + 273 = 288$ K

Velocity of the bullet, V :

Sonic velocity, $C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 228} = 340.2$ m/s

Now, $\sin \alpha = \frac{C}{V}$

or, $\sin 40° = \frac{340.2}{V}$ or $V = \frac{340.2}{\sin 40°} =$ **529.26 m/s (Ans.)**

Example 16.6. *A projectile is travelling in air having pressure and temperature as 88.3 kN/m² and – 2°C. If the Mach angle is 40°, find the velocity of the projectile. Take $\gamma = 1.4$ and $R = 287$ J/kg K.* **[M.U.]**

Sol. Pressure, $p = 88.3$ kN/m^2

Temperature, $T = -2 + 273 = 271$ K

Mach angle, $M = 40°$

$\gamma = 1.4$, $R = 287$ J/kg K

Velocity of the projectile, V :

Sonic velocity, $C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 271} \simeq 330$ m/s

Now, $\sin \alpha = \frac{C}{V}$ or $\sin 40° = \frac{340}{V}$

or, $V = \frac{330}{\sin 40°} =$ **513.4 m/s (Ans.)**

Example 16.7. *A supersonic aircraft flies at an altitude of 1.8 km where temperature is 4°C. Determine the speed of the aircraft if its sound is heard 4 seconds after its passage over the head of an observer. Take $R = 287$ J/kg K and $\gamma = 1.4$.*

Sol. Altitude of the aircraft = 1.8 km = 1800 m

Temperature, $T = 4 + 273 = 277$ K

Time, $t = 4$ s

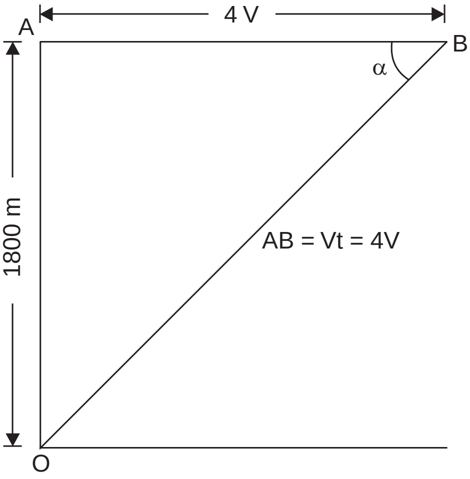


Fig. 16.3

Speed of the aircraft, V :

Refer Fig. 16.3. Let O represent the observer and A the position of the aircraft just vertically over the observer. After 4 seconds, the aircraft reaches the position represented by the point B. Line AB represents the wave front and α the Mach angle.

From Fig. 16.3, we have

$$\tan \alpha = \frac{1800}{4V} = \frac{450}{V} \qquad ...(i)$$

But, Mach number, $M = \frac{C}{V} = \frac{1}{\sin \alpha}$

or,
$$V = \frac{C}{\sin \alpha} \qquad ...(ii)$$

Substituting the value of V in eqn. (i), we get

$$\tan \alpha = \frac{450}{(C/\sin \alpha)} = \frac{450 \sin \alpha}{C}$$

or,
$$\frac{\sin \alpha}{\cos \alpha} = \frac{450 \sin \alpha}{C} \text{ or } \cos \alpha = \frac{C}{450} \qquad ...(iii)$$

But $C = \sqrt{\gamma RT}$, where C is the sonic velocity.

$R = 287$ J/kg K and $\gamma = 1.4$ ...(Given)

$\therefore$ $C = \sqrt{1.4 \times 287 \times 277} = 333.6$ m/s

Substituting the value of C in eqn. (ii), we get

$$\cos \alpha = \frac{333.6}{450} = 0.7413$$

$$\therefore \quad \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - 0.7413^2} = 0.6712$$

Substituting the value of sin α in eqn. (ii), we get

$$V = \frac{C}{\sin \alpha} = \frac{333.6}{0.6712} = 497 \text{ m/s} = \frac{497 \times 3600}{1000} = \mathbf{1789.2 \text{ km/h}} \text{ (Ans.)}$$

16.6. STAGNATION PROPERTIES

The point on the immersed body where the velocity is zero is called **stagnation point.** At this point velocity head is converted into pressure head. The values of pressure (p_s), temperature (T_s) and density (ρ_s) at stagnation point are called *stagnation properties.*

16.6.1. Expression for Stagnation Pressure (p_s) in Compressible Flow

Consider the flow of compressible fluid past an immersed body where the velocity becomes zero. Consider *frictionless adiabatic* (*isentropic*) condition. Let us consider two points, O in the free stream and the stagnation point S as shown in Fig. 16.4.

Let, p_0 = pressure of compressible fluid at point O,
V_0 = velocity of fluid at O,
ρ_0 = density of fluid at O,
T_0 = temperature of fluid at O,

and p_s, V_s, ρ_s and T_s are corresponding values of pressure, velocity density, and temperature at point S.

Applying Bernoulli's equation for adiabatic (frictionless) flow at points O and S, (given by eqn. 16.7), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0 g}+\frac{V_0^2}{2g}+z_0=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s g}+\frac{V_s^2}{2g}+z_s$$

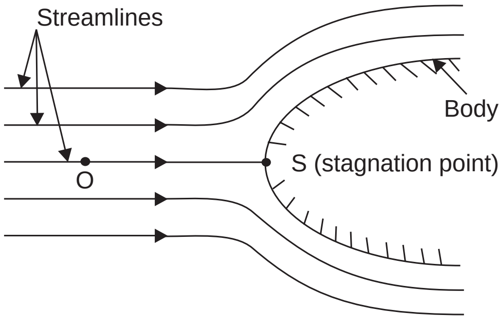


Fig. 16.4. Stagnation properties.

But $z_0 = z_s$; the above equation reduces to

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0 g}+\frac{V_0^2}{2g}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s g}+\frac{V_s^2}{2g}$$

Cancelling 'g' on both the sides, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}+\frac{V_0^2}{2}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s}+\frac{V_s^2}{2}$$

At point S the velocity is zero, *i.e.*, $V_s = 0$; the above equation becomes

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}-\frac{p_s}{\rho_s}\right)=-\frac{V_0^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1-\frac{p_s}{\rho_s}\times\frac{\rho_0}{p_0}\right)=-\frac{V_0^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1-\frac{p_s}{p_0}\times\frac{\rho_0}{\rho_s}\right)=-\frac{V_0^2}{2} \quad ...(i)$$

For adiabatic process : $\frac{p_0}{\rho_0^{\gamma}}=\frac{p_s}{\rho_s^{\gamma}}$ or $\frac{p_0}{p_s}=\frac{\rho_0^{\gamma}}{\rho_s^{\gamma}}$ or $\frac{\rho_0}{\rho_s}=\left(\frac{p_0}{p_s}\right)^{\frac{1}{\gamma}}$ ...(*ii*)

Substituting the value of $\frac{\rho_0}{\rho_s}$ in eqn. (*i*), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left[1-\frac{p_s}{p_0}\times\left(\frac{p_0}{p_s}\right)^{\frac{1}{\gamma}}\right]=-\frac{V_0^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left\{1-\left(\frac{p_s}{p_0}\right)^{1-\frac{1}{\gamma}}\right\}=-\frac{V_0^2}{2}$$

or,
$$\left[1-\left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]=-\frac{V_0^2}{2}\left(\frac{\gamma-1}{\gamma}\right)\frac{\rho_0}{p_0}$$

or,
$$1+\frac{V_0^2}{2}\left(\frac{\gamma-1}{\gamma}\right)\frac{\rho_0}{p_0}=\left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \quad ...(iii)$$

For adiabatic process, the sonic velocity is given by,

$$C = \sqrt{\gamma RT} = \sqrt{\gamma \frac{p}{\rho}} \qquad \left(\because \ \frac{p}{\rho} = RT \right)$$

For point O,

$$C_0 = \sqrt{\gamma \frac{p_0}{\rho_0}} \text{ or } C_0^2 = \gamma \frac{p_0}{\rho_0}$$

Substituting the value of $\frac{\gamma p_0}{\rho_0} = C_0^2$ in eqn. (*iii*), we get

$$1 + \frac{V_0^2}{2} (\gamma - 1) \times \frac{1}{C_0^2} = \left(\frac{p_s}{p_0} \right)^{\frac{\gamma - 1}{\gamma}}$$

or,

$$1 + \frac{V_0^2}{2C_0^2} (\gamma - 1) = \left(\frac{p_s}{p_0} \right)^{\frac{\gamma - 1}{\gamma}}$$

$$1 + \frac{M_0^2}{2} (\gamma - 1) = \left(\frac{p_s}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} \qquad \left(\because \ \frac{V_0^2}{C_0^2} = M_0^2 \right)$$

or,

$$\left(\frac{p_s}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} = \left[1 + \left(\frac{\gamma - 1}{2} \right) M_0^2 \right]$$

or,

$$\frac{p_s}{p_0} = \left[1 + \left(\frac{\gamma - 1}{2} \right) M_0^2 \right]^{\frac{\gamma}{\gamma - 1}} \qquad \text{...}(iv)$$

or,

$$p_s = p_0 \left[1 + \left(\frac{\gamma - 1}{2} \right) M_0^2 \right]^{\frac{\gamma}{\gamma - 1}} \qquad \text{...(16.17)}$$

Eqn. (16.17) gives the value of **stagnation pressure.**

Compressibility correction factor :

If the right hand side of eqn. (16.17) is expanded by the binomial theorem, we get

$$p_s = p_0 \left[1 + \frac{\gamma}{2} M_0^2 + \frac{\gamma}{8} M_0^4 + \frac{\gamma(2 - \gamma)}{48} M_0^6 \right]$$

$$= p_0 \left[1 + \frac{\gamma M_0^2}{2} \left(1 + \frac{M^2}{4} + \frac{2 - \gamma}{24} M_0^4 + \ldots \right) \right]$$

or,

$$p_s = p_0 + \frac{p_0 \, \gamma M_0^2}{2} \left(1 + \frac{M_0^2}{4} + \frac{2 - \gamma}{24} M_0^4 + \ldots \right) \qquad \text{...(16.18)}$$

But,

$$M_0^2 = \frac{V_0^2}{C_0^2} = \frac{V_0^2}{\left(\frac{\gamma p_0}{\rho_0} \right)} = \frac{V_0^2 \, \rho_0}{\gamma p_0} \qquad \left(\because \ C_0^2 = \frac{\gamma p_0}{\rho_0} \right)$$

Substituting the value of M_0^2 in eqn. (16.18), we get

$$p_s = p_0 + \frac{p_0\gamma}{2} \times \frac{V_0^2\rho_0}{\gamma p_0}\left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right)$$

or, $$p_s = p_0 + \frac{\rho_0 V_0^2}{2}\left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right) \quad \ldots(16.19)$$

Also, $p_s = p_0 + \dfrac{\rho_0 V_0^2}{2}$ (when compressibility effects are neglected) ...(16.20)

The comparison of eqns. (16.19) and (16.20) shows that the effects of compressibility are isolated in the bracketed quantity and that these effects *depend only* upon the *Mach number*. The bracketed quantity $\left[i.e., \left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right)\right]$ may thus be considered as a **compressibility correction factor.** It is worth noting that :

- For $M < 0.2$, the compressibility affects the pressure difference $(p_s - p_0)$ by *less than 1 per cent* and the simple formula for flow at constant density is then sufficiently accurate.
- For larger value of M, as the terms of binomial expansion become significant, the compressibility effect must be taken into account.
- When the Mach number exceeds a value of about 0.3 the *Pitot-static tube used for measuring aircraft speed needs calibration to take into account the compressibility effects.*

16.6.2. Expression for Stagnation Density (ρ_s)

From eqn. (*ii*), we have

$$\frac{\rho_0}{\rho_s} = \left(\frac{p_0}{p_s}\right)^{\frac{1}{\gamma}} \text{ or } \frac{\rho_s}{\rho_0} = \left(\frac{p_s}{p_0}\right)^{\frac{1}{\gamma}} \text{ or } \rho_s = \rho_0 \left(\frac{p_s}{p_0}\right)^{\frac{1}{\gamma}}$$

Substituting the value of $\left(\dfrac{p_s}{p_0}\right)$ from eqn. (*iv*), we get

$$\rho_s = \rho_0 \left[\left\{1 + \left(\frac{\gamma-1}{2}\right) M_0^2\right\}^{\frac{\gamma}{\gamma-1}}\right]^{\frac{1}{\gamma}}$$

or, $$\rho_s = \rho_0 \left[1 + \left(\frac{\gamma-1}{2}\right) M_0^2\right]^{\frac{1}{\gamma-1}} \quad \ldots(16.21)$$

16.6.3. Expression for Stagnation Temperature (T_s)

The equation of state is given by : $\dfrac{p}{\rho} = RT$

For stagnation point, the equation of state may be written as :

$$\frac{p_s}{\rho_s} = RT_s \text{ or } T_s = \frac{1}{R}\frac{p_s}{\rho_s}$$

Substituting the values of p_s and ρ_s from eqns. (16.17) and (16.18), we get

$$T_s = \frac{1}{R}\frac{p_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{\gamma}{\gamma-1}}}{\rho_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{1}{\gamma-1}}}$$

$$= \frac{1}{R}\frac{p_0}{\rho_0}\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\left(\frac{\gamma}{\gamma-1}-\frac{1}{\gamma-1}\right)}$$

$$= \frac{1}{R}\frac{p_0}{\rho_0}\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\left(\frac{\gamma-1}{\gamma-1}\right)}$$

or,
$$T_s = T_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right] \qquad \text{...(16.22)} \qquad \left(\because \frac{p_0}{\rho_0} = RT_0\right)$$

Example 16.8. *An aeroplane is flying at 1000 km/h through still air having a pressure of 78.5 kN/m*^{2} *(abs.) and temperature – 8°C. Calculate on the stagnation point on the nose of the plane :*

(*i*) *Stagnation pressure,* (*ii*) *Stagnation temperature, and*

(*iii*) *Stagnation density.*

Take for air : $R = 287$ *J/kg K and* $\gamma = 1.4$.

Sol. Speed of aeroplane, $V = 1000 \text{ km/h} = \frac{1000 \times 1000}{60 \times 60} = 277.77$ m/s

Pressure of air, $p_0 = 78.5$ kN/m^2

Temperature of air, $T_0 = -8 + 273 = 265$ K

For air : $R = 287$ J/kg K, $\gamma = 1.4$

The sonic velocity for adiabatic flow is given by,

$$C_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \times 287 \times 265} = 326.31 \text{ m/s}$$

∴ Mach number,
$$M_0 = \frac{V_0}{C_0} = \frac{277.77}{326.31} = 0.851$$

(*i*) **Stagnation pressure, p_s :**

The stagnation pressure (p_s) is given by the relation,

$$p_s = p_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{...[Eqn. (16.17)]}$$

or,
$$p_s = 78.5\left[1+\left(\frac{1.4-1}{2}\right)\times 0.851^2\right]^{\frac{1.4}{1.4-1}}$$

$$= 78.5\,(1.145)^{3.5} = \mathbf{126.1\ kN/m^2} \quad \textbf{(Ans.)}$$

(*ii*) **Stagnation temperature, T_s :**

The stagnation temperature is given by,

$$T_s = T_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right] \quad \text{...[Eqn. (16.22)]}$$

$$= 265 \left[1 + \frac{1.4 - 1}{2} \times 0.851^2\right] = 303.4 \text{ K or } \mathbf{30.4°C} \quad \textbf{(Ans.)}$$

(*iii*) **Stagnation density, ρ_s :**

The stagnation density (ρ_s) is given by,

$$\frac{p_s}{\rho_s} = RT_s \quad \text{or} \quad \rho_s = \frac{p_s}{RT_s}$$

or,

$$\rho_s = \frac{126.1 \times 10^3}{287 \times 303.4} = \mathbf{1.448\ kg/m^3} \quad \textbf{(Ans.)}$$

Example 16.9. *Air has a velocity of 1000 km/h at a pressure of 9.81 kN/m² in vacuum and a temperature of 47°C. Compute its stagnation properties and the local Mach number. Take atmospheric pressure = 98.1 kN/m², R = 287 J/kg K and γ = 1.4.*

What would be the compressibility correction factor for a pitot-static tube to measure the velocity at a Mach number of 0.8.

Sol. Velocity of air, $V_0 = 1000 \text{ km/h} = \frac{1000 \times 1000}{60 \times 60} = 277.78 \text{ m/s}$

Temperature of air, $T_0 = 47 + 273 = 320 \text{ K}$

Atmospheric pressure, $p_{atm} = 98.1 \text{ kN/m}^2$

Pressure of air (static), $p_0 = 98.1 - 9.81 = 88.29 \text{ kN/m}^2$

$R = 287 \text{ J/kg K}, \gamma = 1.4$

Sonic velocity, $C_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \times 287 \times 320} = 358.6 \text{ m/s}$

$\therefore$ Mach number, $M_0 = \frac{V_0}{C_0} = \frac{277.78}{358.6} = 0.7746$

Stagnation pressure, p_s :

The stagnation pressure is given by,

$$p_s = p_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right]^{\frac{\gamma}{\gamma - 1}} \quad \text{...[Eqn. (16.17)]}$$

or,

$$p_s = 88.29 \left[1 + \frac{1.4 - 1}{2} \times 0.7746^2\right]^{\frac{1.4}{1.4 - 1}}$$

$$= 88.29\ (1.12)^{3.5} = \mathbf{131.27\ kN/m^2} \quad \textbf{(Ans.)}$$

Stagnation temperature, T_s :

$$T_s = T_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right] \quad \text{...[Eqn. (16.22)]}$$

or,

$$T_s = 320 \left[1 + \frac{1.4 - 1}{2} \times 0.7746^2\right] = 358.4 \text{ K or } \mathbf{85.4°C} \quad \textbf{(Ans.)}$$

Stagnation density, ρ_s :

$$\rho_s = \frac{p_s}{RT_s} = \frac{131.27 \times 10^3}{287 \times 358.4} = \mathbf{1.276\ kg/m^3} \quad \textbf{(Ans.)}$$

Compressibility factor at M = 0.8 :

$$\text{Compressibility factor} = 1 + \frac{M_0{}^2}{4} + \frac{2-\gamma}{24} M_0{}^4 + \ldots$$

$$= 1 + \frac{0.8^2}{4} + \frac{2-1.4}{24} \times 0.8^4 = \mathbf{1.1702} \quad \textbf{(Ans.)}$$

Example 16.10. *Air at a pressure of 220 kN/m² and temperature 27°C is moving at a velocity of 200 m/s. Calculate the stagnation pressure if*

(*i*) *Compressibility is neglected ;* (*ii*) *Compressibility is accounted for.*

For air take R = 287 J/kg K, γ = 1.4.

Sol. Pressure of air, p_0 = 200 kN/m²

Temperature of air, T_0 = 27 + 233 = 300 K

Velocity of air, V_0 = 200 m/s

Stagnation pressure, p_s :

(*i*) **Compressibility is neglected :**

$$p_s = p_0 + \frac{\rho_0 V_0{}^2}{2}$$

where $\rho_0 = \dfrac{p_0}{RT_0} = \dfrac{220 \times 10^3}{287 \times 300} = 2.555$ kg/m³

$$\therefore \quad p_s = 220 + \frac{2.555 \times 200^2}{2} \times 10^{-3}\ (\text{kN/m}^2) = \mathbf{271.1\ kN/m^2}.\ \textbf{Ans.}$$

(*ii*) **Compressibility is accounted for :**

The stagnation pressure, when compressibility is accounted for, is given by,

$$p_s = p_0 + \frac{\rho_0 V_0{}^2}{2}\left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0{}^4 + \ldots\right) \qquad \text{...[Eqn. (16.19)]}$$

Mach number, $M_0 = \dfrac{V_0}{C_0} = \dfrac{200}{\sqrt{\gamma R T_0}} = \dfrac{200}{\sqrt{1.4 \times 287 \times 300}} = 0.576$

Whence, $p_s = 220 + \dfrac{2.555 \times 200^2}{2} \times 10^{-3}\left(1 + \dfrac{0.576^2}{4} + \dfrac{2-1.4}{24} \times 0.576^4\right)$

or, p_s = 220 + 51.1 (1 + 0.0829 + 0.00275) = **275.47 kN/m² (Ans.)**

Example 16.11. *In aircraft flying at an altitude where the pressure was 35 kPa and temperature – 38°C, stagnation pressure measured was 65.4 kPa. Calculate the speed of the aircraft. Take molecular weight of air as 28.* **(UPSC, 1998)**

Sol. Pressure of air, p_0 = 35 kPa = 35 × 10³ N/m²

Temperature of air, T_0 = – 38 + 273 = 235 K

Stagnation pressure, p_s = 65.4 kPa = 65.4 × 10³ N/m²

Speed of the aircraft, V_a :

$$p_0V = mRT_0 = m \times \left(\frac{R_0}{M}\right) T_0 \text{ or } \rho_0 = \frac{m}{V} = \frac{p_0M}{R_0T_0}$$

where R = characteristic gas constant,

R_0 = universal gas constant = 8314 Nm/mole K.

M = molecular weight for air = 28, and

ρ_0 = density of air.

Substituting the values, we get

$$\rho_0 = \frac{(35 \times 10^3) \times 28}{8314 \times 235} = 0.5 \text{ kg/m}^3$$

Now, using the relation : $p_s = p_0 + \frac{\rho_0 V_a^2}{2}$ ...[Eqn. (16.20)]

or,
$$V_a = \sqrt{\frac{2(p_s - p)}{\rho_0}} = \sqrt{\frac{2(65.4 \times 10^3 - 35 \times 10^3)}{0.5}} = \mathbf{348.7\ m/s} \quad \textbf{(Ans.)}$$

16.7. AREA-VELOCITY RELATIONSHIP AND EFFECT OF VARIATION OF AREA FOR SUBSONIC, SONIC AND SUPERSONIC FLOWS

For an incompressible flow the continuity equation may be expressed as :

AV = constant, which when differentiated gives

$$AdV + VdA = 0 \quad \text{or} \quad \frac{dA}{A} = -\frac{dV}{V} \qquad \text{...(16.23)}$$

But in case of compressible flow, the continuity equation is given by,

ρAV = constant, which can be differentiated to give

$$\rho d(AV) + AVd\rho = 0 \quad \text{or} \quad \rho(AdV + VdA) + AVd\rho = 0$$

or,
$$\rho AdV + \rho VdA + AVd\rho = 0$$

Dividing both sides by ρAV, we get

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \qquad \text{...(16.24)}$$

or,
$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho} \qquad \text{...[16.24 (a)]}$$

The Euler's equation for compressible fluid is given by,

$$\frac{dp}{\rho} + VdV + gdz = 0$$

Neglecting the z terms the above equation reduces to, $\frac{dp}{\rho} + VdV = 0$

This equation can also be expressed as :

$$\frac{dp}{\rho} \times \frac{d\rho}{d\rho} + VdV = 0 \quad \text{or} \quad \frac{dp}{d\rho} \times \frac{d\rho}{\rho} + VdV = 0$$

But
$$\frac{dp}{d\rho} = C^2 \qquad \text{...[Eqn. (16.10)]}$$

$$\therefore \quad C^2 \times \frac{d\rho}{\rho} + VdV = 0 \quad \text{or} \quad C^2 \frac{d\rho}{\rho} = -VdV \quad \text{or} \quad \frac{d\rho}{\rho} = -\frac{VdV}{C^2}$$

Substituting the value of $\frac{d\rho}{\rho}$ in eqn. (16.24), we get

$$\frac{dV}{V} + \frac{dA}{A} - \frac{VdV}{C^2} = 0$$

or,

$$\frac{dA}{A} = \frac{VdV}{C^2} - \frac{dV}{V} = \frac{dV}{V}\left(\frac{V^2}{C^2} - 1\right)$$

$$\therefore \quad \frac{dA}{A} = \frac{dV}{V}(M^2 - 1) \qquad \left(\because \; M = \frac{V}{C}\right) \qquad ...(16.25)$$

This important equation is due to *Hugoniot*.

Eqns. (16.23) and (16.25) give variation of $\left(\frac{dA}{A}\right)$ for the flow of incompressible and compressible fluids respectively. The ratios $\left(\frac{dA}{A}\right)$ and $\left(\frac{dV}{V}\right)$ are respectively fractional variations in the values of area and flow velocity in the flow passage.

Further, in order to study the variation of pressure with the change in flow area, an expression similar to eqn. (16.25), as given below, can be obtained.

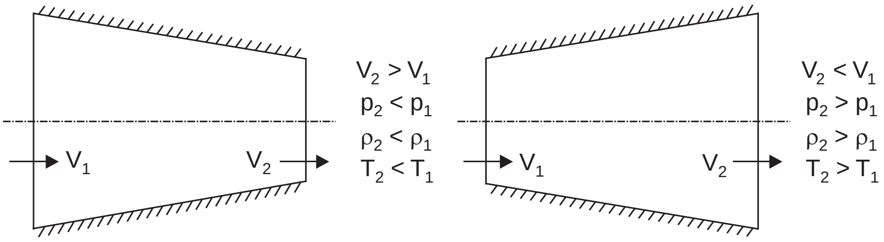
$$dp = \rho V^2 \left(\frac{1}{1 - M^2}\right)\frac{dA}{A} \qquad ...(16.26)$$

From eqns. (16.25) and (16.26), it is possible to formulate the following conclusions of practical significance.

