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Topics below are additional and supplementary to the course

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Topic 1: Statics I - Principles

Topic 1.1: Algebra/Trigonometry/Vectors

1.1a. Algebra/Trigonometry:

It is anticipated that the student will have introductory level college algebra skills, including the ability to solve simple algebraic equations, quadratics, and simultaneous equations. An appropriate level of skill would be one which a student would be expected to acquire in a one-year College Algebra sequence. Minimal trigonometric skills should include being able to find sides and angles in both right and non-right triangle. See Basic Trigonometric Review.

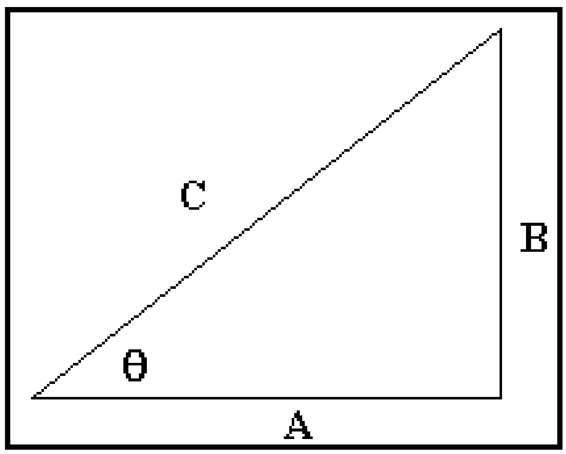
1.1b. Vectors:

The student should have a working knowledge of vectors (as, of course, forces are vectors and we will be summing forces in many of our problems.) A working knowledge of vectors would include vector addition, subtraction and resolving vectors into perpendicular components. We will be using the component method for vector addition. For a short review of this method see: Basic Vector Review. Also, Vector Review Problems are available.

Basic Trigonometric Review

For Right Triangles:

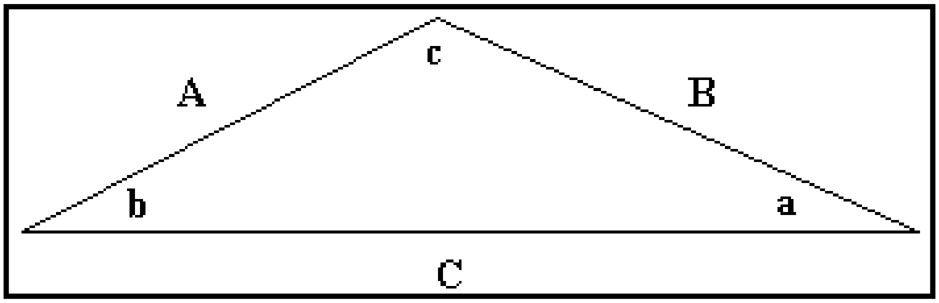
1. **Pythagorean Law**: **$C^2 = A^2 + B^2$**
2. **Sine Ø** = **opposite side / hypotenuse = B / C**
3. **Cosine Ø** = **adjacent side / hypotenuse = A / C**
4. **Tangent Ø** = **opposite side / adjacent side = B / A**



For Non-Right Triangles:

1. **Law of Cosines**:
   $C^2 = A^2 + B^2 - 2AB \cos c$
2. **Law of Sines**:
   $(A / \sin a) = (B / \sin b) = (C / \sin c)$

Where A,B,C are length of the sides, and a, b, c are the corresponding angles opposite the sides.



Some general **Trigonometric Identities** are shown below.

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\cos(-\theta) = \cos\theta$$

$$\sin(-\theta) = \sin\theta$$

$$\tan(-\theta) = \tan\theta$$

$$\tan(A-B) = \frac{\tan A + \tan B}{1 + \tan A \tan B}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos\theta}{2}}$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = 2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

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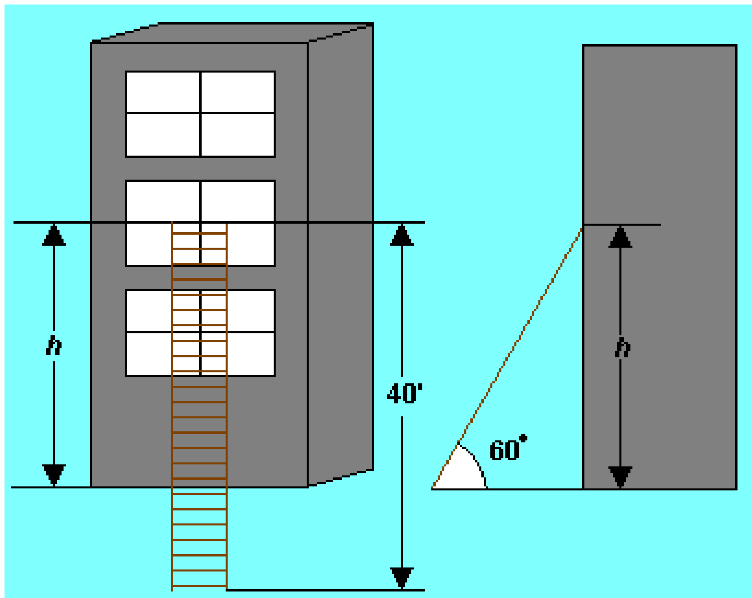
Strength of Materials

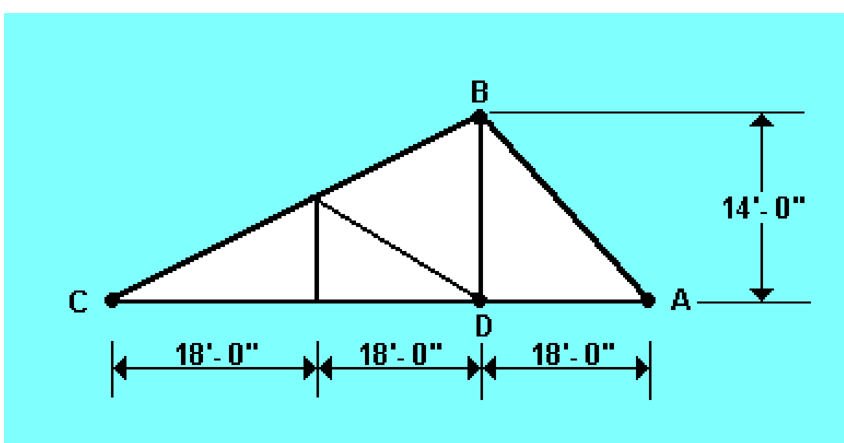
Problem Assignment - Trigonometry

1. A 40 ft long ladder leaning against a wall makes an angle of 60° with the ground. Determine the vertical height to which the ladder will reach. (answers at bottom of problem set.)



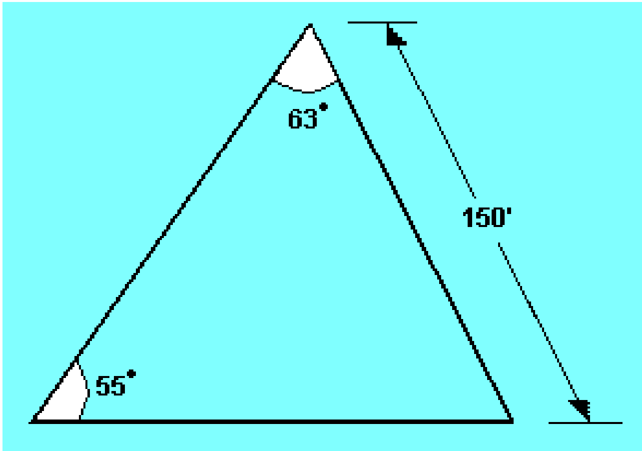
Problem #1

2. **In the roof truss shown, the bottom chord members *AD* and *DC* have lengths of 18ft. and 36ft respectively. The height *BD* is 14 ft. Determine the lengths of the top chords AB and BC and find the angles at A and C.**



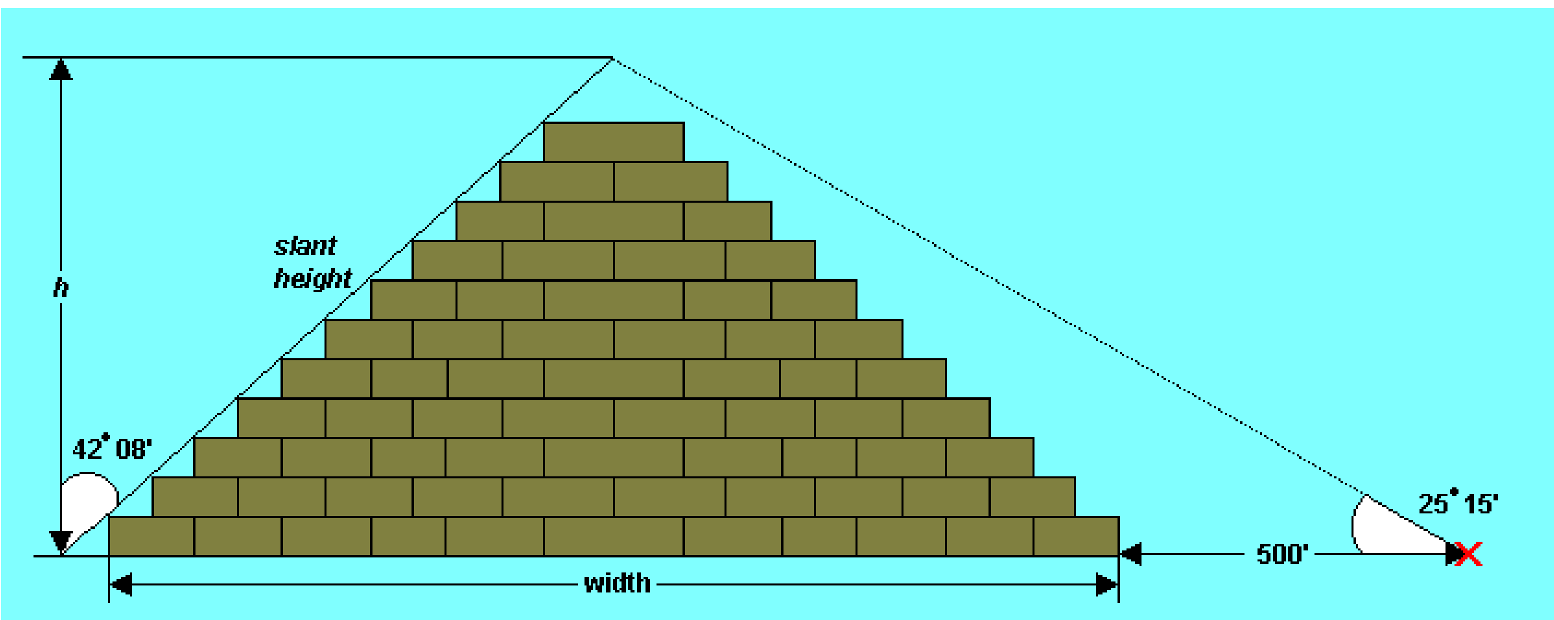
Problem #2

3. One side of a triangular lot is 150 ft. and the angle opposite this side is 55°. Another angle is 63°. Sketch the shape of the lot and determine how much fencing is needed to enclose it.



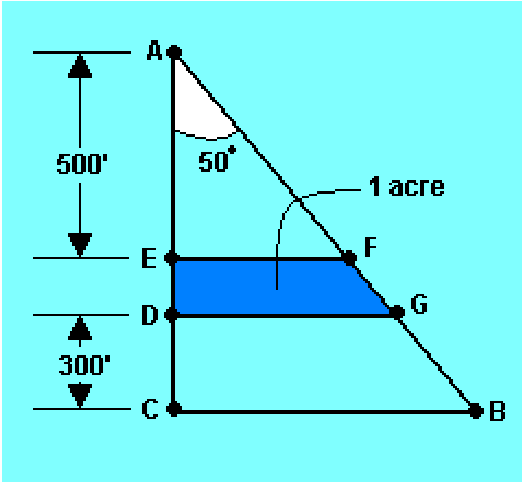
Problem #3

4. An Egyptian pyramid has a square base and symmetrical sloping faces. The inclination of a sloping face is 42° 08'. At a distance of 500 ft. from the base, on level ground the angle of inclination to the apex is 25° 15'. Find the vertical height(*h*), the slant height of the pyramid, and the width of the pyramid at its base.



Problem #4

#5. Triangle ABC shown is a triangular tract of land. The one acre tract DEFG is to be subdivided. AE is 500ft and DC is 300 ft. Determine the lengths of DG and CB. [This is a harder problem and may be skipped.]



Problem #5

Some Answers 1. H =34.64'; 2. AB=22.8', BC=38.6', A = 37.9°, C = 21.25°; 3. L=474.8'; 4. h=411.3', slant = 554.6', width=744.2' 5. ED=68.4', DG=677.4', CB=1035'

Basic Vector Review

1. Definitions:

- **Scalar**: Any quantity possessing magnitude (size) only, such as mass, volume, temperature
- **Vector**: Any quantity possessing both magnitude and direction, such as force, velocity, momentum

2. Vector Addition:

Vector addition may be done several ways including, **Graphical Method**, **Trigonometric Method**, and **Component Method**. We will be reviewing only the Component Method, as that is the method which will be used in the course. Other methods are detailed in your textbook.

3. Vector Addition - Component Method: (2-dimensional)

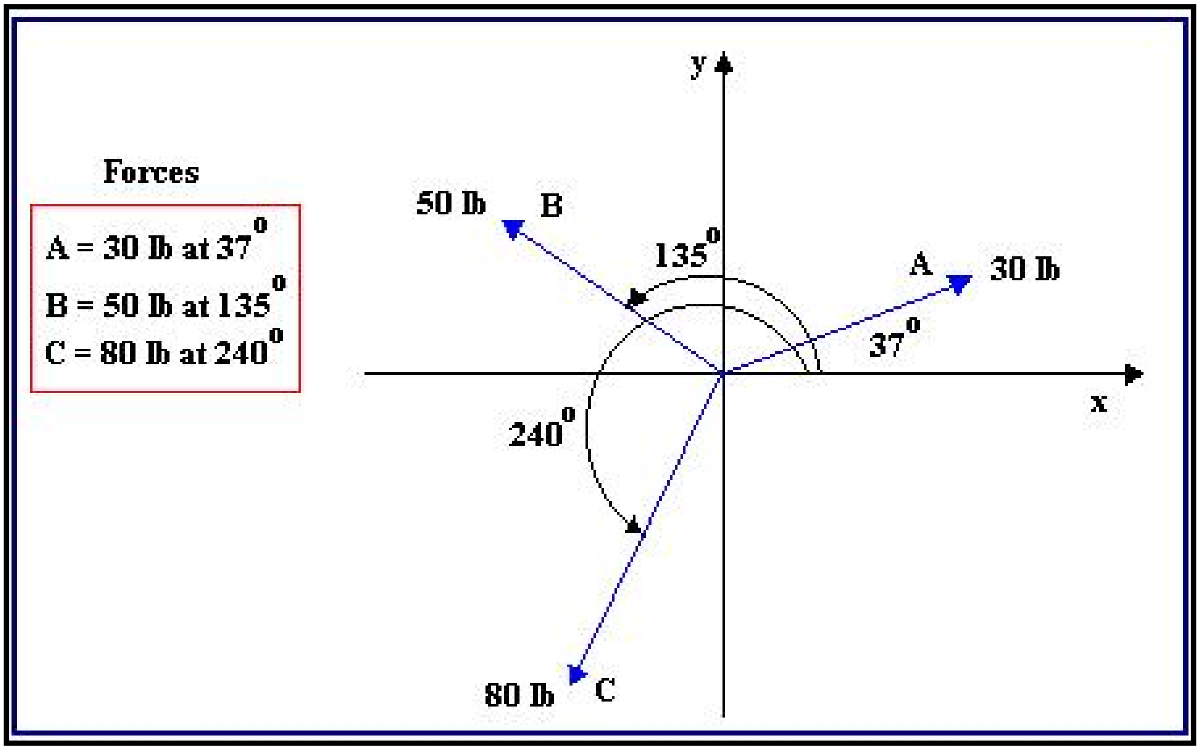
The component method will follow the procedure shown below:

- **1. Choose an origin, sketch a coordinate system, and draw the vectors to be added (or summed).**
- **2. Break (resolve) each vector into it's "x" and "y" components, using the following relationships:**
  - **A_x = A cosine Ø, and, A_y = A sine Ø, where A is the vector, and Ø is the vector's angle.**
- **3. Sum all the x-components and all the y-components obtaining a net resultant R_x, and R_y vectors.**
  - **$R_x = A_x + B_x + C_x + \ldots$, &, $R_y = A_y + B_y + C_y + \ldots$**
- **4. Recombine Rx and Ry to obtain the final resultant vector (magnitude and direction) using**

  $$R = \sqrt{R^2_x + R^2_y} \quad \text{and Tangent } \theta = R_y / R_x$$

  See Example below

Example - Vector Addition



Three ropes are tied to a small metal ring. At the end of each rope three students are pulling, each trying to move the ring in their direction. If we look down from above the students, the forces and directions they are applying the forces are as follows: (See diagram to the right)

Find the net (resultant) force (magnitude and direction) on the ring due to the three applied forces.

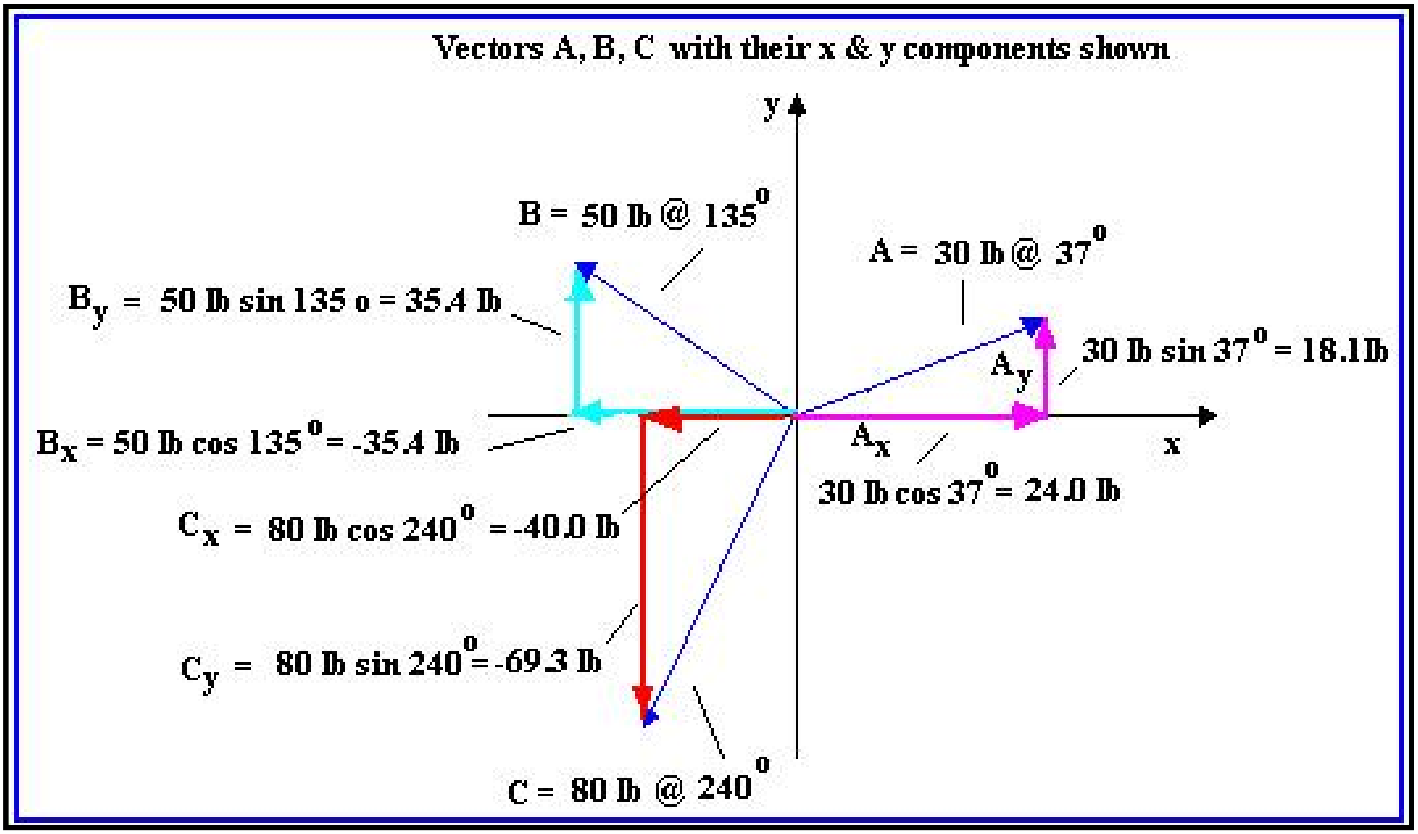
Choose origin, sketch coordinate system and vectors (done above)

Resolve vectors into x & y components (See Diagram)

$\mathbf{A_x}$ = 30 lb cos 37º = **+ 24.0 lbs** ; $\mathbf{A_y}$ = 30 lb sin 37º = **+ 18.1 lb**

$\mathbf{B_x}$ = 50 lb cos135º = **- 35.4 lbs** ; $\mathbf{B_y}$ = 50 lb sin135º = **+ 35.4 lb**

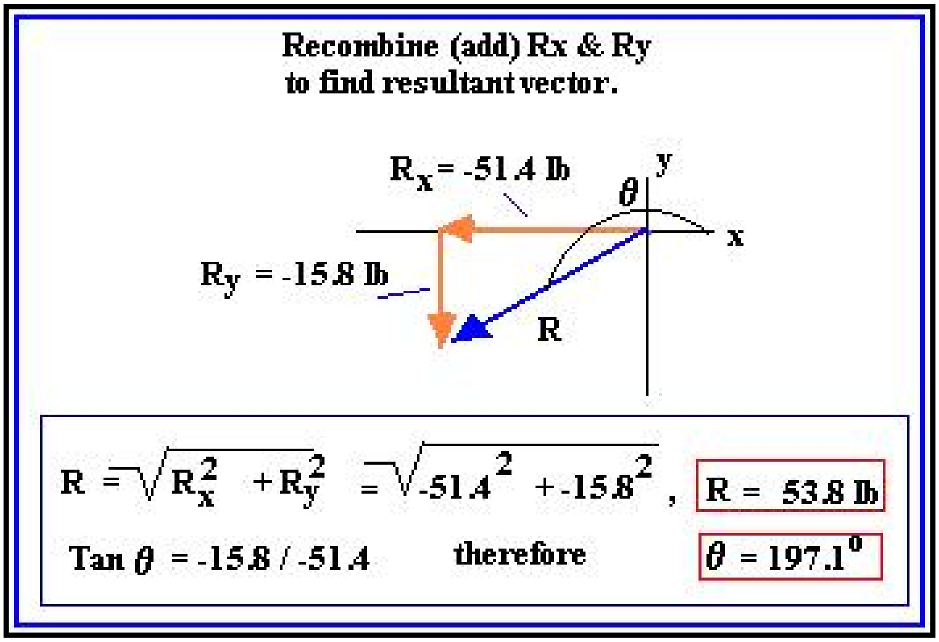
$\mathbf{C_x}$ = 80 lb cos240º = **- 40.0 lbs** ; $\mathbf{C_y}$ = 80 lb sin240º = **- 69.3 lb**



Sum x & y components to find resultant R_x and R_y forces.

R_x = 24.0 lbs - 35.4 lbs - 40.0 lbs = **-51.4 lbs**

R_y =18.1 lbs + 35.4 lbs - 69.3 lbs = **-15.8 lbs**



'Recombine' (add) Rx and Ry to determine final resultant vector.

Thus the resultant force on the ring is 53.8 pounds acting at an angle of 197.1

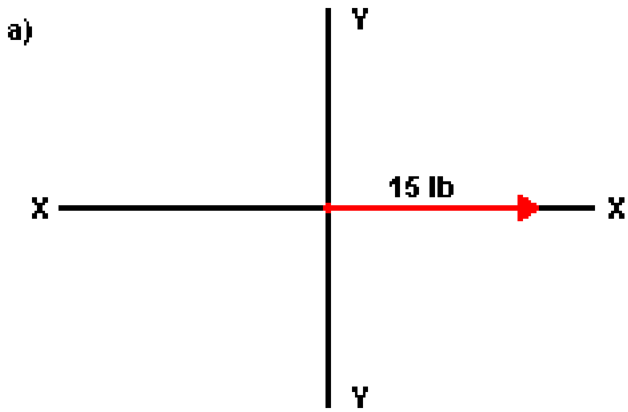
degrees.

Statics & Strength of Materials

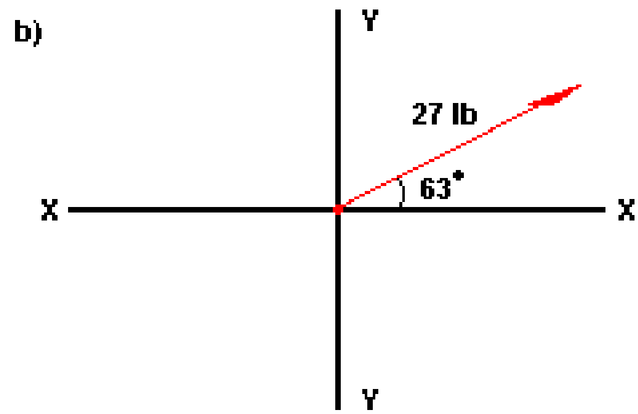
Problems Assignment - Vector Review Problems

1. Find the x and y components of the following vectors, specifically stating each component.

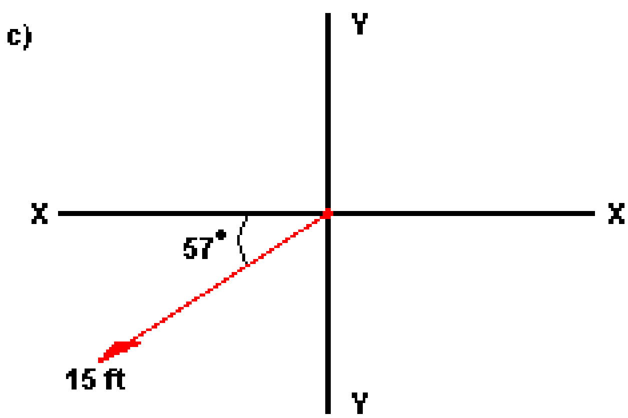
a) 15# at 0°



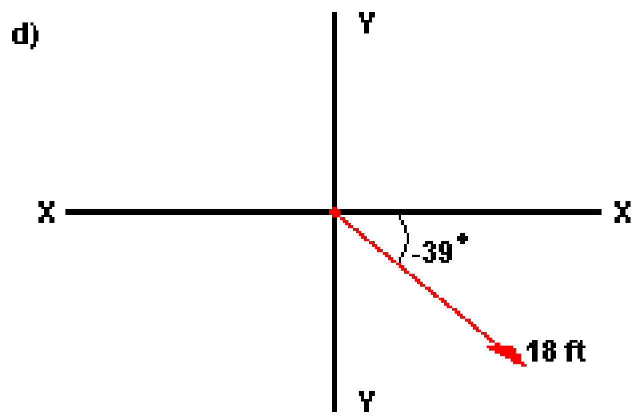
b) 27# at 63°



c) 15 ft at 237°



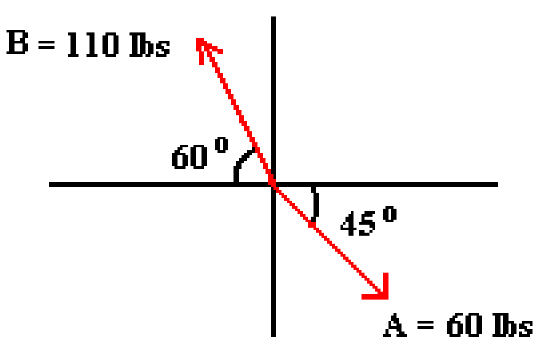
d) 18 ft at -39°



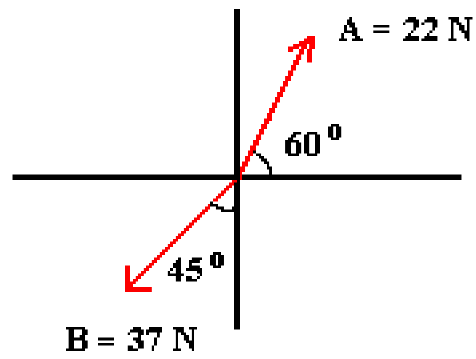
2. Find the vectors (magnitude and direction) that have the following components.
a) x = 5 feet y = 17 feet; b) x = -8 # y = 3 #; c)x = 3# y = -8#; d)x = -13ft = -24ft

3. Solve the following vector problems by adding vectors A and B to find the resultant vector for problems a,b, and c. (Answers at bottom of page.)

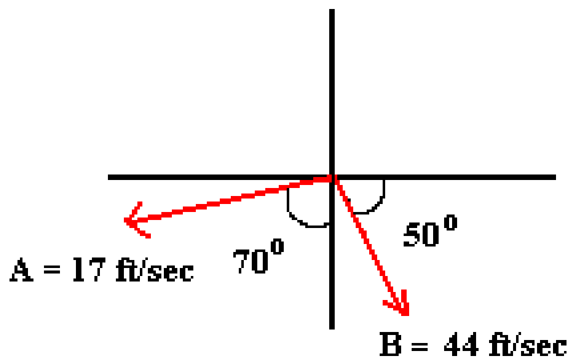
(a).



(b).

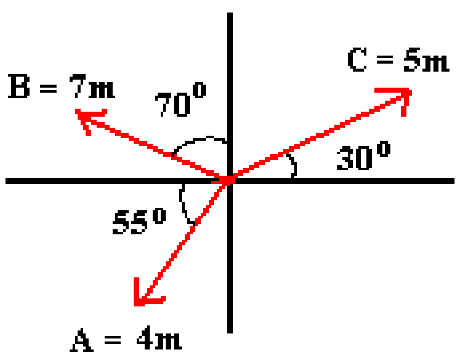


(c).

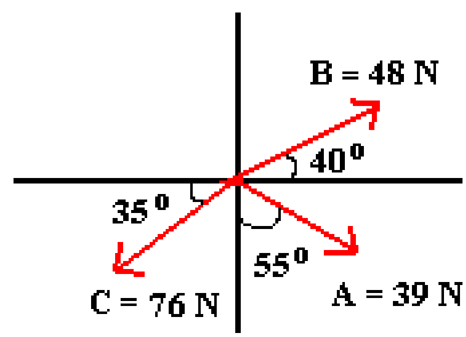


4. Add vectors A, B, and C and find the resultant vector for problems a,b,c, and d.

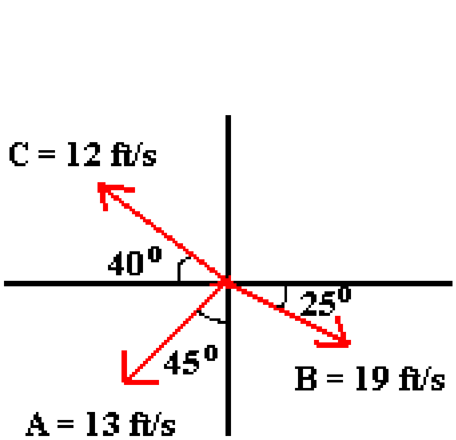
(a).



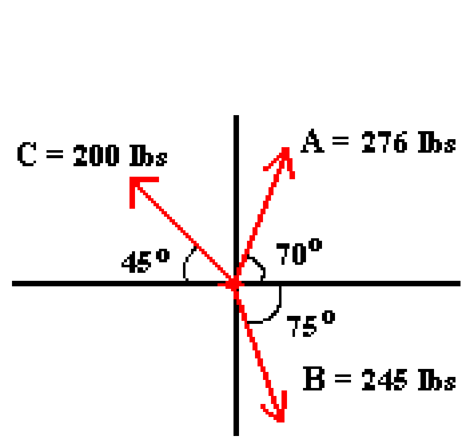
(b).



(c).



(d).



5. Using diagram (a) above, find the following resultants:

a) **$R = 2 * A - B + 3 * C$** **b)** **$R = -3 * A + 2 * B - C$**

Some Answers:

3a. Rx = -12.57 lb., Ry = 52.83 lb., R = 54.3 lb at 103.4º

3b. Rx = -15.16 n., Ry = -7.11n., R = 16.74 n at 205º

3c. Rx = 12.31 ft/s, Ry = -39.51 ft/s, R = 41.38 ft/s at 287.3º

4a. Rx = -4.54 m., Ry = 1.61 m., R = 4.82 m. at 160.5º

4b. Rx = 6.46 n., Ry = -35.11 n., R = 35.7 n at 280.4º

4c. Rx = -1.16 ft/s., Ry = -9.5 ft/s., R = 9.57 n at 263º

4d. Rx = 16.39 lb., Ry = 164.13 lb., R = 164.95 lb. at 84.3º

5a. Rx = 14.99 m., Ry = - 1.45 m., R = 15.07 m. at 354º.

5b. Rx = -10.62 m., Ry = 12.12 m., R = 16.11 m. at 131º.

Topic 1.2 - Translational Equilibrium

The topic of statics deal with objects or structures which are in equilibrium, that is structures that are at rest or in uniform, (non-accelerated) motion. We will be normally looking at structures which are at rest. For these structures we will be interested in determining the forces (loads and support reactions) acting on the structure and forces acting within members of the structure (internal forces). To determine forces on and in structures we will proceed carefully, using a well defined methodology. This is important as most problems in statics and strength of materials are not the kind of problem in which we can easily see the answer, but rather we must relay on our problem solving techniques.

For static equilibrium problems, we will be able to apply the **Conditions of Equilibrium** to help us solve for the force in and on the structures. There are two general equilibrium conditions: **Translational Equilibrium,** and **Rotational Equilibrium**.

The **Translational Equilibrium** condition states that for an object or a structure to be in translational equilibrium (which means that the structure as a whole will not experience linear acceleration) the vector sum of all the external forces acting on the structure must be zero. Mathematically this may be expressed as:

$\sum \mathbf{F_{all}} = 0$ or, in 3-dimensions: $\sum \mathbf{F_x} = 0$, $\sum \mathbf{F_y} = 0$, $\sum \mathbf{F_z} = 0$

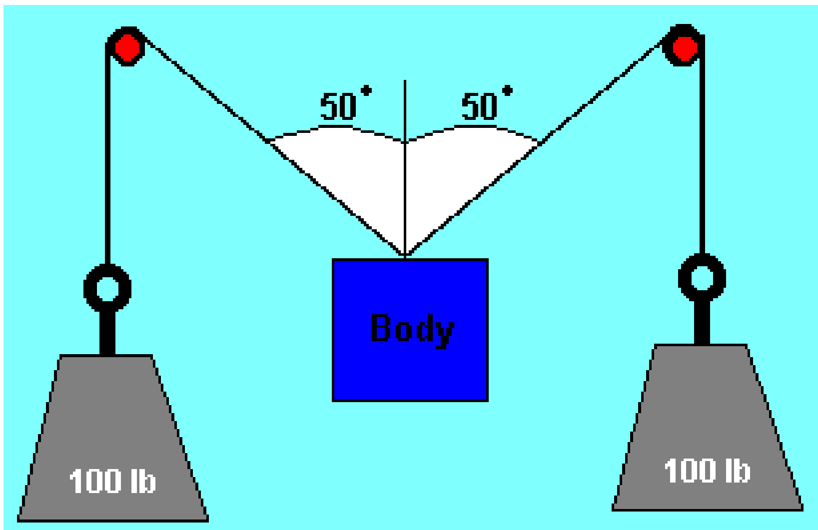
That is, forces in the x-direction must sum to zero, for translational equilibrium in the x-direction, and, the forces in the y-direction must sum to zero, for translational equilibrium in the y-direction, and, the forces in the z-direction must sum to zero, for translational equilibrium in the z-direction.

To see the application of the first condition of equilibrium and also the application of a standard problem solving technique, let's look carefully at introductory examples. **Select:** Example 1- Concurrent Forces**.**

Select: Example 2- Concurrent Forces

Example 1 - Concurrent, Coplanar Forces

In this relatively simple structure, we have a weight supported by two cables, which run over pulleys (which we will assume are very low friction) and are attached to 100 lb. weights as shown in the diagram. The two cords each make an angle of 50º with the vertical. Determine the weight of the body. (The effect of the pulleys is just to change the direction of the force, it may be considered to not effect the value of the tensions in the ropes.)

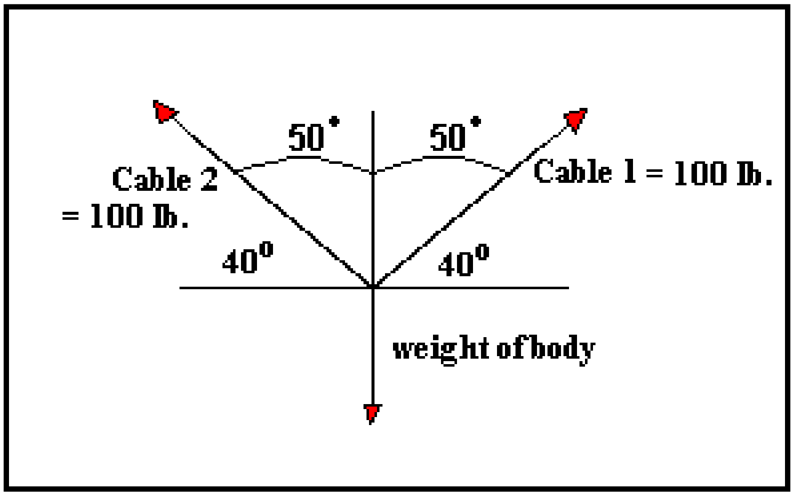


If we examine the first diagram for a moment we observe this problem may be classified as a problem involving **Concurrent**, **Coplanar** Forces. That is, the vectors representing the two support forces in Cable 1 and Cable 2, and the vector representing the load force will all intersect at one point, just above the body. When the force vectors all intersect at one point, the forces are said to be **Concurrent**. Additionally, we note that this is a two-dimensional problem, that forces lie in the x-y plane only. When the problem involves forces in two dimensions only, the forces are said to be **Coplanar.**

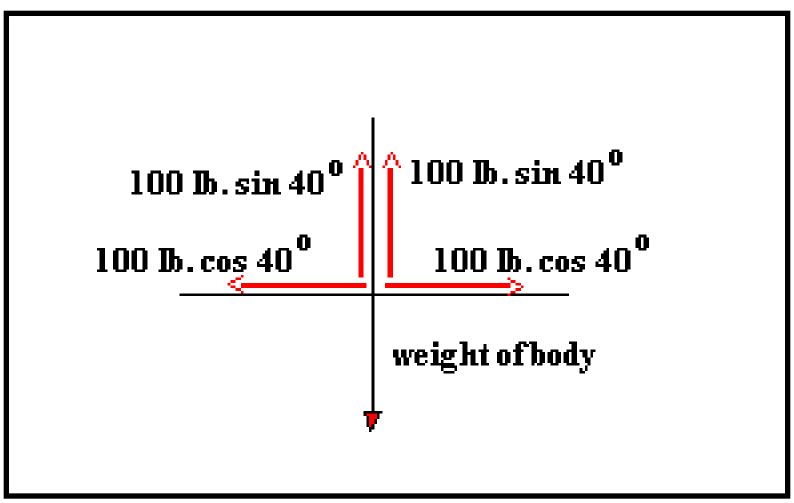
(Notice in this problem, that since the two supporting members are cables, and cables can only be in tension, the directions the support forces act are easy to determine. In later problems this will not necessarily be the case, and will be discussed later.)

To "Solve" this problem, that is to determine the weight supported by forces (tensions) in cable 1 and cable 2, we will now follow a very specific procedure or technique, as follows:

1. **Draw a Free Body Diagram** (**FBD**) of the structure or a portion of the structure. This Free Body Diagram should include a coordinate system and vectors representing all the **external** forces (which include support forces and load forces) acting on the structure. These forces should be labeled either with actual known values or symbols representing unknown forces. The second diagram 2 is the Free Body Diagram of point just above the weight where with all forces come together.



2. **Resolve (break) forces** not in x or y direction into their x and y components. Notice for Cable 1, and Cable 2, the vectors representing the tensions in the cables were acting at angles with respect to the x-axis, that is, they are not simply in the x or y direction. Thus the forces Cable 1, Cable 2, we must be replaced with their horizontal and vertical components. In the third diagram, the components of Cable 1 and Cable 2 are shown.



3. **Apply the Equilibrium Conditions** and solve for unknowns. In this step we

will now apply the actual equilibrium equations. Since the problem is in two dimensions only (coplanar) we have the following two equilibrium conditions: The sum of the forces in the x direction, and the sum of the forces in the y direction must be zero. We now place our forces into these equations, remembering to put the correct sign with the force, that is if the force acts in the positive direction it is positive and if the force acts in the negative direction, it is negative in the equation.

$\sum \mathbf{F_x} = 0$ or, -100 cos 40º + 100 cos 40º = 0 (Just as we would expect, the x-forces balance each other.)

$\sum \mathbf{F_y} = 0$ or, 100 sin 40º + 100 sin 40º - weight of body = 0

In this instance, it is very easy to solve for the weigth of the body from the y-equation; and find:

Weight of body = 128.56 lb.

Example 2 - Concurrent, Coplanar Forces

In this relatively simple structure, we have a 500 lb. load supported by two cables, which in turn are attached to walls. Let's say that we would like to determine the forces (tensions) in each cable.

If we examine Diagram-1 for a moment we observe this problem may be classified as a problem involving **Concurrent**, **Coplanar** Forces. That is, the vectors representing the two support forces in Cable 1 and Cable 2, and the vector representing the load force will all intersect at one point (Point C, See Diagram 2). When the force vectors all intersect at one point, the forces are said to be **Concurrent**. Additionally, we note that this is a two-dimensional problem, that forces lie in the x-y plane only. When the problem involves forces in two dimensions only, the forces are said to be **Coplanar.**

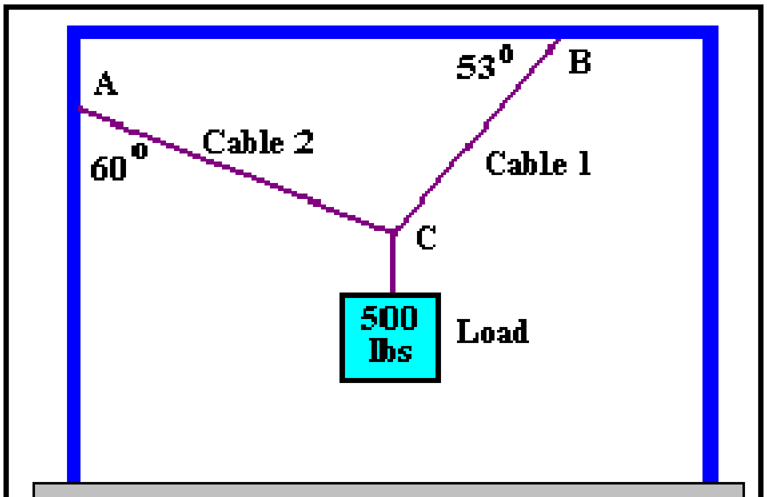


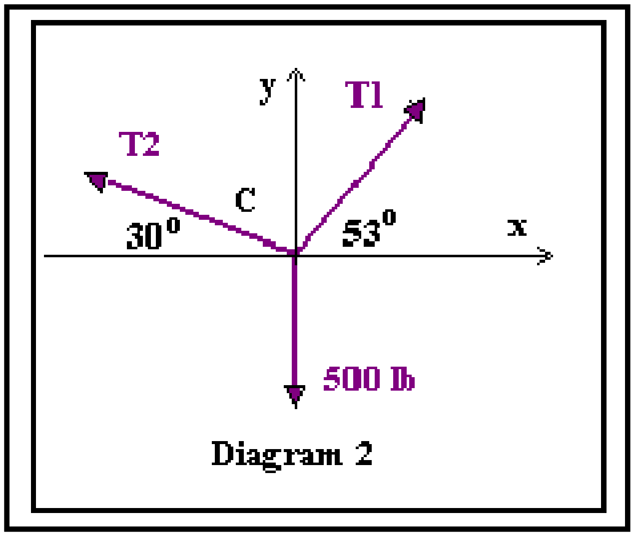
Diagram 1

(Notice in this problem, that since the two supporting members are cables, and cables can only be in tension, the directions the support forces act are easy to determine. In later problems this will not necessarily be the case, and will be discussed later.)

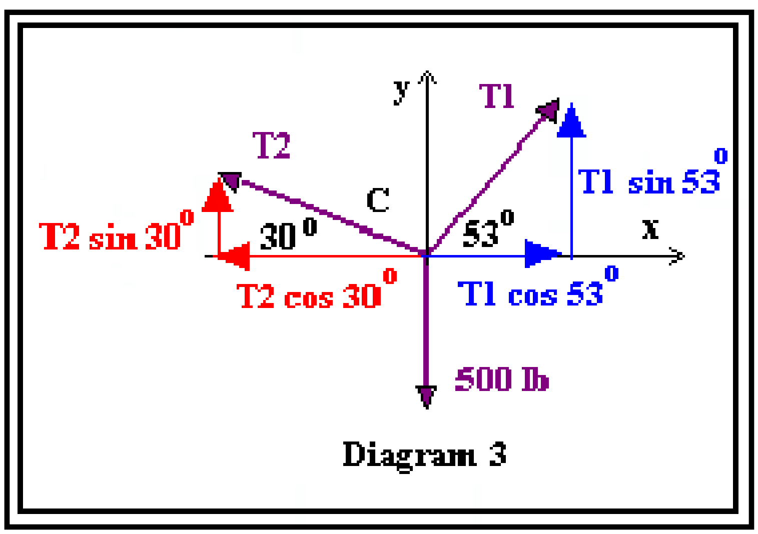
To "Solve" this problem, that is to determine the forces (tensions) in cable 1 and cable 2, we will now follow a very specific procedure or technique, as follows:

1. **Draw a Free Body Diagram** (**FBD**) of the structure or a portion of the structure. This Free Body Diagram should include a coordinate system and vectors representing all the **external** forces (which include support forces and load forces) acting on the structure. These forces should be labeled either with actual known values or symbols representing unknown forces. Diagram 2 is the Free Body

Diagram of point C with all forces acting on point C shown and labeled.



2. **Resolve (break) forces** not in x or y direction into their x and y components. Notice that T1, and T2, the vectors representing the tensions in the cables are acting at angles with respect to the x-axis, that is, they are not simply in the x or y direction. Thus for the forces T1, T2, we must replace them with their horizontal and vertical components. In Diagram 3, the components of T1 and T2 are shown.



Since the components of T1 and T2 (T1 sin 53°, T1 cos 53°, T2 sin 30°, T2 cos 30°) are equivalent to T1 and T2, in the final diagram 1d, we remove T1 and T2 which are now represented by their components. Notice that we do not have to do this for the load force of 500 lb., since it is already acting in the y-direction only.

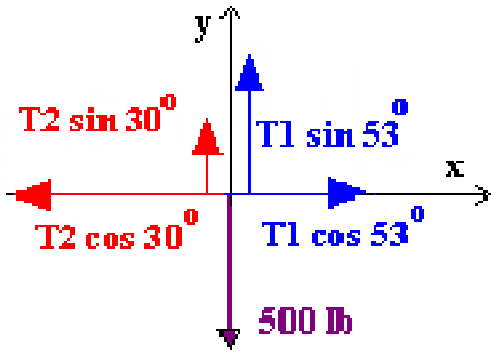


Diagram 4

3. **Apply the Equilibrium Conditions** and solve for unknowns. In this step we will now apply the actual equilibrium equations. Since the problem is in two dimensions only (coplanar) we have the following two equilibrium conditions: The sum of the forces in the x direction, and the sum of the forces in the y direction must be zero. We now place our forces into these equations, remembering to put the correct sign with the force, that is if the force acts in the positive direction it is positive and if the force acts in the negative direction, it is negative in the equation.

$\sum F_x = 0$ or, T1 cos53º - T2 cos30º = 0

$\sum F_y = 0$ or, T1 sin53º + T2 sin30º - 500 lb= 0

Notice we have two equations and two unknowns (T1 and T2), and therefore can solve for the unknowns. There are several ways to solve these two 'simultaneous' equations. We could solve the first equation for T1 in terms of T2, (T1 = T2 cos 30º/cos 53º), and substitute the expression for T1 into the second equation [(T2 cos30º/cos53º)sin53º + T2 sin30º - 500 lb= 0], giving us only one equation and one unknown.

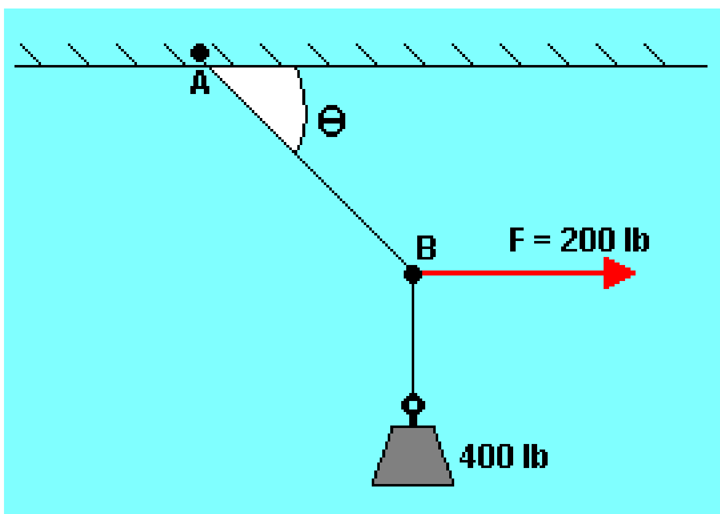
On solving the equations for T1 and T2 we obtain: **T1 = 436 lb.; T2 = 302 lb.** Thus, if the structure is to be in equilibrium, if the cables, acting at the angles given, are to support the 500 lb. load, then the forces in the cable must be as found above, 436 lb. and 302 lb., respectively. So when we go to purchase cables for our structure, we must be sure they will support loads at least equal to the tensions we found.

Statics & Strength of Materials

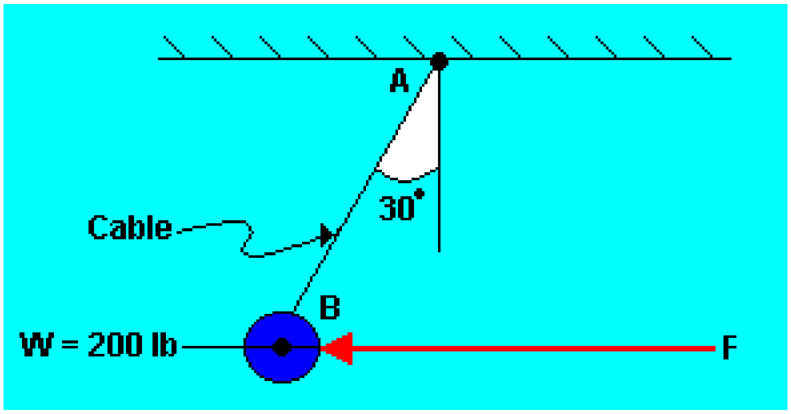
Problem Assignment - Coplanar

Draw a free body diagram and a force diagram as a part of the solution for each problem.
All problems are coplanar with concurrent forces.

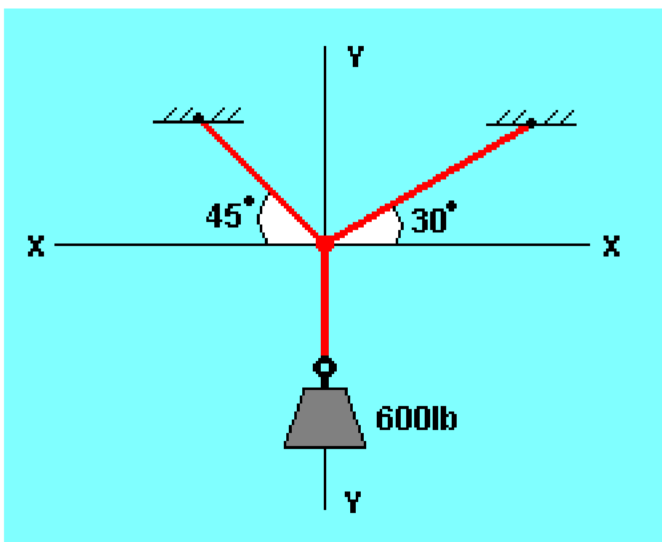
1. Calculate the force in cable AB and the angle θ for the support system shown. (447.2 lb. 63.4º)



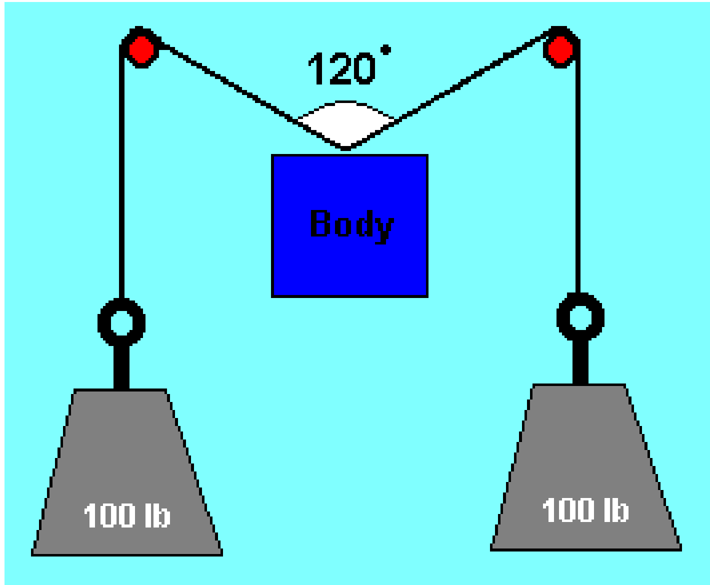
2. Calculate the horizontal force F that should be applied to the 200lb weight shown in order that the cable AB be inclined at an angle of 30° with the vertical. (115 lb.)



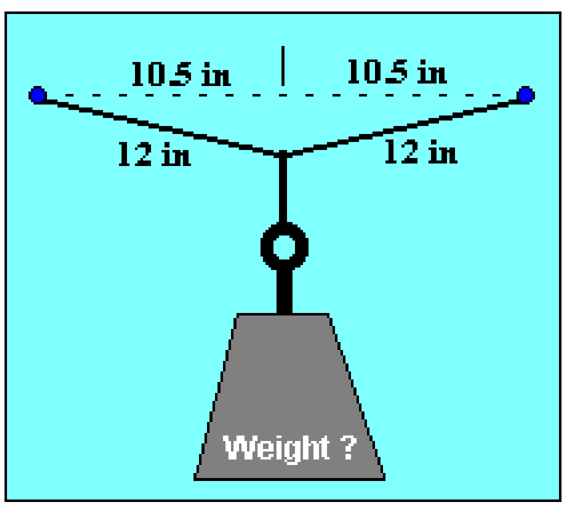
3. Calculate the force in each cable for the suspended weight shown. (439 lb., 538 lb.)



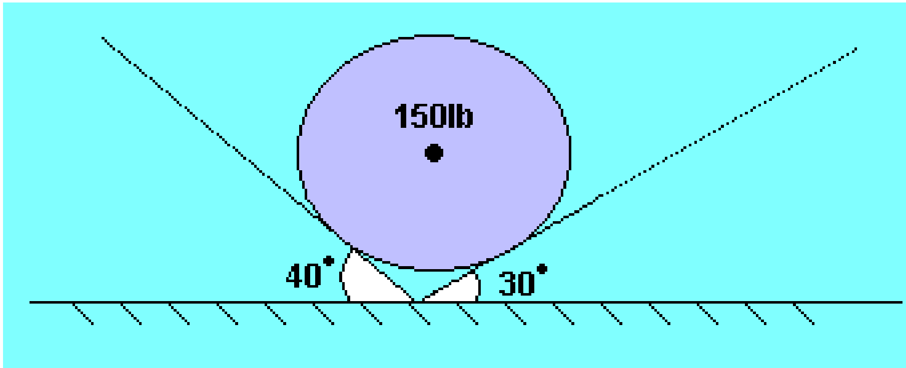
4. Two forces of 100 lb. each act on a body at an angle of 120º with each other. What is the weight of the body the two forces are supporting? (100 lb.)



5. A wire 24 inches long will stand a straight pull of 100 lb. The ends are fastened to two points 21 inches apart on the same level. What weight suspended from the middle of the wire will break it? (96.8 lb.)



6. Calculate the reactions of the two smooth inclined planes against the cylinder shown. The cylinder weighs 150 lbs. (79.8 lb., 102.6 lb.)



Topic 1.3 - Rotational Equilibrium

The second condition for equilibrium is rotational equilibrium. We can see the need for this second condition if we look at the diagram 1.3a. In this diagram, if we apply the 1st condition of equilibrium and sum the forces in the y-direction, we obtain zero. (+100 lb. - 100 lb. = 0). This would indicate that the object is in translational equilibrium. However, we almost instinctively recognize that the object certain will not remain at rest, and will experience rotational motion (and rotational acceleration).
Please notice that the object actually is in translational equilibrium. That is, even though it rotates, it rotates about the center of mass of the bar, and the center of mass of the bar will not move.

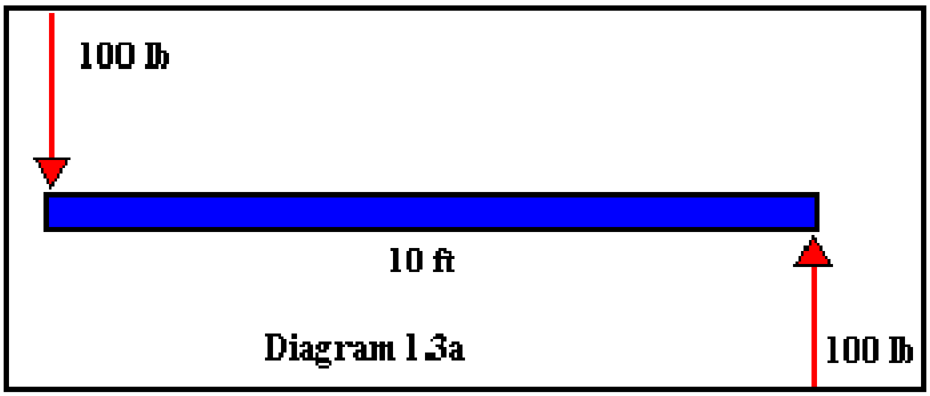


Diagram 1.3a

The second condition for equilibrium states that if we are to have rotational equilibrium, the sum of the Torque acting on the structure must be zero. Torque (or Moments) is normally covered in the first semester of a General College Physics course. (For an overview and review of Torque, please select: **Subtopic 1.31 Torque**)

The 2nd condition for equilibrium may be written:
$\sum \tau = 0$ or in three dimensions: $\sum \tau_x = 0$, $\sum \tau_y = 0$, $\sum \tau_z = 0$

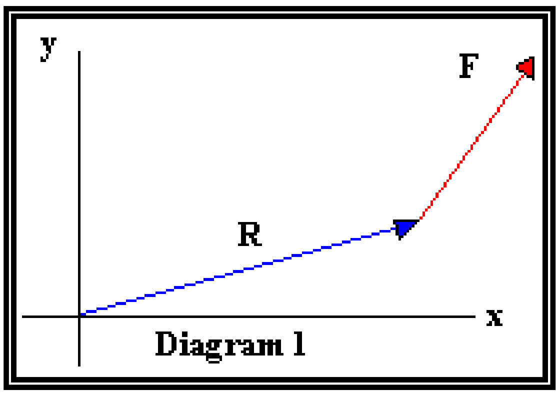
Since, most of our problems will be dealing with structures in two dimensions, our normal rotational equilibrium condition will be: $\sum \tau_{point} = 0$. That is, the sum of the torque acting on a structure with respect to any point selected must equal zero. Where, in two dimensions, one can only have counterclockwise(+) or clockwise(-) torque.

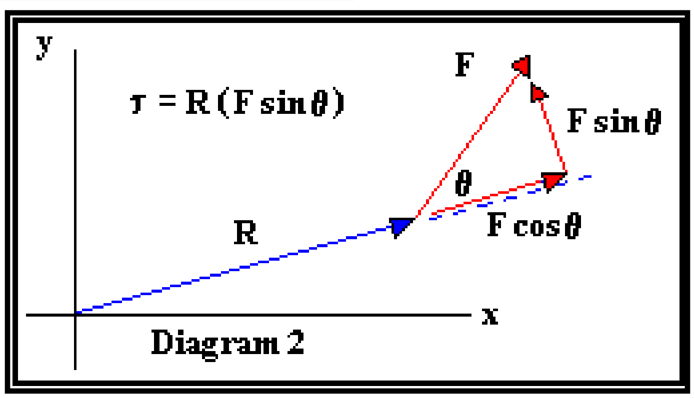
Now let's take a moment to go very slowly through an introductory problem involving both conditions of equilibrium, and applying our statics problem solving

techniques. Select **Torque Example 1**.
Select **Torque Example 2**; Select **Torque Example 3**

Subtopic 1.31 - Torque

Formally, a torque (also know as the Moment of Force) is a vector cross product, that is, given a distance vector **R** (from origin, as shown in diagram 1) and a force vector **F**, the torque is defined as: $\tau = \mathbf{R} \times \mathbf{F}$. This cross product is a vector and has a resultant magnitude (size) of $\tau = RF \sin \theta$, where theta is the angle between the distance vector and force vector. As diagram 2 shows, what this cross product effectively does is to take the component of the force vector perpendicular to the distance vector and then multiply it by the distance vector.

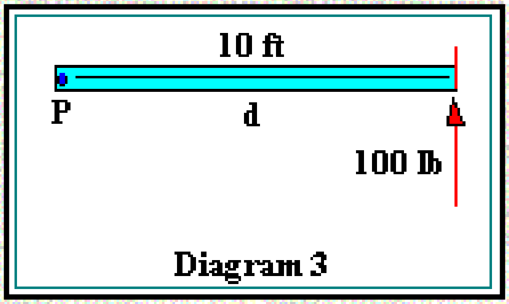




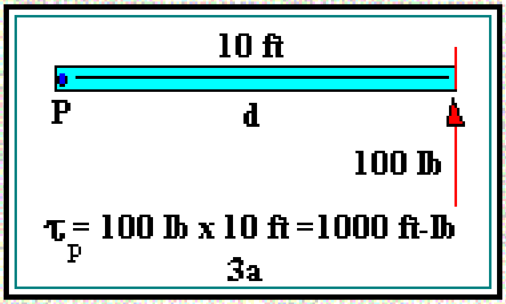
The direction of the torque vector is perpendicular to both the distance and force vectors, and is given by the right hand rule. There are a variety of ways of expressing the right hand rule. In this case it could be expressed this way. If vector R were a rod, pinned at the origin, notice that force component $F \sin \theta$ would cause R to start to rotate counterclockwise about the origin. If we now curl the fingers of our right hand in the same way as the rotation, our thumb will point the direction or the torque vector - in this case, out of the screen. (+ z direction.)

If this all seems confusing at this point, don't panic. We will use a somewhat less formal approach to determining torque acting on structures, as shown in the following example(s).

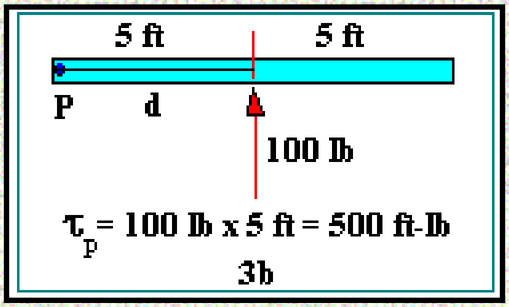
Example 1. In Diagram 3, a 10 foot long beam, pinned at point P, is shown with 100 lb. force acting upward on the beam. We wish to determine the torque on the beam due to the 100 lb. force.
The very first thing we must do when determining torque is to **PICK A POINT**. Torque is always calculated with respect to a point (or axis), and so before we can proceed we first choose a point (sometimes called the pivot point) to calculate torque about. In this example we will use point P, at the left end of the beam.



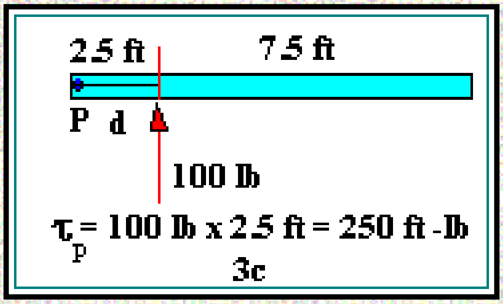
We next calculate the torque caused by the force by **Torque(about P) = Force x perpendicular distance**. That is, the torque caused by the 100 lb. force with respect to point P will be the 100 lb. force times the perpendicular distance from point P to the line of action of force (d = 10 ft.). By perpendicular distance, we mean, that there is only one line we can draw which begins at point P and intersects the line of action (direction) of the force at 90 degrees, and the length of this line is the perpendicular distance. In diagram 3a, it is the thin dark line with 'd' below it, and its value is 10 ft., so the torque due to the 100 lb. force with respect to point P will be: **Torque = F x d = 100 lb. x 10 ft = 1000 ft-lb.**



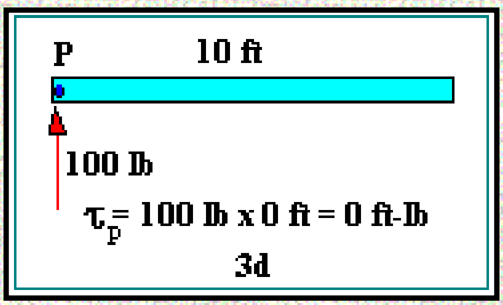
In diagram 3b, we have moved the 100 lb. force to halfway down the beam toward point P, and so the perpendicular distance is changed to 5 ft., and the torque due to the 100 lb. force with respect to point P now becomes: **Torque = F x d = 100 lb. x 5 ft. = 500 ft-lb**.



In diagram 3c, we have moved the 100 lb. force three quarters length down the beam toward point P, and so the perpendicular distance is now 2.5 ft., and the torque due to the 100 lb. force with respect to point P now becomes: **Torque = F x d = 100 lb. x 2.5 ft. = 250 ft-lb**.



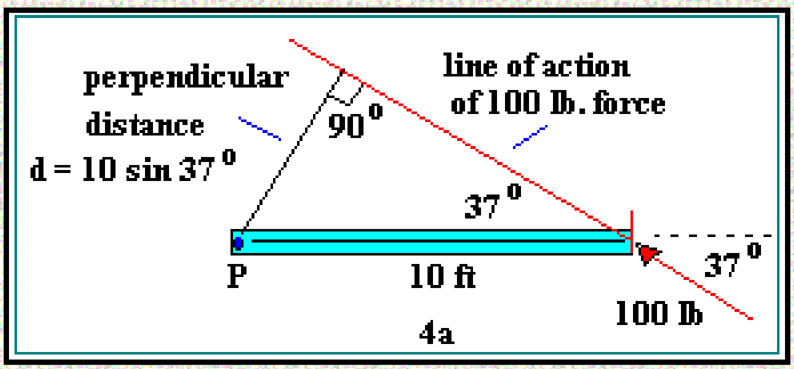
Finally in diagram 3d, the 100 lb. force acts directly below point P, and so the perpendicular distance is 0 ft, and therefore there is zero torque due to the 100 lb. force with respect to point P. This does not mean the 100 lb. force does not push on point P, it does, however it produces no torque (no rotation) with respect to point P.



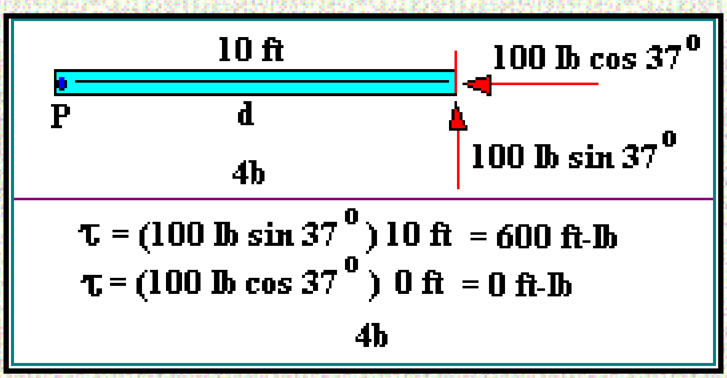
To see what happens if we do not have a force acting a 'nice' direction as in example 1 above, we will do a second example.

Example 2: In this example we use the same beam as above but now the 100 lb. force acts at an angle of 37^o with respect to the horizontal (below) as shown in diagram 4.
Once again we wish to calculate the torque produced by the 100 lb. force with respect to point P. We will first do this directly from our less formal definition of torque: **Torque = Force (times) perpendicular distance**.

Notice in Diagram 4a, we have extended the line of action of the force, and then we have started at point P and drawn a line from point P which intersects the force line at 90°. This is the perpendicular distance 'd' from the pivot point to the line of the force. We can find the value of the perpendicular distance d by noting that the force, perpendicular distance d, and beam length form a right triangle with the beam length as the hypotenuse. Therefore from trigonometry, **d = 10 sin 37° = 6 ft**, and the torque will be: **Torque = 100 lb. x 6 ft = 600 ft-lb**. (Positive torque, since if the beam were actually pinned at point P, the 100 lb. force would start the beam rotating in the counterclockwise direction.)



While this method of calculating the torque is fine and does work, it actually is more effective in problems, certainly in problems with a number of forces, to first break the force into it's equivalent x and y-components before calculating torque. This is the process we will normally use, and we have shown this process in Diagram 4b.



We first found the x and y-components of the 100 lb. force as shown in the diagram. Then the resultant torque about point P will be the sum of the torque produced by each component force. Notice the y-component force results in **Torque = 100 lb. sin 37º x 10 ft. = 600 ft-lb**., while the x-component force results in **Torque = 100 lb. cos 37º x 0 ft. = 0 ft-lb** since its line of action passes through point P, and thus its perpendicular distance is zero, resulting in zero torque. Summing the two torque we get a total of 600 ft-lb., just as we found in the first method.

As we continue with examples of statics problems, the applications of torque in solving problems will be shown in complete detail. While we have determined torque here in only 2-dimensions, the procedure is the same 3-dimensions, with the exception that there are 3-axis of rotation possible. Since the type of problems we will consider are mainly 2-dimensional, we will not go into calculating torque in 3-dimensions at this point.

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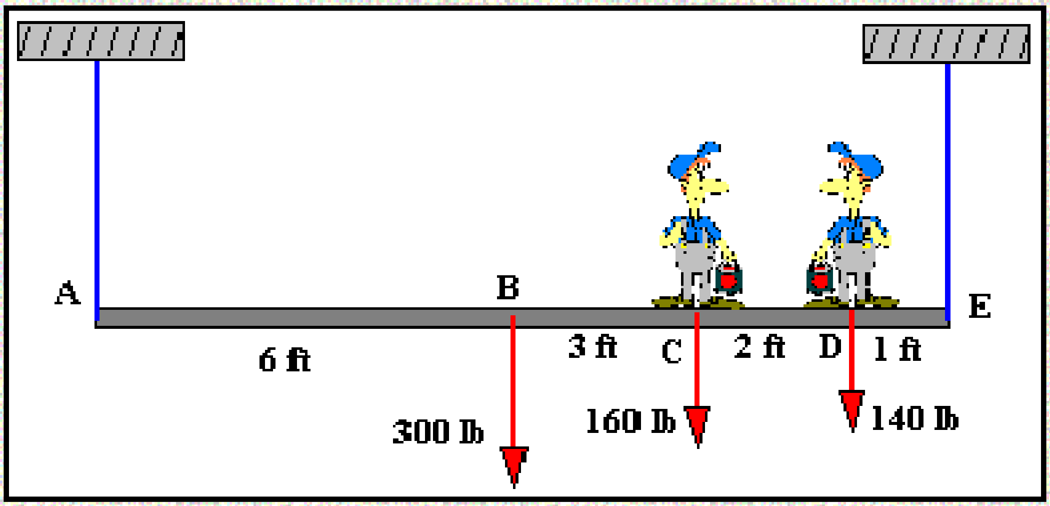
Example 1 - Torque review problems

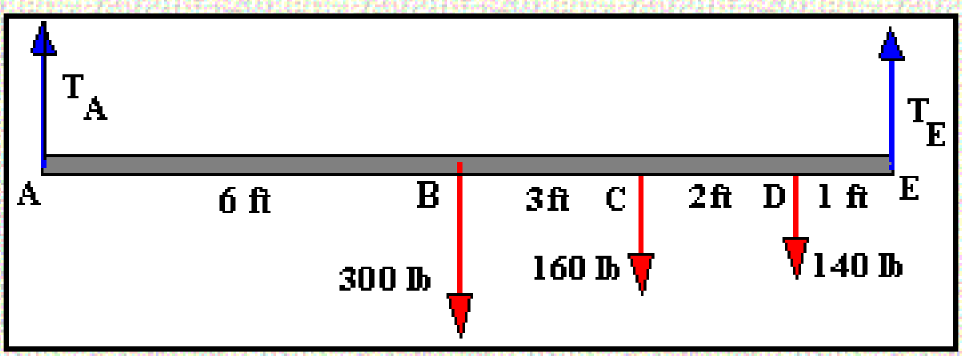
In this example, two painters are standing on a 300 lb. scaffolding (beam) which is 12 ft. long. One painter weighs 160 lb. and the second painter weighs 140 lb. The scaffolding is supported by two cables, one at each end. As they paint, the painters begin wondering what force (tension) is in each cable.

The question is, what is the force (tension) in each cable when the painters are standing in the positions shown. Notice that for a uniform beam or bar, as far as equilibrium conditions are concerned, the beam weight may be considered to act at the center (of mass) of the bar.



We proceed with this problem with a well establish statics technique.

1. Draw a Free Body Diagram (FBD) showing and labeling all external forces acting on the structure, and including a coordinate system.
Notice in the diagram to the right, we have shown the forces in the cables supporting the beam as arrows upward, and labeled these forces T_A and T_B.



2. Resolve all forces into x and y-components. In this example, all the forces are already acting in the y-direction only, so nothing more needs to be done.

3. Apply the (2-dimensional) equilibrium conditions:

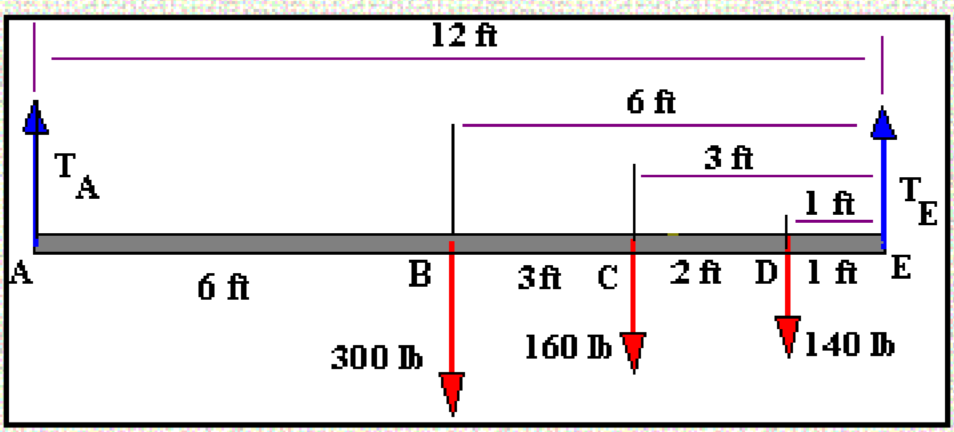
$\sum F_x = 0; \sum F_y = 0 ; \sum \tau_P = 0$

$\sum F_x = 0$ (No external x-forces acting on the structure, so this equation gives no information.) $\sum F_y = 0$ **T_A - 300 lb - 160 lb - 140 lb + T_E = 0**

Here we have summed the external y-forces. We can't solve this equation yet, as there are two unknowns (T_A and T_E) and only one equation so far. We obtain our second equation from the 2nd condition of equilibrium - sum of torque must equal zero.

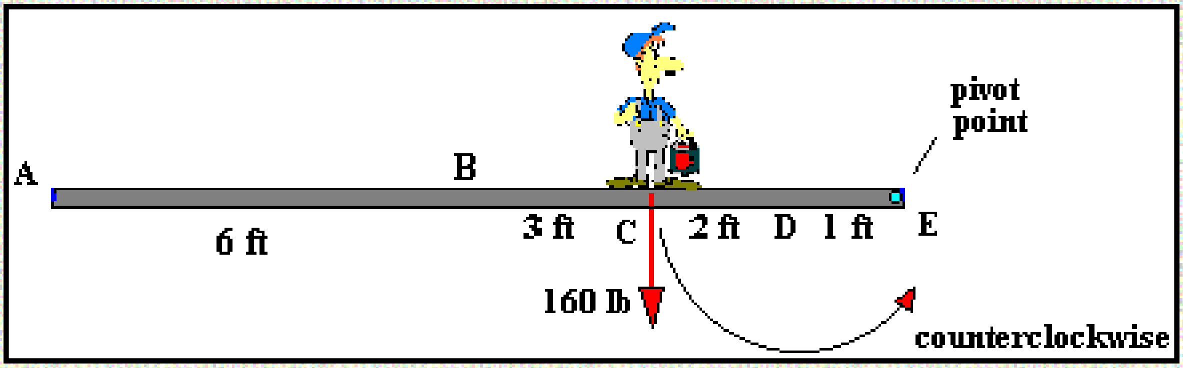
Before we can sum torque we must first **PICK A POINT**, as we always calculate torque with respect to a point (or axis). Any point on (or off) the structure will work, however some points result in an easier equation(s) to solve. As an example, if we sum torque with respect to point E, we notice that unknown force T_E acts through point E, and, if a force acts through a point, that force does not produce a torque with respect to the point (since the perpendicular distance is zero). Thus summing torque about point E will result in an equation with only one unknown, as shown

$\sum \tau_E = 0$ or **$-T_A$(12) +300 lb(6ft) + 160 lb(3 ft) + 140 lb(1 ft) = 0**



Where we determined each term by looking at the forces acting on the beam one by one, and calculating the torque produced by each force with respect to the chosen point E from: **Torque = Force x perpendicular distance** (from the point E to the line of action of the force). For a review of torque, select Torque.

The sign of the torque is determined by considering which way the torque would cause the beam to rotate, if the beam were actually pinned at the chosen point. That is, if we look at the 160 lb. weight of painter one (and ignore the other forces), the 160 lb. weight would cause the beam to start rotating counterclockwise (+), if the beam were pinned at point E.



It is important to note that the sign of the torque depends on the direction of the rotation it would produce with respect to the chosen pivot point, not on the direction of the force. That is, the 160 lb. force is a negative force (downward) in the sum of forces equation, but it produces a positive torque with respect to point E in the sum of torque equation. See equations below:

$\sum F_y = 0$ or, $\mathbf{T_A - 300\ lb - 160\ lb - 140\ lb + T_E = 0}$

$\sum \tau_E = 0$ or, $\mathbf{-T_A(12) + 300\ lb(6ft) + 160\ lb(3\ ft) + 140\ lb(1\ ft) = 0}$

We now solve the torque equation for T_A, finding $\mathbf{T_A = 201.67\ lb}$ (round off to **202 lb**). We then place the value for T_A back into the y-force equation and find the value of $\mathbf{T_E = 398\ lb}$.

We have now found the forces in the cables when the painters are in the positions shown in the problem. As an additional thought problem, one might consider the question of what is the minimum cable strength required so that the painters could move anywhere on the scaffolding safely. (Answer = 450 lb.)

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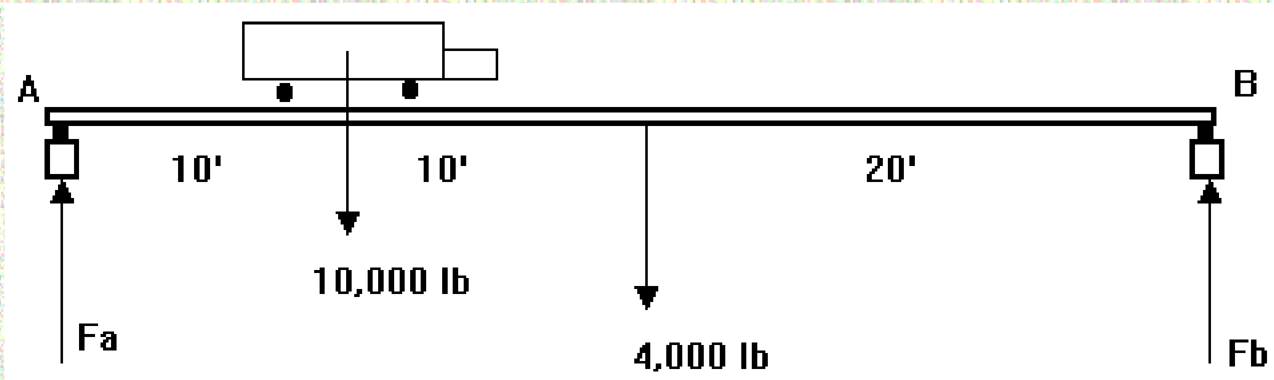
Statics & Strength of Materials

Example 2 - Torque Review Problems

A simply supported 40 foot, 4000 lb bridge, shown above, has a 5 ton truck parked 10 feet from the left end of the bridge. We would like to determine the compression force in each support. The weight of the bridge may be considered to act at its center.



Statics Problems: Techniques
1. Draw Free Body Diagram of entire structure, showing and labeling all external forces, including support forces and loads. Choose an appropriate coordinate system. Determine needed dimensions and angles.
2. Resolve all forces into their x and y components.
3. Apply the equilibrium conditions and solve for unknown external forces and torques as completely as possible.

1. Free body diagram is shown above.
2. All forces are in x or y direction
3. Eq. Cond:

Σ Fx = 0 (no external x forces acting on structure.)
Σ Fy = + Fa + Fb - 10,000 lb - 4,000 lb = 0
Σ τa = -10,000 lb x 10 ft - 4,000 lb x 20 ft + Fb x 40 ft = 0
Solving for unknowns: Fa = 9500 lb, Fb = 4500 lb

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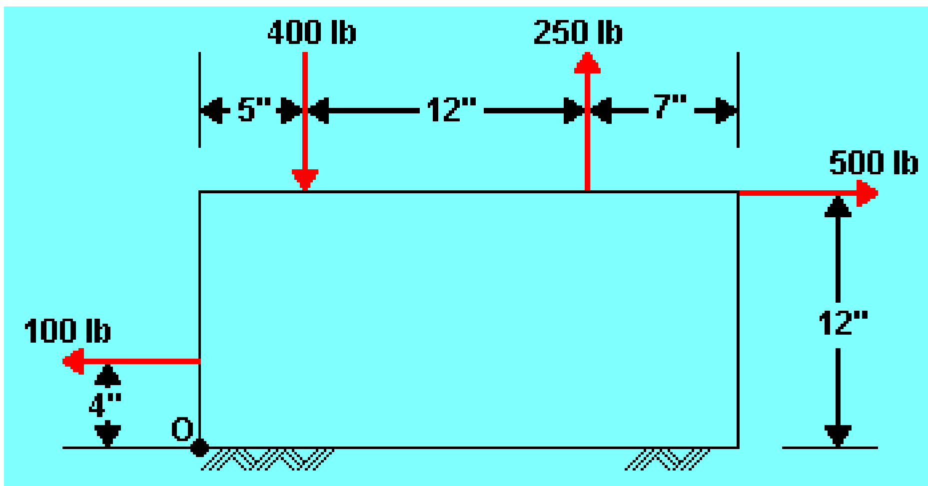
Statics & Strength of Materials

Example 3 - Torque Review Problems

Determine the magnitude, direction, and sense of the resultant forces shown. Determine where the resultant intersects the bottom of the body with respect to point O.

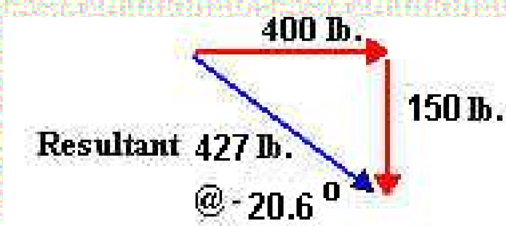


Problem #1

For the first part of this problem we sum the forces in both the x and y direction to determine the resultant components our final force vector will have.

Sum Fx = +500 lb. - 100 lb. = 400 lb. =Rx
Sum Fy = -400 lb. + 250 lb. = -150 lb. =Ry

We next find the resultant vector from it's components: R = square root ($Rx^2 + Ry^2$) = Sqrt ($400^2 + (-150)^2$)
R = 427 lb., and then direction from Tan ϕ = Ry/Rx = -150/400 = -.375 The negative y-component and the positive x-component tell us that the resultant must be in the fourth quadrant. Solving for ϕ we find ϕ = -20.6 or 339.4 degrees.



To determine where the resultant intersects the bottom of the body with respect to point O, we realize that our resultant vector must produce the same torque with respect to point O as the orginal forces produce. Thus we must determine the resultant torque of our original forces.
Torque = + 100 lb. x 4" -400 lb. x 5" + 250 lb. x 17" - 500 lb. * 12" = -3350 in-lb. (clockwise direction)

Our resultant must produce the same torque when it is applied or acts at the bottom of the body. This actually simplifies the solution. At the bottom, the x-component of our vector (400 lb.) produces no torque as its line of action passes through point O. Only the y-component (-150 lb.,downward) produces a torque, and so we may write:
-150 lb. x (d") = -3350 in-lb., and therefore d = 22.3"

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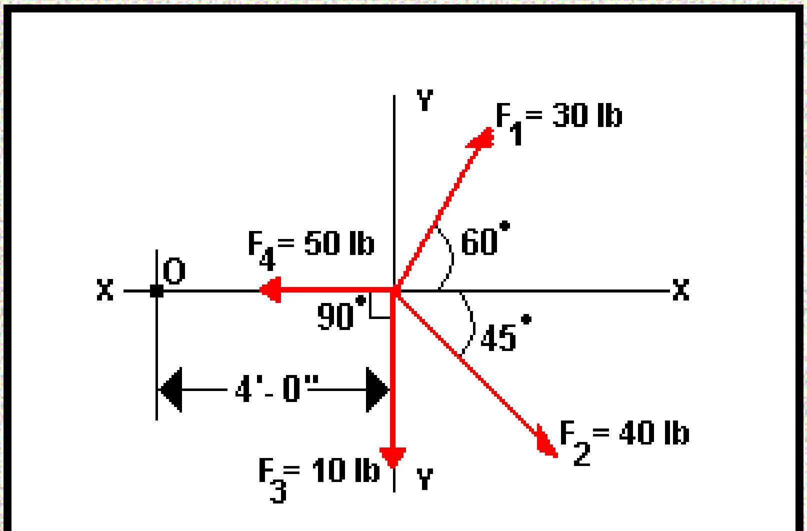
Problem Assignment - Torque

1. Four coplanar concurrent forces act as shown.
a) Calculate the moment of each force about point O that lies in the line of action of the F_4 force.

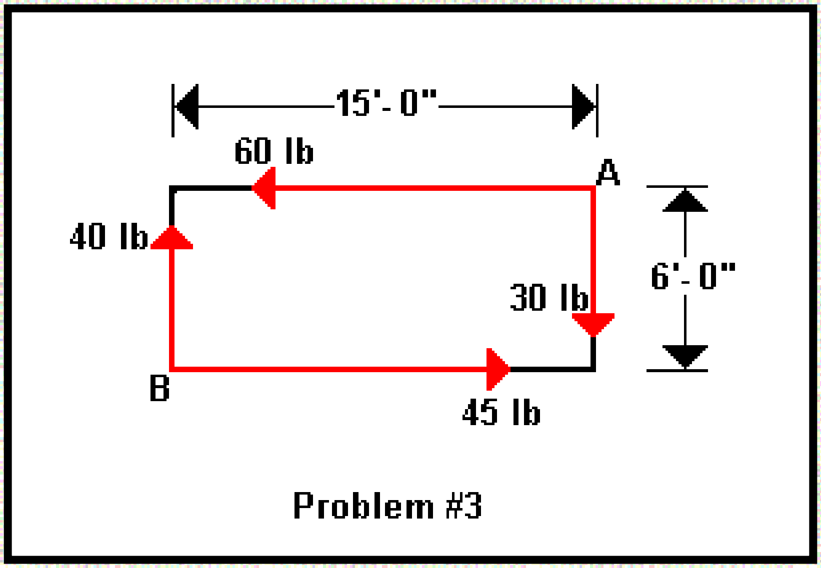
b) Calculate the algebraic summation of the four moments and determine the direction of rotation. (F1: 104 ft-lb., F2: -113 ft-lb.,F3: -40 ft-lb., F4 = 0, Net = -49 ft-lb.)



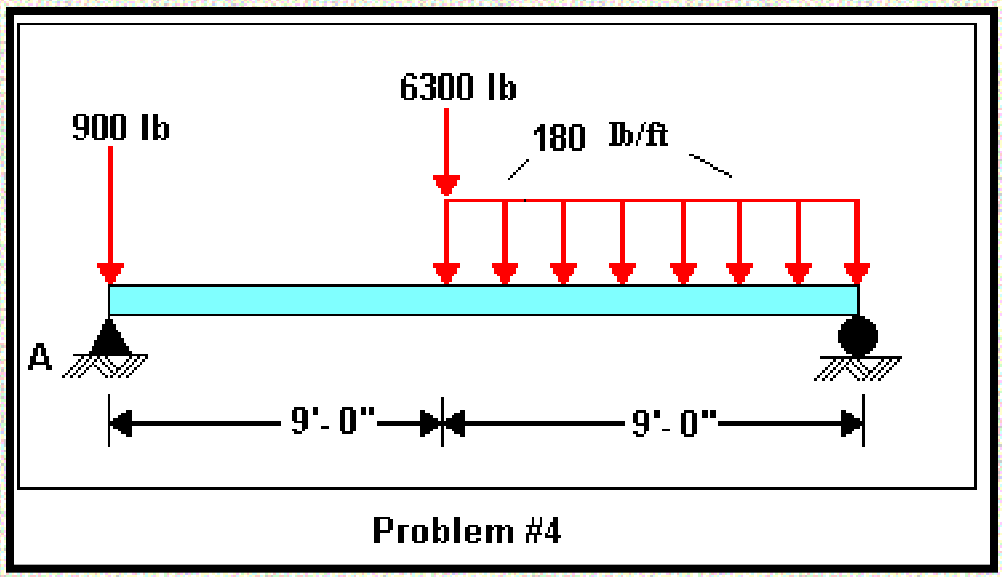
Problem #1

2. Determine the resultant of the four forces in problem #1 (magnitude and angle of inclination with respect to the X axis). Compute the moment of the resultant with respect to point O and compare with the results of problem 1. (13.89 lb @ 240°), Moment of resultant should be same as in previous problem.

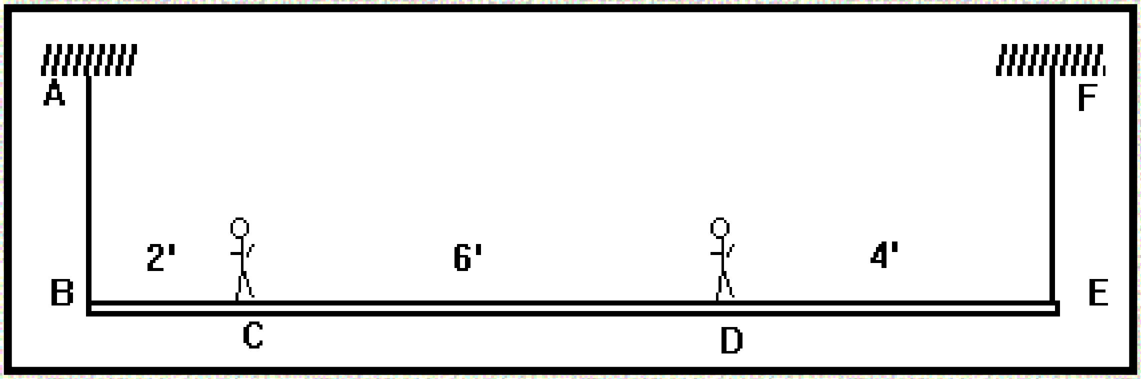
3. The rectangular body shown measures 6 ft. by 15 ft.
a) Calculate the algebraic summation of the moments of the forces shown about point A. (-330 ft-lb.)
b) Calculate the algebraic summation of the moments of the forces about point B. (-90 ft-lb.)



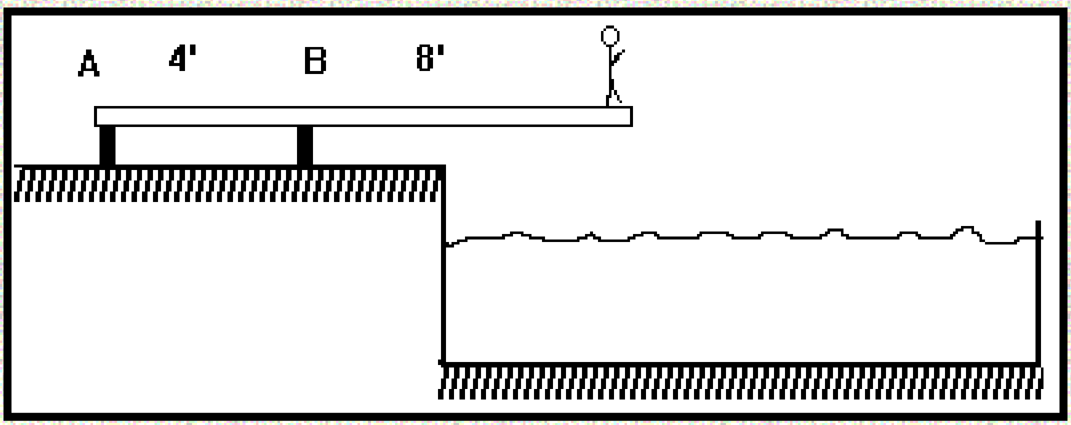
4. Determine the magnitude of a vertical resultant force which would be equivalent to the load system shown. (8820 lb.) Next determine where that force would have to act to give the same torque about the center of the beam as the loads produce. (8.91 ft left of point A) [Do not calculate the support forces in this problem.]



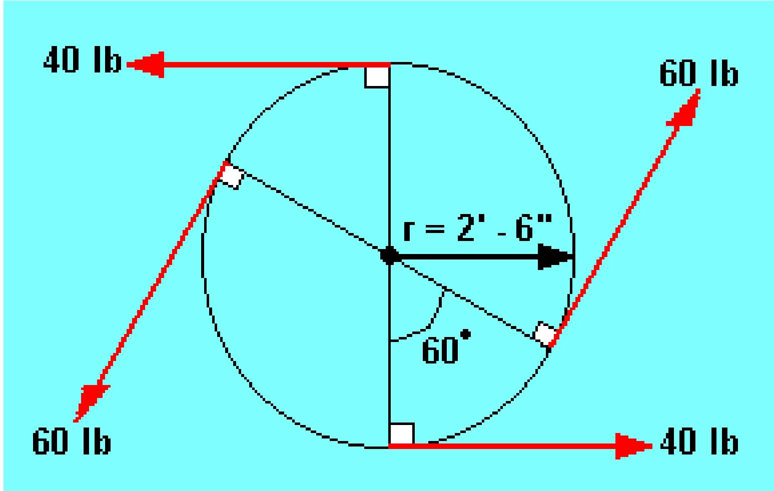
5. A beam is supporting two painters as shown below. Each painter weighs 180 lbs. The beam weights 100 pounds. Determine the tension in each rope (AB & FE). (260 lb., 200 lb.)



6. A 160 lb person is standing at the end of a diving board as shown below. The diving board weighs 140 lbs, and this weight may be considered to act at the center of the board. Calculate the vertical forces acting at each support, A & B. (Include the directions of the forces) (-390 lb., 690 lb.)



7. Compute the magnitude and direction of the resultant couples action on the bodies shown. (500 ft-lb, ccw)



Problem #5

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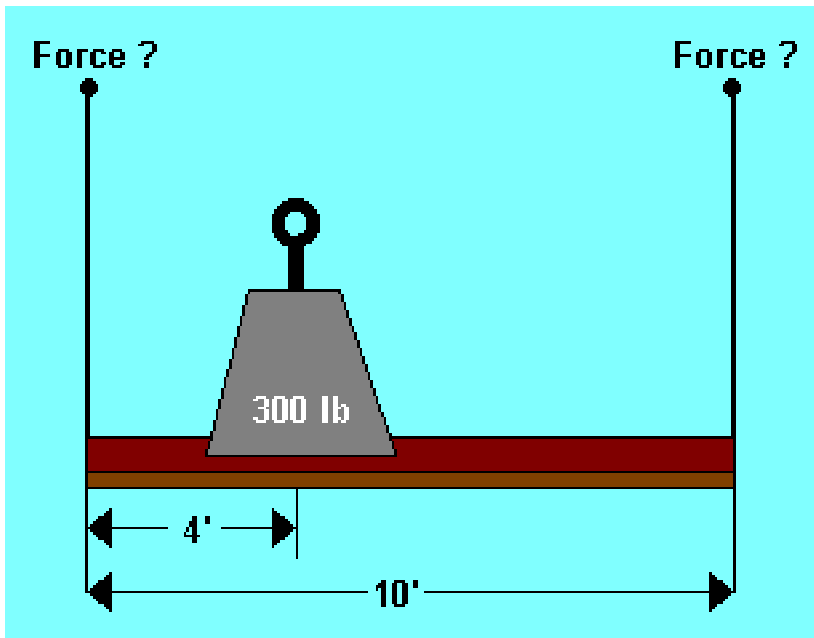
Statics & Strength of Materials

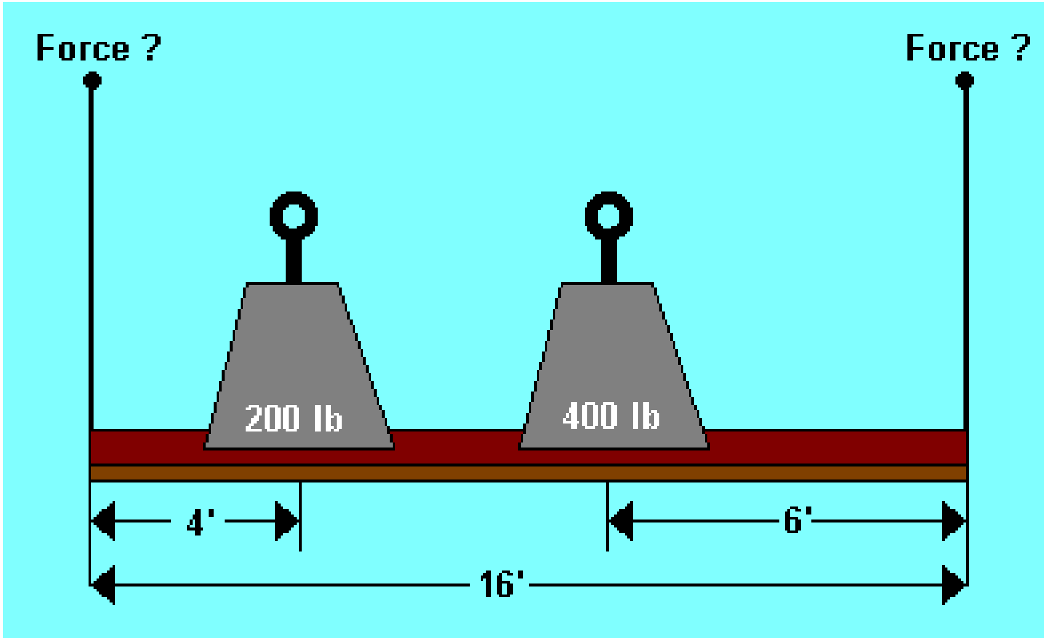
Problems

1. A weightless horizontal plank is ten feet long. It is simply supported at its two ends. A single load of 300 pounds rests at 4 feet from the left end. Determine the support forces at each of the two ends. (120 lb., 180 lb.)



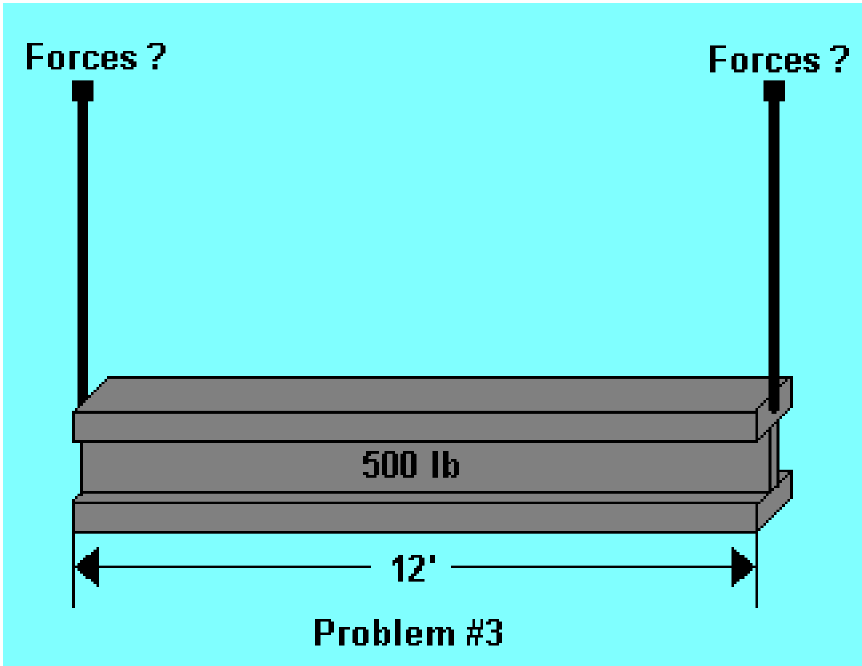
Problem #1

2. A weightless horizontal plank is 16 feet long and is simply supported at its two ends. A load of 200 pounds is 4 feet from the left end and a load of 400 pounds is 6 feet from the right end. Determine the support forces at each of the two ends. (300 lb., 300 lb.)

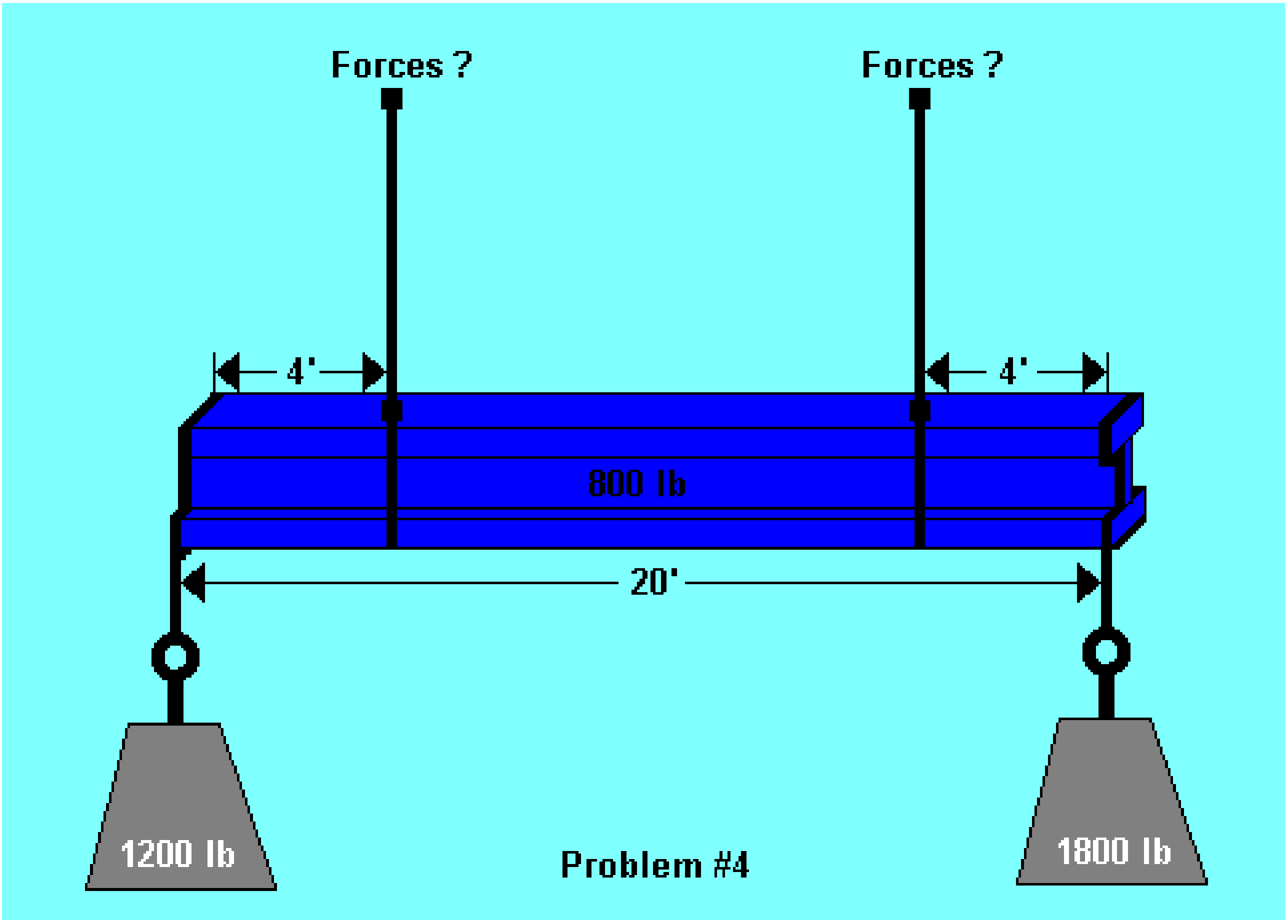


Problem #2

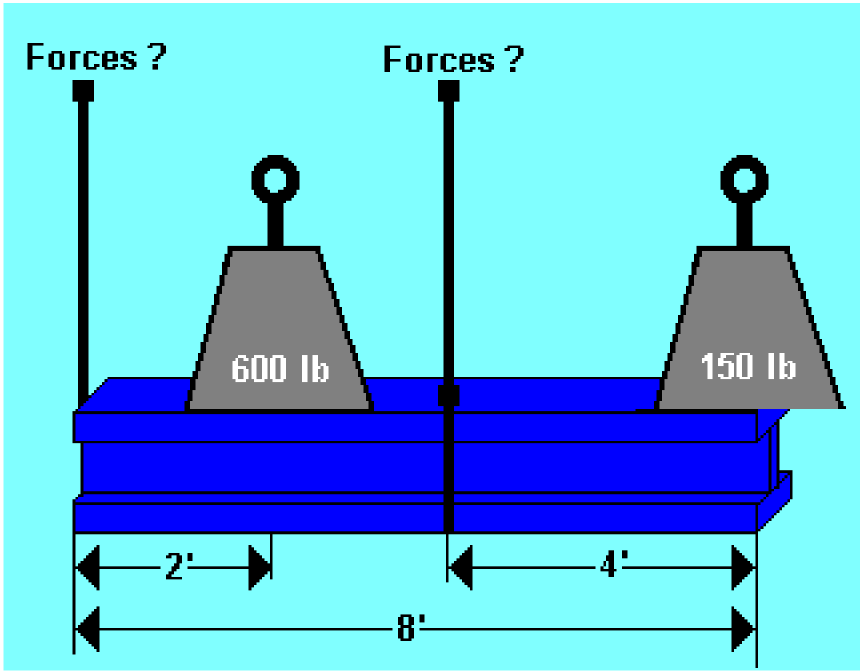
3. A 12 foot long uniform steel beam weighs 500 pounds and has no additional loads. (For static equilibrium calculations the entire weight of a uniform beam can be considered to act at its midpoint.) The beam is simply supported at its two ends. Determine the forces acting at each end. (250 lb., 250 lb.)



4. A 20 foot long uniform steel beam weighs 800 pounds. The beam is simply supported 4 feet from the ends. A load of 1200 pounds is at one end of the beam and 1800 pounds at the other end. Determine the support forces acting on the beam.(1400 lb., 2400 lb.)

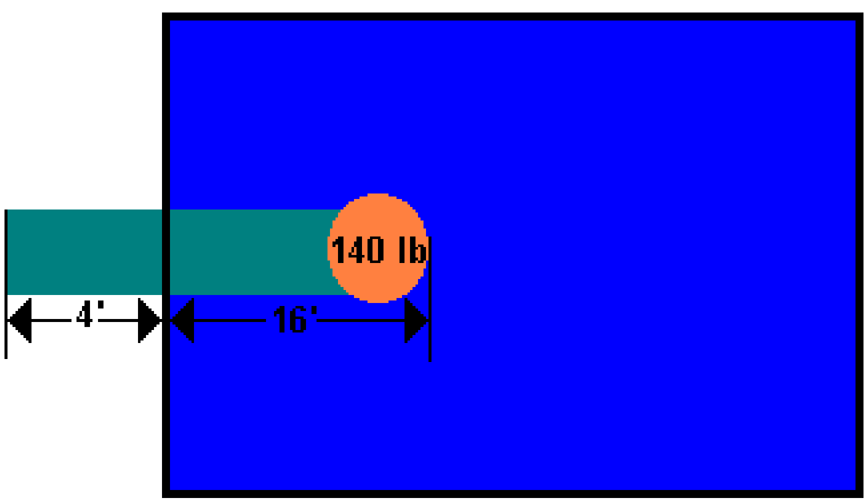


5. An 8 foot beam (weightless) is simply supported at its left end and its midpoint. There is a load of 150 pounds placed at the free end and a load of 600 pounds placed midway between the supports. Determine the support forces acting on the beam.(600 lb., 150 lb.)



Problem #5

6. A diving board is 16 feet long and is supported at its left end and a point 4 feet from the left end. A 140 lb. diver stands at the right end of the board. Determine the support forces acting on the diving board. (560 lb., - 420 lb.)



Problem #6

Statics - Review / Summary / Problem Sheet

I Equilibrium Conditions 3-dimensions

Translational: $\Sigma \mathbf{F} = 0$ or $\Sigma \mathbf{F_x} = 0, \Sigma \mathbf{F_y} = 0, \Sigma \mathbf{F_z} = 0$

Rotational $\Sigma \tau = 0$ or $\Sigma \tau_x = 0$ (right hand rule +), $\Sigma \tau_y = 0$ (right hand rule +), $\Sigma \tau_z = 0$ (right hand rule +)

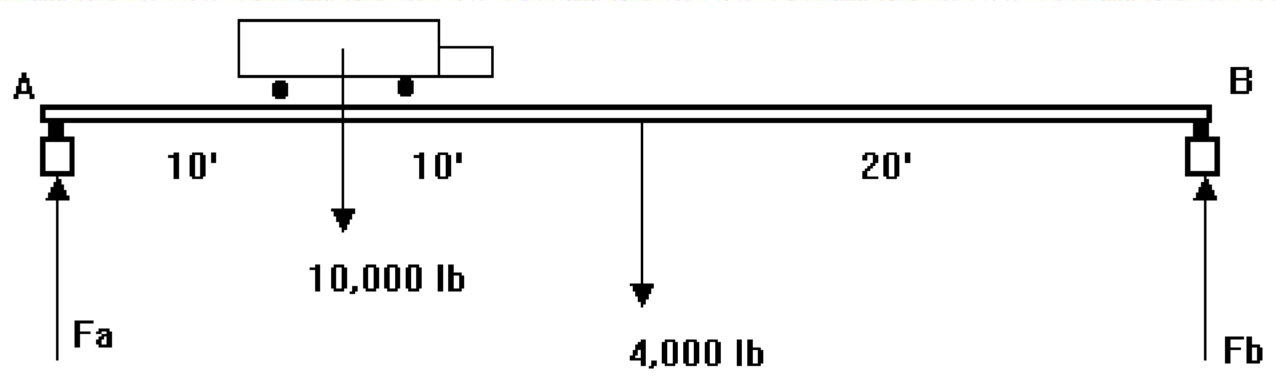
where τ **= R x F** or (τ = force x perpendicular distance to pivot point)

II Equilibrium Conditions 2-dimensions

Translational: $\Sigma \mathbf{F} = 0$ or $\Sigma \mathbf{F_x} = 0, \Sigma \mathbf{F_y} = 0$

Rotational $\Sigma \tau = 0$ or $\Sigma \tau_p = 0$ (ccw = +), where τ **= R x F** or (τ = force x perpendicular distance to pivot point)

Example 1



A simply supported 40 foot, 4000 lb bridge, shown above, has a 5 ton truck parked 10 feet from the left end of the bridge. We would like to determine the compressional force in each support. The weight of the bridge may be considered to act at its center.

1. Free body diagram is shown above.
2. All forces are in x or y direction
3. Eq. Cond: Σ Fx = 0 (no external x forces acting on structure.)

Σ Fy = + Fa + Fb - 10,000 lb - 4,000 lb = 0

Σ τa = -10,000 lb x 10 ft - 4,000 lb x 20 ft + Fb x 40 ft = 0

Solving: Fa = 9500 lb, Fb = 4500 lb

III Statics Problems: Techniques

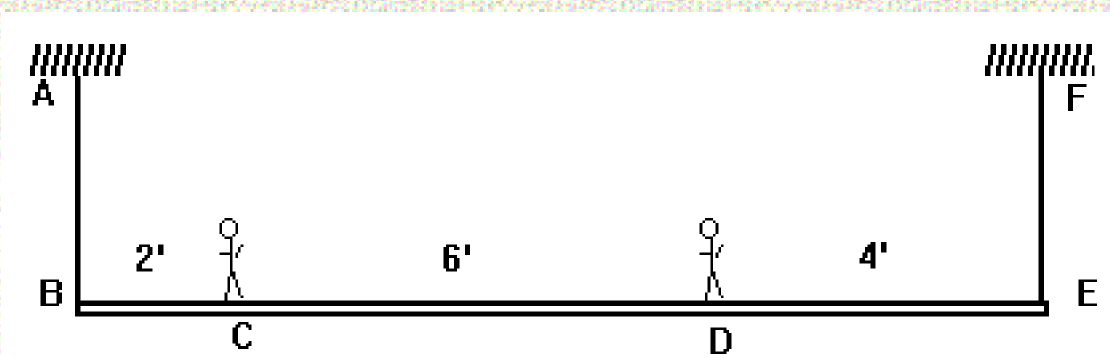
1. Draw Free Body Diagram of entire structure, showing and labeling all external forces, including support forces and loads. Choose an appropriate coordinate system. Determine needed dimensions and angles.
2. Resolve all forces into their x and y components.
3. Apply the equilibrium conditions and solve for unknown external forces and torques as completely as possible.

Often we wish to know the internal forces (tension & compression) in each **member** of the structure in addition to the external support forces. To do this (with non-truss problems) we continue the procedure above, but with members of the structure, not the entire structure.

4. Draw Free Body Diagram of a member (s) of the structure of interest. Show and label all external forces and loads acting on the selected member. Choose an appropriate coordinate system. Determine needed dimensions and angles.
5. Resolve all forces into their x and y components.
6. Apply the equilibrium conditions and solve for unknown external forces and torques acting on the member as completely as possible.

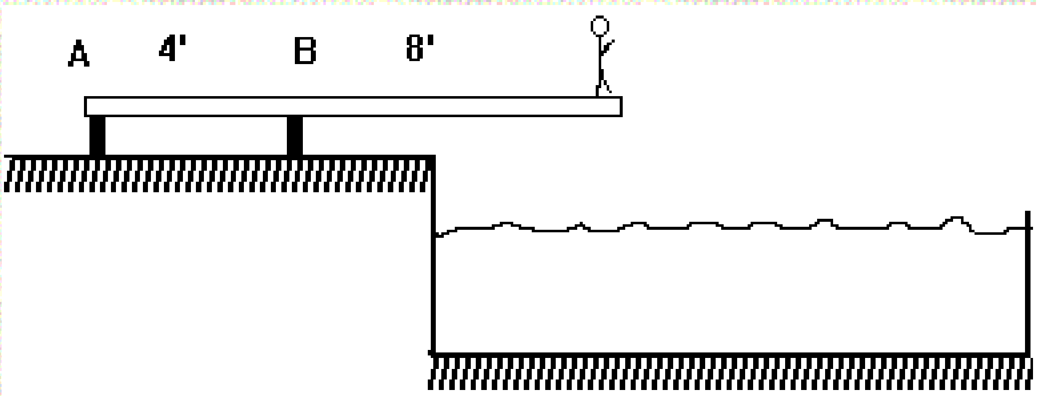
In certain problems, the equilibrium equations for both the entire structure and for its members may have to be written and solved simultaneously before all the forces on the structure and in its members can be determined.

Problem #1. A beam is supporting two painters as shown below. Each painter weighs 180 lbs. Determine the tension in each rope (AB & FE). (Neglect the weight of the beam.) (210 lb., 150 lb.)

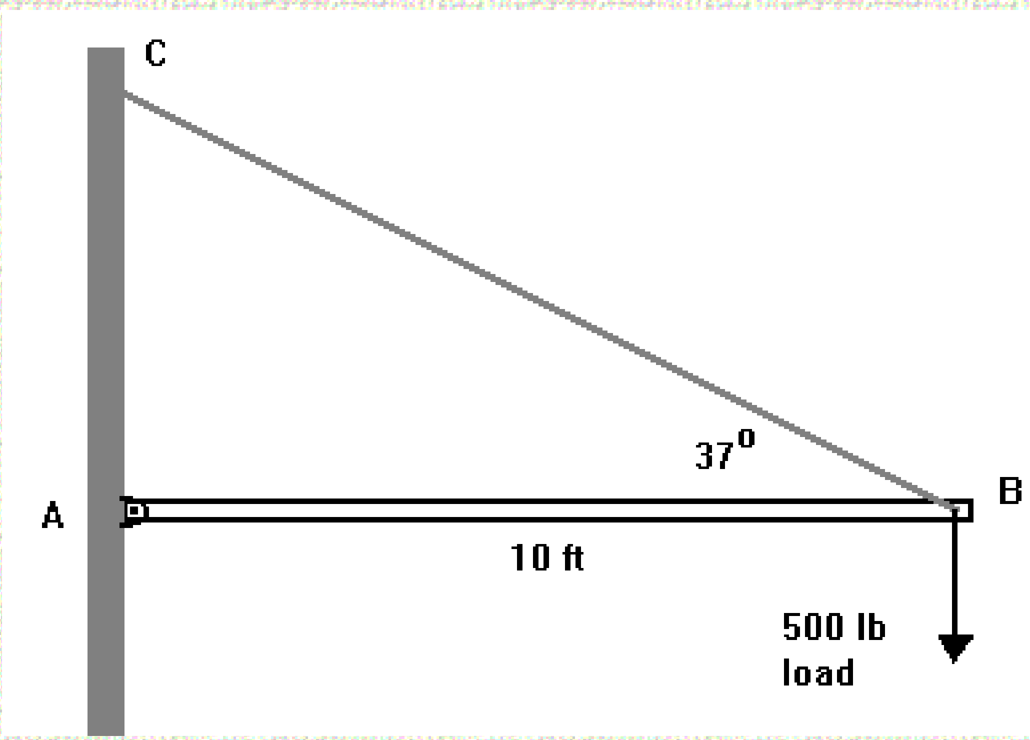


Probem #2. Rework problem #1. Assume that the plank weighs 100 pounds and that this weight may be considered to act at the center of the span.(260 lb., 200 lb.)

Problem #3. A 160 lb person is standing at the end of a diving board as shown below. The diving board weighs 140 lbs, and this weight may be considered to act at the center of the board. Calculate the vertical forces acting at each support, A & B. (Include the directions of the forces) (-390 lb., 690 lb.)



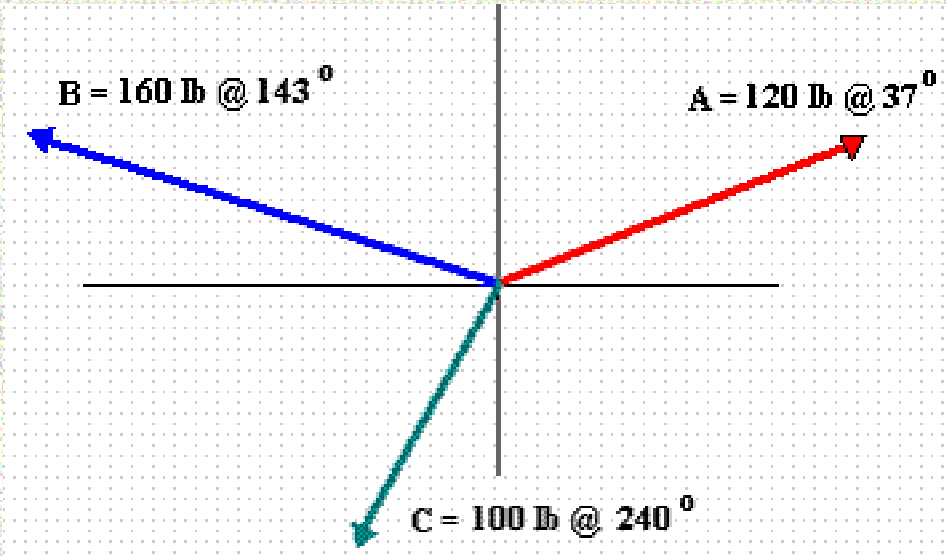
Problem #4. A 500 pound sign is supported by a beam and cable as shown below. The beam is attached to a wall by a hinge, and has a uniformly distributed weight of 100 lbs. Determine the tension in the cable and the forces acting at the hinge. (BC=917 lb., Ax = 733 lb., Ay = 50lb.)



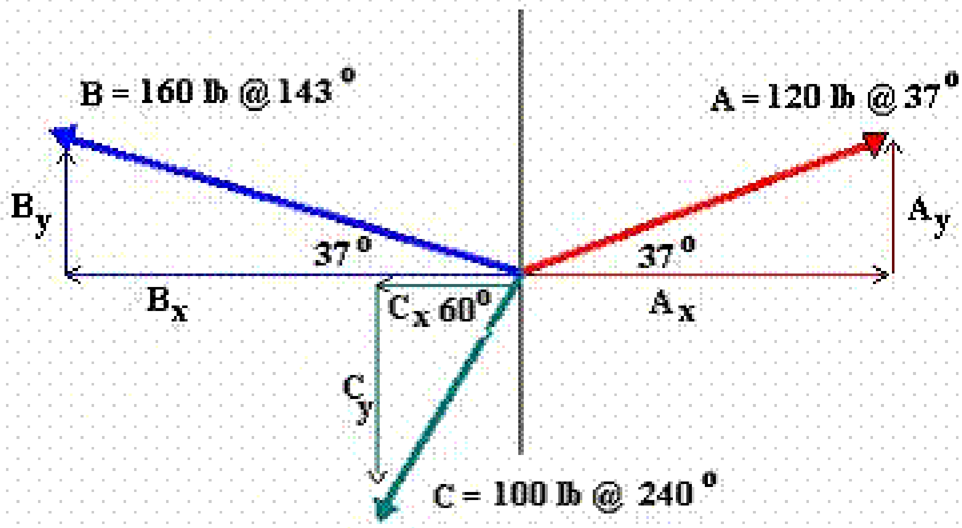
Additional Important Examples

Some additional examples with brief solutions

1. Three vectors are shown in the diagram below. Find the resultant vector (magnitude and direction):
R = -2 **A** - 2 **B** -3 **C**



Solution



$A_x = 120 \cos 37^0$ $\quad B_x = -160 \cos 37^0$ $\quad C_x = -100 \cos 60^0$

$A_y = 120 \sin 37^0$ $\quad B_y = 160 \sin 37^0$ $\quad C_y = -100 \sin 60^0$

$$R_x = -2 * 120 \cos 37^0 \; -2 * -160 \cos 37^0 \; -3 * -100 \cos 60^0$$

$$R_x = -2(96) -2(-128) -3(-50) = +214 \text{ lb}$$

$$R_y = -2 * 120 \sin 37^0 \; -2 * 160 \sin 37^0 \; -3 * -100 \sin 60^0$$

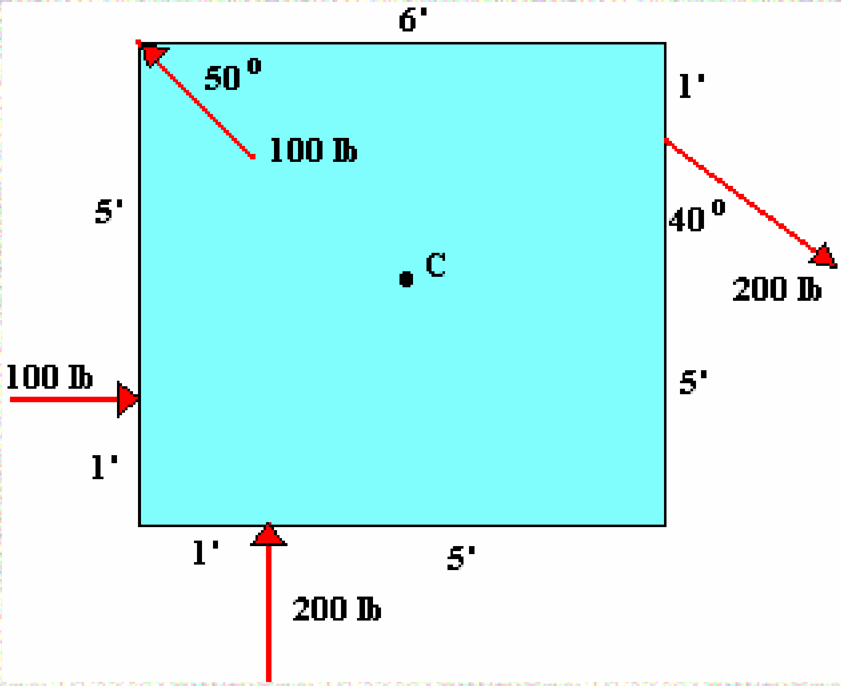
$$Ry = -2(72) -2(96) - 3(-86.7) = -75.6 \text{ lb}$$

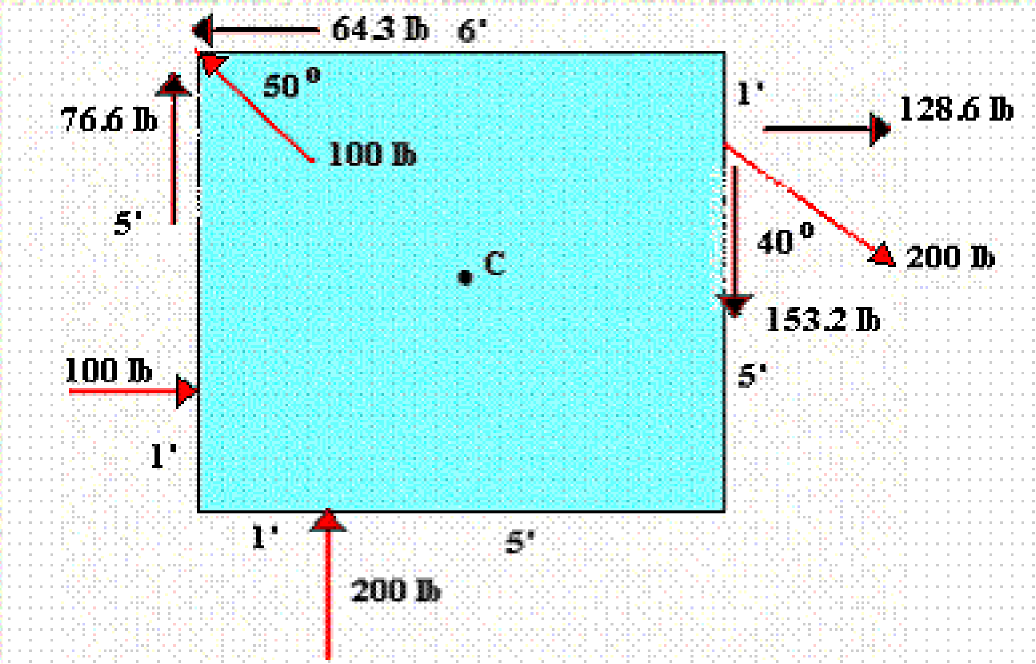
$$R = \text{sqrt}(R_x^2 + R_y^2) = 227 \text{ lb}$$

Tan (Theta) = - 75.6 lb /214 lb

Theta = -19.5^0 or 340.5^0

2. A block has four forces acting on it as shown in the diagram.
Determine the resultant (net) torque (magnitude & direction) about the center of mass (point O).
Determine the resultant force (magnitude & direction) and where along the base it would act to produce the same torque about point C



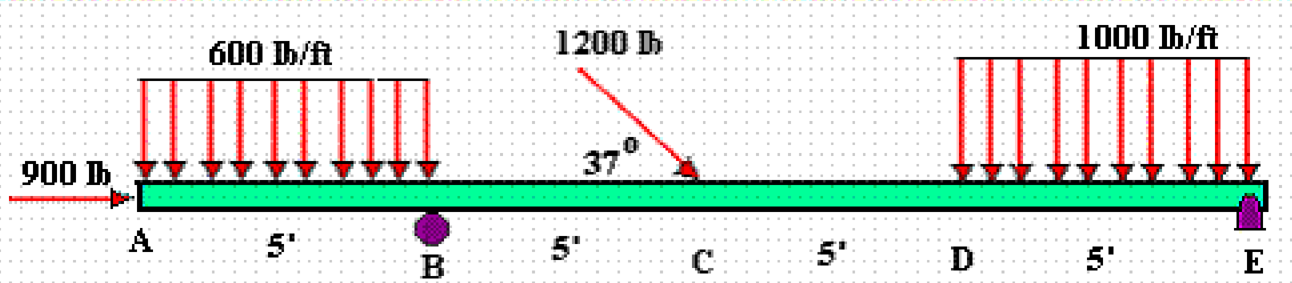


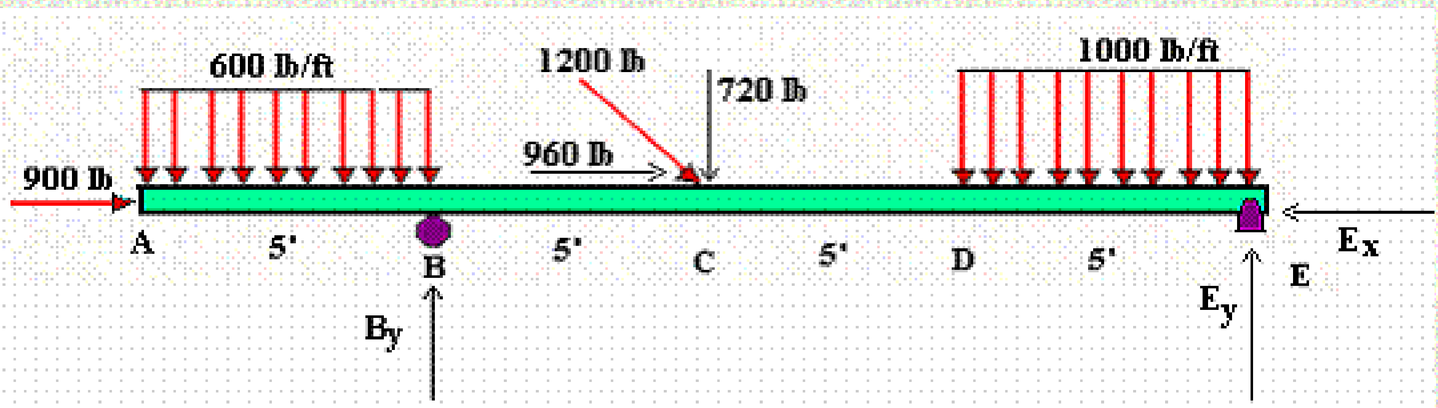
Sum Torque-c: +64.3 lb * 3 ft – 76.6 lb * 3 ft -128.6 lb * 2 ft – 153 lb * 3 ft + 100 lb * 2ft – 200 lb * 2 ft = - 953 ft-lb
Sum Force x = -64.3 lb + 126.6 lb + 100 lb = 162.3 lb
Sum Force y = 76.6 lb – 153.2 lb + 200 lb = 123. 4 lb

R = sqrt(162.3 ^2 + 123.4^2) = 204 lb. Tan (Theta) = 123.4/162.3 ; Theta = 37.2 deg

123.4 lb (x ft) – 162.3 lb * (3 ft) = -953 ft-lb, so x = - 3.78 ft (to left of center of bottom)

3. The beam shown below is supported by a roller at point B and by a pinned joint at point E. Determine the support forces at points B and E





Solution: Sum Fx: 900 lb + 960 lb – Ex = 0

Sum Fy: - (600 lb/ft * 5ft) + By – 720 lb – (1000 lb/ft * 5 ft) + Ey = 0

Sum Torque B: +(3000 lb * 2.5 ft) – (720 lb * 5 ft) – (5000 lb * 12.5 ft) + Ey * 15 ft = 0

Ex = 1860 lb (negative x direction); Ey = + 3907 lb; By = + 4813 lb

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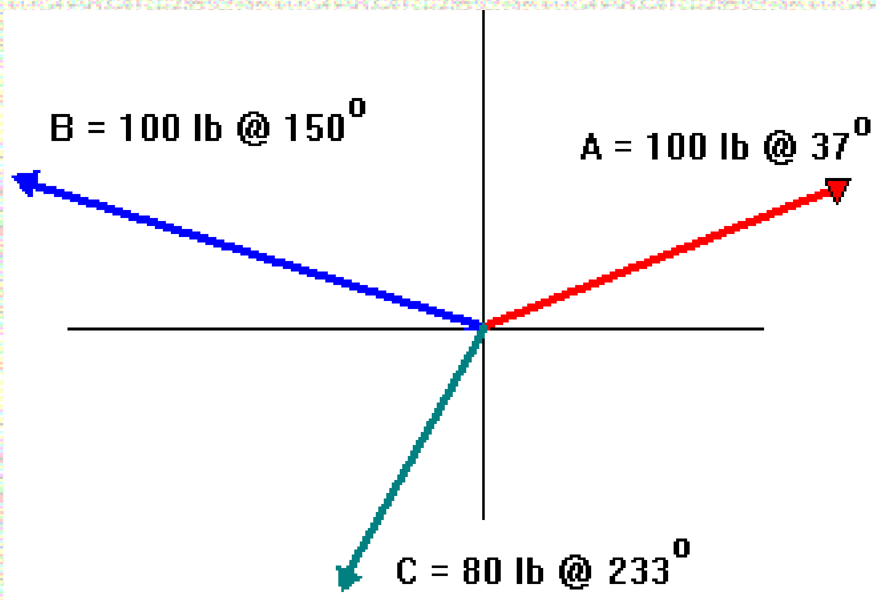
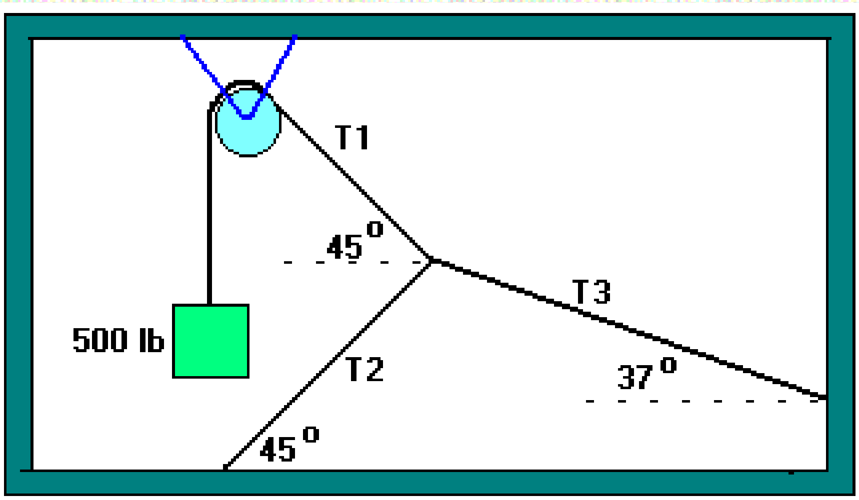
Topic 1: Statics I - Exam

1. Three vectors (A, B, & C) are shown in the diagram below. Find one vector (magnitude and direction) that will have the same effect as the three vectors shown below. (Remember to show your work.) (71.7 lb. @ 140º)



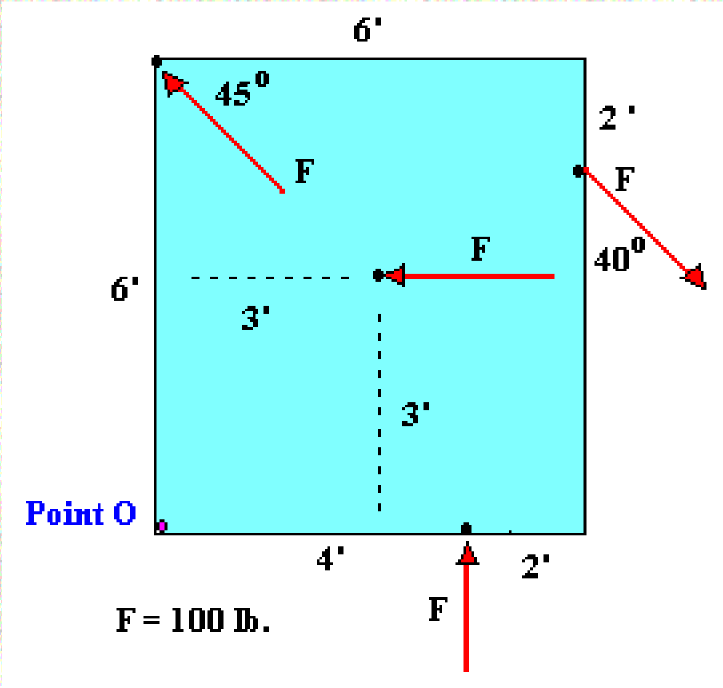
2. Determine the tensions (T1, T2, T3) in each cable. (T1 = 500 lb., T2 = 71 lb., T3 = 505 lb.)



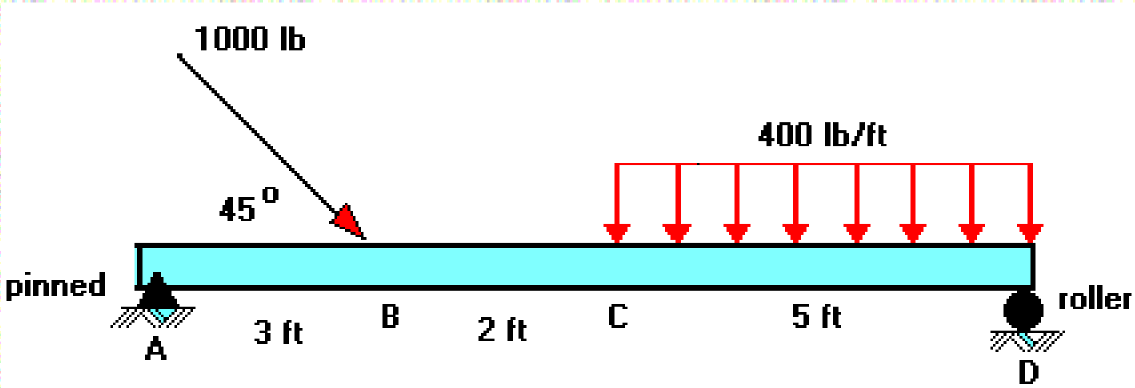
3. A square block has four forces acting on it as shown in the diagram. Each force has strength of 100 lb.

a) Determine the resultant torque due to all the forces with respect to Point O. (+ 407.2 ft-lb)

b) Determine a resultant force – magnitude, direction and location (acting at the base) which would be equivalent to the four forces shown below. (142 lb @ 132 deg, x = 4.33 ft)



4. Determine the support forces acting on the beam at points A & D. (Ax = -707 lb., Ay = 995 lb., Dy = 1712 lb.)



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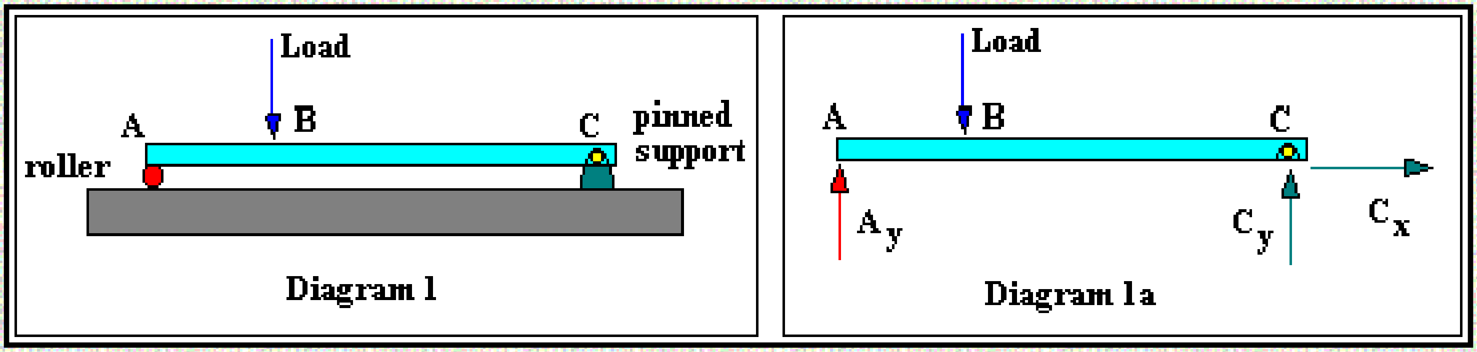
Topic 2: Statics II - Applications

Topic 2.1: Frames (non-truss, rigid body structures)

We are now ready to begin looking at somewhat more involved problems in statics. As we do so there will be a number of concepts we need to keep in mind (and apply) as we approach these problems. We will begin by looking at problems involving **rigid bodies**. This simply means that at this point we will not be concerned with the fact that a body (or member of a structure) may actually bend or deform (change length) length under the applied loads. Bending and deformation effects will be considered in later materials. At this point in the course we will also ignore the weight of the members, which are often small compared to the loads applied to the structures.

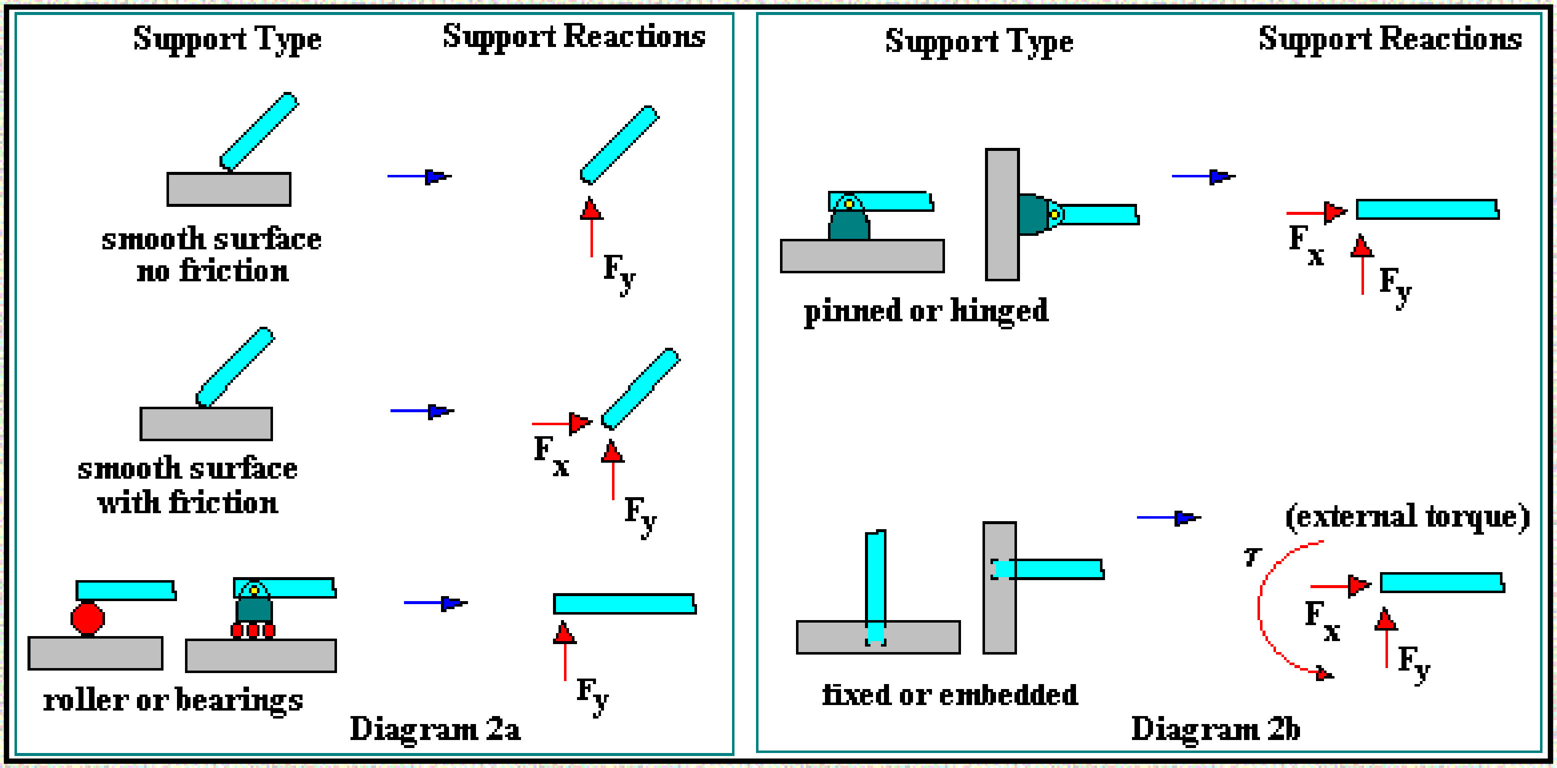
As we begin to analyze structures there are two important considerations to keep in mind, especially as we draw our free body diagrams. The first concern will be

Structure Supports. Different types of supports will result in different types of support forces (reactions) acting on the structure. For example, a roller or bearing can only be placed in compression, thus the force it will exert on a structure will be a normal or perpendicular force only, while a hinged or pinned support point may exert both a horizontal and vertical force. [Or to be more specific, a hinged or pinned support (or joint) will exert a support force acting at a particular angle, this force may then always be broke into an x and y-component. The net effect is that a hinged or pinned support may be replaced by x and y support forces.] See Diagram 1 below.



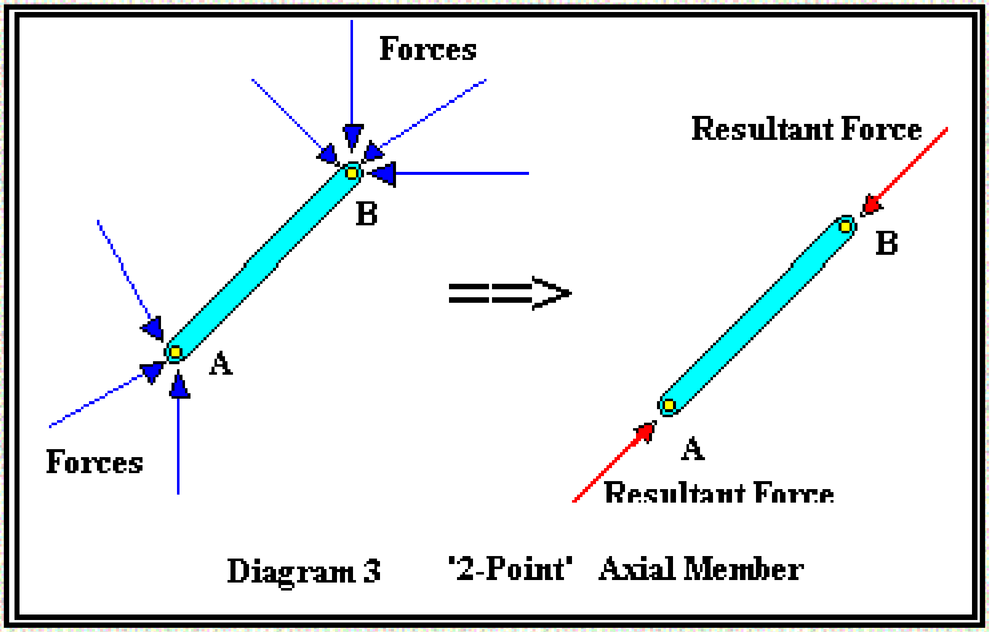
In Diagram 1 we have shown a horizontal beam supported at point A by a roller and at point C by a pinned support. Diagram 1a is the Free Body Diagram of the beam with the roller and the pinned joint now replaced by the support forces which they apply on the beam. The roller applies the vertical force A_y and the pinned support applies the forces C_x and C_y. If we knew the value of the load, we could apply statics principles to find the actual value of the support forces. We shall do this process in great detail later for a somewhat more complex example.

Diagram 2 below shows examples of supports and the types of forces and/or torque which they may exert on a structure.

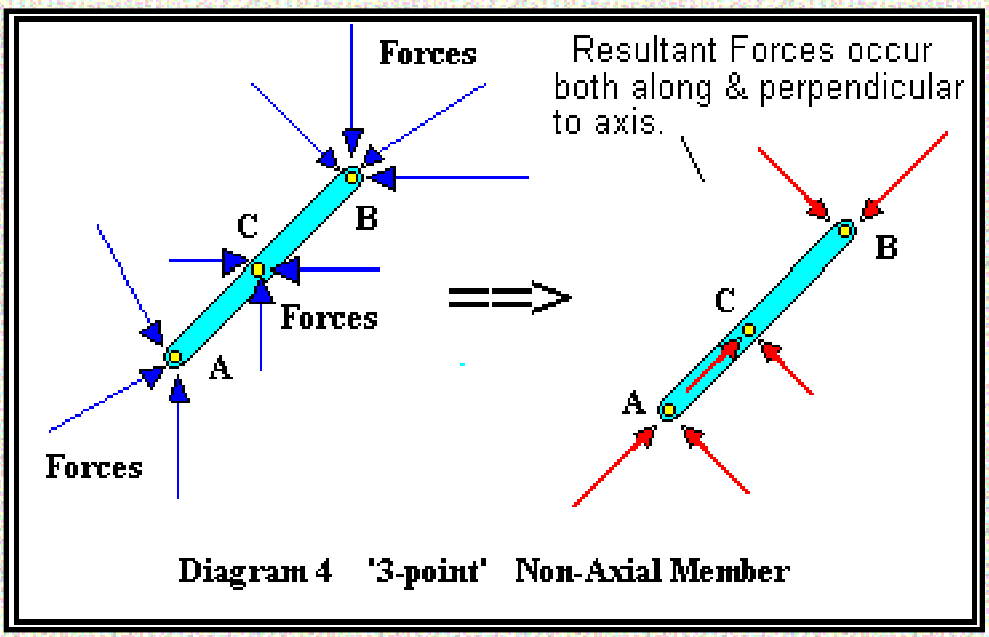


The second important concept to keep in mind as we begin looking at our structure is the type of members the structure is composed of - **Axial or Non-Axial Members**. (The importance of this will be seen in more detail when we look at our first extended example.)

In equilibrium or statics problems, an **Axial Member** is a member which is only in simple tension or compression. The internal force in the member is constant and acts only along the axis of the member. A simple way to tell if a member is an axial member is by the number of **Points** at which forces act on a member. If forces (no matter how many) act at only **Two Points** on the member - it is an axial member. That is, the resultant of the forces must be two single equal forces acting in opposite directions along axis of the member. See Diagram 3.



A **Non-Axial Member** is a member which is not simply in tension or compression. It may have shear forces acting perpendicular to the member and/or there may be different values of tension and compression forces in different parts of the member. A member with forces acting at **More Than Two Points** (locations) on the member is a non-axial member. See Diagram 4.



EXAMPLES: To show the application of the concepts discussed above and of our general statics problem-solving technique, we will now look in careful detail at several statics problems.

In **Example 1** we will concentrate on finding the values of the external support forces acting on the structure. Select Example 1

In **Example 2**, we will examine a relatively straight forward problem which points out several features concerning torques and beam loading. Select Example 2

In **Example 3**, we will see how **both** the **external support reactions** and also the **internal forces in a member** of the structure may be found. Select Example 3

In **Example 4**, we will look at a problem which seems to be a statically indeterminate problem. Select Example 4

Additional Examples: The following additional examples all demonstrate different aspects and features of a variety of non-truss, or frame problems.

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Example 1: Frames (non-truss, rigid body) problems

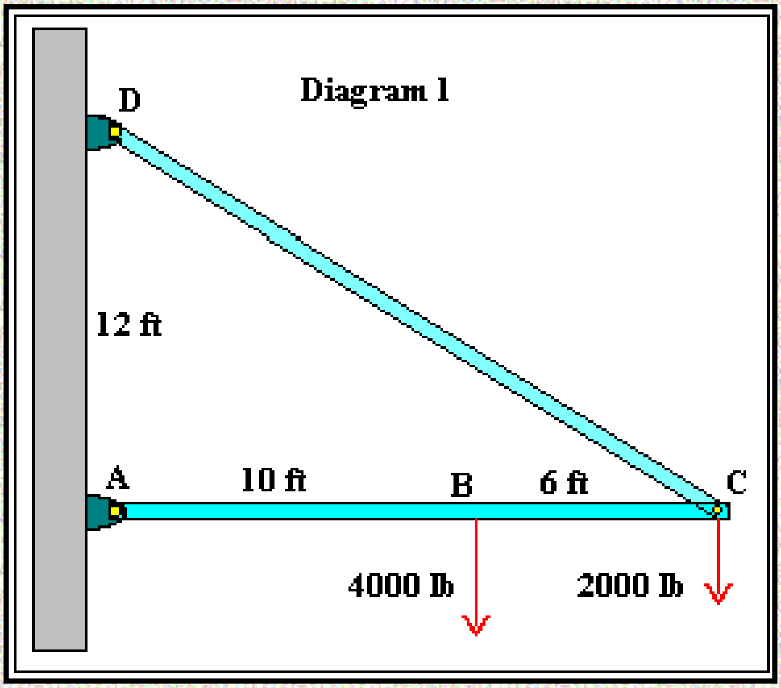
In this first example, we will proceed very carefully and methodically. It is important to get the method and concepts we need to keep in mind firmly established.

In this problem we wish to determine all the external support forces (reactions) acting on the structure shown in Diagram 1 below. Once again our procedure consists basically of three steps.

1. **Draw a Free Body Diagram of the entire structure showing and labeling all external load forces and support forces, include any needed dimensions and angles.**
2. **Resolve (break) all forces into their x and y-components.**
3. **Apply the Equilibrium Equations ($\sum Fx = 0; \sum Fy = 0; \sum \tau_P = 0$) and solve for the unknown forces.**

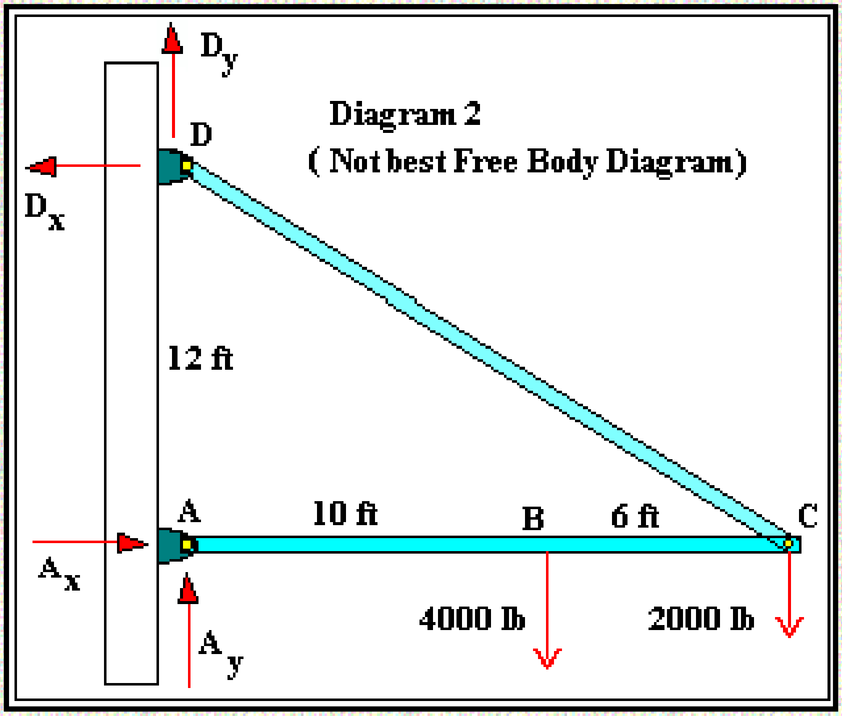
Step 1: Free Body Diagram (FBD). Making the FBD is probably the most important part of the problem. A correct FBD usually leads to a quick solution, while an inaccurate FBD can leave a student investing frustrating unsuccessful hours on a problem. With this in mind we will discuss in near excruciating detail the process of making a good FBD.

We note that the structure is composed of members ABC, and CD. These two members are pinned together at point C, and are pinned to the wall at points A and D. Loads of 4000 lb. and 2000 lb. are applied to member ABC as shown in Diagram 1.



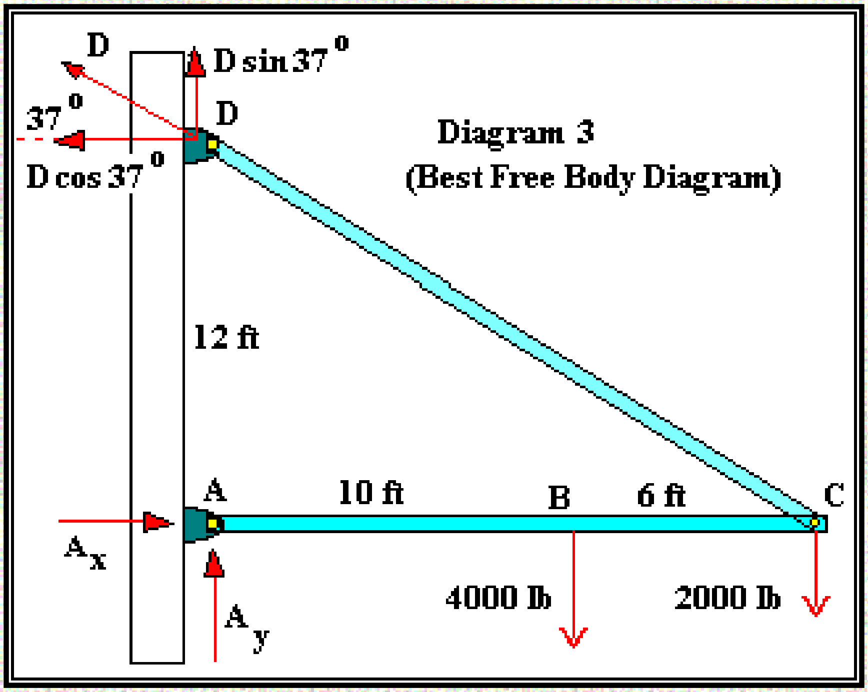
In our example, the load forces are already shown by the downward arrows. We next look at the forces exerted on the structure by the supports. Since each support is a pinned joint, the worst case we could have is an unknown x and y-force acting on the structure at each support point. We also must choose directions for the x and y support forces. In some problems the directions of the support forces are clear from the nature of the problem. In other problems the directions the support forces act is not clear at all. However, this is not really a problem. We simple make our best guess for the directions of the support reactions. If our guess is wrong, when we solve for the value of the support forces, that value will be negative. **This is important. A negative value when solving for a force does not mean the force necessarily acts in the negative direction, rather it means that the force acts in the direction OPPOSITE to the one we initially chose**.

Thus, in our first FBD on the right (Diagram 2), we have shown unknown x and y support forces acting on the structure at each support point.



This is an accurate FBD, but it is not th $\sum Fx = 0; \; \sum Fy = 0; \; \sum \tau_P = 0$ problem, we have three equilibrium conditions ( ), but we have four unknowns (**A_x, A_y, D_x, D_y**) in this FBD. And as we are well aware, we can not solve for more unknowns than we have independent equations.

We can draw a better FBD by reflecting on the concept of axial and non-axial members. Notice in our structure that member ABC is a non-axial member (since forces act on it at more than two points), while member CD is an axial member (since if we drew a FBD of member CD we would see forces act on it at only two points, D and C). This is important. Since CD is an axial member the force acting on it from the wall (and in it) must act along the direction of the member. This means that at point D, rather than having two unknown forces, we can draw one unknown force acting at a known angle (force D acting at angle of 37°, as shown in Diagram 3). This means we have only three unknowns, A_x, A_y, and D. In Diagram 3, we have also completed Step II, breaking any forces not in the x or y-direction into x and y-components. Thus, in Diagram 3, we have shown the two components of D (which act at 37°), D cos 37° being the x-component, and D sin 37° being the y-component. [**Please notice that there are not three forces at point D, there is either D acting at 37° or its two equivalent components, D cos 37° and D sin 37°. In Diagram 3 at this point we really should cross out the D force, which has been replaced by its components.**]



Now before we proceed with the final step and determine the values of the support reactions, we should deal with several conceptual questions which often arise at this point. First, why can't we do at point A what we did at point D, that is put in one force acting at a known angle. Member ABC is a horizontal member, doesn't the wall just push horizontally on member ABC, can't we just drop the A_y force? The answer is NO, because member ABC is not an axial member, it is not simply in compression or tension, and the wall does not just push horizontally on member ABC (as we will see in our solution). Thus the best we can do at point A is unknown forces A_x and A_y.

A second question is often, what about the wall, aren't there forces acting on the wall that we should consider? Well, yes and no. YES, there are forces acting on the wall (as a matter of fact they are exactly equal and opposite to the forces acting on the members, in compliance with Newton's Third Law). But NO we should not consider them, because we are making a FBD of the STRUCTURE, not of the wall, so we want to consider forces which act on the structure due to the wall, not forces on the wall due to the structure.

Now **Step III.** Apply the Equilibrium conditions. $\sum Fx = 0: \quad \sum Fy = 0: \quad \sum \tau_P = 0$

$$\sum Fx = 0: \quad Ax - D\cos 37° = 0$$

Here we sum the x-forces, keeping track of their direction signs, forces to right, +, to left, -

$$\sum F_y = 0: \quad A_y + D \sin 37^\circ - 4000\ \text{lb} - 2000\ \text{lb} = 0$$

Sum of y-forces, including load forces. Again keeping track of direction signs.

$$\sum \tau_A = 0: \quad D \cos 37^\circ (12\text{ft}) - 4000\ \text{lb}\ (10\text{ft}) - 2000\ \text{lb}\ (16\text{ft}) = 0$$

Sum of Torque about a point. We choose point A. Point D is also a good point to sum torque about since unknowns act through both points A and D, and if a force acts through a point, it does not produce a torque with respect to that point. Thus our torque equation will have less unknowns in it, and will be easier to solve. Notice that with respect to point A, forces A_x, A_y, and D sin 37º do not produce torque since their lines of action pass through point A. Thus in this problem the torque equation has only one unknown, D. We can solve for force D, and then use it in the two force equations to find the other unknowns, A_x and A_y. (Completing the calculations, we arrive at the following answers.) **D = +7500 lb. Ax = +6000 lb. Ay = +1500 lb**.

Note that all the support forces we solved for are positive, which means the directions we choose for them initially are the actual directions they act. We have now solved our problem. The support force at point D is 7500 lb. acting at 37º. The support forces at A can be left as the two components, Ax = 6000 lb. and Ay = 1500 lb., or may be added (as vectors) obtaining one force at a known angle, as shown in Diagram 4.

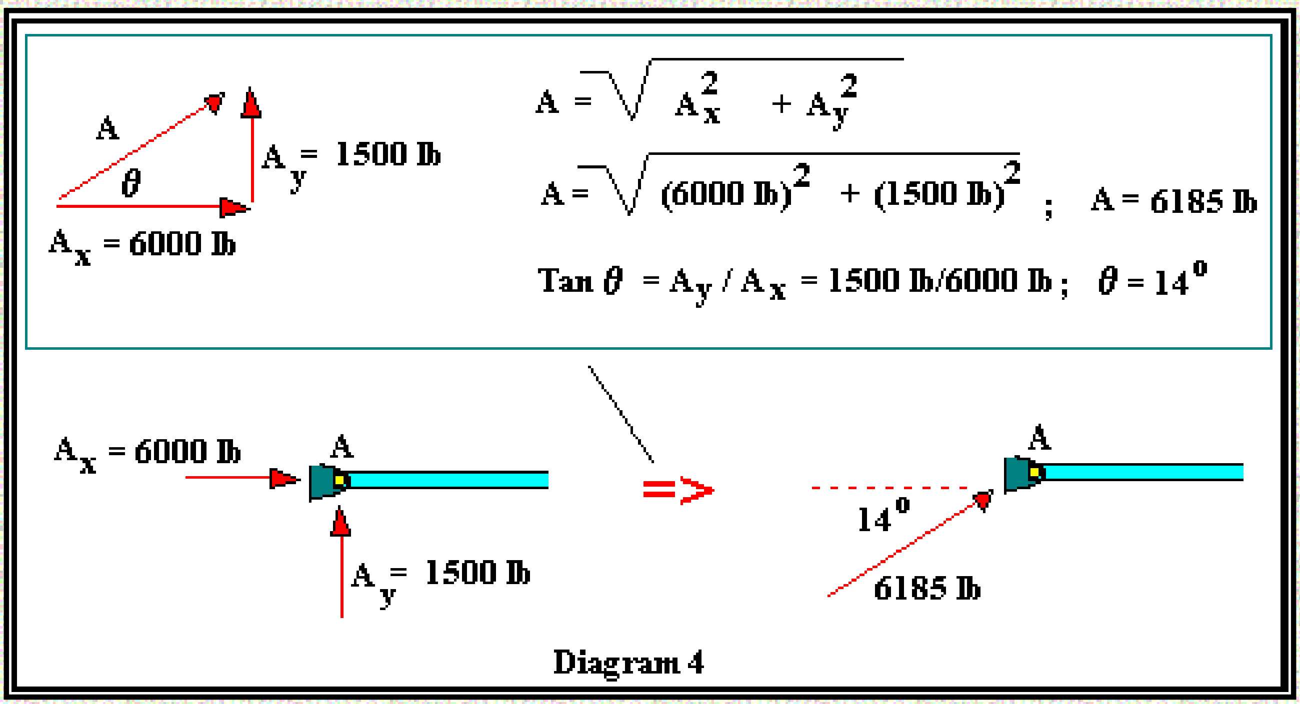


Diagram 4

Thus the force at point A is 6185 lb. acting at 14º, as shown. This information would help us purchase the correct size hinge (able to support 6185 lb. at A, and able to support 7500 lb. at D), or estimate if the wall is strong enough to support the structure. All very useful and interesting information.

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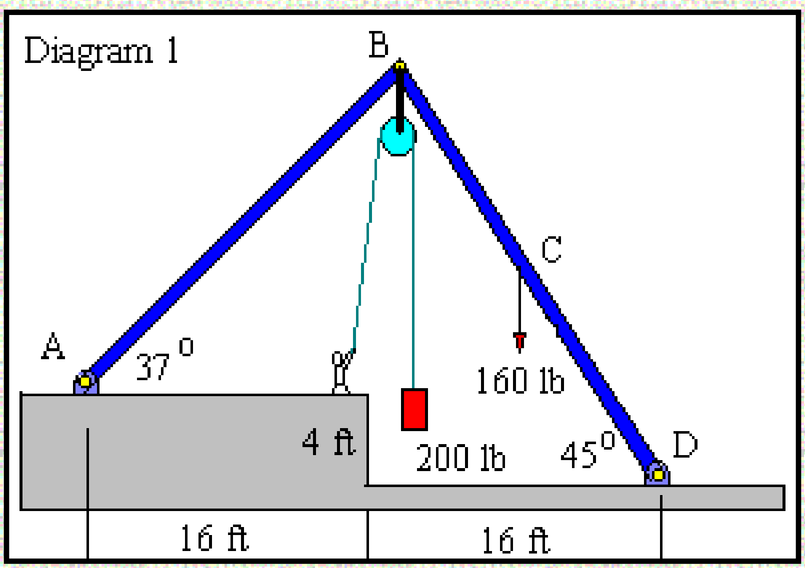
Example 2: Frame (non-truss, rigid body) Problems

Again in this second example, we will continue to proceed very carefully and methodically. This second example is reasonably straight forward, but points out some aspects of axial / non-axial forces, and torque. In this second example (Diagram 1, below) we will want to determine the value of the external support reactions. The general procedure to find the external support reactions consists of three basic steps.

- **I.** Draw a Free Body Diagram of the entire structure showing and labeling all external load forces and support forces, include any needed dimensions and angles.
- **II.** Resolve (break) all forces into their x and y-components.
- **III.** Apply the Equilibrium Equations ($\sum Fx = 0; \ \sum Fy = 0; \ \sum \tau_P = 0$) and solve for the unknown forces.

We observe, as shown in Diagram 1, that the structure is composed of members AB and BCD. These members are pinned together at point B, and are pinned to the floor at points A and D. Additionally, point B supports a pulley with which a person is hoisting a 200 lb. load. Member BCD has a weight of 160 lb., which may be considered to act at the center of member BCD.



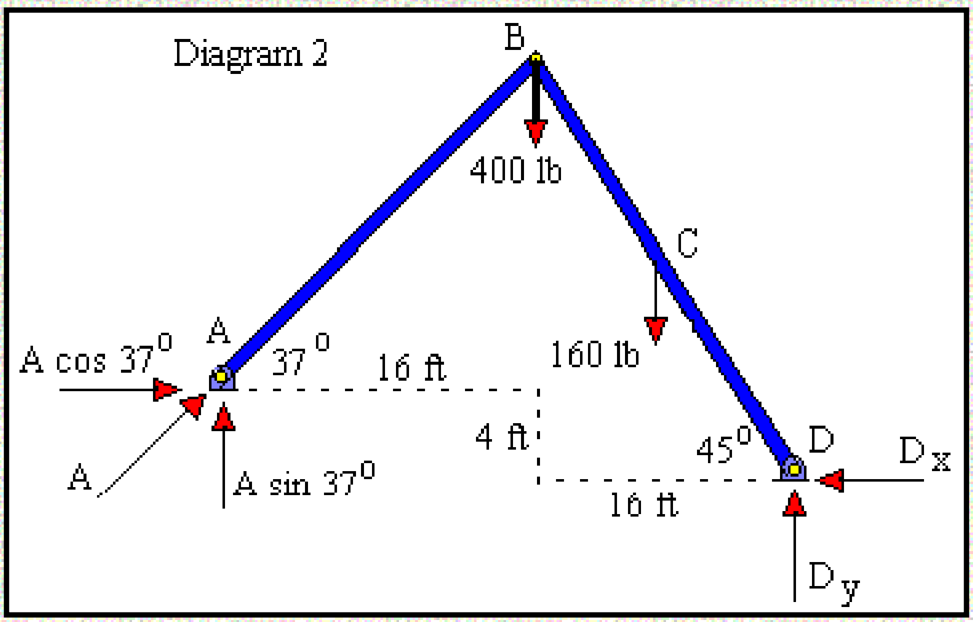
Step 1: Free Body Diagram (FBD).
We now proceed normally, that is we first draw our FBD (Diagram 2), showing and labeling all loads and support reactions acting on the structure. As we do so we will note several items: that member AB is an axial member (only in tension or compression), and that the person / load / pulley combination at point B produces a net 400 lb. downward load at point B. (**This results since both sides of the rope have 200 lb. force in them - one side due to the load, and the other**

side due to the pull of the person. Point B must support both the load and the pull of the person which results in a total force of 400 lb. acting on point B)

In the FBD (Diagram 2), at point A we have shown one unknown support force 'A' acting at a known angle (37º). We can do this at point A since we know member AB is an axial member. In an axial member the force is along the direction of the member, thus the floor must exert a force on the member also along the direction of the member (due to equal and opposite forces principle). However, at point D, since member D is a non-axial member, the best we can do is to show an unknown D_x and D_y support forces acting on the structure at point D

[We simply make our best guess for the directions of the support reactions. If our guess is wrong, when we solve for the value of the support forces, that value will be negative - indicating our original direction was incorrect].

In Diagram 2, we have also included **Step II: Resolve (break) any forces not in the x or y-direction into x and y-components.** Thus, we have shown the two components of A (which act at 37º) - A cos 37º being the x-component, and A sin 37º being the y-component. .



Step III. Apply the Equilibrium conditions.

$$\sum Fx = 0; \quad \sum Fy = 0; \quad \sum \tau_P = 0$$

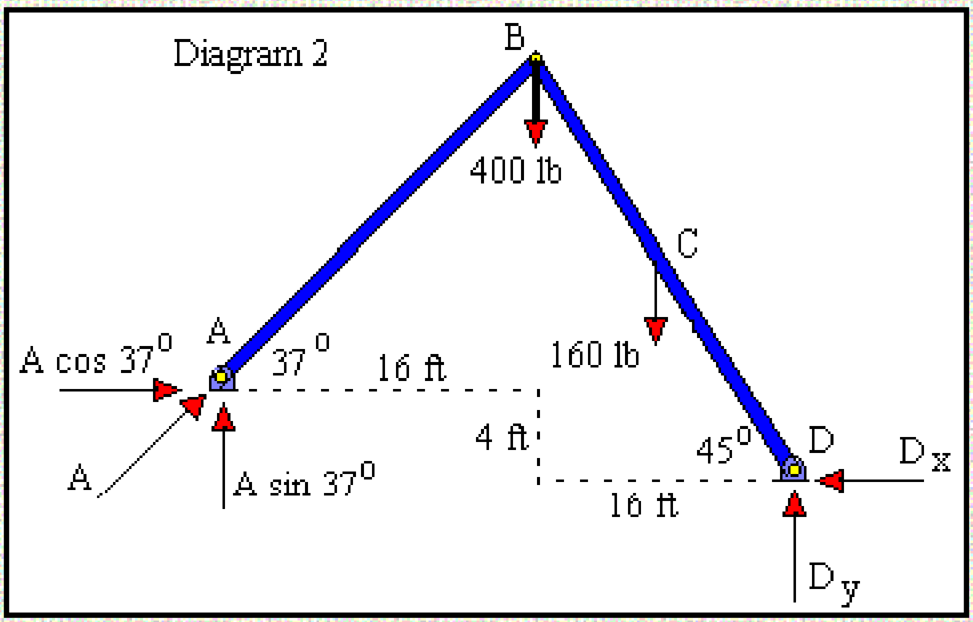
$$\sum Fx = 0: \quad A \cos 37° - D_x = 0$$

(Here we sum the x-forces, keeping track of their direction signs, forces to right, +, to left, -) $\sum Fy = 0: \quad A \sin 37° + D_y - 400 \text{ lb} - 160 \text{ lb} = 0$

(Sum of y-forces, including load forces. Again keeping track of direction signs.)

$$\sum \tau_D = 0: \quad -A\cos 37^\circ (4ft) - A\sin 37^\circ (32ft) + 400lb\,(16ft) + 160\,lb\,(8ft) = 0$$

Sum of Torque about a point. We chose Point D to calculate torque. Since two unknown forces (D_x, D_y) are acting at Point D, and if a force acts through a point, it does not produce a torque with respect to that point; thus our torque equation will have fewer unknowns in it, and will be easier to solve. We now proceed through the structure, looking a each force and calculating the torque due to that force with respect to the chosen Point D, and entering it in our torque equation (above) with the correct sign (+ for counterclockwise acting torque, - for clockwise acting torque). In this example, we must be careful to use the correct distance in the torque relationship - Torque = Force x Perpendicular Distance. (See Torque Review if needed.)



Finally, solving for our unknowns we obtain: **A = +343 lb. D_x = +274 lb. D_y = +354 lb**.

We observe that all the support forces we solved for are positive, which means the **directions we chose for them initially are the actual directions they act**. (Notice that means that A acts at 37º as shown, D_x act in the negative x-direction, and D_y acts in the positive y - direction.)

We have now solved our problem - finding the support reactions (forces) acting on the structure. We could add (as vectors) D_x & D_y to find one resultant force acting a some angle on point D, as follows:

D = Square Root [$Dx^2 + Dy^2$] = Square Root [$(-274\ lb)^2 + (354\ lb)^2$] = 447.7 lb.

Tangent (angle) = Dy / Dx = 354lb / -274 lb = -1.29, so Angle = ArcTangent (-1.29) = 127.8º (from x -axis)
so support force at Point D could also be expressed as: **D = 447.7 lb. @ 127.8º**

(Please note that the force at D does not act along the direction of member BCD, which it would do if BCD were an axial member.

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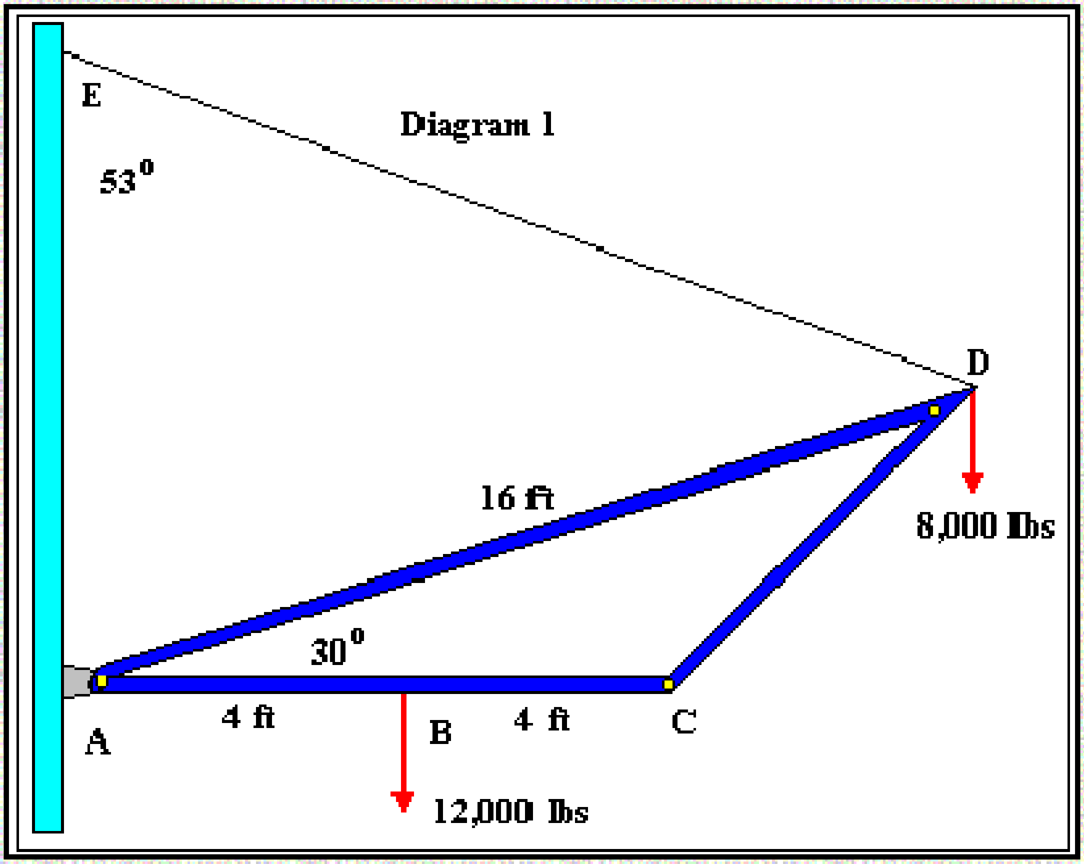
Example 3: Frame (non-truss, rigid body) Problems

This third example is somewhat similar to example one, but we will extend the problem by not only determining the external support reactions acting on the structure, but we will also determine the internal force in member CD of the structure shown in Diagram 1.

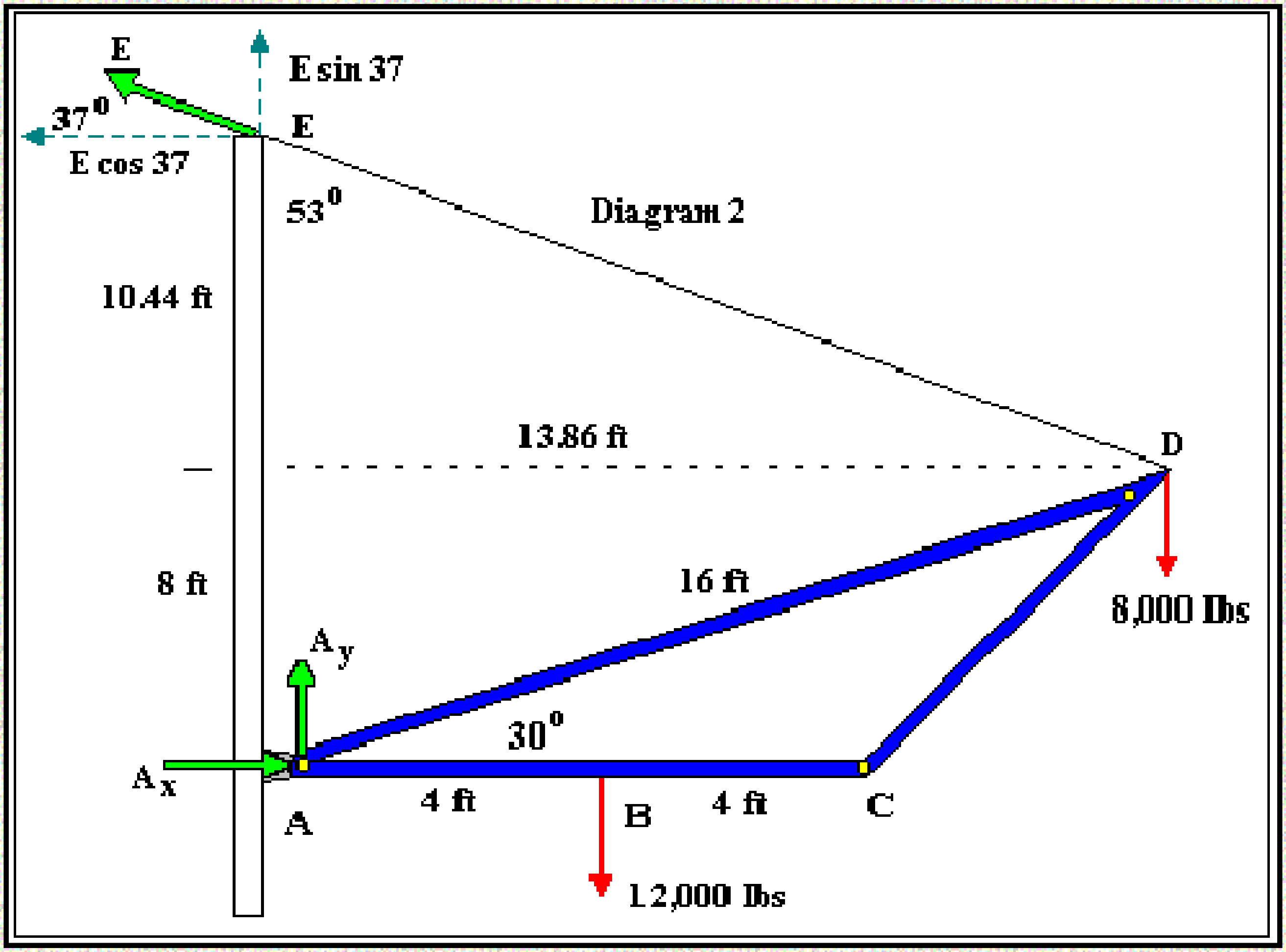
We first calculate the support forces. The procedure to find the external support reactions consists of our basic statics procedure..

I. Draw a Free Body Diagram of the entire structure showing and labeling all external load forces and support forces, include any needed dimensions and angles.
II. Resolve (break) all forces into their x and y-components.
III. Apply the Equilibrium Equations ($\sum Fx = 0; \quad \sum Fy = 0; \quad \sum \tau_p = 0$) and solve for the unknown forces.

We note that the structure is composed of members ABC, CD, AD, and cable DE. These members are pinned together at several points as shown in Diagram 1. A load of 12,000 lb. is acting on member ABC at point B, and a load of 8000 lb. is applied at point C. These forces are already shown by the downward arrows. We next look at the forces exerted on the structure by the supports. Since each support is a pinned joint, the worst case we could have is an unknown x and y-force acting on the structure at each support point. We also must choose directions for the x and y support forces. In some problems the directions of the support forces are clear from the nature of the problem. In other problems the directions the support forces act is not clear at all. However, this is not really a problem. We simply make our best guess for the directions of the support reactions. If our guess is wrong, when we solve for the value of the support forces, that value will be negative. **This is important. A negative value when solving for a force does not mean the force necessarily acts in the negative direction, rather it means that the force acts in the direction OPPOSITE to the one we initially chose**.



Step 1: **Draw a Free Body Diagram of the entire structure.** In the FBD (Diagram 2), we have shown unknown x and y support forces acting on the structure at point A, however, at point E we have shown one unknown force 'E' acting at a known angle (37º).



We can do this at point E since we know that ED is a cable, and a cable is an axial member which can only be in tension. Since the cable pulls axially on the wall, the wall pulls equally and in the opposite direction on the structure., as shown in Diagram 2.

In Diagram 2, we have also included **Step II, Resolve any forces not in the x or y-direction into x and y-components**. Thus, we have shown the two components of E (which act at 37º) - E cos 37º being the x-component, and E sin 37º being the y-component. We have also used given angles and dimensions to calculate some distance, as shown, which may be needed when we apply the equilibrium equations.

We also note that at point A we have two members pinned together to the wall, axial member AD, and non-axial ABC. Because of these two members (as opposed to a single axial member, such as at point E), the best we can do at point A is to replace the hinged joint by an unknown A_x and A_y support forces acting on the structure as shown in Diagram 2. However, we have a good FBD since we have only three unknown forces, and we have three independent equations from our equilibrium conditions.

Step III. Apply the Equilibrium conditions. $\sum Fx = 0; \ \sum Fy = 0; \ \sum \tau_P = 0$

$$\sum Fx = 0: \quad Ax - E\cos 37^\circ = 0$$

(Here we sum the x-forces, keeping track of their direction signs, forces to right, +, to left, -)

$$\sum Fy = 0: \quad Ay + E\sin 37^\circ - 12000\,\text{lb} - 8000\,\text{lb} = 0$$

(Sum of y-forces, including load forces. Again keeping track of direction signs.)

$$\sum \tau_A = 0: \quad E\cos 37^\circ(18.44\text{ft}) - 12000\,\text{lb}\,(4\text{ft}) - 8000\,\text{lb}\,(13.86\text{ft}) = 0$$

Sum of Torque about a point. We choose point A. Point E is also a good point to sum torque about since unknowns act through both points A and D, and if a force acts through a point, it does not produce a torque with respect to that point - thus, our torque equation will have less unknowns in it, and will be easier to solve. Notice that with respect to point A, forces A_x, A_y, and E sin 37º do not produce torque since their lines of action pass through point A. Thus, in this problem the torque equation has only one unknown - E. We can solve for E, and then use it in the two force equations to find the other unknowns, A_x and A_y. Doing the mathematics we arrive at the following answers.) **E = +10,800 lb. Ax = +8620 lb. Ay = +13500 lb**.

We see that all the support forces we solved for are positive, which means the directions we chose for them initially are the actual directions they act. We have now solved part one of our problem. The support force at point **E is 10,800 lb. acting at 37º**. The support force(s) at A can be left as the two components, **A_x = 8,620 lb. and A_y = 13,500 lb**., or may be added (as vectors) obtaining one force at a known angle.

The second part of the problem is to determine the force in axial member CD. (We know member CD is axial as there are only two points at which forces acts on CD, point C and point D.) To determine the force in an internal member of a structure we use a procedure similar to that used to find the external support reactions. That is, we draw a FBD, **not of the entire structure, but of a member of the structure**, (choosing not the member we wish to find the force in, but a member it acts on). Thus, if we wish to find the force in member CD, we draw a FBD - not of member CD, but a member CD acts on, such as member ABC, or member AD. In this example we will use member ABC to find the force in member CD.

To find the force in a member of the structure we will use the following steps:
First, determine the external support reactions acting on the structure (as we did in

the first part of this example). Then continue with steps below

- I. Draw a Free Body Diagram **of a member** of the structure showing and labeling all external load forces and support forces acting on that member, include any needed dimensions and angles. (The member selected should be one acted on by the member in which we wish to find the force.)
- II. Resolve (break) all forces into their x and y-components.
- III. Apply the Equilibrium Equations ($\sum Fx = 0; \sum Fy = 0; \sum \tau_P = 0$) and solve for the unknown forces.

Step 1: FBD of Member ABC. There are actually two good FBD for member ABC. In Diagram 3 we have shown the first of these. Notice at the left end we have shown both the wall support reactions at A, and also the force from axial member AD which acts on member ABC. At point C we have shown the force from axial member CD which acts on member ABC. That is, we have isolated member ABD and indicated the forces on it due to the other members (and the wall) attached to it.

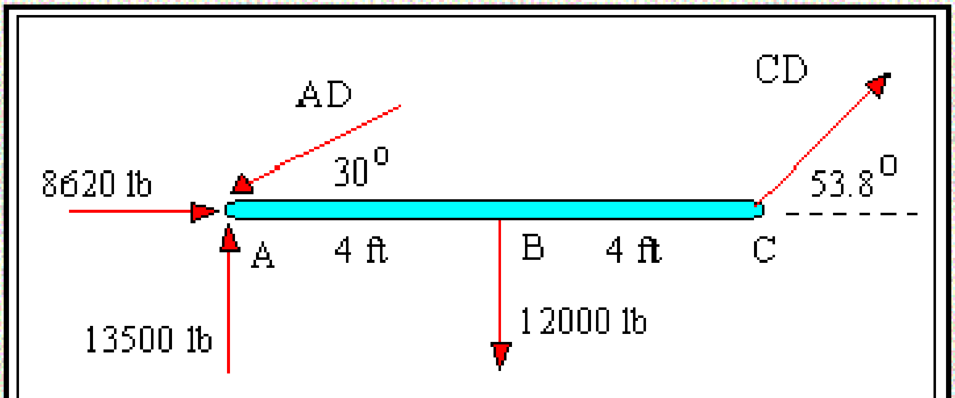
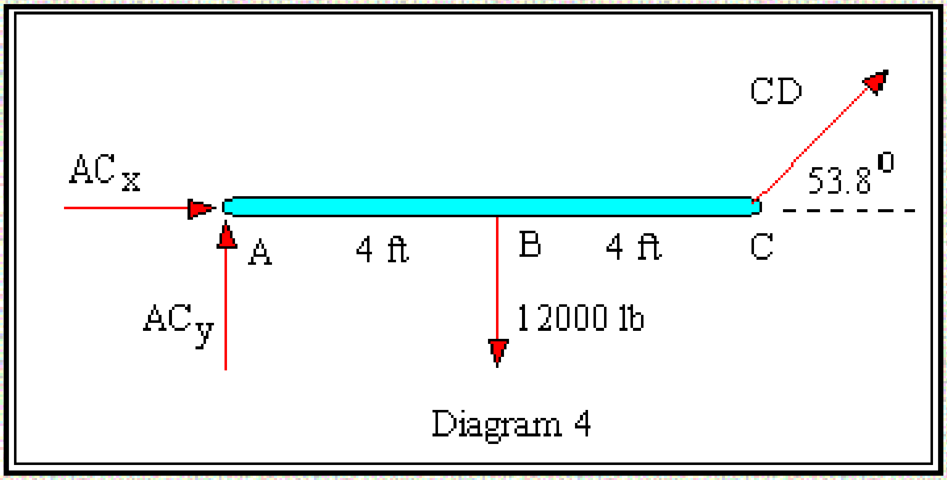
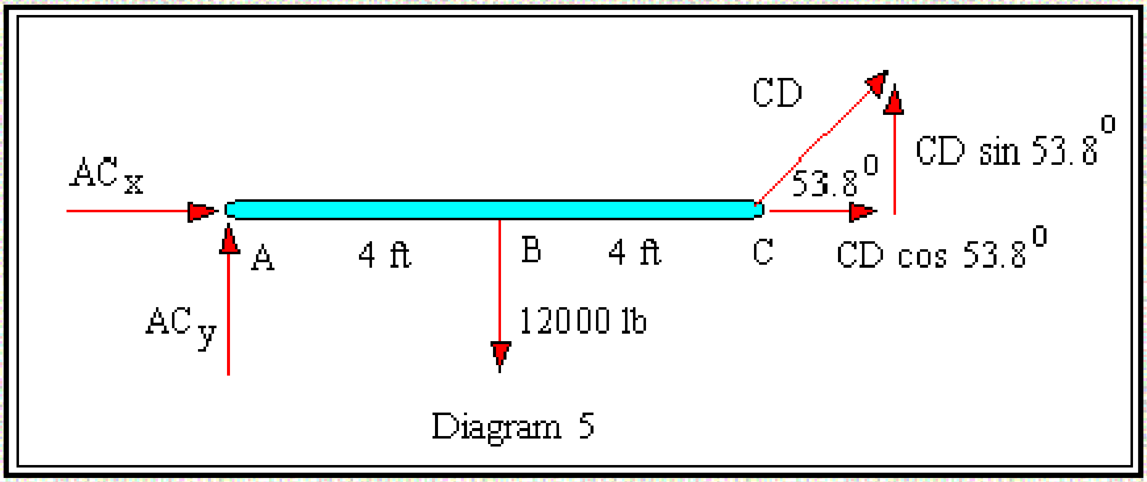


Diagram 3

The second good FBD of member ABC is shown in Diagram 4. What we have done in this diagram is to look more closely at the left end of member ABC and observe that the effect of the wall forces and the effect of member AD, is to give some net x and y-force acting on member ABC. Thus, rather than show both the wall forces and the force due to AD on ABC, we simple show an AC_x and an AC_y force which is the net horizontal and vertical force acting on ABC at the left end. This is fine to do, as we are looking for force CD, and that is still present in our FBD. This second FBD is slightly easier than the first in that it will result in one less force (AD) in the equilibrium equations. We will use the second FBD in the rest of the problem.



Step 2: Resolve forces into their x and y-components (This is done in Diagram 5.) Notice we have chosen directions earlier for the forces. These may not be the correct directions, but our solution will tell us if we have the right or wrong directions for the unknown forces.



Step 3: Apply the equilibrium conditions and solve for unknown forces.

Sum F_x: AC_x + CD cos 53.8º = 0

Sum F_y: AC_y -12,000 lb. + CD sin 53.8º = 0

Sum T_A: -12,000 lb. (4 ft) + CD sin 53.8º (8 ft) =0

Solving: CD = 7,440 lb., AC_x = - 4390 lb. (- sign shows force acts opposite direction chosen), AC_y = 6000 lb.

We have determined the force in CD to be 7,440 lb. Since the force is positive, this indicates that we have chosen the correct direction for the force CD (which indicates it is in tension). This solves our problem. (For a little further analysis of forces at point A, select MORE.)

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Point A: As an aside, notice that the forces AC_x and AC_y (The horizontal and vertical forces acting on member ABC at end A.) are not the same as the forces A_x and A_y acting on the entire structure at joint A. This results since the forces of the wall at point A are not just acting on member ABC, but are distributed to both members ABC and AD, as shown in Diagram 6. Note in the diagram that if the forces AC_x and AD_x are summed (13010 lb. - 4390 lb. = 8620 lb.), and if forces AC_y and AD_y are summed (6000 lb. + 7500 lb. = 13500 lb.), that their vector sums equal the external forces (A_x and A_y) acting on point A, as we expect they should.

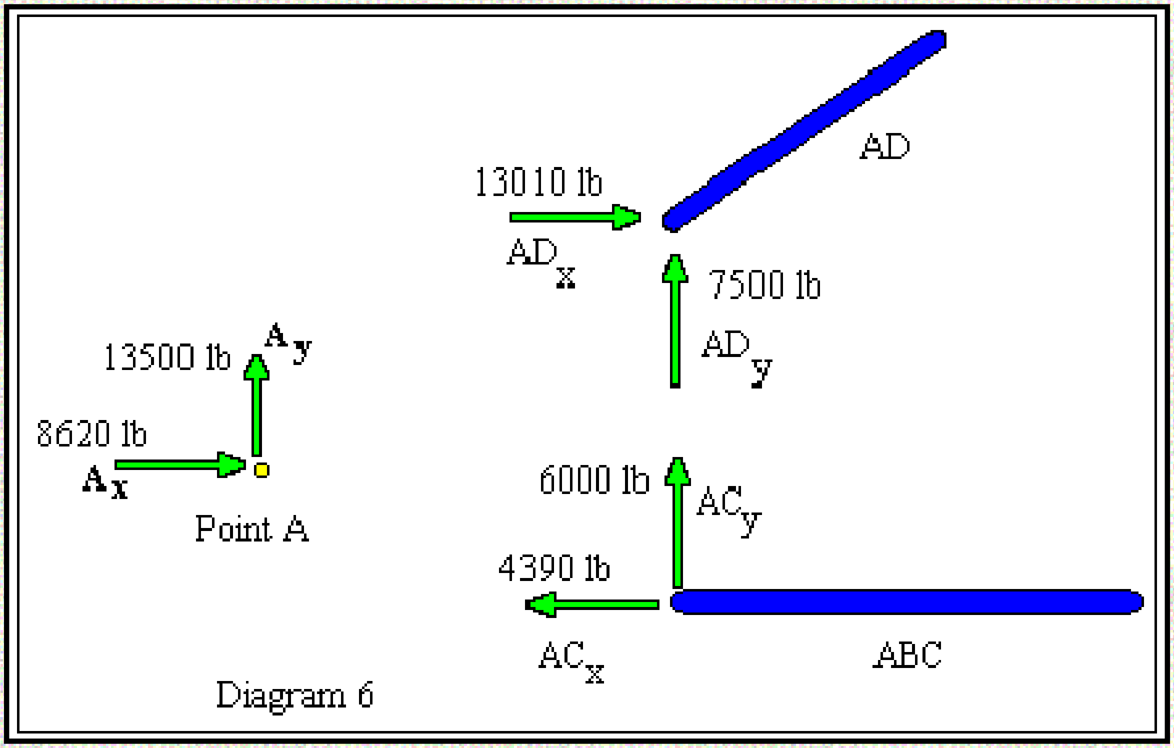

Diagram 6

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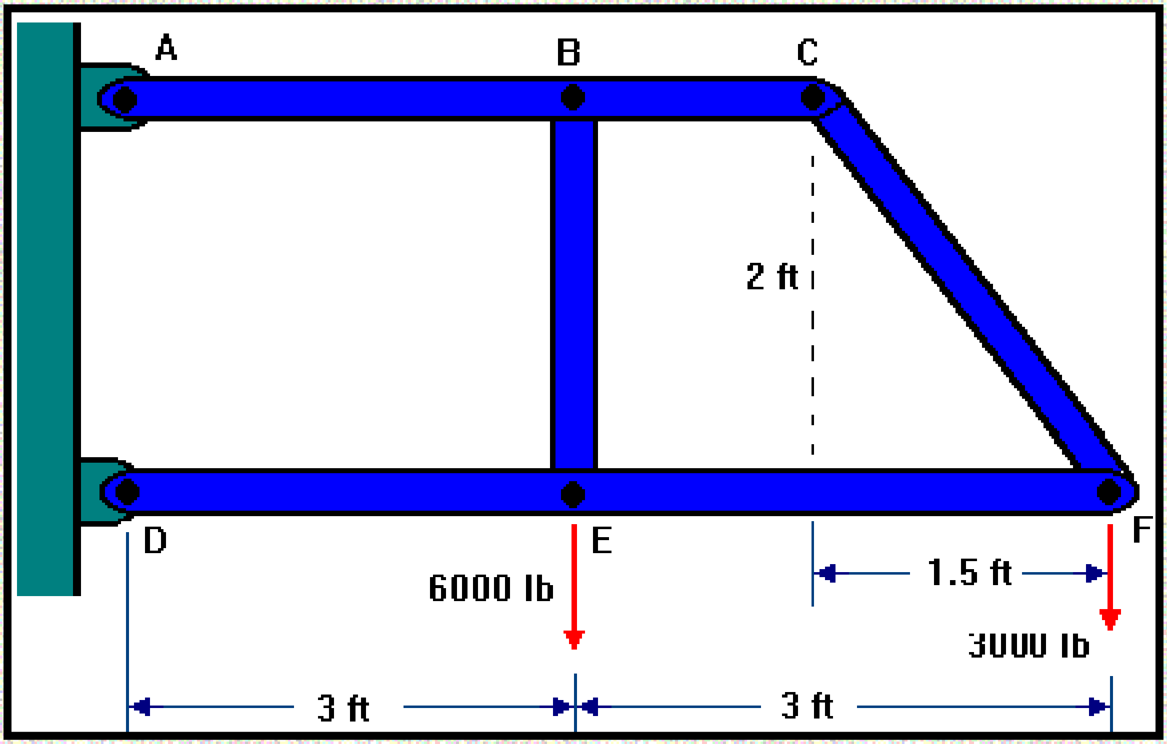
Example 4: Frame (non-truss, rigid body) Problems

In our fourth example, we will examine a problem in which it will initially seem that there are too many unknowns to allow us to determine the external support forces acting on the structure, however, by a slight variation of our approach we will find that we can determine all the unknowns. The problem then is this, for the structure shown in Diagram 1 below, determine the external support forces acting on the structure, and additionally, determine the force in member CF. Our usual procedure to find the external support reactions consists of three basic steps.

- **I.** Draw a Free Body Diagram of the entire structure showing and labeling all external load forces and support forces, include any needed dimensions and angles.
- **II.** Resolve (break) all forces into their x and y-components.
- **III.** Apply the Equilibrium Equations ($\sum Fx = 0;\ \sum Fy = 0;\ \sum \tau_A = 0$) and solve for the unknown forces.

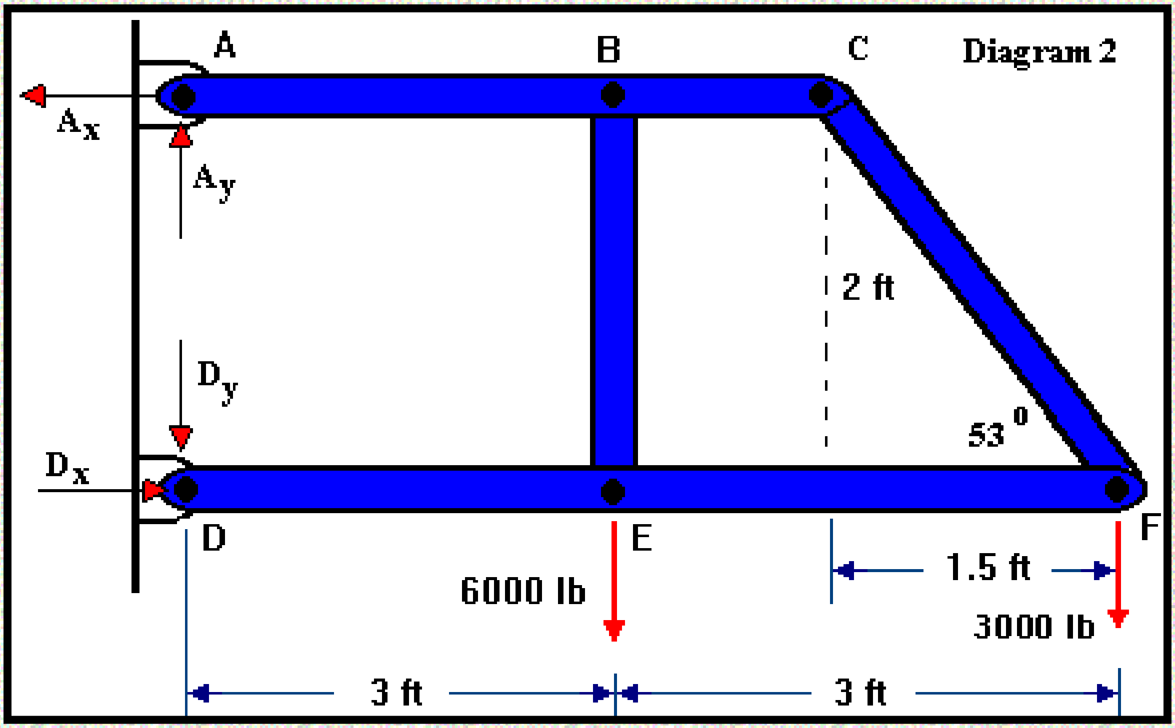
We note that the structure is composed of members ABC, DEF, DE, and CF. These members are pinned together at several points as shown in Diagram 1. A load of 6000 lb. is acting on member DEF at point E, and a load of 3000 lb. is applied at point F. These forces are already shown by the downward arrows. We next look at the forces exerted on the structure by the supports. Since each support is a pinned joint, the worst case we could have is an unknown x and y-force acting on the structure at each support point. We also must choose directions for the x and y support forces. In some problems the directions of the support forces are clear from the nature of the problem. In other problems the directions the support forces act is not clear at all. However, this is not really a problem. We simple make our best guess for the directions of the support reactions. If our guess is wrong, when we solve for the value of the support forces, that value will be negative.



This is important. A negative value when solving for a force does not mean the force necessarily acts in the negative direction, rather it means that the force acts in the direction OPPOSITE to the one we initially chose.

Step 1: Free Body Diagram (FBD).

In the FBD (Diagram 2), we have shown unknown x and y support forces acting on the structure at pinned support points A and D, (A_x, A_y, D_x, D_y). If we think ahead somewhat, we realize that there could be a problem. We have four unknown forces supporting the structure, but there are only three equations in our Equilibrium Conditions, $\sum Fx = 0;\ \sum Fy = 0;\ \sum \tau_A = 0$.



Normally, one can not solve for more unknowns then there are independent equations. Our first reaction should be to see if we can draw a better FBD. Perhaps we can replace the two unknowns at either point A or D by one unknown acting at a known angle (which is possible if we have a single axial member acting at the support point). However in this case both member ABC and member DEF are non-axial members, and the forces in them (and on them from the wall) do not act along the axis of the member. Thus, we already have the best FBD possible - we can not reduce the number of external unknowns acting on the structure.

At this point we will simply continue with our normal analysis procedure and see what results.

Step II: Resolve any forces not in the x or y-direction into x and y-components. (All forces are already in either the x or y-direction.

Step III. Apply the Equilibrium conditions.

$$\sum Fx = 0; \quad \sum Fy = 0; \quad \sum \tau_A = 0$$

$\sum Fx = 0: \quad -Ax + Dx = 0$ **(Here we sum the x-forces, keeping track of their direction signs, forces to right, +, to left, -)**

$\sum Fy = 0: \quad Ay - Dy - 6000\text{ lb} - 3000\text{ lb} = 0$ **(Sum of y-forces, including load forces. Again keeping track of direction signs.)**

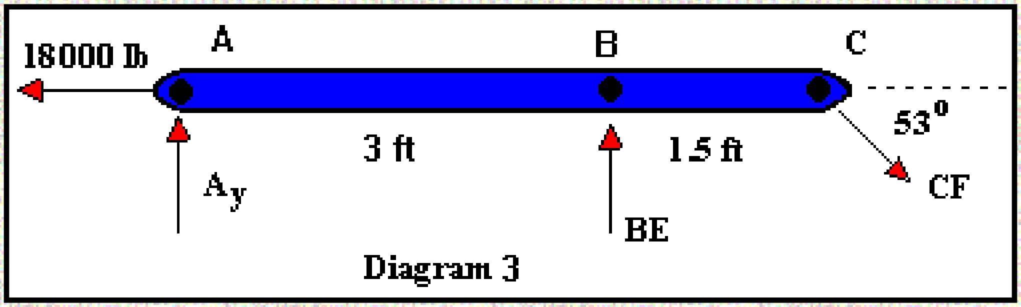
$\sum \tau_D = 0: \quad Ax\,(2\text{ ft}) - 6000\text{ lb}\,(3\text{ft}) - 3000\text{ lb}\,(6\text{ft}) = 0$ **Sum of Torque about a**

point. We choose point D. Forces A_y, D_x and D_y do not produce torque since their lines of action pass through point A. Thus, the torque equation has only one unknown, A_x. We solve for A_x, and then use it in the sum of forces in the x-direction equation to find the unknown, D_x . And if we do so, we find: A_x = +18000 lb. D_x = +18000 lb. (The positive signs indicate we initially chose the correct direction for the forces.) However, please notice that while we found Ax and Dx, we can not find Ay and Dy. There are still two unknowns in the y-equation and not enough information to determine then at this point. Thus, analysis of the structure as a whole has enabled us to determine several of the external support forces, but not all of them. What now?

First, an overview. There are problems for which the static equilibrium conditions are not enough to enable one to solve the problem. They are called **Statically Indeterminate Problems**, and we will be considering these a bit later. Then there are problems which, on first glance, appear to be statically indeterminate, but are not. That is the case here. To find the remaining unknown support forces (and at the same time, determining the force in member CF), we will now take out a member of the structure and apply our statics analysis procedure to the selected member of the structure (rather than the entire structure). We will select member ABC to analyze. (See Diagram 3)

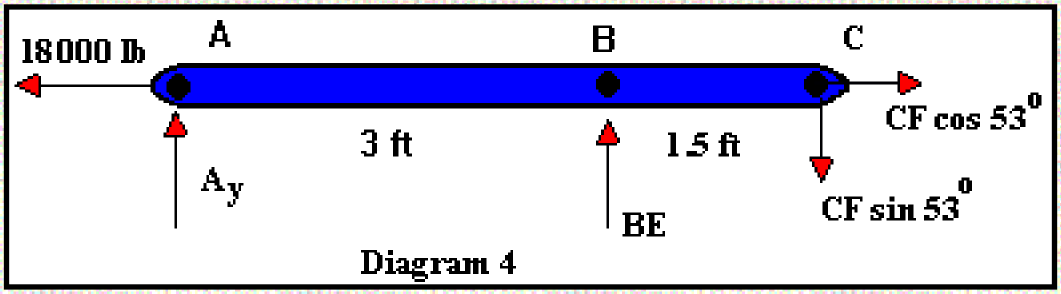
Step 1: Free Body Diagram (FBD)

In Diagram 3, we have drawn a FBD of member ABC, showing and labeling all forces external to member ABC which act on it. That is at point A we have the forces of the wall acting on ABC, at point B we have an axial force on ABC due to member BE, and at point C we have a axial force on ABC due to the member CF. Both member BE and CF are axial members and so we know the directions their forces act - along their axis. We do have to guess if they push or pull on member ABC, and we have chosen those directions as shown, (if the direction chosen is wrong, the force's value will be negative when we solve). We also are happy with the FBD as it has only three unknowns acting on the member, which indicates that we should be able to solve completely for the unknowns.



Step 2: Resolve forces into x/y components.

Here we have resolved force CF into its horizontal and vertical components as shown in Diagram 4. All other forces are already in x or y-direction.



Step 3. Apply the Equilibrium conditions.

$$\sum Fx = 0; \quad \sum Fy = 0; \quad \sum \tau_A = 0$$

$$\sum Fx = 0: \quad -18{,}000\ \text{lb} + CF\cos 53^\circ = 0$$

$$\sum Fy = 0: \quad Ay + BE - CF\sin 53^\circ = 0$$

$$\sum \tau_A = 0: \quad BE\,(3\ \text{ft}) - CF\sin 53^\circ\,(4.5\text{ft}) = 0$$

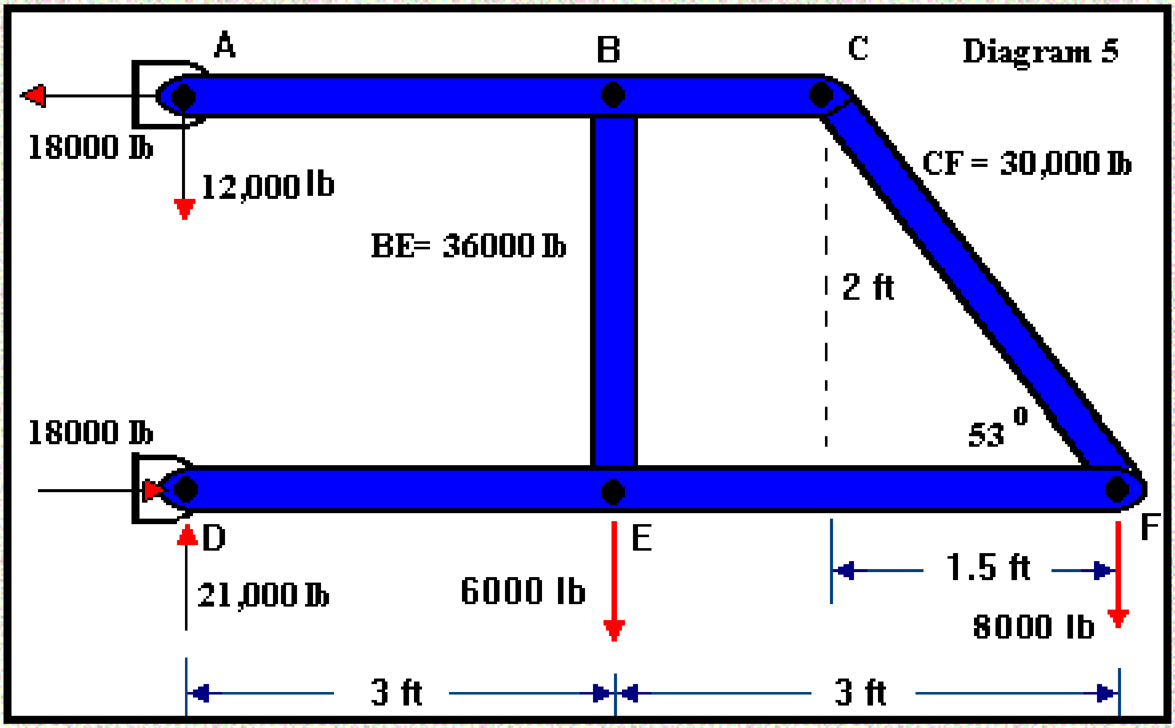
We now solve the first equation for CF, then use that value in the last equation to find BE, and use both values in the middle equation to find A_y, giving us**: CF = 30,000 lb., BE = 36,000 lb., Ay = -12, 000 lb.** Please note that the value of A_y is negative, which indicates the direction we chose was incorrect. A_y acts downward rather than in the positive y-direction we initial chose. Additionally, we now can return to the equations for the entire structure (see below), and knowing the value for A_y, we can use it in the y-forces equation to solve for the value of D_y.

Equilibrium Conditions for entire structure (from first part of problem)

- $\sum Fx = 0: \quad -Ax + Dx = 0$
- $\sum Fy = 0: \quad Ay - Dy - 6000\ \text{lb} - 3000\ \text{lb} = 0$
- $\sum \tau_D = 0: \quad Ax\,(2\ \text{ft}) - 6000\ \text{lb}\,(3\text{ft}) - 3000\ \text{lb}\,(6\text{ft}) = 0$

From the y-forces equation: (-12,000 lb.) - D_y -6000 lb. -3000 lb = 0;

Solving **D_y = -21,000 lb.**



Once again, the negative sign indicates we selected the wrong direction for D_y, rather than acting downward it actually acts upward. See Diagram 5 for final force values and directions.

A_x = 18000 lb.

A_y = 12,000 lb. D_x = 18000 lb. D_y = 21,000 lb. CF = 30,000 lb. BE = 36,000 lb.

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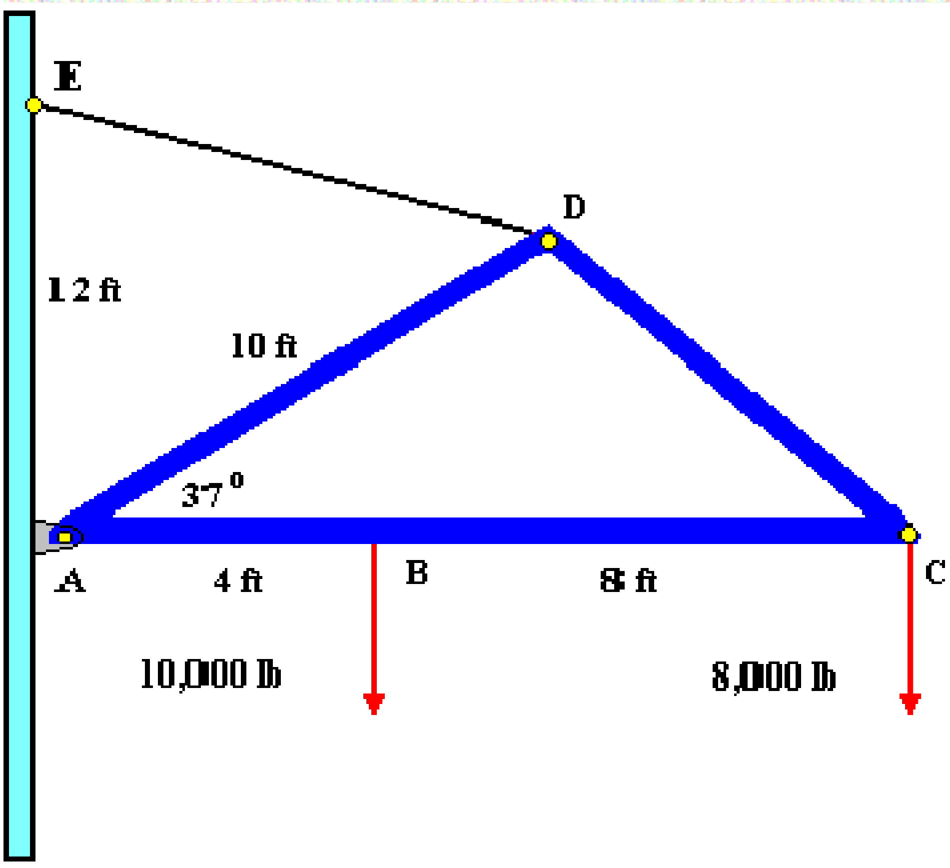
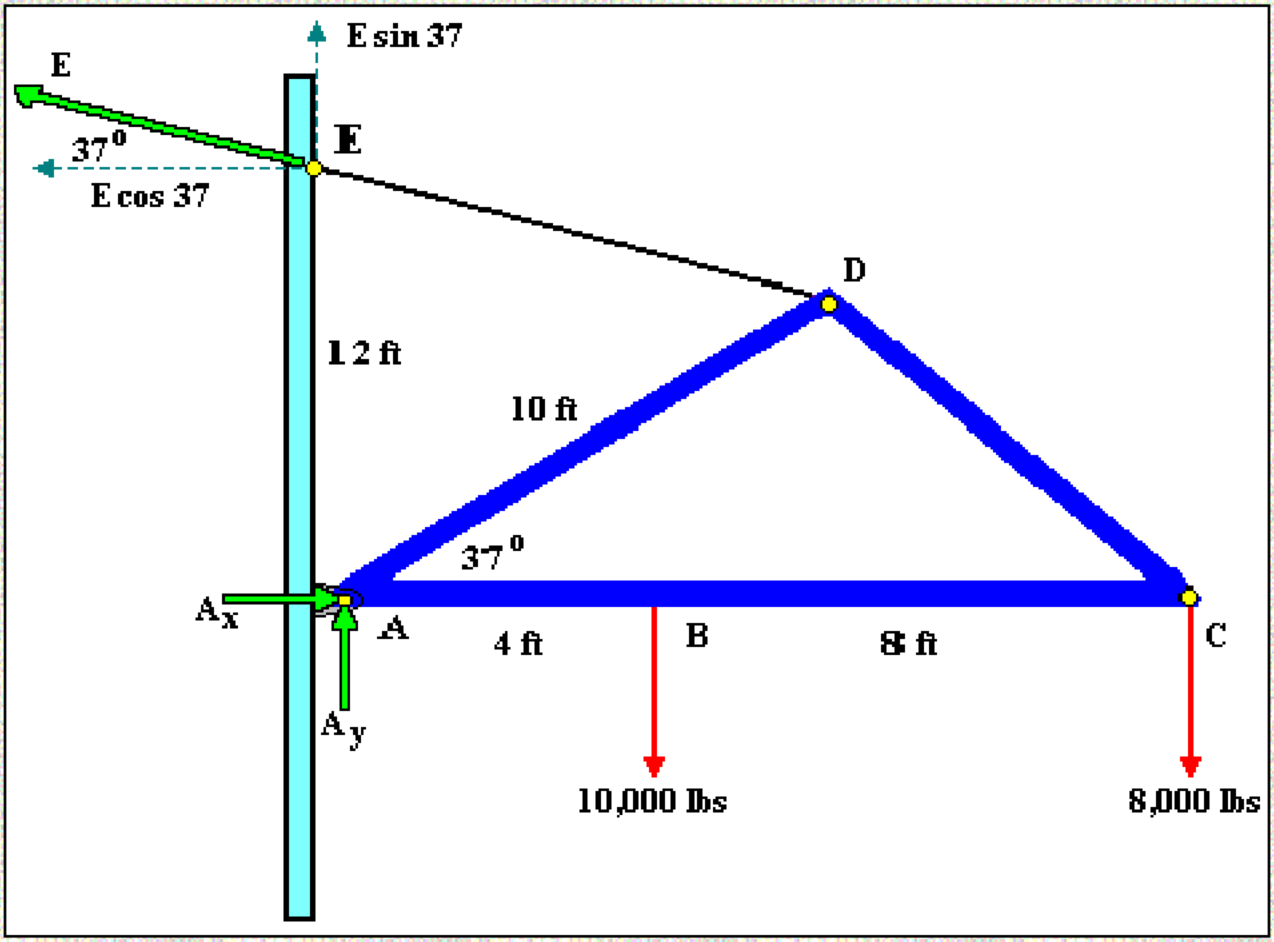
STATICS / STRENGTH OF MATERIALS - Example

In the structure shown below members AD, DC, and ABC are assumed to be solid rigid members. Member ED is a cable. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the values of all the support forces acting on the structure.
C. Determine the force (tension or compression) in member DC.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
PARTS A & B:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

Sum $F_x = -E\cos(37^o) + A_x = 0$

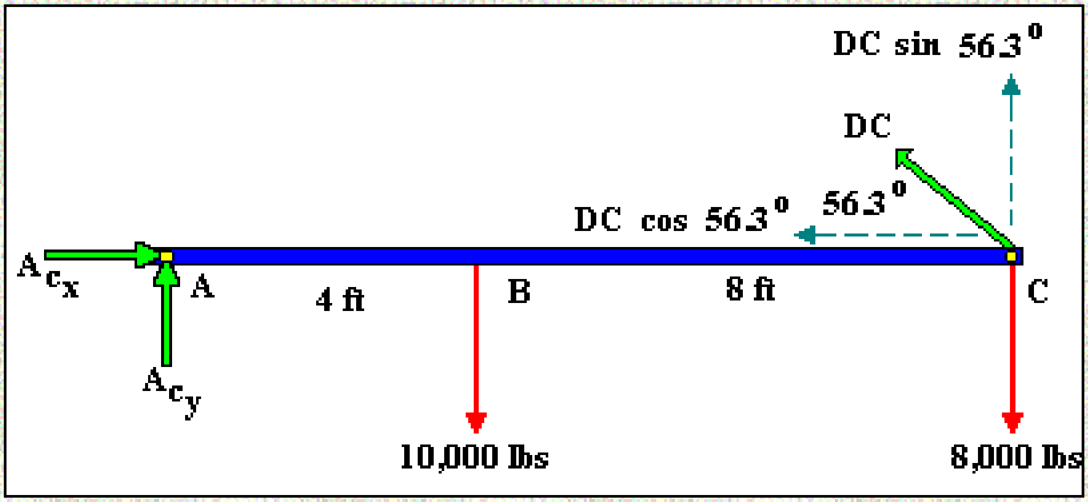
Sum $F_y = A_y + E\sin(37^o) - 10{,}000\text{ lbs} - 8{,}000\text{ lbs} = 0$

Sum $T_A = E\cos(37^o)(12\text{ft}) - (10{,}000\text{ lbs})(4\text{ ft}) - (8{,}000\text{ lbs})(12\text{ft}) = 0$

Solving for the unknowns: E = 14,200 lbs; A_y = 9,480 lbs; A_x = 11,400 lbs

PART C: Now find internal force in member DC

STEP 1: Draw a free body diagram of a member that DC acts on - member ABC.



STEP 2: Resolve all forces into x and y components (see diagram).

STEP 3: Apply the equilibrium conditions.

Sum $F_x = A_{cx}$ - DC cos (56.3º) = 0

Sum $F_y = A_{cy}$ - 10,000 lbs - 8,000 lbs + DC sin (56.3º) = 0

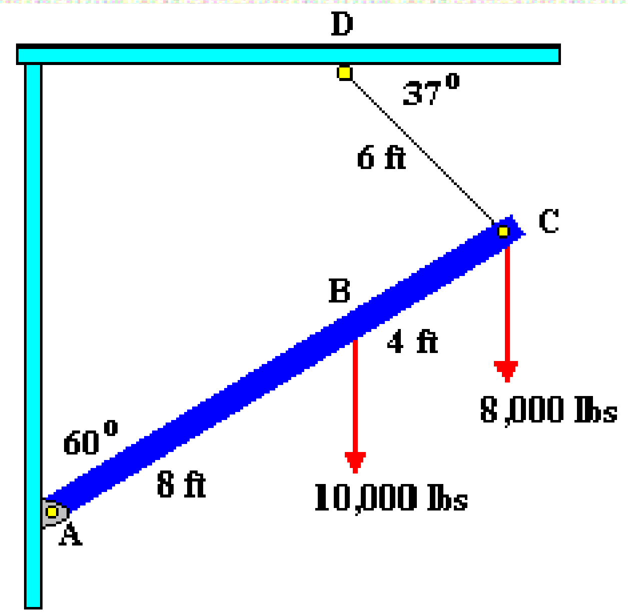
Sum T_A = (-10,000 lbs)(4 ft) - (8,000 lbs)(12 ft) +DC sin (56.3º)(12 ft) = 0

Solving for the unknowns: DC = 13,600 lbs; A_{cx} = 7,560 lbs; A_{cy} = 6,670 lbs

These are external forces acting on member ABC. The force in DC is 13,600 lbs (c).

STATICS / STRENGTH OF MATERIALS - Example

In the structure shown below member ABC is assumed to be a solid rigid member. Member CD is a cable. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force (tension) in member CD.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

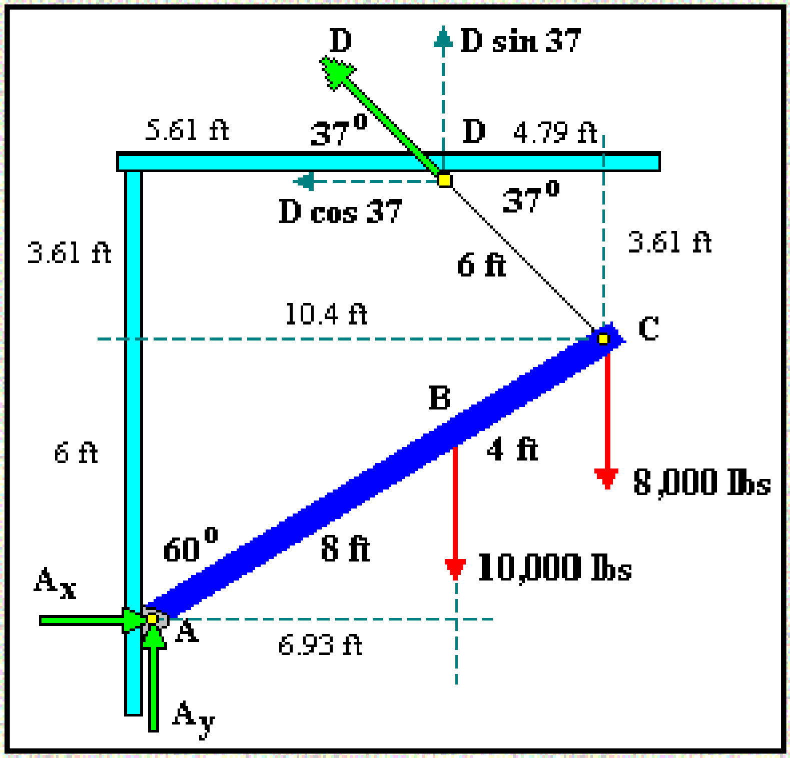


Solution:

PARTS A & B

STEP 1: Draw a free body diagram showing and labeling all load forces and

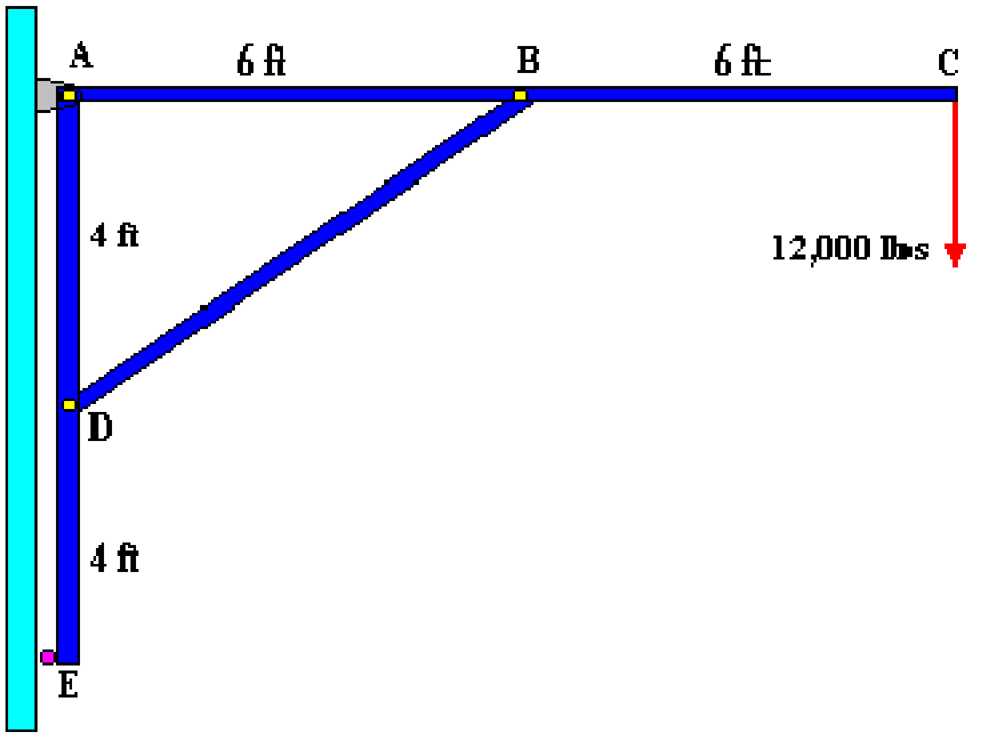
support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.
Sum F_x = -D cos (37^o) + A_x = 0
Sum F_y = D sin (37^o) + A_y - 10,000 lbs - 8,000 lbs = 0
Sum T_A = (-10,000 lbs)(6.93 ft) - (8,000 lbs)(10.4 ft) + D sin (37^o)(5.61 ft) + D cos (37^o)(9.61 ft) = 0
Solving for the unknowns:
D = 13,800 lbs; A_x = 11,100 lbs; A_y = 9,710 lbs



PART C - Now find internal force in member DC.
In this problem member DC is a single axial member connected to the wall at point D. Therefore, the force in member DC is equal and opposite the force exerted on DC by the wall. From parts A and B, the force on DC due to the wall is 13,800 lbs. Therefore, force in DC is also 13,800 lbs (in tension since DC is a cable).

STATICS / STRENGTH OF MATERIALS - Example

In the structure shown below members ABC, ADE, and DB are assumed to be solid rigid members. Members ABC and ADE are pinned to the wall at point A. Member ADE is supported by a roller at point E. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force (tension or compression) in member DB.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

PARTS A & B

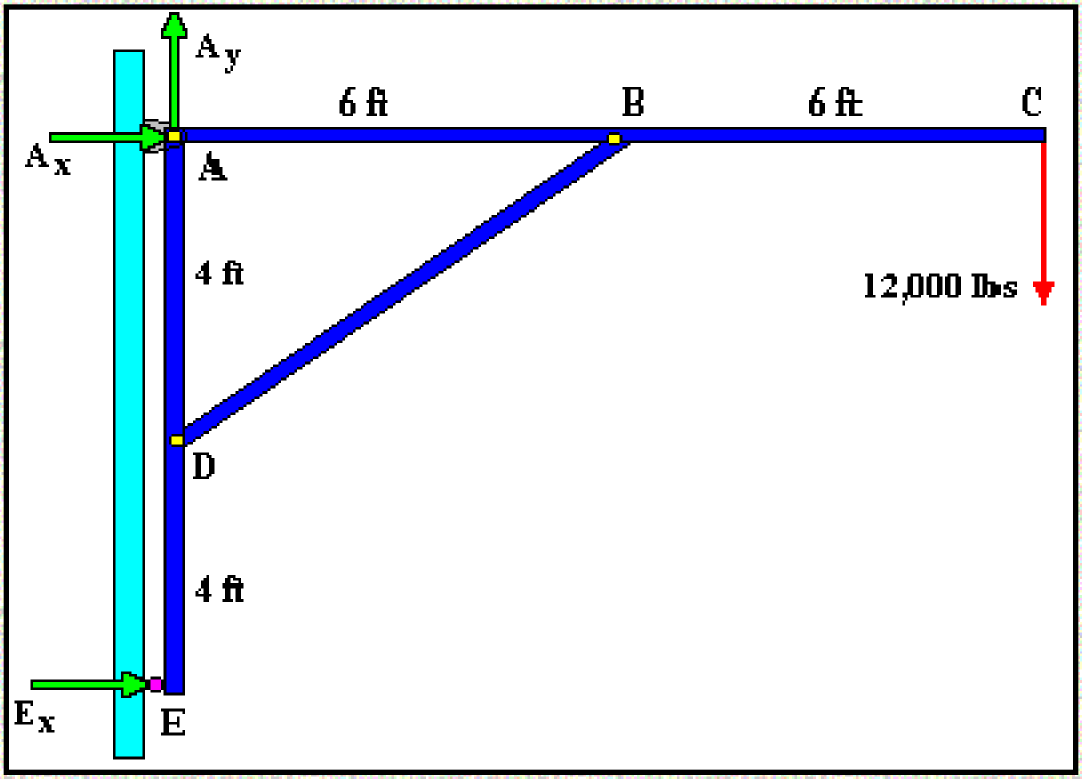
STEP 1: Draw a free body diagram showing and labeling all load forces and

support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.
Sum $F_x = E_x + A_x = 0$

Sum $F_y = A_y$ - 12,000 lbs = 0

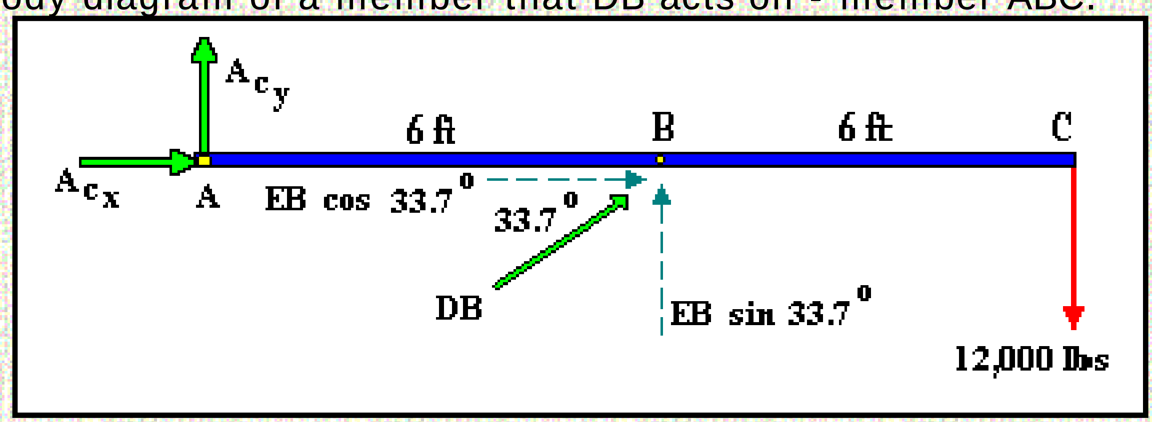
Sum $T_A = E_x$(8 ft) - (12,000 lbs)(12 ft) = 0

Solving for the unknowns:
E_x= 18,000 lbs; A_x= -18,000 lbs; A_y = 12,000 lbs



PART C - Now find internal force in member DB

STEP 1: Draw a free body diagram of a member that DB acts on - member ABC.
STEP 2: Resolve all forces into x and y components (see diagram)
STEP 3: Apply the equilibrium conditions.
Sum $F_x = A_{cx}$ + DB cos (33.7°) = 0
Sum $F_y = A_{cy}$ + DB sin (33.7°) - 12,000 lbs = 0



Sum T_A = DB sin (33.7°)(6 ft) - (12,000 lbs)(12 ft) = 0

Solving for the unknowns:

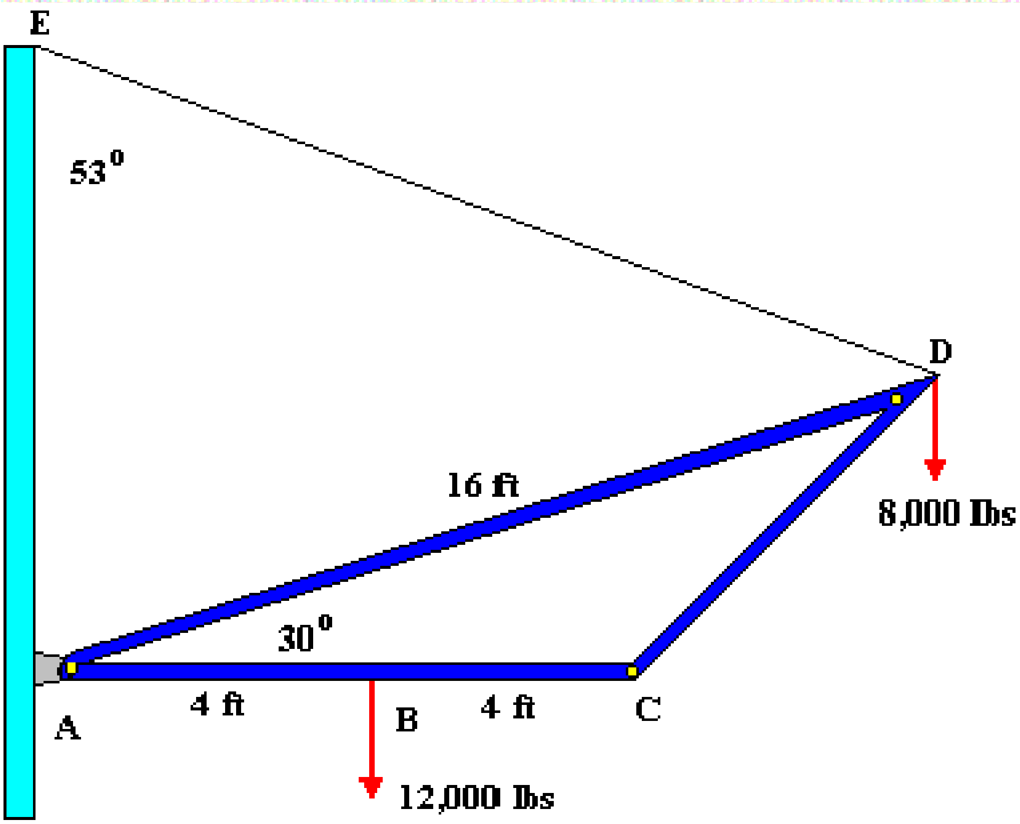
DB = 43,300 lbs; A_{cx} = -36,000 lbs; A_{cy} = -12,000 lbs

These are external forces acting on member ABC.

The force in DB is 43,300 lbs (c).

STATICS / STRENGTH OF MATERIALS - Example

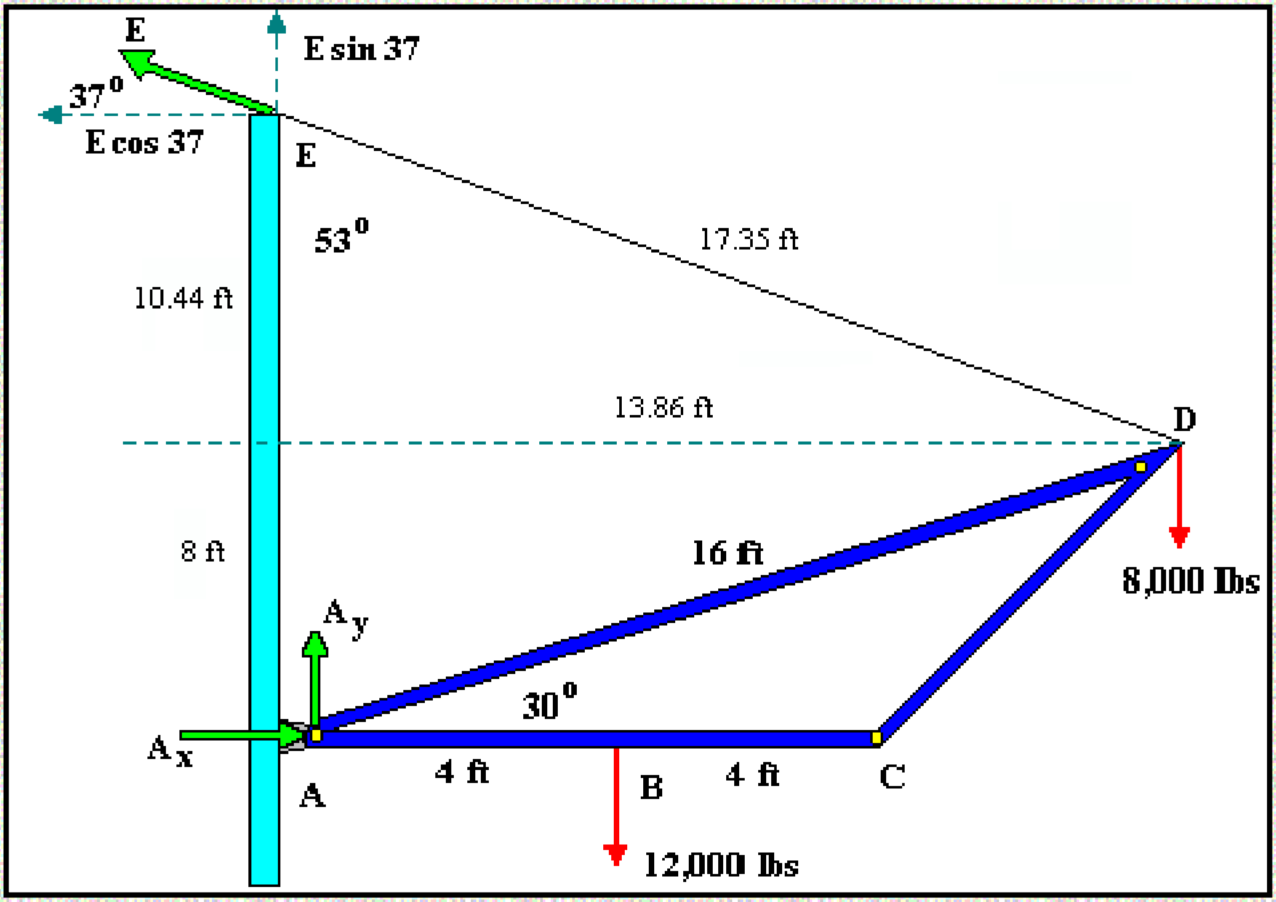
In the structure shown below members ABC, CD, and AD are assumed to be solid rigid members. Member DE is a cable. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force (tension or compression) in member CD.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

PARTS A & B:

STEP 1: Draw a free body diagram showing and labeling all load forces and

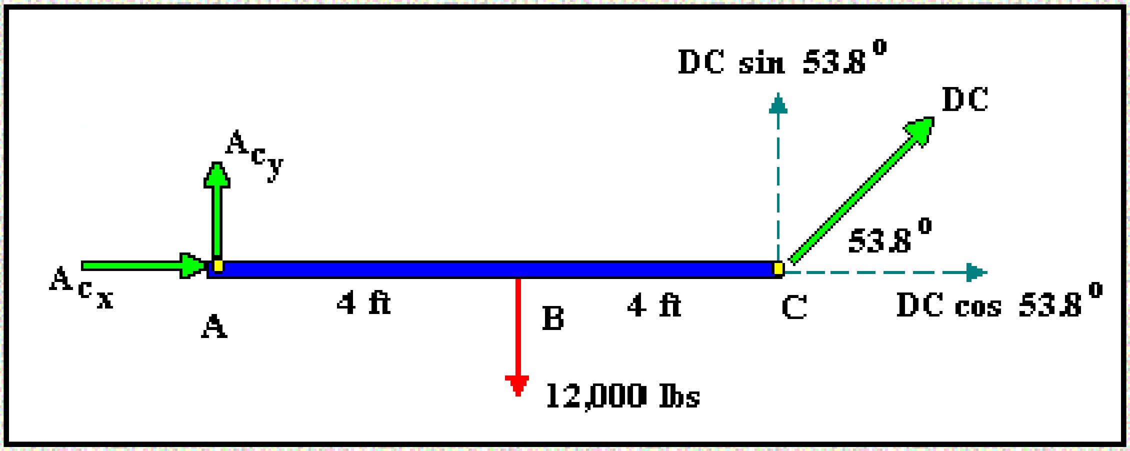


support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.
Sum F_x = -E cos (37º) + A_x = 0
Sum F_y = E sin (37º) + A_y - 12,000 lbs - 8,000 lbs = 0
Sum T_A = E cos(37º)(18.44 ft) - (12,000 lbs)(4 ft) - (8,000 lbs)(13.86 ft) = 0

Solving for the unknowns:
E = 10,800 lbs; A_x = 8,600 lbs; A_y = 13,400 lbs

PART C - Now find internal force in member DC.
STEP 1: Draw a free body diagram of a member that DC acts on - member ABC.

STEP 2: Resolve all forces into x and y components (see diagram).



STEP 3: Apply the equilibrium conditions.

Sum $F_x = A_{cx}$ + DC cos (53.8º) = 0

Sum $F_y = A_{cy}$ - 12,000 lbs + DC sin (53.8º) = 0

Sum T_A = (-12,000 lbs)(4 ft) + DC sin(53.8º)(8 ft) = 0

Solving for the unknowns:
DC = 7,440 lbs; A_{cx} = -4,390 lbs; A_{cy} = 6,000 lbs

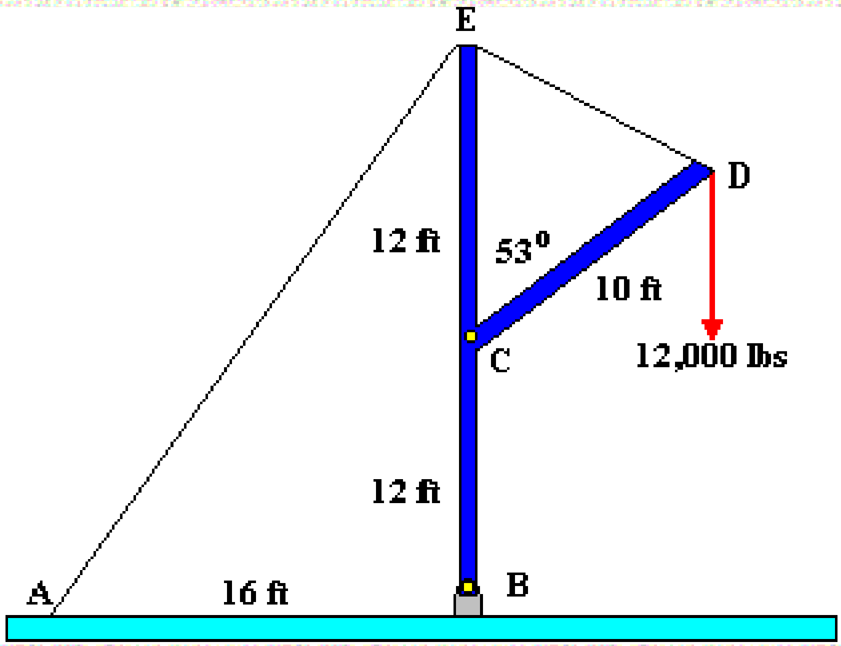
These are the external forces acting on member ABC.
The force in DC is 7,440 lbs (t).

STATICS / STRENGTH OF MATERIALS - Example

In the structure shown below members BCE, and CD are assumed to be solid rigid members. Members AE and DE are cables. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force (tension or compression) in member CD.

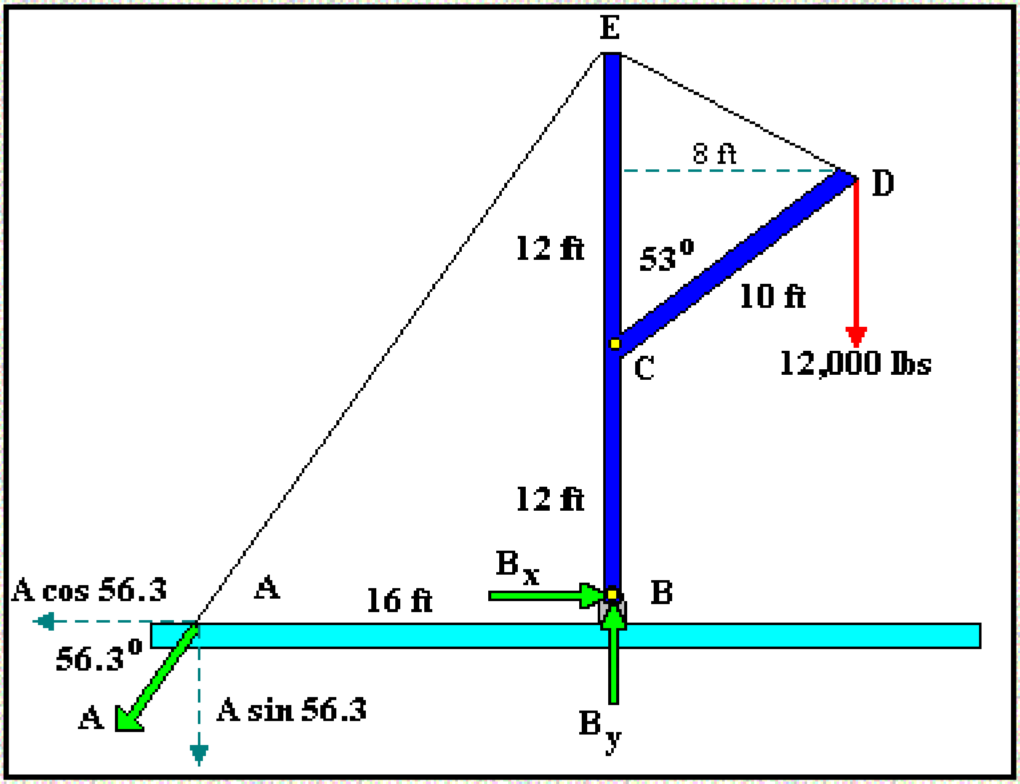
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

PARTS A & B:

STEP 1: Draw a free body diagram showing and labeling all load forces and



support (reaction) forces, as well as any needed angles and dimensions.

STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

Sum F_x = -A cos(56.3°) + B_x = 0

Sum F_y = - A sin(56.3°) + B_y - 12,000 lbs = 0

Sum T_A = B_y(16 ft) - (12,000 lbs)(24 ft) = 0

Solving for the unknowns:

A = 7210 lbs; B_y = 18,000 lbs; B_x = 4,000 lbs

PART C - Now find internal force in member CD.

STEP 1: Draw a free body diagram of a member that CD acts on - member BCE.

STEP 2: Resolve all forces into x and y components (see diagram).

STEP 3: Apply the equilibrium conditions:

Sum $F_x = E_{dx} - CD \cos(37^o) + 4{,}000 \text{ lbs} = 0$

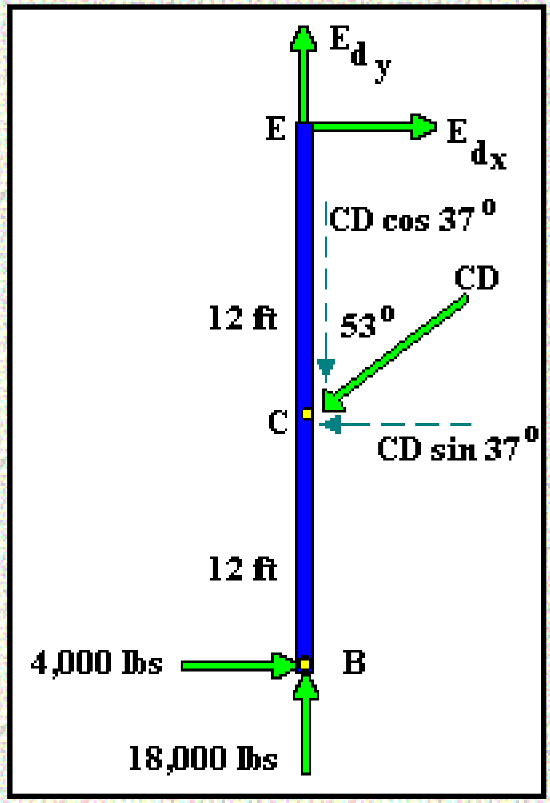
Sum $F_y = E_{dy} - CD \sin(37^o) + 18{,}000 \text{ lbs} = 0$

Sum $T_E = -CD \cos(37^o)(12 \text{ ft}) + (4{,}000 \text{ lbs})(24 \text{ ft}) = 0$

Solving for the unknowns:

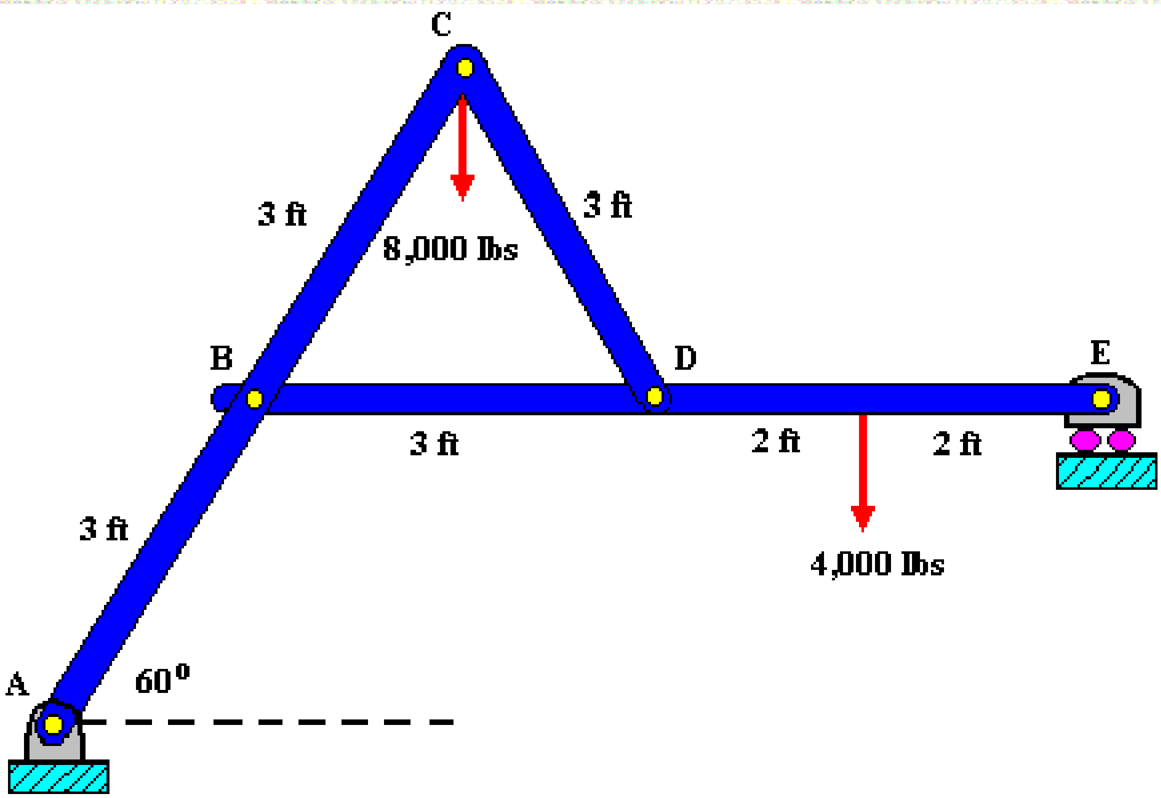
CD = 10,000 lbs; E_{dx} = 4,000 lbs; E_{dy} = -12,000 lbs

These are the external forces acting on member BCE. The force in CD is 10,000 lbs (c).

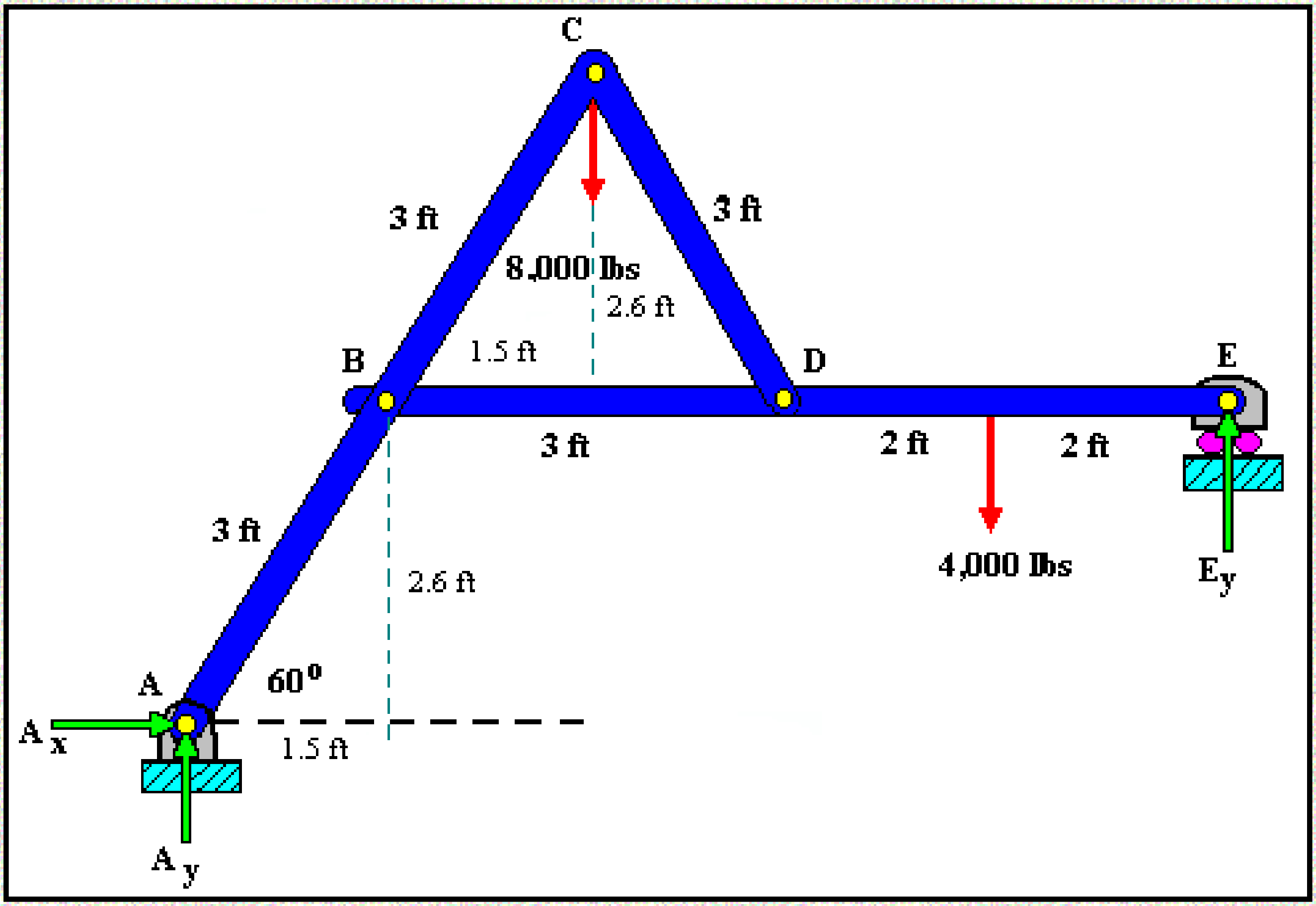


STATICS / STRENGTH OF MATERIALS - Example

In the structure shown below members ABC , BDE and CD are assumed to be solid rigid members. The structure is pinned at A and supported by a roller at E. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force (tension or compression) in member CD.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
PARTS A & B:
STEP 1: Draw a free body diagram showing and labeling all load forces and



support (reaction) forces, as well as any needed angles and dimensions.

STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

Sum $F_x = A_x = 0$

Sum $F_y = A_y + E_y$ - 8,000 lbs - 4,000 lbs = 0

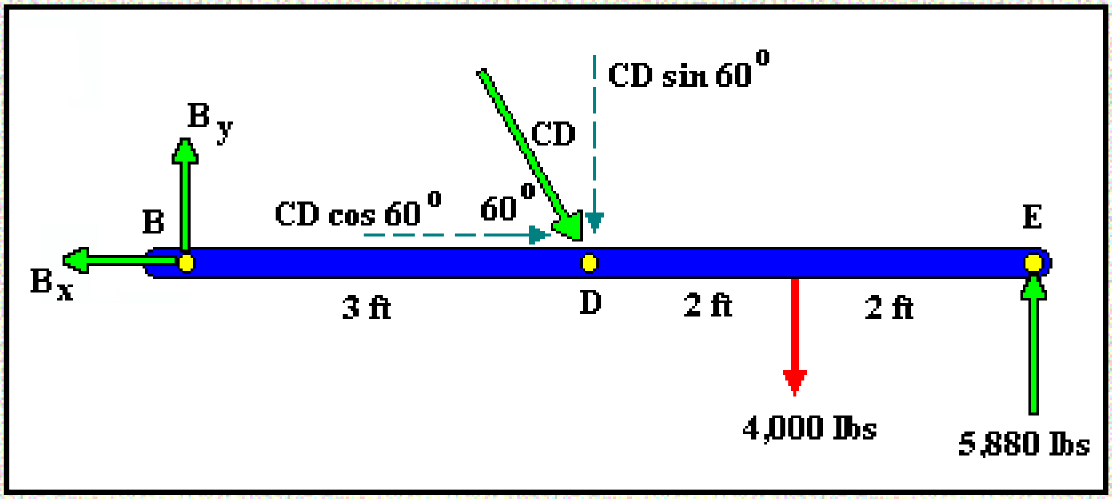
Sum T_A = (-8,000 lbs)(3 ft) - (4,000 lbs)(6.5 ft) + E_y(8.5 ft) = 0

Solving for the unknowns:

E_y = 5,880 lbs; A_y = 6,120 lbs

PART C - Now find internal force in member CD.

STEP 1: Draw a free body diagram of a member that CD acts on - member BDE.



STEP 2: Resolve all forces into x and y components (see diagram).
STEP 3: Apply the equilibrium conditions:
Sum F_x = $-B_x$ + CD cos (60^o) = 0
Sum F_y = B_y - CD sin (60^o) - 4,000 lbs + 5,880 lbs = 0
Sum T_B = -CD sin (60^o)(3 ft) - (4,000 lbs)(5 ft) + (5,880 lbs)(7 ft) = 0

Solving for the unknowns:
CD = 8,140 lbs; B_x = 4,070 lbs; B_y = 5,170 lbs

These are the external forces acting on member BDE.
The force in CD is 8,140 lbs (c).

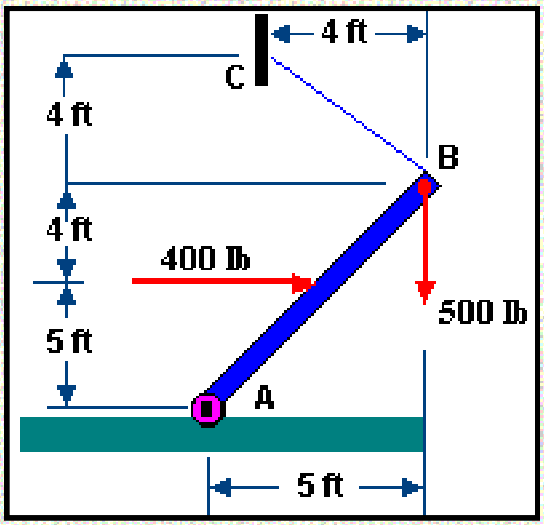
Statics & Strength of Materials

Problems Assignment - Frames 1

Draw a complete free body diagram as a part of the solution for each problem.

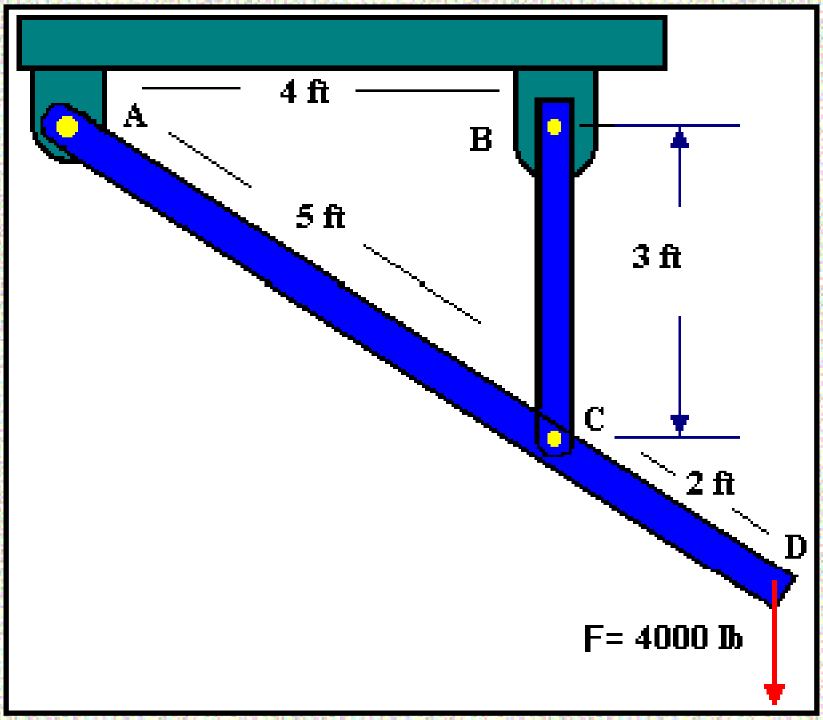
1. In the structure shown member AB is pinned at the floor, and member CB is a cable pinned at the wall. Determine the values of the external support reactions, and the force in member CB.

(Ax = -78.3 lb., Ay = +178 lb., C = 455 lb., CB = 455 lb. (T))



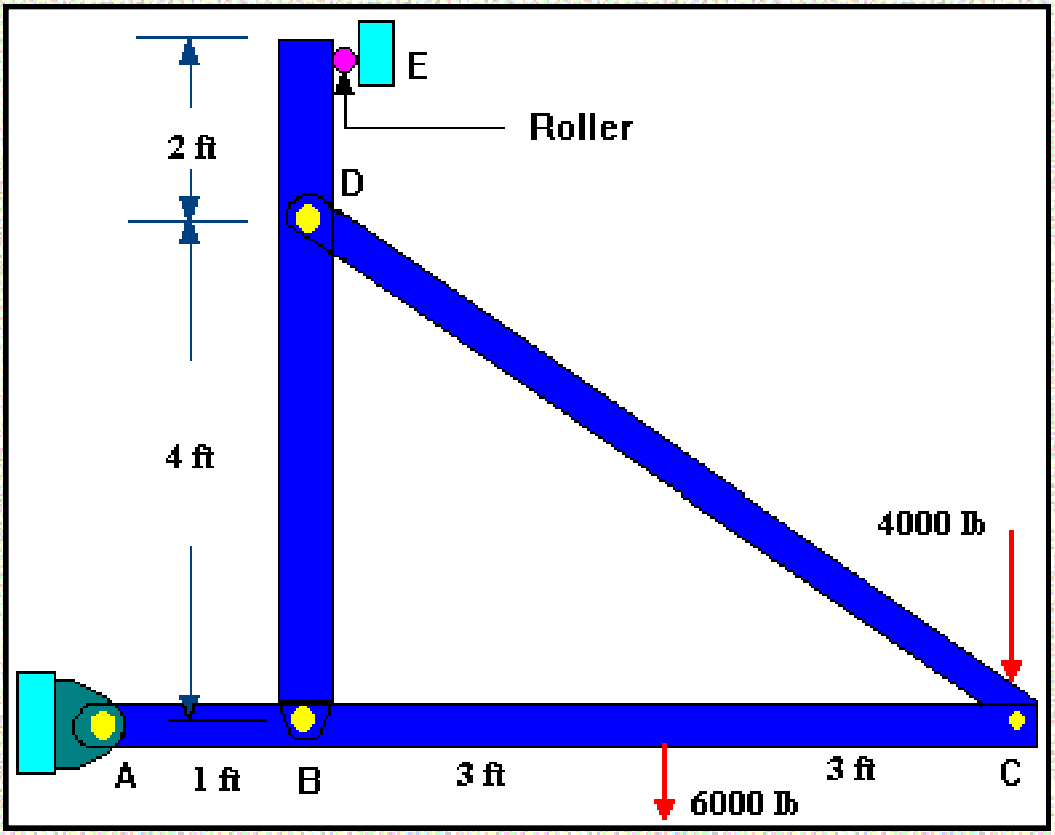
2. The structure shown is composed of members ACD and BC pinned together at point C.. The structure is pinned to ceiling at points A and B. Determine the values of the external support reactions, and the force in member BC.

(Ax = 0 lb., Ay = -1600 lb., By = 5600 lb., BC = 5600 lb.T)



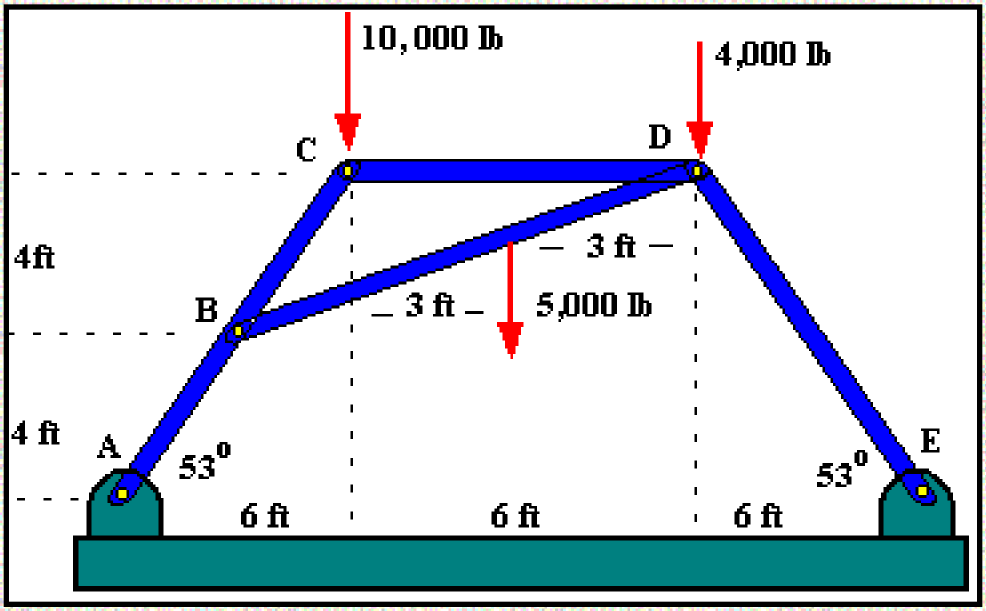
3. The structure shown is composed of solid rigid members ABC, CD, and BDE pinned together at points B, C, and D. The structure is supported by a roller at point E, and pinned to the wall at point A.For the structure shown, determine the values of the external support reactions, and the force in member DC.

(Ax = 8,670 lb., Ay = 10,000 lb.,
Ex =- 8,670 lb., DC = 15,600 (T))



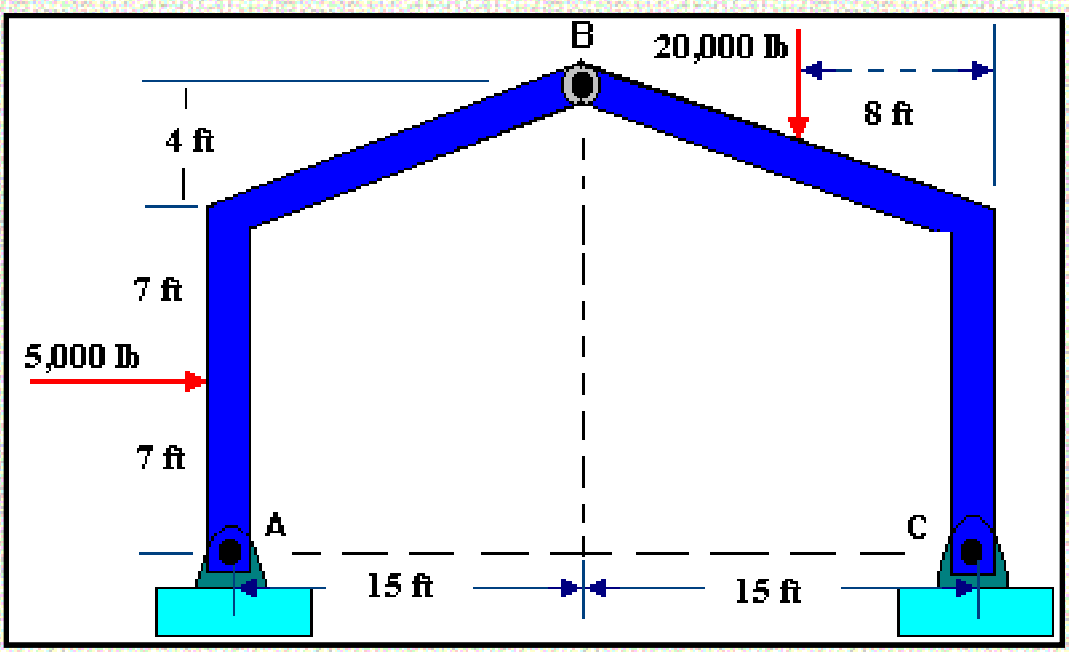
4. The structure shown is composed of solid rigid members ABC, CD, BD, and DE pinned together at points B, C, and D. The structure is pinned to the floor at points A and E. For the structure shown, determine the values of the external support reactions, and the force in member CD.

(Ax = 6375 lb., Ay = 10500 lb., E = 10,625 lb.
CD = 9000 lb. C)



5. The structure shown is composed of solid rigid members AB and BC pinned together at point B. The structure is pinned to the floor at points A and C. Determine the values of the external support reactions

(Ax = +419 lb., Ay = 4170 lb.
Cx = - 5419 lb., Cy = 15,830 lb.)



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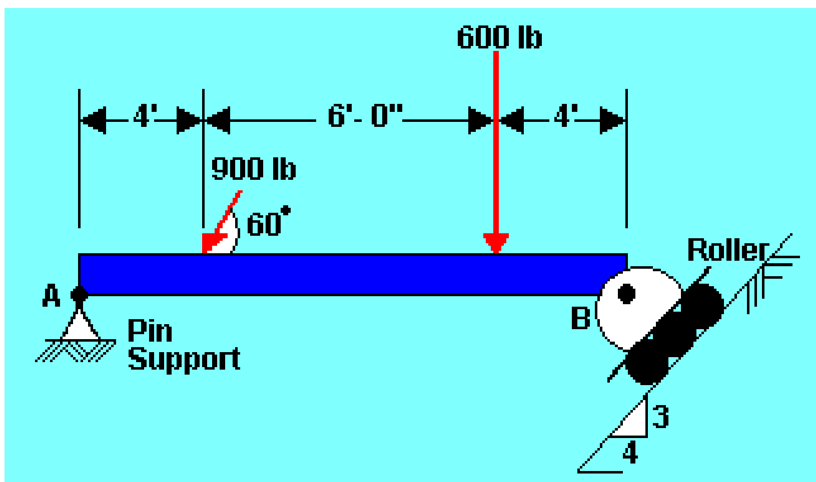
Statics & Strength of Materials

Problems Assignment - Frames 2

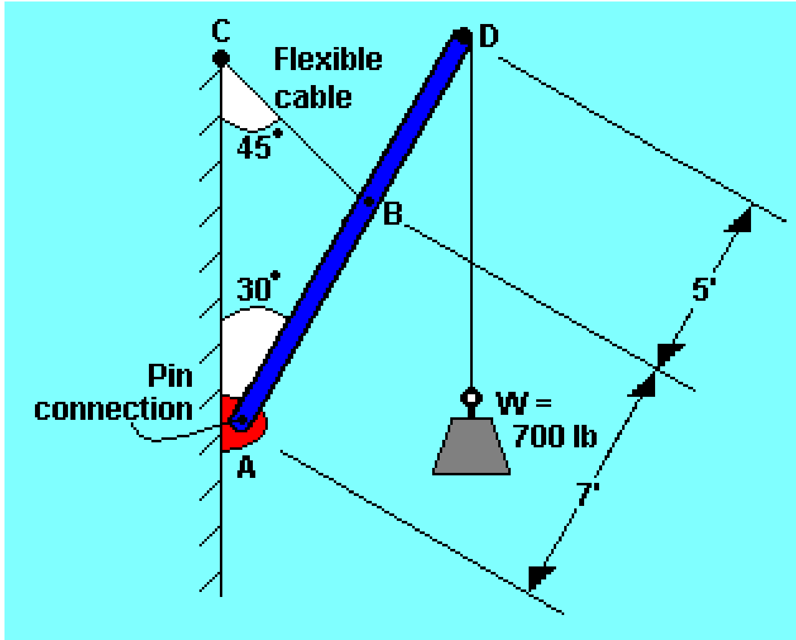
Draw a complete free body diagram as a part of the solution for each problem.

1. Determine the reaction at supports A and B of the beam in the diagram shown. Neglect the weight of the beam. (Ax = 938 lb., Ay = 728 lb., B = 814 lb.)



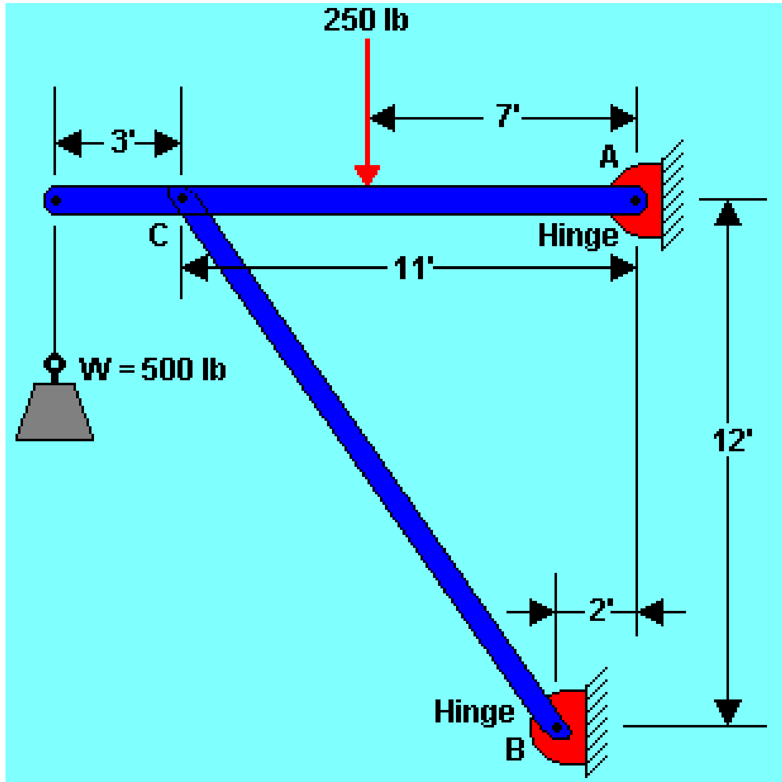
Problem #2

2. A 700 lb. weight is carried by a boom-and-cable arrangement, as shown in the diagram. Determine the force in the cable and the reactions at point A. (Ax = 404 lb., Ay = 296 lb., C = 572 lb.)



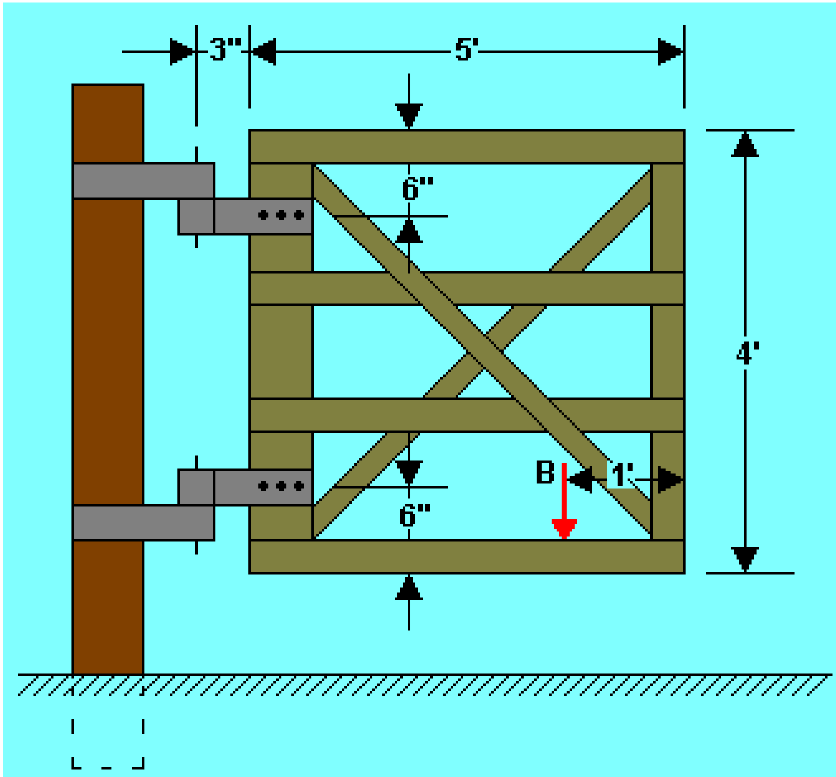
Problem #3

3. A brace is hinged at one end to a vertical wall and at the other end to a beam 14ft long.   The beam weighs 250 lb. and is also hinged to a vertical wall as shown. The beam carries load of 500 lb. at the free end. What will be the compressive force in the brace, and what will be the values of the vertical and horizontal components of the reaction at hinge A? (Ax = 596 lb., Ay = 45 lb., B = 994 lb.)



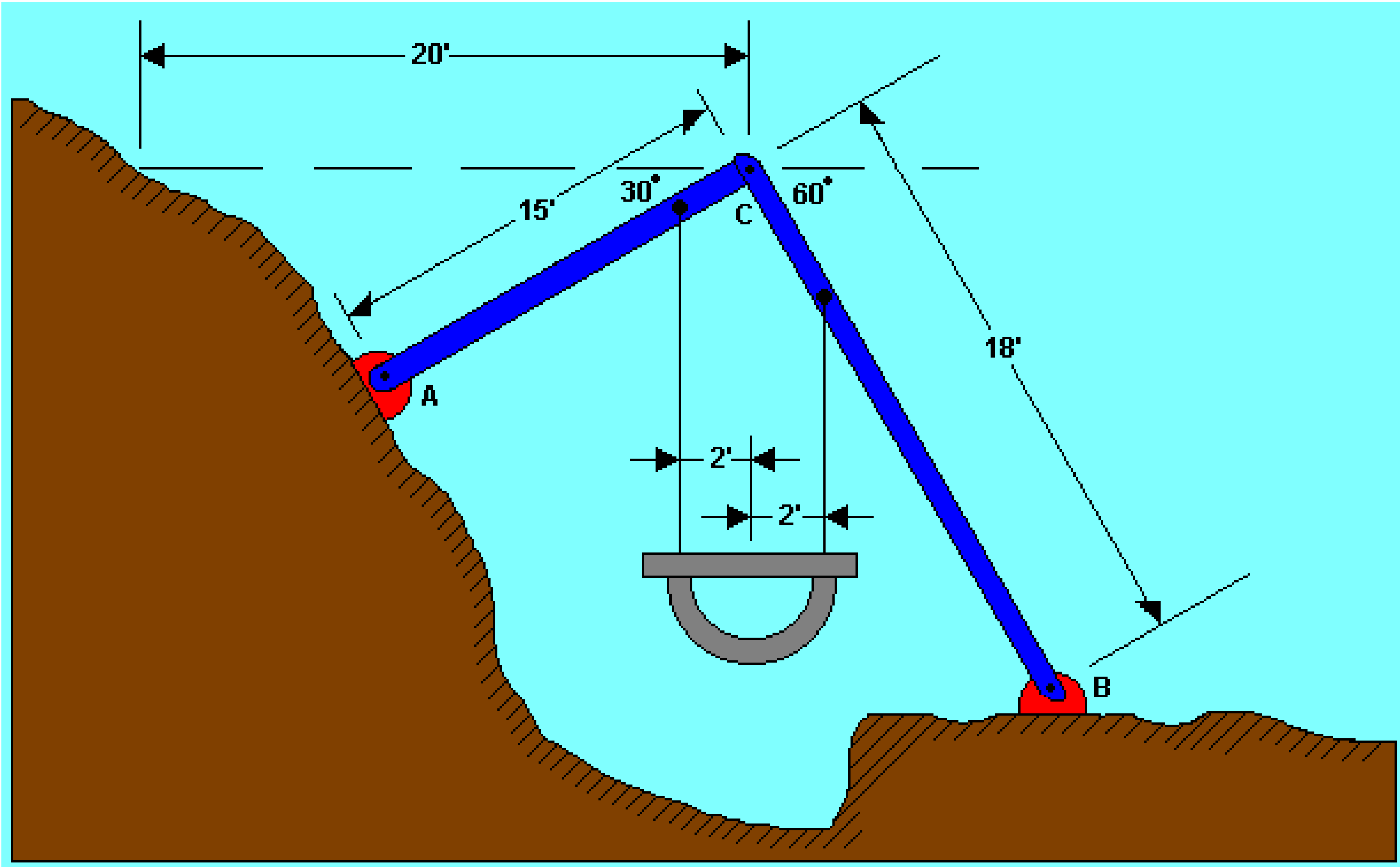
Problem #2

4. A gate has a weight of 200 lb., which may be considered as uniformly distributed, see the diagram shown. A small boy weighing 95 lb. climbs up on the gate at the point B. What will be the reactions on the hinges? (Upper hinge horizontal force only = 318 lb., lower hinge horizontal force = 318 lb., vertical force = 295 lb.)



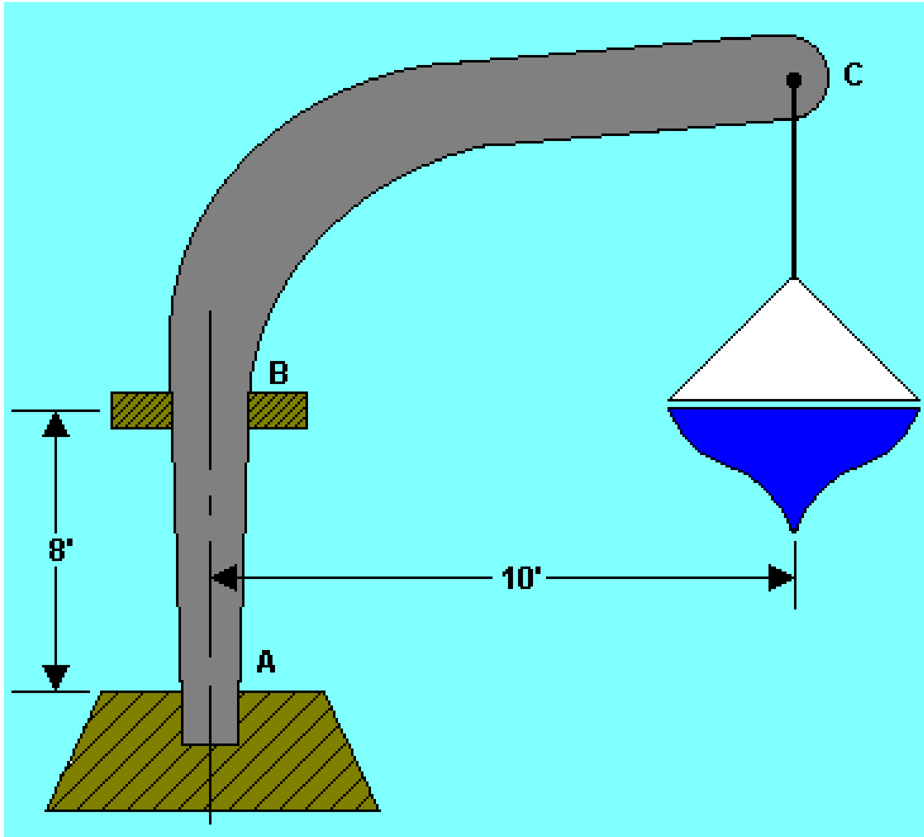
Problem #3

5. In an irrigation project, it was found necessary to cross low ground or else swing the canal to the left by cutting into the solid rock. It was decided to run the canal as a flume and support it on a number of frames as shown in the diagram. The two members rest in sockets in solid rock at points A and B. These sockets may be considered as hinges. What will be the vertical and horizontal components of the reactions at A and B? The weight of the water in the flume supported by each frame is estimated as 18,200 lb. (This is a somewhat more complicated problem then the others in this problem set, and it may be skipped. Or contact your instructor for hints.) (Ax = 6389 lb., Ay = 5093 lb., Bx = 6389 lb., By = 13, 107 lb.)



Problem #4

6. An Ocean liner has an arrangement for supporting lifeboats and for lowering them over the side as shown in the diagram. There is a socket at A and a smooth hole through the deck rail at B. If the boat and its load weigh 2,000 lb., what are the reactions at A and B? Two identical davits support each lifeboat. (Ay = 1000 lb., Ax = 1250 lb., Bx = 1250 lb.)



Problem #1

Answers: 1) Ax = 938 lb., Ay = 728 lb., B = 814 lb.; 2) Ax = 404 lb., Ay = 296 lb., C = 572 lb.; 3) Ax = 596 lb., Ay = 45 lb., B = 994 lb.; 4) Upper hinge horizontal force only = 318 lb., lower hinge horizontal force = 318 lb., vertical force = 295 lb.; 5) Ax = 6389 lb., Ay = 5093 lb., Bx = 6389 lb., By = 13, 107 lb.; 6) Ay = 1000 lb., Ax = 1250 lb., Bx = 1250 lb.

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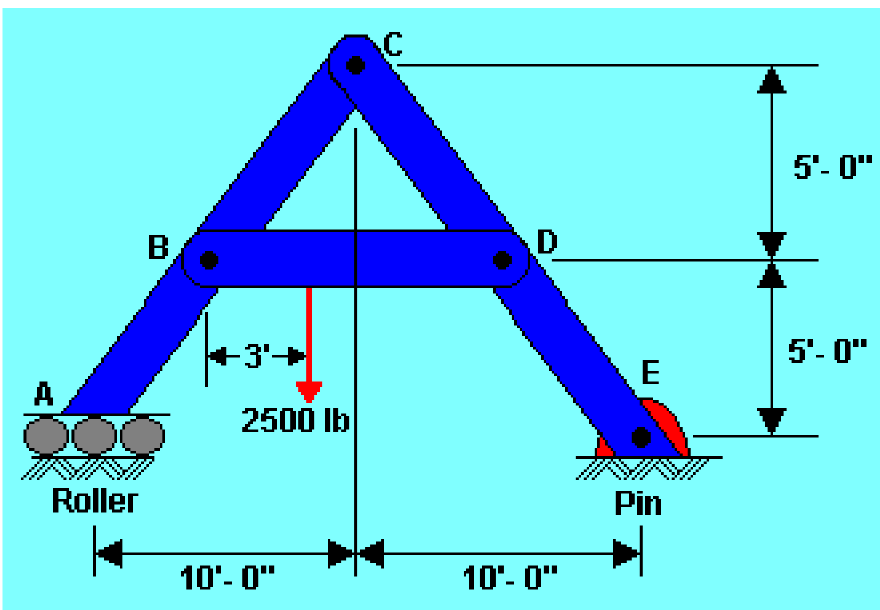
Statics & Strength of Materials

Problems Assignment - Frames 3

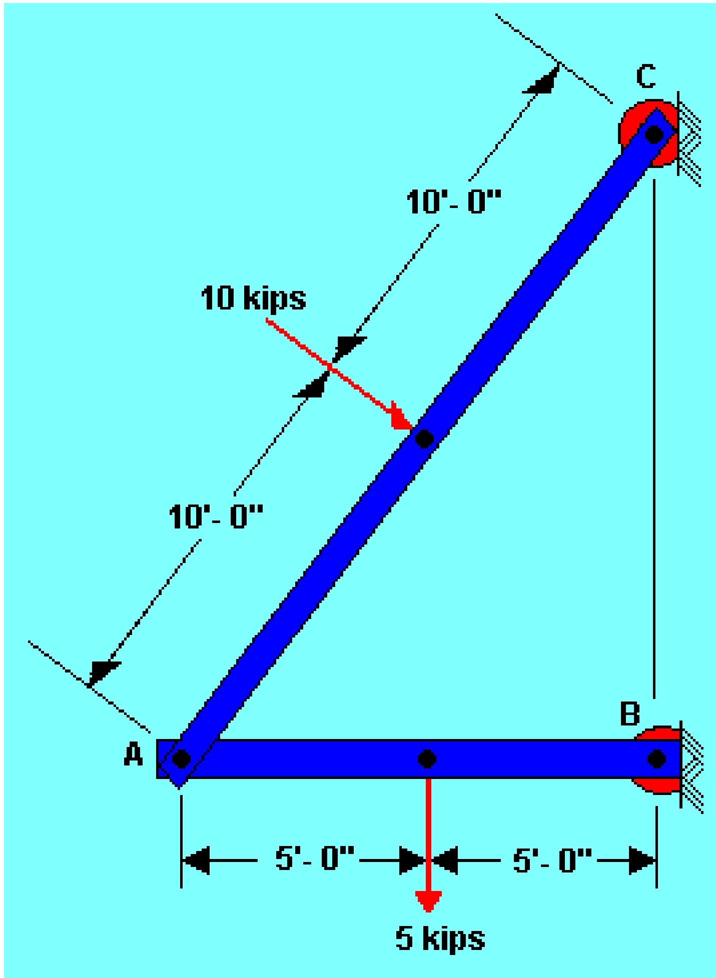
Draw a complete free body diagram as a part of the solution for each problem.

1. A pin-connected A-frame supports a load as shown. Compute the pin reactions at all of the pins. Neglect the weight of the members. (Ay=1500 lb., Ey=1000 lb., Bx=1250 lb., By=1750 lb., Cx=1250 lb., Cy=250 lb., Dx = 1250 lb., Dy=750 lb., directions not indicated)



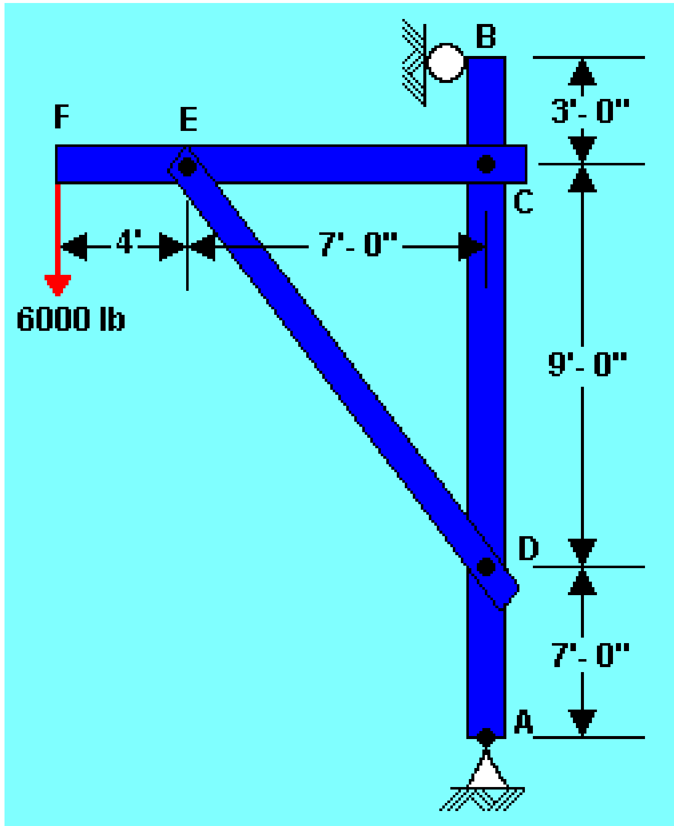
Problem #1

2. A simple frame is pin connected at points A, B, and C and is subjected to loads as shown. Compute the pin reactions at A, B, and C. Neglect the weight of the members. (Ax=10,100 lb., Ay=2500 lb,. Bx=10,100 lb., By=2500 lb., Cx=1443 lb., Cy=7500 lb., directions not indicated)



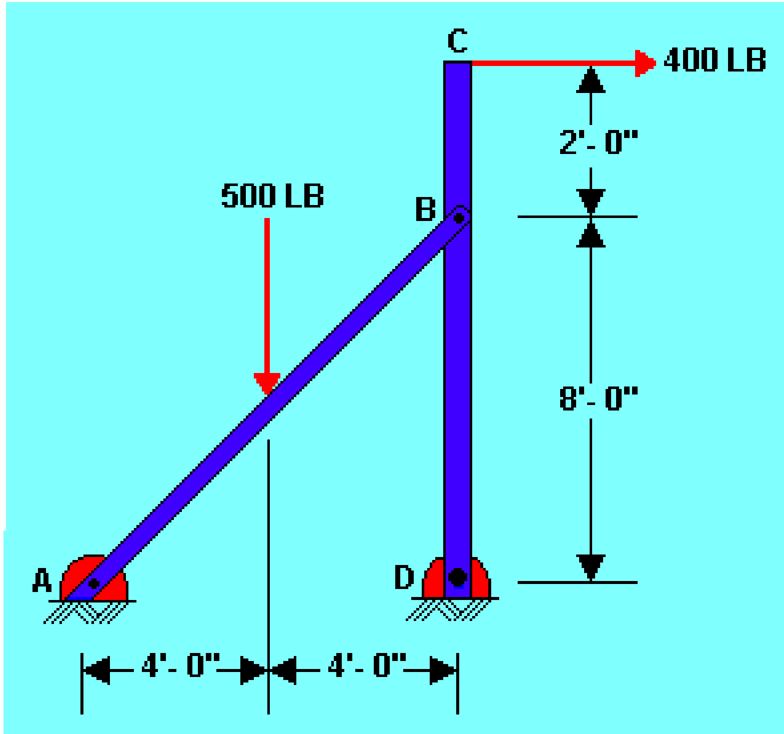
Problem #2

3. A pin-connected crane framework is loaded and supported as shown. The member weights are: post, 600 lbs.; boom, 700 lbs.; and brace, 800lbs. These weights may be considered to be acting at the midpoint of the respective members. Calculate the pin reactions at pins A, B, C, D, and E. (Ax=3824 lb., Ay=8100 lb., Bx=3824 lb., By=0, Cx=7768 lb., Cy=3279 lb., ED = 12, 650 lb @ 52.1º , directions not indicated)



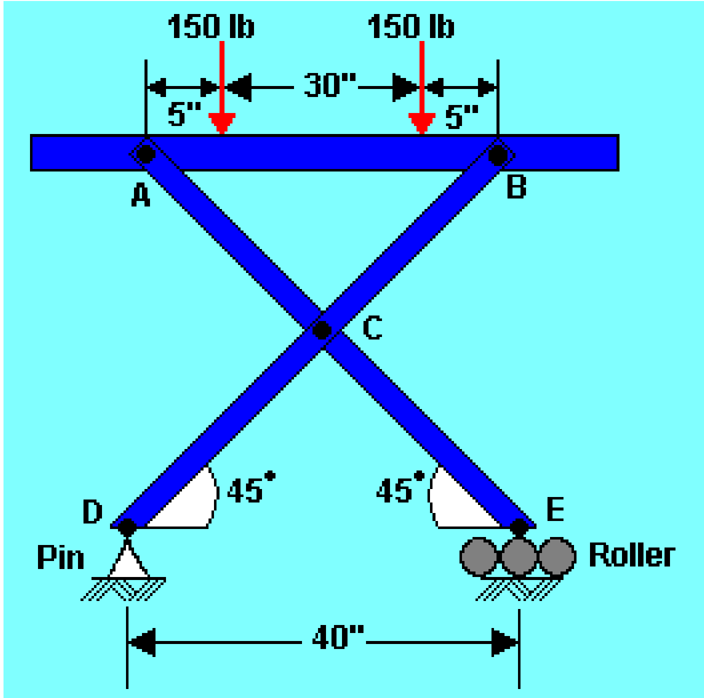
Problem #3

4. Sketch the structure and draw a free body diagram for the relevant member or the entire structure. Write the appropriate moment or force equations and solve them for the unknown forces. (Ax=500 lb., Ay=250 lb., Bx=500 lb., By=750 lb., Dx=100 lb., Dy=750 lb., directions not indicated)



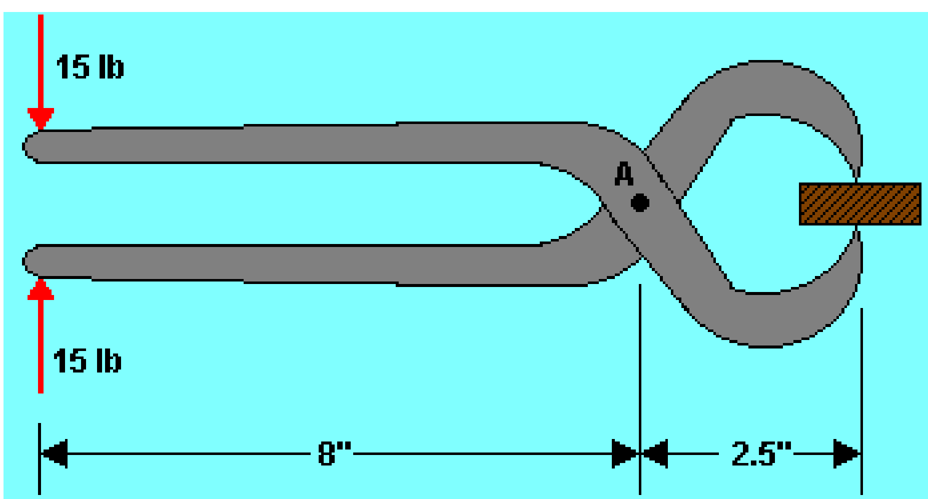
Problem #2

5. Calculate the pin reactions at each point of the pins in the frame shown below. (Ax=300 lb., Ay=150 lb., Bx=300 lb., By=150 lb., Cx=300 lb., Cy=0, Dx =0, Dy=150 lb., Ey=150 lb., directions not indicated)



Problem #4

6. The tongs shown are used to grip an object. For an input force of 15 lb. on each handle, determine the forces exerted on the object and the forces exerted on the pin at A. (F = 48 lb. on object from each jaw., Ax = 0, Ay=63 lb.)



Problem #5

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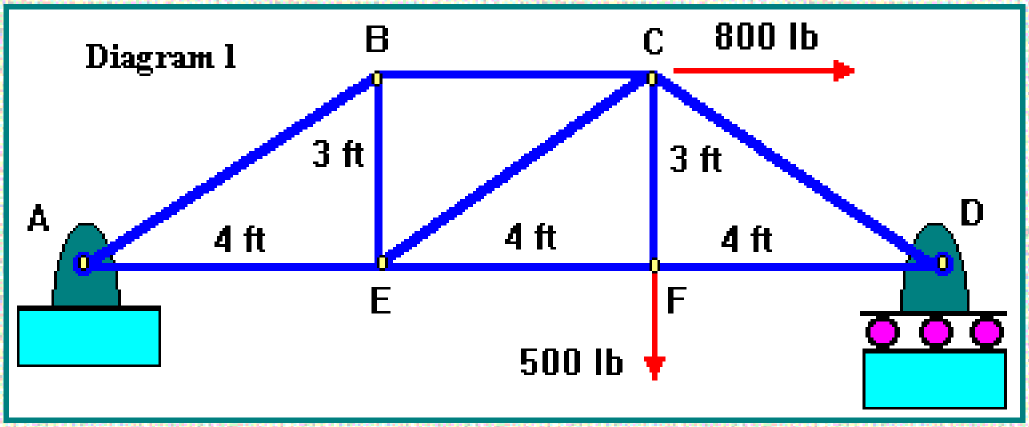
Topic 2.2: Rigid Body Structures - Trusses

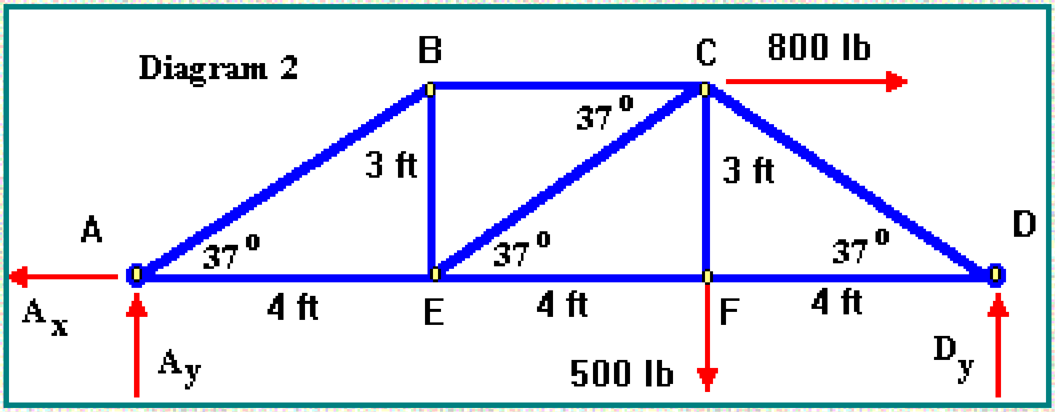
A very common structure used in construction is a truss. An ideal truss is a structure which is composed completely of (weightless) Axial Members that lie in a plane, connected by pinned (hinged) joints, forming triangular substructures (within the main structure), and with the external loads applied only at the joints. See Diagram 1. In real trusses, of course, the members have weight, but it is often much less than the applied load and may be neglected with little error. Or the weight maybe included by dividing the weight in half and allowing half the weight to act at each end of the member. Also in actual trusses the joints may be welded, riveted, or bolted to a gusset plate at the joint. However as long as the centerline of the member coincide at the joint, the assumption of a pinned joint maybe used. In cases where there are distributed loads on a truss, these may be transmitted to a joint by use of a support system composed of stringers and cross beams, which is supported at the joints and transmits the load to the joints.



The procedure for determining the external support reactions acting on a truss is exactly the same as the procedure for determining the support forces in non-truss problems, however the method for determining the internal forces in members of a truss is not the same. The procedures for finding internal forces in truss members are **Method of Sections** and **Method of Joints** (either of which may be used), and in fact, one must be very careful not to use these methods with non-truss problems as they will not give correct results. Perhaps the best way to clarify these concepts is to work very slowly and carefully through a truss example.

Example 1: In Diagram 1 we have a truss supported by a pinned joint at Point A and supported by a roller at point D. A vertical load of 500 lb. acts at point F, and a horizontal load of 800 lb. acts at Point C. For this structure we wish to determine the values of the support reactions, and the force (tension/compression) in members BE, BC, and EF.

For the first part, determining the external support reactions, we apply the normal static equilibrium procedure:



I. **Draw a Free Body Diagram of the entire structure showing and labeling all external load forces and support forces, include any needed dimensions and angles**. Note that at the pinned support point A, the best we can do is to put both an unknown x and y support force, however at point D we only need a unknown y support force since a roller can only be in compression and so must support vertically in this problem.

II. Resolve (break) all forces into their x and y-components.

III. Apply the Equilibrium Equations ($\sum Fx = 0; \ \sum Fy = 0; \ \sum \tau_P = 0$ **) and solve for the unknown forces.**

$\sum Fx = 0: \ -Ax + 800 \text{ lb} = 0$ (Sum of x-forces)

$\sum Fy = 0: \ Ay + Dy - 500 \text{ lb} = 0$ (Sum of y-forces)

$\sum \tau_A = 0: \ Dy\,(12 \text{ ft}) - 500 \text{ lb}\,(8\text{ft}) - 800 \text{ lb}\,(3\text{ft}) = 0$ (Sum of Torque about A.)

Solving we obtain: **A_x = 800 lb, A_y = -33 lb, D_y = 533 lb** (The negative sign for force A_y means that we initially chose it in the incorrect direction, A_y acts downward, not upward as shown in FBD.)

Part 2

Once we have determined the values of the external support reactions, we may proceed to determining the values of the forces in the members themselves, the internal forces. In this first example, we will use **Method of Joints** to determine the force in the selected members. In **Method of Joints,** rather then analyze the entire structure, or even a member of the structure, we rather examine the joint (pin or hinge) where members come together. As the structure is in static equilibrium, so the pin or joint will be in static equilibrium, and we may apply the static equilibrium conditions (and procedure) to solve for the forces on the joint(s) due to the members, which will also equal the forces in the axial members - due to Newton's third law of equal and opposite reactions (forces).

There are several points to keep in mind as we use method of joints. One is that since we are analyzing a point (joint) rather then an extended body, our sum of torque equation will be of no help. That is, since all the forces pass through the same point, they have no perpendicular distance to that point and so produce no torque. **This means that to solve completely for the forces acting on a joints we must have a joint which has, at most, two unknown forces acting**. In our example (Diagram 2), we notice that there are only two joints which initially have only two unknowns acting - Joint A and Joint D. Thus, we start our process at one of these. We will begin with Joint A.

Step 1. FBD of the Joint A, showing and labeling all forces acting on the joint. Include needed angles. In Diagram 3 we have shown Joint A with the all the forces which act on the joint. The forces on Joint A, due to members AB and AE, act along the directions of the members (since the members are axial). We choose directions for AB and AE (into or out of the joint). When we solve for the forces AB and AE, if these values are negative it means that our chosen directions were incorrect and the forces act in the opposite direction. A force acting into the joint due to a member means that member is in compression. That is, if a member of a truss is in compression, it will push outward on it ends - pushing into the joint. And likewise, if a truss member is in tension, it will pull outward on the joint. In Diagram 3, we have assumed member AB is in compression, showing its direction into the joint, and that member AE is in tension, showing it acting out of the joint.

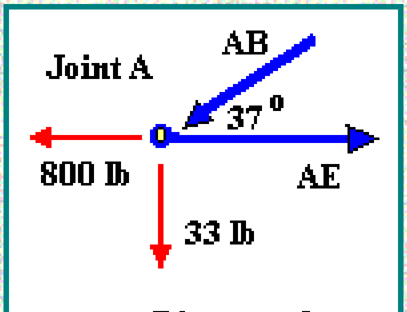


Diagram 3

[A little consideration will show that we have actually chosen AB in an incorrect direction. We can see this if we consider the y-component of AB, which clearly will be in the - y direction. However, the y-support reaction, 33 lb., is also in the -y direction. There are no other y-forces on the joint, so it can not be in equilibrium is both forces act in the same direction. We will leave AB as chosen to see, if indeed, that the solution will tell us that AB is in the wrong direction.]

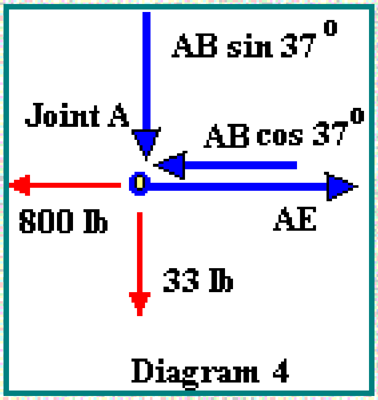
Step 2: Resolve (Break) all forces into their x and y components.

Step 3: Apply the Equilibrium Conditions: $\sum F_x = 0 \; \sum F_y = 0$

$\sum F_x = 0: \; -800 \text{ lb} - AB\cos 37^\circ + AE = 0$ (Sum of x-forces)

$\sum F_y = 0: \; -33 \text{ lb} - AB\sin 37^\circ = 0$ (Sum of y-forces)

Solving: AE = 756 lb. (Tension), AB = -55 lb. (The negative sign indicates we selected an incorrect initial direction for AB, AB is in Tension, not Compression.)



Remember, we are trying to find the forces in members BE, BC, and EF. Now that we have the forces in members AE (756 lb. tension) and AB (55 lb. tension), we can move unto a second joint (joint B) and find two of the unknowns we are looking for. We could not have solved for the forces acting at joint B initially, since there were three unknowns (initially) at joint B (AB, BC, and BE). However now that we have analyzed joint A, we have the value of the force in member AB, and can proceed to joint B where we will now have only two unknowns to determine (BE & BC).

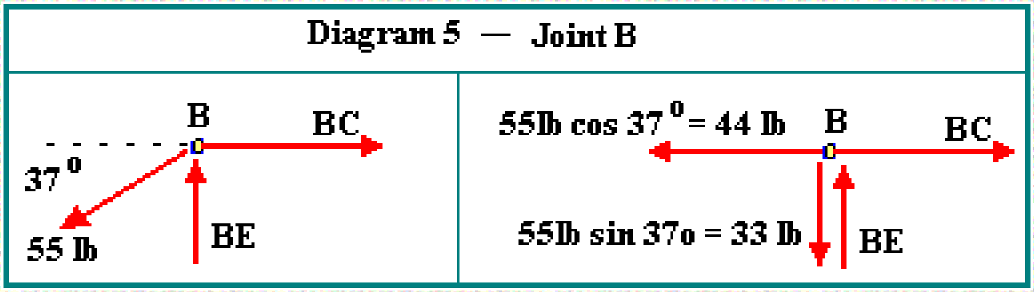
Joint B: Procedure - Method of Joints

Step 1. FBD of the Joint B, showing and labeling all forces acting on the joint. Include needed angles.

Step 2: Resolve (Break) all forces into their x and y components.

Step 3: Apply the Equilibrium Conditions: $\sum F_x = 0 \; \sum F_y = 0$

In Diagram 5, we have on the left the FBD of joint B with all external forces acting on the joint shown, and our initial direction for the forces. If the directions we chose for the unknowns are correct, their values will be positive in the solution. If a value is negative it means the force acts in the opposite direction. On the right side of Diagram 5 is the FBD with all forces resolved into x and y-components. We now apply the Equilibrium Conditions (for joints).



$\sum Fx = 0: \ -44 \text{ lb} - BC = 0$ (Sum of x-forces)

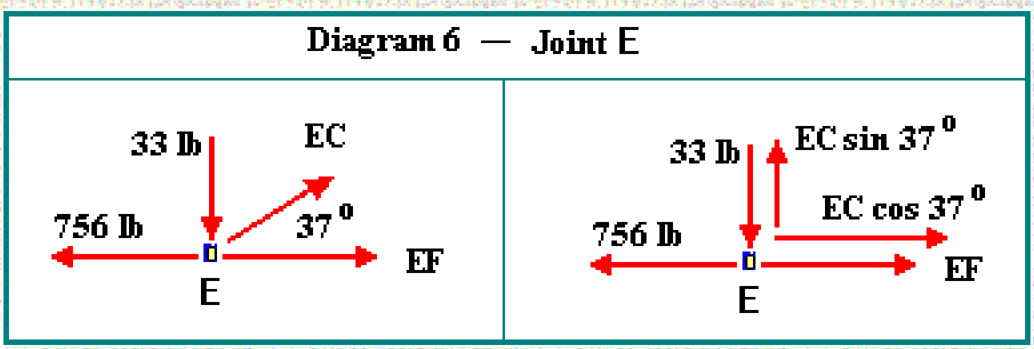
$\sum Fy = 0: \ -33 \text{ lb} + BE = 0$ (Sum of y-forces)

Solving : **BC = 44 lb. (Tension) BE = 33 lb. (Compression)**

Finally, we can now proceed to analyze joint E and determine the force in member EF.

Joint E: Procedure - Method of Joints

Step 1. FBD of the Joint E, showing and labeling all forces acting on the joint. Include needed angles.
Step 2: Resolve (Break) all forces into their x and y components.
Step 3: Apply the Equilibrium Conditions: $\sum F_x = 0 \ \sum F_y = 0$
In Diagram 6, we have on the left, the FBD of joint E with all external forces acting on the joint shown, and our initial direction for the forces. If the directions we chose for the unknowns are correct, their values will be positive in the solution. If a value is negative it means the force acts in the opposite direction. The right hand drawing in Diagram 6 is the FBD of joint E with all forces resolved into x and y-components. We now apply the Equilibrium Conditions (for joints).

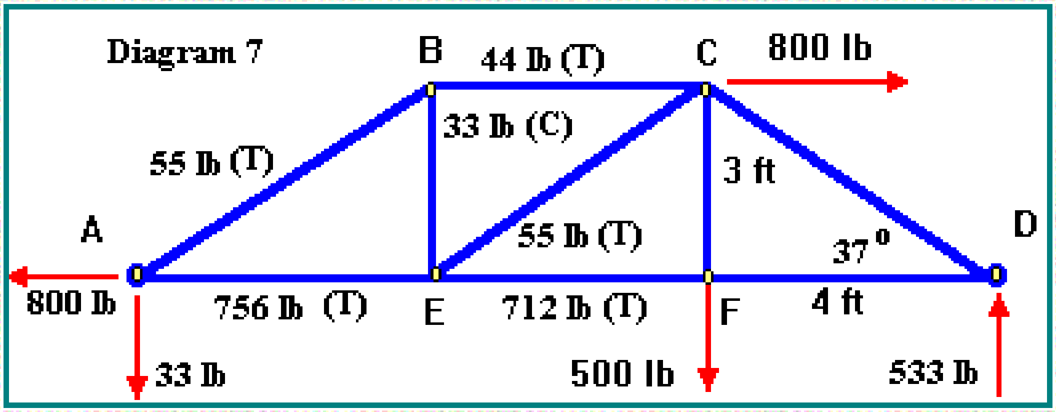


$\sum Fx = 0: \ -756 \text{ lb} + EC \cos 37^{\circ} + EF = 0$ (Sum of x-forces)

$\sum Fy = 0: \ -33 \text{ lb} + EC \sin 37^{\circ} = 0$ (Sum of y-forces)

Solving : **EC = 55 lb. (Tension) EF = 712 lb. (Tension)** Since both force values came out positive in our solution, this means that the initial directions selected for the forces were correct.

We have now solved our problem, finding both the external forces and forces in members BE, BC, and EF (and along the way, the forces in several other members - See Diagram 7)



Additional Examples

To see an example of finding internal forces in a truss using **Method of Sections**, select Example 1

For additional Examples of Truss using Method of Joints, select Example 2

For additional Examples of Truss using Method of Sections, select Example 3

Select:

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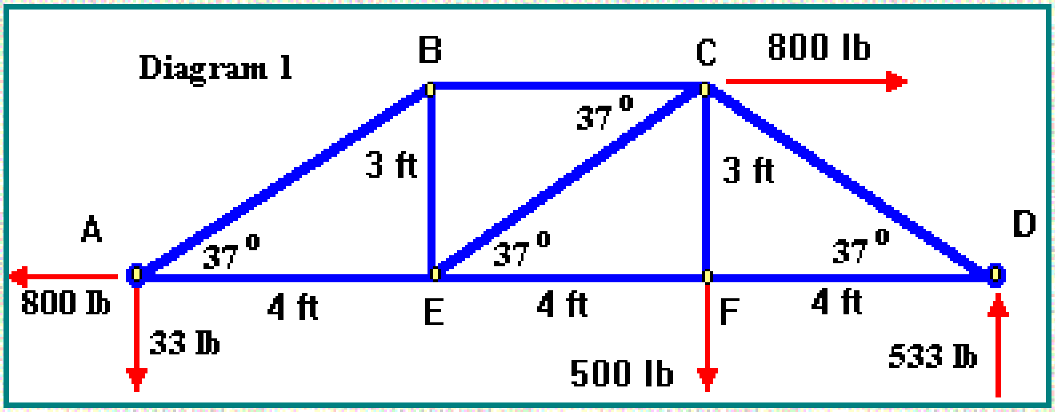
Topic 2.2 Trusses - Example 1

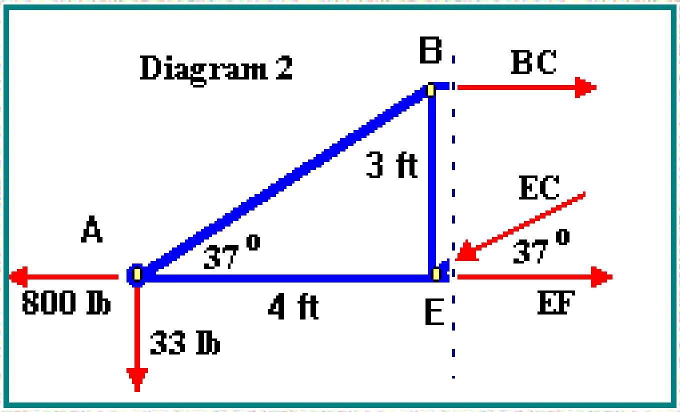
In the **Statics - Truss Page**, we ended with a sample truss example in which we determined the external forces acting on the truss and the internal forces in several members by **Method of Joints**. For example 2, we would like to use the same truss and solve for several internal forces by **Method of Sections**. And since we previously solved for the external support reactions, we will not repeat that portion, but begin with the external support forces given and move to determine the internal forces in the selected members.

Example 1: In Diagram 1 we have a truss supported by a pinned joint at Point A and supported by a roller at point D. A vertical load of 500 lb. acts at point F, and a horizontal load of 800 lb. acts at Point C. The support reactions acting on the structure at points A and D are shown. For this structure we wish to determine the values of the internal force (tension/compression) in members BC, EC, and EF.

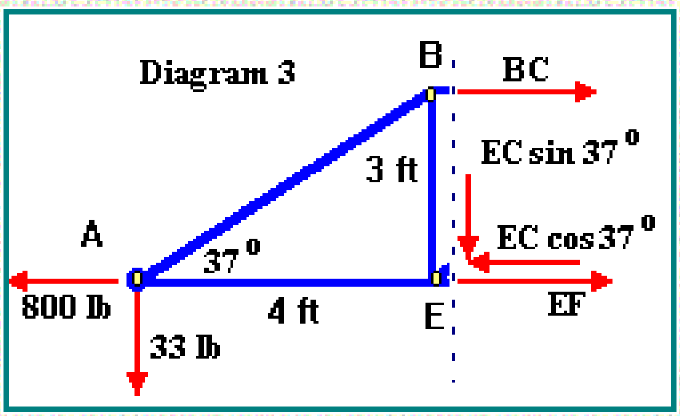


In **Method of Sections**, we will 'cut' the truss into two sections by drawing a line through the truss. This line may be vertical, horizontal, at some angle, or even curved depending on the problem. The criteria for this line is that we would like to cut through the unknown members (whose internal force value we wish to determine), but not to cut through more than three unknowns (since we will have three equilibrium conditions equations, we can only solve for three unknowns). In this example, cutting the truss once will enable us to find our selected unknowns, however, in some trusses, or for finding more internal forces, one may have to repeat Method of Sections several times to determine all the unknowns.

In Diagram 2, we have cut through the original truss with a vertical line just to the right of member BE. This vertical line cuts through members BC, EC, and EF (the selected members whose internal forces we wish to determine). We have shown the section of the truss to the left of the cut. We now treat this section of the truss as if it were a completely new structure. The internal forces in members BC, EC, and EF now become external forces with respect to this section. We have represented these forces with the arrows shown. The forces must act along the direction of the cut member (since all members in a truss are axial members), and we have selected an initial direction either into or away from the section for each of the forces. If we have selected an incorrect initial direction for a force, when we solve for the value of the force, the value will be negative indicating the force acts in the opposite direction of the one chosen initially. We may now proceed with the analysis of this structure using standard Static's techniques.



I. Draw a Free Body Diagram of the structure (section), showing and labeling all external forces, and indicating needed dimensions and angles. (Diagram 3)



II. Resolve (break) all forces into their x and y-components.

III. Apply the Equilibrium Equations : $\sum Fx = 0$: $\sum Fy = 0$: $\sum \tau_P = 0$ and solve for the unknown forces.

$\sum Fx = 0$: $-800\text{ lb} + EF - EC\cos 37^{\circ} + BC = 0$ (Here we sum the x-forces)

$\sum Fy = 0$: $-33\text{ lb} - EC\sin 37^{\circ} = 0$ (Sum of y-forces, including load forces.)

$\Sigma \tau_E = 0{:}\ 33\,\text{lb}(4\,\text{ft}) - BC(3\,\text{ft}) = 0$ (Sum of Torque with respect to point E.)

Solving we obtain: **BC = 44 lb. (T), EC = -50 lb. (T), EF = 712 lb. (T)** (The negative sign for force EC means that we initially chose it in the incorrect direction. EC acts out of the section and so is in tension, not into the section as shown in the FBD). Thus, we have solved for the internal forces in the members BC, EC, and EF by method of sections.

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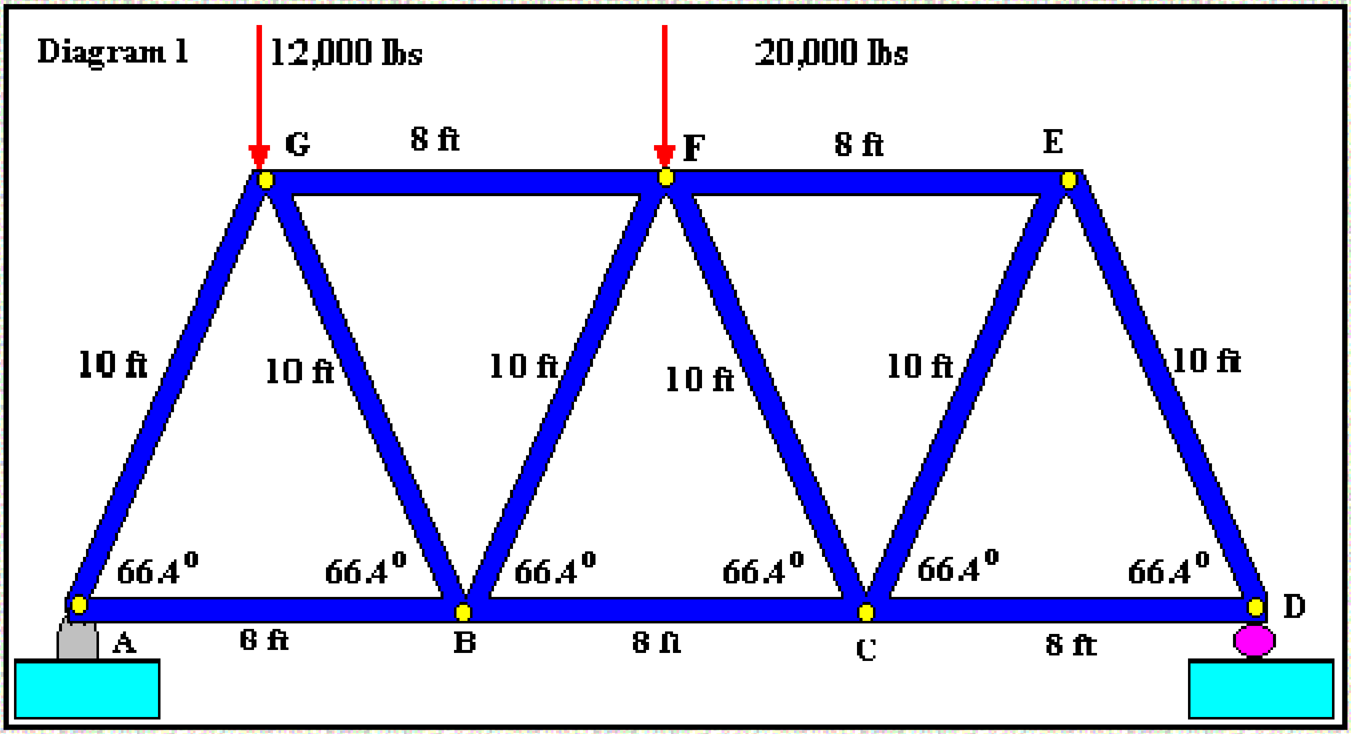
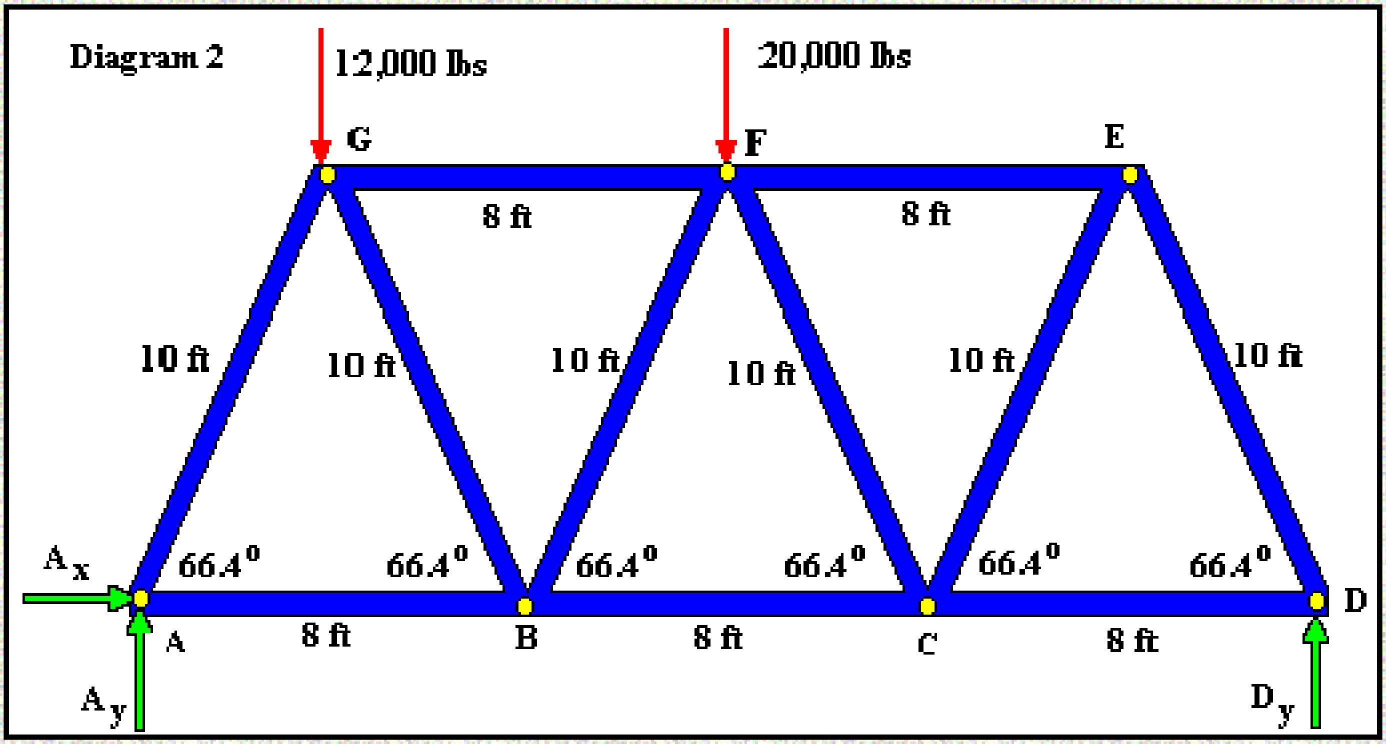
Topic 2.2 Trusses - Example 2

The structure shown in Diagram 1 is a truss which is pinned to the floor at point A, and supported by a roller at point D. For this structure we wish to determine the value of all the support forces acting on the structure, and to determine the force in member FC by method of joints.



For the first part of the problem we proceed using our normal static's procedure.

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces as well as any needed angles and dimensions. (Note in Diagram 2, we have replaced the pinned support by an unknown x and y force (A_x , A_y), and replaced the roller support by the vertical unknown force D_y.



STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions $\sum Fx = 0;\ \sum Fy = 0;\ \sum \tau_P = 0$

$\sum Fx = 0$: **$A_x = 0$**

$\sum Fy = 0$: **$A_y + D_y$ - 12,000 lbs - 20,000 lbs = 0**

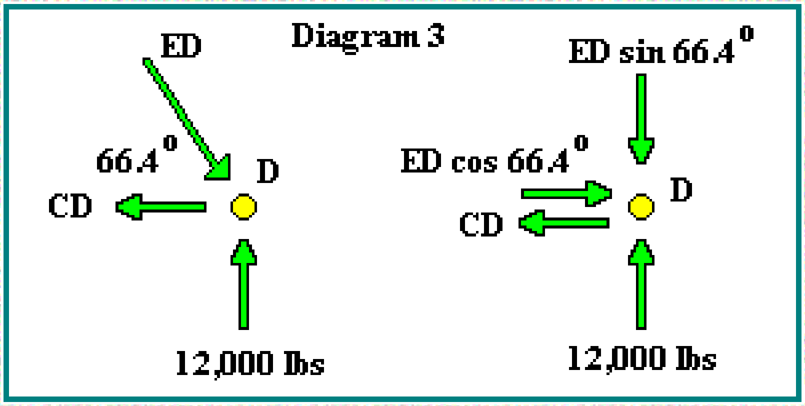
$\sum \tau_A = 0$: **(-12,000 lbs)(4 ft) - (20,000 lbs)(12 ft) + D_y(24 ft) = 0**

Solving for the unknowns: **D_y = 12,000 lbs; A_y = 20,000 lbs.** These are the external support reactions acting on the structure.

PART 2: Determine the internal force in member FC by method of joints. We begin at a joint with only two unknowns acting, joint D.

JOINT D:

STEP 1: Draw a free body diagram of the joint, showing and labeling all external forces and load, and including any needed angles.(Diagram 3) We select an initial direction for the unknowns, if their solution value is negative they act in a direction opposite to the direction initially selected.



STEP 2: Resolve all forces into x and y components. (Diagram 3).

STEP 3: Apply equilibrium conditions: $\sum F_x = 0 \; \sum F_y = 0$

$\sum F_x = 0$: **-CD + ED cos (66.4º) = 0**

$\sum F_y = 0$: **12,000 lbs - ED sin (66.4º)= 0**

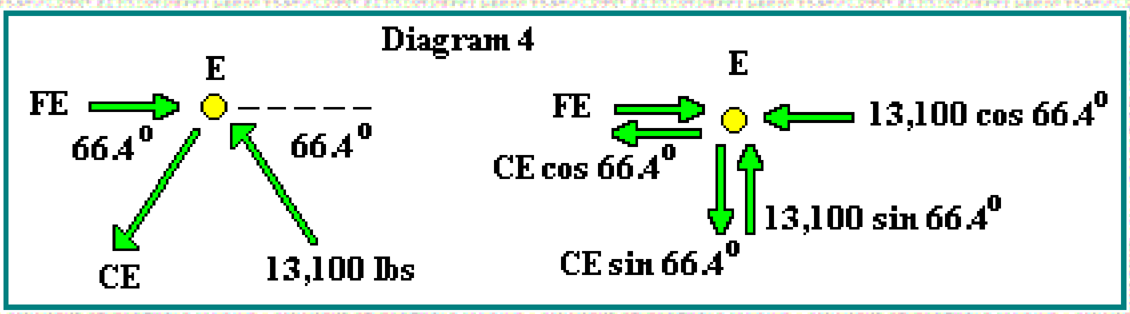
Solving for the unknowns: **ED = 13,100 lbs (C); CD = 5,240 lbs (T)**

Now that we have calculated the values for ED and CD we can move to joint E. We could not solve joint E initially as it had too many unknowns forces acting on it.

JOINT E:

STEP 1: Draw a free body diagram of the joint, showing and labeling all external forces and loads, and including any needed angles. (Diagram 4) We select an initial direction for the unknowns, if their solution value is negative they act in a direction opposite to the direction initially selected.

STEP 2: Resolve all forces into x and y components. (Diagram 4).



STEP 3: Apply equilibrium conditions: $\sum F_x = 0 \; \sum F_y = 0$

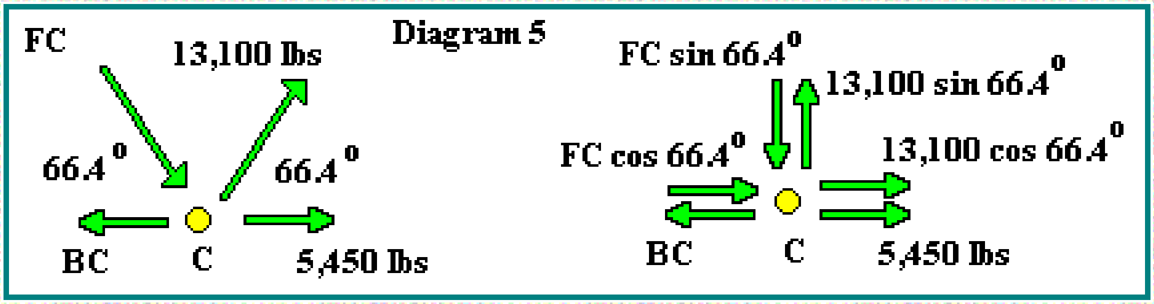
$\sum F_x = 0$ **FE -(13,100 lbs) cos (66.4º) - CE cos (66.4º) = 0**

$\sum F_y = 0$:**(13,100 lbs) sin (66.4º) - CE sin (66.4º) = 0**

Solving for the unknowns: **FE = 10,500 lbs (C); CE = 13,100 lbs (T)** (Since force values were positive, the initial direction chosen for the forces was correct.)

Now that we have calculated the values for FE and CE we move to joint C. We could not solve joint C initially as it had too many unknowns forces acting on it.

JOINT C:
STEP 1: Draw a free body diagram of the joint, showing and labeling all external forces and loads, and including any need angles. (Diagram 5) We select an initial direction for the unknowns, if their solution value is negative they act in a direction opposite to the direction initially selected.
STEP 2: Resolve all forces into x and y components. (Diagram 5).



STEP 3: Apply equilibrium conditions: $\sum F_x = 0 \; \sum F_y = 0$

$\sum F_x = 0$: **5,450 + (13,100 lbs) cos (66.4°) + FC cos (66.4°) - BC = 0**

$\sum F_y = 0$: **13,100 lbs sin (66.4°) - FC sin (66.4°) = 0**

Solving for the unknowns: **FC = 13,100 lbs (c); BC = 15,950 lbs (t)** Thus, member FC is in compression with a force of 13, 100 lbs.

Return to [Topic 2.2 - Trusses]
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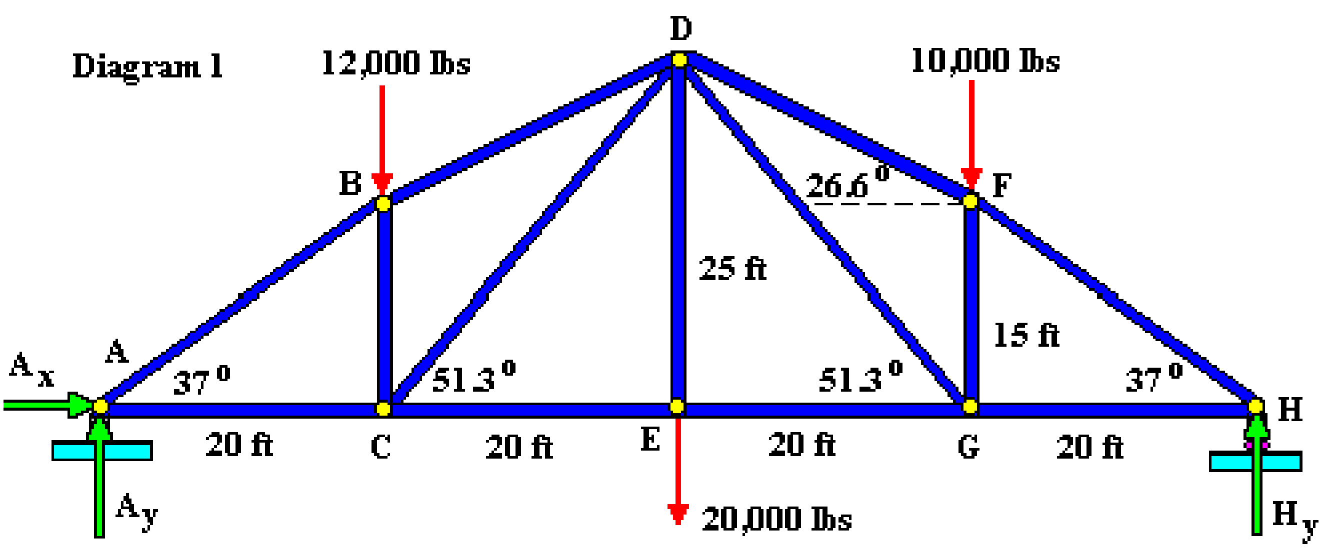
Topic 2.2 Trusses -Example 3

The structure shown in Diagram 1 is a truss which is pinned to the floor at point A, and supported by a roller at point H. For this structure we wish to determine the value of all the support forces acting on the structure, and to determine the force in member DG by method of sections.



We begin by determining the external support reactions acting on the structure.

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions. (Note in Diagram 1, we have replaced the pinned support by an unknown x and y force (A_x , A_y), and replaced the roller

support by the vertical unknown force H_y

STEP 2: Break any forces not already in x and y direction into their x and y components. (All forces in x/y directions.)

STEP 3: Apply the equilibrium conditions. $\sum Fx = 0; \ \sum Fy = 0; \ \sum \tau_P = 0$

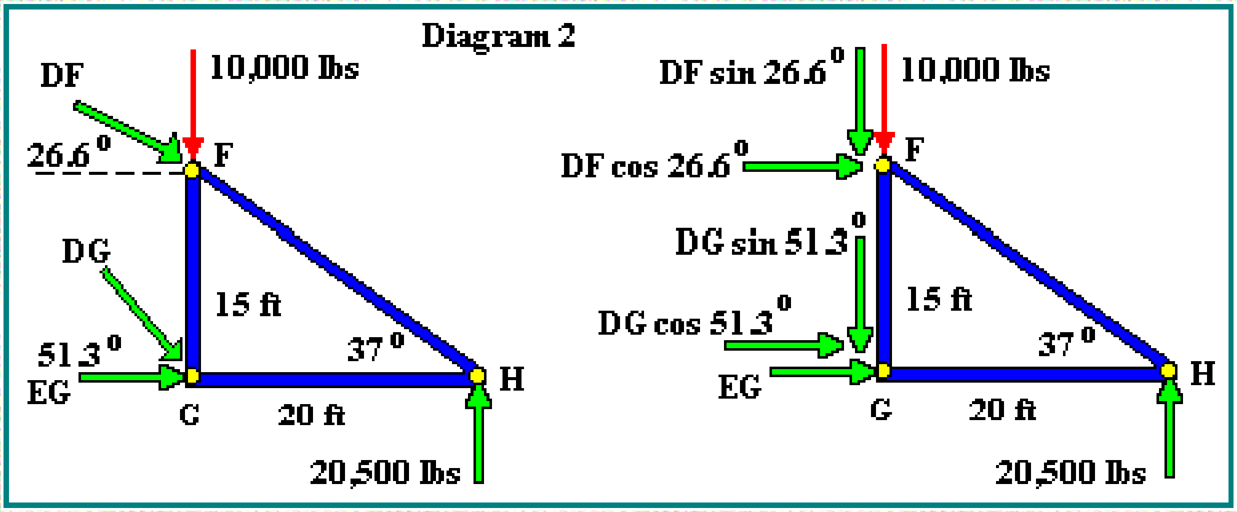
$\sum F_x = 0$ **: $A_x = 0$**

$\sum F_y = 0$**: $A_y + H_y$ -12,000 lbs - 20,000 lbs - 10,000 lbs = 0**

$\sum \tau_A = 0$**: (-12,000 lbs)(20 ft) - (20,000 lbs)(40 ft) - (10,000 lbs)(60 ft) + H_y(80 ft) = 0**

Solving for the unknowns: **A_y = 21,500 lbs; H_y = 20,500 lbs.** These are the external support reactions acting on the structure.

Part 2: Now we will find internal force in member DG by method of sections. Cut the truss vertically with a line passing through members DF, DG, and EG. We have shown the section of the truss to the right of the cut. We now treat this section of the truss as if it were a completely new structure. The internal forces in members DF, DG, and EG now become external forces with respect to this section. We have represented these forces with the arrows shown. The forces must act along the direction of the cut member (since all members in a truss are axial members), and we have selected an initial direction either into or away from the section for each of the forces. If we have selected an incorrect initial direction for a force, when we solve for the value of the force, the value will be negative indicating the force acts in the opposite direction of the one chosen initially. We may now proceed with the analysis of this structure using standard Static's techniques.



I. Draw a Free Body Diagram of the structure (section), showing and labeling all external forces, and indicating needed dimensions and angles. (Diagram 2)

II. Resolve (break) all forces into their x and y-components. (Diagram 2)

III. Apply the Equilibrium Equations ($\sum Fx = 0;\ \sum Fy = 0;\ \sum \tau_P = 0$)

$\sum F_x = 0$ **:EG + DG cos (51.3°) + DF cos (22.6°) = 0**

$\sum F_y = 0$ **: -10,000 lbs + 20,500 lbs - DG sin (51.3°) - DF sin (26.6°) = 0**

$\sum \tau_G = 0$ **: -DF cos (26.6°)(15 ft) + (20,500 lbs)(20 ft) = 0**

Solving for the unknowns: **DF = 30,600 lbs (C)**; **DG = -4,090 (opposite direction)= 4,090 lbs (T); EG = -24,800(opposite direction)= 24,800 lbs (T)**

Return to Topic 2.2 - Trusses

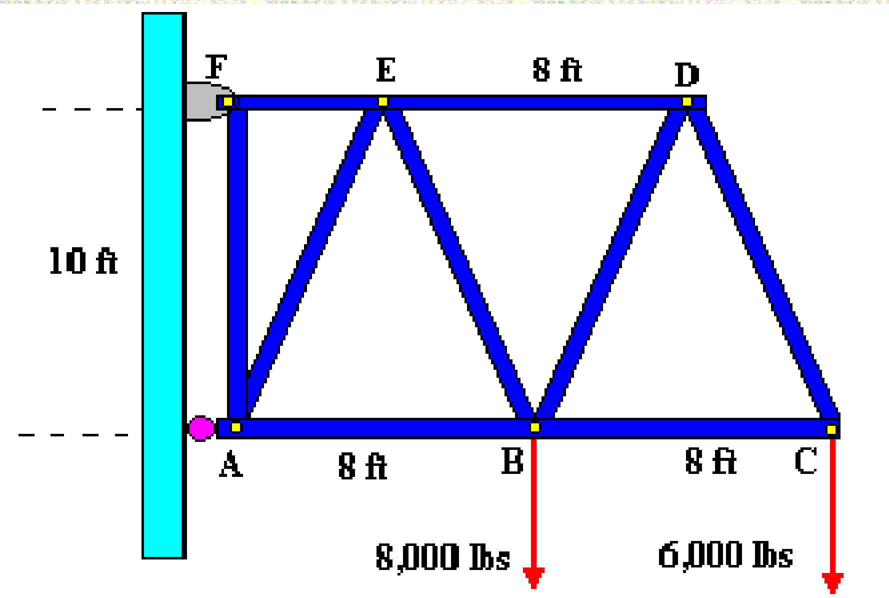
or select:

Topic 2: Statics II - Applications-Topic Table of Contents

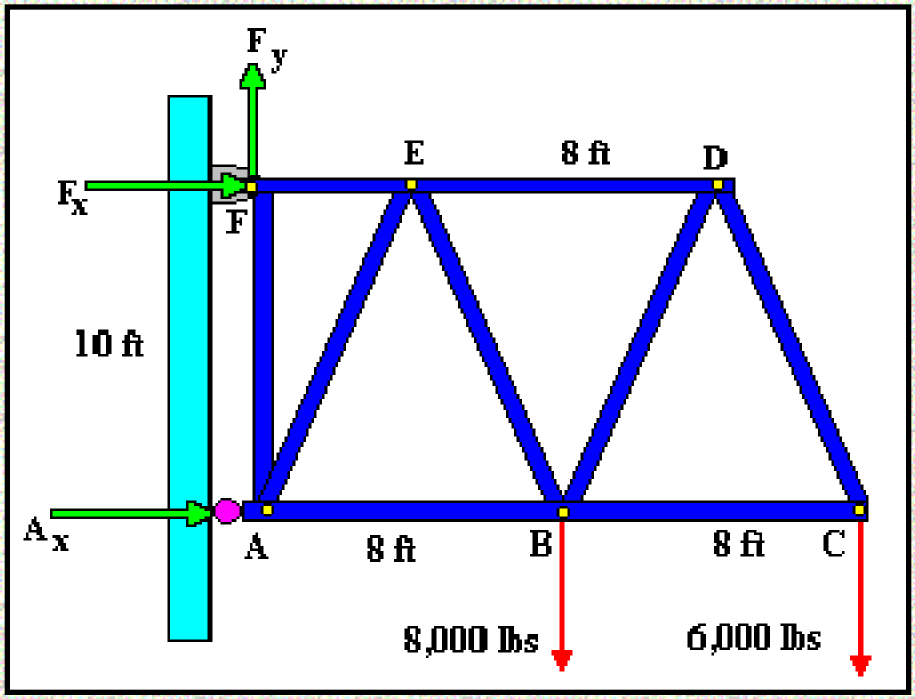
[Strength of Materials Home Page]

STATICS / STRENGTH OF MATERIALS - Example

The structure shown below is a truss which is pinned to the wall at point F, and supported by a roller at point A. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force (tension or compression) in member EB by method of joints.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
PARTS A & B:
STEP 1: Draw a free body diagram showing and labeling all load forces and



support (reaction) forces, as well as any needed angles and dimensions.

STEP 2: Break any forces not already in x and y direction into their x and y components

STEP 3: Apply the equilibrium conditions.

Sum $F_x = A_x + F_x = 0$

Sum $F_y = F_y - 8{,}000$ lbs - 6,000 lbs = 0

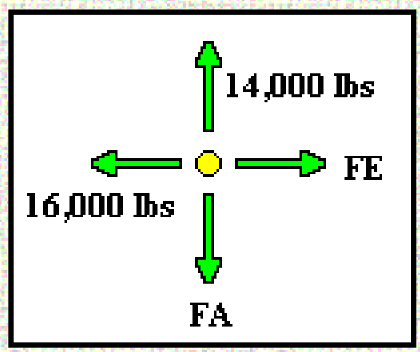
Sum T_A = (-8,000 lbs)(8 ft) - (6,000 lbs)(16 ft) - F_x(10 ft) = 0

Solving for the unknowns:

A_x = 16,000 lbs; F_x = -16,000 lbs; F_y = 14,000 lbs

PART C - Now find internal force in member EB by method of joints.

JOINT F:



STEP 1: Draw a free body diagram of the joint.

STEP 2: Resolve all forces into x and y components (see diagram).

STEP 3: Apply equilibrium conditions:

Sum F_x = FE - 16,000 lbs = 0

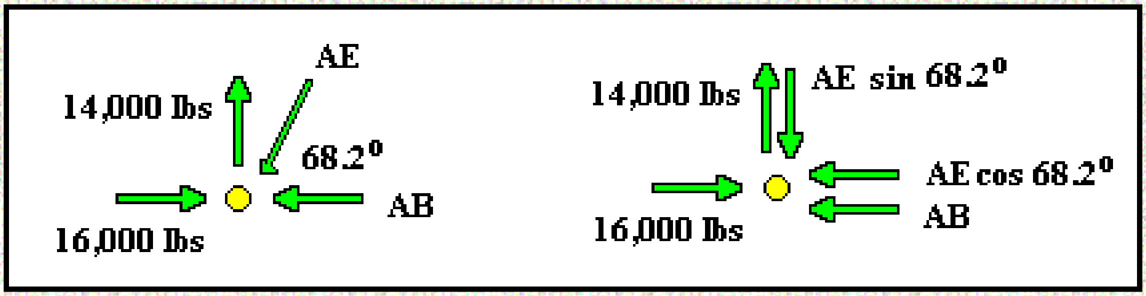
Sum F_y = 14,000 lbs - FA = 0

Solving for the unknowns:

FE = 16,000 lbs (t); FA = 14,000 lbs (t)

JOINT A:

STEP 1: Draw a free body diagram of the joint.
STEP 2: Resolve all forces into x and y components (see diagram).



STEP 3: Apply equilibrium conditions:

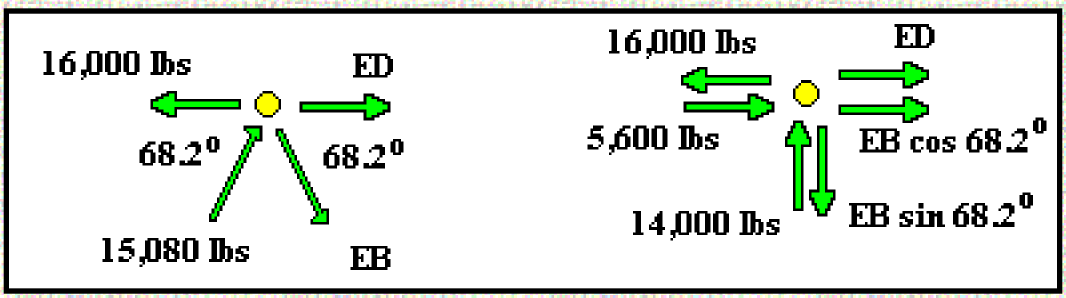
Sum F_x = 16,000 lbs - AB -AE cos (68.2°) = 0

Sum F_y = 14,000 lbs - AE sin (68.2°) = 0

Solving for the unknowns: AE = 15,080 lbs (c); AB = 10,400 lbs (c)

JOINT E:

STEP 1: Draw a free body diagram of the joint.
STEP 2: Resolve all forces into x and y components (see diagram).



STEP 3: Apply equilibrium conditions:

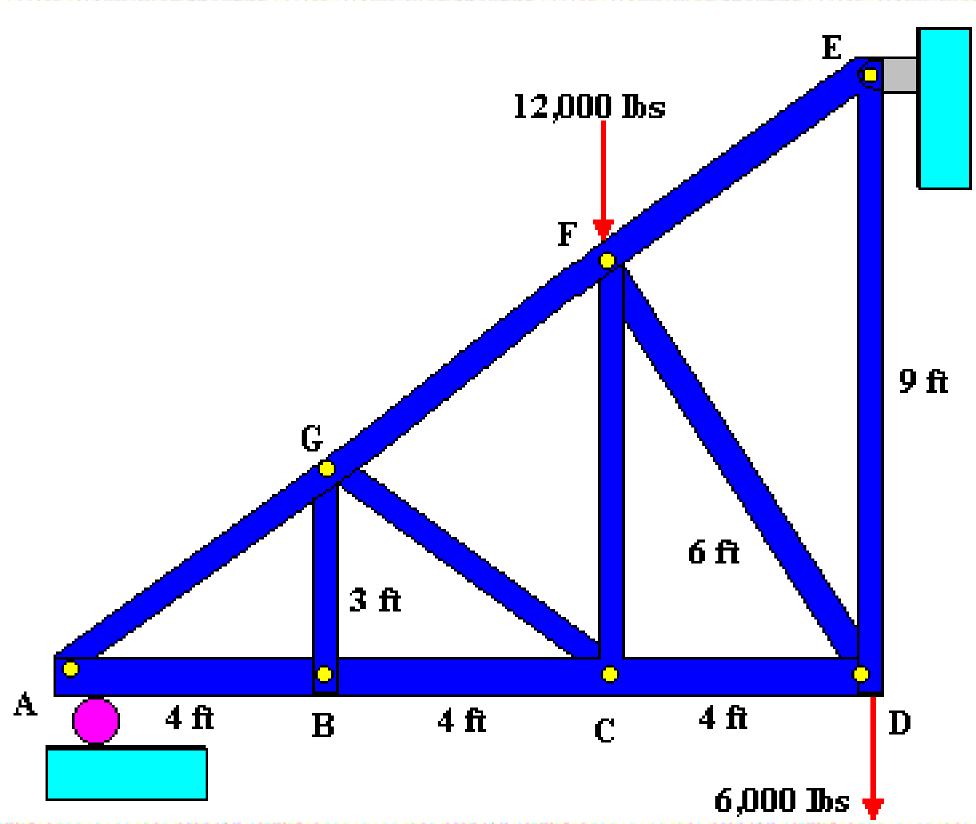
Sum F_x = -16,000 lbs + ED + 5,600 lbs + EB cos (68.2°) = 0

Sum F_y = 14,000 lbs - EB sin (68.2°) = 0

Solving for the unknowns: ED = 4,800 lbs (t); EB = 15,080 lbs (t)

STATICS / STRENGTH OF MATERIALS - Example

The structure shown below is a truss which is pinned to the wall at point E, and supported by a roller at point A. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force in member GC by method of sections.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



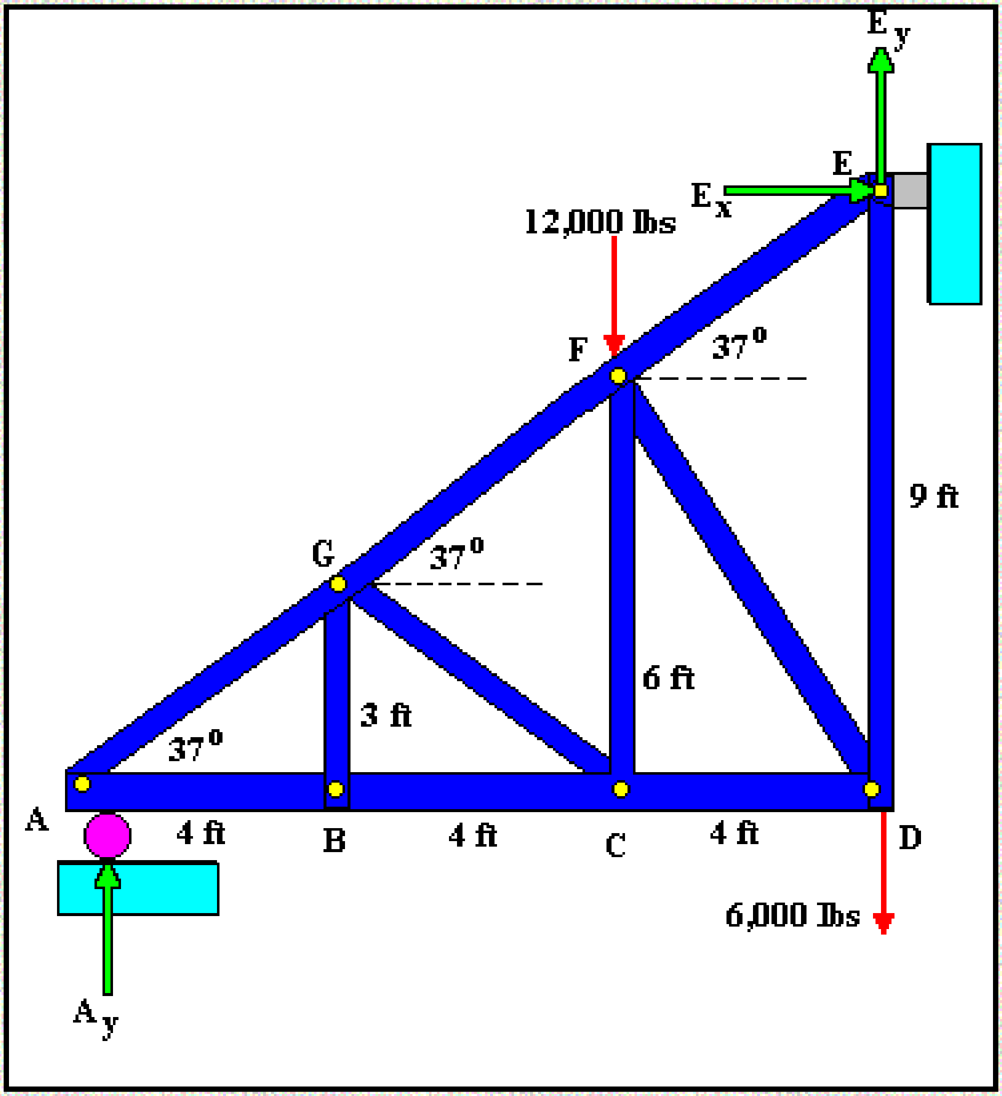
Solution:
PARTS A & B:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.
Sum $F_x = E_x = 0$

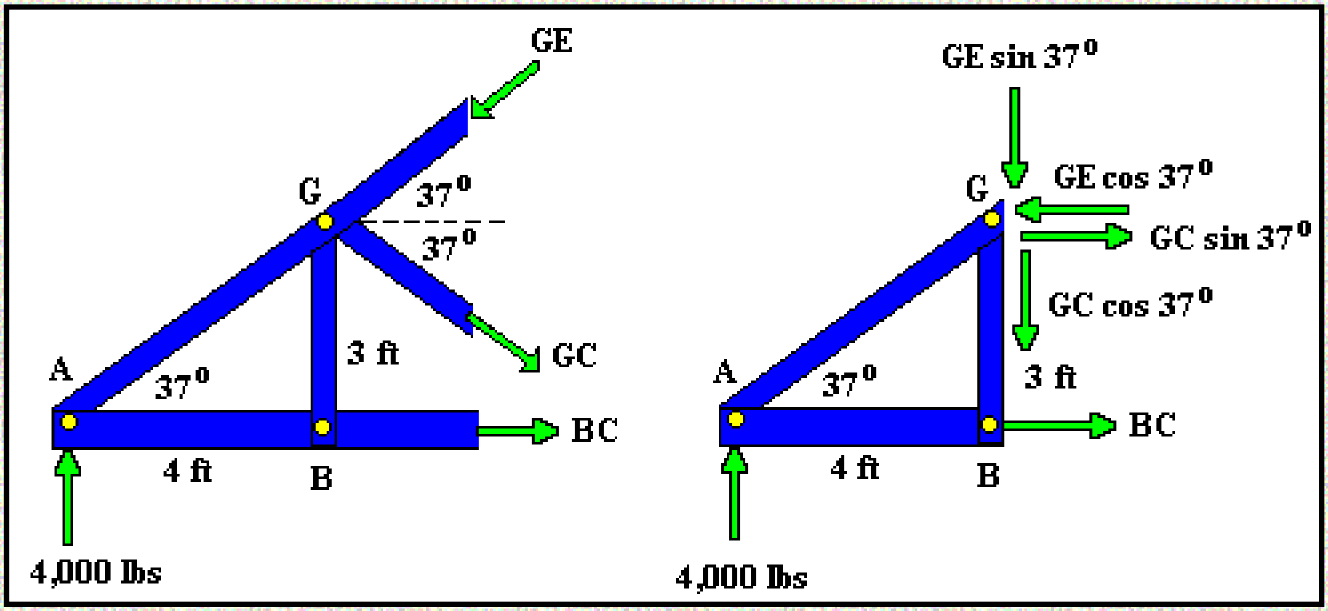
Sum $F_y = A_y + E_y - 12{,}000$ lbs - 6,000 lbs = 0

Sum T_E = (12,000 lbs)(4 ft) - A_y(12 ft) = 0

Solving for the unknowns:
E_y = 14,000 lbs; A_y = 4,000 lbs



PART C: - Now find the internal force in member GC by method of section
STEP 1: Cut the structure into "2 sections" with a vertical line which cuts through



members GE, GC and BC, and is just to the right of point G (see diagram). The internal forces in members GE, GC and BC now become external forces acting on the left hand section as shown. (We chose directions for these forces which may or may not be correct, but which will become clear when we solve for their values.)

STEP 2: Now treat the section shown as a new structure and apply statics procedure

- Draw a free body diagram of the left hand section.
- Resolve all forces into x an y components (see diagram).
- Apply equilibrium conditions:

Sum F_x = - GE cos (37º) + GC cos (37º) + BC = 0

Sum F_y = 4,000 lbs - GE sin (37º) - GC sin (37º) = 0

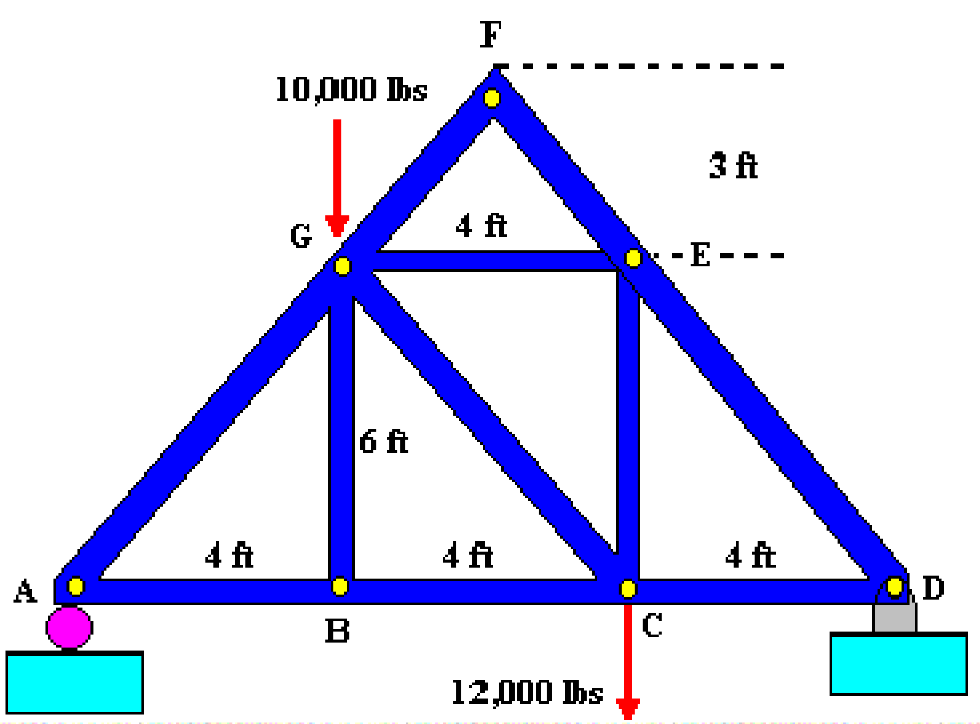
Sum T_G = (-4,000 lbs)(4 ft) + BC(3 ft) = 0

Solving for the unknowns:

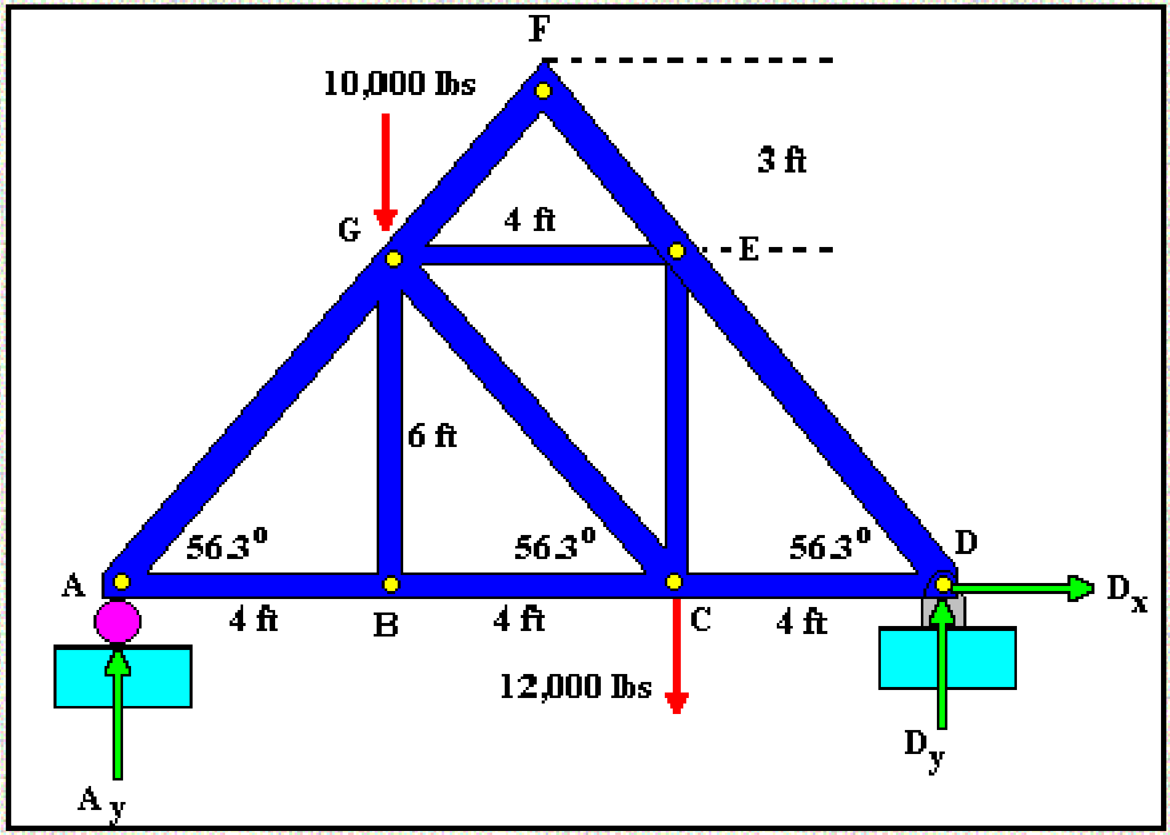
BC = 5,330 lbs; GE = 6,670 lbs; GC = 0 lbs

STATICS / STRENGTH OF MATERIALS - Example

The structure shown below is a truss which is pinned to the floor at point D, and supported by a roller at point A. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force in member GC by method of joints.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
PARTS A & B:
STEP 1: Draw a free body diagram showing and labeling all load forces and



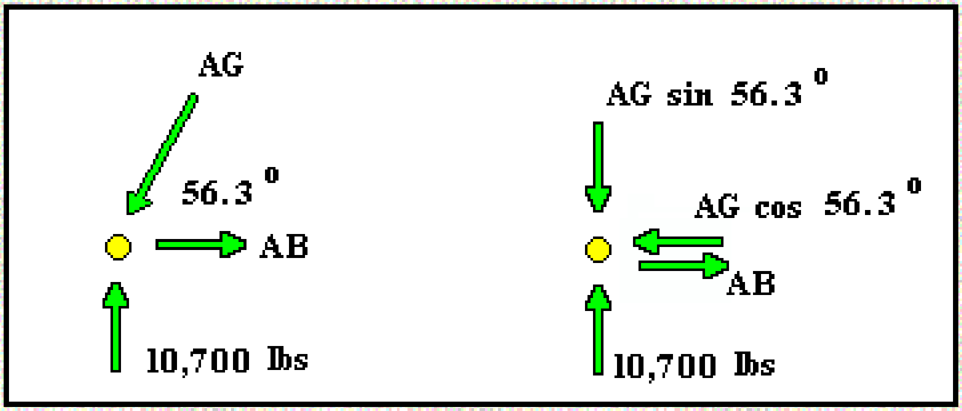
support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.
Sum $F_x = D_x = 0$

Sum $F_y = A_y + D_y$ - 10,000 lbs - 12,000 lbs = 0
Sum $T_A = D_y$(12 ft) - (12,000 lbs)(8 ft) - (10,000 lbs)(4 ft) = 0
Solving for the unknowns:
D_y = 11,300 lbs; A_y = 10,700 lbs

PART C - Now find internal force in member GC by method of joints.