(*i*) **For subsonic flow (M < 1) :**

$$\frac{dV}{V} > 0 \; ; \; \frac{dA}{A} < 0 \; ; \; dp < 0 \text{ (convergent nozzle)}$$

$$\frac{dV}{V} < 0 \; ; \; \frac{dA}{A} > 0 \; ; \; dp > 0 \text{ (divergent diffuser)}$$



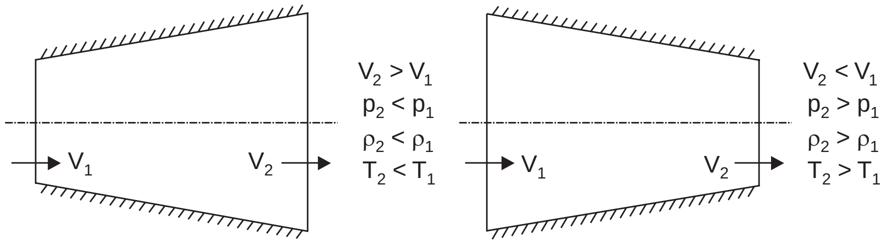
(*a*) Convergent nozzle. (*b*) Divergent diffuser.

Fig. 16.5. Subsonic flow ($M < 1$).

(*ii*) **For supersonic flow (M > 1) :**

$$\frac{dV}{V} > 0 \; ; \; \frac{dA}{A} > 0 \; ; \; dp < 0 \text{ (divergent nozzle)}$$

$$\frac{dV}{V} < 0 \; ; \; \frac{dA}{A} < 0 \; ; \; dp > 0 \text{ (convergent diffuser)}$$



(*a*) Divergent nozzle. (*b*) Convergent nozzle.

Fig. 16.6. Supersonic flow ($M > 1$).

(*iii*) **For sonic flow (M = 1) :**

$$\frac{dA}{A} = 0 \text{ (straight flow passage}$$

since dA must be zero)

and dp = (zero/zero) *i.e.*, indeterminate, but when evaluated, the change of pressure $p = 0$, since $dA = 0$ and the flow is frictionless.

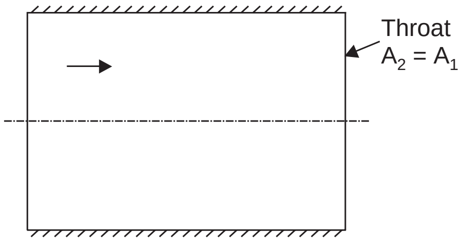


Fig. 16.7. Sonic flow ($M = 1$).

16.8. FLOW OF COMPRESSIBLE FLUID THROUGH A CONVERGENT NOZZLE

Fig. 16.8 shows a large tank/vessel fitted with a short convergent nozzle and containing a compressible fluid. Consider two points 1 and 2 inside the tank and exit of the nozzle respectively.

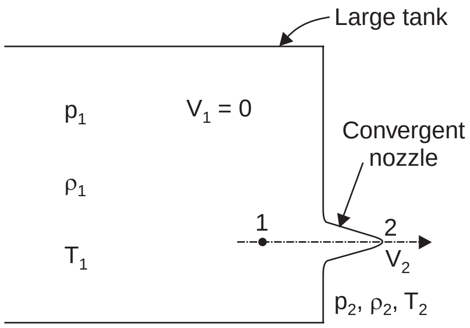


Fig. 16.8. Flow of fluid through a convergent nozzle.

Let p_1 = pressure of fluid at the point 1,

V_1 = velocity of fluid in the tank (= 0),

T_1 = temperature of fluid at point 1,

ρ_1 = density of fluid at point 1, and p_2, V_2, T_2 and ρ_2 are corresponding values of pressure, velocity, temperature and density at point 2.

Assuming the flow to take place *adiabatically*, then by using Bernoulli's equation (for adiabatic flow), we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 \qquad \text{[Eqn. (16.7)]}$$

But $z_1 = z_2$ and $V_1 = 0$

$$\therefore \qquad \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1 g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g}$$

or,

$$\left(\frac{\gamma}{\gamma-1}\right)\left[\frac{p_1}{\rho_1 g} - \frac{p_2}{\rho_2 g}\right] = \frac{V_2^2}{2g} \quad \text{or} \quad \frac{\gamma}{\gamma-1}\left[\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right] \frac{V_2^2}{2}$$

or
$$V_2 = \sqrt{\frac{2\gamma}{(\gamma - 1)}\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right)}$$

or
$$V_2 = \sqrt{\frac{2\gamma}{(\gamma - 1)}\,\frac{p_1}{\rho_1}\left(1 - \frac{p_2}{\rho_2} - \frac{\rho_1}{p_1}\right)} \qquad ...(1)$$

For adiabatic flow :
$$\frac{p_1}{{\rho_1}^{\gamma}} = \frac{p_2}{{\rho_2}^{\gamma}} \text{ or } \frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} \text{ or } \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \qquad ...(i)$$

Substituting the value of $\frac{\rho_1}{\rho_2}$ in eqn. (1), we get

$$V_2 = \sqrt{\frac{2\gamma}{(\gamma - 1)}\,\frac{p_1}{\rho_1}\left[1 - \frac{p_2}{p_1} \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}\right]} = \sqrt{\frac{2\gamma}{(\gamma - 1)}\,\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{1 - \frac{1}{\gamma}}\right]}$$

or
$$V_2 = \sqrt{\frac{2\gamma}{(\gamma - 1)}\,\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]} \qquad ...(16.27)$$

The mass rate of flow of the compressible fluid,

$m = \rho_2 A_2 V_2$, A_2 being the area of the nozzle at the exit

$$= \rho_2 A_2 \sqrt{\frac{2\gamma}{(\gamma - 1)}\,\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]}\,, \quad \text{[substituting } V_2 \text{ from eqn. (16.27)]}$$

or
$$m = A_2 \sqrt{\frac{2\gamma}{(\gamma - 1)}\,\frac{p_1}{\rho_1} \times {\rho_2}^2 \left[1 - \left(\frac{p_2}{\rho_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$$

From eqn. (*i*), we have
$$\rho_2 = \frac{\rho_1}{(p_1/p_2)^{1/\gamma}} = \rho_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

∴
$${\rho_2}^2 = {\rho_1}^2 \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}$$

Substituting this value in the above equation, we get

$$m = A_2 \sqrt{\frac{2\gamma}{\gamma - 1}\,\frac{p_1}{\rho_1} \times {\rho_1}^2 \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$$

$$= A_2 \sqrt{\frac{2\gamma}{\gamma - 1}\, p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma} + \frac{2}{\gamma}}\right]}$$

$$m = A_2 \sqrt{\frac{2\gamma}{\gamma - 1} p_1\rho_1 \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right]} \quad \text{...(16.28)}$$

The mass rate of flow (m) depends on the value of $\frac{p_2}{p_1}$ (for the given values of p_1 and ρ_1 at point 1).

Value of $\frac{\mathbf{p_2}}{\mathbf{p_1}}$ for maximum value of mass rate of flow :

For maximum value of m, we have $\frac{d}{d\left(\frac{p_2}{p_1}\right)}(m) = 0$

As other quantities except the ratio $\frac{p_2}{p_1}$ are constant

$$\therefore \quad \frac{d}{d\left(\frac{p_2}{p_1}\right)}(m) = \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right] = 0$$

or,
$$\frac{2}{\gamma}\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}-1} - \left(\frac{\gamma+1}{\gamma}\right)\left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}-1} = 0$$

or,
$$\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}-1} = \left(\frac{\gamma+1}{2}\right)\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \text{ or } \left(\frac{p_2}{p_1}\right)^{\frac{2-\gamma}{\gamma}} = \frac{\gamma+1}{2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

or,
$$\left(\frac{p_2}{p_1}\right)^{2-\gamma} = \left(\frac{\gamma+1}{2}\right)^{\gamma}\left(\frac{p_2}{p_1}\right)$$

or,
$$\left(\frac{p_2}{p_1}\right)^{2-\gamma-1} = \left(\frac{\gamma+1}{2}\right)^{\gamma} \text{ or } \left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{\gamma+1}{2}\right)^{\gamma}$$

or,
$$\left(\frac{p_2}{p_1}\right)^{\gamma-1} = \left(\frac{2}{\gamma+1}\right)^{\gamma}$$

or,
$$\left(\frac{p_2}{p_1}\right) = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{...(16.29)}$$

Eqn. (16.29) is the *condition for maximum mass flow rate through the nozzle.*

It may be pointed out that a convergent nozzle is employed when the exit pressure is equal to or more than the critical pressure, and a convergent-divergent nozzle is used when the discharge pressure is less than the critical pressure.

For air with $\gamma = 1.4$, the *critical pressure ratio,*

$$\frac{p_2}{p_1} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} = 0.528 \quad \text{...(16.30)}$$

Relevant relations for critical density and temperature are :

$$\frac{\rho_2}{\rho_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \qquad \text{...[16.30 (a)]}$$

$$\frac{T_2}{T_1} = \frac{2}{\gamma+1} \qquad \text{...[16.30 (b)]}$$

Value of V_2 for maximum rate of flow of fluid :

Substituting the value of $\frac{p_2}{p_1}$ from eqn. (16.29) in eqn. (16.27), we get

$$V_2 = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left[1-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}\times\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left(1-\frac{2}{\gamma+1}\right)}$$

$$= \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left(\frac{\gamma+1-2}{\gamma+1}\right)} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left(\frac{\gamma-1}{\gamma+1}\right)}$$

or

$$V_2 = \sqrt{\frac{2\gamma}{\gamma+1}\frac{p_1}{\rho_1}} \; (= C_2) \qquad \text{...(16.31)}$$

Maximum rate of flow of fluid through nozzle, m_{max} :

Substituting the value of $\frac{p_2}{p_1}$ from eqn. (16.30) in eqn. (16.28), we get

$$m_{max} = A_2\sqrt{\left(\frac{2\gamma}{\gamma-1}\right)p_1\rho_1\left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1}\times\frac{2}{\gamma}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}\times\frac{\gamma+1}{\gamma}}\right]}$$

$$= A_2\sqrt{\left(\frac{2\gamma}{\gamma-1}\right)p_1\rho_1\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

For air, $\gamma = 1.4$,

$$\therefore \qquad m_{max} = A_2\sqrt{\frac{2\times1.4}{(1.4-1)}p_1\rho_1\left[\left(\frac{2}{1.4+1}\right)^{\frac{2}{1.4-1}} - \left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{1.4-1}}\right]}$$

$$= A_2\sqrt{7\,p_1\rho_1\,(0.4018-0.3348)}$$

or

$$m_{max} = 0.685\,A_2\sqrt{p_1\rho_1} \qquad \text{...(16.32)}$$

Variation of mass flow rate of compressible fluid with pressure ratio $\left(\frac{p_2}{p_1}\right)$:

A passage in which the sonic velocity has been reached and thus in which the mass flow rate is maximum, is often said to be **choked** or in **choking conditions**. It is evident from eqn. (16.28) that for a fixed value of inlet pressure the mass flow depends on nozzle exit pressure.

Fig. 16.9. depicts the variation of actual and theoretical mass flow rate versus $\frac{p_2}{p_1}$. Following points are *worthnoting* :

(*i*) The flow rate increases with a decrease in the pressure ratio $\frac{p_2}{p_1}$ and attains the maximum value of the critical pressure ratio $\frac{p_2}{p_1}$ = 0.528 for air.

(*ii*) With further decrease in exit pressure below the critical value, the theoretical mass flow rate decreases. This is contrary to the actual results where the mass flow rate remains constant after attaining the maximum value. This may be explained as follows :

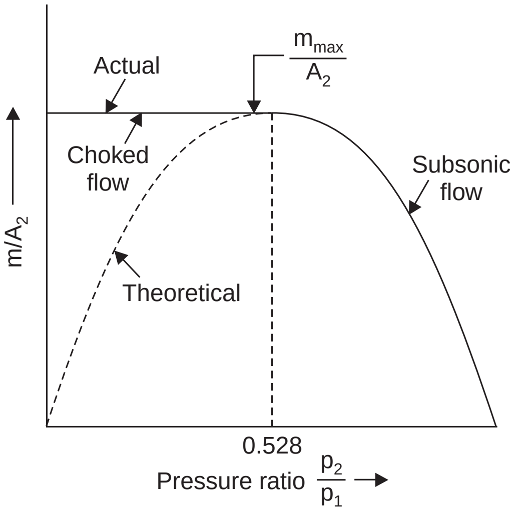


Fig. 16.9. Mass flow rate through a convergent nozzle.

At critical pressure ratio, the velocity V_2 at the throat is equal to the sonic speed (derived below). For an accelerating flow of a compressible fluid in a convergent nozzle the velocity of flow within the nozzle is subsonic with a maximum velocity equal to the sonic velocity at the throat : Thus once the velocity V_2 at the throat has attained the sonic speed at the critical pressure ratio, it remains at the same value for all the values of $\left(\frac{p_2}{p_1}\right)$ less than critical pressure ratio, since the flow in the nozzle is being continuously accelerated with the reduction in the throat pressure below the critical values and hence the velocity cannot reduce. Thus, the mass flow rate for all values of $\left(\frac{p_2}{p_1}\right)$ less than critical pressure ratio remains constant at the maximum value (indicated by the solid horizontal line in Fig. 16.9). This fact has been verified experimentally too.

Velocity at outlet of nozzle for maximum flow rate :

The velocity at outlet of nozzle for maximum flow rate is given by,

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right)\frac{p_1}{\rho_1}} \qquad \text{...[Eqn. (16.31)]}$$

Now pressure ratio,
$$\frac{p_2}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

∴
$$p_1 = \frac{p_2}{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}} = p_2\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

For *adiabatic flow* :
$$\frac{p_1}{{\rho_1}^{\gamma}} = \frac{p_2}{{\rho_2}^{\gamma}} \quad \text{or} \quad \frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} \quad \text{or} \quad \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}}$$

$$\therefore \qquad \rho_1 = \rho_2 \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}} \text{ or } \rho_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}\times\frac{1}{\gamma}} \text{ or } \rho_2 \left(\frac{2}{\gamma+1}\right)^{\frac{1}{1-\gamma}}$$

Substituting the values of p_1 and ρ_1 in the above eqn. (16.31), we get

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times p_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{1-\gamma}} \times \left\{\frac{1}{\rho_2} \times \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\right\}}$$

$$= \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \times \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{1-\gamma}+\frac{1}{\gamma-1}}} = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \left(\frac{2}{\gamma+1}\right)^{-1}}$$

or
$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \left(\frac{\gamma+1}{2}\right)} = \sqrt{\frac{\gamma p_2}{\rho_2}} = C_2$$

i.e.,
$$V_2 = C_2 \qquad \text{...(16.33)}$$

Hence the *velocity* at the outlet of nozzle for maximum flow rate equals sonic velocity.

16.9. VARIABLES OF FLOW IN TERMS OF MACH NUMBER

In order to obtain relationship involving change in velocity, pressure, temperature and density in terms of the Mach number use is made of the continuity, perfect gas, isentropic flow and energy equations.

For *continuity equation*, we have

$$\rho AV = \text{constant}$$

Differentiating the above equation, we get

$$\rho[AdV + VdA] + AVd\rho = 0$$

Dividing throughout by ρAV, we have

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$$

From *isentropic flow*, we have $\frac{p}{\rho^\gamma}$ = constant or $\frac{dp}{p} = \gamma \frac{dp}{p}$

For *perfect gas*, we have $p = \rho RT$ or $\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$

From *energy equation*, we have $c_pT + \frac{V^2}{2}$ + constant

Differentiating throughout, we get

$$c_p dT + VdV = 0 \quad \text{or} \quad \left(\frac{\gamma R}{\gamma - 1}\right) dT + VdV = 0 \qquad \left(\because\ c_p = \frac{\gamma R}{\gamma - 1}\right)$$

or,
$$\frac{\gamma R}{\gamma - 1} \frac{dT}{V^2} + \frac{dV}{V} = 0 \qquad \text{...}(i)$$

Also, sonic velocity, $C = \sqrt{\gamma RT}$ $\quad \therefore \gamma R = \frac{C^2}{T}$

Substituting the value of $\gamma R = \frac{C^2}{T}$ in eqn. (*i*), we get

$$\frac{C^2}{(\gamma - 1)T} \times \frac{dT}{V^2} + \frac{dV}{V} = 0$$

or,
$$\frac{1}{(\gamma - 1)\, M^2} \times \frac{dT}{T} + \frac{dV}{V} = 0 \qquad \left(\because \; M = \frac{V}{C}\right) \qquad \text{...(16.34)}$$

From the Mach number relationship

$$M = \frac{V}{\sqrt{\gamma RT}} \text{ (where } \sqrt{\gamma RT} = C)$$

$$\frac{dM}{M} = \frac{dV}{V} - \frac{1}{2}\,\frac{dT}{T} \qquad \text{...(16.35)}$$

Substituting the value of $\frac{dT}{T}$ from eqns. (16.34) in eqn. (16.35), we get

$$\frac{dM}{M} = \frac{dV}{V} - \frac{1}{2}\left[-\frac{dV}{V} \times (\gamma - 1)\, M^2\right]$$

$$= \frac{dV}{V} + \frac{1}{2}\,\frac{dV}{V} \times (\gamma - 1)\, M^2$$

or,
$$\frac{dM}{M} = \frac{dV}{V}\left[1 + \frac{\gamma - 1}{2} M^2\right] \text{ or } \frac{dV}{V} = \frac{1}{\left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]}\,\frac{dM}{M} \qquad \text{...(16.36)}$$

Since the quantity within the bracket is *always positive*, the *trend of variation of velocity and Mach number is similar. For temperature variation, one can write*

$$\frac{dT}{T} = \left[\frac{-(\gamma - 1)\, M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}\right] \frac{dM}{M} \qquad \text{...(16.37)}$$

Since the right hand side is *negative the temperature changes follow an opposite trend to that of Mach number*. Similarly for *pressure and density*, we have

$$\frac{dp}{p} = \left[\frac{-\gamma M^2}{1 + \frac{\gamma - 1}{2} M^2}\right] \frac{dM}{M} \qquad \text{...(16.38)}$$

and,
$$\frac{d\rho}{\rho} = \left[\frac{-M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}\right] \frac{dM}{M} \qquad \text{...(16.39)}$$

For changes in *area*, we have

$$\frac{dA}{A} = \left[\frac{-(1 - M^2)}{1 + \frac{\gamma - 1}{2} M^2}\right] \frac{dM}{M} \qquad \text{...(16.40)}$$

The quantity within the brackets may be *positive or negative* depending upon the *magnitude of Mach number*. By integrating eqn. (16.40), we can obtain a relationship between the critical throat area A_c, where Mach number is *unity* and the area A at any section where $M \gtrless 1$

$$\frac{A}{A_c} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad \text{...(16.41)}$$

Example 16.12. *The pressure leads from Pitot-static tube mounted on an aircraft were connected to a pressure gauge in the cockpit. The dial of the pressure gauge is calibrated to read the aircraft speed in m/s. The calibration is done on the ground by applying a known pressure across the gauge and calculating the equivalent velocity using incompressible Bernoulli's equation and assuming that the density is 1.224 kg/m³.*

The gauge having been calibrated in this way the aircraft is flown at 9200 m, where the density is 0.454 kg/m³ and ambient pressure is 30 kN/m². The gauge indicates a velocity of 152 m/s. What is the true speed of the aircraft ? **(UPSC)**

Sol. Bernoulli's equation for an *incompressible flow* is given by,

$$p + \frac{\rho V^2}{2} = \text{constant}$$

The stagnation pressure (p_s) created at Pitot-static tube,

$$p_s = p_0 + \frac{\rho_0 V_0^2}{2} \text{ (neglecting compressibility effects)} \quad \text{...}(i)$$

Here $p_0 = 30$ kN/m², $V_0 = 152$ m/s, $\rho_0 = 1.224$ kg/m³ ...(*Given*)

$$\therefore \quad p_s = 30 + \frac{1.224 \times 152^2}{2} \times 10^{-3} = 44.139 \text{ kN/m}^2$$

Neglecting compressibility effect, the speed of the aircraft when

$\rho_0 = 0.454$ kg/m³ is given by [using eqn. (*i*)],

$$44.139 \times 10^3 = 30 \times 10^3 + \frac{0.454 \times V_0^2}{2}$$

or

$$V_0^2 = \frac{(44.139 - 30) \times 10^3 \times 2}{0.454} = 62286.34$$

$$\therefore \quad V_0 = 249.57 \text{ m/s}$$

Sonic velocity,

$$C_0 = \sqrt{\gamma R T_0} = \sqrt{\gamma \frac{p_0}{\rho_0}} = \sqrt{1.4 \times \frac{30 \times 10^3}{0.454}} = 304.16 \text{ m/s}$$

Mach number,

$$M = \frac{V_0}{C_0} = \frac{249.57}{304.16} = 0.82$$

Compressibility correction factor $= \left(1 + \frac{M_0^2}{4}\right)$, neglecting the terms containing higher powers of M_0 (from eqn 16.19).

$$= \left(1 + \frac{0.82}{4}\right) = 1.168$$

$\therefore$ True speed of aircraft $= \dfrac{249.57}{\sqrt{1.168}} = 230.9$ m/s

Hence true speed of aircraft = **230.9 m/s (Ans.)**

Example 16.13. (*a*) *In case of isentropic flow of a compressible fluid through a variable duct, show that*

$$\frac{A}{A_c} = \frac{1}{M}\left[\frac{1+\frac{1}{2}(\gamma-1)M^2}{\frac{1}{2}(\gamma+1)}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

where γ is the ratio of specific heats, M is the Mach number at a section whose area is A and A_c is the critical area of flow.

(*b*) *A supersonic nozzle is to be designed for air flow with Mach number 3 at the exit section which is 200 mm in diameter. The pressure and temperature of air at the nozzle exit are to be 7.85 kN/m^2 and 200 K respectively. Determine the reservoir pressure, temperature and the throat area. Take : $\gamma = 1.4$.* **(U.P.S.C. Exam.)**

Sol. (*a*) Please Ref. to Art. 16.9.

(*b*) Mach number, $M = 3$

Area at the exit section, $A = \pi/4 \times 0.2^2 = 0.0314$ m^2

Pressure of air at the nozzle, $(p)_{\text{nozzle}} = 7.85$ kN/m^2

Temperature of air at the nozzle, $(T)_{\text{nozzle}} = 200$ K

Reservoir pressure, $(p)_{\text{res.}}$:

From eqn. (16.17),
$$(p)_{\text{res.}} = (p)_{\text{nozzle}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)}$$

or,
$$(p)_{\text{res.}} = 7.85\left[1+\left(\frac{1.4-1}{2}\right)\times 3^2\right]^{\left(\frac{1.4}{1.4-1}\right)} = \mathbf{288.35\ kN/m^2} \quad \textbf{(Ans.)}$$

Reservoir temperature, $(T)_{\text{res}}$. :

From eqn. (16.22),
$$(T)_{\text{res.}} = (T)_{\text{nozzle}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]$$

or,
$$(T)_{\text{res.}} = 200\left[1+\left(\frac{1.4-1}{2}\right)\times 3^2\right] = \mathbf{560\ K} \quad \textbf{(Ans.)}$$

Throat area (critical), A_c :

From eqn. (16.41),
$$\frac{A}{A_c} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

or,
$$\frac{0.0314}{A_c} = \frac{1}{3}\left[\frac{2+(1.4-1)3^2}{1.4+1}\right]^{\frac{1.4+1}{2(1.4-1)}} \quad \text{or} \quad \frac{0.0314}{A_c} = \frac{1}{3}(2.333)^3 = 4.23$$

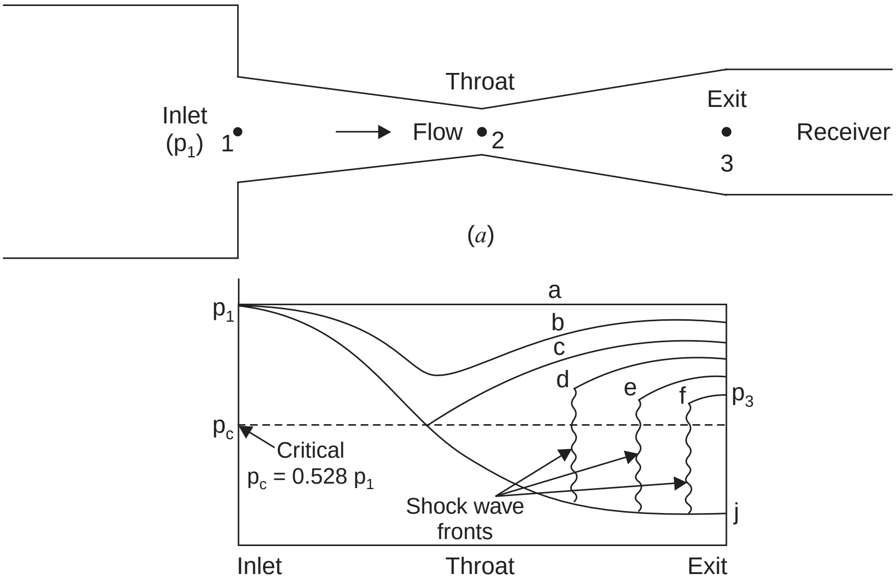
or,
$$A_c = \frac{0.0314}{4.23} = \mathbf{0.00742\ m^2} \quad \textbf{(Ans.)}$$

16.10. FLOW THROUGH LAVAL NOZZLE (CONVERGENT-DIVERGENT NOZZLE)

Laval nozzle is a convergent-divergent nozzle (named after de Laval, the swedish scientist who invented it) in which *subsonic flow prevails in the converging section, critical or transonic conditions in the throat* and *supersonic flow in the diverging section.*

Let p_2 (= p_c) = pressure in the throat when the flow is sonic for given pressure p_1.

- When the pressure in the receiver, $p_3 = p_1$, there will be no flow through the nozzle, this is shown by line *a* in Fig. 16.10 (*b*).



(*a*)

(*b*)

Fig. 16.10. (*a*) Laval nozzle (convergent-divergent nozzle) ; (*b*) Pressure distribution through a convergent-divergent nozzle with flow of compressible fluid.