JOINT A:

STEP 1: Draw a free body diagram of the joint.
STEP 2: Resolve all forces into x and y components (see diagram).
STEP 3: Apply equilibrium conditions:



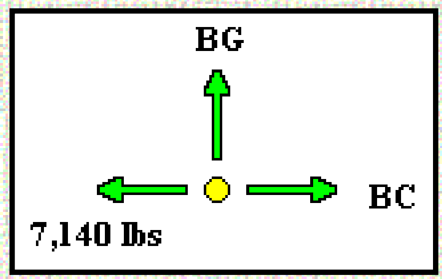
Sum F_x = AB - AG cos (56.3º) = 0

Sum F_y = 10,700 lbs - AG sin (56.3º) = 0

Solving for the unknowns: AG = 12,900 lbs (c); AB = 7,140 lbs (t)

JOINT B:

STEP 1: Draw a free body diagram of the joint.
STEP 2: Resolve all forces into x and y components (see diagram).
STEP 3: Apply equilibrium conditions:



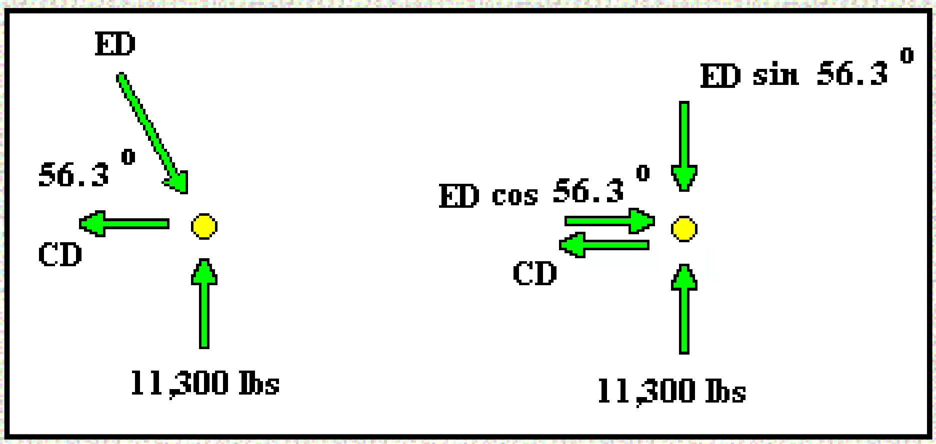
Sum F_x = -7,140lbs + BC = 0

Sum F_y = BG = 0

Solving for the unknowns: BG = 0 lbs ; BC = 7,140 lbs (t)

JOINT D:

STEP 1: Draw a free body diagram of the joint.
STEP 2: Resolve all forces into x and y components (see diagram).
STEP 3: Apply equilibrium conditions:

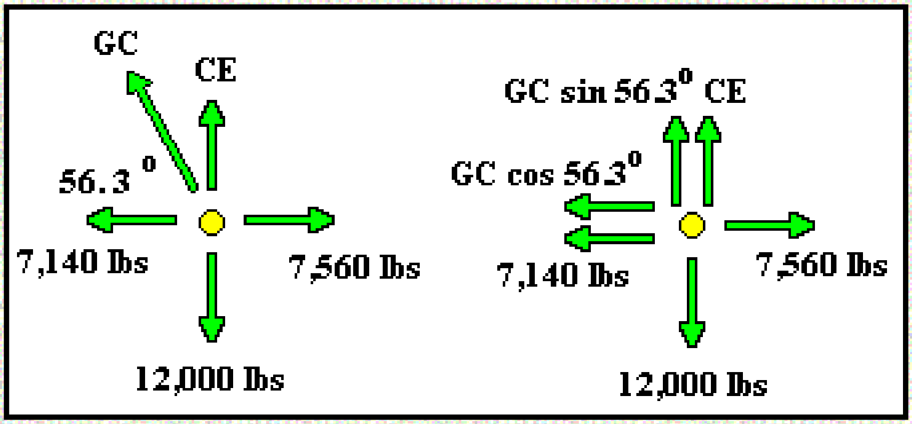


Sum F_x = -CD +ED cos (56.3º) = 0

Sum F_y = 11,300 lbs - ED sin (56.3º) = 0

Solving for the unknowns: CD = 7,560 lbs (t); ED = 13,600 lbs (c)

JOINT C:



STEP 1: Draw a free body diagram of the joint.
STEP 2: Resolve all forces into x and y components (see diagram).
STEP 3: Apply equilibrium conditions:
Sum F_x = 7,560 lbs - 7,140 lbs - GC cos (56.3º) = 0
Sum F_y = CE - 12,000 lbs - GC sin (56.3º) = 0
Solving for the unknowns: CE = 11,400 lbs (t); GC = 730 lbs (t)

STATICS / STRENGTH OF MATERIALS - Example

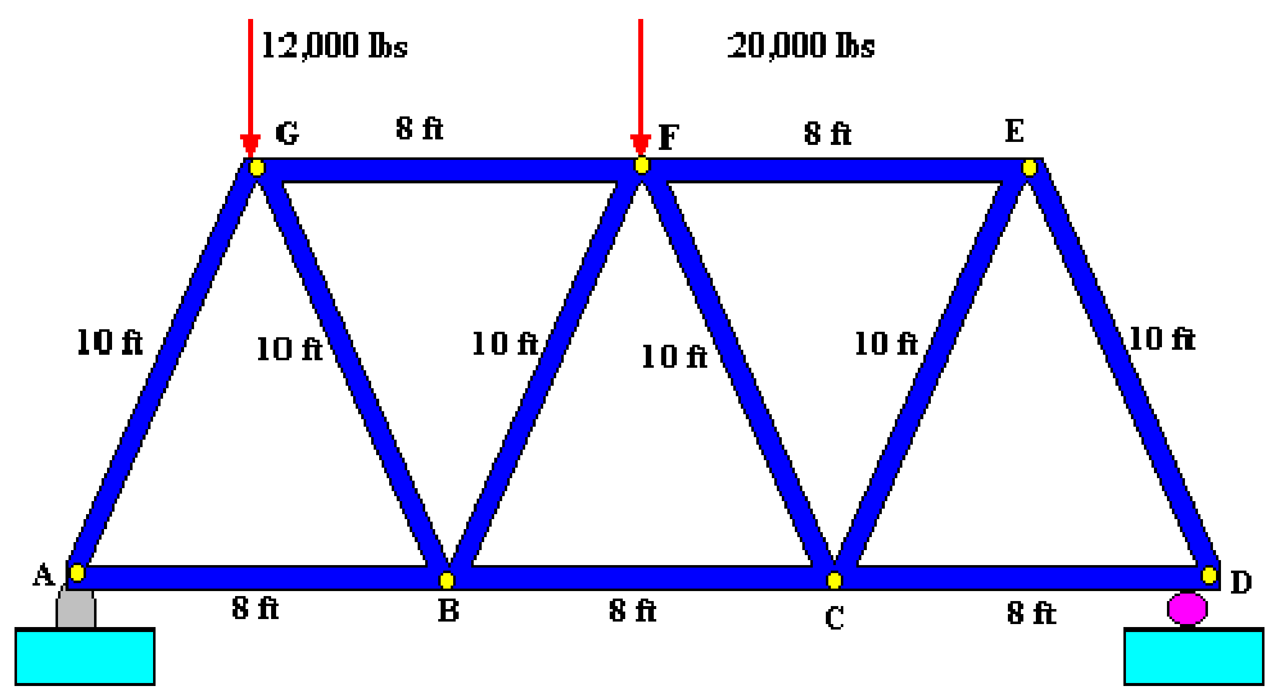
The structure shown below is a truss which is pinned to the floor at point A, and supported by a roller at point D. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.

B. Determine the value of all the support forces acting on the structure.

C. Determine the force in member FC by method of joints.

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

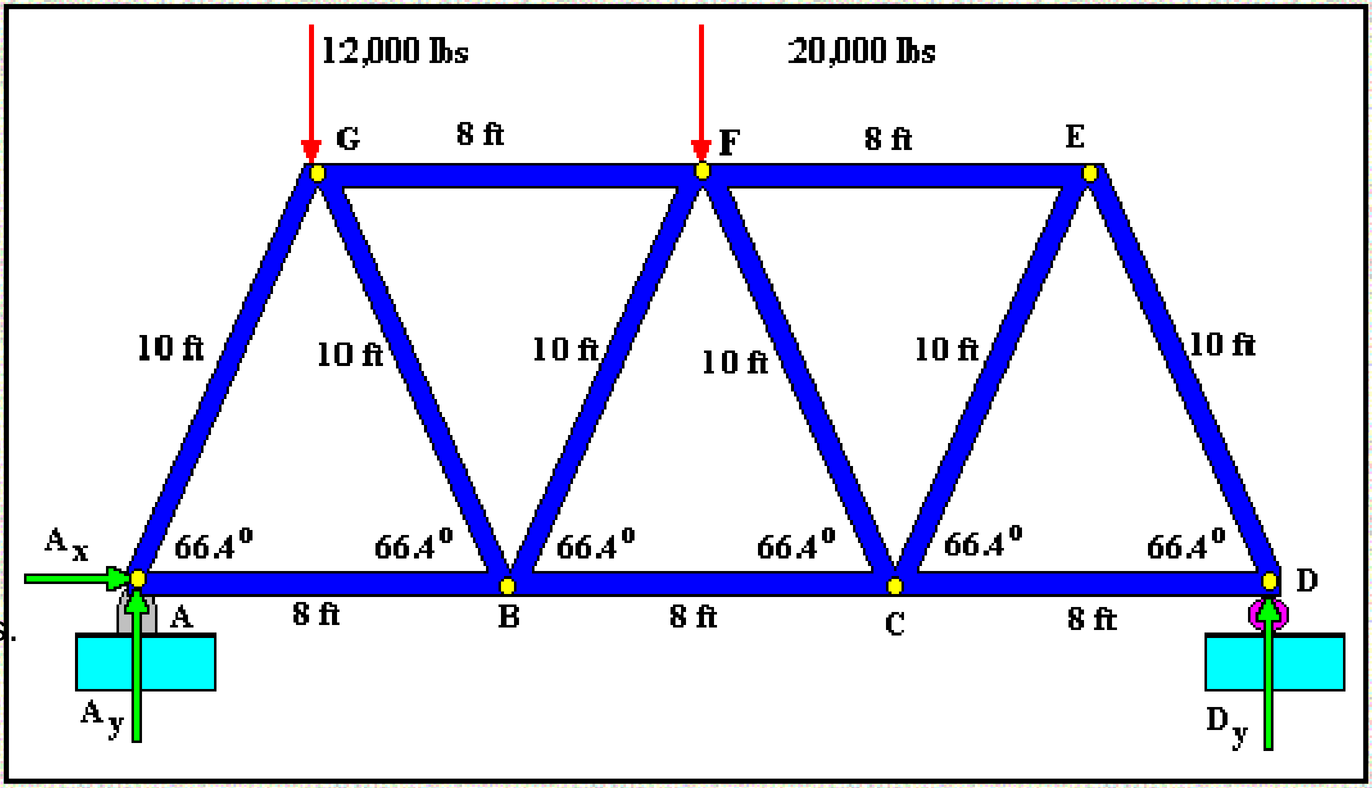


Solution:

PARTS A & B:

STEP 1: Draw a free body diagram showing and labeling all load forces and support

(reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



Sum $F_x = A_x = 0$

Sum $F_y = A_y + D_y - 12{,}000 \text{ lbs} - 20{,}000 \text{ lbs} = 0$

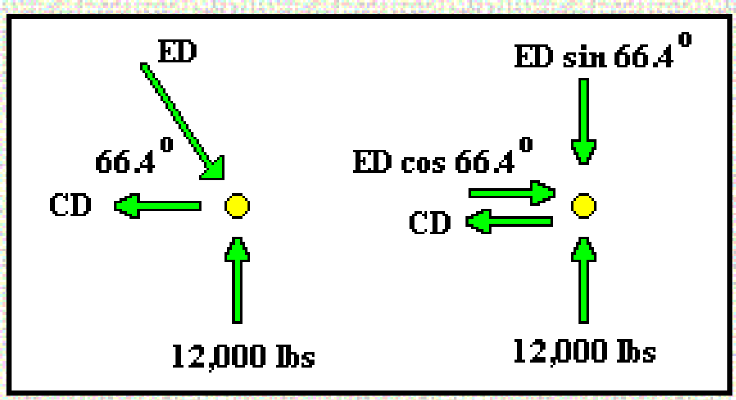
Sum $T_A = (-12{,}000 \text{ lbs})(4 \text{ ft}) - (20{,}000 \text{ lbs})(12 \text{ ft}) + D_y(24 \text{ ft}) = 0$

Solving for the unknowns: $D_y = 12{,}000$ lbs; $A_y = 20{,}000$ lbs

PART C - Now find internal force in member FC by method of joints.

JOINT D:

STEP 1: Draw a free body diagram of the joint.
STEP 2: Resolve all forces into x and y components (see diagram).
STEP 3: Apply equilibrium conditions:



Sum $F_x = -CD + ED \cos(66.4^o) = 0$

Sum $F_y = 12{,}000 \text{ lbs} - ED \sin(66.4^o) = 0$

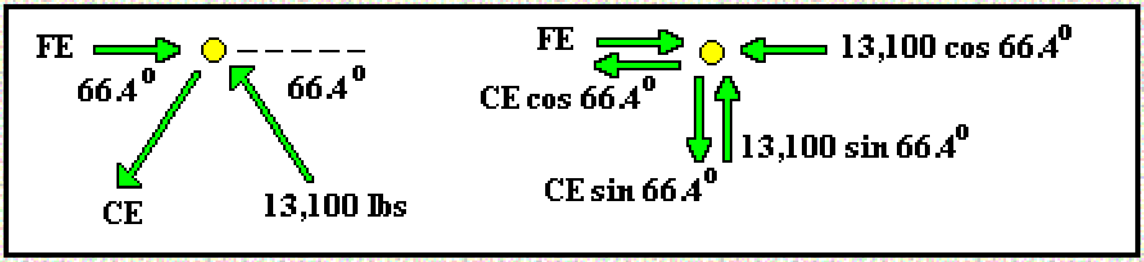
Solving for the unknowns:

ED = 13,100 lbs (c); CD = 5,450 lbs (t)

JOINT E:

STEP 1: Draw a free body diagram of the joint.

STEP 2: Resolve all forces into x and y components (see diagram).



STEP 3: Apply equilibrium conditions:

Sum F_x = (-13,100 lbs) cos (66.4°) + CE cos (66.4°) = 0

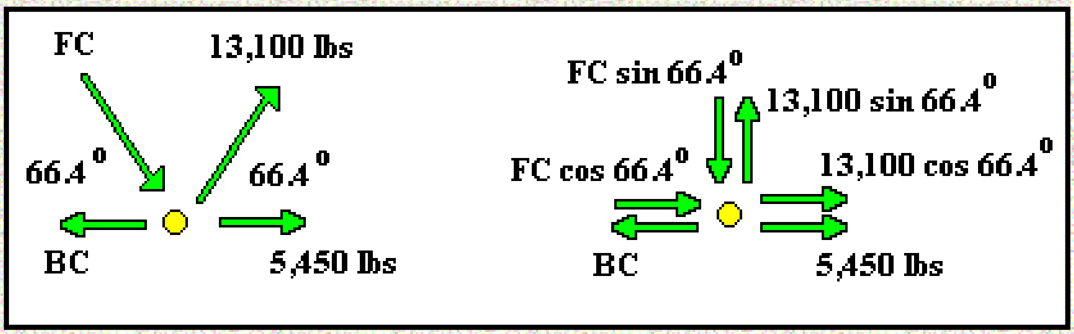
Sum F_y = (13,100 lbs) sin (66.4°) - CE sin (66.4°) = 0

Solving for the unknowns: FE = 10,500 lbs (c); CE = 13,100 lbs (t)

JOINT C:

STEP 1: Draw a free body diagram of the joint.

STEP 2: Resolve all forces into x and y components (see diagram).



STEP 3: Apply equilibrium conditions:

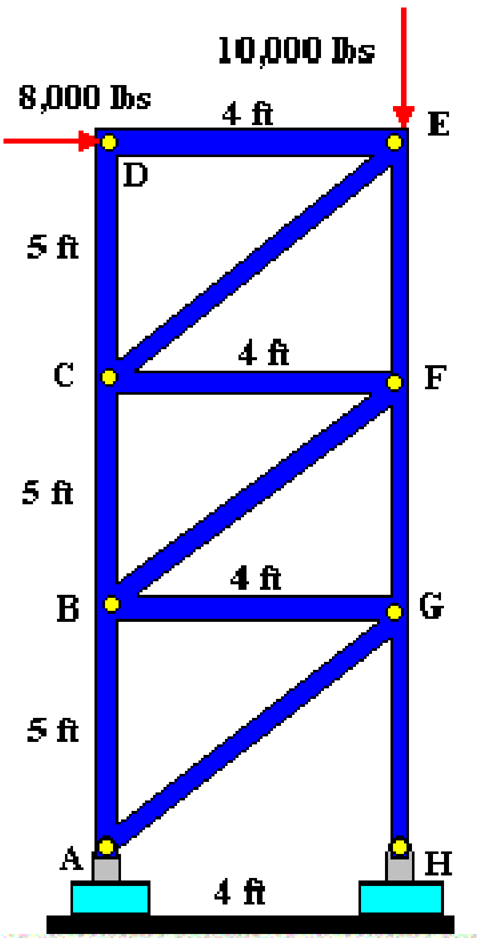
Sum F_x = 5,450 + (13,100 lbs) cos (66.4°) + FC cos (66.4°) - BC = 0

Sum F_y = (13,100 lbs) sin (66.4°) - FC sin (66.4°) = 0

Solving for the unknowns: FC = 13,100 lbs (c); BC = 15,950 lbs (t)

STATICS / STRENGTH OF MATERIALS - Example

The structure shown below is a truss which is pinned to the floor at point A and also at point H. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force in member FB by any method.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
PARTS A & B:
STEP 1: Draw a free body diagram showing and labeling all load forces and

support (reaction) forces, as well as any needed angles and dimensions.

STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

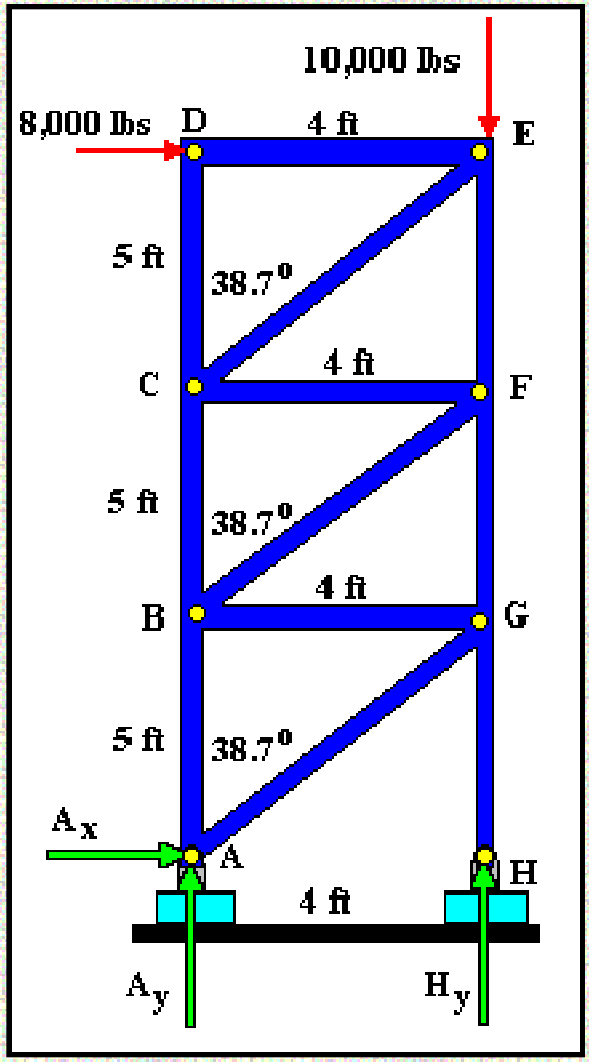
Sum $F_x = A_x + 8{,}000$ lbs $= 0$

Sum $F_y = A_y + H_y - 10{,}000$ lbs $= 0$

Sum $T_A = H_y(4 \text{ ft}) - (10{,}000 \text{ lbs})(4 \text{ ft}) - (8{,}000 \text{ lbs})(15 \text{ ft}) = 0$

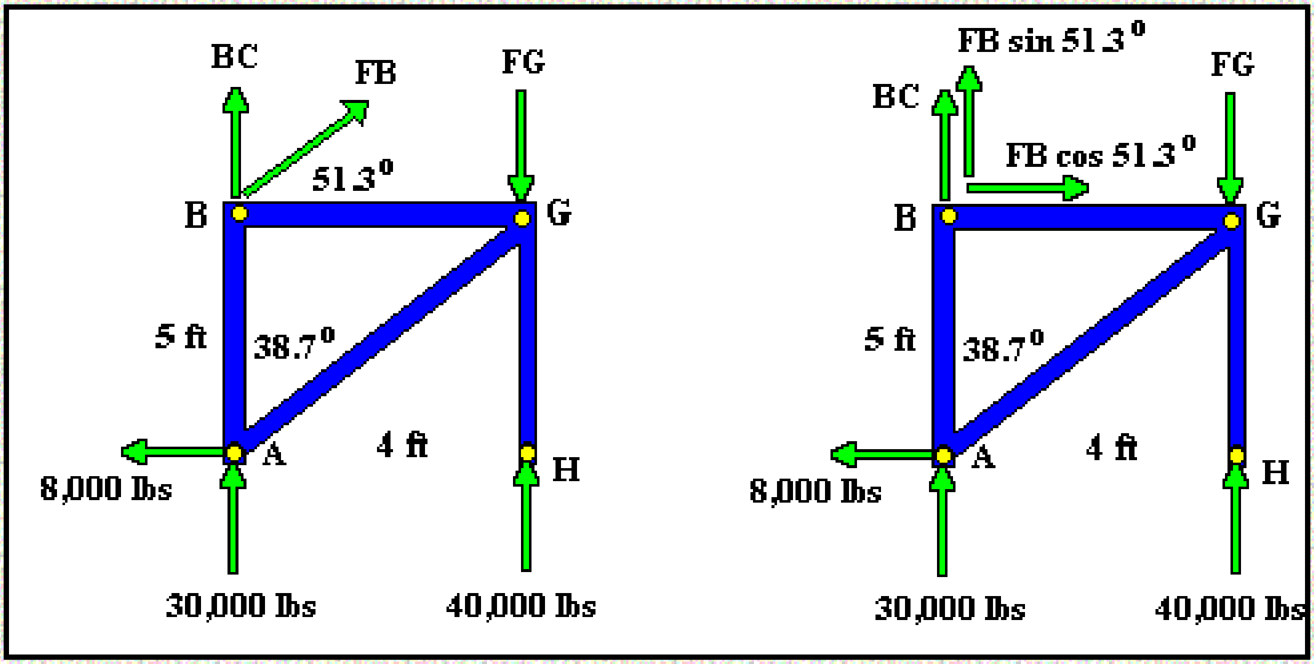
Solving for the unknowns:

$H_y = 40{,}000$ lbs; $A_x = -8{,}000$ lbs; $A_y = -30{,}000$ lbs



PART C - Now find internal force in member FB by section method. Cut horizontally through members BC, FB, and FG. Analyze lower section.

STEP 1: Draw a free body diagram of the lower section (see diagram).



STEP 2: Resolve all forces into x and y components (see diagram).

STEP 3: Apply equilibrium conditions:

Sum F_x = FB cos (51.3º) - 8,000 lbs = 0

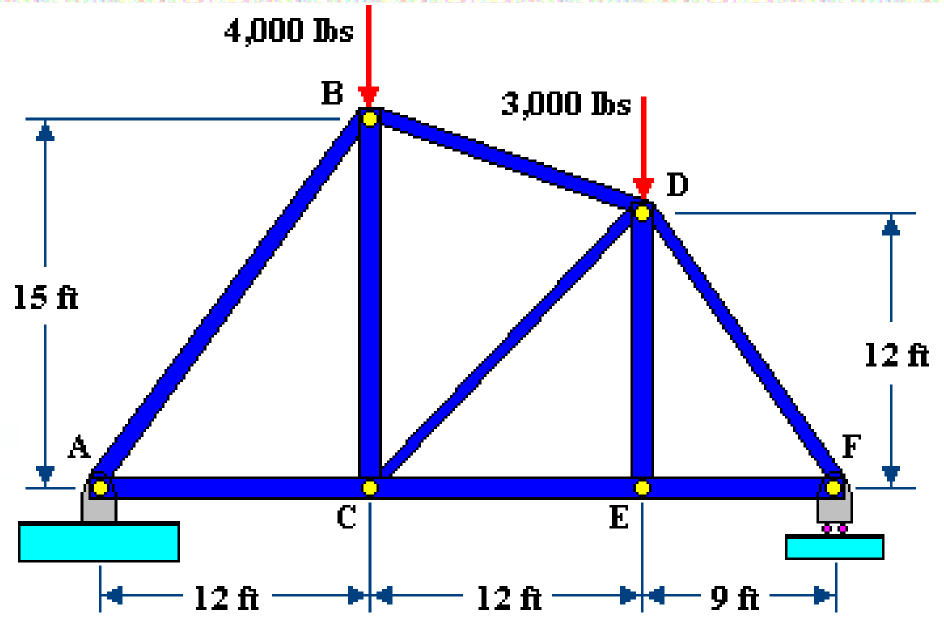
Sum F_y = BC - 30,000 lbs + FB sin (51.3º) - GF + 40,000 lbs = 0

Sum T_B = (-8,000 lbs)(5 ft) - GF(4 ft) + (40,000 lbs)(4 ft) = 0

Solving for the unknowns: FB = 12,8000 lbs (t); GF = 30,000 lbs (c); BC = 10,000 lbs (t)

STATICS / STRENGTH OF MATERIALS - Example

The structure shown below is a truss which is pinned to the floor at point A and supported by a roller at point F. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the value of all the support forces acting on the structure.
C. Determine the force in member CD by any method.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
PARTS A & B:
STEP 1: Draw a free body diagram showing and labeling all load forces and support

(reaction) forces, as well as any needed angles and dimensions.

STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

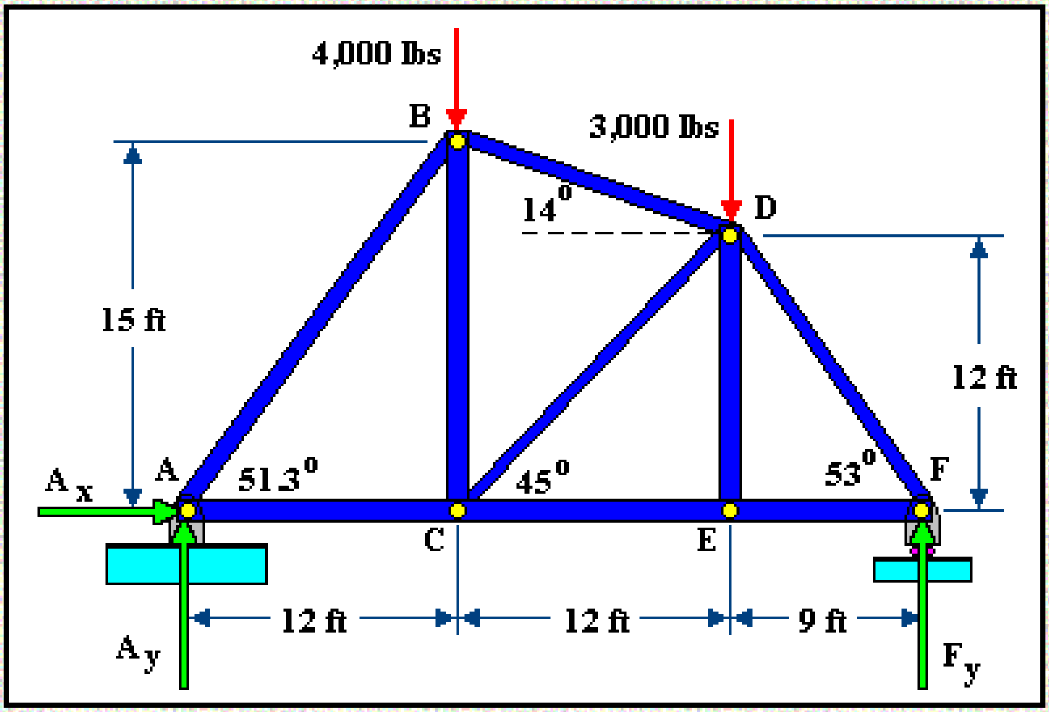
Sum $F_x = A_x = 0$

Sum F_y = -4,000 lbs - 3,000 lbs + A_y + F_y = 0

Sum T_A = (-4,000 lbs)(12 ft) - (3,000 lbs)(24 ft) + F_y (33 ft) = 0

Solving for the unknowns:

F_y = 3,640 lbs; A_y = 3,360 lbs



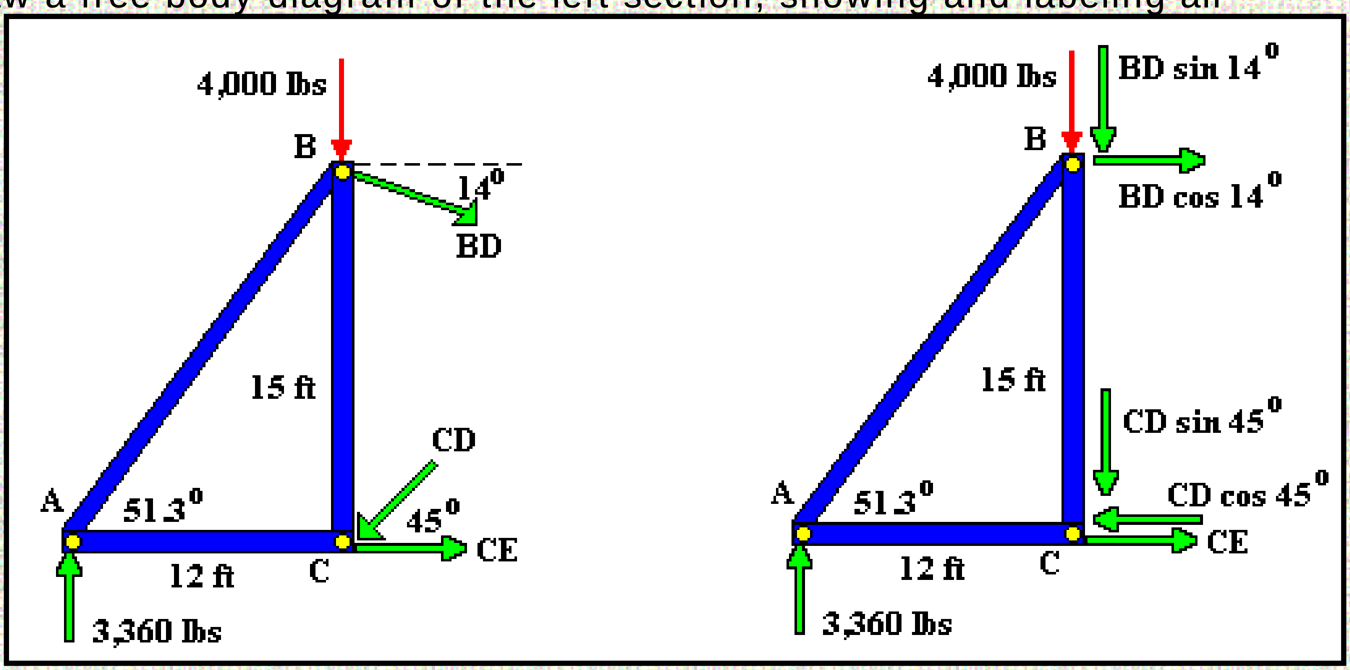
PART C - Now find internal force in member CD by sections. Cut vertically through members BD, CD, and CE near to points B and C. Analyze left section as shown in diagram.

STEP 1: Draw a free body diagram of the left section, showing and labeling all external loads and forces.

STEP 2: Resolve all forces into x and y components.

STEP 3: Apply the equilibrium conditions:

Sum F_x = BD cos (14°) - CD



cos (45º) - CE = 0
Sum F_y = -4,000 lbs + 3,360 lbs - BD sin (14º) - CD sin (45º) = 0

Sum T_C = (-3,360 lbs)(12 ft) - BD cos (14º)(15 ft) = 0

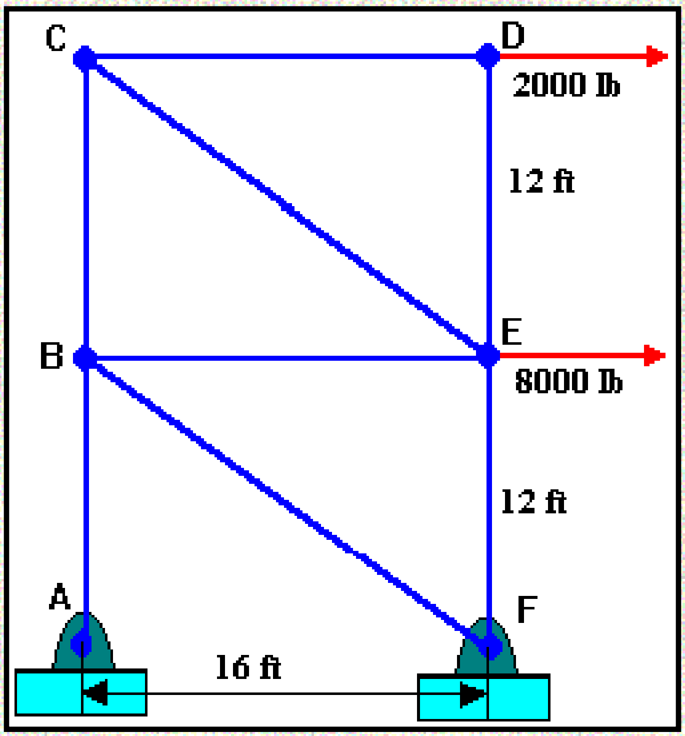
Solving for the unknowns:
BD = -2,770 lbs (opposite direction), 2,770 lbs (c); CD = 43 lbs (c); CE =2,720 lbs (t)

Statics & Strength of Materials

Problem Assignment - Trusses 1

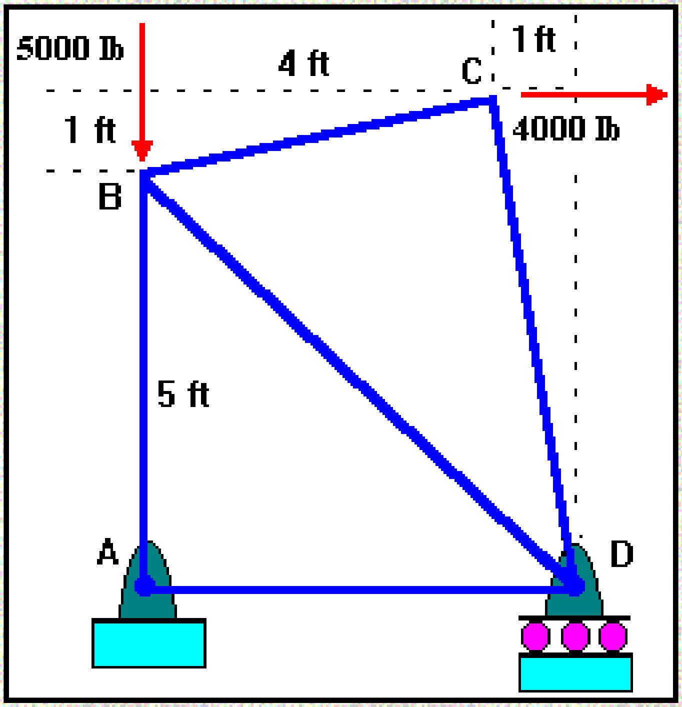
1. The structure shown is a truss composed of axial members pinned together at the joints. The stucture is pinned to the floor at points A and F. Determine the external force support reactions, and the force in member BE by method of joints.

(Ay = -9,000 lb, Fx = - 10,000 lb, Fy = 9,000 lb
BE = 10,000 lb (T))



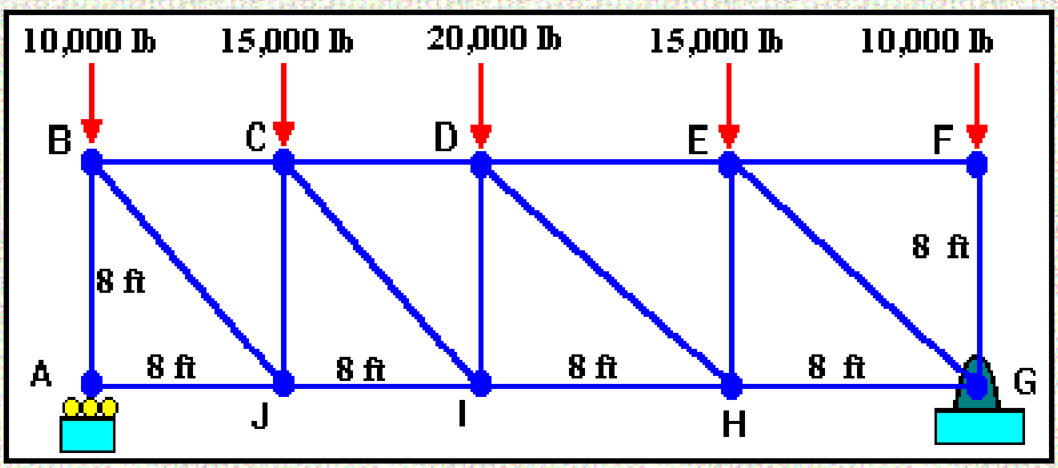
2. The structure shown is a truss composed of axial members pinned together at the joints. The stucture is pinned to the floor at point A and supported by a roller at point D. Determine the external force support reactions, and the force in member CD by method of joints.

(Ax = -4000 lb, Ay = 200 lb, Dy = 4800 lb
CD = 970 lb (C))



3. The structure shown is a truss composed of axial members pinned together at the joints. The stucture is supported by a roller at point A and pinned to the floor at point G. Determine the external force support reactions, and the force in members DE, DH and IH by method of sections

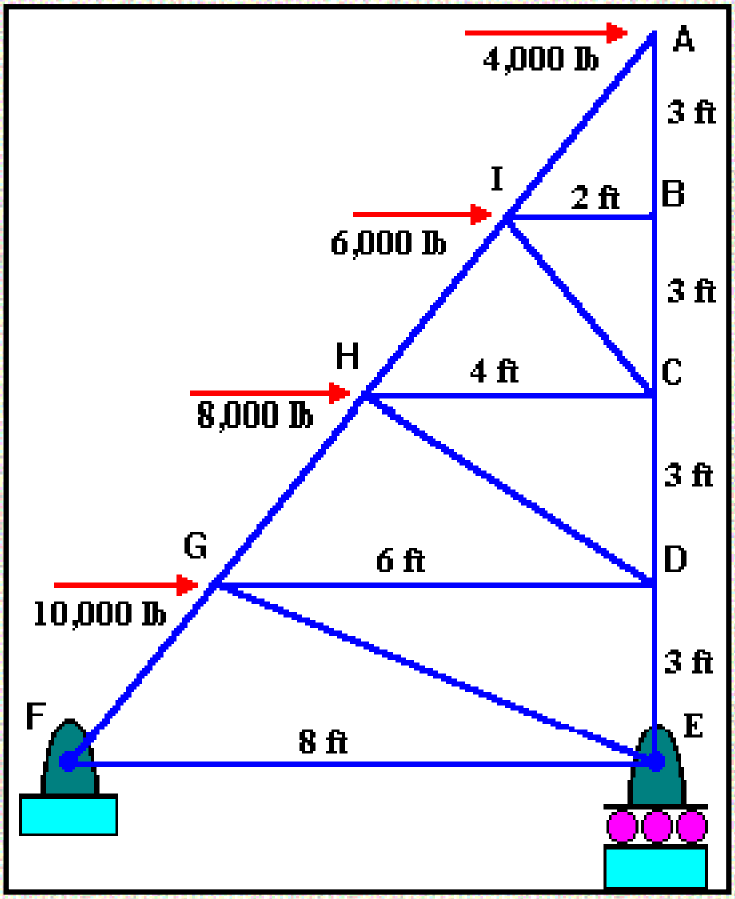
(Ay = 35,000 lb, Gy = 35,000 lb, DE = 25,000 lb (C), DH = 14,140 (C), IH = 35,000 lb (T))



4. The structure shown is a truss composed of axial members pinned together at the joints. The stucture is pinned to the floor at point E and supported by a roller at point F. Determine the external force support reactions and the force in

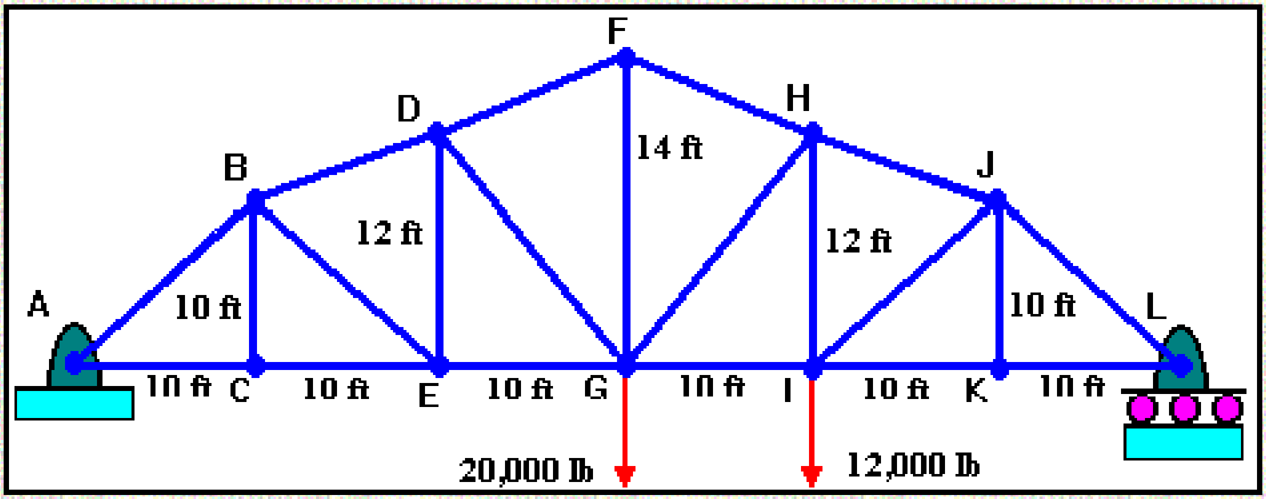
members HI, HC and DC by method of sections.

(Fx = -28,000 lb, Fy = - 22,500 lb,Ey = 22,500 lb
HI = 12,650 lb (T), HC = 3000 lb (T)
CD = 10,500 lb (C))



5. The structure shown is a truss composed of axial members pinned together at the joints. The stucture is pinned to the floor at point A and supported by a roller at point L.Determine the external force support reactions and the force in members DF, DG and EG by any method.

(Ay = 14,000 lb, Ly = 18,000 lb, DF = 30,600 (C), DG = 10, 420 (C), EG = 23,330 (T))



Select:

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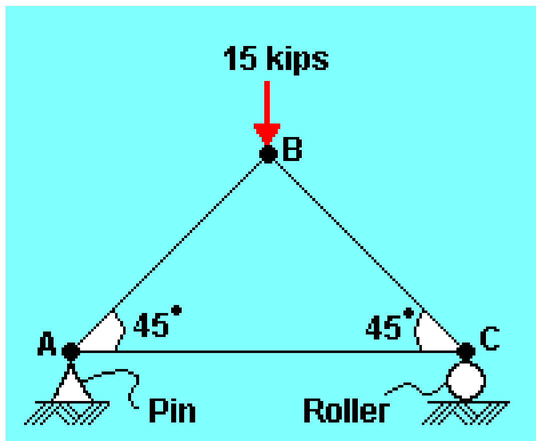
Statics & Strength of Materials

Problem Assignment - Trusses 2

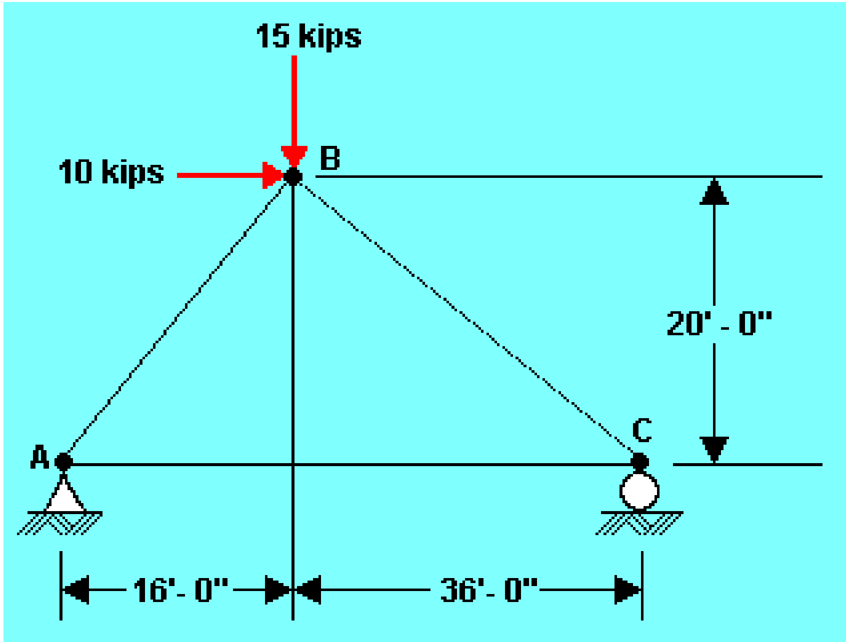
Note the typical designation of pin and roller supports in the diagrams shown.

1. Calculate the forces in all members of the truss shown in the following diagram using the method of joints. [AB =10,600 lb (C), CB = 10,600 lb (C)]



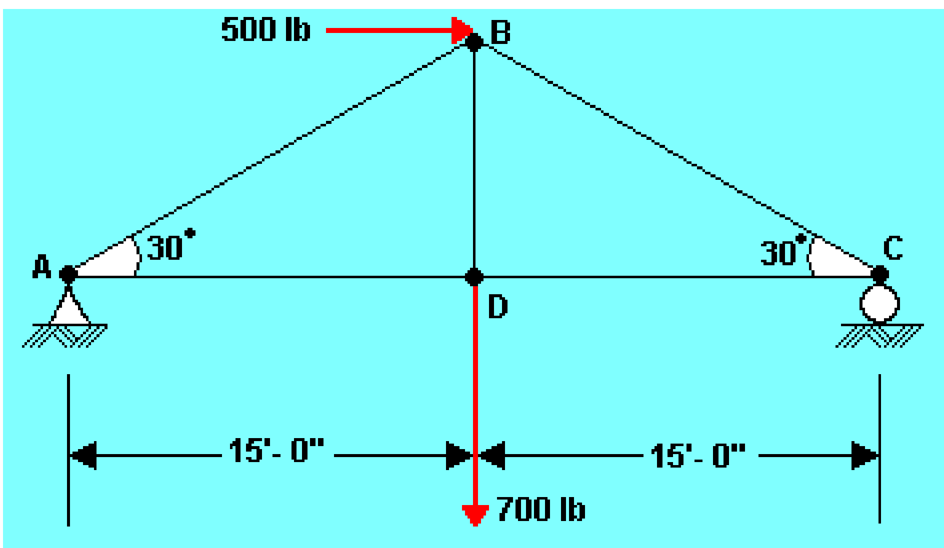
Problem #1

2. Calculate the forces in all member of the truss shown in the following diagram using the method of joints. [Ay = 6,540 lb., Ax = -10,000 lb, Cy = 8460 lb., AB = 8,375 lb, AC = 15,230 lb, BC=17,420 lb)



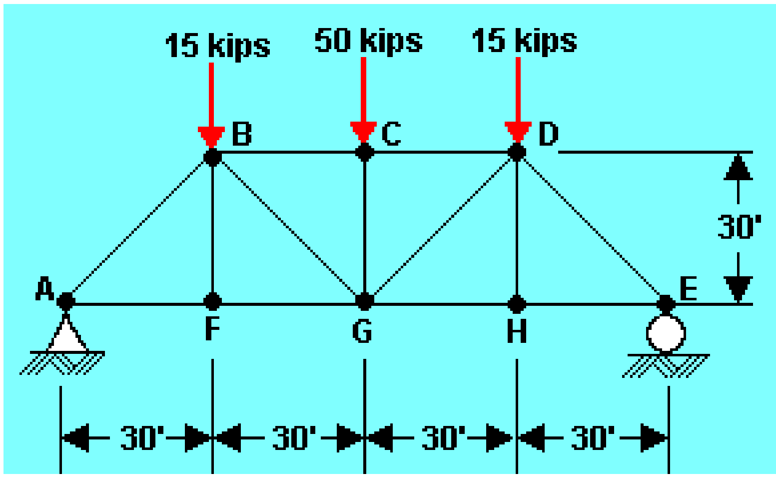
Problem #2

3. Calculate the forces in all members of the truss shown in the following diagram using the method of joints. (Ay = 225 lb, Ax = -500 lb., Cy = 475 lb., AB = 450 lb., AD = 890 lb., BC = 950 lb.)



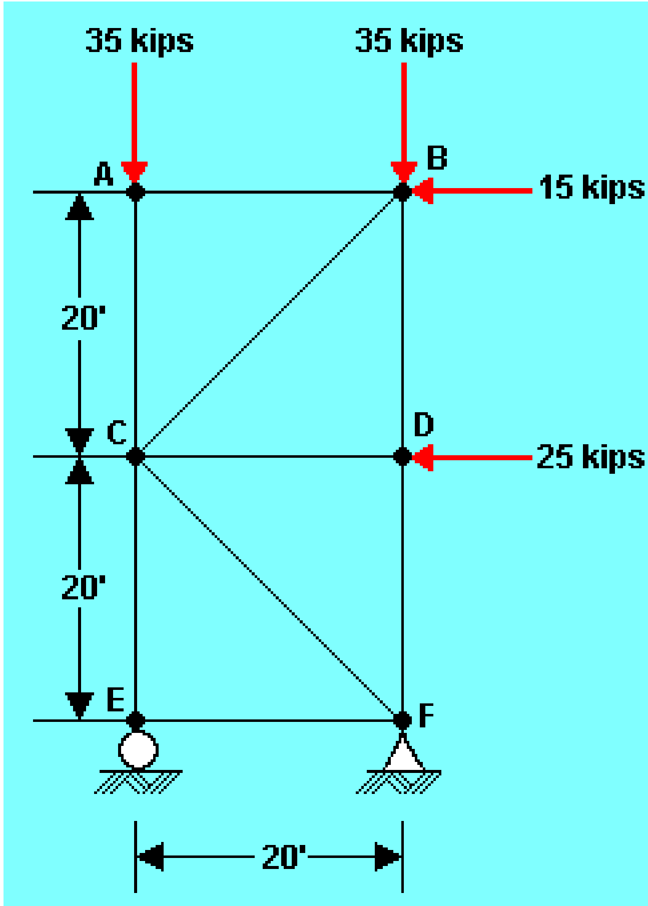
Problem #3

4. Calculate the forces in all members of the truss shown in the following diagram using the method of joints. [Ay=Ey=40,000 lb., AB =56,580 lb. (C), AF = FG = 40,000 lb. (T), BF=0, BC = CD =65,000 lb (T), BG = 35,360 (T), CG = 50,000 lb, right side same by symmetry]



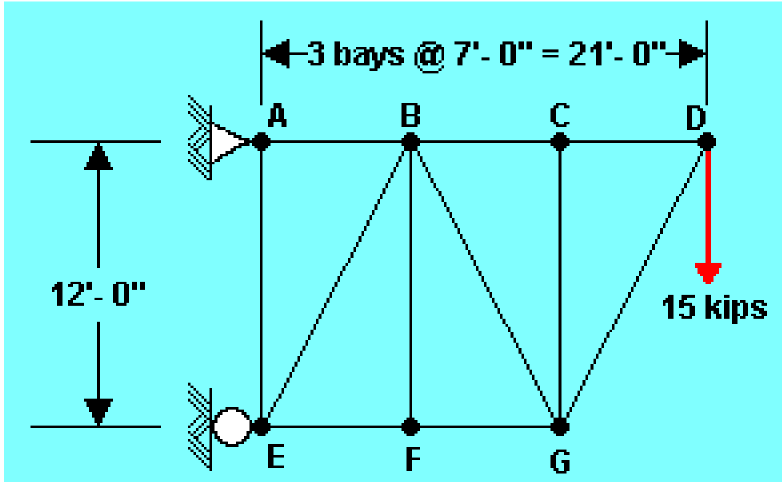
Problem #4

5. Calculate the forces in all members of the trusses shown in the following diagram using the method of joints. [Ey =90,000 lb., Fy=-20,000 lb., Fx=40,000 lb., AB = 0, AC = 35,000 lb. (C), BC = 21,220 lb. (C), BD=20,000 lb. (C), CD = 25,000 lb. (C), DF = 20,000 lb. (C), CE = 90,000 lb. (C), EF = 0, CF = 56,580 lb. (T)]



Problem #5

6. Calculate the forces in members BC, BG, and FG, by method of joints, for the cantilever truss shown below. [Ax=-26,250 lb., Ay=15,000 lb., Ex=26,250 lb., BC = 8,750 lb. (T), BG=17,366 lb. (T), FG=17,500 lb (C)]



Problem #3

Select:

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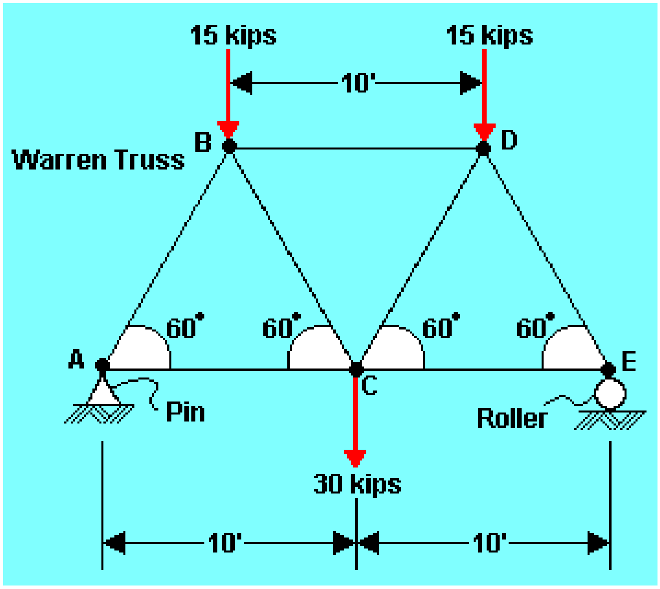
Statics & Strength of Materials

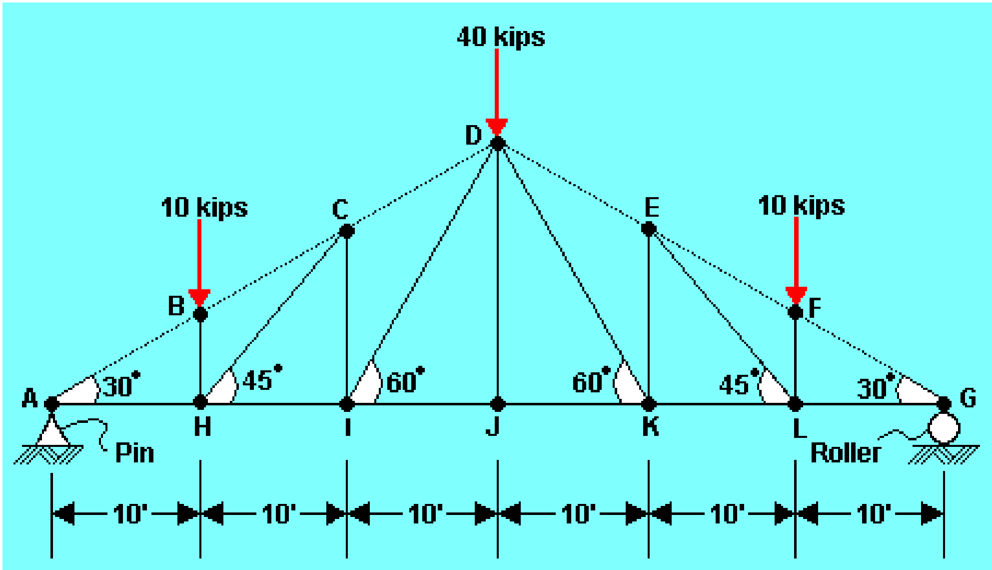
Problem Assignment - Trusses 3

1. Solve the following truss by the method of sections.   Determine the truss reactions and the force in members BD, BC, and AC.



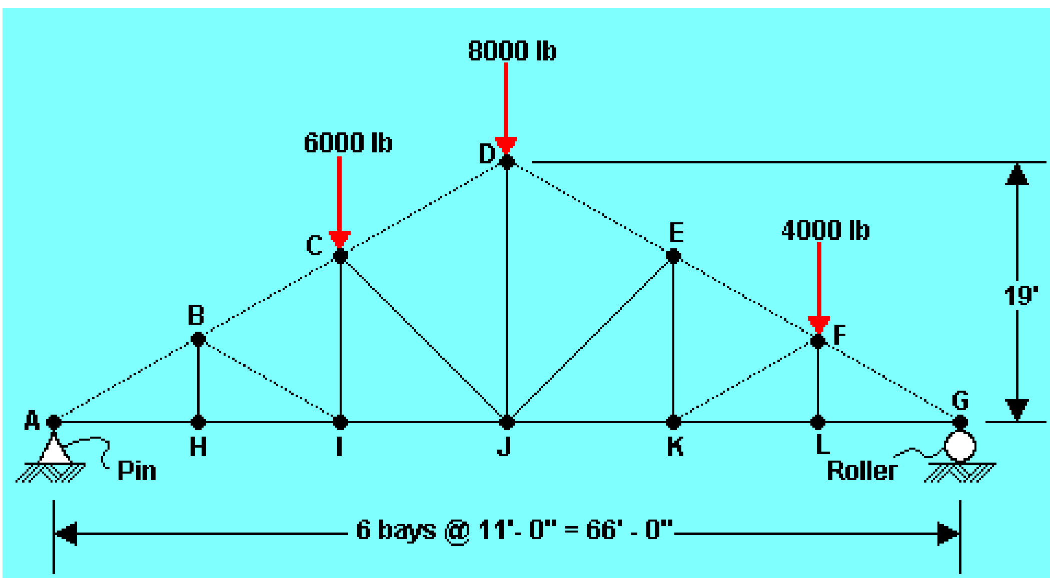
Problem #1

2. Solve the following truss by the method of sections.   Determine the truss reactions and the forces in members CD, ID, and IJ.



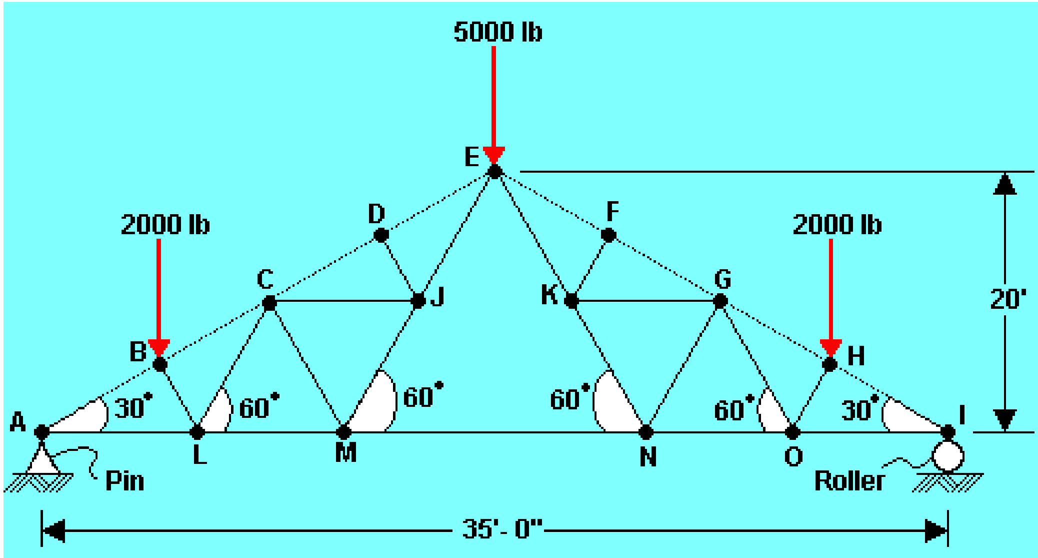
Problem #2

3. Solve the following truss by the method of sections. Determine the truss reactions and the forces in members EF, FK, and KL.



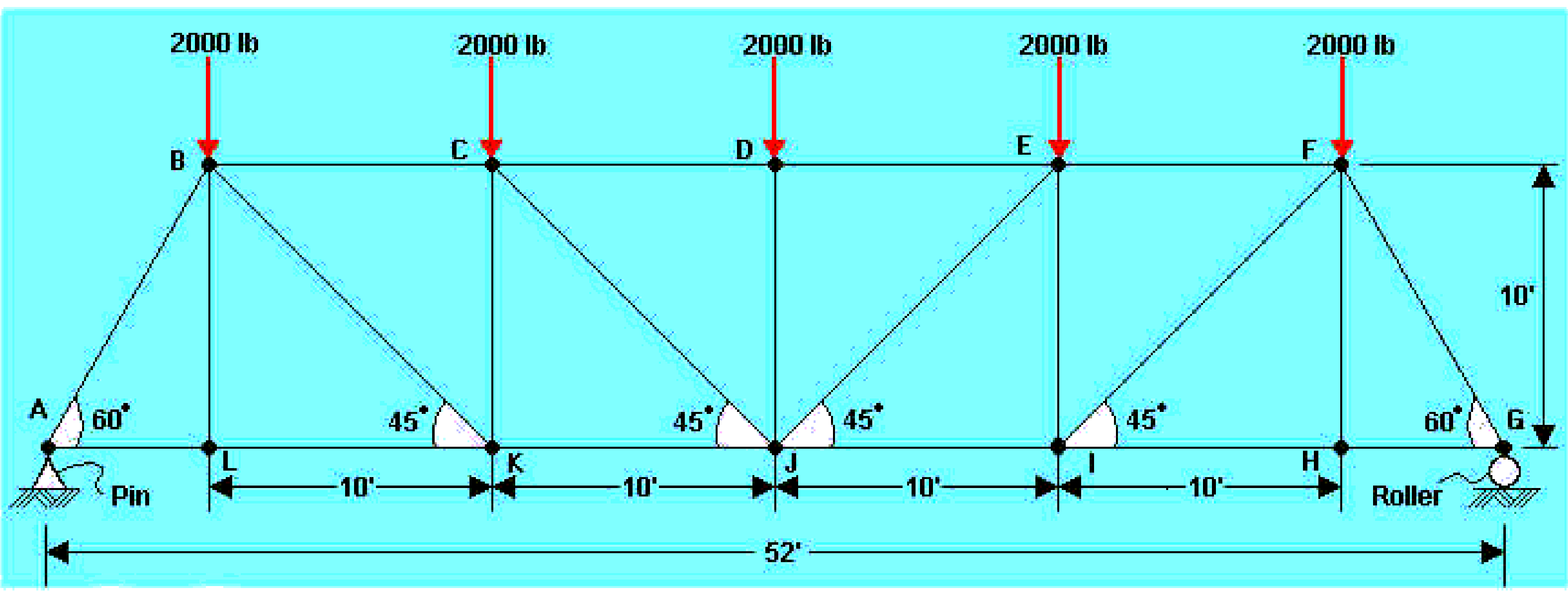
Problem #3

4. Solve the following truss by the method of sections. Determine the truss reactions and the forces in members DE, JE, and MN.

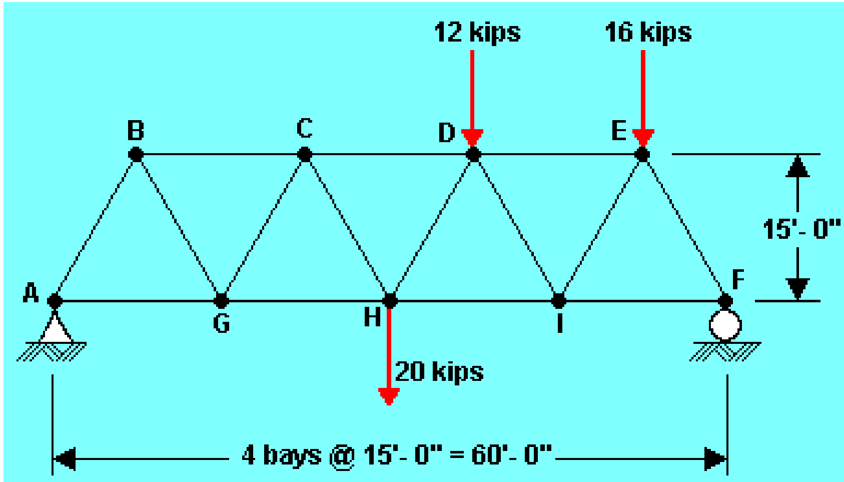


Problem #4

5. Solve the following truss by the method of sections. Determine the truss reactions and the force in members DE, JE, AND JI.

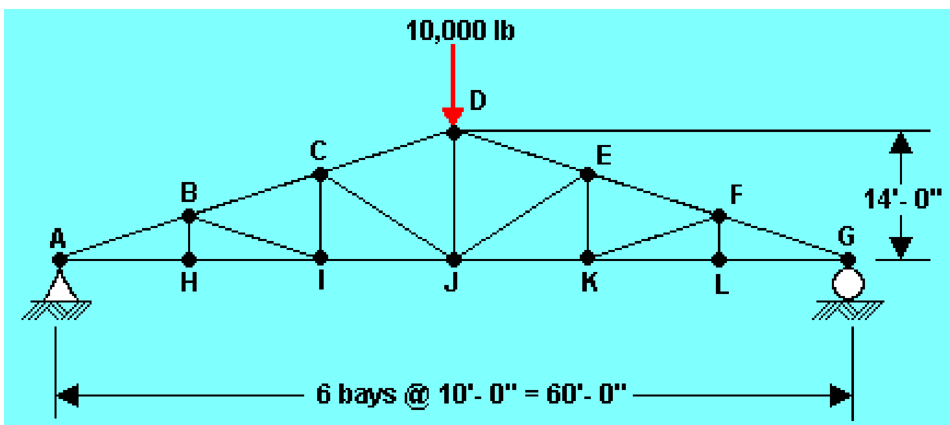


6. Solve the following truss by the method of sections. Determine the truss reactions and the force in members CD, DH, and HI.



Problem #1

7. For the Howe roof truss shown below, determine the support reactions and the forces in members BC, CI, and IJ by the method of sections.



Problem #2

Select:
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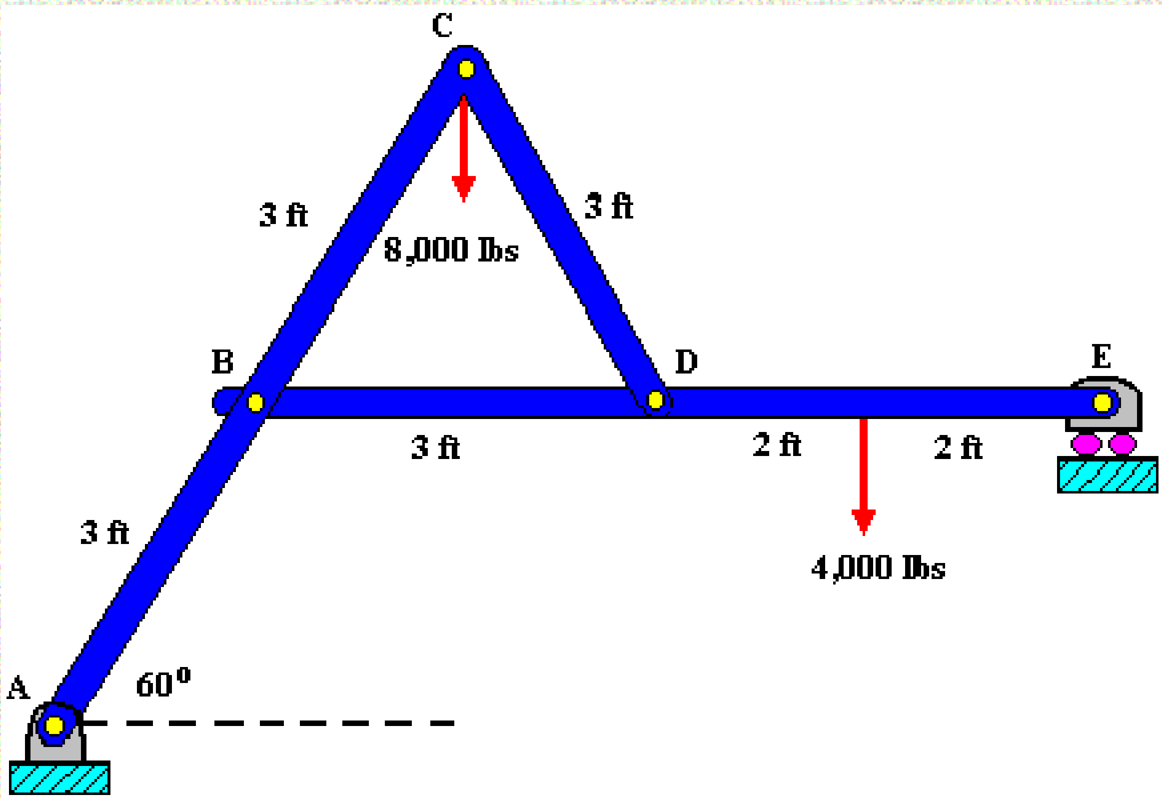
Topic 2: Statics II - Applications

SAMPLE EXAMINATION

1.) In the structure shown below, members ABC , BDE and CD are assumed to be solid rigid members. The structure is pinned at A and supported by a roller at E. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.



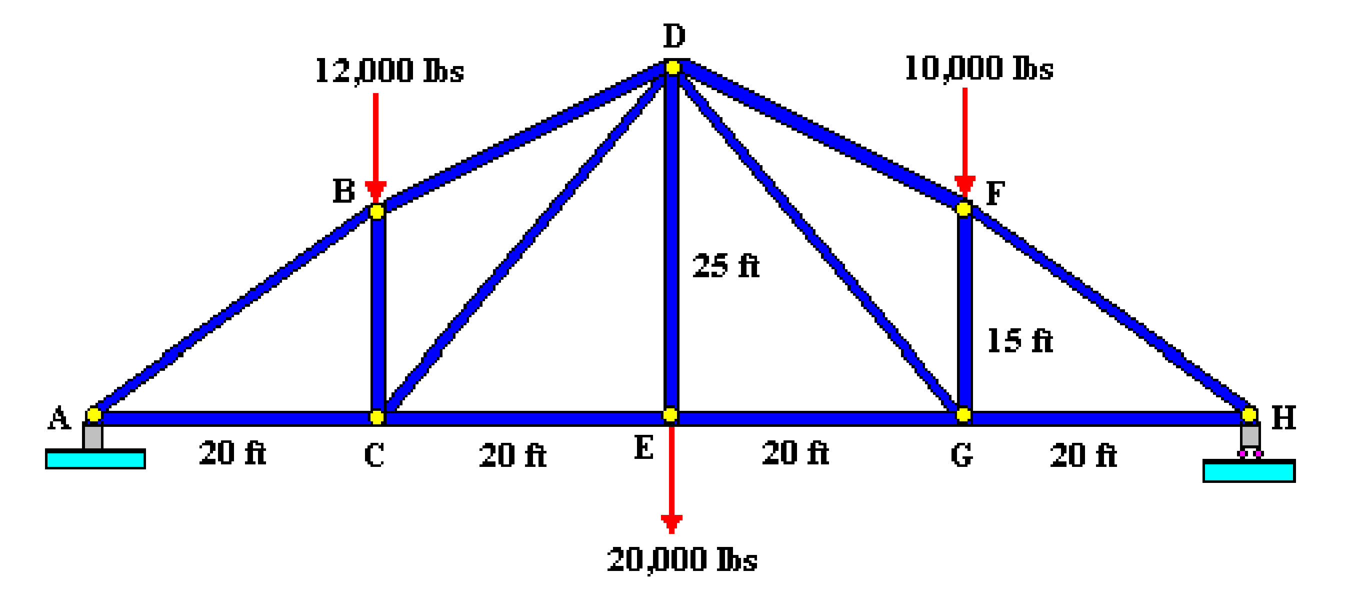
B. Determine the value of all the support forces acting on the structure.

C. Determine the force (tension or compression) in member CD.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

For answers to Problem 1 select **Solution-Exam Problem 1**

2.) The structure shown below is a truss which is pinned to the floor at point A and supported by a roller at point H. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.



B. Determine the value of all the support forces acting on the structure.

C. Determine the force in member DG by method of sections.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

For answers to Problem 2 select **Solution-Exam Problem 2**

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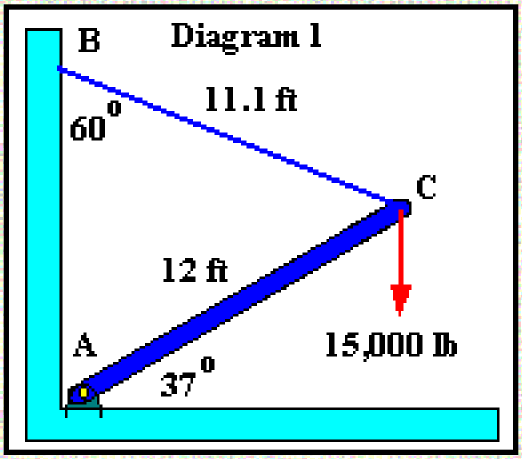
Topic 3: Stress, Strain & Hooke's Law

Topic 3.1: Stress, Strain & Hooke's Law - I

In the first general topic (Statics) we examined the process of determining both the external support forces acting on a structure and the internal forces acting in members of a structure (particularly axial members). While this is important and in fact indispensable when designing structures or determining the safety of a loaded structure, knowing the force values is not enough. We can see that from a simple example.

In Diagram 1, the structure shown is composed of axial member AC which is pinned to the floor at point A, and cable BC which is pinned to the wall at point B. In addition, a load of 15,000 lb is attached to the structure at point C. If we solve for the forces acting on and in the structure we will find that at point A there is a support force of 14,180 lb. acting at 37º (along the direction of the member); which is also the internal force in member AC, 14,180 lb (compression). At point B, the external support force of the wall on the cable has a value of 13,090 lb., acting at an angle of 150º (from +x-axis) This is also the value of the internal tension in the cable, 13,090 lb. **Now, we could ask the question; Is this structure safe? Are members BC and AC strong enough to support the load**?



We recognize right away that knowing the force in the cable BC is not enough to tell us if the cable is safe or if it will break. Clearly it depends on several other factors in addition to the force in the cable. It depends on the size of the cable. A 1" diameter steel cable will carry more load than a ¼ " diameter steel cable. It also depends on what the cable is made of. A steel cable will clearly support more than an aluminum cable. To address the first consideration, we will turn to the concept of **STRESS**.

AXIAL STRESS

What is known as **Axial (or Normal) Stress**, often symbolized by the Greek letter sigma, is defined as the force perpendicular to the cross sectional area of the member divided by the cross sectional area. Or

$$\boxed{\sigma = F / A \;\; (\text{units} = \text{lb} / \text{in}^2, \text{ or N} / \text{m}^2)}$$

In diagram 2, a solid rod of length L, is under simple tension due to force F, as shown. If we divide that axial force, F, by the cross sectional area of the rod (A), this quotient would be the axial stress in the member. Axial stress is the equivalent of pressure in a gas or liquid. As you remember, pressure is the force/ unit area. So axial stress is really the 'pressure' in a solid member. Now the question becomes, how much 'pressure' can a material bear before it fails.

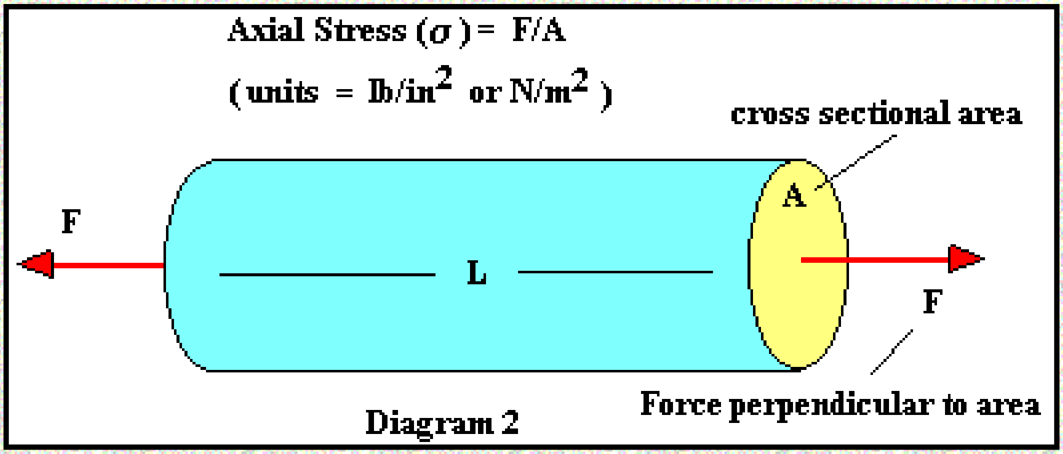


Diagram 2

Well, we will examine that question in some detail in a bit, but to give an example, a normal operating stress for carbon steel might be 30,000 lb/in^2. Now let's return to our example shown in Diagram 1 (repeated in Diagram 3). In our structure , if we assume both the member and the cable are made of steel, and if the diameter of the cable is .5 inches, and if the cross sectional area of the member is 1.2 in^2, are the stresses in the cable BC and in member AC within the 'allowable' stress for steel of 30,000 lb/in^2?

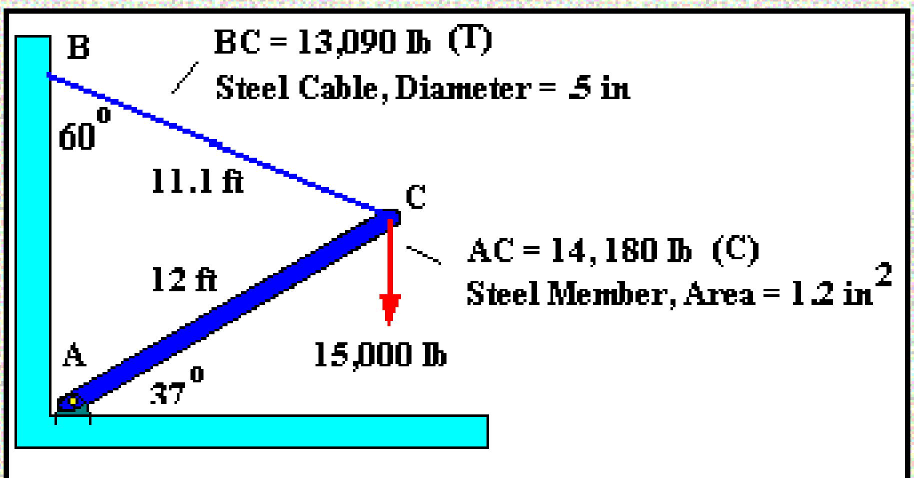


Diagram 3

For the cable BC: **Axial Stress = F / A = 13,090 lb. / (π * .25"^{2}) = 66,700 lb / in^2**

For the member AC: **Axial Stress = F / A = 14,180 lb. / (1.2 in^2) = 11,820 lb / in^2**

These are interesting results. We see from the calculations that the stress in member AC (11,820 lb/in^2) is well within the allowable stress of 30,000 lb/in^2, however, we also see clearly that the stress in the cable AC (66,700 lb/in^2) is over twice the allowable stress of 30,000 lb/in^2. This means that the ½ inch diameter cable is much too small to support the load.

Well, what size cable should we use? Another interesting question whose answer we find by simply reversing our process, using the stress equation to find the minimum size cable for the allowable stress of 30,000 lb/in^2. That is, we set the stress value to the allowable stress of 30,000 lb/in^2, put in the axial force in the cable, and solve for the cable area needed.

Axial Stress = F/A : 30,000 lb/in^2 = 13,090 lb./A; solving for A = .436 in^2. Since the area of cable = 3.14 (r^2), we can solve for the radius **r = square root (.436 in^2/3.14) = .373 inches**. So the minimum diameter steel cable which would safely support the load is **d = .746 inches (or ¾ inch diameter cable)**. This is an important process. We checked the members in the structure, found one was not safe according to the allowable stress for the material, and then calculated the size member needed so that the structure would be safe.

Our next step is to examine an associated property of stress, strain, and to examine the stress and strain in materials in somewhat more detail. Select **Topic 3.2: Stress, Strain & Hooke's Law - II**

or Select:
Topic 3: Stress, Strain & Hooke's Law - Table of Contents
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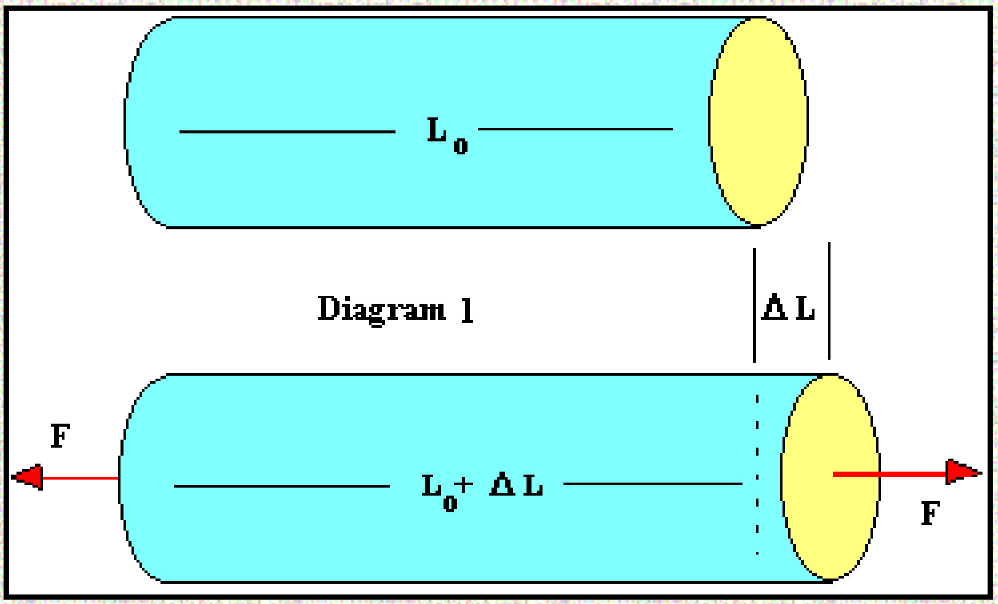
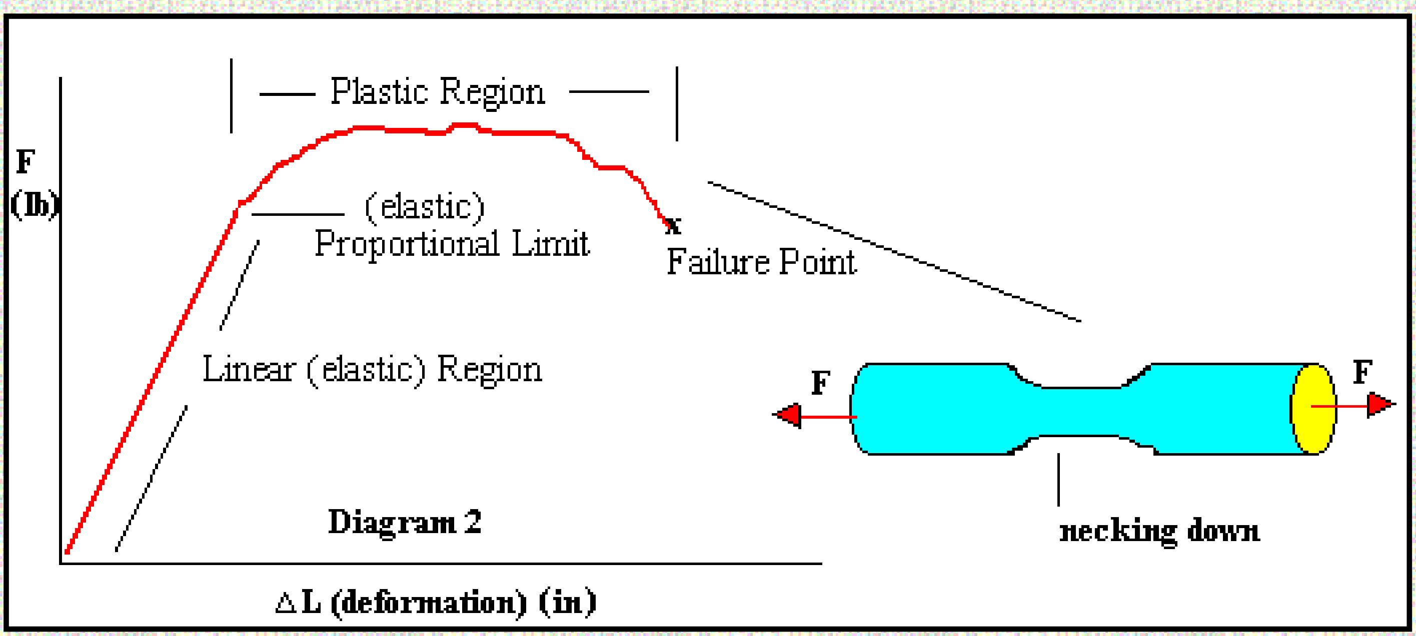
Topic 3.2: Stress, Strain & Hooke's Law - II

In our first topic, Static Equilibrium, we examined structures in which we assumed the members were rigid - rigid in the sense that we assumed that the member did not deform due to the applied loads and resulting forces. In real members, of course, we have deformation. That is, the length (and other dimensions) change due to applied loads and forces. In fact, if we look at a metal rod in simple tension as shown in diagram 1, we see that there will be an elongation (or deformation) due to the tension. If we then graph the tension (force) verses the deformation we obtain a result as shown in diagram 2.





In diagram 2, we see that, if our metal rod is tested by increasing the tension in the rod, the deformation increases. In the first region the deformation increases in proportion to the force. That is, if the amount of force is doubled, the amount of deformation is doubled. This is a form of Hooke's Law and could be written this way: F = k (deformation), where k is a constant depending on the material (and is sometimes called the spring constant). After enough force has been applied the material enters the elastic region - where the force and the deformation are not proportional, but rather a small amount of increase in force produces a large amount of deformation. In this region, the rod often begins to 'neck down', that is, the diameter becomes smaller as the rod is about to fail. Finally the rod actually breaks.

The point at which the **Elastic Region** ends is called the elastic limit, or the proportional limit. In actuality, these two points are not quite the same. The **Elastic Limit** is the point at which permanent deformation occurs, that is, after the elastic limit, if the force is taken off the sample, it will not return to its original size and shape, permanent deformation has occurred. The **Proportional Limit** is the point at which the deformation is no longer directly proportional to the applied force (Hooke's Law no longer holds). Although these two points are slightly different, we will treat them as the same in this course.

Next, rather than examining the applied force and resulting deformation, we will instead graph the **axial stress verses the axial strain** (diagram 3). We have defined the axial stress earlier. The axial strain is defined as the fractional change in length or **Strain = (deformation of member) divided by the (original length of member)** , Strain is often represented by the Greek symbol epsilon(ε), and the deformation is often represented by the Greek symbol delta(δ), so we may write: **Strain** $\varepsilon = \delta / L_o$ (where L_o is the original length of the member) Strain has no units - since its length divided by length, however it is sometimes expressed as 'in./in.' in some texts.

As we see from diagram 3, the Stress verses Strain graph has the same shape and regions as the force verses deformation graph in diagram 2. In the elastic (linear) region, since stress is directly proportional to strain, the ratio of stress/strain will be a constant (and actually equal to the slope of the linear portion of the graph). This constant is known as Young's Modulus, and is usually symbolized by an **E** or **Y**. We will use **E** for Young's modulus. We may now write **Young's Modulus = Stress/Strain, or:** $E = \sigma / \varepsilon$. (This is another form of Hooke's Law.)

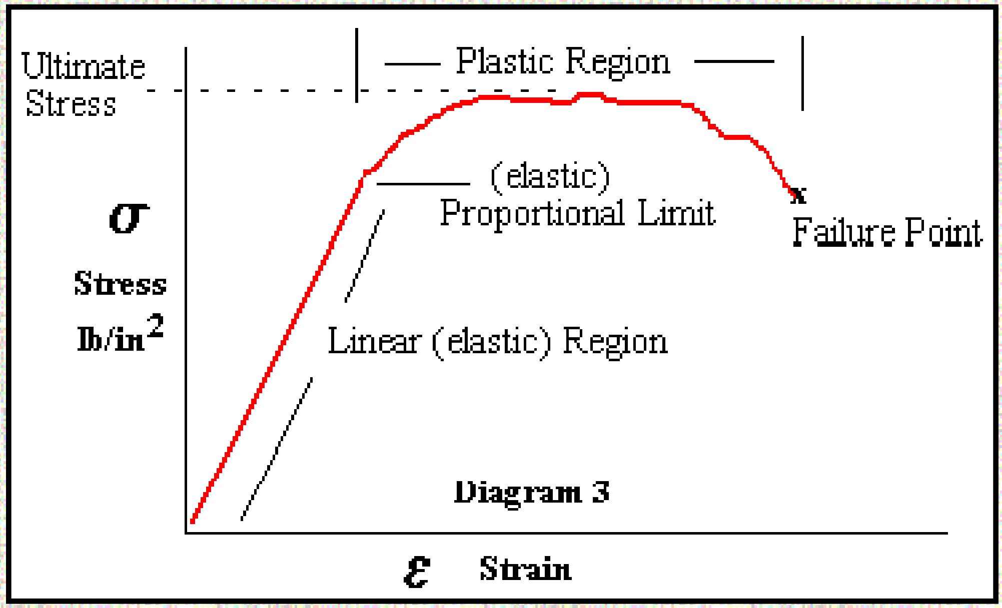


Diagram 3

The value of Young's modulus - which is a measure of the amount of force needed to produce a unit deformation - depends on the material. Young's Modulus for **Steel is 30 x 10^6 lb/in^2**, for **Aluminum E = 10 x 10^6 lb/in^2**, and for **Brass E = 15 x 10^6 lb/in^2**. For more values, select: **Young's Modulus - Table**.
To summarize our stress/strain/Hooke's Law relationships up to this point, we have:

Stress: $\sigma = F / A$ (lb / in^2, or N / m^2)

Strain: $\varepsilon = \delta / L_o$ (no units)

Hooke's Law: $E = \sigma / \varepsilon$ (lb / in^2, or N / m^2)

Deformation: $\delta = FL / EA$ (in. or m)

The last relationship is just a combination of the first three, and says simply that the amount of deformation which occurs in a member is equal to the product of the force in the member and the length of the member (usually in inches) divided by Young's Modulus for the material, and divided by the cross sectional area of the member. To see applications of these relationships, we now will look at several examples.

Continue to:
Example 1 ; Example 2 ; Example 3
or Select:
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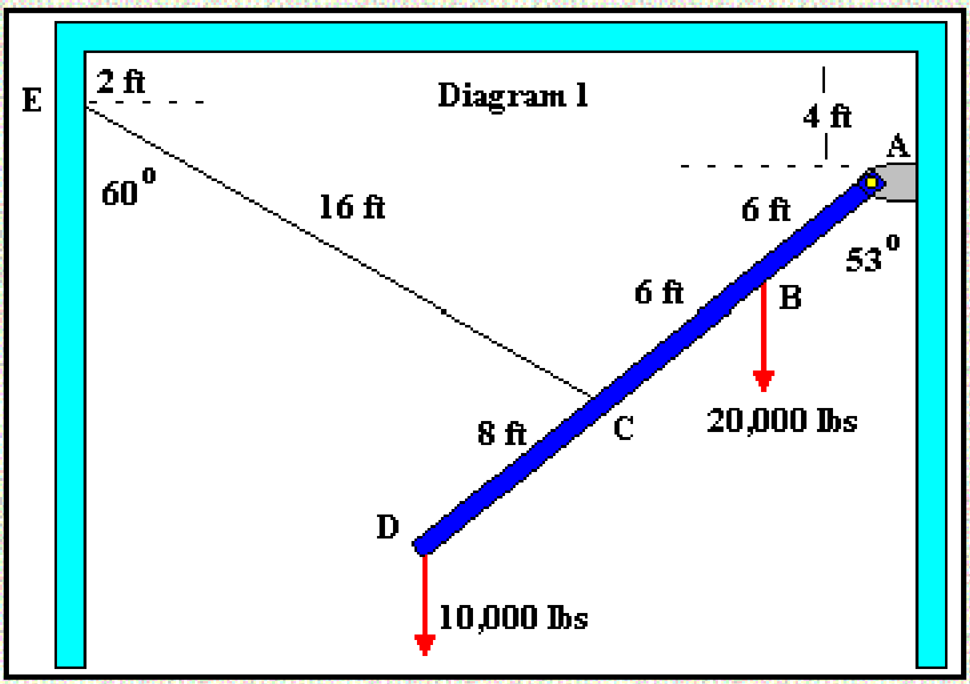
Topic 3.2a: Statically Determinate - Example 1

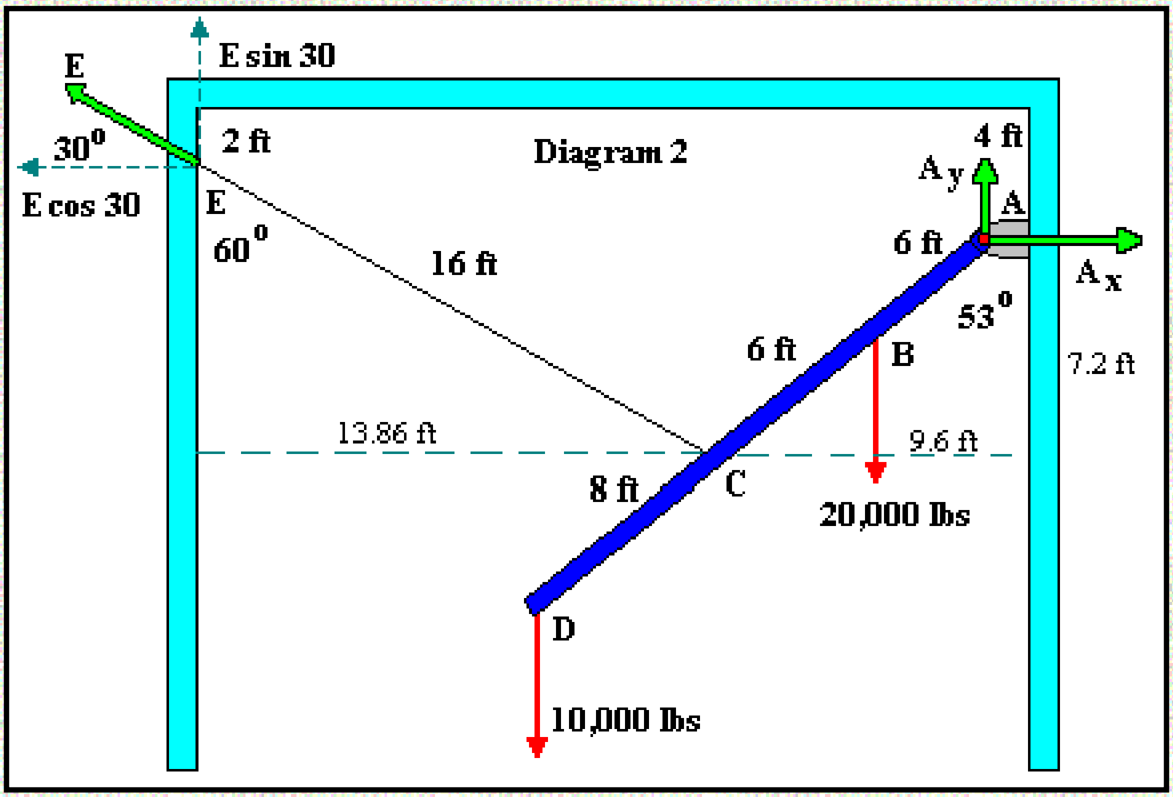
This example may be seen as streaming media or in its text version.

Select one of the following: Text Version Streaming Media Version

In the structure shown in Diagram 1, member ABCD is a solid rigid member pinned to the wall at A, and supported by steel cable CE. Cable CE is pinned to the wall at E and has a diameter of 1 inch. For this structure we would like to determine the axial stress in cable CE, the deformation of member CE, the strain in member CE, and finally to determine the movement of point D due to the applied loads. (Young's Modulus for steel = 30 x 10^6 lb/in^2.)



Step I: As the first step in our solution, we apply static equilibrium conditions to determine the value of the external support reactions (the process we studied in Topic 1 - Statics). Our, hopefully familiar, procedure is as follows:



1: Draw a free body diagram showing and labeling all load forces and support forces, as well as any needed angles and dimensions (Diagram 2).

2: Resolve all forces into their x and y components.

3: Apply the static equilibrium conditions.

Sum $F_x = A_x$ - E cos (30º) = 0

Sum $F_y = A_y$ + E sin (30º) - 10,000 lbs - 20,000 lbs = 0

Sum T_A = (20,000 lbs)(4.8 ft) + (10,000 lbs)(16 ft) + E cos (30º)(2 ft) - E sin (30º)(23.46 ft) = 0

Solving for the unknowns: **E = 25,600 lb.; A_x = 22,170 lb.; A_y = 17,200 lb.**

Step II. Now that we have the values of the external support forces, we determine the value of the force in the internal member in which we would like to find the stress. In this particular problem, this is quite easy once we recognize that at point E, there is only one axial member (CE) attached to the wall. Therefore the force of the wall acting on member CE is equal to the internal force in the member itself: **Force in CE = 25,600 lb.**

Stress: $\sigma = F / A$ (lb / in^2, or N / m^2)

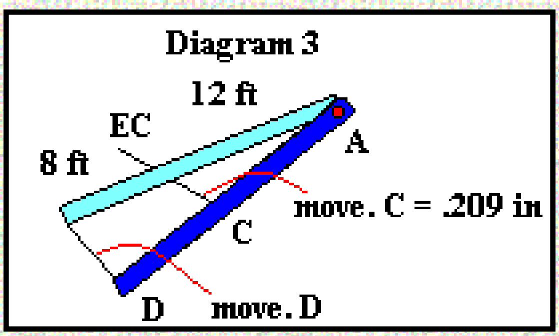
Strain: $\varepsilon = \delta / Lo$ (no units)

Hooke's Law: $E = \sigma / \varepsilon$ (lb / in^2, or N / m^2)

Deformation: $\delta = FL / EA$ (in. or m)

Once we have the force in member CE, we can apply the appropriate stress/strain relationships and solve for the quantities of interest.

A.) Stress in CE = F / A = 25,600 lbs / (3.14*(.5 in)^2= 32,600 psi.
B.) Deformation of CE = (FL/ EA)$_{CE}$ = (25,600 lbs)(16 ft * 12 in / ft) / (30*10^6 lbs / in^2)(3.14*(.5 in)^2) = .209 in
C.) Strain in CE = (Deformation of CE) / (Length of CE) = (.209" / 192") = .00109
D.) Movement of point D. This part of the problem requires a bit of reflection on the geometry of the problem. As is shown in diagram 3, when member CE elongates, member ACD - which is pinned at point A - rotates downward. The amount that point D moves is related to the amount point C moves, and point C moves the same amount that cable CE deforms. We can relate the movement of the two points from geometry by:
[Movement of C / 12 ft = Movement of D / 20 ft] or [.209 in / 12 ft = Movement of D / 20 ft], and solving gives us **Movement of D = .348 in.**



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Topic 3.2 - Stress,Strain & Hooke's Law - II
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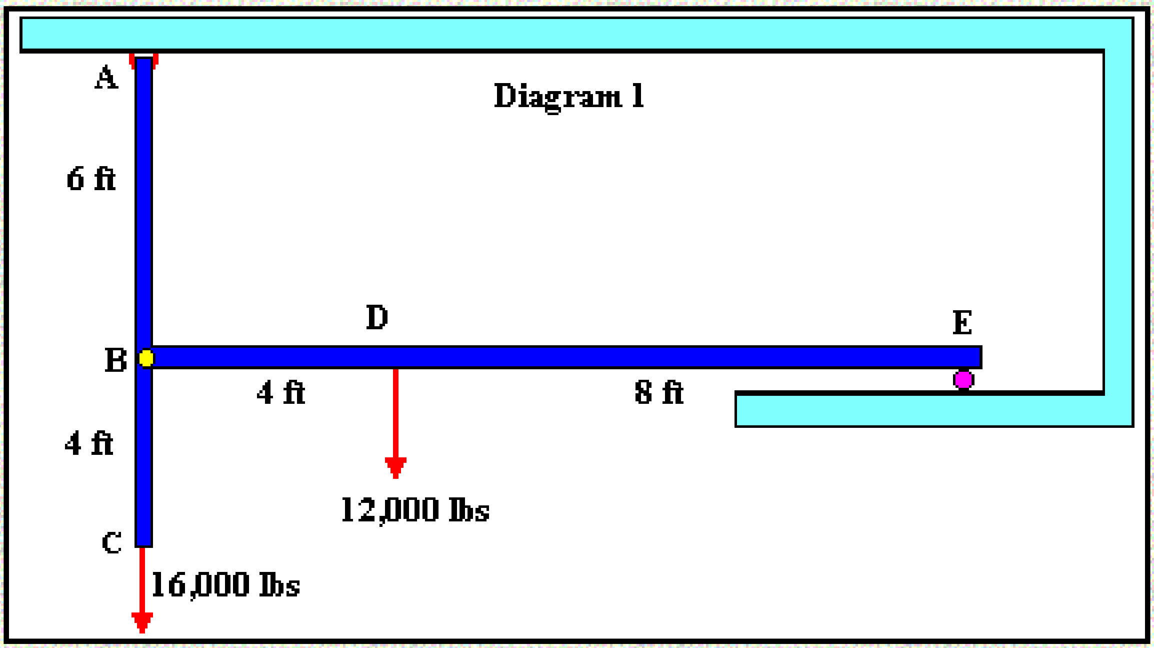
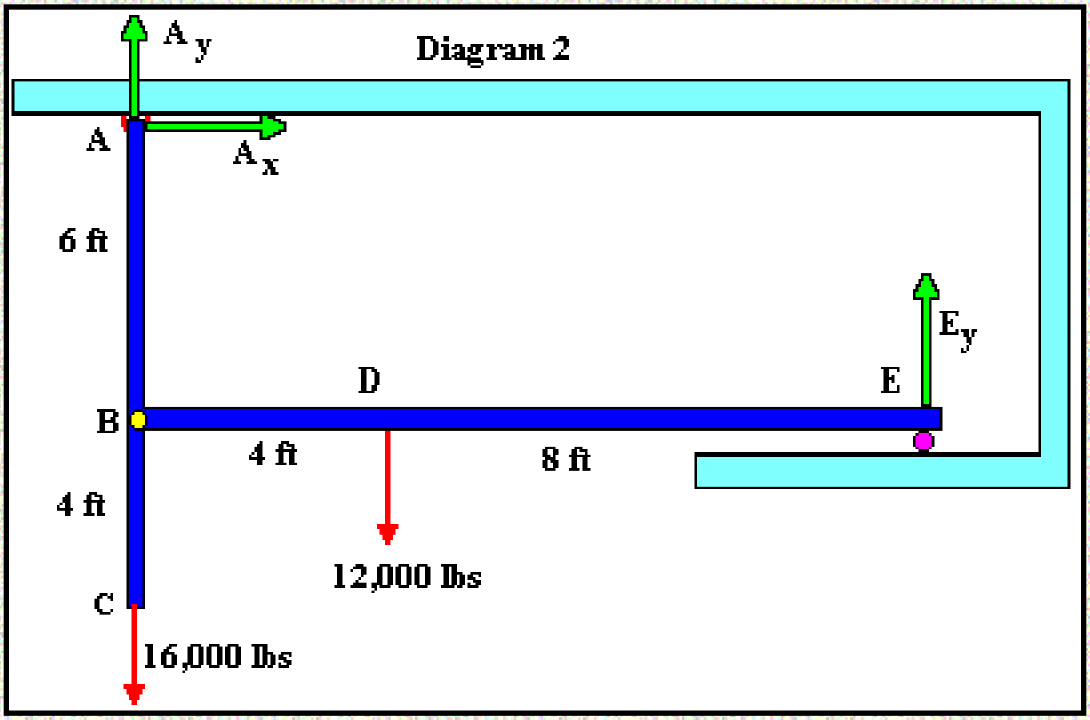
Topic 3.2b: Statically Determinate - Example 2

In the structure shown in Diagram 1, members ABC and BDE are assumed to be solid rigid members. Member BDE is supported by a roller at point E, and is pinned to member ABC at point B. Member ABC is pinned to the wall at point A. Member ABC is an aluminum rod with a diameter of 1 inch. (Young's Modulus for Aluminum is 10×10^6 lb/in^2)
For this structure we would like to determine the axial stress in member ABC both in section AB and section BC, and to determine the movement of point C due to the applied loads.



Part I. To solve the problem we first need to determine the external support forces acting on the structure. We proceed using our static equilibrium procedure (from Topic 1 - Statics)

1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions. (See Diagram 2)



2: Resolve forces into x and y components.

3: Apply the equilibrium conditions.

Sum $F_x = A_x = 0$

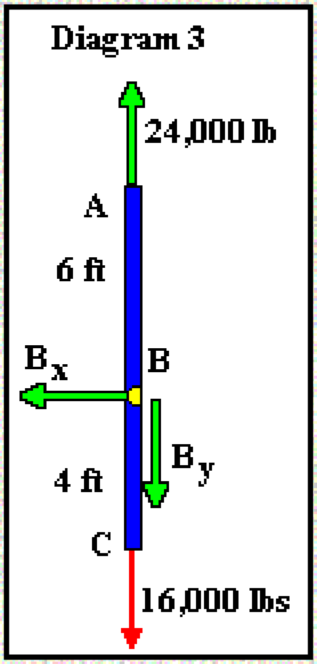
Sum $F_y = A_y + E_y$ -16,000 lbs - 12,000 lbs = 0

Sum T_E = (16,000 lbs)(12 ft) + (12,000 lbs)(8 ft) - A_y(12 ft) = 0

Solving for the unknowns: **A_y = 24,000 lbs; E_y = 4000 lbs**

Part II. An interesting aspect to this problem is that member ABC is not an axial member, and so it is not in simple uniform tension or compression. However, we are fortunate in that it is not a complex non-axial member. It is not in shear, but rather simply is in different amounts of tension above and below point B. Therefore to determine the amount of stress in each part of ABC, we first make a free body diagram of member ABC and apply static equilibrium principles.

1: FBD of member ABC. (Diagram 3)



2. Resolve all forces into x and y components.

3. Apply equilibrium conditions.

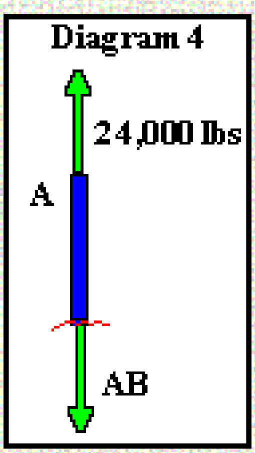
Sum $F_x = B_x = 0$

Sum F_y = 24,000 lb. - B_y - 16,000 lbs = 0

Solving for the unknowns: **B_y = 8,000 lb.**

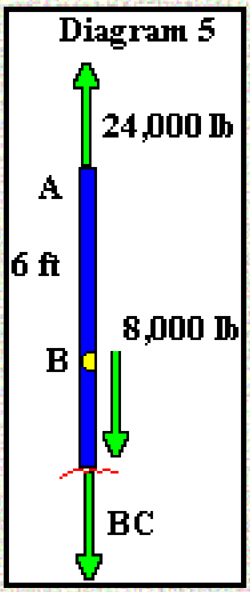
Now to find the force in section AB of member ABC. Cut the member between points A and B,and analyze the top section. We can do this since if a member is in static equilibrium, then any portion of the member is also in static equilibrium.

Looking at Diagram 4 (which is the free body diagram of the upper section of member ABC), we see that for the section of AB shown to be in equilibrium, the internal force (which becomes external when we cut the member) must be equal and opposite to the 24,000 lb force of the wall on the member at point A.



Once we know the tension in section AB of member ABD, we find the stress from our relationship **Stress = F / A = 24,000 lb / (3.14 x $.5^2$) = 30,600 psi.**

We then use the same approach with section BC of member ABC. We cut member ABC between point B and point C, and apply static equilibrium principles to the top section. Diagram 5 is the free body diagram of that section, and by simply summing forces in the y-direction, we see that the internal force BC (which becomes an external force when we cut the member) must be 16,000 lb. for equilibrium.



The stress in section BC is then given by Stress = F/A = 16,000 lb/(3.14 x $.5^2$)
Stress (BC) = 20,400 psi.

Part III. To determine the movement of point C is a relatively simple problem when we realize that the movement of point C will be equal to the deformation (elongation) of section AB plus the deformation (elongation) of section BC. That is, the Movement of C = Def_{AB} + Def_{BC}

Movement of. C = [$(FL / EA)_{AB}$ + $(FL / EA)_{BC}$]

Movement of C = [(24,000 lbs)(72 in) / ($10*10^6$ psi)($3.14*(.5\text{ in})^2$)$]_{AB}$ +

[(16,000 lbs)(48 in) /
($10*10^6$ psi)($3.14*(.5\text{ in})^2$)$]_{BC}$

Movement of C = (.220 in) + (.0978 in) = .318 in

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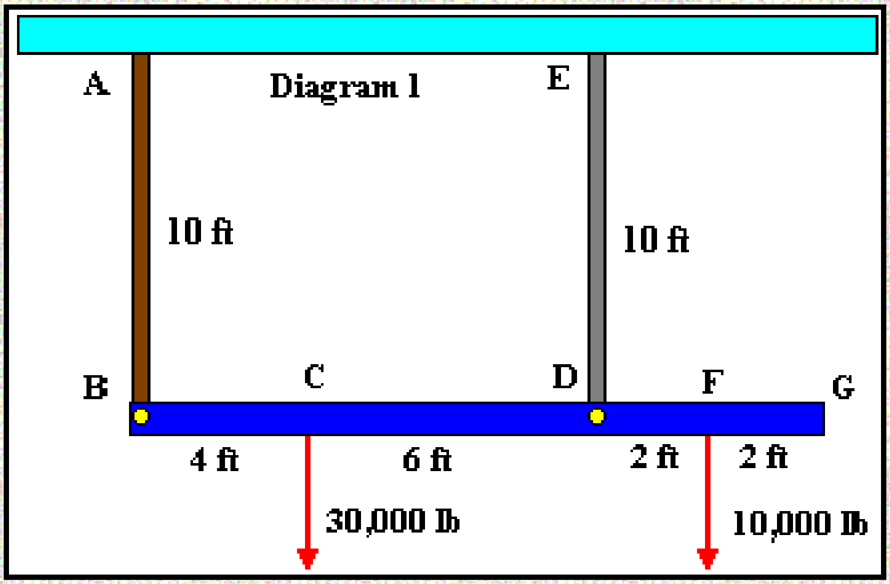
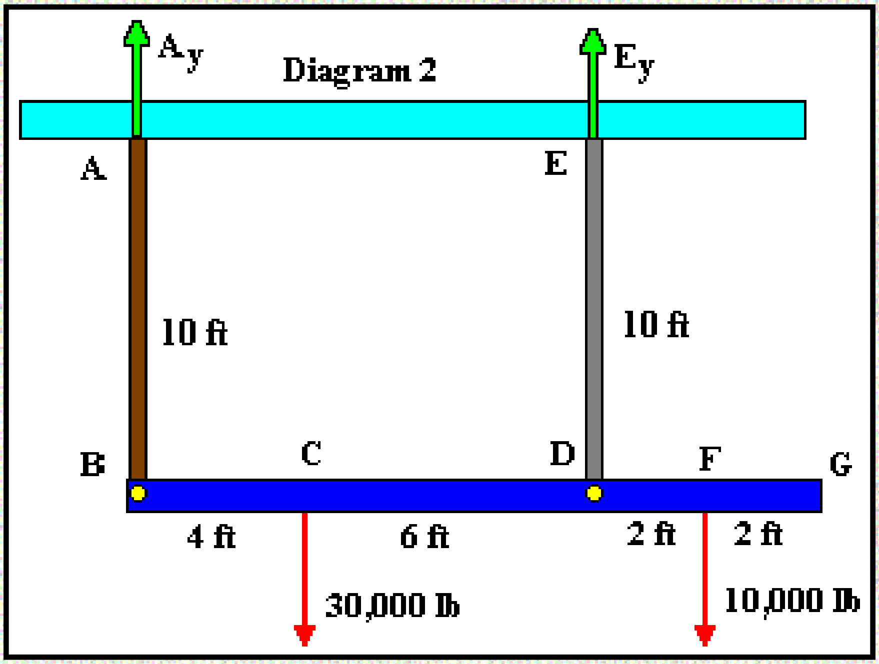
Topic 3.2c: Statically Determinate - Example 3

In the structure shown in Diagram 1, member BCDFG is assumed to be a solid rigid member. It is supported by a two cables, AB & DE. Cable AB is steel and cable DE is aluminum. Both cables have a cross sectional area of .5 in^2. For this structure we would like to determine the axial stress in cable AB and DE. We would also like to determine the movement of point F due to the applied loads. [Young's Modulus for steel : E_{st} = 30 x 10^6 psi, Young's Modulus for Aluminum: E_{al} = 10 x 10^6 psi.]



Part I. To solve the problem we first need to determine the external support forces acting on the structure. We proceed using our static equilibrium procedure (from Topic 1 - Statics)

1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



2: Resolve forces into x and y components. All the forces are acting in the y-direction, as is shown in the FBD.

3: Apply the equilibrium conditions.

Sum $F_y = A_y + E_y - 30{,}000$ lb. $- 10{,}000$ lb. $= 0$

Sum $T_B = (-30{,}000$ lb.$)(4$ ft$) + E_y(10$ ft$) - (10{,}000$ lb.$)(12$ft$) = 0$

Solving for the unknowns: **$E_y = 24{,}000$ lb.; $A_y = 16{,}000$ lb.**

Part II. Since both the steel member AB and the aluminum member DE are single axial members connected to the supporting ceiling, the external forces exerted by the ceiling on the members is also equal to the internal forces in the members. Thus **$F_{AB} = 16{,}000$ lb. (tension), $F_{DE} = 24{,}000$ lb. (tension)**. To find the stress in each cable is now straight forward. We apply the stress equation

Stress: $\sigma = F / A$ $(\text{lb}/\text{in}^2, \text{ or N}/\text{m}^2)$

Strain: $\varepsilon = \delta / L_o$ (no units)

Hooke's Law: $E = \sigma / \varepsilon$ $(\text{lb}/\text{in}^2, \text{ or N}/\text{m}^2)$

Deformation: $\delta = FL / EA$ (in. or m)

(from the Stress / Strain / Hooke's Law relationships shown to the right).

So, **Stress AB = F / A = 16,000 lbs / .5 in^2 = 32,000 psi., Stress DE = F / A = 24,000 lbs / .5 in^2 = 48,000 psi.**

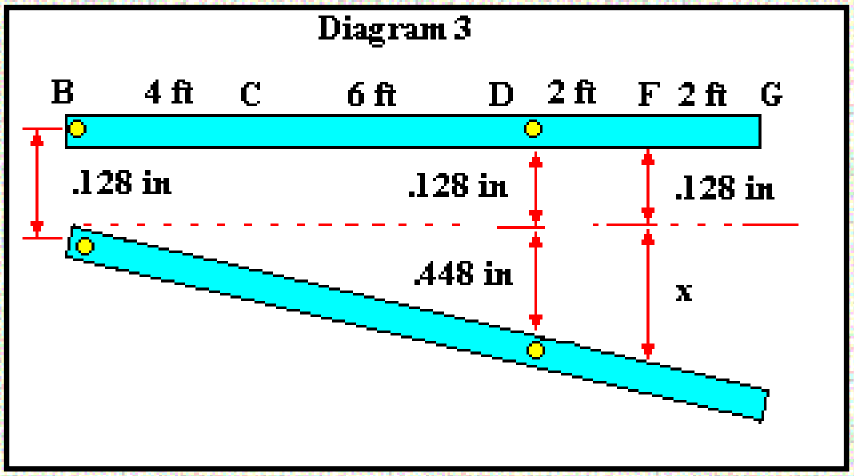
Part III To find the movement of point F requires us to use a bit of geometry. Point F moves since both member AB and ED deform and member BCDFG moves downward according to these deformations.

1: Calculate deformation of members AB and ED.

Def_{AB} = $(FL / AE)_{AB}$ = (16,000 lbs)(120 in) / ($30*10^6$ lbs/in^2)(.5 in^2) = **.128 in**

Def_{ED} = $(FL / AE)_{ED}$ = (24,000 lbs)(120 in) / ($10*10^6$ lbs/in^2)(.5 in^2) = **.576 in**

2: Movement of point F. In Diagram 3 we have shown the initial and final position (exaggerated) of member BCDFG.



Point B moves down .128 inches (the deformation of member AB), point D moves down .576 inches (the deformation of member DE). Point F moves down an intermediate amount. To determine this we have drawn a horizontal line from the final position of point B across to the right side of the beam as shown. From this we see that the distance point F moves down is .128 inches + "x" (where x is the distance below the horizontal line as shown in the diagram). We can determine the value of x from proportionality (since similar triangles are involved), and write: (x / 12 ft) = (.448 in / 10 ft). Solving for x we find: x = .5376 inches.

So the Movement of F = .128 inches + .5376 inches = .666 inches

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STATICS / STRENGTH OF MATERIALS - Example

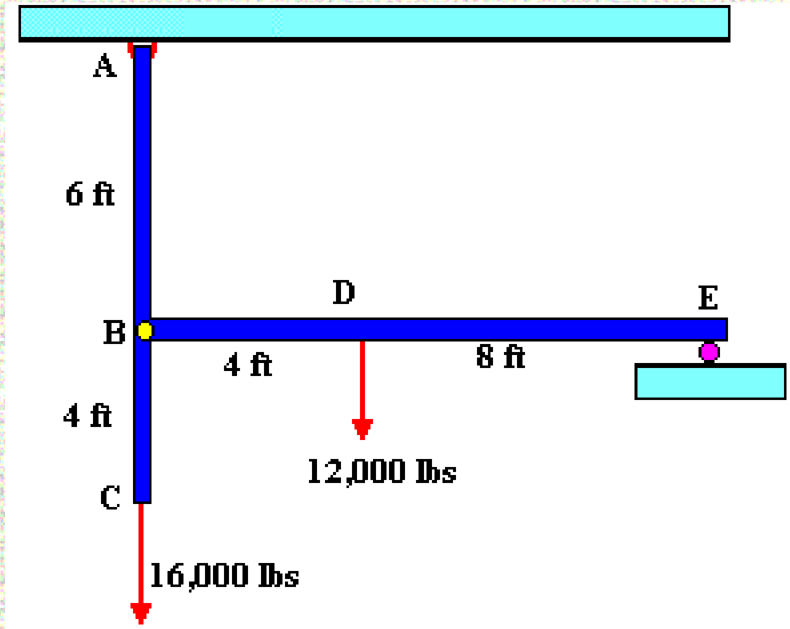
In the structure shown below members ABC and BDE are assumed to be solid rigid members. Member BDE is supported by a roller at point E, and is pinned to member ABC at point B. Member ABC is pinned to the wall at point A. Member ABC is a aluminum rod with a diameter of 1 inch. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.

B. Determine the axial stress in member ABC both in section AB and section BC.

C. Determine the movement of point C due to the two applied loads.

E_{al} = 10 x 10^6 psi



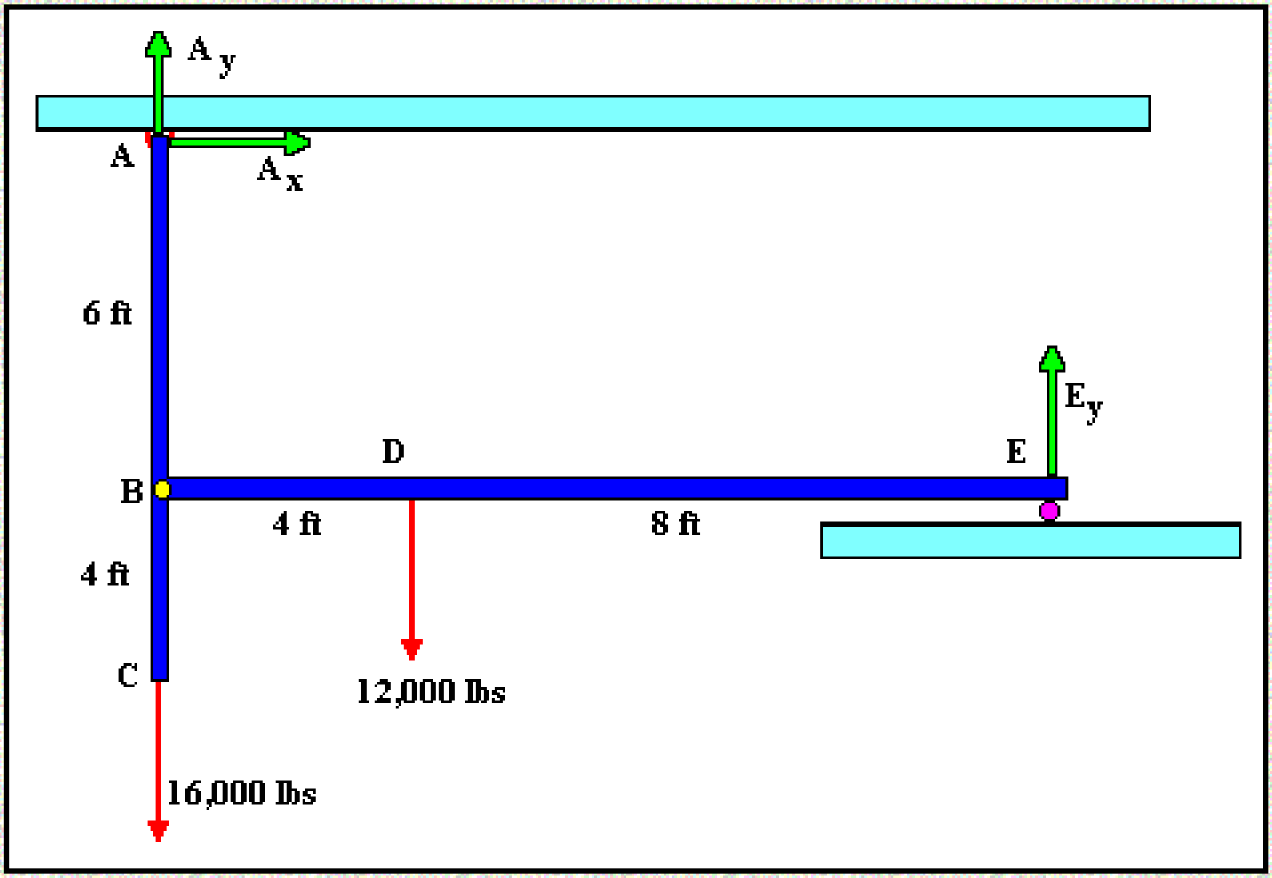
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

PART A:

External support reaction - Statics:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

Sum F_x = A_x = 0

Sum F_y = A_y + E_y -16,000 lbs - 12,000 lbs = 0

Sum T_E = (16,000 lbs)(12 ft) + (12,000 lbs)(8 ft) - A_y(12 ft) = 0

Solving for the unknowns:

A_y = 24,000 lbs; E_y = 4,000 lbs

PART B:

STEP 1: Take out member ABC. Analyze force acting on it. Draw a free body diagram of ABC

STEP 2: Resolve all forces into x and y components

STEP 3: Apply equilibrium conditions.

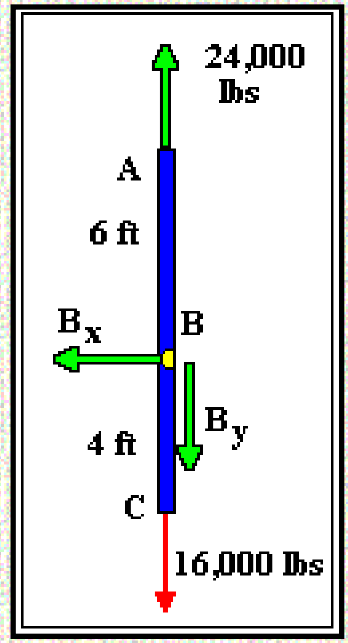
Sum $F_x = B_x = 0$

Sum F_y = 24,000 lbs - B_y - 16,000 lbs = 0

Solving for the unknowns:

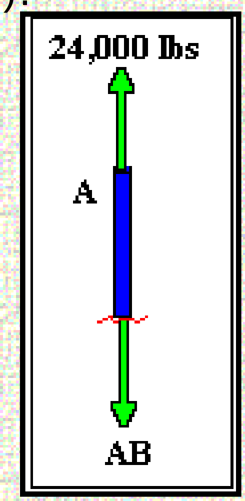
B_y = 8,000 lbs

Now to find the force in section AB of member ABC.

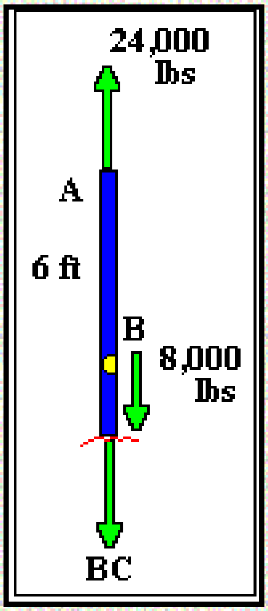


Cut member between points A and B. Look at top section (see diagram).
The internal force in section AB must be 24,000 lbs (equal to A) for equilibrium.

$Stress_{AB}$ = F / A = 24,000 lbs / .785 in^2 = 30,600 psi



Now to find the force in section BC of member ABC.



Cut member between points B and C. Look at top section (see diagram).
The internal force in section BC must be 16,000 lbs for equilibrium.
$Stress_{BC}$ = F/A = 16,000 lbs/.785 in^2 = 20,400 psi

PART C:

C. Def = Deformation
To find the total movement of C = Def_{AB} + Def_{BC}

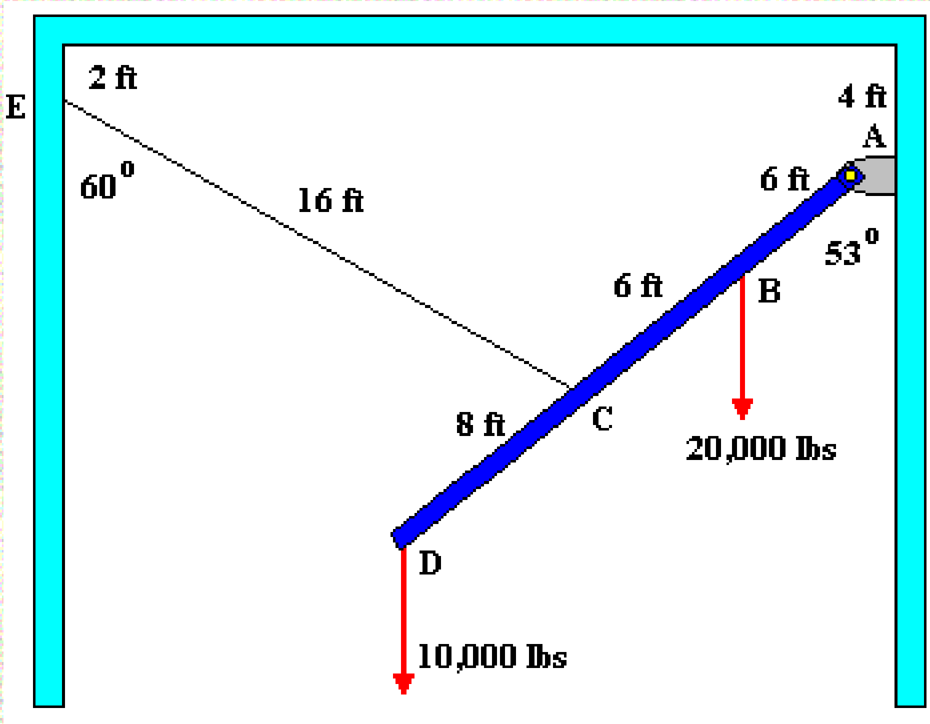
move.C=[$(FL / EA)_{AB}$ + $(FL / EA)_{BC}$]

move.C=[(24,000 lbs)(72 in)/(10*10^6 psi)(3.14*(.5 $in)^2$)$]_{AB}$+[(16,000 lbs)(48 in)/(10*10^6 psi)(3.14*(.5 $in)^2$)$]_{BC}$

move.C=(.22 in) + (.0978 in) = .318 in

STATICS / STRENGTH OF MATERIALS - Example

In the structure shown below member ABCD a solid rigid member pinned to the wall at A, and supported by steel cable CE. Cable CE is pinned to the wall at E and has a diameter of 1 inch. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress in cable CE.
C. Determine the movement of point D due to the applied load.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



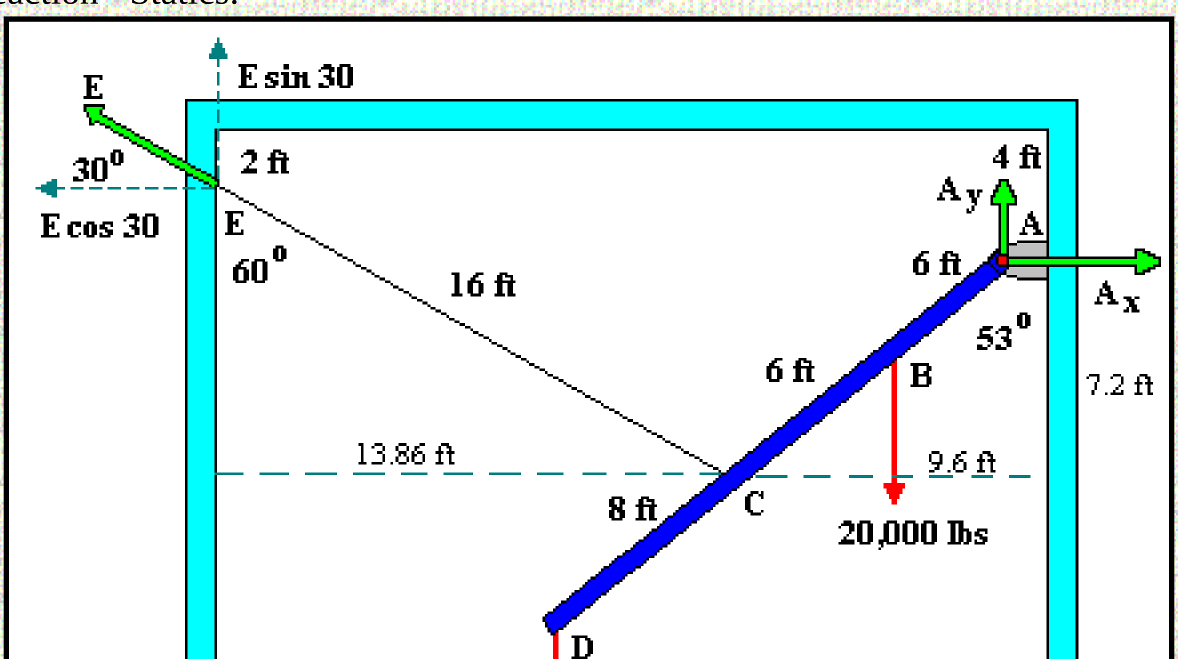
Solution:

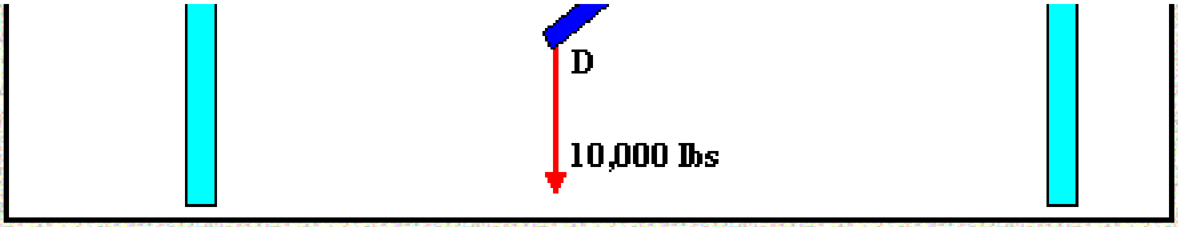
Part A. External support reaction - Statics:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.

STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the



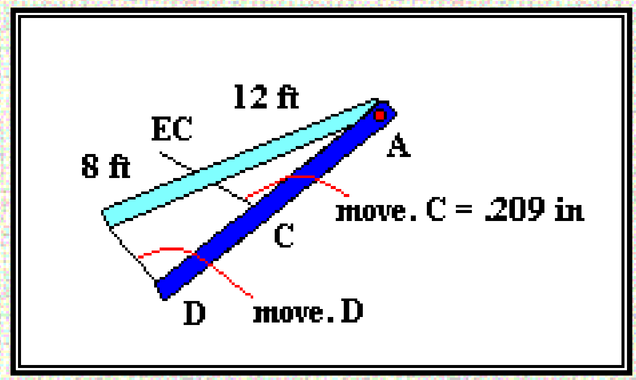


equilibrium conditions.
Sum $F_x = A_x$ - E cos (30º) = 0
Sum $F_y = A_y$ + E sin (30º) - 10,000 lbs - 20,000 lbs = 0
Sum T_A = (20,000 lbs)(4.8 ft) + (10,000 lbs)(16 ft) + E cos (30º)(2 ft) - E sin (30º)(23.46 ft)

Solving for the unknowns:
E = 25,600 lbs; A_x = 22,170 lbs; A_y = 17,200 lbs

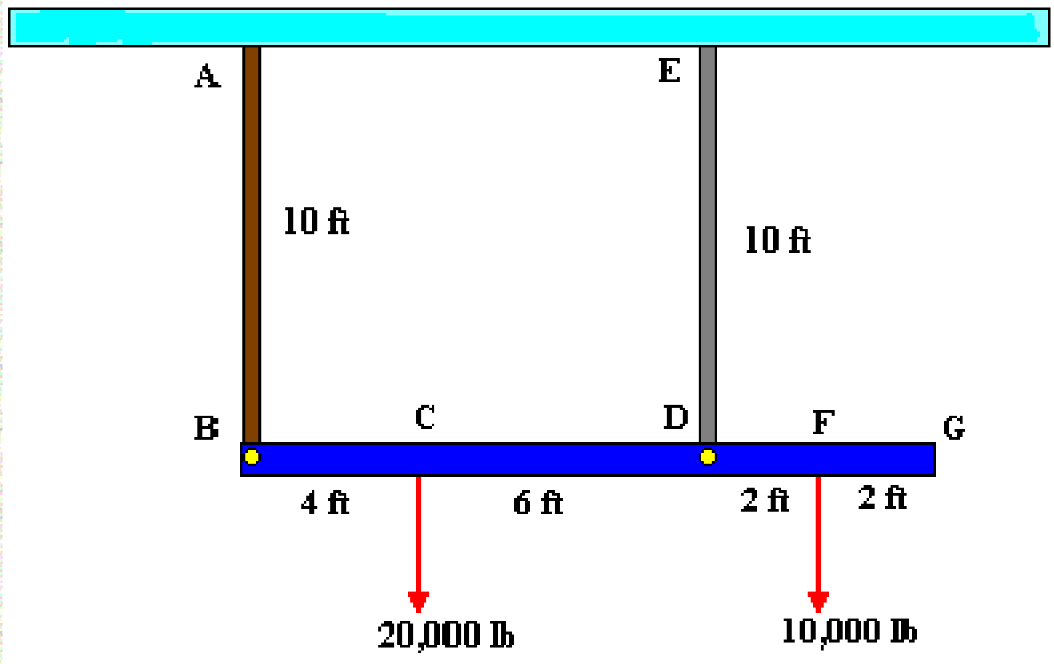
Part B. $Stress_{CE}$ = F/A = 25,600 lbs/.785 in^2 = 32,600 psi

Part C. Def = Deformation
Movement of D: First find deformation of cable CE
STEP 1: Def_{CE} = $(FL / EA)_{CE}$ = (25,600 lbs)(16 ft * 12 in/ft) / (30*10^6 lbs/in^2) (3.14*(.5 in)^2) = .209 in
STEP 2: Point C moves the same amount that cable CE deforms, and point D moves a proportional amount compared to point C (see diagram).



move. C / 12 ft = move. D / 16 ft
.209 in / 12 ft = move. D / 16 ft
Solving for movement of D = .348 in

STATICS / STRENGTH OF MATERIALS - Example



In the structure shown below member BCDFG is assumed to be a solid rigid member. It is supported by a two cables, AB, and DE. Cable AB is brass, and cable DE is steel. Both cables have a cross sectional area of .5 square inches. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.

B. Determine the axial stress in cable AB.

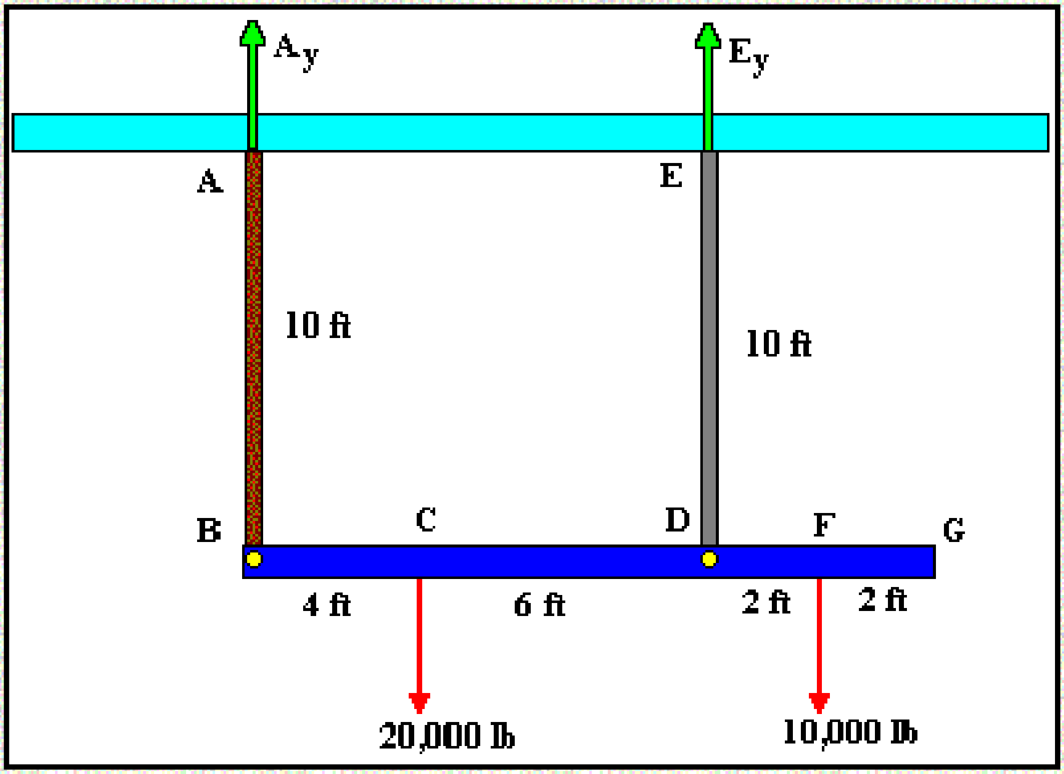
C. Determine the movement of point F due to the applied loads.

E_{st} = 30 x 10^6 psi

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

Part A. External support reaction - Statics:
STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.
Sum $F_y = A_y + E_y$ - 20,000 lbs - 10,000 lbs = 0
Sum T_B = (-20,000 lbs)(4 ft) + E_y(10 ft) - (10,000 lbs)(12ft) = 0



Solving for the unknowns: **E_y = 20,000 lbs; A_y = 10,000 lbs**

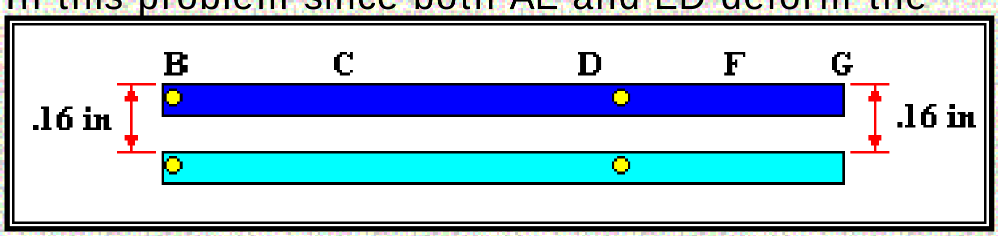
Part B. $Stress_{AB}$ = F / A = 10,000 lbs / .5 in^2 = 20,000 psi

Part C. Def = Deformation
Movement of point F. Point F moves since both member AB and ED deform and member BCDFG moves downward according to these deformations.
STEP 1: Calculate deformation of members AB and ED.

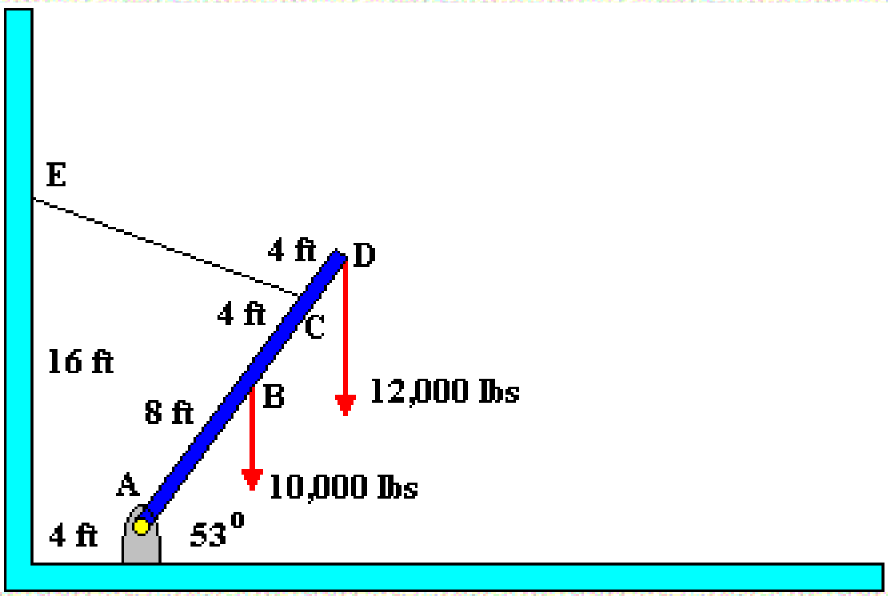
$Def_{AB} = (FL / AE)_{AB}$ = (10,000 lbs)(120 in) / ($15*10^6$ lbs/in^2)(.5 in^2) = .16 in

$Def_{ED} = (FL / AE)_{ED}$ = (20,000 lbs)(120 in) / ($30*10^6$ lbs/in^2)(.5 in^2) = .16 in

STEP 2: Movement of point F. In this problem since both AE and ED deform the same amount (.16 in), then cross member BCDFG simply moves downward that amount, as does point F. So, movement of **F = .16 in**



STATICS / STRENGTH OF MATERIALS - Example

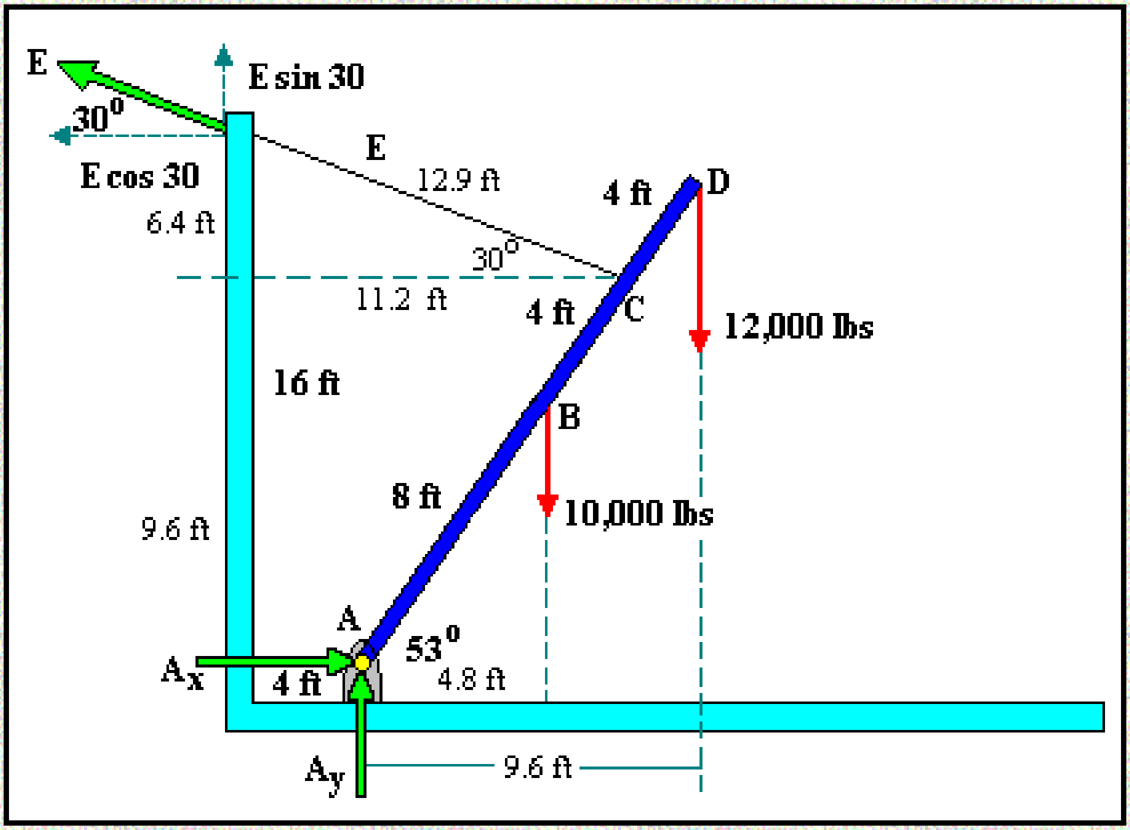


In the structure shown below member ABCD is assumed to be a solid rigid member. It is pinned to the floor at point A, and is supported by cable CE. Cable CE is made of steel and has a diameter of 1 inch. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress in cable CE.
Determine the movement of point D due to the applied loads.
Est = 30 x 10 6 psi; Ebr = 15 x 10 6 psi; Eal = 10 x 10 6 psi
αst = 12 x 10-6 /oC; αbr = 20 x 10-6 /oC; αal = 23 x 10-6 /oC
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

Part A. External support reaction - Statics:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

Sum $F_x = A_x$ - E cos $(30^o) = 0$

Sum $F_y = A_y$ + E sin (30^o) - 10,000 lbs - 12,000 lbs = 0

Sum T_A = -E sin (30^o)(4 ft) + E cos (30^o)(16 ft) - (10,000 lbs)(4.8 ft)- (12,000 lbs)(9.6 ft)= 0

Solving for the unknowns:

E = 13,790 lbs; A_x = 11940 lbs; A_y = 15,100 lbs

Part B. Stress$_{CE}$ = F / A = 13,790 lbs / .785 in^2 = 17,600 psi

Part C. Def = Deformation

STEP 1: Point D moves due to the deformation of cable EC. So we first determine the deformation of EC.

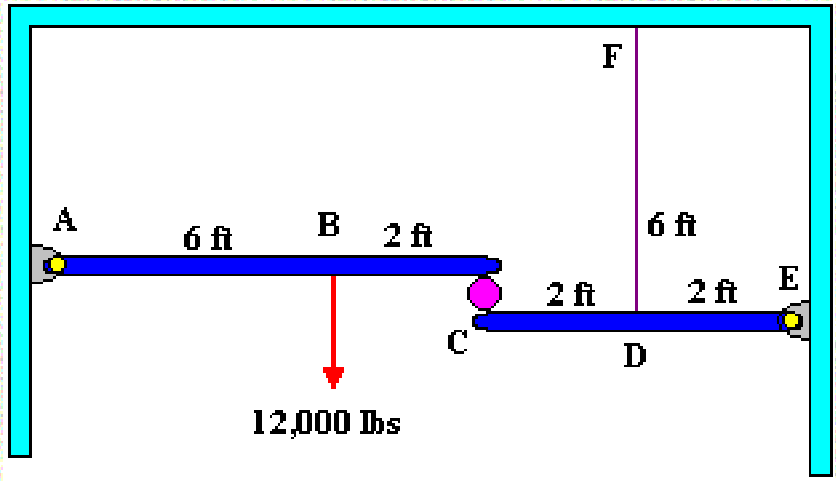
$Def_{EC} = (FL / EA)_{EC}$ = (13,790 lbs)(12.9 ft)(12 in/ft) / ($30*10^6$ lbs/in^2)(3.14 * .5 in^2) = .0912 in

STEP 2: Point D moves in proportion to how much point C moves. And point C moves the amount cable EC stretches. So we can write:

Mov. C / 12 ft = Mov. D / 16 ft

.0912 in / 12 ft = Mov. D / 16 ft and so **Mov. D = .122 in**

STATICS / STRENGTH OF MATERIALS - Example



In the structure shown below members ABC and CDE are assumed to be solid rigid members. Member ABC is pinned to the wall at A and is supported by a roller at point C. Member CDE is pinned to the wall at point E, and is supported by steel cable DF. Cable DF has a diameter of .75 inch. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.

B. Determine the axial stress in cable DF.

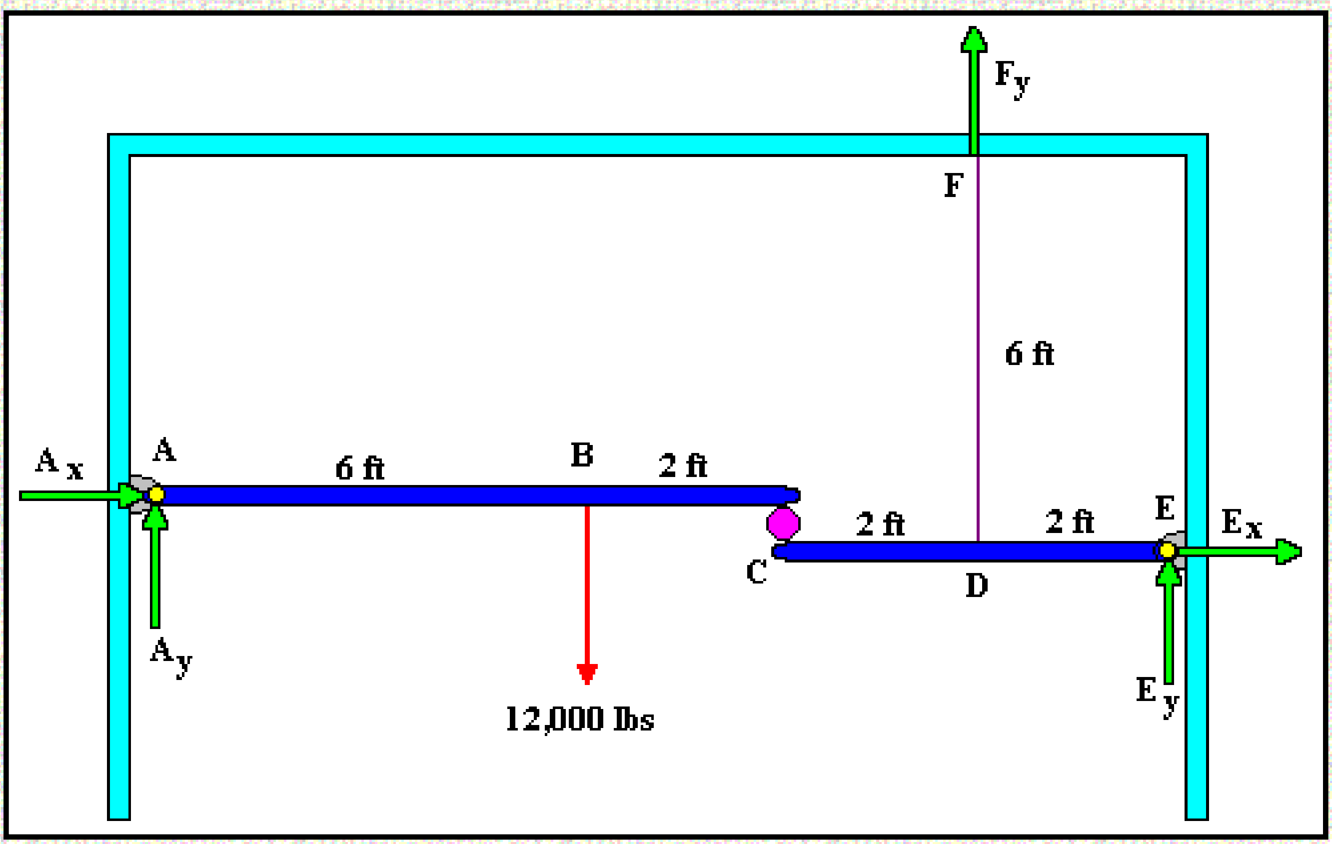
C. Determine the movement of point B due to the applied load.

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

Part A.

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



STEP 2: Break any forces not already in x and y direction

into their x and y components.

STEP 3: If we were to go about this problem in the normal fashion, we would have five unknowns and only three equations. There would be no way to solve this problem using that approach. Therefore, we must take a different perspective. By taking the member apart and analyzing member ABC there will only be three unknowns.

Apply the equilibrium conditions:

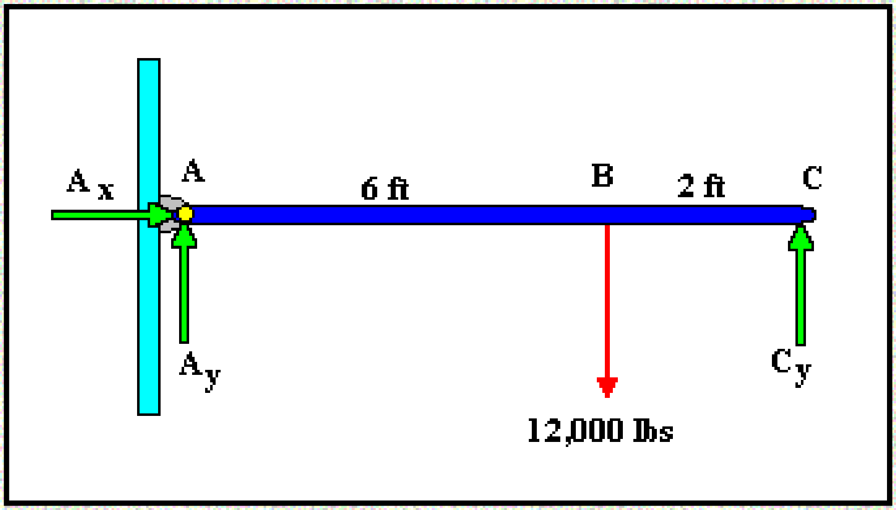
Sum $F_x = A_x = 0$

Sum $F_y = A_y + C_y - 12{,}000$ lbs $= 0$

Sum $T_A = (-12{,}000 \text{ lbs})(6 \text{ ft}) + C_y (8 \text{ ft}) = 0$

Solving for the unknowns:

$C_y = 9{,}000$ lbs; $A_y = 3{,}000$ lbs



Now we can analyze member CDE in a similar fashion.

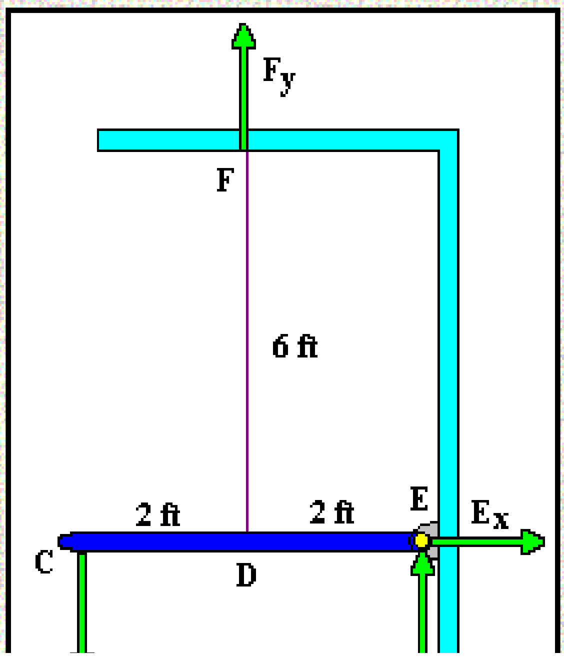
Apply the equilibrium conditions:

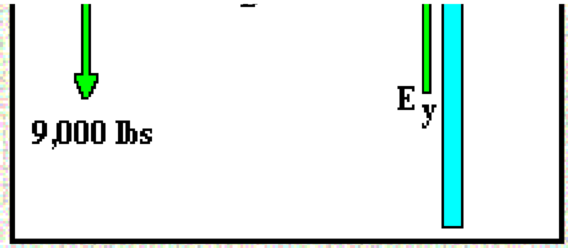
Sum $F_x = E_x = 0$

Sum $F_y = F_y + E_y - 9{,}000$ lbs $= 0$

Sum $T_E = -F_y (2 \text{ ft}) + (9{,}000 \text{ lbs})(4 \text{ ft}) = 0$

Solving for the unknowns:





F_y = 18,000 lbs; E_y = -9,000 lbs

Part B. $Stress_{DF}$ = F / A = 18,000 lbs / .4418 in^2

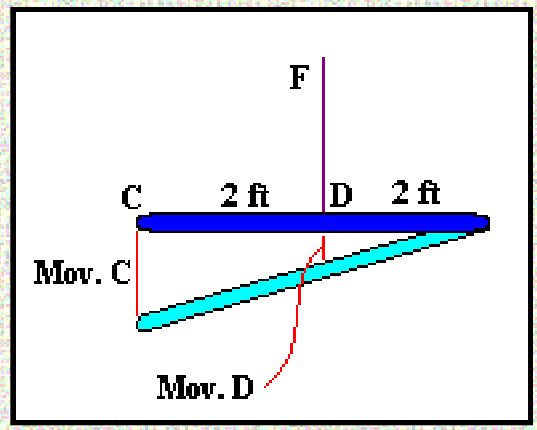
= 40,740 psi

Part C. Def = Deformation

STEP 1: Point C moves due to the deformation of cable FD. So we first determine the deformation of FD.

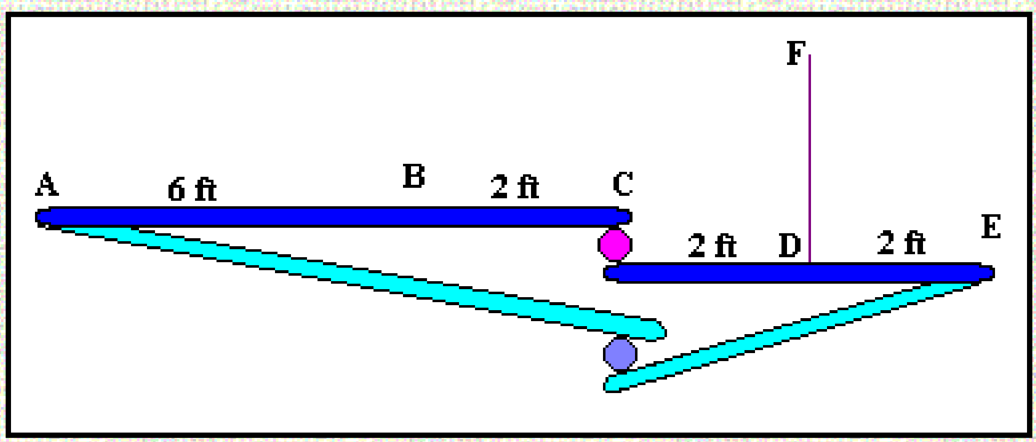
Def_{FD} = (FL / EA) = (18,000 lbs)(6 ft)(12 in/ft) / (30 *10^6)(.4417 in^2) = **.0978 in**

STEP 2: Point C moves in proportion to how much point D moves (see diagram).

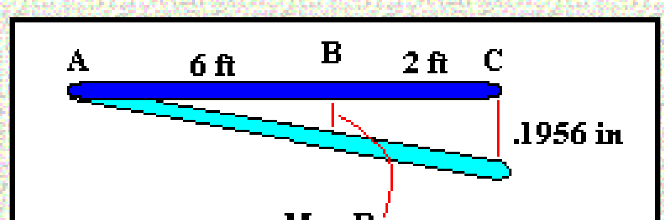


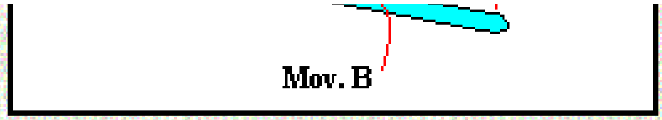
Mov. C / 4 ft = Mov. D / 2 ft
Mov. C / 4 ft = .0978 in/ 2 ft
Mov. C = .1956 in

STEP 3: Member ABC is resting on member CDE therefore, at point C the movement is the same for members ABC and CDE (see diagram).

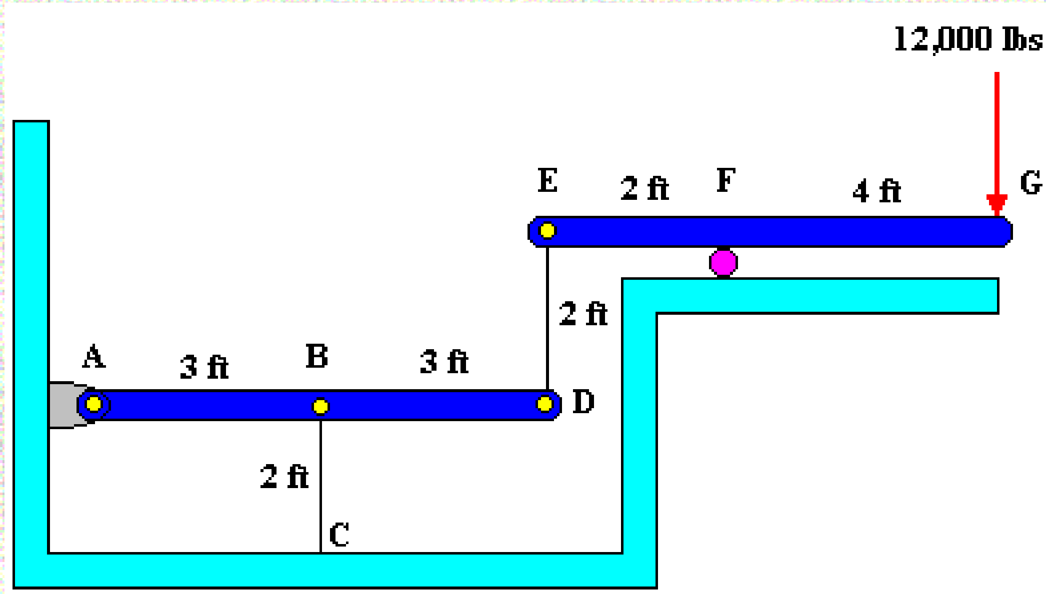


Mov. B / 6 ft = Mov. C / 8 ft
Mov. B / 6 ft = .1956 in/ 8 ft
Mov. B = .1467 in




Mov. B

STATICS / STRENGTH OF MATERIALS - Example



In the structure shown below member ABD is a solid rigid member pinned to the wall at A, supported by steel cable BC, and connected to member EFG by steel cable DE.
(Cables BC and DE each have a cross sectional area of .5 square inches.)
Member EFG is supported by a roller at F and is loaded with 12000 lbs at G.

For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.

B. Determine the axial stress in cable BC.

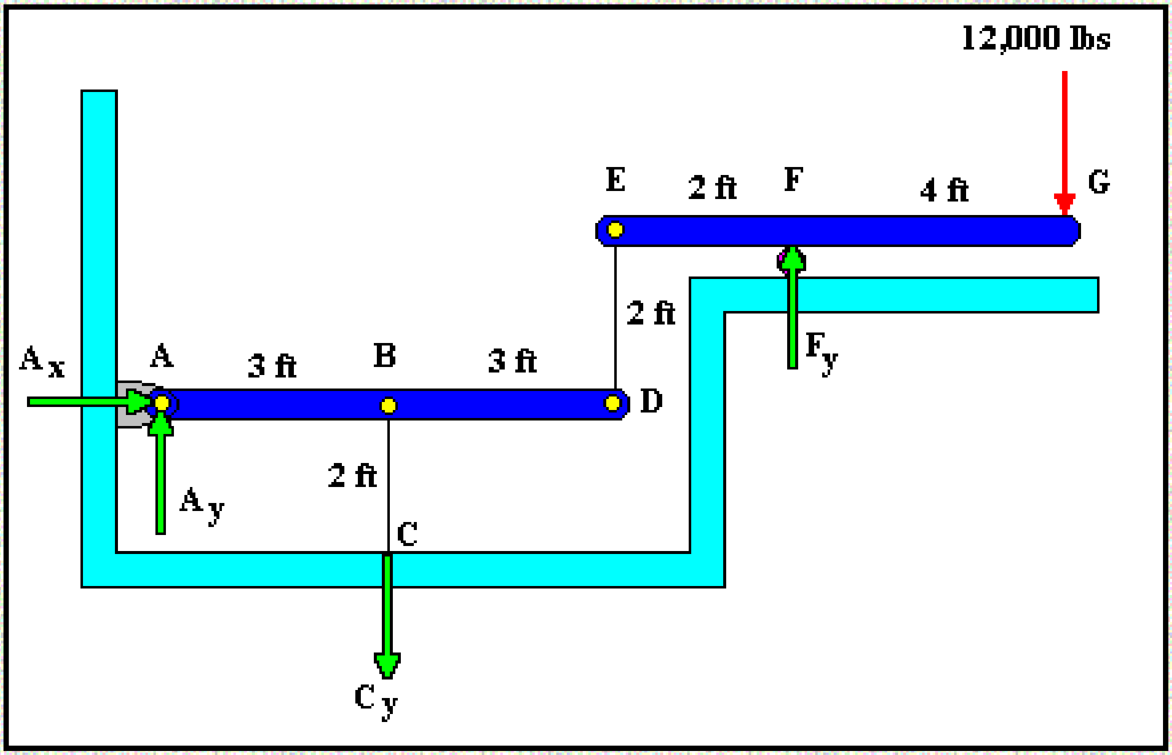
C. Determine the movement of point G due to the applied load.

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

Part A:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



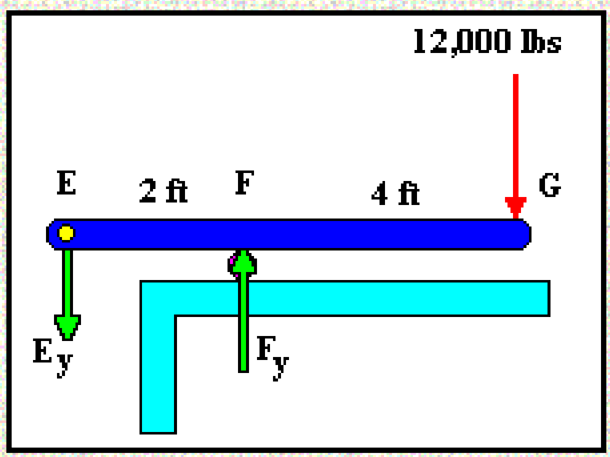
STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: If we were to go about this problem in the normal fashion, we would have four unknowns and only three equations. There would be no way to solve this problem using that approach. Therefore, we must take a different perspective. By taking the member apart and analyzing member EFG there will only be two unknowns.

Apply the equilibrium conditions:

Sum $F_y = F_y - E_y - 12{,}000$ lbs $= 0$

Sum $T_E = (-12{,}000 \text{ lbs})(6 \text{ ft}) + F_y\,(2 \text{ ft}) = 0$

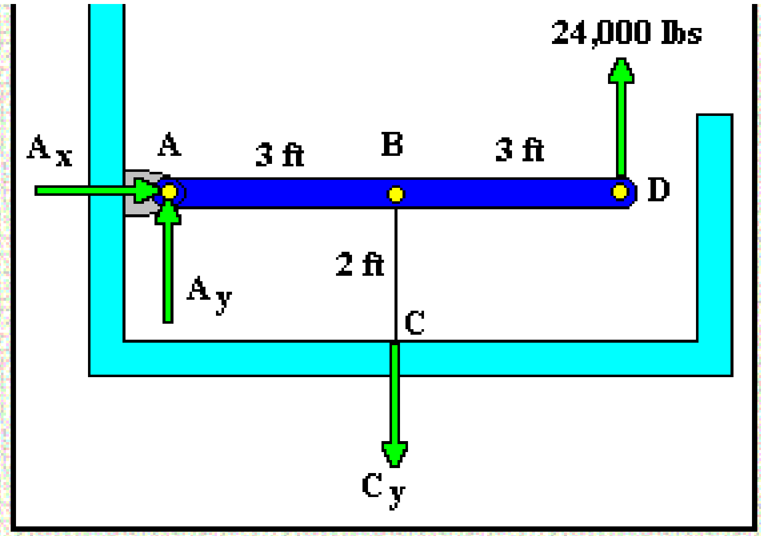


Solving for the unknowns:

F_y = 36,000 lbs; E_y = 24,000 lbs

Now we can analyze member ABD in a similar fashion.

Apply the equilibrium conditions:



Sum $F_x = A_x = 0$

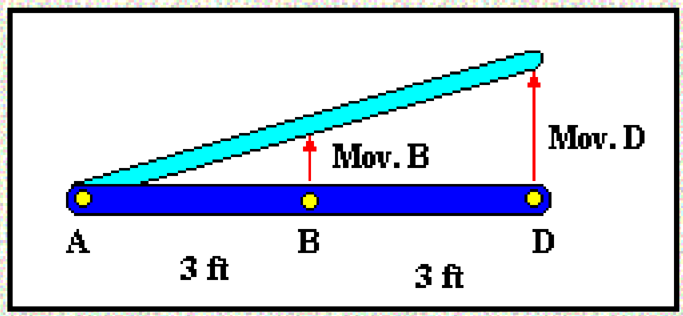
Sum $F_y = A_y - C_y + 24{,}000$ lbs $= 0$

Sum $T_B = -A_y$ (3 ft) + (24,000 lbs)(3 ft) = 0

Solving for the unknowns:
A_y = 24,000 lbs; C_y = 48,000 lbs

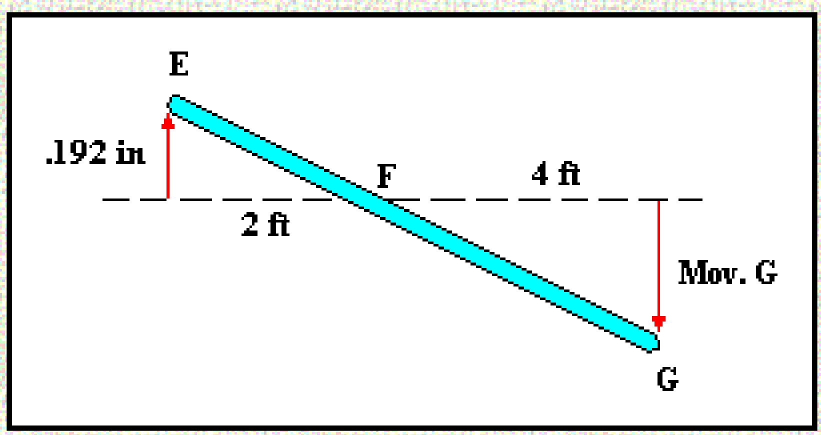
Part B. Stress_{BC} = F/A = 48,000 lbs/ .5 in^2 = 96,000 psi

Part C. Def = Deformation
Point G moves due to the deformation of cable BC and Cable DE.
STEP 1: Mov. B = Deformation cable BC
Mov. B = (FL / EA) = (48,000 lbs)(24 in) / (30*106 lbs/in^2)(.5 in^2) = **.0768 in**



STEP 2: Movement of point D is proportional to movement of point B, and we can write:
Mov. D / 6 ft = Mov. B / 3 ft
Mov. D / 6 ft = .0768 in / 3 ft
Mov. D = .1536 in

STEP 3: Movement of point E is equal to movement of point D plus the elongation of cable DE.
Mov. E = Mov. D + $(FL / EA)_{DE}$ = .1536 in + (24,000 lbs)(24 in) / (30*10^6 lbs/in^2)(.5 in^2)
Mov. E = .1536 in + .0384 in = .192 in

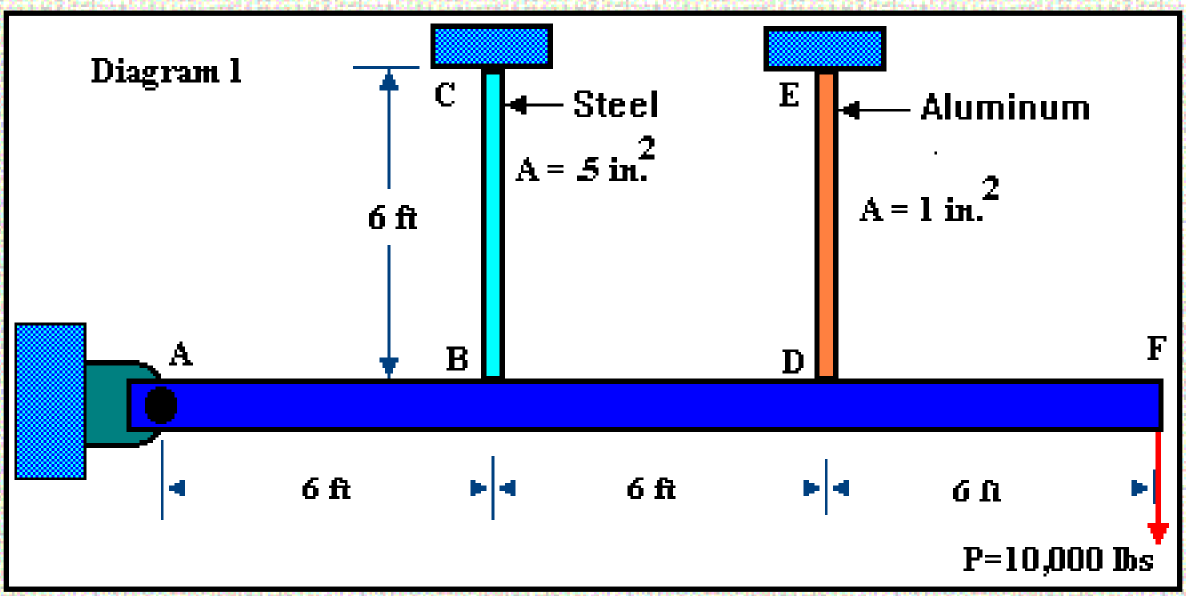


STEP 4: Finally the movement of point G is proportional to the movement of point E and we may write:
Mov. G / 4 ft = Mov. E / 2 ft
Mov. G / 4 ft = .192 in / 2 ft
Mov. G = .384 in

Topic 3.3: Statically Indeterminate Structures

The fact that real members deform under external forces and loads seems to add a complication when analyzing structures, but for certain types of structures, known as **Statically Indeterminate Structures**, it is the effect of the deformation that allows use to solve for the external forces on the structure and in the members of the structure. Perhaps the best way to illustrate this is to examine a relatively simple statically indeterminate structure.

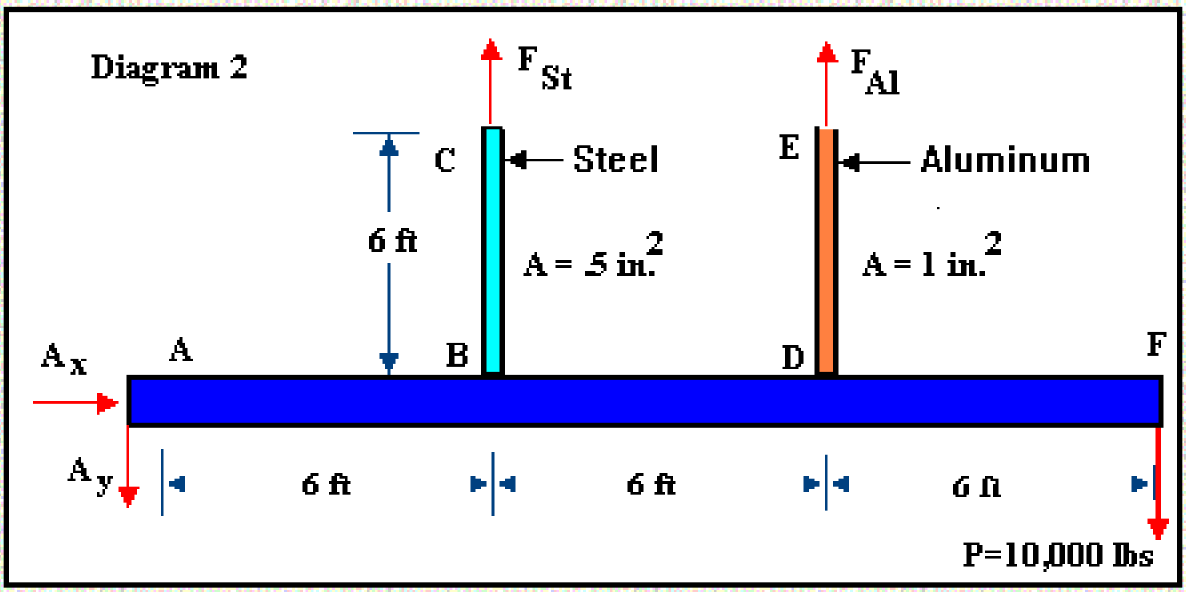
Example 1:
The structure shown in Diagram 1 is formed by member ABDF (which is pinned to the wall at point A), steel member BC, and aluminum member DE, both of which are pinned to and support member ABDF, and both of which are pinned to the ceiling as shown. An external load of 10,000 lb. is applied at point F. For this structure some of the things we might wish to know might be: after the 10,000 lb. load is applied, what are the external support forces acting on the structure, what is the stress in the steel and aluminum members, what is the movement of point F due to the load. As we analyze this structure, we are going to ignore the fact that member ABDF will experience some bending (which in a sense is a deformation, and which we will deal with when the topic of beams is discussed). Ignoring the bending in this case effects the result only to a small degree. As is also the case when the weight of the structural member is ignored in analyzing the structure. This often effects the result only slightly, especially when the external force and loads are much larger than the weight of the members.



To understand what we mean by a statically indeterminate structure, let us first try to analyze the structure using our standard static equilibrium procedure.

Part I - Static Equilibrium Analysis

1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions. In Diagram 2, a FBD of the structure is shown. At point A, where member ABDF is hinged to the wall, we replace the hinge by support forces A_x and A_y. The steel and aluminum members are pinned to the ceiling. Normally we would replace the pins by horizontal and vertical support forces, however in this case we can do better (meaning less unknowns). Since both the steel and aluminum members have forces acting on them at only two points (each end), they are axial members and are in simple tension or compression - in this problem, simple tension. Therefore, the ceiling simply pulls vertically upward on each member with force, F_{St} and F_{Al}, as shown in Diagram 2.



2: Resolve forces into x and y components. (All forces are either in x or y direction.)

3: Apply the equilibrium conditions.