- When the receiver pressure is reduced, flow will occur through the nozzle. As long as the value of p_3 is such that throat pressure p_2 is greater than the critical pressure 0.528 p_1, the flow in the converging and diverging sections will be subsonic. This condition is shown by line '*b*'.
- With further reduction in p_3, a stage is reached when p_2 is equal to critical pressure $p_c = 0.528\, p_1$, at this line $M = 1$ in the throat. This condition is shown by line '*c*'. Flow is subsonic on the upstream as well the downstream of the throat. The flow is also *isentropic*.
- If p_3 *is further reduced, it does not effect the flow in convergent section. The flow in throat is sonic, downstream it is supersonic*. Somewhere in the diverging section a shock wave occurs and flow changes to subsonic (curve *d*). The flow *across the shock is non-isentropic. Downstream of the shock wave the flow is subsonic and decelerates.*
- If the value of p_3 is further reduced, the shock wave forms somewhat downstream (curve *e*).
- For p_3 equal to p_j, the shock wave will occur just at the exit of divergent section.
- If the value of p_3 lies before p_f and p_j oblique waves are formed at the exit.

Example 16.14. *A large tank contains air at 284 kN/m² gauge pressure and 24°C temperature. The air flows from the tank to the atmosphere through a convergent nozzle. If the diameter at the outlet of the nozzle is 20 mm, find the maximum flow rate of air.*

Take : R = 287 J/kg K, γ = 1.4 and atmospheric pressure = 100 kN/m².

(Punjab University)

Sol. Pressure in the tank, $p_1 = 284 \text{ kN/m}^2$ (gauge)

$= 284 + 100 = 384 \text{ kN/m}^2$ (absolute)

Temperature in the tank, $T_1 = 24 + 273 = 297 \text{ K}$

Diameter at the outlet of the nozzle, $D = 20 \text{ mm} = 0.02 \text{ m}$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.02^2 = 0.0003141 \text{ m}^2$$

$$R = 287 \text{ J/kg K}, \ \gamma = 1.4$$

(Two points are considered. Point 1 lies inside the tank and point 2 lies at the exit of the nozzle).

Maximum flow rate, m_{max} :

Equation of state is given by $p = \rho RT$ or $\rho = \dfrac{p}{RT}$

$$\therefore \quad \rho_1 = \frac{p_1}{RT_1} = \frac{384 \times 10^3}{287 \times 297} = 4.5 \text{ kg/m}^3$$

The fluid parameters in the tank correspond to the stagnation values, and maximum flow rate of air is given by,

$$m_{max} = 0.685 \ A_2 \sqrt{p_1 \rho_1} \qquad \text{...[Eqn. (16.32)]}$$

$$= 0.685 \times 0.0003141 \sqrt{384 \times 10^3 \times 4.5} = 0.283 \text{ kg/s}$$

Hence maximum flow rate of air = **0.283 kg/s (Ans.)**

Example 16.15. *A large vessel, fitted with a nozzle, contains air at a pressure of 2500 kN/m^2 (abs.) and at a temperature of 20°C. If the pressure at the outlet of the nozzle is 1750 kN/m^2, find the velocity of air flowing at the outlet of the nozzle.*

Take : R = 287 J/kg K and γ = 1.4.

Sol. Pressure inside the vessel, $p_1 = 2500 \text{ kN/m}^2$ (abs.)

Temperature inside the vessel, $T_1 = 20 + 273 = 293 \text{ K}$

Pressure at the outlet of the nozzle, $p_2 = 1750 \text{ kN/m}^2$ (abs.)

$$R = 287 \text{ J/kg K}, \ \gamma = 1.4$$

Velocity of air, V_2 :

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right) \frac{p_1}{\rho_1} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]} \qquad \text{...[Eqn. (16.27)]}$$

where,
$$\rho_1 = \frac{p_1}{RT_1} \left(\text{From equation of state} : \frac{p}{\rho} = RT\right)$$

$$= \frac{2500 \times 10^3}{287 \times 293} = 29.73 \text{ kg/m}^3$$

Substituting the values in the above equation, we get

$$V_2 = \sqrt{\left(\frac{2 \times 1.4}{1.4 - 1}\right) \times \frac{2500 \times 10^3}{29.73} \left[1 - \left(\frac{1750}{2500}\right)^{\frac{1.4 - 1}{1.4}}\right]}$$

$$= \sqrt{7 \times 84090\,(1 - 0.903)} = 238.9 \text{ m/s}$$

i.e., $V_2 =$ **238.9 m/s (Ans.)**

Example 16.16. *A tank fitted with a convergent nozzle contains air at a temperature of 20°C. The diameter at the outlet of the nozzle is 25 mm. Assuming adiabatic flow, find the mass rate of flow of air through the nozzle to the atmosphere when the pressure in the tank is :*

(*i*) *140 kN/m² (abs.),* (*ii*) *300 kN/m²*

Take for air : R = 287 J/kg K and γ = 1.4, Barometric pressure = 100 kN/m².

Sol. Temperature of air in the tank, $T_1 = 20 + 273 = 293$ K

Diameter at the outlet of the nozzle, $D_2 = 25$ mm $= 0.025$ m

Area, $A_2 = \pi/4 \times 0.025^2 = 0.0004908 \text{ m}^2$

$R = 287$ J/kg K, $\gamma = 1.4$

(*i*) **Mass rate of flow of air when pressure in the tank is 140 kN/m² (abs.) :**

$$\rho_1 = \frac{p_1}{RT_1} = \frac{140 \times 10^3}{287 \times 293} = 1.665 \text{ kg/m}^3$$

$p_1 = 140 \text{ kN/m}^2$ (abs.)

Pressure at the nozzle, $p_2 =$ atmospheric pressure $= 100 \text{ kN/m}^2$

∴ Pressure ratio, $\dfrac{p_2}{p_1} = \dfrac{100}{140} = 0.7143$

Since the pressure ratio is more than the critical value, flow in the nozzle will be *subsonic,* hence mass rate of flow of air is given by eqn. 16.28, as

$$m = A_2 \sqrt{\frac{2\gamma}{\gamma - 1}\, p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$= 0.0004908 \sqrt{\left(\frac{2 \times 1.4}{1.4 - 1}\right) \times 140 \times 10^3 \times 1.665 \left[(0.7143)^{\frac{2}{1.4}} - (0.7143)^{\frac{1.4+1}{1.4}}\right]}$$

$$= 0.0004908 \sqrt{1631700\,(0.7143)^{1.4285} - (0.7143)^{1.7142}}$$

or $m = 0.0004908 \sqrt{1631700\,(0.6184 - 0.5617)} =$ **0.1493 kg/s (Ans.)**

(*ii*) **Mass rate of flow of air when pressure in the tank is 300 kN/m² (abs.) :**

$p_1 = 300 \text{ kN/m}^2$ (abs.)

$p_2 =$ pressure at the nozzle $=$ atmospheric pressure $= 100 \text{ kN/m}^2$

∴ Pressure ratio, $\dfrac{p_2}{p_1} = \dfrac{100}{300} = 0.33.$

The pressure ratio being less than the critical ratio 0.528, the flow in the nozzle will be *sonic*, the flow rate is maximum and is given by eqn. (16.32), as

$$m_{\max} = 0.685\, A_2 \sqrt{p_1 \rho_1}$$

where, $\rho_1 = \dfrac{p_1}{RT_1} = \dfrac{300 \times 10^3}{287 \times 293} = 3.567 \text{ kg/m}^3$

∴ $m_{\max} = 0.685 \times 0.0004908 \sqrt{300 \times 10^3 \times 3.567} =$ **0.3477 kg/s (Ans.)**

Example 16.17. *At some section in the convergent-divergent nozzle, in which air is flowing, pressure, velocity, temperature and cross-sectional area are 200 kN/m², 170 m/s, 200°C and 1000 mm² respectively. If the flow conditions are isentropic, determine :*

(*i*) *Stagnation temperature and stagnation pressure,*

(*ii*) *Sonic velocity and Mach number at this section,*

(*iii*) *Velocity, Mach number and flow area at outlet section where pressure is 110 kN/m²,*

(*iv*) *Pressure, temperature, velocity and flow area at throat of the nozzle.*

Take for air : $R = 287$ *J/kg K,* $c_p = 1.0$ *kJ/kg K* and $\gamma = 1.4$.

Sol. Let subscripts 1, 2 and t refers to the conditions at given section, outlet section and throat section of the nozzle respectively.

Pressure in the nozzle, $p_1 = 200 \text{ kN/m}^2$

Velocity of air, $V_1 = 170 \text{ m/s}$

Temperature, $T_1 = 200 + 273 = 473 \text{ K}$

Cross-sectional area, $A_1 = 1000 \text{ mm}^2 = 1000 \times 10^{-6} = 0.001 \text{ m}^2$

For air : $R = 287 \text{ J/kg K}, c_p = 1.0 \text{ kJ/kg K}, \gamma = 1.4$

(*i*) **Stagnation temperature (T_s) and stagnation pressure (p_s) :**

Stagnation temperature,
$$T_s = T_1 + \frac{V_1^2}{2 \times c_p}$$
$$= 473 + \frac{170^2}{2 \times (1.0 \times 1000)} = \mathbf{487.45 \text{ K (or } 214.45°\text{C)}} \quad \textbf{(Ans.)}$$

Also,
$$\frac{p_s}{p_1} = \left(\frac{T_s}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{487.45}{473}\right)^{\frac{1.4}{1.4-1}} = 1.111$$

∴ Stagnation pressure, $p_s = 200 \times 1.111 =$ **222.2 kN/m²** **(Ans.)**

(*ii*) **Sonic velocity and Mach number at this section :**

Sonic velocity, $C_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 473} =$ **435.9 m/s** **(Ans.)**

Mach number, $M_1 = \dfrac{V_1}{C_1} = \dfrac{170}{435.9} =$ **0.39** **(Ans.)**

(*iii*) **Velocity, Mach number and flow area at outlet section where pressure is 110 kN/m² :**

Pressure at outlet section, $p_2 = 110 \text{ kN/m}^2$ ...(*Given*)

From eqn (16.17),
$$\frac{p_s}{p_1} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_2^2\right]^{\frac{\gamma}{\gamma - 1}}$$
$$\frac{222.2}{110} = \left[1 + \left(\frac{1.4 - 1}{2}\right) M_0^2\right]^{\frac{1.4}{1.4-1}} = (1 + 0.2\, M_0^2)^{3.5}$$

or,
$$(1 + 0.2\, M_2^2) = \left(\frac{222.2}{110}\right)^{\frac{1}{3.5}} = 1.222$$

or,
$$M_2 = \left(\frac{1.222 - 1}{0.2}\right)^{1/2} = \mathbf{1.05} \quad \textbf{(Ans.)}$$

Also, $$\frac{T_2}{T_s} = \left(\frac{p_2}{p_s}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{110}{222.2}\right)^{\frac{1.4-1}{1.4}} = 0.818$$

or, $$T_2 = 0.818 \times 487.45 = 398.7 \text{ K}$$

Sonic velocity at outlet section,

$$C_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \times 287 \times 398.7} = 400.25 \text{ m/s}$$

∴ Velocity at outlet section, $V_2 = M_2 \times C_2 = 1.05 \times 400.25 =$ **420.26 m/s. Ans.**

Now, mass flow at the given section = mass flow at outlet section (exit)

......continuity equation

i.e., $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{or} \quad \frac{p_1}{RT_1} A_1 V_1 = \frac{p_2}{RT_2} A_2 V_2$$

∴ Flow area at the outlet section,

$$A_2 = \frac{p_1 A_1 V_1 T_2}{T_1 p_2 V_2} = \frac{200 \times 0.001 \times 170 \times 398.7}{473 \times 110 \times 420.26} = 6.199 \times 10^{-4} \text{ m}^2$$

Hence, $A_2 = 6.199 \times 10^{-4}$ m^2 or **619.9 mm^2. Ans.**

(*iv*) **Pressure (p_t), temperature (T_t), velocity (V_t), and flow area (A_t) at throat of the nozzle :**

At throat, *critical conditions prevail, i.e.* the flow velocity becomes *equal to the sonic velocity* and *Mach number attains* a unit value.

From eqn. (16.22), $$\frac{T_s}{T_t} = \left[1 + \left(\frac{\gamma-1}{2}\right) M_t^2\right]$$

or, $$\frac{487.45}{T_t} = \left[1 + \left(\frac{1.4-1}{2}\right) \times 1^2\right] = 1.2 \text{ or } T_t = 406.2 \text{ K}$$

Hence $T_t =$ **406.2 K (or 133.2°C). Ans.**

Also, $$\frac{p_t}{T_s} = \left(\frac{T_t}{T_s}\right)^{\frac{\gamma}{\gamma-1}} \text{ or } \frac{p_t}{222.2} = \left(\frac{406.2}{487.45}\right)^{\frac{1.4}{1.4-1}} = 0.528$$

or, $p_t = 222.2 \times 0.528 =$ **117.32 kN/m^2. Ans.**

Sonic velocity (corresponding to throat conditions),

$$C_t = \sqrt{\gamma R T_t} = \sqrt{1.4 \times 287 \times 406.2} = 404 \text{ m/s}$$

∴ Flow velocity, $V_t = M_t \times C_t = 1 \times 404 = 404$ m/s

By continuity equation, we have : $\rho_1 A_1 V_1 = \rho_t A_t V_t$

or, $$\frac{p_1}{RT_1} A_1 V_1 = \frac{p_t}{RT_t} A_t V_t$$

∴ Flow area at throat, $$A_t = \frac{p_1 A_1 V_1 T_t}{T_1 p_t V_t} = \frac{200 \times 0.001 \times 170 \times 406.2}{473 \times 117.32 \times 404} = 6.16 \times 10^{-4} \text{ m}^2$$

Hence, $A_t = 6.16 \times 10^{-4}$ m^2 or **616 mm^2 (Ans.)**

16.11. SHOCK WAVES

Whenever a supersonic flow (compressible) abruptly changes to subsonic flow, a shock *wave* (analogous to hydraulic jump in an open channel) is produced, *resulting in a sudden rise in pressure, density, temperature and entropy.* This occurs due to pressure differentials and when the Mach number of the approaching flow $M_1 > 1$. A shock wave is a pressure wave of finite thickness, of the order of 10^{-2} to 10^{-4} mm in the atmospheric pressure. A shock wave takes place in the *diverging section of a nozzle*, in a *diffuser, throat of a supersonic wind tunnel*, in *front of sharp nosed bodies.*

Shock waves are of two types :

1. Normal shocks which are almost perpendicular to the flow.
2. Oblique shocks which are inclined to the flow direction.

16.11.1. Normal Shock Wave

Consider a duct having a compressible sonic flow (see Fig. 16.11).

Let p_1, ρ_1, T_1, and V_1 be the pressure, density, temperature and velocity of the flow ($M_1 > 1$) and p_2, ρ_2, T_2 and V_2 the corresponding values of pressure, density, temperature and velocity after a shock wave takes place ($M_2 < 1$).

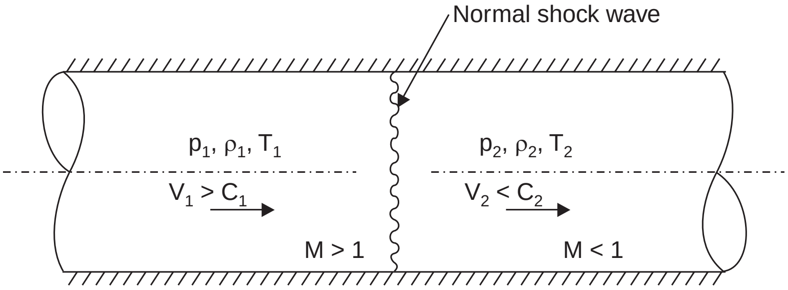


Fig. 16.11. Normal shock wave.

For analysing a normal shock wave, use will be made of the *continuity, momentum* and *energy equations*.

Assume unit area cross-section, $A_1 = A_2 = 1$.

Continuity equation : $m = \rho_1 V_1 = \rho_2 V_2$ ...(*i*)

Momentum equation : $\Sigma F_x = p_1A_1 - p_2A_2 = m\,(V_2 - V_1) = \rho_2 A_2 V_2^2 - \rho_1 A_1 V_1^2$

for $A_1 = A_2 = 1$, the pressure drop across the shock wave,

$$p_1 - p_2 = \rho_2 V_2^2 - \rho_1 V_1^2 \qquad \text{...}(ii)$$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

Consider the flow across the shock wave as adiabatic.

Energy equation : $$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1} + \frac{V_1^2}{2} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2} + \frac{V_2^2}{2} \qquad \text{...[Eqn. (16.7)]}$$

($z_1 = z_2$, duct being in horizontal position)

or, $$\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) = \frac{V_1^2 - V_2^2}{2} \qquad \text{...}(iii)$$

Combining continuity and momentum equations [refer eqns. (*i*) and (*ii*)], we get

$$p_1 + \frac{(\rho_1 V_1)^2}{\rho_1} = p_2 + \frac{(\rho_2 V_2)^2}{\rho_2} \qquad \text{...(16.42)}$$

This equation is 'known as **Rankine Line Equation.**

Now combining continuity and energy equations [refer eqns. (*i*) and (*iii*)], we get

$$\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_1}\right)+\frac{(\rho_1 V_1)^2}{2\rho_1^{\,2}}=\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2}\right)+\frac{(\rho_2 V_2)^2}{2\rho_2^{\,2}} \quad \text{...(16.43)}$$

This equation is called **Fanno Line Equation.**

Further combining eqns. (*i*), (*ii*) and (*iii*) and solving for $\frac{p_2}{p_1}$, we get

$$\frac{p_2}{p_1}=\frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_2}{\rho_1}-1}{\left(\frac{\gamma+1}{\gamma-1}\right)-\frac{\rho_2}{\rho_1}} \quad \text{...(16.44)}$$

Solving for density ratio $\frac{\rho_2}{\rho_1}$, the same equations yield

$$\frac{\rho_2}{\rho_1}=\frac{V_1}{V_2}=\frac{1+\left(\frac{\gamma+1}{\gamma-1}\right)\frac{p_2}{p_1}}{\left(\frac{\gamma+1}{\gamma-1}\right)+\frac{p_2}{p_1}} \quad \text{...(16.45)}$$

The eqns. (16.44) and (16.45) are called **Ranking-Hugoniot equations.**

One can also express $\frac{p_2}{p_1}$, $\frac{V_2}{V_1}$, $\frac{\rho_2}{\rho_1}$ and $\frac{T_2}{T_1}$ in terms of Mach number as follows :

$$\frac{p_2}{p_1}=\frac{2\gamma M_1^{\,2}-(\gamma-1)}{(\gamma+1)} \quad \text{...(16.46)}$$

$$\frac{V_1}{V_2}=\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)M_1^{\,2}}{(\gamma-1)M_1^{\,2}+2} \quad \text{...(16.47)}$$

$$\frac{T_2}{T_1}=\frac{[(\gamma-1)M_1^{\,2}+2]\,[2\gamma M_1^{\,2}-(\gamma-1)]}{(\gamma+1)^2 M_1^{\,2}} \quad \text{...(16.48)}$$

By algebraic manipulation the following equation between M_1 and M_2 can be obtained.

$$M_2^{\,2}=\frac{(\gamma-1)M_1^{\,2}+2}{2\gamma M_1^{\,2}-(\gamma-1)} \quad \text{...(16.49)}$$

Example 16.18. *For a normal shock wave in air Mach number is 2. If the atmospheric pressure and air density are 26.5 kN/m² and 0.413 kg/m³ respectively, determine the flow conditions before and after the shock wave. Take γ = 1.4.*

Sol. Let subscripts 1 and 2 represent the flow conditions before and after the shock wave.

Mach number, $M_1 = 2$

Atmospheric pressure, $p_1 = 26.5\ \text{kN/m}^2$

Air density, $\rho_1 = 0.413\ \text{kg/m}^3$

Mach number, M_2 :

$$M_2^2 = \frac{(\gamma - 1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)} \qquad \text{...[Eqn. (16.49)]}$$

$$= \frac{(1.4 - 1) \times 2^2 + 2}{2 \times 1.4 \times 2^2 - (1.4 - 1)} = \frac{3.6}{11.2 - 0.4} = 0.333$$

$\therefore$ M_2 = **0.577 (Ans.)**

Pressure, p_2 :

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{(\gamma + 1)} \qquad \text{...[Eqn. (16.46)]}$$

$$= \frac{2 \times 1.4 \times 2^2 - (1.4 - 1)}{(1.4 + 1)} = \frac{11.2 - 0.4}{2.4} = 4.5$$

$\therefore$ $p_2 = 26.5 \times 4.5$ = **119.25 kN/m² (Ans.)**

Density, ρ_2 :

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2}{(\gamma - 1) M_1^2 + 2} \qquad \text{...[Eqn. (16.47)]}$$

$$= \frac{(1.4 + 1)2^2}{(1.4 - 1)2^2 + 2} = \frac{9.6}{1.6 + 2} = 2.667$$

$\therefore$ $\rho_2 = 0.413 \times 2.667$ = **1.101 kg/m³ (Ans.)**

Temperature, T_1 :

Since $p_1 = \rho_1 RT_1$, $\therefore$ $T_1 = \frac{p_1}{\rho_1 R} = \frac{26.5 \times 10^3}{0.413 \times 287}$ = 223.6 K or – **49.4°C (Ans.)**

Temperature, T_2 :

$$\frac{T_2}{T_1} = \frac{[(\gamma - 1)M_1^2 + 2]\,[2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2\, M_1^2} = \frac{[(1.4 - 1)2^2 + 2]\,[2 \times 1.4 \times 2^2 - (1.4 - 1)]}{(1.4 + 1)^2\, 2^2}$$

$$= \frac{(1.6 + 2)(11.2 - 0.4)}{23.04} = 1.6875$$

$\therefore$ $T_2 = 223.6 \times 1.6875$ = **377.3 K** or **104.3°C (Ans.)**

Velocity, V_1 :

$$C_1 = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 223.6} = 299.7 \text{ m/s}$$

Since $\frac{V_1}{C_1} = M_1 = 2$ $\therefore$ $V_1 = 299.7 \times 2$ = **599.4 m/s (Ans.)**

Velocity, V_2 :

$$C_2 = \sqrt{\gamma RT_2} = \sqrt{1.4 \times 287 \times 377.3} = 389.35 \text{ m/s}$$

Since $\frac{V_2}{C_2} = M_2 = 0.577$ $\therefore$ $V_2 = 389.35 \times 0.577$ = **224.6 m/s (Ans.)**

16.11.2. Oblique Shock Wave

As shown in Fig. 16.12, when a supersonic flow undergoes a sudden turn through a small angle α (positive), an oblique wave is established at the corner. In comparison with normal shock waves, the oblique shock waves, being weaker, are preferred.

The shock waves should be avoided or made as weak as possible, since during a shock wave conversion of mechanical energy into heat energy takes place.

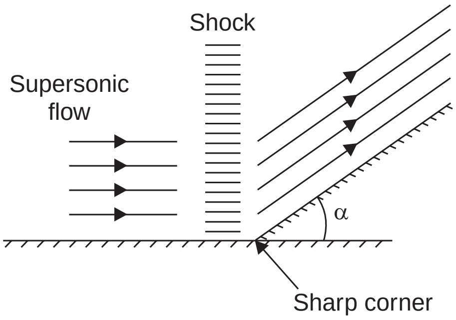


Fig. 16.12. Oblique shock wave.

16.11.3. Shock Strength

The *strength of shock* is defined as *the ratio of pressure rise across the shock to the upstream pressure.*

i.e. Strength of shock
$$= \frac{p_2 - p_1}{p_1} = \frac{p_2}{p_1} - 1$$
$$= \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} - 1 = \frac{2\gamma M_1^2 - (\gamma - 1) - (\gamma + 1)}{\gamma + 1}$$
$$= \frac{2\gamma M_1^2 - \gamma + 1 - \gamma - 1}{\gamma + 1} = \frac{2\gamma M_1^2 - 2\gamma}{\gamma + 1} = \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)$$

Hence, strength of shock $= \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)$ ...(16.50)

Example 16.19. *In a duct in which air is flowing, a normal shock wave occurs at a Mach number of 1.5. The static pressure and temperature upstream of the shock wave are 170 kN/m² and 23°C respectively. Determine :*

(*i*) *Pressure, temperature and Mach number downstream of the shock, and*

(*ii*) *Strength of shock.*

Take $\gamma = 1.4$.

Sol. Let subscripts 1 and 2 represent flow conditions upstream and downstream of the shock wave respectively.

Mach number, $M_1 = 1.5$

Upstream pressure, $p_1 = 170$ kN/m²

Upstream temperature, $T_1 = 23 + 273 = 296$ K

$\gamma = 1.4$

(*i*) **Pressure, temperature and Mach number downstream of the shock :**

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} \quad \text{...[Eqn. (16.46)]}$$

$$= \frac{2 \times 1.4 \times 1.5^2 - (1.4 - 1)}{1.4 + 1} = \frac{6.3 - 0.4}{2.4} = 2.458$$

∴ $p_2 = 170 \times 2.458 =$ **417.86 kN/m². Ans.**

$$\frac{T_2}{T_1} = \frac{[(\gamma - 1)M_1^2 + 2]\,[2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_1^2} \quad \text{...[Eqn. (16.48)]}$$

$$= \frac{[(1.4-1)\times 1.5^2 + 2][2\times 1.4\times 1.5^2 - (1.4-1)]}{(1.4+1)^2\times 1.5^2} = \frac{2.9\times 5.9}{12.96} = 1.32$$

$\therefore$ $T_2 = 296 \times 1.32 =$ **390.72 K** or **117.72°C. Ans.**

$$M_2^{\,2} = \frac{(\gamma-1)\,M_1^{\,2} + 2}{2\gamma M_1^{\,2} - (\gamma-1)} \qquad \text{...[Eqn. (16.49)]}$$

$$= \frac{(1.4-1)\times 1.5^2 + 2}{2\times 1.4\times 1.5^2 - (1.4-1)} = \frac{2.9}{5.9} = 0.49$$

$\therefore$ **M$_2$ = 0.7. Ans.**

(*ii*) **Strength of shock :**

Strength of shock $= \frac{p_2}{p_1} - 1 = 2.458 - 1 =$ **1.458. Ans.**

HIGHLIGHTS

1. A compressible flow is that flow in which the density of the fluid changes during flow.
2. The characteristic equation of state is given by :

$$\frac{p}{\rho} = RT$$

where p = absolute pressure, N/m^2,
ρ = density of gas, kg/m^3,
R = characteristic gas constant, J/kg K, and
R = absolute temperature (= $t°C + 273$).

3. The pressure and density of a gas are related as :

For isothermal process : $\frac{p}{\rho}$ = constant

For adiabatic process : $\frac{p}{\rho^\gamma}$ = constant.

4. The continuity equation for compressible flow is given as :

ρAV = constant

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad \text{... in differential form.}$$

5. For compressible fluids Bernoulli's equation is given as :

$$\frac{p}{\rho g}\log_e p + \frac{V^2}{2g} + z = \text{constant} \qquad \text{... for isothermal process}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \qquad \text{... for adiabatic process.}$$

6. Sonic velocity is given by :

$$C = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \qquad \text{... in terms of bulk modulus}$$

$$C = \sqrt{\frac{p}{\rho}} = \sqrt{RT} \qquad \text{... for isothermal process}$$

$$C = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT} \qquad \text{... for adiabatic process.}$$

7. Mach number, $M = \frac{V}{C}$

(*i*) Subsonic flow : $M < 1, V < C$... disturbance always moves ahead of the projectile

(*ii*) Sonic flow : $M = 1, V = C$... disturbance moves along the projectile

(*iii*) Supersonic flow : $M > 1, V > C$... The projectile always moves ahead of the disturbance.

Mach angle is given by : $\sin \alpha = \frac{C}{V} = \frac{1}{M}$.

8. The pressure, temperature and density at a point where velocity is zero are called stagnation pressure (p_s), temperature, (T_s) and stagnation density ρ_s. Their values are given as :

$$p_s = p_o \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\rho_s = \rho_o \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right]^{\frac{1}{\gamma - 1}}$$

$$T_s = T_o \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right]$$

where p_0, ρ_0 and T_o are the pressure, density and temperature at any point O in the flow.

9. Area-velocity relationship for compressible fluid is given as :

$$\frac{dA}{A} = \frac{dV}{V} (M^2 - 1)$$

(*i*) Subsonic flow ($M < 1$) : $\frac{dV}{V} > 0 \, ; \frac{dA}{A} < O \, ; dp < 0$ (convergent nozzle)

$\frac{dV}{V} < 0 \, ; \frac{dA}{A} > 0 \, ; dp > 0$ (divergent diffuser)

(*ii*) Supersonic flow ($M > 1$) : $\frac{dV}{V} > 0 \, ; \frac{dA}{A} > 0 \, ; dp < 0$ (divergent nozzle)

$\frac{dV}{V} < 0 \, ; \frac{dA}{A} < 0 \, ; dp > 0$ (convergent diffuser)

(*iii*) Sonic flow ($M = 1$) : $\frac{dA}{A} = 0$ (straight flow passage since dA must be zero)

$dp = \frac{\text{zero}}{\text{zero}}$ *i.e.* indeterminate, but when evaluated,

the change of pressure $dp = 0$, since $dA = 0$ and the flow is frictionless.

10. Flow of compressible fluid through a convergent nozzle :

(*i*) Velocity through a nozzle or orifice fitted to a large tank :

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right) \frac{p_1}{\rho_1} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$$

(*ii*) The mass rate of flow is given by :

$$m = A_2 \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right) p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma + 1}{\gamma}}\right]}$$

(*iii*) Value of $\left(\frac{p_2}{p_1}\right)$ for maximum value of mass rate of flow is given by :

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} = 0.528 \qquad \text{(when } \gamma = 1.4\text{)}$$

(*iv*) Value of V_2 for maximum rate of flow of liquid is given as,

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma + 1}\right) \frac{p_1}{\rho_1}} \quad (= C_2)$$

(*v*) Maximum rate of flow of fluid through nozzle,

$$m_{\max} = A_2 \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right) p_1 \rho_1 \left[\left(\frac{2\gamma}{\gamma + 1}\right)^{\frac{2}{\gamma - 1}} - \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{\gamma - 1}}\right]}$$

For air, substituting $\gamma = 1.4$, we get

$$m_{\max} = 0.685 A_2 \sqrt{p_1 \rho_1}$$

If the pressure ratio is less than 0.528, the mass rate of flow of the fluid is always corresponding to the pressure ratio of 0.528. But if the pressure ratio is more than 0.528, the mass rate of flow of fluid is corresponding to the given pressure ratio.

11. Whenever a supersonic flow (compressible) changes to subsonic flow, a shock wave (analogous to hydraulic jump in an open channel) is produced, resulting in a sudden rise in pressure, density, temperature and entropy.

$$p_1 + \frac{(\rho_1 V_1)^2}{\rho_1} = p_2 + \frac{(\rho_2 V_2)^2}{\rho_2} \qquad \text{... Ranking Line Equation}$$

$$\frac{\gamma}{\gamma - 1}\left(\frac{p_1}{\rho_1}\right) + \frac{(\rho_1 V_1)^2}{2\rho_1^2} = \frac{\gamma}{\gamma - 1}\left(\frac{p_2}{\rho_2}\right) + \frac{(\rho_2 V_2)^2}{2\rho_2^2} \qquad \textit{... Fanno line Equation}$$

$$\left.\begin{aligned} \frac{p_2}{p_1} &= \frac{\left(\frac{\gamma + 1}{\gamma - 1}\right)\frac{\rho_2}{\rho_1} - 1}{\left(\frac{\gamma + 1}{\gamma - 1}\right) - \frac{\rho_2}{\rho_1}} \\ \frac{\rho_2}{\rho_1} &= \frac{V_1}{V_2} = \frac{1 + \left(\frac{\gamma + 1}{\gamma - 1}\right)\frac{p_2}{p_1}}{\left(\frac{\gamma + 1}{\gamma - 1}\right) + \frac{p_2}{p_1}} \end{aligned}\right\} \qquad \textit{...Rankinge-Hugoniot Equations}$$

One can also express $\frac{p_2}{p_1}$, $\frac{V_2}{V_1}$, $\frac{\rho_2}{\rho_1}$ and $\frac{T_2}{T_1}$ in terms of Mach number as follows :

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} \qquad ...(i)$$

$$\frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)\, M_1^2 + 2} \quad ...(ii)$$

$$\frac{T_2}{T_1} = \frac{[(\gamma - 1)M_1^2 + 2]\,[2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2\, M_1^2} \quad ...(iii)$$

Also $$M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}\,.$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. All real fluids are
(*a*) incompressible (*b*) compressible to some extent
(*c*) compressible to any extent (*d*) none of the above.

2. A change in the state of a system at constant volume is called
(*a*) isobaric process (*b*) isochoric process
(*c*) isothermal process (*d*) adiabatic process.

3. A process during which no heat is transferred to or from the gas is called an
(*a*) isochoric process (*b*) isobaric process
(*c*) adiabatic process (*d*) isothermal process.

4. An adiabatic process is one which follows the relation
(*a*) $\frac{p}{\rho}$ = constant (*b*) $\frac{p}{\rho^\gamma}$ = constant
(*c*) $\frac{p}{\rho^n}$ = constant $(n \neq \gamma)$ (*d*) v = constant.

5. An isentropic flow is one which is
(*a*) isothermal (*b*) adiabatic
(*c*) adiabatic and irreversible (*d*) adiabatic and reversible.

6. Indica upto what Mach number can a fluid flow be considered incompressible ?
(*a*) 0.1 (*b*) 0.3
(*c*) 0.8 (*d*) 1.0.

7. Which of the following is the basic equation of compressible fluid flow ?
(*a*) Continuity equation (*b*) Momentum equation
(*c*) Energy equation (*d*) Equation of state
(*e*) All of the above.

8. The velocity of disturbance in case of fluids is ...... the velocity of the disturbance in solids.
(*a*) less than (*b*) equal to
(*c*) more than (*d*) none of the above.

9. Sonic velocity (C) for adiabatic process is given as
(*a*) $C = \sqrt{\gamma R T^3}$ (*b*) $C = \sqrt{\gamma R T}$
(*c*) $C = \sqrt{\gamma^2 R T}$ (*d*) $C = \gamma R T$.
where γ = ratio of specific heats, R = gas constant, T = temperature.

10. The flow is said to be subsonic when Mach number is
(*a*) equal to unity (*b*) less than unity
(*c*) greater than unity (*d*) none of the above.

11. The region outside the Mach cone is called
(*a*) zone of action (*b*) zone of silence
(*c*) control volume (*d*) none of the above.

12. A stagnation point is the point on the immersed body where the magnitude of velocity is
(*a*) small (*b*) large
(*c*) zero (*d*) none of the above.

13. A convergent-divergent nozzle is used when the discharge pressure is
(*a*) less than the critical pressure (*b*) equal to the critical pressure
(*c*) more than the critical pressure (*d*) none of the above.

14. At critical pressure ratio, the velocity at the throat of a nozzle is
(*a*) equal to the sonic speed (*b*) less than the sonic speed
(*c*) more than the sonic speed (*d*) none of the above.

15. Laval nozzle is a
(*a*) convergent nozzle (*b*) divergent nozzle
(*c*) convergent-divergent nozzle (*d*) any of the above.

16. A shock wave is produced when
(*a*) a subsonic flow changes to sonic flow (*b*) a sonic flow changes to supersonic flow
(*c*) a supersonic flow changes to subsonic flow (*d*) none of the above.

17. The sonic velocity in a fluid medium is directly proportional to
(*a*) Mach number (*b*) pressure
(*c*) square root of temperature (*d*) none of the above.

18. The stagnation pressure (p_s) and temperature (T_s) are
(*a*) less than their ambient counterparts (*b*) more than their ambient counterparts
(*c*) the same as in ambient flow (*d*) none of the above.

19. Across a normal shock
(*a*) the entropy remains constant (*b*) the pressure and temperature rise
(*c*) the velocity and pressure decrease (*d*) the density and temperature decrease.

20. A normal shock wave
(*a*) is reversible (*b*) is irreversible
(*c*) is isentropic (*d*) occurs when approaching flow is supersonic.

21. The sonic speed in an ideal gas varies
(*a*) inversely as bulk modulus (*b*) directly as the absolute pressure
(*c*) inversely as the absolute temperature (*d*) none of the above.

22. In a supersonic flow, a diffuser is a conduit having
(*a*) gradually decreasing area (*b*) converging-diverging passage
(*c*) constant area throughout its length (*d*) none of the above.

23. Choking of a nozzle fitted to a pressure tank containing gas implies
(*a*) sonic velocity at the throat (*b*) increase of the mass flow rate
(*c*) obstruction of flow (*d*) all of the above.

24. A shock wave which occurs in a supersonic flow represents a region in which
(*a*) a zone of silence exists
(*b*) there is no change in pressure, temperature and density
(*c*) there is sudden change in pressure, temperature and density
(*d*) velocity is zero.

25. Which of the following statements regarding a normal shock is *correct* ?
(*a*) It occurs when an abrupt change takes place from supersonic into subsonic flow condition
(*b*) It causes a disruption and reversal of flow pattern
(*c*) It may occur in sonic or supersonic flow
(*d*) None of the above.

26. For compressible fluid flow the area-velocity relationship is

(*a*) $\frac{dA}{A} = \frac{dV}{V}(1 - M^2)$ (*b*) $\frac{dA}{A} = \frac{dV}{V}(C^2 - 1)$

(*c*) $\frac{dA}{A} = \frac{dV}{V}(M^2 - 1)$ (*d*) $\frac{dA}{A} = \frac{dV}{V}(1 - V^2)$.

27. The sonic velocity is largest in which of the following ?

(*a*) Water (*b*) Steel

(*c*) Kerosene (*d*) Air.

28. Which of the following expressions does not represent the speed of sound in a medium ?

(*a*) $\sqrt{\frac{K}{\rho}}$ (*b*) $\sqrt{\gamma RT}$

(*c*) $\sqrt{K\frac{p}{\rho}}$ (*d*) $\sqrt{\frac{dp}{d\rho}}$.

29. The differential equation for energy in isentropic flow is of the form

(*a*) $\frac{dV}{V} + \frac{dp}{\rho} + \frac{dA}{A} = 0$ (*b*) $VdV + \frac{dp}{\rho} = 0$

(*c*) $2VdV + \frac{dp}{\rho} = 0$ (*d*) $dp + d(\rho V^2) = 0$.

30. Which of the following statements is *incorrect* ?

(*a*) A shock wave occurs in divergent section of a nozzle when the compressible flow changes abruptly from supersonic to subsonic state

(*b*) A plane moving at supersonic state is not heard by the stationary observer on the ground until it passes him because zone of disturbance in Mach cone trails behind the plane

(*c*) A divergent section is added to a convergent nozzle to obtain supersonic velocity at the throat

(*d*) none of the above.

ANSWERS

1.	(*b*)	**2.**	(*b*)	**3.**	(*c*)	**4.**	(*b*)	**5.**	(*d*)	**6.**	(*b*)	**7.**	(*e*)
8.	(*a*)	**9.**	(*b*)	**10.**	(*b*)	**11.**	(*b*)	**12.**	(*c*)	**13.**	(*a*)	**14.**	(*a*)
15.	(*c*)	**16.**	(*c*)	**17.**	(*c*)	**18.**	(*b*)	**19.**	(*b*)	**20.**	(*d*)	**21.**	(*d*)
22.	(*a*)	**23.**	(*d*)	**24.**	(*c*)	**25.**	(*a*)	**26.**	(*c*)	**27.**	(*b*)	**28.**	(*c*)
29.	(*b*)	**30.**	(*c*).										

THEORETICAL QUESTIONS

1. Differentiate between compressible and incompressible flows.

2. Give the examples when liquid is treated as a compressible fluid.

3. When is the compressibility of fluid important ?

4. What is the difference between isentropic and adiabatic flows ?

5. What is the relation between pressure and density of a compressible fluid for (*a*) isothermal process (*b*) adiabatic process ?

6. Obtain an expression in differential form for continuity equation for one-dimensional compressible flow.

7. Derive an expression for Bernoulli's equation when the process is adiabatic.

8. How are the disturbances in compressible fluid propagated ?

9. What is sonic velocity ? On what factors does it depend ?

10. What is Mach number ? Why is this parameter so important for the study of flow of compressible fluids ?

11. Prove that velocity of sound wave in a compressible fluid is given by : $C = \sqrt{k/\rho}$, where k and ρ are the bulk modulus and density of fluid respectively.

12. Define the following terms :
(*i*) Subsonic flow (*ii*) Sonic flow (*iii*) Supersonic flow, (*iv*) Mach cone.

13. What is silence zone during the disturbance which propagates when an object moves in still air ?

14. What is stagnation point of an object immersed in fluid ?

15. What is stagnation pressure ?

16. What are static and stagnation temperatures ?

17. Derive an expression for mass flow rate of compressible fluid through an orifice or nozzle fitted to a large tank. What is the condition for maximum rate of flow ?

18. What is the critical pressure ratio for a compressible flow through a nozzle ? On what factors does it depend ?

19. Describe compressible flow through a convergent-divergent nozzle. How and where does the shock wave occur in the nozzle ?

20. What do you mean by compressibility correction factor ?

21. How is a shock wave produced in a compressible fluid ? What do you mean by the term "Shock strength" ?

UNSOLVED EXAMPLES

1. A 100 mm diameter pipe reduces to 50 mm diameter through a sudden contraction. When it carries air at 20.16°C under isothermal conditions, the absolute pressures observed in the two pipes just before and after the contraction are 400 kN/m^2 and 320 kN/m^2 respectively. Determine the densities and velocities at the two sections. Take R = 290 J/kg K. [**Ans.** 4.7 kg/m^3 ; 3.76 kg/m^3 ; 39.7 m/s ; 198.5 m/s]

2. A gas with a velocity of 300 m/s is flowing through a horizontal pipe at a section where pressure is 60 kN/m^2 (*abs.*) and temperature 40°C. The pipe changes in diameter and at this section the pressure is 90 kN/m^2. If the flow of gas is adiabatic find the velocity of gas at this section.
Take : R = 287 J/kg K and γ = 1.4. [**Ans.** 113 m/s]

3. An aeroplane is flying at 21.5 m/s at a low altitude where the velocity of sound is 325 m/s. At a certain point just outside the boundary layer of the wings, the velocity of air relative to the plane is 305 m/s. If the flow is frictionless adiabatic determine the pressure drop on the wing surface near this position.
Assume γ = 1.4, pressure of ambient air = 102 kN/m^2. [**Ans.** 28.46 kN/m^2]

4. A jet propelled aircraft is flying at 1100 km/h. at sea level. Calculate the Mach number at a point on the aircraft where air temperature is 20°C.
Take : R = 287 J/kg K and γ = 1.4. [**Ans.** 0.89]

5. An aeroplane is flying at an height of 20 km where the temperature is – 40°C. The speed of the plane is corresponding to M = 1.8. Find the speed of the plane.
Take : R = 287 J/kg K, γ = 1.4. [**Ans.** 1982.6 km/h]

6. Find the velocity of bullet fired in standard air if its Mach angle is 30°. [**Ans.** 680.4 m/s]

7. Air, thermodynamic state of which is given by pressure p = 230 kN/m^2 and temperature = 300 K is moving at a velocity V = 250 m/s. Calculate the stagnation pressure if (*i*) compressibility is neglected and (*ii*) compressibility is accounted for.
Take γ = 1.4 and R = 287 J/kg K. [**Ans.** 313 kN/m^2, 323 kN/m^2]

8. A large vessel, fitted with a nozzle, contains air at a pressure of 2943 kN/m^2 (*abs.*) and at a temperature of 20°C. If the pressure at the outlet of the nozzle is 2060 kN/m^2 (*abs.*) find the velocity of air flowing at the outlet of the nozzle.
Take : R = 287 J/kg K and γ = 1.4 [**Ans.** 239.2 m/s]

9. Nitrogen gas (γ = 1.4) is released through a 10 mm orifice on the side of a large tank in which the gas is at a pressure of 10 bar and temperature 20°C. Determine the mass flow rate if (*i*) the gas escapes to atmosphere (1 bar) ; (*ii*) the gas is released to another tank at (*a*) 5 bar, (*b*) 6 bar.
[**Ans.** (*i*) 0.183 kg/s ; (*ii*) 0.183 kg/s ; 0.167 kg/s]

10. Air is released from one tank to another through a convergent-divergent nozzle at the rate of 12 N/s. The supply tank is at a pressure of 400 kN/m^2 and temperature 110°C, and the pressure in the receiving tank is 100 kN/m^2. Determine : (*i*) The pressure, temperature, and Mach number in the constriction, (*ii*) The required diameter of constriction, (*iii*) The diameter of the nozzle at the exit for full expansion, and the Mach number. [**Ans.** (*i*) 210 kN/m^2 ; 319 K, (*ii*) 43.5 mm ; (*iii*) 48 mm ; 1.56]

11. Oxygen flows in a conduit at an absolute pressure of 170 kN/m^2. If the absolute pressure and temperature at the nose of small object in the stream are 200 kN/m^2 and 70.16°C respectively, determine the velocity in the conduit. Take $\gamma = 1.4$ and $R = 281.43$ J/kg K. [**Ans.** 175.3 m/s]

12. Air at a velocity of 1400 km/h has a pressure of 10 kN/m^2 vacuum and temperature of 50.16°C. Calculate local Mach number and stagnation pressure, density and temperature. Take $\gamma = 1.4$, $R = 281.43$ J/kg K and barometric pressure = 101.325 kN/m^2. [**Ans.** 1.089 ; 192.358 kN/m^2 ; 1.708 kg/m^3 ; 399.8 K]

13. A normal shock wave occurs in a diverging section when air is flowing at a velocity of 420 m/s, pressure 100 kN/m^2, and temperature 10°C. Determine : (*i*) The Mach number before and after the shock, (*ii*) The pressure rise, and (*iii*) The velocity and temperature after the shock.

[**Ans.** (*i*) 1.25 ; 0.91 ; (*ii*) 66 kN/m^2, (*iii*) 292 m/s ; 54°C]

14. A normal shock wave occurs in air flowing at a Mach number of 1.5. The static pressure and temperature of the air upstream of the shock wave are 100 kN/m^2 and 300 K. Determine the Mach number, pressure and temperature downstream of the shockwave. Also estimate the shock strength.

[**Ans.** 0.7 ; 246 kN/m^2 ; 396.17 K ; 1.46]

COMPETITIVE EXAMINATIONS QUESTIONS

(Including ESE and CSE Questions, from 1996 onwards)

Match List I with List II or Choose the Correct Answer :

1. Match List I with List II and select the correct answer using the codes given below the lists :

List I	List II
A. Work done in a polytropic process	1. $\int v\,dp$
B. Work done in a steady flow process	2. Zero
C. Heat transfer in a reversible adiabatic process	3. $\dfrac{p_1V_1 - p_2V_2}{\gamma - 1}$
D. Work done in an isentropic process	4. $\dfrac{p_1V_1 - p_2V_2}{n - 1}$

Codes :

	A	*B*	*C*	*D*
(*a*)	4	1	3	2
(*b*)	1	4	2	3
(*c*)	4	1	2	3
(*d*)	1	2	3	4.

2. Match the curves in Fig. 1 with the curves in Fig. 2 and select the correct answer using the codes given below the diagrams :

Diagram I (Process on *p*-*V* plane)

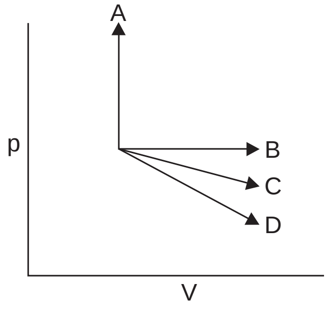


Fig. 1

Diagram II (Process on *T*-*s* plane)

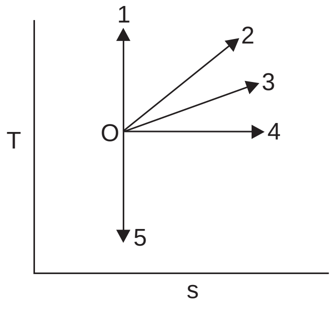


Fig. 2

Codes :

	A	*B*	*C*	*D*
(*a*)	3	2	4	5
(*b*)	2	3	4	5
(*c*)	2	3	4	1
(*d*)	1	4	2	3.

3. The heat transfer Q, the work done W and the change in internal energy ΔU are all zero in the case of

(*a*) a rigid vessel containing steam at 150°C left in the atmosphere which is at 25°C.

(*b*) 1 kg of gas contained in an insulated cylinder expanding as the piston move slowly outwards.

(*c*) a rigid vessel containing ammonia gas connected through a valve to an evacuated rigid vessel, the vessel, the valve and the connecting pipes being well insulated and the valve being opened and after a time, conditions through the two vessel becoming uniform.

(*d*) 1 kg of air flowing adiabatically from the atmosphere into a previously evacuated bottle.

4. Zeroth Law of thermodynamics states that

(*a*) two thermodynamic system are always in thermal equilibrium with each other.

(*b*) if two systems are in thermal equilibrium, then the third system will also be in thermal equilibrium.

(*c*) two systems not in thermal equilibrium with a third system are also not in thermal equilibrium with each other.

(*d*) when two systems are in thermal equilibrium with a third system, they are in thermal equilibrium with each other.

5. Which one of the following statements applicable to a perfect gas will also be true for an irreversible process ? (Symbols have the usual meanings)

(*a*) $dQ = du + pdV$

(*b*) $dQ = Tds$

(*c*) $Tds = du + pdV$

(*d*) None of the above.

6. The throttling process undergone by a gas across an orifice is shown by its states in Fig. 3.

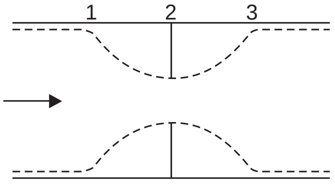
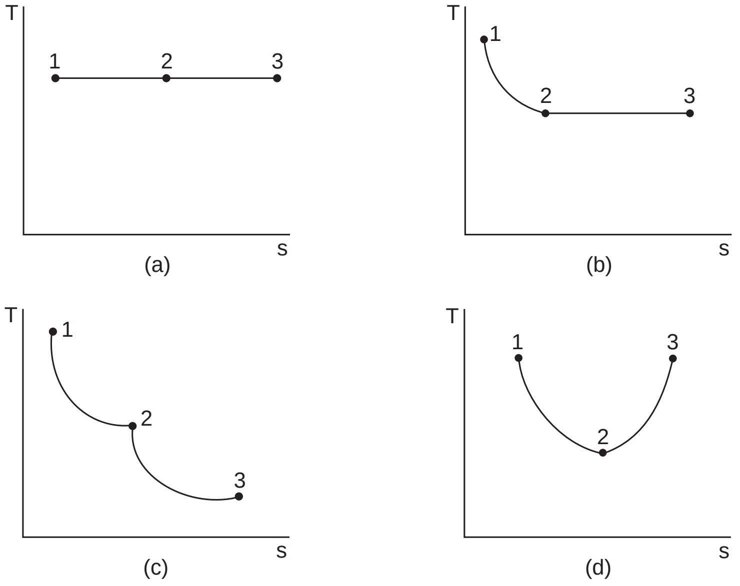


Fig. 3

It can be represented on the T-s diagram as



(a) (b) (c) (d)

Fig. 4

7. Which one of the following temperature entropy diagrams of steam shows the reversible and irreversible processes correctly ?