$\sum F_x = 0$: $A_x = 0$

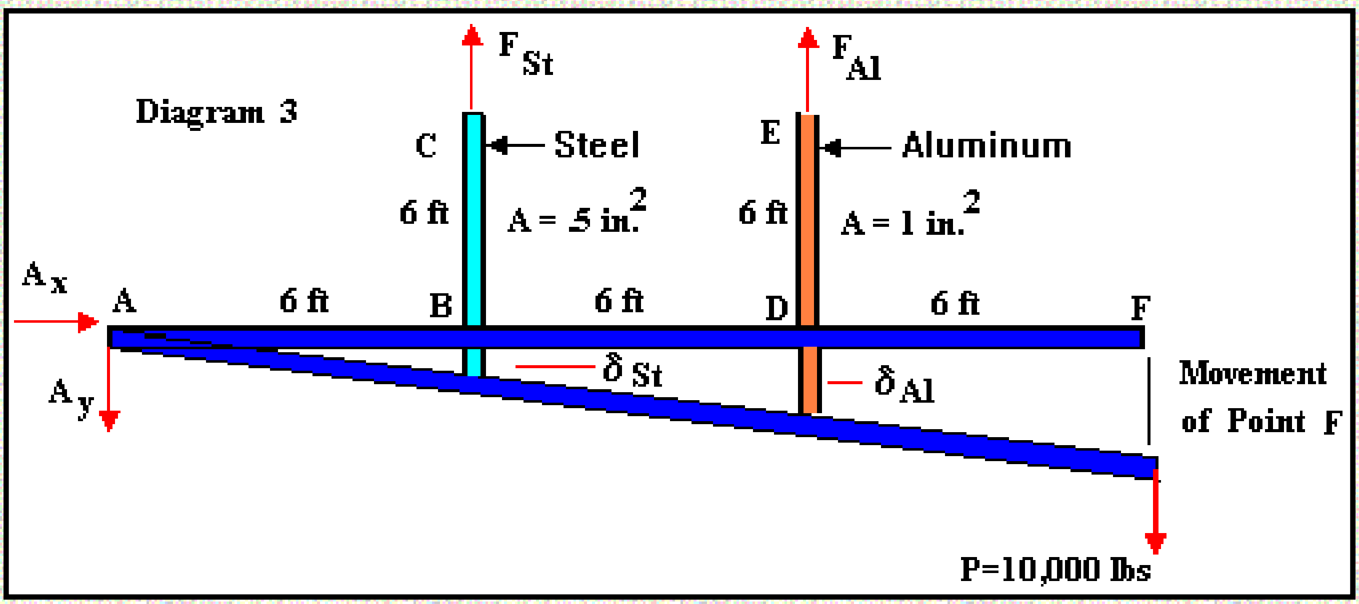
$\sum F_y = 0$: $-A_y + F_{St} + F_{Al} - 10{,}000 \text{ lb.} = 0$

$\sum \tau_A = 0$: $+ F_{St} (6 \text{ ft.}) + F_{Al} (12 \text{ ft.}) - 10{,}000 \text{ lb.} (18 \text{ ft.}) = 0$

At this point in a statically determinate problem, we would, in most cases, be able to solve for the external support reactions. However, in this case, we observe that we have three unknowns and only two independent equations - and can not solve. (We do see that A_x must be zero, from the first equation, but that is no help with the other two equations, in finding A_y, F_{St}, F_{Al}.) We might try to take the structure apart in some way, or redraw the FBD, but none of this will help. **Static equilibrium conditions alone are not enough to solve this problem - it is statically indeterminate.** Another way to state this difficulty is that we need another independent equation to solve for the unknowns. The deformations of the steel and aluminum members will give us this additional equation..

Part II. - Deformation Equation

Step 1 is to find some general relationship between the deformations of the members of the structure. We may get this from the way the problem is stated, or often from the geometry of the structure - as in this case. The effect of the 10,000 lb. load will be to elongate both the steel and aluminum members which will cause member ABDF to rotate downward about hinged point A. We diagram this, showing and labeling the deformations involved. This is somewhat like a Free Body Diagram, but with deformation effects shown. See Diagram 3. We now write a general relationship between the deformations of the members which we can get from the geometry of the problem. Since member ABDF is pinned at point A, as it rotates downward we see we can write (from similar triangles):



Deformation of Steel / 6 ft = Deformation of Aluminum / 12 ft
or we can rewrite as: **2 * Deformation of Steel = Deformation of Aluminum**
or symbolically: $2\delta_{St} = \delta_{Al}$

This is our additional equation which we will use in combination with the static equilibrium equations to find the external forces acting on the structure. At first it may not be clear how we can use this equation, since it involves deformations, not forces as in the equilibrium equations. However, if we recall our stress/strain/

deformation relationships, we see we can write the deformation of a member as:

Stress: $\sigma = F/A$ (lb/in^2, or N/m^2)
Strain: $\varepsilon = \delta/Lo$ (no units)
Hooke's Law: $E = \sigma/\varepsilon$ (lb/in^2, or N/m^2)
Deformation: $\delta = FL/EA$ (in. or m)

deformation = [force in member * length of member] / [young's modulus of member * area of member], or def = FL/EA. Substituting this relationship into our deformation equation we have:

$2 * [FL/EA]_{St} = [FL/EA]_{Al}$

We now have our additional relationship between the forces. After substituting in the values for the members we have:

$(2 * F_{St} * 72") / (30 \times 10^6\ lb/in^2 * .5\ in^2) = (F_{Al} * 72") / (10 \times 10^6\ lb/in^2 * 1\ in^2)$

If we simplify this equation we obtain: **$F_{St} = .75\ F_{Al}$** We now substitute this into our torque equation from static equilibrium equations (shown below)

$\sum F_x = 0$: **$A_x = 0$**

$\sum F_y = 0$: **$-A_y + F_{St} + F_{Al} - 10{,}000$ lb. = 0**

$\sum \tau_A = 0$: **$+ F_{St}$ (6 ft.) + F_{Al} (12 ft.) - 10,000 lb. (18 ft.) = 0**

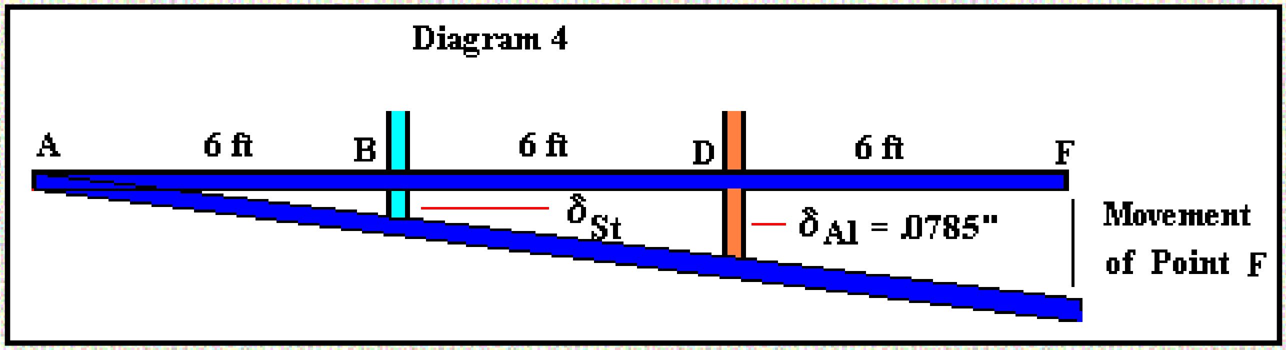
and obtain: **$(.75\ F_{Al})$(6 ft.) + F_{Al} (12) - 10,000 lb. (18 ft.) = 0**; and solving we have:

F_{Al} = 10,900 lb., and F_{St} = 8175 lb. We can now also solve for Ay, finding **Ay = 10,075 lb**

We find the stress from: **Stress Steel = 8175 lb/ .5 in^2 = 16, 350 lb/in^2, Stess Aluminum = 10,900 lb/1 in^2 = 10,900 lb/in^2.**

And finally we can find the movement of point F by first finding the elongation of member DE, the aluminum member, from **Def. = FL/EA = (10,900 lb * 72 in.)/ (10×10^6 $lb.in^2$ * 1 in^2) = .0785 in**. Point F moves in proportion to the elongation of member DE, and we may write: **.0785 in./12 ft = Move. F/18 ft**, solving **Move. F = .118 in.**

We have now solved our statically indeterminate problem and determined the values of the external support reactions acting on the structure, the stresses, and the movement of point F.



Summary: As a brief review, our method for solving statically indeterminate problems consists of two basic steps: Step 1 is to apply static equilibrium conditions and write the static equilibrium equations. Step 2 is to find a general relationship between the deformations of the members in the structure, and then to rewrite this relationship using deformation = FL/EA to obtain an additional equation between the forces acting on the structure. Finally using the equations from Steps 1 and 2, we should be able to solve for all the external support forces acting on the structure. To see applications of these relationships, we now will look at several examples.

Continue to:
Example 1 ; **Example 2**
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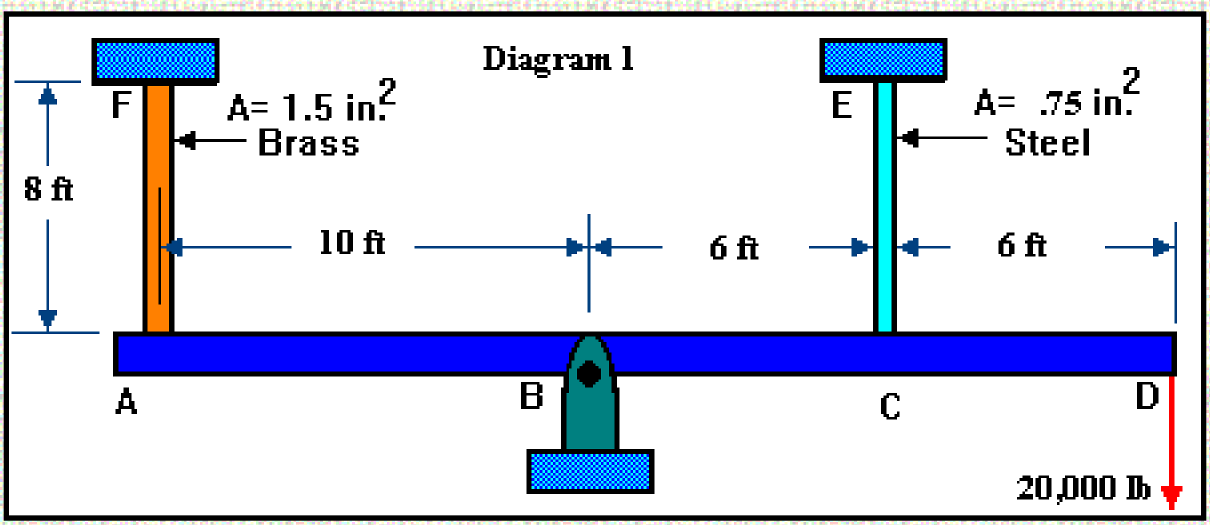
Topic 3.3a: Statically Indeterminate Structures - Example 1

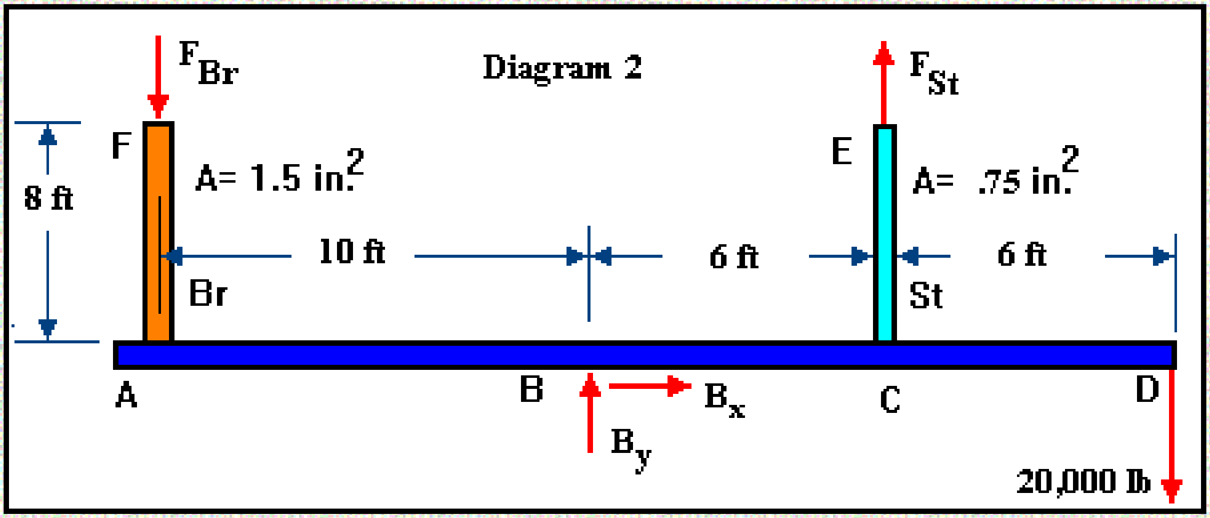
The structure shown in Diagram 1 is formed by member ABCD (which is pinned to the floor at point B), Brass member AF, and Steel member CE, both of which are pinned to and support member ABCD, and both of which are pinned to the ceiling as shown. A downward external load of 20,000 lb. is applied at point D. For this structure we would like to determine the stress in the steel and brass members, and the movement of point D due to the load. As we analyze this structure, we ignore the fact that member ABCD will experience some bending (which in a sense is a deformation, and which we will deal with when the topic of beams is discussed). Ignoring the bending in this case effects the result only to a small degree.



Our first step is to apply our static equilibrium procedure to our structure

Part I - Static Equilibrium Analysis

1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions. In Diagram 2, a FBD of the structure is shown. At point B, where member ABCD is pinned to the floor, we replace the pin by support forces B_x and B_y. The brass and steel members are pinned to the ceiling. Normally we would replace the pins by horizontal and vertical support forces, however in this case we can do better (meaning less unknowns). Since both the steel and brass members have forces acting on them at only two points (each end), they are axial members and are in simple tension or compression. The 20,000 lb. load acting at end D tries to rotate the bar ABCD downward about point B. Thus putting the brass member (AF) in compression and the steel member (CE) in tension. As a result, the ceiling acts on the members as shown in the free body diagram.



2: Resolve forces into x and y components. (All forces are either in x or y direction.)

3: Apply the equilibrium conditions.

$\sum F_x = 0$: $B_x = 0$

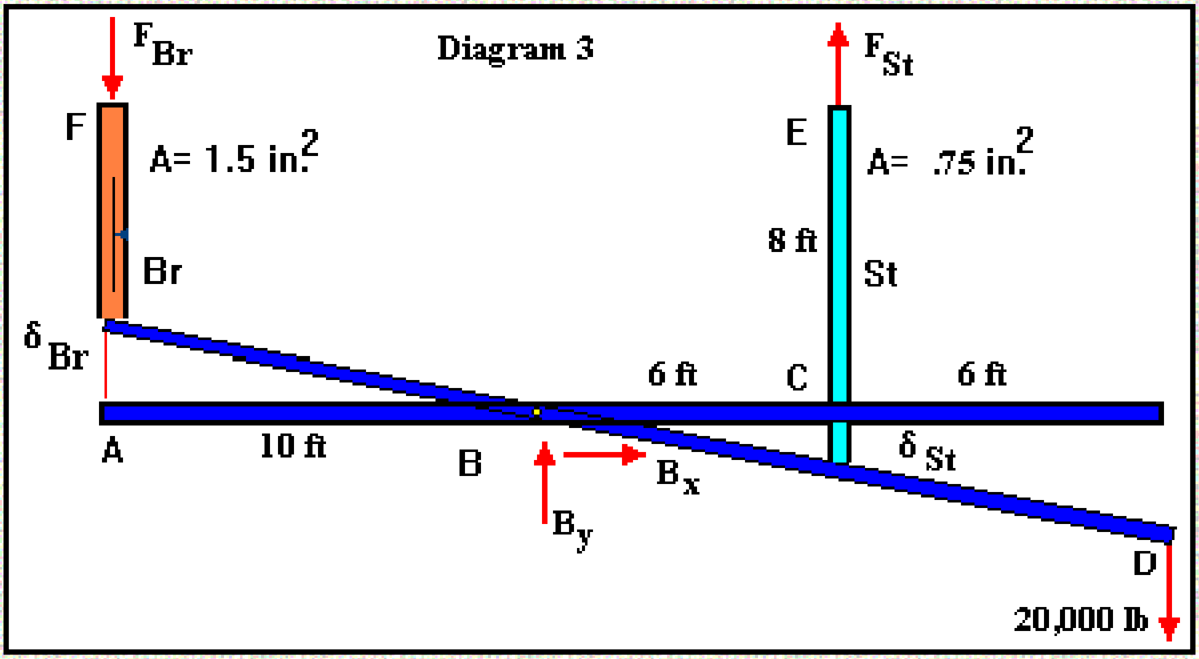
$\sum F_y = 0$: $B_y - F_{Br} + F_{St} - 20{,}000$ lb. $= 0$

$\sum \tau_B = 0$: $+ F_{Br}$ (10 ft.) $+ F_{St}$ (6 ft.) - 20,000 lb. (12 ft.) = 0

At this point in a statically determinate problem, we would, in most cases, be able to solve for the external support reactions. However, in this case, we observe that we have three unknowns and only two independent equations - and can not solve. (We do see that B_x must be zero, from the first equation, but that is no help with the other two equations, in finding B_y, F_{Br}, F_{St}.) We might try to take the structure apart in some way, or redraw the FBD, but none of this will help. **Static equilibrium conditions alone are not enough to solve this problem - it is statically indeterminate.** Another way to state the problem is that we need another independent equation to solve for the unknowns. The deformations of the brass and steel members will give us this additional equation..

Part II. - Deformation Equation

Step 1 is to find some general relationship between the deformations of the members of the structure. We may get this from the way the problem is stated, or often from the geometry of the structure - as in this case. The effect of the 20,000 lb. load will be to compress the brass member and to elongate the steel member which will cause member ABCD to rotate downward about hinged point B. We diagram this, showing and labeling the deformations involved. See Diagram 3. We now write a general relationship between the deformations of the members which we can get from the geometry of the problem. Since member ABDF is pinned at point A, as it rotates downward we see we can write (from similar triangles):



Deformation of Steel / 6 ft = Deformation of Brass / 10 ft
or we can rewrite as: **Deformation of Steel = .6 * Deformation of Brass**

or symbolically: $\delta_{St} = .6\,\delta_{Br}$

This is our additional equation which we will use in combination with the static equilibrium equations to find the external forces acting on the structure. At first it may not be clear how we can use this equation, since it involves deformations, not forces as in the equilibrium equations. However, if we recall our stress/strain/deformation relationships, we see we can write the deformation of a member as:

Stress: $\sigma = F/A$ (lb / in^2, or N / m^2)
Strain: $\varepsilon = \delta / Lo$ (no units)
Hooke's Law: $E = \sigma / \varepsilon$ (lb / in^2, or N / m^2)
Deformation: $\delta = FL/EA$ (in. or m)

deformation = [force in member * length of member] / [young's modulus of member * area of member], or def = FL/EA. Substituting this relationship into our deformation equation we obtain: **$[FL/EA]_{St} = .6\ [FL/EA]_{Br}$**

We now have our additional relationship between the forces. After substituting in the values for the members we get:

$(F_{St} * 96")/(30 \times 10^6\ lb/in^2 * .75\ in^2) = .6[(F_{Br} * 96")/(15 \times 10^6\ lb/in^2 * 1.5\ in^2)]$

If we simplify this equation we obtain: **F_{St} = .6 F_{Br}** We now substitute this into our torque equation from static equilibrium equations (shown on right)

$\sum F_x = 0$: **B_x = 0**

$\sum F_y = 0$: **B_y -F_{Br} + F_{St} - 20,000 lb. = 0**

$\sum \tau_B = 0$: **+ F_{Br} (10 ft.) + F_{St} (6 ft.) - 20,000 lb. (12 ft.) = 0**

and obtain: **(F_{Br})(10 ft.) + (.6 F_{Br}) (6 ft.) - 20,000 lb**. (12 ft.) = 0; and solving we have:

F_{Br} = 17,650 lb., and F_{St} = 10,600 lb. We can now also solve for By, finding **By = 27,050 lb**

We find the stress from: **Stress Steel = 10,600 lb/ .75 in^2 = 14,130 lb/in^2, Stress Brass = 17,650 lb/1.5 in^2 = 11,770 lb/in^2.**

And finally we can find the movement of point D by first finding the elongation of member CE, the steel member, from **Def. = FL/EA = (10,600 lb * 96 in.)/(30 x 10^6 lb.in^2 * .75 in^2) = .0452** in. Point D moves in proportion to the elongation of member CE, and we may write: **.0452 in./6ft = Move. D/12 ft**, solving **Move. D = .0904 in.**

We have now solved our statically indeterminate problem and determined the values of the external support reactions acting on the structure, the stresses, and the movement of point D.

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Topic 3.3b: Statically Indeterminate Structures - Example 2

The structure shown in Diagram 1 consists of horizontal member BCE, which is supported by two steel cables, AB and EF, and a Brass cable, BD pinned to the ceiling as shown. A downward external load of 50,000 lb. is applied at point C. For this structure we would like to determine the stress in the steel and brass cables, and the movement of point C due to the load. As we analyze this structure, we ignore the fact that member BCE will experience some bending (which in a sense is a deformation, and which we will deal with when the topic of beams is discussed). Ignoring the bending in this case effects the result only to a small degree.
Our first step is to apply our static equilibrium procedure to our structure

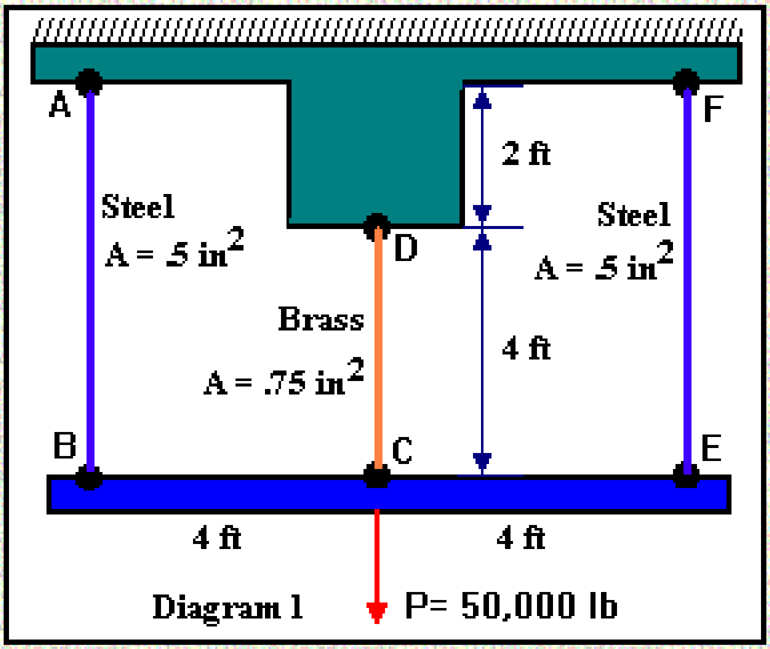
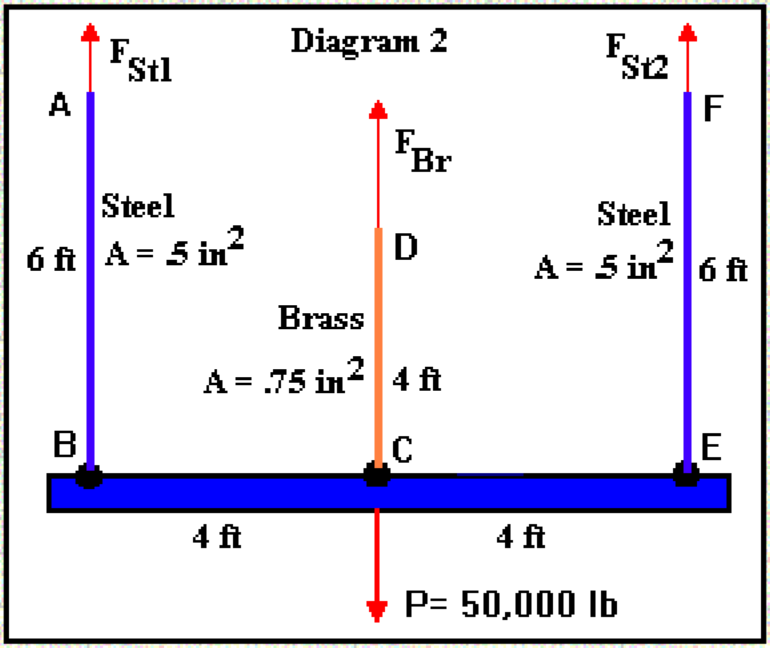


Diagram 1

Part I - Static Equilibrium Analysis
1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions. In Diagram 2, a FBD of the structure is shown. In this structure the load simply puts all the cables in tension, and the ceiling acts on the structure as shown in Diagram 2. Notice that at this point we do not assume the force the ceiling exerts on each steel cable is the same (although the symmetry of problem indicates that should be true), but rather we indicate two different forces on the steel cables, F_{st1} and F_{st2}. Our static equilibrium equations will make clear the relationship between the forces in the two steel cables.



2: Resolve forces into x and y components. (All forces are either in x or y direction.)

3: Apply the equilibrium conditions.

$\sum F_x = 0$: (no external x forces.)

$\sum F_y = 0$: **$F_{St1} + F_{St2} + F_{Br}$ - 50,000 lb. = 0**

$\sum \tau_C = 0$: **- F_{St1} (4 ft.) + F_{St2} (4 ft.) = 0**

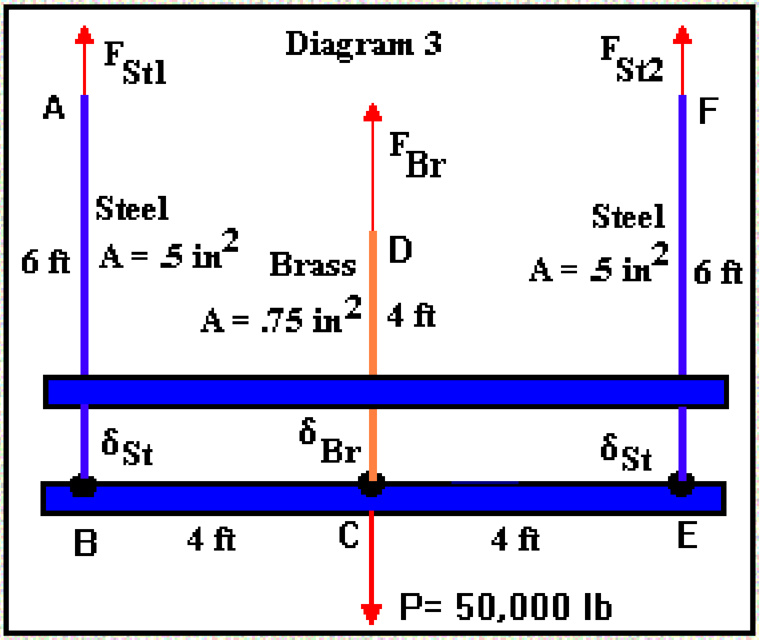
From the torque equation, we see that $F_{st1} = F_{st2}$, and so we will now call the force in the steel cables just F_{St}. And we can rewrite the sum of y- forces equation as:

$F_{St} + F_{St} + F_{Br}$ - 50,000 lb. = 0, or 2 $F_{St} + F_{Br}$ -50,000 lb. = 0

However, we still have too many unknowns. We have two unknowns at this point, and only one independent equations left - and we cannot solve. **Static equilibrium conditions alone are not enough to solve this problem - it is statically indeterminate.** Another way to state the problem is that we need another independent equation to solve for the unknowns. The deformations of the brass and steel members will give us this additional equation..

Part II. - Deformation Equation

Step 1 is to find some general relationship between the deformations of the members of the structure. We may get this from the way the problem is stated, or often from the geometry of the structure - as in this case. The effect of the 50,000 lb. load, because of the symmetry of the problem, will be to elongate both the steel and the brass members equally, and cause horizontal member BCE to move downward as shown in Diagram 3.



We now write a general relationship between the deformations of the members which we can get from the geometry of the problem. For our structure:
Deformation of Steel = Deformation of Brass

or symbolically: $\delta_{St} = \delta_{Br}$

This is our additional equation which we will use in combination with the static equilibrium equations to find the external forces acting on the structure. At first it may not be clear how we can use this equation, since it involves deformations, not forces as in the equilibrium equations. However, if we recall our stress/strain/deformation relationships, we see we can write the deformation of a member as:

Stress: $\sigma = F/A$ $(lb/in^2$, or $N/m^2)$
Strain: $\varepsilon = \delta/Lo$ (no units)
Hooke's Law: $E = \sigma/\varepsilon$ $(lb/in^2$, or $N/m^2)$
Deformation: $\delta = FL/EA$ (in. or m)

deformation = [force in member * length of member] / [young's modulus of member * area of member], or def = FL/EA. Substituting this relationship into our deformation equation we have: **$[FL/EA]_{St} = [FL/EA]_{Br}$** We

now have our additional relationship between the forces. After substituting in the values for the members we have:
$(F_{St} * 72")/(30 \times 10^6\ lb./in^2 * .5\ in^2) = (F_{Br} * 48")/(15 \times 10^6\ lb./in^2 * .75\ in^2)$

If we simplify this equation we obtain: **$F_{St} = .89\ F_{Br}$** We now substitute this into our sum of y-forces equation from static equilibrium equations.

$\sum F_x = 0$: (no external x forces.)

$\sum F_y = 0$: 2 F_{St} + F_{Br} -50,000 lb. = 0

$\sum \tau_C = 0$: - F_{St1} (4 ft.) + F_{St2} (4 ft.) = 0

and obtain: **2 (.89 F_{Br}) + F_{Br} - 50,000 lb. = 0**; and solving we have:

F_{Br} = 18,000 lb., and F_{St} = 16,000 lb.

We find the stress from: **Stress Brass = 18,000 lb./ .75 in^2 = 24,000 lb./in^2, Stress Steel = 16,000 lb./.5 in^2 = 32,000 lb./in^2.**
And finally we can find the movement of point C simply by finding the elongation of member CD, the Brass member, from **Def. = FL/EA = (18,000 lb. * 48 in.)/ (15 x 10^6 lb.in^2 * .75 in^2) = .0768 in.**

We have now solved our statically indeterminate problem and determined the values of the external support reactions acting on the structure, the stresses, and the movement of point C.

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Topic 3.4: Shear Stress & Strain

SHEAR STRESS

In additional to Axial (or normal) Stress and Strain (discussed in topic 2.1and 2.2), we may also have what is known as Shear Stress and Shear Strain.In Diagram 1 we have shown a metal rod which is solidly attached to the floor. We then exert a force, F, acting at angle theta with respect to the horizontal, on the rod. The component of the Force perpendicular to the surface area will produce an Axial Stress on the rod given by **Force perpendicular to an area divided by the area**, or:

$\sigma = F \sin \theta \ / A$

The component of the Force parallel to the area will also effect the rod by producing a Shear Stress, defined as **Force parallel to an area divided by the area**, or:

$\tau = F \cos\theta / A$ where the Greek letter, Tau, is used to represent Shear Stress. The units of both Axial and Shear Stress will normally be lb/in^2 or N/m^2.

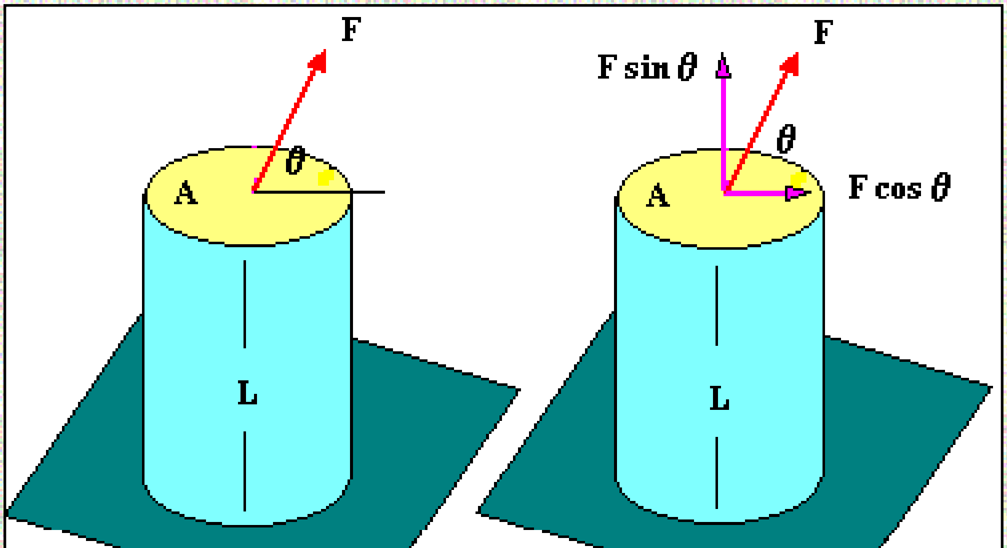


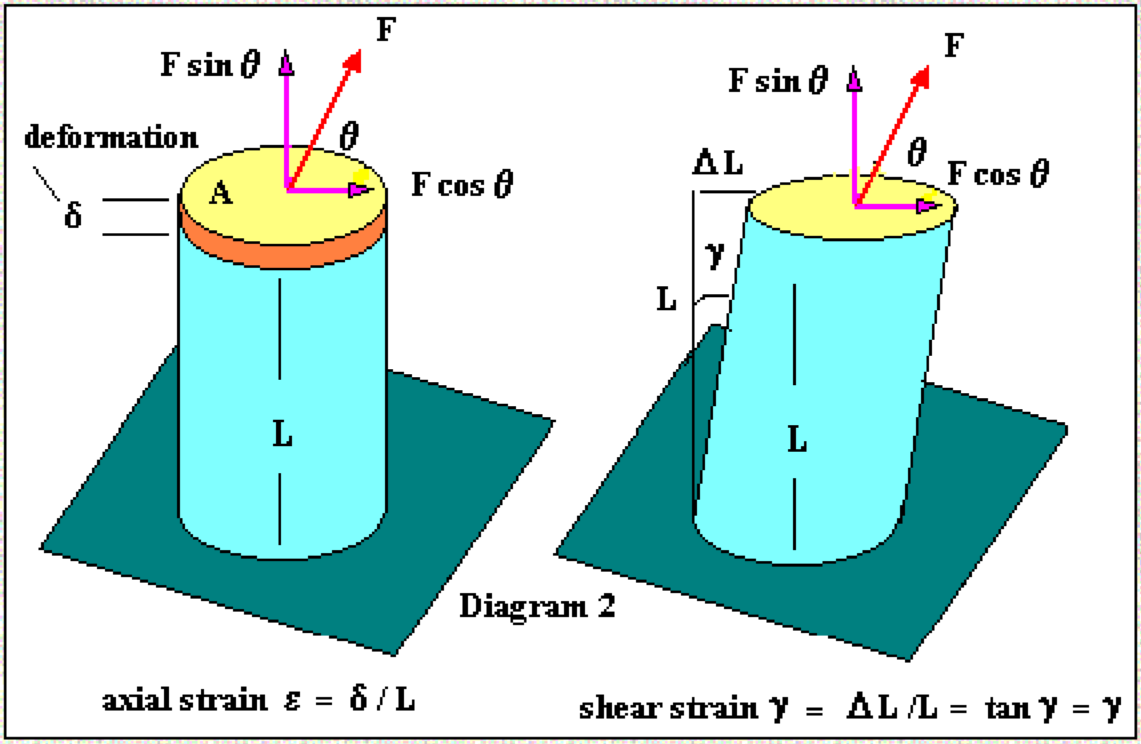
Diagram 1

Shear Strain:

Just as an axial stress results in an axial strain, which is the change in the length divided by the original length of the member, so does shear stress produce a shear strain. Both **Axial Strain** and **Shear Strain** are shown in Diagram 2. The shear stress produces a displacement of the rod as indicated in the right drawing in Diagram 2. The edge of the rod is displaced a horizontal distance, ΔL from its initial position. This displacement (or horizontal deformation) divided by the length of the rod L is equal to the **Shear Strain**. Examining the small triangle made by

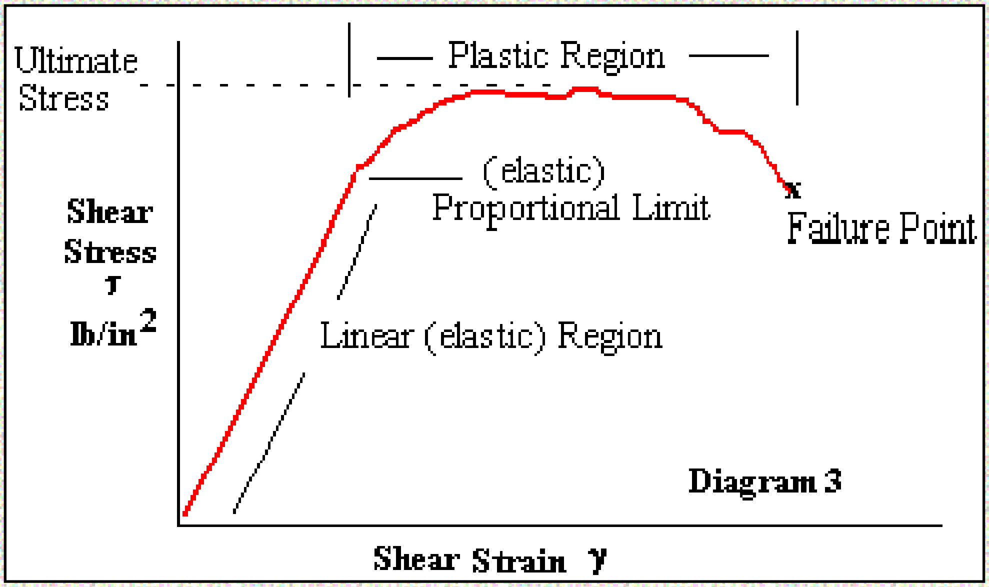
ΔL, L and the side of the rod, we see that the Shear Strain, **ΔL / L** , is also equal to the tangent of the angle gamma, and since the amount of displacement is quite small the tangent of the angle is approximately equal to the angle itself. Or we may write:

Shear Strain = $\Delta L / L = \tan\gamma = \gamma$



As with Axial Stress and Strain, Shear Stress and Strain are proportional in the elastic region of the material. This relationship may be expressed as **G = Shear Stress / Shear Strain,** where G is a property of the material and is called the Modulus of Rigidity (or at times, the Shear Modulus) and has units of lb/in^2. The Modulus of Rigidity for Steel is approximately 12 x 10^6 lb/in^2.

If a graph is made of **Shear Stress versus Shear Strain**, it will normally exhibit the same characteristics as the graph of **Axial Stress versus Axial Strain**. There is an **Elastic Region** in which the Stress is directly proportional to the Strain. The point at which the **Elastic Region** ends is called the **elastic limit**, or the **proportional limit**. In actuality, these two points are not quite the same. The **Elastic Limit** is the point at which permanent deformation occurs, that is, after the elastic limit, if the force is taken off the sample, it will not return to its original size and shape, permanent deformation has occurred. The **Proportional Limit** is the point at which the deformation is no longer directly proportional to the applied force (Hooke's Law no longer holds). Although these two points are slightly different, we will treat them as the same in this course. There is a **Plastic Region**, where a small increase in the Shear Stress results in a larger increase in Shear Strain, and finally there is a **Failure Point** where the sample fails in shear.



To summarize our axial stress/strain/Hooke's Law relationships up to this point, we have:

Shear Stress: $\tau = (F_{parallel}) / A$ $\quad (lb/in^2, \text{ or } N/m^2)$
Shear Strain: $\gamma = \Delta L / L = \tan \gamma = \gamma$
Hooke's Law: $G = \tau / \gamma$ $\quad (lb/in^2, \text{ or } N/m^2)$

While we will not go in any great depth, at this point, with respect to Shear Stress and Strain, we will look at several relative easy examples. Please Select:

Example 1 - Shear Stress & Strain

Example 2 - Shear Stress & Strain.

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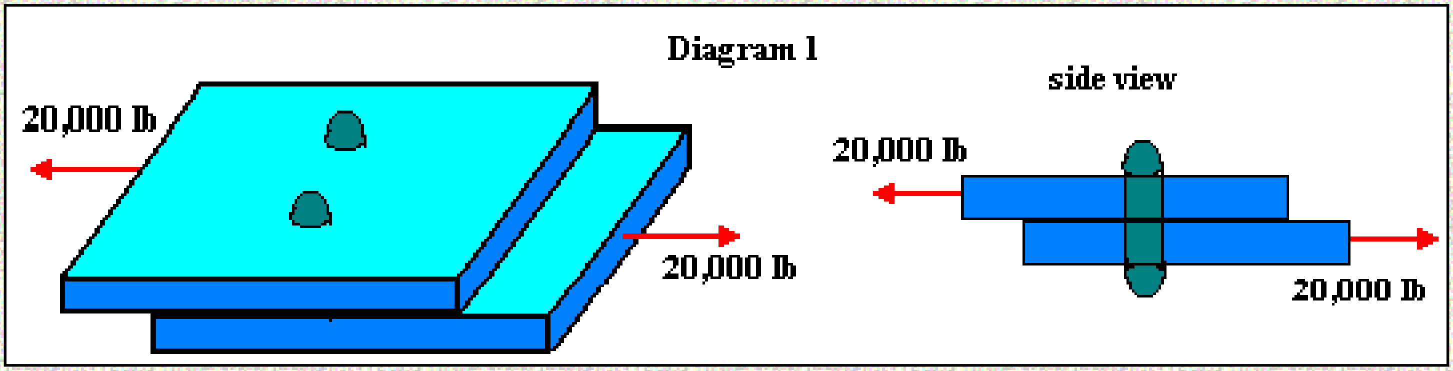
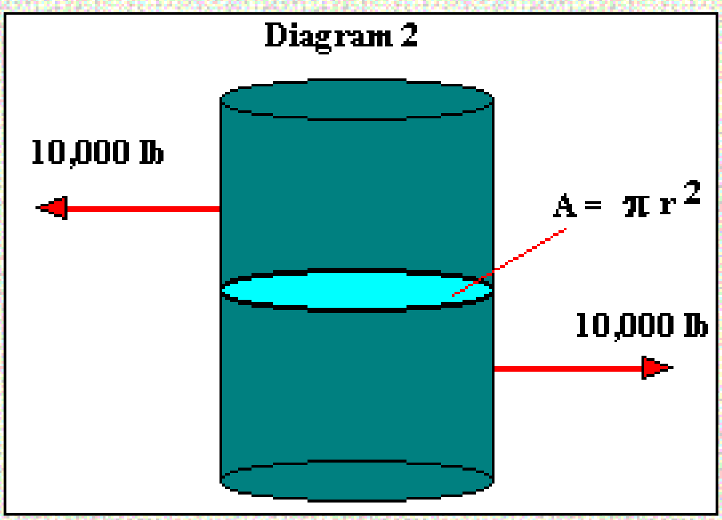
Topic 3.4a: Shear Stress & Strain - Example 1

Example 1
Two metal plates, as shown in Diagram 1, are bolted together with two 3/4" inch diameter steel bolts. The plates are loaded in tension with a force of 20,000 lb., as shown. What is the shearing stress that develops in the steel bolts? If the Modulus of Rigidity for Steel in 12×10^6 lb/in^2, what shear strain develops in the steel bolts?



As we examine the structure we see the area of the bolt where the two plates surfaces come together are in shear. That is, if we examine one bolt, the top of the bolt experiences a force to the left while the bottom of the bolt experiences and an equal force to the right. (See Diagram 2.) The surface area between the top and bottom interface is in shear. We assume the bolts carry the load equally, and so each bolt "carries" 10,000 lb. From this we can calculate the shear stress in a straight forward manner from one of our Shear Stress/Strain Relationships:



Stress = Force parallel to area/ area
Stress = F / (π * r^2) = 10,000 lb/ (3.14 * .375"^{2}) = 22, 650 lb/in^2.

In similar manner the Shear Strain can be found from the appropriate form of Hooke's Law:

Shear Stress: $\tau = (F_{parallel})/A$ $(lb/in^2, \text{or } N/m^2)$

Shear Strain: $\gamma = \Delta L / L = \tan \gamma = \gamma$

Hooke's Law: $G = \tau / \gamma$ $(lb/in^2, \text{or } N/m^2)$

G = (Shear Stress) / (Shear Strain), or (Shear Strain) = (Shear Stress) / G = (22, 650 lb/in^2) / (12 x 106 lb/in^2) = .00189

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Topic 3.4b: Shear Stress & Strain - Example 2

Example 2

In our second example we have a inner shaft (which perhaps drives a piece of machinery) and an outer driving wheel connected by spokes to an inner ring which is connected by means of a shear key to the inner shaft. That is, the wheel, spokes, and inner ring are one structure which is connected to the inner shaft by a shear key. (See Diagram 3.) When force is applied to the wheel (through a driving belt perhaps), the wheel begins to rotate, and through the shear key causes the inner shaft to rotate. We are interested in determining the shear stress on the shear key. We will also determine the compressive stress (or bearing stress) acting on the shear key. [**The purpose of a shear key is to protect machinery connected to a shaft or gear. If the driving forces and/or torque become too large the shear key will "shear off " (fail in shear) and thus disconnect the driving force/torque before it can damage the connected machinery.]**

To determine the shear stress we first need to determine the force trying to shear the key. We do this by realizing, after a little thought, that the driving force is really producing a torque about the center of the shaft, and that the torque produced by the driving force(s) must equal the torque produced by the force (of the inner ring) acting on the shear key. In our problem the two 500 lb. driving forces are acting a distance of 2 ft from the center of the 1 ft diameter shaft. Calculating torque about the center of the shaft we have:

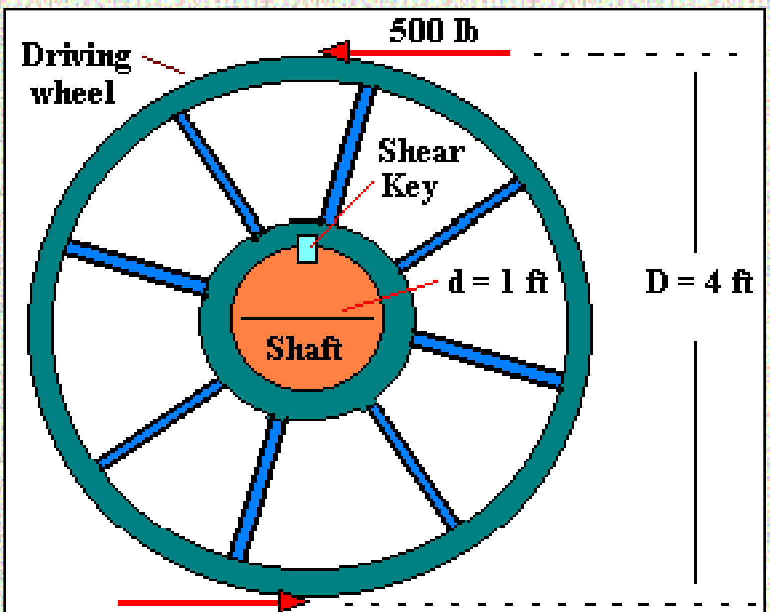

Diagram 3

500 lb.(2 ft) + 500 lb.(2 ft) = 2000 ft-lb. This must also be the torque produced by the force acting on the upper half of the shear key (shown in Diagram

4). So we may write: **500 lb.(2 ft) + 500 lb.(2 ft) = 2000 ft-lb. = F (.5 ft), Solving for F = 4000 lb.** This is the force acing on the top half of the shear key. There is a equal force in the opposite direction acting on the bottom half of the shear key. These two forces place the horizontal cross section of the key in shear. The key is 1/2 inch wide, by 3/4 inch high, by 1 inch deep as shown in Diagram 4. We calculate the shear stress by:
Shear Stress = Force parallel to area / area = 4000 lb./ (1/2" * 1") = 8000 lb/in^2.

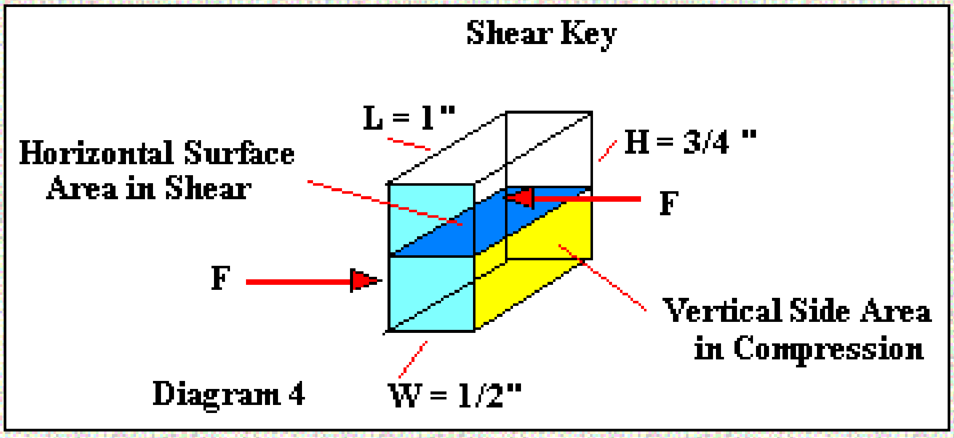

Diagram 4

In addition to the shear stress on the horizontal cross sectional area, the forces acting on the key also place the key itself into compression. The compressive stress (also called the bearing stress) on the top half of the shear key will be given by:
Compression (Bearing) Stress = Force normal to the area / area = 4000 lb. / (3/8" * 1") = 10, 700 lb/in^2. There is an equal compressive stress on the bottom of the shear key.

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Statics & Strength of Materials

Problem Assignment (determinate) Stress/Strain 1

1. A 3/4 inch diameter steel cable hangs vertically and supports a 750 pound ball. Determine the stress in the cable. (1700 psi)

2. Find the stress in a 1.5 inch square cast iron rod that has a tensile force of 15,000 pounds applied to it. (6667 psi)

3. A concrete post has a 20 inch diameter. It supports a compressive load of 8.4 tons. Determine the bearing stress of the post in pounds per square inch. (53.5 psi)

4. A rectangular concrete column is 1.5 feet by 2.5 feet. It supports a load of 72,000 pounds. Determine the bearing stress of the post. (133 psi)

5. A cast iron pipe with a 6 inch outer diameter and a 1/4 inch wall thickness carries a load of 3200 pounds. Determine the compressive stress in the pipe. (708 psi)

6. A construction crane has a 3/4 inch diameter cable. The allowable working stress for the cable is 20,000 psi. Determine the maximum load that the crane can lift. (8840 lb)

7. A short fir 4x4 has an allowable compressive stress of 1200 psi. parallel to the grain. Determine the maximum load that the 4x4 can carry. (19,200 lb)

8. A 24’ x 36’ house with full basement rests on 18" wide concrete footings. The entire house weighs 65,000 pounds. Determine the bearing stress on the soil beneath the footings. (~ 2.5 psi)

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Statics & Strength of Materials

Problem Assignment (determinate) Stress/Strain 2

1. Find the size cable required for a crane. Two cranes lift a concrete bridge beam. The beam is 80 feet long with a rectangular cross section 8 inches wide and 24 inches deep. The density of concrete is 150 pounds per cubic foot. Each crane lifts an equal share of the weight. The allowable tensile stress in the cable is 25,000 psi. Cable size is by 16ths to 1/2 inch, by 1/8ths to 2 inches and by 1/4 thereafter. (3/4")

2. The boom on the crane is 100 feet long. Determine the weight of cable hanging from the boom when it is fully extended. That is, how much does 100 feet of cable weigh. The density of steel is 490 pounds per cubic foot. (150 lb)

3. The ends of the bridge beams in problem 1 rest on 8 inch wide steel plates. How long should the plates be if the allowable bearing stress is 45,000 psi.? (.0223")

4. At one point while the cranes in problem 1 are lifting the bridge beams in problem 1, their booms are at an angle of 53 degrees. The cable at the top of the boom runs over a pulley held in place with an axle. Assuming that the axle is solid steel, determine the minimum diameter of axle required so that the shear stress in the axle material does not exceed the allowable shear stress for hardened steel, 20,000 psi. The tension in the cable is constant. Determine the shear force acting on the axle from static equilibrium. It requires a drawing of the crane and then a free body diagram of the pulley assembly. (d = .98")

5. A hammer strikes a 12 penny common nail with a maximum force of 500 pounds. What is the compressive stress in the nail? (29,064 psi., using nominal diameter)

6. A construction crane has a 3/4 inch diameter cable. The allowable working stress for the cable is 20,000 psi. Determine the maximum load that the crane can lift.(8830 lb)

Select:

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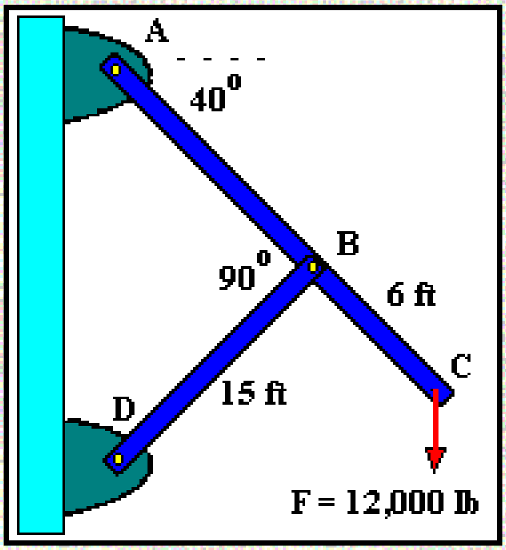
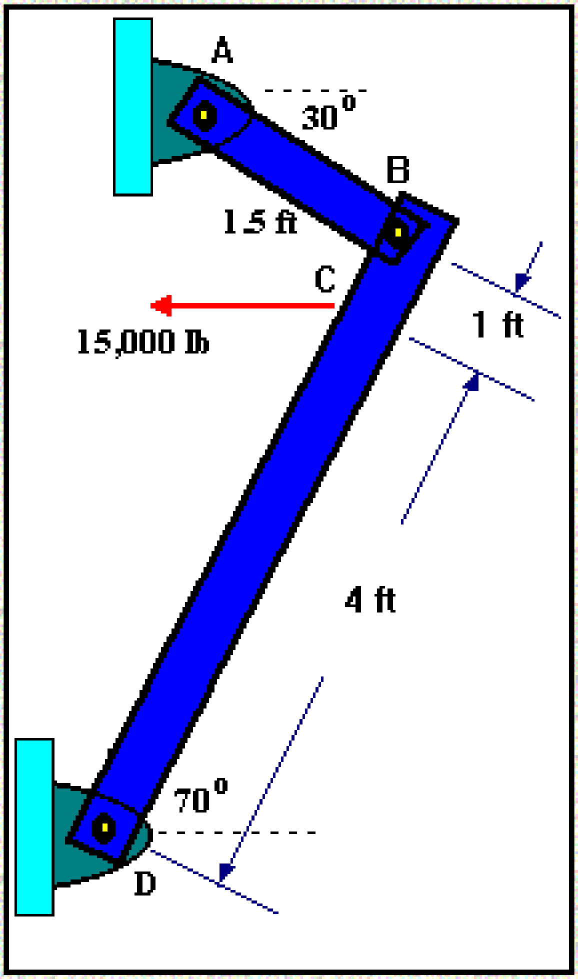
Statics & Strength of Materials

Problem Assignment (determinate) Stress/Strain 3

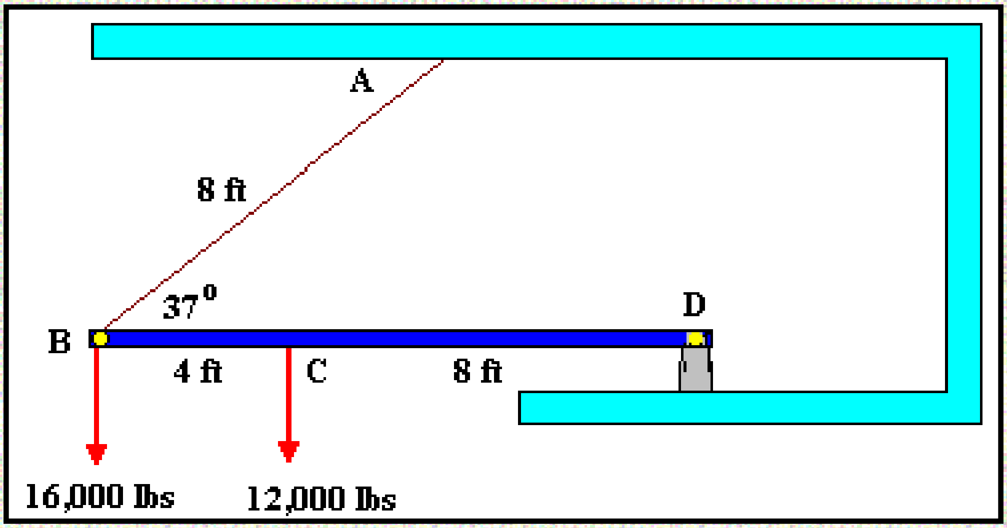
1. In the structure shown to the right, member BD is a steel rod with a diameter of 1 inch. Member ABC is also a steel member. Both members are attached to the wall by pinned joints. If we assume that member AB does not bend, determine the stress in member BD and the deformation of member BD.
(answers: Ax = 8723 lb., Ay = 1605 lb., D = 13, 570 lb)
(answers: Stress BD = 17, 280 psi, Deform BD = .104")



2. In the structure shown to the right, member AB is a Brass rod with a diameter of 1.5 inches. Member BCD is also a Brass member. Both members are pinned to the wall. If we assume that member BCD does not bend, determine the stress in member AB and the deformation of member AB.
(answers: AB = 11, 450 lb., Dx = 5,084 lb., Dy = 5,725 lb)
(answers: Stress AB = 6,480 psi, Deform AB = .0081")

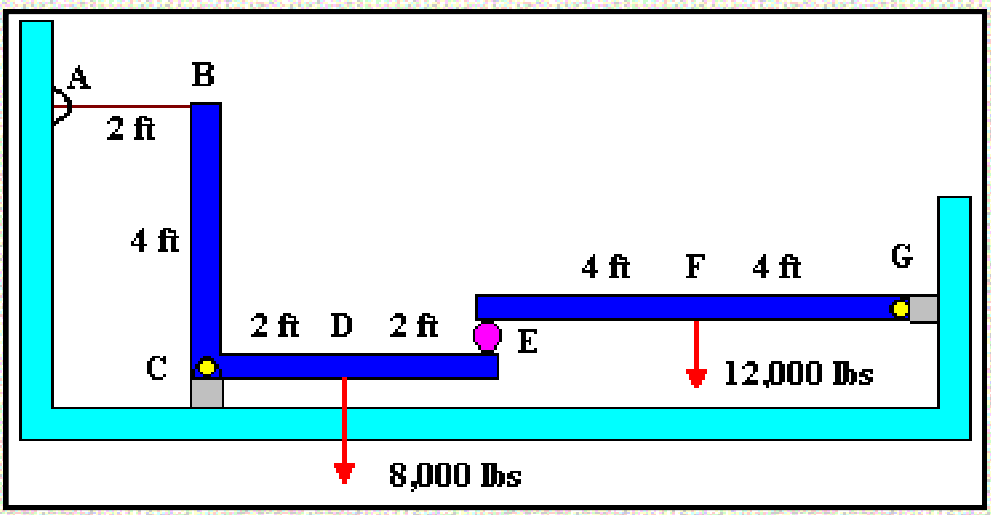


3 In the structure shown to the right, member AB is a cable and BCD is a solid rigid member (that is, it does not bend). Member BCD is pinned at point E, and is supported by cable AB at point B. Member AB is pinned to the wall at point A. Member AB is a steel cable with a diameter of 1 inch. You may assume that point B moves "down" the amount cable AB deforms. For this structure, determine the stress in cable AB, and the movement of point C.
(answers: A = 40,000 lb., Dx = 32,000 lb., Dy = 4,000 lb)
(answers: Stress AB = 50,930 psi, Movement of C = .109")



4. In the structure shown to the right L-shaped member BCDE is connect by a steel cable, AB, to the wall. Member EFG is pinned to the wall at G, and is resting on top of member BCDE, supported by a roller. The members are loaded as shown. The steel cable has an area of .5 in^2. Assume members BCE and EFG do not bend. Determine the stress which develops in cable AB, and the movement of point F.

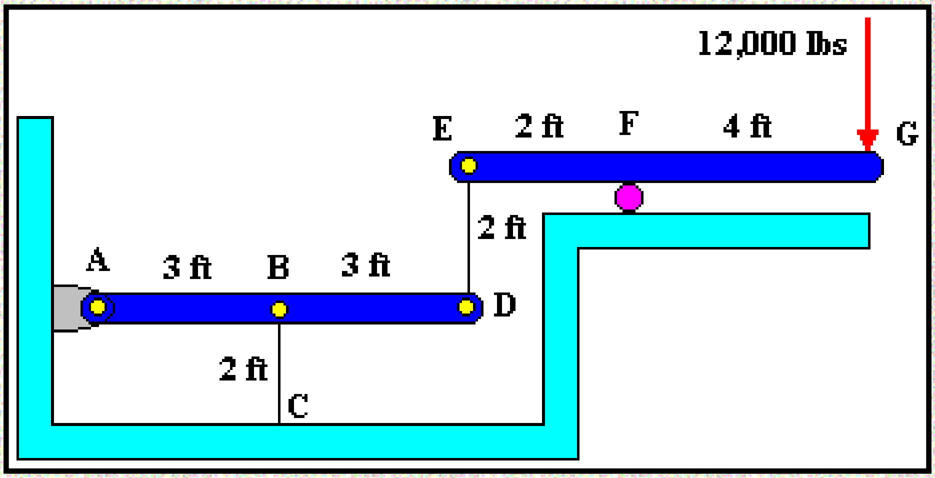
(answers: A = 10,000 lb., Cx = 10,000 lb., Cy = 14,000 lb)
(answers: Stress AB = 20,000 psi, Movement of F = .008")



5. In the structure shown to the right member ABD is a solid rigid member pinned to the wall at A, supported by steel cable BC, and connected to member EFG by

steel cable DE. (Cables BC and DE each have a cross sectional area of .5 square inches.) Member EFG is supported by a roller at F and is loaded with 12000 lb. at G. For this structure determine the stress in cable BC, and the movement of point G.

(answers: Fy = 36,000 lb., ED = 24,000 lb., Ay = 24,000 lb., BC = 48,000 lb.)
(answers: Stress BC = 96,000 psi, Movement of G = .384")



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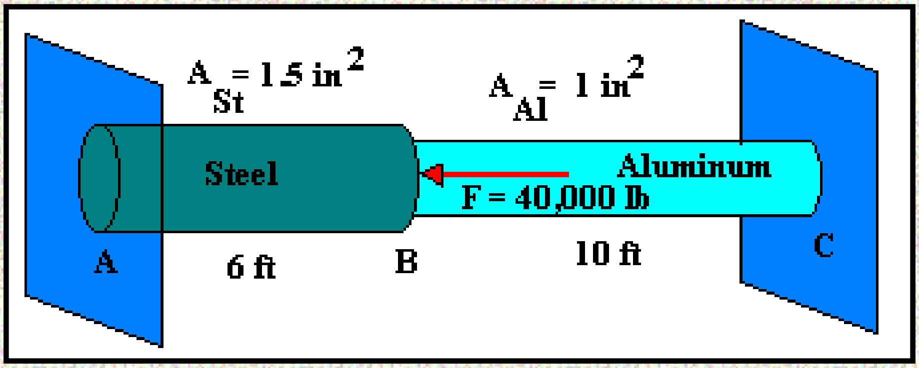
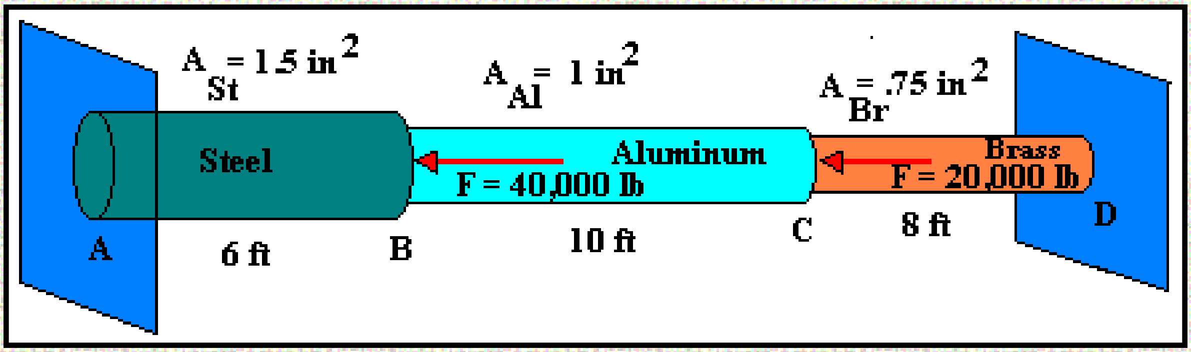
Topic 3.6: Assignment Problems - Statically Indeterminate

1. In the structure shown, steel rod AB and aluminum rod BC are joined together and then held between two rigid walls as shown. A force of 40,000 lb acting to the left is applied at junction B. This force will compress the steel member while stretching the aluminum member. The cross sectional areas and lengths of the steel and aluminum rods are respectively 1.5 in^2, 6 ft, 1 in^2, 10 ft as shown in the diagram. Young's modulus for steel is 30 x 10^6 lb/in^2; Young's modulus for aluminum is 10 x 10^6 lb/in^2. Determine the stress that develops in each member, and determine the deformation of the aluminum rod.
(answers: $F_{aluminum}$ = 4,700 lb., F_{steel} = 35,300, Stress-Al = 4700 psi, Stress-St = 23, 500 psi)
(answers: Deformation-Al = .0565")

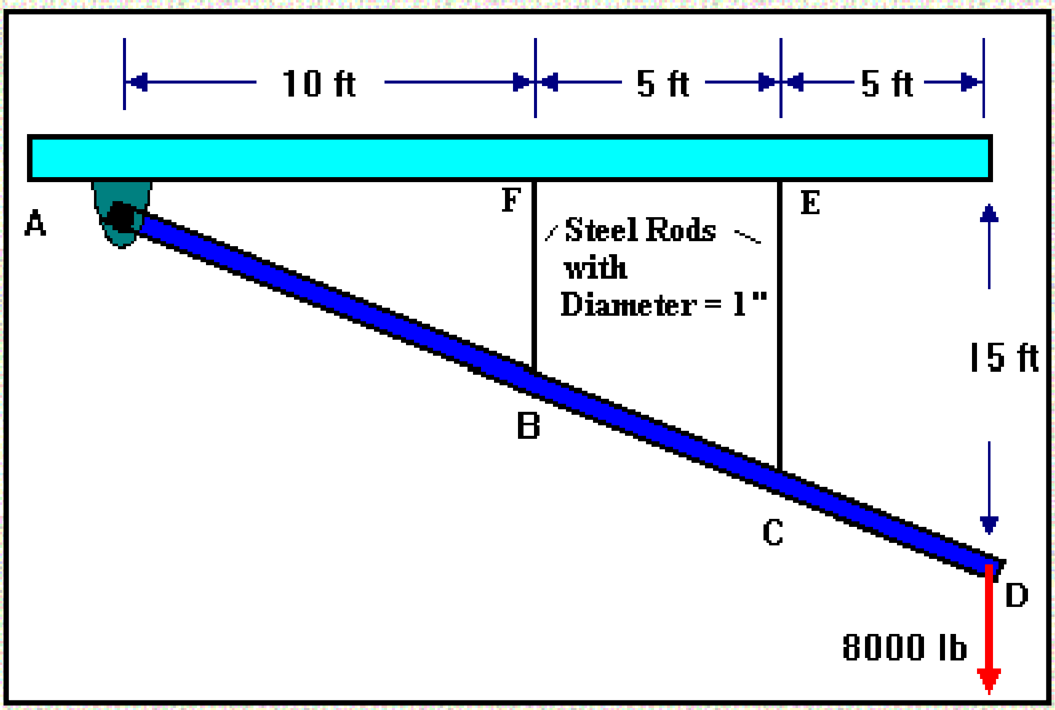


2. In the structure shown, steel rod AB, aluminum rod BC, and brass rod CD are joined together and then held between two rigid walls as shown. A force of 40,000 lb acting to the left is applied at junction B, and a force of 20,000 lb acting to the left is applied at junction C. The cross sectional areas and lengths of the steel, aluminum, and brass rods are respectively 1.5 in^2, 6 ft; 1 in^2, 10 ft; .75 in^2, 8 ft as shown in the diagram. Young's modulus for steel is 30 x 10^6 lb/in^2; Young's modulus for aluminum is 10 x 10^6 lb/in^2; Young's modulus for brass is 15 x 10^6 lb/in^2. Determine the stress that develops in each member, and determine the deformation of the aluminum rod.
(answers: F_{st} = 44,800 lb. comp., F_{al} = 4,800 lb.comp., F_{br} = 15,000 lb. tens.)

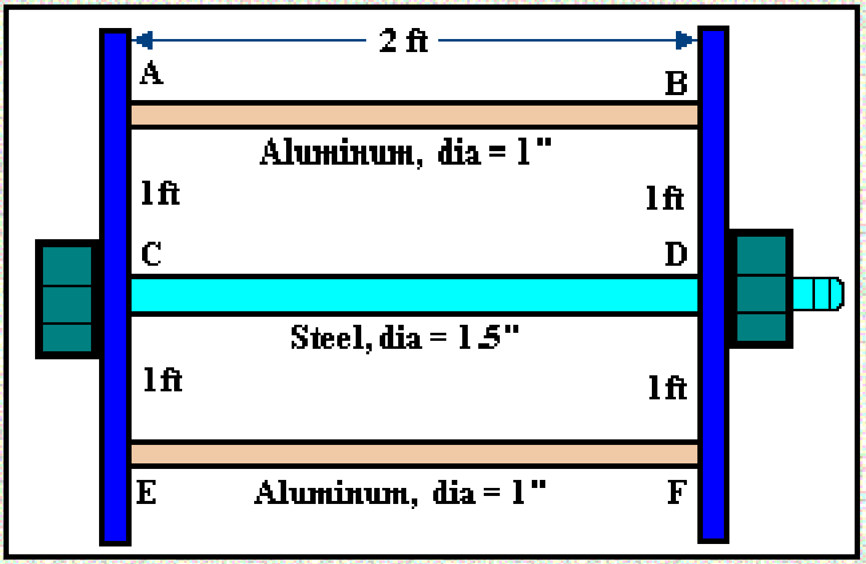
(answers: $Stress_{st}$ = 29,900 psi., $Stress_{al}$ = 4,800 psi., $Stress_{br}$ = 20,000 psi, def.-al = -.0576")



3 In the structure shown, member ABCD is pinned to the ceiling at point A, and is supported by two steel rods, FB and EC, as shown in the Diagram. A load of 8000 lb. is applied at point D. Young's modulus for Steel is 30 x 10^6 lb/in^2. Determine the stress in each rod, and the movement of point D.(answers: $Stress_{FB}$ = $Stress_{EC}$ = 8150 psi; M_d=.0448")

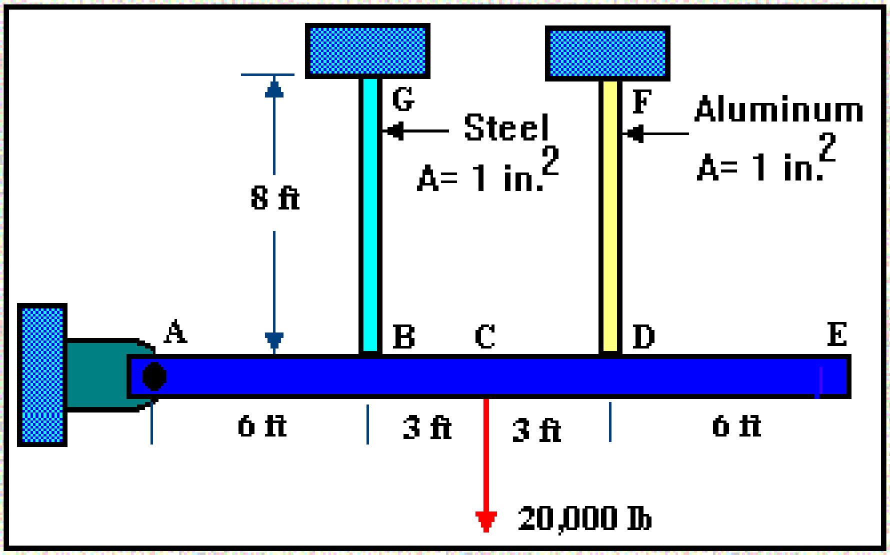


4. In the structure shown, two aluminum rods and a threaded steel rod connect to vertical plates, as shown in the diagram. If the structure is initially unstressed and the nut on the threaded steel rod in screwed in .25 inches, what is the stress that develops in the aluminum and steel members?
(answers: Stress$_{ST}$ = 71,300 psi., Stress$_{Al}$ = 80,200 psi.)



5. In the structure shown, solid rigid member ABCDE is pinned to the wall at point A, and supported by steel member BG and aluminum member DF, with lengths and dimensions as shown. A load of 20,000 lb. is applied a point C. Determine the stress that develops in the steel and aluminum members, and the movement of point E. (answers: Stress$_{ST}$ = 12,860 psi., Stress$_{Al}$ = 8570 psi.)

(Movement of E = .123")



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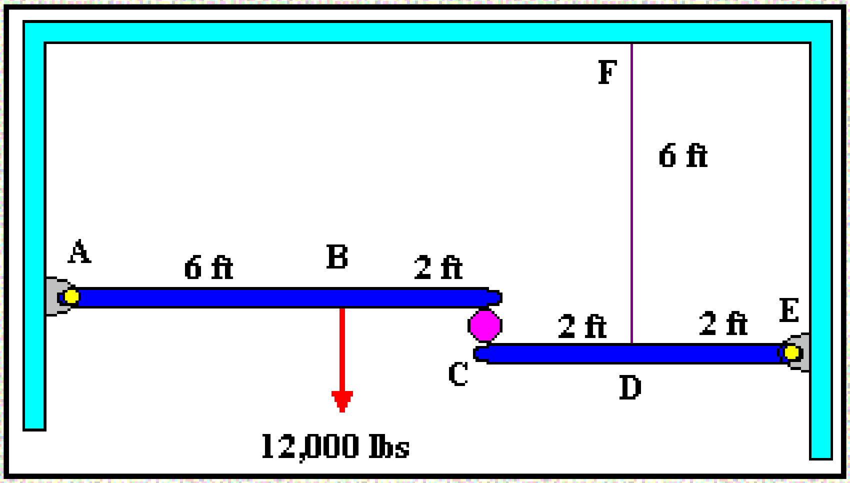
Topic 3.7 Stress, Strain & Hooke's Law - Topic Examination

1.) In the structure shown members ABC and CDE are assumed to be solid rigid members. MemberABC is pinned to the wall at A and is supported by a roller at point C. Member CDE is pinned to the wall at point E, and is supported by steel cable DF. Cable DF has a diameter of .75 inch. For this structure:



A. Draw a Free Body Diagram showing all support forces and loads.

B. Determine the axial stress in cable DF.

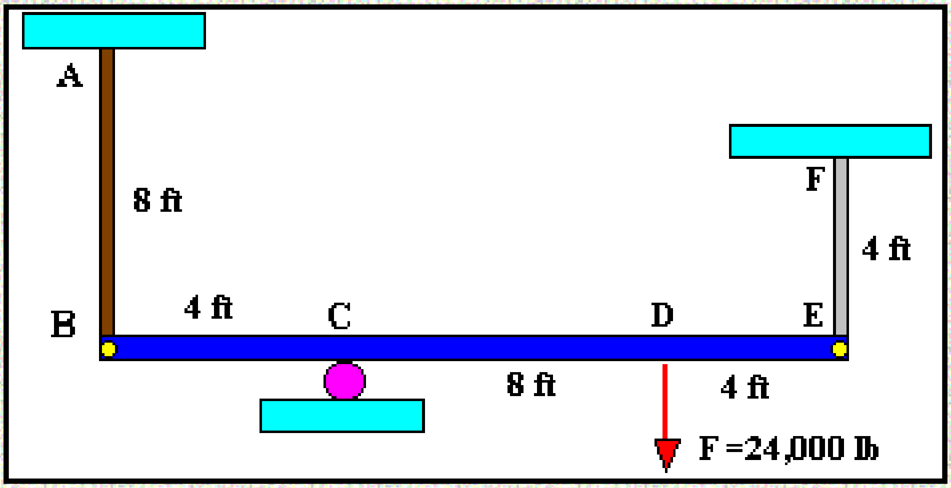
C. Determine the movement of point B due to the applied load.

E_{st} = 30 x 10 6 psi; E_{br} = 15 x 10 6 psi; E_{al} = 10 x 10 6 psi

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

(For Solution, Select: Solution Problem 1)

2.) In the structure shown horizontal member BCDE is supported by vertical brass member, AB, an aluminum member EF, and by a roller at point C. Both AB and EF have cross sectional areas of .5 in^2. The structure is initially unstressed and then a load of 24,000 lb. is applied at point D. For this structure:



A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress that develops in aluminum member EF.
C. Determine the resulting movement of point E.
E_{st} = 30 x 10 6 psi; E_{br} = 15 x 10 6 psi; E_{al} = 10 x 10 6 psi

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.
(For Solution, Select: **Solution Problem 2**)

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Topic 3.8: Thermal Stress, Strain & Deformation I

Thermal Stress, Strain & Deformation - I

Changes in temperatures causes thermal effects on materials. Some of these thermal effects include thermal stress, strain, and deformation. The first effect we will consider is thermal deformation. Thermal deformation simply means that as the "thermal" energy (and temperature) of a material increases, so does the vibration of its atoms/molecules; and this increased vibration results in what can be considered a stretching of the molecular bonds - which causes the material to expand. Of course, if the thermal energy (and temperature) of a material decreases, the material will shrink or contract. For a long rod the main thermal deformation occurs along the length of the rod, and is given by:

$$\delta_{th} = \alpha(\Delta T)L$$

where α (alpha) is the linear coefficient of expansion for the material, and is the fractional change in length per degree change in temperature. **[Some values of the linear coefficient of expansion are: Steel = 12 x 10^{-6}/ºC = 6.5 x 10^{-6}/ºF; Brass = 20 x 10^{-6}/ºC = 11 x 10^{-6}/ºF; Aluminum = 23 x 10^{-6}/ºC = 13 x 10^{-6}/ºF.]** The term ΔT is the temperature change the material experiences, which represents (T_f - T_o), the final temperature minus the original temperature.

If the change in temperature is positive we have thermal expansion, and if negative, thermal contraction. The term 'L' represents the initial length of the rod.

Example 1

A twelve foot steel rod is initially at a temperature of 0ºF and experiences a temperature increase to a final temperature of 80ºF. What is the resultant change in length of the steel?

Solution: **Deformation =** $\alpha\Delta TL$ **= 6.5 x 10^{-6}/ºF (80ºF- 0ºF) (144 inches)**

= .075 inches

(The length of the rod was converted into inches in the equation since the deformations are normally quite small.) We see the deformation is indeed quite small, and in many cases the thermal deformation has no significant effect on the structure. However, if the structure or members of the structure are constrained such that the thermal expansion can not occur, then a significant thermal stress may arise which can effect the structure substantially - and which we will address shortly.

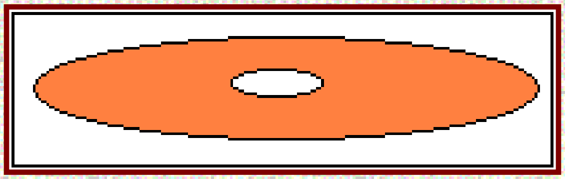
In addition to the length, both the area and volume of a material will change with a corresponding change in temperature. The resulting changes in area and volume are given by:

$\Delta A \cong 2\alpha(\Delta T)A$; and $\Delta V = 3\alpha(\Delta T)V$

These formulas, as written, are not exact. In the derivations [using $(L + \Delta L)^2$ for area, and $(L + \Delta L)^3$ for Volume] there are cross terms involving the linear coefficient of expansion squared in the area formula, and the coefficient of expansion squared and cubed in the volume formula. These terms are very small and can be ignored, resulting in the two equations above.

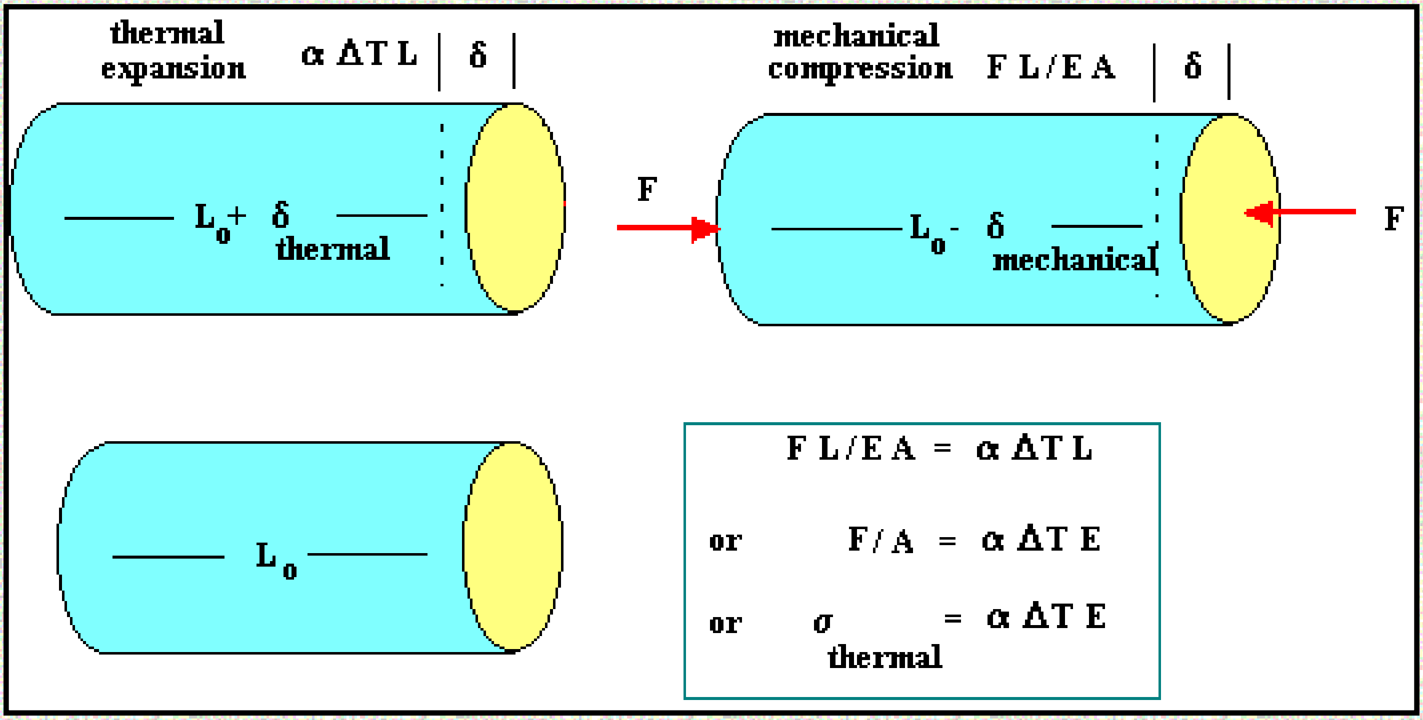
While unconstrained thermal expansion is relatively straight forward effect, it still requires a bit of thought, such as in the following question.

A flat round copper plate has a hole in the center. The plate is heated and expands. What happens to the hole in the center of the plate - expands, stays the same, or shrinks?



When I ask this question in my classroom it is not unusual for the majority of the answers to be incorrect. Our first thought often is that since the plate is expanding, the hole is the center must be getting smaller. However, this is not the case. The atoms/molecules all move away from each other with the result that the hole expands just as if it were made of the same material as the plate. This is also true of volume expansion. The inside volume of a glass bottle expands as if it were made of glass.

A somewhat more interesting aspect of thermal expansion is when it "can't" - that is, what happens when we constrain a structure or member so it can not expand. (or contract)? When this happens a force and resulting stress develop in the structure. A simple way to determine the amount of stress is to let the material expand freely due to thermal expansion, and then compress it back to its original length (a mechanical deformation) . See diagram below.



If we equate these two effects (deformations) we have: $\alpha \Delta T L$ = FL/EA ;note that we can cancel the length L from each side of the equation, and then cross multiply by E, arriving at: $\alpha(\Delta T)E$ = F/A , however, F/A is stress and we can finally write:

$\sigma_{th} = \alpha(\Delta T)E$; The thermal stress which develops if a structure or member is completely constrained (not allowed to move at all) is the product of the coefficient of linear expansion and the temperature change and Young's modulus for the material.

Select:Thermal Stress, Strain & Deformation II
Or
Topic 3: Stress, Strain & Hooke's Law - Table of Contents
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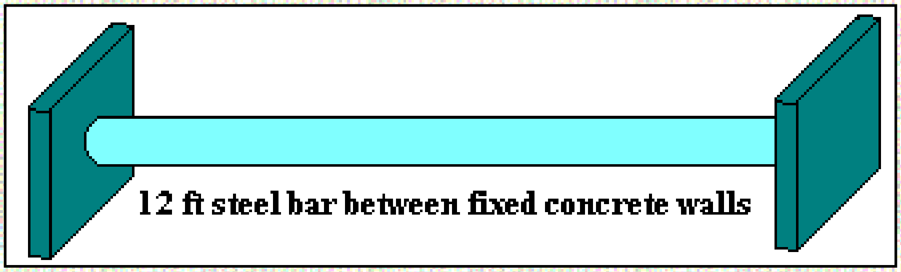
Topic 3.81: Thermal Stress, Strain & Deformation II

1. Completely Constrained Thermal Deformation
The thermal stress which develops if a structure or member is completely constrained (not allowed to move at all) is the product of the coefficient of linear expansion and the temperature change and Young's modulus for the material, or

$\sigma_{th} = \alpha(\Delta T)E$

Example:
A twelve foot horizontal steel rod is fixed between two concrete walls. The rod is initially at temperature of 0ºF and experiences a temperature increase to a final temperature of 80ºF. If the steel rod was initially unstressed, what is the stress in the steel at 80ºF? [Young's modulus for steel is 30×10^6 lb/in^2, and the coefficient of linear expansion of steel is 6.5×10^{-6}/ºF.]



Solution:
In a completely constrained problem, where the member can not move at all, the thermal stress which develops is given by: $\sigma_{th} = \alpha(\Delta T)E$ **= (6.5 x 10^{-6}/ºF) (80ºF - 0ºF)(30 x 10^6 lb/in^2) = 15,600 lb/in^2.**

Notice that this is quite a sizable stress. In this case there was no initial stress, so the stress which developed is well within the range of allowable stresses for steel. However, there are many cases where structures and materials are near or at their allowable stresses. In that case, if a thermal stress develops, the total stress may well exceed the allowable stress and cause the structure to fail. This, of course, is the reason bridges are built with expansion joints which allow the structure to expand and contract freely and thus avoid thermal stresses. Additionally, this is why concrete sidewalks are built with spaces separating adjacent slabs, allowing expansions to avoid thermal stresses. Concrete highways used to also have expansion spaces built-in, however modern concrete highways are designed without expansion spaces to withstand thermal stresses which develop. Normally they do withstand these stresses, but occasionally long hot periods will allow stresses to built up until the highway actually exploded in a area, producing a large hole in the concrete.

2. Partially Constrained Thermal Deformation:

A more normal situation in a structure, rather than completely constrained or completely free thermal deformation, is a partially constrained thermal deformation. This means a member may expand (or contract) but not as much as it would if unconstrained. Perhaps the best way to demonstrate this situation is to work slowly through an example(s). **Please select the following examples:**
Example 1 ; Example 2 ; Example 3

or Select:
Topic 3. Stress, Strain & Hooke's - Table of Contents
Statics & Srength of Materials Home Page

Topic 3.82a: Mixed Mechanical/Thermal - Example 1

Partially Constrained Thermal Deformation:

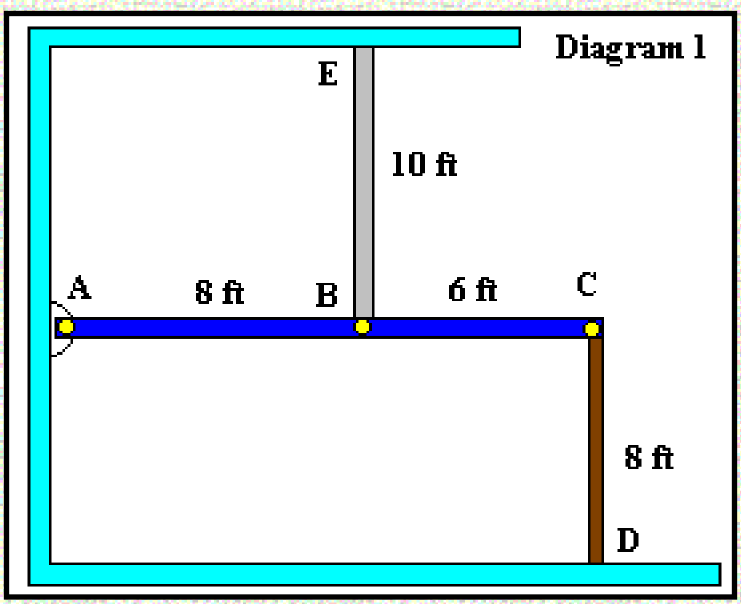
In a structure, rather than being completely constrained or completely free, members more often are partially constrained. This means a member may expand (or contract) but not as much as it would if unconstrained.

Example 1

In the structure shown in Diagram 1 horizontal member ABC is pinned to the wall at point A, and supported by a Aluminum member, BE, and by a Brass member, CD. Member BE has a .75 in^2 cross sectional area and Member CD has .5 in^2 cross sectional area.

The structure is initially unstressed and then experiences a temperature increase of 40 degrees Celsius. For this structure we wish to determine the stress which develops in the aluminum and brass members. We would also like to determine the deformation of the brass member. (At this point we will assume that the horizontal member ABC does not bend due to forces acting on it. We will consider beam bending at a later point.) The linear coefficient of expansion, and Young's modulus for brass and aluminum are :

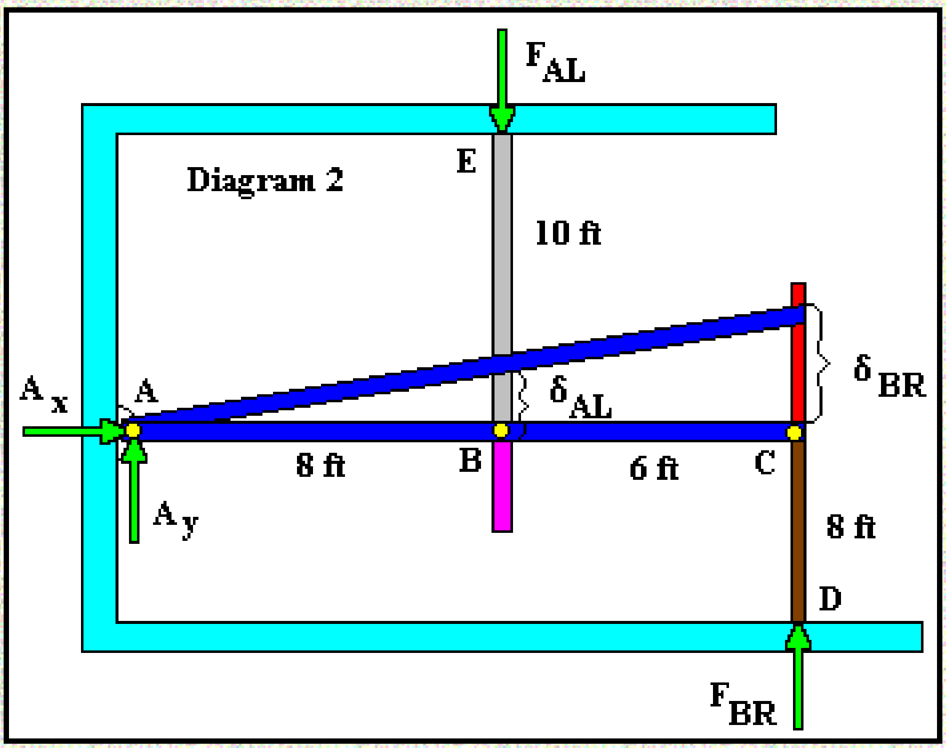
br = 20 x 10-6 /oC; al = 23 x 10-6 /oC; Ebr = 15 x 10 6 psi; Eal = 10 x 10 6 psi



Solution:

The first part of the solution will be to consider the static equilibrium conditions for the structure. However to do this effectively we first need to consider the physical effects of the temperature change to determine the directions of the forces acting on the structure. We do this by first considering how much the brass and aluminum members would expand if they were free to expand, due to the temperature increase.

Both the brass and aluminum members try to expand. In Diagram 2 we have shown the amount each member would expand if free to do so. However both can not expand - one will "win" causing the other to contract. We will assume the brass "wins" forcing the aluminum to compress. The horizontal member, ABC, will rotate upward about point A as shown in Diagram 2. That is, the brass will expand, but not as much as it would if unconstrained - since it is compressing the aluminum which is in turn pushing back on the brass, putting it into compression also. Thus both the brass and aluminum members are in compression. We have shown in Diagram 2 the direction the external support forces will act on the structure. Notice the forces at E and D are along the direction of the members. This is due to the fact that the brass and aluminum are axial members, and thus simply in tension or compression. At point A where the structure is pinned to the wall, we have put in horizontal and vertical support forces A_x and A_y. We are now ready to apply static equilibrium conditions.



PART I: STATICS

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions. (Already done in Diagram 2) Apply equilibrium conditions:

Sum $F_x = A_x = 0$ (There is no external horizontal force acting at point A)

Sum $F_y = F_{br} - F_{al} + A_y = 0$

Sum $Torque_A = -F_{br}$ (14 ft) + F_{al} (8 ft) = 0

At this point we see we have too many unknowns (**F_{br}, F_{al}, A_y**) and not enough equations to solve for our forces. We need an additional equation to solve for the unknowns. We will obtain the additional equation from the deformation is the problem.

PART II: DEFORMATION

We first write a general relationship between the deformations, which we find from the geometry of the problem. We see (from Diagram 2) that: δ_{br} / 14 ft = - δ_{al} / 8 ft or simplifying:

$\delta_{br} = -1.75\ \delta_{al}$

The negative sign in front of the deformation of aluminum term comes from the fact that, in our assumption, the aluminum gets shorter, and in problems involving mixed thermal and mechanical deformations (as in this problem) we need to keep signs associated with deformations, with elongation being positive, and contractions being negative.

Once we have this general deformation relationship, we now substitute in our deformation expressions for mechanical plus thermal deformation. That is, the net deformation is the sum of the thermal deformation ($\alpha \Delta TL$) due to the temperature change, and the mechanical deformation (**FL / EA**) due to the forces which develop in the members. So the total deformation of a member may be written as δ_{total} = [$\alpha \Delta TL \pm$ **FL / EA**] where the mechanical deformation term is positive (use + sign) if the member is in tension, and the mechanical deformation term is negative (use - sign) if the member is in compression. If we substitute this expression for the deformations into our general relationship ($\delta_{br} = -1.75\ \delta_a$) we obtain:

[$\alpha \Delta TL$ **- FL / EA]$_{br}$ = -1.75[** $\alpha \Delta TL$ **- FL / EA**]$_{al}$ (Notice that we use a negative sign for each mechanical term since both members are in compression. We now substitute in numeric values given in our problem and get:

$[(20\text{x}10^{-6} / {}^oC)(40^oC)(96 \text{ in}) - F_{br} (96 \text{ in}) / (15\text{x}10^6 \text{ lbs./in}^2)(.5 \text{ in}^2)] = -1.75[(23\text{x}10^{-6} / {}^oC)(40^oC)(120 \text{ in}) - F_{al} (120 \text{ in}) / (10\text{x}10^6 \text{ lbs./in}^2)(.75 \text{ in}^2)]$.

(Notice that we need not be concerned that the temperature is in Celsius rather than Fahrenheit, since the linear coefficient of expansion is also per ^{o} Celsius, the units cancel.) Then after combining terms we have:

$(0.0768 - 1.28\text{x}10^{-5} F_{br}) = (-0.193 + 2.8\text{x}10^{-5} F_{al})$ or **$1.28\text{x}10^{-5} F_{br} + 2.8\text{x}10^{-}$**

^{5} $F_{al} = 0.27$

This is our additional equation. Now from our torque equation from static equilibrium in Part I we have **$F_{al} = 1.75\ F_{br}$** We now substitute this into our deformation equation above obtaining:

$1.28\times10^{-5}\ F_{br} + 2.8\times10^{-5}\ (1.75\ F_{br}) = 0.27$ or

6.18×10^{-5} (in./lb.) F_{br} = .27 in.

solving:

F_{br} = 4,370 lbs.; F_{al} = 7,650 lbs. **The stress in brass = F/A = 4,370 lbs./.5 in^2 = 8,740 lbs./in^2 The stress in aluminum = F/A = 7,650 lbs./.75 in^2 = 10,200 lbs./in^2**

PART III: Deformation of Brass Member

The deformation of the brass member may now be found from the general expression for the deformation: $[\alpha \Delta T L - FL / EA]_{br}$ or putting in values

$\delta_{Brass} = [(20\times10^{-6} / {}^oC)(40^oC)(96\ in) - (4{,}370\ lbs.)(96\ in) / (15\times10^6\ lbs./in^2)(.5\ in^2)]$ = **0.210 in**. (Since this deformation is positive, it means brass member CD does expand as we assumed. If the value were negative it would have meant that the brass member was actually compressed by the aluminum member, which then would have expanded. However the values found for the forces and the stress would have been correct in either case.)

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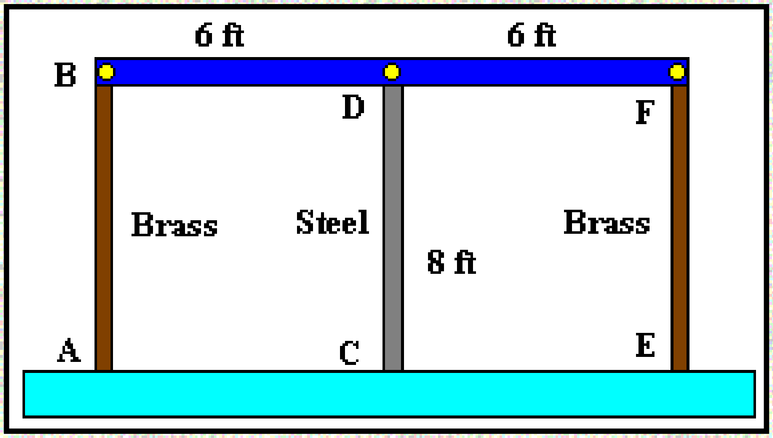
Topic 3.82b: Mixed Mechanical/Thermal - Example 2

Partially Constrained Thermal Deformation:

In a structure, rather than being completely constrained or completely free, members more often are partially constrained. This means a member may expand (or contract) but not as much as it would if unconstrained.

Example 2

In the structure shown below horizontal member BDF is supported by two brass members, AB and EF, and a steel member CD. Both AB and EF have cross sectional areas of .5 in2. Member CD has a cross sectional area of .5 in2.
The structure is initially unstressed and then experiences a temperature increase of 40 degrees celsius. For this structure, determine the axial stress that develops in steel member CD, and the resulting movement of point D. (At this point we will assume that the horizontal member BDF does not bend due to forces acting on it. We will consider beam bending at a later point.) The linear coefficient of expansion, and Young's modulus for brass and steel are :
αbr = 20 x 10-6 /ºC; αst = 12 x 10-6 /ºC; Ebr = 15 x 10 6 psi; Est = 30 x 10 6 psi

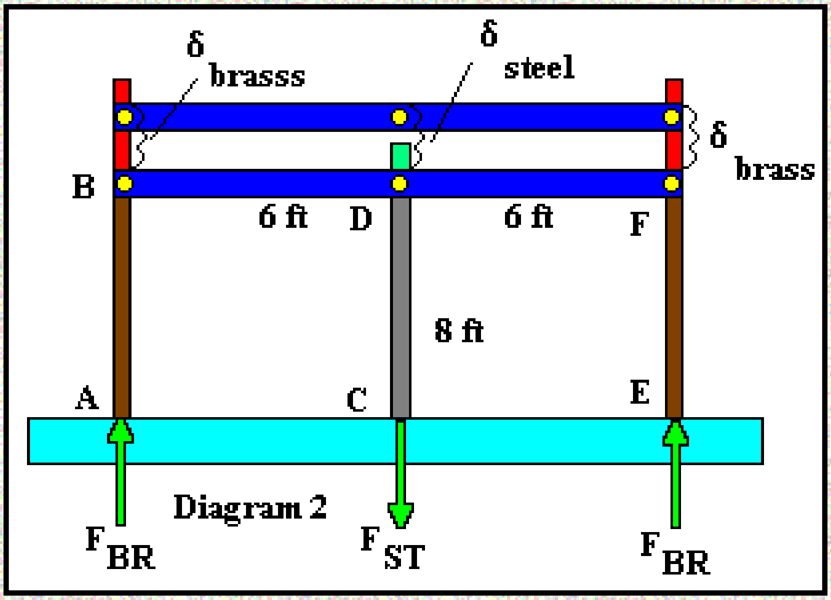


Solution:

PART I: STATICS

In this problem we first consider how much the brass and steel member would expand, if free to do so, due to the temperature increase. As the coefficient of expansion of the brass is larger than the coefficient of expansion of the steel, the brass would expand more if free to do so. (See Diagram 2)
As the brass expands it pulls on the steel placing the steel in tension. The steel pulls back on the brass placing the brass in compression. The support reactions, reflecting these forces, are shown in diagram 2. Notice that the brass and steel members have forces acting on them at only two points, so they are axial

members. This also means that the external forces on the members due to the floor are equal to the forces in the members.



Additionally we note that the forces in the brass member are equal from symmetry of the structure. (or if we mentally sum torque about point D, the center of the member BDF, we see that forces in the brass members would produce opposing torque which would need to be equal and opposite for equilibrium. Since the distances are equal, the forces in the brass members need to be equal to produce equal amounts of torque.)

The result of the brass and steel members working against each other is that the horizontal member BDF moves to an intermediate position, as shown in Diagram 2. We are now ready to proceed with the Statics.

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions. (Diagram 2)

We now write the equilibrium equations:

Sum $F_x = 0$

Sum $F_y = F_{br} - F_{st} + F_{br} = 0$

Sum $Torque_B = -F_{st}\,(6\text{ ft}) + F_{br}\,(12\text{ ft}) = 0$

The torque equation gives us: **$F_{st} = 2\,F_{b_r}$** and the sum of y forces also gives us:

$F_{st} = 2\,F_{br}$

We do not have enough equations at this point to solve the problem. We need an additional equation, which we will obtain from the deformation relationships.

PART II: DEFORMATION

From the geometry of the problem, we see that the final net deformation of the brass and steel members will be equal, or $\delta_{br} = \delta_{st}$ Notice that both deformations are positive since the members expand. Next using our expression for combined thermal and mechanical deformations: $\delta_{total} = [\alpha \Delta TL \pm FL / EA]$, we substitute into the general deformation relationship and obtain:

$$[\alpha\Delta TL - FL / EA]_{br} = [\alpha\Delta TL + FL / EA]_{st}$$

The sign of the mechanical deformation term for the brass is negative (-) since the brass is in compression. The sign of the mechanical deformation term for the steel is positive (+) since the steel is in tension. We now substitute values from our problem into the equation and obtain:

$$[(20\times10^{-6} / {}^oC)(40^oC)(96 \text{ in}) - F_{br} (96 \text{ in}) / (15\times10^6 \text{ lbs/in}^2)(.5 \text{ in}^2)] =$$

$$[(12\times10^{-6} / {}^oC)(40^oC)(96 \text{ in}) + F_{st} (96 \text{ in}) / (30\times10^6 \text{ lbs/in}^2)(.5 \text{ in}^2)]$$

or

$$(0.0768 - 1.28\times10^{-5} F_{br}) = (0.0461 + 0.64\times10^{-5} F_{st})$$

or

$$1.28\times10^{-5} F_{br} + 0.64\times10^{-5} F_{st} = 0.0307$$

This is our additional equation. We can now substitute our relationship between the brass and steel force from our static equilibrium equations in part I, which gave us : $F_{st} = 2 F_{br}$ Substituting, we obtain:

$1.28\times10^{-5} (F_{br}) + 0.64\times10^{-5} (2 F_{br}) = 0.0307$, or $2.56 \times 10^{-5} \text{ (in./lb) } F_{br} = .0307 \text{ in.}$

Then solving **$F_{br} = 1{,}200$ lb.; and $F_{st} = 2 F_{br} = 2{,}400$ lbs**, and so the **Stress in steel = F/A = 2,400 lbs/.5 in^2 = 4,800 lbs/in^2**

PART III: MOVEMENT

Movement of point D. Point D is connected to steel member CD and so moves the amount CD deforms, or **Movement of D =** $[\alpha\Delta TL + FL / EA]_{st}$, or

Movement of D $= [(12\times10^{-6} / {}^oC)(40^oC)(96 \text{ in}) + (2{,}400 \text{ lbs})(96 \text{ in}) / (30\times10^6 \text{ lbs/in}^2)(.5 \text{ in}^2)] = 0.06144 \text{ in.}$

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Topic 3.82c: Mixed Mechanical/Thermal - Example 3

Partially Constrained Thermal Deformation:

In a structure, rather than being completely constrained or completely free, members more often are partially constrained. This means a member may expand (or contract) but not as much as it would if unconstrained.

Example 3

In the structure shown three metal rods (steel, aluminum, brass) are attached to each other and constrained between two rigid walls. The rods are initially unstressed and then experience a temperature increase of 80º F. We would like to determine the stress which develops in each rod, and the amount of deformation of the aluminum rod.

The steel, aluminum and brass rod have areas and lengths respectively of 1.5 in², 6 ft. , 1 in², 10 ft., .75 in², 8 ft. as shown in the diagram. The linear coefficient of expansion for the materials are as follows: Steel = 6.5 x 10^{-6} / ºF; Brass = 11 x 10^{-6} / ºF; Aluminum = 13 x 10^{-6} / ºF. And Young's modulus for the materials are Est = 30 x 10 6 psi; Eal = 10 x 10 6 psi; Ebr = 15 x 10 6 psi.

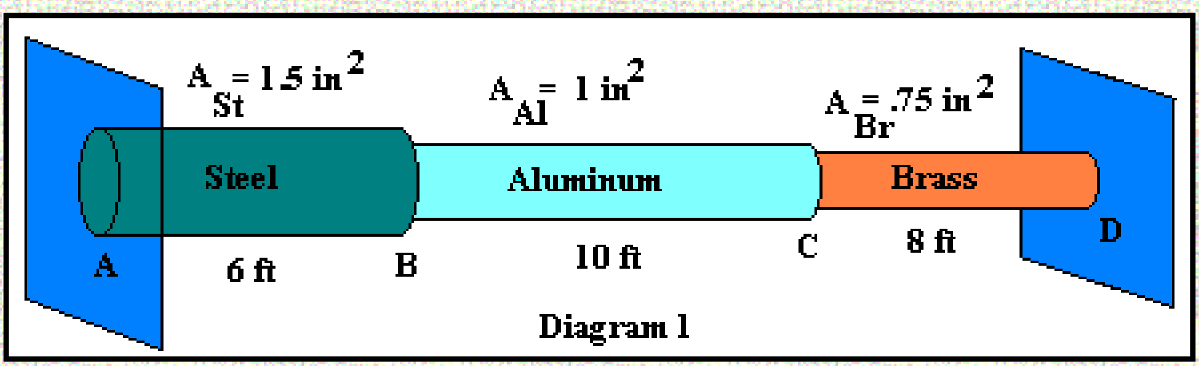
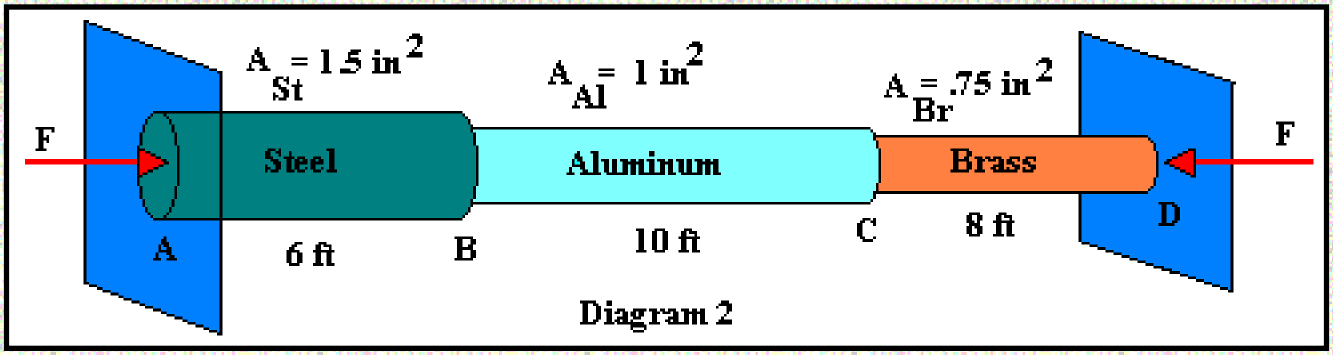


Diagram 1

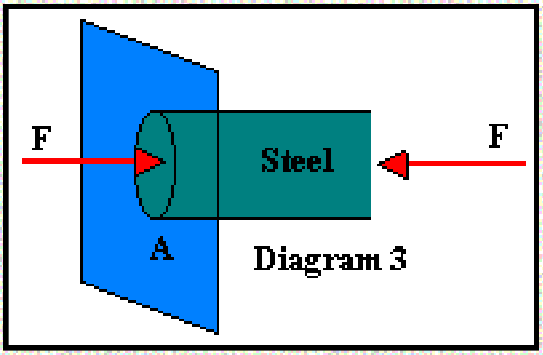
Solution:

PART I: STATICS

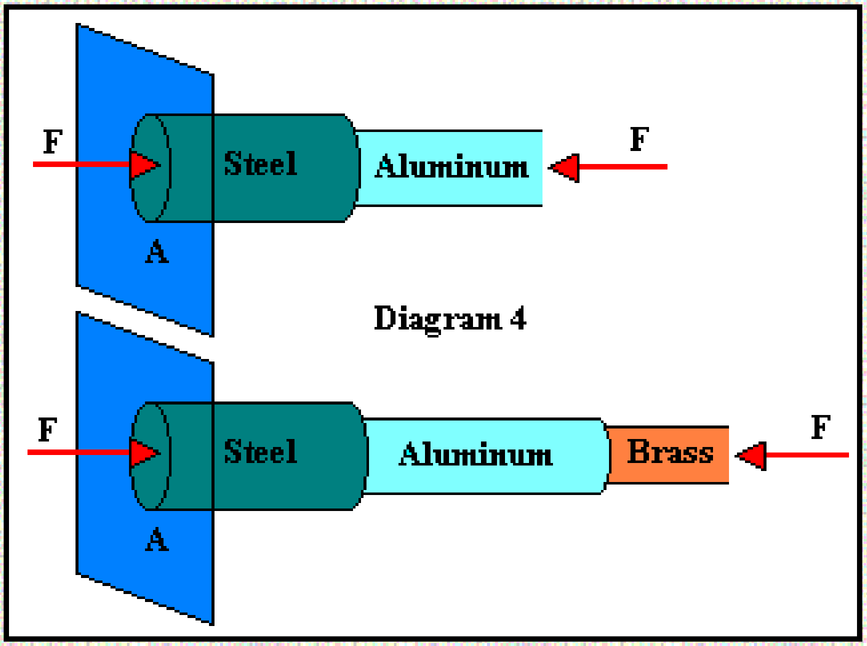
In this problem we first consider the effect of the change in temperature. Even though the total length of the three members is fixed, the members may expand or contract against each other. Since all three members are trying to expand they put each other into compression, and also push outward on the walls. In response an external force due to the wall acts inward on the members at each end as shown in Diagram 2.



With a little thought we realize that the forces in each member are the same and equal to the force exerted by the wall on the structure. This may be seen in Diagram 3, where we have cut the structure through the steel member. Since the structure is in equilibrium, this section of the structure must also be in equilibrium. But we can see that this is only possible if there is an internal force in the steel member which is equal and opposite to the force exerted by the wall.



And, of course, this same argument would hold if we cut the structure through the aluminum member, or through the brass member (as shown in Diagram 4)
So we can write (from static equilibrium) that $\mathbf{F_{st} = F_{al} = F_{br} = F}$



PART II: DEFORMATION

We now consider the geometry of the problem to obtain an additional equation to use in determining the force in the member.

After a little consideration we see that following deformation relationship must be true: $\delta_{st} + \delta_{al} + \delta_{st} = 0$ That is, the sum of the deformations (some of which may be positive expansions and some of which may be negative contractions) must be zero, since the rods are fixed between two walls. On substituting our expression for the total deformation (thermal +/- mechanical) we obtain:

[$\alpha\Delta TL$ - FL / EA]$_{st}$ + [$\alpha\Delta TL$ - FL / EA]$_{al}$ + [$\alpha\Delta TL$ - FL / EA]$_{br}$ = 0

The sign of the mechanical deformation term is negative (-) for all the rods since they are all in compression.. We now substitute values from our problem into the equation and obtain:

[(6.5 x 10^{-6}/ºF)(80ºF)(72 in) - F (72 in) / (30x10^6 lbs/in^2)(1.5 in^2)] + [(13 x 10^{-6}/ºF)(80ºF)(120 in) - F (120 in) / (10x10^6 lbs/in^2)(1 in^2)] + [(11 x 10^{-6}/ºF)(80ºF)(96 in) - F (96 in) / (15x10^6 lbs/in^2)(.75 in^2)] = 0

Rewriting the equation and combining terms we can write:

.0374 in. + .1248 in. + .0845 in. = [1.6 x 10^{-6} in/lb (F) + 12 x 10^{-6} in/lb (F) + 8.53 x 10^{-6} in/lb (F)], or

.2467 in = [22.13 x 10^{-6} in/lb]F

Now solving for **F = 11,150 lb.** This is the force in each rod, and we then calculate the stress in each rod from Stress = Force/Area. Finding: **Steel Stress = 11,150 lb/1.5 in^2 = 7,430 lb/in^2; Aluminum Stress = 11,150 lb/1 in^2 = 11, 150 lb/in^2; Brass Stress = 11,150 lb/.75 in^2 = 14,870 lb/in^2.**

PART III: DEFORMATION

Now that we have the amount of force in the aluminum member, its deformation may be calculated from [$\alpha \Delta T L$ - FL / EA], or

δ_{al} = [(13 x 10^{-6}/ºF)(80ºF)(120 in) - 11,150 lb (120 in) / (10x10^{-6} * 1 in^2] =

-.009 in. The negative deformation means that the aluminum is forced to shrink by that amount due to the effects of the steel and brass acting on it.

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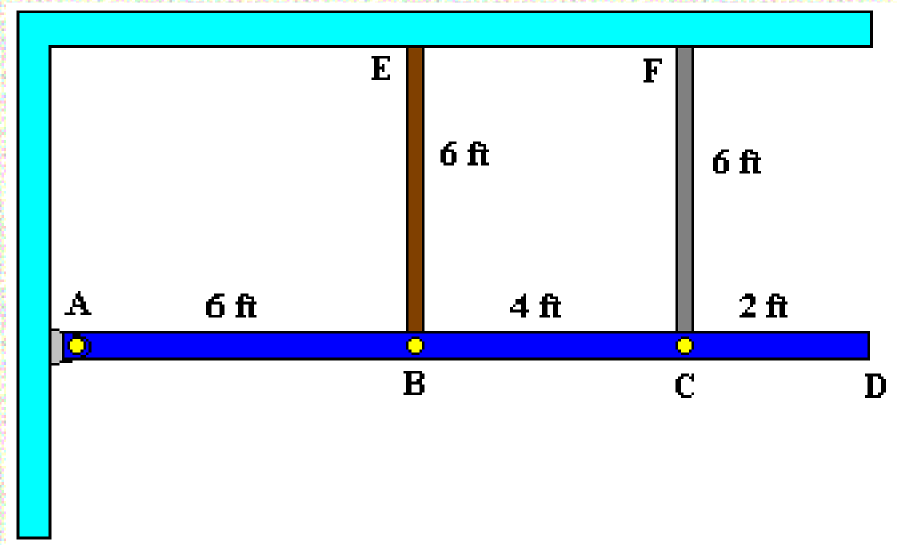
STATICS / STRENGTH OF MATERIALS - Example



In the structure shown on right member ABCD is pinned to the wall at point A, and supported by a brass member, BE, and by a steel member, CF. Both BE and CF have cross sectional areas of .5 in^2.
The structure is initially unstressed and then experiences a temperature increase of 50 degrees Celsius. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress that develops in brass member BE.
C. Determine the resulting movement of point D .

E_{st} = 30 x 10 ^{6} psi; E_{br} = 15 x 10 ^{6} psi; E_{al} = 10 x 10 ^{6} psi

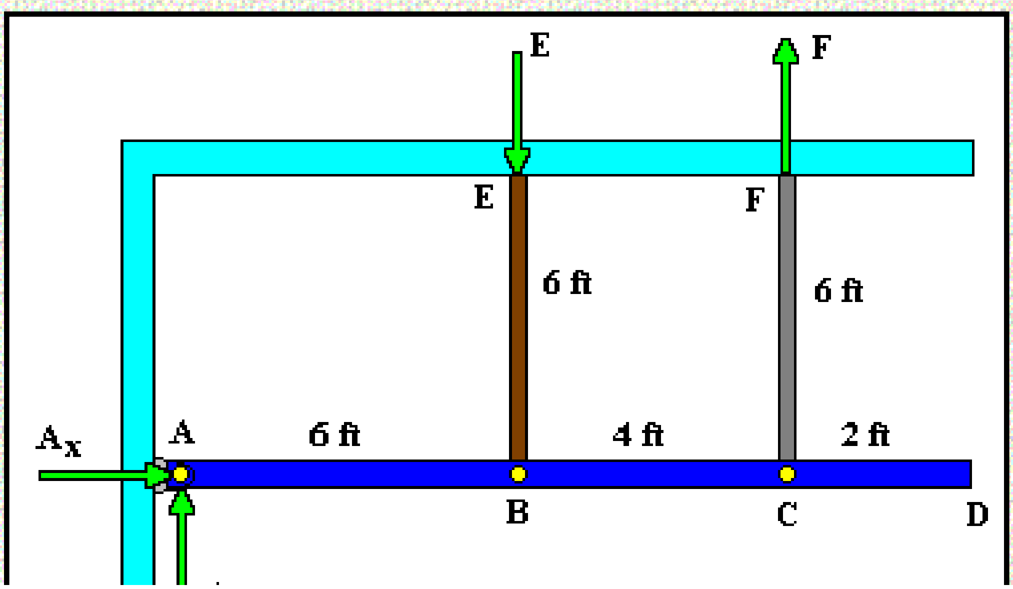
α_{st} = 12 x 10^{-6} /^{o}C; α_{br} = 20 x 10^{-6} /^{o}C; α_{al} = 23 x 10^{-6} /^{o}C

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

PART A: **STATICS**

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



First determine the direction of support forces by examining what thermal deformation are trying to occur and how the structure will respond.

Diagram 1

As shown in Diagram 2, the brass member (BE) would like to expand more than the steel member (CF) due to thermal effects. (Since the thermal coefficient of expansion of brass is larger than the thermal coefficient of expansion of steel.) Since we assume member ABCD will not bend, it will rotate about point A to a middle position as shown in Diagram 3.

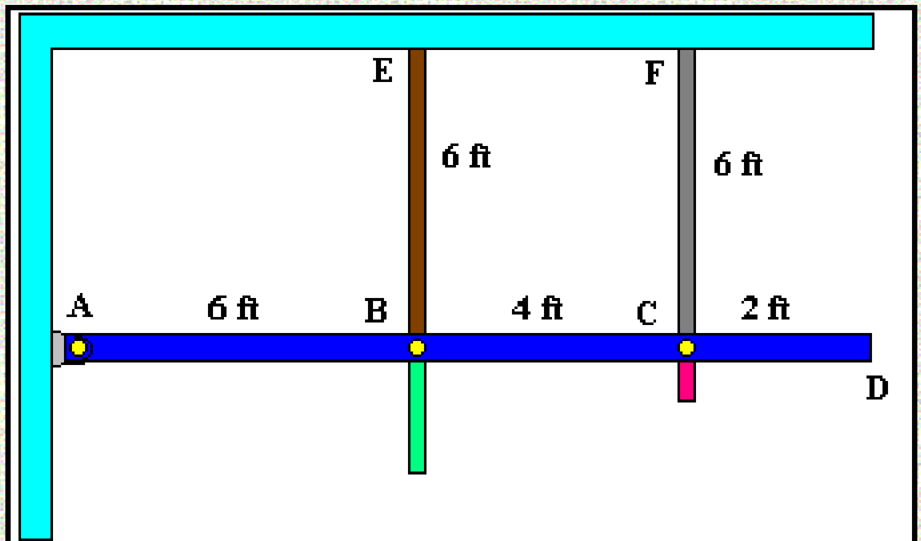


Diagram 2

That is, as the brass member expands, the steel member want to expand less and pulls back on the brass member putting it into compression, and stopping it from expanding as much as it would like to. From the steel member's point of view, it expands to a point (thermal expansion) and wants to stop,

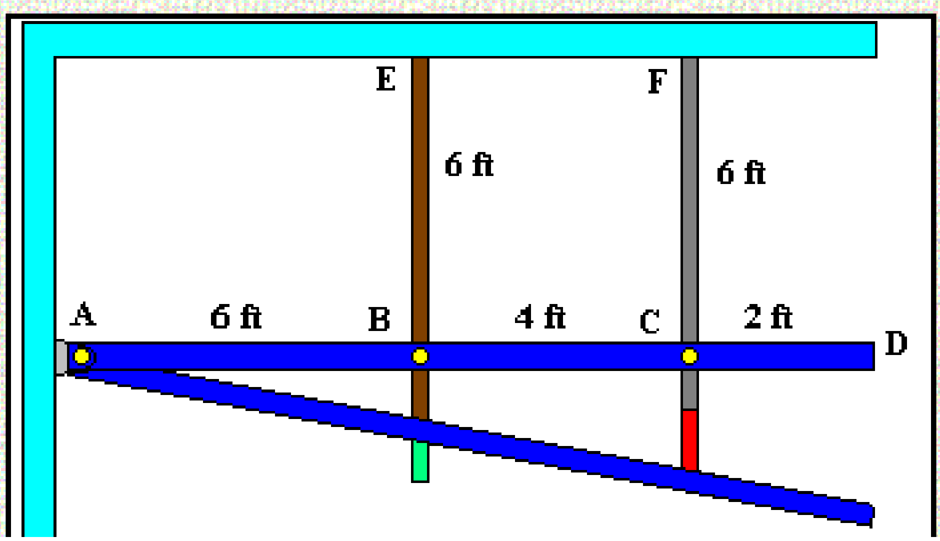


Diagram 3

but the brass member continues to expand, pulling on the steel member causing it to expand more than it would like (putting the steel member in tension).

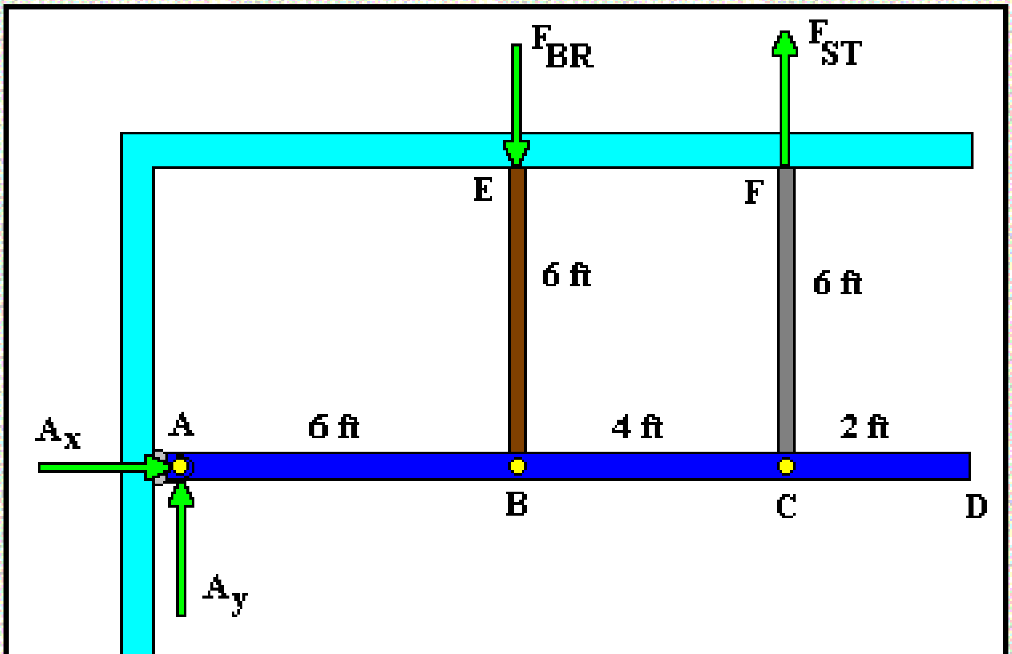


Diagram 4

Thus the brass member is in compression and the steel member is in tension, and the free body diagram may now be drawn as shown in Diagram 4.

Apply equilibrium conditions: Sum $F_x = A_x = 0$

Sum $F_y = A_y - F_{BR} + F_{ST} = 0$

Sum $T_A = F_{ST}$ (10 ft) - F_{BR} (6 ft) = 0

PART B: DEFORMATION

Too many unknowns; we now find a relation ship between the deformations to develop an additional equation. From the geometry of the problem, we have:

$$+\delta_{BR}/6ft = +\delta_{ST}/10ft \text{ or } \delta_{BR\ total} = .6\ \delta_{ST\ total}$$

The total deformation depends on the thermal deformation and the mechanical deformation and can be expressed as:

$$\delta_{total} = (\alpha\,\Delta TL \pm FL/EA);$$

substituting this expression into our deformation relationship gives us:

$$(\alpha\,\Delta TL - FL/EA)_{BR} = .6(\alpha\,\Delta TL + FL/EA)_{ST}$$

Substituting in values, we have:

[(20x10^{-6}/^{o}C) (+50^{o}C) (72 in) - F_{BR} (72 in) / (15x10^6 lbs/in^2) (.5 in^2)] = .6 [(12x10^{-6}/^{o}C) (+50^{o}C) (72 in) + F_{ST} (72 in) / (30x10^6 lbs/in^2) (.5 in^2)] OR 0.072 in - 9.6x10^{-6} F_{BR} = 0.026 in + 2.88x10^{-6} F_{ST} OR 2.88x10^{-6} F_{ST} + 9.6x10^{-6} F_{BR} = 0.046

From our statics torque equation we have: F_{ST} (10 ft) - F_{BR} (6 ft) = 0 OR F_{ST} = .6F_{BR}

We now substitute into our deformation expression
2.88x10^{-6} (.6 F_{BR}) + 9.6x10^{-6} F_{BR} = 0.046

Solving for F_{BR} = **4,060 lbs F_{ST} = 2,440 lbs**

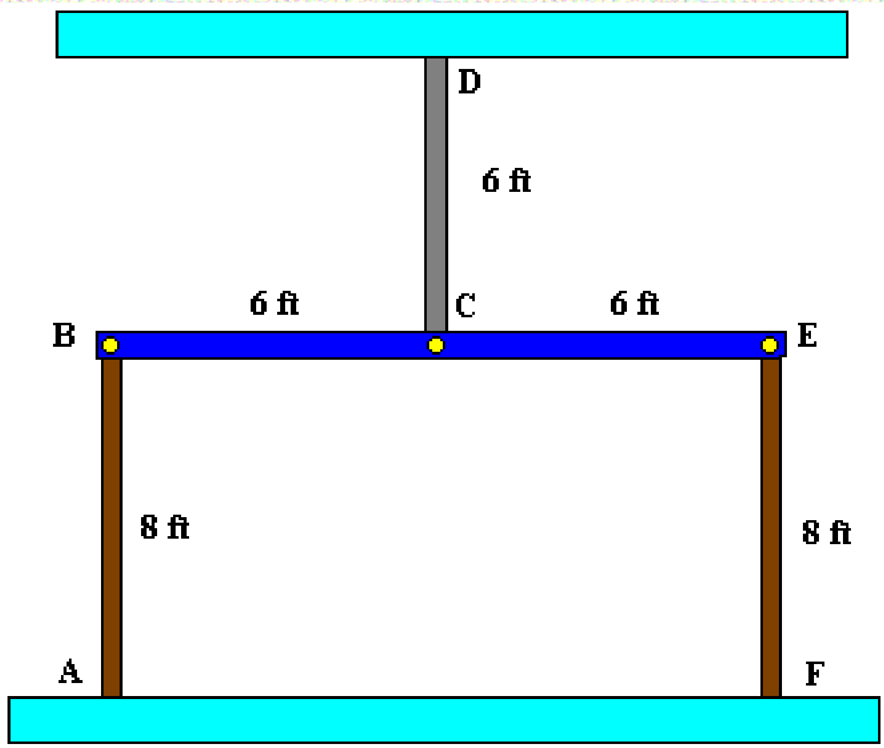
Then the stress in brass member BE is σ **= F/A = 4,060 lbs/ .5 in^2 = 8,120 lbs/ in^2**

PART C: MOVEMENT

Finally, point D moves in proportion to the movement of point C (which is equal to the deformation of member CF), and we can write:
Mov. D / 12 ft = δ_{CF} / 10 ft

Mov. D / 12 ft = [(12x10^{-6}/^{o}C) (+50^{o}C) (72 in) + (2,440 lbs) (72 in) / (30x10^6 lbs/in^2) (.5 in^2)] / 10 ft
or Mov. D = (12 ft) [0.0549 in / 10 ft] = 0.0659 in

STATICS / STRENGTH OF MATERIALS - Example



In the structure shown on right horizontal member BCF is supported by vertical brass members, AB, and EF, and by steel member, CD. Both AB and EF have cross sectional areas of .5 in^2. Member CD has a cross sectional area of .75 in^2. The structure is initially unstressed and then experiences a temperature decrease of 50 degrees celsius. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.

B. Determine the axial stress that develops in steel member CD.

C. Determine the resulting movement of point E .

E_{st} = 30 x 10 ^{6} psi; E_{br} = 15 x 10 ^{6} psi; E_{al} = 10 x 10 ^{6} psi

α_{st} = 12 x 10^{-6} /^{o}C; α_{br} = 20 x 10^{-6} /^{o}C; α_{al} = 23 x 10^{-6} /^{o}C

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

PART A: STATICS

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.

STEP 2: Break any forces into x and y components

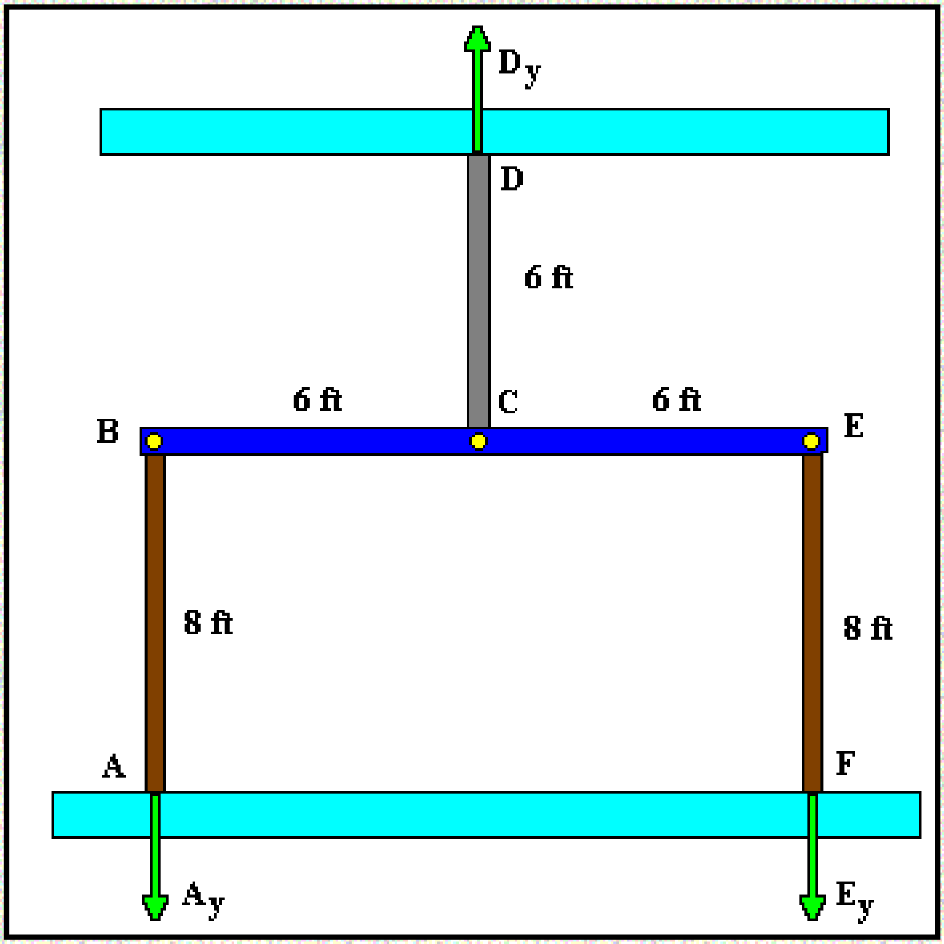
STEP 3: Apply equilibrium conditions:

Sum $F_x = 0$

Sum $F_y = D_y - A_y - F_y = 0$

Sum $T_c = A_y (6 \text{ ft}) - F_y (6 \text{ ft}) = 0$

From torque equation we get $A_y = F_y$; If we put this into the force equation we get: $D_y - 2A_y = 0$ or $A_y = D_y / 2$



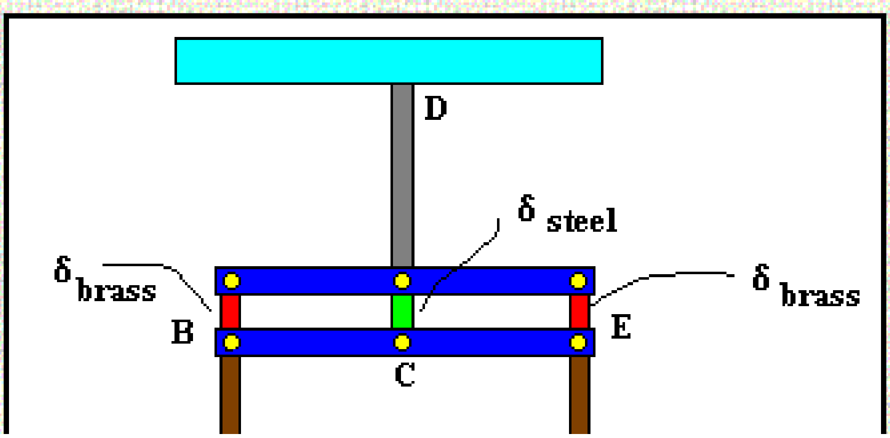
To solve for forces A_y, D_y and F_y, we need a third equation - which we obtain from the relationship between the deformations.

PART B: DEFORMATION

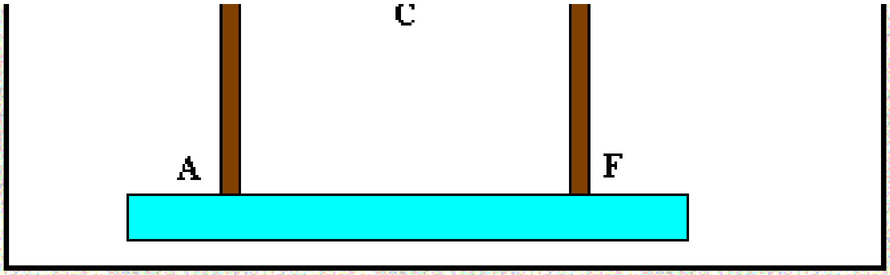
Since the forces in members AB and FE are equal, since they are made of the same material - brass, and since they are the same length and area, they will deform the same amount (from $\delta = FL / EA$).

Both the bottom brass member (AB and FE) and the top steel member (CD) try to contract (because the change in temperature is negative).

However clearly both top and bottom members can not contract, one will "win," contracting while forcing the other member(s) to expand. We will assume the steel member "wins" actually shrinking a certain amount. The brass members will elongate the same



amount. The steel contracts, as shown in diagram 2.



We write the deformation relationship as: - δ (steel) = + δ (brass)
(negative indicates that the deformation of the steel is a contraction)

The deformation of the members depend on both the thermal deformation and the mechanical deformation due to the forces that develop in the members.
This may be written as follows:

$\delta = [\alpha \Delta T L \pm (FL/EA)]_{material}$

("+" sign if member is in tension, "-" sign if member is in compression)
where:
α = thermal coefficient of expansion
ΔT = change in temperature
L = length of member
F = force in member
E = Young's modulus for material
A = cross sectional area of member

We now substitute the expression into our deformation relationship so:
- δ (steel) = + δ (brass) becomes
$-[\alpha \Delta T L + (FL/EA)]_{steel} = [\alpha \Delta T L + (FL/EA)]_{brass}$

We now substitute in values and get the expression:
$-[(12\times10^{-6}/^{o}C)(-50^{o}C)(72\text{ in}) + D_y(72\text{ in}) / (30\times10^{6}\text{ lbs/in}^2)(.75\text{ in}^2)] =$

$+[(12\times10^{-6}/^{o}C)(-50^{o}C)(96\text{ in}) + A_y(96\text{ in}) / (15\times10^{6}\text{ lbs/in}^2)(.5\text{ in}^2)]$

Combining terms, the equation becomes:
$-[-0.0432 + (3.2\times10^{-6})D_y] = [-0.096 + (12.8\times10^{-6})A_y]$ OR

$3.2\times10^{-6} D_y + 12.8\times10^{-6} A_y = 0.139$

This is our third equation, we can now substitute our relationship from statics: $A_y = D_y / 2$, into the above expression and get:

$3.2\times10^{-6} D_y + 12.8\times10^{-6} (D_y / 2) = 0.139$

Solving for

D_y = 14,500 lbs (force in steel member); A_y = 7,250 lbs (force in brass member(s))

Then to find stress in member CD

$\sigma = D_y$ / A = 14,500 lbs / 0.75 in^2 = 19,300 lbs/in^2

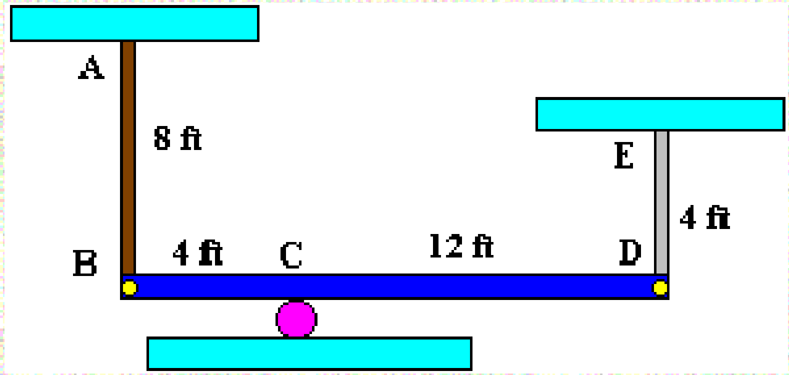
PART C: <u>MOVEMENT</u>

Point E moves since it is attached to member FE, and its movement is equal to the deformation of member FE.

δ_{FE} = [(20x10^{-6}/^{o}C) (-50^{o}C) (96 in) + (7,250 lbs) (96 in) / (15x10^6 lbs/in^2) (.5 in^2)] = 0.0496 in

STATICS / STRENGTH OF MATERIALS -Example

In the structure shown on right horizontal member BCD is supported by vertical brass member, AB, an aluminum member DE, and by a roller at point C. Both AB and ED have cross sectional areas of .5 in^2.



The structure is initially unstressed and then experiences a temperature increase of 60 degrees Celsius. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress that develops in aluminum member DE.
C. Determine the resulting movement of point B .

E_{st} = 30 x 10 ^{6} psi; E_{br} = 15 x 10 ^{6} psi; E_{al} = 10 x 10 ^{6} psi

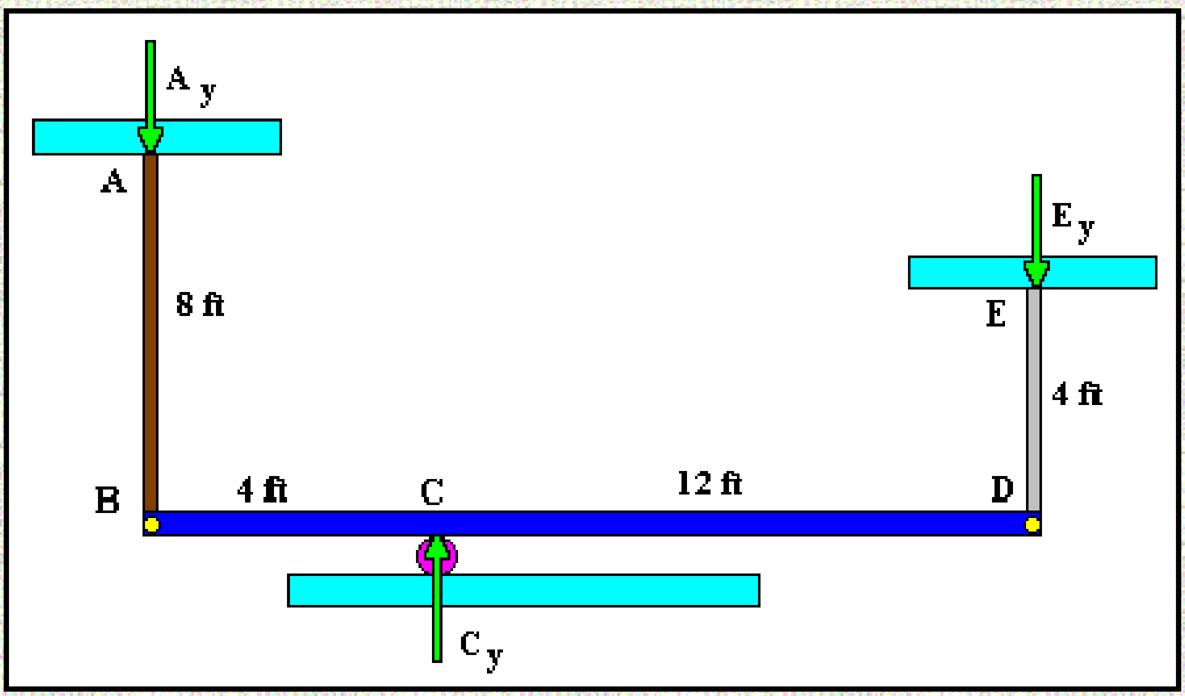
α_{st} = 12 x 10^{-6} / ^oC; α_{br} = 20 x 10^{-6} / ^oC; α_{al} = 23 x 10^{-6} / ^oC

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

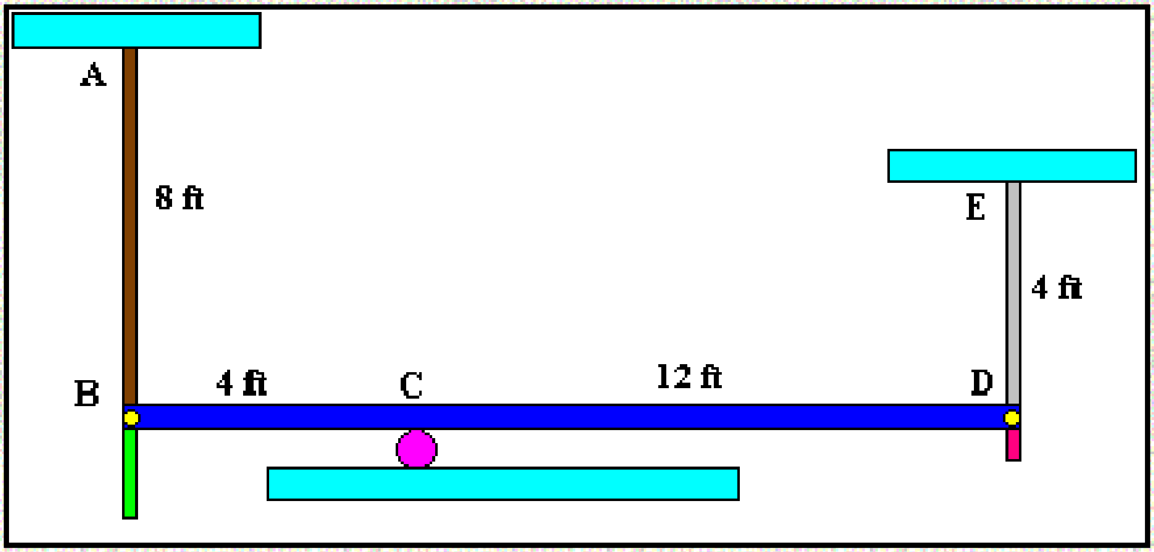
Solution:

PART A: **STATICS**

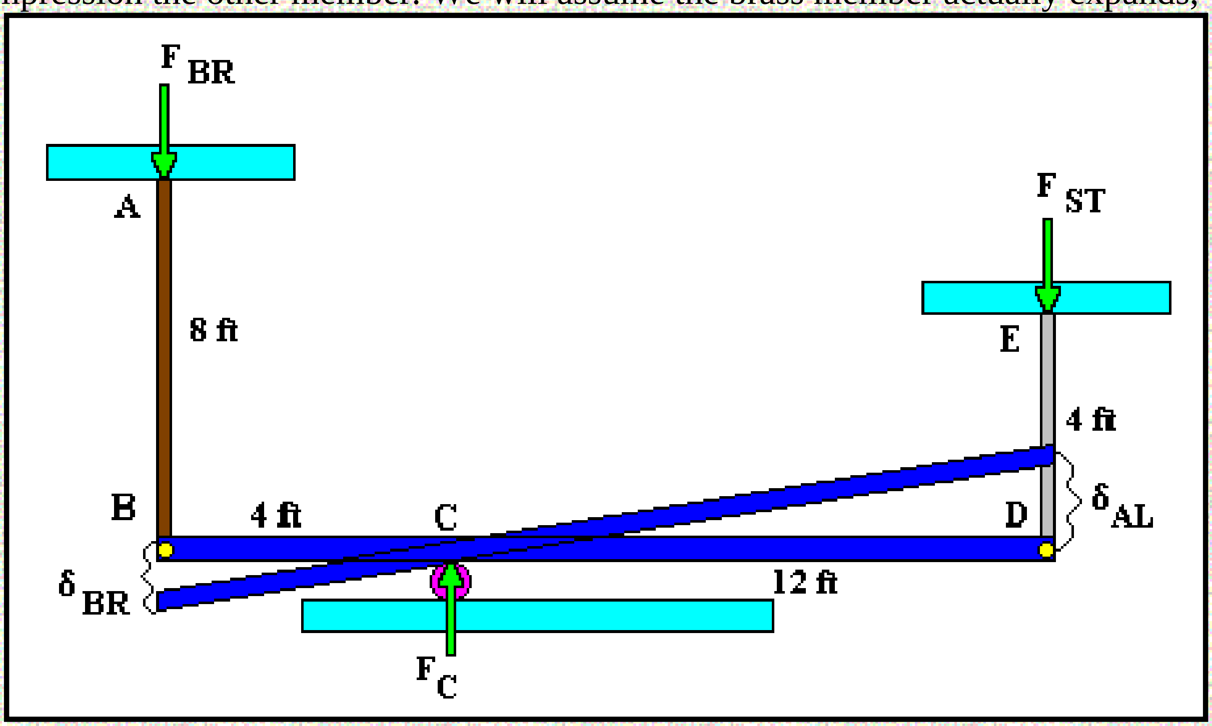
STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



We first examine how much the members would expand freely, due to temperature increase, if they were free to expand (Diagram 2).



Since both members can not expand as shown (assuming member BCD does not bend), one member will "win," expanding and compression the other member. We will assume the brass member actually expands, compression the aluminum member (Diagram 3).



Since the two members are pushing against each other, both members are put into compression, and the support reaction are as shown in diagram 3.

Now we write the equilibrium equations:

Sum $F_x = 0$ (None)

Sum $F_y = -F_{BR} + F_C - F_{AL} = 0$

Sum $T_C = F_{BR}$ (4 ft) - F_{AL} (12 ft) = 0

Not enough independent equations to solve for our three unknowns, so we need another equation - which we will obtain from the deformation relationship.

PART B: **DEFORMAION**

General relationship from geometry of structure:

δ_{BR} / 4 ft = - δ_{AL} / 12 ft ; or 3 δ_{BR} = - δ_{AL}

(negative sign is due to our assumption that the aluminum got shorter (is compressed).)

We now expand the expression for our deformations using the fact that the total deformation is given by:

$\delta_{total} = [\alpha \Delta TL \pm FL/EA]_{material}$ so,

$3\ [\alpha \Delta TL - FL/EA]_{brass} = -\ [\alpha \Delta TL - FL/EA]_{aluminum}$

or, using known values:

$3[\ (20\times10^{-6}/^{o}C)\ (+60^{o}C)\ (96\ in) - F_{BR}\ (96\ in)\ /\ (15\times10^{6}\ lbs/in^{2})\ (.5\ in^{2})\ ] =$

$[\ (23\times10^{-6}/^{o}C)\ (+60^{o}C)\ (48\ in) - F_{AL}\ (48\ in)\ /\ (10\times10^{6}\ lbs/in^{2})\ (.5\ in^{2})\ ]$

or $+\ (0.346 - 3.84\times10^{-5}\ F_{BR}) = -\ (0.0662 - 0.96\times10^{-5}\ F_{AL})$ **or**

$3.84\times10^{-5}\ F_{BR} + 0.96\times10^{-5}\ F_{AL} = 0.412$ in

From our statics, we had

F_{BR} (4 ft) - F_{AL} (12 ft) = 0 or $F_{BR} = 3\ F_{AL}$,

which we now substitute into our deformation relationship

$3.84\times10^{-5}\ (3\ F_{AL}) + 0.96\times10^{-5}\ F_{AL} = 0.412$ in

Solving for

F_{AL} = 3,330 lbs ; F_{BR} = 9,900 lbs ; σ_{AL} = F / A = 3,300 lbs / .5 in^2 = 6,600 lbs / in^2

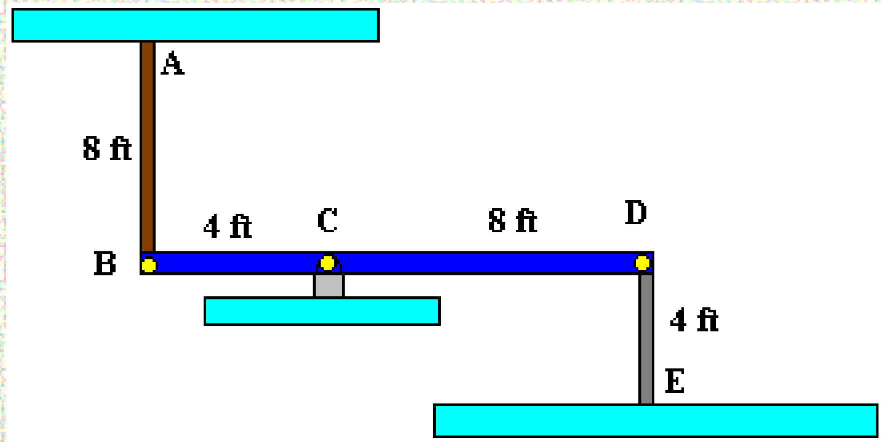
PART C: **MOVEMENT**

Mov. B = the deformation of member AB

So, Mov. B = $[(20\times10^{-6}\ /\ ^{o}C)(60^{o}C)(96\ in) - (9{,}900\ lbs)(96\ in)\ /\ (15\times10^{6}\ lbs/in^{2})\ (.5\ in^{2})]$

Mov. B = -.01152" (This indicates that the brass member is compressed this amount.)

STATICS / STRENGTH OF MATERIALS - Example



In the structure shown on right horizontal member BCD is supported by vertical brass member, AB, a steel member DE, and is pinned to the floor at point C. Both AB and DE have cross sectional areas of .5 in^2. The structure is initially unstressed and then experiences a temperature decrease of 60 degrees celsius. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress that develops in steel member DE.
C. Determine the resulting movement of point B .

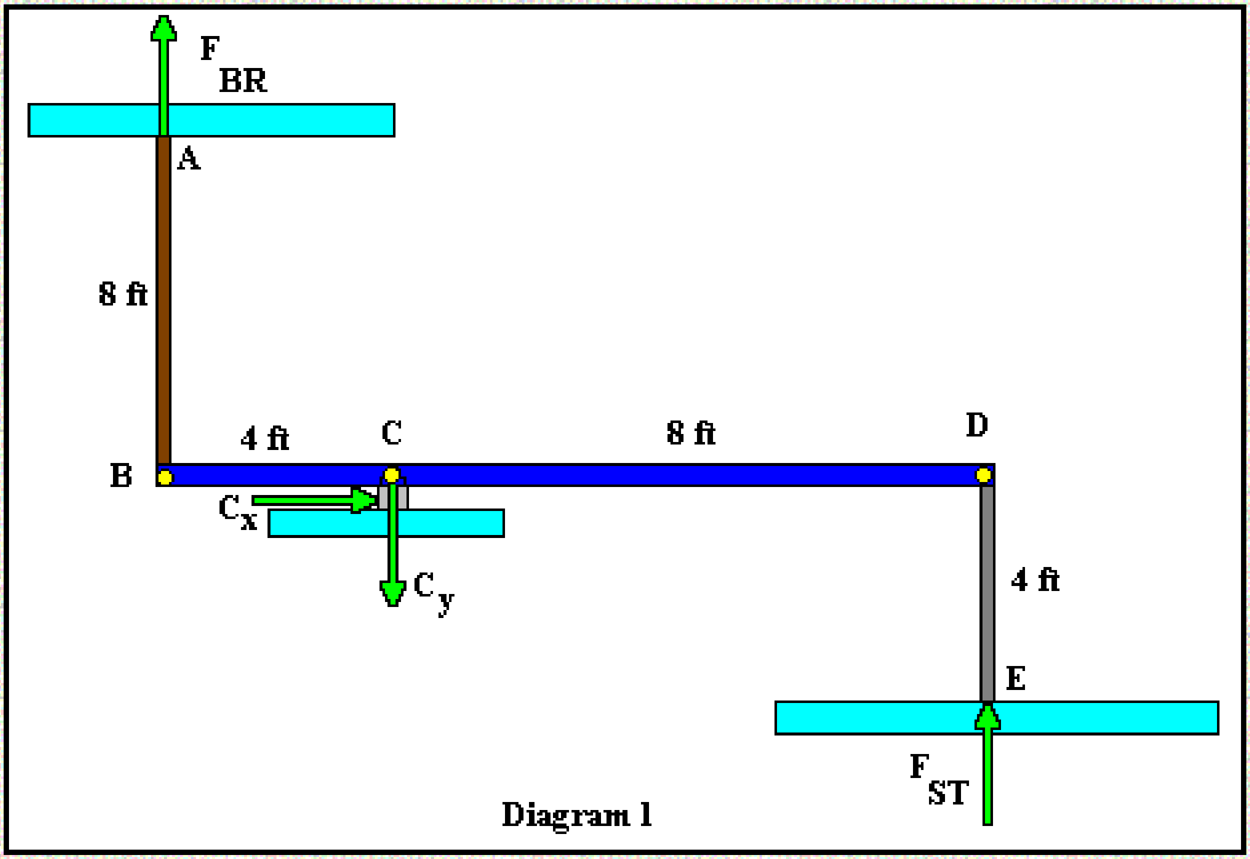
E_{st} = 30 x 10 ^{6} psi; E_{br} = 15 x 10 ^{6} psi; E_{al} = 10 x 10 ^{6} psi

α_{st} = 12 x 10^{-6} /^{o}C; α_{br} = 20 x 10^{-6} /^{o}C; α_{al} = 23 x 10^{-6} /^{o}C

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

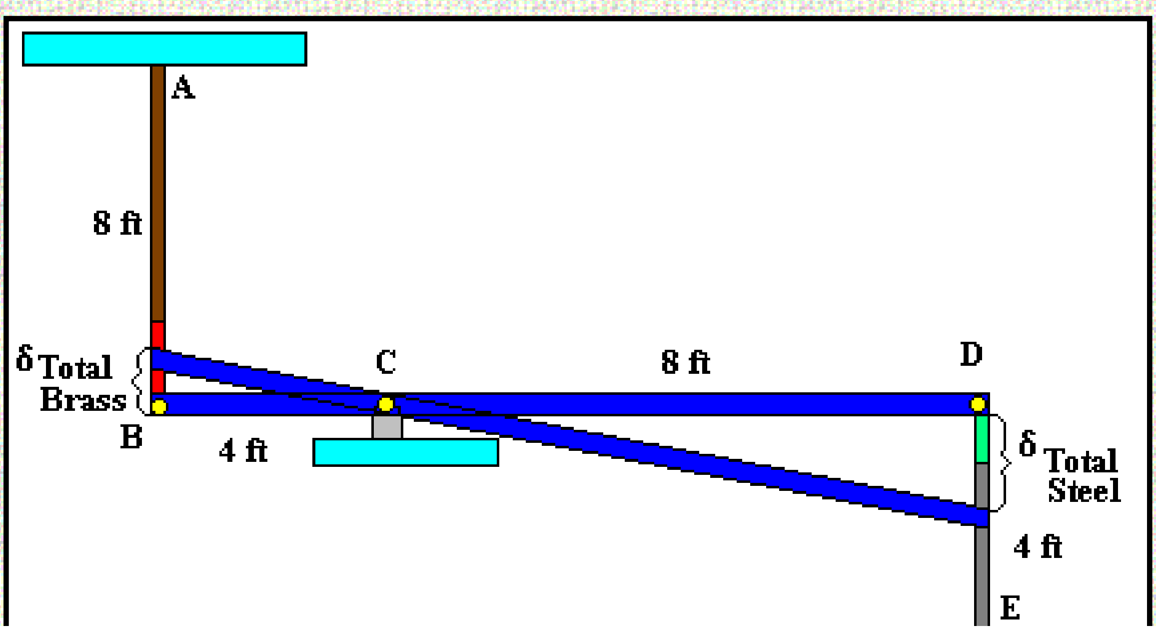
Solution:

PART A : STATICS



STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.

In this problem we first examine how much the brass and steel members would shrink if they were free to do so due to the temperature decrease. The brass member, AB, would shrink much more than the steel member since it's coefficient of linear expansion is nearly twice that of steel. (see diagram 2)



As the brass shrinks, it causes the horizontal member BCD to rotate about point C. As it does so, point D moves downward - which is all right to a certain point, as the steel is also shrinking. At a certain pint the steel is done shrinking due to the temperature decrease and "wants" to stop; however

Diagram 2

the brass "wants" to shrink more and continues to pull upward on point B causing more rotation of member BCD about point C, and causing point D to move downward an additional amount, putting steel member DE into compression. The steel member DE pushes back on point D, stopping the brass from shrinking any more and putting the brass into tension. This thought analysis process is how the directions of the support reactions direction in diagram 1 were arrives at.

The horizontal member BCD ends up in an intermediate position as shown in diagram 2. We now write the equilibrium equations:

Sum $F_x = C_x = 0$

Sum $F_y = F_{br} - C_y + F_{st} = 0$

Sum $T_C = -F_{br}(4\text{ ft}) + F_{st}(8\text{ ft}) = 0$

There are not enough independent equations to solve for our three unknowns, so we need another independent equation - which we will obtain from the deformation relationship.

PART B: DEFORMATION

From the geometry of the problem we see that

$-\delta_{brass} / 4\text{ ft} = -\delta_{steel} / 8\text{ ft}$

(negative signs since deformation and contractions - members get shorter.)

$2\delta_{brass} = \delta_{steel}$

We now expand our expression for our deformations using:

$\delta_{total} = [\alpha\,\Delta TL \pm FL / EA]_{material}$;

so, we obtain

$2[\alpha\,\Delta TL + FL / EA]_{brass} = [\alpha\,\Delta TL - FL / EA]_{steel}$

and putting values of materials and the structure into this equation we obtain:

$2[(20\times10^{-6} / {}^oC)(60^oC)(96\text{ in}) + F_{br}(96\text{ in}) / (15\times10^6\text{ lbs/in}^2)(.5\text{ in}^2)] =$

$[(12\times10^{-6} / {}^oC)(60^oC)(48\text{ in}) + F_{st}(48\text{ in}) / (30\times10^6\text{ lbs/in}^2)(.5\text{ in}^2)]$

or $(-0.2304 + 2.56\times10^{-5} F_{br}) = (-0.0346 - 0.32\times10^{-5} F_{st})$

or $2.56\times10^{-5} F_{br} + 0.32\times10^{-5} F_{st} = 0.196$

from our static equilibrium torque equilibrium we had:

$-F_{br}(4) + F_{st}(8) = 0$ or $F_{br} = 2F_{st}$, we now substitute this into our deformation equation.

and obtain:

$2.56x10^{-5}$ (2 F_{st}) + $0.32x10^{-5}$ F_{st} = 0.196

solving:

F_{st} = 3,600 lbs ; F_{br} = 7,200 lbs

and stress in steel = F/A = 3,600 lbs / .5 in^2 = 7,200 lbs/in^2

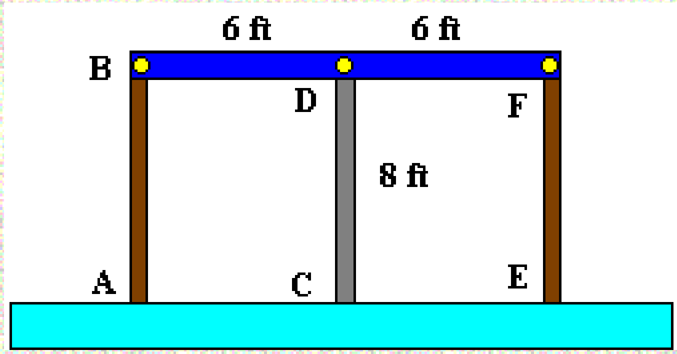
PART C: **MOVEMENT**

Point B is attached to member AB and so movement of point B is equal to the deformation of brass member AB.

Mov. B = [($20x10^{-6}$ / ºC)(60ºC)(96 in) +(7,200 lbs)(96 in) / ($15x10^6$ lbs/in^2)(.5 in^2)]

Mov. B = -0.023 in

STATICS / STRENGTH OF MATERIALS - Example



In the structure shown on the right, horizontal member BDF is supported by two brass members, AB and EF, and a steel member CD. Both AB and EF have cross sectional areas of .5 in^2. Member CD has a cross sectional area of .75 in^2.
The structure is initially unstressed and then experiences a temperature increase of 40 degrees celsius. For this structure:

A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress that develops in steel member CD.
C. Determine the resulting movement of point D .

E_{st} = 30 x 10 ^{6} psi; E_{br} = 15 x 10 ^{6} psi; E_{al} = 10 x 10 ^{6} psi

α_{st} = 12 x 10^{-6} /^{o}C; α_{br} = 20 x 10^{-6} /^{o}C; α_{al} = 23 x 10^{-6} /^{o}C

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:
PART A: **STATICS**

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.

In this problem we first examine how much the brass and steel member would expand, if free to do so, due to the temperature increase. As the coefficient of expansion of the brass in larger than the coefficient of expansion of the steel, the brass would expand more if free to do so. (see diagram 2)

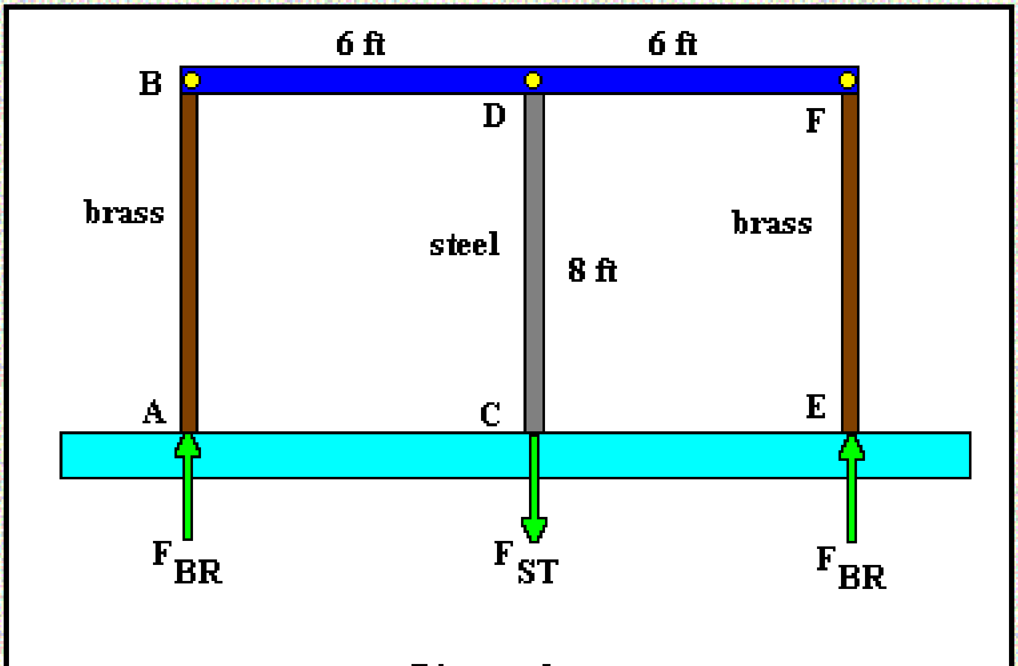


Diagram1

As the brass expands it pulls on the steel placing the steel in tension. The steel pulls back on the brass placing the brass in compression. The support reactions, reflecting these forces, are shown in diagram 2. The forces in the brass member are equal from symmetry of the structure.

The horizontal member BDF moves to a middle position, as shown in diagram 2.

We now write the equilibrium equations:

Sum $F_x = 0$

Sum $F_y = F_{br} - F_{st} + F_{br} = 0$

Sum $T_D = -F_{br}\,(6\text{ ft}) + F_{br}\,(6\text{ ft}) = 0$

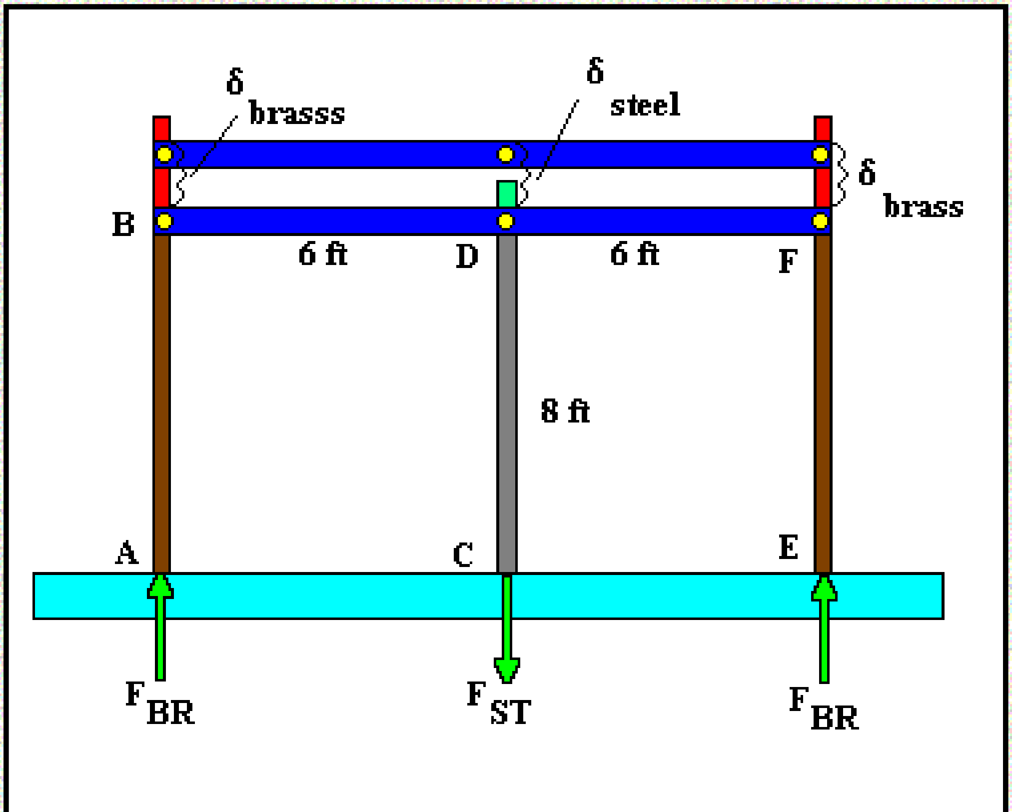


Diagram 2

Diagram 2

The torque equation simply gives us: $F_{br} = F_{br}$

and **the sum of y forces gives us:** $F_{st} = 2\,F_{br}$

We do not have enough equation to solve the problem. We obtain an additional independent equation for the formation relationship.

PART B: **DEFORMATION**

$\delta_{br} = \delta_{st}$ from the geometry of the structure...expanding using:

$\delta_{total} = [\alpha\,\Delta TL \pm FL / EA]$, we obtain:

$[(20\text{x}10^{-6} / {}^{o}C)(40^{o}C)(96\text{ in}) - F_{br}(96\text{ in}) / (15\text{x}10^{6}\text{ lbs/in}^2)(.5\text{ in}^2)] =$

$[(12\text{x}10^{-6} / {}^{o}C)(40^{o}C)(96\text{ in}) + F_{st}(96\text{ in}) / (30\text{x}10^{6}\text{ lbs/in}^2)(.5\text{ in}^2)]$

or $(-0.0768 - 1.28\text{x}10^{-5}\,F_{br}) = (0.0461 + 0.64\text{x}10^{-5}\,F_{st})$

or $1.28\text{x}10^{-5}\,F_{br} + 0.64\text{x}10^{-5}\,F_{st} = 0.0307$

From our static equilibrium equation in part I, we have : $F_{st} = 2\,F_{br}$

Substituting, we obtain:

$0.64\text{x}10^{-5}(2\,F_{br}) + 1.28\text{x}10^{-5}(F_{br}) = 0.0307$

solving: F_{br} = 1,200 lbs ; F_{st} = 2,400 lbs

Stress in steel = F/A = 2,400 lbs/.75 in^2 = 3,200 lbs/in^2

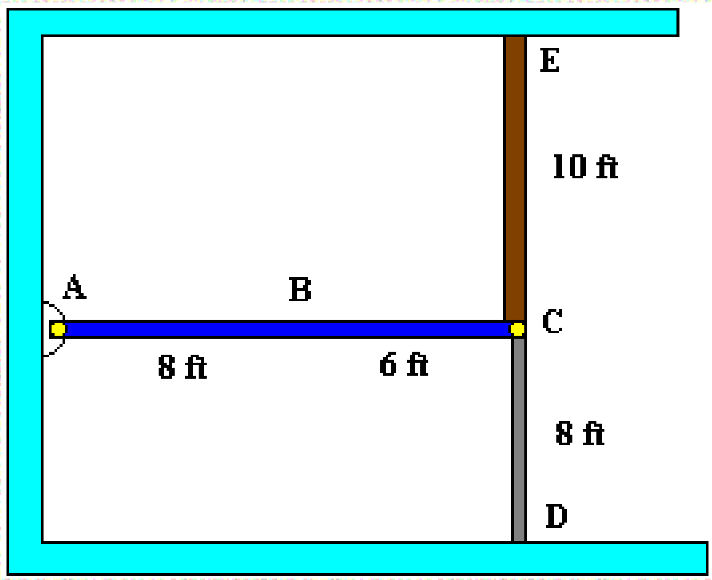
PART C: **MOVEMENT**

Movement of point D**.** Point D is connected to steel member CD and so moves the amount CD deforms or

Mov. D = $[(12\text{x}10^{-6} / {}^{o}C)(40^{o}C)(96\text{ in}) + (2{,}400\text{ lbs})(96\text{ in}) / (30\text{x}10^{6}\text{ lbs/in}^2)(.75\text{ in}^2)]$

Mov. D = 0.05632 in

STATICS / STRENGTH OF MATERIALS - Example



In the structure shown on the right horizontal member ABC is supported by steel member CD, and brass member CE. Member CE has a cross sectional area of .75 in^2. Member CD has a .5 in^2 cross sectional area.
The structure is initially unstressed and then experiences a temperature decrease of 50 degrees celsius. For this structure:
A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress that develops in member CE.
C. Determine the resulting movement of point B .

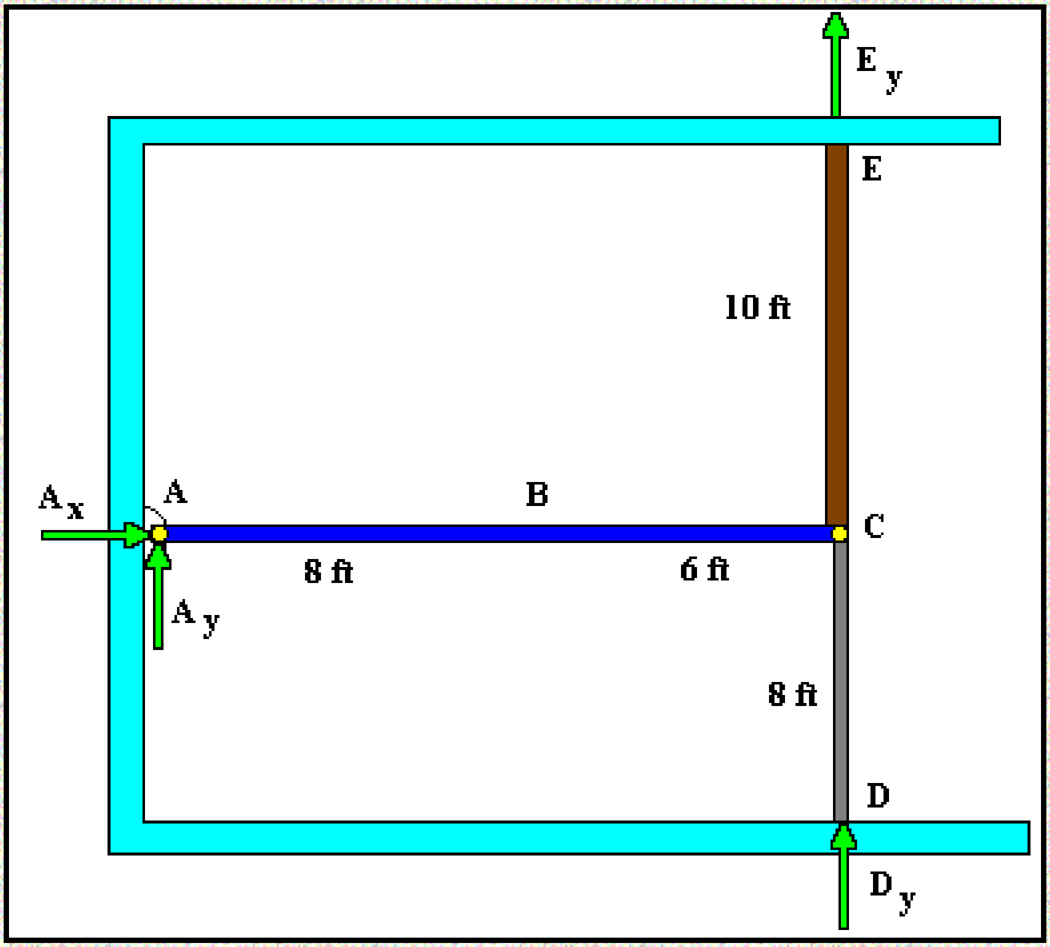
E_{st} = 30 x 10 ^{6} psi; E_{br} = 15 x 10 ^{6} psi; E_{al} = 10 x 10 ^{6} psi

α_{st} = 12 x 10^{-6} / ^oC; α_{br} = 20 x 10^{-6} / ^oC; α_{al} = 23 x 10^{-6} / ^oC

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:
Part A: STATICS

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



Topic 3.83: Thermal Stress, Strain, & Deformation - Problems

The following values will be need for the Problems below

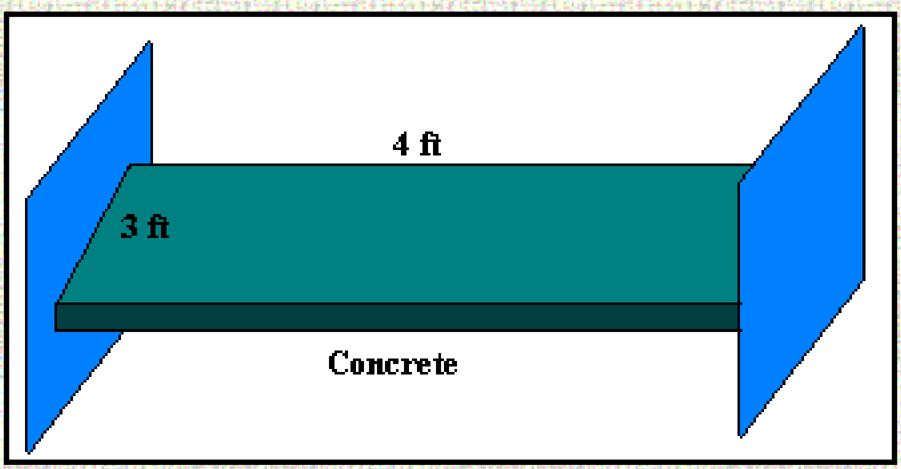
Linear coefficient of Expansion:

$\alpha_{Steel} = 6.5 \times 10^{-6} / {}^oF$; $\alpha_{Aluminum} = 13 \times 10^{-6} / {}^oF$; $\alpha_{Brass} = 11 \times 10^{-6} / {}^oF$; $\alpha_{concrete} = 6 \times 10^{-6} / {}^oF$

Young's Modulus:

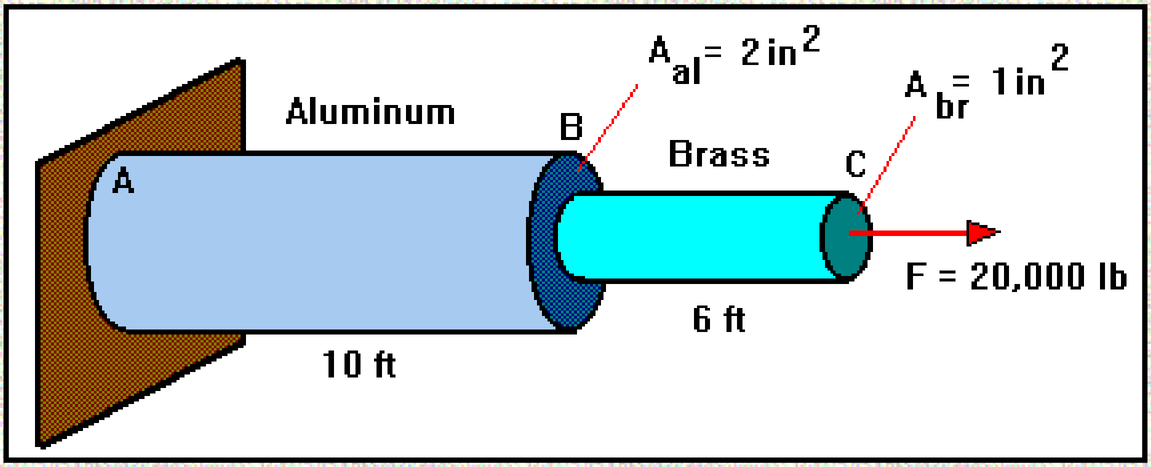
$E_{Steel} = 30 \times 10^6$ lb/in^2; $E_{Aluminum} = 10 \times 10^6$ lb/in^2; $E_{Brass} = 15 \times 10^6$ lb/in^2; $E_{Concrete} = 5 \times 10^6$ lb/in^2

1. A concrete sidewalk slab is 3 ft. wide by 4 ft. long. If the concrete slab is constrained so it can not expand, what stress would develop in it due to thermal effects if it experienced a temperature increase of 60^o F? (Stress = 1800 psi.)