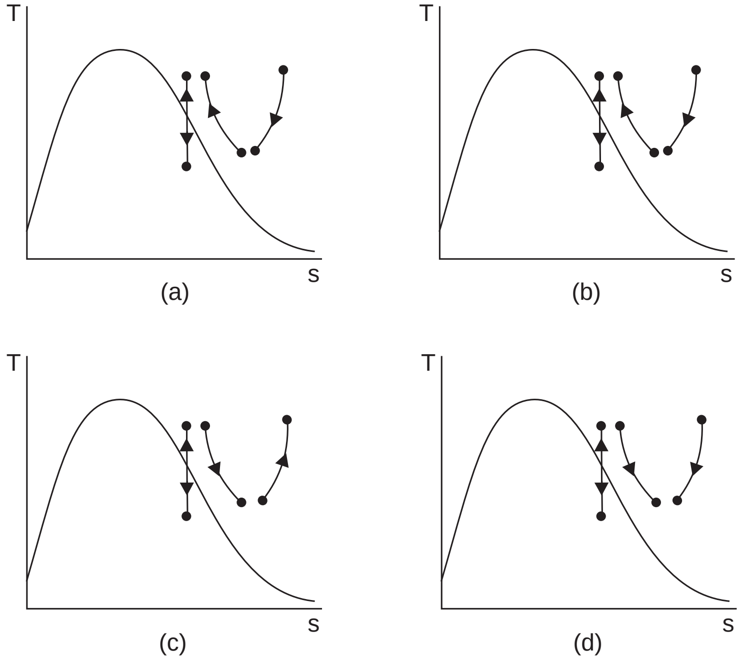


Fig. 5

8. Consider the following statements :

1. Availability is generally conserved.
2. Availability can either be negative or positive.
3. Availability is the maximum theoretical work obtainable.
4. Availability can be destroyed in irreversibilities.

Of these statements

(*a*) 3 and 4 are correct

(*b*) 1 and 3 are correct

(*c*) 1 and 4 are correct

(*d*) 2 and 4 are correct.

9. For a given volume of dry saturated steam, Clapeyron's equation is given by

(*a*) $V_g - V_t = \frac{dT_s}{dp} \times \frac{T}{h_g - h_f}$

(*b*) $V_g - V_f = \frac{dT_s}{dp} \times \frac{h_g - h_f}{T_s}$

(*c*) $V_g - V_f = \frac{dp}{dT_s} \times \frac{h_g - h_f}{T_s}$

(*d*) $V_g - V_f = \frac{dp}{dT_s} \times \frac{T_s}{h_g - h_f}$.

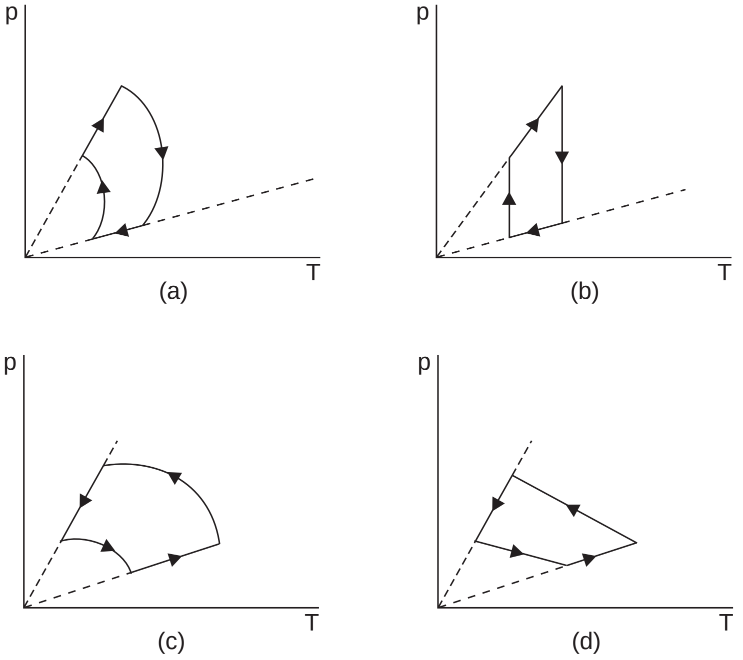
10. The Joule-Thomson coefficient is the

(*a*) $\left(\frac{\partial T}{\partial p}\right)_h$ of pressure-temperature curve of real gases

(*b*) $\left(\frac{\partial T}{\partial s}\right)_v$ of temperature entropy curve of real gases (*c*) $\left(\frac{\partial h}{\partial s}\right)_T$ of enthalpy entropy curve of real gases

(*d*) $\left(\frac{\partial v}{\partial T}\right)_p$ of pressure volume curve of real gases.

11. Which one of the following p-T diagrams illustrates the Otto cycle of an ideal gas ?



(a) (b)

(c) (d)

Fig. 6

12. When a system is taken from state A to state B along the path A-C-B, 180 kJ of heat flows into the system and it does 130 kJ of work (see Fig. 7 given).

How much heat will flow into the system along the path A-D-B if the work done by it along the path is 40 kJ ?

(*a*) 40 kJ (*b*) 60 kJ

(*c*) 90 kJ (*d*) 135 kJ.

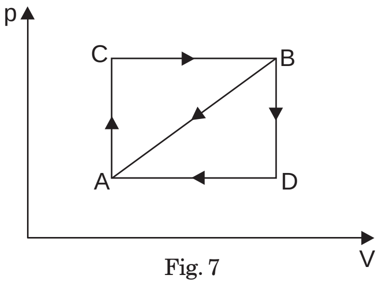


Fig. 7

13. A gas expands from pressure p_1 to pressure p_2 $\left(p_2 = \frac{p_1}{10}\right)$. If the process of expansion is isothermal, the volume at the end of expansion is 0.55 m^3. If the process of expansion is adiabatic, the volume at the end of expansion will be closer to

(*a*) 0.45 m^3 (*b*) 0.55 m^3

(*c*) 0.65 m^3 (*d*) 0.75 m^3.

14. A standard vapour is compressed to half its volume without changing is temperature. The result is that :

(*a*) All the vapour condenses to liquid

(*b*) Some of the liquid evaporates and the pressure does not change

(*c*) The pressure is double its initial value

(*d*) Some of the vapour condenses and the pressure does not change.

15. A system of 100 kg mass undergoes a process in which its specific entropy increases from 0.3 kJ/kg-K to 0.4 kJ/kg-K. At the same time, the entropy of the surroundings decreases from 80 kJ/K to 75 kJ/K. The process is

(*a*) Reversible and isothermal (*b*) Irreversible

(*c*) Reversible (*d*) Impossible.

16. The thermodynamic parameters are :

I. Temperature II. Specific volume

III. Pressure IV. Enthaply

V. Entropy

The Clapeyron equation of state provides relationship between

(*a*) I and II (*b*) II, III and V

(*c*) III, IV and V (*d*) I, II, III and IV.

17. The work done in compressing a gas isothermally is given by :

(*a*) $\frac{\gamma}{\gamma - 1} \cdot p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$ (*b*) $mRT_1 \log_e \frac{p_2}{p_1}$ N.m

(*c*) $mc_p\ (T_2 - T_1)$ kJ (*d*) $mRT_1\left(1 - \frac{T_2}{T_1}\right)$ kJ.

18. An ideal air standard cycle is shown in the given temperature entropy diagram (Fig. 8).

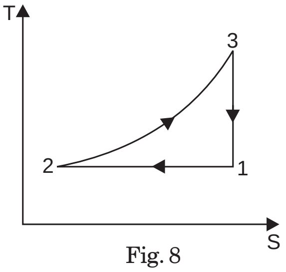
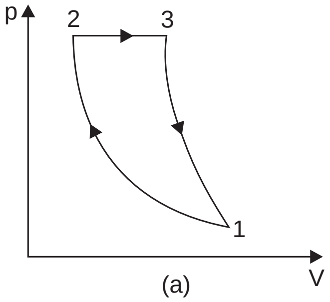
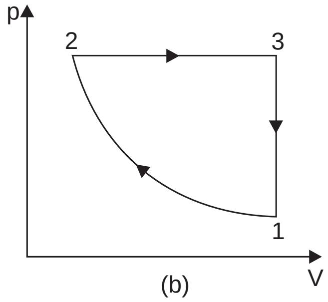


Fig. 8

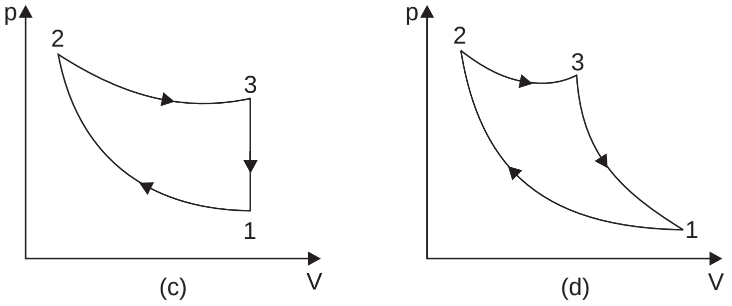
The same cycle, when represented on the pressure-volume co-ordinates, takes the form.



(a)



(b)



(c) (d)

Fig. 9

19. In a Rankine cycle, with the maximum steam temperature being fixed from metallurgical considerations, as the boiler pressure increases

(*a*) the condenser load will increase

(*b*) the quality of turbine exhaust will decrease

(*c*) the quality of turbine exhaust will increase

(*d*) the quality of turbine exhaust will remain unchanged.

20. Match List I (details of the processes of the cycle) with List II (name of the cycle) and select the correct answer using the code given below the Lists :

List I	**List II**
A. Two isothermals and two adiabatics	1. Otto
B. Two isothermals and two constant volumes	2. Joule
C. Two adiabatics and two constant pressures	3. Carnot
D. Two adiabatics and two constant pressures	4. Stirling

Codes :

(*a*)	*A*	*B*	*C*	*D*	(*b*)	*A*	*B*	*C*	*D*
	4	3	1	2		4	3	2	1
(*c*)	*A*	*B*	*C*	*D*	(*d*)	*A*	*B*	*C*	*D*
	3	4	1	2		3	4	2	1.

21. Two blocks which are at different states are brought into contact with each other and allowed to reach a final state of thermal equilibrium. The final temperature attained is specified by the

(*a*) Zeroth law of thermodynamics (*b*) First law of thermodynamics

(*c*) Second law of thermodynamics (*d*) Third law of thermodynamics.

22. A control mass undergoes a process from state 1 to state 2 as shown in Fig. 10. During this process, the heat transfer to the system is 200 kJ. If the control mass returned adiabatically from state 2 to state 1 by another process, then the work interaction during the return process (in kN.m) would be

(*a*) – 400 (*b*) – 200

(*c*) 200 (*d*) 400.

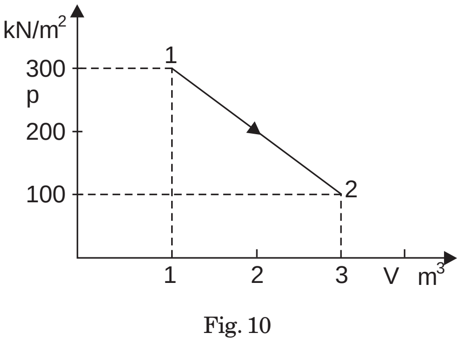


Fig. 10

23. Four processes of a thermodynamic cycle are shown above in Fig. 11 on the T-s plane in the sequence 1—2—3—4. The corresponding correct sequence of these processes in the p-V plane as shown in Fig. 12 will be

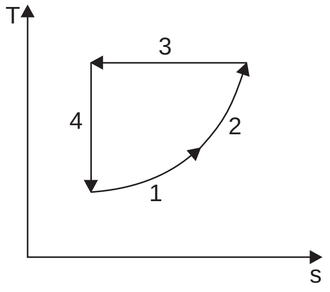


Fig. 11

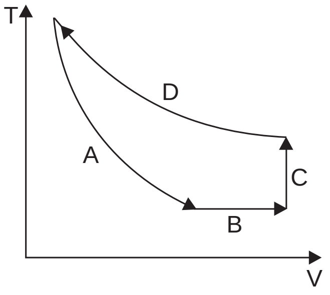


Fig. 12

(*a*) (*C*—*D*—*A*—*B*)

(*b*) (*D*—*A*—*B*—*C*)

(*c*) (*A*—*B*—*C*—*D*)

(*d*) (*B*—*C*—*D*—*A*).

24. The Fig. 13 shows an isometric cooling process 1-2 of a pure substance. The ordinate and abscissa are respectively

(*a*) pressure and volume

(*b*) enthalpy and entropy

(*c*) temperature and entropy

(*d*) pressure and enthalpy.

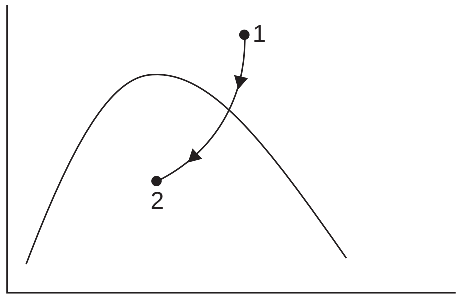


Fig. 13

25. For a thermodynamic cycle to be irreversible, it is necessary that

(*a*) $\oint \frac{\delta Q}{T} = 0$

(*b*) $\oint \frac{\delta Q}{T} < 0$

(*c*) $\oint \frac{\delta Q}{T} > 0$

(*d*) $\oint \frac{\delta Q}{T} \geq 0$.

26. Neglecting changes in kinetic energy and potential energy, for unit mass the availability in a non-flow process becomes $a = \phi - \phi_0$ where ϕ is the availability function of the

(*a*) open system

(*b*) closed system

(*c*) isolated system

(*d*) steady flow process.

27. It can be shown that for a simple compressible substance, the relationship

$$c_p - c_v = -T\left(\frac{\partial V}{\partial T}\right)_P^2 \left(\frac{\partial p}{\partial v}\right)_T \quad \text{exists}$$

where c_p and c_v are specific heats at constant pressure and constant volume respectively, T is temperature, V is volume and p is pressure.

Which one of the following statements is NOT true ?

(*a*) c_p is always greater than c_v

(*b*) The right side of the equation reduces to R for an ideal gas (*c*) Since $\left(\frac{\partial p}{\partial V}\right)_T$ can be either positive or negative, and $\left(\frac{\partial V}{\partial T}\right)_p$ must be positive, T must have a sign which is opposite to that of $\left(\frac{\partial p}{\partial V}\right)_T$

(*d*) c_p is very nearly equal to c_v for liquid water.

28. Consider the following statements : In an irreversible process

1. entropy always increases.
2. the sum of the entropy of all the bodies taking part in a process always increases.
3. once created, entropy cannot be destroyed.

Of these statements :

(*a*) 1 and 2 are correct (*b*) 1 and 3 are correct

(*c*) 2 and 3 are correct (*d*) 1, 2 and 3 are correct.

29. An ideal cycle is shown in the Fig. 14. Its thermal efficiency is given by

(*a*) $1-\dfrac{\left(\dfrac{v_3}{v_1}-1\right)}{\left(\dfrac{p_2}{p_1}-1\right)}$ (*b*) $1-\dfrac{1}{\gamma}\dfrac{\left(\dfrac{v_3}{v_1}-1\right)}{\left(\dfrac{p_2}{p_1}-1\right)}$

(*c*) $1-\gamma\,\dfrac{(v_3-v_1)}{(p_2-p_1)}\,\dfrac{p_1}{v_1}$ (*d*) $1-\dfrac{1}{\gamma}\,\dfrac{(p_3-p_1)}{(v_3-v_1)}\,\dfrac{v_1}{p_1}$.

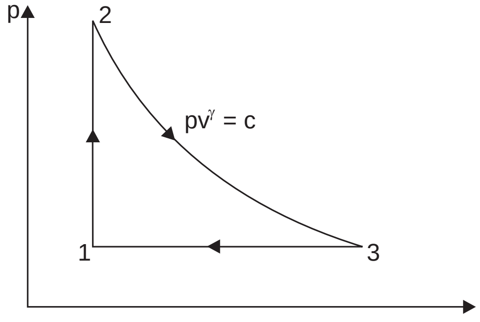


Fig. 14

30. Consider the following statements regarding Otto cycle :

1. It is not a reversible cycle.
2. Its efficiency can be improved by using a working fluid of higher value of ratio of specific heats.
3. The practical way of increasing its efficiency is to increase the compression ratio.
4. Carburetted gasoline engines working on Otto cycle can work with compression ratios more than 12.

Of these statements :

(*a*) 1, 3 and 4 are correct (*b*) 1, 2 and 3 are correct

(*c*) 1, 2 and 4 are correct (*d*) 2, 3 and 4 are correct.

31. Consider the following statements : The difference between higher and lower heating values of the fuels is due to

1. heat carried by steam from the moisture content of fuel.
2. sensible heat carried away by the flue gases.
3. heat carried away by steam from the combustion of hydrogen in the fuel.
4. heat lost by radiation.

Of these statements :

(*a*) 2, 3 and 4 are correct (*b*) 1 and 2 are correct

(*c*) 3 alone is correct (*d*) 1, 2, 3 and 4 are correct.

32. Match List I (Gadgets undergoing a thermodynamic process) with List II (Property of the system that remains constant) and select the correct answer using the codes given below the Lists :

List I	**List II**
A. Bomb calorimeter	1. Pressure
B. Exhaust gas calorimeter	2. Enthalpy
C. Junker gas calorimeter	3. Volume
D. Throttling calorimeter	4. Specific heats

Codes :

	A	*B*	*C*	*D*
(*a*)	3	4	1	2
(*b*)	2	4	1	3
(*c*)	3	1	4	2
(*d*)	4	3	2	1

33. Consider the following statements :

The maximum temperature produced by the combustion of an unit mass of fuel depends upon

1. LCV 2. ash content 3. mass of air supplied 4. pressure in the furnace.

Of these statements :

(*a*) 1 alone is correct
(*b*) 1 and 3 are correct
(*c*) 2 and 4 are correct
(*d*) 3 and 4 are correct.

34. The graph shown in the Fig. 15 represents the emission of a pollutant from an SI engine for different fuel/air ratios. The pollutant in question is

(*a*) CO
(*b*) CO_2
(*c*) hydrocarbon
(*d*) NO_x.

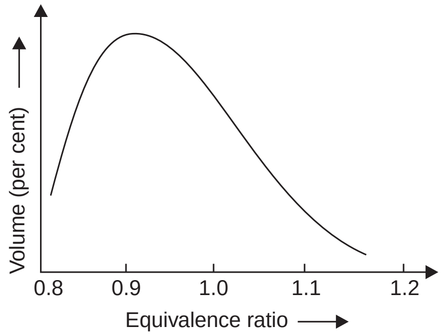


Fig. 15

35. Which of the following are the assumptions involved in the auto-ignition theory put forth for the onset of knock in SI engines ?

1. Flame velocity is normal before the onset of auto-ignition.
2. A number of end-gas elements autoignite simultaneously.
3. Preflame reactions are responsible for preparing the end-gas to ignite.

Select the correct answer using the codes given below :

(*a*) 1 and 2
(*b*) 1 and 3
(*c*) 2 and 3
(*d*) 1, 2 and 3.

36. In a vapour compression refrigeration system, a throttle valve is used in place of an expander because

(*a*) it considerably reduces the system weight
(*b*) it improves the COP, as the condenser is small
(*c*) the positive work in isentropic expansion of liquid is very small
(*d*) it leads to significant cost reduction.

37. A cube at high temperature is immersed in a constant temperature bath. It loses heat from its top, bottom and side surfaces with heat transfer coefficient of h_1, h_2 and h_3, respectively. The average heat transfer coefficient for the cube is

(*a*) $h_1 + h_2 + h_3$ (*b*) $(h_1 h_2 h_3)^{1/3}$

(*c*) $\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$ (*d*) None of the above.

38. Match items in List I with those in Lists II and III and select the correct answer using the codes given the lists :

List I	**List II**	**List III**
A. Reversed Carnot engine	1. Condenser	6. Generator
B. Sub-cooling	2. Evaporator	7. Increase in refrigeration effect
C. Superheating	3. Vortex refrigerator	8. Highest COP
D. Constant enthalpy	4. Throttling	9. Adiabatic
	5. Heat pump	10. Dry compression

Codes :

	A	*B*	*C*	*D*
(*a*)	3, 10	1, 7	2, 9	4, 6
(*b*)	5, 8	1, 7	2, 10	4, 9
(*c*)	4, 10	3, 8	3, 10	1, 6
(*d*)	2, 7	5, 8	4, 6	1, 9

39. Consider the following statements :

In ammonia refrigeration systems, oil separator is provided because

1. oil separation in evaporator would lead to reduction in heat transfer coefficient.
2. oil accumulation in the evaporator causes choking of evaporator.
3. oil is partially miscible in the refrigerant.
4. oil causes choking of expansion device.

Of these statements :

(*a*) 1 and 2 are correct (*b*) 2 and 4 are correct

(*c*) 2, 3 and 4 are correct (*d*) 1, 3 and 4 are correct.

40. Consider the following statements :

Moisture should be removed from refrigerants to avoid :

1. compressor seal failure 2. freezing at the expansion valve

3. restriction to refrigerant flow 4. corrosion of steel parts.

On these statements :

(*a*) 1, 2, 3 and 4 are correct (*b*) 1 and 2 are correct

(*c*) 2, 3 and 4 are correct (*d*) 1, 3 and 4 are correct.

41. Consider the following statements :

1. Practically all common refrigerants have approximately the same COP and power requirement.
2. Ammonia mixes freely with lubricating oil and this helps lubrication of compressors.

3. Dielectric strength of refrigerants is an important property in hermetically sealed compressor units.
4. Leakage of ammonia can be detected by halide torch method.

Of these statements :

(*a*) 1, 2 and 4 are correct (*b*) 2 and 4 are correct
(*c*) 1, 2 and 4 are correct (*d*) 1 and 3 are correct.

42. The most commonly used method for the design of duct size is the

(*a*) velocity reduction method (*b*) equal friction method
(*c*) static regain method (*d*) dual or double method.

43. The refrigerant used for absorption refrigerators working on heat from solar collectors is a mixture of water and

(*a*) carbon dioxide (*b*) sulphur dioxide
(*c*) lithium bromide (*d*) freon 12.

44. During the adiabatic cooling of moist air

(*a*) DBT remains constant (*b*) specific humidity remains constant
(*c*) relative humidity remains constant (*d*) WBT remains constant.

45. When a stream of moist air is passed over a cold and dry cooling coil such that no condensation takes place, then the air stream will get cooled along the line of

(*a*) constant wet bulb temperature (*b*) constant dew point temperature
(*c*) constant relative humidity (*d*) constant enthalpy.

46. For large tonnage (more than 200 tons) air-conditioning applications, which one of the following types of compressors is recommended ?

(*a*) Reciprocating (*b*) Rotating
(*c*) Centrifugal (*d*) Screw.

47. In a cooling tower, "approach" is the temperature difference between the

(*a*) hot inlet water and cold outlet water (*b*) hot inlet water and WBT
(*c*) cold outlet water and WBT (*d*) DBT and WBT.

48. When the discharge pressure is too high in a refrigeration system, high pressure control is installed to

(*a*) stop the cooling fan (*b*) stop the water circulating pump
(*c*) regulate the flow of cooling water (*d*) stop the compressor.

49. A refrigerating machine working on reversed Carnot cycle takes out 2 kW per minute of heat from the system while working between temperature limits of 300 K and 200 K. C.O.P. and Power consumed by the cycle will be respectively

(*a*) 1 and 1 kW (*b*) 1 and 2 kW
(*c*) 2 and 1 kW (*c*) 2 and 2 kW.

50. Consider the following statements :

In the case of a vapour compression machine, if the condensing temperature of the refrigerant is closer to the critical temperature, then there will be

1. excessive power consumption 2. high compression
3. large volume flow.

Of these statements :

(*a*) 1, 2 and 3 are correct (*b*) 1 and 2 are correct
(*c*) 2 and 3 are correct (*d*) 1 and 3 are correct.

51. Hydrogen is essential in an Electrolux refrigeration system, because

(*a*) it acts as a catalyst in the evaporator

(*b*) the reaction between hydrogen and ammonia is endothermic in evaporator and exothermic in absorber

(*c*) the cooled hydrogen leaving the heat exchanger cools the refrigerant entering the evaporator

(*d*) it helps in maintaining a low partial pressure for the evaporating ammonia.

52. In an ideal refrigeration (reversed Carnot) cycle, the condenser and evaporator temperatures are 27°C and – 13°C respectively. The COP of this cycle would be

(*a*) 6.5 (*b*) 7.5

(*c*) 10.5 (*d*) 15.0.

53. A single-stage vapour compression refrigeration system cannot be used to produce ultra low temperatures because

(*a*) refrigerants for ultra-low temperatures are not available

(*b*) lubricants for ultra-low temperatures are not available

(*c*) volumetric efficiency will decrease considerably

(*d*) heat leakage into the system will be excessive.

54. Vapour absorption refrigeration system works using the

(*a*) ability of a substance to get easily condensed or evaporated

(*b*) ability of a vapour to get compressed or expanded

(*c*) affinity of a substance for another substance

(*d*) absorptivity of a substance.

55. Which one of the following statements regarding ammonia absorption system is correct ? The solubility of ammonia in water is

(*a*) a function of the temperature and pressure of the solution

(*b*) a function of the pressure of the solution irrespective of the temperature

(*c*) a function of the temperature of the solution alone

(*d*) independent of the temperature and pressure of the solution.

56. Consider the following statements :

In thermoelectric refrigeration, the coefficient of performance is a function of

1. electrical conductivity of materials.
2. Peltier coefficient.
3. Seebeck coefficient.
4. temperature at cold and hot junctions.
5. thermal conductivity of materials.

Of these statements :

(*a*) 1, 3, 4 and 5 are correct (*b*) 1, 2, 3 and 5 are correct

(*c*) 1, 2, 4 and 5 are correct (*d*) 2, 3, 4 and 5 are correct.

57. Air cooling is used for freon compressors whereas water jacketing is adopted for cooling ammonia compressors. This is because

(*a*) latent heat of ammonia is higher than that of a freon

(*b*) thermal conductivity of water is higher than that of air

(*c*) specific heat of water is higher than that of air

(*d*) of the larger superheat horn of ammonia compression cycle.

58. Consider the following statements :

A psychrometer measures

1. wet bulb temperature
2. dew point temperature
3. dry bulb temperature.

Of these statements :

(*a*) 1 alone is correct (*b*) 2 and 3 are correct

(*c*) 1 and 3 are correct (*d*) 1, 2 and 3 are correct.

59. Hot coffee in a cup is allowed to cool. Its cooling rate is measured and found to be greater than the value calculated by conduction, convection and radiation measurements. The difference is due to

(*a*) properties of coffee changing with temperature

(*b*) currents of air flow in the room (*c*) underestimation of the emissivity of coffee

(*d*) evaporation.

60. For an air-conditioning plant above 300 ton, which one of the following systems would normally be preferred ?

(*a*) Ammonia reciprocating compressor (*b*) Centrifugal chiller

(*c*) Absorption refrigeration system (*d*) Hermetic compressor.

61. Fresh air intake (air change per hour) recommended for ventilation purposes in the air-conditioning system of an office building is

(*a*) 1/2 (*b*) 3/2

(*c*) 9/2 (*d*) 25/2.

62. Give that

Nu = Nusselt number, Re = Reynolds number,

Pr = Prandtl number, Sh = Sherwood number,

Sc = Schmidt number, and Gr = Grashoff number,

the functional relationship for free convective mass transfer is given as :

(*a*) Nu = *f* (Gr, Pr) (*b*) Sh = *f* (Sc, Gr)

(*c*) Nu = *f* (Re, Pr) (*d*) Sh = *f* (Re, Sc).

63. Air refrigeration cycle is used in

(*a*) commercial refrigerators (*b*) domestic refrigerators

(*c*) gas liquification (*d*) air-conditioning.

64. The flash chamber in single-stage simple vapour compression cycle

(*a*) increases the refrigerating effect (*b*) decrease the refrigerating effect

(*c*) increases the work of compression (*d*) has no effect on refrigerating effect.

65. Consider the following statements :

In a vapour compression system, a thermometer placed in the liquid line can indicate whether the

1. refrigerant flow is too low 2. water circulation is adequate

3. condenser is fouled 4. pump is functioning properly.

Of these statements :

(*a*) 1, 2 and 3 are correct (*b*) 1, 2 and 4 are correct

(*c*) 1, 3 and 4 are correct (*d*) 2, 3 and 4 are correct.

66. Match List with List II and select the correct answer using the codes given below the Lists :

List I	**List II**
A. Bell Colemen refrigeration	1. Compressor
B. Vapour compression refrigeration	2. Generator
C. Absorption refrigeration	3. Flash chamber
D. Jet refrigeration	4. Expansion cylinder

Codes :

	A	*B*	*C*	*D*
(*a*)	1	4	3	2
(*b*)	4	1	3	2
(*c*)	1	4	2	3
(*d*)	4	1	2	3.

67. The maximum C.O.P. for the absorption cycle is given by (T_G = generator temperature, T_C = environment temperature, T_E = refrigerated space temperature)

(*a*) $\dfrac{T_E(T_G - T_C)}{T_G(T_C - T_E)}$

(*b*) $\dfrac{T_G(T_C - T_E)}{T_E(T_G - T_C)}$

(*c*) $\dfrac{T_C(T_G - T_E)}{T_G(T_C - T_E)}$

(*d*) $\dfrac{T_G(T_C - T_E)}{T_C(T_G - T_E)}$.

68. In milk chilling plants, the usual secondary refrigerant is

(*a*) ammonia solution

(*b*) sodium silicate

(*c*) glycol

(*d*) brine.

69. The desirable combination of properties for a refrigerant include

(*a*) high specific heat and low specific volume

(*b*) high heat transfer coefficient and low latent heat

(*c*) high thermal conductivity and low freezing point

(*d*) high specific heat and high boiling point.

70. Which of the following method(s) is/are adopted in the design of air duct system ?

1. Velocity reduction method 2. Equal friction method 3. Static regain method.

Select the correct answer using the codes given below :

Codes :

(*a*) 1 alone

(*b*) 1 and 2

(*c*) 2 and 3

(*d*) 1, 2 and 3.

71. To fix the state point in respect of air-vapour mixtures, three intrinsic properties are needed. Yet, the psychrometric chart requires only two because

(*a*) water vapour is in the superheated state

(*b*) the chart is for a given pressure

(*c*) the chart is an approximation to true values

(*d*) the mixtures can be treated as a perfect gas.

72. During sensible cooling of air,

(*a*) its wet bulb temperature increases and dew point remains constant

(*b*) its wet bulb temperature decreases and the dew point remains constant

(*c*) its wet bulb temperature increases and the dew point decreases

(*d*) its wet bulb temperature decreases and dew point increases.

73. The expression $\dfrac{0.622\,p_v}{p_1 - p_v}$ is used to determine

(*a*) relative humidity (*b*) specific humidity

(*c*) degree of saturation (*d*) partial pressure.

74. The effective temperature is a measure of the combined effects of

(*a*) dry bulb temperature and relative humidity

(*b*) dry bulb temperature and air motion

(*c*) wet bulb temperature and air motion

(*d*) dry bulb temperature, relative humidity and air motion

75. In air-conditioning design for summer months, the condition inside a factory where heavy work is performed as compared to a factory in which light work is performed should have

(*a*) lower dry bulb temperature and lower relative humidity

(*b*) lower dry bulb temperature and higher relative humidity

(*c*) lower dry bulb temperature and same relative humidity

(*d*) same dry bulb temperature and same relative humidity.

ANSWERS

1.	(*c*)	**2.**	(*b*)	**3.**	(*c*)	**4.**	(*d*)	**5.**	(*a*)	**6.**	(*d*)	**7.**	(*c*)
8.	(*a*)	**9.**	(*b*)	**10.**	(*a*)	**11.**	(*a*)	**12.**	(*c*)	**13.**	(*a*)	**14.**	(*c*)
15.	(*b*)	**16.**	(*d*)	**17.**	(*b*)	**18.**	(*a*)	**19.**	(*a*)	**20.**	(*c*)	**21.**	(*a*)
22.	(*c*)	**23.**	(*d*)	**24.**	(*c*)	**25.**	(*b*)	**26.**	(*a*)	**27.**	(*b*)	**28.**	(*c*)
29.	(*c*)	**30.**	(*b*)	**31.**	(*c*)	**32.**	(*a*)	**33.**	(*b*)	**34.**	(*d*)	**35.**	(*d*)
36.	(*c*)	**37.**	(*d*)	**38.**	(*b*)	**39.**	(*d*)	**40.**	(*c*)	**41.**	(*d*)	**42.**	(*c*)
43.	(*c*)	**44.**	(*d*)	**45.**	(*b*)	**46.**	(*c*)	**47.**	(*c*)	**48.**	(*d*)	**49.**	(*c*)
50.	(*b*)	**51.**	(*d*)	**52.**	(*a*)	**53.**	(*d*)	**54.**	(*c*)	**55.**	(*a*)	**56.**	(*b*)
57.	(*c*)	**58.**	(*a*)	**59.**	(*b*)	**60.**	(*c*)	**61.**	(*b*)	**62.**	(*c*)	**63.**	(*d*)
64.	(*a*)	**65.**	(*a*)	**66.**	(*d*)	**67.**	(*a*)	**68.**	(*d*)	**69.**	(*d*)	**70.**	(*d*)
71.	(*b*)	**72.**	(*a*)	**73.**	(*b*)	**74.**	(*d*)	**75.**	(*a*).				

SOLUTIONS—COMMENTS

6. Because throttling is not actually an isothermal process. The expansion of gas causes a fall in temperature and increased kinectic energy increases it to the initial level.

13. For isothermal process, $T_1 = T_2$

or $$p_1V_1 = p_2V_2 \quad \Rightarrow \quad V_1 = \frac{p_2V_2}{p_1}$$

Given : $$\frac{p_1}{p_2} = 10 \text{ and } V_2 = 0.55 \text{ m}^3$$

$$\therefore \quad V_1 = \frac{0.55}{10} = 0.055 \text{ m}^3$$

For adiabatic expansion, pV^γ = constant

or $$p_1V_1^\gamma = p_2V_2^\gamma \quad \Rightarrow \quad V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \times V_1 = (10)^{\frac{1}{1.4}} \times 0.055 = 0.2848 \text{ m}^3$$

14. $$T_1 = T_2\ ;\ V_2 = \frac{V_1}{2} \quad \text{...(Given)}$$

From $$p_1V_1 = p_2V_2, \quad p_1V_1 = p_2 \times \frac{V_1}{2} \quad \Rightarrow \quad \mathbf{p_2 = 2p_1}$$

15. Since $(\Delta s)_{\text{system}} > (\Delta s)_{\text{surrounding}}$, where both $(\Delta s)_{\text{system}} > 0$ and $(\Delta s)_{\text{surrounding}} > 0$.

40. The compressors seal cannot fail due to moisture, all the other conditions do occur due to presence of moisture.

44. All common refrigerants like F_{11}, F_{12}, F_{22}, NH_3 etc. have approximately the same C.O.P. ranging from 4.76 to 5.09 and H.P./ton varies from 0.99 to 1.01.

The electric resistance of the refrigerants is an important factor when it is used in hermetically sealed unit where the motor is exposed to the refrigerant.

46. The reason why centrifugal compressors are used to large tonnage is that they can handle larger volumes of refrigerant, also the part load efficiency of this kind is higher.

49. $$\text{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{200}{300 - 200} = 2$$

$$\text{Power consumed} = \frac{R_n \text{ (net refrigerating effect)}}{\text{C.O.P.}} = \frac{2 \text{ kW}}{2} = 1 \text{ kW.}$$

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STEAM TABLES

and

Mollier Diagram

(*S.I. Units*)

CONTENTS

SYMBOLS AND UNITS USED IN THE TABLES

t = Temperature, °C
t_s = Saturation temperature, °C
p = Pressure, bar
h_f = Specific enthalpy of saturated liquid, kJ/kg
h_{fg} = Specific enthalpy of evaporation (latent heat), kJ/kg
h_g = Specific enthalpy of saturated vapour, kJ/kg
s_f = Specific entropy of saturated liquid, kJ/kg K
s_{fg} = Specific entropy of evaporation, kJ/kg K
s_g = Specific entropy of saturated vapour, kJ/kg K
v_f = Specific volume of saturated liquid, m^3/kg
v_g = Specific volume of saturated steam, m^3/kg

TABLE I

Saturated Water and Steam (Temperature) Tables

Temp. (°C)	*Absolute pressure (bar)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume* (m^3/kg)	
t	p	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
0	0.0061	– 0.02	2501.4	2501.3	– 0.0001	9.1566	9.1565	0.0010002	206.3
0.01	0.0061	0.01	2501.3	2501.4	0.000	9.156	9.156	0.0010002	206.2
1	0.0065	4.2	2499.0	2503.2	0.015	9.115	9.130	0.0010002	192.6
2	0.0070	8.4	2496.7	2505.0	0.031	9.073	9.104	0.0010001	179.9
3	0.0076	12.6	2494.3	2506.9	0.046	9.032	9.077	0.0010001	168.1
4	0.0081	16.8	2491.9	2508.7	0.061	8.990	9.051	0.0010001	157.2
5	0.0087	21.0	2489.6	2510.6	0.076	8.950	9.026	0.0010001	147.1
6	0.0093	25.2	2487.2	2512.4	0.091	8.909	9.000	0.0010001	137.7
7	0.0100	29.4	2484.8	2514.2	0.106	8.869	8.975	0.0010002	129.0
8	0.0107	33.6	2482.5	2516.1	0.121	8.829	8.950	0.0010002	120.9
9	0.0115	37.8	2480.1	2517.9	0.136	8.789	8.925	0.0010003	113.4
10	0.0123	42.0	2477.7	2519.7	0.151	8.750	8.901	0.0010004	106.4
11	0.0131	46.2	2475.4	2521.6	0.166	8.711	8.877	0.0010004	99.86
12	0.0140	50.4	2473.0	2523.4	0.181	8.672	8.852	0.0010005	93.78
13	0.0150	54.6	2470.7	2525.3	0.195	8.632	8.828	0.0010007	88.12
14	0.0160	58.8	2468.3	2527.1	0.210	8.595	8.805	0.0010008	82.85
15	0.0170	63.0	2465.9	2528.9	0.224	8.557	8.781	0.0010009	77.93
16	0.0182	67.2	2463.6	2530.8	0.239	8.519	8.758	0.001001	73.33
17	0.0194	71.4	2461.2	2532.6	0.253	8.482	8.735	0.001001	69.04
18	0.0206	75.6	2458.8	2534.4	0.268	8.444	8.712	0.001001	65.04
19	0.0220	79.8	2456.5	2536.3	0.282	8.407	8.690	0.001002	61.29
20	0.0234	84.0	2454.1	2538.1	0.297	8.371	8.667	0.001002	57.79
21	0.0249	88.1	2451.8	2539.9	0.311	8.334	8.645	0.001002	54.51
22	0.0264	92.3	2449.4	2541.7	0.325	8.298	8.623	0.001002	51.45
23	0.0281	96.5	2447.0	2543.5	0.339	8.262	8.601	0.001002	48.57
24	0.0298	100.7	2444.7	2545.4	0.353	8.226	8.579	0.001003	45.88
25	0.0317	104.9	2442.3	2547.2	0.367	8.191	8.558	0.001003	43.36
26	0.0336	109.1	2439.9	2549.0	0.382	8.155	8.537	0.001003	40.99
27	0.0357	113.2	2437.6	2550.8	0.396	8.120	8.516	0.001004	38.77
28	0.0378	117.4	2435.2	2552.6	0.409	8.086	8.495	0.001004	36.69
29	0.0401	121.6	2432.8	2554.5	0.423	8.051	8.474	0.001004	34.73
30	0.0425	125.8	2430.5	2556.3	0.437	8.016	8.453	0.001004	32.89

Temp. *(°C)*	*Absolute pressure (bar)*	*Specific enthalpy* *(kJ/kg)*			*Specific entropy* *(kJ/kg K)*			*Specific volume* *(m^3/kg)*	
t	p	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
31	0.0450	130.0	2428.1	2558.1	0.451	7.982	8.433	0.001005	31.17
32	0.0476	134.2	2425.7	2559.9	0.464	7.948	8.413	0.001005	29.54
33	0.0503	138.3	2423.4	2561.7	0.478	7.915	8.393	0.001005	28.01
34	0.0532	142.5	2421.0	2563.5	0.492	7.881	8.373	0.001006	26.57
35	0.0563	146.7	2418.6	2565.3	0.505	7.848	8.353	0.001006	25.22
36	0.0595	150.9	2416.2	2567.1	0.519	7.815	8.334	0.001006	23.94
37	0.0628	155.0	2413.9	2568.9	0.532	7.782	8.314	0.001007	22.74
38	0.0663	159.2	2411.5	2570.7	0.546	7.749	8.295	0.001007	21.60
39	0.0700	163.4	2409.1	2572.5	0.559	7.717	8.276	0.001007	20.53
40	0.0738	167.6	2406.7	2574.3	0.573	7.685	8.257	0.001008	19.52
41	0.0779	171.7	2404.3	2576.0	0.586	7.652	8.238	0.001008	18.57
42	0.0821	175.9	2401.9	2577.8	0.599	7.621	8.220	0.001009	17.67
43	0.0865	180.1	2399.5	2579.6	0.612	7.589	8.201	0.001009	16.82
44	0.0911	184.3	2397.2	2581.5	0.626	7.557	8.183	0.001010	16.02
45	0.0959	188.4	2394.8	2583.2	0.639	7.526	8.165	0.001010	15.26
46	0.1010	192.6	2392.4	2585.0	0.652	7.495	8.147	0.001010	14.54
47	0.1062	196.8	2390.0	2586.8	0.665	7.464	8.129	0.001011	13.86
48	0.1118	201.0	2387.6	2588.6	0.678	7.433	8.111	0.001011	13.22
49	0.1175	205.1	2385.2	2590.3	0.691	7.403	8.094	0.001012	12.61
50	0.1235	209.3	2382.7	2592.1	0.704	7.372	8.076	0.001012	12.03
52	0.1363	217.7	2377.9	2595.6	0.730	7.312	8.042	0.001013	10.97
54	0.1502	226.0	2373.1	2599.1	0.755	7.253	8.008	0.001014	10.01
56	0.1653	234.4	2368.2	2602.6	0.781	7.194	7.975	0.001015	9.149
58	0.1817	242.8	2363.4	2606.2	0.806	7.136	7.942	0.001016	8.372
60	0.1994	251.1	2358.5	2609.6	0.831	7.078	7.909	0.001017	7.671
62	0.2186	259.5	2353.6	2613.1	0.856	7.022	7.878	0.001018	7.037
64	0.2393	267.9	2348.7	2616.5	0.881	6.965	7.846	0.001019	6.463
66	0.2617	276.2	2343.7	2619.9	0.906	6.910	7.816	0.001020	5.943
68	0.2859	284.6	2338.8	2623.4	0.930	6.855	7.785	0.001022	5.471
70	0.3119	293.0	2333.8	2626.8	0.955	6.800	7.755	0.001023	5.042
75	0.3858	313.9	2321.4	2635.3	1.015	6.667	7.682	0.001026	4.131
80	0.4739	334.9	2308.8	2643.7	1.075	6.537	7.612	0.001029	3.407
85	0.5783	355.9	2296.0	2651.9	1.134	6.410	7.544	0.001033	2.828
90	0.7014	376.9	2283.2	2660.1	1.192	6.287	7.479	0.001036	2.361
95	0.8455	397.9	2270.2	2668.1	1.250	6.166	7.416	0.001040	1.982
100	1.0135	419.0	2257.0	2676.0	1.307	6.048	7.355	0.001044	1.673

TABLE II

Saturated Water and Steam (Pressure) Tables

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
0.006113	0.01	0.01	2 501.3	2 501.4	0.000	9.156	9.156	0.0010002	206.14
0.010	7.0	29.3	2 484.9	2 514.2	0.106	8.870	8.976	0.0010000	129.21
0.015	13.0	54.7	2 470.6	2 525.3	0.196	8.632	8.828	0.0010007	87.98
0.020	17.0	73.5	2 460.0	2 533.5	0.261	8.463	8.724	0.001001	67.00
0.025	21.1	88.5	2 451.6	2 540.1	0.312	8.331	8.643	0.001002	54.25
0.030	24.1	101.0	2 444.5	2 545.5	0.355	8.223	8.578	0.001003	45.67
0.035	26.7	111.9	2 438.4	2 550.3	0.391	8.132	8.523	0.001003	39.50
0.040	29.0	121.5	2 432.9	2 554.4	0.423	8.052	8.475	0.001004	34.80
0.045	31.0	130.0	2 428.2	2 558.2	0.451	7.982	8.433	0.001005	31.13
0.050	32.9	137.8	2 423.7	2 561.5	0.476	7.919	8.395	0.001005	28.19
0.055	34.6	144.9	2 419.6	2 565.5	0.500	7.861	8.361	0.001006	25.77
0.060	36.2	151.5	2 415.9	2 567.4	0.521	7.809	8.330	0.001006	23.74
0.065	37.6	157.7	2 412.4	2 570.1	0.541	7.761	8.302	0.001007	22.01
0.070	39.0	163.4	2 409.1	2 572.5	0.559	7.717	8.276	0.001007	20.53
0.075	40.3	168.8	2 406.0	2 574.8	0.576	7.675	8.251	0.001008	19.24
0.080	41.5	173.9	2 403.1	2 577.0	0.593	7.636	8.229	0.001008	18.10
0.085	42.7	178.7	2 400.3	2 579.0	0.608	7.599	8.207	0.001009	17.10
0.090	43.8	183.3	2 397.7	2 581.0	0.622	7.565	8.187	0.001009	16.20
0.095	44.8	187.7	2 395.2	2 582.9	0.636	7.532	8.168	0.001010	15.40
0.10	45.8	191.8	2 392.8	2 584.7	0.649	7.501	8.150	0.001010	14.67
0.11	47.7	199.7	2 388.3	2 588.0	0.674	7.453	8.117	0.001011	13.42
0.12	49.4	206.9	2 384.2	2 591.1	0.696	7.390	8.086	0.001012	12.36
0.13	51.0	213.7	2 380.2	2 593.9	0.717	7.341	8.058	0.001013	11.47
0.14	52.6	220.0	2 376.6	2 596.6	0.737	7.296	8.033	0.001013	10.69
0.15	54.0	226.0	2 373.2	2 599.2	0.754 9	7.254 4	8.009 3	0.001014	10.022
0.16	55.3	231.6	2 370.0	2 601.6	0.772 1	7.214 8	7.986 9	0.001015	9.433
0.17	56.6	236.9	2 366.9	2 603.8	0.788 3	7.177 5	7.965 8	0.001015	8.911
0.18	57.8	242.0	2 363.9	2 605.9	0.803 6	7.142 4	7.945 9	0.001016	8.445
0.19	59.0	246.8	2 361.1	2 607.9	0.818 2	7.109 0	7.927 2	0.001017	8.027
0.20	60.1	251.5	2 358.4	2 609.9	0.832 1	7.077 3	7.909 4	0.001017	7.650
0.21	61.1	255.9	2 355.8	2 611.7	0.845 3	7.047 2	7.892 5	0.001018	7.307
0.22	62.2	260.1	2 353.3	2 613.5	0.858 1	7.018 4	7.876 4	0.001018	6.995
0.23	63.1	264.2	2 350.9	2 615.2	0.870 2	6.990 8	7.861 1	0.001019	6.709
0.24	64.1	268.2	2 348.6	2 616.8	0.882 0	6.964 4	7.846 4	0.001019	6.447

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
0.25	65.0	272.0	2 346.4	2 618.3	0.893 2	6.939 1	7.832 3	0.001020	6.205
0.26	65.9	275.7	2 344.2	2 619.9	0.904 1	6.914 7	7.818 8	0.001020	5.980
0.27	66.7	279.2	2 342.1	2 621.3	0.914 6	6.891 2	7.805 8	0.001021	5.772
0.28	67.5	282.7	2 340.0	2 622.7	0.924 8	6.868 5	7.793 3	0.001021	5.579
0.29	68.3	286.0	2 338.1	2 624.1	0.934 6	6.846 6	7.781 2	0.001022	5.398
0.30	69.1	289.3	2 336.1	2 625.4	0.944 1	6.825 4	7.769 5	0.001022	5.229
0.32	70.6	295.5	2 332.4	2 628.0	0.962 3	6.785 0	7.747 4	0.001023	4.922
0.34	72.0	301.5	2 328.9	2 630.4	0.979 5	6.747 0	7.726 5	0.001024	4.650
0.36	73.4	307.1	2 325.5	2 632.6	0.995 8	6.711 1	7.707 0	0.001025	4.408
0.38	74.7	312.5	2 322.3	2 634.8	1.011 3	6.677 1	7.688 4	0.001026	4.190
0.40	75.9	317.7	2 319.2	2 636.9	1.026 1	6.644 8	7.670 9	0.001026	3.993
0.42	77.1	322.6	2 316.3	2 638.9	1.040 2	6.614 0	7.654 2	0.001027	3.815
0.44	78.2	327.3	2 313.4	2 640.7	1.053 7	6.584 6	7.638 3	0.001028	3.652
0.46	79.3	331.9	2 310.7	2 642.6	1.066 7	6.556 4	7.623 1	0.001029	3.503
0.48	80.3	336.3	2 308.0	2 644.3	1.079 2	6.529 4	7.608 6	0.001029	3.367
0.50	81.3	340.6	2 305.4	2 646.0	1.091 2	6.503 5	7.594 7	0.001030	3.240
0.55	83.7	350.6	2 299.3	2 649.9	1.119 4	6.442 8	7.562 3	0.001032	2.964
0.60	86.0	359.9	2 293.6	2 653.6	1.145 4	6.387 3	7.532 7	0.001033	2.732
0.65	88.0	368.6	2 288.3	2 656.9	1.169 6	6.336 0	7.505 5	0.001035	2.535
0.70	90.0	376.8	2 283.3	2 660.1	1.192 1	6.288 3	7.480 4	0.001036	2.369
0.75	92.0	384.5	2 278.6	2 663.0	1.213 1	6.243 9	7.457 0	0.001037	2.217
0.80	93.5	391.7	2 274.0	2 665.8	1.233 0	6.202 2	7.435 2	0.001039	2.087
0.85	95.1	398.6	2 269.8	2 668.4	1.251 8	3.162 9	7.414 7	0.001040	1.972
0.90	96.7	405.2	2 265.6	2 670.9	1.269 6	6.125 8	7.395 4	0.001041	1.869
0.95	98.2	411.5	2 261.7	2 673.2	1.286 5	6.090 6	7.377 1	0.001042	1.777
1.0	99.6	417.5	2 257.9	2 675.4	1.302 7	6.057 1	7.359 8	0.001043	1.694
1.1	102.3	428.8	2 250.8	2 679.6	1.333 0	5.994 7	7.327 7	0.001046	1.549
1.2	104.8	439.4	2 244.1	2 683.4	1.360 9	5.937 5	7.298 4	0.001048	1.428
1.3	107.1	449.2	2 237.8	2 687.0	1.386 8	5.884 7	7.271 5	0.001050	1.325
1.4	109.3	458.4	2 231.9	2 690.3	1.410 9	5.835 6	7.246 5	0.001051	1.236
1.5	111.3	467.1	2 226.2	2 693.4	1.433 6	5.789 8	7.233 4	0.001053	1.159
1.6	113.3	475.4	2 220.9	2 696.2	1.455 0	5.746 7	7.201 7	0.001055	1.091
1.7	115.2	483.2	2 215.7	2 699.0	1.475 2	5.706 1	7.181 3	0.001056	1.031
1.8	116.9	490.7	2 210.8	2 701.5	1.494 4	5.667 8	7.162 2	0.001058	0.977
1.9	118.6	497.8	2 206.1	2 704.0	1.512 7	5.631 4	7.144 0	0.001060	0.929