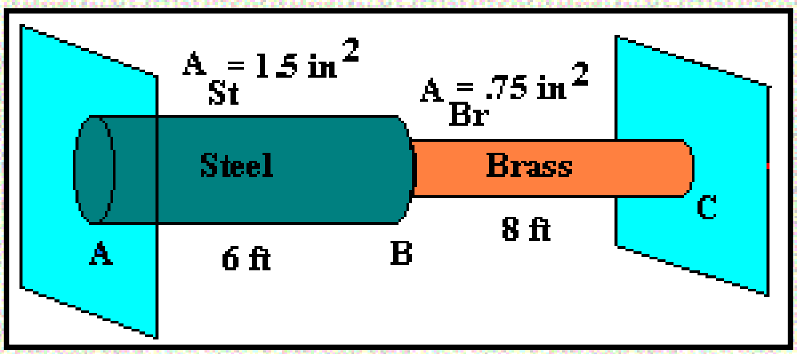


2. An aluminum rod and a brass rod are attached to each other, and the aluminum rod is attached to a wall as shown in the diagram. The rods are initially unstressed and then a 20, 000 lb horizontal force is applied to the end of the brass rod as shown. Additionally, the rods experience a temperature increase of 80 ^{o} F. Determine the final stress that develops in each rod, and the total movement of

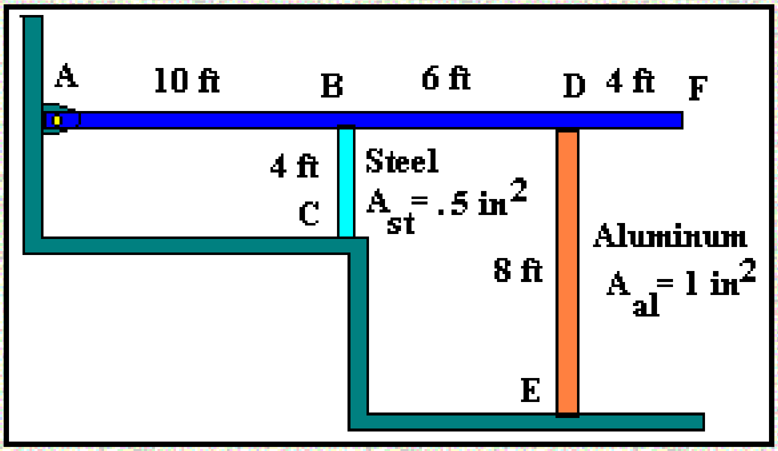
point C. (Al. stress = 10,000 psi.; Br. stress = 20,000 psi.; Total def. = .384")



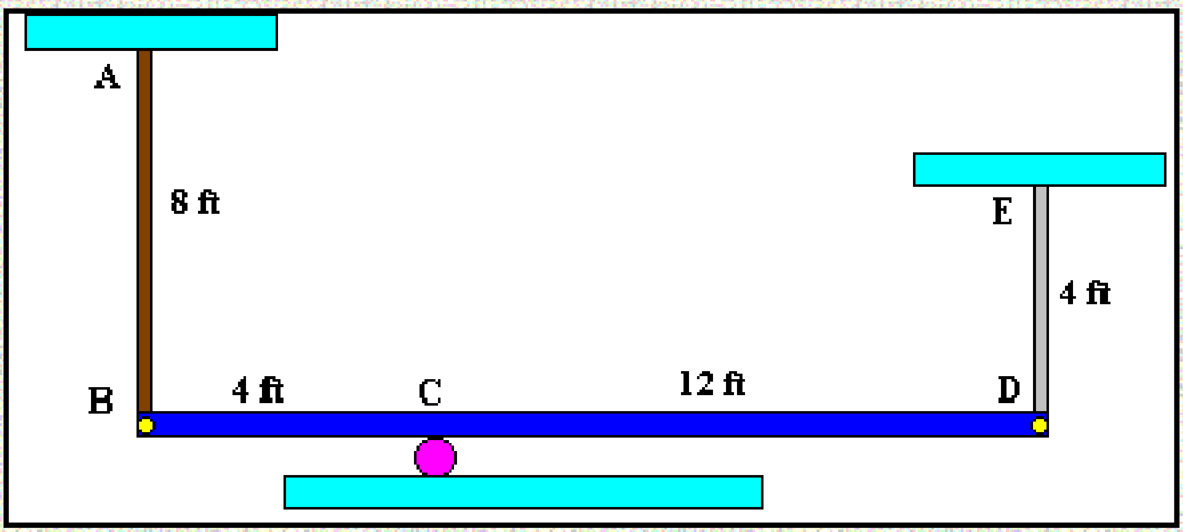
3. A steel rod and a brass rod are attached to each other and mounted between two walls as shown in the diagram. If the structure is initially unstressed and then experiences a temperature increase of 80º F., determine the stress which develops both in the steel and in the brass rod, and the amount of deformation of the brass rod. (St. stress = 8000 psi.; Br. stress = 16000 psi.; Def. of Br.=.0182")



4. A horizontal bar ABDF is pinned to the wall at point A, and supported by steel member BC and aluminum member DE, as shown in the diagram. If the structure is initially unstressed and then experiences a temperature decrease of 70º F., determine the stress which develops in the steel and aluminum members, and the movement of point F.(St.stress =9444 psi.; Al stress = 2951 psi.; Move. of F = -.074")



5. In the structure shown below horizontal member BCD is supported by vertical brass member, AB, an aluminum member DE, and by a roller at point C. Both AB and ED have cross sectional areas of .5 in^2. If the structure is initially unstressed and then experiences a temperature increase of 90 degrees, determine the stress that develops in aluminum member DE, and the movement of Point B. (Al. stress = 5470 psi.; Move B.= -.01")



Select:

[Topic 3. Stress, Strain & Hooke's - Table of Contents]

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Topic 3.84: Thermal Stress, Strain & Deformation - Topic Examination

The following values will be needed for the problems below

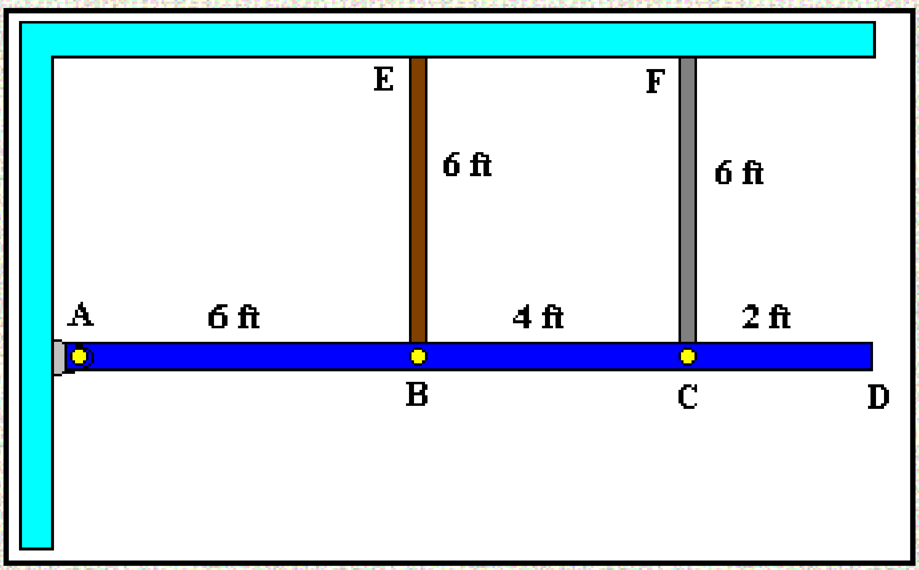
Linear coefficient of Expansion:

α_{Steel} **= 6.5 x 10^{-6} / °F;** $\alpha_{Aluminum}$ **= 13 x 10^{-6} / °F ;** α_{Brass} **= 11 x 10^{-6} / °F;** $\alpha_{concrete}$ **= 11 x 10^{-6} / °F**

Young's Modulus:

E_{Steel} = 30 x 10^{6} lb / in^2; $E_{Aluminum}$ = 10 x 10^{6} lb / in^2; E_{Brass} = 15 x 10^{6} lb / in^2; $E_{Concrete}$ = 5 x 10^{6} lb / in^2

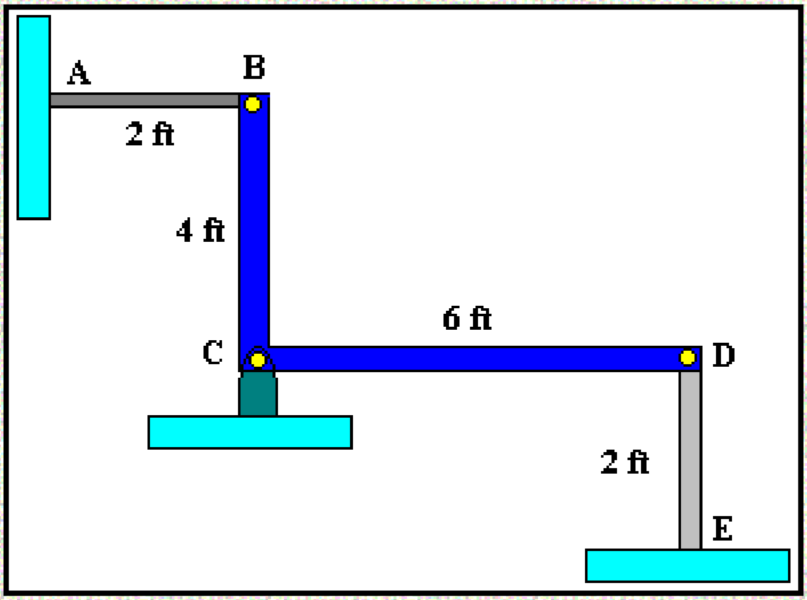
1. In the structure shown member ABCD is pinned to the wall at point A, and supported by a brass member, BE, and by a steel member, CF. Both BE and CF have cross sectional areas of .5 in^2. The structure is initially unstressed and then experiences a temperature increase of 50 degrees Celsius.
For this structure:



A. Draw a Free Body Diagram showing all support forces and loads.
B. Determine the axial stress that develops in brass member BE.
C. Determine the resulting movement of point D.
(Select **Solution Problem 1** for solution)

2. In the structure shown the L-shaped member BCD is supported by Steel rod AB and Aluminum member DE, and pinned at point C, as shown. Member DE has a cross sectional area of 1 in^2 and member AB has a cross sectional area of .5 in^2.

The structure is initially unstressed and then experiences a temperature decrease of 60 degrees Celsius.
For this structure:



A. Draw a Free Body Diagram showing all support forces and loads
B. Determine the axial stress that develops in steel rod AB.
C. Determine the resulting movement of point D.
(Select **Solution Problem 2** for solution)

or Select:

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Physics Department
University of Wisconsin-Stout

STRENGTH OF MATERIALS

[UW-Stout - Physics 372-325]

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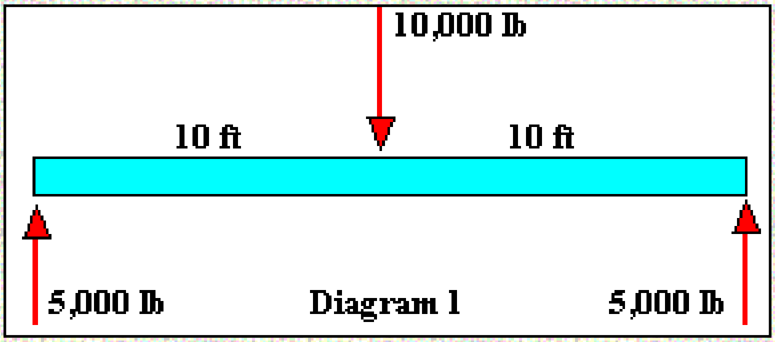
Topic 4: Beams I

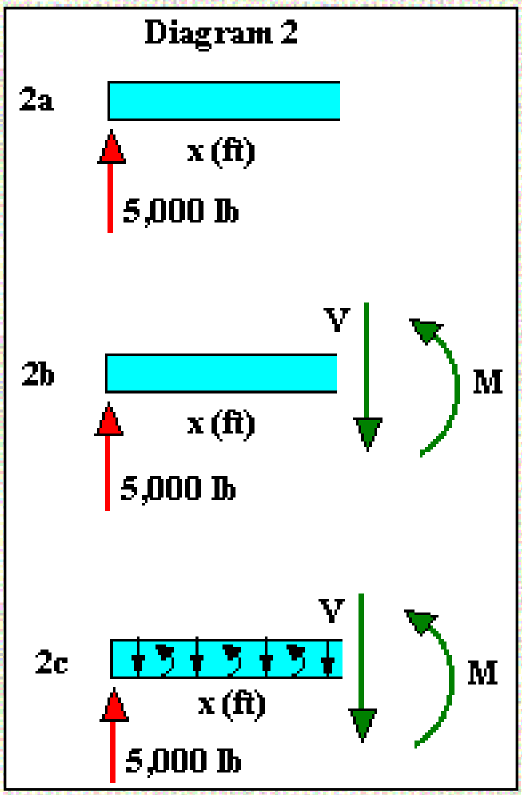
Topic 4.1: Shear Forces & Bending Moments I

We will now turn our attention to the forces and torque which develop in a loaded beam. Up to this point we have generally looked at only axial members - members in simple tension or compression; and have considered the forces, stresses, and deformations which occur in such members. We will now look at a particular type of non-axial member - loaded horizontal beams, and will begin the process of determining the forces, toque, stresses, and deformations which occur in these beams. And as we proceed on we will also consider the problem of beam design and/or beam selection. In this first topic, we will focus only on the **SHEAR FORCES** and **BENDING MOMENTS** (internal torque) which occur in loaded beams. These quantities are very important, as we shall see, since the axial and shear stresses which will develop in the beam depend on the values of the shear forces and bending moments in the beam.

To understand the shear forces and bending moments in a beam, we will look at a simple example. In Diagram 1, we have shown a simply supported 20 ft. beam with a load of 10,000 lb. acting downward right at the center of the beam. Due to symmetry the two support forces will be equal, with a value of 5000 lb. each. This is the static equilibrium condition for the whole beam.



Next let's examine a section of the beam. We will cut the beam a arbitrary distance (x) between 0 and 10 feet, and apply static equilibrium conditions to the left end section as shown in Diagram 2.. We can do this since as the entire beam is in static equilibrium, then a section of the beam must also be in equilibrium.



In Diagram 2a, we have shown left section of the beam, x feet, long - where x is an arbitrary distance greater than 0 ft. and less than 10 ft. Notice if we just include the 5000 lb. external support force, the section of the beam is clearly not in equilibrium. Neither the sum of forces (translational equilibrium), nor the sum of torque (rotational equilibrium) will sum to zero - as required for equilibrium. Therefore, since we know the **beam section is in equilibrium**, there must be some forces and/or torque not accounted for.

In diagram 2b, we have shown the missing force and torque. The 10,000 lb. load which we originally applied to the beam, and the support force cause internal "shearing forces" and internal torque called "bending moments" to develop. (We have symbolically shown these in Diagram 2c.) When we cut the beam, the internal shear force and bending moment at that point then become an external force and moment (torque) acting on the section. We have shown these in Diagram 2b, and labeled them **V (shear force)** and **M (bending moment)**.

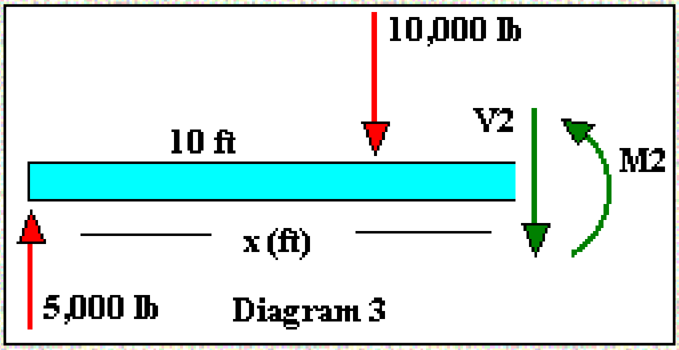
Please note that M is a moment or torque - not a force. It does not appear in the sum of forces equation when we apply static equilibrium to the section - which will be our next step.

Equilibrium Conditions:
Sum of Forces in y-direction: **+ 5000 lb. - V = 0** , solving **V = 5000 lb.**
Sum of Toque about left end: **-V * x + M = 0** , we next substitute the value of V from the force equation into the torque equation: **- 5000 lb. * x + M = 0** , then solving for **M = 5000x (ft-lb.)**

These are the equations for the shear force and bending moments for the section of the beam from 0 to 10 feet. Notice that the internal shear force is a constant value of 5000 lb. for the section, but that the value of the internal torque (bending moment) varies from 0 ft-lb. at x = 0, to a value of 50,000 ft-lb. at x = 10 ft.

[We really should not put exactly 0 ft., and 10 ft. into our equation for the bending moment. The reason is that at 0 and 10 ft., there are 'point loads/forces' acting. That is, we have our forces acting at point - and a point has zero area, so the stress (F/A) at these points would in theory be infinite. Of course, a stress can not be infinite, and we can not apply a force at a point - it is actually applied over some area (even if the area if small). However, in 'book' problems we normally apply forces at a point. To deal with this difficulty, we actually skip around these points. We cut our section at 0' < x < 6'. Still when we put values into our expressions we put in values such as x = 9.99999999 ft, and round it off (numerically) to 10 ft. This is, in effect, cheating a bit. We are putting in the value x = 10 ft., but only because the number we actually put in was rounded off to 10 ft. It all may sound confusing, but it works, and will become clear as we do several examples.]

First, however we will finish analyzing our simple beam. So far we have found expressions for the shear force and bending moments **(V1 = 5000 lb, M1 = 5000x ft-lb)** for section 1 of the beam, between 0 and 10 ft. Now we will look at the next section of the beam. We cut the beam at distance x (ft) from the left end, where x is now greater than 10 ft. and less then 20 ft. and then look at entire section to the left of where we cut the beam (See Diagram 3). Where the beam was cut, we have an internal shear force and bending moment - which now become external. These are shown in Diagram 3 as V2 and M2. (We add the '2', to indicate we are looking at section two of the beam.)



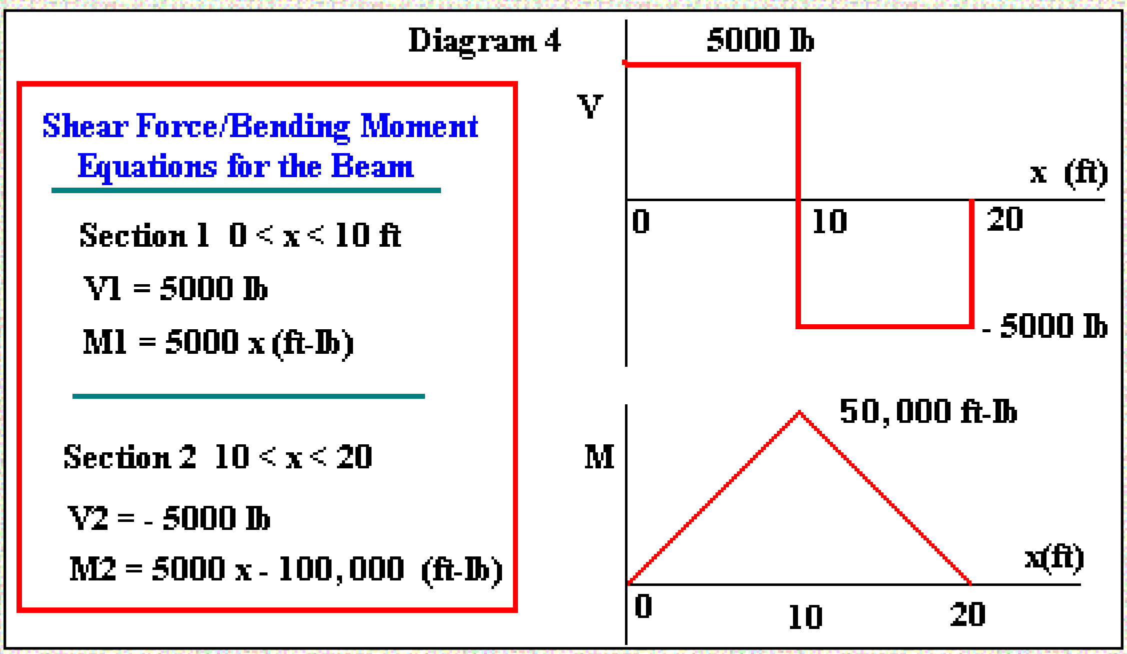
We next apply static equilibrium conditions to the beam section, and obtain:

Equilibrium Conditions:

Sum of Forces in y-direction: **+ 5000 lb. -10,000 lb. - V = 0** , solving **V2 = - 5000 lb.**

Sum of Toque about left end: **-10,000 lb * 10 (ft) -V * x (ft) + M = 0** , we next substitute the value of V from the force equation into the torque equation : - 10,000 lb * 10 ft. - (-5000 lb) * x (ft) + M = 0 , then solving for **M2 = -[5000x (ft-lb.) - 100,000] ft-lb.**

The two expressions above give the value of the internal shear force and bending moment in the beam, between the distances of the 10 ft. and 20 ft. A useful way to visualize this information is to make **Shear Force and Bending Moment Diagrams** - which are really the graphs of the shear force and bending moment expressions over the length of the beam. (See Diagram 4.)



These are a quite useful way of visualizing how the shear force and bending moments vary through out the beam. We have completed our first Shear Force/ Bending Moment Problem. We have determined the expressions for the shear forces and bending moments in the beam, and have made accompanying shear force and bending moment diagrams.

Now that we have the general concepts concerning shear forces and bending moments, we want to step back for a moment and become a little more specific concerning some details, such as choosing the direction of the shear forces and bending moments.

Continue to:
Topic 4.2: Beams - Shear Force and Bending Moments II
or Select:
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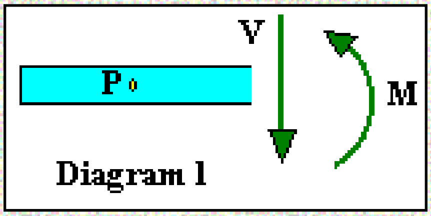
Topic 4.2: Shear Forces and Bending Moments II

Before continuing with a second example of determining the shear forces and bending moments in a loaded beam, we need to take a moment to discuss the sign associated with the shear force and bending moment. The signs associated with the shear force and bending moment are defined in a different manner than the signs associated with forces and moments in static equilibrium.

- **The Shear Force is positive if it tends to rotate the beam section clockwise with respect to a point inside the beam section.**
- **The Bending Moment is positive if it tends to bend the beam section concave facing upward. (Or if it tends to put the top of the beam into compression and the bottom of the beam into tension.)**

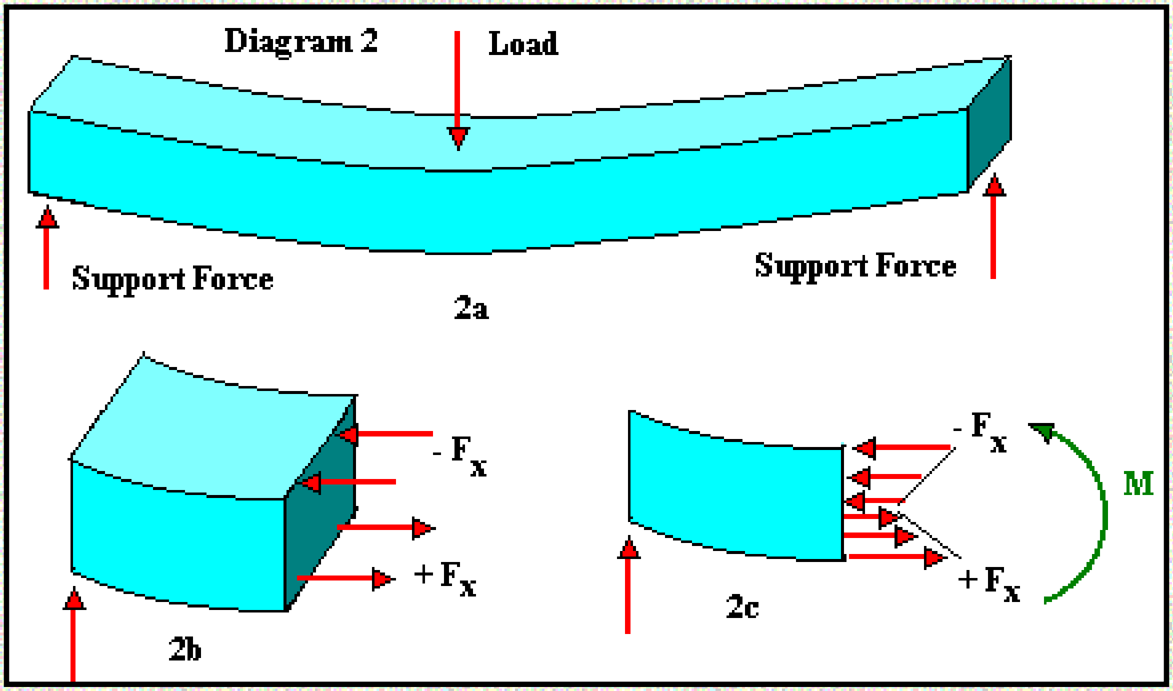
In the beam section shown in Diagram 1, we have shown the **Shear Force V** and **Bending Moment M** acting in positive directions according to the definitions above.



Notice that there is a possibility for a degree of confusion with sign notation. When summing forces, the direction of V shown in the diagram is in the negative y-direction, yet it is a positive shear force. This can lead to some confusion unless we are careful. We will deal with possible confusion by always working from the left for our beam sections, and always choosing V & M in a positive direction according to the shear force and bending moments conventions defined above. That is, we will always select the V & M directions as shown in Diagram 1. This approach will simplify the sign conventions, as we will see in the next example.

However before the next example, we will look at the causes of the internal bending moment in a little greater detail.

In Diagram 2a, we have shown a simply supported loaded beam, and have indicated in an exaggerated way the bending caused by the load. If we then cut the beam and look at a left end section, we have the Diagram 2b.



In this diagram we have, for the sake of clarity, left out the vertical shear force which develops, but have shown horizontal forces ($-F_x$ and $+ F_x$). These forces develop since, as the beam bends, the top region of the beam is put into compression and the bottom region of the beam is put into tension. As a result there are internal horizontal (x-direction) forces acting in the beam; however for every positive x-force, there is an equal and opposite negative x-force. **Thus the net horizontal (x-direction) internal force in the beam section is zero. However, even though the actual x-forces cancel each other, the torque produced by these x-forces is not zero**. Looking at Diagram 2c and mentally summing torque about the center of the beam, we see that the horizontal x-forces cause a net toque - which we call the internal bending moment, M. This is the cause of the internal bending moment (torque) inside a loaded beam.

We now continue by proceeding very slowly and carefully through a somewhat extended example(s). We will also examine an alternate method for determining the bending moments in a beam.
Please select: Example 1 ; Example 2 ; Example 3

Or:
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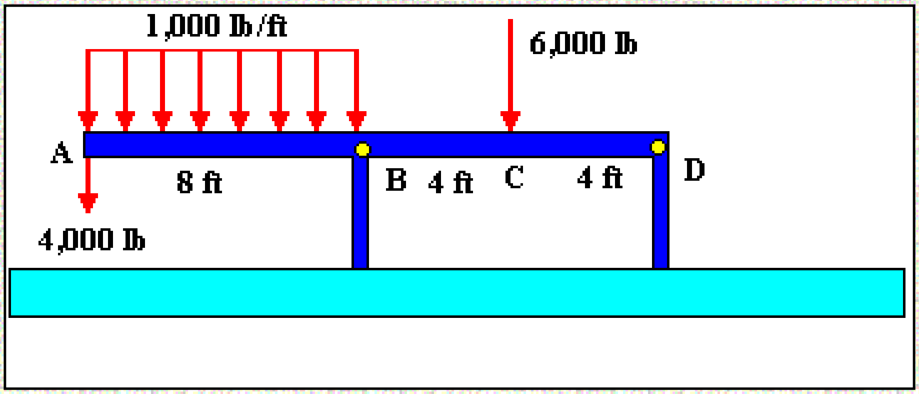
Topic 4.3a: Simply Supported Beam - Example 1

Example 1

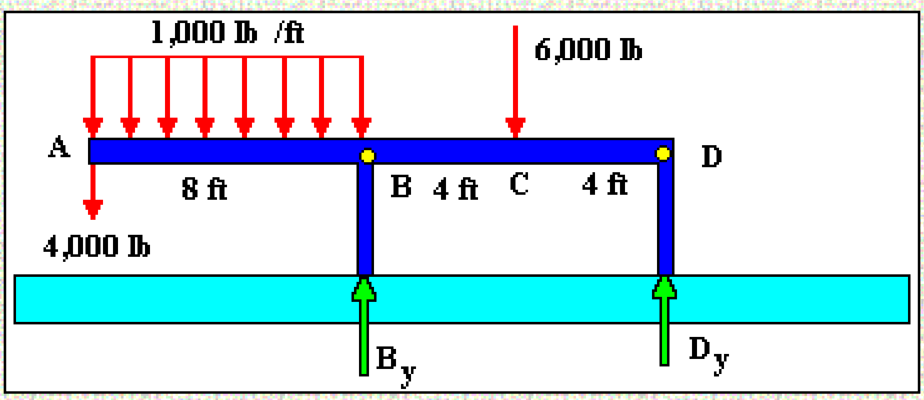
A loaded, simply supported beam is shown. For this beam we would like to determine expressions for the internal shear forces and bending moments in each section of the beam, and to make shear force and bending moment diagrams for the beam.



We will work very slowly and carefully, step by step, through the solution for this example.

Solution: Part A. We first find the support forces acting on the structure. We do this in the normal way, by applying static equilibrium conditions for the beam.

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.
Sum F_y = -4,000 lb. - (1,000 lb./ft)(8 ft) - 6,000 lb. + B_y + D_y = 0

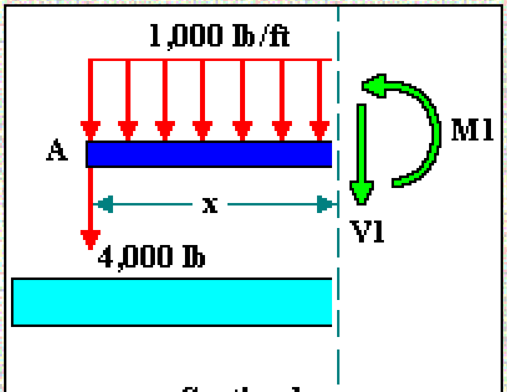
Sum T_B = (D_y)(8 ft) - (6,000 lb.)(4 ft) + (1,000 lb./ft)(8 ft)(4 ft) + (4,000 lb.)(8 ft) = 0

Solving for the unknowns: **B_y =23,000 lb.; D_y = -5,000 lb.** (The negative sign indicates that D_y acts the opposite of the initial direction we chose.)

Part B: Now we will determine the **Shear Force** and **Bending Moment** expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use the translational equilibrium condition for the beam section (Sum of Forces = zero) to determine the **Shear Force** expressions in each section. Determining the **Bending Moment** expression for each section of the beam may be done in two ways.

- **1) By applying the rotational equilibrium condition for the beam section (Sum of Torque = zero), and solving for the bending moment.**
- **2) By Integration. The value of the bending moment in the beam may be found from $M = \int V\, dx$. That is, the bending moment expression is the integral of the shear force expression for the beam section.**

We now continue with the example. We begin by starting at the left end of the beam, and cutting the beam a distance "x" from the left end - where x is a distance greater than zero and less the position where the loading of the beam changes in some way. In this problem we see that from zero to eight feet there is a uniformly distributed load of 1000 lb./ft. However this ends at eight feet (the loading changes). Thus for section 1, we will cut the beam at distance x from the left end, where x is greater than zero and less then eight feet.

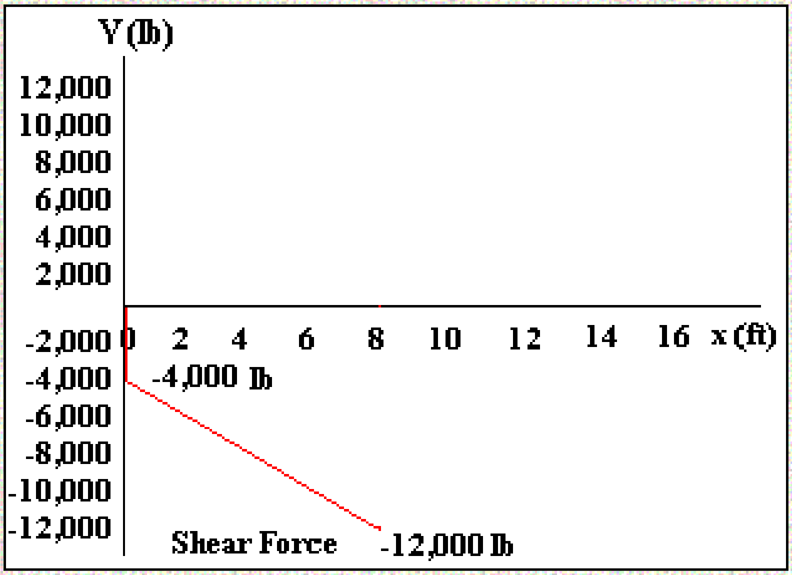


Section 1:
Cut beam between 0 and 8 ft

Section 1: Cut the beam at x, where 0 < x < 8 ft., and analyze left hand section.
1. Draw a FBD of the beam section shown and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) (See Diagram - Section 1) Notice we have drawn the shear force and bending moment in their positive directions according to the defined sign convention discussed earlier, and have labeled them as V1 and M1, as this is section 1 of the beam.
2. We check that we have all forces in x & y components (yes)
3. Apply translational equilibrium conditions to determine the shear force expression.
Sum Fx = 0 (no net external x- forces)
Sum Fy = -4,000 lb. - 1,000 lb./ft *(x) ft - V1 = 0 ; and solving: **V1 = [-4,000 -**

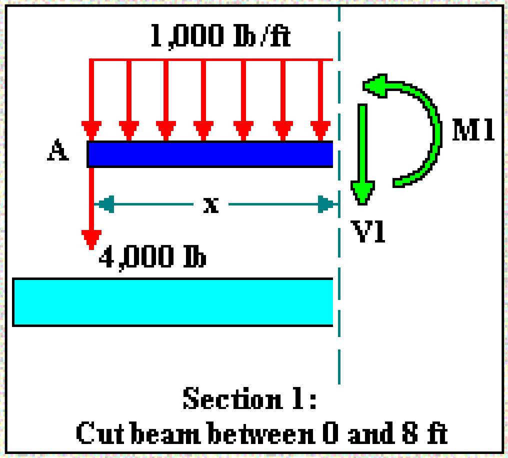
1,000x] lb.

This expression gives us the values of the internal shear force in the beam between 0 and 8 ft. Notice as x nears zero, the shear force value in the beam goes to - 4000 lb., and as x approaches 8 ft., the shear force value becomes -12,000 lb., and that is negative everywhere between 0 and 8 ft. Let's think for a moment what this negative sign tells us. Since we found the shear force (V) by static equilibrium conditions, the negative sign tells us that we choose the incorrect direction for the shear force - that the shear force acts in the opposite direction. However, we choose the positive direction of the shear force (by its definition) and so the negative sign also tells us we have a negative shear force.
To try to simplify a somewhat confusing sign situation we may say this: As long as we work from the left end of the beam, and choose the initial direction of the shear force and bending moment in the positive direction (by their definition), then when we solve for the shear force and bending moment, the sign which results is the correct sign as applies to the shear force and bending moment values.



If we graph the shear force expression above, we obtain the graph shown of the internal shearing force in the beam for the first eight feet. We next will determine the bending moment expression for this first beam section.

4. We can find the bending moment from static equilibrium principles; summing torque about the left end of the beam.

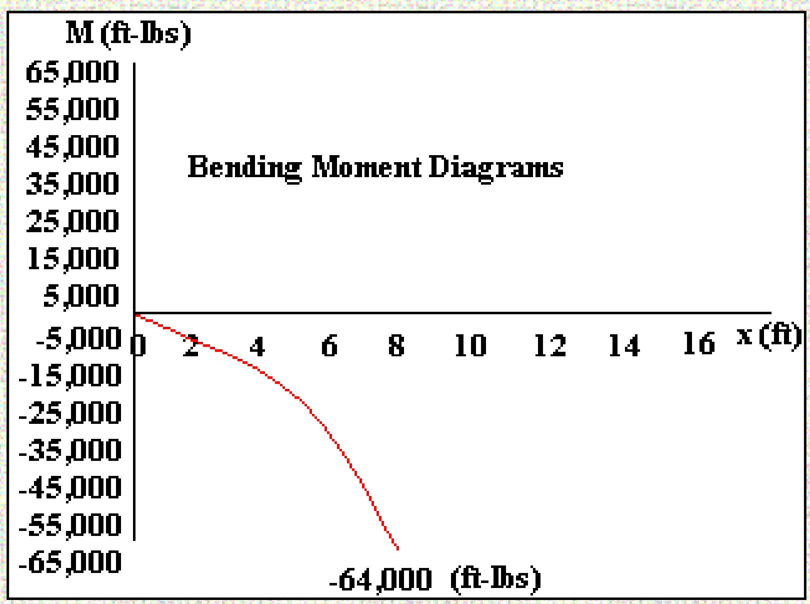


Referring to the free body diagram for beam section 1, we can write:
Sum Torque $_{\text{left end}}$ = **-1000 lb/ft * (x) * (x/2) - V1 (x) + M1 = 0**

To make sure we understand this equation, let's examine each term. The first term is the torque due to the uniformly distributed load - 1000 lb./ft * (x) ft (this is the load) times (x/2) which is the perpendicular distance, since the uniform load may be considered to act in the center, which is x/2 from the left end. Then we have the shear force V1 times x feet to the left end, and finally we have the bending moment M1 (which needs no distance since it is already a torque).
Next we substitute the expression for V1 (V1 = [-4,000 - 1,000x] lb.) from our sum of forces result above into the torque equation to get:
Sum Torque $_{\text{left end}}$ = -1000 lb/ft * (x) * (x/2) -[-4,000 - 1,000x] (x) + M1 = 0 ;

and solving for **M1 = [-500x^2 - 4,000x] ft-lb.**

This is our expression for the internal torque inside the load beam for section 1, the first eight feet, which is graphed in the diagram below.



5. Finally, we may also obtain the expression for the bending moment by integration of the shear force expression. The integrals we will be using are basic types.

For a simple, brief review and/or introduction to basic calculus concepts,
Please Select: Simple Derivatives/Integrals.

Continuing with our example:
Integration:
$M1 = \int V1\,dx = \int[-1{,}000x - 4{,}000]dx = \int 1{,}000x\,dx - \int 4{,}000dx = 1000\int x\,dx - 4000\int dx$ = 1000(1/2 x^2) - 4000 (x) + C1; so **M1 = -500x^2 - 4,000x + C1**

As we the results above show, when we do an indefinite integral, the result include an arbitrary constant, in this case called C1. To determine the correct value for C1 for our problem we must apply a boundary condition: That is, we must know the value of the bending moment at some point on our interval into to find the constant.

For simply supported beams (with no external torque applied to the beam) the value of the bending moment will be zero at the ends of the beam.

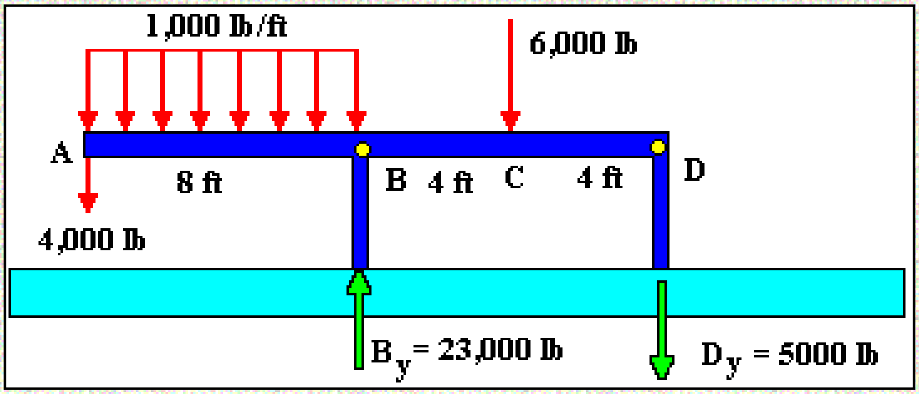
(There are many ways to explain why this must be so. One of the easiest explanations is to remember that the bending moment value at a point in a simply supported beam is equal to the total area under the shear force diagram up to that point. However, at the left end, as x goes to zero, the area under the shear force diagram would also go to zero, and thus so would the bending moment value.)

So we have for our "boundary condition" that at **x = 0, M1 = 0**. We put these values into our expression for the bending moment (M1 = -500x^2 - 4,000x + C1), and solve for the value of the integration constant, C1; that is:

0 = -500$(0)^2$ - 4,000(0) + C1, and solving: C1 = 0

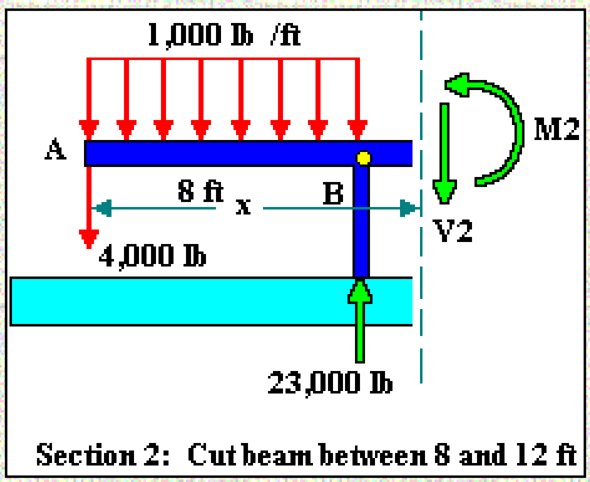
Therefore: **M1 = [-500x^2 - 4,000x] ft-lb.** for 0 < x < 8 ft., is our final expression for the bending moment over the first section. (Note, it is the same as found above by summing torque for the beam section.)

We now continue with the next section of the beam. Referring to the beam diagram, we see that at a location just greater than 8 ft., there is no loading, and that this continues until 12 ft. where there is a point load of 6,000 lb. So for our second section, we cut the beam at a location "x", where x is greater than 8 ft., and less than 12 ft - and then analyze the entire left hand section of the beam.



Section 2: We cut the beam at x, where 8 < x <12 ft., and analyze the entire section left of where we cut the beam.

1. Draw a FBD of the beam section shown and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) (See Diagram - Section 2) Notice we have drawn the shear force and bending moment in their positive directions according to the defined sign convention discussed earlier, and have labeled them as V2 and M2, as this is section 2 of the beam.



Section 2: Cut beam between 8 and 12 ft

2. We check that we have all forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):
Sum Fx = 0 (no net external x- forces)
Sum Fy = -4,000 lb. - 1,000 lb./ft(8 ft) + 23,000 lb. - V2 = 0, Solving: **V2 = 11,000 lb.**

4. We may determine the bending moment expression by applying rotational equilibrium conditions, or by integration. Once more we will do it both ways for this section.
Rotational Equilibrium:
Sum of Toque $_{\text{left end}}$ = - (1000 lb./ft * 8 ft) * 4 ft. + 23,000 lb. * 8 ft. - V2 * x + M2 = 0; then we substitute the value for **V2 (V2 = 11,000 lb)** from above and obtain: **- (1000 lb./ft * 8 ft) * 4 ft. + 23,000 lb. * 8 ft. - (11,000 lb.) * x + M2 = 0**; and then solving for M2 we find: **M2 = [11,000x - 152,000] ft-lb.**

From integration of the shear force, we find:

Integration: $M2 = \int V2\, dx = \int 11{,}000\, dx$, or **M2 = 11,000x + C2**

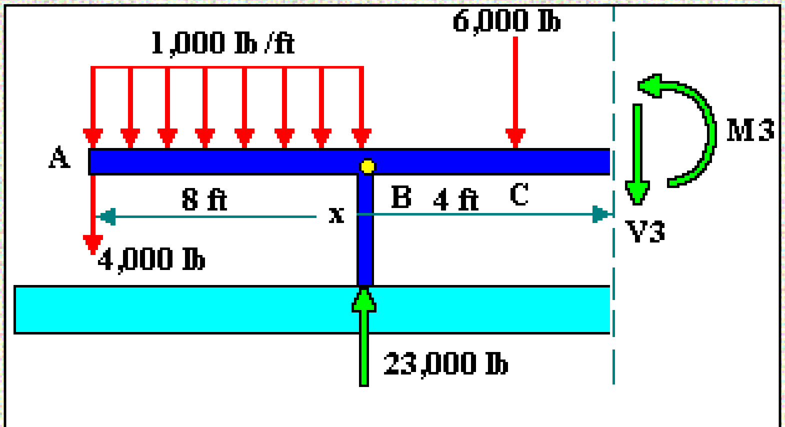
We get our boundary condition from another characteristic of the bending moment expression - which is that the bending moment must be continuous. That is, the value of the bending moment at the end of the first beam section, and the value of the bending moment at the beginning of the second beam section must agree - they must be equal. We determine the value of the bending moment from our M1 equation as x approaches 8 ft. (M1 = [-500 $(8)^2$ - 4,000(8)] = -64,000 ft-lb.)

Then our boundary condition to find C2 is: at **x=8 ft M=-64,000 ft-lb.** We apply our boundary condition to find C2.
Apply BC: -64,000 ft-lb. = 11,000 lb. (8) + C2, Solving: **C2 = -152,000 ft-lb**.

Therefore: **M2 = [11,000x - 152,000] ft-lb.** for 8 < x < 12

In like manner we proceed with section 3 of the beam, cutting the beam at a location greater than 12 ft. and less 16 ft., and then analyzing the entire section left of where we cut the beam.

Section 3: Cut the beam at x, where 12 < x < 16 ft. Analyze left hand section.
1. FBD. (See Diagram Section 3)



Section 3: Cut beam between 12 and 16 ft.

2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):
Sum Fx = 0 (no net external x- forces)Sum Fy = -4,000 lb. - 1,000 lb./ft(8 ft) + 23,000 lb. -6,000 lb. - V3 = 0, and Solving:
V3 = 5,000 lb.

4. We may determine the bending moment expression by applying rotational equilibrium conditions, or by integration. Once more we will do it both ways for this section.
Rotational Equilibrium:
Sum of Toque $_{\text{left end}}$ = - (1000 lb./ft * 8 ft) * 4 ft. + 23,000 lb. * 8 ft. - 6,000 lb.

*** 12 ft -V3 * x + M3 = 0**; then we substitute the value for **V3 (V3 = 5,000 lb)** from above and obtain:
- (1000 lb./ft * 8 ft) * 4 ft. + 23,000 lb. * 8 ft. - 6,000 lb. * 12 ft - (5,000 lb.) * x + M3 = 0; and then solving for M3 we find: **M3 = [5,000x - 80,000] ft-lb.**

We will find the bending moment expression for this section using integration only.

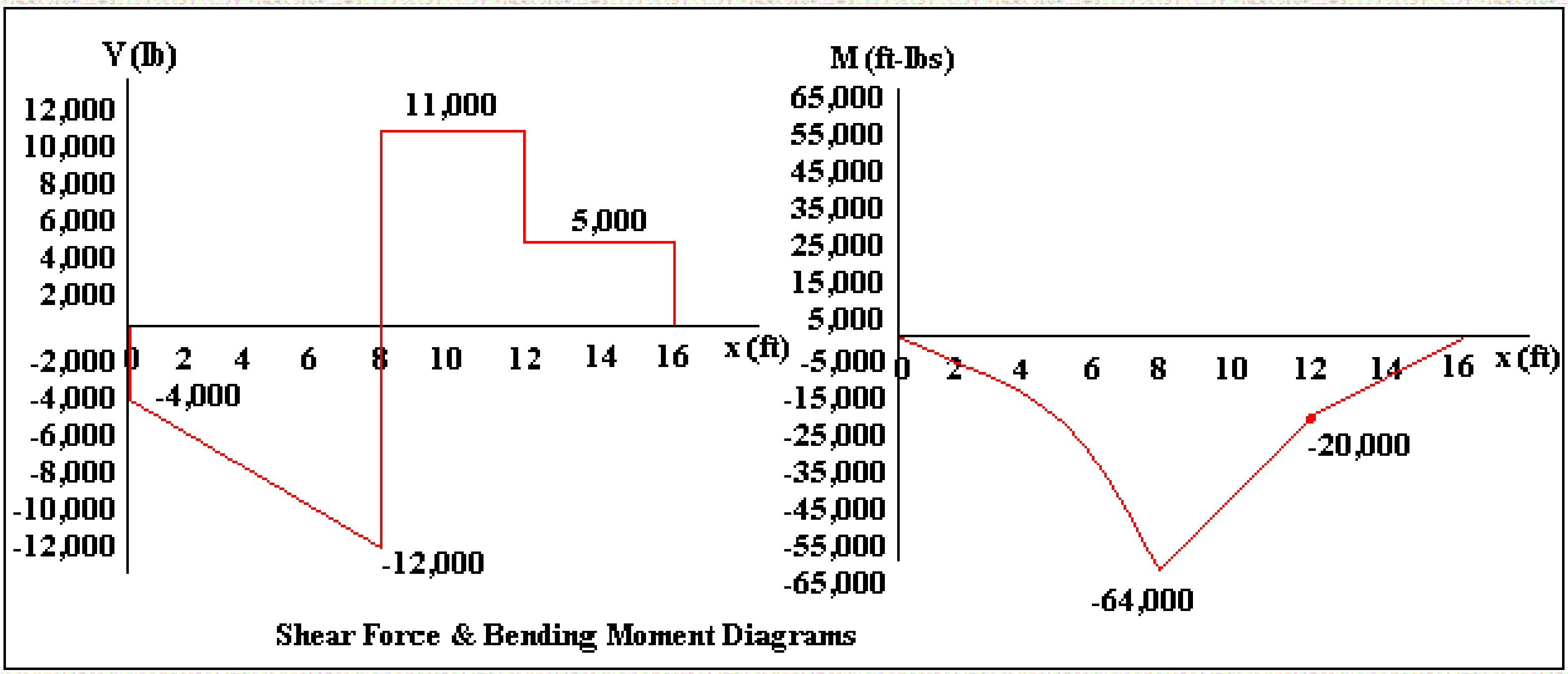
Integration $M3 = \int V3\,dx = \int 5{,}000\,dx$, or **M3 = 5,000x + C3**

We obtain a boundary condition for section 3 by remembering that at a free end or simply supported (no external torque) end, the bending moment must go to zero, thus we have the boundary condition to find C3: **at x = 16 ft., M = 0 ft-lb**.
Apply BC: 0 = 5,000(16) + C3, and Solving: C3 = -80,000 ft-lb.
Therefore: **M3 = [5,000x - 80,000] ft-lb.** for 12 < x < 16

We now have our expressions for the shear forces and bending moments in each section of the loaded beam (summarized below). Additional, we have shown the shear force and bending moment diagrams for the entire beam - which is a visual representation of the internal shear forces and internal torque in the beam due to the loading.

Part C: Shear Force and Bending Moment Diagrams: Using the expressions found above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = -1,000x+4,000 lb.; **V2 = 11,000** lb.; **V3 = 5,000** lb
M1 = $\mathbf{-500x^2+4{,}000x}$ ft-lb.; **M2 = 11,000x -152,000** ft-lb.; **M3 = 5,000x-80,000** ft-lb



Shear Force & Bending Moment Diagrams

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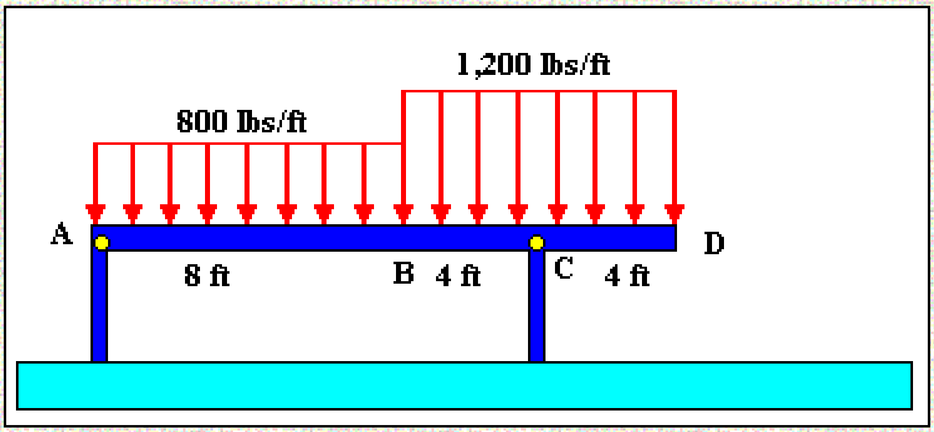
Topic 4.3b: Simply Supported Beam - Example 2

Example 2

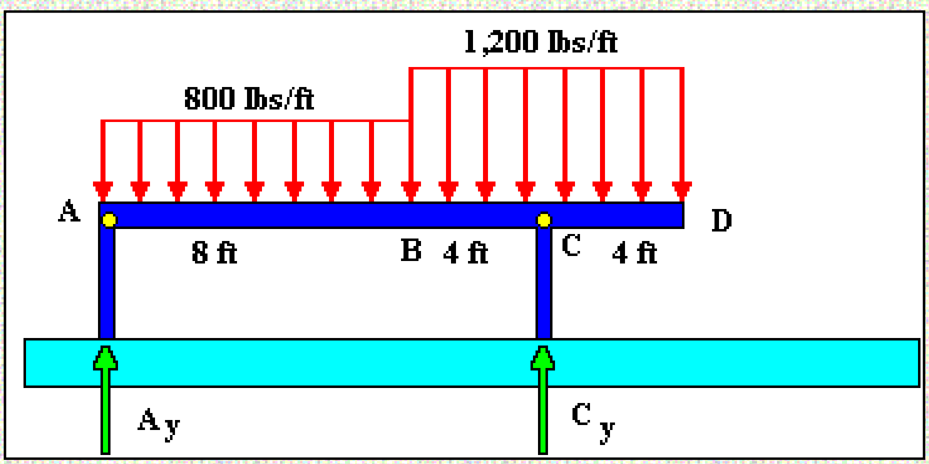
For the loaded, simply supported beam shown, determine expressions for the internal shear forces and bending moments in each section of the beam, and draw shear force and bending moment diagrams for the beam.



Solution: We first need to determine the external support reaction by applying our standard static equilibrium conditions and procedure..

PART A

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



STEP 2: Resolve any forces not already in x and y direction into their x and y components.

STEP 3: Apply the equilibrium conditions.

Sum F_x = 0 (no external x-forces acting on structure.)

Sum F_y = (-800 lbs/ft)(8 ft) - (1,200 lbs/ft)(8 ft) + A_y + C_y = 0

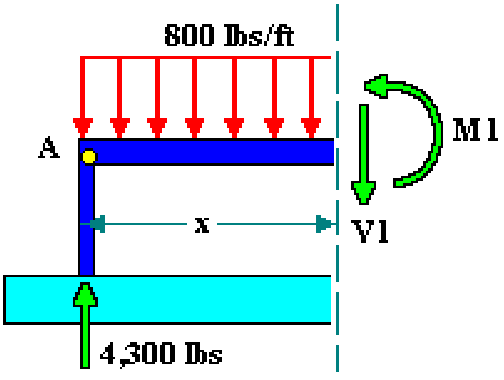
Sum T_A = (C_y)(12 ft) - (800 lbs/ft)(8 ft)(4 ft) - (1,200 lbs/ft)(8 ft)(12 ft) = 0

Solving for the unknowns: C_y = 11,700 lbs; A_y = 4,270 lbs

Part B: Determine the Shear Force and Bending Moment expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: We note that the loading of the beam (800 lb./ft) remains uniform until 8 feet, where it changes to 1200 lb./ft. As a result, for our first beam section, we cut the beam at an arbitary position x, where 0 < x < 8 ft. Then we analyze the left hand beam section.

1. Draw a FBD of the beam section showing and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) (See Diagram - Section 1) Notice we have drawn the shear force and bending moment in their positive directions according to the defined sign convention discussed earlier, and have labeled them as V1 and M1, as this is section 1 of the beam.



Section 1:
Cut beam between 0 and 8 ft.

2. Resolve all forces into x & y components (yes)

3. Apply translational equilibrium conditions (forces only) to the section 1 of the beam:
Sum Fx = 0 (no net external x- forces)
Sum Fy = 4,300 lbs - 800 lbs/ft(x)ft - V1 = 0;
Solving for the shear force: V1 = [4,300 - 800x] lbs

4. We can find the bending moment from static equilibrium principles; summing torque about the left end of the beam. Referring to the free body diagram for beam section 1, we can write:

Sum Torque $_{\text{left end}}$ = **-800 lb/ft * (x) * (x/2) - V1 (x) + M1 = 0**

To make sure we understand this equation, let's examine each term. The first term is the torque due to the uniformly distributed load - 800 lb./ft * (x) ft (this is the load) times (x/2) which is the perpendicular distance, since the uniform load may be considered to act in the center, which is x/2 from the left end. Then we have the shear force V1 times x feet to the left end, and finally we have the bending moment M1 (which needs no distance since it is already a torque).

Next we substitute the expression for V1 (V1 = [-4,300 - 800x] lb.) from our sum of forces result above into the torque equation to get:

Sum Torque $_{\text{left end}}$ = -1000 lb/ft * (x) * (x/2) -[-4,300 - 800x] (x) + M1 = 0 ;

and solving for **M1 = [$-400x^2$ + 4,300x] ft-lbs** for 0 < x < 8 ft.

We will now also find the bending moment expression by integration of the shear force equation. Integration $M1 = \int V1\,dx = \int [-800x + 4{,}300]\,dx$, solving **M1 = $-400x^2$ + 4,300x + C1**

Our boundary condition to find the integration constant, C1, is **at x = 0, M = 0** (since this is a simply support beam end.)

Applying the boundary condition: **0 = $-400(0)^2$ + 4,300(0) + C1**, and solving gives us: **C1 = 0**.

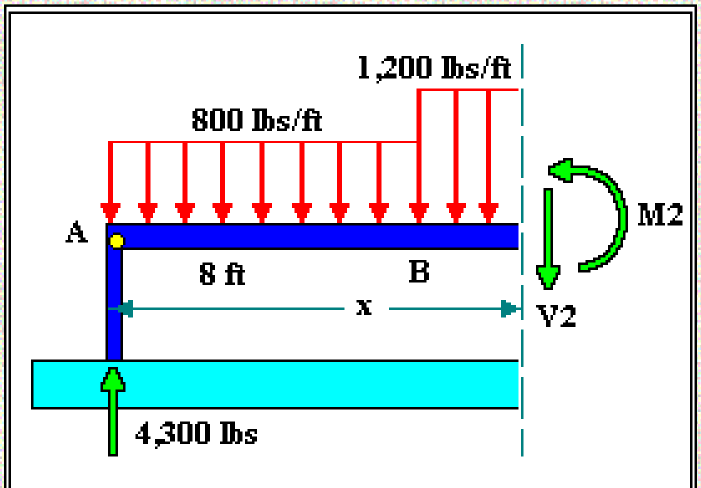
Therefore the bending moment expression for section 1 of the beam is:

M1 = [$-400x^2$ + 4,300x] ft-lbs for 0 < x < 8 ft.

(The shear force and bending moment diagrams are shown at the bottom of this example page.)

Section 2: We continue in the same manner with beam section 2. We note that the loading changes once more at 12 ft, due to the upward support force acting at that point. So for beam section 2, we cut the beam at location x, where 8 < x <12 ft., and then analyze left hand beam section from x to the end of the beam.

1. Draw a FBD of the beam section showing and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) We have labeled them as V2 and M2, as this is section 2 of the beam.



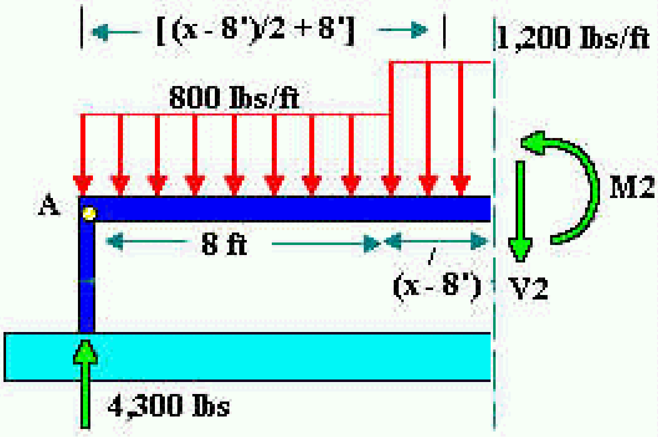
Section 2: Cut beam between 8 and 12 ft.

2. Resolve all forces into x & y components (yes).

3. Apply translational equilibrium conditions (forces only):
Sum Fx = 0 (no net external x- forces)
Sum Fy = 4,300 lbs - 800 lbs/ft*(8 ft) - 1,200lbs/ft *(x - 8)ft - V2 = 0; and solving for the shear force expression: V2 = [7,500 - 1,200x] lbs

4. We may determine the bending moment expression by applying rotational equilibrium conditions, or by integration. We will do it both ways for this section.
Rotational Equilibrium:
Sum of Toque $_{\text{left end}}$ = - (800 lb./ft * 8 ft) * 4 ft. -1200 lb/ft * (x - 8')*[(x-8')/2 +8'] - V2 * x + M2 = 0;

Please notice the second term. In that term the quantity (1200 * (x-8')) is load due to the 1200 lb/ft acting over the distance (x-8). However we still need to multiply the force expression times the distance to obtain the torque. The uniform load of 1200 lb/ft acts at the center of its distance (x-8'), so the lever arm to point A would be [(x-8')/2 + 8'] (See diagram.)



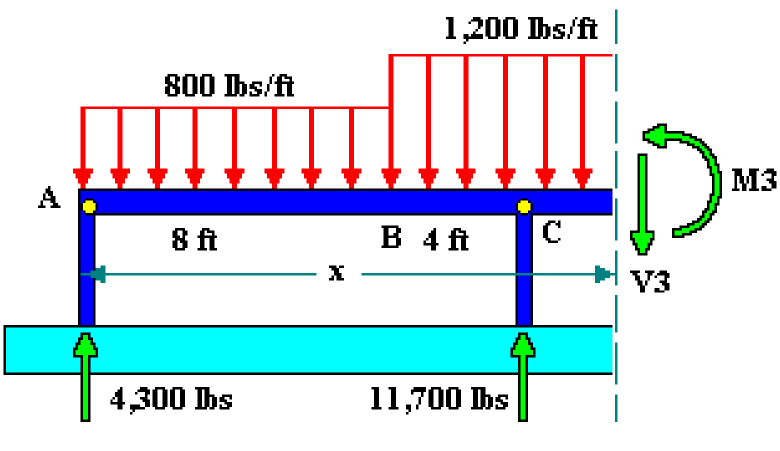
Section 2: Cut beam between 8 and 12 ft.

We next substitute the value for **V2** (**V2 = [7,500 - 1,200x] lb.**) from above and obtain:
- (800 lb./ft * 8 ft) * 4 ft. -1200 * (x - 8')*[(x-8')/2+8'] - [7,500 - 1,200x]* x + M2 = 0;
and then solving for M2 we find:
M2 = [-600x^2 + 7,500x - 12,800] ft-lbs for 8 < x < 12

Next we find the bending moment, M2, from integration of shear force expression,V2.
Integration: $M2 = \int V2\, dx = \int [-1{,}200x + 7{,}500]\, dx$, and solving, **M2 = -600x^2 + 7,500x + C2**
We obtain our boundary condition for beam section 2 by remembering that the bending moment must be continuous along the beam. This means that value of the bending moment at the end of section 1 (at x = 8 ft.) must also be the value of the bending moment at the beginning of section 2 (at x = 8 ft.). Thus our boundary condition to find C2 is: **at x = 8 ft M = 8,560 ft-lbs** (from equation M1). Now applying the boundary condition and solving for the integration constant, C2, we have:
8560 ft-lbs = -600(8)^2 +7500(8) + C2, and solving: **C2 = -13,000 ft-lbs**
Therefore our bending moment expression is:
M2 = [-600x^2 + 7,500x - 12,800] ft-lbs for 8 < x < 12
(The shear force and bending moment diagrams are shown at the bottom of this example page.)

Section 3: Finally, we continue with the last section of the beam, cutting the beam at location x, where 12 < x < 16 ft., and analyzing the left hand beam section from x to the left end of the beams.

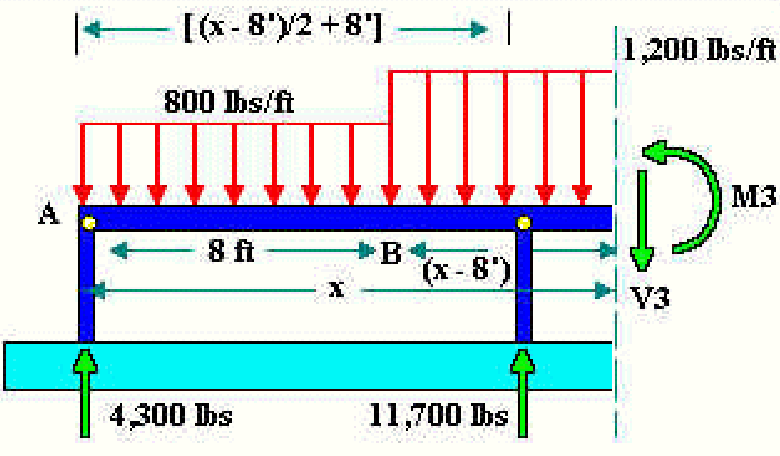
1. Draw a FBD of the beam section showing and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) We have labeled them as V3 and M3, as this is section 3 of the beam.



Section 3: Cut beam between 12 and 16 ft.

2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):
Sum Fx = 0 (no net external x- forces)
Sum Fy = 4,300 lbs - 800 lbs/ft(8 ft) + 11,700 lbs - 1,200lbs/ft (x - 8)ft - V3 = 0
Solving for the shear force expressing: V3 = [-1,200x + 19,200] lbs

4. We may determine the bending moment expression by applying rotational equilibrium conditions, or by integration. Once more we will do it both ways for this section.
Rotational Equilibrium:
Sum of Toque $_{\text{left end}}$ = - (800 lb./ft * 8 ft) * 4 ft. - 1200 lb. *(x- 8 ft.)*[(x-8')/2 +8'] + 11,700 lb. * 12 ft -V3 * x + M3 = 0;



Section 3: Cut beam between 12 and 16 ft.

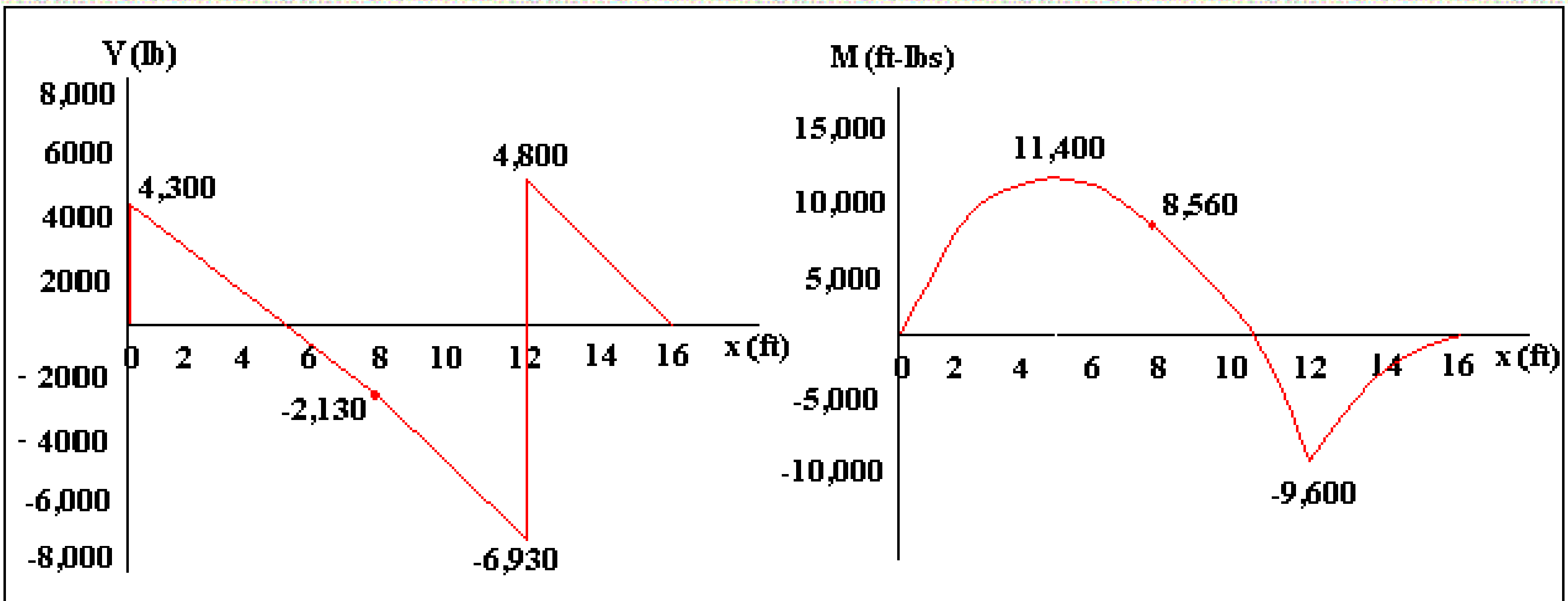
Once again the distance from end A at which the effective load due to the uniform load of 1200 lb/ft **[1200 lb/ft * (x-8')]** may be considered to act must be determined carefully. That distance **[(x-8')/2 + 8')]** is shown at the top of the adjacent diagram.

We next substitute the value for **V3 (V3 = [-1,200x + 19,200] lb.)** from above and obtain:
- (800 lb./ft * 8 ft) * 4 ft. - 1200 lb. *(x- 8 ft.)*[(x-8')/2 +8'] + 11,700 lb. * 12 ft - [-1,200x + 19,200] lb* x + M3 = 0; and then solving for M3 we find: **M3 = [-600x^2 + 19,200x - 153,000] ft-lbs** for 12 < x < 16**.**

Determine the bending moment by Integration: $M3 = \int V3\,dx = \int[-1{,}200x + 19{,}200]dx$,
or, **M3 = -600x^2 + 19,200x + C3**

We find our boundary condition for beam section 3, by realizing at the end of the beam (a free end) the bending moment must go to zero, so our boundary condition to find C3 is: at **x = 16 ft M = 0 ft-lb.**
Appling the boundary condition, and solving for the integration constant C3, we have:
BC: 0 = -600(16)^2 + 19,200(16) + C3; and then **C3 = -153,000 ft-lbs**
So the final expressionn for the bending moment on section 3 will be:
M3 = [-600x^2 + 19,200x - 153,000] ft-lbs for 12 < x < 16

PART C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.
V1 = -800x + 4,300 lb.; **V2 = -1,200x + 7,500** lb.; **V3 = -1,200x + 19,200** lb
M1 = -400x^2 + 4,300x ft-lb.; **M2 = -600x^2 + 7,500 - 12,800** ft-lb.; **M3 = -600x^2 + 19,200x - 153,000**ft-lb



Shear Force & Bending Moment Diagrams

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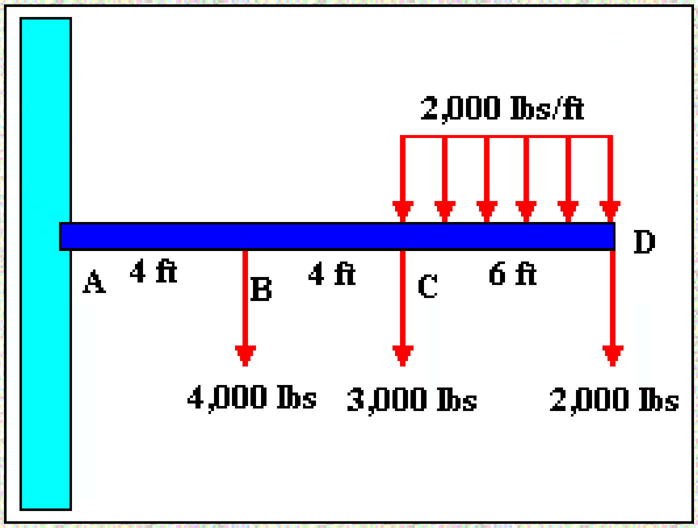
Topic 4.3c: Cantilever Beam - Example 3

Example 3

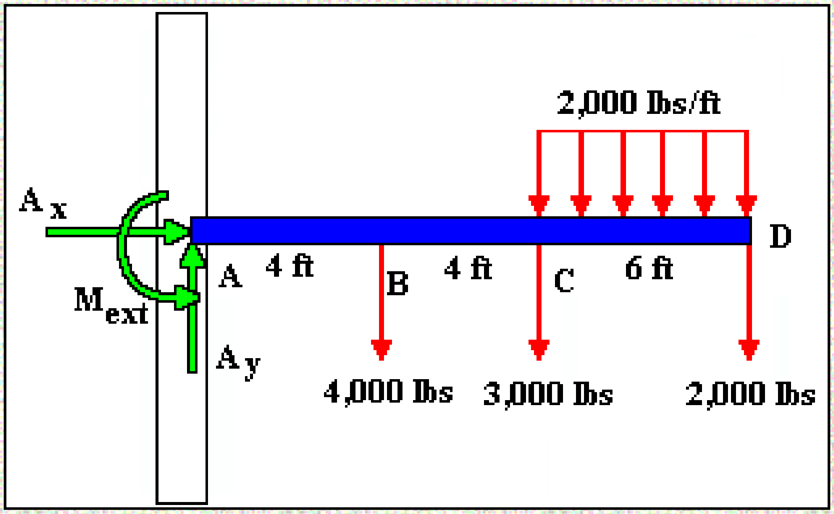
In this example we have a loaded, cantilever beam, as shown . For this beam we would like to determine expressions for the internal shear forces and bending moments in each section of the beam, and to draw the shear force and bending moment diagrams for the beam.



Solution:

Part A: Our first step will be to determine the support reactions and external torque acting on the loaded beam. For a cantilevered beam (with one end embedded or rigidly fixed at the wall), the wall may exert horizontal and vertical forces and an external torque (which we will call an external moment, and label $\mathbf{M_{ext}}$) acting on the beam - as we have shown in the free body diagram of the beam.

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



STEP 2: Break any forces not already in x and y direction into their x and y components.

STEP 3: Apply the static equilibrium conditions.

Sum $F_x = A_x = 0$

Sum F_y = -4,000 lbs - 3,000 lbs - (2,000 lbs/ft)(6 ft) - 2,000 lbs + A_y = 0

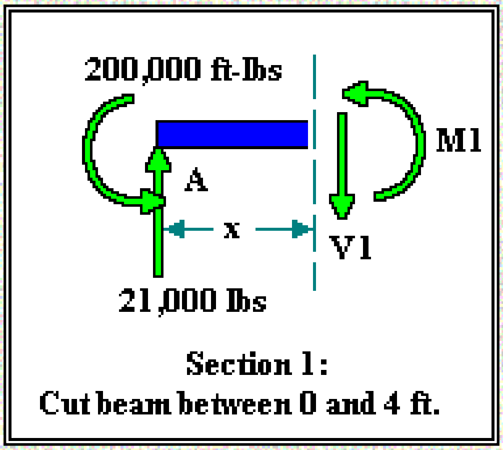
Sum T_A = (-4,000 lbs)(4 ft) - (3,000 lbs)(8 ft) -(2,000 lbs/ft)(6 ft)(11 ft) - (2,000 lbs)(14 ft) +M_{ext} = 0

Solving for the unknowns: A_y = 21,000 lbs; M_{ext} = 200,000 ft-lbs

Part B: Determine the Shear Force and Bending Moment expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use static equilibrium condition - sum of forces to determine the shear force expressions, and Integration to determine the bending moment expressions in each section of the beam.

Section 1: Cut the beam at an arbitrary location x, where 0 < x < 4 ft. (since at 4 ft. the beam loading changes.), and analyze left hand beam section, from x to the left end of the beam.

1. Draw a FBD of the beam section showing and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) (See Diagram - Section 1) Notice we have drawn the shear force and bending moment in their positive directions according to the defined sign convention for the shear force and bending moment discussed earlier, and have labeled them as V1 and M1, as this is section 1 of the beam.



2. Resolve all forces into x & y components (yes)

3. Apply translational equilibrium conditions (forces only) to determine the shear force expression:
Sum Fx = 0 (no net external x- forces)
Sum Fy = 21,000 lbs - V1 = 0 ; Solving for shear force: **V1 = 21,000 lbs**

4. We now find the bending moment expression by summing torque about the left end. That equation would be as follows:
Sum of Torque left end = -V1 * x + 200,000 ft-lb + M1 = 0;
when we substitute in the value for **V1 = 21,000 lb**. we obtain the equation:
Sum of Torque left end = -21,000 lb * x + 200,000 ft-lb + M1 = 0.
And solving for **M1 = 21,000 x -200,000 ft-lb.**
[This is the same result we will find by integration. In general, particularly for non-point loads, integration is the faster method.]

Now we find the bending moment equation by integration of the shear force expression.
Integration $M1 = \int V1\, dx = \int 21{,}000\, dx$, and then **M1 = 21,000x + C1**

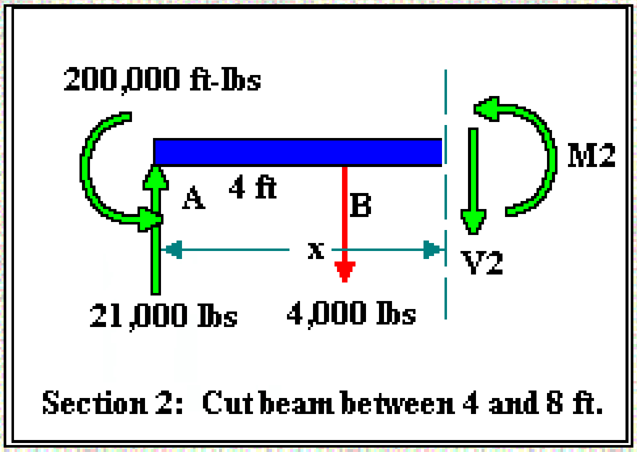
The boundary condition, which we will use to determine the integration constant C1, is different for the cantilevered end of a beam as compared with the end of a simply supported beam (or a free end). Since we have an external moment acting at the end of the cantilevered beam, **the value of the bending moment must become equal to the negative of the external moment at the cantilevered end of the beam, in order for the beam to be in rotational equilibrium** (sum of torque = 0). Thus our boundary condition to determine C1 is: **at x = 0, M = -200,000 ft-lb** (That is, for a cantilever beam, the value of the bending moment at the wall is equal to the negative of the external moment.)

Applying the boundary condition and solving for the integration constant:
-200,000 = 21000(0) + C1; and so C1 = -200,000

Then the bending moment expression is: **M1 = [21,000x - 200,000] ft-lb** for 0 < x < 4 ft.

Section 2: Since the loading changes at 8 ft, due to the point load and the beginning of the uniformly distributed load, for section 2 we cut the beam at location x, where 4 < x < 8 ft.; and analyze the left hand beam section.

1. Draw a FBD of the beam section showing and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) We label them as V2 and M2, as this is section 2 of the beam.



Section 2: Cut beam between 4 and 8 ft.

2. Resolve all forces into x & y components (yes).

3. Apply translational equilibrium conditions (forces only):
Sum Fx = 0 (no net external x- forces)
Sum Fy = 21,000 lbs - 4,000 lbs - V2 = 0 ; and solving: **V2 = 17,000 lb**

4. We now find the bending moment expression by summing torque about the left end. That equation would be as follows:
Sum of Torque left end = -4000 lb.* 4 ft. -V2 * x + 200,000 ft-lb + M2 = 0;
when we substitute in the value for **V2 = 17,000 lb**. we obtain the equation:
Sum of Torque left end = -4000 lb.* 4 ft. -17,000 lb. * x + 200,000 ft-lb + M2 = 0
And solving for **M2 = [17,000x - 184,000] ft-lb** for 4 < x < 8 ft**.**

Next we also determine the bending moment expression by integration of the shear force equation from above.
Integration: $M2 = \int V2\,dx = \int 17{,}000\,dx$, and so **M2 = 17,000x + C2**

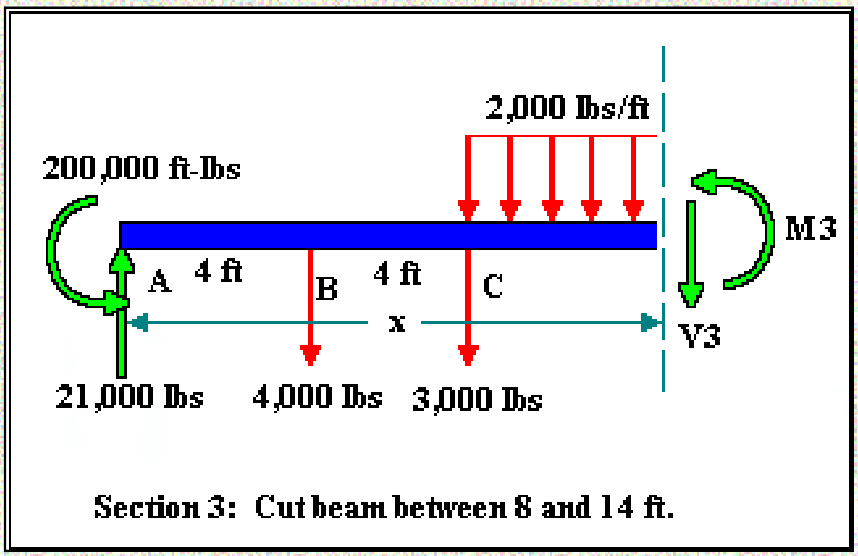
We obtain our boundary condition for beam section 2 by remembering that the bending moment must be continuous along the beam. This means that value of the bending moment at the end of section 1 (at x = 4 ft.) must also be the value of the

bending moment at the beginning of section 2 (at x = 4 ft.). Thus our boundary condition to find the integration constant C2 is: at **x = 4 ft., M = -116,000 ft-lb**. (from equation M1).
Now applying the boundary condition and solving for the integration constant, C2, we have:
-116,000 ft-lbs = 17,000(4) + C2, and so **C2 = -184,000 ft-lb**., and our expression for the bending moment on beam section 2 is: **M2 = [17,000x - 184,000] ft-lb** for 4 < x < 8 ft.

We continue in like manner for the last section of the beam

Section 3: Cut the beam at x, where 8 < x < 14 ft. and analyze left hand beam section, from x to the left end of the beam.

1. Draw a FBD of the beam section showing and labeling all forces and toque acting - including the shear force and bending moment (which act as an external force and torque at the point where we cut the beam.) We label them as V3 and M3, as this is section 3 of the beam.



Section 3: Cut beam between 8 and 14 ft.

2. Resolve all forces into x & y components (yes)

3. Apply translational equilibrium conditions (forces only):
Sum Fx = 0 (no net external x- forces)
Sum Fy = 21,000 lbs - 4,000 lbs - 3,000 lbs - (2,000 lbs/ft)(x-8)ft - V3 = 0
Solving for the bending moment: **V3 = [-2,000x + 30,000] lb**

4. We now find the bending moment expression by summing torque about the left end. That equation would be as follows:
Sum of Torque left end = -4000 lb.* 4 ft. -3000 lb.* 8 ft. - 2000 lb/ft *(x-8') *[(x-8)/2 + 8'] -V3 * x + 200,000 ft-lb + M3 = 0; [If you are unsure how the

third term in the equation was obtained, please see example 2.}

When we substitute in the value for **V3 = [-2,000x + 30,000] lb**. we obtain the equation:

Sum of Torque= -4000 lb.* 4 ft. -3000 lb.* 8 ft. - 2000 lb/ft *(x-8')*[(x-8)/2 + 8'] - [-2,000x + 30,000] lb* x + 200,000 ft-lb + M2 = 0

And solving for **M3 = [-1,000x^2+30,000x - 224,000] ft-lb.**

By Integration: $M3=\int V3\,dx = \int[-2{,}000x + 30{,}000]\,dx$, and then **M3 =-1,000x^2 + 30,000x + C3**

Since the right hand end of the beam is a free end, the bending moment must go to zero as x goes approaches 14 ft. So our boundary condition to find the integration constant, C3, is: **at x = 14 ft M = 0 ft-lb**

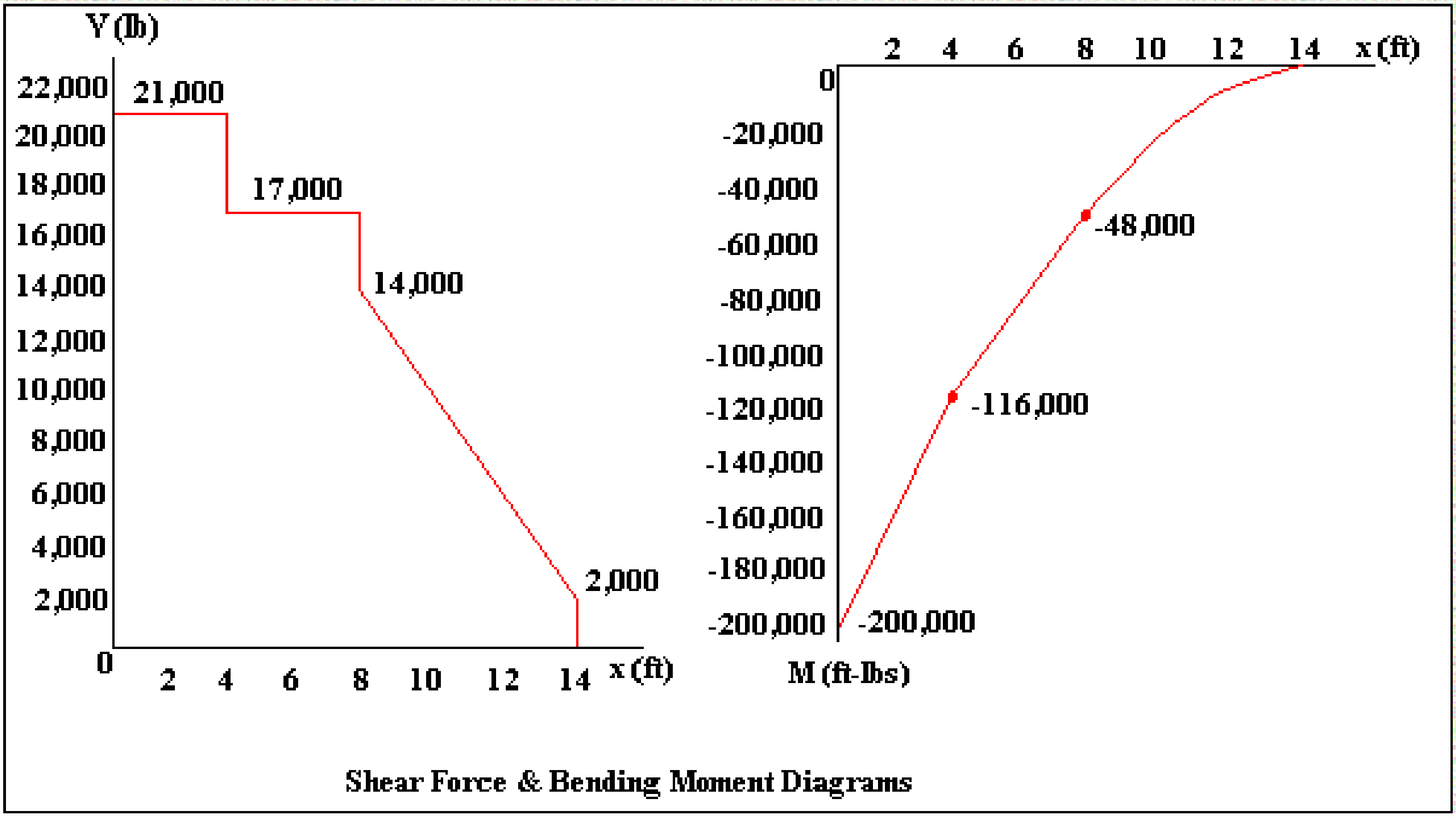
Applying the boundary condition: **0 = -1,000(14)^2 + 30,000(14) + C3** , and solving **C3 = -224,000 ft-lbs**

So **M3 = [-1,000x^2 +30,000x - 224,000] ft-lbs** for 8 < x < 14 ft.

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = 21,000 lb., V2 = 17,000 lb., V3 = [-2,000x+30,000] lb.

M1 =[21,000x-200,000] ft-lb., M2 = [17,000x-184,000] ft-lb., M3 = [-1,000x^2+30,000x - 224,000] ft-lb.



Return to:
Topic 4.2: Shear Forces and Bending Moments II

or Select:
Topic 4: Beams - Table of Contents
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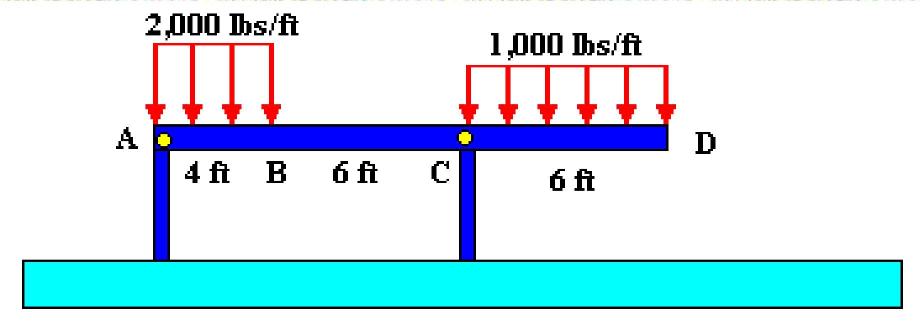
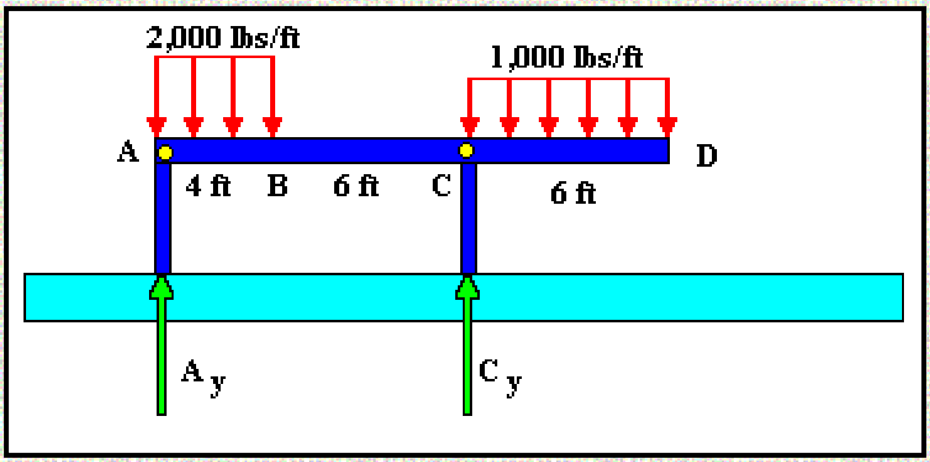
STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported beam is shown below. For this beam:
A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each sections of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:



STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.

Sum F_y = (-2,000 lbs/ft)(4 ft) - (1,000 lbs/ft)(6 ft) + A_y + C_y = 0

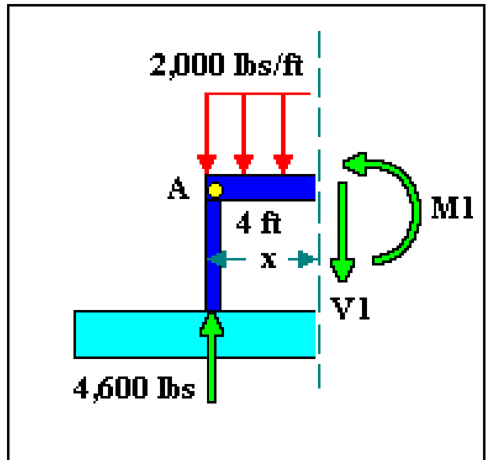
Sum T_A = (C_y)(10 ft) - (2,000 lbs/ft)(4 ft)(2 ft) - (1,000 lbs/ft)(6 ft)(13 ft) = 0

Solving for the unknowns:
C_y = 9,400 lbs; A_y = 4,600 lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use

Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where 0 < x < 4 ft.



Section 1:
Cut Beam between 0 and 4 ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum F_x = 0 (no net external x- forces)

Sum F_y = 4,600 lb - 2,000 lb/ft *x - V1 = 0

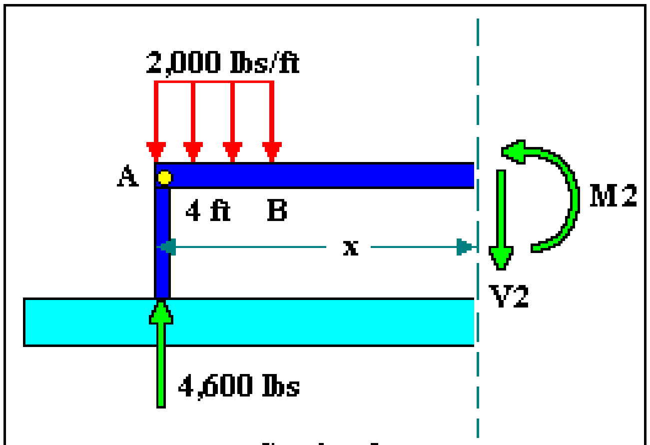
Solving: V1 = (4,600 - 2,000x) lbs

4. Integration $\boxed{M1 = \int V1\,dx = \int[-2{,}000x + 4{,}600]\,dx}$

M1 = -1000 x^2 + 4600x + C1

a) Boundary condition to find C1: at x = 0; M = 0
Apply BC: 0 = $-1000(0)^2$ + 4600(0) + C1, Solving: C1 = 0
Therefore... **M1 = [-1000x^2 + 4600x] ft-lbs** for 0 < x < 4 ft.

Section 2: Cut the beam at x, where 4 < x < 10 ft.



Section 2:
Cut beam between 4 and 10 ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum F_x = 0 (no net external x- forces)

Sum F_y = 4600 lbs - 2,000 lbs/ft(4 ft) - V2 = 0

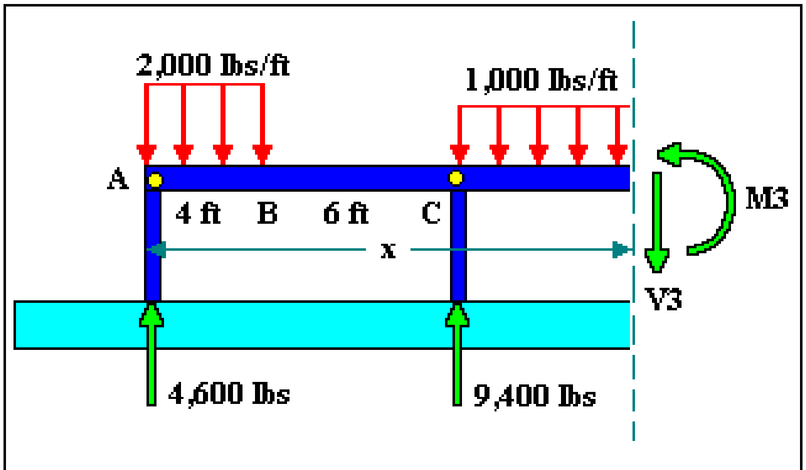
Solving: V2 = -3400 lbs

4. Integration $\boxed{M2 = \int V2\,dx = \int -3400dx}$

M2 = -3400x + C2

a) Boundary condition to find C2: at x=4 ft M=2400 ft-lbs (from equation M1)
Apply BC: 2400 ft lbs = -3400(4) + C2
Solving: C2 = 16,000 ft-lbs
Therefore... **M2 = [-3400x + 16,000] ft-lbs** for 4 < x < 10

Section 3: Cut the beam at x, where 10 < x < 16 ft.



Section 3: Cut beam between 10 and 16 ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum F_x = 0 (no net external x- forces)

Sum F_y = 4,600 lbs - 2,000 lb/ft(4 ft) + 9,400 lbs - 1,000 lb/ft(x-10)ft - V3 = 0

Solving: **V3 = [-1000 x + 16,000] lbs**

4. Integration $M3 = \int V3\,dx = \int [-1{,}000x + 16{,}000]\,dx$

M3 = -500x^2 + 16,000x + C3

a) Boundary condition to find C3: at x=16 ft M=0 ft-lbs (end of beam, no external torque so M3=0)

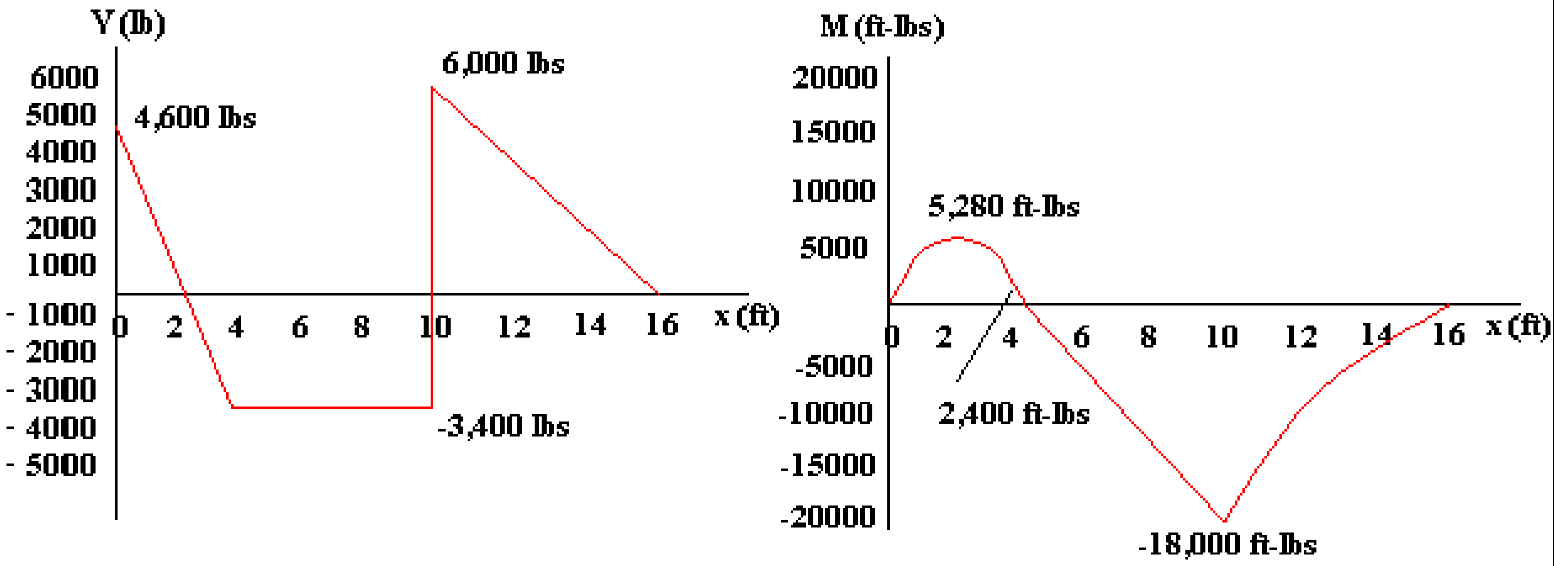
Apply BC: 0 = -500(16)^2 + 16,000(16) + C3

Solving: C3 = -128,000 ft-lbs

Therefore... **M3 = [-500x^2 + 16,000x - 128,000] ft-lbs** for 10 < x < 16

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in **Part B** above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = 4,600-2,000x lb, V2 = -3,400 lb, V3 = -1,000x+16,000 lb
M1 = -1,000x^2+4,600 ft-lb, M2 = -3,400x+16,000 ft-lb, M3 = -500x^2+16,000x-128,000 ft-lb

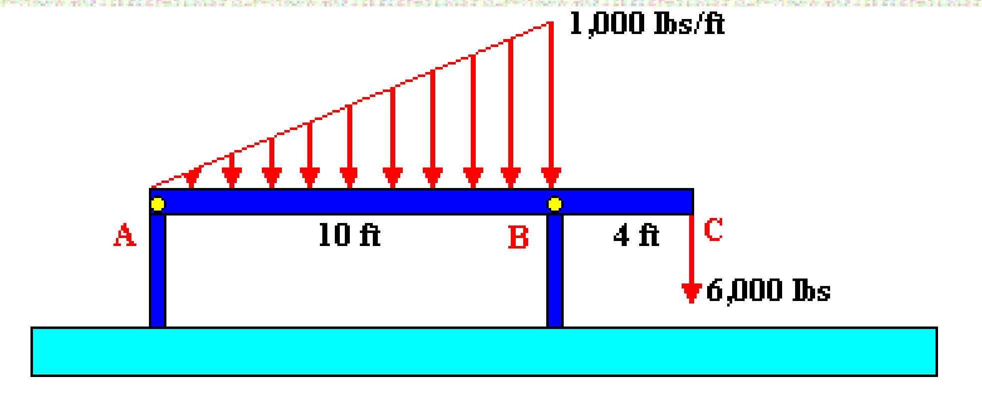


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

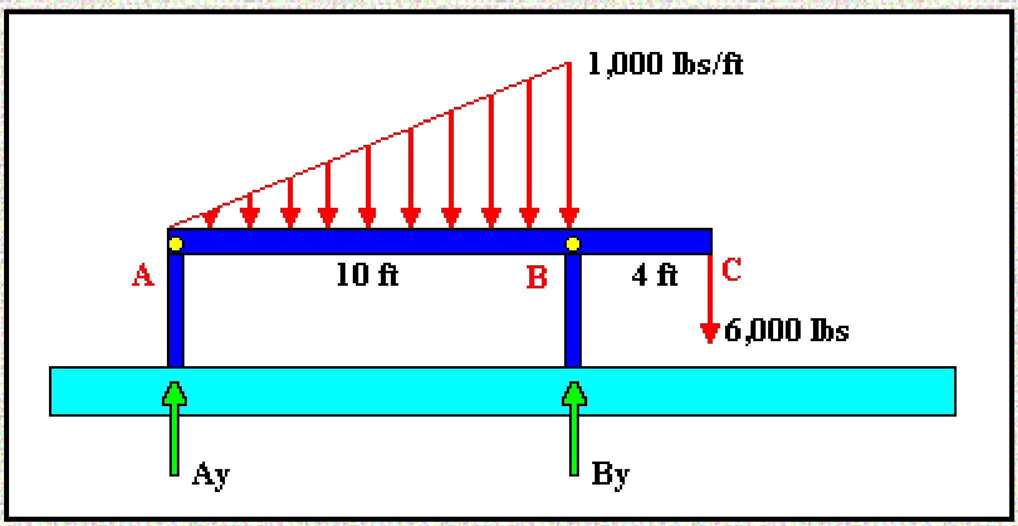
A loaded, simply supported beam is shown below. For this beam:
A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each sections of the beam.
C. Construct the shear force and bending moment diagrams for the beam.

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A: Statics



STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.

$\Sigma F_x = 0$

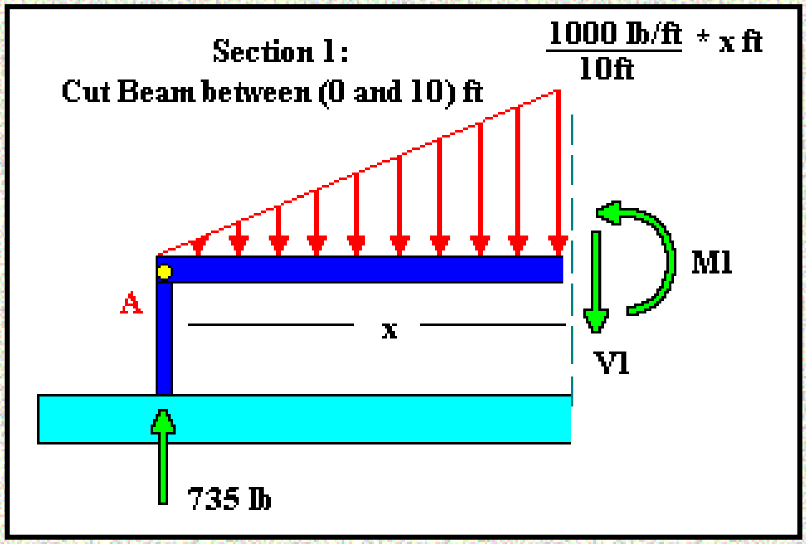
$\Sigma F_y = A_y + B_y - 1/2(10\text{ ft})(1{,}000\text{ lb/ft}) - 6{,}000\text{ lb} = 0$

$\Sigma T_A = -(5{,}000\text{ lbs})(6.67\text{ ft}) + (B_y)(10\text{ ft}) - (6{,}000\text{ lbs})(14\text{ ft}) = 0$

Solving for the unknowns:
B_y = 11,735 lbs; A_y = -735 lbs (acts downward)

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where (0 < x < 10) ft.



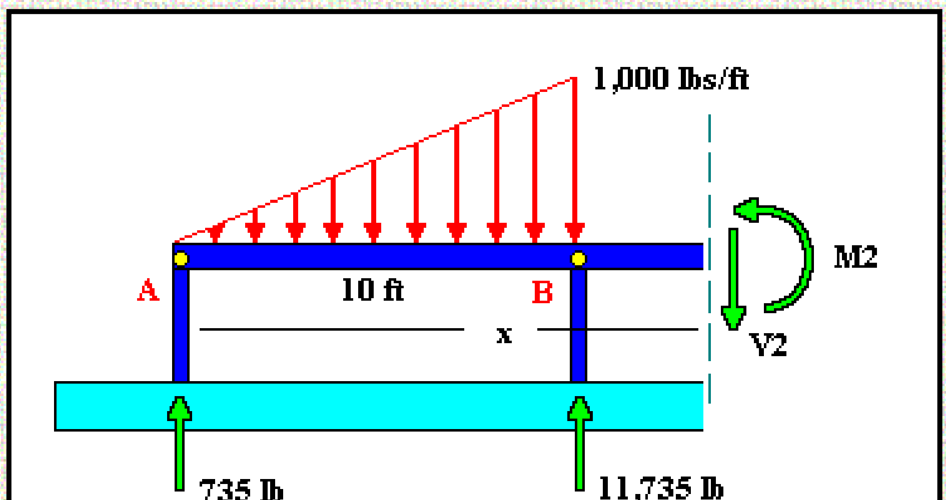
1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):
$\Sigma F_x = 0$ (no net external x- forces)
$\Sigma F_y = -735$ lb - 1/2(**x** ft)[(1,000(lb/ft)/10ft)](x ft) - V1 = 0
Solving: V1 = [-50x² - 735] lbs

4. Integration $M1 = \int V1\,dx = \int (-50x^2 - 735)\,dx$

M1 = -16.67x³ - 735x + C1

Boundary condition to find C1: at x = 0; M = 0. (simply supported beam)
Apply BC: $0 = -16.67(0)^3 - 735(0) + C1$, **Solving: C1 = 0**
Therefore... **M1 = [-16.67x³ - 735x] ft-lbs** for (0 < x < 10) ft.

Section 2: Cut the beam at x, where (10 < x < 14) ft.



Section 2:
Cut Beam between (10 and 14) ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):
$\Sigma F_x = 0$ (no net external x- forces)
$\Sigma F_y = -735$ lb - 1/2(10 ft)(1,000 lb/ft) + 11,735 lb - V2 = 0
Solving: V2 = +6,000 lbs

4. Integration

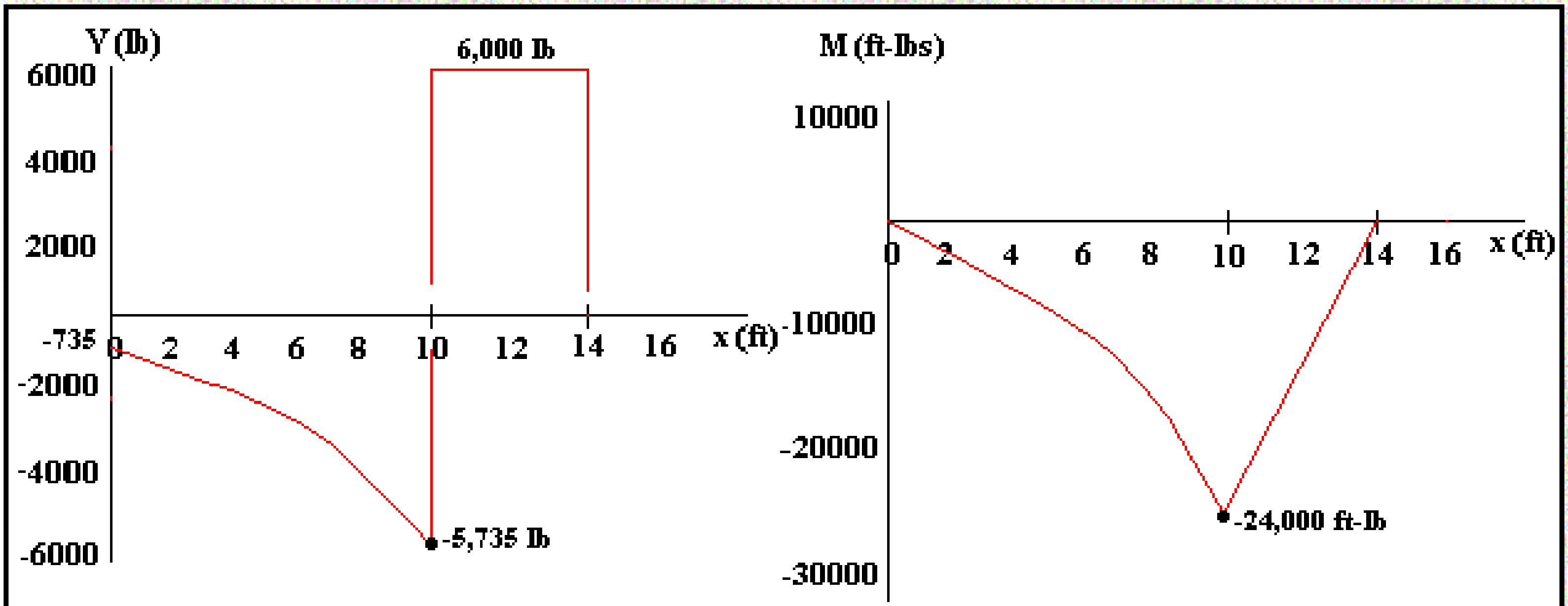
$$M2 = \int V2\,dx = \int 6{,}000\,dx$$

M2 = 6,000x + C2

Boundary condition from Section 1 at (**x = 14 ft**) ; M = 0. (end of simply supported beam):
Apply BC: 0 = 6,000(14 ft) + C2
Solving: C2 = -84,000 ft-lbs
Therefore... **M2 = [6,000x - 84,000] ft-lbs.**

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in **Part B** above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = [$-50x^2$ - 735] lbs, V2 = +6,000 lbs
M1 = [$-16.67x^3$ - 735x] ft-lbs, M2 = [6,000x - 84,000] ft-lbs.

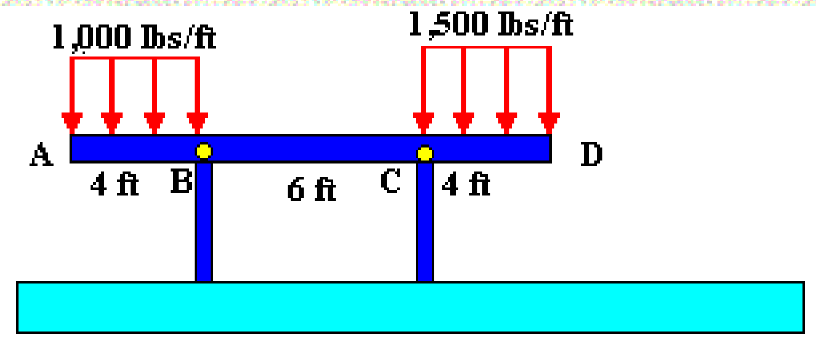


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported beam is shown below. For this beam:

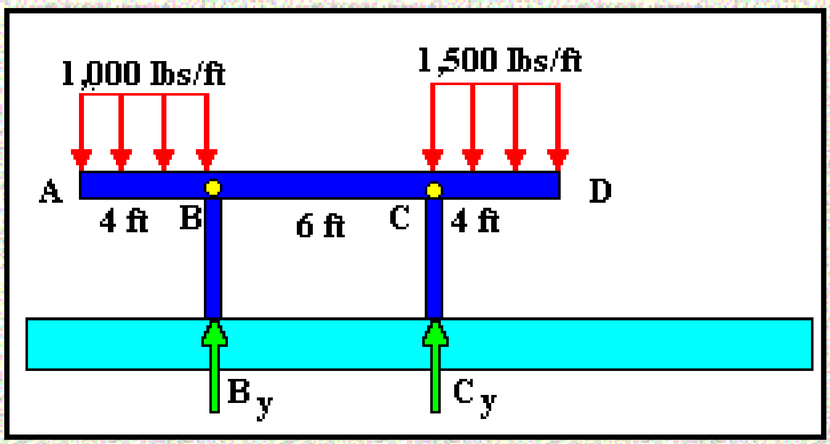
A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each sections of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.

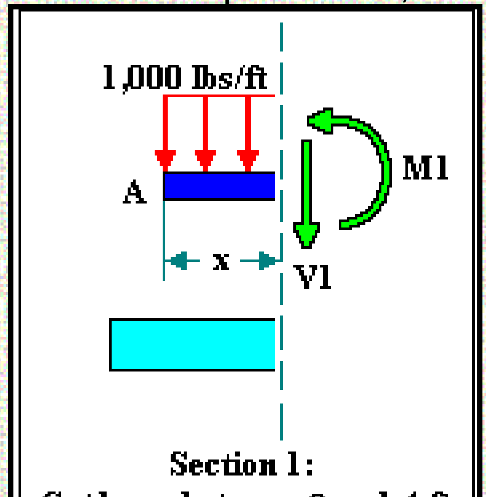


Sum F_y = (-1,000 lbs/ft)(4 ft) - (1,500 lbs/ft)(4 ft) + B_y + C_y = 0

Sum T_B = (C_y)(6 ft) + (1,000 lbs/ft)(4 ft)(2 ft) - (1,500 lbs/ft)(4 ft)(8 ft) = 0

Solving for the unknowns:
C_y = 6,670 lbs; B_y = 3,330 lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.



Section 1:
Cut beam between 0 and 4 ft

Section 1: Cut the beam at x, where 0 < x < 4 ft. Analyze left hand section.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = -1,000 lbs/ft(x) - V1 = 0

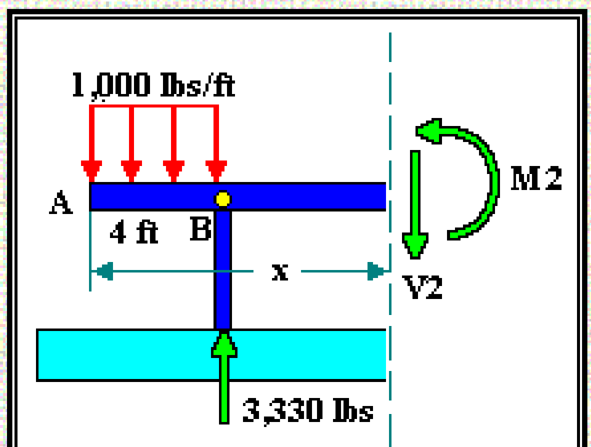
Solving: **V1 = -1,000x lbs**

4. Integration $M1 = \int V1\, dx = \int [-1{,}000x]\, dx$

$$\mathbf{M1 = -500\, x^2 + C1}$$

a) Boundary condition to find C1: at x=0 M=0
Apply BC: $0 = -500(0)^2 + C1$
Solving: **C1 = 0**
Therefore... **M1 = [$-500x^2$] ft-lbs** for 0 < x < 4 ft.

Section 2: Cut the beam at x, where 4 < x < 10 ft. Analyze left hand section.



Section 2:
Cut beam between 4 and 10 ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = -1,000 lbs/ft (4 ft) + 3,330 lbs - V2 = 0
Solving: **V2 = -667 lbs**

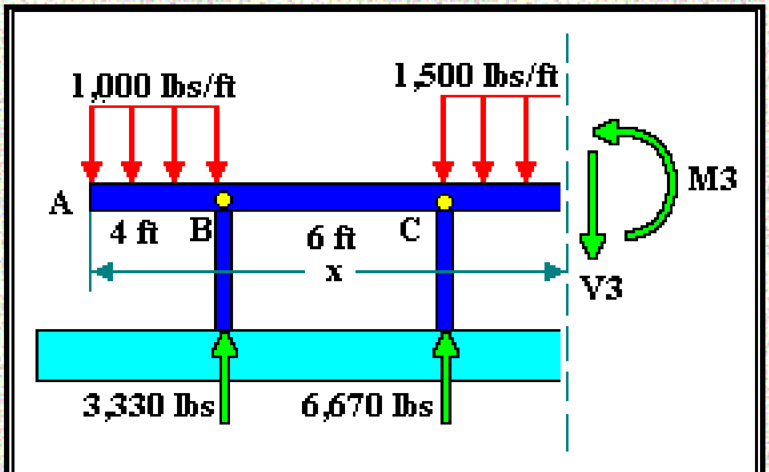
4. Integration $M2 = \int V2\, dx = \int -667 dx$

M2 = -667x + C2

a)Boundary condition to find C2: at x=4 ft M=-8000 ft-lbs (from equation M1)
Apply BC: 8000 ft-lbs = -667(4) + C2

Solving: **C2 = -5,330 ft-lbs**
Therefore... **M2 = [-667x - 5,330] ft-lbs** for 4 < x < 10

Section 3: Cut the beam at x, where 10 < x < 14 ft. Analyze left hand section.



Section 3: Cut beam between 10 and 14 ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = -1,000 lbs/ft(4 ft) + 3,330 lbs + 6,670 lbs -1500lbs/ft(x-10)ft - V3 = 0

Solving: **V3 = [-1,500x + 21,000] lbs**

4. Integration $M3 = \int V3\, dx = \int [-1{,}500x + 21{,}000] dx$

M3 = -750x^2 + 21,000x + C3

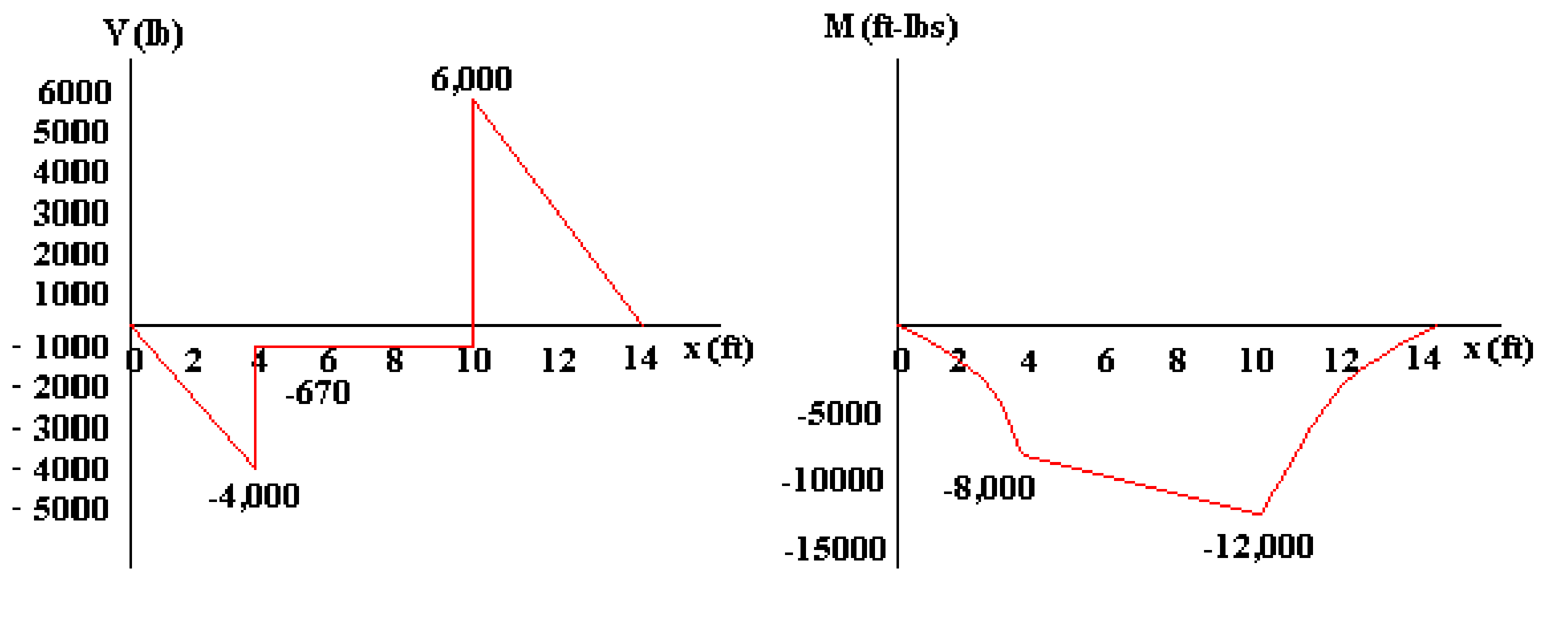
a)Boundary condition to find C3: at x=14 ft M=0 ft-lbs (end of beam, no external torque so M3=0)
Apply BC: 0 = -750(14)^2 + 21,000(14) + C3

Solving: C3 = -147,000 ft-lbs
Therefore... **M3 = [-750x^2 + 21,000x - 147,000] ft-lbs** for 10 < x < 14

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = -1,000x lb, V2 = -667 lb, V3 = -1,500x+21,000 lb

M1 = -500x^2 ft-lb, M2 = -667x-5,330 ft-lb, M3 = -750x^2+21,000x-147,000 ft-lb

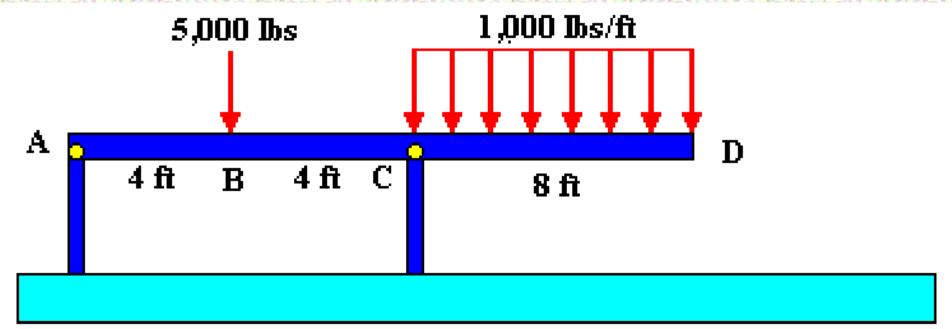


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported beam is shown below. For this beam:

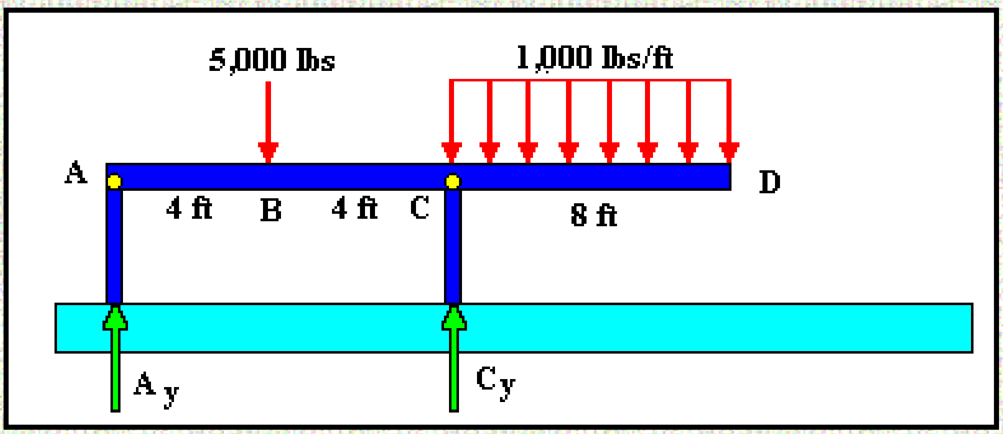
A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each sections of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



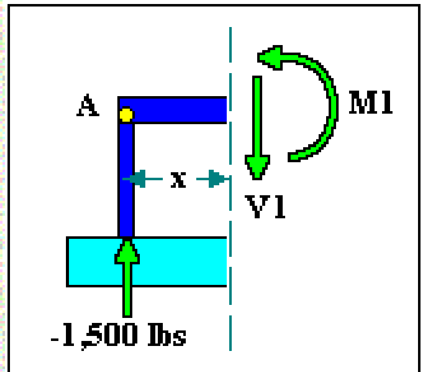
Sum F_y = (-5,000 lbs) - (1,000 lbs/ft)(8 ft) + A_y + C_y = 0

Sum T_A = (C_y)(8 ft) - (5,000 lbs)(4 ft) - (1,000 lbs/ft)(8 ft)(12 ft) = 0

Solving for the unknowns:
C_y = 14,500 lbs; A_y = -1,500 lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where 0 < x < 4 ft. Analyze left hand section.



Section 1:
Cut beam between 0 and 4 ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = -1,500 lbs - V1 = 0

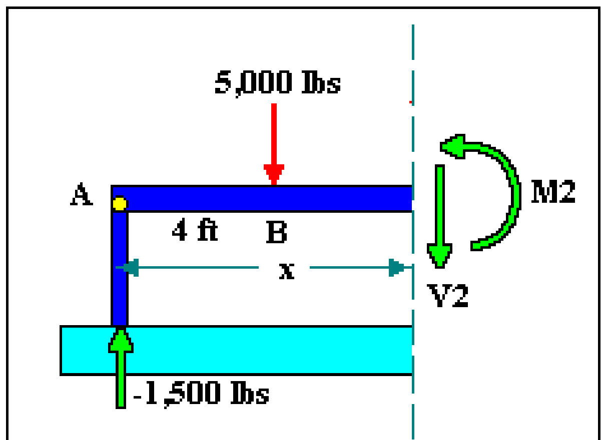
Solving: V1 = -1,500 lbs

4. Integration $M1 = \int V1\,dx = \int [-1{,}500]\,dx$

M1 = -1,500x + C1

a) Boundary condition to find C1: at x=0 M=0
Apply BC: 0 = -1,500(0) + C1
Solving: C1 = 0
Therefore... **M1 = [-1,500x] ft-lbs** for 0 < x < 4 ft.

Section 2: Cut the beam at x, where 4 < x < 8 ft. Analyze left hand section.



Section 2: Cut beam between 4 and 8 ft

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

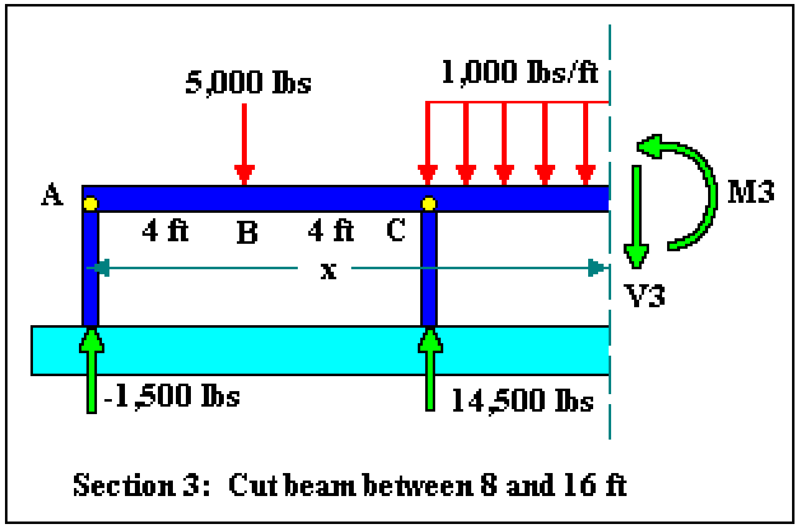
Sum Fx = 0 (no net external x- forces)
Sum Fy = -1,500 lbs - 5,000 lbs - V2 = 0
Solving: V2 = -6,500 lbs

4. Integration $M2 = \int V2\,dx = \int -6{,}500\,dx$

M2 = -6,500x + C2

a) Boundary condition to find C2: at x=4 ft M=-8,000 ft-lbs (from equation M1)
Apply BC: 8000 ft-lbs = -6,500(4) + C2
Solving: C2 = 20,000 ft-lbs
Therefore... **M2 = [-6,500x + 20,000] ft-lbs** for 4 < x < 8

Section 3: Cut the beam at x, where 8 < x < 16 ft. Analyze left hand section.



1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = -1,500 lbs - 5,000 lbs + 14,500 lbs -1,000lbs/ft(x-8)ft - V3 = 0

Solving: V3 = [-1,000x + 16,000] lbs

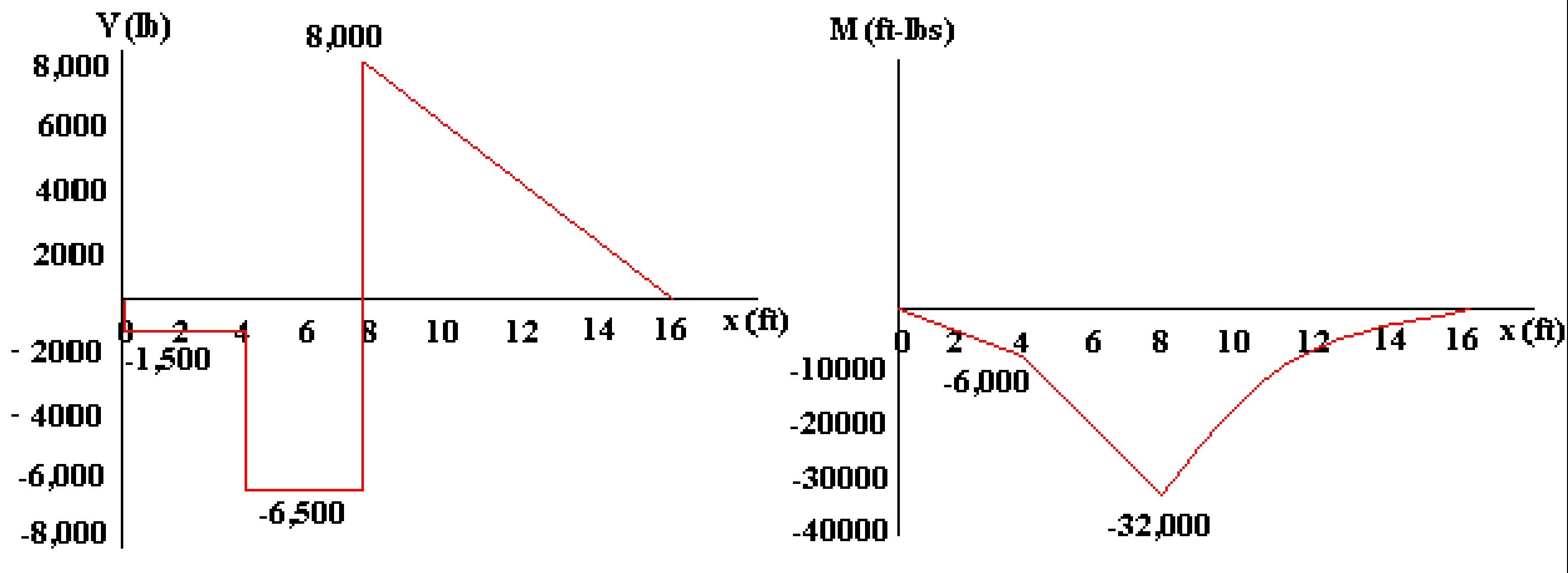
4. Integration $M3 = \int V3\,dx = \int [-1{,}000x + 16{,}000]\,dx$

M3 = -500x^2 + 16,000x + C3

a) Boundary condition to find C3: at x=16 ft M=0 ft-lbs (end of beam, no external torque so M3=0)
Apply BC: 0 = -500(16)^2 + 16,000(16) + C3
Solving: C3 = -128,000 ft-lbs
Therefore... **M3 = [-500x^2 +16,000x - 128,000] ft-lbs** for 8 < x < 16

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = -1,500 lb, V2 = -6,500 lb, V3 = -1,000x+16,000 lb
M1 = -1,500x ft-lb, M2 = -6,500x+20,000ft-lb, M3 = -500x^2+16,000x-128,000 ft-lb

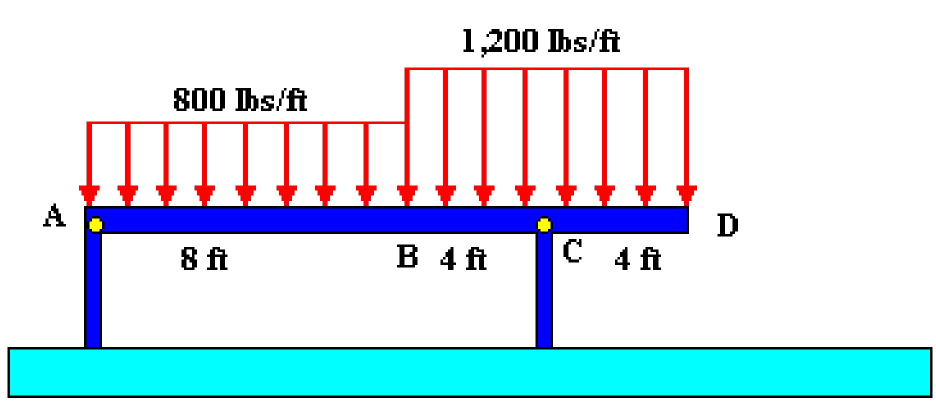


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

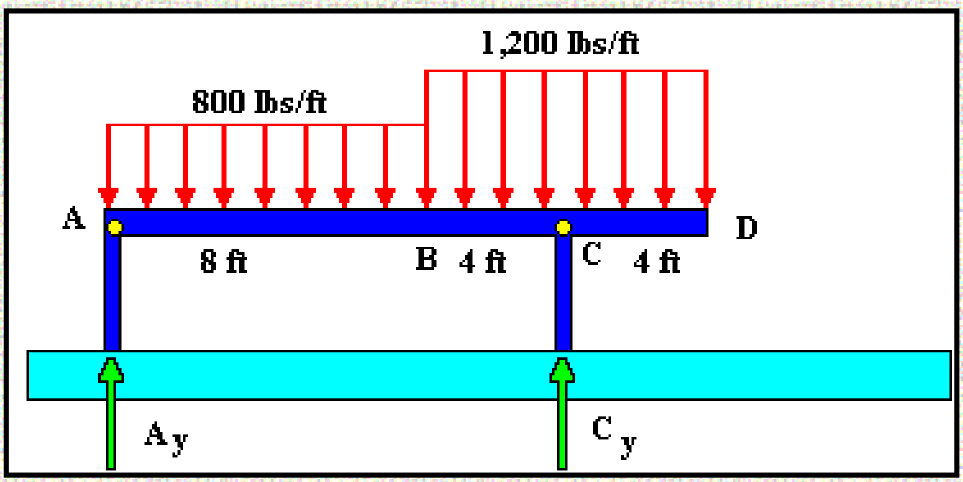
A loaded, simply supported beam is shown below. For this beam:

A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each section of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.

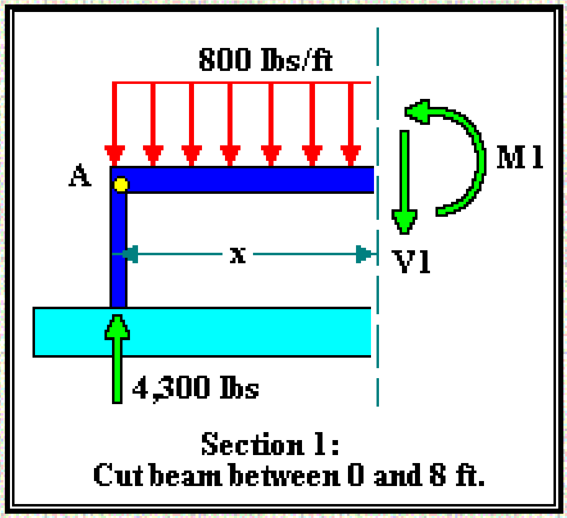


Sum $F_y = (-800 \text{ lbs/ft})(8 \text{ ft}) - (1{,}200 \text{ lbs/ft})(8 \text{ ft}) + A_y + C_y = 0$

Sum $T_A = (C_y)(12 \text{ ft}) - (800 \text{ lbs/ft})(8 \text{ ft})(4 \text{ ft}) - (1{,}200 \text{ lbs/ft})(8 \text{ ft})(12 \text{ ft}) = 0$

Solving for the unknowns:
C_y = 11,700 lbs; A_y = 4,300 lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.



Section 1: Cut the beam at x, where 0 < x < 8 ft. Analyze left hand section.

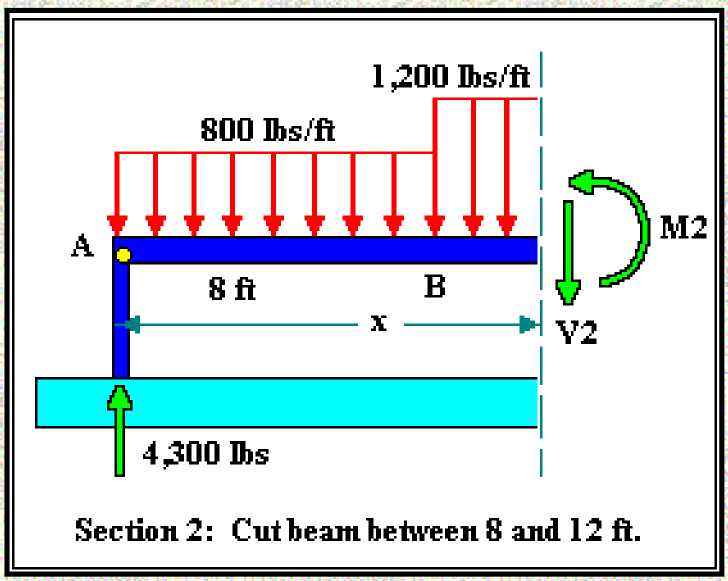
1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 4,270 lbs - 800 lbs/ft(x)ft - V1 = 0
Solving: V1 = [4,270 - 800x] lbs

4. Integration $\boxed{M1 = \int V1\, dx = \int [-800x + 4{,}270]\, dx}$

M1 = -400x^2 + 4,270x + C1

a)Boundary condition to find C1: at x=0 M=0
Apply BC: 0 = -400$(0)^2$ + 4,270(0) + C1
Solving: C1 = 0
Therefore... **M1 = [-400x^2 + 4,270x] ft-lbs** for 0 < x < 8 ft.



Section 2: Cut the beam at x, where 8 < x <12 ft. Analyze left hand section.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

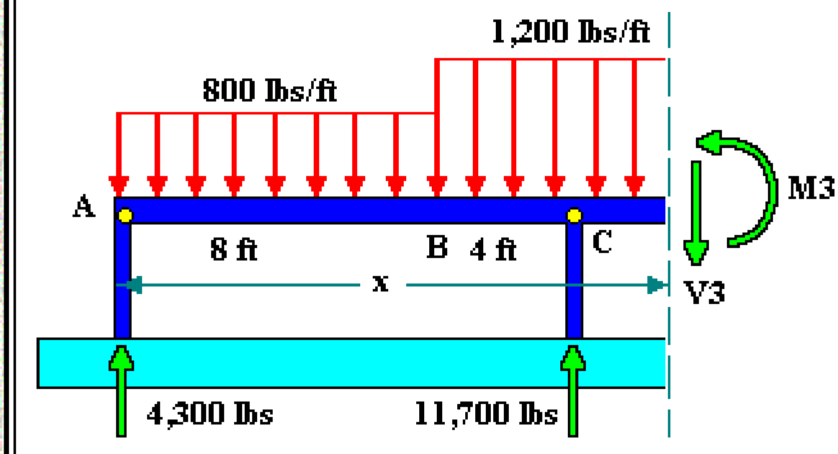
Sum Fx = 0 (no net external x- forces)
Sum Fy = 4,270 lbs - 800 lbs/ft(8 ft) - 1,200lbs/ft (x-8)ft - V2 = 0
Solving: V2 = [7,470 - 1,200x] lbs

4. Integration $\boxed{M2 = \int V2\, dx = \int [-1{,}200x + 7{,}470]\, dx}$

$M2 = -600x^2 + 7{,}470x + C2$

a) Boundary condition to find C2: at x=8 ft M=-8,560 ft-lbs (from equation M1)
Apply BC: -8560 ft-lbs = $-600(8)^2 + 7470(8) + C2$
Solving: C2 = -128,000 ft-lbs
Therefore... **$M2 = [-600x^2 + 7{,}470x - 128{,}000]$ ft-lbs** for 8 < x < 12

Section 3: Cut the beam at x, where 12 < x < 16 ft. Analyze left hand section.



Section 3: Cut beam between 12 and 16 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 4,270 lbs - 800 lbs/ft(8 ft) + 11,730 lbs - 1,200lbs/ft (x-8)ft - V3 = 0

Solving: $V3 = [-1{,}200x + 19{,}200]$ lbs

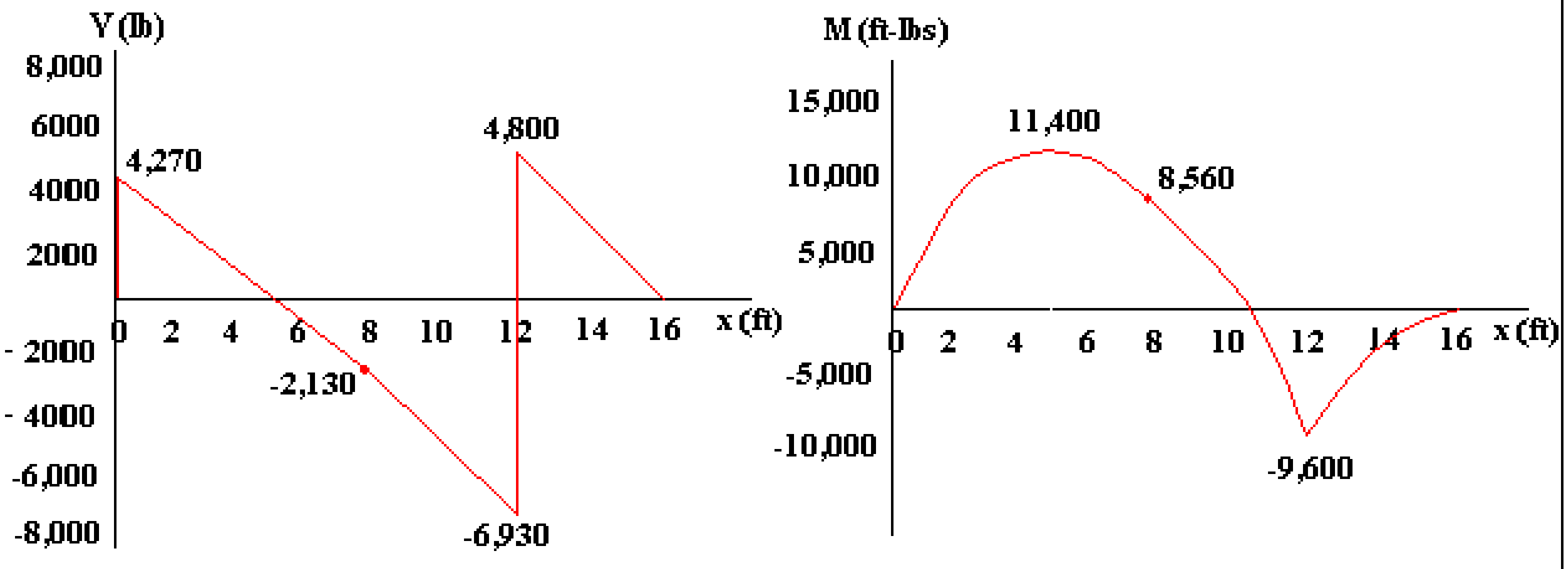
4. Integration $M3 = \int V3\,dx = \int [-1{,}200x + 19{,}200]\,dx$

$M3 = -600x^2 + 19{,}200x + C3$

a) Boundary condition to find C3: at x=16 ft M=0 ft-lbs (end of beam, no external torque so M3=0)
Apply BC: 0 = $-600(16)^2 + 19{,}200(16) + C3$
Solving: C3 = -153,000 ft-lbs
Therefore... **$M3 = [-600x^2 + 19{,}200x - 153{,}000]$ ft-lbs** for 12 < x < 16

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = $-800x+4{,}270$ lb, V2 = $-1{,}200x+7{,}470$ lb, V3 = $-1{,}200x+19{,}200$ lb
M1 = $-400x^2+4{,}270x$ ft-lb, M2 = $-600x^2+7{,}470-12{,}800$ ft-lb, M3 = $-600x^2+19{,}200x-153{,}000$ft-lb

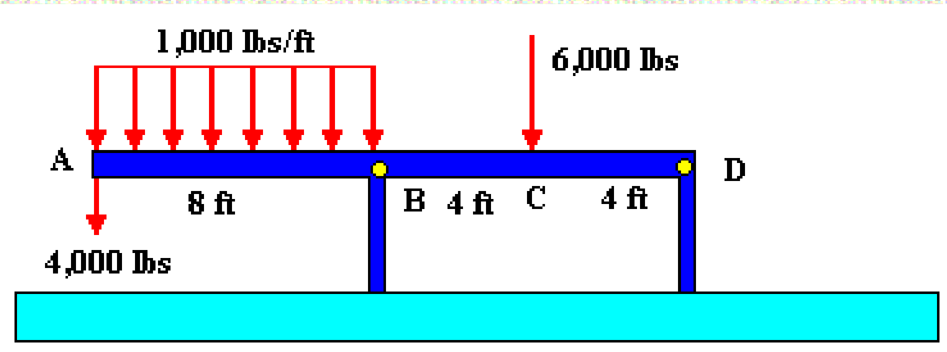


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported beam is shown below. For this beam:

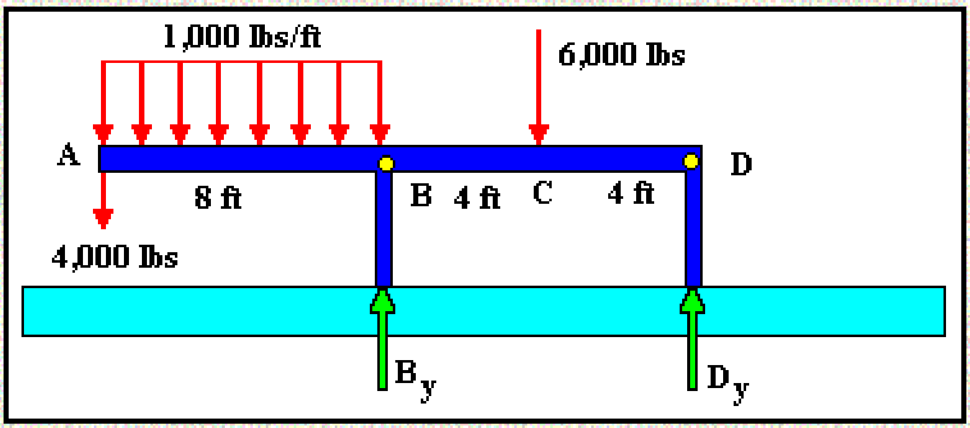
A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each section of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



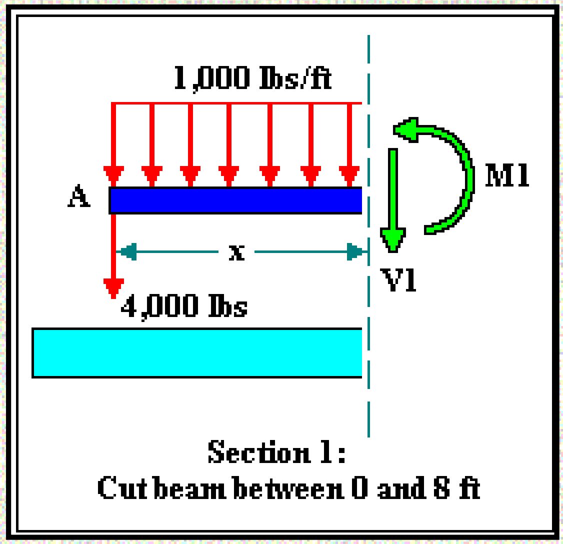
Sum F_y = -4,000 lbs - (1,000 lbs/ft)(8 ft) - 6,000 lbs + B_y + D_y = 0

Sum T_B = (D_y)(8 ft) - (6,000 lbs)(4 ft) + (1,000 lbs/ft)(8 ft)(4 ft) + (4,000 lbs)(8 ft) = 0

Solving for the unknowns:

B_y =23,000 lbs; D_y = -5,000 lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.



Section 1: Cut the beam at x, where 0 < x < 8 ft. Analyze left hand section.

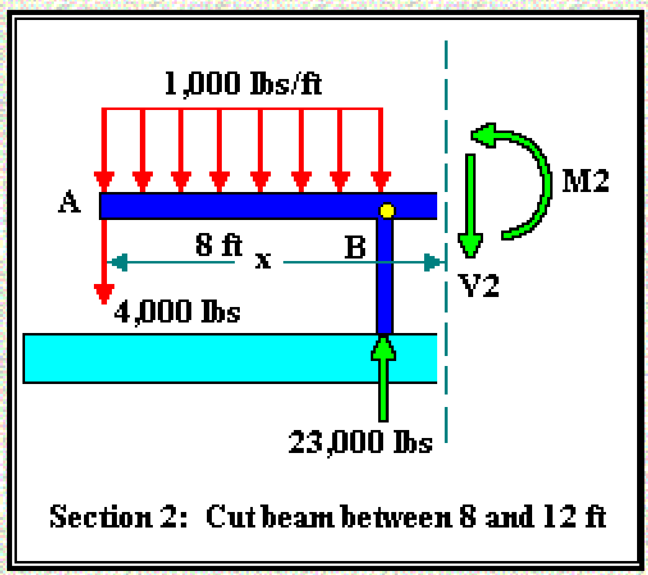
1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = -4,000 lbs - 1,000 lbs/ft(x)ft - V1 = 0
Solving: V1 = [-4,000 - 1,000x] lbs

4. Integration $\boxed{M1 = \int V1\,dx = \int [-1{,}000x - 4{,}000]\,dx}$

M1 = $-500x^2$ - 4,000x + C1

a) Boundary condition to find C1: at x=0 M=0
Apply BC: 0 = $-500(0)^2$ - 4,000(0) + C1
Solving: **C1 = 0**
Therefore... **M1 = [$-500x^2$ - 4,000x] ft-lbs** for 0 < x < 8 ft.



Section 2: Cut the beam at x, where 8 < x <12 ft. Analyze left hand section.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

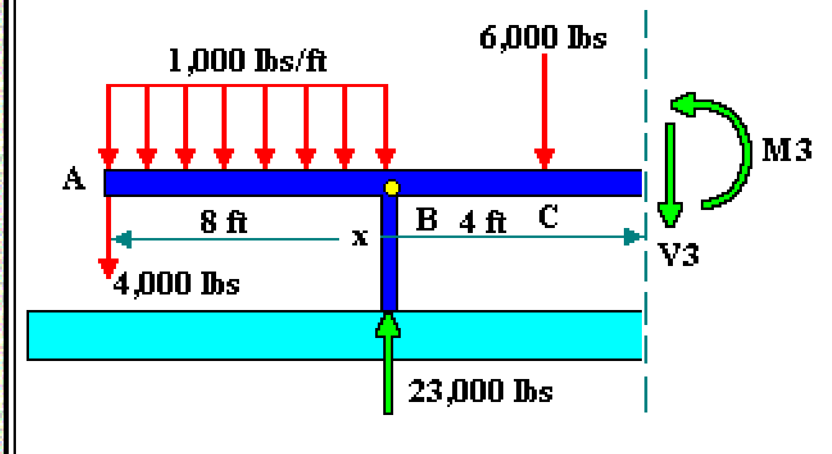
Sum Fx = 0 (no net external x- forces)
Sum Fy = -4,000 lbs - 1,000 lbs/ft(8 ft) + 23,000 lbs - V2 = 0
Solving: V2 = 11,000 lbs

4. Integration $\boxed{M2 = \int V2\,dx = \int 11{,}000\,dx}$

M2 = 11,000x + C2

a)Boundary condition to find C2: at x=8 ft M=-64,000 ft-lbs (from equation M1)
Apply BC: -64,000 ft-lbs = 11,000 lbs (8) + C2
Solving: C2 = -152,000 ft-lbs
Therefore... **M2 = [11,000x - 152,000] ft-lbs** for 8 < x < 12

Section 3: Cut the beam at x, where 12 < x < 16 ft. Analyze left hand section.



Section 3: Cut beam between 12 and 16 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = -4,000 lbs - 1,000 lbs/ft(8 ft) + 23,000 lbs -6,000 lbs - V3 = 0
Solving: **V3 = 5,000 lbs**

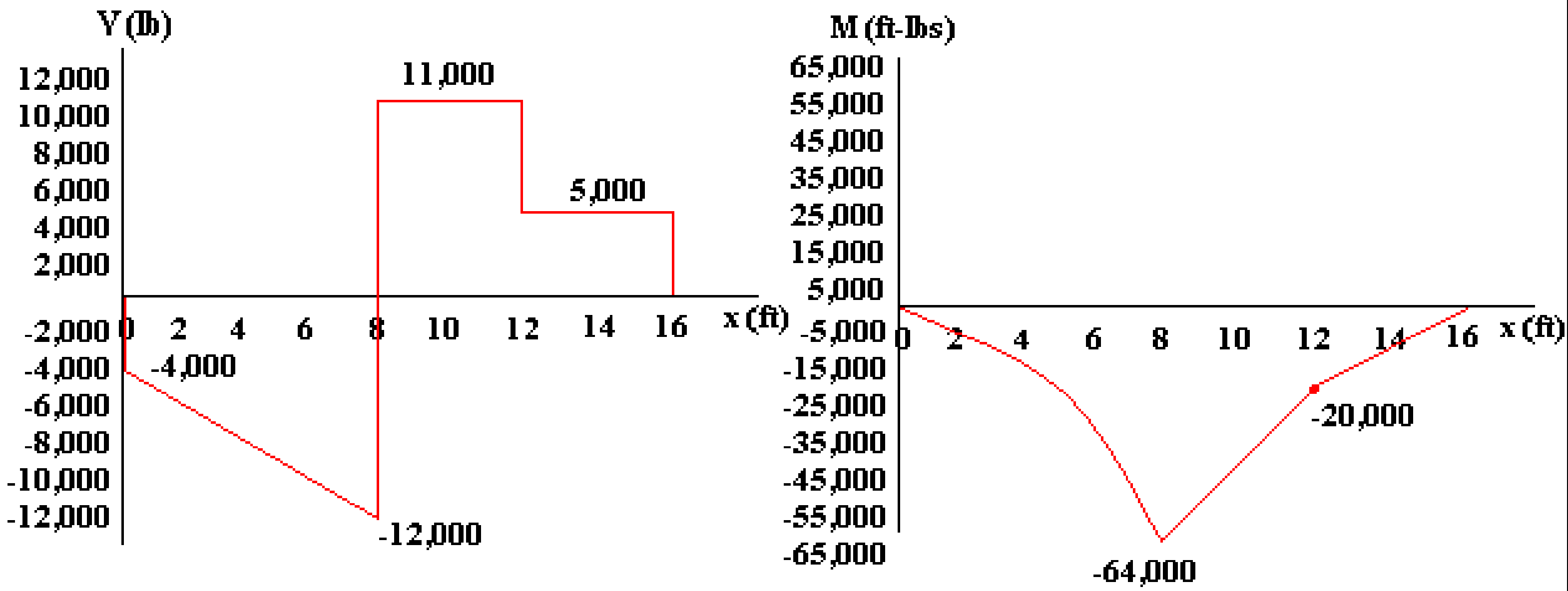
4. Integration $M3 = \int V3\,dx = \int 5{,}000\,dx$

M3 = 5,000x + C3

a)Boundary condition to find C3: at x=16 ft M=0 ft-lbs (end of beam, no external torque so M3=0)
Apply BC: 0 = 5,000(16) + C3
Solving: C3 = -80,000 ft-lbs
Therefore... **M3 = [5,000x - 80,000] ft-lbs** for 12 < x < 16

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = -1,000x+4,000 lb, V2 = 11,000 lb, V3 = 5,000 lb
M1 = $-500x^2$**+4,000x ft-lb, M2 = 11,000x -152,000 ft-lb, M3 = 5,000x-80,000 ft-lb**

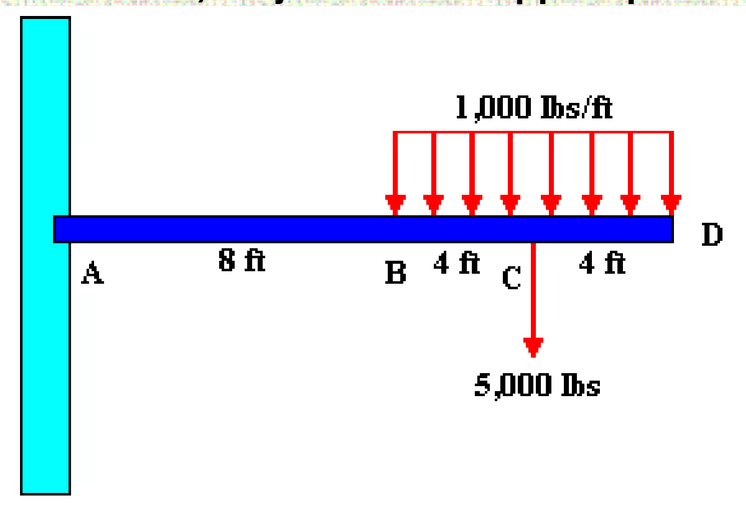


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

A loaded, cantilever beam is shown below. For this beam:

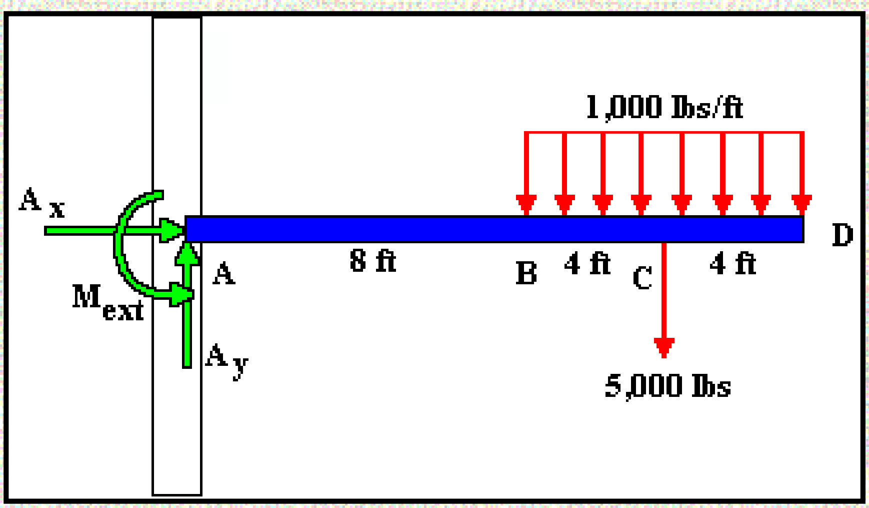
A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each sections of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



Sum $F_x = A_x = 0$

Sum F_y = -5,000 lbs - (1,000 lbs/ft)(8 ft) + $A_y = 0$

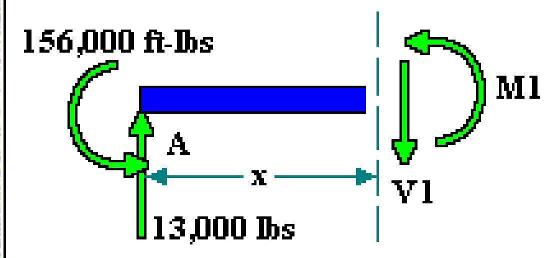
Sum T_A = - (5,000 lbs)(12 ft) - (8,000 lbs)(12 ft) + $M_{ext} = 0$

Solving for the unknowns:

A_y = 13,000 lbs; M_{ext} = 156,000 ft-lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where 0 < x < 8 ft. Analyze left hand section.



Section 1:
Cut beam between 0 and 8 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 13,000 lbs - V1 = 0
Solving: **V1 = 13,000 lbs**

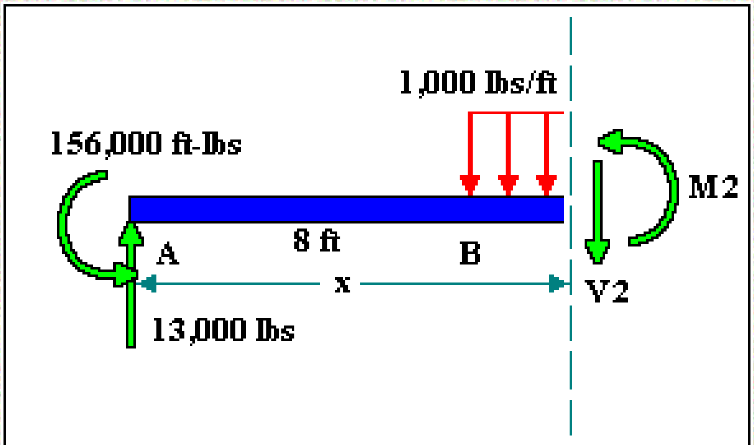
4. Integration $M1 = \int V1\,dx = \int 13{,}000dx$

M1 = 13,000x + C1

a)Boundary condition to find C1: at x=0 M=-156,000 ft-lbs (That is, for a cantilever beam, the value of the bending moment at the wall is equal to the negative of the external moment.)
Apply BC: -156,000 = 13000(0) + C1

Solving: **C1 = -156,000**
Therefore... **M1 = [13,000x - 156,000] ft-lbs** for 0 < x < 8 ft.

Section 2: Cut the beam at x, where 8 < x < 12 ft. Analyze left hand section.



Section 2: Cut beam between 8 and 12 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 13,000 lbs - (1,000 lbs/ft)((x - 8) ft) - V2 = 0

Section 2: Cut beam between 8 and 12 ft.

Solving: **V2 = [-1,000x + 21,000] lbs**

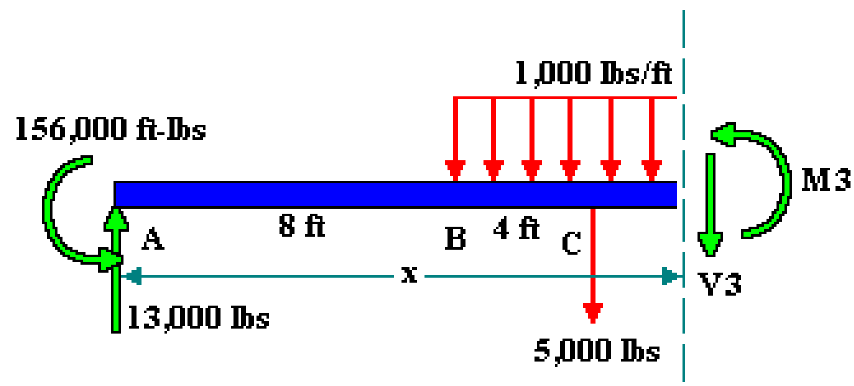
4. Integration $M2 = \int V2\,dx = \int [-1{,}000x + 21{,}000]\,dx$

M2 = $-500x^2$ + 21,000x + C2

a)Boundary condition to find C2: at x=8 ft M=-52,000 ft-lbs (from equation M1)
Apply BC: -52,000 ft-lbs = $-500(8)^2$ + 21,000(8) + C2

Solving: **C2 = -188,000 ft-lbs**
Therefore... **M2 = [$-500x^2$ + 21,000x - 188,000] ft-lbs** for 8 < x < 12

Section 3: Cut the beam at x, where 12 < x < 16 ft. Analyze left hand section.



Section 3: Cut beam between 12 and 16 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x-forces)
Sum Fy = 13,000 lbs - (1,000 lbs/ft)((x - 8)ft) - 5,000 lbs - V3 = 0

Solving: **V3 = [-1,000x + 16,000] lbs**

4. Integration $M3 = \int V3\,dx = \int [-1{,}000x + 16{,}000]\,dx$

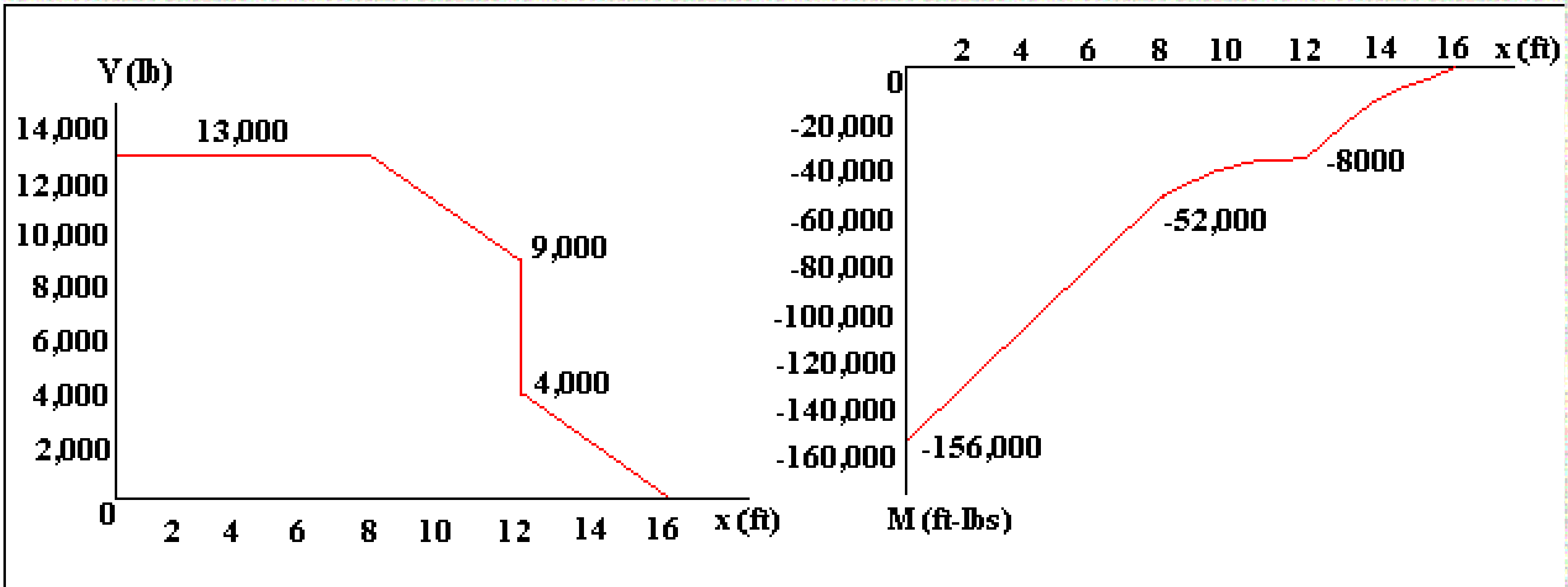
M3 = $-500x^2$ + 16,000x + C3

a)Boundary condition to find C3: at x=16 ft M=0 ft-lbs (free end of beam, no external torque so M3=0)
Apply BC: 0 = $-500(16)^2$ + 16,000(16) + C3

Solving: **C3 = -128,000 ft-lbs**
Therefore... **M3 = [$-500x^2$ + 16,000x - 128,000] ft-lbs** for 12 < x < 16

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = 13,000 lb, **V2 = [-1,000x+21,000]** lb, **V3 = [-1,000x + 16,000] lb**
M1 =[13,000x-156,000] ft-lb, **M2 = [-500x^2+21,000x-188,000] ft-lb**, **M3 = [-500x^2 + 16,000x - 128,000] ft-lb**

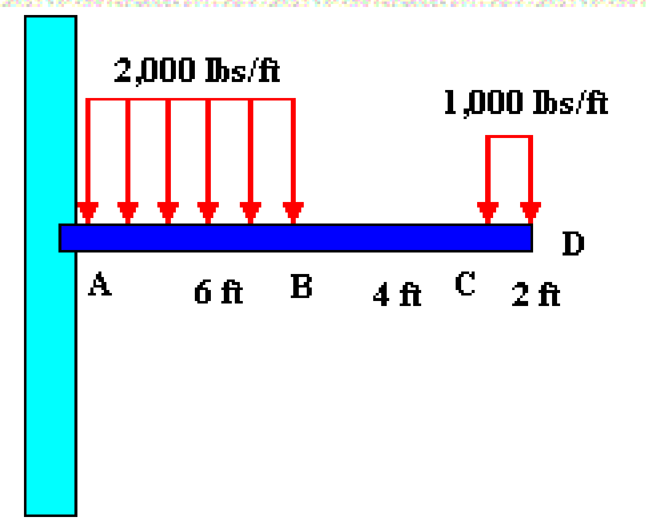


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

A loaded, cantilever beam is shown below. For this beam:

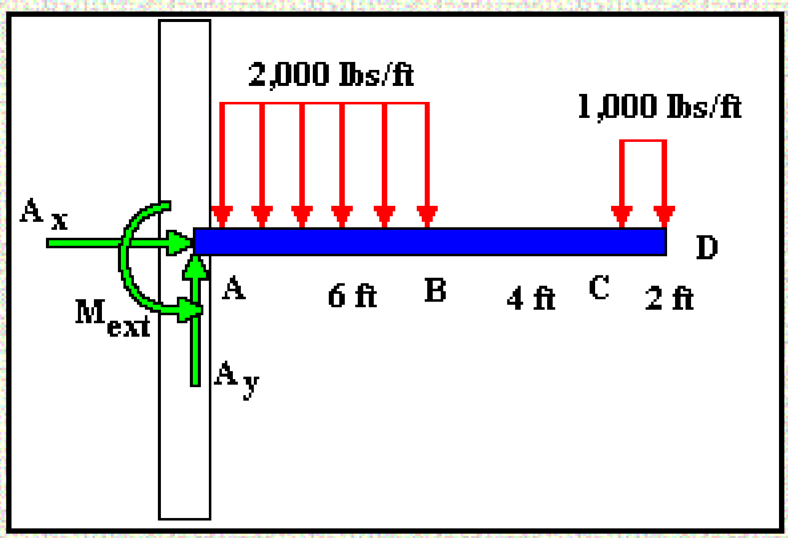
A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each sections of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:

STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



Sum $F_x = A_x = 0$

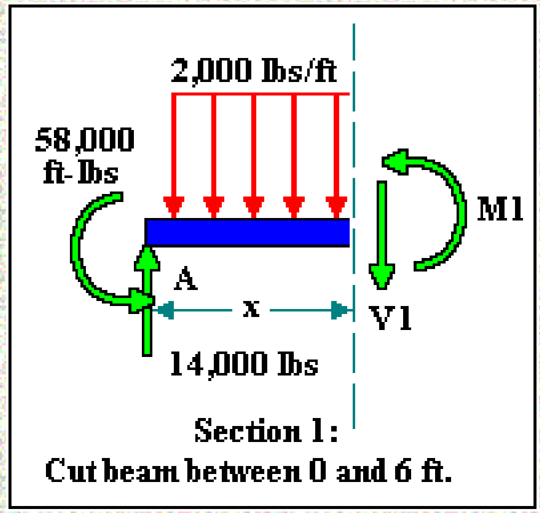
Sum $F_y = (-2{,}000 \text{ lbs/ft})(6 \text{ ft}) - (1{,}000 \text{ lbs/ft})(2 \text{ ft}) + A_y = 0$

Sum T_A = (-2,000 lbs/ft)(6 ft)(3 ft) - (1,000 lbs/ft)(2 ft)(11 ft) + M_{ext} = 0

Solving for the unknowns:
A_y = 14,000 lbs; M_{ext} = 58,000 ft-lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where 0 < x < 6 ft. Analyze left hand section.



1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 14,000 lbs - (2,000 lbs/ft)x ft - V1 = 0

Solving: **V1 = [-2,000x + 14,000] lbs**

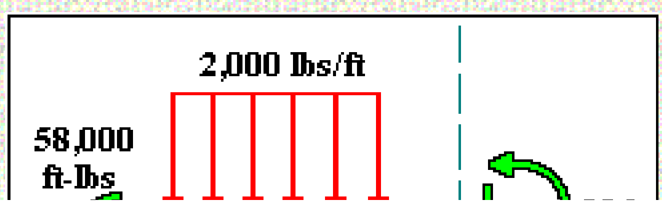
4. Integration $M1 = \int V1\,dx = \int [-2{,}000x + 14{,}000]\,dx$

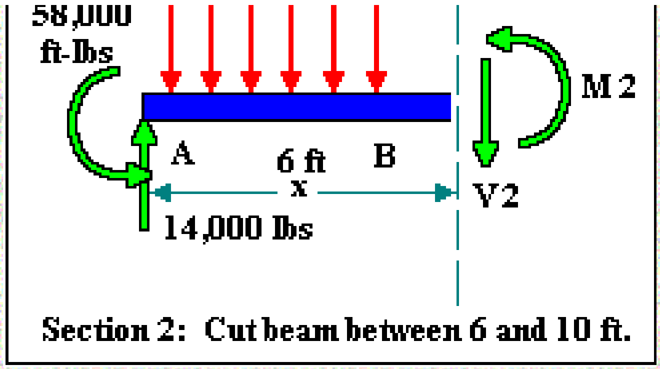
$M1 = -1{,}000x^2 + 14{,}000x + C1$

a)Boundary condition to find C1: at x=0 M=-58,000 ft-lbs (That is, for a cantilever beam, the value of the bending moment at the wall is equal to the negative of the external moment.)
Apply BC: $-58{,}000 = -1{,}000(0)^2 + 14000(0) + C1$

Solving: **C1 = -58,000**
Therefore... **$M1 = [-1{,}000x^2 + 14{,}000x - 58{,}000]$ ft-lbs** for 0 < x < 6 ft.

Section 2: Cut the beam at x, where 6 < x < 10 ft. Analyze left hand section.





Section 2: Cut beam between 6 and 10 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 14,000 lbs - (2,000 lbs/ft)(6 ft) - V2 = 0
Solving: **V2 = 2,000 lbs**

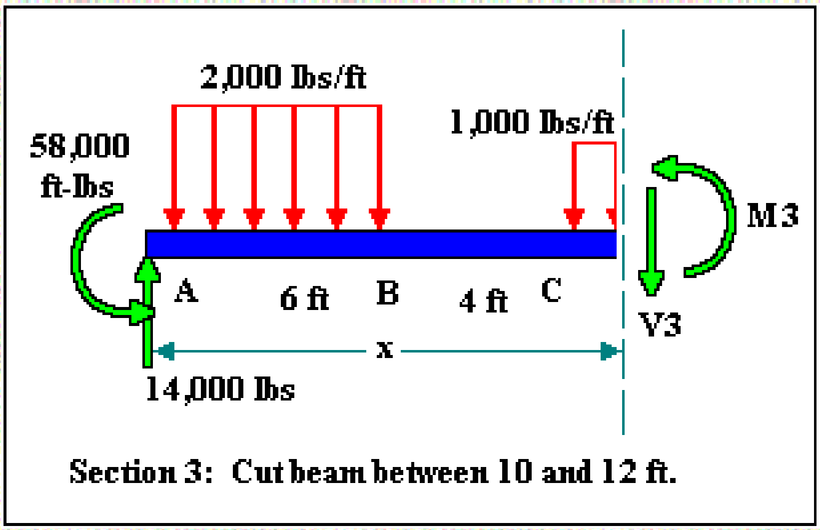
4. Integration $\boxed{M2 = \int V2\,dx = \int 2{,}000\,dx}$

M2 = 2,000x + C2

a)Boundary condition to find C2: at x=6 ft M=-10,000 ft-lbs (from equation M1)
Apply BC: -10,000 ft-lbs = 2,000(6) + C2

Solving: **C2 = -22,000 ft-lbs**
Therefore... **M2 = [2,000x - 22,000] ft-lbs** for 6 < x < 10

Section 3: Cut the beam at x, where 10 < x < 12 ft. Analyze left hand section.



Section 3: Cut beam between 10 and 12 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 14,000 lbs - (2,000 lbs/ft)(6 ft) - (1,000 lbs/ft)(x-10)ft - V3 = 0
Solving: **V3 = [-1,000x + 12,000] lbs**

4. Integration $\boxed{M3 = \int V3\,dx = \int [-1{,}000x + 12{,}000]\,dx}$

M3 = -500x^2 + 12,000x + C3

a)Boundary condition to find C3: at x=12 ft M=0 ft-lbs (free end of beam, no

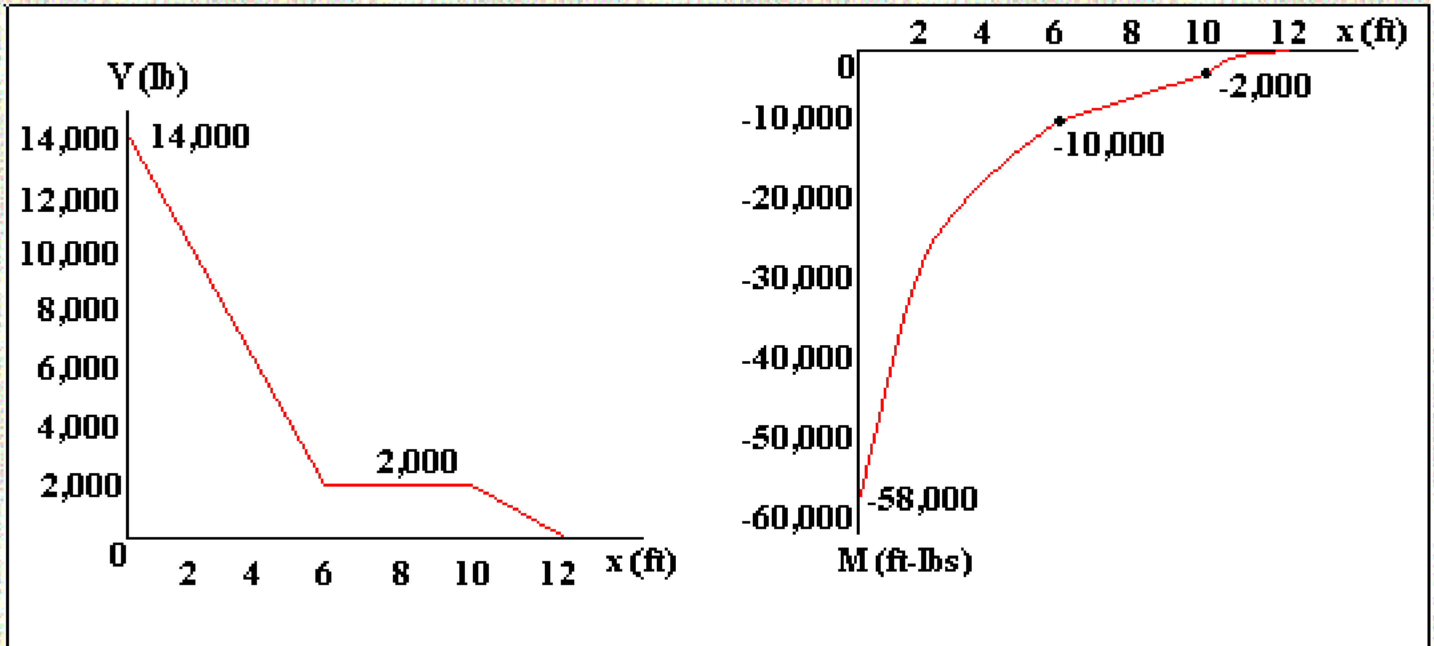
external torque so M3=0)
Apply BC: $0 = -500(12)^2 + 12{,}000(12) + C3$

Solving: **C3 = -72,000 ft-lbs**

Therefore... **M3 = [$-500x^2$ + 12,000x - 72,000] ft-lbs** for $10 < x < 12$

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = [-2,000x+14,000] lb V2 = 2,000 lb V3 = [-1,000x + 12,000] lb
M1 =[$-1{,}000x^2$+14,000x-58,000] ft-lb M2 = [2,000x-22,000] ft-lb M3 = [$-500x^2$ + 12,000x - 72,000] ft-lb

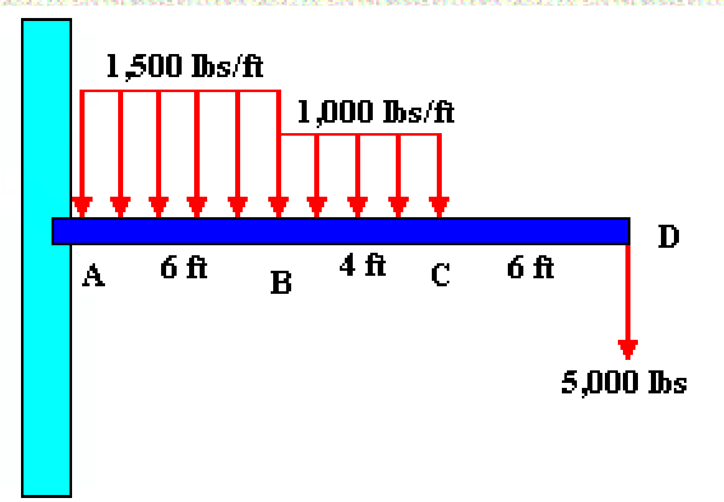


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

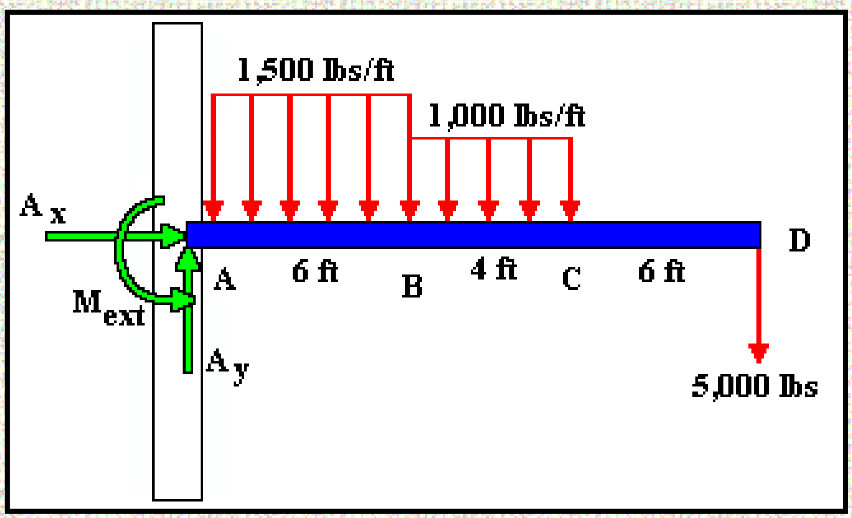
A loaded, cantilever beam is shown below. For this beam:

A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each section of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:



STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.

Sum F_y = A_x = 0

Sum F_y =(-1,500 lbs/ft)(6 ft) - (1,000 lbs/ft)(4 ft) - 5,000 lbs + A_y = 0

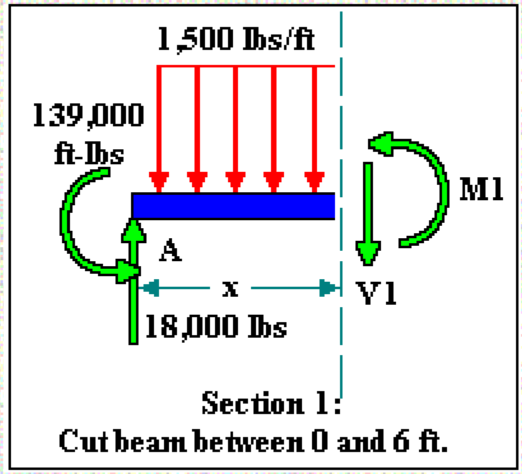
Sum T_A = (-1,500 lbs/ft)(6 ft)(3 ft) - (1,000 lbs/ft)(4 ft)(8 ft) - (5,000 lbs)(16 ft) +

$M_{ext} = 0$

Solving for the unknowns:
A_y = 18,000 lbs; M_{ext} = 139,000 ft-lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where 0 < x < 6 ft. Analyze left hand section.



Section 1: Cut beam between 0 and 6 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 18,000 lbs - 1,500 lbs/ft(x) - V1 = 0
Solving: **V1 = [-1,500x + 18,000] lbs**

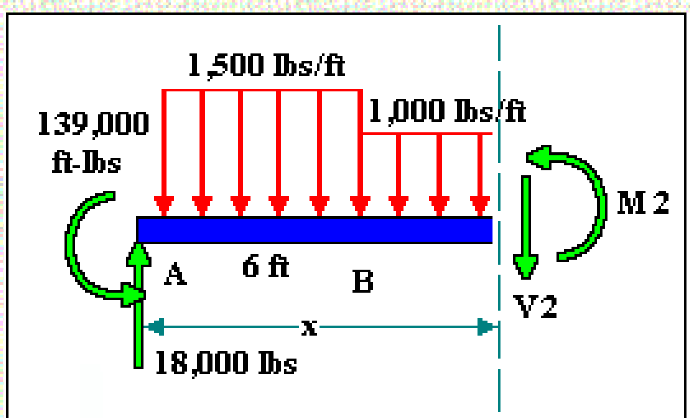
4. Integration $M1 = \int V1\,dx = \int [-1{,}500x + 18{,}000]\,dx$

$M1 = -750x^2 + 18{,}000x + C1$

a)Boundary condition to find C1: at x=0 M=-139,000 ft-lbs (That is, for a cantilever beam, the value of the bending moment at the wall is equal to the negative of the external moment.)
Apply BC: $-139{,}000 = -750(0)^2 + 18000(0) + C1$

Solving: **C1 = -139,000**
Therefore... **$M1 = [-750x^2 + 18{,}000x - 139{,}000]$ ft-lbs** for 0 < x < 6 ft.

Section 2: Cut the beam at x, where 6 < x < 10 ft. Analyze left hand section.



Section 2: Cut beam between 6 and 10 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)

Section 2: Cut beam between 6 and 10 ft.

Sum Fy = 18,000 lbs - (1,500 lbs/ft)(6 ft) - (1,000 lbs/ft)(x - 6)ft - V2 = 0
Solving: **V2 = [-1,000x + 15,000] lbs**

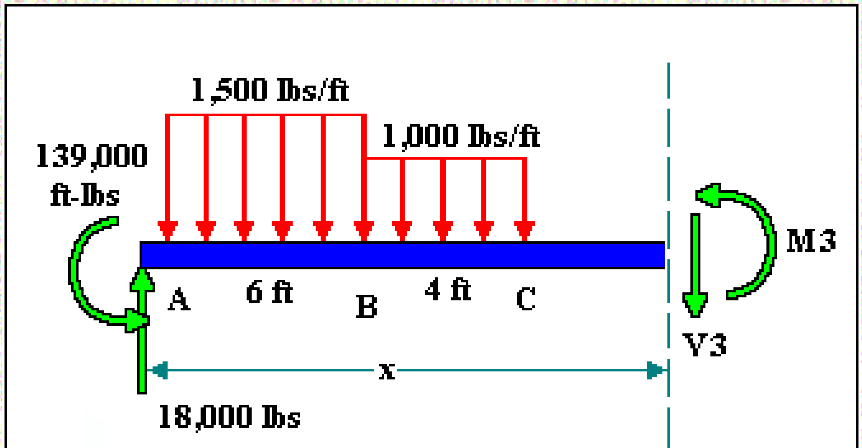
4. Integration $M2 = \int V2\,dx = \int [-1{,}000x + 15{,}000]\,dx$

M2 = -500x² + 15,000x + C2

a) Boundary condition to find C2: at x=6 ft M=-58,000 ft-lbs (from equation M1)
Apply BC: -58,000 ft-lbs = $-500(6)^2$ + 15,000(6) + C2

Solving: **C2 = -130,000 ft-lbs**
Therefore... **M2 = [$-500x^2$ + 15,000x - 130,000] ft-lbs** for 6 < x < 10

Section 3: Cut the beam at x, where 10 < x < 16 ft. Analyze left hand section.



Section 3: Cut beam between 10 and 16 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 18,000 lbs - (1,500 lbs/ft)(6 ft) - (1,000 lbs/ft)(4 ft) - V3 = 0
Solving: **V3 = 5,000 lbs**

4. Integration $M3 = \int V3\,dx = \int 5{,}000\,dx$

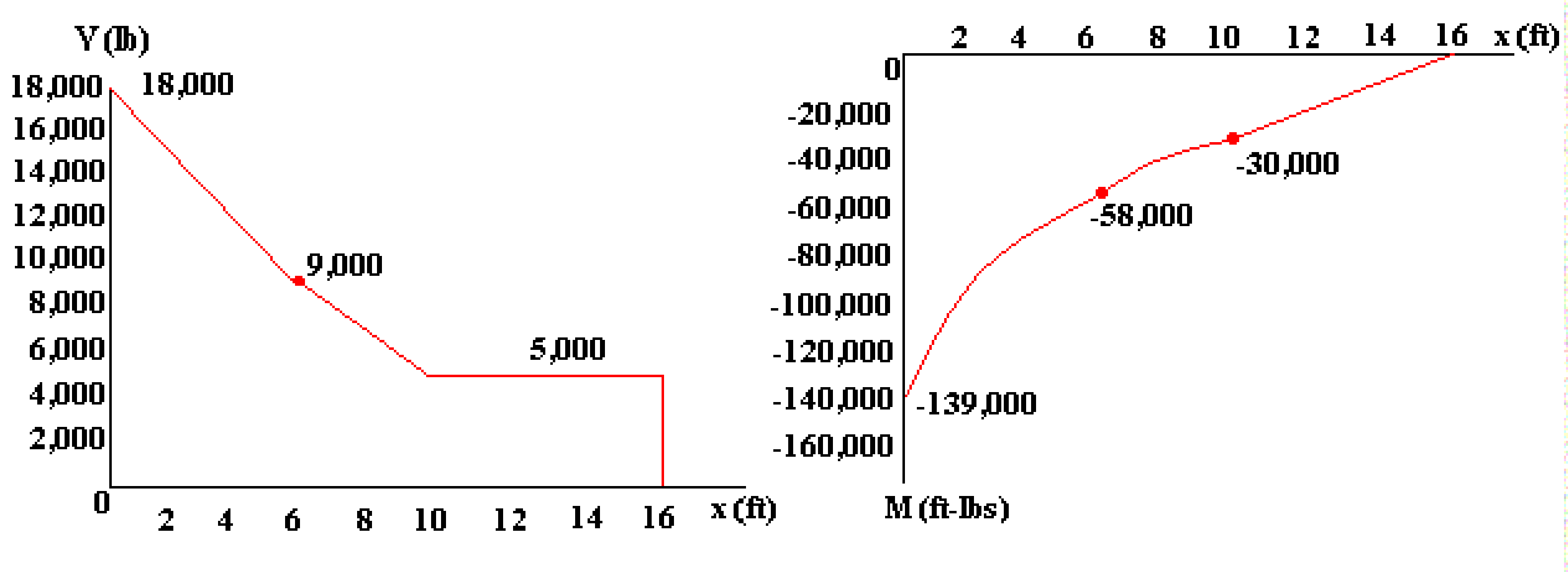
M3 =5,000x + C3

a) Boundary condition to find C3: at x=16 ft M=0 ft-lbs (free end of beam, no external torque so M3=0)
Apply BC: 0 = 5,000(16) + C3

Solving: **C3 = -80,000 ft-lbs**
Therefore... **M3 = [5,000x - 80,000] ft-lbs** for 10 < x < 16

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = [-1,500x+18,000] lb V2 = [-1,000x+15,000] lb V3 = 5,000 lb
M1 =[$-750x^2$+18,000x-139,000] ft-lb M2 = [$-500x^2$+15,000x-130,000] ft-lb
M3 = [5,000x - 80,000] ft-lb

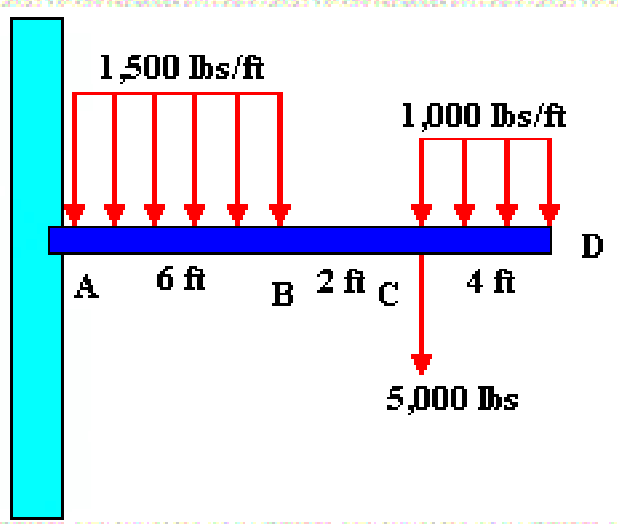


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

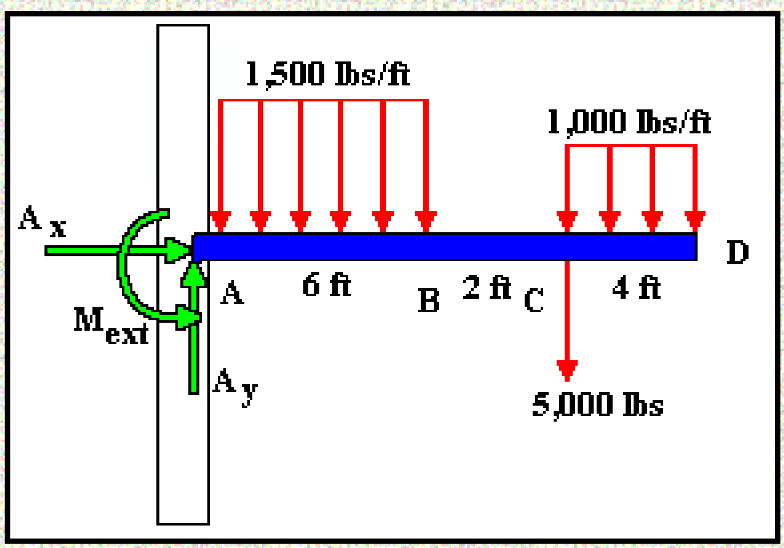
A loaded, cantilever beam is shown below. For this beam:

A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each section of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:
STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



Sum F_x = + A_x = 0

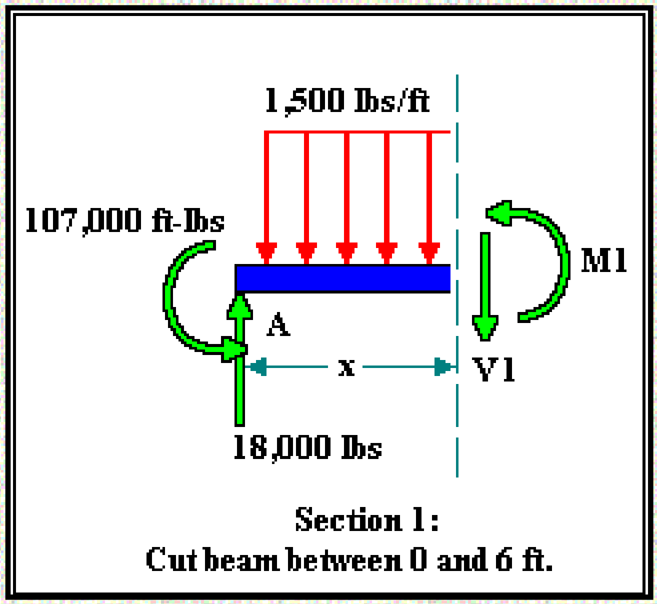
Sum F_y =(-1,500 lbs/ft)(6 ft) - (1,000 lbs/ft)(4 ft) - 5,000 lbs + A_y = 0

Sum T_A = (-1,500 lbs/ft)(6 ft)(3 ft) - (1,000 lbs/ft)(4 ft)(10 ft) - (5,000 lbs)(8 ft) + M_{ext} = 0

Solving for the unknowns:
A_y = 18,000 lbs; M_{ext} = 107,000 ft-lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where 0 < x < 6 ft. Analyze left hand section.



Section 1:
Cut beam between 0 and 6 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 18,000 lbs - 1,500 lbs/ft(x) - V1 = 0

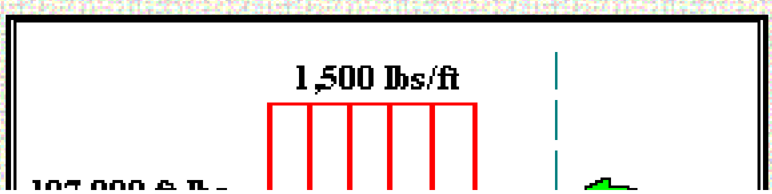
Solving: **V1 = [-1,500x + 18,000] lbs**

4. Integration $M1 = \int V1\,dx = \int [-1{,}500x + 18{,}000]\,dx$

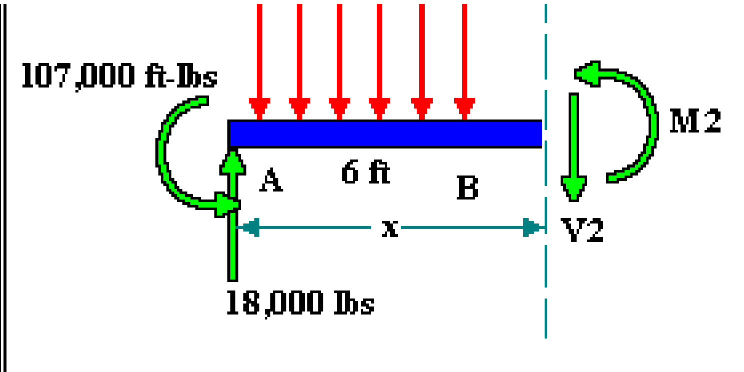
$M1 = -750x^2 + 18{,}000x + C1$

a) Boundary condition to find C1: at x=0 M=-107,000 ft-lbs (That is, for a cantilever beam, the value of the bending moment at the wall is equal to the negative of the external moment.)
Apply BC: $-107{,}000 = -750(0)^2 + 18000(0) + C1$
Solving: **C1 = -107,000**
Therefore... **$M1 = [-750x^2 + 184{,}000x - 107{,}000]$ ft-lbs** for 0 < x < 6 ft.

Section 2: Cut the beam at x, where 6 < x < 8 ft. Analyze left hand section.



1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):



Section 2: Cut beam between 6 and 8 ft.

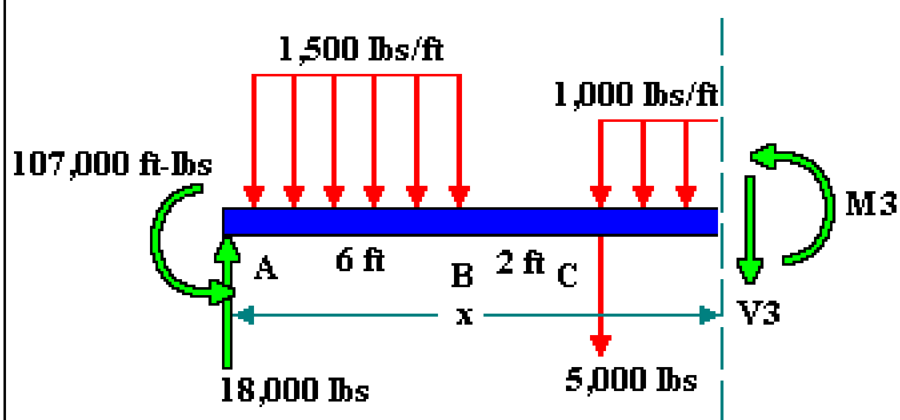
Sum Fx = 0 (no net external x- forces)
Sum Fy = 18,000 lbs - (1,500 lbs/ft)(6 ft) - V2 = 0
Solving: **V2 = 9,000 lbs**

4. Integration $M2 = \int V2\, dx = \int 9{,}000\, dx$

M2 = 9,000x + C2

a)Boundary condition to find C2: at x=6 ft M=-26,000 ft-lbs (from equation M1)
Apply BC: -26,000 ft-lbs = 9,000(6) + C2
Solving: **C2 = -80,000 ft-lbs**
Therefore... **M2 = [9,000x - 80,000] ft-lbs** for 6 < x < 8

Section 3: Cut the beam at x, where 8 < x < 12 ft. Analyze left hand section.



Section 3: Cut beam between 8 and 12 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 18,000 lbs - (1,500 lbs/ft)(6 ft) -5,000 lbs - (1,000 lbs/ft)(x - 8)ft - V3 = 0
Solving: **V3 = [-1,000x + 12,000] lbs**

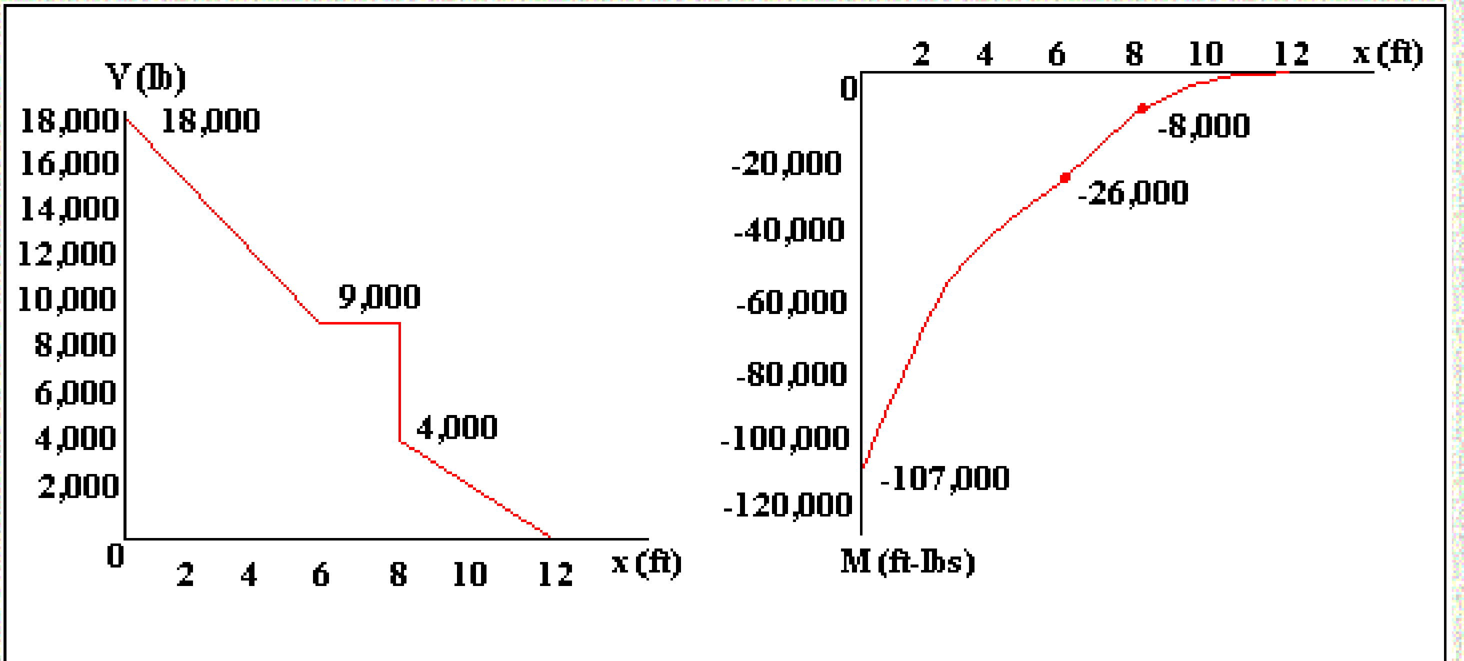
4. Integration $M3 = \int V3\, dx = \int [-1{,}000x + 12{,}000]\, dx$

M3 =-500x^2 + 12,000x + C3

a) Boundary condition to find C3: at x=12 ft M=0 ft-lbs (free end of beam, no external torque so M3=0)
Apply BC: $0 = -500(12)^2 + 12{,}000(12) + C3$
Solving: **C3 = -72,000 ft-lbs**
Therefore... **$M3 = [500x^2 + 12{,}000x - 72{,}000]$ ft-lbs** for $8 < x < 12$

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

$V1 = [-1{,}500x+18{,}000]$ lb $V2 = 9{,}000$ lb $V3 = [-1{,}000x+12{,}000]$ lb
$M1 = [-750x^2+18{,}000x-107{,}000]$ ft-lb $M2 = [9{,}000x-80{,}000]$ ft-lb $M3 = [-500x^2+12{,}000x-72{,}000]$ ft-lb

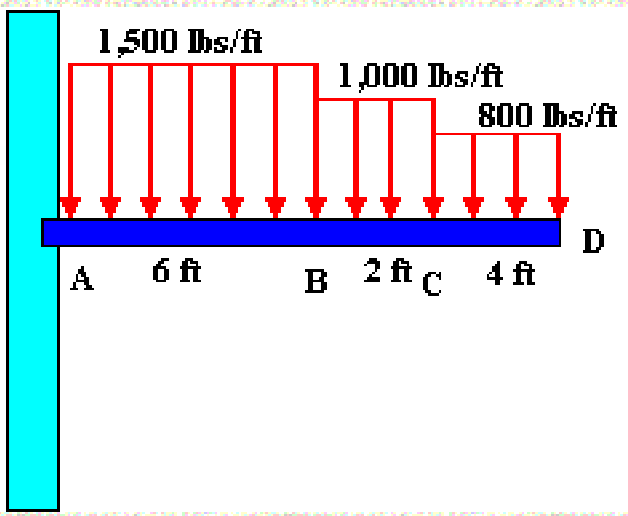


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

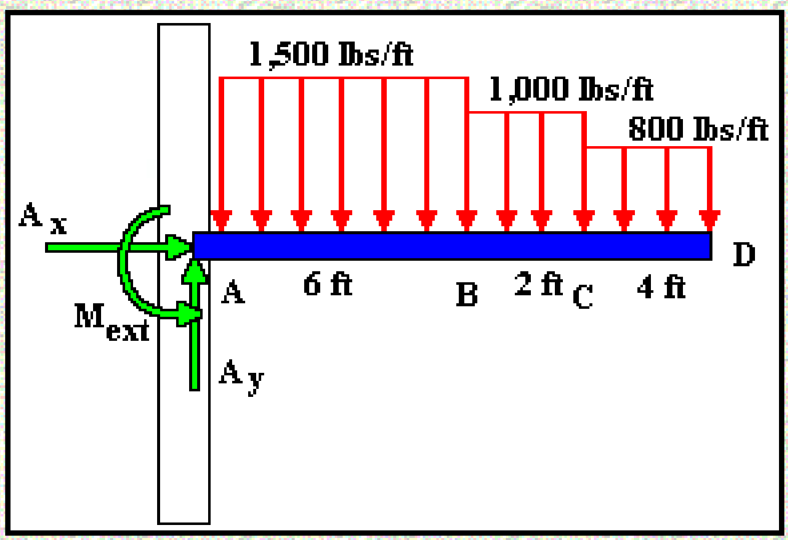
A loaded, cantilever beam is shown below. For this beam:

A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each section of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:
STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



Sum $F_x = A_x = 0$

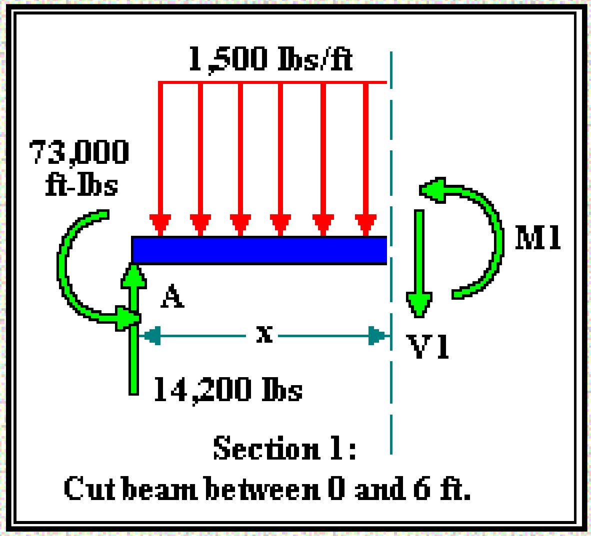
Sum F_y = (-1,500 lbs/ft)(6 ft) - (1,000 lbs/ft)(2 ft) - (800 lbs/ft)(4 ft) + A_y = 0

Sum T_A = (-1,500 lbs/ft)(6 ft)(3 ft)-(1,000 lbs/ft)(2 ft)(7 ft)-(800 lbs/ft)(4 ft)(10 ft) + M_{ext} = 0

Solving for the unknowns:
A_y = 14,200 lbs; M_{ext} = 73,000 ft-lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.

Section 1: Cut the beam at x, where 0 < x < 6 ft. Analyze left hand section.



1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

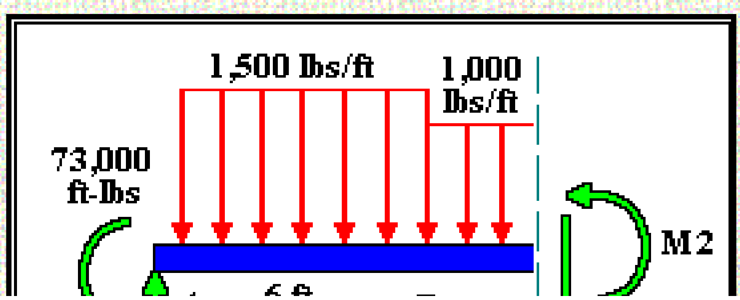
Sum Fx = 0 (no net external x- forces)
Sum Fy = 14,200 lbs - 1,500 lbs/ft(x) - V1 = 0
Solving: **V1 = [-1,500x + 14,200] lbs**

4. Integration $\boxed{M1 = \int V1\,dx = \int [-1{,}500x + 14{,}200]\,dx}$

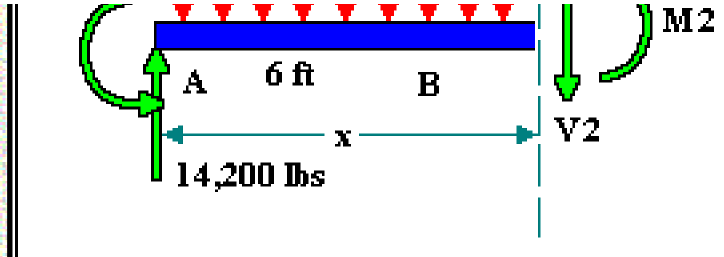
$M1 = -750x^2 + 14{,}200x + C1$

a) Boundary condition to find C1: at x=0 M=-73,000 ft-lbs (That is, for a cantilever beam, the value of the bending moment at the wall is equal to the negative of the external moment.)
Apply BC: $-73{,}000 = -750(0)^2 + 14200(0) + C1$
Solving: **C1 = -73,000**
Therefore... **$M1 = [-750x^2 + 14{,}200x - 73{,}000]$ ft-lbs** for 0 < x < 6 ft.

Section 2: Cut the beam at x, where 6 < x < 8 ft. Analyze left hand section.



1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)



Section 2: Cut beam between 6 and 8 ft.

3. Apply translational equilibrium conditions (forces only):

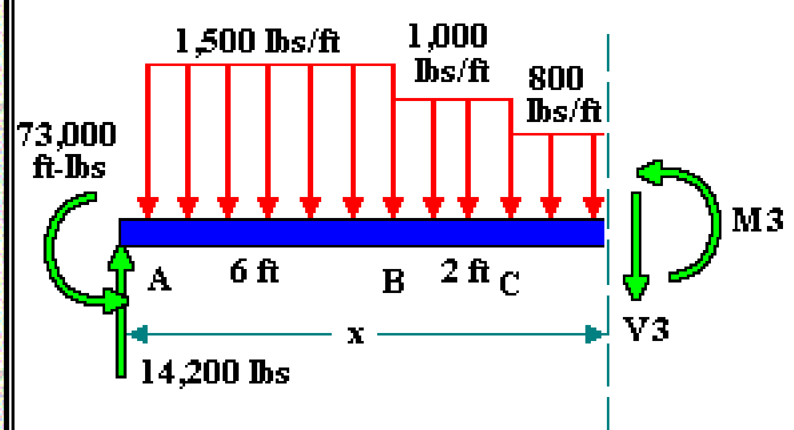
Sum Fx = 0 (no net external x- forces)
Sum Fy = 14,200 lbs - (1,500 lbs/ft)(6 ft) - (1,000 lbs/ft)(x - 6)ft - V2 = 0
Solving: **V2 = [-1,000x + 11,200] lbs**

4. Integration $M2 = \int V2\, dx = \int [-1{,}000x + 11{,}200]\, dx$

M2 = -500x^2 + 11,200x + C2

a)Boundary condition to find C2: at x=6 ft M=-14,800 ft-lbs (from equation M1)
Apply BC: -14,800 ft-lbs = -500(6)^2 + 11,200(6) + C2
Solving: **C2 = -64,000 ft-lbs**
Therefore... **M2 = [-500x^2 + 11,200x - 64,000] ft-lbs** for 6 < x < 8

Section 3: Cut the beam at x, where 8 < x < 12 ft. Analyze left hand section.



Section 3: Cut beam between 8 and 12 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 14,200 lbs - (1,500 lbs/ft)(6 ft) - (1,000 lbs/ft)(2 ft) - (800 lbs/ft)(x-8)ft - V3 = 0
Solving: **V3 = [-800x + 9,600] lbs**

4. Integration $M3 = \int V3\, dx = \int [-800x + 9{,}600]\, dx$

M3 =-400x^2 + 9,600x + C3

a)Boundary condition to find C3: at x=12 ft M=0 ft-lbs (free end of beam, no external torque so M3=0)
Apply BC: 0 = -400(12)^2 + 9,600(12) + C3
Solving: **C3 = -57,600 ft-lbs**

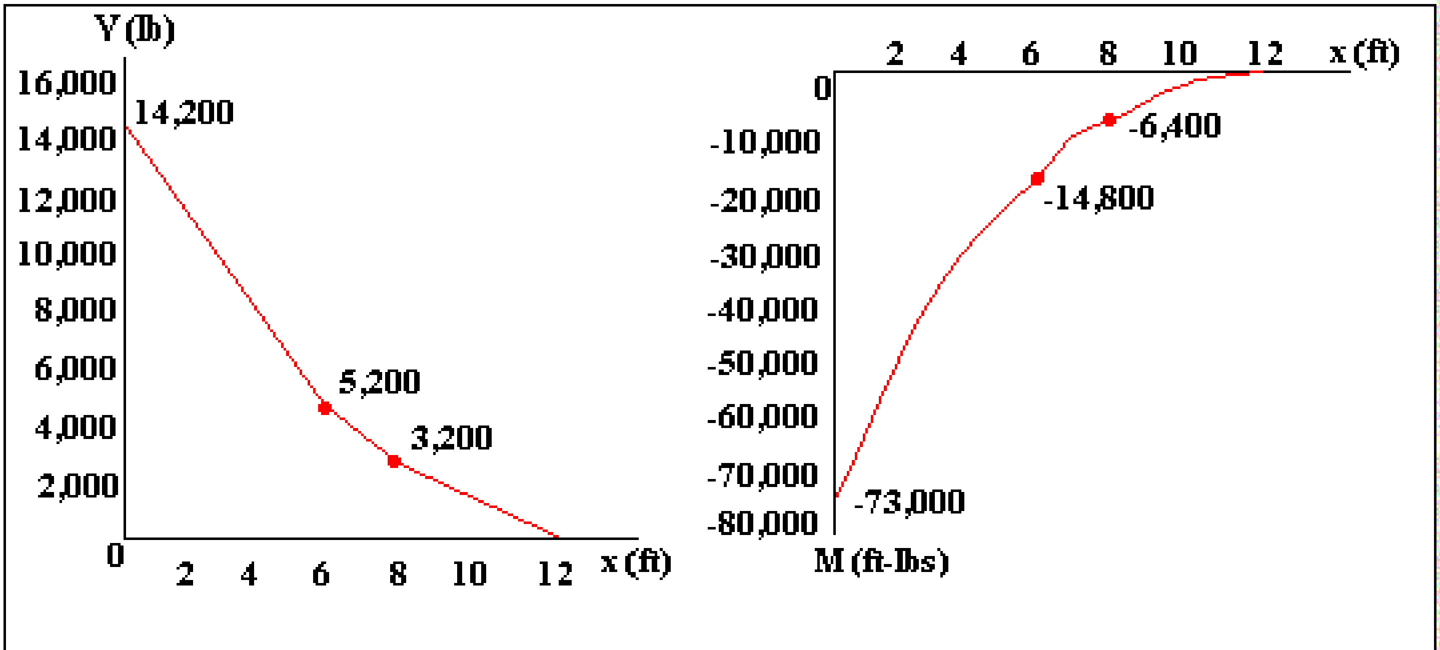
Therefore... **$M3 = [-400x^2 + 9,600x - 57,600]$ ft-lbs** for $8 < x < 12$

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

$V1 = [-1,500x + 14,200]$ lb $V2 = [-1,000x + 11,200]$ lb $V3 = [-800x + 9,600]$ lb

$M1 = [-750x^2 + 14,200x - 73,000]$ ft-lb $M2 = [-500x^2 + 11,200x - 64,000]$ ft-lb

$M3 = [-400x^2 + 9,600x - 57,600]$ ft-lb

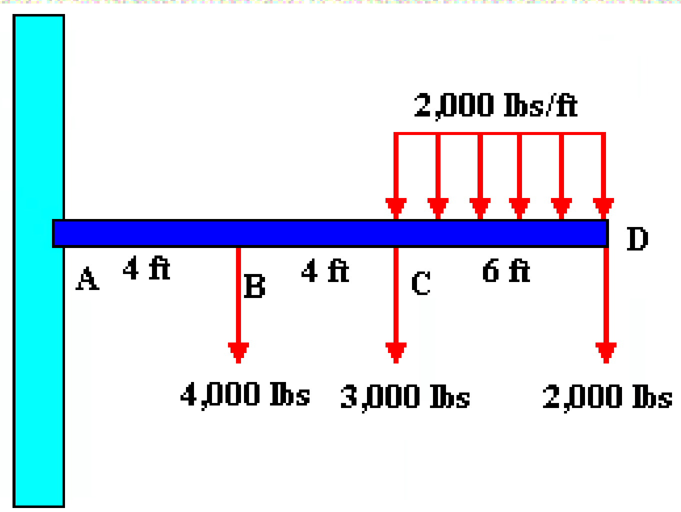


Shear Force & Bending Moment Diagrams

STATICS & STRENGTH OF MATERIALS - Example

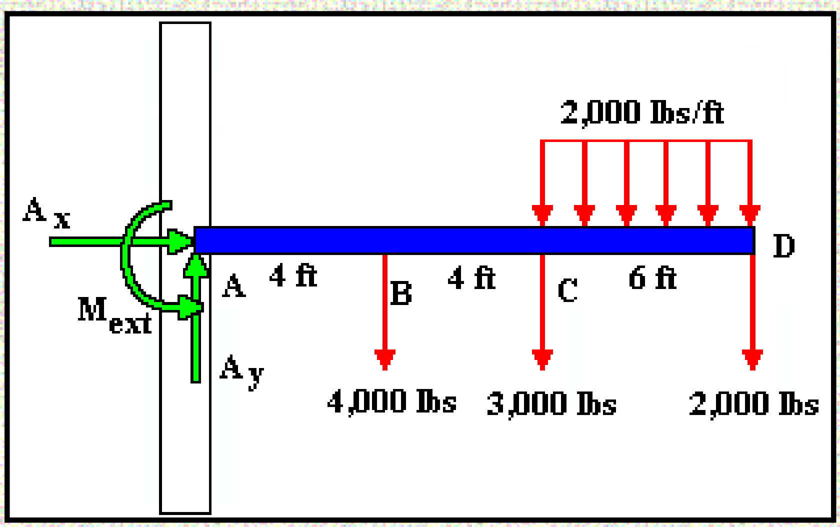
A loaded, cantilever beam is shown below. For this beam:

A. Draw a Free Body Diagram of the beam, showing all external loads and support forces (reactions).
B. Determine expressions for the internal shear forces and bending moments in each section of the beam.
C. Make shear force and bending moment diagrams for the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:

Part A:
STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
STEP 2: Break any forces not already in x and y direction into their x and y components.
STEP 3: Apply the equilibrium conditions.



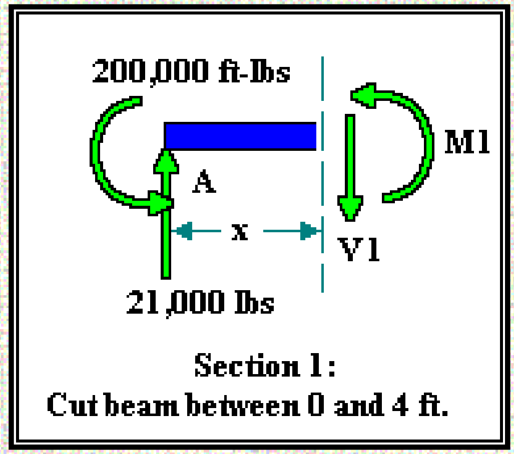
Sum $F_x = A_x = 0$

Sum F_y = -4,000 lbs - 3,000 lbs - (2,000 lbs/ft)(6 ft) - 2,000 lbs + A_y = 0

Sum T_A = (-4,000 lbs)(4 ft) - (3,000 lbs)(8 ft) -(2,000 lbs/ft)(6 ft)(11 ft) - (2,000 lbs)(14 ft) + M_{ext} = 0

Solving for the unknowns:
A_y = 21,000 lbs; M_{ext} = 200,000 ft-lbs

Part B: Determine the Shear Forces and Bending Moments expressions for each section of the loaded beam. For this process we will 'cut' the beam into sections, and then use Statics - Sum of Forces to determine the Shear Force expressions, and Integration to determine the Bending Moment expressions in each section of the beam.



Section 1: Cut the beam at x, where 0 < x < 4 ft. Analyze left hand section.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

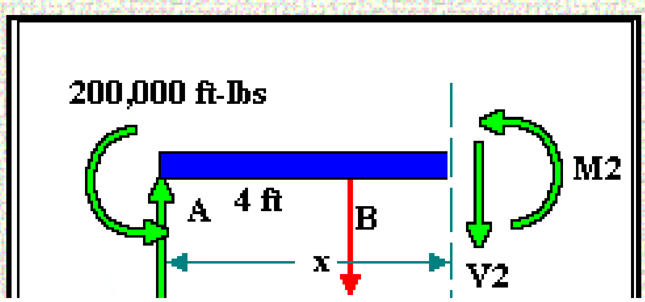
Sum Fx = 0 (no net external x- forces)
Sum Fy = 21,000 lbs - V1 = 0
Solving: **V1 = 21,000 lbs**

4. Integration $\boxed{M1 = \int V1 \, dx = \int 21{,}000 \, dx}$

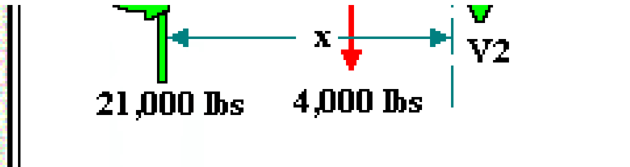
M1 = 21,000x + C1

a) Boundary condition to find C1: at x=0 M=-200,000 ft-lbs (That is, for a cantilever beam, the value of the bending moment at the wall is equal to the negative of the external moment.)
Apply BC: -200,000 = 21000(0) + C1
Solving: C1 = -200,000
Therefore... **M1 = [21,000x - 200,000] ft-lbs** for 0 < x < 4 ft.

Section 2: Cut the beam at x, where 4 < x < 8 ft. Analyze left hand section.



1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)



Section 2: Cut beam between 4 and 8 ft.

3. Apply translational equilibrium conditions (forces only):

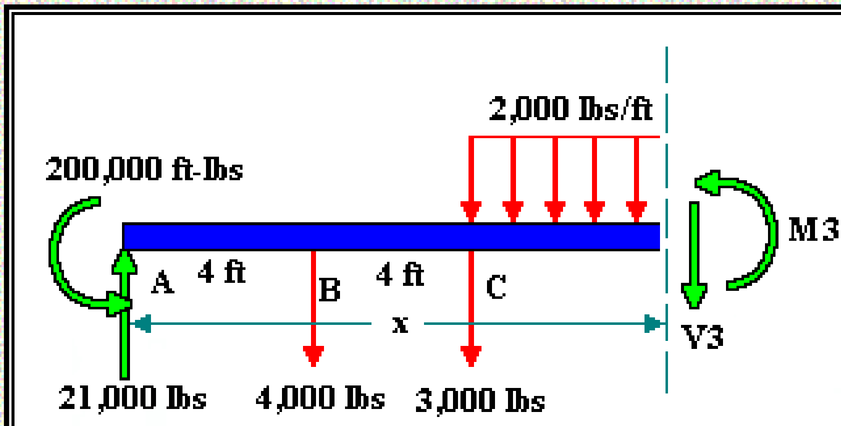
Sum Fx = 0 (no net external x- forces)
Sum Fy = 21,000 lbs - 4,000 lbs - V2 = 0
Solving: **V2 = 17,000 lbs**

4. Integration $M2 = \int V2\, dx = \int 17{,}000\, dx$

M2 = 17,000x + C2

a)Boundary condition to find C2: at x=4 ft M=-116,000 ft-lbs (from equation M1)
Apply BC: -116,000 ft-lbs = 17,000(4) + C2
Solving: **C2 = -184,000 ft-lbs**
Therefore... **M2 = [17,000x - 184,000] ft-lbs** for 4 < x < 8

Section 3: Cut the beam at x, where 8 < x < 14 ft. Analyze left hand section.



Section 3: Cut beam between 8 and 14 ft.

1. FBD. (Shown in Diagram)
2. All forces in x & y components (yes)
3. Apply translational equilibrium conditions (forces only):

Sum Fx = 0 (no net external x- forces)
Sum Fy = 21,000 lbs - 4,000 lbs - 3,000 lbs - (2,000 lbs/ft)(x-8)ft - V3 = 0
Solving: **V3 = [-2,000x + 30,000] lbs**

4. Integration $M3 = \int V3\, dx = \int [-2{,}000x + 30{,}000]\, dx$

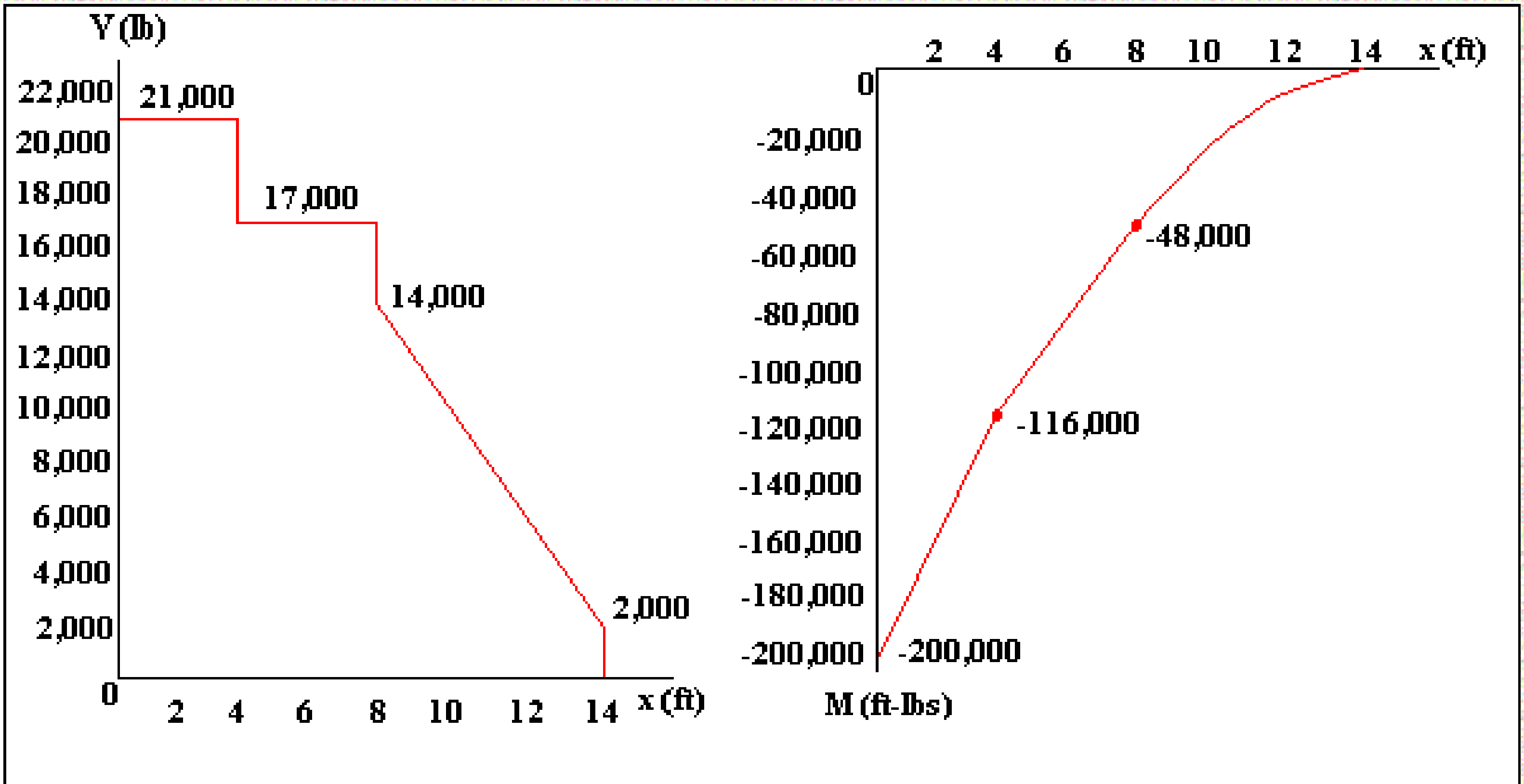
M3 =-1,000x^2 + 30,000x + C3

a)Boundary condition to find C3: at x=14 ft M=0 ft-lbs (free end of beam, no external torque so M3=0)
Apply BC: 0 = -1,000$(14)^2$ + 30,000(14) + C3
Solving: **C3 = -224,000 ft-lbs**
Therefore... **M3 = [-1,000x^2 +30,000x - 224,000] ft-lbs** for 8 < x < 14

Part C: Shear Force and Bending Moment Diagrams: Now using the expressions found in Part B above, we can draw the shear force and bending moment diagrams for our loaded beam.

V1 = 21,000 lb V2 = 17,000 lb V3 = [-2,000x+30,000] lb

M1 =[21,000x-200,000] ft-lb M2 = [17,000x-184,000] ft-lb M3 = [-1,000x^2 +30,000x - 224,000] ft-lb



Shear Force & Bending Moment Diagrams

Statics & Strength of Materials

Topic 4.4a1 Problem Assignment - Shear Forces & Bending Moments 1

For the loaded, simply supported beams described below:
A. Draw a Free Body Diagram of the beam, and determine the external support reactions.
B. Find expressions for the Shear Forces and Bending Moments in each section of the beams.
C. Draw the Shear Force and Bending Moment Diagrams for each beam. Use a sheet of graph paper, use topic half for shear diagram and the bottom half for the bending moment diagram.
Beams in these problems are considered weightless.

1. A 10 foot beam is simply supported at its ends. A load of 500 pounds is placed 4 feet from the left end.
V1=300 lb, M1= 300x ft-lb; V2=-200 lb, M2 = -200x + 2000 ft-lb

2. An 8 foot beam is simply supported at its ends. It has a uniformly distributed load of 200 pounds per foot along its entire length. V1 = 800 - 200 x lb, M1 = 800x -100 x^2 ft-lb.

3. A 24 foot beam is simply supported at the quarter points (6 feet and 18 feet). It has point loads of 800 pounds at each end and a point load of 1000 points at its mid point. V1= - 800 lb, M1= - 800x ft-lb; V2 = 500 lb, M2 = 500 x -7800 ft-lb; V3 = - 500 lb, M3 = - 500x +4200 ft-lb; V4 = 800 lb, M4 = 800x - 19,200 ft-lb

4. A 16 foot beam is simply supported at its ends. It has a uniformly distributed load of 400 pounds per foot over the first 6 feet and a point load of 1200 pounds at the 10 foot mark. V1 = 2400 - 400x lb, M1 = 2400 x - 200x^2 ft-lb; V2 = 0 lb, M2 = 7200 ft-lb; V3 = -1200 lb, M3 = 1200 x + 19200 ft-lb

5. A diving board is 16 feet long and pinned at the left end. It has a roller support at the 6 foot mark. A 160 pound person stands at the right end of the board. V1 = -267 lb, M1 = -267 x ft-lb; V2 = 160 lb, M2 = 160 x -2560 ft-lb

6. A 16 foot beam is simply supported at its left end and its midpoint. It has a distributed load of 500 pounds per foot between the supports and a point load of 2000 pounds at the right end.
V1 = - 500 x lb, M1 = -250x^2 ft-lb; V2 = 2000 lb, M2 = 2000 x - 32,000 ft-lb.

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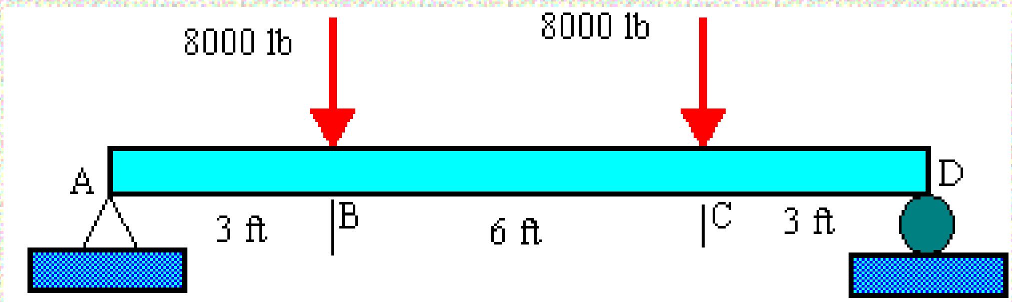
Topic 4.4a2 Problem Assignment - Shear Forces & Bending Moments 2

For the loaded, simply supported beams shown in the diagrams below:
A. Draw a Free Body Diagram of the beam, and determine the external support reactions.
B. Find expressions for the Shear Forces and Bending Moments in each section of the beams.
C. Draw the Shear Force and Bending Moment Diagrams for each beam. Use a sheet of graph paper, use topic half for shear diagram and the bottom half for the bending moment diagram.

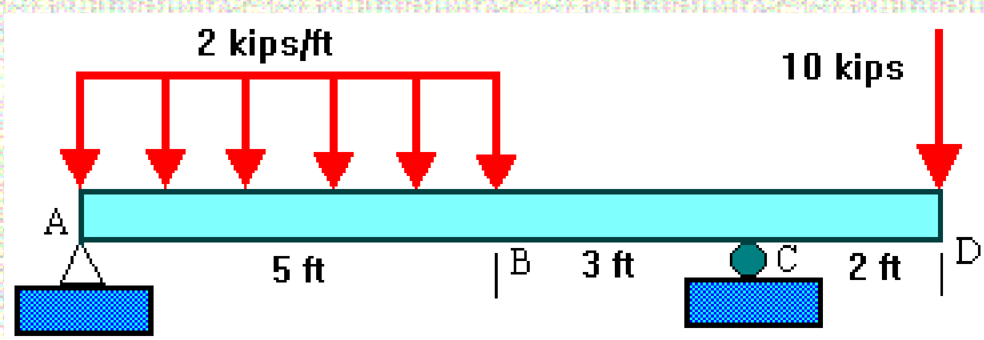
1.



A_y = 8,000 lb, D_y = 8,000 lb.

V1 = 8,000 lb., M1 = 8,000 x
V2 = 0 lb., M2 = 24,000 ft-lb
V3 = -8,000 lb.; M3 = - 8,000x + 96,000 ft.-lb.

2.

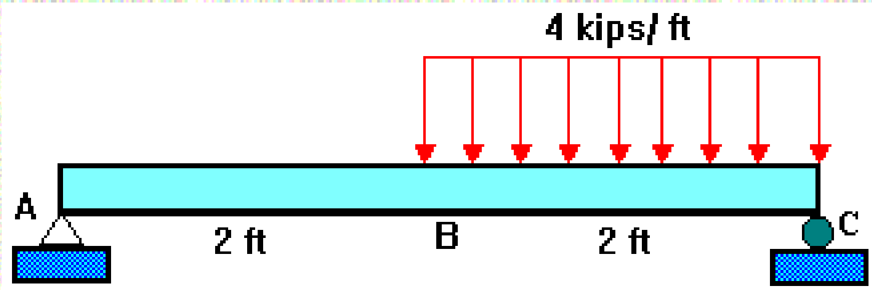


A_y = 4,400 lb. C_y = 15,600 lb.

V1 = -2,000x + 4,400 lb.; M1 = -1,000 x^2 + 4,400x + 0
V2 = -5,600 lb., M2 = - 5,600x + 25,000 ft.-lb.

V3 = 10,000 lb., M3 = 10,000x - 99,800 ft.-lb.

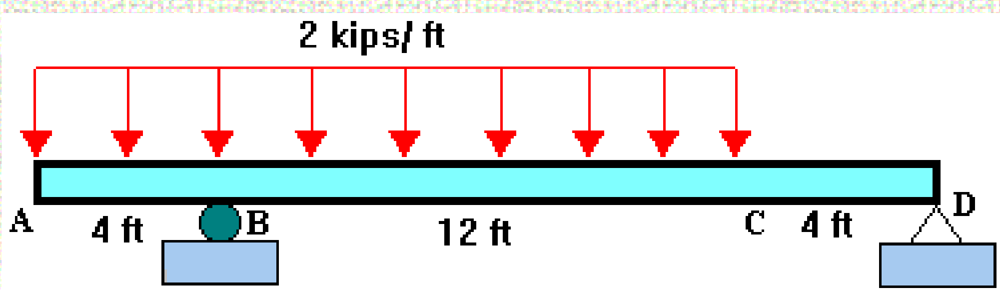
3.



A_y = 2,000 lb. C_y = 6,000 lb.

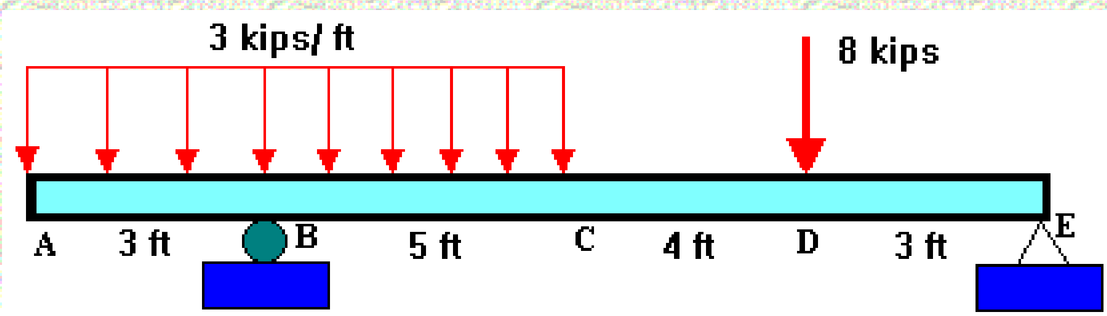
V1 = 2,000 lb., M1 = 2,000x ft.-lb.
V2 = -4,000x + 10,000 lb., M2 = $-2{,}000x^2$ + 10,000x - 8,000 ft.lb.

4.



B_y = 24,000 lb. D_y = 8,000 lb.

V1 = -2,000x lb.,M1 = $-1{,}000\ x^2$ ft.-lb.
V2 = -2,000x + 24,000 lb.; M2 = $-1{,}000\ x^2$ + 24,000x - 96,000 ft.-lb.
V3 = 8,000 lb.,M3 = -8,000x + 160,000 ft.-lb.5.



B_y = 24,000 lb. E_y = 8,000 lb.

V1 = -3,000x lb.,M1 = $-1{,}500\ x^2$ ft.-lb.
V2 = -3,000x + 24,000 lb., M2 = $-1{,}500\ x^2$ + 24,000x - 72,000 ft.-lb.

$V3 = 0$, $M3 = 24,000$ ft.-lb.
$V4 = -8,000$lb.,$M4 = -8,000x + 120,000$ ft.-lb.

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Statics & Strength of Materials

Topic 4.4b1 Problem Assignment - Shear Forces & Bending Moments 1

For the loaded, simply supported beams shown in the diagrams below:
A. Draw a Free Body Diagram of the beam, and determine the external support reactions.
B. Find expressions for the Shear Forces and Bending Moments in each section of the beams.
C. Draw the Shear Force and Bending Moment Diagrams for each beam. Use a sheet of graph paper, use topic half for shear diagram and the bottom half for the bending moment diagram.
Beams in these problems are considered weightless.

1. A 10 foot cantilever carries a point load of 800 pounds at its free end. [V= 800 lb., M = 800x -8000 ft-lb.]

2. A 12 foot cantilever carries a load of 1200 pounds 4 feet from the wall and a load of 600 pounds at its end, 12 feet from the wall. [V1 = 1800 lb., M1 = 1800x - 12,000 ft-lb., V2 = 600 lb., M2 = 600x - 7200 ft-lb.]

3. A 10 foot cantilever has a uniformly distributed load of 400 pounds per foot over its entire length. [V = 4000 - 400x lb., M = $-2000x^2$ +4000x - 20,000 ft-lb.]

4. A 16 foot cantilever has a uniformly distributed load of 400 pounds per foot from the wall to the 10 foot mark and a load of 200 pounds per foot from that point to the end of the beam. [V1 = 5200 - 400x lb., M1 = $-200x^2$ +5200x - 35,600 ft-lb., V2 = 3200-200x lb., M2 = $-100x^2$+3200x - 25,600 ft-lb.]

5. A 6 foot cantilever has a point load of 600 pounds at its mid point and a uniformly distributed load of 100 pounds per foot from the midpoint to the free end. [V1 = 900 lb., M1 = 900x - 3150 ft-lb., V2=600-100x lb., M2 = -50 x^2 + 600x - 1800 ft-lb.]

6. An 8 foot cantilever was designed to carry a point load of 800 pounds at its free end. The beam joint of the beam with the wall was found to be failing and a prop was placed 4 feet from the wall that exerted a 400 pound upward point force on the beam. [V1 = 400 lb., M1 = 400x -4800 ft-lb., V2=800 lb., M2 = 800x - 6400 ft-lb.]

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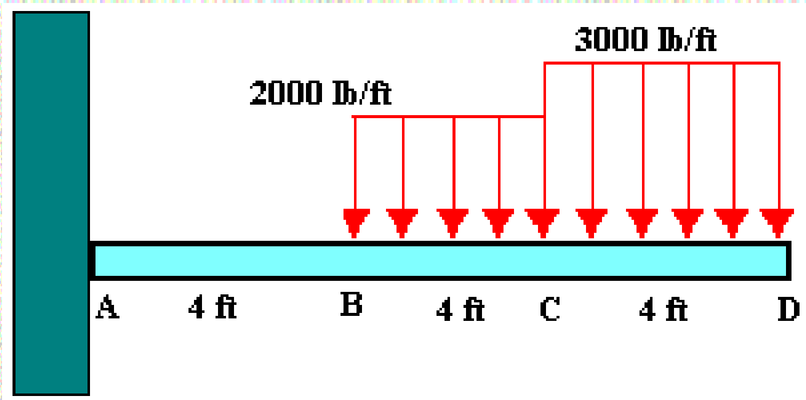
Topic 4.4b2 Problem Assignment - Shear Forces & Bending Moments 2

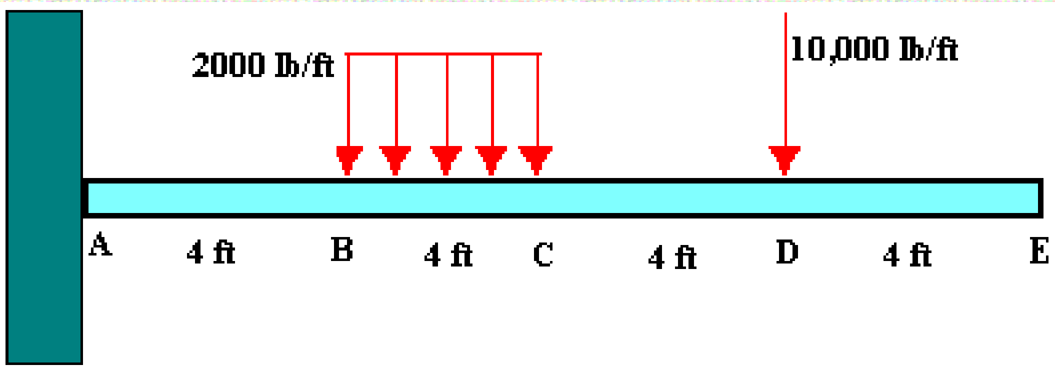
For the loaded, cantelever beams shown in the diagrams below:
A. Draw a Free Body Diagram of the beam, and determine the external support reactions.
B. Find expressions for the Shear Forces and Bending Moments in each section of the beams.
C. Draw the Shear Force and Bending Moment Diagrams for each beam. Use a sheet of graph paper, use topic half for shear diagram and the bottom half for the bending moment diagram.

1.



A_y = 20,00 lb. Mext= 168,000 ft-lb., V1 = 20,000 lb., M1 = 20,000x - 168,000 ft.-lb.
V2 = 28,000 - 2,000x lb., M2 = 28,000x - 1,000x^2 - 184,000 ft.-lb.
V3 = 36,000-3000x lb., M3 = 36,000 x -1500x^2-216,000 ft.-lb.

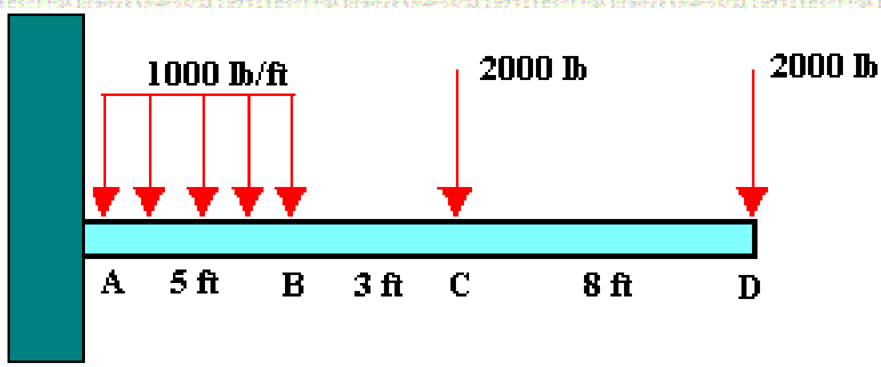
2



A_y = 18,000 lb. M_{ext} = 168,000 ft.-lb.

V1 = 18,000 lb., M1 = 18,000x - 168,000 ft.-lb.
V2 = 26,000 - 2,000x lb.,M2 = 26,000x - 1,000 x^2 - 184,000 ft.-lb.
V3 = 10,000 lb.,M3 = 10,000x -120,000 ft.-lb.
V4 = 0, M4 = 0

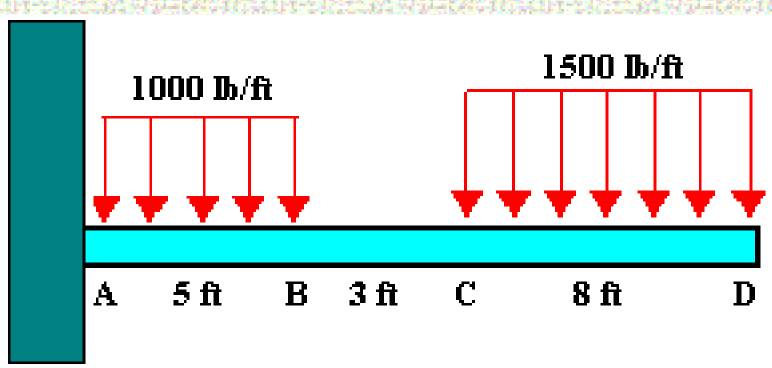
3.



A_y = 9,000 lb., M_{ext} = 60,500 ft/lb.

V1 = - 1,000x + 9,000 lb.,M1 = -500x^2 + 9,000x - 60,500 ft/lb.
V2 = 4,000 lb.,M2 = 4,000x - 48,000 ft/lb.
V3 = 2,000 lb.,M3 = 2,000x - 32,000 ft/lb.

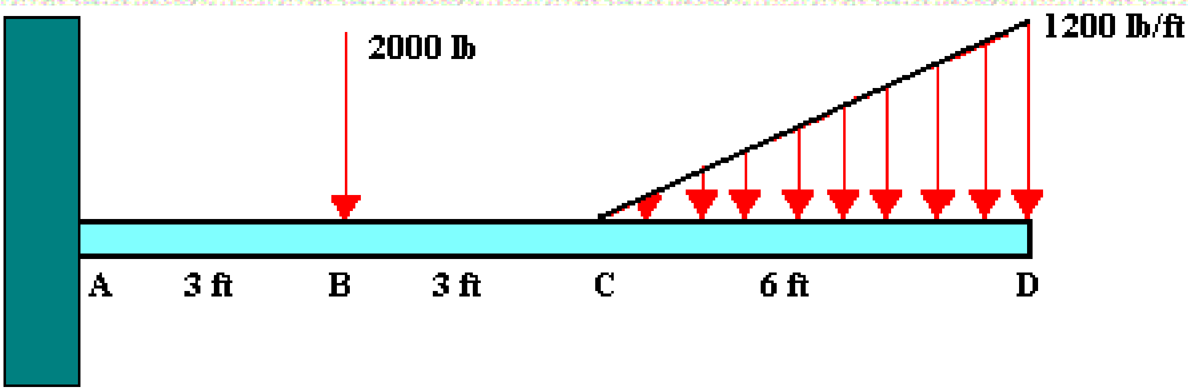
4.



A_y = 17,000 lb. M_{ext} = 156,500 ft.-lb.

V1 = 1,000x + 17,000 lb.,M1 = - 500x^2 + 17,000x - 156,000 ft.-lb. ft.-lb.
V2 = 12,000 lb.,M2 = 12,000x - 143,500 ft.-lb.
V3 = -1,500 + 24,000 lb.,M3 = -750x^2 + 24,000 - 192,000 ft.-lb.

5.



A_y = 5,600 lb. M_{ext} = 42,000 ft.-lb.

V1 = 5,600,M1 = [5,600x - 42,000] ft.-lb.
V2 = 3,600 lb., M2 = [3,600x - 36,000] ft.-lb.
V3 = [3,600 - 100$(x-6)^2$] or V3 = [-100x^2 + 1,200x]lb., M3 = [-33.3x^3 + 600x^2 - 28,800] ft.-lb.

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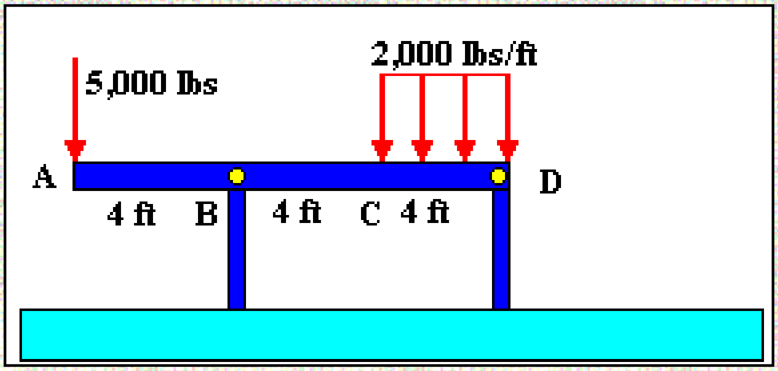
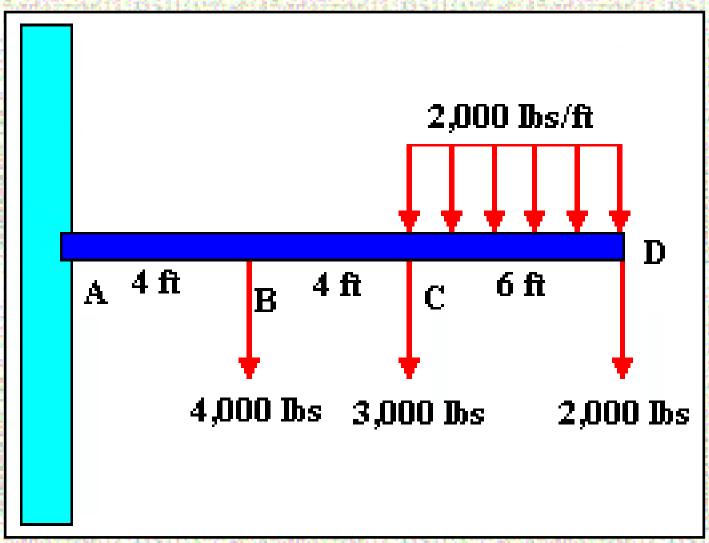
Topic 4.5: Shear Forces & Bending Moments - Topic Exam

1.) A loaded, simply supported beam is shown. For this beam:



A. Draw a Free Body Diagram of the beam, showing all external loads and support forces.
B. Determine expressions for the internal Shear Forces and Bending Moments in each section of the beam.
C. Draw the Shear Force and Bending Moment Diagrams for the beam.
(For Solution Select: Solution Beams 4.5a)

2.) A loaded, cantilever beam is shown below. For this beam:



A. Draw a Free Body Diagram of the beam, showing all external loads and support

forces and moments.
B. Determine expressions for the internal Shear Forces and Bending Moments in each section of the beam.
C. Make Shear Force and Bending Moment Diagrams for the beam.

(For Solution Select: Solution Beams 4.5b)

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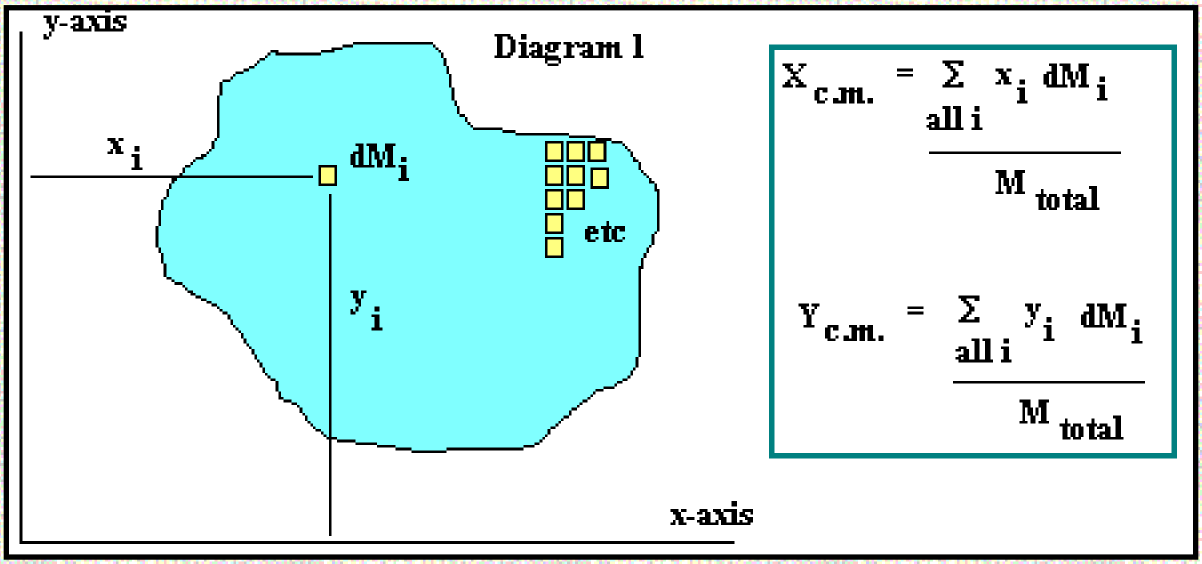
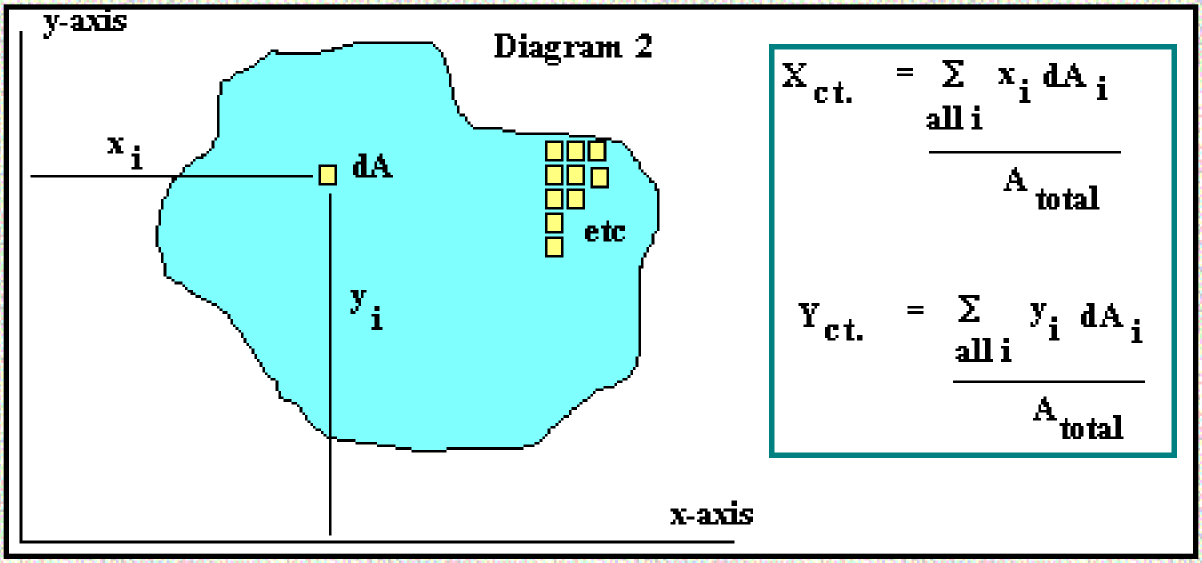
Topic 5.1a: Centroids and The Moment of Inertia

Centroids: The concept of the **centroid** is nearly the same as the **center of mass** of an object in two dimensions, as in a very thin plate. The **center of mass** is obtained by breaking the object into very small bits of mass dM, multiplying these bits of mass by the distance to the x (and y) axis, summing over the entire object, and finally dividing by the total mass of the object to obtain the **Center of Mass** - which may be considered to be the point at which the entire mass of the object may be considered to "act". See Diagram 1.



The only difference between the center of mass and the centroid is that rather than summing the product of each bit of mass dM and the distance x_i (and y_i) to an axis then dividing by the total mass, we instead divided the object into small bits of areas dA, and then take the sum of the product of each bit of area dA and the distance x_i (and y_i) to an axis then divide by the total area of the object. This results in an $X_{ct.}$ and Y_{ct} location for the Centroid (center of area) of the object. See Diagram 2

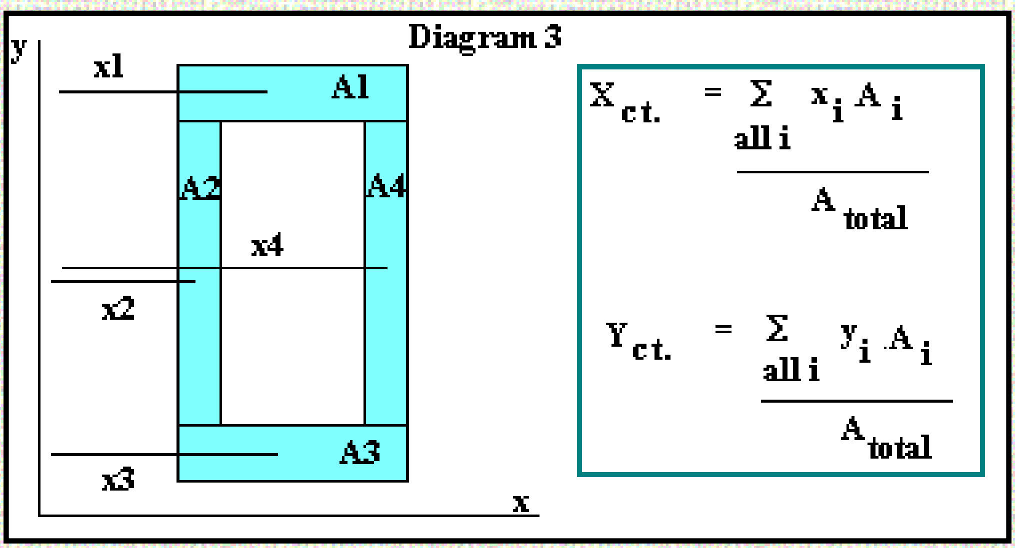


Several points to mention. We will assume all our beams have uniform density and will not consider the case of non-uniform density beams. We will also point out that for any beam cross section (or object) which is symmetry, the centroid will simply be at the geometric center of the cross section. Thus for rectangular beam and I-beams, the centroid is located at the exact center of the beam. This is not the case for T-beams.

Centroids of Composite Areas:

Some objects or beams may be formed from several simple areas, such as rectangles, triangles, etc. (See Diagram 3) In this case the centroid of the compose area may be found by taking the sum of the produce of each simple area and the distance it's centroid is from the axis, divided by the sum of the areas. For the composite area shown in Diagram 3, the location of it's x - centroid would be given by:

X ct = (A1 * x1 + A2 * x2 + A3 * x3 + A4 * x4) / (A1 +A2 +A3 + A4)

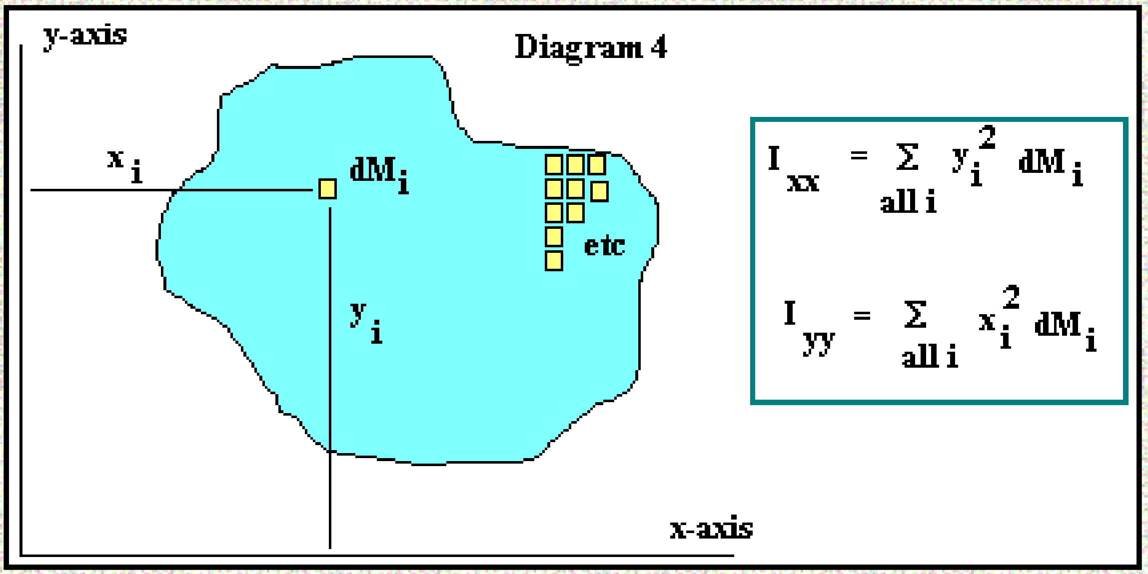


where x1, x2, x3, and x4 are the distances from the centroid of each simple area to the y-axis as shown in the Diagram 3. The location of the y - centroid would be given in like manner, although the y distances are not shown in Diagram 3:

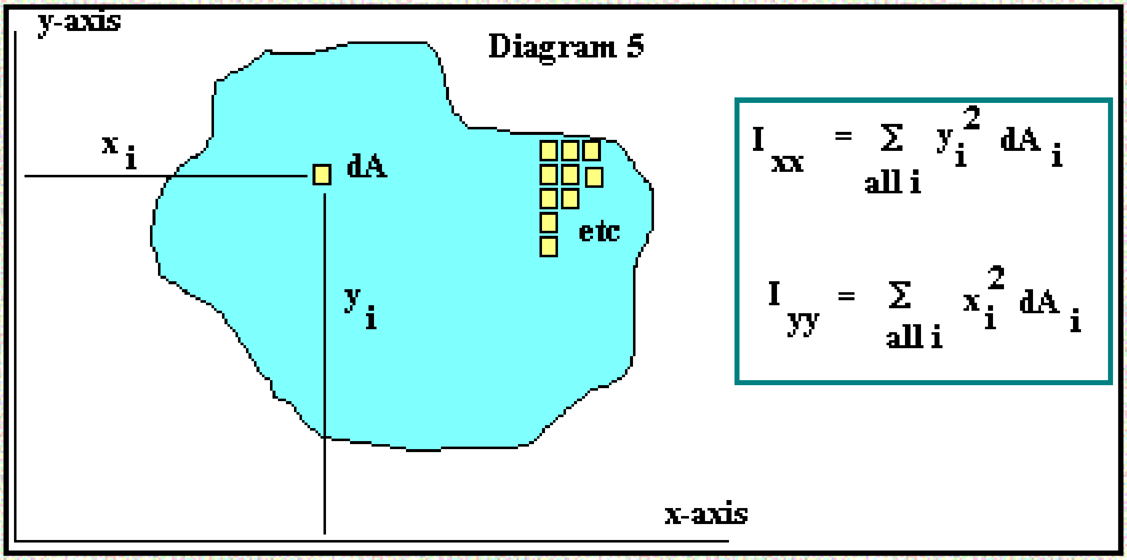
Y ct = (A1 * y1 + A2 * y2 + A3 * y3 + A4 * y4) / (A1 +A2 +A3 + A4)

Moment of Inertia

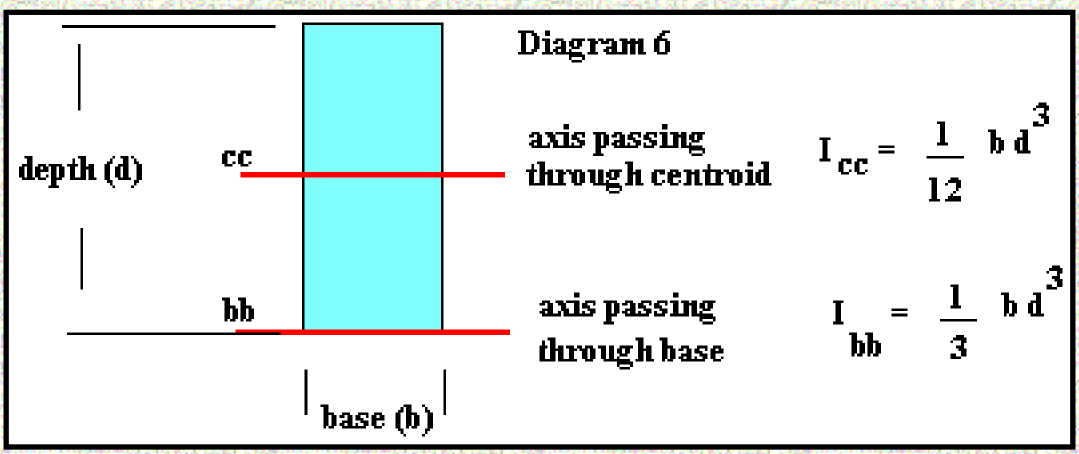
A second quantity which is of importance when considering beam stresses is the **Moment of Inertia**. Once again, the **Moment of Inertia** as used in Physics involves the mass of the object. The **Moment of Inertia** is obtained by breaking the object into very **small bits of mass dM**, multiplying these bits of mass by the **square of the distance** to the x (and y) axis and summing over the entire object. See Diagram 4.



For use with beam stresses, rather than using the Moment of Inertia as discussed above, we will once again use an Area Moment of Inertia. This **Area Moment of Inertia** is obtained by breaking the object into very **small bits of area dA**, multiplying these bits of area by the **square of the distance** to the x (and y) axis and summing over the entire object. See Diagram 5.



The actual value of the moment of inertia depends on the axis chosen to calculate the moment of the inertia with respect to. That is, for a rectangular object, the moment of inertia about an axis passing through the centroid of the rectangle is: I = 1/12 (base * depth^3) with units of inches^4., while the moment of inertia with respect to an axis through the base of the rectangle is: I = 1/3 (base * depth^3) in^4. See Diagram 6. Note that the moment of inertia of any object has its smallest value when calculated with respect to an axis passing through the centroid of the object.



Parallel Axis Theorem:
Moments of inertia about different axis may calculated using the **Parallel Axis Theorem**, which may be written: $I_{xx} = I_{cc} + Ad_{c-x}^2$ This says that the moment of inertia about any axis (I_{xx}) parallel to an axis through the centroid of the object is equal to the moment of inertia about the axis passing through the centroid (I_{cc}) plus the product of the area of the object and the distance between the two

parallel axis (**Ad_{c-x}^2**).

We lastly take a moment to define several other concepts related to the Moment of Inertia.

Radius of Gyration: $r_{xx} = (I_{xx}/A)^{1/2}$ The radius of gyration is the distance from an axis which, if the entire area of the object were located at that distance, it would result in the same moment of inertia about the axis that the object has.

Polar Moment of Inertia $J = \Sigma\ r^2\ dA$ The polar moment of inertia is the sum of the produce of each bit of area dA and the radial distance to an origin squared. In a case as shown in Diagram 7, the polar moment of inertia in related to the x & y moments of inertia by: **$J = I_{xx} + I_{yy}$.**

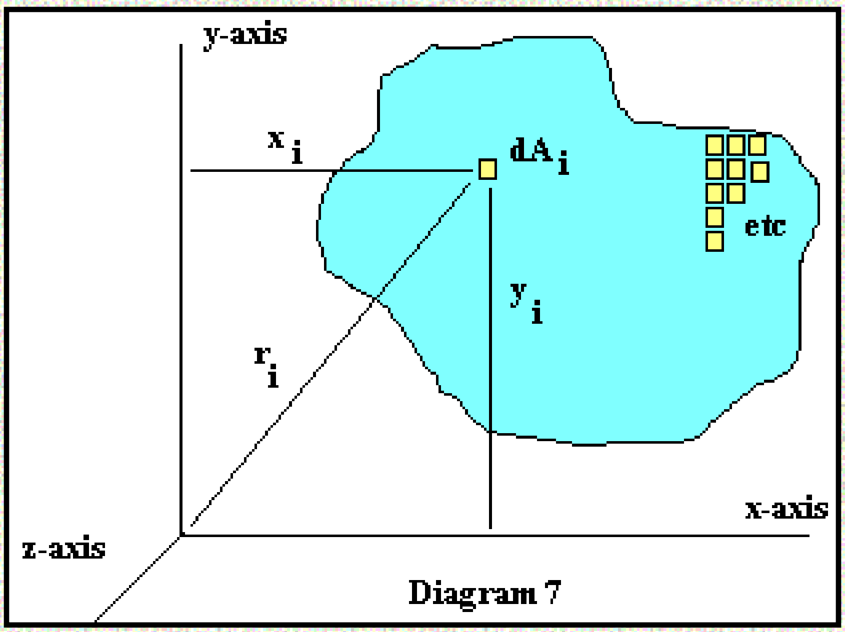


Diagram 7

One final comment - all the summations shown above become integrations as we let the dM's and dA's approach zero. And, while this is important and useful when calculating Centroids and Moments of Inertia, the summation method is just as useful for understanding the concepts involved

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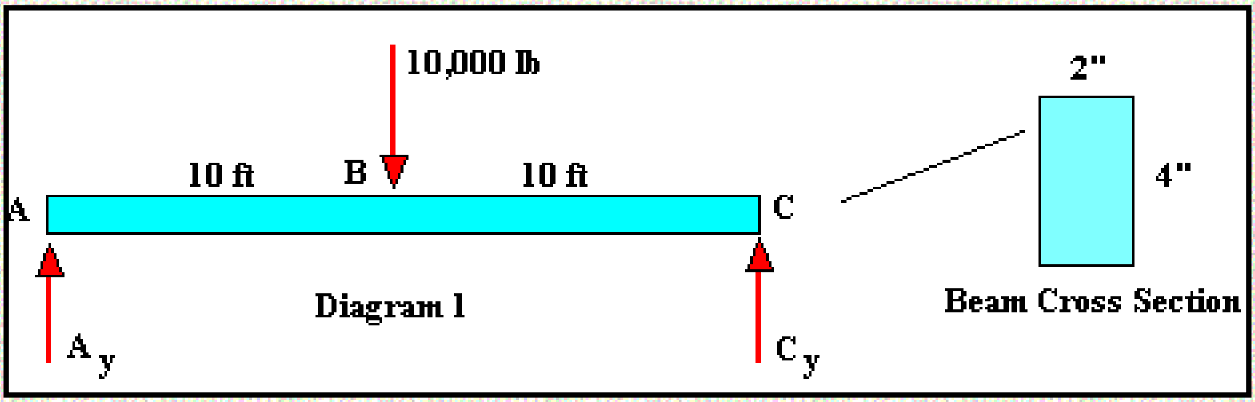
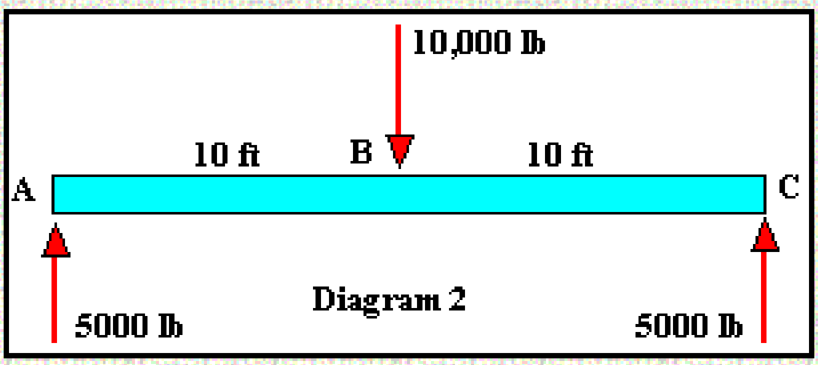
Topic 5.1b: Bending Stress - Example 1

In Diagram 1, we have shown a simply supported 20 ft. beam with a load of 10,000 lb. acting downward at the center of the beam. The beam used is a rectangular 2" by 4" steel beam. We would like to determine the maximum bending (axial) stress which develops in the beam due to the loading.

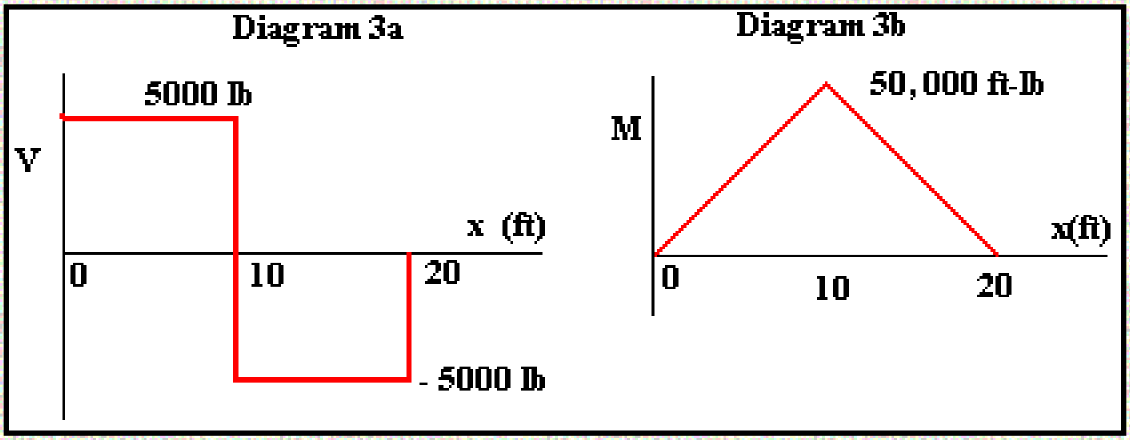


Step 1: Out first step in solving this problem is, of course, to apply static equilibrium conditions to determine the external support reactions. In this particular example, because of the symmetry of the problem, we will not go through the statics in detail, but point out that the two support forces will support the load at the center equally with forces of 5000 lb. each as shown in Diagram 2.



Step 2: The second step is to draw the shear force and bending moment diagrams for the beam. We really don't need the shear force diagram at this point, except we will use it to make the bending moment diagram. We will normally be able to draw the shear force diagram by simply looking at the load forces and the support reactions. If necessary we will determine the shear force and bending moment expressions and make the shear force and bending moment diagrams from these expressions, as we did in the proceeding topic.

We first draw the shear force diagram. Due to the 5000 lb. support force, the shear force value begins at +5000 lb., and since there is no additional loads for the next 10 feet, the shear force remains constant at 5000 lb. between 0 and 10 feet. At 10 feet, the 10,000 lb. downward load drives the shear force down by that amount, from + 5000 lb. to a value of - 5000 lb. Then as there are no additional loads for the next 10 feet, the shear force will remain constant over the remainder of the beam. Graphing the shear force values produces the result in Diagram 3a.



Then using the fact that for non-cantilevered beams the bending moment values are equal to the area under the shear force diagram, we develop the bending moment graph shown in Diagram 3b.

Step 3. We now apply the flexure formula: **Bending Stress = M y / I**
We wish to find the **maximum** bending stress, which occurs at the **outer edge of the beam** so:
M = maximum bending moment = 50,000 ft-lb. = 600,000 in-lb. (from bending moment diagram)
y = distance from the neutral axis of the cross section to outer edge of beam = 2 inches
I = moment of inertia of cross section; for rectangle I = (1/12) bd^3 = 1/12 (2" * 4"^{3}) = 10.67 in^4.
Then, **Maximum Bending Stress = M y / I = (600,000 in-lb)*(2 in)/(10.67 in^4) = 112, 500 lb/in^2**
This is the correct value, but it is clearly excessive for normal steel. Thus if we tried to use a rectangular 2"x4" steel beam, it would fail under the load. We will have to use a stronger beam.
Notice that the **maximum bending moment** does not depend on the type of beam. The values of "y" and "I" in the flexure formula do depend on the beam used. Thus, if we had used a rectangular 2"x6" beam (instead of a 2" x 4" beam), the value of y would be: y = 3", and the value of I would be: I = (1/12)(2" * 6"^{3}) = 36 in^4. Then the maximum bending stress for this beam would be:
Maximum Bending Stress = M y / I = (600,000 in-lb)*(3 in)/(36in^4) = 50,000 lb/in^2 . This value is in a more reasonable range for acceptable axial

stresses in steel.

When finished with this example; Continue to:

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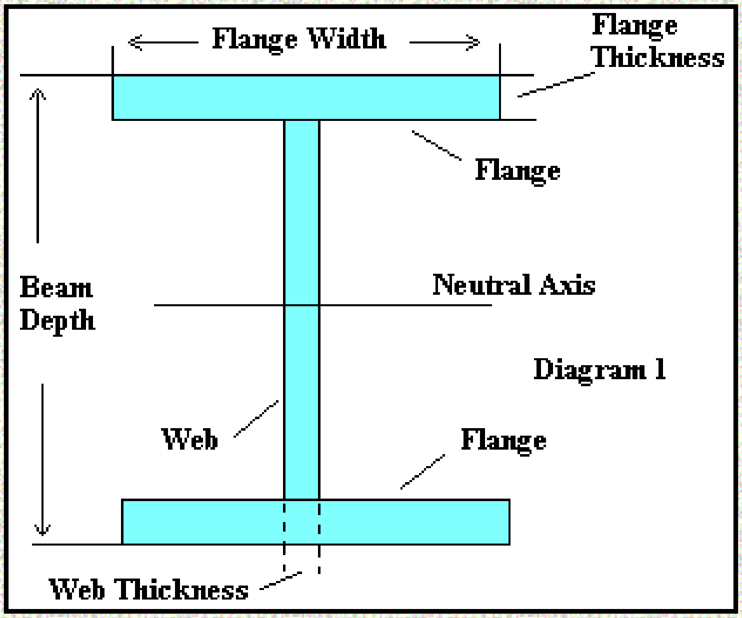
Topic 5.2: Beams - Bending Stress (cont)

We continue our discussion of the **Flexure Formula**., σ = **M y / I**, shown in its general form. In **Bending Stress Example 1**, we applied the flexure formula to a loaded rectangular beam. Rectangular metal beams **are not** normally used in large building construction, as they are not very efficient, rather metal **I beams**, and **T beams** are used instead. Rectangular beams are used in wood construction where the cost of using a metal I or T beam is excessive, and the lower cost of wood rectangular beams offsets their lower efficiency.

I-Beams
We will now consider an I-beam example. In Diagram 1, we have shown an I-beam cross section. The top and bottom sections are known was the flanges, and the connecting region is known as the web. The horizontal line passing through the center (centroid) of the beam is the neutral axis.



In the sample **Beam Data Table** shown below, beam information is shown. The Beam Designation, such as **W 8 x 20**, gives us the following information. The "**W**" indicates a Wide Flange beam. The "**8**" gives the approximate depth of the beam in inches. (Notice the actual depth of the beam is 8.14"). The "**20**" indicates the weight per foot of the beam in a standard type of steel. **Thus the first beam in the table, W 8 x 20, is a wide flange, eight inch deep I-beam, with a weight of 20 lb/ft.** Additional information given in the table includes the flange width and depth, the web thickness, the moment of inertia, **I**, about the x-x neutral axis of the beam (shown in Diagram 1), something called the section modulus, **S**, which we will discuss shortly, and the radius of gyration, **r**, with respect to the x-x neutral axis. Additionally, there is also a moment of inertia, section modulus, and radius of gyration about the y-y neutral axis. (This would apply if we flipped the beam on its side and loaded it in that orientation. This is not usually done and so we will not expect to use the y-y axis information.). (You may wish to print out this topic and the beam tables to study them more effectively.)

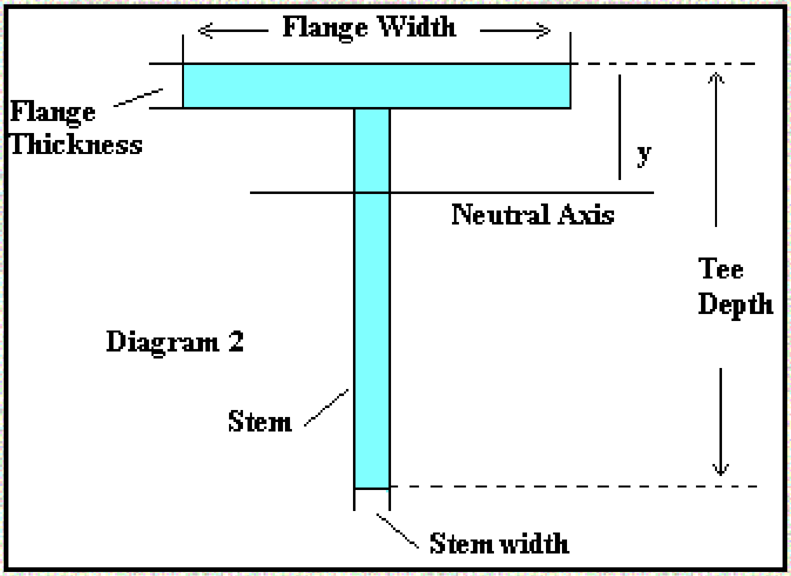
I-Beams	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.

Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	b_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 8x20	5.89	8.14	5.268	0.378	0.248	69.4	17.0	3.43	9.22	3.50	1.25
W 8x17	5.01	8.00	5.250	0.308	0.230	56.6	14.1	3.36	7.44	2.83	1.22
W 10x45	13.20	10.12	8.022	0.618	0.350	249.0	49.1	4.33	53.20	13.30	2.00
W 10x39	11.50	9.94	7.990	0.528	0.318	210.0	42.2	4.27	44.90	11.20	1.98
W 10x33	9.71	9.75	7.964	0.433	0.292	171.0	35.0	4.20	36.50	9.16	1.94
W 12x22	6.47	12.31	4.030	0.424	0.260	156.0	25.3	4.91	4.64	2.31	0.85
W 12x19	5.59	12.16	4.007	0.349	0.237	130.0	21.3	4.82	3.76	1.88	0.82
W 12x31	9.13	12.09	6.525	0.465	0.265	239.0	39.5	5.12	21.60	6.61	1.54
W 14x38	11.20	14.12	6.776	0.513	0.313	386.0	54.7	5.88	26.60	7.86	1.54
W 14x34	10.00	14.00	6.750	0.453	0.287	340.0	48.6	5.83	23.30	6.89	1.52
W 16x50	14.70	16.25	7.073	0.628	0.380	657.0	80.8	6.68	37.10	10.50	1.59
W 16x40	11.80	16.00	7.000	0.503	0.307	517.0	64.6	6.62	28.80	8.23	1.56
W 16x36	10.60	15.85	6.992	0.428	0.299	447.0	56.5	6.50	24.40	6.99	1.52
W 24x94	27.70	24.29	9.061	0.872	0.516	2690.0	221.0	9.86	108.00	23.90	1.98
W 24x76	22.40	23.91	8.985	0.682	0.400	2100.0	176.0	9.69	82.60	18.40	1.92
W 24x68	20.00	23.71	8.961	0.582	0.416	1820.0	153.0	9.53	70.00	15.60	1.87

We now look at an example of a loaded I-beam. Please select **Topic 5.2a: Bending Stress - Example 2**

T-Beams

We will next consider a T-beam example. In Diagram 2, we have shown a T-beam cross section. The top horizontal section is known as the flange, and the vertical section is known as the stem. The horizontal line passing through a portion of the T is the neutral axis. In the diagram, y is the distance from the top of the T to the neutral axis of the beam.



In the Beam Data Table shown below, beam information is given. The Beam Designation, such as **WT 6 x 11**, gives us the following information. The "**WT**" indicates a Wide Flange Tee beam. The "**6**" gives the approximate depth of the beam in inches. (Notice the actual depth of the beam is 6.16"). The "**11**" indicates the weight per foot of the beam in a standard type of steel. **Thus the first beam in the table, W 6 x 11, is a wide flange, six inch deep T-beam, with a weight of 11 lb./ft.** Additional information given in the table includes the flange width and depth, the stem thickness, the moment of inertia, **I**, about the x-x neutral axis of the beam (shown in Diagram 2), the section modulus, **S**, the radius of gyration, **r**, with respect to the x-x neutral axis, and **y**, the distance from the top of the tee to the neutral axis of the beam.

T-Beams	-	Depth	Flange	Flange	Stem	-	Cross	Section	Info.	-
Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A	d	b_f	t_f	tw	d / t_w	I	S	r	y
-	in^2	in	in	in	in	-	in^4	in^3	in	in
WT 6x11	3.24	6.16	4.030	0.424	0.260	23.70	11.70	2.590	1.900	1.630
WT 6x9.5	2.80	6.08	4.007	0.349	0.237	25.70	10.20	2.300	1.910	1.650
WT 7x26.5	7.79	6.97	8.062	0.658	0.370	18.80	27.70	4.960	1.880	1.380
WT 7 x24	7.06	6.91	8.031	0.593	0.339	20.40	24.90	4.490	1.880	1.350
WT 8x25	7.36	8.13	7.073	0.628	0.380	21.40	42.20	6.770	2.400	1.890
WT 8x20	5.89	8.00	7.000	0.503	0.307	26.10	33.20	5.380	2.370	1.820
WT 8x18	5.30	7.93	6.992	0.428	0.299	26.50	30.80	5.110	2.410	1.890
WT10.5 x34	10.00	10.57	8.270	0.685	0.430	24.60	103.00	12.900	3.200	2.590
WT 10x31	9.13	10.50	8.240	0.615	0.400	26.20	93.80	11.900	3.210	2.580
WT 12x47	13.80	12.15	9.061	0.872	0.516	23.50	186.00	20.300	3.670	3.000
WT 12x42	12.40	12.05	9.015	0.772	0.470	25.60	166.00	18.300	3.660	2.970
WT 12x38	11.20	11.96	8.985	0.682	0.440	27.20	151.00	16.900	3.680	2.990

WT 15x66	19.40	15.15	10.551	1.000	0.615	24.60	421.00	37.400	4.650	3.900
WT 15x58	17.10	15.00	10.500	0.850	0.564	26.60	372.00	33.600	4.670	3.930
WT 15x54	15.90	14.91	10.484	0.760	0.548	27.20	350.00	32.100	4.690	4.020
WT18x97	28.60	18.24	12.117	1.260	0.770	23.70	905.00	67.400	5.630	4.810
WT 18x75	22.10	17.92	11.972	0.940	0.625	28.70	698.00	53.100	5.620	4.780

We now look at an example of a loaded T-beam. **Please Select:** Topic 5.2b: Bending Stress - Example 3

When finished with Bending Stress Example 3,
Continue to:
Topic 5.3: Beams II - Horizontal Shear Stress
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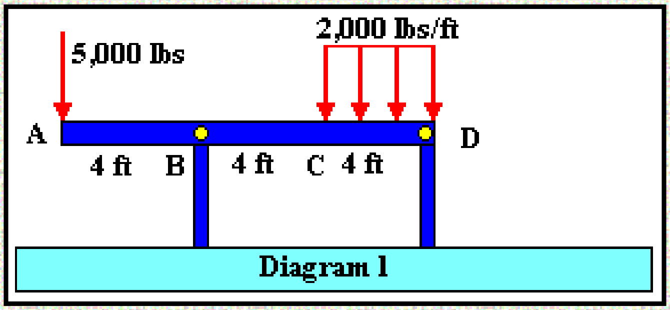
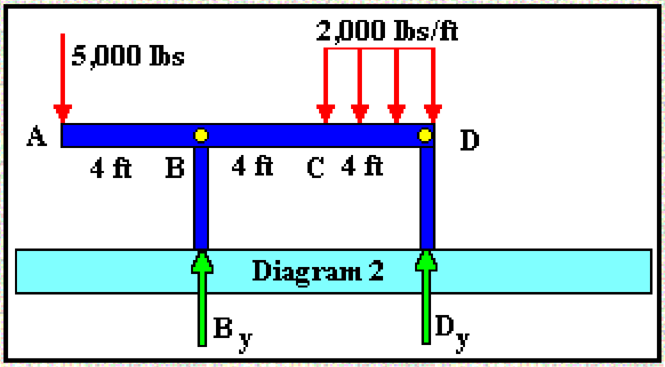
Topic 5.2a: Bending Stress - Example 2

A loaded, simply supported W 10 x 45 beam is shown in Diagram 1. For this beam we will first determine the maximum bending moment (and where it occurs in the beam). Then we will determine the maximum bending stress at that location, and also the bending stress at that location along the beam and 8 inches from the bottom of the beam cross section.



STEP 1: Apply Static Equilibrium Principles and determine the external support reactions:
1.) Draw Free Body Diagram of structure (See Diagram 2)



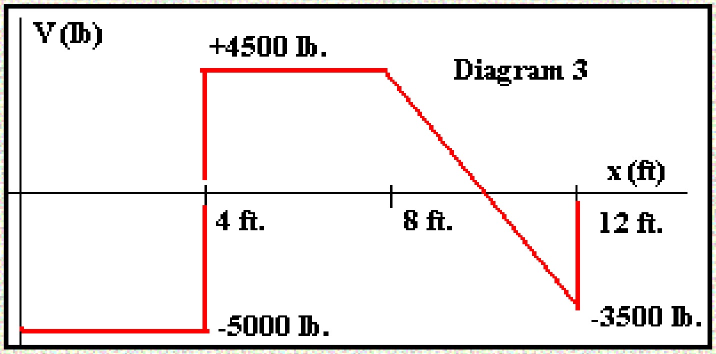
2.) Resolve all forces into x & y components
3.) Apply equilibrium conditions:
Sum F_x = 0 none

Sum $F_y = B_y + D_y$ - 2,000 lbs/ft (4 ft) - 5,000 lbs = 0

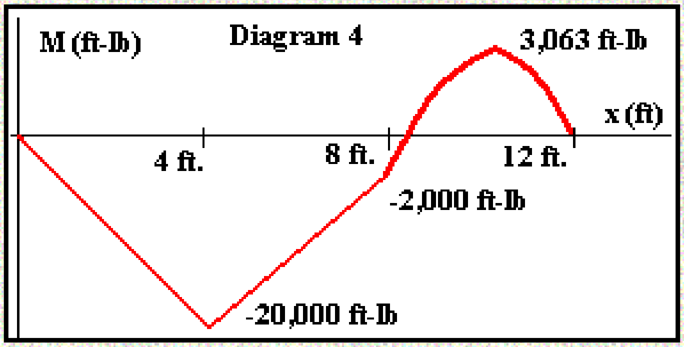
Sum T_B = 5,000 lbs (4 ft) - 2,000 lbs/ft (4 ft) (6 ft) + D_y(8 ft) = 0

Solving: **B_y = 9,500 lbs; D_y = 3,500 lbs**

STEP 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam. We can normally do this for the Shear Force Diagram, reasonably accurately, by simple looking at the loading and the support reactions. That is, for this example, the shear force begins with a value of -5000 lb. (at x = 0') due to the downward acting load of 5000 lb. at x= 0'. Next nothing happens (no loading) for the next 4 ft., so the shear force remains constant. Then at x = 4 ft there is an upward support force of 9,500 lb. which drives the shear force value up this amount, from a value of -5000 lb. to + 4,500 lb. Again there is no change in loading for the next 4 ft., so the shear force remains constant until x = 8 ft. At that point, a uniform load of 2,000 lb./ft. is applied, driving the shear force downward at a rate of 2000 lb. per each foot for the 4 ft (at total of - 8000 lb.), until at x = 12 ft. the value of shear force is -3,500 lb. Also at x = 12 ft. is the upward support reaction of +3,500 lb., which can be considered to bring the shear force back to zero.(See Diagram 3).



The Bending Moment Diagram may be drawn using the Shear Force Diagram, by remembering that (for non-cantilever beams) the value of the bending moment at a given location, x feet from the left end, is equal to the area under the shear force diagram up to that point. Applying this, we obtain the Bending Moment Diagram shown in Diagram 4.



STEP 3: We will now Apply the Flexure Formula to determine the maximum bending stress for the beam. We may use Flexure Formula: σ = **M y / I,** or a special form of the Flexure Formula: σ_{max} = **M / s**, where s is what is known as the section modulus. If we rewrite the standard flexure formula several times as follows for the maximum stress:
$\sigma_{max} = M\,(y_{max} / I) = M / (I / y_{max}) = M / s$, we then see that the section modulus is defined as $s = I / y_{max}$. That is, this special form of the flexure formula can only be used to find the maximum bending stress, and uses the section modulus, where the section modulus is equal to the moment of inertia of the beam cross section divided by the **maximum distance** from the neutral axis of the beam to an outer edge of the beam.

As an example we apply this form to determine the maximum bending stress in our beam. First we determine the maximum bending moment from our bending moment diagram - which we observe from Diagram 4 is: M_{max} **= 20,000 ft.-lb.,** and occurs at x = 4 ft. (We will drop the negative sign which simply tells us that the beam is bent concave facing downward at this point. This means the top of the beam is in tension and the bottom of the beam is in compression.) We also then find the x-x axis section modulus of the beam as listed in the beam table below, **s = 49.1 in^3**

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis

-	A	d	b_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 10x45	13.20	10.12	8.022	0.618	0.350	249.0	49.1	4.33	53.20	13.30	2.00

Now σ_{max} = **M / s = (20,000 ft-lb.)(12 in./ft.)/ 49.1 in^3 = 4, 890 lb./in^2**. (Notice that we had to convert the bending moment in **ft.-lb.** to **in.-lb**. for the units to be consistent) We have thus determined the maximum bending stress (axial stress) in the beam. Since the beam is symmetric about the neutral axis, the stress at the top of the beam and at the bottom of the beam are equal in value (4,890 psi.) with the top in tension and the bottom in compression.

Finally, we will determine the bending stress at 4 ft (where the maximum bending moment occurs), and 8 inches above the bottom of the beam. For this we need to use the flexure formula in the form σ = **M y / I**, where **M = 20,000 ft-lb. = 240,000 in-**lb., **I = moment of inertia of beam = 249 in^4**, and **y = distance from the neutral axis to point at which we wish to find the bending stress**. Since we wish to find the bending stress 8 inches above the bottom of the beam, and since the neutral axis is 5.06 inches above the bottom of the beam (at the beam center), then **y = 8 - 5.06 = 2.94 inches**. Then σ = **240,000 in-lb. * 2.94 in. / 249 in4. = 2830 lb./in^2**. And since the location is above the beam centroid (and the bending moment is positive), this is a tensile stress.

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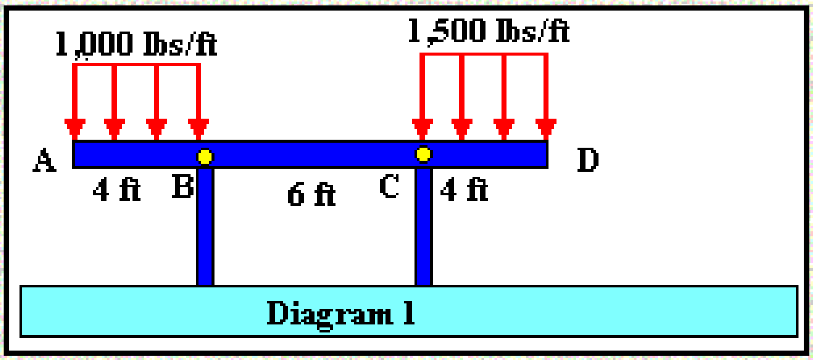
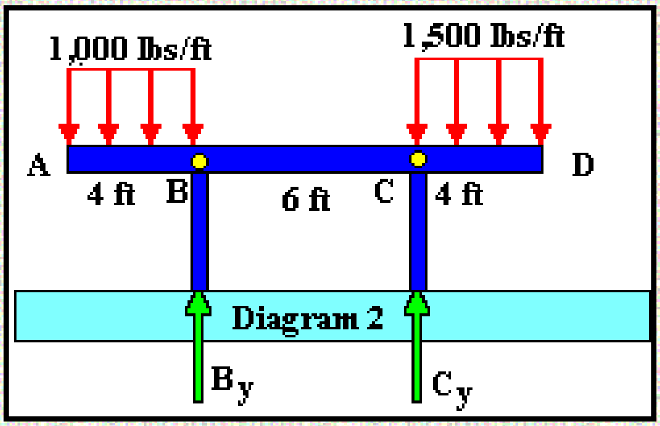
Topic 5.2b: Bending Stress - Example 3

A simply supported WT 8 x 25 T-beam is loaded is shown in Diagram 1. For this beam we will determine the maximum bending stress in the beam. We will also determine the bending stress 4 ft from the left end of the beam and 2 inches above the bottom of the beam.



STEP 1: Apply Static Equilibrium Principles and determine the external support reactions:
1.) FBD of structure (See Diagram 2)



2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

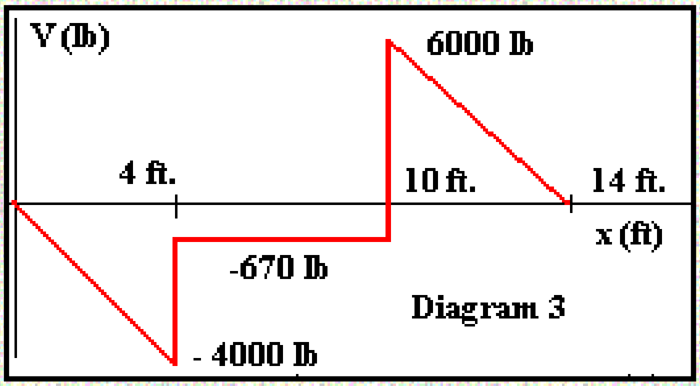
Sum F_x = 0 none

Sum $F_y = B_y + C_y$ - 1,000 lbs/ft (4 ft) - 1,500 lbs/ft (4 ft) = 0

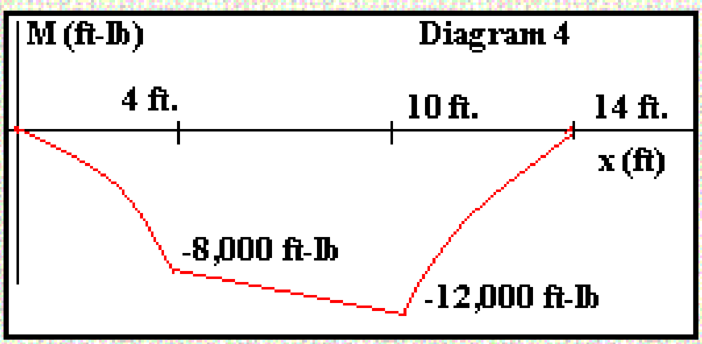
Sum T_B = 1,000 lbs/ft (4 ft) (2 ft) - 1,500 lbs/ft (4 ft) (8 ft) + C_y(6 ft) = 0

Solving: B_y = 3,330 lbs; C_y = 6,670 lbs

STEP 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam. We can normally do this for the Shear Force Diagram, reasonably accurately, by simple looking at the loading and the support reactions. That is, for this example, the shear force begins with a value of zero (since there is not point load or reaction at x = 0), and then decreases at a rate of 1000 lb. per foot for the first 4 feet due to the uniform load, resulting in a shear force value of -4000 lb. at x = 4 ft. Then, also at x = 4 ft., there is an upward support reaction of 3,330 lb. which drives the value of the shear force upward (from -4000 lb.) by that amount to a value of -670 lb. Next nothing happens (no loading) for the next 6 ft., so the shear force remains constant. Then at x = 10 ft there is an upward support force of 6,670 lb. which drives the shear force value up this amount, from a value of -670 lb. to +6000 lb. Then from 10 ft. to 14 ft., a uniform load of 1,500 lb./ft. is applied, driving the shear force downward at a rate of 1500 lb. per each foot for the last 4 ft (for total of - 6000 lb), Which brings the shear force value down to zero at x = 14 ft.(See Diagram 3).



The Bending Moment Diagram may be drawn using the Shear Force Diagram, by remembering that (for non-cantilever beams) the value of the bending moment at a given location, x feet from the left end, is equal to the area under the shear force diagram up to that point. Applying this, we obtain the Bending Moment Diagram shown in Diagram 4.



STEP 3: We will now Apply the Flexure Formula to determine the maximum bending stress for the beam. We may use Flexure Formula: σ = **M y / I,** or a special form of the Flexure Formula: σ_{max} = **M / s**, where s is the section modulus. This special form of the flexure formula can only be used to find the maximum bending stress, and uses the section modulus, where the section modulus is equal to the moment of inertia of the beam cross section divided by the **maximum**

distance from the neutral axis of the beam to an outer edge of the beam.

As an example we apply this form to determine the maximum bending stress in our beam. First we determine the maximum bending moment from our bending moment diagram - which we observe from Diagram 4 is: **M_{max} = 12,000 ft.-lb,** and occurs at x = 10 ft. (We will drop the negative sign which simply tells us that the beam is bent concave facing downward at this point. This means the top of the beam is in tension and the bottom of the beam is in compression.) We also then find the x-x axis section modulus of the beam as listed in the beam table, **s = 6.77 in^3**

Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A	d	b_f	t_f	tw	d/t_w	I	S	r	y
-	in^2	in	in	in	in	-	in^4	in^3	in	in
WT8x25	7.36	8.13	7.073	0.628	0.380	21.40	42.20	6.770	2.400	1.890

Now σ_{max} = **M / s = (12,000 ft-lb)(12 in./ft.)/ 6.77 in^3 = 21,270 lb/in^2**. (Notice that we had to convert the bending moment in **ft.-lb.** to **in.-lb**. for the units to be consistent) We have thus determined the maximum bending stress (axial stress) in the beam. Please note that this maximum stress is compressive and occurs at the bottom of the stem, since that is the outer edge of the beam which is farthest from the neutral axis. (The stress at the top of the tee is less than that at the bottom of the stem, since the top of the tee is closer to the neutral axis than the bottom of the stem.)

Finally, we will determine the bending stress at 4 ft from the left end of the beam and 2 inches above the bottom of the beam. For this we need to use the flexure formula in the form σ = **M y / I.**, where **M = -8,000 ft-lb = -96,000 in-lb** (which we determine from the bending moment graph shown in Diagram 4), **I = moment of inertia of beam = 42.2 in^4**, and **y = distance from the neutral axis to point at which we wish to find the bending stress**. The neutral axis is 1.89 inches below the top of the beam (from the beam data table), then the neutral axis is **8.13"-1.89" = 6.24"** from the bottom of the beam. Since we wish to find bending stress 2 inches above the bottom of the beam, then y = 6.24" - 2" = 4.24" from the neutral axis to where we wish to determine the bending stress. Thus σ = **96,000 in-lb * 4.24 in. / 42.2 in4. = 9,450 lb/in^2.** Since the bending moment is negative, meaning the beam is bent concave facing downward, and since the location is below the beam centroid, then this stress is compressive.

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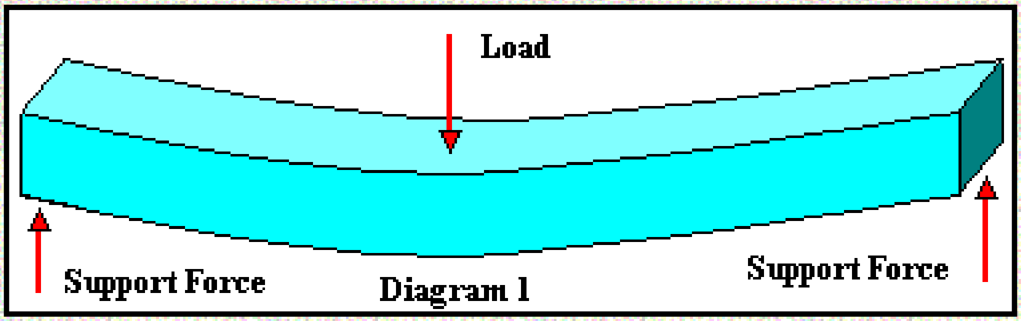
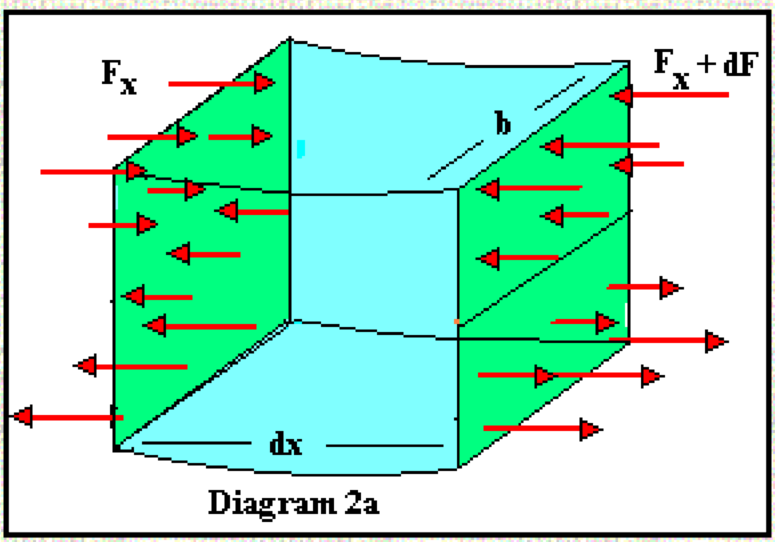
Topic 5.3: Beams -Horizontal Shear Stress

In addition to the bending (axial) stress which develops in a loaded beam, there is also a shear stress which develops, including both a **Vertical Shear Stress**, and a **Horizontal (longitudinal) Shear Stress.** It can be shown that **at any given point in the beam**, the values of vertical shear stress and the horizontal shear stress must be equal, at that point, for static equilibrium. As a result it is usual to discuss and calculate the horizontal shear stress in a beam (and simply remember that the vertical shearing stress is equal in value to the horizontal shear stress at any given point).We will take a moment to derive the formula for the Horizontal Shear Stress. In Diagram 1, we have shown a simply supported loaded beam.



In Diagram 2a, we have cut a section dx long out of the left end of the beam, and have shown the internal horizontal forces acting on the section.



In Diagram 2b, we have shown a side view of section dx. Notice that the bending moment is larger on the right hand face of the section by an amount dM. (This is clear if we make the bending moment diagram for the beam, in which we see the bending moment increases from a value of zero at the left end to a maximum at the center of the beam.)

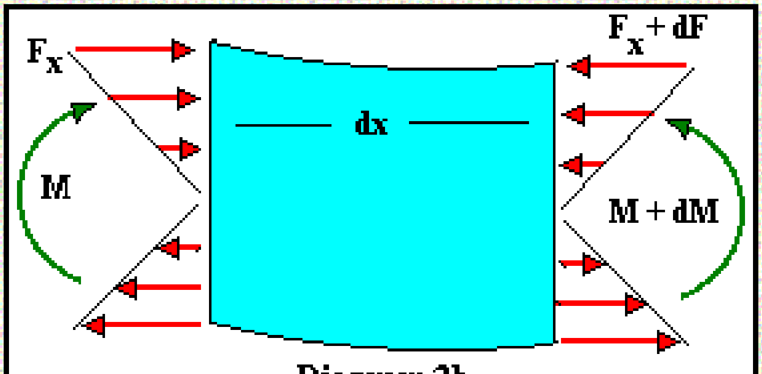


Diagram 2b

In Diagram 2c, we have shown a top slice of section dx. Since the forces are different between the top of the section and the bottom of the section (less at the bottom) there is a differential (shearing) force which tries to shear the section, shown in Diagram 2c, horizontally. This means there is a shear stress on the section, and in terms of the shear stress, the differential shearing force, ΔF, can be written as $\Delta F = \tau$ times the longitudinal area of the section (**b dx**). A second way of expressing the shear force is by expressing the forces in terms of the bending stress, that is $F_1 = \sum (My/I)\, dA$, and $F2 = \sum (M+dM)y/I\, dA$, then the differential force is $\sum (dM\, y/I)dA$. If we now combine the two $\Delta F =$ expressions, we have:

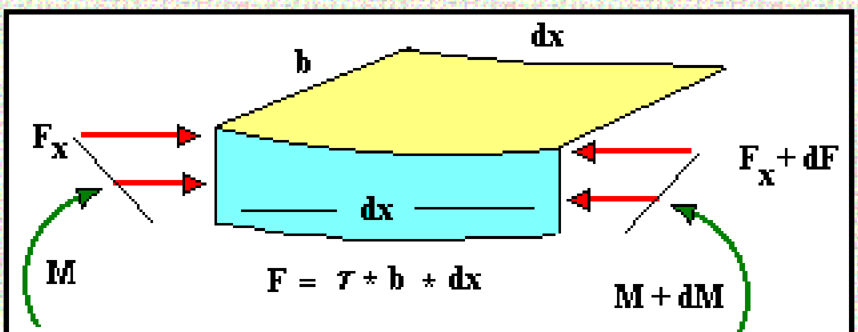


Diagram 2c

$\Delta F = \tau * b\, dx = \sum (dM\, y/I)dA$, and then rewriting to solve for the shear stress: $\tau = [(dM/dx)/Ib] \sum y\, dA$, however dM/dx is equal to the shear force V (as discussed in the previous topic), and $\sum y\, dA$ is the first moment of the area of the section, and may be written as A y', where A is the area of the section and y' is the distance from the centroid of the area A to the neutral axis of the beam cross section. Rewriting in a final form we have:

Horizontal Shear Stress: τ = VAy' / Ib, where

V = Shear force at location along the beam where we wish to find from the horizontal shear stress
A = cross sectional area, from point where we wish to find the shear stress at, to an outer edge of the beam cross section (top or bottom)
y' = distance from neutral axis to the centroid of the area A.
I = moment of inertia for the beam cross section.
b = width of the beam at the point we wish to determine the shear stress.
(In some texts, the product Ay' is given the symbol Q and used in the shear stress equation)

If we consider our shear relationship a little, we observe that the **Horizontal Shear Stress is zero at the outer edge of the beam** - since the area A is zero there. **The Horizontal Shear Stress is (normally) a maximum at the neutral axis of the beam.** This is the opposite of the behavior of the Bending Stress which is maximum at the other edge of the beam, and zero at the neutral axis.

To help clarify the Horizontal Shear Stress equation we will now look at at several example of calculating the Horizontal Shear Stress.

Please select:
Topic 5.3a: Horizontal Shear Stress - Example 1
Topic 5.3b: Horizontal Shear Stress - Example 2

When finished with the above examples, **Continue to:**
Topic 5.4: Beams II - Beam Selection
or Select:
Topic 5: Beams II - Table of Contents
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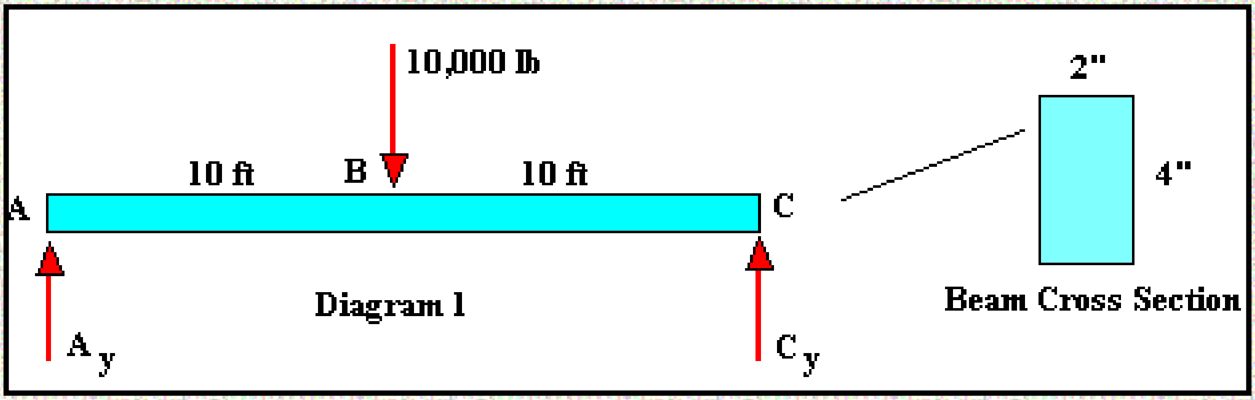
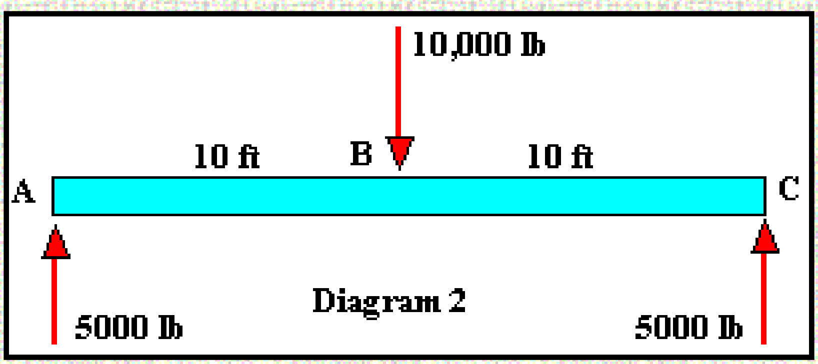
Topic 5.3a: Horizontal Shear Stress - Example 1

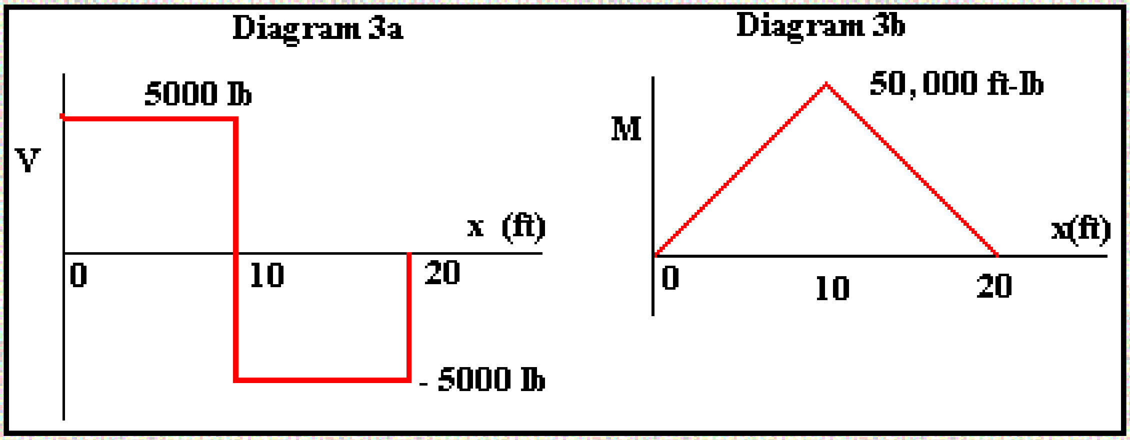
In Diagram 1, we have shown a simply supported 20 ft. beam with a load of 10,000 lb. acting downward at the center of the beam. The beam used is a rectangular 2" by 4" steel beam. We would like to determine the maximum Horizontal Shear Stress which develops in the beam due to the loading. We will also determine the Horizontal Shear Stress 3 inches above the bottom of the beam at the position in the beam where the shear force is a maximum.



Step 1: Out first step in solving this problem is to apply static equilibrium conditions to determine the external support reactions. In this particular example, because of the symmetry of the problem, we will not go through the statics in detail, but point out that the two support forces will support the load at the center equally with forces of 5000 lb. each as shown in Diagram 2.

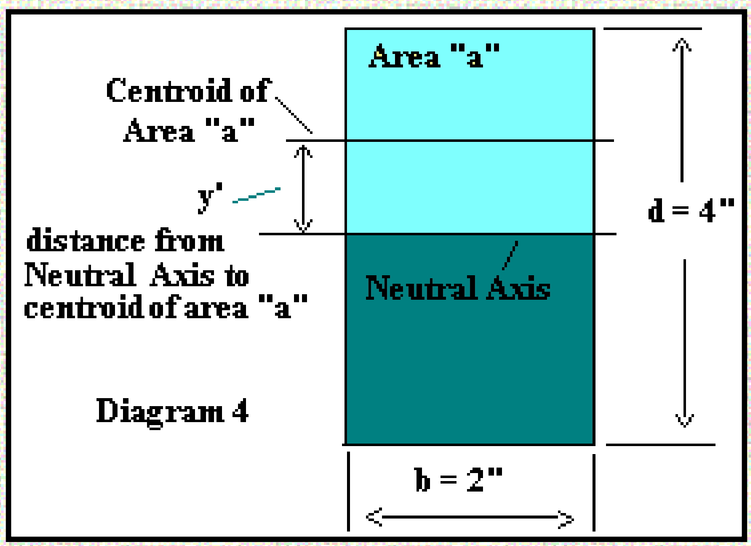


Step 2: The second step is to draw the shear force and bending moment diagrams for the beam. We really don't need the bending moment diagram, but will include it for completeness. We have shown the shear force and bending moment graphs in Diagram 3a and 3b. This beam is the same beam used in **Topic 4.7b:Bending Stress - Example 1** Please see that example, if needed, for a more complete explanation of how the shear force and bending moment diagrams were made.



Step 3. We will now apply the **Horizontal Shear Stress formula**: **Shear Stress = Vay' / Ib**
We wish to find the **maximum shear stress**, which occurs at the **neutral axis of the beam:**

V = maximum shear force = 5,000 ft-lb. (from the shear force diagram)
I = moment of inertia of cross section; for rectangle
I = (1/12) bd^3 = 1/12 (2" * 4"^{3}) = 10.67 in^4.
b = width of beam section where we wish to find shear stress at; b= 2 in.
a = area from point we wish to find shear stress at (neutral axis) to an outer edge of beam
a= (2" x 2")= 4 in^2.
y' = distance from neutral axis to the centroid of the area "a" which we used; y'= 1 in.
(See Diagram 4)



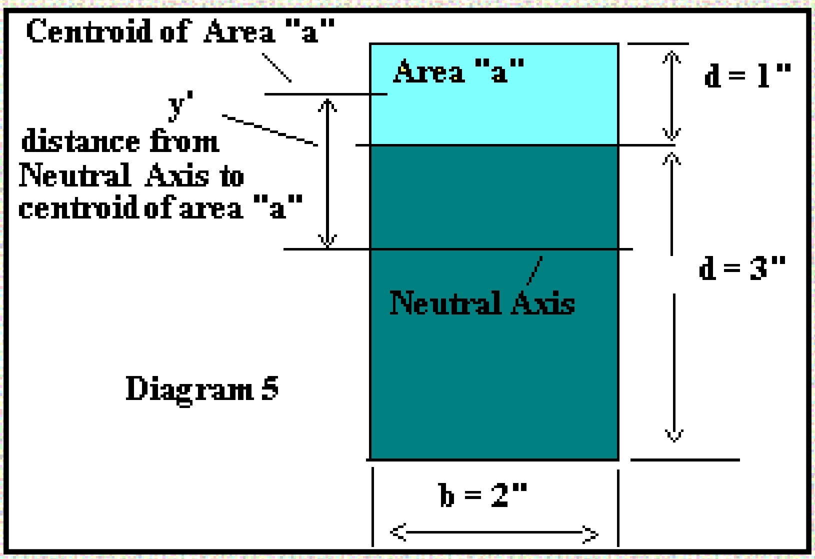
Placing the values into the equation, we find:
Maximum Horizontal Shear Stress = Vay' / Ib = (5000 lb)*(4 in^2)*(1 in) / (10.67 in^4)(2 in)= 937 lb/in^2

This is the correct value; we notice it is not very large. The beam is clearly able to carry the load without failing in shear.

Part II We now would also like to determine the Horizontal Shear Stress 3 inches above the bottom of the beam at the position in the beam where the shear force is a maximum (which is actually through out the beam, since the value of shear force is either +5000 lb., or - 5000 lb. through out the beam.)

We again apply the Horizontal Shear Stress formula: **Horizontal Shear Stress = Vay' / Ib**

We wish to find the shear stress 3 inches above the bottom of the beam cross section. (See Diagram 5)
V = shear force = 5,000 ft-lb. (from the shear force diagram)



I = moment of inertia of cross section; for rectangle I = (1/12) bd^3 = 1/12 (2" * 4"^{3}) = 10.67 in^4.
b = width of beam section where we wish to find shear stress at; b = 2 in.
a = area from point we wish to find shear stress at (3" above bottom of the beam) to an outer edge of beam. We will go to the top edge of the beam, then a = (2" x 1")= 2 in^2.
y' = distance from neutral axis to the centroid of the area "a" which we used; y'= 1.5 in. (See Diagram 5)
Then the horizontal shear stress 3 inches above the bottom of the beam is:
Horizontal Shear Stress = Vay' / Ib = (5000 lb)*(2 in^2)*(1.5 in) / (10.67

in^4)(2 in) = 703 lb/in^2

Notice, as we expect, the horizontal shear stress value becomes smaller as we move toward the outer edge of the beam cross section.

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Topic 5.3: Beams - Horizontal Shear Stress
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Topic 5: Beams II - Table of Contents
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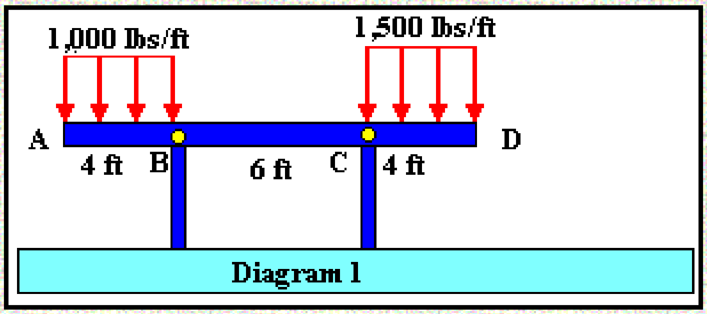
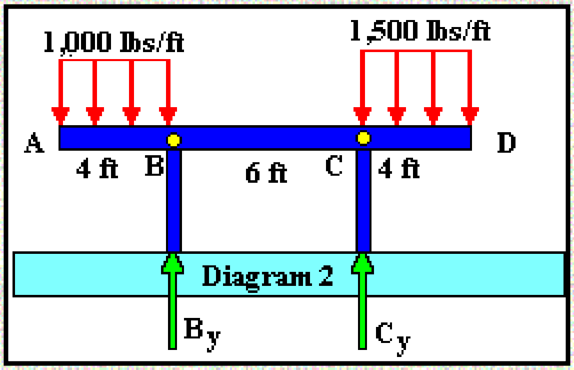
Topic 5.3b: Horizontal Shear Stress - Example 2

A loaded, simply supported beam is shown in Diagram 1. For two different beam cross sections (a WT 8 x 25 T-beam, and a W 10 x 45 beam) we will determine the maximum Horizontal Shear Stress which would develop in the beam due to the loading. We will also determine the Horizontal Shear Stress 3 inches above the bottom of the beam at the position in the beam where the shear force is a maximum.



STEP 1: Apply Static Equilibrium Principles and determine the external support reactions:
1.) FBD of structure (See Diagram 2)



2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

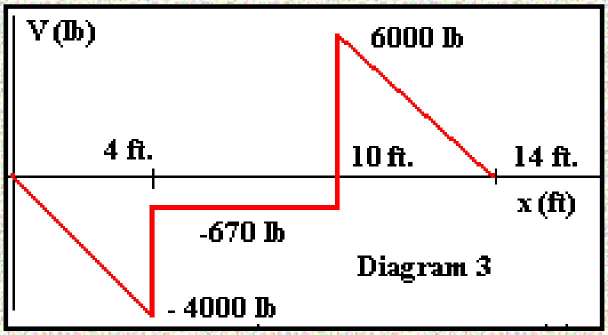
Sum F_x = 0 none

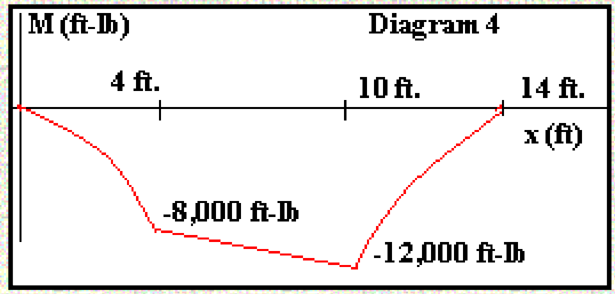
Sum F_y = B_y + C_y - 1,000 lbs/ft (4 ft) - 1,500 lbs/ft (4 ft) = 0

Sum T_B = 1,000 lbs/ft (4 ft) (2 ft) - 1,500 lbs/ft (4 ft) (8 ft) + C_y(6 ft) = 0

Solving: B_y = 3,330 lb.; C_y = 6,670 lb.

Step 2: The second step is to draw the shear force and bending moment diagrams for the beam. We really don't need the bending moment diagram, but will include it for completeness. We have shown the shear force and bending moment graphs in Diagram 3 and 4. This beam is the same beam used in **Beams - Bending Stress Example III**. Please see that example, if needed, for a more complete explanation of how the shear force and bending moment diagrams were made.

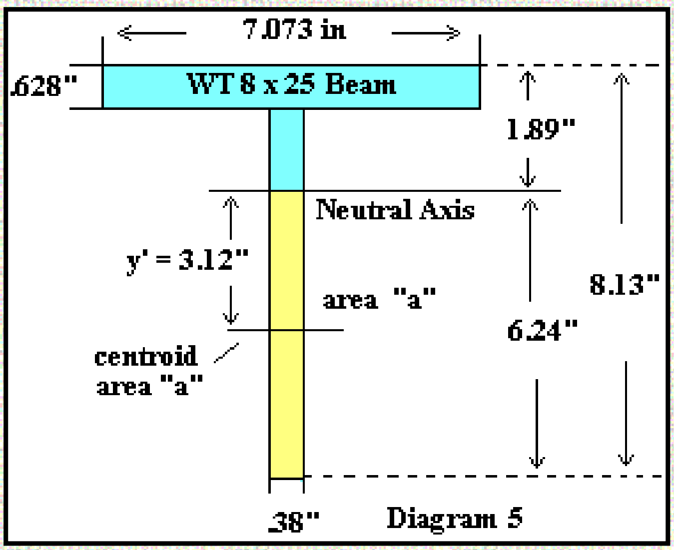




Beam Table for WT 8 x 25

Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A	d	b_f	t_f	tw	d / t_w	I	S	r	y
-	in^2	in	in	in	in	-	in^4	in^3	in	in
WT 8x25	7.36	8.13	7.073	0.628	0.380	21.40	42.20	6.770	2.400	1.890

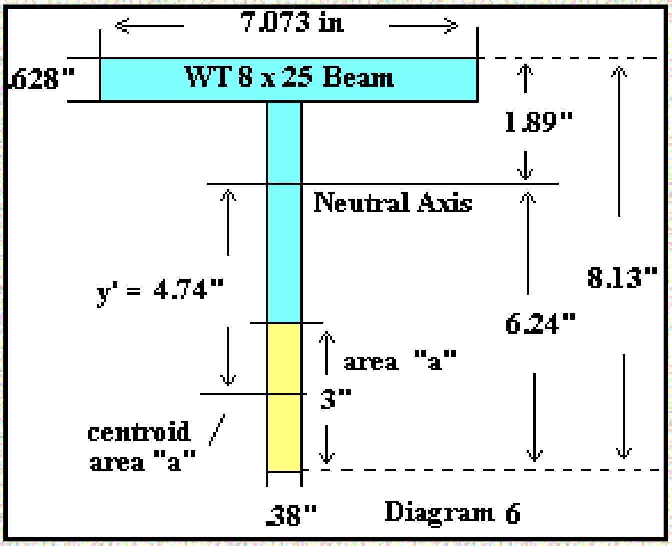
Step 3. For the **WT 8 x 25 T-beam** (table above) we will now apply the Horizontal Shear Stress formula:



Shear Stress = Vay' / Ib, to find the **maximum** shear stress, which occurs at the **neutral axis of the beam:**
V = maximum shear force = 6,000 lb. (from the shear force diagram)
I = moment of inertia of cross section, from beam table; I = 42.20 in^4.
b = width of beam where we wish to find shear stress (neutral axis for maximum) from table; b = .38 in.
a = area from point we wish to find shear stress at (neutral axis) to an outer edge of beam. In this case we will go to bottom of beam. Then a = (.38" * 6.24") = 2.37 in^2.
y' = distance from neutral axis to the centroid of the area "a" ; y' = 3.12 in. (See Diagram 5) Then placing values into our expression we find:
Maximum Horizontal Shear Stress = Vay' / Ib = (6000 lb) * (2.37 in^2) * (3.12 in) / (42.20 in^4) (.38 in) = 2770 lb / in^2

We now would also like to determine the Horizontal Shear Stress 3 inches above the bottom of the beam at the position in the beam where the shear force is a maximum
We again apply the Horizontal Shear Stress formula: **Shear Stress = Vay' / Ib**

We wish to find the shear stress 3 inches above the bottom of the beam cross section, where the shear force is a maximum. (See Diagram 6)\



V = maximum shear force = 6,000 lb. (from the shear force diagram)
I = moment of inertia of cross section, from beam table; I = 42.20 in^4.
b = width of beam where we wish to find shear stress (3" above bottom of beam) from table; b = .38 in.
a = area from point we wish to find shear stress at (neutral axis) to an outer edge of beam. In this case we will go to bottom of beam. Then a = (.38" * 3")= 1.14 in^2. (See Diagram 6)
y' = distance from neutral axis to the centroid of the area "a" ; y' = 4.74 in. (See Diagram 6)
Then the horizontal shear stress 3 inches above the bottom of the beam is:
Horizontal Shear Stress = Vay' / Ib = (6000 lb)*(1.14 in^2)*(4.74 in) / (42.2 in^4)(.38 in) = 2020 lb / in^2
Notice, as we expect, the horizontal shear stress value becomes smaller as we move toward an outer edge of the beam cross section.

Part 2: **W 10 x 45 beam**: We would like to again determine the maximum horizontal shear stress, and the shear stress 3 inches above the bottom of the beam (at the point where the shear force is a maximum), but now find these values for a **W 10 x 45 I-beam**.

Beam Table for W 10 x 45 I-Beam

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	b_f	t_f	t_w	I	S	r	I	S	r

-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 10 x 45	13.20	10.12	8.022	0.618	0.350	249.0	49.1	4.33	53.20	13.30	2.00

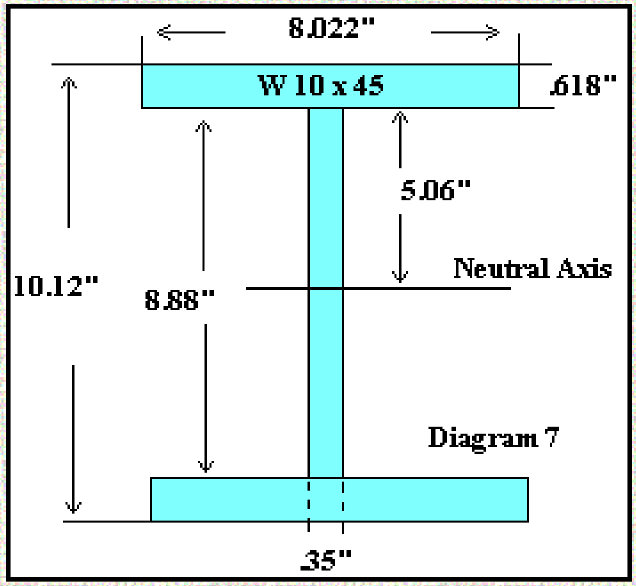
We have already done the statics, and the shear force and bending moment diagrams are shown in the first part of this example above, so we continue at the point where we apply the horizontal shear stress formula to find the values we desire.

For the **WT 8 x 25 T-beam** we apply the Horizontal Shear Stress formula: **Shear Stress = Vay'/Ib**, however since we are looking for the **maximum shear stress** in the I-Beam, we can use the approximate formula for I-beam, **Maximum Horizontal Shear Stress = V_{max}/A_{web}**.

This says the approximate maximum shear stress in an I - Beam is equal to the maximum shear force divided by the area of the web of the I-Beam. Applying this we have:

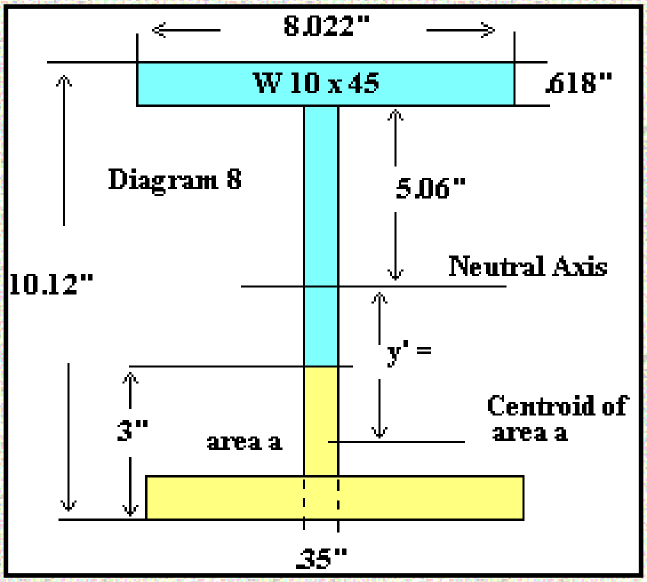
V_{max} = maximum shear force = **6,000 lb**. (from the shear force diagram)

A_{max} = area of web: **A = (.35" * 8.88")= 3.11 in^2**. (See Diagram 7)

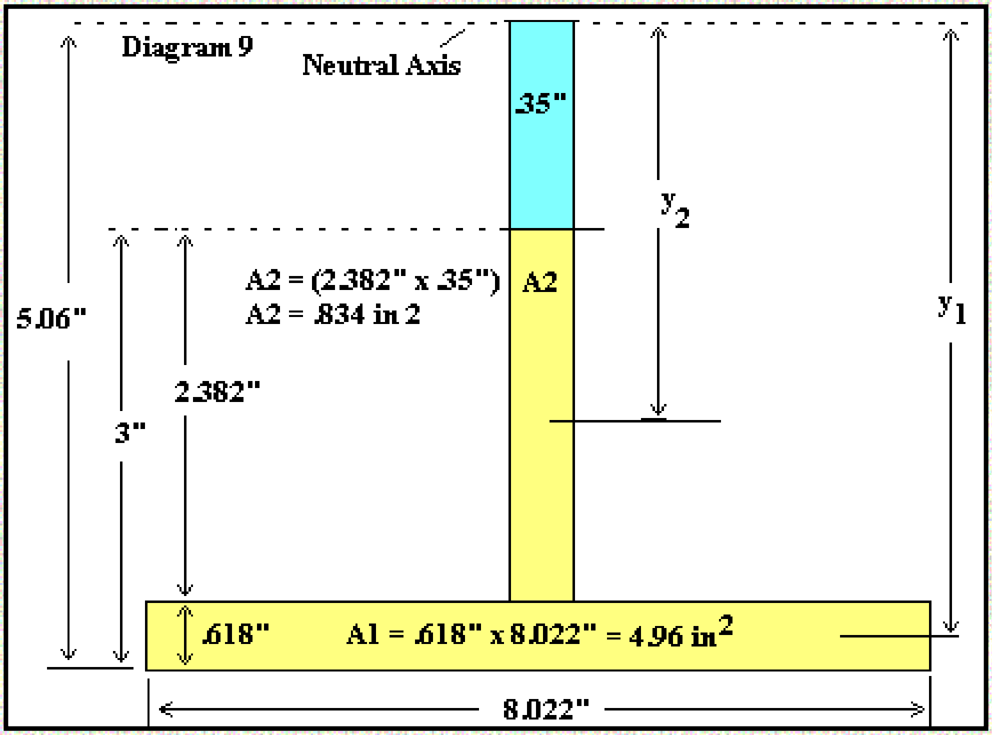


Then **Maximum Horizontal Shear Stress = (6000 lb)/(3.11 in^2) = 1930 lb/in^2** As long as this approximate value is reasonably below the allowable shear stress for the beam material there is no need to use the exact formula for the maximum shear stress. Please remember, however, the approximate formula is only for the maximum horizontal shear stress (which occurs are the neutral axis) in an I-Beam. If we need to know the shear stress at any other location, we must use the standard formula – as we will do in the next part.

We now wish to find the shear stress 3 inches above the bottom of the beam cross section, where the shear force is a maximum. (See Diagram 8). To do so, we apply the standard horizontal shear stress formula: **Shear Stress = Vay'/Ib**



V = maximum shear force = **6,000 lb**. (from the shear force diagram)
I = moment of inertia of cross section, from beam table; **I = 249.0 in^4.**
b = width of beam where we wish to find shear stress (3" above bottom of beam) from table; **b = .35 in**.
a = area from point we wish to find shear stress at (3" from the bottom) to an outer edge of beam. In this case we will go to bottom of beam. Notice that the area is composed of the area of the flange (A1) and part of the area of the web (A2). (See Diagrams 8 and 9.) Then **a = (A1 + A2)= (.618" x 8.022") + (2.383 in^2 x .35in^2) = 4.96 in^2 + .834 in^2= 5.794 in^2** (See Diagram 9)
y' = distance from neutral axis to the centroid of the area "**a**" Notice that in this case, for an I-Beam, this is not a entirely simple matter. The area we wish to find the centroid of is not a simple rectangle, but rather two rectangles. To find the centroid of this compound area we use: **y' = (A1 y1 + A2 y2) / (A1 +A2)**; where A1 and A2 are the two areas, and y1 and y2 are the distances from the neutral axis of the beam to the centroid of each of the respective areas. (Which is simply the distance from the neutral axis to the center of each of the respective areas, since for a rectangle the centroid is at the center.) Using the values shown in Diagram 9, we have:



$y' = (A1\ y1 + A2\ y2) / (A1 + A2) = (4.96\ in^2 \times 4.75\ in + .834\ in^2 \times 2.94\ in) / (4.96\ in^2 + .834\ in^2) = 4.49\ in.$

Then the horizontal shear stress 3 inches above the bottom of the beam is:

Horizontal Shear Stress = $Vay'/Ib = (6000\ lb)*(5.794\ in^2)*(4.49\ in) / (249.0\ in^4)(.35\ in) = 1790\ lb/in^2$

Notice, as we expect, the horizontal shear stress value becomes smaller as we move toward an outer edge of the beam cross section.

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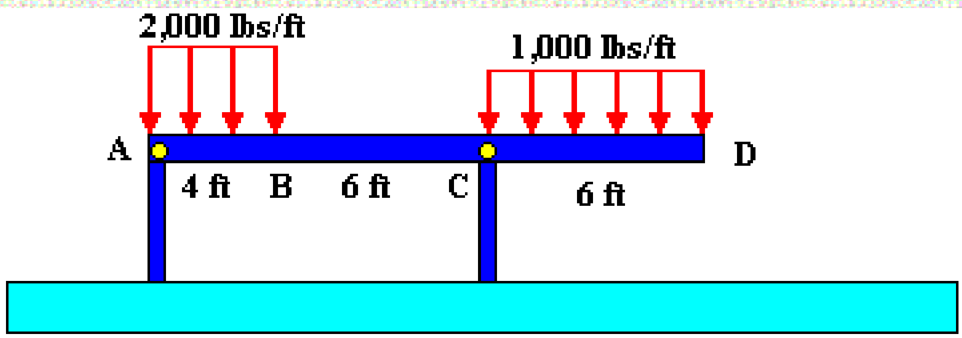
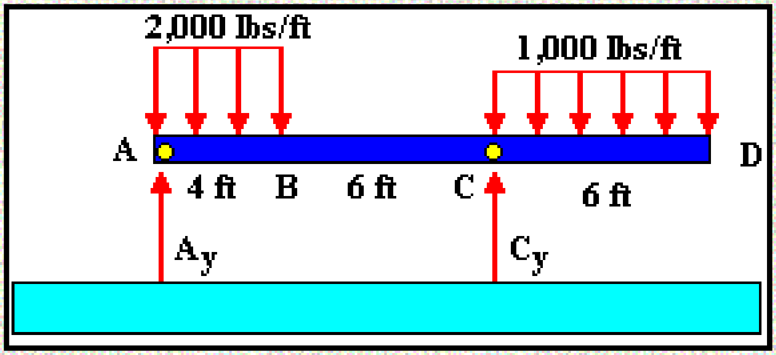
STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported W T 12 x 50 beam is shown below. For this beam:
A. Determine the maximum bending stress 12 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 7 inches above the bottom of the beam cross section and 12 feet from the left end of the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Part A:
STEP 1: Determine the external support reactions:
1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply Equilibrium Conditions:



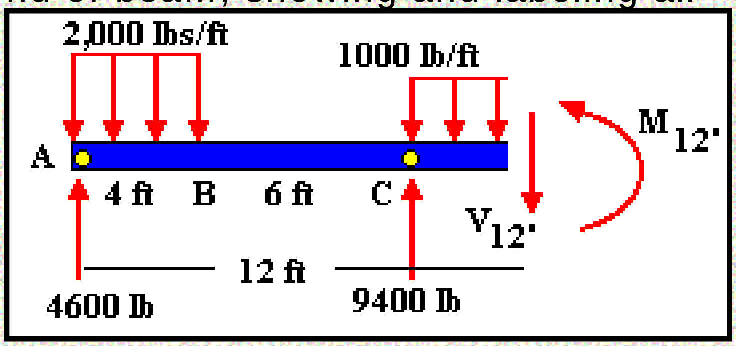
Sum F_x: none

Sum F_y: $A_y + C_y$ - 2000 lb/ft(4 ft) - 1000 lb/ft(6 ft) = 0

Sum Torque(A) = T_A = -(8000 lb)(2 ft) + C_y (10 ft) - (6000 lb)(13 ft) = 0

Solving: **C_y = 9400 lb. A_y = 4600 lb**

STEP 2: Determine the shear force and bending moment at x = 12 ft.
1.) Cut Beam at 12 ft. Draw the FBD of left end of beam, showing and labeling all external forces.
2.) Resolve all forces into x/y directions
3.) Apply Equilibrium Conditions:



Sum F_x: none

Sum F_y: 9400 lb - 2000 lb/ft(4 ft) + 4600 -

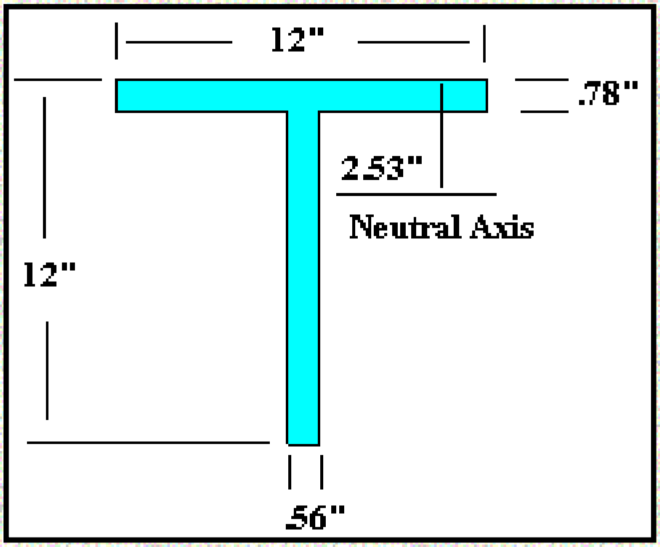
1000 lb/ft(2 ft) - $V_{12'}$ = 0

Sum Torque(A) = T_A = -(8000 lb)(2 ft) + 9400 lb (10 ft) - (2000 lb)(11 ft) - V(12) + $M_{12'}$= 0

Solving: **$V_{12'}$ = 4000 lb. $M_{12'}$ = -8000 ft-lb**

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress at 12 ft.

Maximum Bending Stress **(MBS) = $M_{12'}$/S** (Where $M_{12'}$ is the bending moment at 12 ft., and S is the section modulus for the beam. The section modulus is available from the Beam Tables. The WT 12 X 50 beam is shown in the diagram to the right. The section modulus for this beam from the beam tables is 18.7 in^3.)



MBS = -8000 ft-lb. (12in./ft)/18.7 in^3 = -5130 lb/in^2.

The Maximum bending stress occurs at the bottom of the T-beam (since it is the outer edge most distance from the neutral axis) The negative sign means that the beam is bent concave downward at this point, so the bottom of the beam is in compression.

Part B:

STEP 4: To determine the Horizontal Shear Stress (HSS) at 12 ft from the left end of the beam and 7 inches above the bottom of the beam, apply the horizontal shear stress formula. The form we will use is: **HSS = Vay'/Ib**

Where:

V = Shear force 12 ft from the end of the beam

a = cross sectional area from 7 in above the bottom of the beam to bottom of beam

y' = distance from neutral axis to the centroid of area **a**

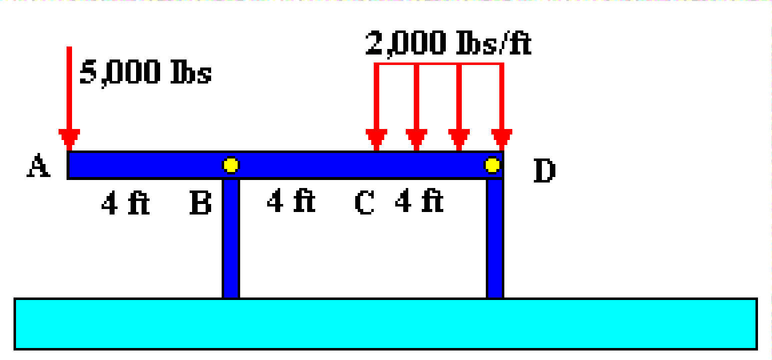
I = moment of inertia of the beam (177 in^4 for W 12 x 50 beam)

b = width of beam a 7 in above the bottom of the beam

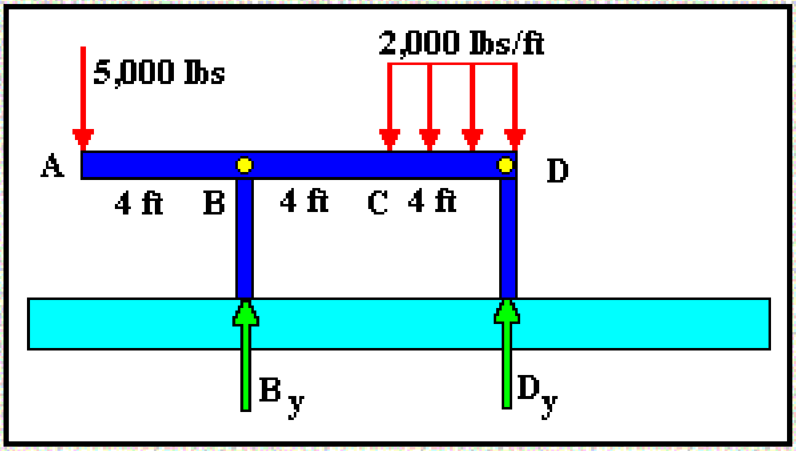
HSS = [(4000 lb.)(7" x ..47")(5.97")]/[(177 in^4)(.47)] = 944 lb/in^2

STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported W 10 x 45 beam is shown below. For this beam:
A. Determine the maximum bending stress 6 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 4 inches above the bottom of the beam cross section and 6 feet from the left end of the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Part A:
STEP 1: Determine the external support reactions:
1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

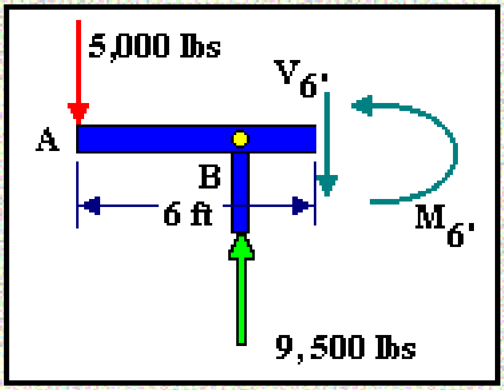


Sum F_x = 0 none

Sum F_y = B_y + D_y - 2,000 lbs/ft (4 ft) - 5,000 lbs = 0
Sum T_B = 5,000 lbs (4 ft) - 2,000 lbs/ft (4 ft) (6 ft) + D_y(8 ft) = 0
Solving: **B_y = 9,500 lbs; D_y = 3,500 lbs**

STEP 2: Determine the shear force and bending moment at x=6 ft.
1.) Cut beam at 6 ft. Draw the FBD of left end of beam, showing and labeling all external forces.
2.) Resolve all forces into x/y directions.

3.) Apply equilibrium conditions:



Sum F_x = 0 none

Sum F_y = -5,000 lbs + 9,500 lbs - V_6 = 0

Sum T_A = 9,500 lbs (4 ft) - 4,500 lbs (6 ft) + M_6 = 0

Solving: **V_6 = 4,500 lbs; M_6 = -11,000 ft-lbs**

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress (MBS) at 6'.

MBS = $M_{6'}$/S (Where $M_{6'}$ is the bending moment at 6 ft, and S is the section modulus for the beam. The section modulus is available from the Beam Tables. The W 10 x 45 beam has a section modulus for the beam from the beam tables is 49.1 in^3.)

MBS = -11,000 ft-lbs(12 in/ft)/49.1 in^3 = -2,688 lbs/in^2

Part B:

STEP 4: To determine the Horizontal Shear Stress (HSS) at 6 ft from the end of the beam and 4 inches above the bottom of the beam, apply the horizontal shear stress formula.

The form we will use is: **HSS = Vay'/Ib**

Where:

V = Shear force 6 ft from the end of the beam

a = cross sectional area from 4 in above the bottom of the beam to bottom of beam

y' = distance from neutral axis to the centroid of area **a**

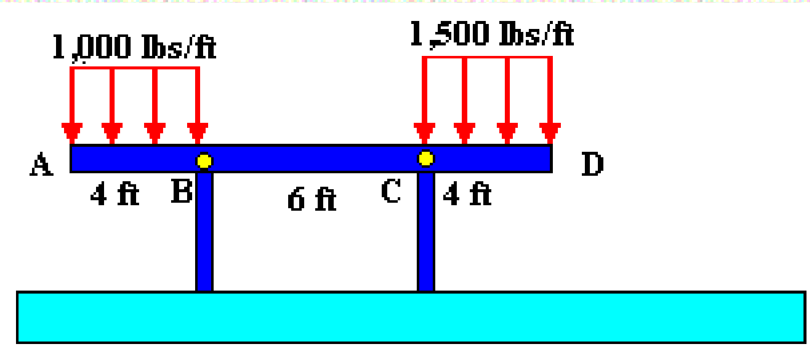
I = moment of inertia of the beam (249 in^4 for W 10 x 45 beam)

b = width of beam a 4 in above the bottom of the beam

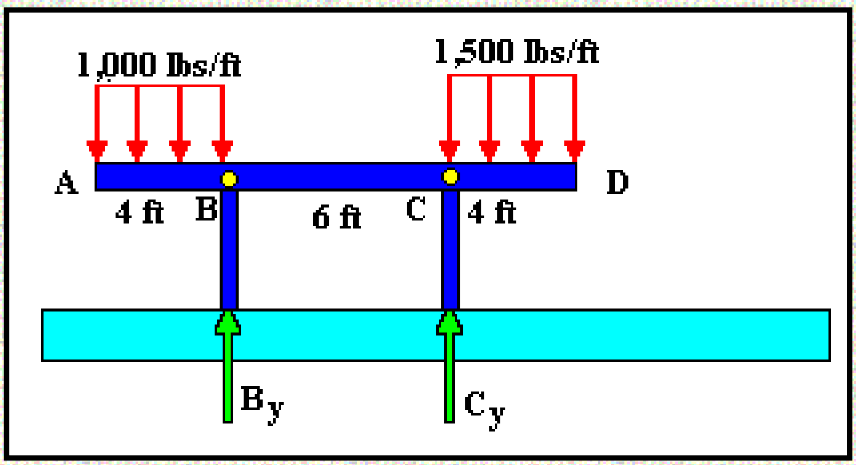
HSS = [(4,500 lbs)(6.153 in^2)(4.37 in)]/[(249 in^4)(.35 in)] = 1,388 psi

STATICS & STRENGTH OF MATERIALS - Example

A simply supported rectangular 2 x 12 beam is loaded as shown below. For this beam:
A. Determine the maximum bending stress 8 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 3 inches above the bottom of the beam cross section and 8 feet from the left end of the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Part A:
STEP 1: Determine the external support reactions:
1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:



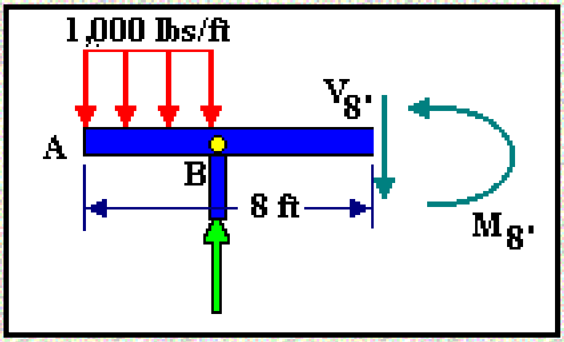
Sum F_x = 0 none

Sum $F_y = B_y + C_y$ - 1,000 lbs/ft (4 ft) - 1,500 lbs/ft (4 ft) = 0

Sum T_B = 1,000 lbs/ft (4 ft) (2 ft) - 1,500 lbs/ft (4 ft) (8 ft) + C_y(6 ft) = 0

Solving: **B_y = 3,330 lbs; C_y = 6,670 lbs**

STEP 2: Determine the shear force and bending moment at x=8 ft.
1.) Cut beam at 8 ft. Draw the FBD of left end of beam, showing and labeling all



external forces.
2.) Resolve all forces into x/y directions.
3.) Apply equilibrium conditions:

Sum F_x = 0 none

Sum F_y = -1,000 lbs/ft (4 ft) + 3,330 lbs - V_8 = 0

Sum T_A = -1,000 lbs/ft (4 ft) (2 ft) + 3,330 lbs (4 ft) + M_8 = 0

Solving: **V_8 = 667 lbs; M_8 = 10,670 ft-lbs**

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress (MBS) at 8'.

MBS = My / I (Where $M_{8'}$ is the bending moment at 8 ft, and S is the section modulus for the beam. The section modulus is available from the Beam Tables. This is a 2 x 12 beam.)

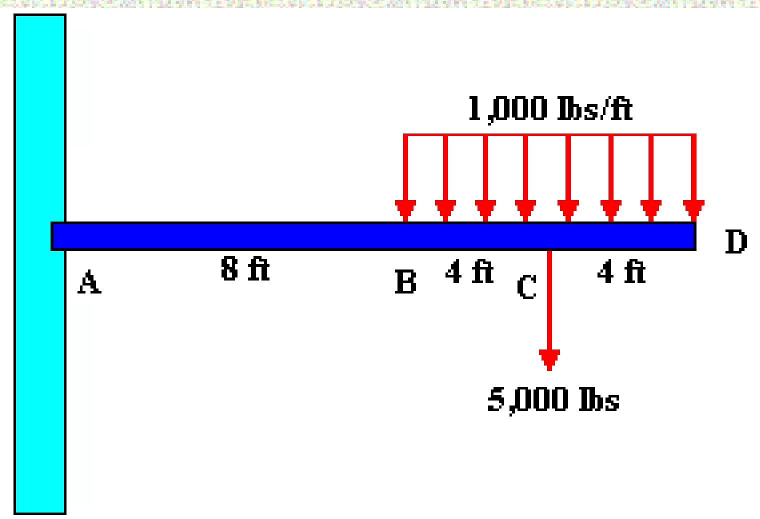
MBS = -10,670 ft-lbs(12 in / ft)(6 in) / [(1 / 12)(2 in)(12 in)^3] = 2,667 lbs/ in^2

Part B:
STEP 4: To determine the Horizontal Shear Stress (HSS) at 8 ft from the end of the beam and 3 inches above the bottom of the beam, apply the horizontal shear stress formula.
The form we will use is: **HSS = Vay' / Ib**
Where:
V = Shear force 8 ft from the end of the beam
a = cross sectional area from 3 in above the bottom of the beam to bottom of beam
y' = distance from neutral axis to the centroid of area a
I = moment of inertia of the beam (288 in^4 for 2 x 12 beam)
b = width of beam a 3 in above the bottom of the beam

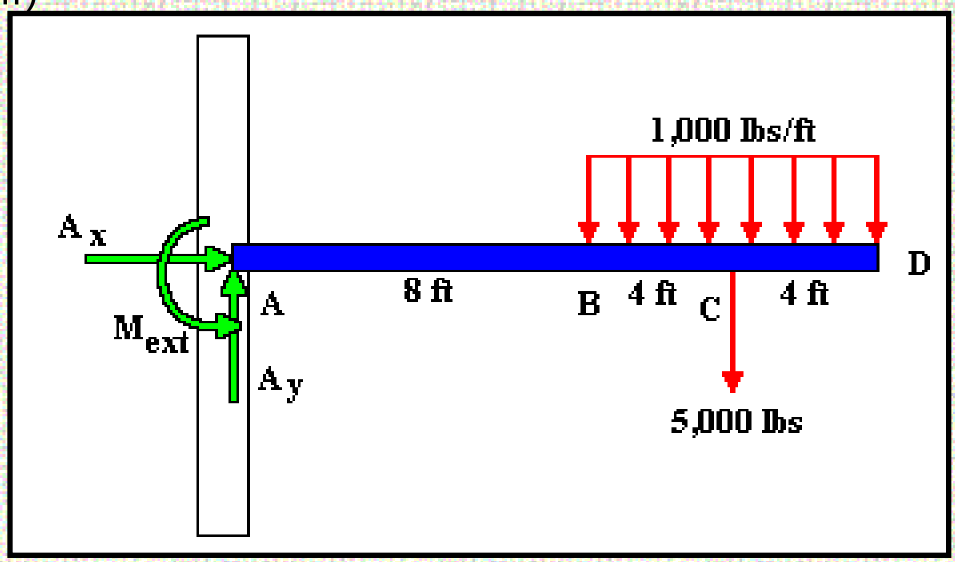
HSS = [(667 lbs)(6 in^2)(4.5 in)] / [(288 in^4)(2 in)] = 31.3 psi

STATICS & STRENGTH OF MATERIALS - Example

A loaded, cantilever 2 x 10 rectangular beam is shown below. For this beam:
A. Determine the maximum bending stress 4 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 6 inches above the bottom of the beam cross section and 4 feet from the left end of the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



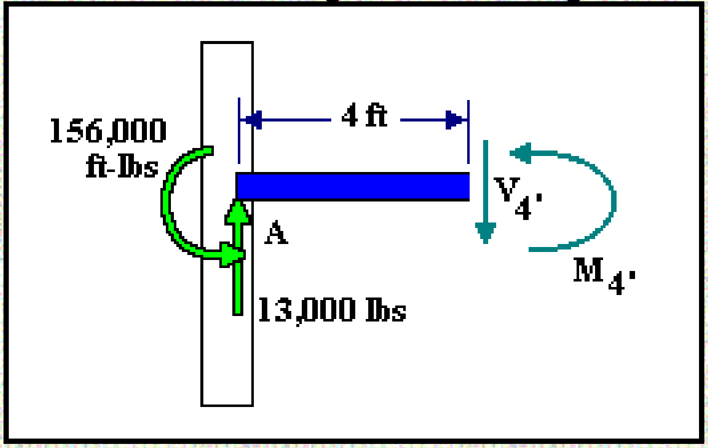
Solution:
Part A:
STEP 1: Determine the external support reactions:
1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:



Sum $F_x = A_x = 0$
Sum $F_y = A_y$ - 1,000 lbs/ft (8 ft) - 5,000 lbs = 0
Sum T_A = -5,000 lbs (12 ft) - 1,000 lbs/ft (8 ft) (12 ft) + M_{ext} = 0
Solving: **A_y = 13,000 lbs;**

M_{ext} = 156,000 ft-lbs

STEP 2: Determine the shear force and bending moment at x=4 ft.
1.) Cut beam at 4 ft. Draw the FBD of left end of beam, showing and labeling all external forces.
2.) Resolve all forces into x/y directions.
3.) Apply equilibrium conditions:



Sum F_x = 0 none

Sum F_y = 13,000 lbs - V_4 = 0

Sum T_A = $-V_4$(4 ft) +156,000 ft-lbs + M_4 = 0

Solving: **V_4 = 13,000 lbs; M_4 = 104,000 ft-lbs**

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress (MBS) at 4'.

MBS = My / I (Where $M_{4'}$ is the bending moment at 4 ft, and S is the section modulus for the beam. The section modulus is available from the Beam Tables. This is a 2 x 10 beam.)

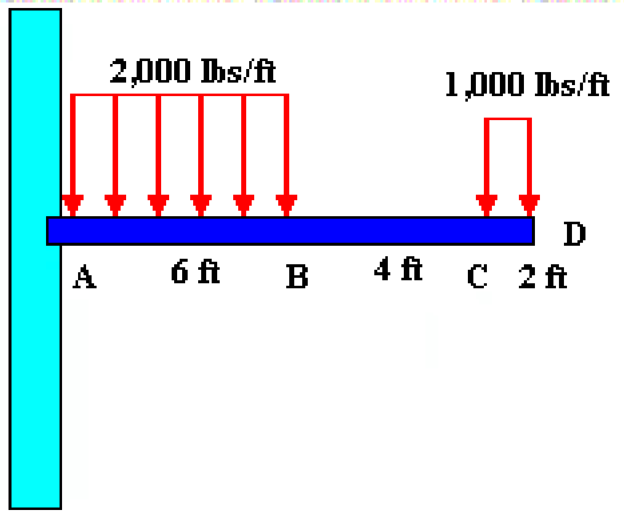
MBS = 104,000 ft-lbs(12 in / ft)(5 in) / [(1 / 12)(2 in)(10 in)^3] = 37,400 lbs / in^2

Part B:
STEP 4: To determine the Horizontal Shear Stress (HSS) at 4 ft from the end of the beam and 6 inches above the bottom of the beam, apply the horizontal shear stress formula.
The form we will use is: **HSS = Vay' / Ib**
Where:
V = Shear force 4 ft from the end of the beam
a = cross sectional area from 6 in above the bottom of the beam to bottom of beam
y' = distance from neutral axis to the centroid of area a
I = moment of inertia of the beam (167 in^4 for 2 x 10 beam)
b = width of beam a 6 in above the bottom of the beam

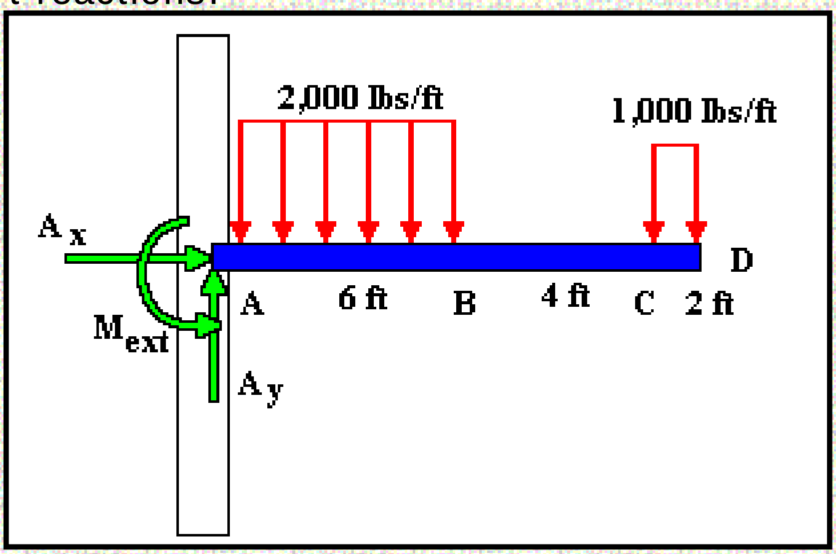
HSS = [(13,000 lbs)(12 in^2)(2 in)] / [(167 in^4)(2 in)] = 934 psi

STATICS & STRENGTH OF MATERIALS - Example

A loaded, cantilever WT 8 x 29 beam is shown below. For this beam:
A. Determine the maximum bending stress 5 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 6 inches above the bottom of the beam cross section and 5 feet from the left end of the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Part A:
STEP 1: Determine the external support reactions:
1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:



Sum $F_x = A_x = 0$

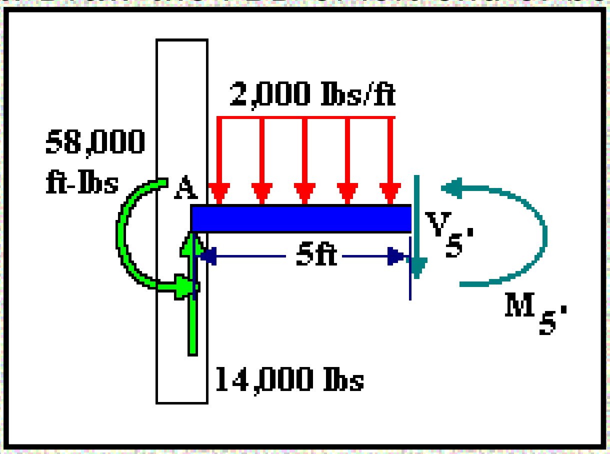
Sum $F_y = A_y$ - 2,000 lbs/ft (6 ft) - 1,000 lbs/ft (2 ft) = 0

Sum T_A = 2,000 lbs/ft (6 ft) (3 ft) - 1,000 lbs/ft (2 ft) (11 ft) + M_{ext} = 0

Solving: **A_y = 14,000 lbs; M_{ext} = 58,000 ft-lbs**

STEP 2: Determine the shear force and bending moment at x=5 ft.

1.) Cut beam at 5 ft. Draw the FBD of left end of beam, showing and labeling all external forces.



2.) Resolve all forces into x/y directions.
3.) Apply equilibrium conditions:

Sum F_x = 0 none

Sum F_y = 14,000 lbs - 2,000 lbs/ft (5 ft) - V_5 = 0

Sum T_A = -2,000 lbs/ft (5 ft) (2.5 ft) + 58,000 ft-lbs $-V_5$(5 ft) + M_5 = 0

Solving: **V_5 = 4,000 lbs; M_5 = -13,000 ft-lbs**

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress (MBS) at 5'.

MBS = $M_{5'}$ / S (Where $M_{5'}$ is the bending moment at 5 ft, and S is the section modulus for the beam. The section modulus is available from the Beam Tables. The WT 8 x 29 beam has a section modulus for the beam from the beam tables is 7.0 in^3.)

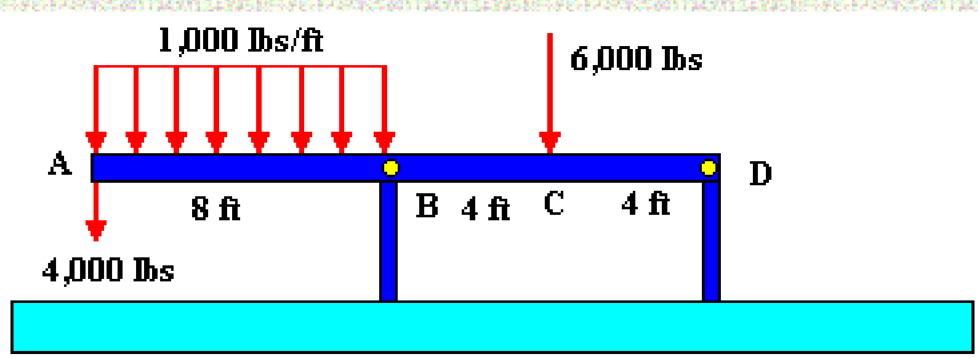
MBS = 13,000 ft-lbs(12 in / ft) / 7.0 in^3 = 21,900 lbs / in^2

Part B:
STEP 4: To determine the Horizontal Shear Stress (HSS) at 5 ft from the end of the beam and 6 inches above the bottom of the beam, apply the horizontal shear stress formula.
The form we will use is: **HSS = Vay' / Ib**
Where:
V = Shear force 5 ft from the end of the beam
a = cross sectional area from 6 in above the bottom of the beam to bottom of beam
y' = distance from neutral axis to the centroid of area a
I = moment of inertia of the beam (44 in^4 for WT 8 x 29 beam)
b = width of beam a 6 in above the bottom of the beam

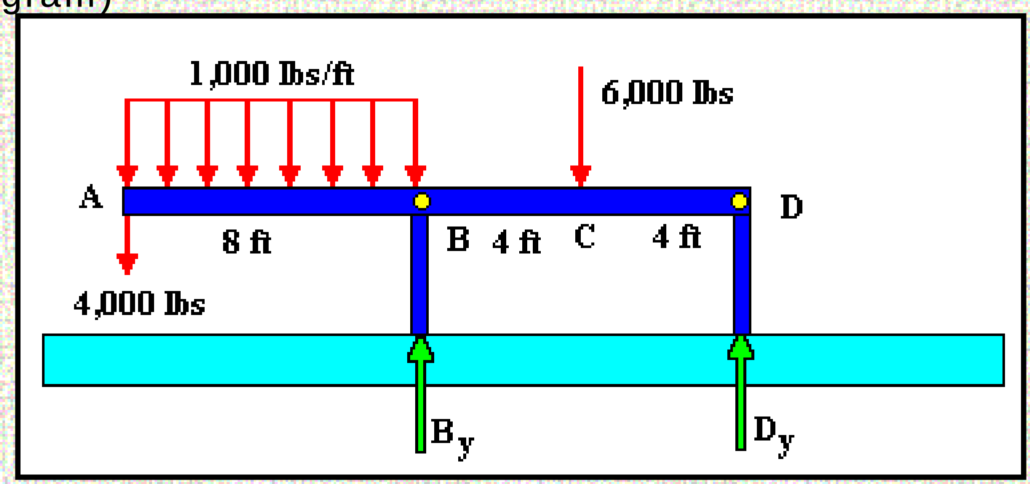
HSS = [(4,000 lbs)(2.46 in^2)(3.19 in)] / [(44 in^4)(.41 in)] = 7,740 psi

STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported W 8 x 28 beam is shown below. For this beam:
A. Determine the maximum bending stress 10 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 6 inches above the bottom of the beam cross section and 10 feet from the left end of the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Part A:
STEP 1: Determine the external support reactions:
1.) FBD of structure (See Diagram)



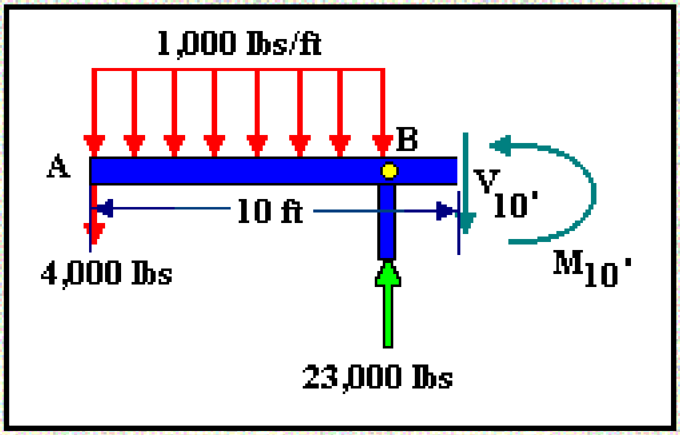
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

Sum F_x = 0 none

Sum $F_y = B_y + D_y$ - 1,000 lbs/ft (8 ft) - 4,000 lbs - 6,000 lbs = 0
Sum T_B = 1,000 lbs/ft (8 ft) (4 ft) + 4,000 lbs (8 ft) - 6,000 lbs (4 ft) + D_y (8 ft) = 0

Solving: **B_y = 23,000 lbs; D_y = -5,000 lbs**

STEP 2: Determine the shear force and bending moment at x=10 ft.