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
2.0	120.2	504.7	2 201.6	2 706.3	1.530 1	5.596 7	7.126 8	0.001061	0.885
2.1	121.8	511.3	2 197.2	2 708.5	1.546 8	5.563 7	7.110 5	0.001062	0.846
2.2	123.3	517.6	2 193.0	2 710.6	1.562 7	5.532 1	7.094 9	0.001064	0.810
2.3	124.7	523.7	2 188.9	2 712.6	1.578 1	5.501 9	7.080 0	0.001065	0.777
2.4	126.1	529.6	2 184.9	2 714.5	1.592 9	5.472 8	7.065 7	0.001066	0.746
2.5	127.4	535.3	2 181.0	2 716.4	1.607 1	5.444 9	7.052 0	0.001068	0.718
2.6	128.7	540.9	2 177.3	2 718.2	1.620 9	5.418 0	7.038 9	0.001069	0.693
2.7	129.9	546.2	2 173.6	2 719.9	1.634 2	5.392 0	7.026 2	0.001070	0.668
2.8	131.2	551.4	2 170.1	2 721.5	1.647 1	5.367 0	7.014 0	0.001071	0.646
2.9	132.4	556.5	2 166.6	2 723.1	1.659 5	5.342 7	7.002 3	0.001072	0.625
3.0	133.5	561.4	2 163.2	2 724.7	1.671 6	5.319 3	6.990 9	0.001074	0.606
3.1	134.6	566.2	2 159.9	2 726.1	1.683 4	5.296 5	6.979 9	0.001075	0.587
3.2	135.7	570.9	2 156.7	2 727.6	1.694 8	5.274 4	6.969 2	0.001076	0.570
3.3	136.8	575.5	2 153.5	2 729.0	1.705 9	5.253 0	6.958 9	0.001077	0.554
3.4	137.8	579.9	2 150.4	2 730.3	1.716 8	5.232 2	6.948 9	0.001078	0.538
3.5	138.8	584.3	2 147.4	2 731.6	1.727 3	5.211 9	6.939 2	0.001079	0.524
3.6	139.8	588.5	2 144.4	2 732.9	1.737 6	5.192 1	6.929 7	0.001080	0.510
3.7	140.8	592.7	2 141.4	2 734.1	1.747 6	5.172 9	6.920 5	0.001081	0.497
3.8	141.8	596.8	2 138.6	2 735.3	1.757 4	5.154 1	6.911 6	0.001082	0.486
3.9	142.7	600.8	2 135.7	2 736.5	1.767 0	5.135 8	6.902 8	0.001083	0.473
4.0	143.6	604.7	2 133.0	2 737.6	1.776 4	5.117 9	6.894 3	0.001084	0.462
4.2	145.4	612.3	2 127.5	2 739.8	1.794 5	5.083 4	6.877 9	0.001086	0.441
4.4	147.1	619.6	2 122.3	2 741.9	1.812 0	5.050 3	6.862 3	0.001088	0.423
4.6	148.7	626.7	2 117.2	2 743.9	1.828 7	5.018 6	6.847 3	0.001089	0.405
4.8	150.3	633.5	2 112.2	2 745.7	1.844 8	4.988 1	6.832 9	0.001091	0.390
5.0	151.8	640.1	2 107.4	2 747.5	1.860 4	4.958 8	6.819 2	0.001093	0.375
5.2	153.3	646.5	2 102.7	2 749.3	1.875 4	4.930 6	6.805 9	0.001094	0.361
5.4	154.7	652.8	2 098.1	2 750.9	1.889 9	4.903 3	6.793 2	0.001096	0.348
5.6	156.2	658.8	2 093.7	2 752.5	1.904 0	4.876 9	6.780 9	0.001098	0.337
5.8	157.5	664.7	2 089.3	2 754.0	1.917 6	4.851 4	6.769 0	0.001099	0.326
6.0	158.8	670.4	2 085.0	2 755.5	1.930 8	4.826 7	6.757 5	0.001101	0.315
6.2	160.1	676.0	2 080.9	2 756.9	1.943 7	4.802 7	6.746 4	0.001102	0.306
6.4	161.4	681.5	2 076.8	2 758.2	1.956 2	4.779 4	6.735 6	0.001104	0.297
6.6	162.6	686.8	2 072.7	2 759.5	1.968 4	4.756 8	6.725 2	0.001105	0.288
6.8	163.8	692.0	2 068.8	2 760.8	1.980 2	4.734 8	6.715 0	0.001107	0.280

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
7.0	165.0	697.1	2 064.9	2 762.0	1.991 8	4.713 4	6.705 2	0.001108	0.273
7.2	166.1	702.0	2 061.1	2 763.2	2.003 1	4.692 5	6.695 6	0.001110	0.265
7.4	167.2	706.9	2 057.4	2 764.3	2.014 1	4.672 1	6.686 2	0.001111	0.258
7.6	168.3	711.7	2 053.7	2 765.4	2.024 9	4.652 2	6.677 1	0.001112	0.252
7.8	169.4	716.3	2 050.1	2 766.4	2.035 4	4.632 8	6.668 3	0.001114	0.246
8.0	170.4	720.9	2 046.5	2 767.5	2.045 7	4.613 9	6.659 6	0.001115	0.240
8.2	171.4	725.4	2 043.0	2 768.5	2.055 8	4.595 3	6.651 1	0.001116	0.235
8.4	172.4	729.9	2 039.6	2 769.4	2.065 7	4.577 2	6.642 9	0.001118	0.229
8.6	173.4	734.2	2 036.2	2 770.4	2.075 3	4.559 4	6.634 8	0.001119	0.224
8.8	174.4	738.5	2 032.8	2 771.3	2.084 8	4.542 1	6.626 9	0.001120	0.219
9.0	175.4	742.6	2 029.5	2 772.1	2.094 1	4.525 0	6.619 2	0.001121	0.215
9.2	176.3	746.8	2 026.2	2 773.0	2.103 3	4.508 3	6.611 6	0.001123	0.210
9.4	177.2	750.8	2 023.0	2 773.8	2.112 2	4.492 0	6.604 2	0.001124	0.206
9.6	178.1	754.8	2 019.8	2 774.6	2.121 0	4.475 9	6.596 9	0.001125	0.202
9.8	179.0	758.7	2 016.7	2 775.4	2.129 7	4.460 1	6.589 8	0.001126	0.198
10.0	179.9	762.6	2 013.6	2 776.2	2.138 2	4.444 6	6.582 8	0.001127	0.194
10.5	182.0	772.0	2 005.9	2 778.0	2.158 8	4.407 1	6.565 9	0.001130	0.185
11.0	184.1	781.1	1 998.5	2 779.7	2.178 6	4.371 1	6.549 7	0.001133	0.177
11.5	186.0	789.9	1 991.3	2 781.3	2.197 7	4.336 6	6.534 2	0.001136	0.170
12.0	188.0	798.4	1 984.3	2 782.7	2.216 1	4.303 3	6.519 4	0.001139	0.163
12.5	189.8	806.7	1 977.4	2 784.1	2.233 8	4.271 2	6.505 0	0.001141	0.157
13.0	191.6	814.7	1 970.7	2 785.4	2.251 0	4.240 3	6.491 3	0.001144	0.151
13.5	193.3	822.5	1 964.2	2 786.6	2.267 6	4.210 4	6.477 9	0.001146	0.146
14.0	195.0	830.1	1 957.7	2 787.8	2.283 7	4.181 4	6.465 1	0.001149	0.141
14.5	196.7	837.5	1 951.4	2 788.9	2.299 3	4.153 3	6.452 6	0.001151	0.136
15.0	198.3	844.7	1 945.2	2 789.9	2.314 5	4.126 1	6.440 6	0.001154	0.132
15.5	199.8	851.7	1 939.2	2 790.8	2.329 2	4.099 6	6.428 9	0.001156	0.128
16.0	201.4	858.6	1 933.2	2 791.7	2.343 6	4.073 9	6.417 5	0.001159	0.124
16.5	202.8	865.3	1 927.3	2 792.6	2.357 6	4.048 9	6.406 5	0.001161	0.120
17.0	204.3	871.8	1 921.5	2 793.4	2.371 3	4.024 5	6.395 7	0.001163	0.117
17.5	205.7	878.3	1 915.9	2 794.1	2.384 6	4.000 7	6.385 3	0.001166	0.113
18.0	207.1	884.6	1 910.3	2 794.8	2.397 6	3.977 5	6.375 1	0.001168	0.110
18.5	208.4	890.7	1 904.7	2 795.5	2.410 3	3.954 8	6.365 1	0.001170	0.107
19.0	209.8	896.8	1 899.3	2 796.1	2.422 8	3.932 6	6.355 4	0.001172	0.105
19.5	211.1	902.8	1 893.9	2 796.7	2.434 9	3.911 0	6.345 9	0.001174	0.102

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
20.0	212.4	908.6	1 888.6	2 797.2	2.446 9	3.889 8	6.336 6	0.001177	0.0995
20.5	213.6	914.3	1 883.4	2 797.7	2.458 5	3.869 0	6.327 6	0.001179	0.0971
21.0	214.8	920.0	1 878.2	2 798.2	2.470 0	3.848 7	6.318 7	0.001181	0.0949
21.5	216.1	925.5	1 873.1	2 798.6	2.481 2	3.828 8	6.310 0	0.001183	0.0927
22.0	217.2	931.0	1 868.1	2 799.1	2.492 2	3.809 3	6.301 5	0.001185	0.0907
22.5	218.4	936.3	1 863.1	2 799.4	2.503 0	3.790 1	6.293 1	0.001187	0.0887
23.0	219.5	941.6	1 858.2	2 799.8	2.513 6	3.771 3	6.284 9	0.001189	0.0868
23.5	220.7	946.8	1 853.3	2 800.1	2.524 1	3.752 8	6.276 9	0.001191	0.0849
24.0	221.8	951.9	1 848.5	2 800.4	2.534 3	3.734 7	6.269 0	0.001193	0.0832
24.5	222.9	957.0	1 843.7	2 800.7	2.544 4	3.716 8	6.261 2	0.001195	0.0815
25.0	223.9	962.0	1 839.0	2 800.9	2.554 3	3.699 3	6.253 6	0.001197	0.0799
25.5	225.0	966.9	1 834.3	2 801.2	2.564 0	3.682 1	6.246 1	0.001199	0.0783
26.0	226.0	971.7	1 829.6	2 801.4	2.573 6	3.665 1	6.238 7	0.001201	0.0769
26.5	227.1	976.5	1 825.1	2 801.6	2.583 1	3.648 4	6.231 5	0.001203	0.0754
27.0	228.1	981.2	1 820.5	2 801.7	2.592 4	3.632 0	6.224 4	0.001205	0.0740
27.5	229.1	985.9	1 816.0	2 801.9	2.601 6	3.615 8	6.217 3	0.001207	0.0727
28.0	230.0	990.5	1 811.5	2 802.0	2.610 6	3.599 8	6.210 4	0.001209	0.0714
28.5	231.0	995.0	1 807.1	2 802.1	2.619 5	3.584 1	6.203 6	0.001211	0.0701
29.0	232.0	999.5	1 802.6	2 802.2	2.628 3	3.568 6	6.196 9	0.001213	0.0689
29.5	233.0	1 004.0	1 798.3	2 802.2	2.637 0	3.553 3	6.190 2	0.001214	0.0677
30.0	233.8	1 008.4	1 793.9	2 802.3	2.645 5	3.538 2	6.183 7	0.001216	0.0666
30.5	234.7	1 012.7	1 789.6	2 802.3	2.653 9	3.523 3	6.177 2	0.001218	0.0655
31.0	235.6	1 017.0	1 785.4	2 802.3	2.662 3	3.508 7	6.170 9	0.001220	0.0645
31.5	236.5	1 021.2	1 781.1	2 802.3	2.670 5	3.494 2	6.164 7	0.001222	0.0634
32.0	237.4	1 025.4	1 776.9	2 802.3	2.678 6	3.479 9	6.158 5	0.001224	0.0624
32.5	238.3	1 029.6	1 772.7	2 802.3	2.686 6	3.465 7	6.152 3	0.001225	0.0615
33.0	239.2	1 033.7	1 768.6	2 802.3	2.694 5	3.451 8	6.146 3	0.001227	0.0605
33.5	240.0	1 037.8	1 764.4	2 802.2	2.702 3	3.438 0	6.140 3	0.001229	0.0596
34.0	240.9	1 041.8	1 760.3	2 802.1	2.710 1	3.424 4	6.134 4	0.001231	0.0587
34.5	241.7	1 045.8	1 756.3	2 802.1	2.717 7	3.410 9	6.128 6	0.001233	0.0579
35.0	242.5	1 049.8	1 752.2	2 802.0	2.725 3	3.397 6	6.122 8	0.001234	0.0570
35.5	243.3	1 053.7	1 748.2	2 801.8	2.732 7	3.384 4	6.117 1	0.001236	0.0562
36.0	244.2	1 057.6	1 744.2	2 801.7	2.740 1	3.371 4	6.111 5	0.001238	0.0554
36.5	245.0	1 061.4	1 740.2	2 801.6	2.747 4	3.358 5	6.105 9	0.001239	0.0546
37.0	245.7	1 065.2	1 736.2	2 801.4	2.754 7	3.345 8	6.100 4	0.001242	0.0539

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ / kg)*			*Specific entropy (kJ / kg K)*			*Specific volume (m^3 / kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
37.5	246.5	1 069.0	1 732.3	2 801.3	2.761 8	3.333 2	6.095 0	0.001243	0.0531
38.0	247.3	1 072.7	1 728.4	2 801.1	2.768 9	3.320 7	6.089 6	0.001245	0.0524
38.5	248.1	1 076.4	1 724.5	2 800.9	2.775 9	3.308 3	6.084 2	0.001247	0.0517
39.0	248.8	1 080.1	1 720.6	2 800.8	2.782 9	3.296 1	6.078 9	0.001249	0.0511
39.5	249.6	1 083.8	1 716.8	2 800.5	2.789 7	3.284 0	6.073 7	0.001250	0.0504
40.0	250.3	1 087.4	1 712.9	2 800.3	2.796 5	3.272 0	6.068 5	0.001252	0.0497
41.0	251.8	1 094.6	1 705.3	2 799.9	2.809 9	3.248 3	6.058 2	0.001255	0.0485
42.0	253.2	1 101.6	1 697.8	2 799.4	2.823 1	3.225 1	6.048 2	0.001259	0.0473
43.0	254.6	1 108.5	1 690.3	2 798.8	2.836 0	3.202 3	6.038 3	0.001262	0.0461
44.0	256.0	1 115.4	1 682.9	2 798.3	2.848 7	3.179 9	6.028 6	0.001266	0.0451
45.0	257.4	1 122.1	1 675.6	2 797.7	2.861 2	3.157 9	6.019 1	0.001269	0.0440
46.0	258.7	1 128.8	1 668.3	2 797.0	2.873 5	3.136 2	6.009 7	0.001272	0.0430
47.0	260.1	1 135.3	1 661.1	2 796.4	2.885 5	3.114 9	6.000 4	0.001276	0.0421
48.0	261.4	1 141.8	1 653.9	2 795.7	2.897 4	3.093 9	5.991 3	0.001279	0.0412
49.0	262.6	1 148.2	1 646.8	2 794.9	2.909 1	3.073 3	5.982 3	0.001282	0.0403
50.0	263.9	1 154.5	1 639.7	2 794.2	2.920 6	3.052 9	5.973 5	0.001286	0.0394
51.0	265.1	1 160.7	1 632.7	2 793.4	2.931 9	3.032 8	5.964 8	0.001289	0.0386
52.0	266.4	1 166.8	1 625.7	2 792.6	2.943 1	3.013 0	5.956 1	0.001292	0.0378
53.0	267.6	1 172.9	1 618.8	2 791.7	2.954 1	2.993 5	5.947 6	0.001296	0.0371
54.0	268.7	1 178.9	1 611.9	2 790.8	2.965 0	2.974 2	5.939 2	0.001299	0.0363
55.0	269.9	1 184.9	1 605.0	2 789.9	2.975 7	2.955 2	5.930 9	0.001302	0.0356
56.0	271.1	1 190.8	1 598.2	2 789.0	2.986 3	2.936 4	5.922 7	0.001306	0.0349
57.0	272.2	1 196.6	1 591.4	2 788.0	2.996 7	2.917 9	5.914 6	0.001309	0.0343
58.0	273.3	1 202.3	1 584.7	2 787.0	3.007 1	2.899 5	5.906 6	0.001312	0.0336
59.0	274.4	1 208.0	1 578.0	2 786.0	3.017 2	2.881 4	5.898 6	0.001315	0.0330
60.0	275.5	1 213.7	1 571.3	2 785.0	3.027 3	2.863 5	5.890 8	0.001318	0.0324
61.0	276.6	1 219.3	1 564.7	2 784.0	3.037 2	2.845 8	5.883 0	0.001322	0.0319
62.0	277.7	1 224.8	1 558.0	2 782.9	3.047 1	2.828 3	5.875 3	0.001325	0.0313
63.0	278.7	1 230.3	1 551.5	2 781.8	3.056 8	2.810 9	5.867 7	0.001328	0.0308
64.0	279.8	1 235.7	1 544.9	2 780.6	3.066 4	2.793 8	5.860 1	0.001332	0.0302
65.0	280.8	1 241.1	1 538.4	2 779.5	3.075 9	2.776 8	5.852 7	0.001335	0.0297
66.0	281.8	1 246.5	1 531.9	2 778.3	3.085 3	2.760 0	5.845 2	0.001338	0.0292
67.0	282.8	1 251.8	1 525.4	2 777.1	3.094 6	2.743 3	5.837 9	0.001341	0.0287
68.0	283.8	1 257.0	1 518.9	2 775.9	3.103 8	2.726 8	5.830 6	0.001345	0.0283
69.0	284.8	1 262.2	1 512.5	2 774.7	3.112 9	2.710 5	5.823 3	0.001348	0.0278

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
70.0	285.8	1 267.4	1 506.0	2 773.5	3.121 9	2.694 3	5.816 2	0.001351	0.0274
71.0	286.7	1 272.5	1 499.6	2 772.2	3.130 8	2.678 2	5.809 0	0.001355	0.0269
72.0	287.7	1 277.6	1 493.3	2 770.9	3.139 7	2.662 3	5.802 0	0.001358	0.0265
73.0	288.6	1 282.7	1 486.9	2 769.6	3.148 4	2.646 5	5.794 9	0.001361	0.0261
74.0	289.6	1 287.7	1 480.5	2 768.3	3.157 1	2.630 9	5.788 0	0.001364	0.0257
75.0	290.5	1 292.7	1 474.2	2 766.9	3.165 7	2.615 3	5.781 0	0.001368	0.0253
76.0	291.4	1 297.6	1 467.9	2 765.5	3.174 2	2.599 9	5.774 2	0.001371	0.0249
77.0	292.3	1 302.5	1 461.6	2 764.2	3.182 7	2.584 6	5.767 3	0.001374	0.0246
78.0	293.2	1 307.4	1 455.3	2 762.8	3.191 1	2.569 5	5.760 5	0.001378	0.0242
79.0	294.1	1 312.3	1 449.1	2 761.3	3.199 4	2.554 4	5.753 8	0.001381	0.0239
80.0	294.9	1 317.1	1 442.8	2 759.9	3.207 6	2.539 5	5.747 1	0.001384	0.0235
81.0	295.8	1 321.9	1 436.6	2 758.4	3.215 8	2.524 6	5.740 4	0.001387	0.0232
82.0	296.7	1 326.6	1 430.3	2 757.0	3.223 9	2.509 9	5.733 8	0.001391	0.0229
83.0	297.5	1 331.4	1 424.1	2 755.5	3.232 0	2.495 2	5.727 2	0.001394	0.0225
84.0	298.4	1 336.1	1 417.9	2 754.0	3.239 9	2.480 7	5.720 6	0.001397	0.0222
85.0	299.2	1 340.7	1 411.7	2 752.5	3.247 9	2.466 3	5.714 1	0.001401	0.0219
86.0	300.1	1 345.4	1 405.5	2 750.9	3.255 7	2.451 9	5.707 6	0.001404	0.0216
87.0	300.9	1 350.0	1 399.3	2 749.4	3.263 6	2.437 6	5.701 2	0.001408	0.0213
88.0	301.7	1 354.6	1 393.2	2 747.8	3.271 3	2.423 5	5.694 8	0.001411	0.0211
89.0	302.5	1 359.2	1 387.0	2 746.2	3.279 0	2.409 4	5.688 4	0.001414	0.0208
90.0	303.3	1 363.7	1 380.9	2 744.6	3.286 7	2.395 3	5.682 0	0.001418	0.0205
91.0	304.1	1 368.3	1 374.7	2 743.0	3.294 3	2.381 4	5.675 7	0.001421	0.0202
92.0	304.9	1 372.8	1 368.6	2 741.4	3.301 8	2.367 6	5.669 4	0.001425	0.0199
93.0	305.7	1 377.2	1 362.5	2 739.7	3.309 3	2.353 8	5.663 1	0.001428	0.0197
94.0	306.4	1 381.7	1 356.3	2 738.0	3.316 8	2.340 1	5.656 8	0.001432	0.0194
95.0	307.2	1 386.1	1 350.2	2 736.4	3.324 2	2.326 4	5.650 6	0.001435	0.0192
96.0	308.0	1 390.6	1 344.1	2 734.7	3.331 5	2.312 9	5.644 4	0.001438	0.0189
97.0	308.7	1 395.0	1 338.0	2 733.0	3.338 8	2.299 4	5.638 2	0.001442	0.0187
98.0	309.4	1 399.3	1 331.9	2 731.2	3.346 1	2.285 9	5.632 1	0.001445	0.0185
99.0	310.2	1 403.7	1 325.8	2 729.5	3.353 4	2.272 6	5.625 9	0.001449	0.0183
100.0	311.1	1 408.0	1 319.7	2 727.7	3.360 5	2.259 3	5.619 8	0.001452	0.0181
102.0	312.4	1 416.7	1 307.5	2 724.2	3.374 8	2.232 8	5.607 6	0.001459	0.0176
104.0	313.8	1 425.2	1 295.3	2 720.5	3.388 9	2.206 6	5.595 5	0.001467	0.0172
106.0	315.3	1 433.7	1 283.1	2 716.8	3.402 9	2.180 6	5.583 5	0.001474	0.0168
108.0	316.6	1 442.2	1 270.9	2 713.1	3.416 7	2.154 8	5.571 5	0.001481	0.0164

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
110.0	318.0	1 450.6	1 258.7	2 709.3	3.430 4	2.129 1	5.559 5	0.001488	0.0160
112.0	319.4	1 458.9	1 246.5	2 705.4	3.444 0	2.103 6	5.547 6	0.001496	0.0157
114.0	320.7	1 467.2	1 234.3	2 701.5	3.457 4	2.078 3	5.535 7	0.001504	0.0153
116.0	322.1	1 475.4	1 222.0	2 697.4	3.470 8	2.053 1	5.523 9	0.001511	0.0149
118.0	323.4	1 483.6	1 209.7	2 693.3	3.484 0	2.028 0	5.512 1	0.001519	0.0146
120.0	324.6	1 491.8	1 197.4	2 689.2	3.497 2	2.003 0	5.500 2	0.001527	0.0143
122.0	325.9	1 499.9	1 185.0	2 684.9	3.510 2	1.978 2	5.488 4	0.001535	0.0139
124.0	327.1	1 508.0	1 172.6	2 680.6	3.523 2	1.953 3	5.476 5	0.001543	0.0137
126.0	328.4	1 516.0	1 160.1	2 676.1	3.536 0	1.928 6	5.464 6	0.001551	0.0134
128.0	329.6	1 524.0	1 147.6	2 671.6	3.548 8	1.903 9	5.452 7	0.001559	0.0131
130.0	330.8	1 532.0	1 135.0	2 667.0	3.561 6	1.879 2	5.440 8	0.001567	0.0128
132.0	332.0	1 540.0	1 122.3	2 662.3	3.574 2	1.854 6	5.428 8	0.001576	0.0125
134.0	333.2	1 547.9	1 109.5	2 657.4	3.586 8	1.830 0	5.416 8	0.001584	0.0123
136.0	334.3	1 555.8	1 096.7	2 652.5	3.599 3	1.805 3	5.404 7	0.001593	0.0120
138.0	335.5	1 563.7	1 083.8	2 647.5	3.611 8	1.780 7	5.392 5	0.001602	0.0117
140.0	336.6	1 571.6	1 070.7	2 642.4	3.624 2	1.756 0	5.380 3	0.001611	0.0115
142.0	337.7	1 579.5	1 057.6	2 637.1	3.636 6	1.731 3	5.367 9	0.001619	0.0112
144.0	338.8	1 587.4	1 044.4	2 631.8	3.649 0	1.706 6	5.355 5	0.001629	0.0110
146.0	339.9	1 595.3	1 031.0	2 626.3	3.661 3	1.681 8	5.343 1	0.001638	0.0108
148.0	341.1	1 603.1	1 017.6	2 620.7	3.673 6	1.656 9	5.330 5	0.001648	0.0106
150.0	342.1	1 611.0	1 004.0	2 615.0	3.685 9	1.632 0	5.317 9	0.001658	0.0103
152.0	343.2	1 618.9	990.3	2 609.2	3.698 1	1.607 0	5.305 1	0.001668	0.0101
154.0	344.2	1 626.8	976.5	2 603.3	3.710 3	1.581 9	5.292 2	0.001678	0.00991
156.0	345.3	1 634.7	962.6	2 597.3	3.722 6	1.556 7	5.279 3	0.001689	0.00971
158.0	346.3	1 642.6	948.5	2 591.1	3.734 8	1.531 4	5.266 3	0.001699	0.00951
160.0	347.3	1 650.5	934.3	2 584.9	3.747 1	1.506 0	5.253 1	0.001710	0.00931
162.0	348.3	1 658.5	920.0	2 578.5	3.759 4	1.480 6	5.239 9	0.001721	0.00911
164.0	349.3	1 666.5	905.6	2 572.1	3.771 7	1.455 0	5.226 7	0.001733	0.00893
166.0	350.3	1 674.5	891.0	2 565.5	3.784 2	1.429 0	5.213 2	0.001745	0.00874
168.0	351.3	1 683.0	875.6	2 558.6	3.797 4	1.402 1	5.199 4	0.001757	0.00855
170.0	352.3	1 691.7	859.9	2 551.6	3.810 7	1.374 8	5.185 5	0.001769	0.00837
172.0	353.2	1 700.4	844.0	2 544.4	3.824 0	1.347 3	5.171 3	0.001783	0.00819
174.0	354.2	1 709.0	828.1	2 537.1	3.837 2	1.319 8	5.157 0	0.001796	0.00801
176.0	355.1	1 717.6	811.9	2 529.5	3.850 4	1.292 2	5.142 5	0.001810	0.00784
178.0	356.0	1 726.2	795.6	2 521.8	3.863 5	1.264 3	5.127 8	0.001825	0.00767