1.) Cut beam at 10 ft. Draw the FBD of left end of beam, showing and labeling all external forces.
2.) Resolve all forces into x/y directions.
3.) Apply equilibrium conditions:

Sum F_x = 0 none

Sum F_y = -4,000 lbs - 8,000 lbs + 23,000 lbs - V_{10} = 0

Sum T_A = -8,000 lbs (4 ft) + 23,000 lbs (8 ft) -V_{10}(10 ft) + M_{10} = 0

Solving: **V_{10} = 11,000 lbs; M_{10} = -42,000 ft-lbs**

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress (MBS) at 10'.

MBS = $M_{10'}$/S (Where $M_{10'}$ is the bending moment at 10 ft, and S is the section modulus for the beam. The section modulus is available from the Beam Tables. The W 8 x 28 beam has a section modulus for the beam from the beam tables is 24.3 in^3.)

MBS = 42,000 ft-lbs(12 in/ft)/24.3 in^3 = 20,740 lbs/in^2

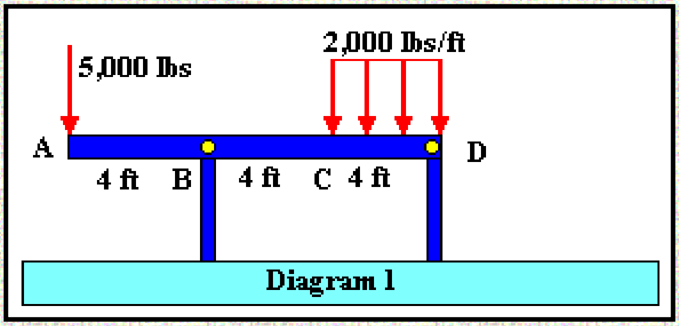
Part B:
STEP 4: To determine the Horizontal Shear Stress (HSS) at 10 ft from the end of the beam and 6 inches above the bottom of the beam, apply the horizontal shear stress formula.
The form we will use is: **HSS = Vay'/Ib**
Where:
V = Shear force 10 ft from the end of the beam
a = cross sectional area from 6 in above the bottom of the beam to bottom of beam
y' = distance from neutral axis to the centroid of area a
I = moment of inertia of the beam (97.8 in^4 for W 8 x 28 beam)
b = width of beam a 6 in above the bottom of the beam

HSS = [(11,000 lbs)(4.62 in^2)(2.76 in)]/[(97.8 in^4)(.29 in)] = 4,950 psi

Topic 5.4: Beams -Beam Selection

Another very important use of the flexure formula is in Beam Selection. That is, how does one decide on the best (safe and least expensive) beam to use with a particular loading. Perhaps the best way to explain this process is to work carefully through an example of the procedure.

In Diagram 1 we have shown a loaded beam (same loading as in **Beams II - Bending Stress Example II)**. For this beam and loading we would like to select the best I-Beam to use (from a selection of I-Beams, which we will discuss shortly)



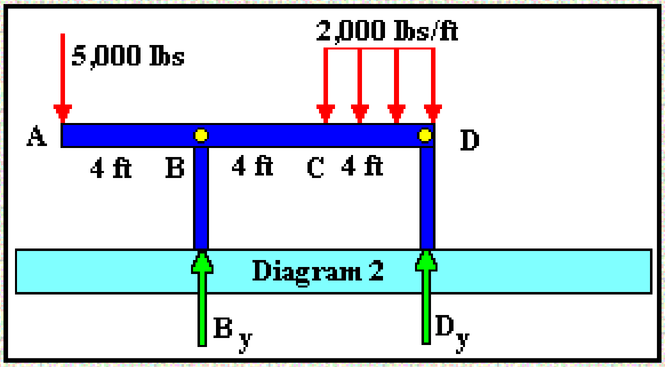
STEP 1: Apply Static Equilibrium Principles and determine the external support reactions.
1.) Draw Free Body Diagram of structure (See Diagram 2)
2.) Resolve all forces into x & y components
3.) Apply equilibrium conditions:

Sum F_x = 0 none

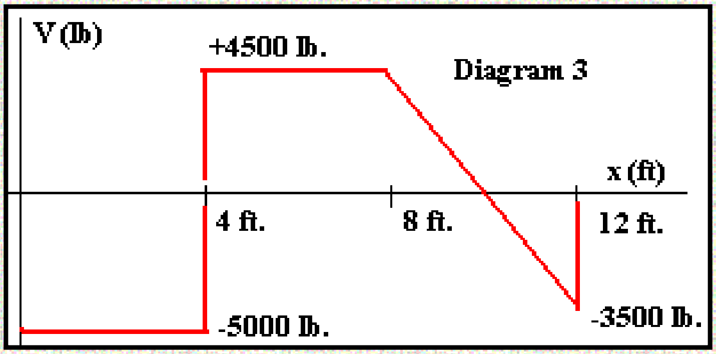
Sum $F_y = B_y + D_y$ - 2,000 lbs/ft (4 ft) - 5,000 lbs = 0

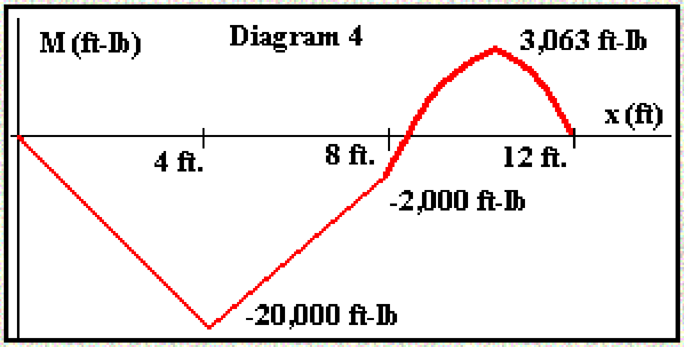
Sum T_B = 5,000 lbs (4 ft) - 2,000 lbs/ft (4 ft) (6 ft) + D_y(8 ft) = 0

Solving: **B_y = 9,500 lbs; D_y = 3,500 lbs**



STEP 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. The graphs are shown in Diagram 3 and Diagram 4. (We determine these diagrams in some detail in [Topic 4.8a: Bending Stress - Example 2](). If you need more detail please see that example.)





From the Diagrams we observe that **M_{max} = -20,000 ft-lb.**; and **V_{max} = -5000 lb**.

Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. For this example we will use the following allowable stresses for the beam material:

$\sigma_{tension} = 20{,}000$ psi ; $\sigma_{compression} = 20{,}000$ psi ; $\tau_{shear} = 16{,}000$ psi

We now use the flexure formula form: **σ_{max} = M_{max} / S**, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:

20,000 lb/in^2 = (20,000 ft-lb.)(12 in./ft.) / S; and then **S = 240,000 in-lb./ 20,000 lb/ in^2 = 12 in^3** .

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam). After examining the selections, we determine **W 8 x 17** is the best beam from the selection listed. It has a section modulus of 14.1 in^3 (greater than the minimum section modulus of 12 in^3), and a weight of 17 lb/ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.

Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	b_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 6x25	7.35	6.37	6.080	0.456	0.320	53.3	16.7	2.69	17.10	5.62	1.53
W 6x20	5.88	6.20	6.018	0.367	0.258	41.5	13.4	2.66	13.30	4.43	1.51
W 6x15.5	4.56	6.00	5.995	0.269	0.235	30.1	10.0	2.57	9.67	3.23	1.46
W 8x20	5.89	8.14	5.268	0.378	0.248	69.4	17.0	3.43	9.22	3.50	1.25
W 8x17	5.01	8.00	5.250	0.308	0.230	56.6	14.1	3.36	7.44	2.83	1.22
W 8x67	19.70	9.00	8.287	0.933	0.575	272.0	60.4	3.71	88.60	21.40	2.12
W 8x40	11.80	8.25	8.077	0.558	0.365	146.0	35.5	3.53	49.00	12.10	2.04
W 8x35	10.30	8.12	8.027	0.493	0.315	126.0	31.1	3.50	42.50	10.60	2.03
W 8x31	9.12	8.00	8.000	0.433	0.288	110.0	27.4	3.47	37.00	9.24	2.01

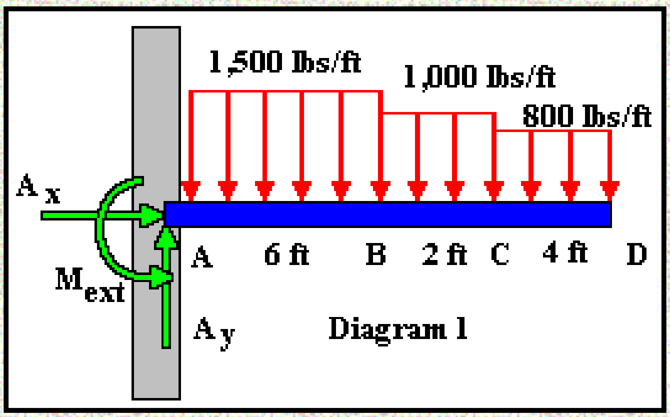
Step 4. The last step is to now check that the beam we have selected is also safe with respect to the horizontal shear stress – that is, that the maximum horizontal shear stress for the selected beam is within the allowable shear stress for the beam material. We therefore now apply our formula for the maximum horizontal shear stress in an I-Beam: $\tau_{max} = V_{max} / A_{web} = 5000$ **lb / (.230 x 7.384) = 2944 lb / in^2.** We see that this value is well within the allowable shear stress of 16,000 lb/in2 given above. Thus we have selected the best beam to use from the given list of possible beam.

We now look at an example selecting the **best T-Beam** to use for a particular loading.
Please Select **Topic 5.4a: Beam Selection Example 1**

When finished with Example of Beam Selection,
Select:
Topic 5: Beams II - Table of Contents
Strength of Materials Home Page

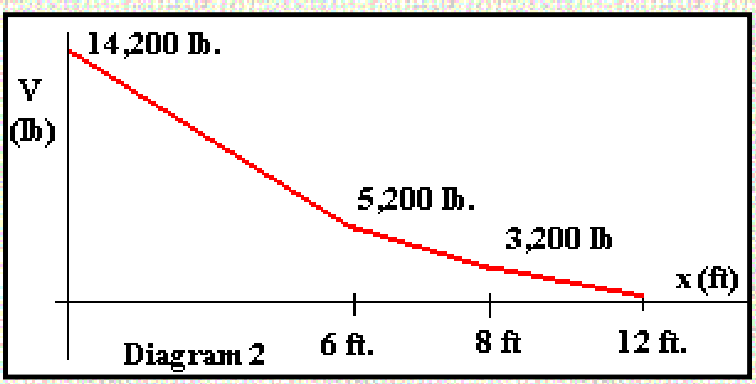
Topic 5.4a: Beam Selection - Example 1

A loaded, cantilever beam is shown in Diagram 1. We would like to choose the best T-beam to use from the T-Beam selection shown at the lower part of this page.
The **maximum allowable bending stress = 35,000 lb./in2**. (both in tension and compression), and the **maximum allowable shear stress = 15,000 lb/in2** for the beam material used.

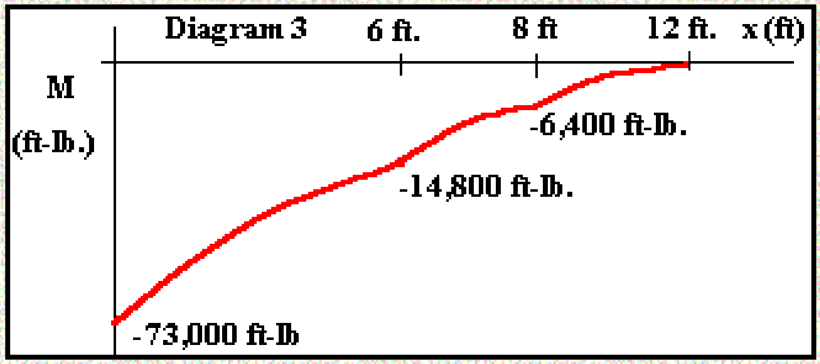


STEP 1: Apply Static Equilibrium Principles and determine the external support reactions:
1.) FBD of structure (See Diagram 1)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:
Sum $F_x = A_x = 0$
Sum $F_y = A_y$ - (1,500 lb./ft.)(6 ft) - (1,000 lb./ft.)(2 ft) - (800 lb./ft.)(4 ft) = 0
Sum T_B = -(1,500 lb./ft.)(6 ft)(3 ft.)-(1,000 lb./ft.)(2 ft)(7 ft.)-(800 lb./ft.)(4 ft)(10 ft.) + M_{ext} = 0
Solving: A_y = 14,200 lb.; M_{ext} = 73,000 ft-lb.

STEP 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. (See Diagrams 2 and 3.)



From the Diagrams we observe that **M_{max} = -73,000 ft-lb.; and V_{max} = 14,200 lb**.



Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. The allowable stresses were given at the beginning of this problem as: **Maximum Allowable Bending Stress = 35,000 psi.; and Maximum Allowable Shear Stress = 15,000 psi.**

We now use the flexure formula form: $\sigma_{max} = M_{max} / S$, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:
35,000 lb/in^2 = (73,000 ft-lb.)(12 in./ft.)/ S; and then **S = 876,000 in-lb./ 35,000 lb/in^2 = 25 in^3** .

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam).

After examining the selections, we determine **WT 15 x 49.5** is the best beam from the selection listed. It has a section modulus of 30.1 in^3 (greater than the minimum section modulus of 25 in^3), and a weight of 49.5 lb./ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

-	-	Depth	Flange	Flange	Stem	-	Cross	Section	Info.	-
Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A	d	b_f	t_f	tw	d/t_w	I	S	r	y
-	in^2	in	in	in	in	-	in^4	in^3	in	in

WT 7 x157	46.20	8.60	16.235	2.283	1.415	6.07	179.00	27.000	1.970	1.980
WT 7x143.5	42.20	8.41	16.130	2.093	1.310	6.42	157.00	24.100	1.930	1.870
WT 7x132	38.80	8.25	16.025	1.938	1.205	6.85	139.00	21.500	1.890	1.780
WT 7x123	36.20	8.13	15.945	1.813	1.125	7.22	126.00	19.600	1.860	1.710
WT 13.5x57	16.80	13.64	10.070	0.932	0.570	23.90	289.00	28.300	4.150	3.410
WT 13.5x51	15.00	13.54	10.018	0.827	0.518	26.10	258.00	25.400	4.140	3.380
WT 13.5x47	13.80	13.46	9.990	0.747	0.490	27.50	239.00	23.800	4.150	3.410
WT 13.5 x42	12.40	13.35	9.963	0.636	0.463	28.80	216.00	22.000	4.180	3.500
WT 15x66	19.40	15.15	10.551	1.000	0.615	24.60	421.00	37.400	4.650	3.900
WT 15x62	18.20	15.08	10.521	0.930	0.585	25.80	395.00	35.300	4.650	3.890
WT 15x58	17.10	15.00	10.500	0.850	0.564	26.60	372.00	33.600	4.670	3.930
WT 15x54	15.90	14.91	10.484	0.760	0.548	27.20	350.00	32.100	4.690	4.020
WT 15x49.5	14.60	14.82	10.458	0.670	0.522	28.40	323.00	30.100	4.710	4.100

Step 4. The last step is to now check that the beam we have selected is also safe with respect to the horizontal shear stress – that is, that the maximum horizontal shear stress for the selected beam is within the allowable shear stress for the beam material. We therefore now apply our formula for the horizontal shear stress for the **WT 15 x 49.5 T-beam: Shear Stress = Vay' / Ib**

The **maximum** shear stress occurs at the **neutral axis of the beam:**

V = maximum shear force = **14,200 lb**. (from the shear force diagram)

I = moment of inertia of cross section, from beam table; **I = 323 in^4.**

b = width of beam where we wish to find shear stress (neutral axis for maximum) from table; **b = .522 in**.

a = area from point we wish to find shear stress at (neutral axis) to an outer edge of beam. In this case we will go to bottom of beam. Then **a = (.522" * 10.72")= 5.6 in^2**.

y' = distance from neutral axis to the centroid of the area "**a**" ; y' = **5.36 in**.

Maximum Horizontal Shear Stress = Vay' / Ib = (14,200 lb) * (5.6 in^2) * (5.36 in) / (323 in^4)(.522 in) = 2530 lb / in^2

We see that this value is well within the allowable shear stress of 15,000 lb / in2 given above. Thus we have selected the best beam to use from the given list of possible beam.

Return to:
[Topic 5.4: Beams II - Beam Selection]
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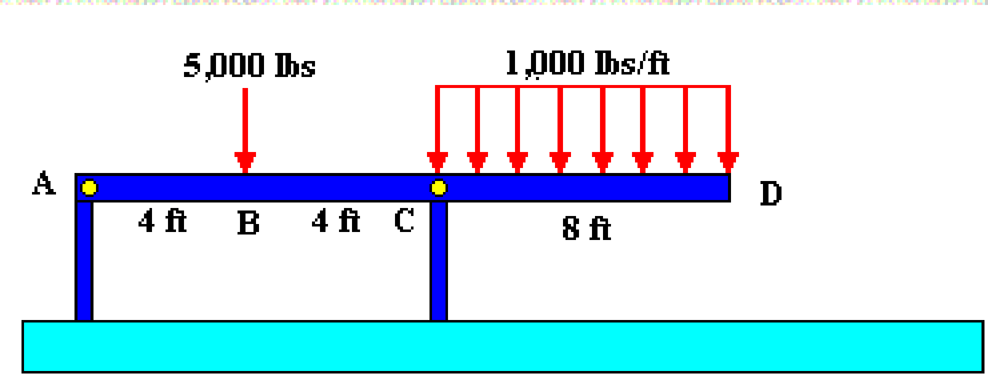
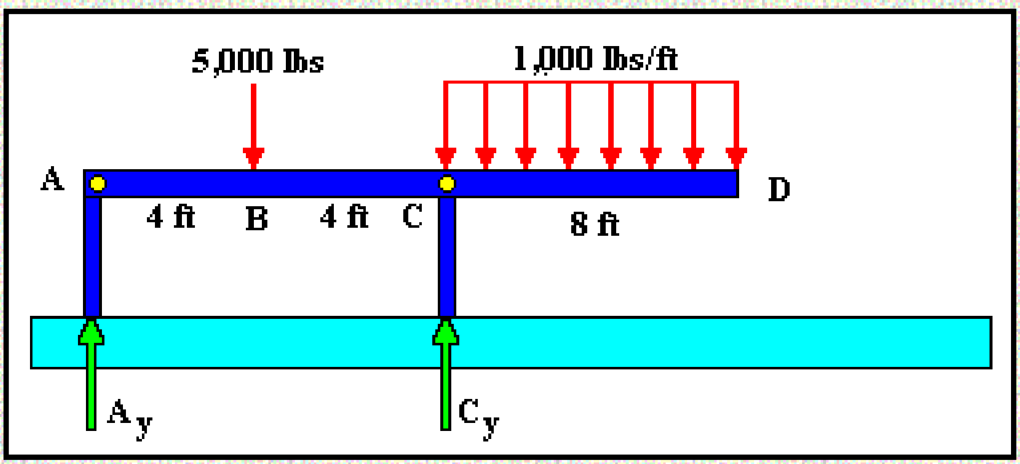
STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported beam is shown below. For this beam:

Select the best T-beam to use if:
1). The maximum allowable bending stress = 30,000 lb/in^2.
2). The maximum allowable shear stress = 12,000 lb/in^2.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Step 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

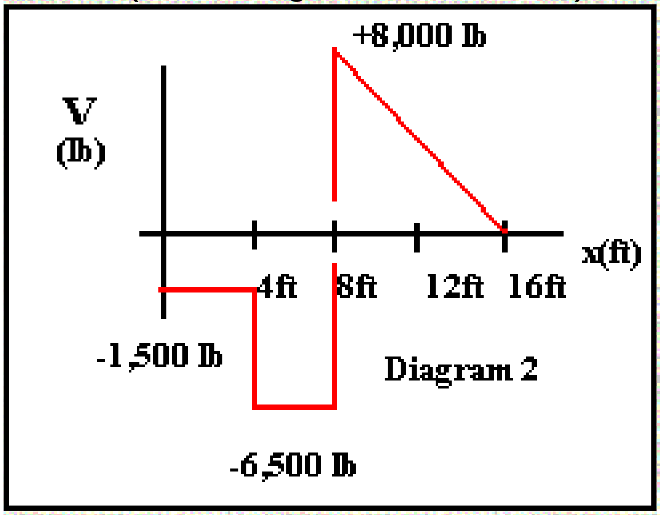
Sum F_x = 0 none

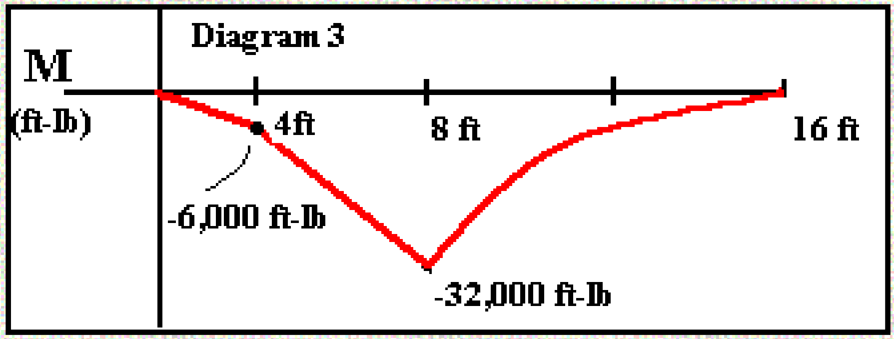
Sum $F_y = A_y + C_y$ - 5,000 lbs - 1,000 lbs/ft (8 ft)= 0

Sum T_A = -5,000 lbs (4 ft) + C_y (8 ft) - 1,000 lbs/ft (8 ft)(12 ft)= 0

Solving: **A_y = -1500 lbs; C_y = 14,500 lbs**

Step 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. (See Diagrams 2 and 3.)





From the Diagrams we observe that **M_{max} = -32,000 ft-lb.; and V_{max} = 8,000 lb**.

Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. The allowable stresses were given at the beginning of this problem as: **Maximum Allowable Bending Stress = 30,000 psi.; and Maximum Allowable Shear Stress = 12,000 psi.**

We now use the flexure formula form: σ_{max} = **M_{max} / S**, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:
30,000 lb/in^2 = (32,000 ft-lb.)(12 in./ft.) / S; and then **S = 384,000 in-lb./ 30,000 lb/in^2 = 12.8 in^3** .

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam).

Step 4. After examining the selections, we determine **WT 12 x 34** is the best beam from the selection listed. It has a section modulus of 15.6 in^3 (greater than the minimum section modulus of 12.8 in^3), and a weight of 34 lb./ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

T-Beam Data

	-	Depth			Stem				Info.	-
Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A	d	b_f	t_f	tw	d/t_w			r	y
-	in^2	in	in	in	in				in	in
WT 12x34	10.00	11.86	8.961	0.582	0.416		137.00	15.600	3.700	3.070

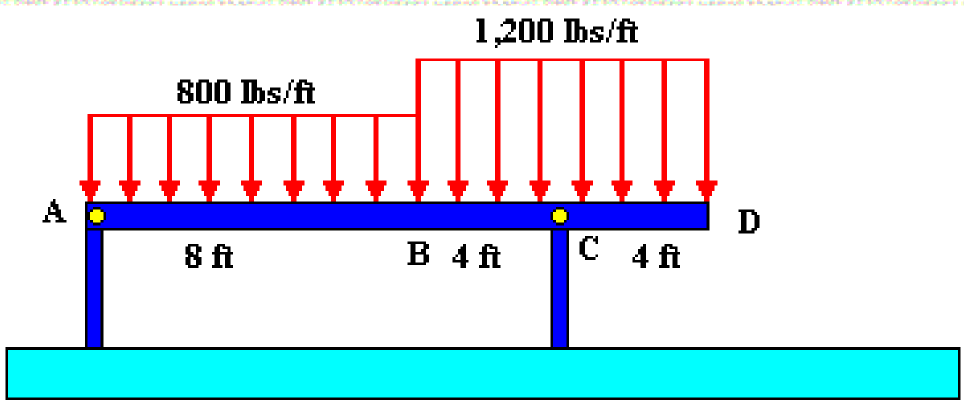
Step 5: Now using the selected beam we check that the maximum horizontal shear stress (HSS) in the beam is within the allowable stress.

τ **= Vay' / Ib = (8000 lb.)(8.83" * .42")(4.42") / (137 in^4 * .42") = 2280 lb/in^2** which is lower than the allowable shear stress of 12,000 lb/in^2. So the beam is safe and is our best choice of the beams given to choose from.

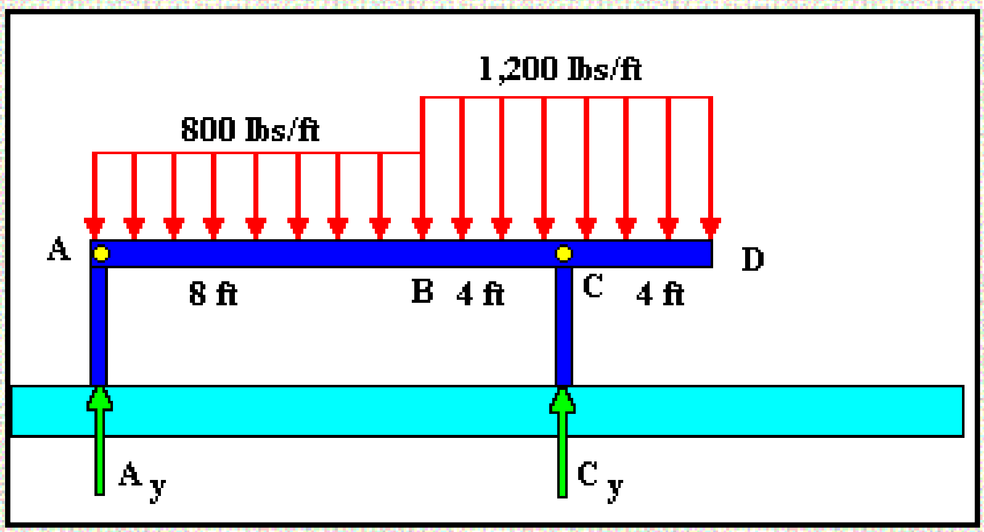
STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported beam is shown below. For this beam:

Select the best I-beam to use if:
1). The maximum allowable bending stress = 35,000 lb/in^2.
2). The maximum allowable shear stress = 10,000 lb/in^2.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
STEP 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

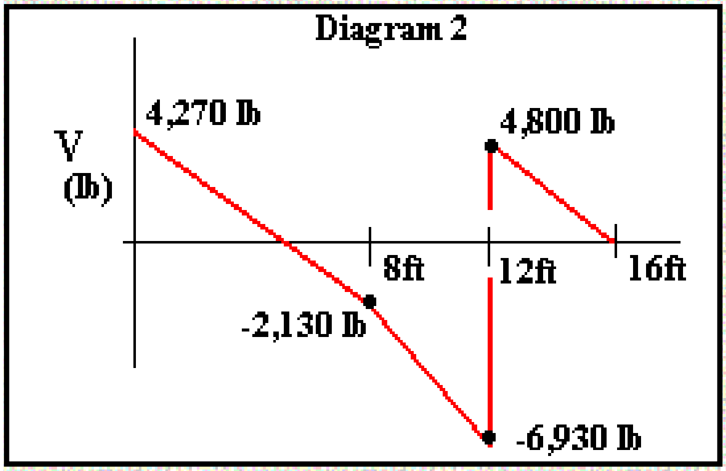
Sum F_x = 0 none

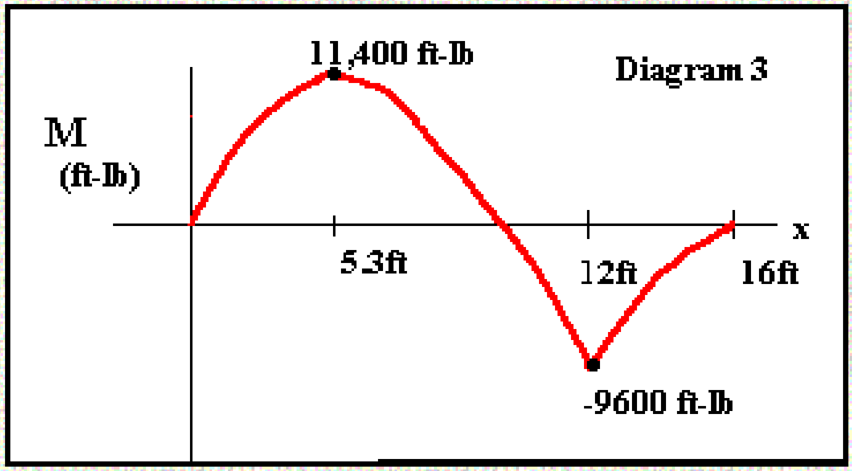
Sum F_y = A_y + C_y - 800 lbs/ft (8 ft) - 1,200 lbs/ft (8 ft) = 0

Sum T_A = -800 lbs (8 ft)(4 ft) + C_y (12 ft) - 1,200 lbs/ft (8 ft)(12 ft) = 0

Solving: **A_y = 4270 lbs; C_y = 11,730 lbs**

STEP 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. (See Diagrams 2 and 3.)





From the Diagrams we observe that M_{max} **= 11,400 ft-lb.; and** V_{max} **= 6,930 lb**.

Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. The allowable stresses were given at the beginning of this problem as: **Maximum Allowable Bending Stress = 35,000 psi.; and Maximum Allowable Shear Stress = 10,000 psi.**

We now use the flexure formula form: $\sigma_{max} = M_{max} / S$, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:

35,000 lb/in^2 = (11,400 ft-lb.)(12 in./ft.) / S; and then **S = 136,800 in-lb./ 35,000 lb/in^2 = 3.91 in^3** .

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam).

Step 4. After examining the selections, we determine **W 6 x 8.5** is the best beam from the selection listed. It has a section modulus of 5.1 in^3 (greater than the minimum section modulus of 3.91 in^3), and a weight of 8.5 lb./ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

I-Beam Data

-	-	-	Flange	Flange	Web						
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	b_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 6x8.5	2.51	5.83	3.940	0.194	0.170	14.8	5.1	2.43	1.98	1.01	0.89

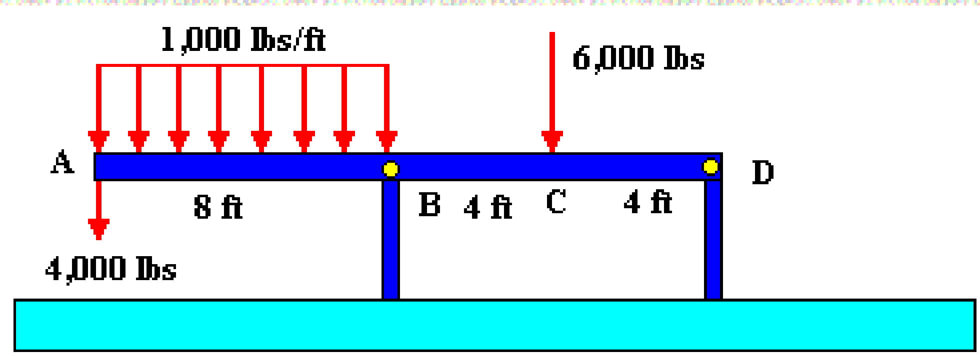
Step 5: Now using the selected beam we check that the maximum horizontal shear stress (HSS) in the beam is within the allowable stress.

$\tau_{max} = V_{max} / a_{web}$ **= 6930 lb / (5.442 in. * .170 in.)= 7,490 lb/in^2** which is lower than the allowable shear stress of 12,000 lb/in^2. So the beam is safe and is our best choice of the beams given to choose from.

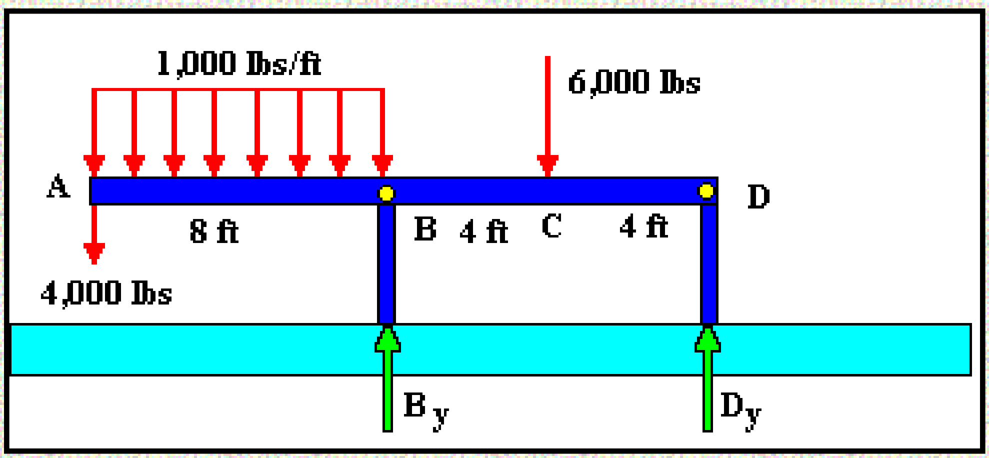
STATICS & STRENGTH OF MATERIALS - Example

A loaded, simply supported beam is shown below. For this beam:

Select the best T-beam to use if:
1). The maximum allowable bending stress = 20,000 lb/in^2.
2). The maximum allowable shear stress = 10,000 lb/in^2.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Step 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

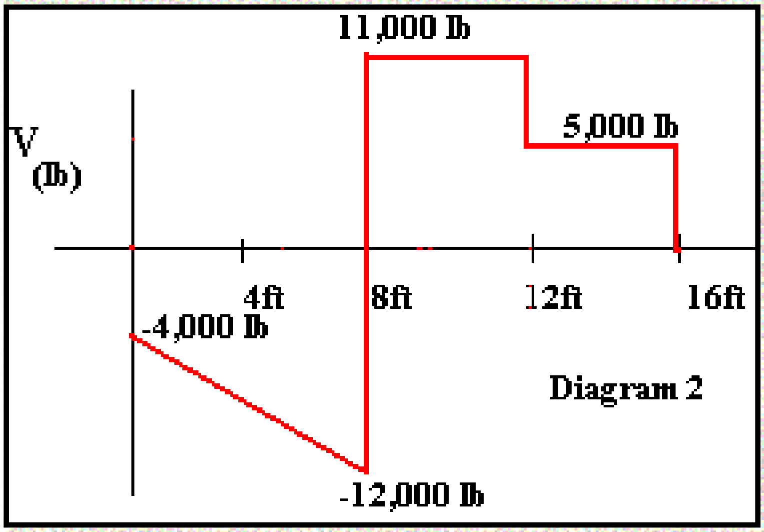
Sum F_x = 0 none

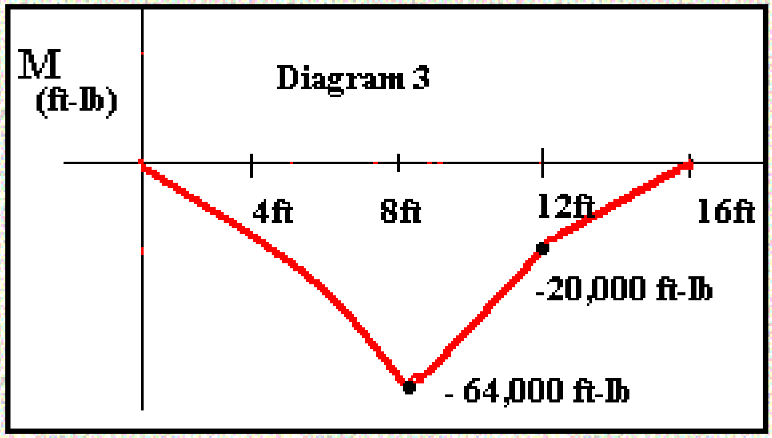
Sum F_y = B_y + D_y - 4,000 lbs - 1,000 lbs/ft (8 ft) - 6,000 lbs = 0

Sum T_D = 4,000 lbs (16 ft) + 1,000 lbs/ft (8 ft)(12 ft) - B_y (8 ft) + 6,000 lbs/ft (4 ft)= 0

Solving: **B_y = 23,000 lbs; D_y = - 5,000 lbs**

Step 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. (See Diagrams 2 and 3.)





From the Diagrams we observe that **M_{max} = -64,000 ft-lb.; and V_{max} = 12,000 lb**.

Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. The allowable stresses were given at the beginning of this problem as: **Maximum Allowable Bending Stress = 20,000 psi.; and Maximum Allowable Shear Stress = 10,000 psi.**

We now use the flexure formula form: σ_{max} = **M_{max} / S**, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:

20,000 lb/in^2 = (64,000 ft-lb.)(12 in./ft.) / S; and then **S = 768,000 in-lb./ 20,000 lb/in^2 = 38.4 in^3**.

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam).

Step 4. After examining the selections, we determine **W T 16.5 x 65** is the best beam from the selection listed. It has a section modulus of 42.2 in^3 (greater than the minimum section modulus of 38.4 in^3), and a weight of 65 lb./ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

T-Beam Data

-	-	Depth	Flange	Flange	Stem	-	Cross	Section	Info.	-
Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A	d	b_f	t_f	tw	d/t_w	I	S	r	y
-	in^2	in	in	in	in	-	in^4	in^3	in	in
WT 16.5x65	19.20	16.55	11.510	0.855	0.580	28.50	514.00	42.200	5.180	4.370

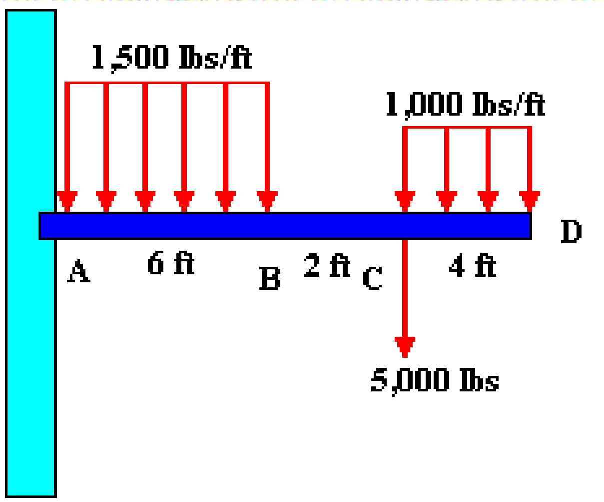
Step 5: Now using the selected beam we check that the maximum horizontal shear stress (HSS) in the beam is within the allowable stress.

τ = Vay' / Ib = (12,000 lb.)(12.18" * .58")(6.09") / (514 in^4 * .58") = 1,732 lb/in^2 which is lower than the allowable shear stress of 10,000 lb/in^2. So the beam is safe and is our best choice of the beams given to choose from.

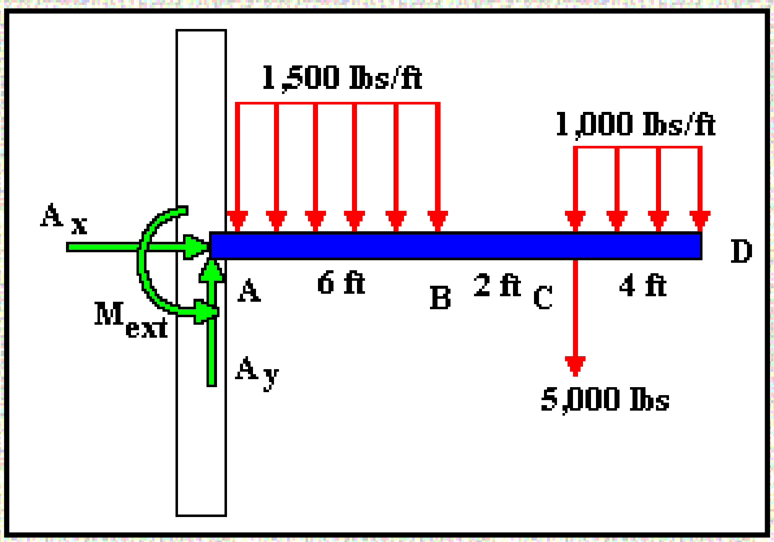
STATICS & STRENGTH OF MATERIALS - Example

A loaded, cantilever beam is shown below. For this beam:

Select the best I-beam to use if:
1). The maximum allowable bending stress = 25,000 lb/in^2.
2). The maximum allowable shear stress = 12,000 lb/in^2.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Step 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.



1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

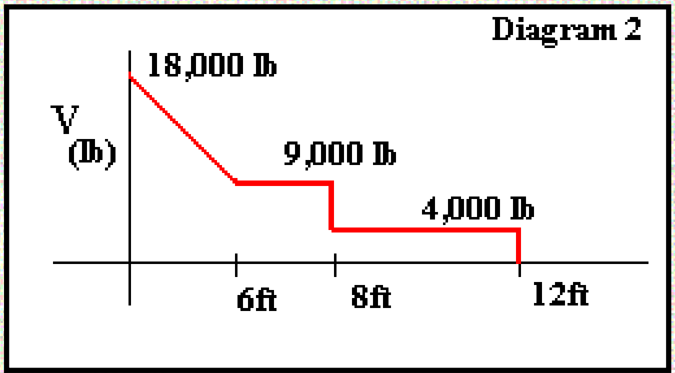
Sum F_x = 0 none

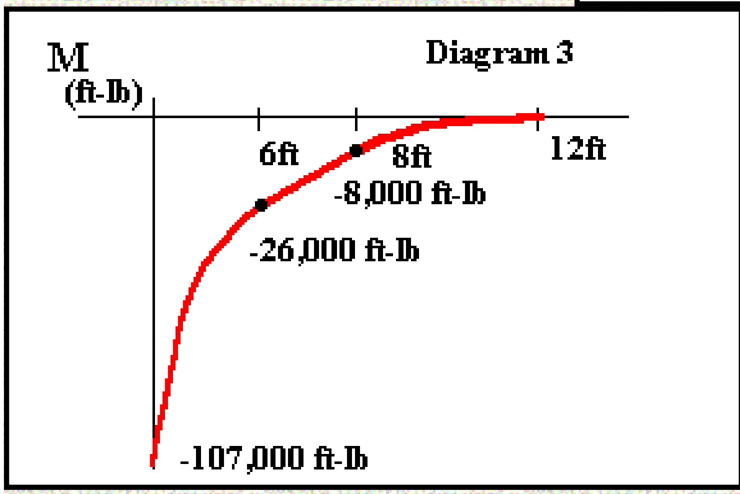
Sum F_y = A_y - 1,500 lb/ft (6 ft) - 5,000 lbs - 1,000 lb/ft (4 ft) = 0

Sum T_A = M_{ext} - 9,000 lbs (3 ft) - 5,000 lbs (8 ft) - 4,000 lbs (10 ft)= 0

Solving: **A_y = 18,000 lbs; M_{ext} = 107,000 ft-lbs**

Step 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. (See Diagrams 2 and 3.)





From the Diagrams we observe that **M_{max} = -107,000 ft-lb.; and V_{max} = 18,000 lb**.

Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. The allowable stresses were given at the beginning of this problem as: **Maximum Allowable Bending Stress = 25,000 psi.; and Maximum Allowable Shear Stress = 12,000 psi.**

We now use the flexure formula form: $\sigma_{max} =$ **M_{max} / S**, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:

25,000 lb/in^2 = (107,000 ft-lb.)(12 in./ft.) / S; and then **S = 1,284,000 in-lb./ 25,000 lb/in^2 = 51.4 in^3**.

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam).

Step 4. After examining the selections, we determine **W 14 x 36** is the best beam from the selection listed. It has a section modulus of 56.5 in^3 (greater than the minimum section modulus of 51.4 in^3), and a weight of 36 lb./ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

I-Beam Data

-	-	-	Flange	Flange	Web						
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	b_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 14x36	10.60	15.85	6.992	0.428	0.299	447.0	56.5	6.50	24.40	6.99	1.52

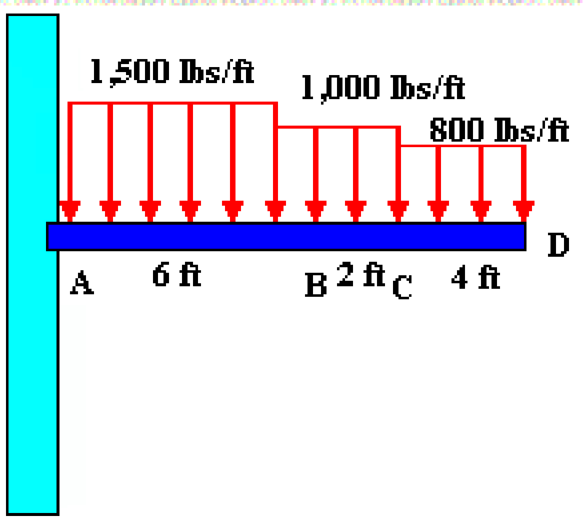
Step 5: Now using the selected beam we check that the maximum horizontal shear stress (HSS) in the beam is within the allowable stress.

$\tau_{max} = V_{max} / a_{web} = 18{,}000 \text{ lb} / (14.99 \text{ in.} * .299 \text{ in.}) = 4{,}020 \text{ lb/in}^2$ which is lower than the allowable shear stress of 12,000 lb/in^2. So the beam is safe and is our best choice of the beams given to choose from.

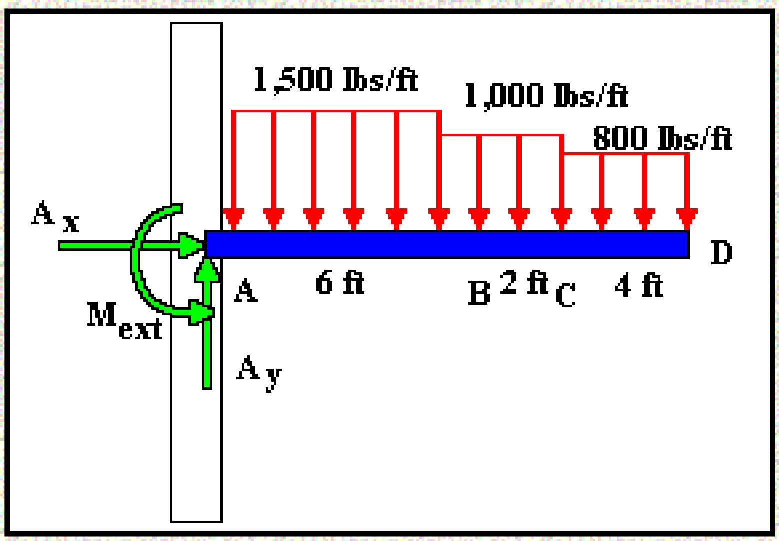
STATICS & STRENGTH OF MATERIALS - Example

A loaded, cantilever beam is shown below. For this beam:

Select the best T-beam to use if:
1). The maximum allowable bending stress = 35,000 lb/in^2.
2). The maximum allowable shear stress = 15,000 lb/in^2.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Step 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.

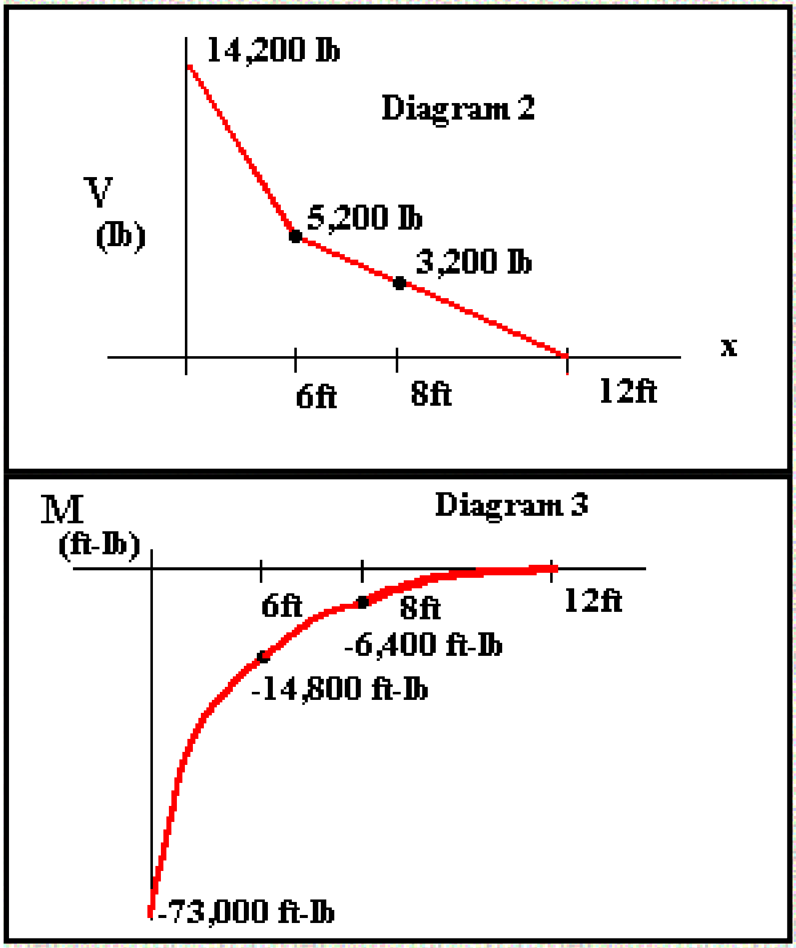


1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

Sum F_x = 0 none

Sum F_y = A_y - 1,500 lb/ft (6 ft) - 1,000 lbs/ft (2 ft) - 800 lb/ft (4 ft) = 0
Sum T_A = M_{ext} - 9,000 lbs (3 ft) - 2,000 lbs (7 ft) - 3,200 lbs (10 ft)= 0
Solving: **A_y = 14,200 lbs; M_{ext} = 73,000 ft-lbs**

Step 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. (See Diagrams 2 and 3.)



From the Diagrams we observe that **M_{max} = -73,000 ft-lb.; and V_{max} = 14,200 lb**.

Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. The allowable stresses were given at the beginning of this problem as: **Maximum Allowable Bending Stress = 35,000 psi.; and Maximum Allowable Shear Stress = 15,000 psi.**

We now use the flexure formula form: $\sigma_{max} =$ **M_{max} / S**, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:

$35{,}000\ lb/in^2 = (73{,}000\ ft\text{-}lb.)(12\ in./ft.)\ /\ S$; and then **$S = 876{,}000\ in\text{-}lb./35{,}000\ lb/in^2 = 25\ in^3$**.

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam).

Step 4. After examining the selections, we determine **WT 13.5 x 51** is the best beam from the selection listed. It has a section modulus of 25.4 in^3 (greater than the minimum section modulus of 25 in^3), and a weight of 51 lb./ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

T-Beam Data

-	-	Depth	Flange	Flange	Stem	-	Cross	Section	Info.	-
Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A	d	b_f	t_f	tw	d/t_w	I	S	r	y
-	in^2	in	in	in	in	-	in^4	in^3	in	in
WT 13.5x51	15.00	13.54	10.018	0.827	0.518	26.10	258.00	25.400	4.140	3.380

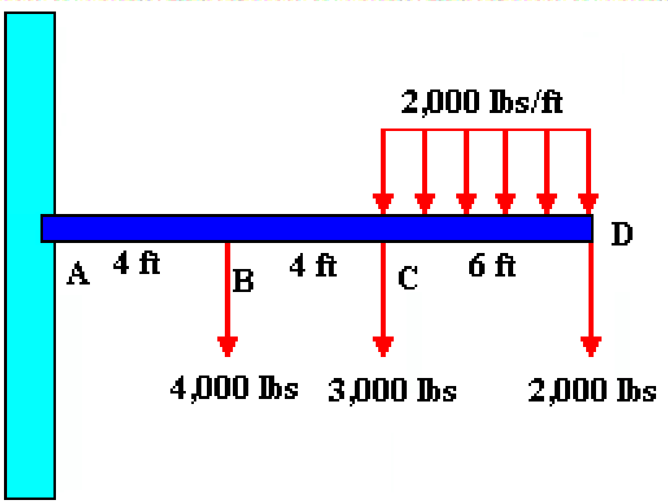
Step 5: Now using the selected beam we check that the maximum horizontal shear stress (HSS) in the beam is within the allowable stress.

$\tau = Vay'\ /\ Ib = (14{,}200\ lb.)(10.16" * .518")(5.08")\ /\ (258\ in^4 * .518") = 2{,}480\ lb/in^2$ which is lower than the allowable shear stress of 15,000 lb/in^2. So the beam is safe and is our best choice of the beams given to choose from.

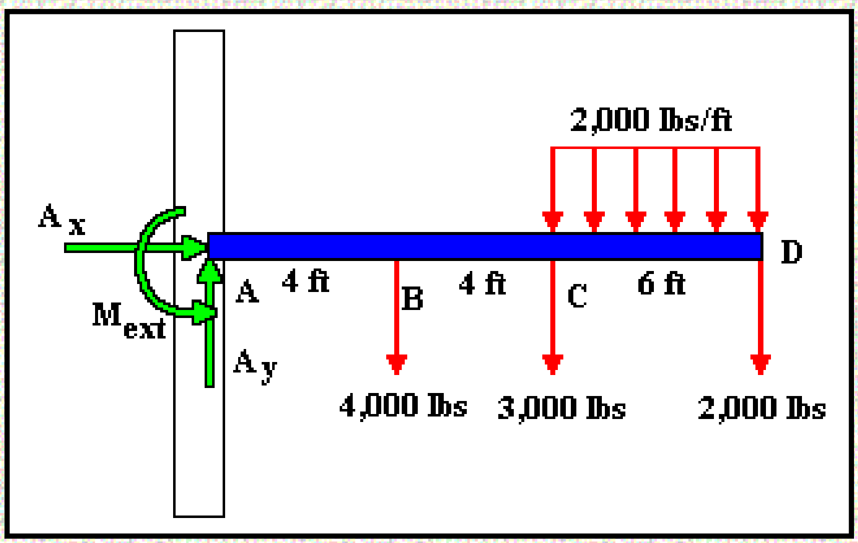
STATICS & STRENGTH OF MATERIALS - Example

A loaded, cantilever beam is shown below. For this beam:

Select the best I-beam to use if:
1). The maximum allowable bending stress = 40,000 lb/in^2.
2). The maximum allowable shear stress = 15,000 lb/in^2.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



Solution:
Step 1: Draw a free body diagram showing and labeling all load forces and support (reaction) forces, as well as any needed angles and dimensions.
1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:



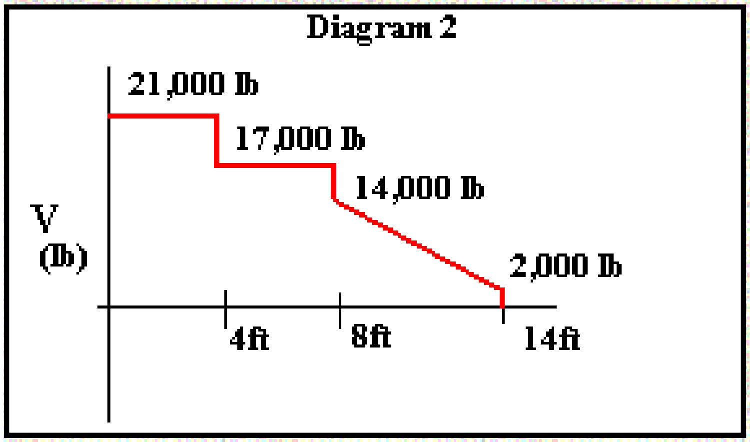
Sum F_x = 0 none

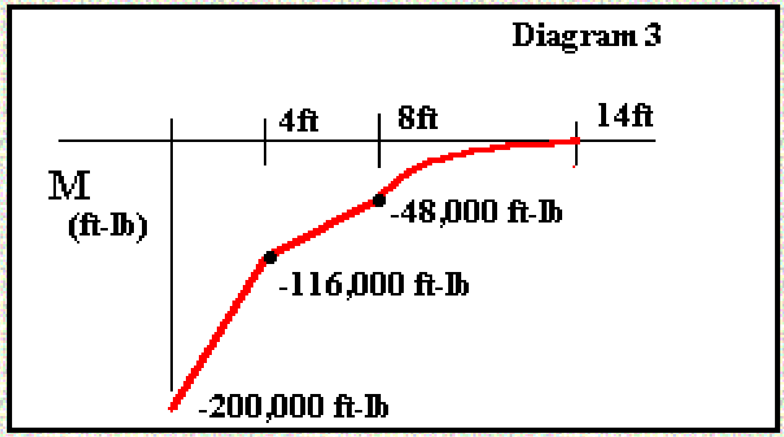
Sum F_y = A_y - 4,000 lbs - 3,000 lbs - 2,000 lbs/ft (6 ft) - 2000 lbs = 0

Sum T_A = Mext - 4,000 lbs (4 ft) - 3,000 lbs (8 ft) - 12,000 lbs (11 ft) - 2000 lb (14 ft) = 0

Solving: **A_y = 21,000 lbs; M_{ext} = 200,000 ft-lbs**

Step 2: Draw both the Shear Force and Bending Moment Diagrams for the Beam, and determine the values of the maximum bending moment and maximum shear force. (See Diagrams 2 and 3.)





From the Diagrams we observe that **M_{max} = -200,000 ft-lb.; and V_{max} = 21,000 lb**.

Step 3: Use the Flexure Formula for maximum bending stress and the specifications for the beam material to determine the **minimum Section Modulus** needed to carry the load. By material specification we mean the allowable stresses (tensile, compressive, and shear) for the beam material. This information is normally furnished by the beam supplier with their selection of beams. The allowable stresses were given at the beginning of this problem as: **Maximum Allowable Bending Stress = 40,000 psi.; and Maximum Allowable Shear Stress = 15,000 psi.**

We now use the flexure formula form: σ_{max} = **M_{max} / S**, and use the lowest allowable axial stress for the maximum bending stress, and solve for the value of the section modulus. Placing values into the equation we have:

40,000 lb/in^2 = (200,000 ft-lb.)(12 in./ft.)/ S; and then **S = 2,400,000 in-lb./ 40,000 lb/in^2 = 60 in^3**.

This value for the section modulus is the smallest value possible if the maximum bending stress is not to exceed the allowable axial stress for the beam material. Shown below is a selection of beams. We would like to now selected the best beam based on the minimum value of the section modulus determined above. We select the beam but find the one with a section modulus equal or greater than the minimum section modulus and with the least pounds per foot weight (which normally means the least expensive beam).

Step 4. After examining the selections, we determine **W 14 x 40** is the best beam from the selection listed. It has a section modulus of 64.6 in^3 (greater than the minimum section modulus of 60 in^3), and a weight of 40 lb./ft, which is the least weight for beams with a section modulus greater than the minimum from the beam selection listed below.

I-Beam Data

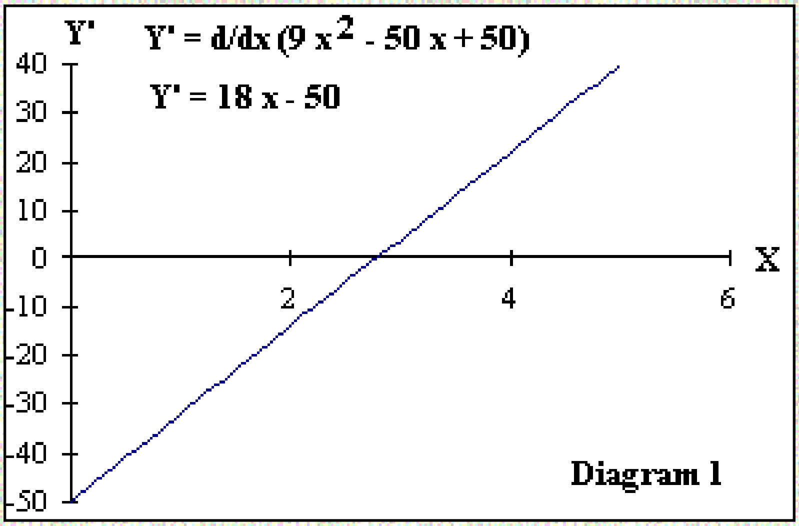
-	-	-	Flange	Flange	Web						
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	b_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 14x40	11.80	16.00	7.000	0.503	0.307	517.0	64.6	6.62	28.80	8.23	1.56

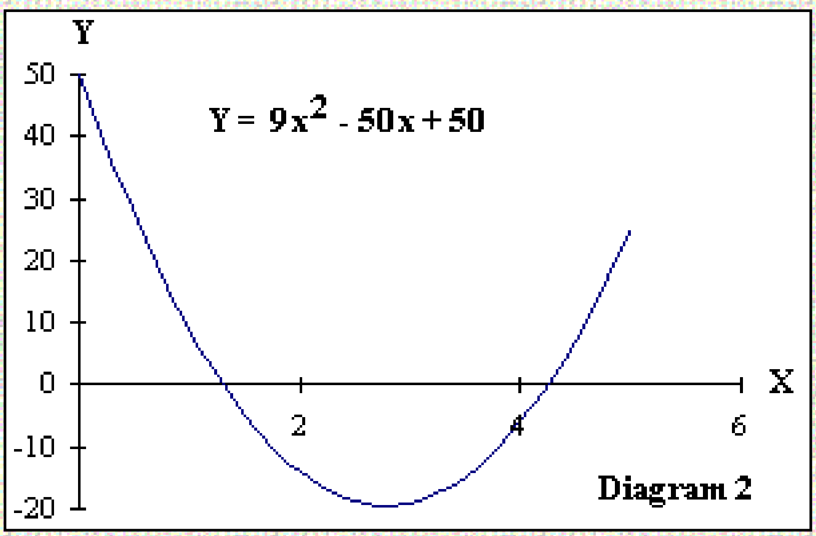
Step 5: Now using the selected beam we check that the maximum horizontal shear stress (HSS) in the beam is within the allowable stress.

$\tau_{max} = V_{max} / a_{web}$ **= 21,000 lb / (14.94 in. * .307 in.) = 4,580 lb/in^2** which is lower than the allowable shear stress of 15,000 lb/in^2. So the beam is safe and is our best choice of the beams given to choose from.

Review - Basic Calculus Concepts

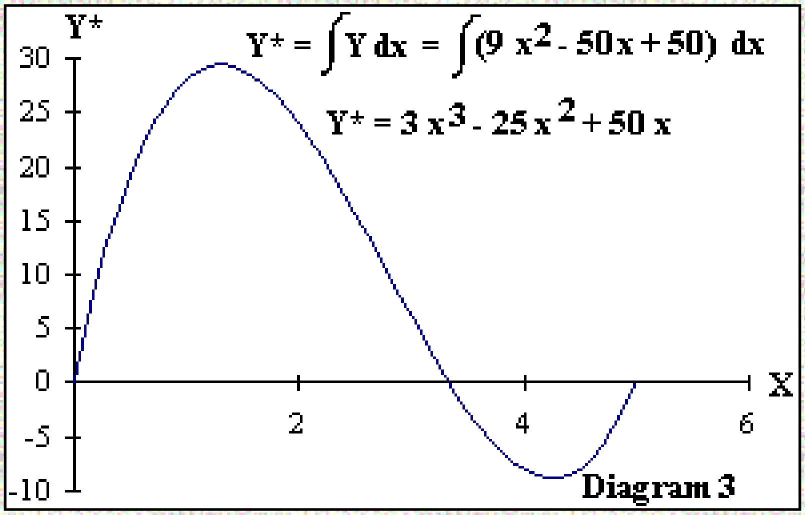
This is a very basic review of introductory Calculus concepts. We first look at a quadratic function: **$y = 9x^2 - 50x + 50$**, which is graphed in Diagram 2. This function has a slope at every point. If we take the **"derivative"** of our quadratic function, we obtain a new function (**$y' = 18x - 50$**), which is graphed in Diagram 1, next to Diagram 2. **The 'derivative' function gives us the value of the slope of our quadratic function at every point.** (**Thus at x = 2, the slope of the quadratic function is 18 * 2 - 50 = -14**)





If on the other hand we **"integrate"** our quadratic function, we obtain a new function (**$y^* = 3x^3 - 25x^2 + 50t$**), which is graphed in Diagram 3 directly below Diagram 2. The **'integrated' function** tells us the net area under the quadratic function curve (between the function curve and the x axis). It actually tells us the area between some beginning x-value and ending x-value (when we do what is

called a definite integral). For the integrated function above, the initial x value is zero and the ending x-value is what ever value we choose.
That is for x = 2, the area under the quadratic function curve between zero and 2 is: A = 3 $(2)^3$ -25 $(2)^2$ + 50 (2) = 24. Thus every y value on the curve in Diagram 3 is equal to the sum of the area under the curve in Diagram 2 up to that point.



We can also do an "indefinite" integral which results in 'integrated' function involving a constant - which is evaluated by applying a boundary condition.

A second way to think of derivatives and integrals is as inverse functions of each other. That is, the integral asks the question - what function must we take the derivative of to obtain what is inside the integral sign. Before we look at an example of this, let's review some basic derivatives (which are normally covered in the first semester of Calculus).

- **d / dx (any constant) = 0**
- **d / dx (x) = 1**
- **d / dx (x^2) = 2x**
- **d / dx (x^3) = $3x^2$**
- **d / dx (x^n) = n x^{n-1}**

Now we look at the indefinite integral $\int x\,dx = ?$. This integral asks the question - what function must we take the derivative of to obtain 'x' (what is inside the integral sign). With a little reflection we see that a possible function is: **½ x^2** . That is, if we take the derivative of **½ x^2**, we do obtain **x**. However, the function **½ x^2 + 5** is also a solution, as is the function **½ x^2 + 1,000,000**, or **½ x^2 + C** (where C is any arbitrary constant). Thus, we see that when one does an indefinite integral, the answer is a function plus a constant which must be determined by an

appropriate boundary condition.
Some basic integrals, (which we will use in determine bending moment expressions in loaded beams) are:

- $\int dx = x + C$
- $\int x\, dx = (1/2)\, x^2 + C$
- $\int x^2\, dx = (1/3)\, x^3 + C$
- $\int x^n\, dx = [1/(n+1)]\, x^{(n+1)}$
- $\int A\, f(x)\, dx = A \int f(x)\, dx$, where A is any constant
- $\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, dx$ that is, the integral of a sum (or difference) is the sum (or difference) of the integrals.

These simple integrals will be enough to solve beam problems involving standard type loads. For an example of the application of integration and use of a boundary condition in solving a problem,

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Beams 4.32 Example 1
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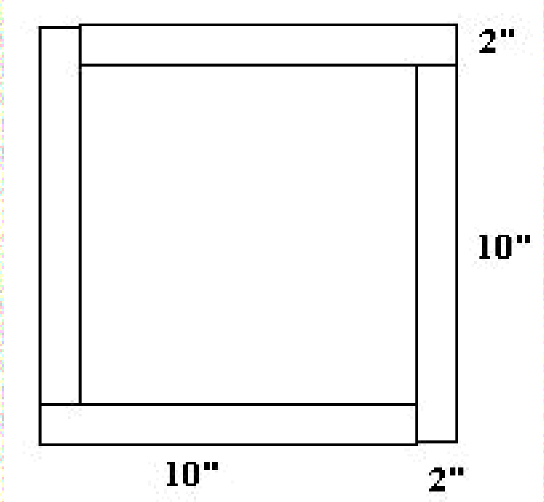
Topic 5.6a - Problem Assignment 1 - Beam Inertia, Bending Moment, Stress

1. Asquare box beam is made of four full size 2"x10"s. (See Image) Calculate the moment of inertia and section modulus of the beam. (1386 in^4, 231 in^3)



2. A book shelf is to be made of a clear pine board 1"x10". Determine the moment of inertia and section modulus of the 1"x10". (.833 in^4, 1.66 in^3)

3. The maximum bending moment in a rectangular 2"x 8", 12 foot long beam is 1150 foot pounds. Determine the maximum bending stress in the beam. (647 psi)

4. Determine the maximum bending moment that can be applied to a rectangular 6"x10" beam if the allowable stress in the member is 1200 psi. (120,000 in-lb = 10,000 ft-lb)

5. A wooden cantilever beam has a maximum bending moment of 3600 foot pounds. A rectangular wood beam, 12" high, is to support the load. If the allowable axial stress in 2000 psi., what is the minimum width of the beam needed to support the load? (.9")

6. Determine the moment of inertia and section modulus of a beam made of a solid rod 3 inches in diameter. Also determine the moment of inertia and section modulus of a beam made of a 3 inch diameter pipe with 1/4 inch walls. (3.98 in^4, 2.65 in^3, 2.06 in^4, 1.38in^3)

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Topic 5.6b - Problem Assignment 2 - Bending Stresses

For the following problems use a maximum allowable bending stress of 1500 psi. for wood, 25,000 psi. for steel and 6000 psi. for concrete.

1. A W10x21 steel cantilever beam is 8 feet long. Determine the maximum point load that a the beam can carry at its end without exceeding the allowable bending stress. (5630 lb.)

2. Find the maximum point load that a 14 foot long steel W24x84 can carry at its midpoint without exceeding the allowable bending stress. The beam is simply supported at its ends. (117,260 lb.)

3. Find the maximum point load that a 30 foot long steel W30x108 can carry at its midpoint without exceeding the allowable bending stress. The beam is simply supported at its ends. (83.330 lb.)

4. Determine the minimum section modulus needed for a 14 foot beam to carry a uniformly distributed load of 160 pounds per foot. The beam is simply supported at its ends. Select the appropriate wood beam to carry this load.(31.4 in^3, 2" x 10" selection may depend on table used)

5. A parking ramp floor is supported by horizontal concrete beams. The horizontal concrete beams are supported at the quarter points, that is 1/4 of the beam length in from each end. The beams are designed to carry a uniformly distributed load of 5000 pounds per foot and are 72 feet long. Determine the minimum section modulus for the beams.(1620 in^3)

6. Find the minimum section modulus for the parking ramp in problem 5 if the beams are supported at their ends. (6480 in^3)

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Topic 5.6c - Problem Assignment 3 - Horizontal Shear Stresses

1. A 6’ long rectangular 2"x 6" beam is simply supported at its ends. It has a point load of 200 pounds at its midpoint. What is the maximum horizontal shear stress in the beam? (12.5 psi)

2. A 14’ long rectangular 2"x 10" beam has a distributed load of 50 pounds per foot. It is simply supported at its ends. Determine the maximum horizontal shear stress in the beam. (26.25 psi)

3. A 10’ long cantilever 4"x 4" beam has a 150 pound load at its extreme end. Find the maximum horizontal shear stress in the beam. (14.1 psi)

4. A 8’ long rectangular beam has point loads of 500 pounds at the 2’, 4’ and 6’ marks. The beam is simply supported at its ends. Determine the depth of beam if it is 2" wide and has maximum horizontal shear stress of 95 psi. (5.92")

5. Determine the maximum point load that can be suspended from the end of a 4’ long 4"x 4" cantilever beam. The maximum allowable horizontal shear stress for construction fir is 95 psi. (1013 lb)

6. Determine the maximum uniformly distributed load that can be carried by an 8’ long 2"x 10" beam which is simply supported at its ends. The allowable bending stress is 1500 psi. What is the maximum horizontal shear stress in the beam? (65 lb/ft, 19.5 psi)

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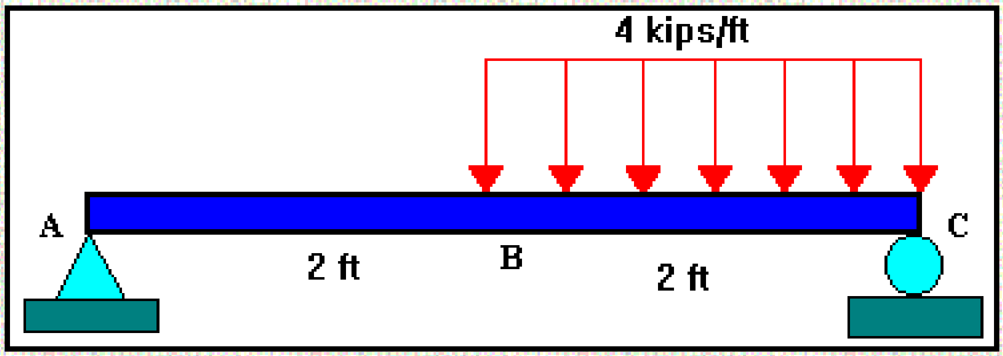
Topic 5.6d - Problem Assignment 4 - Beam Stresses

For the loaded beams shown below, determine the support reactions, and then using the beam designated by the problem:

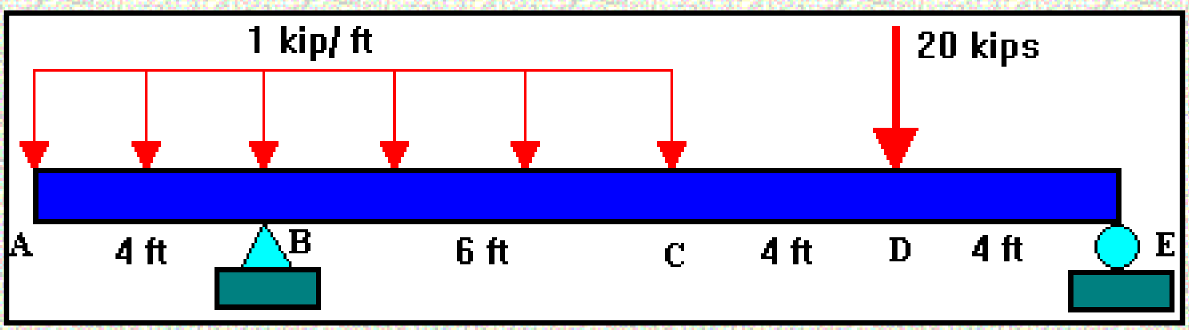
1.) Determine the maximum bending stress (σ_{max})at the location stated.

2.) Determine the shear stress (τ) at the location stated.

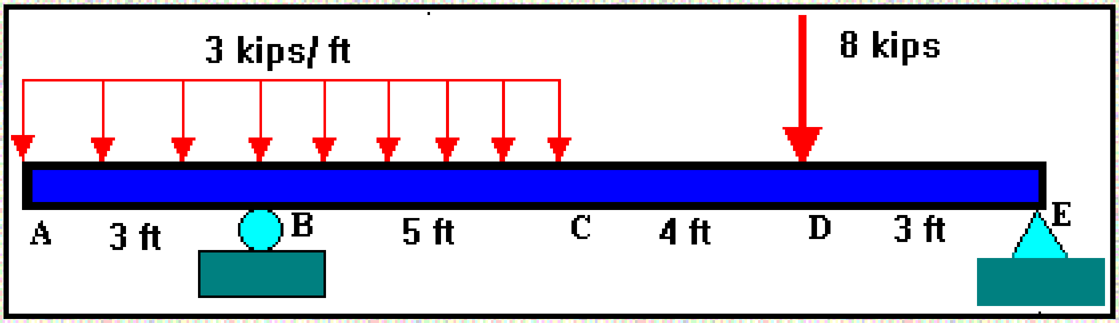
1. For a rectangular 2 in x 8 in beam, find σ_{max} at x = 3 ft, and τ at x = 3 ft, 2 inches above bottom of beam. (2250 psi.; 141 psi.)



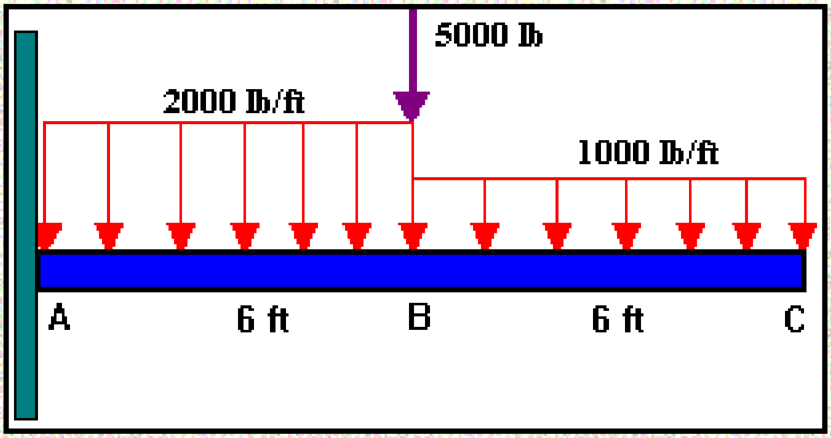
2. For WT 8 x 32 beam, find σ_{max} at x = 9 ft, and τ at x = 9 ft, 4 inches above bottom of beam.(53,630 psi., 2160 psi.)



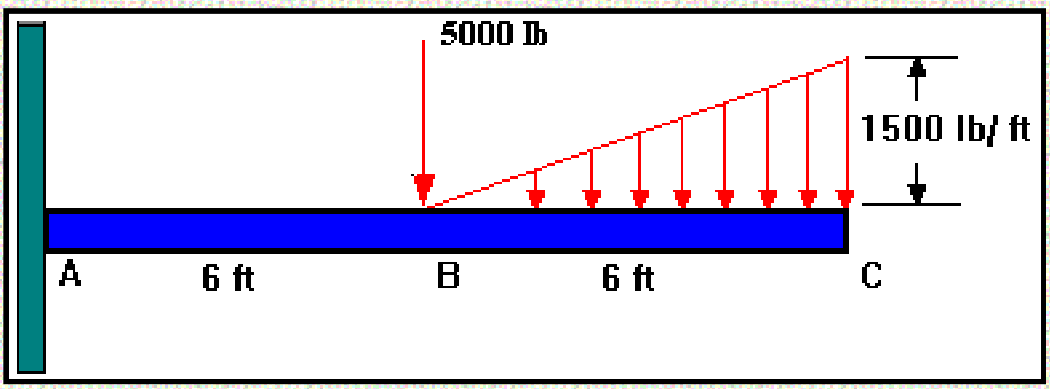
3. For W 8 x 40 beam, find σ_{max} at x = 10 ft, and τ at x = 10 ft, 3 inches above bottom of beam. (8113 psi., 0 psi.)



4. For WT 12 x 60 beam, find σ_{max} at x = 4 ft, and τ at x = 4 ft, 8 inches above bottom of beam. (23,467 psi.; 3092 psi.)



5. For W 10 x 60 beam, find σ_{max} at x = 9 ft, and τ at x = 9 ft, 4 inches above bottom of beam.(1000 psi.; 870 psi.)



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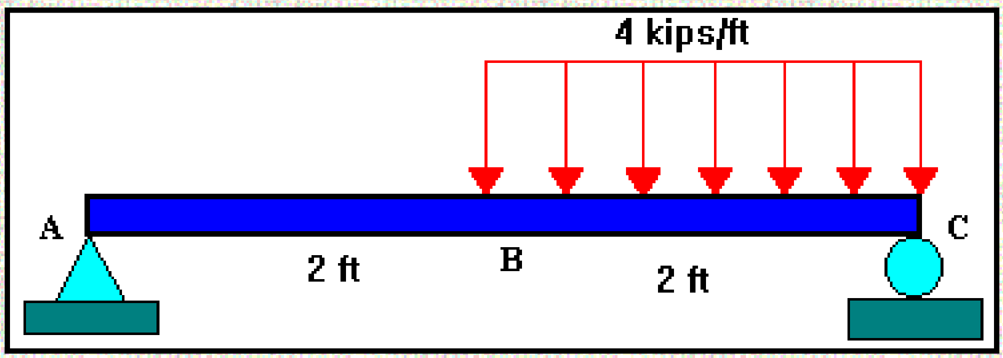
Topic 5.7a - Problem Assignment 5 - Beam Selection / Design

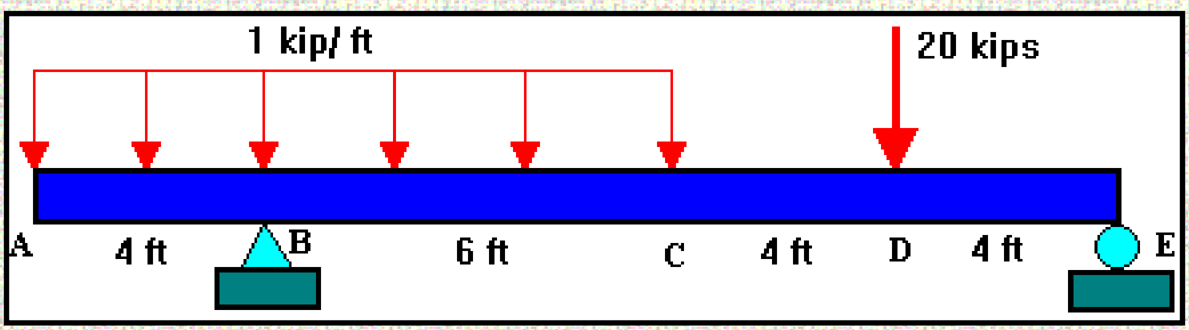
For the loaded beams shown:
A. Determine the support reactions.
B. Make Shear Force and Bending Moment diagrams for each beam.
C. Select the best I-Beam & Best T-Beam to use
D. For the selected Beam, determine the Maximum Shear Stress and see if it is with in the allowable shear stress.

For all the beams use:
Allowable Tensile Stress = 20,000 psi., Allowable Compressive Stress = 25,000 psi., Allowable Shear Stress = 15,000 psi

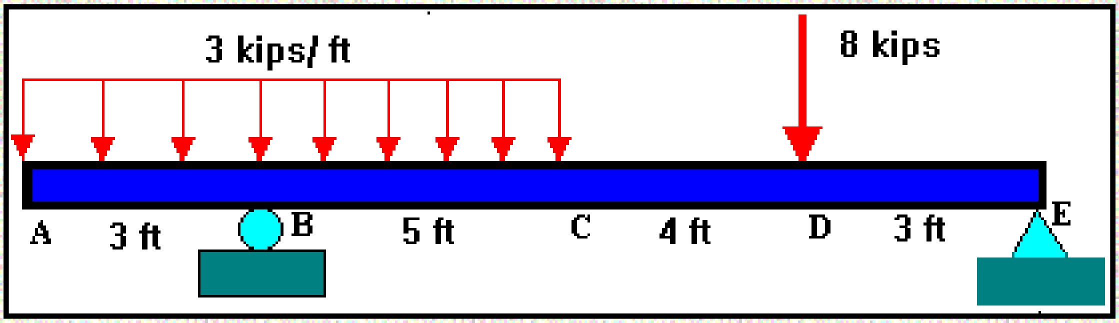
1 (Minimum Section Modulus = 2.7 in^3, bottom of beam in tension, best I-beam, T-beam depends on beam table used)



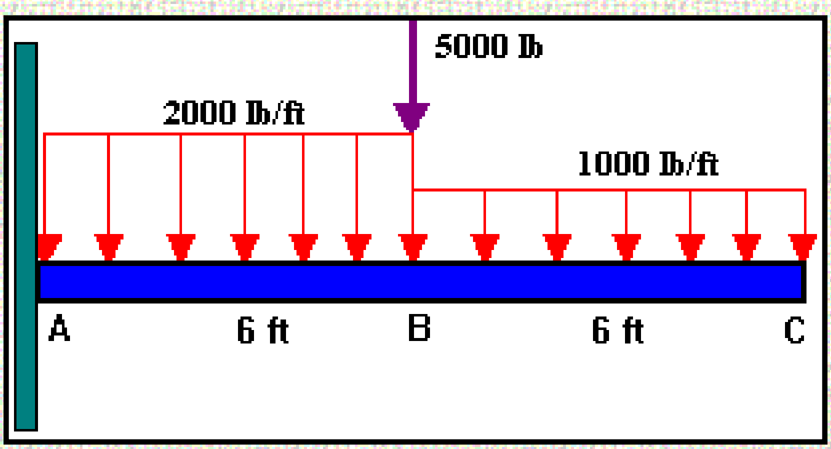
2. (Minimum Section Modulus = 36 in^3, bottom of beam in tension, best I-beam, T-beam depends on beam table used)



3. (Minimum Section Modulus = 14.4 in^3, bottom of beam in tension, best I-beam, T-beam depends on beam table used)

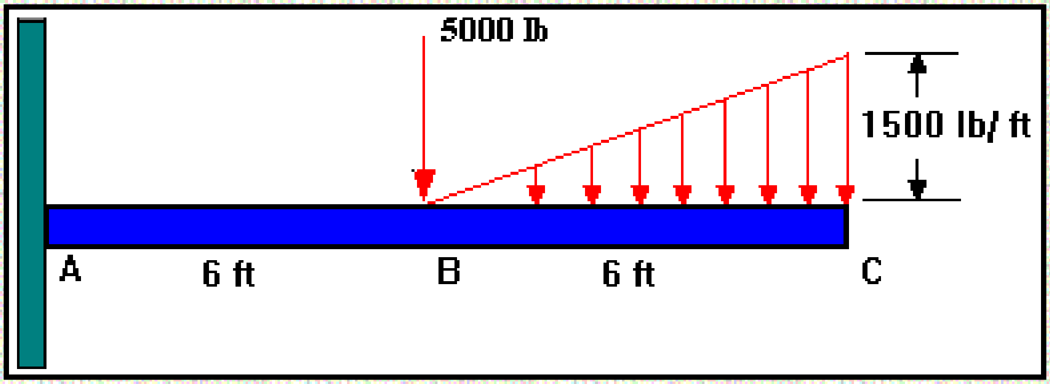


4. (Minimum Section Modulus = 72 in^3 for I-Beam; = 58.6 in^3 for T-Beam; bottom of beam in compression, best I-beam, T-beam depends on beam table used)



5. (Minimum Section Modulus = 45 in^3 for I-Beam; = 36 in^3 for T-Beam; bottom

of beam in compression, best I-beam, T-beam depends on beam table used)



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Topic 5.7b - Problem Assignment 6 - Beam Deflection

The maximum deflection of various beams can be given by specific formulae. The following formulae work only for the specific stated beam configurations.

Beam simply supported at its end points with a point load at its midpoint. Maximum deflection occurs at the mid point and is:

$$y_{max} = FL^3 / (48EI)$$

Beam simply supported at its end points with a uniformly distributed load. Total load is W. Maximum deflection occurs at the mid point and is:

$$y_{max} = 5FL^3 / (384EI)$$

Cantilever beam with a point load at the extreme end. Maximum deflection occurs at the free end and is:

$$y_{max} = FL^3 / (3EI)$$

Cantilever beam with a uniformly distributed load. Total load is W. Maximum deflection occurs at the free end and is:

$$y_{max} = FL^3 / (8EI)$$

In all of the above formulae E is the elasticity of the material from which the beam is made. For steel E = $30\text{x}10^6$ psi. and for Douglas fir or yellow pine E = $1.76\text{x}10^6$ psi.

1. Determine the deflection of the midpoint of a 14 foot long fir 2"x 10" beam which carries a uniformly distributed load of 75 pounds per foot.

2. Find the maximum deflection of a 4"x 6" pine cantilever beam 6 feet long that carries a load of 250 pounds per foot.

3. Find the maximum point load (acting at the end) that can be carried by a 16 foot long steel W14x48 cantilever beam if the beam end is limited to a maximum deflection of 1/4 inch.

4. Find the minimum moment of inertia necessary for a 20 foot long, simply supported, steel beam to deflect no more than 1/2 inch when a load of 20 tons is applied at the center of the beam.

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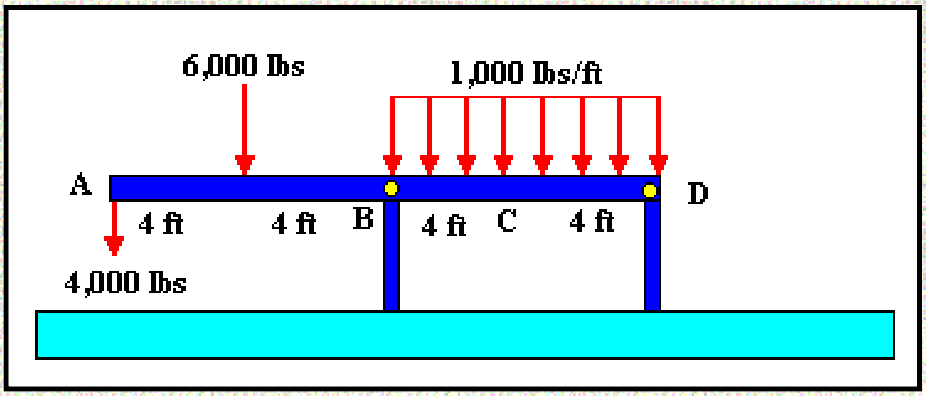
Topic 5.8: Beams - Topic Examination

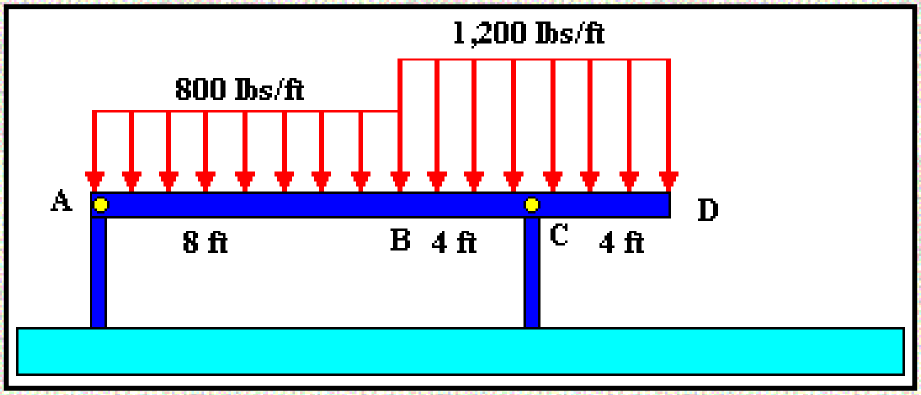
1. A loaded, simply supported WT 6 x 36 T-Beam is shown. For this beam:
A. Determine the maximum bending stress 10 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 4 inches above the bottom of the beam cross section and 10 feet from the left end of the beam.



(For solution select: Topic Exam Solution - Problem 1)

-	-	Depth	Flange	Flange	Stem	-	Cross	Section	Info.	.
Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A - in^2	d - in	b_f -in	t_f - in	tw - in	d/t_w	I - in^4	S - in^3	r - in	y- in
WT 6 x 36	10.60	6.13	12.040	0.671	0.430	14.20	23.20	4.540	1.480	1.02

2. A loaded, simply supported beam is shown. For this beam select the best I-beam to use from the beam table shown below. The **maximum allowable bending stress = 35,000 lb./in2** (for tension and compression), and the **maximum allowable shear stress = 10,000 lb./in2** for the beam material.



(For solution select: Topic Exam Solution - Problem 2)

-	-	-	Flange	Flange	Web	Cross	Section	Cross	Section
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	y-y axis	y-y axis
-	A-in^2	d - in	b_f - in	t_f - in	t_w - in	I - in^4	S - in^3	I - in^4	S - in^3
W 5x16	4.70	5.00	5.000	0.360	0.240	21.3	8.5	7.51	3.00
W 6x20	5.88	6.20	6.018	0.367	0.258	41.5	13.4	13.30	4.43
W 6x25	7.35	6.37	6.080	0.456	0.320	53.3	16.7	17.10	5.62
W 8x17	5.01	8.00	5.250	0.308	0.230	56.6	14.1	7.44	2.83
W 8x20	5.89	8.14	5.268	0.378	0.248	69.4	17.0	9.22	3.50
W 8x31	9.12	8.00	8.000	0.433	0.288	110.0	27.4	37.00	9.24
W 8x35	10.30	8.12	8.027	0.493	0.315	126.0	31.1	42.50	10.60
W 8x40	11.80	8.25	8.077	0.558	0.365	146.0	35.5	49.00	12.10
W 10x25	7.36	10.08	5.762	0.430	0.252	133.0	26.5	13.70	4.76
W 10x29	8.54	10.22	5.799	0.500	0.289	158.0	30.8	16.30	5.61
W 10x33	9.71	9.75	7.964	0.433	0.292	171.0	35.0	36.50	9.16
W 10x39	11.50	9.94	7.990	0.528	0.318	210.0	42.2	44.90	11.20
W 10x45	13.20	10.12	8.022	0.618	0.350	249.0	49.1	53.20	13.30
W 10x54	15.90	10.12	10.028	0.618	0.368	306.0	60.4	104.00	20.70
W 10x60	17.70	10.25	10.075	0.683	0.415	344.0	67.1	116.00	23.10
W 12x19	5.59	12.16	4.007	0.349	0.237	130.0	21.3	3.76	1.88
W 12x22	6.47	12.31	4.030	0.424	0.260	156.0	25.3	4.64	2.31
W 12x27	7.95	11.96	6.497	0.400	0.237	204.0	34.2	18.30	5.63

W 12x31	9.13	12.09	6.525	0.465	0.265	239.0	39.5	21.60	6.61
W 14x34	10.00	14.00	6.750	0.453	0.287	340.0	48.6	23.30	6.89
W 14x38	11.20	14.12	6.776	0.513	0.313	386.0	54.7	26.60	7.86
W 12x53	15.60	12.06	10.000	0.576	0.345	426.0	70.7	96.10	19.20
W 14x61	17.90	13.91	10.000	0.643	0.378	641.0	92.5	107.00	21.50
W 14x87	25.60	14.00	14.500	0.688	0.420	967.0	138.0	350.00	48.20
W 16x36	10.60	15.85	6.992	0.428	0.299	447.0	56.5	24.40	6.99
W 16x50	14.70	16.25	7.073	0.628	0.380	657.0	80.8	37.10	10.50
W 18x45	13.20	17.86	7.477	0.499	0.335	706.0	79.0	34.80	9.32
W 18x60	17.70	18.25	7.558	0.695	0.416	986.0	108.0	50.10	13.30
W 24x68	20.00	23.71	8.961	0.582	0.416	1820.0	153.0	70.00	15.60
W 24x94	27.70	24.29	9.061	0.872	0.516	2690.0	221.0	108.00	23.90
W 36x230	67.70	35.88	16.471	1.260	0.761	15000.0	837.0	940.00	114.00
W 36 x 260	76.50	36.24	16.551	1.440	0.841	17300.0	952.0	1090.00	132.00

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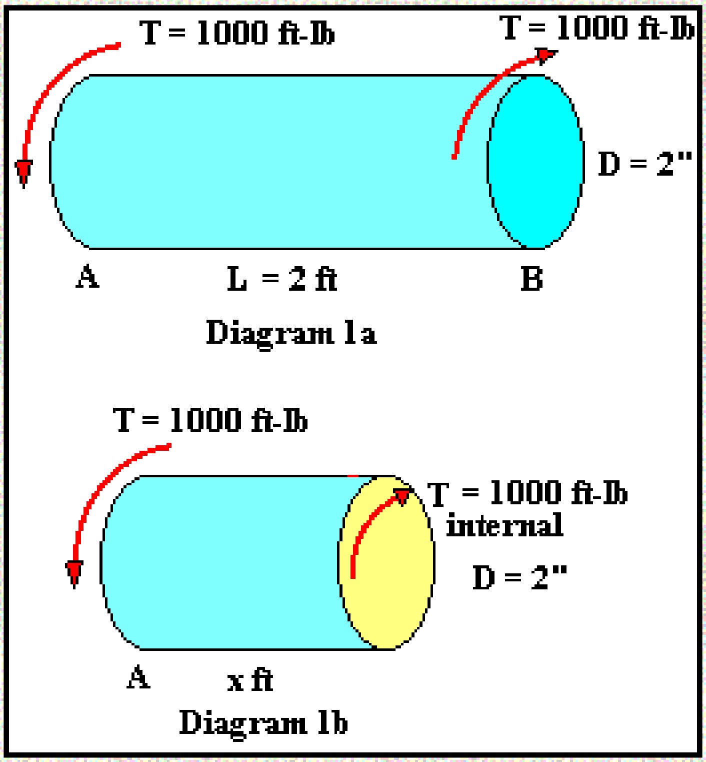
Topic 6.1a: Shear Stress - Example 1

Part I. In Diagram 1a we have shown a solid shaft with what we will call a driving external torque of 1000 ft-lb. at end A, and a load torque of 1000 ft-lb. at end B. The shaft is in equilibrium. We would like to determine the maximum transverse shear stress in the shaft due to the applied torque.

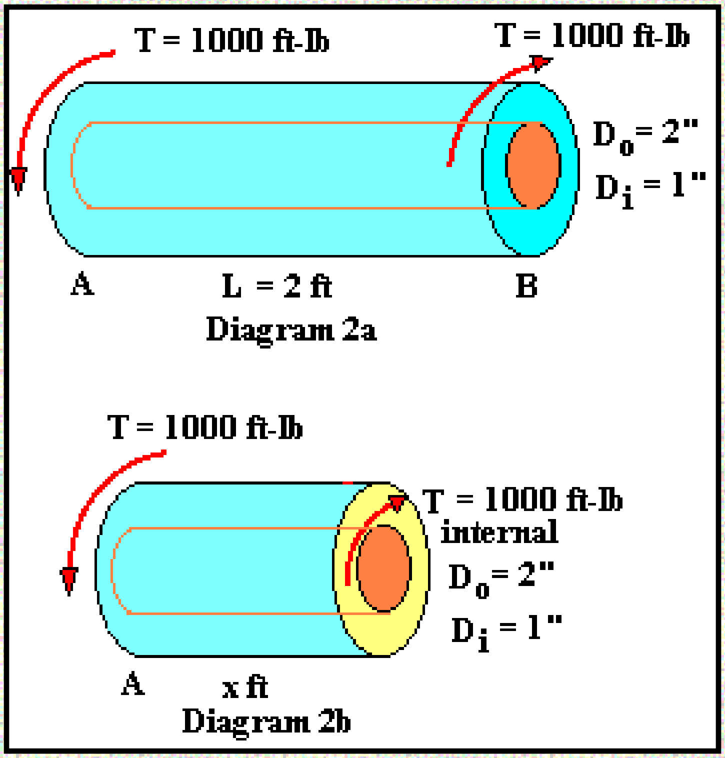


To solve, we first need to determine the internal torque in the shaft. We cut the shaft a distance x from end A and draw a Free Body Diagram of the left end section of the shaft as shown in Diagram 1b. Where we cut the shaft there is an internal torque, which in this case must be equal and opposite to the torque at end A for equilibrium. So for this shaft the value of the internal torque is equal to the value of the externally applied torque.

We next simply apply the torsion formula for the shear stress: **τ = T r / J**; where:
T is the internal torque in that section of the shaft **= 1000 ft-lb = 12,000 in-lb.**
r = the radial distance from the center of the shaft to the point where we wish to find the shear stress. In this problem the outer edge of the shaft since that is where the transverse shear stress is a maximum; **r = 1 in.**
J = polar moment of inertia = **(π/32) d^4 for a solid shaft = (3.1416/32) ($2^4 in^4$) = 1.57 in^4**.
So, **τ = T r / J = 12,000 in-lb. * 1 in./1.57 in^4. = 7,640 lb/in^2.**
This is the Maximum Transverse (and longitudinal) Shear Stress in the shaft.

Part II

We now would like to consider the case where the shaft is not solid, but a hollow shaft with an outer diameter of 2" and an inner diameter of 1", as shown in Diagram 2a. We still apply the same driving and load torque, and still have the same value of the internal torque, as is shown in Diagram 2b.



We next apply the torsion formula for the shear stress for the hollow shaft:
τ **= T r / J**; where we observe that all the values are the same as in part one, except for the value of J, the polar moment of inertia.
T is the internal torque in that section of the shaft **= 1000 ft-lb = 12,000 in-lb.**
r = the radial distance from the center of the shaft to the point where we wish to find the shear stress. In this problem the outer edge of the shaft since that is where the transverse shear stress is a maximum; **r = 1 in.**
J = polar moment of inertia = (3.1416/32) $[d_o^4 - d_i^4]$ for a hollow shaft =

(3.1416/32) $[(2^4 in^4) - (1.0^4 in^4)]$ = **1.47 in^4.**
So, τ **= T r / J = 12,000 in-lb. * 1 in./1.47 in^4. = 8,150 lb/in^2.**

This then is the Maximum Transverse (and longitudinal) Shear Stress in the hollow shaft.

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Continue to: Topic 6.1b: Shear Stress - Example 2
or Select:

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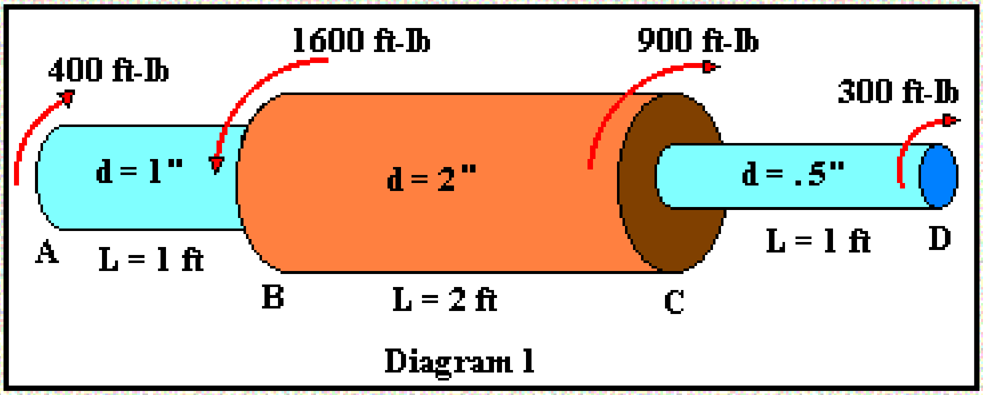
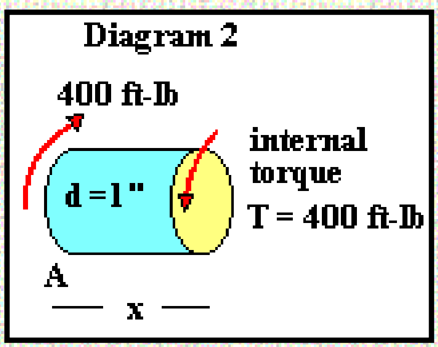
Topic 6.1b: Shear Stress - Example 2

In Diagram 1 we have shown a solid compound shaft with what we will call the driving external torque of 1600 ft-lb. acting at point B, and load torque of 400 ft-lb. at end A, 900 ft-lb. at point C, and 300 ft-lb. at end D. Notice that the shaft is in rotational equilibrium. We would like to determine the maximum transverse shear stress in each section of the shaft due to the applied torque.



To solve, we first need to determine the internal torque in each section of the shaft. We cut the shaft a distance 0' < x < 1' from end A and draw a Free Body Diagram of the left end section of the shaft as shown in Diagram 2. Where we cut the shaft there is an internal torque, which in this case must be equal and opposite to the torque at end A for equilibrium. So for this shaft the value of the internal torque is equal to the value of the externally applied torque.

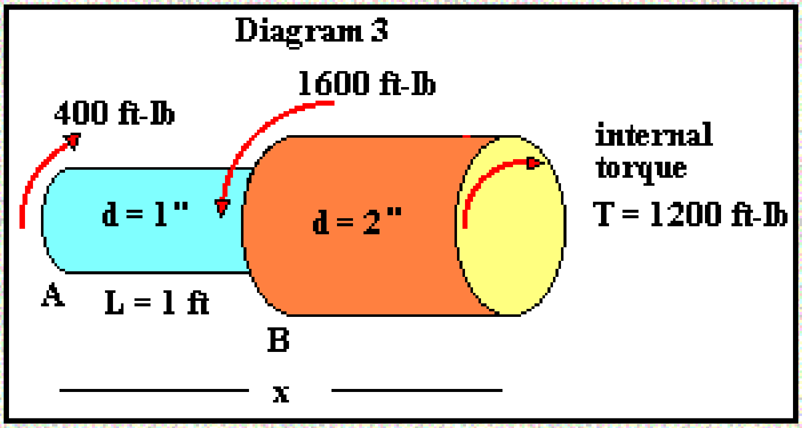


We next apply the torsion formula for the shear stress: **τ = T r / J**; where:
T is the internal torque in that section of the shaft **= 400 ft-lb. = 4,800 in-lb**.
r = the radial distance from the center of the shaft to the point where we wish to find the shear stress. In this problem r is to the outer edge of the shaft since that is where the transverse shear stress is a maximum; **r = .5 in**.
J = polar moment of inertia **= (π/32) d^4 for a solid shaft = (3.1416/32) ($1^4 in^4$) = .098 in^4**.

So, **τ = T r / J = 4,800 in-lb. * .5 in./.098 in^4. = 24,500 lb./in^2.**

This value is the Maximum Transverse Shear Stress in the shaft section AB, and it falls in a reasonable range for allowable shear stresses for metals.

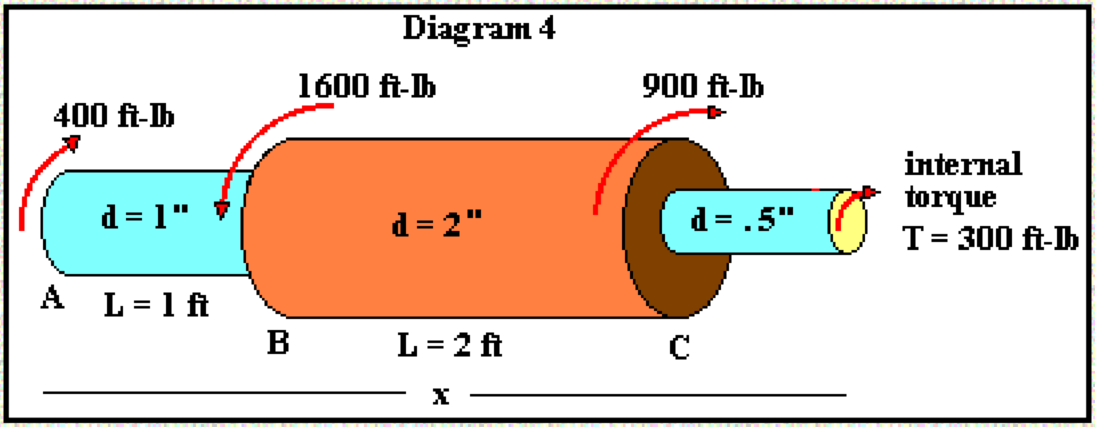
We now determine the internal torque in the next section of the shaft. We cut the shaft a distance 1' < x < 3' from end A and draw a Free Body Diagram of the left end section of the shaft as shown in Diagram 3. Where we cut the shaft there is an internal torque, and by mentally summing torque, we see that in order to have rotational equilibrium we must have an internal torque in section BC of 1200 ft-lb. acting in the direction shown.



We apply the torsion formula for the shear stress once again: **τ = T r / J**; where:
T is the internal torque in that section of the shaft **= 1,200 ft-lb. = 14,400 in-lb**.
r = the radial distance to the outer edge of the shaft since that is where the transverse shear stress is a maximum; **r = 1 in.**
J = polar moment of inertia **= (π/32) d^4 for a solid shaft = (3.1416/32) (2^{4}in^4) = 1.57 in^4.**
So, **τ = T r / J = 14,400 in-lb. * 1 in./ 1.57 in^4. = 9,170 lb./in^2.**

This then is the Maximum Shear Stress in shaft section BC. We note that even though the internal torque is much larger in section BC as compared to section AB, because of the size of the shaft in section BC, the shear stress is much lower in BC.

We now determine the internal torque in the next section of the shaft. We cut the shaft a distance 3' < x < 4' from end A and draw a Free Body Diagram of the left end section of the shaft as shown in Diagram 4. Where we cut the shaft there is an internal torque. From the Free Body Diagram we see that in order to have rotational equilibrium we must have an internal torque in section CD of 300 ft-lb. acting in the direction shown.



We apply the torsion formula for the shear stress: τ **= T r / J**; where:
T is the internal torque in that section of the shaft **= 300 ft-lb. = 3,600 in-lb**.
r = the radial distance from the center of the shaft to the outer edge of the shaft since that is where the transverse shear stress is a maximum; **r = .25 in**.
J = polar moment of inertia = (π/32) d^4 for a solid shaft **= (3.1416/32) ($.5^4 in^4$) = .0061 in^4**.
So, τ **= T r / J = 3,600 in-lb. * .25 in./ .0061 in^4. = 147,500 lb./in^2.**

This is the Maximum Shear Stress in shaft section CD. We note that this is much larger than the ultimate shear stress most metals, thus this section of the shaft would fail - a larger diameter is needed to carry the torque.

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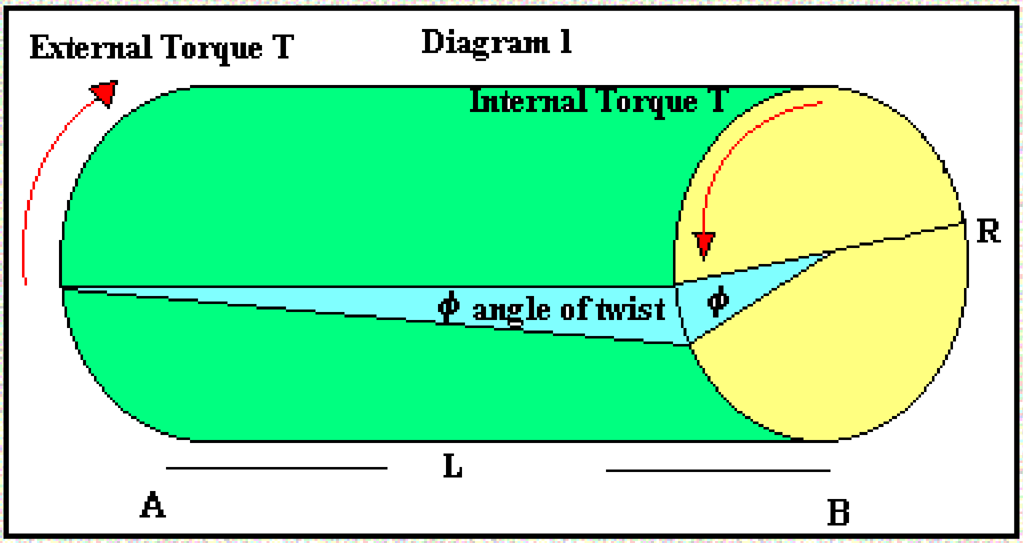
Topic 6.2: Torsion: Deformation - Angle of Twist

Another effect of applying an external torque to a shaft is a resulting deformation or twist as the material is stressed. The resulting shaft deformation is expressed as an Angle of Twist of one end of the shaft with respect to the other.

In Diagram 1 we have shown a section of a solid shaft. An external torque T is applied to the left end of the shaft, and an equal internal torque T develops inside the shaft. Additionally there is a corresponding deformation (angle of twist) which results from the applied torque and the resisting internal torque causing the shaft to twist through an angle, phi, shown in Diagram 1.



The angle of twist may be calculated from:
ϕ **= T L / J G** ; where
T = the internal torque in the shaft
L = the length of shaft being "twisted"
J = the polar moment of inertia of the shaft
G = the Modulus of Rigidity (Shear Modulus) for the material, for example for steel and brass we have, $G_{steel} = 12 \times 10^6$ lb/in^2, $G_{brass} = 6 \times 10^6$ lb/in^2.

We will now look at an example of determining the angle of twist in a shaft. In Diagram 2a we have shown a solid steel circular shaft with an external torque of 1000 ft-lb. being applied at each end of the shaft, in opposite directions.

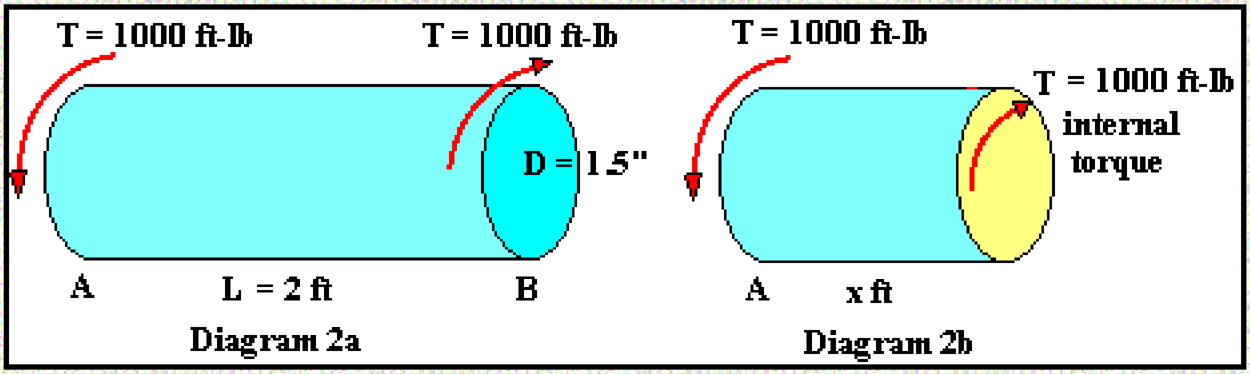


Diagram 2a Diagram 2b

The shaft has a diameter of 1.5 inch. We would like to determine the angle of twist of end B with respect to end A.

To find the angle of twist we first determine the internal torque in the shaft. We cut the shaft a distance x feet from the left end, and make a free body diagram of the left section of the shaft - shown in Diagram 2b. From the free body diagram, we see that the internal torque must be 1000 ft-lb. to satisfy rotational equilibrium.

We next apply the Angle of Twist formula: **ϕ = T L / J G** ; where
T = 1000 ft-lb. = 12,000 in-lb.
L = 2 ft. = 24 inches
J = polar moment of inertia = (π/32) d^4 for a solid shaft = (3.1416/32) ($1.5^4 in^4$) = .5 in^4.
G_{steel} = 12 x 10^6 lb/in^2
Then,
ϕ = T L / J G = (12,000 in-lb.* 24 in) / (.5 in^4 * 12 x 10^6 lb/in^2) = .048 radians = 2.75°.

The angle of twist will have units of radians, and in this problem is clockwise with respect to end A as shown in Diagram 3.

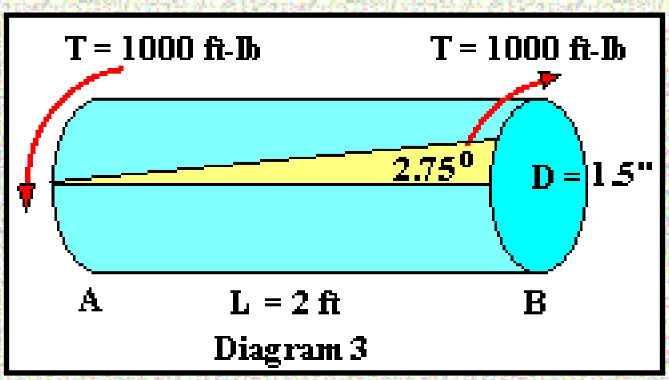


Diagram 3

We also take a moment to calculate the maximum shear stress in the shaft, just out of interest in it's value.

τ = T r / J = 12,000 in-lb. * .75 in. / .5 in^4. = 18,000 lb / in^2.

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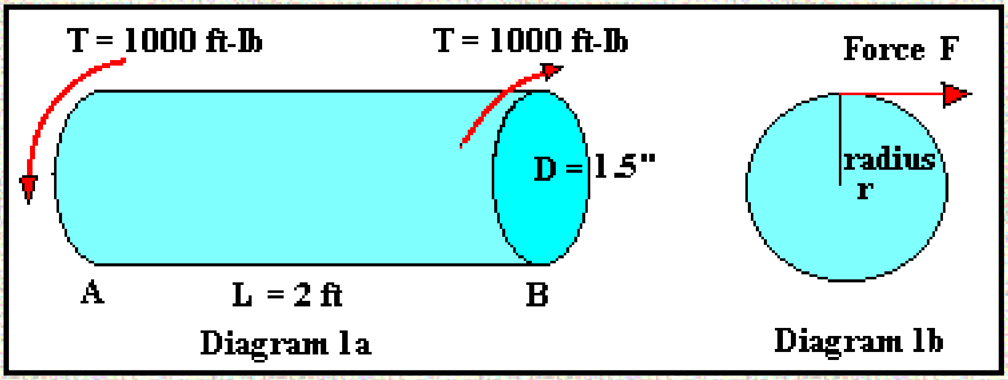
Topic 6.3: Torsion - Power Transmission

One important process which involves applying torque to a shaft is power transmission. To transmit power a drive torque is applied to a rotating shaft. A short derivation will gives us a relationship between Torque, rotation rate, and Power transmitted.

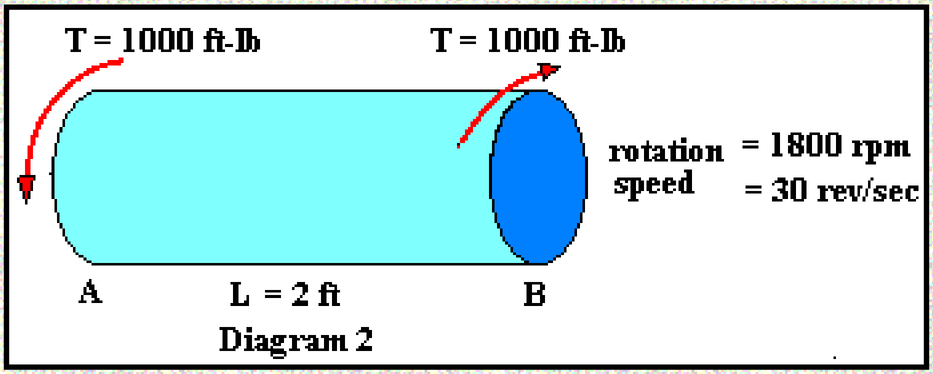
In Diagram 1a we have shown a shaft with a drive torque of 1000 ft-lb. at end A and an equal and opposite load torque at end B.



In Diagram 1b we have shown the shaft from end on. Notice that to apply a torque to a shaft we must exert a force, F, usually at the outer edge of the shaft. This force may be applied through use of a belt or gear. The product of the force (F) and the radius (r) is the applied (or load) torque.
The work done as we rotate the shaft will be the product of the force and the distance the force acts through - which is the circumference. That is, for each revolution of the shaft, the force act through a distance of one circumference, or we may write:
Work = F x d = F * (2 π r) * (# revolutions)
The Power sent down the shaft is then the Work per unit time, or if we divide the equation for Work above by the time we can write:
Power = Work/Time = F * (2 π r) * (# rev/time)
If we now rewrite the above equation slightly, as below:
Power = 2 π (F * r) * (# rev/sec)
Then we recognize the (F * r) term is the torque in the shaft, and we can rewrite as:
Power = 2 π T n (ft-lb./sec) where T = Torque in ft-lb.; n = # rev/sec
This is the formula for power transmitted in foot-pounds/second. It is often more convenient to express it in horsepower (1 hp = 550 ft-lb./sec) as shown below.
Power$_{hp}$ = [2 π T n / 550 ft-lb/sec/hp]

Next we look at a simple example of determining transmitted horsepower.

Example
In Diagram 2, we have shown a solid shaft with an applied driving torque of 1000 ft-lb. and an equal and opposite load torque, rotating at a speed of 1800 rpm (30 rev/sec). We would like to determine the horsepower being transmitted down the shaft.



Solution: First we mentally note that since this is a simple shaft with equal external driving and load torque, then the internal torque will be equal in value to the external torque of 1000 ft-lb.
Next we apply the Horsepower equation:
Power $_{hp}$ **= [2** π T **n / 550 ft-lb/sec/hp]** ; where
T = internal torque in shaft (in foot-pounds) **= 1000 ft-lb**.
n = the number of revolution per second **= 30 rev/sec.**
So we have: **Power** $_{hp}$ **= [2* 3.1416* 1000 ft-lb.* 30 rev/sec / 550 ft-lb/sec/hp] = 343 hp.**

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Topic 6.3a: Power Transmission - Example 1
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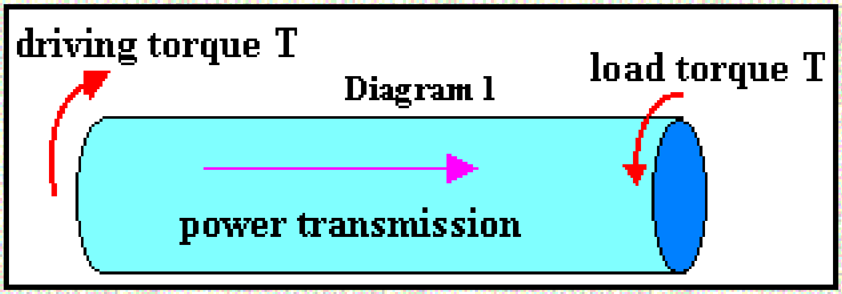
Topic 6.3a: Power Transmission - Example 1

Example 1:
A solid steel shaft, shown in Diagram 1, has a 1 inch diameter and an allowable shear stress of 12,000 lb/in^2. What is the largest amount of power which could safely be transmitted down the shaft if it is to rotate at 2400 rpm?



Solution:
Step 1: First, using the maximum allowable shear stress, we determine the largest torque which may be applied to the shaft. The formula for the shear stress in a shaft is:
τ = T r / J ; then solving for the torque: **T = τ J / r** ; where
J = (3.1416) d^4 / 32 = .098 in^4; r = .5 inch; and τ = 12.000 lb/in^2, (We use the allowable shear stress as the maximum stress in the shaft.) Putting values into the equation and solving:
T = 12,000 lb/in^2 * .098 in^4 / .5 in = 2356 in-lb = 196 ft-lb.

Step 2: Now that we have the maximum torque we can safely apply, we can determine the largest amount of power we can transmitted from the horsepower equation.
P_{hp} = 2 pi T n / 550 = 2 (3.1416) 196 ft-lb. (40 rev/s) / (550 ft-lb/s/hp) = 90 hp

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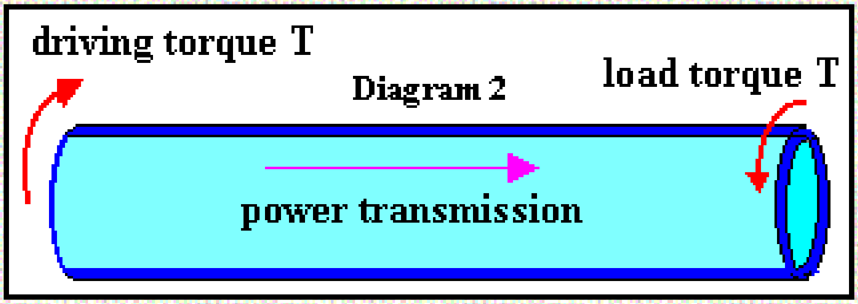
Topic 6.3b: Power Transmission - Example 2

Example 2:
A car has a hollow drive shaft with a two inch outer diameter and a 1/8 inch wall thickness as shown in Diagram 2. The maximum power transmitted down the shaft is 185 hp at a shaft speed of 4400 rpm. What is the maximum shear stress in the shaft?



Solution:
Step 1. We first use the horsepower equation to determine the amount of torque which is being applied to the shaft. **Power $_{hp}$ = [2 pi T n / 550 ft-lb/s/hp]**

Solving for **T = (P$_{hp}$ 550 ft-lb/s/hp) / 2 pi n;** then putting in the values, we have:
T = 185 hp (550 ft-lb/s/hp)/[2 (3.1416) 4400/60 rev/sec] = 221 ft-lb = 2650 in-lb

Step 2. Now that we have the torque being applied to the shaft, we use the transverse shear stress equation for circular shafts to determine the maximum shear stress in the shaft.
τ **= T r / J ;** where
T = 2650 in-lb., r = 1 in.; J = (pi/32)(d_o^4 - d_i^4) = (3.1416/32)(2"^{4} - 1.75"^{4}) = .65 in^4.
τ **= T r / J = 2650 in-lb.* 1 in / .65 in^4 = 4080 lb/in^2.**

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Topic 6.3c: Power Transmission - Example 3
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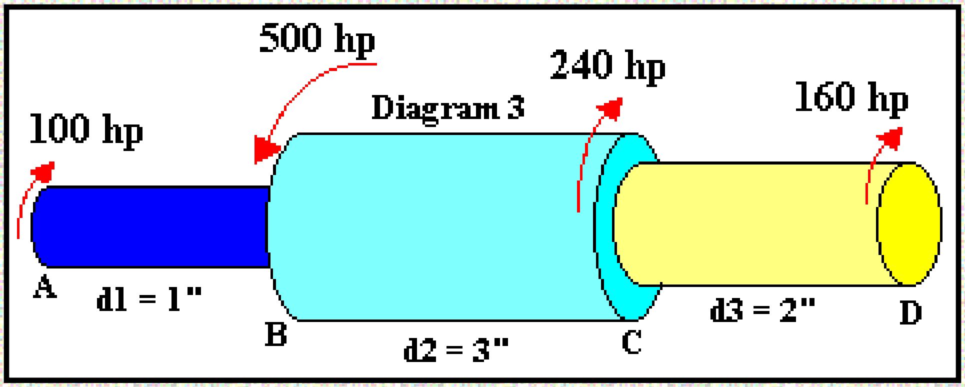
Topic 6.3c: Power Transmission - Example 3

Example 3:
We have shown a compound shaft with 500 horsepower being applied at point B, and with power being taken off the shaft - 100 hp at point A, 240 hp at point C, and 160 hp at point D. Shaft AB has a diameter of 1", shaft BC had a diameter of 3", and shaft CD had a diameter of 2". If the shaft is rotating at 1200 rpm, determine the maximum shear stress in each section of the shaft.



Solution:
Step 1. We first determine the amount of horsepower being transmitted through each section of the compound shaft. In section AB, we can see that 100 hp must being transmitted internally through the shaft from B to A. In section BC, with a little thought, we realize that the amount of power being transmitted through BC must be 400 hp - of which 240 hp is taken off at point C, and the remaining 160 hp. continues through shaft CD, and is taken off at point D.

Step 2. Now that we know the horsepower in each section of the shaft, we use the horsepower equation to determine the amount of torque which is being applied to each shaft.
Power $_{hp}$ = [2 pi T n / 550 ft-lb/s/hp]
Solving for **T = (P_{hp} 550 ft-lb/s/hp) / 2 π n;** then putting in the values, we have:
T_{AB} = 100hp (550 ft-lb/sec/hp)/[2 (3.1416) (1200/60 rev/s)] = 438 ft-lb. = 5256 in-lb.
T_{BC} = 400hp (550 ft-lb/sec/hp)/[2 (3.1416) (1200/60 rev/s)] = 1752 ft-lb. = 21024 in-lb.
T_{CD} = 160hp (550 ft-lb/sec/hp)/[2 (3.1416) (1200/60 rev/s)] = 701 ft-lb. = 8412 in-lb.

Step 3. Finally we use the shear stress formula, τ **= T r / J**, to find maximum shear stress in each section

τ_{AB} = 5256 in-lb * .5 in / [pi(1")^4 / 32] = 26,770 lb/in^2.

τ_{BC} = 21024 in-lb * 1.5 in / [pi(3")^4 / 32] = 3966 lb/in^2.

τ_{CD} = 8412 in-lb * 1 in / [pi(2")^4 / 32] = 5355 lb/in^2.

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Topic 6.3d: Compound Shaft - Example 4
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Topic 6.3d: Compound Shaft - Example 4

This last example looks at determining a number of quantities in a compound shaft.

A solid compound shaft with a driving torque of 1800 ft-lb. and load toque of 600 lf-lb., 800 ft-lb., and 400 ft-lb. is shown in Diagram 1. Section AB is made of steel. Section BC is made of brass. Section CD is made of steel. The shaft is rotating at 2400 rpm. The length and diameters of the shafts are shown in the Diagram 1. For this compound shaft :

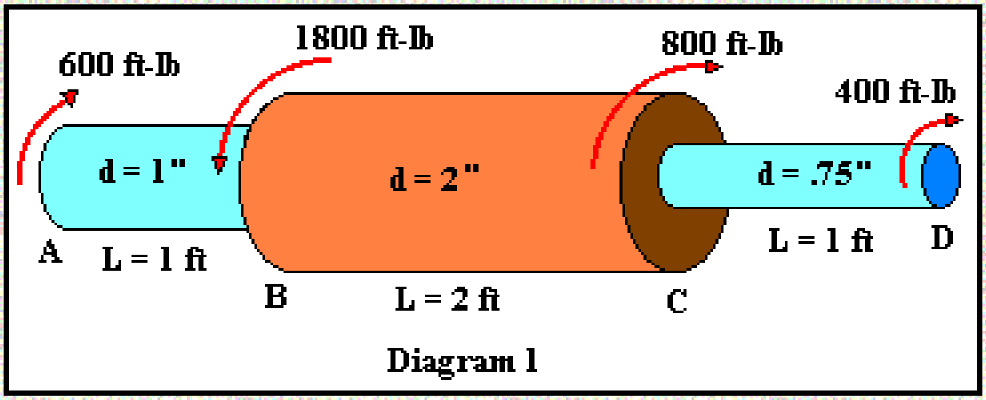


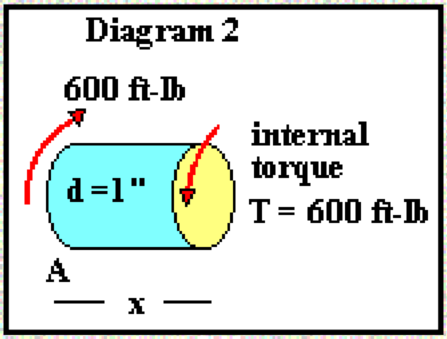
Diagram 1

1. Determine the maximum shear stress in each of the sections of the shaft.
2. Determine the horsepower being transmitted through each section of the shaft.
3. Determine the resultant angle of twist of a point on end D with respect to a point on end A.

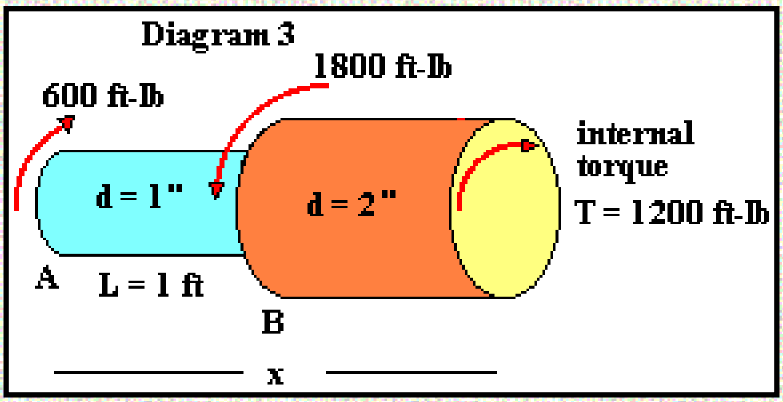
Modulus of Rigidity: Steel = 12 x 106 lb/in^2.; Brass = 6 x 106 lb/in^2.; Aluminum = 4 x 106 lb/in^2.

Step 1. To solve, we first need to determine the internal torque in each section of the shaft. We cut the shaft a distance 0' < x < 1' from end A and draw a Free Body Diagram of the left end section of the shaft as shown in Diagram 2. Where we cut the shaft there is an internal torque, which in this case must be equal and opposite to the torque at end A for equilibrium. So for this shaft the value of the internal torque is equal to the value of the externally applied torque.

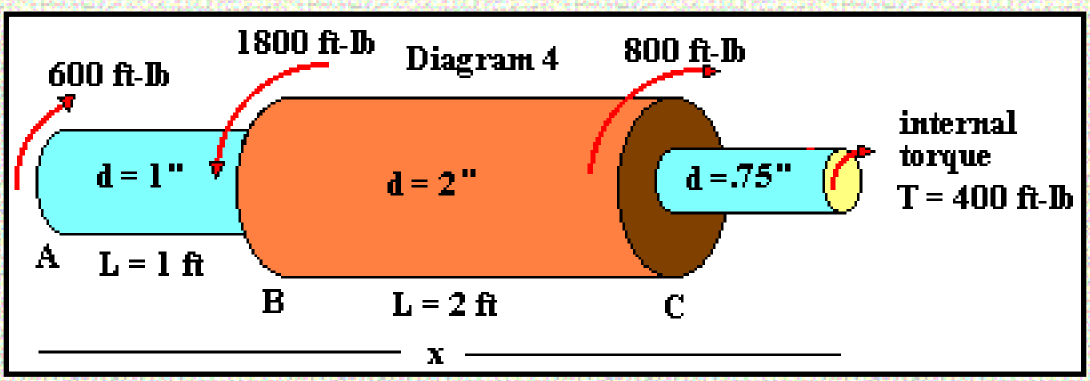
T_{AB} = 600 ft-lb.



We now determine the internal torque in the next section of the shaft. We cut the shaft a distance 1' < x < 3' from end A and draw a Free Body Diagram of the left end section of the shaft as shown in Diagram 3. Where we cut the shaft there is an internal torque, and by mentally summing torque, we see that in order to have rotational equilibrium we must have an internal torque in section BC of **T_{BC} = 1200 ft-lb**. acting in the direction shown.



We now determine the internal torque in the last section of the shaft. We cut the shaft a distance 3' < x < 4' from end A and draw a Free Body Diagram of the left end section of the shaft as shown in Diagram 4. Where we cut the shaft there is an internal torque. From the Free Body Diagram we see that in order to have rotational equilibrium we must have an internal torque in section CD of **T_{CD} = 400 ft-lb**. acting in the direction shown.



Step 2. Using the internal torque in each section (**T_{AB} = 600 ft-lb = 7,200 in-lb.**, **T_{BC} = 1200 ft-lb. = 14,400 in-lb., T_{CD} = 400 ft-lb. = 4,800 in-lb**.), we now apply the torsion formula for the shear stress: τ **= T r / J**

τ_{AB} = 7,200 in-lb * .5 in / [pi(1")^4 / 32] = 36,700 lb/in^2.

τ_{BC} = 14,400 in-lb * 1 in / [pi(2")^4 / 32] = 9,170 lb/in^2.

τ_{CD} = 4,800 in-lb * .375 in / [pi(.75")^4 / 32] = 58,000 lb/in^2.

Step 3. Again using the internal torque in each section, we determine the horsepower transmitted through each shaft section.

P_{AB}=2 pi Tn/(550 ft-lb/s/hp)=2*3.1416*600 ft-lb.*(40 rev/s) /(550 ft-lb/s/hp)= 274 hp

P_{BC}=2 pi Tn/(550 ft-lb/s/hp)=2*3.1416*1200 ft-lb.*(40 rev/s) /(550 ft-lb/s/hp)=548 hp

P_{CD}=2 pi Tn/(550 ft-lb/s/hp)=2*3.1416*400 ft-lb.*(40 rev/s) /(550 ft-lb/s/hp)= 183 hp

Step 4. Apply the Angle of Twist relationship to each section of the shaft.

ϕ_{AB} =T L/J G= (7,200 in-lb.* 12 in)/(.098 in^4 * 12 x 10^6 lb/in^2)=.073 radians = 4.18^o.

ϕ_{BC} =T L/J G= (14,400 in-lb.* 24 in)/(1.57 in^4 * 6 x 10^6 lb/in^2)=.037 radians = 2.12^o.

ϕ_{CD} =T L/J G= (4,800 in-lb.* 12 in)/(.031 in^4 * 12 x 10^6 lb/in^2)=.155 radians = 8.88^o.

Then the total twist of end D with respect to end A will be:

ϕ_{total} = + 4.18 ^{o} - 2.12^o - 8.88^o = - 6.82^o (clockwise); where our signs are taken from the direction of the internal torque in each section. In this example, AB - counter clockwise, BC - clockwise, and CD - clockwise, resulting in the (+, -, -) signs of the angles of twist.

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STATICS & STRENGTH OF MATERIALS - Example

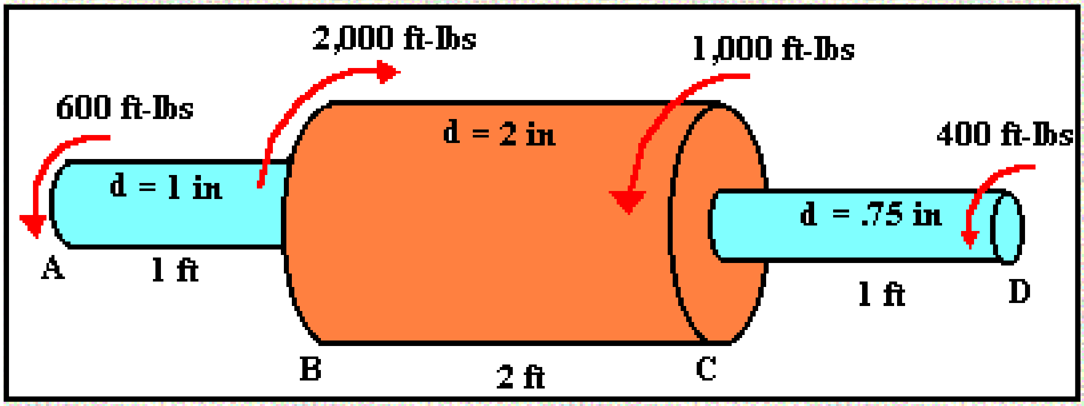
A compound shaft with applied torques and dimensions is shown below. Section AB is made of steel. Section BC is made of brass. Section CD is made of steel. For this shaft:

A. Determine the maximum shear stress in each of the sections of the shaft.

B. Determine the resultant angle of twist of a point on end D with respect to a point on end A.

The modulus of rigidity for steel = 12 x 106 lb/in^2.

The modulus of rigidity for brass = 6 x 106 lb/in^2.



Part A:

Section I:

From equilibrium condtions:

Sum of Torque = 600 ft-lb - T_{AB} = 0; So T_{AB} = 600 ft-lb

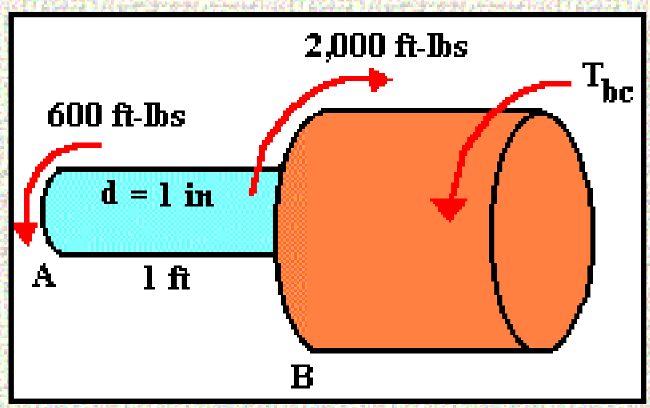
Then τ_{AB} = Tr / J = (600 ft-lb)(12 in/ft)(.5 in) /[3.1416 * (1 in)^4 / 32] = 36,700 psi

Section II:

From equilibrium condtions:

Sum of Torque = 600 ft-lb - 2000 ft-lb + T_{BC} = 0; So T_{BC} = 1400 ft-lb

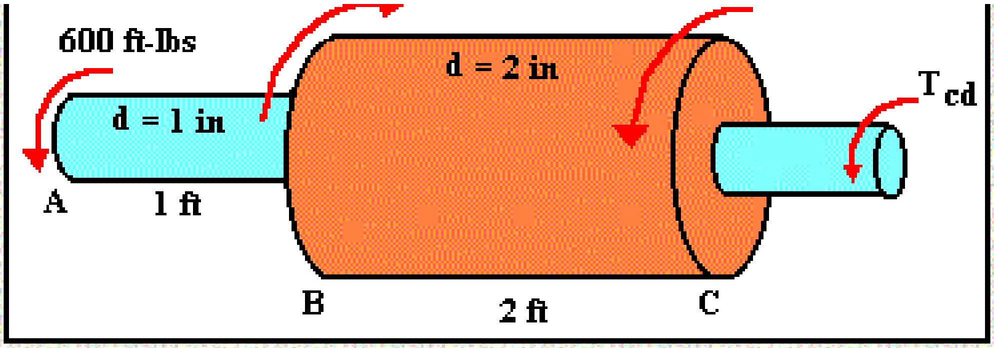
Then τ_{BC} = Tr / J = (1400 ft-lb)(12 in/ft)(1 in) / [3.1416 * (2 in)^4 / 32] = 10,700 psi



Section III:

From equilibrium condtions:

Sum of Torque = 600 ft-lb -



2000 ft-lb + 1000 ft-lb + T_{CD} = 0; So T_{CD} = 400 ft-lb

Then τ_{CD} = Tr / J = (400 ft-lb) (12 in/ft)(.375 in) /[3.1416 * (.75 in)^4 / 32] = 57,900 psi

Part B:

Resultant angle of twist:

ϕ_{AB} = TL/JG = (600 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (1 in)^4 / 32)(12x10^6 lb/in^2)] = .0733 radians (cw)

ϕ_{BC} = TL/JG = (1,400 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * (2 in)^4 / 32) (6x10^6 lb/in^2)] = .0428 radians (ccw)

ϕ_{CD} = TL/JG = (400 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (.75 in)^4 / 32) (12x10^6 lb/in^2)] = .155 radians (ccw)

$\phi_{Total} = -\phi_{AB} + \phi_{BC} + \phi_{CD}$ = .1245 radians (ccw)

STATICS & STRENGTH OF MATERIALS - Example

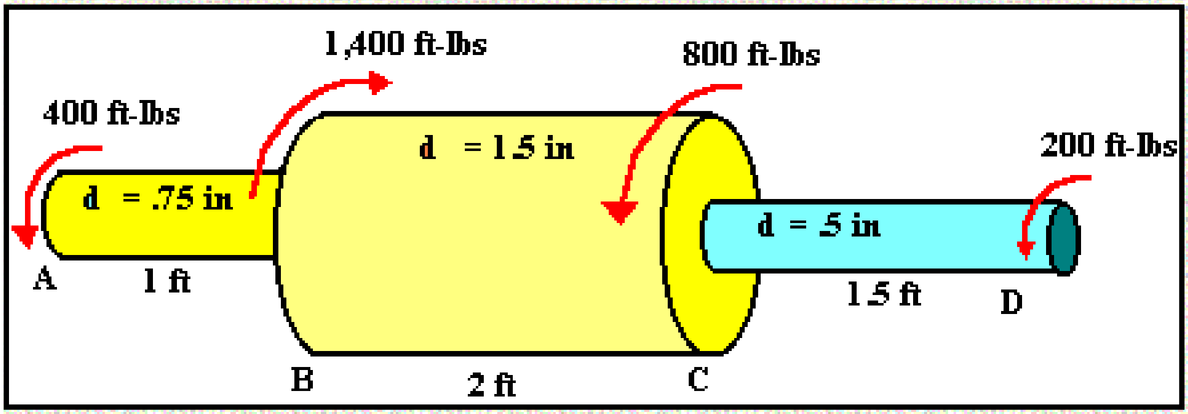
A compound shaft with applied torques and dimensions is shown below. Section AB is made of brass. Section BC is made of brass. Section CD is made of steel. For this shaft:

A. Determine the maximum shear stress in each of the sections of the shaft.

B. Determine the resultant angle of twist of a point on end D with respect to a point on end A.

The modulus of rigidity for steel = 12 x 10^6 lb/sq. in.

The modulus of rigidity for brass = 6 x 10^6 lb/sq. in.



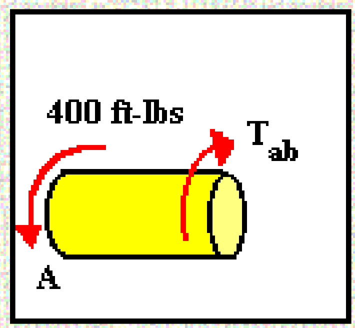
Part A:

Section I:

From equilibrium condtions:

Sum of Torque = 400 ft-lb - T_{AB} = 0; So T_{AB} = 400 ft-lb

Then τ_{AB} = Tr / J = (400 ft-lb)(12 in/ft)(.375in) /[3.1416 * (.75 in)^4 / 32] = 57,900 psi

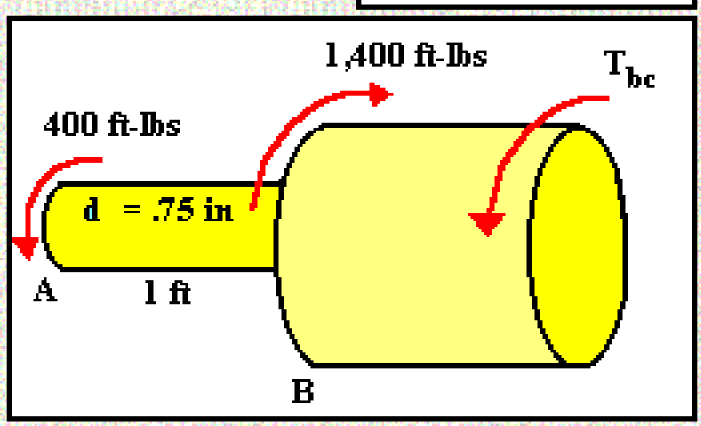


Section II:

From equilibrium condtions:

Sum of Torque = 400 ft-lb - 1,400 ft-lb + T_{BC} = 0; So T_{BC} = 1,000 ft-lb

Then τ_{BC}= Tr / J = (1,000 ft-lb)(12 in/ft)(.75 in) /[3.1416 * (1.5 in)^4 / 32] = 18,100 psi



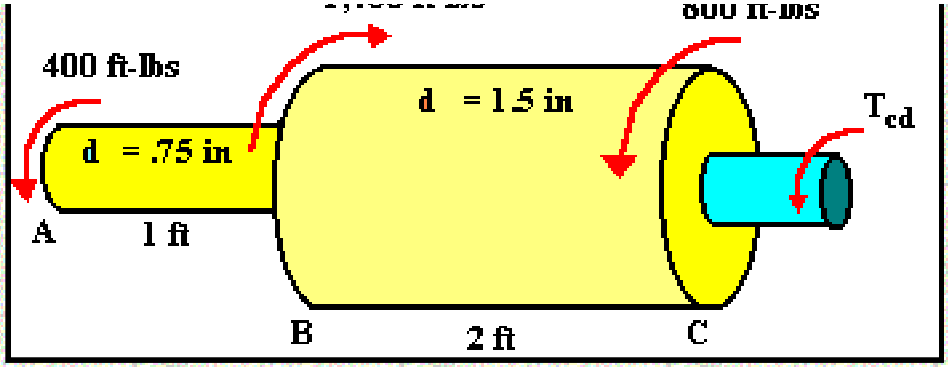
Section III:

From equilibrium condtions:

Sum of Torque = 400 ft-lb -

1,400 ft-lb + 800 ft-lb + T_{CD} = 0; So T_{CD} = 200 ft-lb

Then τ_{CD} = Tr / J = (200 ft-lb)(12 in/ft)(.25 in) /[3.1416 * (.5 in)^4 / 32] = 97,800 psi



Part B:

Resultant angle of twist:

ϕ_{AB} = TL/JG = (400 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (.75 in)^4 / 32)(6x10^6 lb/in^2)] = .309 radians (cw)

ϕ_{BC} = TL/JG = (1,000 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * (1.5 in)^4 / 32)(6x10^6 lb/in^2)] = .0966 radians (ccw)

ϕ_{CD} = TL/JG = (200 ft-lb)(12 in./ft)(1.5 ft)(12 in/ft)/[(3.1416 * (.5 in)^4 / 32)(12x10^6 lb/in^2)] = .587 radians (ccw)

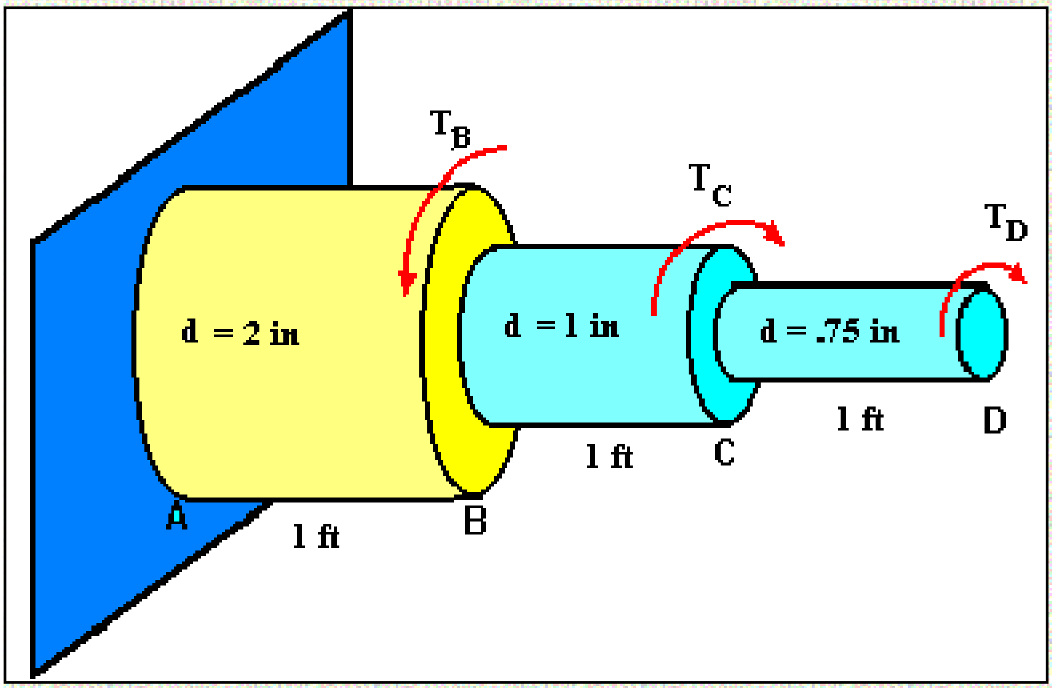
$\phi_{Total} = -\phi_{AB} + \phi_{BC} + \phi_{CD}$ = .375 radians (ccw)

STATICS & STRENGTH OF MATERIALS - Example

A compound shaft is attached to a fixed wall as shown below. Section AB is made of brass. Sections BC and CD are made of steel. If the allowable shear stress for steel and brass are:

τ_{steel} = 18,000 psi τ_{brass} = 12,000 psi

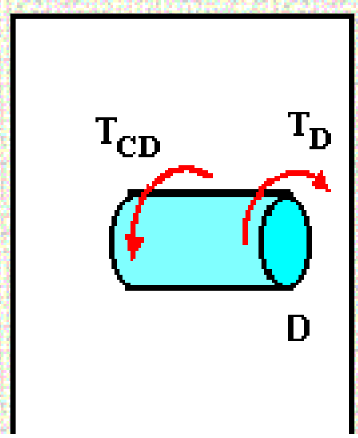
A. Determine the maximum torques which could be applied at points D, C, & B (T_D, T_C, & T_B) without exceeding the allowable shear stress in any section of the shaft.
B. Using the torques found in part A, determine the resultant angle of twist of a point on end D with respect to a point on end A.



The modulus of rigidity for steel = 12 x 10^6 lb/sq. in.
The modulus of rigidity for brass = 6 x 10^6 lb/sq. in.

Part A:

Section I:



From equilibrium condtions:
Sum of Torque = $T_{CD} - T_D = 0$; So $T_D = T_{CD}$

Then: $\tau_{steel} = T_{CD}\, r / J$

$18{,}000 \text{ lb/in}^2 = T_{CD}(.375\text{in}) / [3.1416 * (.75 \text{ in})^4 / 32]$

Solving: **$T_{CD} = T_D$ = 1491 in-lb = 124 ft-lb**

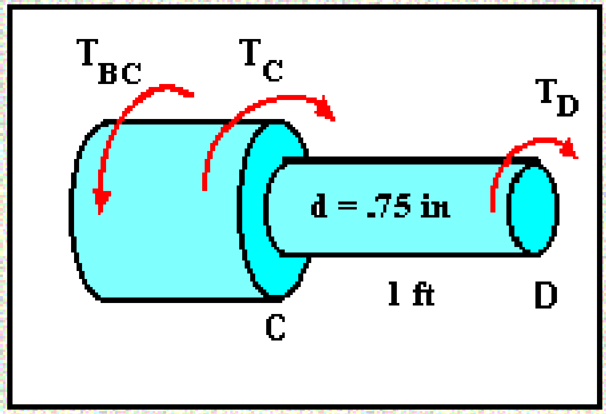
Section II:
From equilibrium condtions:
Sum of Torque = $T_{BC} - T_{CD}$ - 124 ft-lb = 0; So T_C = (T_{BC} - 124 ft-lb)

Then $\tau_{steel} = T_{BC}r / J$

18,000 lb/in^2 = T_{BC}(.5in) /[3.1416 * (1 in)^4 / 32]

Solving: **T_{BC} = 3534 in-lb = 295 ft-lb**

T_C = (295 ft-lb - 124 ft-lb) = 171 ft-lb

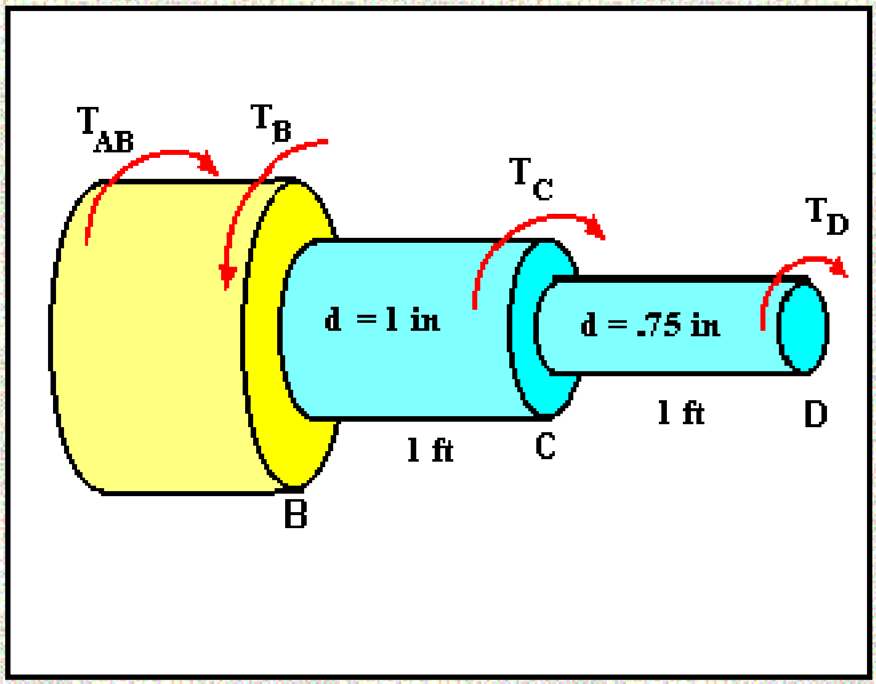


Section III:
From equilibrium condtions:
Sum of Torque = $-T_{AB} + T_B$ - 171 ft-lb - 124 ft-lb = 0; So T_B = (T_{AB} + 171 ft-lb + 124 ft-lb)
Then $\tau_{brass} = T_{AB}r / J$

12,000 lb/in^2 = T_{AB}(1 in) /[3.1416 * (2 in)^4 / 32]

Solving: **T_{AB} = 18,850 in-lb = 1570 ft-lb.**

T_B = (1570 ft-lb + 171 ft-lb + 124 ft-lb) = 1865 ft-lb



Part B:

Resultant angle of twist:

ϕ_{AB} = TL/JG = (1,570 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (2 in)^4 / 32)(6x10^6 lb/in^2)] = .024 radians (ccw)

ϕ_{BC} = TL/JG = (245 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (1 in)^4 / 32)(12x10^6 lb/in^2)] = .0361 radians (cw)

ϕ_{CD} = TL/JG = (124 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (.75 in)^4 / 32)

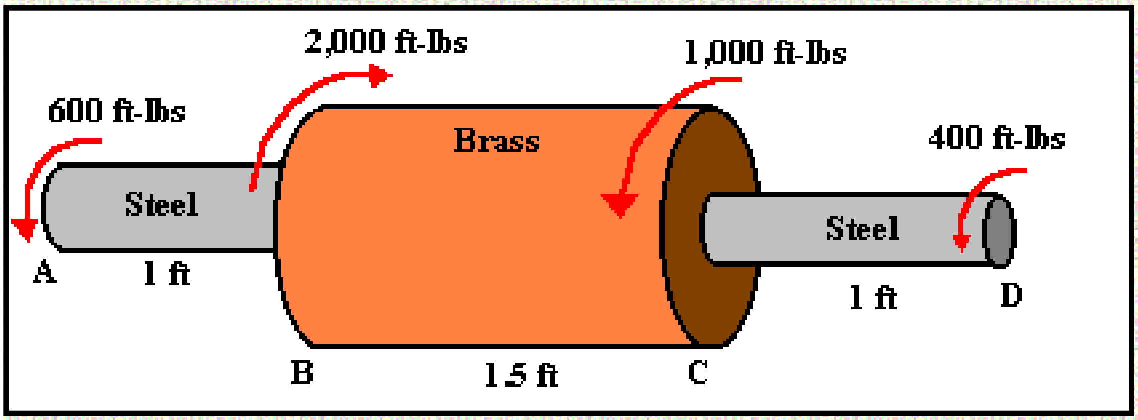
(12x10^6 lb/in^2)] = .0479 radians (cw)

ϕ_{Total} = + ϕ_{AB} − ϕ_{BC} − ϕ_{CD} = .06 radians (cw)

STATICS & STRENGTH OF MATERIALS - Example

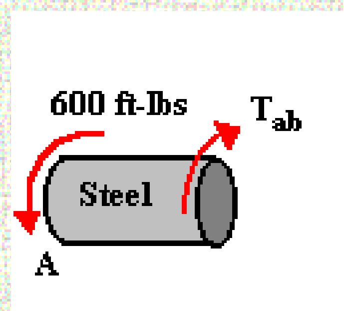
A compound shaft with applied torques is shown below. Section AB is made of steel. Section BC is made of brass. Section CD is made of steel. A driving torque of 2000 ft-lb is applied as shown. Load torques of 600 ft-lbs, 1000 ft-lb, and 400 ft-lbs are also shown. The shaft is rotating at 2400 rev/minute. The allowable shear stress of steel is 20,000 lb/sq.in., and the allowable shear stress for brass in 18,000 lb/sq.in. For this shaft:
A. Determine the horsepower being transfer through each shaft.
B. Determine the minimum diameter for each shaft, such that it can safely carry the transmitted power.

The modulus of rigidity for steel = 12 x 10^6 lb/sq. in.
The modulus of rigidity for brass = 6 x 10^6 lb/sq. in.

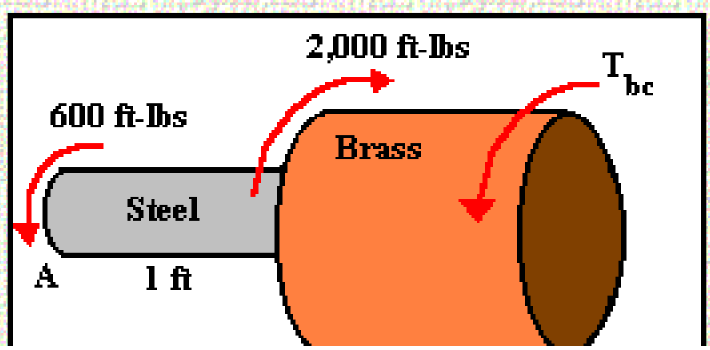


Part A:

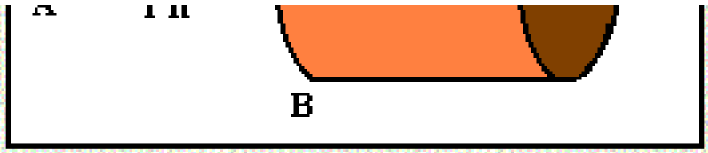
Section I:
From equilibrium condtions:
Sum of Torque = 600 ft-lb - T_{AB} = 0; So T_{AB} = 600 ft-lb
Then: HP = $2\pi\eta T$ / 550 ft-lb/s / hp = 2π (40 Rev/s)(600 ft-lb) / 550 ft-lb/s / hp = 274 hp



Section II:
From equilibrium condtions:
Sum of Torque = 600 ft-lb - 2000 ft-lb + T_{BC} = 0; So T_{BC} = 1400 ft-lb
Then: HP = $2\pi\eta T$ / 550 ft-lb/s / hp = 2π (40

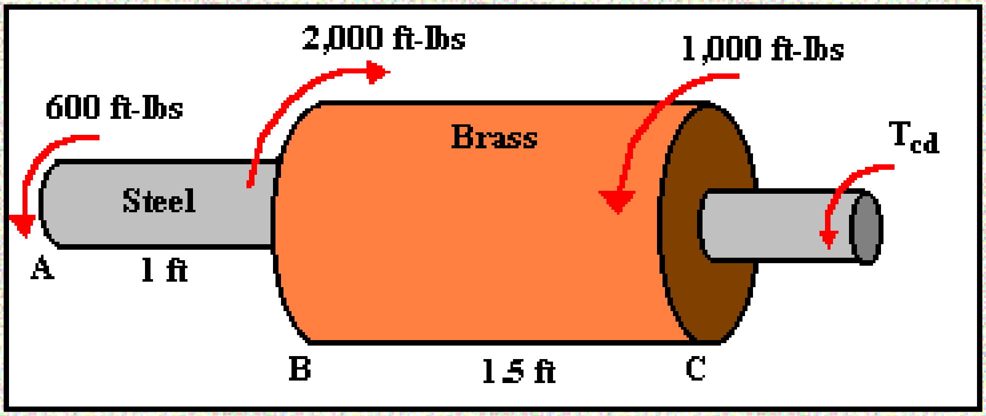


Rev/s)(1,400 ft-lb) / 550 ft-lb/s / hp = 640 hp



Section III:
From equilibrium condtions: Sum of Torque = 600 ft-lb - 2000 ft-lb + 1000 ft-lb + T_{CD} = 0; So T_{CD} = 400 ft-lb

Then: HP = $2\pi\eta T$ / 550 ft-lb/s / hp = 2π (40 Rev/s)(400 ft-lb) / 550 ft-lb/s / hp = 183 hp



Part B:

Section I:

τ_{AB} = Tr / J => 20,000 = (600 ft-lb)(12 in/ft)(d/2) /[3.1416 * $(d)^4$ / 32]

Solve for d^3_{AB} = (600 ft-lb)(12 in/ft)(1/2) /[(3.1416 / 32) (20,000 1b/in^2)] = 1.8335 in^3

so: d_{AB} = 1.22 inches

Section II:

τ_{BC} = Tr / J => 18,000 = (1,400 ft-lb)(12 in/ft)(d/2) /[3.1416 * $(d)^4$ / 32]

Solve for d^3_{BC} = (1,400 ft-lb)(12 in/ft)(1/2) /[(3.1416 / 32) (18,000 1b/in^2)] = 4.7534 in^3

so: d_{BC} = 1.68 inches

Section III:

τ_{CD} = Tr / J => 20,000 = (400 ft-lb)(12 in/ft)(d/2) /[3.1416 * $(d)^4$ / 32]

Solve for d^3_{CD} = (400 ft-lb)(12 in/ft)(1/2) /[(3.1416 / 32) (20,000 1b/in^2)] = 1.2223 in^3

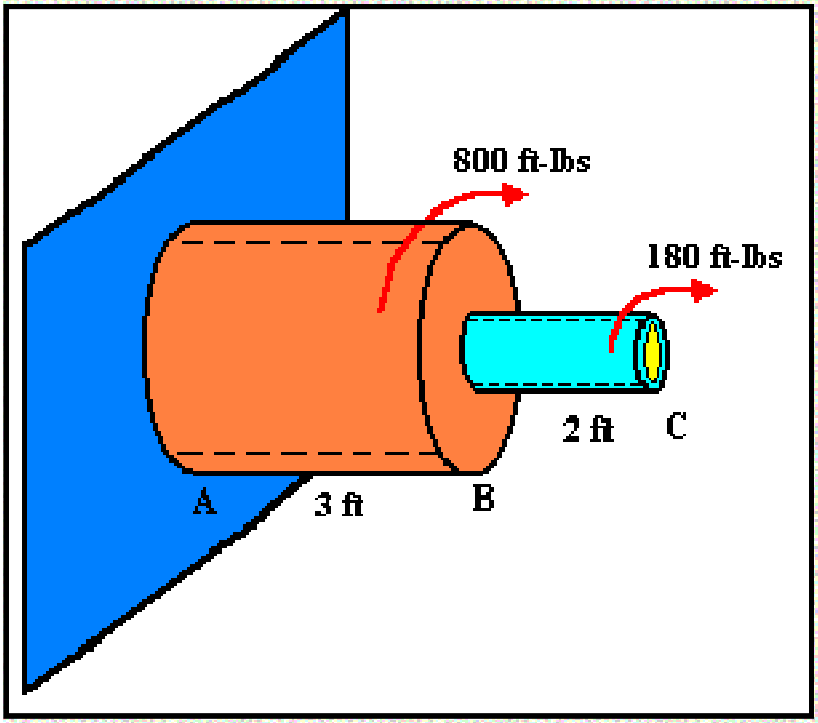
so: d_{CD} = 1.07 inches

STATICS & STRENGTH OF MATERIALS - Example

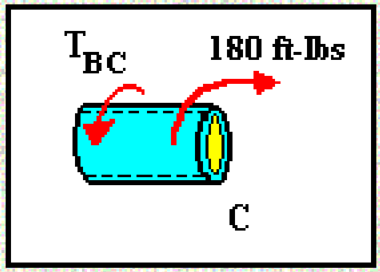
The diagram below represents two hollow brass shafts attached to a solid wall at end A. The torque acting at point B is 800 ft-lbs, and the torque acting at end C is 180 ft-lbs. For this shaft:
A. Determine the maximum shear stress in each shaft.
B. Determine the angle of twist of end C with respect to end A.

The modulus of rigidity for steel = 12 x 10^6 lb / sq. in.
The modulus of rigidity for brass = 6 x 10^6 lb / sq. in.

The diameters of each shaft are as follows:
outer diameter of AB = 2.5 in, inner diameter of AB = 2 in
outer diameter of BC = 1 in , inner diameter of BC = .8 in

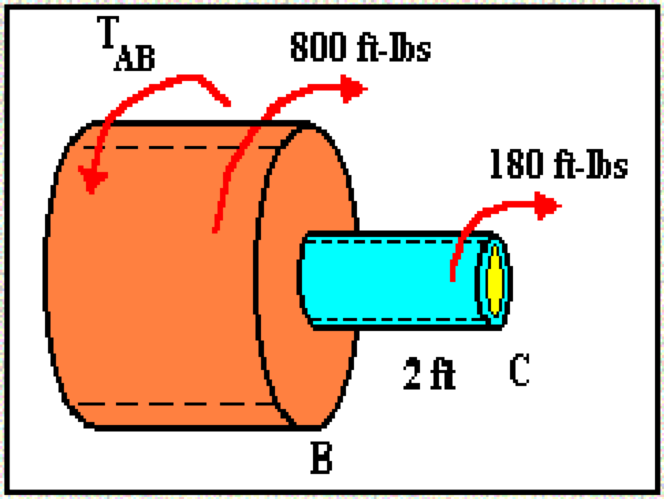


Part A:



Section I:
From equilibrium condtions:
Sum of Torque = T_{BC} - 180 ft-lb = 0; So T_{BC} = 180 ft-lb

Then τ_{AB} = Tr / J = (180 ft-lb)(12 in/ft)(.5 in) /[3.1416 * [(1 in)^4 - (.8 in)^4] / 32] = 18,600 psi



Section II:
From equilibrium condtions:
Sum of Torque = T_{AB} - 800 ft-lb - 180 ft-lb = 0;

So T_{AB} = 980 ft-lb

Then τ_{AB} = Tr / J = (980 ft-lb)(12 in/ft)(1.25 in) /[3.1416 * [(2.5 in)^4 - (2 in)^4] / 32] = 6,490 psi

Part B:

Resultant angle of twist:

ϕ_{AB} = TL/JG = (980 ft-lb)(12 in./ft)(3 ft)(12 in/ft)/[(3.1416 * [(2.5 in)^4 - (2 in)^4] / 32)(6x10^6 lb/in^2)] = .0312 radians (cw)

ϕ_{BC} = TL/JG = (180 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * [(1 in)^4 - (.8 in)^4] / 32)(6x10^6 lb/in^2)] = .0149 radians (cw)

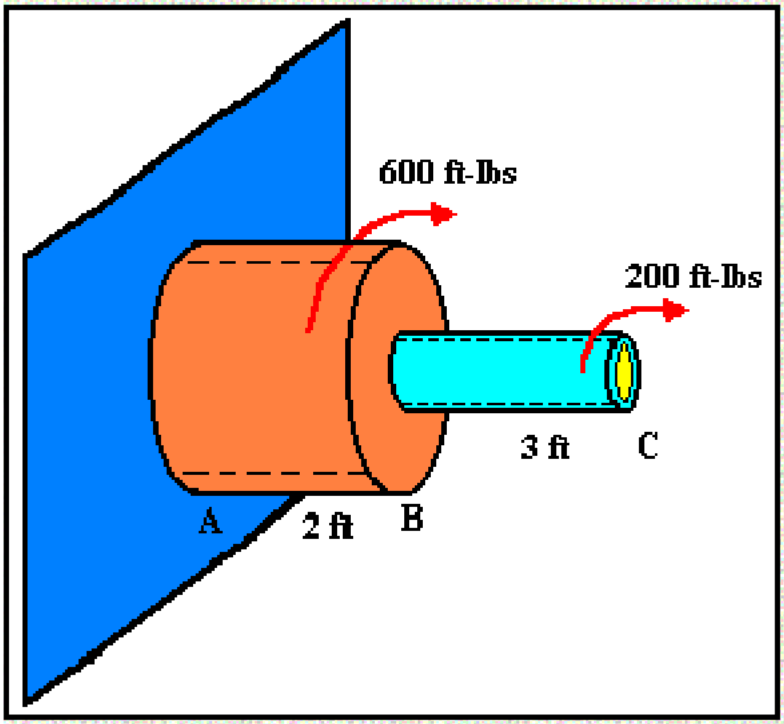
$\phi_{Total} = -\phi_{AB} - \phi_{BC} = -$.0461 radians (cw)

STATICS & STRENGTH OF MATERIALS - Example

The diagram below represents two hollow brass shafts attached to a solid wall at end A. The torque acting at point B is 600 ft-lbs, and the torque acting at end C is 200 ft-lbs. For this shaft:
A. Determine the maximum shear stress in each shaft.
B. Determine the angle of twist of end C with respect to end A.

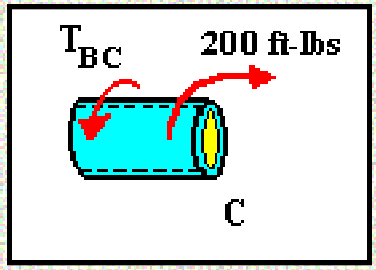
The modulus of rigidity for steel = 12 x 10^6 lb/sq. in.
The modulus of rigidity for brass = 6 x 10^6 lb/sq. in.

The diameters of each shaft are as follows:
outer diameter of AB = 3.5 in, inner diameter of AB = 2.8 in
outer diameter of BC = 2 in , inner diameter of BC = 1.6 in



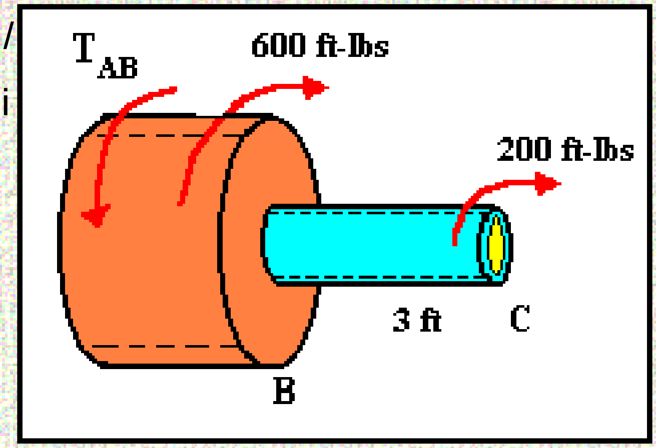
Part A:
Section I:
From equilibrium condtions:
Sum of Torque = T_{BC} - 200 ft-lb = 0; So T_{BC} = 200 ft-lb

Then τ_{AB} = Tr / J = (200 ft-lb)(12 in/ft)(1 in) /[3.1416 * [(2 in)^4 - (1.6 in)^4] / 32] = 2,600 psi



Section II:
From equilibrium condtions:
Sum of Torque = T_{AB} - 600 ft-lb - 200 ft-lb = 0; So T_{AB} = 800 ft-lb

Then τ_{AB} = Tr / J = (800 ft-lb)(12 in/ft)(1.75 in) / [3.1416 * [(3.5 in)^4 - (2.8 in)^4] / 32] = 1,150 psi



Part B:

Resultant angle of twist:

ϕ_{AB} = TL/JG = (800 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * [(3.5 in)^4 - (2.8 in)^4] / 32)(6x10^6 lb/in^2)] = .0044 radians (cw)

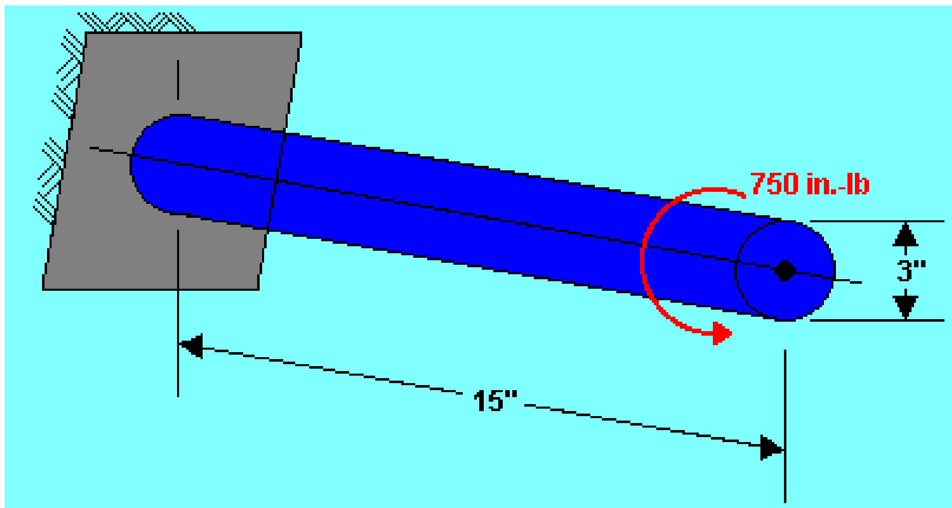
ϕ_{BC} = TL/JG = (200 ft-lb)(12 in./ft)(3 ft)(12 in/ft)/[(3.1416 * [(2 in)^4 - (1.6 in)^4] / 32)(6x10^6 lb/in^2)] = .0155 radians (cw)

$\phi_{Total} = -\phi_{AB} - \phi_{BC} = -$.0199 radians (cw)

Statics & Strength of Materials

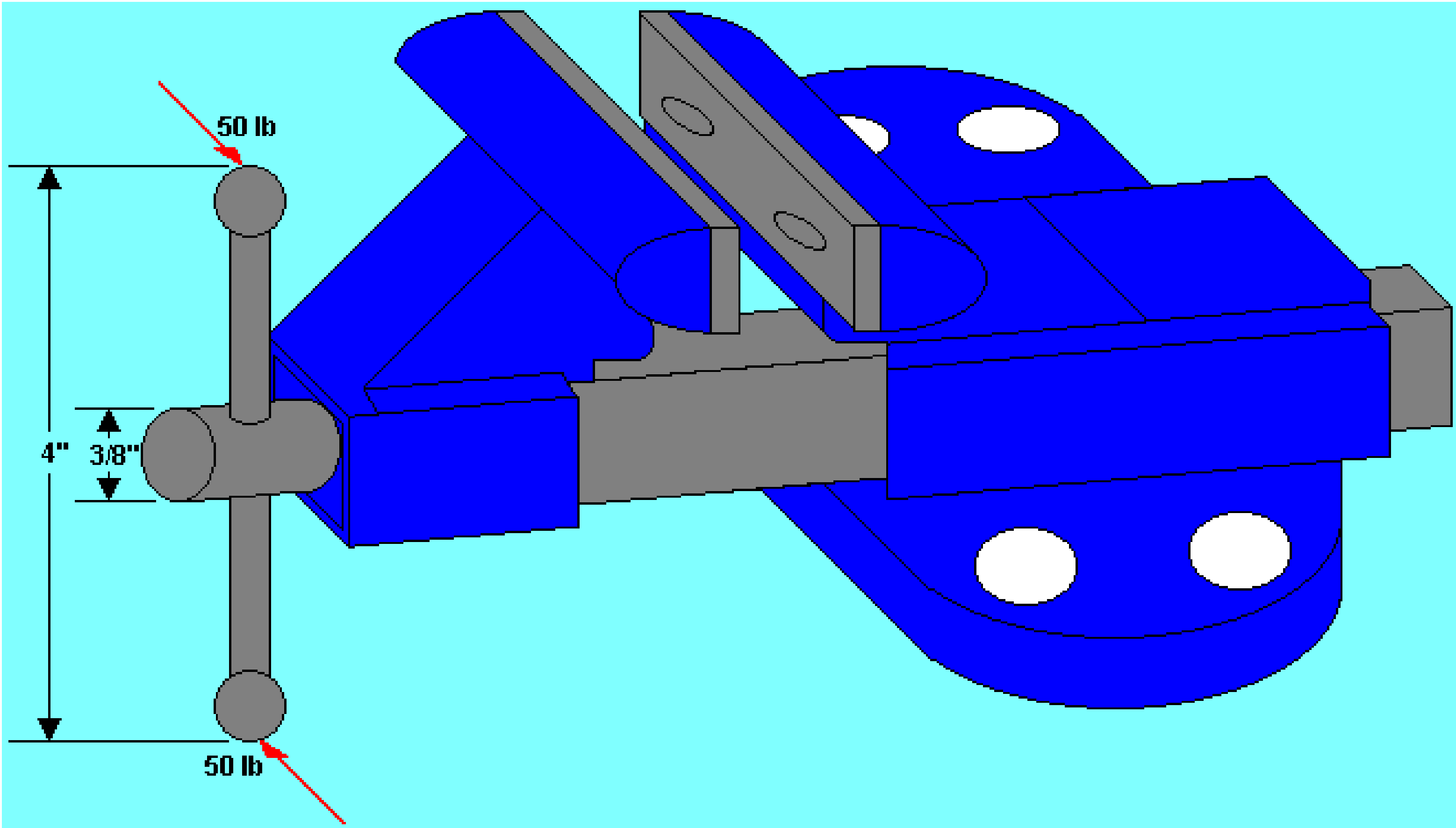
Topic 6.4a - Problem Assignment 1 - Torsion

1. A 3 inch diameter solid steel shaft is 15 inches long. A torque of 750 inch pounds is applied to the shaft. Determine the maximum shear stress in the shaft. Where does the maximum shear stress occur. (142 psi., outer radius)



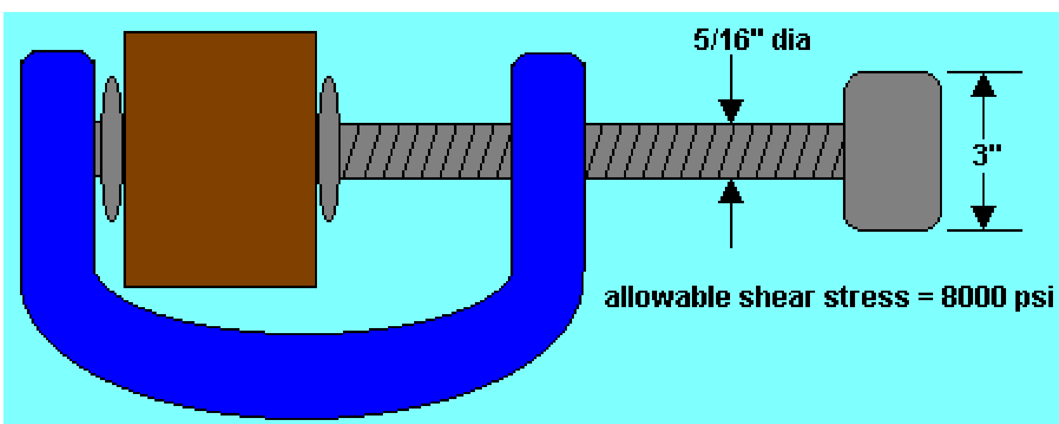
Problem #1

2. A vice is clamped by applying a force of 50 pounds at each end of a 4 inch arm on a shaft. The shaft is solid 3/8 inch diameter steel. Determine the maximum shear stress in the shaft. (19,320 psi)



Problem #2

3. A C-clamp is tightened by applying a couple (equal forces in opposite directions on opposite sides) to a 3" diameter lever at the end of the clamp screw. The clamp screw is effecively a 5/16 inch diameter steel shaft. The maximum allowable shear stress in the shaft is 8000 psi. Find the maximum force that can be applied in the couple. (16 lb)



Problem #3

4. The drive shaft of a car is a hollow steel shaft with an outside diameter of 3 inches and a wall thickness of 1/8 inch. Determine the polar section modulus of the shaft. What is the

maximum shear stress in the shaft when it has a torque applied equal to 2200 inch pounds. (2.34 in^4, 1410 psi)

5. A 3/4 inch solid steel shaft is 8 inches long. The maximum allowable stress for the shaft is 12,000 psi. Determine the maximum torque that can be transmitted by the shaft. What is the angle of twist for the shaft? G= $12x10^6$ psi. (82.84 ft-lb, .0213 rad)

6. A 1/32 inch twist steel drill has a torque of 0.1 inch pounds applied. The free length of the drill is 2 inches long. Find the maximum torsional stress in the drill. Determine the angle of twist of drill. (16.7 x 10^3 psi, .178 rad)

7. The steel drive shaft of a car has a torque applied of 1600 inch pounds. The drive shaft is 4 feet long and 2.5 inches in diameter with a 1/16 diameter wall thickness. Determine the maximum shear stress in the shaft and the angle of twist. (5415 psi, .01733 rad)

8. A hollow steel shaft 2 inches in diameter and 3 feet long with 1/16 inch walls is used in an auger. Determine the angle of twist of the shaft when a torque of 5000 inch pounds is applied to one end of the shaft. What is the maximum shear stress in the shaft? (.042 rad, 13, 990 psi)

9. A three foot wooden dowel 1/4 inch in diameter has a torque of 5 inch pounds applied. Determine the angle of twist of the dowel and the maximum shear stress in the wood. G = $1.04x10^6$ psi. (1630 psi, .4513 rad)

Select:
[Topic 6: Torsion, Rivets & Welds - Table of Contents]
[Strength of Materials Home Page]

Statics & Strength of Materials

Topic 6.4b - Problem Assignment 2 - Torsion

1. An outboard motor has a steel shaft 1/2 inch in diameter and 18 inches long. It develops a torque of 75 inch pounds at 6000 rpm. Determine the horse power that is developed. G = 12×10^6 psi. (7.14 hp)

2. A quarter horsepower electric drill turns at 750 rpm. What is the torque that is developed in the drill bits? (1.75 ft-lb)

3. The drive shaft of a car has a torque applied of 1600 inch pounds. The drive shaft is 4 feet long and 2.5 inches in diameter with a 1/16 diameter wall thickness. Determine the power delivered by the shaft when the angular speed of the shaft is 1500 rpm.(38.1 hp)

4. A motorcycle is shaft driven with a 1 inch solid steel shaft. The maximum allowable transverse shear stress in the shaft is 8,000 psi. Determine the maximum power that can be delivered by the shaft if it is rotating at 6000 rpm. What is the angle of twist of the shaft if the shaft is 24 inches long. (150 hp, .032 rad)

5. The propeller on an ship has a 72 inch pitch and turns at 180 rpm. The shaft on the propeller transmits 3000 horse power. Determine the minimum polar moment of inertia of the shaft if the shaft steel has a maximum allowable shear stress of 12,000 psi. What is the required diameter of the shaft if it is solid?
(334.5 in^4, 7.64 in.)