Absolute pressure (bar)	*Temp. (°C)*	*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*			*Specific volume (m^3/kg)*	
p	t_s	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
180.0	356.9	1 734.8	779.1	2 513.9	3.876 5	1.236 2	5.112 8	0.001840	0.00750
182.0	357.8	1 743.4	762.3	2 505.8	3.889 6	1.207 9	5.097 5	0.001856	0.00733
184.0	358.7	1 752.1	745.3	2 497.4	3.902 8	1.179 2	5.082 0	0.001872	0.00717
186.0	359.6	1 760.9	727.9	2 488.8	3.916 0	1.150 1	5.066 1	0.001889	0.00701
188.0	360.5	1 769.7	710.1	2 479.8	3.929 4	1.120 5	5.049 8	0.001907	0.00684
190.0	361.4	1 778.7	692.0	2 470.6	3.942 9	1.090 3	5.033 2	0.001926	0.00668
192.0	362.3	1 787.8	673.3	2 461.1	3.956 6	1.059 4	5.016 0	0.001946	0.00652
194.0	363.2	1 797.0	654.1	2 451.1	3.970 6	1.027 8	4.998 3	0.001967	0.00636
196.0	364.0	1 806.6	634.2	2 440.7	3.984 9	0.995 1	4.980 0	0.001989	0.00620
198.0	364.8	1 816.3	613.5	2 429.8	3.999 6	0.961 4	4.961 1	0.002012	0.00604
200.0	365.7	1 826.5	591.9	2 418.4	4.014 9	0.926 3	4.941 2	0.002037	0.00588
202.0	366.5	1 837.0	569.2	2 406.2	4.030 8	0.889 7	4.920 4	0.002064	0.00571
204.0	367.3	1 848.1	545.1	2 393.3	4.047 4	0.851 0	4.898 4	0.002093	0.00555
206.0	368.2	1 859.9	519.5	2 379.4	4.065 1	0.809 9	4.875 0	0.002125	0.00538
208.0	368.9	1 872.5	491.7	2 364.2	4.084 1	0.765 7	4.849 8	0.002161	0.00521
210.0	369.8	1 886.3	461.3	2 347.6	4.104 8	0.717 5	4.822 3	0.002201	0.00502
212.0	370.6	1 901.5	427.4	2 328.9	4.127 9	0.663 9	4.791 7	0.002249	0.00483
214.0	371.3	1 919.0	388.4	2 307.4	4.154 3	0.602 6	4.756 9	0.002306	0.00462
216.0	372.1	1 939.9	341.6	2 281.6	4.186 1	0.529 3	4.715 4	0.002379	0.00439
218.0	372.9	1 967.2	280.8	2 248.0	4.227 6	0.434 6	4.662 2	0.002483	0.00412
220.0	373.7	2 011.1	184.5	2 195.6	4.294 7	0.285 2	4.579 9	0.002671	0.00373
221.2	374.1	2 107.4	0.0	2 107.4	4.442 9	0.0	4.442 9	0.003170	0.00317

TABLE III

Superheated Steam at Various Pressures and Temperatures

↓*p (bar)* (t_s)	*t* (°C) →	*50*	*100*	*150*	*200*	*250*	*300*	*400*	*500*
0.01 (7.0)	*v*	149.1	172.2	195.3	218.4	241.5	264.5	310.7	356.8
	u	2445.4	2516.4	2588.4	2661.6	2736.9	2812.2	2969.0	3132.4
	h	2594.5	2688.6	2783.6	2880.0	2978.4	3076.8	3279.7	3489.2
	s	9.242	9.513	9.752	9.967	10.163	10.344	10.671	10.960
0.05 (32.9)	*v*	29.78	34.42	39.04	48.66	48.28	52.9	62.13	71.36
	u	2444.8	2516.2	2588.4	2661.9	2736.6	2812.6	2969.6	3133.0
	h	2593.7	2688.1	2783.4	2879.9	2977.6	3076.7	3279.7	3489.2
	s	8.498	8.770	9.009	9.225	9.421	9.602	9.928	10.218
0.1 (45.8)	*v*	14.57	17.19	19.51	21.82	24.14	26.44	31.06	35.68
	u	2443.9	2515.5	2587.9	2661.3	2736.0	2812.1	2968.9	3132.3
	h	2592.6	2687.5	2783.0	2879.5	2977.3	3076.5	3279.6	3489.1
	s	8.175	8.448	8.688	8.904	9.100	9.281	9.608	9.898
0.5 (81.3)	*v*		34.18	3.889	43.56	4.821	5.284	6.209	7.134
	u		2511.6	2585.6	2659.9	2735.0	2811.3	2968.5	3132.0
	h		2682.5	2780.1	2877.7	2976.0	3075.5	3278.9	3488.7
	s		7.695	7.940	8.158	8.356	8.537	8.864	9.155
0.75 (92.0)	*v*		2.27	2.587	2.900	3.211	3.520	4.138	4.755
	u		2509.2	2584.2	2659.0	2734.4	2810.9	2968.2	3131.8
	h		2679.4	2778.2	2876.5	2975.2	3074.9	3278.5	3488.4
	s		7.501	7.749	7.969	8.167	8.349	8.677	8.967
1.0 (99.6)	*v*		1.696	1.936	2.172	2.406	2.639	3.103	3.565
	u		2506.2	2582.8	2658.1	2733.7	2810.4	2967.9	3131.6
	h		2676.2	2776.4	2875.3	2974.3	3074.3	3278.2	3488.1
	s		7.361	7.613	7.834	8.033	8.216	8.544	8.834
1.01325 (100)	*v*			1.912	2.146	2.375	2.603	3.062	3.519
	u			2582.6	2658.0	2733.6	2810.3	2967.8	3131.5
	h			2776.3	2875.2	2974.2	3074.2	3278.1	3488.0
	s			7.828	7.827	8.027	8.209	8.538	8.828
1.5 (111.4)	*v*			1.285	1.143	1.601	1.757	2.067	2.376
	u			2579.8	2656.2	2732.5	2809.5	2967.3	3131.2
	h			2772.6	2872.9	2972.7	3073.1	3277.4	3487.6
	s			7.419	7.643	7.844	8.027	8.356	8.647

↓*p* (bar) (t_s)	*t* (°C) →	50	100	150	200	250	300	400	500
	v			0.960	1.080	1.199	1.316	1.549	1.781
2.0	*u*			2576.9	2654.4	2731.2	2808.6	2966.7	3130.8
(120.2)	*h*			2768.8	2870.5	2971.0	3071.8	3276.6	3487.1
	s			7.279	7.507	7.709	7.893	8.222	8.513
	v			0.764	0.862	0.957	1.052	1.238	1.424
2.5	*u*			2574.7	2655.7	2734.9	2813.8	2973.9	3139.6
(127.4)	*h*			2764.5	2868.0	2969.6	3070.9	3275.9	3486.5
	s			7.169	7.401	7.604	7.789	8.119	8.410
	v			0.634	0.716	0.796	0.875	1.031	1.187
3.0	*u*			2570.8	2650.7	2728.7	2806.7	2965.6	3130.0
(133.5)	*h*			2761.0	2865.6	2967.6	3069.3	3275.0	3486.1
	s			7.078	7.311	7.517	7.702	8.033	8.325
	v			0.471	0.534	0.595	0.655	0.773	0.889
4.0	*u*			2564.5	2646.8	2726.1	2804.8	2964.4	3129.2
(143.6)	*h*			2752.8	2860.5	2964.2	3066.8	3273.4	3484.9
	s			6.930	7.171	7.379	7.566	7.899	8.191

↓*p* (bar) (t_s)	*t* (°C) →	200	250	300	350	400	450	500	600
	v	0.425	0.474	0.523	0.570	0.617	0.664	0.711	0.804
5.0	*u*	2642.9	2723.5	2802.9	2882.6	2963.2	3045.3	3128.4	3299.6
(151.8)	*h*	2855.4	2960.7	3064.2	3167.7	3271.9	3377.2	3483.9	3701.7
	s	7.059	7.271	7.460	7.633	7.794	7.945	8.087	8.353
	v	0.352	0.394	0.434	0.474	0.514	0.553	0.592	0.670
6.0	*u*	2638.9	2720.9	2801.0	2881.2	2962.1	3044.2	3127.6	3299.1
(158.8)	*h*	2850.1	2957.2	3061.6	3165.7	3270.3	3376.0	3482.8	3700.9
	s	6.967	7.182	7.372	7.546	7.708	7.859	8.002	8.267
	v	0.300	0.336	0.371	0.406	0.440	0.473	0.507	0.574
7.0	*u*	2634.8	2718.2	2799.1	2879.7	2960.9	3043.2	3126.8	3298.5
(165.0)	*h*	2844.8	2953.6	3059.1	3163.7	3268.7	3374.7	3481.7	3700.2
	s	6.886	7.105	7.298	7.473	7.635	7.787	7.930	8.196
	v	0.261	0.293	0.324	0.354	0.384	0.414	0.443	0.502
8.0	*u*	2630.6	2715.5	2797.2	2878.2	2959.7	3042.3	3126.0	3297.8
(170.4)	*h*	2839.3	2950.1	3056.5	3161.7	3267.1	3373.4	3480.6	3699.4
	s	6.816	7.038	7.233	7.409	7.572	7.724	7.867	8.133

↓p (bar) (t_s)	t (°C) →	200	250	300	350	400	450	500	600
	v	0.230	0.260	0.287	0.314	0.341	0.367	0.394	0.446
9.0	u	2626.3	2712.7	2795.2	2876.7	2958.5	3041.3	3125.2	3297.3
(175.4)	h	2833.6	2946.3	3053.8	3159.7	3265.5	3372.1	3479.6	3698.6
	s	6.752	6.979	7.175	7.352	7.516	7.668	7.812	8.078
	v	0.206	0.233	0.258	0.282	0.307	0.330	0.354	0.401
10.0	u	2621.9	2709.9	2793.2	2875.2	2957.3	3040.3	3124.4	3296.8
(179.9)	h	2827.9	2942.6	3051.2	3157.8	3263.9	3370.7	3478.5	3697.9
	s	6.694	6.925	7.123	7.301	7.465	7.618	7.762	8.029
	v	0.132	0.152	0.169	0.187	0.203	0.219	0.235	0.267
15.0	u	2598.8	2695.3	2783.1	2867.6	2951.3	3035.3	3120.3	3293.9
(198.3)	h	2796.8	2923.3	3037.6	3147.5	3255.8	3364.2	3473.1	3694.0
	s	6.455	6.709	6.918	7.102	7.269	7.424	7.570	7.839
	v		0.111	0.125	0.139	0.151	0.163	0.176	0.200
20.0	u		2679.6	2772.6	2859.8	2945.2	3030.5	3116.2	3290.9
(212.4)	h		2902.5	3023.5	3137.0	3247.6	3357.5	3467.6	3690.1
	s		6.545	6.766	6.956	7.127	7.285	7.432	7.702
	v		0.0870	0.0989	0.109	0.120	0.130	0.140	0.159
25	u		2662.6	2761.6	2851.9	2939.1	3025.5	3112.1	3288.0
(223.9)	h		2880.1	3008.8	3126.3	3239.3	3350.8	3462.1	3686.3
	s		6.408	6.644	6.840	7.015	7.175	7.323	7.596
	v		0.0706	0.0811	0.0905	0.0994	0.108	0.116	0.132
30	u		2644.0	2750.1	2843.7	2932.8	3020.4	3108.0	3285.0
(233.8)	h		2855.8	2993.5	3115.3	3230.9	3344.0	3456.5	3682.3
	s		6.287	6.539	6.743	6.921	7.083	7.234	7.509
	v			0.0588	0.0664	0.0734	0.080	0.0864	0.0989
40	u			2725.3	2826.7	2919.9	3010.2	3099.5	3279.1
(250.4)	h			2960.7	3092.5	3213.6	3330.3	3445.3	3674.4
	s			6.362	6.582	6.769	6.936	7.090	7.369
	v			0.0453	0.0519	0.0578	0.0633	0.0686	0.0787
50	u			2698.0	2808.7	2906.6	2999.7	3091.0	3273.0
(263.9)	h			2924.5	3068.4	3195.7	3316.2	3433.8	3666.5
	s			6.208	6.449	6.646	6.819	6.976	7.259

↓*p* (bar) (*t*$_s$)	*t* (°C) →	*200*	*250*	*300*	*350*	*400*	*450*	*500*	*600*
	v			0.0362	0.0422	0.0474	0.0521	0.0567	0.0653
60	*u*			2667.2	2789.6	2892.9	2988.9	3082.2	3266.9
(275.5)	*h*			2884.2	3043.0	3177.2	3301.8	3422.2	3658.4
	s			6.067	6.333	6.541	6.719	6.880	7.168
	v			0.0295	0.0352	0.0399	0.0442	0.0481	0.0557
70	*u*			2632.2	2769.4	2878.6	2978.0	3073.4	3260.7
(285.8)	*h*			2838.4	3016.0	3158.1	3287.1	3410.3	3650.3
	s			5.931	6.228	6.448	6.633	6.798	7.089

↓*p* (bar) (*t*$_s$)	*t* (°C) →	*350*	*375*	*400*	*450*	*500*	*550*	*600*	*700*
80	*v*	0.02995	0.03222	0.03432	0.03817	0.04175	0.04516	0.04845	0.05481
(294.9)	*h*	2987.3	3066.1	3138.3	3272.0	3398.3	3521.0	3642.0	3882.4
	s	6.130	6.254	6.363	6.555	6.724	6.878	7.021	7.281
90	*v*	0.0258	0.02796	0.02993	0.03350	0.03677	0.03987	0.04285	0.04857
(303.3)	*h*	2956.6	3041.3	3117.8	3256.6	3386.1	3511.0	3633.7	3876.5
	s	6.036	6.169	6.285	6.484	6.658	6.814	6.959	7.222
100	*v*	0.02242	0.02453	0.02641	0.02975	0.03279	0.03564	0.03837	0.04358
(311.0)	*h*	2923.4	3015.4	3096.5	3240.9	3373.7	3500.9	3625.3	3870.5
	s	5.944	6.089	6.212	6.419	6.597	6.756	6.903	7.169
110	*v*	0.01961	0.02169	0.02351	0.02668	0.02952	0.03217	0.03470	0.03950
(318.0)	*h*	2887.3	2988.2	3074.3	3224.7	3361.0	3490.7	3616.9	3864.5
	s	5.853	6.011	6.142	6.358	6.540	6.703	6.851	7.120
120	*v*	0.01721	0.01931	0.02108	0.02412	0.02680	0.02929	0.03164	0.03610
(324.6)	*h*	2847.7	2958.9	3051.3	3208.2	3348.2	3480.4	3608.3	3858.4
	s	5.760	5.935	6.075	6.300	6.487	6.653	6.804	7.075
130	*v*	0.01511	0.01725	0.01900	0.02194	0.0245	0.02684	0.02905	0.03322
(330.8)	*h*	2803.3	2927.9	3027.2	3191.3	3335.2	3469.9	3599.7	3852.3
	s	5.663	5.859	6.009	6.245	6.437	6.606	6.759	7.033
140	*v*	0.01322	0.01546	0.01722	0.02007	0.02252	0.02474	0.02683	0.03075
(336.6)	*h*	2752.6	2894.5	3001.9	3174.0	3322.0	3459.3	3591.1	3846.2
	s	5.559	5.782	5.945	6.192	6.390	6.562	6.712	6.994
150	*v*	0.01145	0.01388	0.01565	0.01845	0.02080	0.02293	0.02491	0.02861
(342.1)	*h*	2692.4	2858.4	2975.5	3156.2	3308.6	3448.6	3582.3	3840.1
	s	5.442	5.703	5.881	6.140	6.344	6.520	6.679	6.957

↓*p (bar)* (t_s)	*t* (°C) →	*350*	*375*	*400*	*450*	*500*	*550*	*600*	*700*
160	*v*	0.00975	0.01245	0.01426	0.01701	0.01930	0.02134	0.02323	0.02674
(347.3)	*h*	2615.7	2818.9	2947.6	3138.0	3294.9	3437.8	3573.5	3833.9
	s	5.302	5.622	5.188	6.091	6.301	6.480	6.640	6.922
170	*v*		0.01117	0.01302	0.01575	0.01797	0.01993	0.02174	0.02509
(352.3)	*h*		2776.8	2918.2	3119.3	3281.1	3426.9	3564.6	3827.7
	s		5.539	5.754	6.042	6.259	6.442	6.604	6.889
180	*v*		0.00996	0.01190	0.01462	0.01678	0.01868	0.02042	0.02362
(356.9)	*h*		2727.9	2887.0	3100.1	3267.0	3415.9	3555.6	3821.5
	s		5.448	5.689	5.995	6.218	6.405	6.570	6.858
190	*v*		0.00881	0.01088	0.01361	0.01572	0.01756	0.01924	0.02231
(361.4)	*h*		2671.3	2853.8	3080.4	3252.7	3404.7	3546.6	3815.3
	s		5.346	5.622	5.948	6.179	6.369	6.537	6.828
200	*v*		0.00767	0.00994	0.01269	0.9477	0.01655	0.01818	0.02113
(365.7)	*h*		2602.5	2818.1	3060.1	3238.2	3393.5	3537.6	3809.0
	s		5.227	5.554	5.902	6.140	6.335	6.505	6.799
210	*v*		0.00645	0.00907	0.01186	0.01390	0.01564	0.01722	0.02006
(369.8)	*h*		2511.0	2779.6	3039.3	3223.5	3382.1	3528.4	3802.8
	s		5.075	5.483	5.856	6.103	6.301	6.474	6.772
220	*v*		0.00482	0.00825	0.01110	0.01312	0.01481	0.01634	0.01909
(373.7)	*h*		2345.1	2737.6	3017.9	3208.6	3370.6	3519.2	3796.5
	s		4.810	5.407	5.811	6.066	6.269	6.444	6.745

TABLE IV

Supercritical Steam

p(bar)	*t (°C)* →	*350*	*375*	*400*	*425*	*450*	*500*	*600*	*700*	*800*
230	v	0.00162	0.00221	0.00748	0.00915	0.01040	0.01239	0.01554	0.01821	0.02063
	h	1632.8	1912.2	2691.2	2869.2	2995.8	3193.4	3510.0	3790.2	4056.2
	s	3.137	4.137	5.327	5.587	5.765	6.030	6.415	6.719	6.980
250	v	0.00160	0.00197	0.00600	0.00788	0.00916	0.01112	0.01414	0.01665	0.01891
	h	1623.5	1848.0	2580.2	2806.3	2949.7	3162.4	3491.4	3775.5	4047.1
	s	3.680	4.032	5.142	5.472	5.674	5.959	6.360	6.671	6.934
300	v	0.00155	0.00179	0.00279	0.00530	0.00673	0.00868	0.01145	0.01366	0.01562
	h	1608.5	1791.5	2151.1	2614.2	2821.4	3081.1	3443.9	3745.6	4024.2
	s	3.643	3.930	4.473	5.150	5.442	5.790	6.233	6.561	6.833
350	v	0.00152	0.00110	0.00210	0.00343	0.00496	0.00693	0.00953	0.01153	0.01328
	h	1597.1	1762.4	1987.6	2373.4	2672.4	2994.4	3395.5	3713.5	4001.5
	s	3.612	3.872	4.213	4.775	5.196	5.628	6.118	6.463	6.745
400	v	0.00149	0.00164	0.00191	0.00253	0.00369	0.00562	0.00809	0.00994	0.01152
	h	1588.3	1742.8	1930.9	2198.1	2512.8	2903.3	3346.4	3681.2	3978.7
	s	3.586	3.829	4.113	4.503	4.946	5.470	6.011	6.375	6.666
500	v	0.00144	0.00156	0.00173	0.00201	0.00249	0.00389	0.00611	0.00773	0.00908
	h	1575.3	1716.6	1874.6	2060.0	2284.0	2720.1	3247.6	3616.8	3933.6
	s	3.542	3.764	4.003	4.273	4.588	5.173	5.818	6.219	6.529
600	v	0.00140	0.00150	0.00163	0.00182	0.00209	0.00296	0.00483	0.00627	0.00746
	h	1566.4	1699.5	1843.4	2001.7	2179.0	2567.9	3151.2	3553.5	3889.1
	s	3.505	3.764	3.932	4.163	4.412	4.932	5.645	6.082	6.411
700	v	0.00137	0.00146	0.00157	0.00171	0.00189	0.00247	0.00398	0.00526	0.00632
	h	1560.4	1687.7	1822.8	1967.2	2122.7	2463.2	3061.7	3492.4	3845.7
	s	3.473	3.673	3.877	4.088	4.307	4.762	5.492	5.961	6.307
800	v	0.00135	0.00142	0.00152	0.00163	0.00177	0.00219	0.00339	0.00452	0.00548
	h	1556.4	1679.4	1808.3	1943.9	2086.9	2394.0	2982.7	3434.6	3803.8
	s	3.444	3.638	3.833	4.031	4.232	4.642	5.360	5.851	6.213
900	v	0.00133	0.00139	0.00147	0.00157	0.00169	0.00201	0.00297	0.00397	0.00484
	h	1553.9	1673.4	1797.7	1927.2	2062.0	2346.7	2915.6	3381.1	3763.8
	s	3.419	3.607	3.795	3.984	4.174	4.554	5.247	5.753	6.128
1000	v	0.01308	0.00137	0.00144	0.00152	0.00163	0.00189	0.00267	0.00355	0.00434
	h	1552.7	1669.4	1790.0	1914.8	2043.8	2312.8	2859.8	3332.3	3726.1
	s	3.396	3.579	3.762	3.944	4.126	4.485	5.151	5.664	6.050

TABLE V

Conversion Factors

Force

1 newton = 1 kg-m/sec^2
= 0.012 kgf
1 kgf = 9.81 N

Pressure

1 bar = 750.06 mm Hg
= 0.9869 atm
= 10^5 N/m^2
= 10^3 kg/m-sec^2

1 N/m^2 = 1 pascal
= 10^{-5} bar
= 10^{-2} kg/m-sec^2

1 atm = 760 mm Hg
= 1.03 kgf/cm^2 = 1.01325 bar
= 1.01325×10^5 N/m^2

Work, Energy or Heat

1 joule = 1 newton metre
= 1 watt-sec
= 2.7778×10^{-7} kWh
= 0.239 cal
= 0.239×10^{-3} kcal

1 cal = 4.184 joule
= 1.1622×10^{-6} kWh

1 kcal = 4.184×10^3 joule
= 427 kgfm
= 1.1622×10^{-3} kWh

1 kWh = 8.6×10^5 cal
= 860 kcal
= 3.6×10^6 joule

1 kgfm = $\left(\frac{1}{427}\right)$ kcal = 9.81 joules

Power

1 watt = 1 joule/sec = 0.86 kcal/h
1 h.p. = 75 mkgf/sec = 0.1757 kcal/sec
= 735.3 watt
1 kW = 1000 watts
= 860 kcal/h

Specific heat

1 kcal/kg - °K = 4.18 kJ/kg-K

Thermal conductivity

1 watt/m-K = 0.8598 kcal/h-m-°C
1 kcal/h-m-°C = 1.16123 watt/m-K
= 1.16123 joules/s-m-K

Heat transfer co-efficient

1 watt/m^2-K = 0.86 kcal/m^2-h-°C
1 kcal/m^2-h-°C = 1.163 watt/m^2-K

IMPORTANT ENGINEERING CONSTANTS AND EXPRESSIONS IN SI UNITS

	Engineering constants and expressions	*M.K.S. system*	*S.I. units*
1.	Value of g_0	9.81 kg-m/kgf-sec^2	1 kg-m/N-sec^2
2.	Universal gas constant	848 kgf-m/kg mole-°K	848 × 9.81 = 8314 J/kg-mole-°K ($\because$ 1 kgf-m = 9.81 joules)
3.	Gas constant (R)	29.27 kgf m/kg-°K for air	$\frac{8314}{29}$ = 287 joules/kg-K for air
4.	Specific heats (for air)	c_v = 0.17 kcal/kg-°K c_p = 0.24 kcal/kg-°K	c_v = 0.17 × 4.184 = 0.71128 kJ/kg-K c_p = 0.24 × 4.184 = 1 kJ/kg-K
5.	Flow through nozzle-exit velocity (C_2)	91.5 $\sqrt{U}$ where U is in kcal	44.7 $\sqrt{U}$ where U is in kJ
6.	Refrigeration 1 ton	= 50 kcal/min	= 210 kJ/min
7.	**Heat transfer** The Stefan Boltzman Law is given by :	Q = σT^4 kcal/m^2-h when $\sigma = 4.9 \times 10^{-8}$ kcal/h-m^2-$°K^4$	Q = σT^4 watts/m^2-h when $\sigma = 5.67 \times 10^{-8}$ W/m^2K^4