Select:
[Topic 6: Torsion, Rivets & Welds - Table of Contents]
[Strength of Materials Home Page]

Statics & Strength of Materials

Topic 6.4c - Problem Assignment 3 - Torsion

1. A 3/4 inch diameter steel shaft has a 6 inch diameter pulley attached. The belt on the pulley has a tensile force of 65 pounds on one side and 35 pounds on the other side? Determine the torque applied to the shaft. Determine the power transmitted by the pulley if the shaft is turning at 1200 rpms.

2. A V-belt is attached to a 6 inch diameter pulley. The belt tension on the two sides of the pulley are 60 pounds and 100 pounds. Determine the torque applied to the pulley by the V-belt. Determine the power transmitted by the shaft if it is rotating at 600 rpms.

3. A propeller is attached to the drive shaft of an outboard by means of a brass shear pin. The pin is has an ultimate shear strength of 23,000 psi. Determine the maximum torque that can be transmitted if the shaft is 3/4 inch in diameter and the pin is 1/4 inch in diameter.

4. Two rotating shafts are connected with gears. The first shaft has an 8 inch gear with 40 teeth and the second shaft has a 5 inch gear with 25 teeth. The first shaft is rotating at 300 rpms. Determine the rotational speed of the second shaft.

5. Consider the shafts and gears in problem 4. The teeth of the gear on the first shaft push on the teeth of the gear on the second shaft. These forces are equal and opposite. If this force is 120 pounds, determine the torques acting on each of the two shafts.

6. Determine the power transmitted by the torques in each of the two shafts in the problems above.

7. The chain ring on a bicycle has 42 teeth and the free wheel has 28 teeth. A force of 150 pounds is applied to the 7 inch peddle arm. The chain ring has 3 teeth per inch.

   a. Determine the torque delivered to the chain ring.
   b. Determine the force in the chain.
   c. Determine the torque delivered to the free wheel.
   d. Determine the power delivered if the free wheel rotates at 90 rpms.

8. The chain ring on a bicycle has 42 teeth and the free wheel has 15 teeth. A

force of 150 pounds is applied to the 7 inch peddle arm. The chain ring has 3 teeth per inch.

a. Determine the torque delivered to the chain ring.
b. Determine the force in the chain.
c. Determine the torque delivered to the free wheel.
d. Determine the power delivered if the free wheel rotates at 90 rpms.

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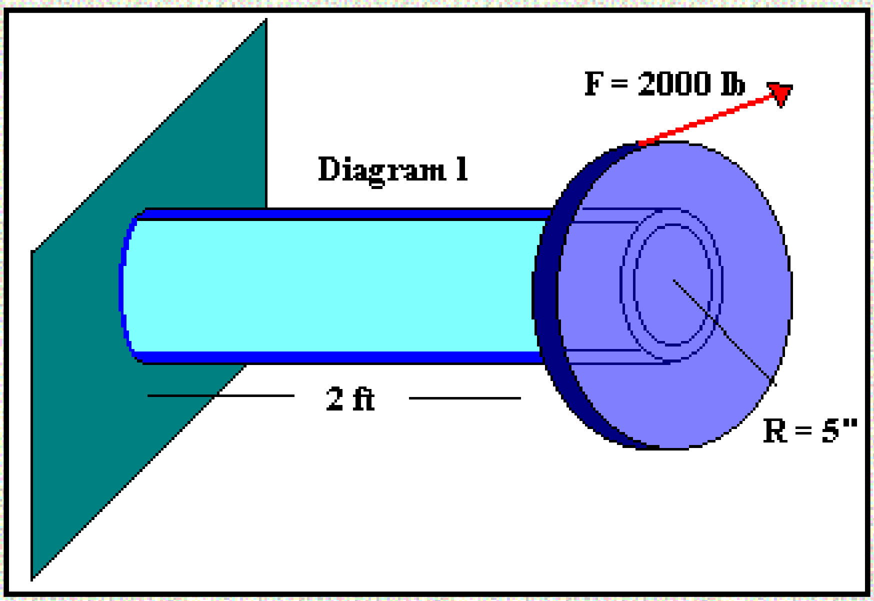
Statics & Strength of Materials

Topic 6.4d - Problem Assignment 4 - Torsion

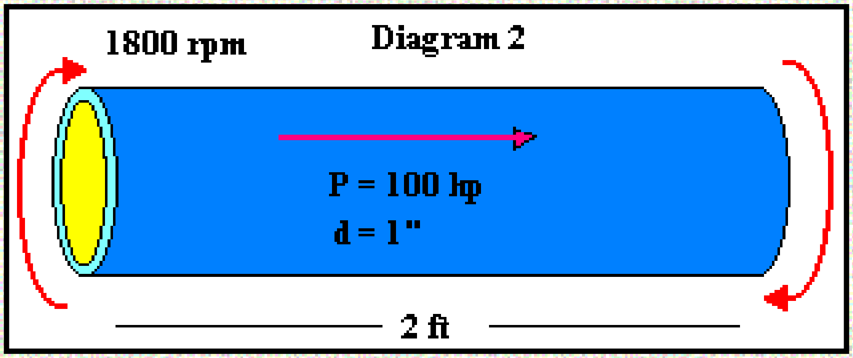
Modulus of Rigidity for several materials:
Steel = 12 x 10^6 lb/in^2.; Brass = 6 x 10^6 lb/in^2.; Aluminum = 4 x 10^6 lb/in^2.

1. A hollow steel shaft with an outer diameter of 2 inches and an inner diameter of 1.75 inches is attached to circular disk with a radius of 5 inches, as shown in Diagram 1. A force of 2,000 lb. is applied to the outer edge of the disk.
A. What is the maximum shear stress which develops in the hollow shaft?
B. What is the angle of twist which develops in the shaft?
C. If the maximum allowable shear stress in the shaft is 20,000 lb/in^2, how large a force could be applied to the outer edge of the disk?



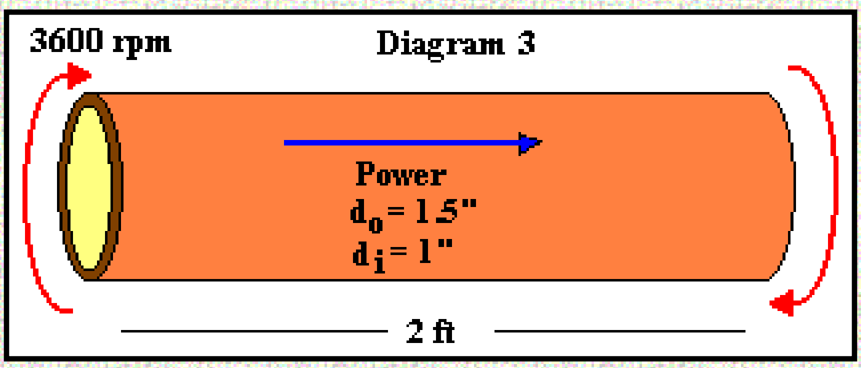
(Answers: A.15,400 lb/in^2, B. .031 radians, C. 2,600 lb.)

2. A 2 foot long hollow steel shaft with an outer diameter of 1 inches is to transmit 100 horsepower while being driven a 1800 rev/min.
A.) If the allowable shear stress in the shaft is 20,000 lb/in^2, what is the maximum possible value of the inner diameter of the shaft.
B.) If we were not given the outer diameter of the shaft, but were told that the inner diameter was to be nine-tenths of the outer diameter, what would be the minimum outer diameter of the shaft which could safely transmit the horse power ? (The allowable shear stress in the shaft is 20,000 lb/in^2)



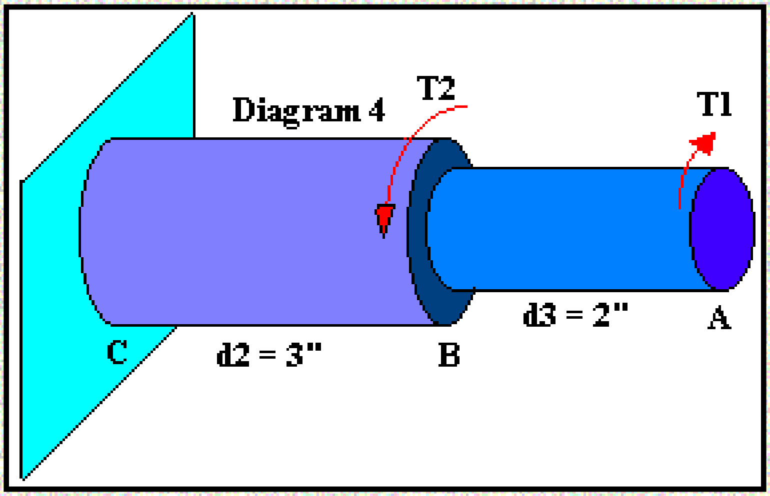
(Answers: A. di = .573" , B. do = 1.37")

3. A 2 foot long hollow brass shaft with an outer diameter of 1.5 inches and an inner diameter of 1 inches is to transmit power while being driven a 3600 rev/min.
A.) If the allowable shear stress in the shaft is 18,000 lb/in^2., what is the maximum horsepower which can be transmitted down the shaft ?
B.) If we were to transmit the maximum power, what would be the resulting angle of twist of the shaft due to the applied torque.
C.) If we were not given the outer diameter of the shaft, but were told that the inner diameter was to be six-tenths of the outer diameter, what would be the minimum outer diameter of the shaft which could safely transmit the horsepower found in part A? (The allowable shear stress in the shaft is 18,000 lb/in^2.)



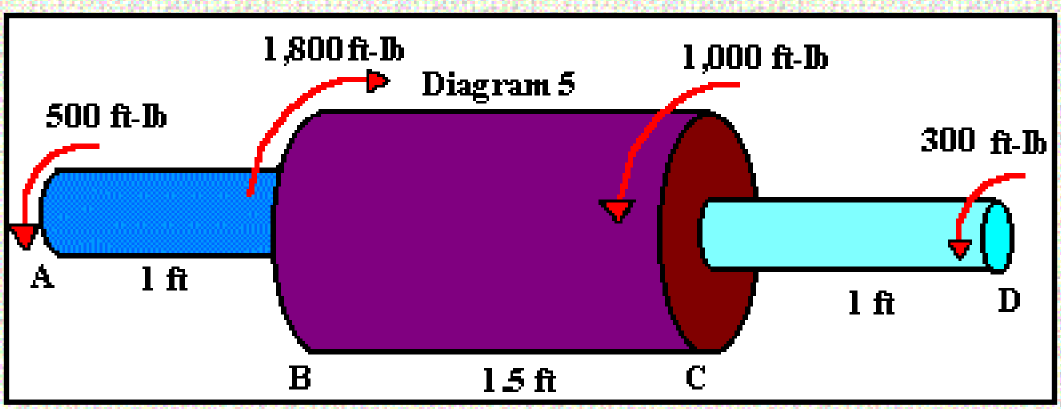
(Answers: A 548 hp., B. .096 radians, C. 1.46")

4. A compound shaft is attached to the wall at point C, as shown in Diagram 4. Shaft section BC is aluminum and has a diameter of 3 inches. Shaft section AB is steel and has a diameter of 2 inches. The allowable shear stress in the aluminum is 14,000 lb/in^2, and the allowable stress in the steel is 18,000 lb/in^2. Torque of T1 and T2 are applied to the shaft in the directions shown in Diagram 4. Determine the maximum values of the applied torque, T1 and T2, such that the allowable shear stress is not exceeded in either shaft section.



(Answers: T1=2360 ft-lb., T2 = 8545 ft-lb.)

5. A compound shaft with applied torque is shown in Diagram 5. Section AB is made of steel. Section BC is made of brass. Section CD is made of aluminum. A driving torque of 1,800 ft-lb. is applied at point B. Load torque of 500 ft-lb., 1000 ft-lb., and 300 ft-lb. act at points A, C and D. The shaft is rotating at 2,400 rev/minute. The allowable shear stress in the steel is 20,000 lb/in^2, the allowable shear stress in the brass is 16,000 lb/in^2, and the allowable shear stress in the aluminum is 14,000 lb/in^2. For this shaft:
A. Determine the horsepower being transferred through each shaft.
B. Determine the minimum diameter for each shaft, such that it can carry the transmitted power within the allowable stress limit.
C. Using the diameters determined in part B, calculate the angle of twist of end D with respect to end A.



(Answers: A. AB=228 hp, BC=594 hp, CD=137 hp; B. AB=1.15", BC=1.71", CD=1.09"; C AB=.035 rad-cw, .BC=056 rad-ccw, CD=078 rad-ccw, Total = .099 rad-ccw)

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Topic 6.5: Rivets & Welds - Riveted Joints

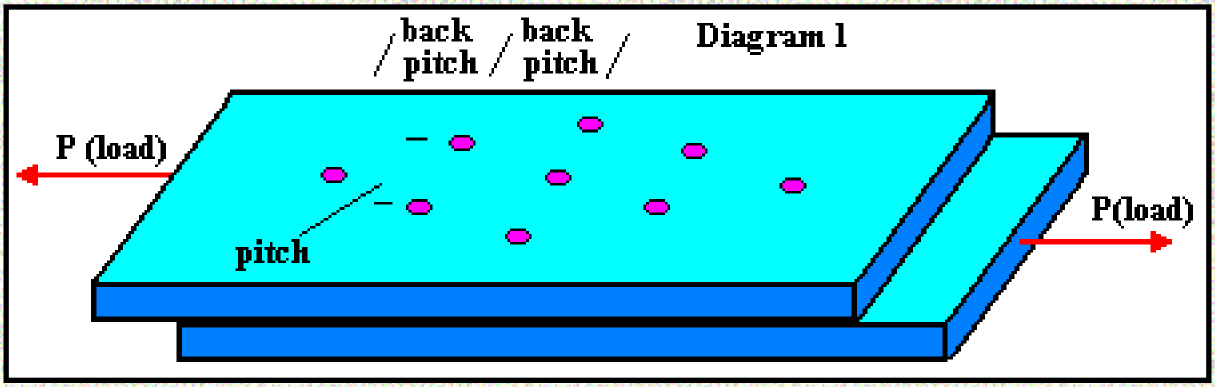
In this topic we will discuss riveted and bolted joints. We will generally treat riveted and bolt joints in the same way and use the same method of analysis. Basically, in a riveted joint a heated rivet is forced into a hole connecting two "plates" (or beams). As the rivet cools, a tension develops in the rivet and the plates are forced together. In bolted joints, high strength bolts are inserted into connecting holes between plates (or beams) and then tightened to a percentage (initially approximately 70 percent) of the allowable bolt tensile strength. Generally for bolts the hole is slightly larger than the bolt, and this is taken into account in a careful analysis. In our discussion, for both rivets and bolts, we will assume the hole diameter is the same as the rivet or bolt diameter.

Before we examine the specific modes of failure, we will list some assumptions used in our discussion.

1. **That the rivets and bolts completely filled the connecting holes.**
2. **That the applied loads are carried equally by the rivets (bolts).**
3. **That the rivet (bolt) shear stress is distributed uniformly over the cross sectional rivet (bolt) area.**
4. **That the tensile load carried by the plate is also distributed equally across the plate material.**

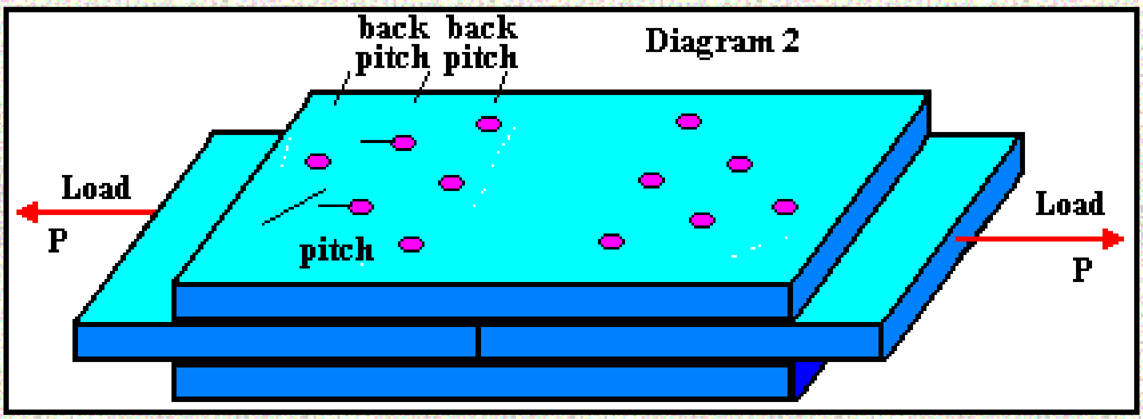
We will also ignore the effect of friction in carrying the load. That is, there is significant friction between plates riveted or bolted together. This friction may play a significant part is the amount of load a joint can carry. Because the friction effect can vary substantially, we will not try to include the contribution of friction to supporting the load. This is a conservative approach in the sense that by ignoring the effect of friction, the joint in actuality will normally carry more than the calculated strength of the joint.

There are two basic types of riveted (bolted) joints - **Lap Joints** and **Butt Joints**. A lap joint is shown in Diagram 1, and a butt joint is shown in Diagram 2.



In a Lap Joint, two plates are overlapped and rivets or bolts penetrate the two plates connecting them together.

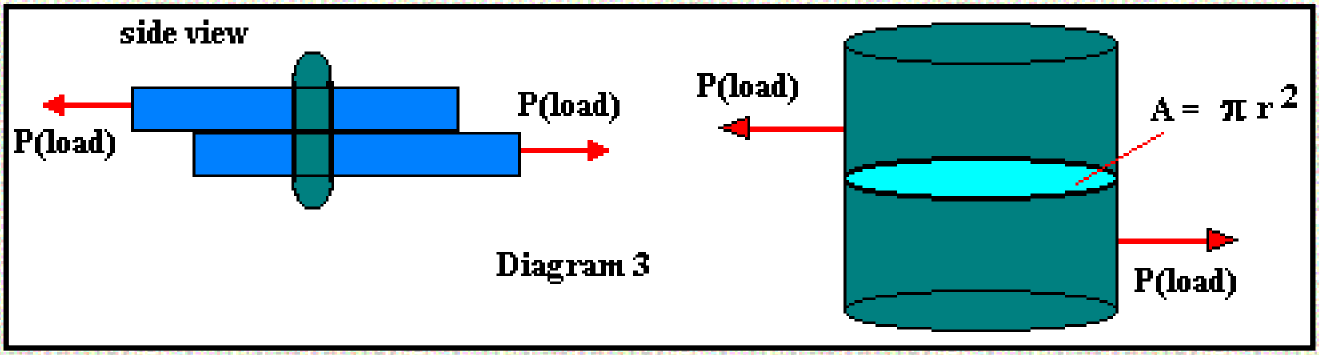
In a Butt Joint, the two main plates are butted up against each other and then covered with one or two cover plates. Then rivets or bolts penetrate the cover plates and the two main plates connecting them together. The load is transferred through one main plate to the cover plates by the rivets and then transferred back to the second main plate. There is a symmetric rivet pattern above each main plate as shown in Diagram 2.



The distance between rivets in a row in riveted (bolted) joint pattern is known as the **Pitch**. The distance between rows in a riveted (bolted) joint pattern is known as the **Back Pitch** (or **Transverse Pitch**, or **Gauge**). The first row (row 1) of a pattern is the row which is closest to the applied load. As a general guideline for steel or aluminum plates, the minimum pitch is three times the diameter of the rivet (bolt), and the edge pitch (distance from the nearest rivet to the edge) is 1.5 times the rivet (bolt) diameter.

Joint Failure: There are a number of ways in which a riveted (bolted) joint may fail.

1. Rivet Shear: As shown in Diagram 3, a side view of a lap joint, the rivet area between the two main plates is in shear. We obtain the formula for the Strength of the Joint in Rivet Shear by simply using the definition of the shear stress - which is the force parallel to the area in shear divided by area. Thus if we take the allowable shear stress for the rivet material times the cross sectional area of the rivet this gives us the load one rivet area could carry in shear before it failed (By failed, we mean having exceeded the allowable stress.)



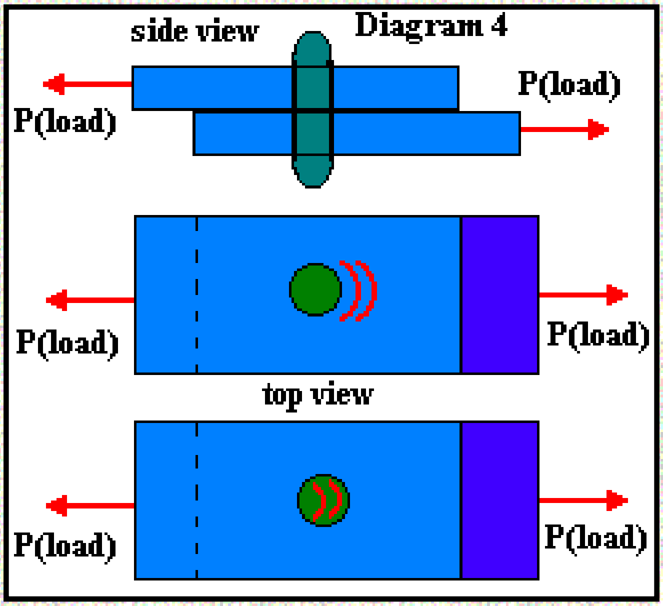
So we can write:

$P_{rivet\ shear} = N\ (pi * d^2 / 4)\ \tau_{all}$. Where:

N = Number of areas in shear. This equals the number of rivets in a lap joint or in a butt joint with one cover plate, and twice the number of rivets in a butt joint with double cover plates.
$A = pi * d^2 / 4$ (or $pi * r^2$) is the cross sectional area of the rivet in shear.
τ_{all} = The allowable shear stress for the rivet material.

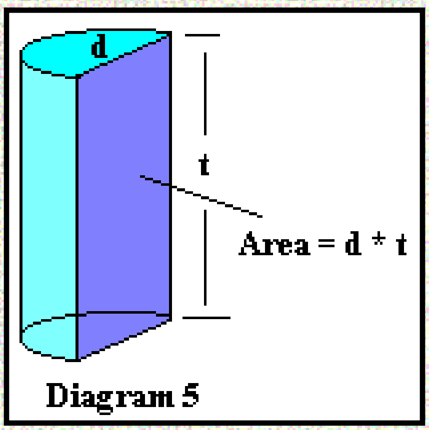
2. Rivet / Plate Bearing Failure: This is compression failure of either the rivet or the plate material behind the rivet.

As shown in middle diagram in Diagram 4, when we consider the top main plate, the top main plate is being pulled into the fixed rivet. This puts the plate material behind the rivet into compression, and if the load is large enough the plate material may fail in compression. From the rivet's perspective, the plate is being pulled into it, and this puts the rivet into compression.



Again if the load is large enough, the rivet material may fail in compression. Which will fail first in compression depends, of course, on the maximum allowable compressive stress for the rivet and plate material - the lowest allowable compressive stress material will fail first.

To determine the load which will cause failure, we again multiply the stress by the area. In this case, it is common practice to take the area in compression as the vertical cross sectional area of the rivet (Diagram 5), for both the area of the rivet in compression and the area of plate in compression.



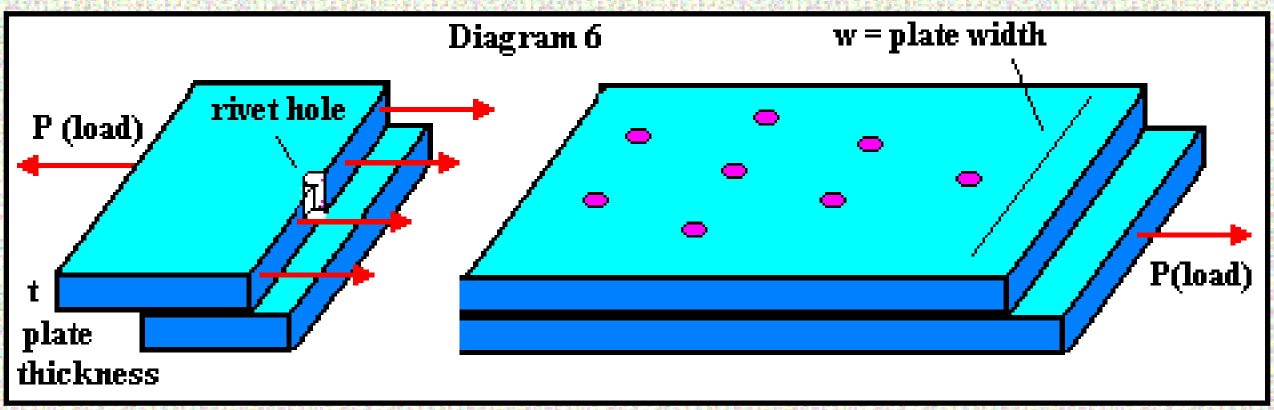
So we can write:

$P_{bearing} = N\ (d*t)\ \sigma_{all}$, where

N = Number of rivets in compression
d = Diameter of rivet
t = Thickness of the main plate

σ_{all} = Allowable compressive stress of the rivet or plate material

3. Plate Tearing: This is a tensile failure of the plate material normally at the rivet row positions, that is, the plate will tear first where the holes are in the plate, just as paper towels tear where the perforations are located.
As is shown in Diagram 6, if we cut the plate material at rivet row 1 and look at the left end section, we see that for equilibrium the plate material is in tension. To determine the applied load, P, the plate can carry before it would fail in tension, we multiply the allowable tensile stress by the area in tension. This area is the cross sectional area of the plate which, if solid, would be the width of the plate times the thickness of the plate (A = w*t). However, we have cut the plate at rivet row 1, and we have to subtract the diameter of the rivet from the width of the plate (since the area of the plate is reduce due to the rivet hole).



Then we can write:

$P_{row1} = (w - n\,d)t\,\sigma_{all}$., where

w = Width of the main plate
n = number of rivets in row (in this example, row 1, 1 rivet)
d = Diameter of rivet
t = Thickness of the main plate
σ_{all} = Maximum allowable tensile stress for the plate material

The formula for plate tearing are rivet rows beyond row 1 have to be modified somewhat, due to the fact that rows beyond row 1 are no longer carrying the entire load, P, since some of the load has already be transferred to the second plate. We will go into this in more detail in a later example.

There are several additional ways the joint may fail, including plate shear - which may occur if a rivet is placed to close to the end of the plate, and the plate material behind the rivet fails in shear. If proper placing of rivets is maintained, this mode is not normally a problem.

We will consider only the three main modes of failure discussed above.

We will now look at several examples of Riveted Joints,
Please select :
Topic 6.5a: Riveted Joints - Example 1
Topic 6.5b: Riveted Joints - Example 2

When finished with Riveted Joints Examples**, Continue to:**
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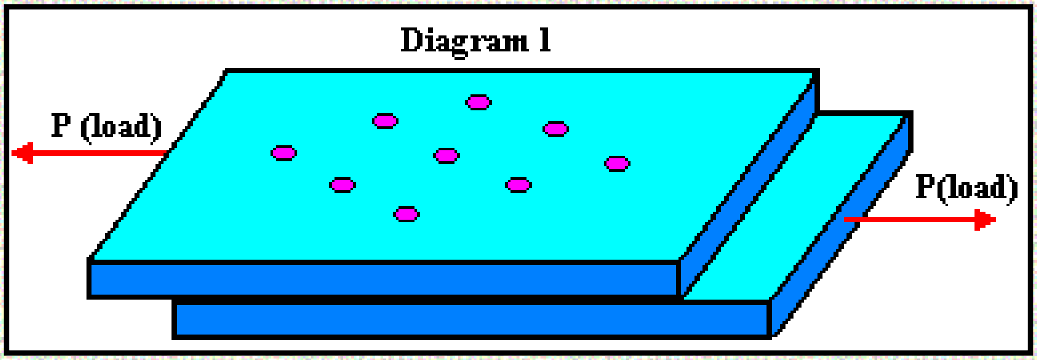
Topic 6.5a: Riveted Joints - Example 1

A riveted lap joint is shown in Diagram 1. The diameter of the rivets is 5/8 inch. The width of the plates is 6 inches, and the thickness of the plates is 1/2 inch. The allowable stresses are as follows:

Rivets: τ = 16, 000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 24, 000 lb/in^2

Plate: τ = 14,000 lb/in^2, σ_t = 20, 000 lb/in^2, σ_c = 23, 000 lb/in^2



We would like to determine the **Strength of the Joint**, and the **Efficiency of the Joint.**

Part 1: To determine the Strength of the Joint, we calculate the load, P, which will cause the joint to fail in each of the main modes of failure (Rivet Shear, Bearing, and Plate Tearing). The lowest load which will cause the joint to fail is known as the **Strength of the Joint**. Please remember that the term failure of the joint in these problems does not mean when the joint actually breaks, but rather we consider the joint to have failed when the stress in the joint exceed the allowable stress. At this point the joint may or may not actually fail depending on the safety factor used in the determining the allowable stress, and of course, depending on how much the allowable stress is exceeded.

1. Rivet Shear: The load the joint can carry before failing in rivet shear is given by:

P = N (pi * d^2/4) τ = (9 rivet areas)* [3.1416 * (5/8)^2/4]*16, 000 lb/in^2 = 44,200 lb.

Thus at a load of 44, 200 lb., the joint will fail in shear. Please note we used the allowable shear stress for the rivet material in our equation, as it is the rivet (not the plate) which fails in shear.

2. Bearing (compression) Failure: We next determine the load the rivet or plate can carry before failing in compression.

$P_{bearing}$ = N (d*t) σ_{all} = (9 rivets) * (5/8" *1/2") * (23,000 lb/in^2) = 64,700 lb.

Thus a load of 64,700 lb. will cause the joint (plate) to fail in compression. Notice we use the smaller of the allowable compressive stress between the rivet and plate.

3. Plate Tearing (Row 1): We now determine the load the joint (plate) can carry before failing in tension - at row 1.

P_{row1} = (w - n d)t σ_{all} = (6"- 1* 5/8") *(1/2") * (20,000 lb/in^2) = 53,750 lb.

This is the load which will cause the plate to fail in tension at row 1.

We see that of the three main modes of failure above, the joint fails first in rivet shear at the lowest load of 44,200 lb. Up to this point, this is the Strength of the Joint, however we still need to check plate tearing failure at rivet row 2 (and perhaps row 3) to see if the joints fails at a lower load there. We do this as follows.

3a. Plate Tearing (Row 2): The main difference between Plate Tearing at row 2 and at row 1 (other than the fact that there are different numbers of rivet in the rows) is that plate material at row 2 does not carry the entire load, P. This is due to the fact that part of the load has been transferred to the bottom plate by the rivet in row 1. Since there are 9 rivets in the pattern and we assume that the rivets share the load equally, then one ninth (1/9) of the load has been transferred to the bottom plate. Thus row 2 carries of 8/9 of the load P, and we may write.

(8/9)P_{row2} = (w - n d)t σ_{all} = (6"- 2* 5/8") *(1/2") * (20,000 lb/in^2) = 47,500 lb., and then:

P_{row2} = (9/8) * 47,500 lb. = 53,400 lb.

This is the load which will cause shear row 2 to fail in tension. We see that it is lower than the load that will cause row 1 to fail (that is, the joint would fail at row 2 before row 1), however the joint will still fail first in rivet shear (44,200 lb.) - since that is the lowest applied load which will cause the joint to exceed the allowable stress.

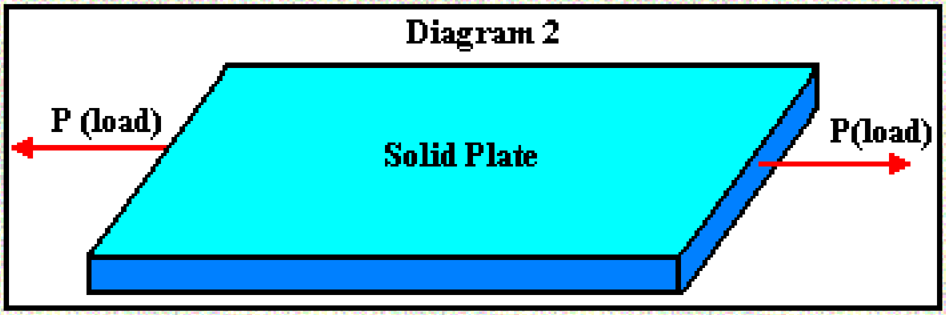
3b. Plate Tearing (Row 3): Since the joint would fail at row 2 before row 1, we need now to continue and determine the load at which row 3 would fail. As there are a total of 3 rivets in row 1 and row 2, this means that 3/9 of the load has been transferred to the bottom plate, and thus the plate material at row 3 carries only 6/9 of the load, P. Then we can write:

$(6/9)P_{row3} = (w - n\ d)t\ \sigma_{all} = (6" - 3 * 5/8") * (1/2") * (20,000\ lb/in^2) =$ 41,250 lb., and then:
$P_{row3} = (9/6) * 41,250$ lb. = 61,900 lb.

Here we see that load which causes row 3 to fail in tension is much larger than the loads which would cause row 1 and row 2 to fail. This means we can stop here.

Of all the loads which will cause the joint to fail (either in shear, compression, or tension) the lowest is still the rivet shear - **44,200 lb**., and this is the final **Strength of the Joint**, the largest load we can safely apply.

Part II. By itself the strength of the joint does not really tell us how 'good' the joint is. The way we determine how good or efficient a joint we have is to compare the strength of the joint with the strength of the plate if it were solid (no riveted or bolted joint). The strength of the joint is 44,200 lb., found above. The strength of the plate we determine by realizing that if it is solid the joint will fail in tension (plate tearing). See Diagram 2. And we can write:
Plate Strength = area of plate cross section times allowable tensile stress for plate material, or
$P_{plate} = (w * t) * \sigma_{all} = (6" * 1/2") * 20,000$ lb/in2 = 60,000 lb.



This is the plate strength, and we then define the joint efficiency as the ratio of the Joint Strength to the Plate Strength, or
Efficiency = Joint Strength / Plate Strength = 44,200 lb. / 60,000 lb. = .737 = 73.7 %. This tells us that the joint can carry 73.7 percent of what the solid plate could carry.

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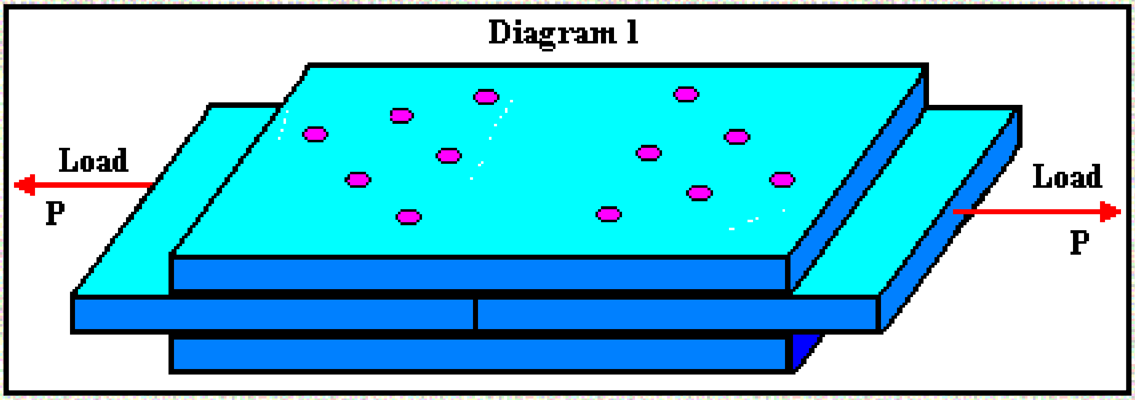
Topic 6.5b: Riveted Joints - Example 2

A riveted butt joint is shown in diagram 1. The diameter of the rivets is 3/4 inch. The width of the plates is 6 inches, and the thickness of the plates is 1/2 inch. The allowable stresses are as follows:

Rivets: τ = 18, 000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 24, 000 lb/in^2

Plate: τ = 16,000 lb/in^2, σ_t = 21, 000 lb/in^2, σ_c = 22, 000 lb/in^2



We would like to determine the **Strength of the Joint**, and the **Efficiency of the Joint.**

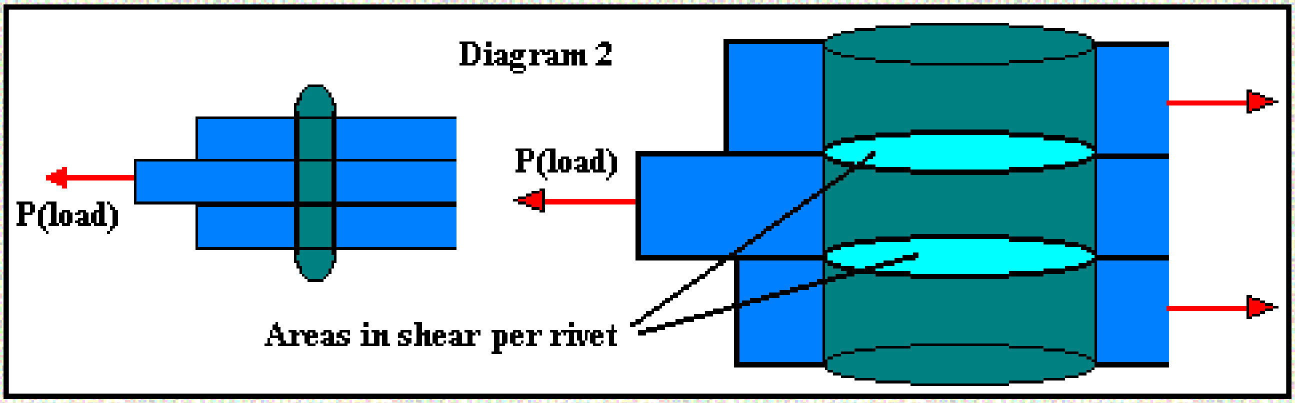
Part 1. To determine the Strength of the Joint, we calculate the load, P, which will cause the joint to fail in each of the main modes of failure (Rivet Shear, Bearing, and Plate Tearing). The lowest load which will cause the joint to fail is known as the **Strength of the Joint**.

For a butt joint we use only one half of the joint pattern, since if one side fails, the main plate on that side has failed and the joint has failed. We will consider the left half of the joint pattern in this example.

1. Rivet Shear: The load the joint can carry before failing in rivet shear is given by:

P = N (pi * d^2/4) τ = (12 rivet areas)* [3.1416 * (3/4)^2/4]*18, 000 lb/in^2 = 95,400 lb.

Notice that we use 12 rivet areas since each of the six rivets in the left half of the pattern has two areas in shear in the double cover plate butt joint. (See Diagram 2) Thus at a load of 95,400 lb., the joint will fail in shear.



2. Bearing (compression) Failure: We next determine the load the rivet or plate can carry before failing in compression.

$P_{bearing}$ = N (d*t) σ_{all} = (6 rivets) * (3/4" *1/2") * (22,000 lb/in^2) = 49,500 lb.

Thus a load of 49,500 lb. will cause the joint (plate) to fail in compression. Note that we use the smaller of the allowable compressive stress between the rivet and plate.

3. Plate Tearing (Row 1): We now determine the load the joint (plate) can carry before failing in tension - at row 1.

P_{row1} = (w - n d)t σ_{all} = (6"- 1* 3/4") *(1/2") * (21,000 lb/in^2) = 55,100 lb.

This is the load which will cause the plate to fail in tension at row 1.

We see that of the three main modes of failure above, the joint fails first in bearing (compression) failure at the lowest load of 49,500 lb. Up to this point, this is the Strength of the Joint, however we still need to check plate tearing failure at rivet row 2 (and perhaps row 3) to see if the joints fails at a lower load there. We do this as follows.

3a. Plate Tearing (Row 2): The main difference between Plate Tearing at row 2 and at row 1 (other than the fact that there are different numbers of rivet in the rows) is that plate material at row 2 does not carry the entire load, P. This is due to the fact that part of the load has been transferred to the bottom plate by the rivet in row 1. Since there are 6 rivets in left side pattern and we assume that the rivets share the load equally, then one sixth (1/6) of the load has been transferred to the cover plates and on the second main plate. Thus row 2 carries of 5/6 of the load P, and we may write.

(5/6)P_{row2} = (w - n d)t σ_{all} = (6"- 2* 3/4") *(1/2") * (21,000 lb/in^2) = 47,250 lb., and then:

P_{row2} = (6/5) * 47,500 lb. = 56,700 lb.

This is the load which will cause shear row 2 to fail in tension. We see that it is larger than the load that will cause row 1 to fail (that is, the joint would fail at row 1 before row 2), however the joint will still fail first in bearing (compression) failure at 49,500 lb. - since that is the lowest applied load which will cause the joint to exceed the allowable stress.

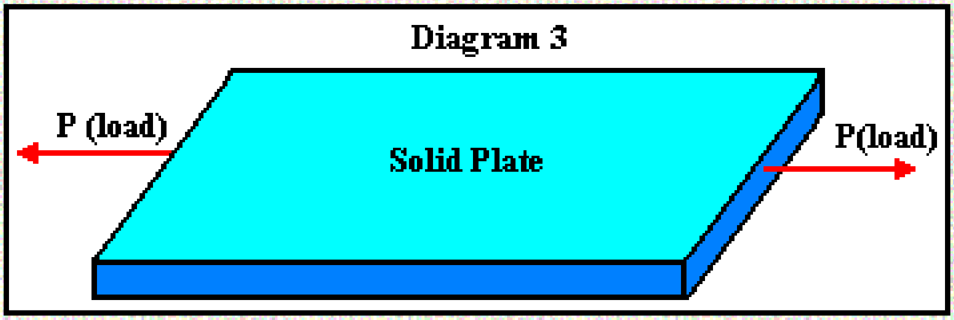
3b. Plate Tearing (Row 3): Since the load which causes row 2 to fail is larger than the load which causes row 1 to fail, we normally do not have to check row 3, since that should fail at a even larger load. However, we will check row 3, simply as an exercise.
As there are a total of 3 rivets in row 1 and row 2, this means that 3/6 of the load has been transferred to the bottom plate, and thus the plate material at row 3 carries only 3/6 of the load, P. Then we can write:

$(3/6)P_{row2} = (w - n\,d)t\,\sigma_{all} = (6" - 3 * 3/4") * (1/2") * (21{,}000\ lb/in^2) =$ **39,400 lb.,** and then:

$P_{row2} = (6/3) * 39{,}400\ lb. = 78{,}800\ lb.$

Here we see that load which causes row 3 to fail in tension is much larger than the loads which would cause row 1 and row 2 to fail. Of all the loads which will cause the joint to fail (either in shear, compression, or tension) the lowest is still the bearing failure, **49,500 lb.**, and this is the final **Strength of the Joint**, the largest load we can safely apply.

Part II. By itself the strength of the joint does not really tell us how 'good' the joint is. The way we determine how good or efficient a joint we have is to compare the strength of the joint with the strength of the plate is it were solid (no riveted or bolted joint). The strength of the joint is 49,500 lb., found above. The strength of the plate we determine by realizing that if it is solid the joint will fail in tension (plate tearing). See Diagram 3.



And we can write:
Plate Strength = area of plate cross section times allowable tensile stress for plate material, or

P_{plate} = **(w * t) * σ_{all} = (6" * 1/2") * 21,000 lb/in^2 = 63,000 lb.** This is the plate strength, and we then define the joint efficiency as the ratio of the Joint Strength to the Plate Strength, or
Efficiency = Joint Strength / Plate Strength = 49,500 lb. / 63,000 lb. = .786 = 78.6 %. This tells us that the butt joint can carry 78.6 percent of what the solid plate could carry.

Return to: Topic 6.5: Rivets & Welds - Riveted Joints
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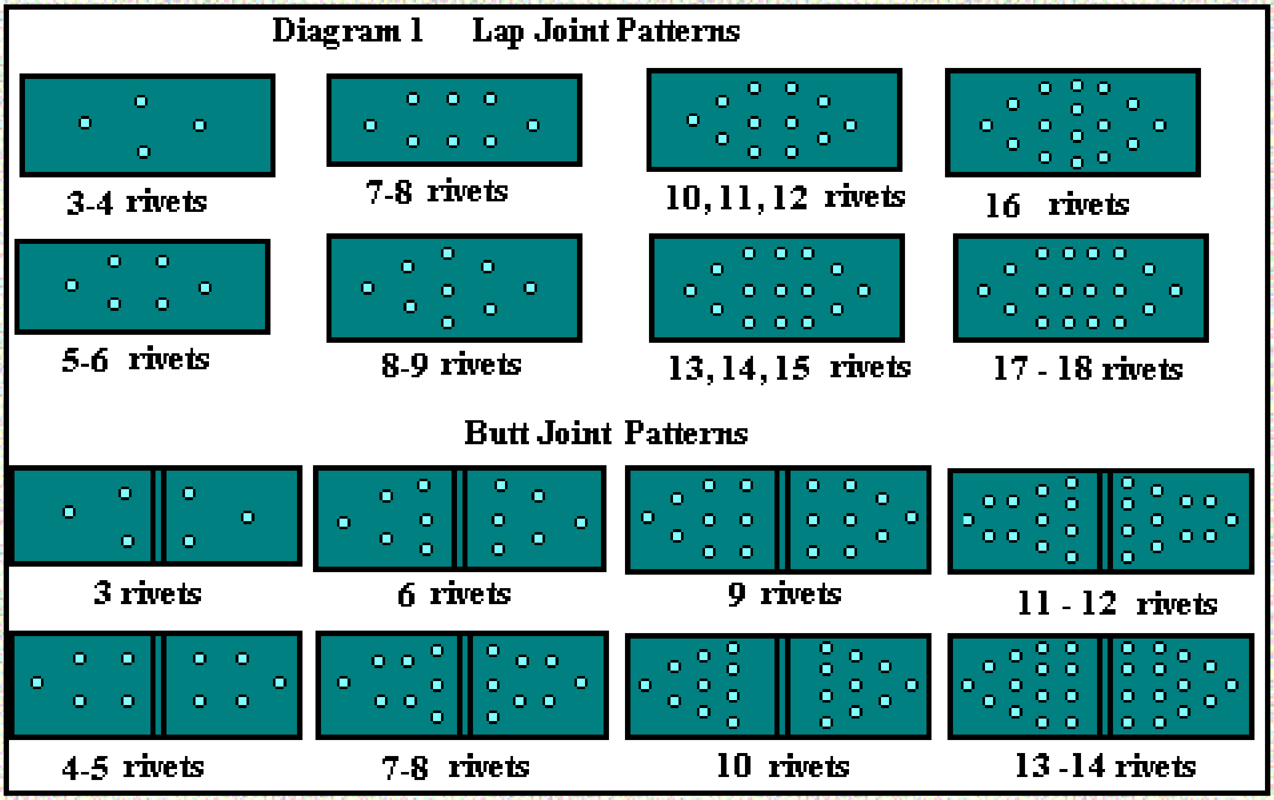
Topic 6.6: Rivets & Welds - Riveted Joint Selection

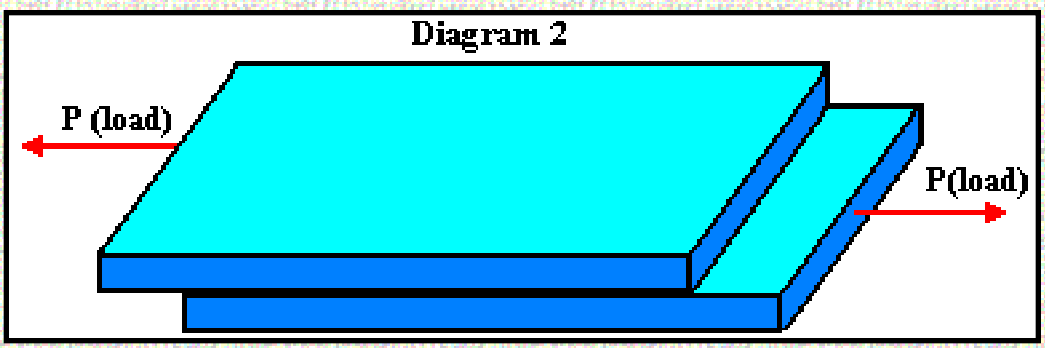
We have so far examined how we determine the strength and efficiency of a given riveted (bolted) joint pattern. A related and interesting question is how do we go about determining (or selecting) the most efficient joint pattern for given main plates.
We will apply the following procedure to this question:

1. **Calculate the load which would cause the joint to fail in plate tearing at row 1 - assuming one rivet in row 1.**
2. **Calculate the maximum load which one rivet could carry in rivet shear.**
3. **Calculate the maximum load which one rivet (or the plate material behind one rivet) could carry in bearing (compression)**
4. **Divide the allowable load in plate tearing, calculated in step 1, by the smallest of the loads one rivet could carry from steps 2 and 3, and round up to determine the number of rivets which will result in the most efficient joint (for the given main plate dimension, rivet dimensions, and allowable stresses).**
5. **Use the number of rivets found in step 4 to design a joint pattern, or more usually, use the number of rivets to select the best joint pattern from a given set of patterns. (See Diagram 1 for example of possible rivet patterns.)**



Example - Joint Selection

A lap joint (See Diagram 2) is to connect two steel plates both with a width of 6 inches and a thickness of 1/2 inch. The rivets to be used have a diameter of 3/4 inch. The maximum allowable stresses for the rivet and weld materials are as follows:



Rivets: τ = 16, 000 lb./in^2, σ_t = 22, 000 lb./in^2, σ_c = 25, 000 lb./in^2

Plate: τ = 17,000 lb./in^2, σ_t = 20, 000 lb./in^2, σ_c = 24, 000 lb./in^2

A. Determine the number of rivets for the most efficient joint.
B. Select the best pattern from Diagram 1 above.
C. Calculate the strength and efficiency of the joint.

Solution: Part A

Step 1: Calculate the load which would cause the joint to fail in plate tearing at row 1 - assuming one rivet in row 1. We use the plate tearing relationship:

$P_{row1} = (w - n\ d)t\sigma_{all}$. , where

w = width of the main plate = 6 inches
n = number of rivets in row (in row 1, 1 rivet)
d = diameter of rivet = 3/4 inch
t = thickness of the main plate = 1/2 inch
σ_{all} = Maximum allowable tensile stress for the plate material = 20, 000 lb./in^2,

P_{row1} = (6" - 1 * 3/4")*(1/2") * 20, 000 lb./in^2 = 52,500 lb.

Step 2. Calculate the maximum load which one rivet could carry in rivet shear. We use the rivet shear relationship:

$P_{rivet\ shear} = N\ (pi * d^2/4)\ \tau_{all}$. Where:

N = Number of areas in shear For 1 rivet in a lap joint this will be 1 area; N = 1
d = 3/4 inch
τ_{all} = 16,000 lb./in^2

$P_{rivet\ shear}$ = 1 [3.1416 * (3/4")^2/4] * 16,000 lb./in^2 = 7070 lb./rivet

Step 3. Calculate the maximum load which one rivet (or the plate material behind one rivet) could carry in bearing (compression)

$P_{bearing} = N\ (d*t)\ \sigma_{all}$. , where

N = Number of rivets in compression = 1
d = Diameter of rivet = 3/4 inch
t = Thickness of the main plate = 1/2 inch
σ_{all} = Lowest allowable compressive stress of the rivet or plate material = 24, 000 lb./in^2

$P_{bearing}$ = 1 * [(3/4") * (1/2")] * 24, 000 lb./in^2 = 9000 lb./rivet

Step 4. Divide the allowable load in plate tearing, calculated in step 1, by the small of the loads one rivet could carry from steps 2 and 3, and round up to determine the number of rivets which **will result in the most efficient joint.**
 # Rivets = 52, 500 lb. / 7070 lb./rivet = 7.43 ; rounding up # rivets = 8

Solution: Part B
We use the number of rivets found in Part A above to select the best joint pattern

from the given set of patterns. From Diagram 1 above we select the a lap joint pattern with correct number of rivets. (See Diagram 3.)

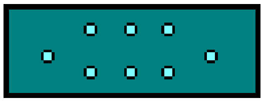


Diagram 3

Solution: Part C

Finally, using the selected joint pattern from part B, we calculate the strength and efficiency of the joint.

1.) Rivet Shear: We have already determined that one rivet can carry 7070 lb./ rivet, so the total load the joint can carry in rivet shear will be the product of the number of rivets and the allowable load per rivet: **P_{shear} = 8 * 7070 lb./rivet = 56,560 lb.**

2.) Bearing Failure: Again, we have already calculated the allowable load per rivet in bearing = 9000 lb./rivet. The total load the joint can carry in bearing (compression) will be the product of the number of rivets and the allowable load per rivet: **$P_{bearing}$ = 8 * 9000 lb./rivet = 72,000 lb.**

3.) Plate Tearing (row 1): We have already calculated plate tearing at row 1, in step 1 above. **P_{row1} = (6" - 1 * 3/4")*(1/2") * 20, 000 lb./in^2 = 52,500 lb.** However, now that we have selected the rivet pattern, we also need to calculate plate tearing at row 2 (and perhaps row 3)

4.) Plate Tearing (row 2): Row 2 carries 7/8 of the load, so we write:
(7/8)P_{row2} = (6" - 2 * 3/4")*(1/2") * 20, 000 lb./in^2 = 45,000 lb., and then
P_{row2} = (8/7) 45,000 lb. = 51, 400 lb. Notice, that this loading is the lowest, up to this point, to cause failure, and so it is currently the strength of the joint. However, we will continue and check the load which causes row 3 to fail.

5.) Plate Tearing (row 3): Row 2 carries 5/8 of the load, so we write:
(5/8)P_{row3} = (6" - 2 * 3/4")*(1/2") * 20, 000 lb./in^2 = 45,000 lb., and then
P_{row3} = (8/5) 45,000 lb. = 72, 000 lb. This loading is much larger than the loading which will cause row 2 to fail, so we can stop here.

Considering all the modes of failure, we see: **Strength of the Joint is 51,400 lb**. (plate tearing row 2), and
Efficiency = Joint Strength / Plate Strength = 51,400 lb./(6"*1/2") *20,000 lb./in^2 = .86 =86%

This is the most efficient joint for the dimensions and allowable stresses of both the plates and rivets. By changing the size of the rivets (and the material and associated allowable stresses) we could arrive at a joint of greater efficiency. This type of problem is easily done using current computer spreadsheet methods.

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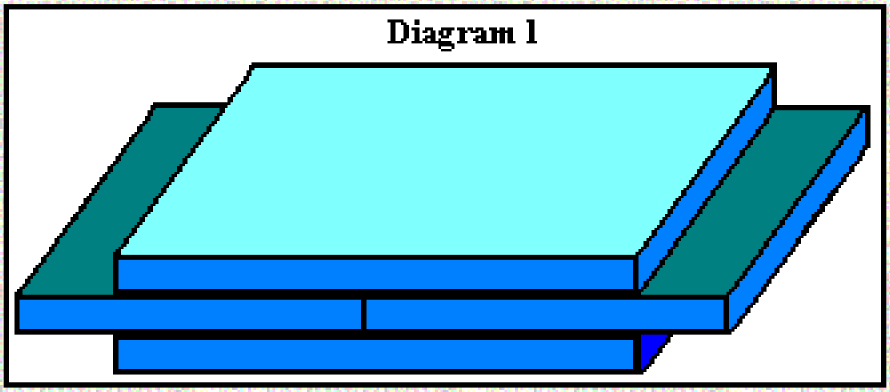
Topic 6.6a: Riveted Joint Selection - Example 1

Riveted Joint Selection - Example 1

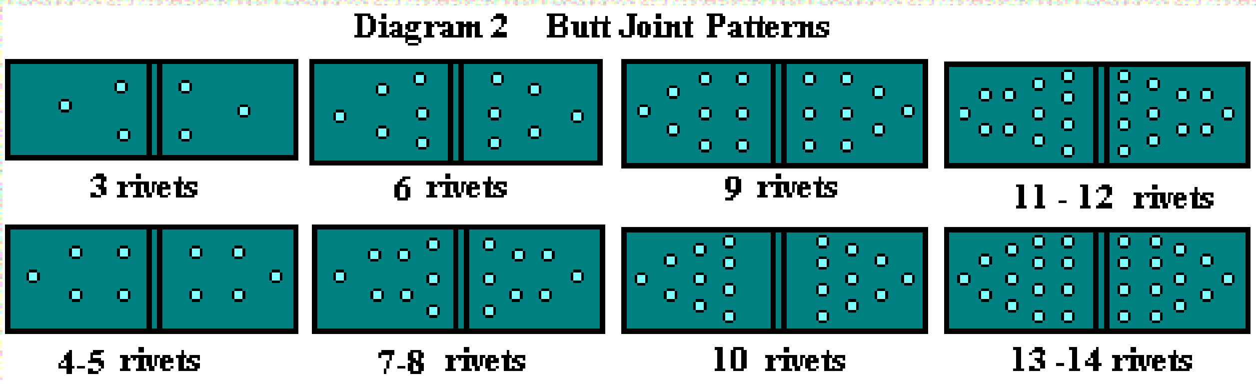
A butt joint (Diagram 1) is to connect two steel plates both with a width of 7 inches and a thickness of 3/4 inch. The rivets to be used have a diameter of 5/8 inch. The maximum allowable stresses for the rivet and weld materials are as follows:



Rivets: τ = 15, 000 lb/in^2, σ_t = 24, 000 lb/in^2, σ_c = 26, 000 lb/in^2

Plate: τ = 16,000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 24, 000 lb/in^2

A. Determine the number of rivets for the most efficient joint.
B. Select the best pattern from the patterns in Diagram 2 .
C. Calculate the strength and efficiency of the joint.



Solution: Part A

Step 1: Calculate the load which would cause the joint to fail in plate tearing at row 1 - assuming one rivet in row 1. We use the plate tearing relationship:

$\mathbf{P_{row1} = (w - n\ d)t}\ \sigma_{all}$. , where

w = width of the main plate = 7 inches
n = number of rivets in row (in row 1, 1 rivet)
d = diameter of rivet = 5/8 inch
t = thickness of the main plate = 3/4 inch
σ_{all} = Maximum allowable tensile stress for the plate material = 22, 000 lb/in^2,

P_{row1} = (7" - 1 * 5/8")*(3/4") * 22, 000 lb/in^2 = 105,200 lb.

Step 2. Calculate the maximum load which one rivet could carry in rivet shear. We use the rivet shear relationship:

$P_{rivet\ shear}$ = N (pi * d^2/4) τ_{all}. Where:

N = Number of areas in shear. For 1 rivet in a double cover plate butt joint this will be 2 area; so N = 2
d = 5/8 inch
τ_{all} = 15,000 lb/in^2

$P_{rivet\ shear}$ = 2 [3.1416 * (5/8")^2/4] * 15,000 lb/in^2 = 9200 lb./rivet

Step 3. Calculate the maximum load which one rivet (or the plate material behind one rivet) could carry in bearing (compression).

$P_{bearing}$ = N (d*t) σ_{all}. , where

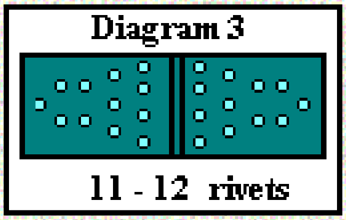
N = Number of rivets in compression = 1
d = Diameter of rivet = 5/8 inch
t = Thickness of the main plate = 3/4 inch
σ_{all} = Lowest allowable compressive stress of the rivet or plate material = 24, 000 lb/in^2

$P_{bearing}$ = 1 * [(5/8") * (3/4")] * 24, 000 lb/in^2 = 11,250 lb./rivet

Step 4. Divide the allowable load in plate tearing, calculated in step 1, by the smallest of the loads one rivet could carry from steps 2 and 3, and round up to determine the number of rivets which will result in the most efficient joint.
 # Rivets = 105, 200 lb. / 9200 lb./rivet = 11.43 ; rounding up # rivets = 12

Solution: Part B

We use the number of rivets found in Part A above to select the best joint pattern from the given set of patterns. From Diagram 1 above we select the a lap joint pattern with correct number of rivets. (See Diagram 3.)



Solution: Part C

Finally, using the selected joint pattern from part B, we calculate the strength and efficiency of the joint.

1.) Rivet Shear: We have already determined that one rivet can carry 9,200 lb./ rivet, so the total load the joint can carry in rivet shear will be the product of the number of rivets and the allowable load per rivet: **P_{shear} = 12 * 9200 lb/rivet = 110,400 lb.**

2.) Bearing Failure: Again, we have already calculated the allowable load per rivet in bearing = 9000 lb./rivet. The total load the joint can carry in bearing (compression) will be the product of the number of rivets and the allowable load per rivet: **$P_{bearing}$ = 12 * 11,250 lb./rivet = 135,000 lb.**

3.) Plate Tearing (row 1): We have already calculated plate tearing at row 1, in step 1 above. **P_{row1} = (7" - 1 * 5/8")*(3/4") * 22, 000 lb/in^2 = 105,200 lb.**

However, now that we have selected the rivet pattern, we also need to calculate plate tearing at row 2 (and perhaps row 3)

4.) Plate Tearing (row 2): Row 2 carries 11/12 of the load, so we write: **(11/12)P_{row2} = (7" - 2 * 5/8")*(3/4") * 22, 000 lb/in^2 = 94,875 lb., and then**

P_{row2} = (12/11) 94,875 lb. = 103,500 lb. Notice, that this loading is the lowest, up to this point, to cause failure, and so it is currently the strength of the joint. However, we will continue and check the load which causes row 3 to fail.

5.) Plate Tearing (row 3): Row 3 carries 9/12 of the load, so we write: **(9/12)P_{row3} = (7" - 2 * 5/8")*(3/4") * 22, 000 lb/in^2 = 94,875 lb., and then**

P_{row3} = (12/9) 94,875 lb. = 126,500 lb. This loading is much larger than the loading which will cause row 2 to fail, so we can stop here.

Considering all the modes of failure, we see: **Strength of the Joint is 103,500 lb**. (plate tearing row 2),and **Efficiency = Joint Strength / Plate Strength =**

$103{,}500$ lb./(7"*3/4")*22,000 lb/in^2 = .896 = 89.6%

This is the most efficient joint for the dimensions and allowable stresses of both the plates and rivets. By changing the size of the rivets (and the material and associated allowable stresses) we could arrive at a joint of greater efficiency. It is usually possible to arrive at a joint efficiency of 90% or greater. This type of problem is easily done using current computer spreadsheet methods.

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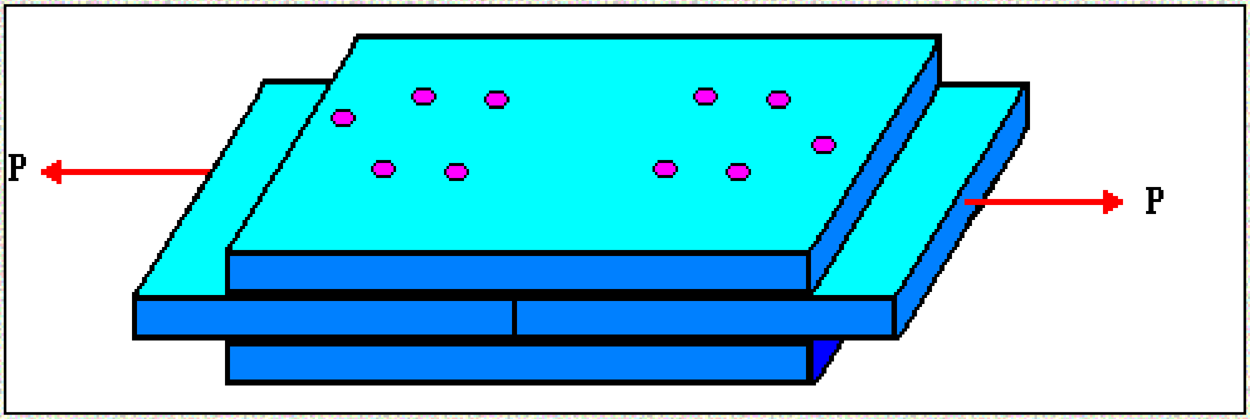
STATICS & STRENGTH OF MATERIALS - Examples

A riveted butt joint is shown below. The diameter of the rivets is 1 inch. The width of the plates is 6 inches, and the thickness of the plates is 1/2 inch. The allowable stresses are as follows:

Rivets: τ **= 15, 000 psi** σt **= 30, 000 psi** σc **= 35, 000 psi**

Plate: τ **= 12,000 psi** σt **= 25, 000 psi** σc **= 30, 000 psi**

A.) Calculate the strength of the joint.
B.) Calculate the efficiency of the joint.



Part A:

1). $P_{shear} = n(\pi d^2/4)\tau_{rivets} = 10(\pi*(1\text{ in})^2/4)(15{,}000\text{ psi}) = 117{,}800\text{ lb}$

2). $P_{bearing} = n(d*t)\sigma_C = 5(1\text{ in}*0.5\text{ in})(30{,}000\text{ psi}) = 75{,}000\text{ lb}$

3). $P_{plate,row\ 1} = (w - nd)t*\sigma_T = (6\text{ in} - (1*1\text{ in}))(0.5\text{ in})(25{,}000\text{ psi}) = 62{,}500\text{ lb}$

$(4/5)\ P_{plate,row\ 2} = (w - nd)t*\sigma_T = (6\text{ in} - (2*1\text{ in}))(0.5\text{ in})(25{,}000\text{ psi}) = 50{,}000\text{ lb}$

$P_{plate,row\ 2} = (5/4)(50{,}000\text{ lb}) = 62{,}500\text{ lb}$

Then strength of joint equals lowest load causing failure = 62,500 lb

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Part B:

efficiency = strength of joint / strength of solid plate = strength / $(w*t)\sigma_{T,plate}$ = 62,500lb / (6in*0.5in)(25,000 psi) = .833

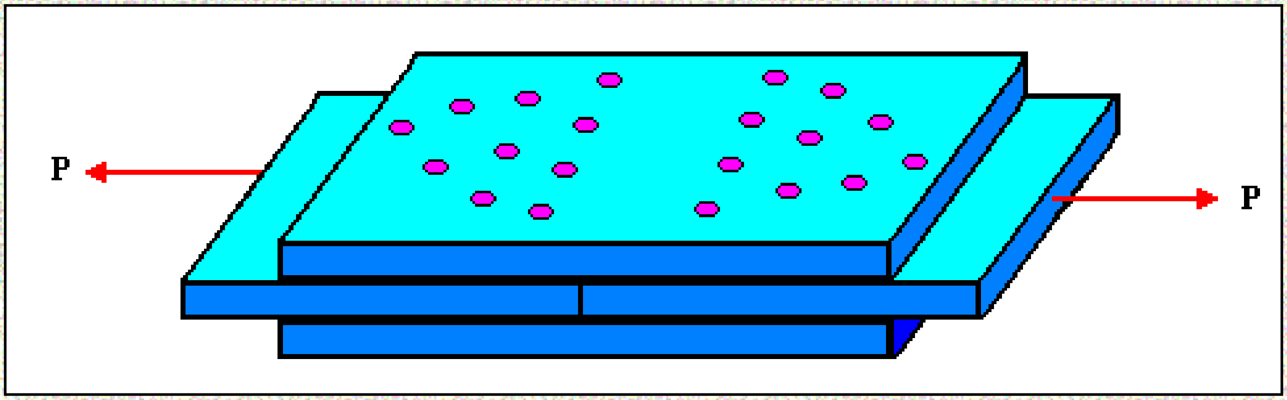
STATICS & STRENGTH OF MATERIALS - Examples

A riveted butt joint is shown below. The diameter of the rivets is 3/4 inch. The width of the plates is 7 inches, and the thickness of the plates is 3/4 inch. The allowable stresses are as follows:

Rivets: τ **= 15, 000 psi** σt **= 32, 000 psi** σc **= 34, 000 psi**

Plate: τ **= 14,000 psi** σt **= 28, 000 psi** σc **= 30, 000 psi**

A.) Calculate the strength of the joint.
B.) Calculate the efficiency of the joint.



Part A:

1). $P_{shear} = n(\pi d^2/4)\tau_{rivets} = 20(\pi*(0.75\text{ in})^2/4)(15{,}000\text{ psi}) = 132{,}550\text{ lb}$

2). $P_{bearing} = n(d*t)\sigma_C = 10(0.75\text{ in}*0.75\text{ in})(30{,}000\text{ psi}) = 168{,}750\text{ lb}$

3). $P_{plate,row\ 1} = (w - nd)t*\sigma_T = (7\text{ in} - (1*0.75\text{ in}))(0.75\text{ in})(28{,}000\text{ psi}) =$

131,250 lb

$(9/10)\ P_{plate,row\ 2} = (w - nd)t*\sigma_T = (7\text{ in} - (2*0.75\text{ in}))(0.75\text{ in})(28{,}000\text{ psi}) =$

115,500 lb

$P_{plate,row\ 2} = (10/9)(115{,}500\text{ lb}) = 128{,}350\text{ lb}$

Then strength of joint equals lowest load causing failure = 128,350 lb

Part B:

efficiency = strength of joint / strength of solid plate = strength / $(w*t)\sigma_T$,

plate = 128,350 lb/ (7in*0.75in)(28,000 psi) = .873

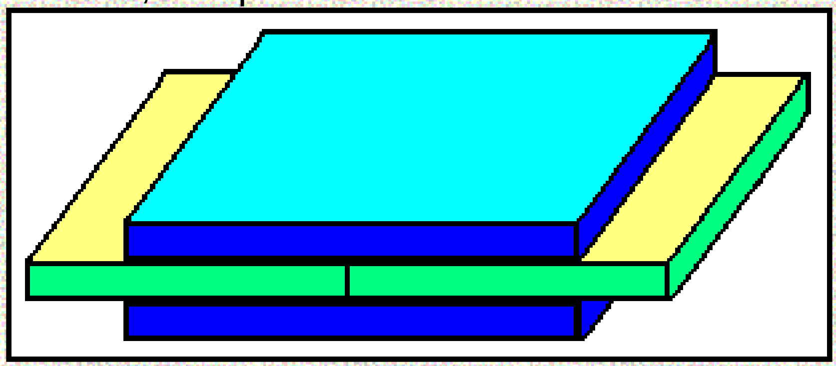
STATICS & STRENGTH OF MATERIALS - Examples

Two steel plates are to be connected using a double cover plate butt joint as shown below. The main plates have a width of 8 inches and a thickness of 1 inch. The rivets to be used have a diameter of 5/8 inch. The allowable stress are:

Rivet: τ = 12,000 psi σt = 24,000 psi σc = 28,000 psi

Plate: τ =15,000 psi σt = 26,000 psi

σc = 30,000 psi



A.) Determine the number of rivets which will result in a joint of maximum efficiency, and select the best pattern to use.

B.) Determine the strength and efficiency of the joint of maximum efficiency, using the selected pattern.

Part A:

1). $P_{plate,row\ 1} = (w - nd)t^*\sigma_T$ = (8 in - (1* 0.625 in))(1 in)(26,000 psi) =

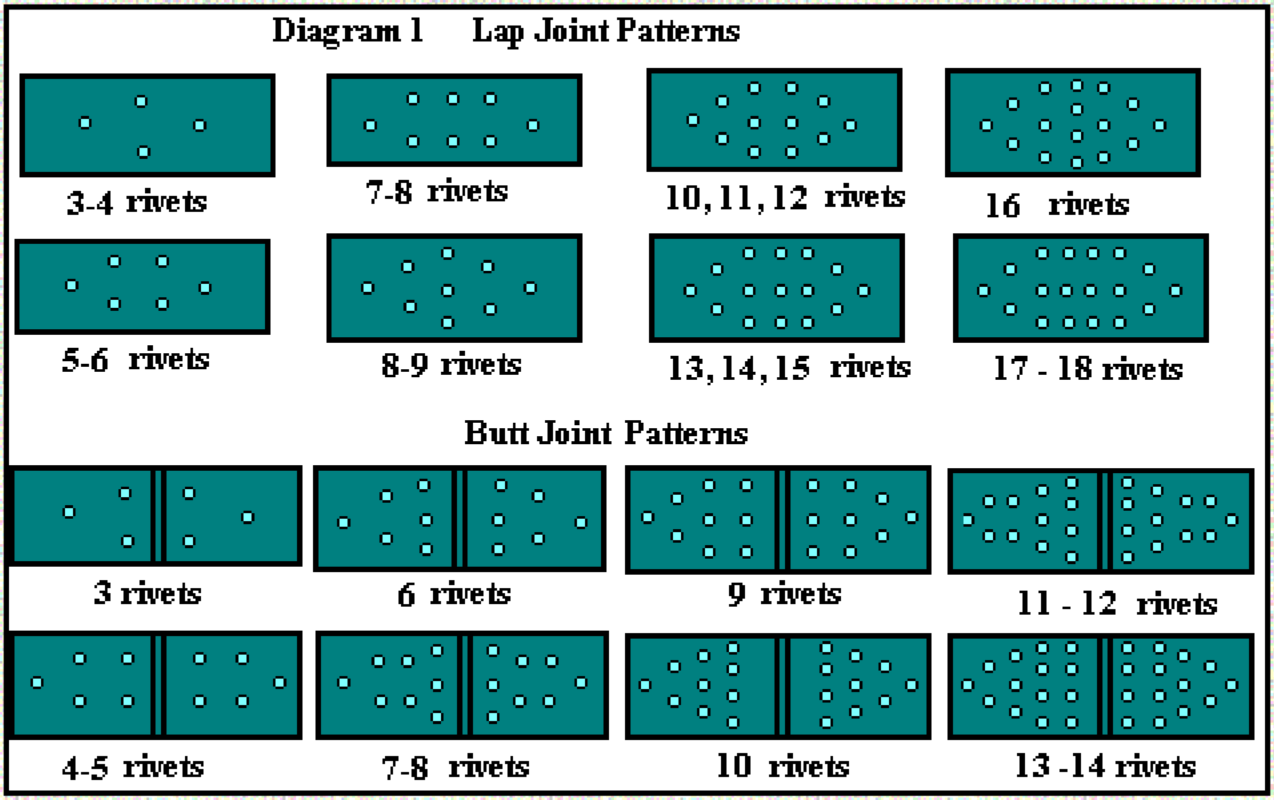
191,750 lb

2). $P_{shear,\ 1\ rivet} = n(\pi d^2/4)\tau_{shear}$ = 2(π*(0.625 in)^2/4)(12,000 psi) = 7,365 lb/rivet

3). $P_{bearing} = n(d^*t)\sigma_C$ = 1(0.625 in* 1 in)(28,000 psi) = 17,500 lb

4). # of Rivets = Plate strength, row 1 / (smallest) load/rivet = 191,750 lb / 7,365 lb/rivet = 26.04 ≅ 27 Rivets.

In this case no shown pattern is large enough. In checking strength of joint in plate tearing, we will use one rivet in row 1, two rivets in row 2, and three rivets in row three.



Part B:

Strength of joint will be Plate Tearing at Row 1 = 191,750 lbs; Unless Row 2 is lower; So check Row 2.

Row 2: (26/27) $P_{plate,\ row\ 2} = (w - nd)t^{*}\sigma_T$ = (8 in - (2* 0.625 in))(1 in)(26,000 psi) = 175,500 lb

$P_{plate,row\ 2}$ = (27/26)(175,500 lb) = 182,250 lb (Since row 2 is lower than row 1, also check row 3.)

Row 3: (24/27) $P_{plate,\ row\ 2} = (w - nd)t^{*}\sigma_T$ = (8 in - (3* 0.625 in))(1 in)(26,000 psi) = 159,250 lb

$P_{plate,row\ 3}$ = (27/24)(159,250 lb) = 179,160 lb

So Strength of joint is plate Tearing Row 2 = 179,160 lbs.

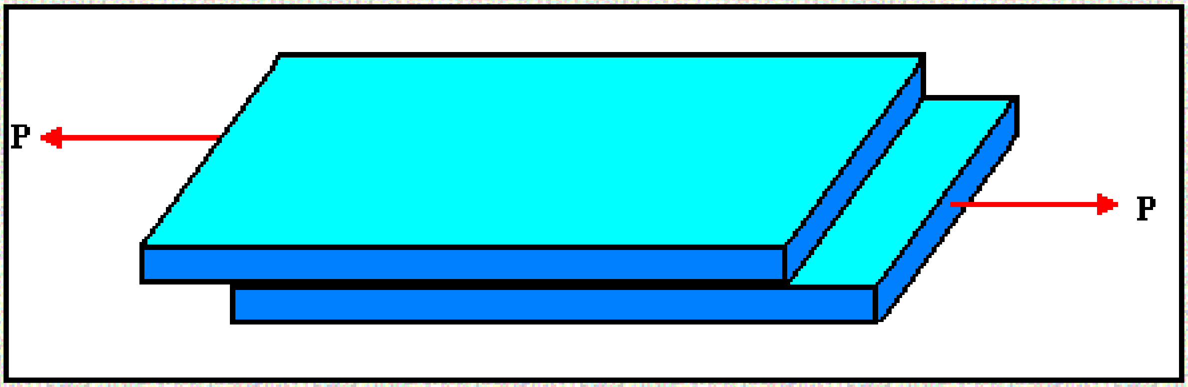
efficiency = strength of joint / strength of solid plate = strength / $(w^{*}t)\sigma_{T,\ plate}$ = 179,160 lb / (8in*1in)(26,000 psi) = .861

STATICS & STRENGTH OF MATERIALS - Examples

Two steel plates are to be connected by a lap joint as shown below. The main plates have a width of 12 inches and a thickness of 3/4 inch. The rivets to be used have a diameter of 1 1/4 inch. Allowable stress are:

Rivet : τ = 14,000 psi σt = 26,000 psi σc = 30,000 psi

Plate: τ =12,000 psi σt = 22,000 psi σc = 26,000 psi



A.) Determine the number of rivets which will result in a joint of maximum efficiency and select the best pattern to use.
B.) Determine the strength and efficiency of the joint of maximum efficiency using the selected pattern.

Part A:

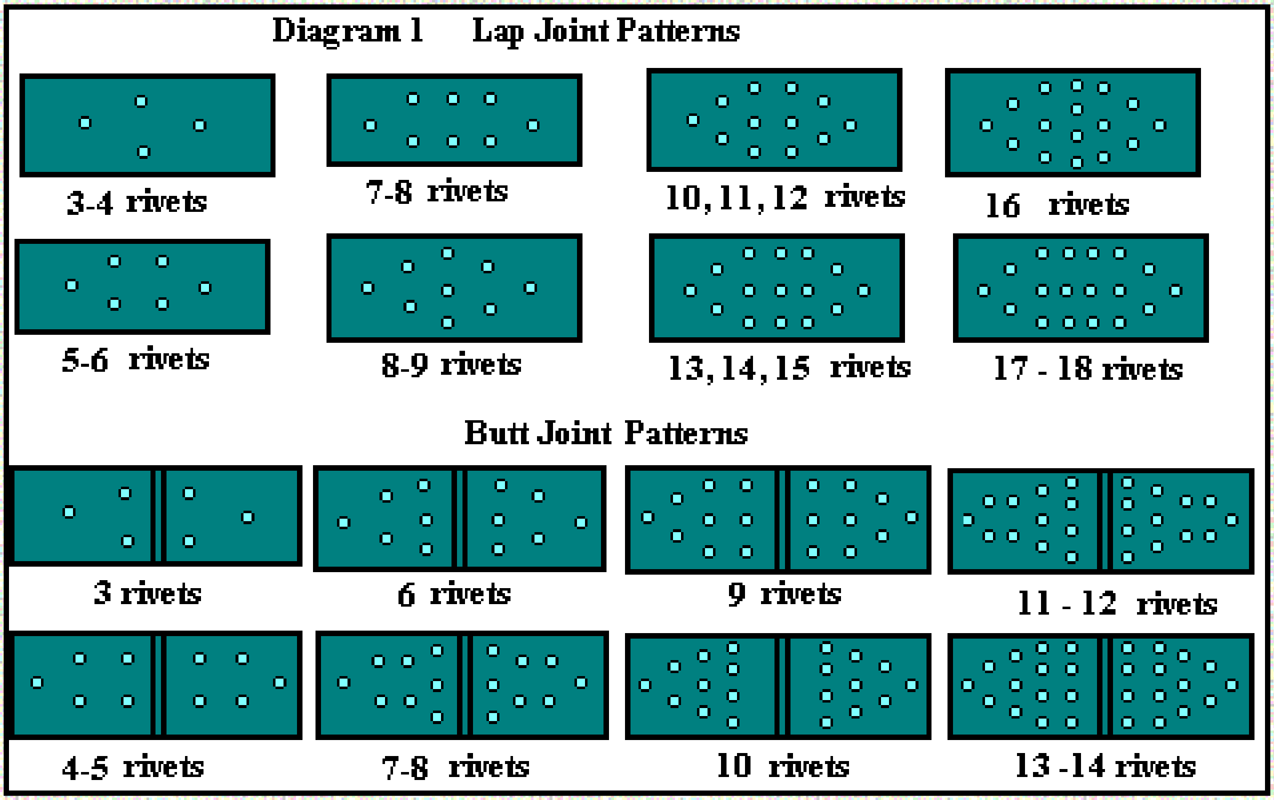
1). $P_{plate,row\ 1} = (w - nd)t*\sigma_T = (12\text{ in} - (1*\ 1.25\text{ in}))(0.75\text{ in})(22{,}000\text{ psi}) =$

177,375 lb

2). $P_{shear,\ 1\ rivet} = n(\pi d^2/4)\tau_{shear} = 1(\pi*(1.25\text{ in})^2/4)(14{,}000\text{ psi}) = 17{,}180$ lb/rivet

3). $P_{bearing} = n(d*t)\sigma_C = 1(1.25\text{ in}*\ 0.75\text{ in})(26{,}000\text{ psi}) = 24{,}375$ lb/rivet

4). # of Rivets = Plate strength, row 1 / (smallest) load/rivet = 177,375 lb / 17,180 lb/rivet = 10.32 ≅ 11 Rivets



Part B:

Strength of joint will be Plate Tearing at Row 1 = 177,375 lbs; Unless Row 2 is lower; So check Row 2.

(11/12) $P_{plate,\ row\ 2} = (w - nd)t^*\sigma_T$ = (12 in - (2* 1 in))(0.75 in)(22,000 psi) = 165,000 lb

$P_{plate,\ row\ 2}$ = (12/11)(165,000 lb) = 180,000 lb

So Strength of joint is plate Tearing Row 1 = 177,375 lbs.

efficiency = strength of joint / strength of solid plate = strength / $(w^*t)\sigma_{T,\ plate}$ = 177,375 lb / (12in*0.75in)(22,000 psi) = .896

STATICS & STRENGTH OF MATERIALS - Examples

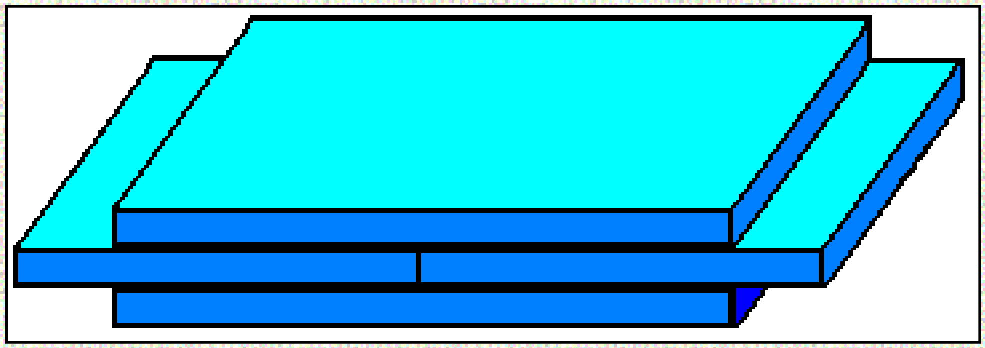
Two steel plates are to be connected using a double cover plate butt joint as shown below. The main plates have a width of 10 inches and a thickness of 3/4 inch. The rivets to be used have a diameter of 7/8 inch. The allowable stress are:

Rivet: τ = 16,000 psi σt = 24,000 psi σc = 30,000 psi

Plate: τ =14,000 psi σt = 20,000 psi σc = 26,000 psi

A.) Determine the number of rivets which will result in a joint of maximum efficiency, and select the best pattern to use.

B.) Determine the strength and efficiency of the joint of maximum efficiency using the selected pattern .



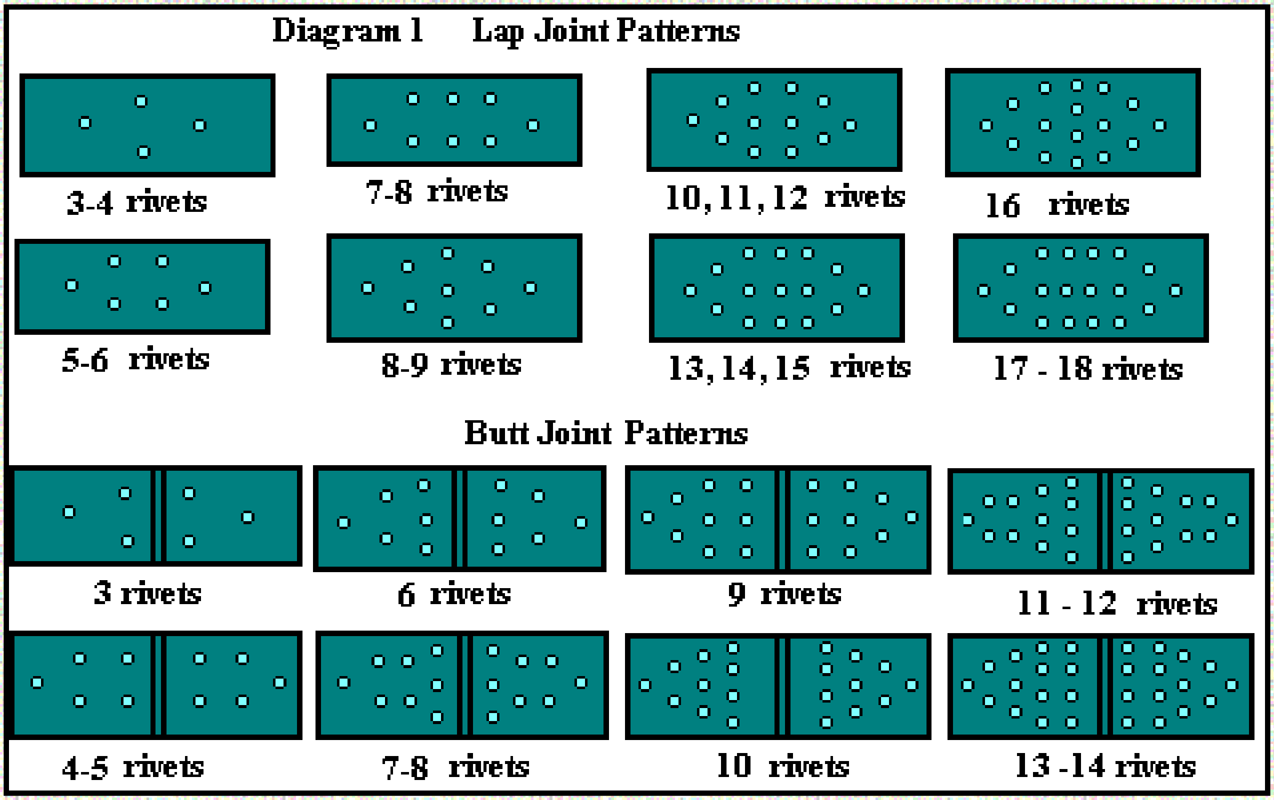
Part A:

1). $P_{plate,row\ 1}$ = (w - nd)t*σ_T = (10 in - (1* 0.875 in))(0.75 in)(20,000 psi) = 136,875 lb

2). $P_{shear,\ 1\ rivet}$ = n($\pi d^2/4$)τ_{shear} = 2(π*(0.875 in)^2/4)(16,000 psi) = 19,250 lb/rivet

3). $P_{bearing}$ = n(d*t)σ_C = 1(0.875 in* 0.75 in)(26,000 psi) = 17,050 lb

4). # of Rivets = Plate strength, row 1 / (smallest) load/rivet = 136,875 lb / 17,050 lb/rivet = 8.03 $\cong$ 9 Rivets.



Part B:

Strength of joint will be Plate Tearing at Row 1 = 136,875 lbs; Unless Row 2 is lower; So check Row 2.

(8/9) $P_{plate,\ row\ 2} = (w - nd)t*\sigma_T$ = (10 in - (2* 0.875 in))(0.75 in)(20,000 psi) =

123,750 lb

$P_{plate,row\ 2}$ = (9/8)(123,750 lb) = 139,220 lb

So Strength of joint is plate Tearing Row 1 = 136,875 lbs.

efficiency = strength of joint / strength of solid plate = strength / $(w*t)\sigma_{T,\ plate}$ = 136,875 lb / (10in*0.75in)(20,000 psi) = .913

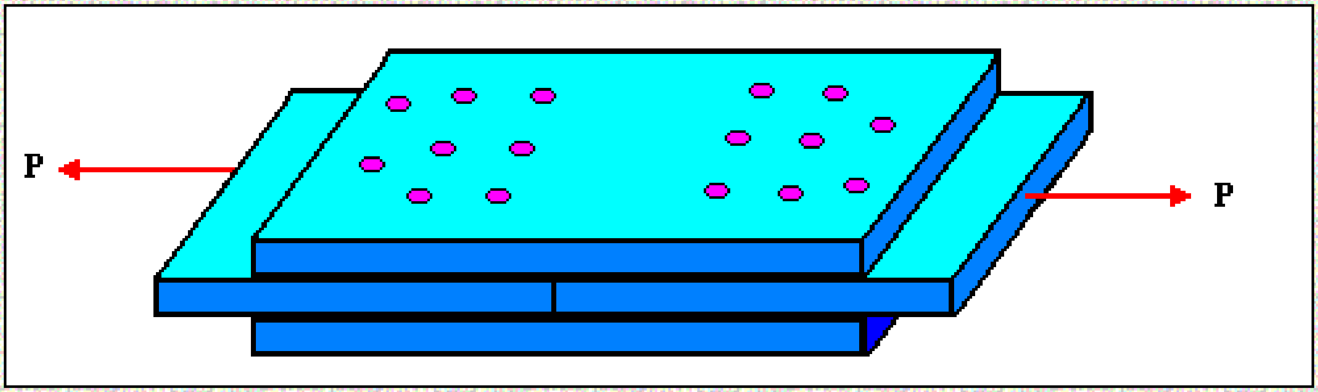
STATICS & STRENGTH OF MATERIALS - Examples

A riveted butt joint is shown below. The diameter of the rivets is 7/8 inch. The width of the plates is 9 inches, and the thickness of the plates is 3/4 inch. The allowable stresses are as follows:

Rivets: τ **= 15, 000 psi** σt **= 32, 000 psi** σc **= 34, 000 psi**

Plate: τ **= 14,000 psi** σt **= 28, 000 psi** σc **= 30, 000 psi**

A.) Calculate the strength of the joint.
B.) Calculate the efficiency of the joint.



Part A:

1). $P_{shear} = n(\pi d^2/4)\tau_{rivets} = 16(\pi*(0.875\text{ in})^2/4)(15{,}000\text{ psi}) = 144{,}300\text{ lb}$

2). $P_{bearing} = n(d*t)\sigma_C = 8(0.875\text{ in}*0.75\text{ in})(30{,}000\text{ psi}) = 157{,}500\text{ lb}$

3). $P_{plate,row\ 1} = (w - nd)t*\sigma_T = (9\text{ in} - (2*0.875\text{ in}))(0.75\text{ in})(28{,}000\text{ psi}) =$ 152,250 lb

$(6/8)\ P_{plate,row\ 2} = (w - nd)t*\sigma_T = (9\text{ in} - (3*0.875\text{ in}))(0.75\text{ in})(28{,}000\text{ psi}) =$ 133,875 lb

$P_{plate,row\ 2} = (8/6)(133{,}875\text{ lb}) = 178{,}500\text{ lb}$

Then strength of joint equals lowest load causing failure = 144,300 lb

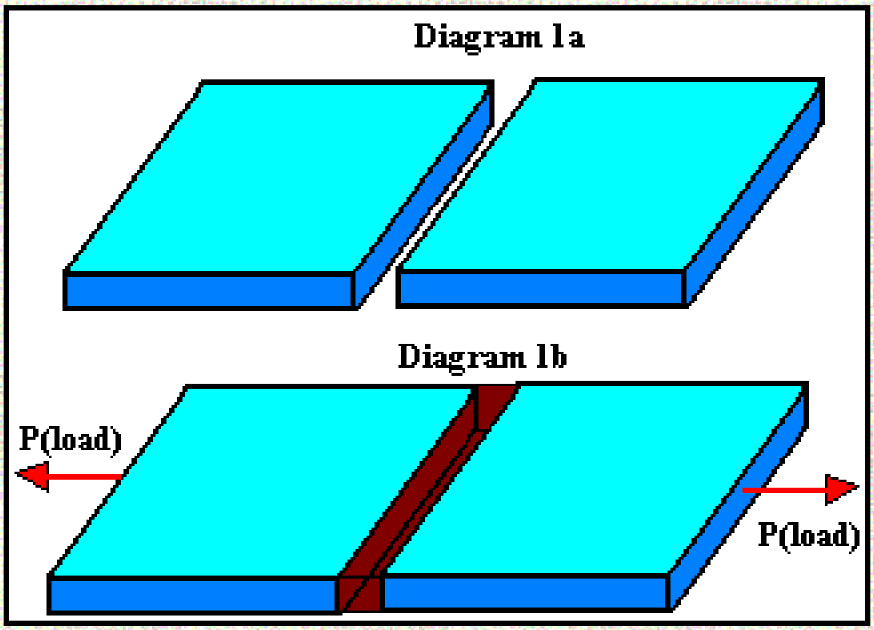
Part B:

efficiency = strength of joint / strength of solid plate = strength / $(w*t)\sigma_T$,

plate = 144,300 lb / (9in*0.75in)(28,000 psi) = .763

Topic 6.7: Rivets & Welds - Welded Joints

There are a number of types of welded joints. We will consider several common types in this discussion. One type of welded joint is the **Butt Joint**, where two plates are brought together with a small gap separating them, as shown in Diagram 1a.



The gap is then filled with weld material as from an arc welder, and the plates are welded together. Depending on the size and thickness of the plates, the end edges to be welded together may by sloped or vee-ed to generate a better weld. In theory, a butt weld can be made as strong or stronger than the plate, depending on the weld material used. In practice, the strength of a weld depends on how well the weld is made, whether or not there are voids or cracks in the weld. In some situations the weld may be x-rayed to determine its quality. For a welded butt joint loaded in tension as shown in Diagram 1b, the load, P, the weld can carry in tension is simply the product of the cross sectional area of the weld and the allowable tensile stress for the weld material, or:

$\mathbf{P_{weld} = (w * t) * \sigma_{tension}}$, where w and t are the width and thickness of the weld (plate).

A second type of weld which we will consider is the **Lap Joint using a 45 degree fillet weld**, shown in Diagrams 2, 3, and 4. In an example of this type of weld a top plate is welded to a bottom plate. The fillet weld runs from the top edge of the top plate to an equal distance horizontally outward on the top of the bottom plate, forming a triangular weld as shown in the cross sectional view in Diagram 2a. Since the two sides of the right triangle formed by the weld are equal forming a

45º right triangle, the weld is known as a 45º fillet weld. The perpendicular line bisecting the 90º angle and intersecting the hypotenuse of the 45º fillet weld is know as the throat of the weld, shown in Diagram 2a. When the weld fails, it is assumed to fail in shear across the throat region. To determine the load a weld can carry before failing we take the product of the area of the weld which fails in shear (the throat distance times the length of weld, Diagram 2b) and the allowable shear stress for the weld material.

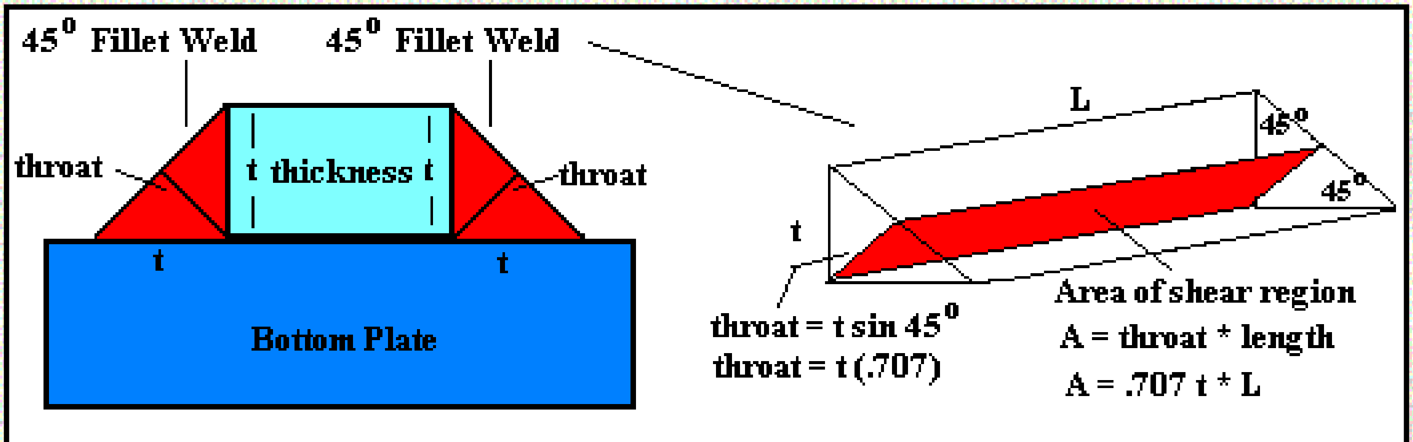


Diagram 2a Diagram 2b

We can write:

P_{weld} = Area* τ_{all} = (throat * length) * τ_{all} = (t sin 45o * L) * τ_{all}, and finally

P_{weld} = (.707 t * L) * τ_{all}, where

t = thickness of plate (and height & base of weld)
L = length of weld.
τ_{all} = allowable shear stress for the weld material

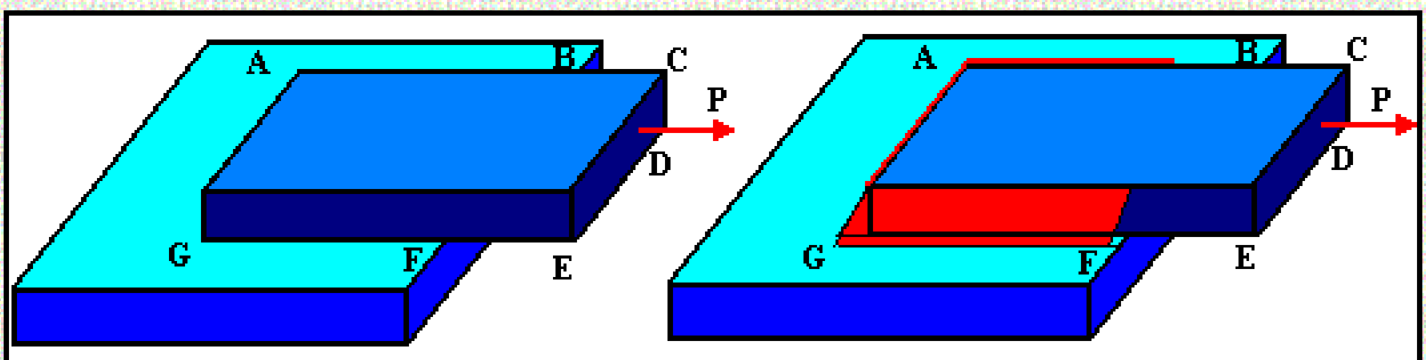


Diagram 3a Diagram 3b

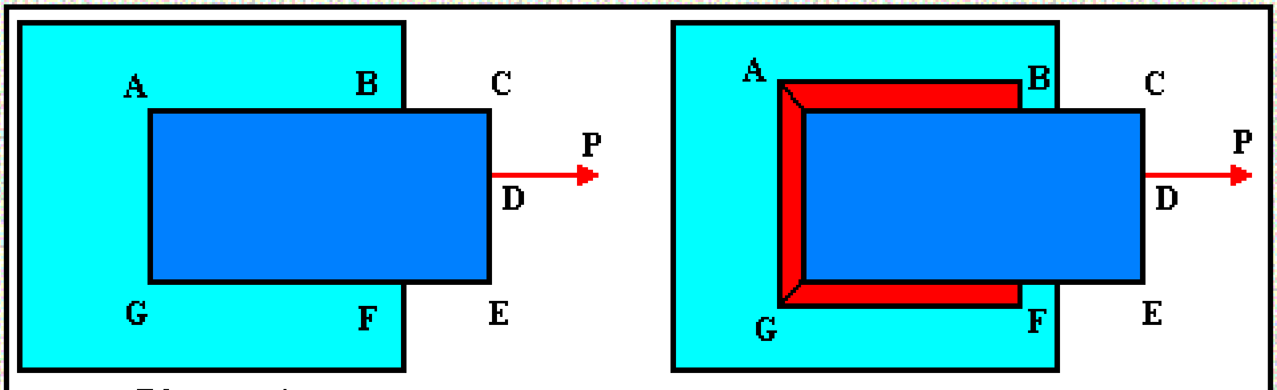


Diagram 4a Diagram 4b

Welded Joint Example

Two steel plates are shown in Diagram 5. The top plate is 3/4 inch thick and 8 inches wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be welded along sides AB and FG. We would first like to determine the minimum inches of weld need to carry the 80,000 lb. load and then to decide how the weld should be distributed along sides AB and FG.(There is only one way for the minimum inches of weld.) Notice also that the load is not applied symmetrically, that is, it is apply closer to one edge of the top plate than the other.

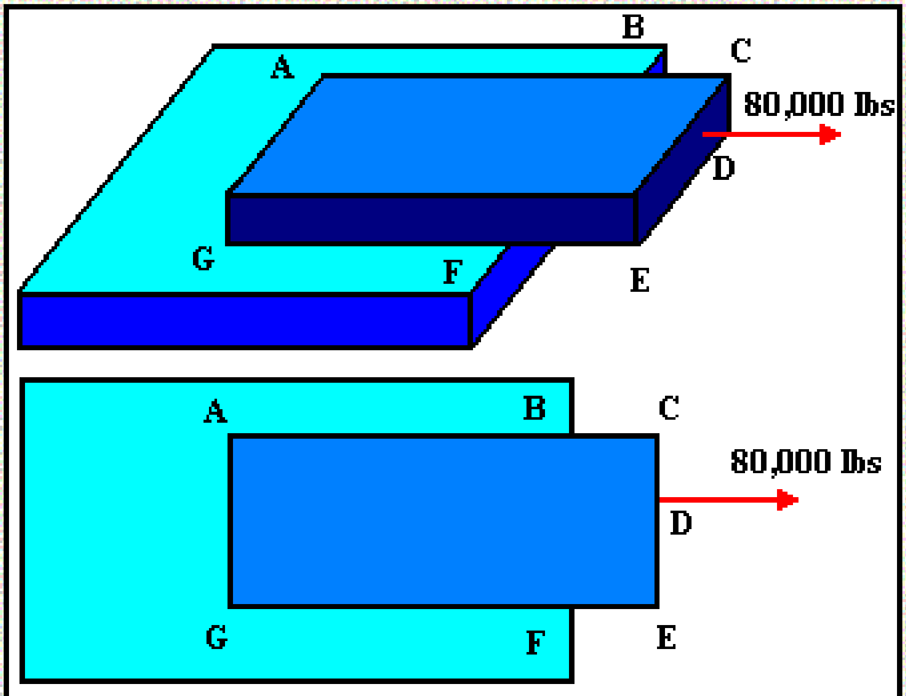


Diagram 5

Following that, we will then determine the minimum number of inches of weld needed to make the weld strength as great as the plate strength.
The allowable stresses are as follows;
Weld Material: τ = 14,000 lb/in^2 ; σ_t = 24,000 lb/in^2

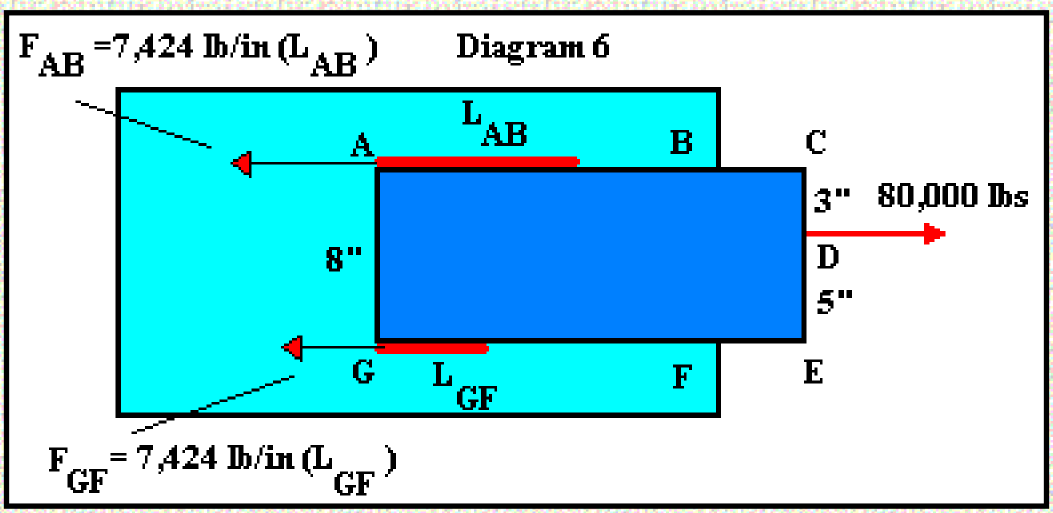
Plate Material: τ = 15,000 lb/in^2 ; σ_t = 30,000 lb/in^2
Dimensions: AB = GF = 20 inches; CD = 3 inches; DE = 5 inches

Solution:
Part 1. We first determine the minimum inches of weld needed to carry the 80,000 lb. load by using the weld formula and setting the load the weld can carry before failing to 80,000 lb..

$$P_{weld} = (.707\ t * L) * \tau_{all}\text{, or}$$

80,000 lb = (.707 * 3/4" * L) * 14,000 lb/in^2 = (7,424 lb./in.)L; then solving for L
L = 80,000 lb./(7,424 lb./in.) = 10.78 inches.
This is the minimum inches of weld needed to carry the load, however since the 80,000 pound load is not applied symmetrically to the plate, we can not apply the weld symmetrically. That is, if we try to put equal amounts of weld on sides AB and GF, the weld will fail in this case. To determine how the weld should be distributed, we once again apply static equilibrium conditions.

Part 2. In Diagram 6 we have shown the weld distributed with an amount L_{AB} on side AB and an amount L_{GF} on side GF. The lengths L_{AB} and L_{GF} must, of course sum to 10.78 inches (the minimum inches of weld needed). The maximum force these weld can resist with is given by F_{AB} = (7,424 lb./in.)* L_{AB} , and F_{GF} = (7,424 lb./in.)* L_{GF} , as shown in Diagram 6.



We now apply static equilibrium conditions to the top plate:
Sum of Forces: 80,000 lb. - (7,424 lb./in.)* L_{AB} - (7,424 lb./in.)* L_{GF} = 0
Sum of Torque $_{\text{about G}}$: -80,000 lb.* (5") + (7,424 lb./in.)* L_{AB} * (8") = 0

(Notice that the force F_{GF} does not produce a torque about point G, since its line of action passes through point G.)

Solving the torque equation for **L_{AB} = 6.73 inches**, we can then solve for L_{GF} since L_{AB} and L_{GF} sum to 10.78 inches. Therefore **L_{GF} = 10.78" - 6.73" = 4.05"**
This is how the weld must be distributed in order to carry the 80,000 lb. load (and satisfy static equilibrium conditions).

Part 3. The other question of interest in welded joints is exactly how much weld is needed to make the welded joint as strong as the plate itself. In this case we assume the plate is loaded in tension, and so the strength of the plate is P_{plate} = product of cross sectional area of plate and the allowable tensile stress for the plate material, or $P_{plate} = (w * t)\ \sigma_t$. We then set this equal to the strength of the riveted joint and solve for the length of weld needed.

$P_{weld} = (.707\ t * L) * \tau_{all} = P_{plate} = (w * t)\ \sigma_t$, or

$(.707\ t * L) * \tau_{all} = (w * t)\ \sigma_t$, and then

(.707 *3/4" * L) * 14,000 lb/in^2 = (8" * 3/4") 30,000 lb/in^2 , then solving for L

L = 24.25 inches

This is the minimum inches of weld needed to make the weld as strong as the plate (in tension). We can not state how the weld should be distributed, since we are not dealing with a specific loading.

Continue to: Topic 6.7a: Welded Joints - Example 1
or Select:
Topic 6: Torsion, Rivets & Welds - Table of Contents
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Topic 6.7a: Welded Joints - Example 1

Two steel plates are shown in Diagram 1. The top plate is .5 inch thick and 10 inches wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be welded completely across end AG and partially along sides AB and FG.

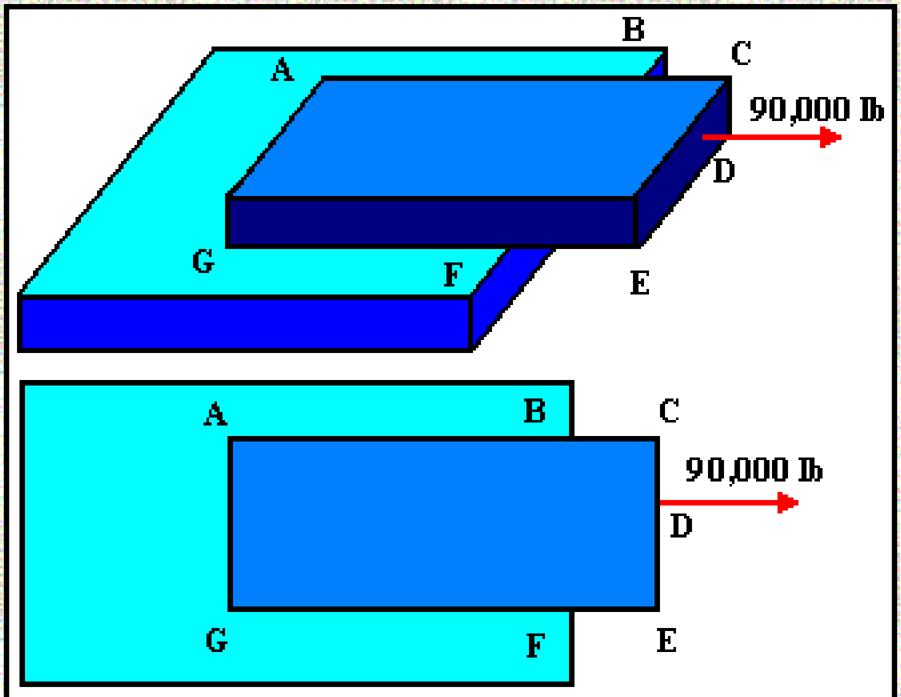


Diagram 1

A.) Determine the minimum inches of weld need to carry the 90,000 lb. load and specify how many inches of weld should be placed along each side AB and FG.
B.) Determine the number of inches of weld needed to make the weld strength as great as the plate strength.

The allowable stresses are as follows;
Weld Material: τ = 15,000 lb/in^2 ; σ**t** = 24, 000 lb/in^2
Plate Material: τ = 14,000 lb/in^2 ; σ**t** = 28,000 lb/in^2
Dimensions: AB = GF = 20 inches; CD = 4 inches; DE = 6 inches

Solution:

Part 1. We first determine the minimum inches of weld needed to carry the 90,000 lb. load by using the weld formula and setting the load the weld can carry before failing to 90,000 lb..

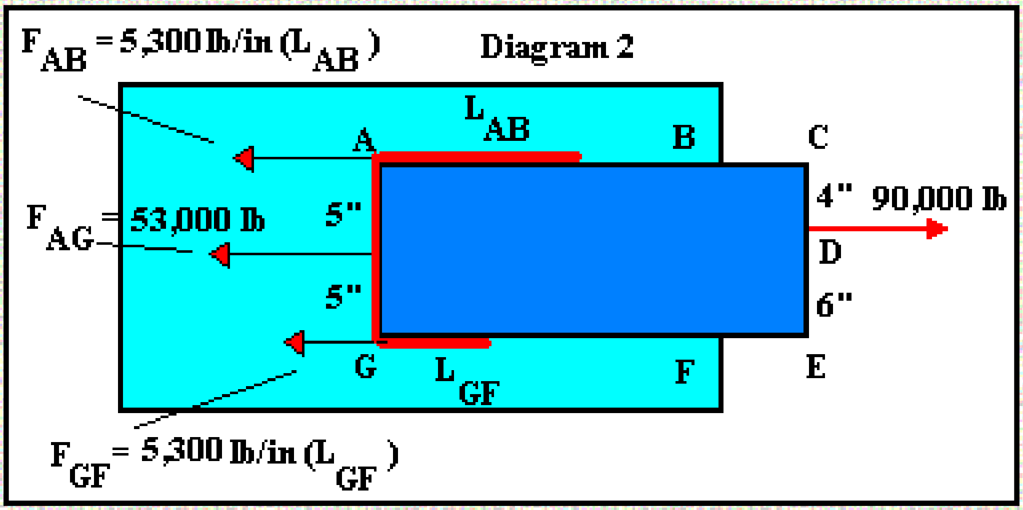
$$P_{weld} = (.707\ t * L) * \tau_{all}\text{, or}$$

90,000 lb = (.707 * .5" * L) * 15,000 lb/in^2 = (5,300 lb./in.)*L; then solving for L

L = 90,000 lb./(5,300 lb./in.) = 16.98 inches.

This is the minimum inches of weld needed to carry the load, however since the 90,000 pound load is not applied symmetrically to the plate, we can not apply the weld symmetrically. That is, if we try to put equal amounts of weld on sides AB and GF, the weld will fail in this case. To determine how the weld should be distributed, we once again apply static equilibrium conditions.

Part 2. In Diagram 2 we have shown the weld distributed with an amount L_{AB} on side AB, L_{GF} on side GF, and 10 inches of weld completely across the end AG. The lengths L_{AB} , L_{GF} , and the 10 inches of weld must sum to 16.98 inches (the minimum inches of weld needed). The maximum force these weld can resist with is given by F_{AB} = (5,300 lb./in.)* L_{AB} , F_{GF} = (5,300 lb./in.)* L_{GF} , and F_{AG} = 5,300 lb/in * 10" = 53,000 lb. as shown in Diagram 2.



We now apply static equilibrium conditions to the top plate:

Sum of Forces: 90,000 lb. - (5,300 lb./in.)* L_{AB} - (5,300 lb./in.)* L_{GF} - 53,000 lb. = 0

Sum of Torque $_{\text{about G}}$: -90,000 lb.* (6") + (5,300 lb./in.)* L_{AB} * (10") + 53,000 lb. * 5" = 0

(Notice that the force F_{GF} does not produce a torque about point G, since its line of action passes through point G.)

Solving the torque equation for **L_{AB} = 5.19 inches**, we can then solve for L_{GF} since L_{AB} + L_{GF} + 10" must sum to 16.98 inches. Therefore **L_{GF} = 16.98" - 5.19" - 10" = 1.79"** This is how the weld must be distributed in order to carry the 80,000 lb. load (and satisfy static equilibrium conditions).

Part 3.. We assume the plate is loaded in tension, and so the strength of the plate is P_{plate} = product of cross sectional area of plate and the allowable tensile stress for the plate material, or $P_{plate} = (w * t)\ \sigma_t$. We then set this equal to the strength of the riveted joint and solve for the length of weld needed.

$$P_{weld} = (.707\ t * L) * \tau_{all} = P_{plate} = (w * t)\ \sigma_t \text{, or}$$

$$(.707\ t * L) * \tau_{all} = (w * t)\ \sigma_t \text{, and then}$$

$$(.707 * .5" * L) * 15{,}000\ lb/in^2 = (10" * .5")\ 28{,}000\ lb/in^2$$, then solving for L

L = 26.4 inches

This is the minimum inches of weld needed to make the weld as strong as the plate (in tension). We can not state how the weld should be distributed, since we are not dealing with a specific loading.

Return to: Topic 6.7: Rivets & Welds - Welded Joints
Continue to: Topic 6.8: Rivets & Welds - Problem Assignment
or Select:
Topic 6: Torsion, Rivets & Welds - Table of Contents
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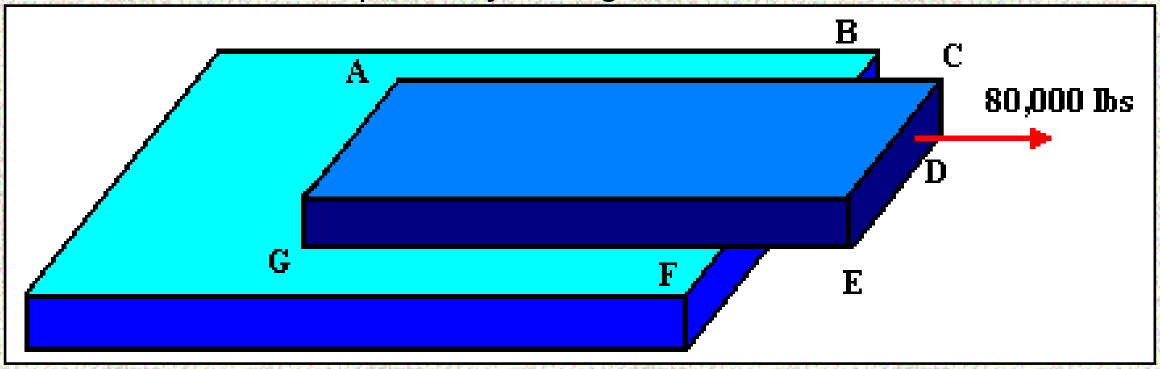
STATICS & STRENGTH OF MATERIALS - Example

Two steel plates are shown below. The top plate is .5 inches thick and 8 inches wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be weld completely across end AG and partially along sides AB and FG.



A.) Determine the minimum inches of weld need to carry the 80,000 lb load and specify how many inches of weld should be placed along sides AB and FG.

B.) Determine the number of inches of weld needed to make the weld strength as great as the plate strength.

The dimension and stresses are as follows; Allowable stresses :

Weld Material: τ=12,000psi, σt= 24,000 psi

Plate Material: τ = 15,000 psi, σt= 30,000 psi

Dimensions:

AB = GF = 20 inches:

CD = 3 inches: DE = 5 inches

Solution:

Part A.

Part I: We first determine the minimum inches of weld needed to carry the 80,000 lb. load by using the weld formula and setting the load the weld can carry before failing to 80,000 lb..

P_{weld} **= (.707 t * L) *** τ_{all} **, or**

80,000 lb = (.707 * .5" * L) * 12,000 lb/in^2 = (4,242 lb./in.)*L; then solving for L

L = 80,000 lb./(4,242 lb./in.) = 18.86 inches.

This is the minimum inches of weld needed to carry the load, however since the 80,000 pound load is not applied symmetrically to the plate, we can not apply the weld symmetrically. That is, if we try to put equal amounts of weld on sides AB and GF, the weld will fail in this case. To determine how the weld should be distributed, we once again apply static equilibrium conditions.

Part II.

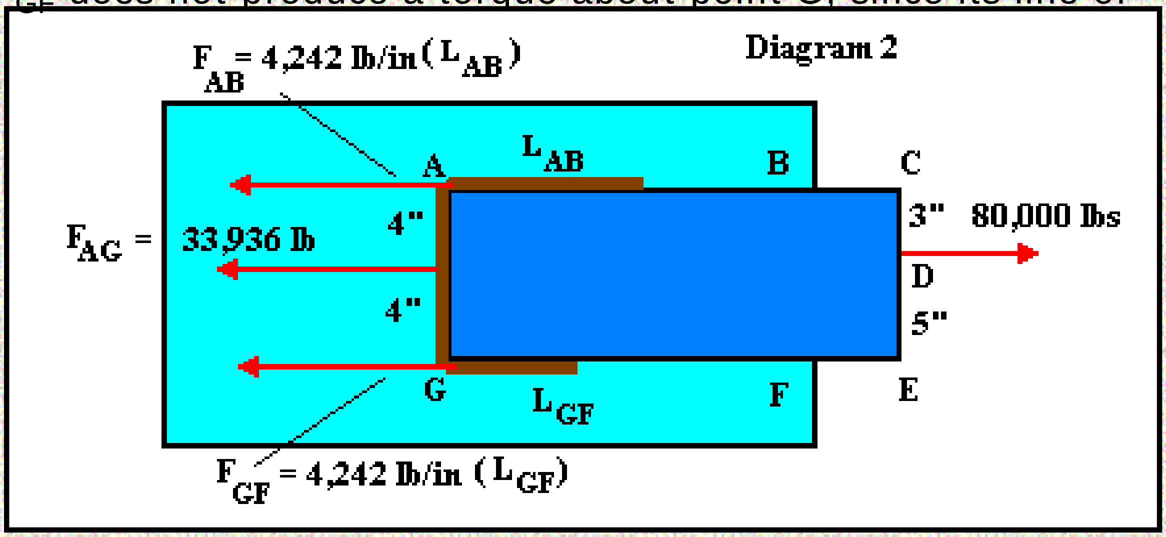
In Diagram 2 we have shown the weld distributed with an amount L_{AB} on side AB,

L_{GF} on side GF, and 8 inches of weld completely across the end AG.The lengths L_{AB} , L_{GF} ,and the 8 inches of weld must sum to 18.86 inches (the minimum inches of weld needed). The maximum force these weld can resist with is given by F_{AB} = (4,242 lb./in.)* L_{AB} , F_{GF} = (4,242 lb./in.)* L_{GF} , and F_{AG} = 4,242 lb/in * 8" = 33,936 lb. as shown in Diagram 2. We now apply static equilibrium conditions to the top plate:

Sum of Forces: 80,000 lb. - (4,242 lb./in.)* L_{AB} - (4,242 lb./in.)* L_{GF} - 33,936 lb. = 0

Sum of Torque $_{\text{about G}}$: -80,000 lb.* (5") + (4,242 lb./in.)* L_{AB} * (8") + 33,936 lb. * 4" = 0

(Notice that the force F_{GF} does not produce a torque about point G, since its line of action passes through point G.)



Solving the torque equation for **L_{AB} = 7.79 inches**, we can then solve for L_{GF} since L_{AB} + L_{GF} + 8" must sum to 18.86 inches. Therefore **L_{GF} = 18.86" - 7.79" - 8" = 3.07"** This is how the weld must be distributed in order to carry the 80,000 lb. load (and satisfy static equilibrium conditions).

Part B:

We assume the plate is loaded in tension, and so the strength of the plate is P_{plate} = product of cross sectional area of plate and the allowable tensile stress for the plate material, or P_{plate} = (w * t) σ_t . We then set this equal to the strength of the riveted joint and solve for the length of weld needed.

P_{weld} = (.707 t * L) * τ_{all} = P_{plate} =(w * t) σ_t , or

(.707 t * L) * τ_{all} = (w * t) σ_t , and then

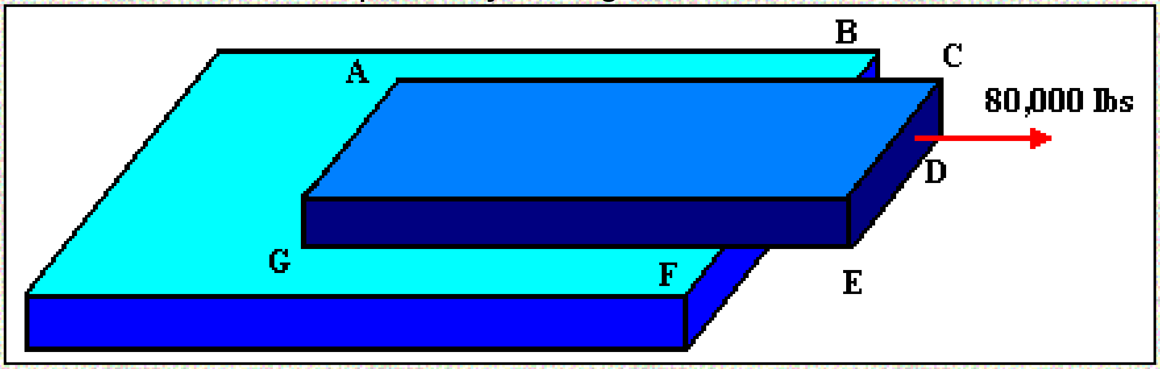
(.707 *.5" * L) * 12,000 lb/in^2 = (8" * .5") 30,000 lb/in^2 , then solving for L

L = 28.29 inches

This is the minimum inches of weld needed to make the weld as strong as the plate (in tension). We can not state how the weld should be distributed, since we are not dealing with a specific loading.

STATICS & STRENGTH OF MATERIALS - Example

Two steel plates are shown below. The top plate in .7 inches thick and 6 inches wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be weld completely across end AG and partially along sides AB and FG.



A.) Determine the minimum inches of weld need to carry the 80,000 lb load and specify how many inches of weld should be placed along each side AB and FG.

B.) Determine the number of inches of weld needed to make the weld strength as great as the plate strength.

The dimension and stresses are as follows; Allowable stresses :

Weld Material: τ = 16, 000 psi, σt= 30, 000 psi

Plate Material: τ = 14,000 psi, σt= 28,000 psi

Dimensions:

AB = GF = 30 inches:
CD = 2 inches: DE = 4 inches

Solution:

Part A.

Part I: We first determine the minimum inches of weld needed to carry the 80,000 lb. load by using the weld formula and setting the load the weld can carry before failing to 80,000 lb..

P_{weld} **= (.707 t * L) *** τ_{all} **, or**

80,000 lb = (.707 * .7" * L) * 16,000 lb/in^2 = (7,918 lb./in.)*L; then solving for L

L = 80,000 lb./(7,918 lb./in.) = 10.1 inches.

This is the minimum inches of weld needed to carry the load, however since the 80,000 pound load is not applied symmetrically to the plate, we can not apply the weld symmetrically. That is, if we try to put equal amounts of weld on sides AB and GF, the weld will fail in this case. To determine how the weld should be distributed, we once again apply static equilibrium conditions.

Part II.

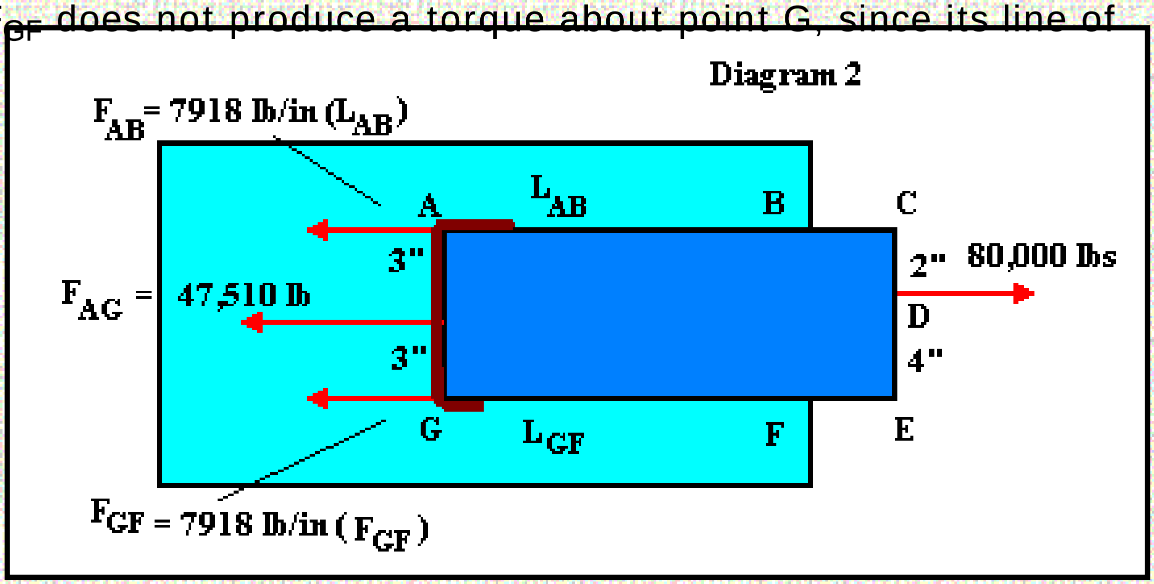
In Diagram 2 we have shown the weld distributed with an amount L_{AB} on side AB,

L_{GF} on side GF, and 6 inches of weld completely across the end AG.The lengths L_{AB} , L_{GF} ,and the 6 inches of weld must sum to 10.1 inches (the minimum inches of weld needed). The maximum force these weld can resist with is given by F_{AB} = (7,918 lb./in.)* L_{AB} , F_{GF} = (7,918 lb./in.)* L_{GF} , and F_{AG} = 7,918 lb/in * 6" = 47,510 lb. as shown in Diagram 2. We now apply static equilibrium conditions to the top plate:

Sum of Forces: 80,000 lb. - (7,918 lb./in.)* L_{AB} - (7,918 lb./in.)* L_{GF} - 47,510 lb. = 0

Sum of Torque $_{\text{about G}}$: -80,000 lb.* (4") + (7,918 lb./in.)* L_{AB} * (6") + 47,510 lb. * 3" = 0

(Notice that the force F_{GF} does not produce a torque about point G, since its line of action passes through point G.)



Solving the torque equation for **L_{AB} = 3.74 inches**, we can then solve for L_{GF} since L_{AB} + L_{GF} + 6" must sum to 10.1 inches. Therefore **L_{GF} = 10.1" - 3.74" - 6" = .36"** This is how the weld must be distributed in order to carry the 80,000 lb. load (and satisfy static equilibrium conditions).

Part B:

We assume the plate is loaded in tension, and so the strength of the plate is P_{plate} = product of cross sectional area of plate and the allowable tensile stress for the plate material, or P_{plate} = (w * t) σ**t** . We then set this equal to the strength of the riveted joint and solve for the length of weld needed.

P_{weld} = (.707 t * L) * τ_{all} = P_{plate} =(w * t) σt , or

(.707 t * L) * τ_{all} = (w * t) σt , and then

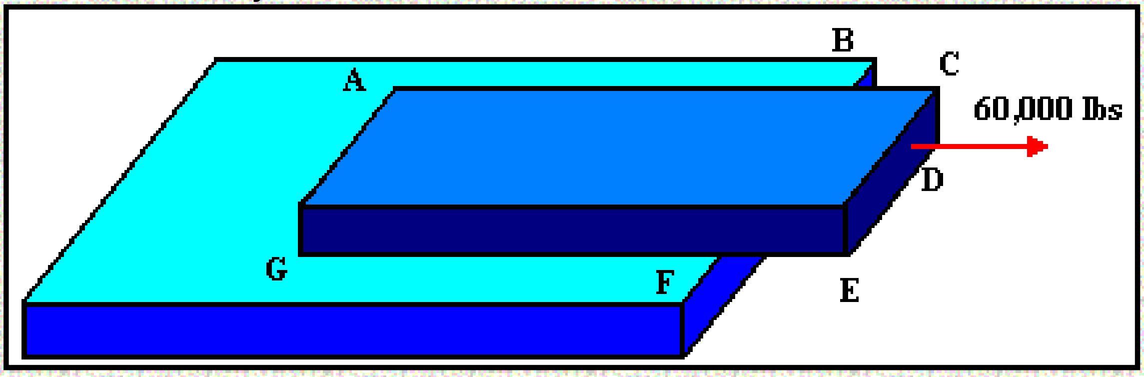
(.707 *.7" * L) * 16,000 lb/in^2 = (6" * .7") 28,000 lb/in^2 , then solving for L

L = 14.85 inches

This is the minimum inches of weld needed to make the weld as strong as the plate (in tension). We can not state how the weld should be distributed, since we are not dealing with a specific loading.

STATICS & STRENGTH OF MATERIALS - Example

Two steel plates are shown below. The top plate in .5 inches thick and 8 inches wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be weld along sides AB and FG only.



A.) Determine the minimum inches of weld need to carry the 60,000 lb load and specify how many inches of weld should be placed along each side AB and FG.

B.) Determine the number of inches of weld needed to make the weld strength as great as the plate strength.

The dimension and stresses are as follows; Allowable stresses :

Weld Material: τ = 15, 000 psi, σt= 26, 000 psi

Plate Material: τ = 14,000 psi, σt= 28,000 psi

Dimensions:

AB = GF = 30 inches: ; CD = 3 inches: ; DE = 5 inches

Solution:

Part A:

Part I. We first determine the minimum inches of weld needed to carry the 60,000 lb. load by using the weld formula and setting the load the weld can carry before failing to 60,000 lb..

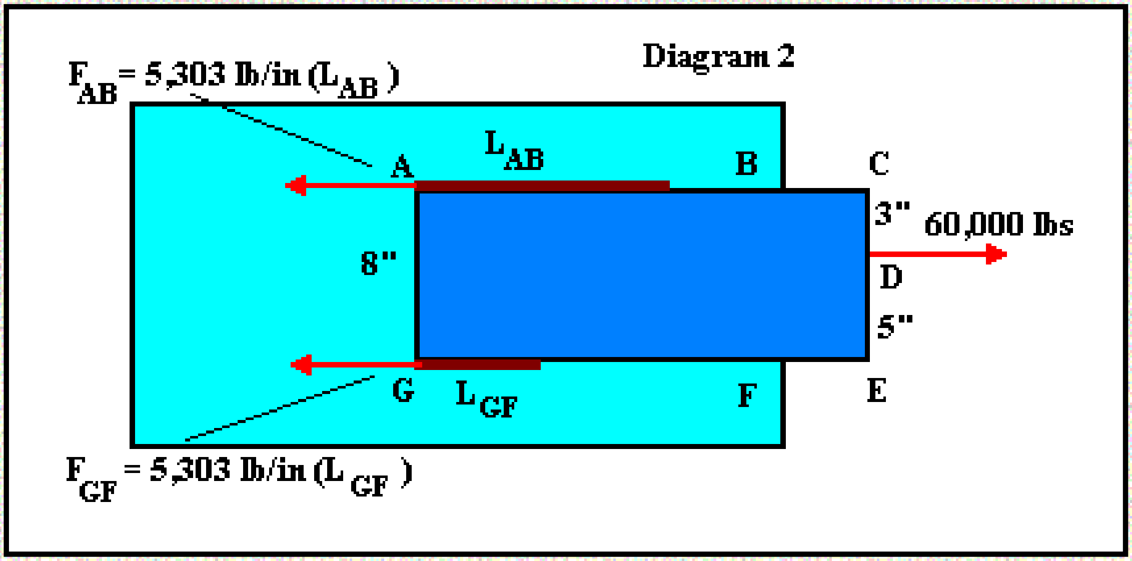
$$P_{weld} = (.707\ t * L) * \tau_{all}\text{ , or}$$

$$60{,}000\ lb = (.707 * .5" * L) * 15{,}000\ lb/in^2 = (5{,}303\ lb./in.)L;$$ then solving for L

$$L = 60{,}000\ lb./(5{,}303\ lb./in.) = 11.31\ inches.$$

This is the minimum inches of weld needed to carry the load, however since the 60,000 pound load is not applied symmetrically to the plate, we can not apply the weld symmetrically. That is, if we try to put equal amounts of weld on sides AB and GF, the weld will fail in this case. To determine how the weld should be distributed, we once again apply static equilibrium conditions.

Part II. In Diagram 2 we have shown the weld distributed with an amount L_{AB} on side AB and an amount L_{GF} on side GF. The lengths L_{AB} and L_{GF} must, of course sum to 11.31 inches (the minimum inches of weld needed). The maximum force these weld can resist with is given by F_{AB} = (5,303 lb./in.)* L_{AB} , and F_{GF} = (5,303 lb./in.)* L_{GF} , as shown in Diagram 2. We now apply static equilibrium conditions to the top plate:



Sum of Forces: 60,000 lb. - (5,303 lb./in.)* L_{AB} - (5,303 lb./in.)* L_{GF} = 0

Sum of Torque $_{\text{about G}}$: -60,000 lb.* (5") + (5,303 lb./in.)* L_{AB} * (8") = 0

(Notice that the force F_{GF} does not produce a torque about point G, since its line of action passes through point G.)

Solving the torque equation for **L_{AB} = 7.07 inches**, we can then solve for L_{GF} since L_{AB} and L_{GF} sum to 11.31 inches. Therefore **L_{GF} = 11.31" - 7.07" = 4.24"** This is how the weld must be distributed in order to carry the 60,000 lb. load (and satisfy static equilibrium conditions).

Part B.

The other question of interest in welded joints is exactly how much weld is needed to make the welded joint as strong as the plate itself. In this case we assume the plate is loaded in tension, and so the strength of the plate is P_{plate} = product of cross sectional area of plate and the allowable tensile stress for the plate material, or P_{plate} = (w * t) σ_t . We then set this equal to the strength of the riveted joint and solve for the length of weld needed.

P_{weld} = (.707 t * L) * τ_{all} = P_{plate} = (w * t) σ_t , or

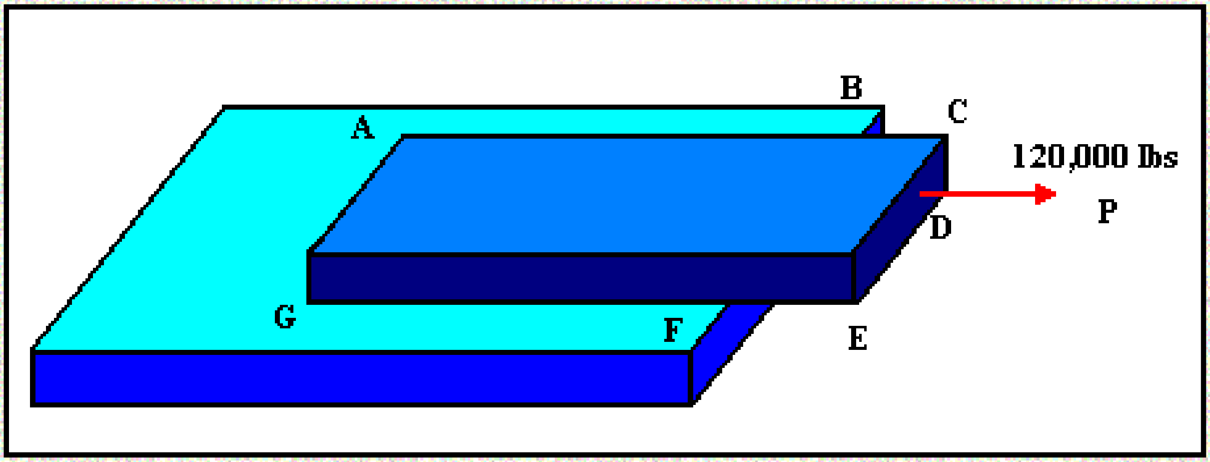
(.707 t * L) * τ_{all} = (w * t) σ_t , and then

(.707 * .5" * L) * 15,000 lb/in^2 = (8" * .5") 28,000 lb/in^2 , then solving for L
L = 21.12 inches

This is the minimum inches of weld needed to make the weld as strong as the plate (in tension). We can not state how the weld should be distributed, since we are not dealing with a specific loading

STATICS & STRENGTH OF MATERIALS - Example

Two steel plates are shown below. The top plate in .7 inches thick and 9 inches wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be welded along sides AB and FG.



A.) Determine the minimum inches of weld need to carry a Load P of 120,000 lb load and specify how many inches of weld should be placed along each side AB and FG.

B.) Determine the number of inches of weld needed to make the weld strength as great as the plate strength.

The dimension and stresses are as follows; Allowable stresses :

Weld Material: τ = 14, 000 psi, σt= 22, 000 psi

Plate Material: τ = 15,000 psi, σt= 35,000 psi

Dimensions:

AB = GF = 30 inches; CD = 3 inches; DE = 6 inches

Solution:

Part A:

Part I. We first determine the minimum inches of weld needed to carry the 120,000 lb. load by using the weld formula and setting the load the weld can carry before failing to 120,000 lb..

P_{weld} = (.707 t * L) * τ_{all} , or

120,000 lb = (.707 * .7" * L) * 14,000 lb/in^2 = (6,929 lb./in.)L; then solving for L

L = 120,000 lb./(6,929 lb./in.) = 17.31 inches.

This is the minimum inches of weld needed to carry the load, however since the 120,000 pound load is not applied symmetrically to the plate, we can not apply the weld symmetrically. That is, if we try to put equal amounts of weld on sides AB and GF, the weld will fail in this case. To determine how the weld should be distributed, we once again apply static equilibrium conditions.

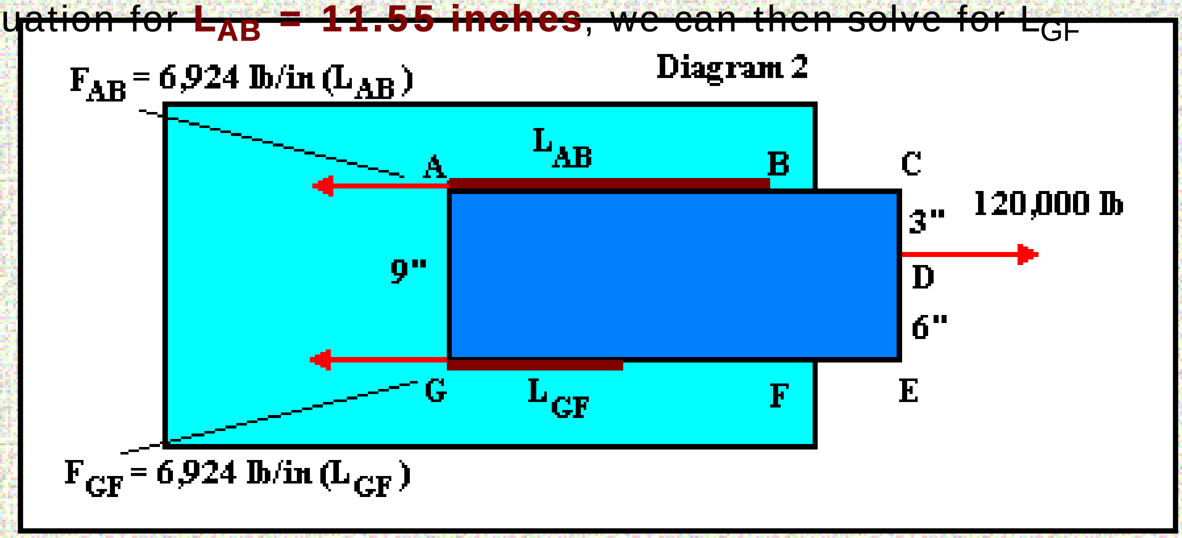
Part II. In Diagram 2 we have shown the weld distributed with an amount L_{AB} on side AB and an amount L_{GF} on side GF.The lengths L_{AB} and L_{GF} must, of course sum to 17.32 inches (the minimum inches of weld needed). The maximum force these weld can resist with is given by F_{AB} = (6,929 lb./in.)* L_{AB} , and F_{GF} = (6,929 lb./in.) * L_{GF} , as shown in Diagram 2. We now apply static equilibrium conditions to the top plate:

Sum of Forces: 120,000 lb. - (6,929 lb./in.)* L_{AB} - (6,929 lb./in.)* L_{GF} = 0

Sum of Torque $_{\text{about G}}$: -120,000 lb.* (6") + (6,929 lb./in.)* L_{AB} * (9") = 0

(Notice that the force F_{GF} does not produce a torque about point G, since its line of action passes through point G.)

Solving the torque equation for **L_{AB} = 11.55 inches**, we can then solve for L_{GF} since L_{AB} and L_{GF} sum to 17.32 inches. Therefore **L_{GF} = 17.32" - 11.55" = 5.77"** This is how the weld must be distributed in order to carry the 120,000 lb. load (and satisfy static equilibrium conditions).



Part B.

The other question of interest in welded joints is exactly how much weld is needed to make the welded joint as strong as the plate itself. In this case we assume the plate is loaded in tension, and so the strength of the plate is P_{plate} = product of cross sectional area of plate and the allowable tensile stress for the plate material, or P_{plate} = (w * t) σ_t . We then set this equal to the strength of the riveted joint and solve for the length of weld needed.

P_{weld} = (.707 t * L) * τ_{all} = P_{plate} =(w * t) σ_t , or

(.707 t * L) * τ_{all} = (w * t) σ_t , and then

(.707 * .7" * L) * 14,000 lb/in^2 = (9" * .7") 35,000 lb/in^2 , then solving for L

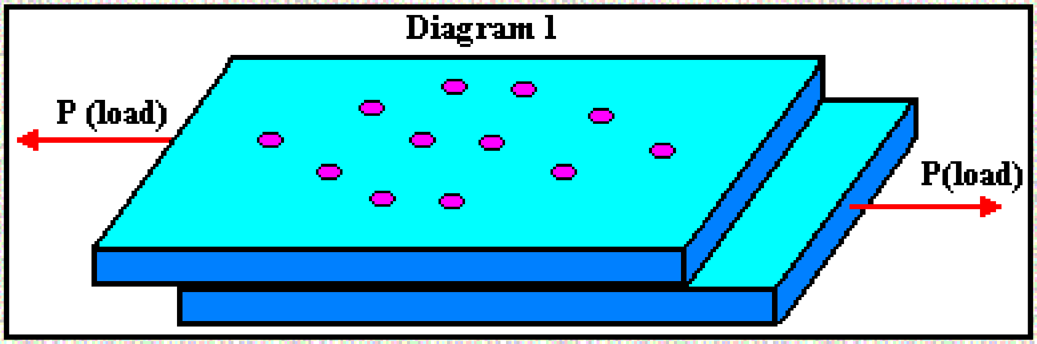
L = 31.82 inches

This is the minimum inches of weld needed to make the weld as strong as the plate (in tension). We can not state how the weld should be distributed, since we are not dealing with a specific loading.

Statics & Strength of Materials

Topic 6.8a - Problem Assignment 1 - Rivets/Welds

1. A riveted lap joint is shown in Diagram 1. The diameter of the rivets is 3/4 inch. The width of the plates is 8 inches, and the thickness of the plates is 1/2 inch. The allowable stresses are as follows:

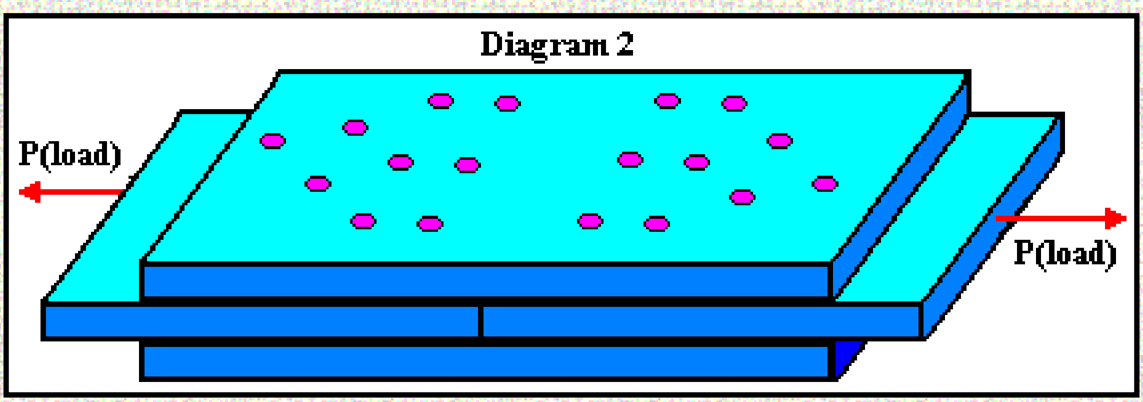


Rivets: τ = 15, 000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 25, 000 lb/in^2

Plate: τ = 14,000 lb/in^2, σ_t = 20, 000 lb/in^2, σ_c = 24, 000 lb/in^2

Determine the **Strength of the Joint**, and the **Efficiency of the Joint.**
(P_s = 79,520 lb., P_b = 108,000 lb, P_{r1} = 72,500 lb, P_{r2} = 70,910 lb, P_{r3} = 76,700 lb)
[**Joint Strength = 70,910** (lowest of failure loads above); **eff. = .886**]

2. A riveted butt joint is shown in Diagram 2. The diameter of the rivets is 1/2 inch. The width of the plates is 6 inches, and the thickness of the plates is 3/4 inch. The allowable stresses are as follows:

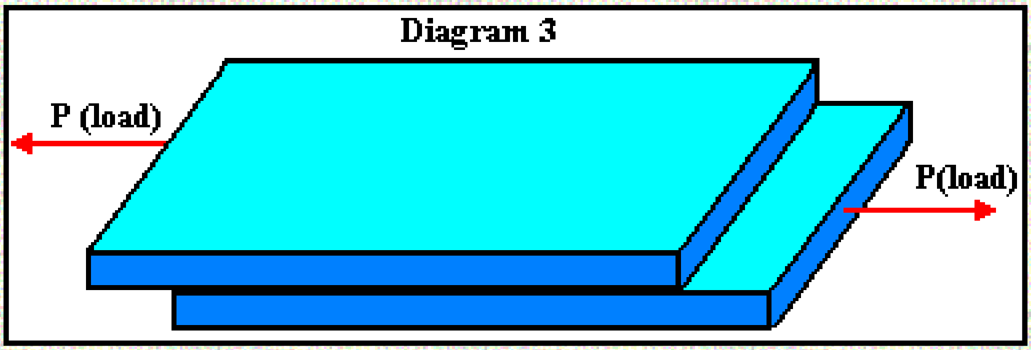


Rivets: τ = 18, 000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 24, 000 lb/in^2

Plate: τ = 17,000 lb/in^2, σ_t = 20, 000 lb/in^2, σ_c = 21, 000 lb/in^2

Determine the **Strength of the Joint**, and the **Efficiency of the Joint.**
(P_s = 63,620 lb., P_b = 70,880 lb, P_{r1} = 82,500 lb, P_{r2} = 84,375 lb, P_{r3} = 101,250 lb)
[**Joint Strength = 63,620** (lowest of failure loads above); **eff. = .707**]

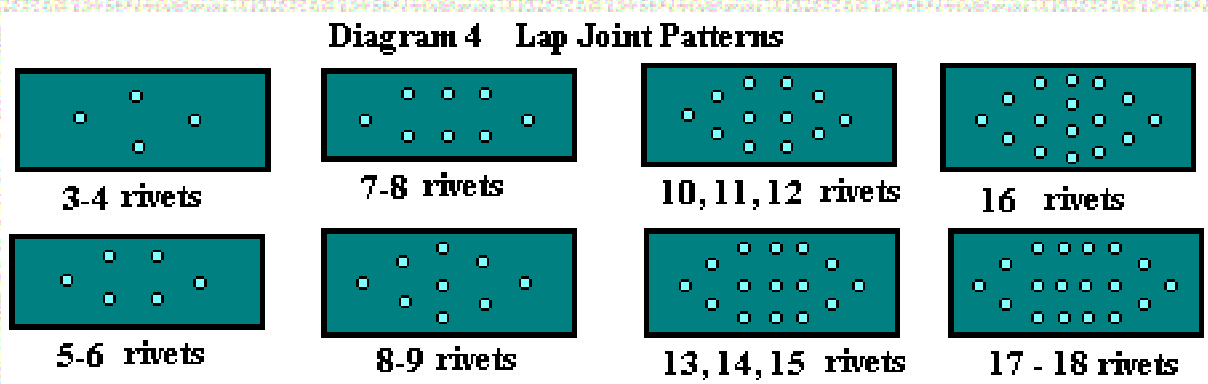
3. A lap joint (Diagram 3) is to connect two steel plates both with a width of 7 inches and a thickness of 5/8 inch. The rivets to be used have a diameter of 3/4 inch. The maximum allowable stresses for the rivet and plate materials are as follows:



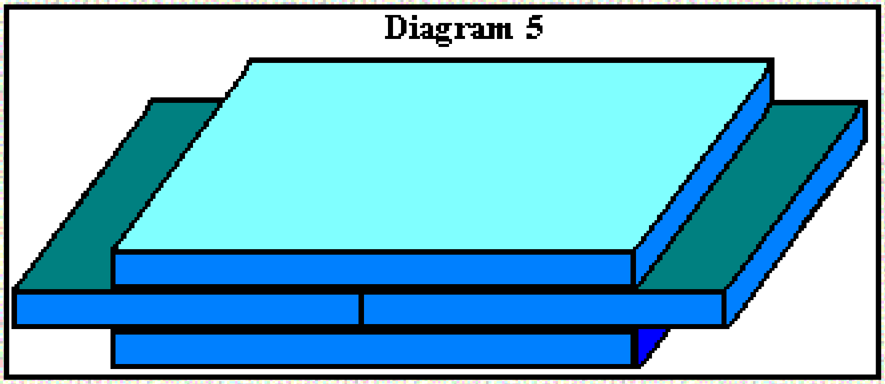
Rivets: τ = 20, 000 lb/in^2, σ_t = 21, 000 lb/in^2, σ_c = 23, 000 lb/in^2

Plate: τ = 17,000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 25, 000 lb/in^2

A. Determine the number of rivets for the most efficient joint. (ans. 9.7 = 10 rivets)
B. Select the best pattern from those shown in Diagram 4. (ans. 10,11,12 pattern)
C. Calculate the strength and efficiency of the joint. (ans. P_{r2} = 82,500, eff = .857)



4. A double cover plate butt joint (Diagram 5) is to connect two steel plates both with a width of 10 inches and a thickness of 5/8 inch.

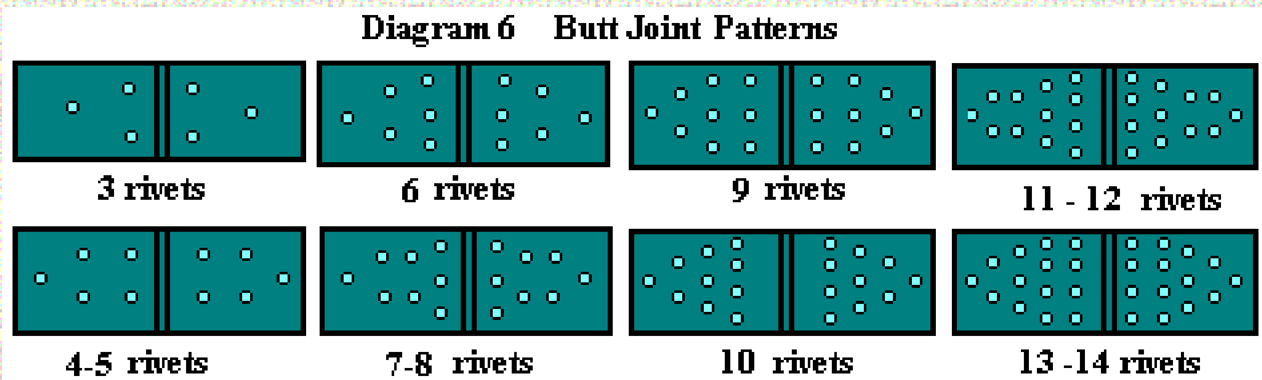


The rivets to be used have a diameter of 3/4 inch. The maximum allowable stresses for the rivet and plate materials are as follows:

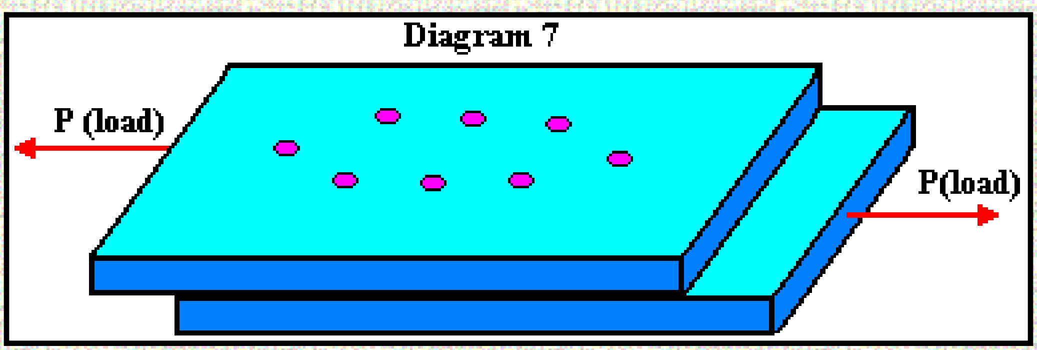
Rivets: τ = 16, 000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 23, 000 lb/in^2

Plate: τ = 14,000 lb/in^2, σ_t = 20, 000 lb/in^2, σ_c = 21, 000 lb/in^2

A. Determine the number of rivets for the most efficient joint. (ans. 11.75 = 12 rivets)
B. Select the best pattern from the patterns in Diagram 6. (11-12 pattern)
C. Calculate the strength and efficiency of the joint. (ans. P_{r1} = 115, 625 lb; eff = .925)



5. A lap joint is to connect two steel plates each with thickness of 3/4 inch (Diagram 7). The rivets to be used have a diameter of 1/2 inch. The maximum allowable stresses for the rivet and plate materials are as follows:



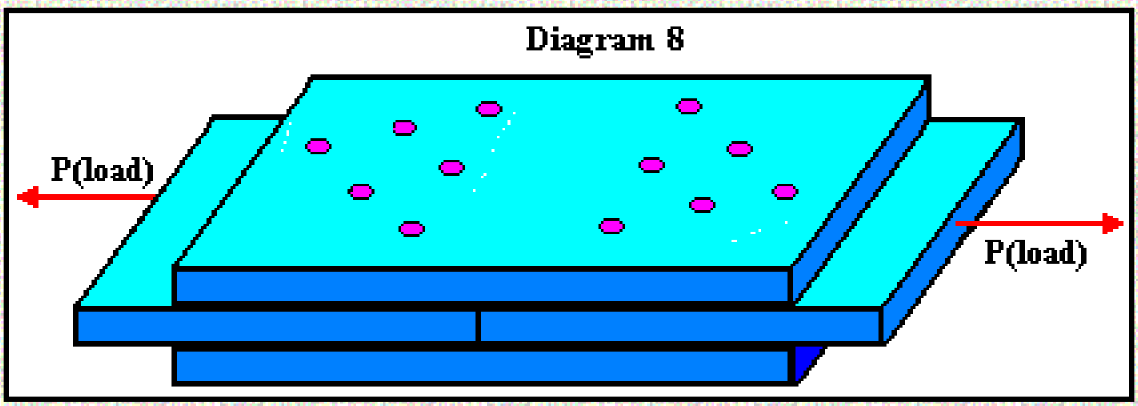
Rivets: τ = 15, 000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 27, 000 lb/in^2

Plate: τ = 14,000 lb/in^2, σ_t = 24, 000 lb/in^2, σ_c = 26, 000 lb/in^2

A. Calculate the width needed for the steel plate such that the joint will fail in bearing and plate tearing at row 1at the same load. (ans. w = 4.83")
B. Using the width found in part A, determine the strength and efficiency of the joint.
(P_s = 23,562 lb., P_b = 78,000 lb, P_{r1} = 77,940 lb, P_{r2} = 78,790 lb)

[**Joint Strength = 23,562** (lowest of failure loads above); **eff. = .27**, not a very good joint]

6. A a double cover plate butt joint is to connect two steel plates each with thickness of 3/4 inch (See Diagram 8). The rivets to be used have a diameter of 1/2 inch. The maximum allowable stresses for the rivet and plate materials are as follows:



Rivets: τ = 16, 000 lb/in^2, σ_t = 24, 000 lb/in^2, σ_c = 28, 000 lb/in^2

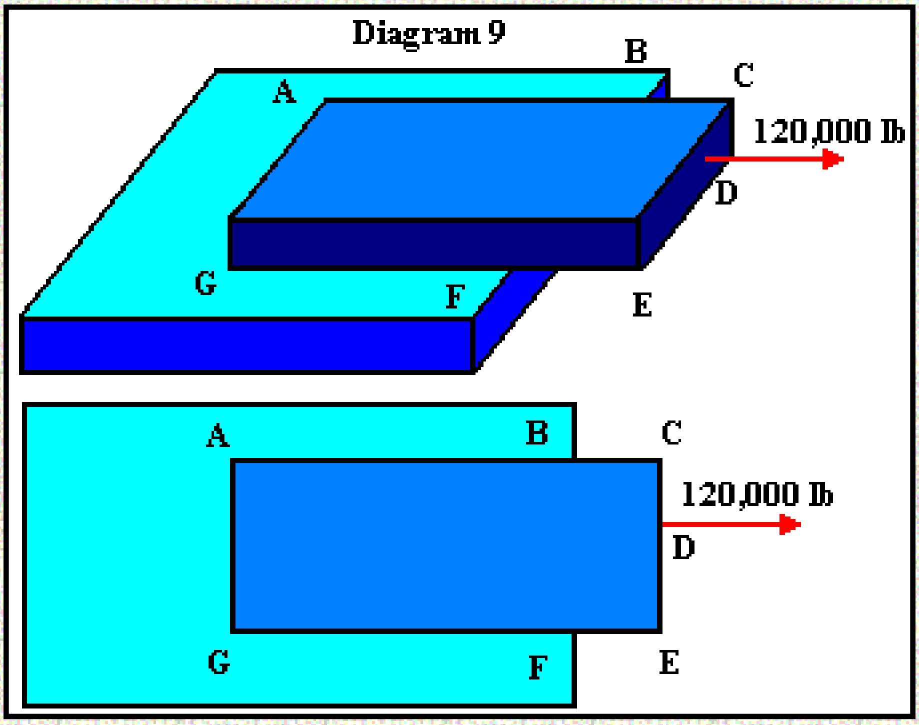
Plate: τ = 15,000 lb/in^2, σ_t = 25, 000 lb/in^2, σ_c = 26, 000 lb/in^2

A. Calculate the width needed for the steel plate such that the joint will fail in rivet shear and plate tearing at row 1 at the same load. (ans. 2.5", very small width, not very realistic.)
B. Using the width found in part A, determine the strength and efficiency of the joint.
(P_s = 37,700 lb., P_b = 58,500 lb, P_{r1} = 37,500 lb, P_{r2} = 33,750 lb, P_{r3} = 37,500 lb)

[**Joint Strength = 33,750** (lowest of failure loads above); **eff. = .72**]

7. Two steel plates are shown below. The top plate is 1/2 inch thick and 10 inches

wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be weld completely across end AG and partially along sides AB and FG.



A.) Determine the minimum inches of weld need to carry the 120,000 lb. load and specify how many inches of weld should be placed along each side AB and FG.
B.) Determine the number of inches of weld needed to make the weld strength as great as the plate strength.

The allowable stresses are as follows;
Weld Material: τ = 16,000 lb/in^2 ; σ_t = 25, 000 lb/in^2
Plate Material: τ = 15,000 lb/in^2 ; σ_t = 26,000 lb/in^2
Dimensions: AB = GF = 20 inches; CD = 3 inches; DE = 7 inches

(ans.Part A. L_{total} =21.2"; L_{AB} = 9.85"; L_{AG} = 10" (given); L_{GF} = 1.35")
(ans.Part B. L_{total} = 22.98")

Select:
[Topic 6: Torsion, Rivets & Welds - Table of Contents](#)
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Statics & Strength of Materials

Topic 6.8b - Problem Assignment - Rivets / Welds

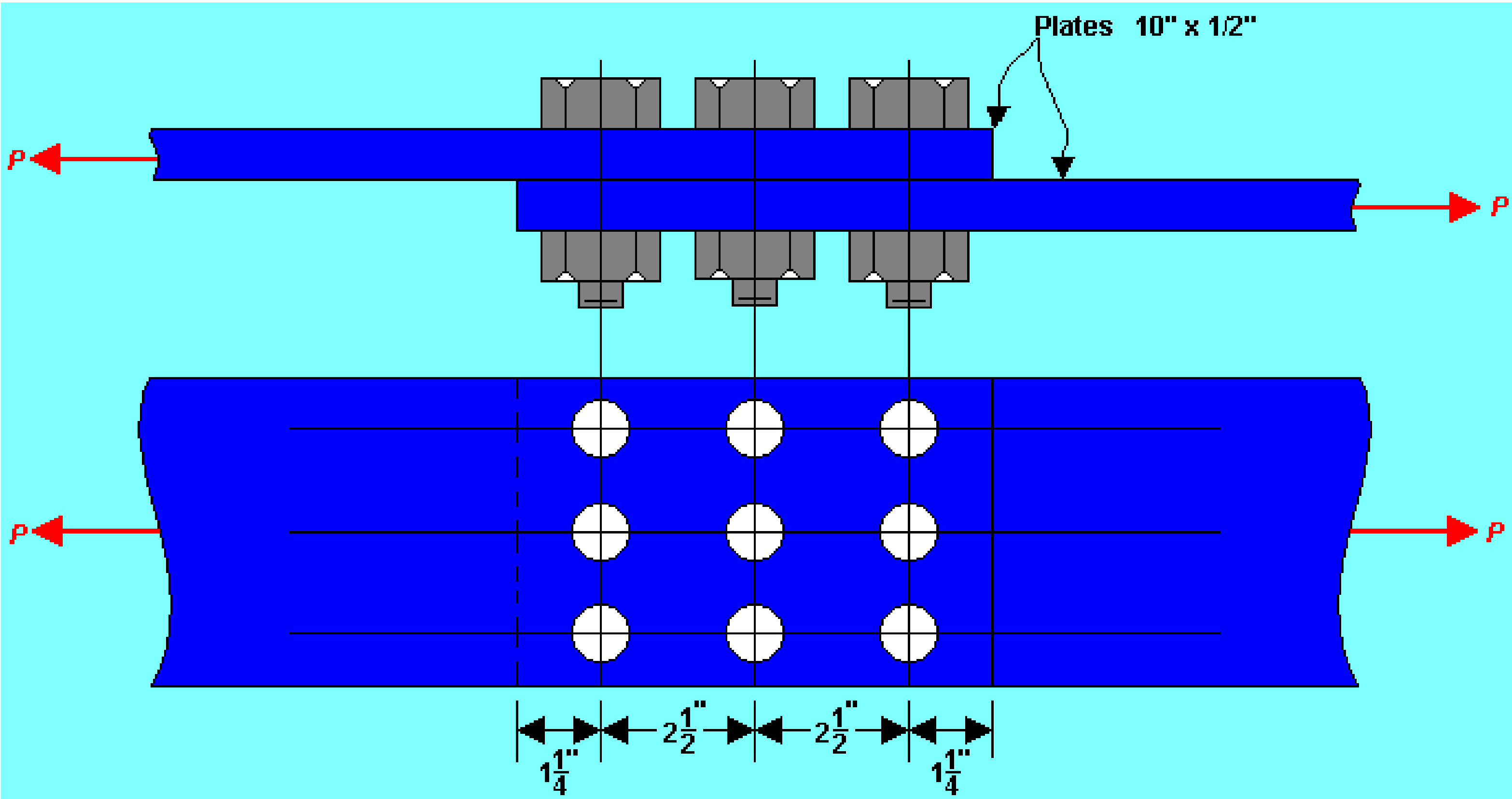
Allowable stresses [ksi(MPa)]

		Allowable Bearing Stress		Allowable Tensile Stress	
Structural steels	$S_{t(ult)}$	$S_{p(all)} = (1.5)S_{t(ult)}$	S_y	(Gross) $S_{t(all)} = (0.60) S_y$	(Net) $S_{t(all)} = (0.50)S_{t(ult)}$
A36 carbon	58 (400)	87 (600)	36 (250)	22 (152)	29*(200)

Allowable shear stress on steel fasteners.

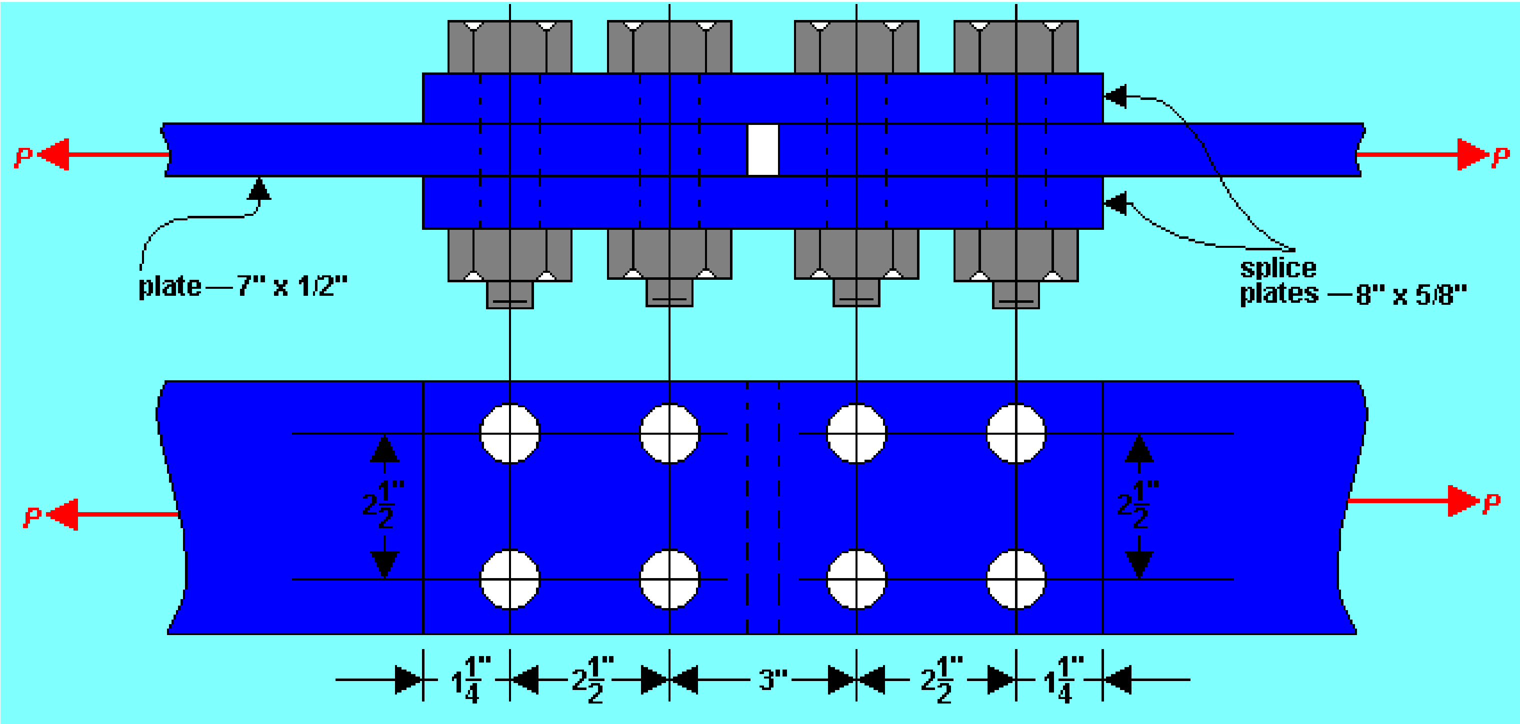
Description of Fastener	Allowable Shear Stress ($S_{s(all)}$)[ksi MPa)]	
	Slip-Critical Connection	Bearing-Type Connection
A307 low-carbon bolts	-	10.0(68.9)
A325 bolts --- threads in shear plane	17.0(117)	21.0(145)
A325 bolts --- threads excluded from shear plane	17.0(117)	30.0(207)
A490 bolts --- threads in shear plane	21.0(145)	28.0(193)
A490 bolts --- threads excludes from shear plane	21.0(145)	40.0(276)
A502 Grade 1 hot-driven rivets	-	17.5(121)
A502 Grade 2 hot-driven rivets	-	22.0(152)

1. Determine the allowable tensile load for the single shear lap joint shown. Assume that the threads are excluded from the shear plane. The plates are ASTM A36 steel and the bolts are A325. The Diameter of the Rivets is 1 inch.
[Use gross allowable tensile stress for ASTM A36]
[For Joint use Bearing Type connection]
(Rivet Shear = 212,000 lb., Plate Bearing = 391,500 lb., Plate Tearing= 77,000 lb)



Problem #1

2. Determine the allowable tensile load for the double shear butt joint shown. Assume that the threads are excluded from the shear plane. The plates are ASTM A36 steel and the bolts are A325. The Diameter of Rivets is 1 inch.
[Use gross allowable tensile stress for ASTM A36]
[For Joint use Bearing Type connection]
(Rivet Shear = 188,500 lb., Plate Bearing = 174,000 lb., Plate Tearing= 68,750 lb)

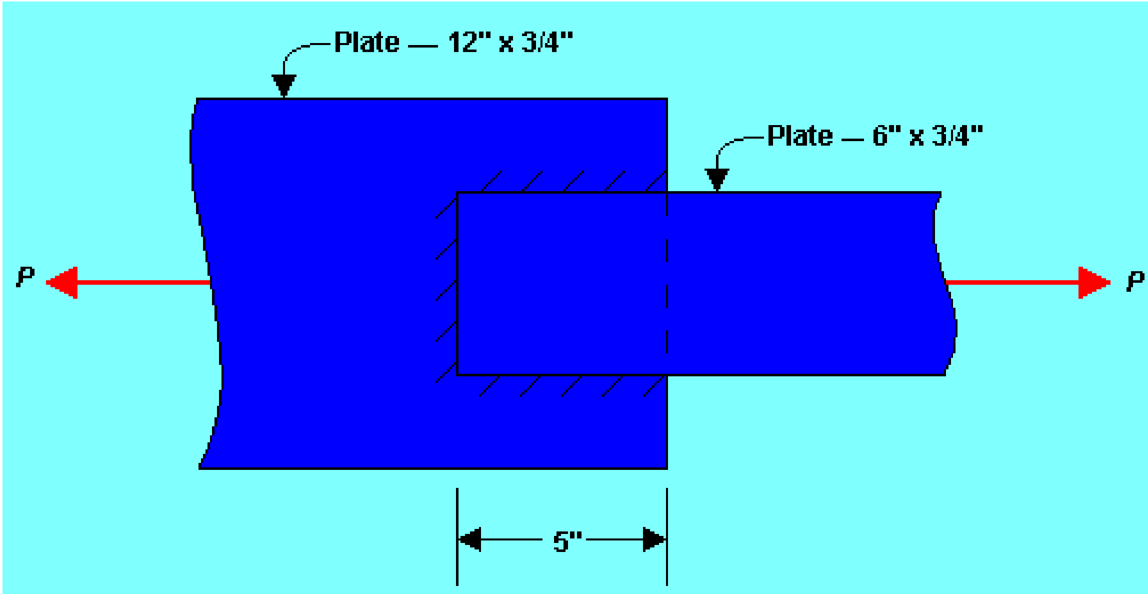


Problem #2

3. Determine the number of 3/4 inch bolts needed for a maximum efficiency lap joint of two 8" x 1/2" plates. The plates are ASTM A36 steel and the bolts are A325.
[Use gross allowable tensile stress for ASTM A36]
[For Joint use Bearing Type connection, and A325 bolts --- threads excluded from shear plane]
(6 rivets)

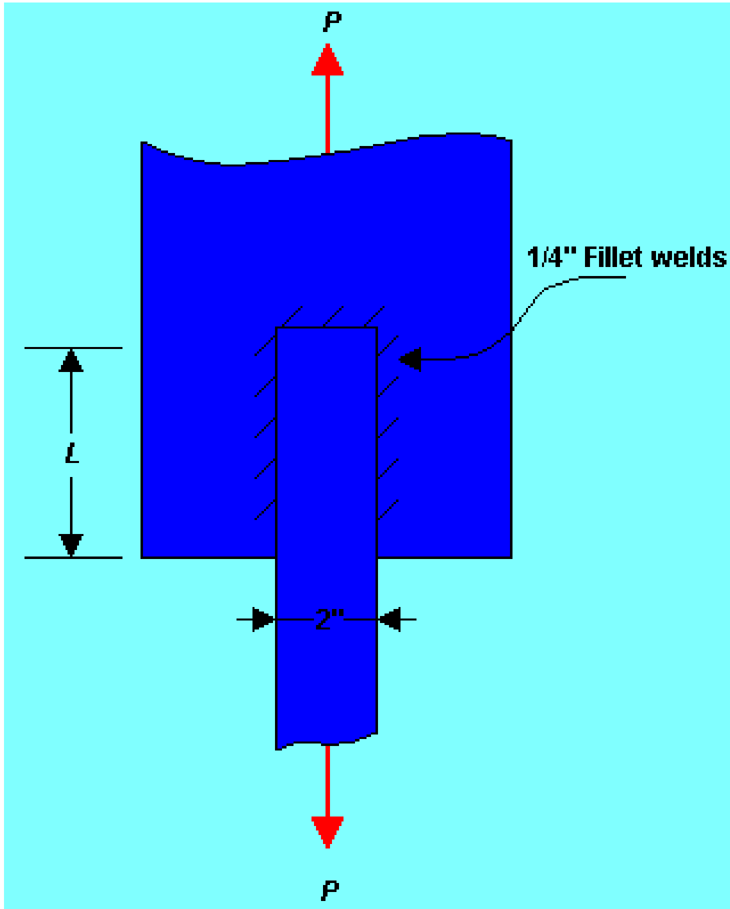
4. Determine the number of 3/4 inch bolts needed for a maximum efficiency butt joint of two 8" x 1/2" plates. The cover plates are 8" x 9/16". The plates are ASTM A36 steel and the bolts are A325.
[Use gross allowable tensile stress for ASTM A36]
[For Joint use Bearing Type connection, and A325 bolts --- threads excluded from shear plane]
(3 rivets)

5. Calculate the allowable tensile load for the connection in the diagram shown. The plates are ASTM A36 steel and the weld is a 3/4 in. fillet weld, which is made using an E70 electrode.
[E70 electrode has allowable tensile stress of 70,000 psi.; and allowable shear stress of 30% of tensile stress]
[Use gross allowable tensile stress for ASTM A36]
(Weld strength (shear) = 178,160 lb.; Plate strength (tension) = 99,000 lb.]



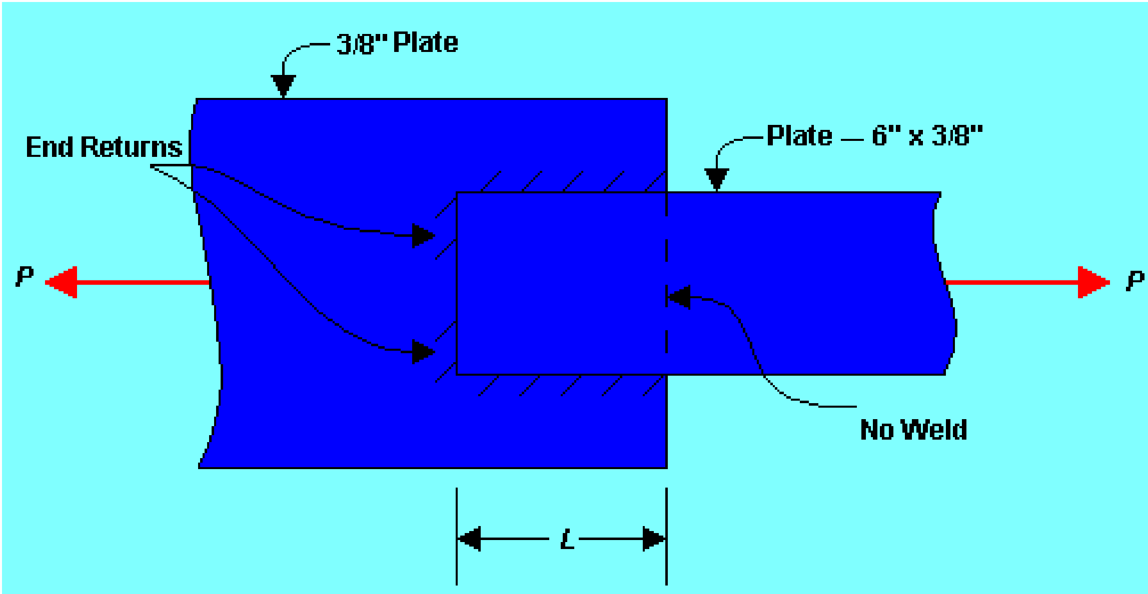
Problem #1

6. In the connection in the diagram shown, 1/4 in. side and end fillet welds are used to connect the 2.0 in. by 1.0 in tension member to the plate. The applied load is 65,000 lb. Find the required dimension *L*. The steel is ASTM A36 and the electrode used is an E70.
[E70 electrode has allowable tensile stress of 70,000 psi.; and allowable shear stress of 30% of tensile stress]
[Use gross allowable tensile stress for ASTM A36]
(L = 7.76"; however joint fails in plate tearing at 44,000 lb)



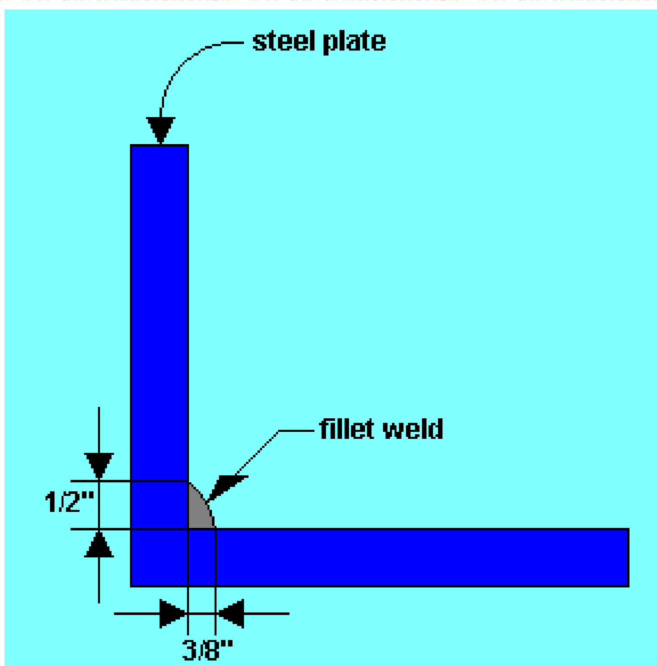
Problem #2

7. Design the fillet welds parallel to the applied load to develop the full allowable tensile load of the 6 in. by 3/8 in. ASTM A36 steel plate in the diagram shown. The electrode is an E70. The Plate is welded along the far end in addition to the sides.
[E70 electrode has allowable tensile stress of 70,000 psi.; and allowable shear stress of 30% of tensile stress]
[Use gross allowable tensile stress for ASTM A36]
(L = 1.45")



Problem #3

8.A fillet weld between two steel plates intersecting at right angles was made with one 3/8 in. leg and one 1/2 in leg using an E70 electrode. Determine the strength of the weld in kips/in. [E70 electrode has allowable tensile stress of 70,000 psi.; and allowable shear stress of 30% of tensile stress]
(5570 lb/in = 5.57kips/in.)



Problem #4

Select:
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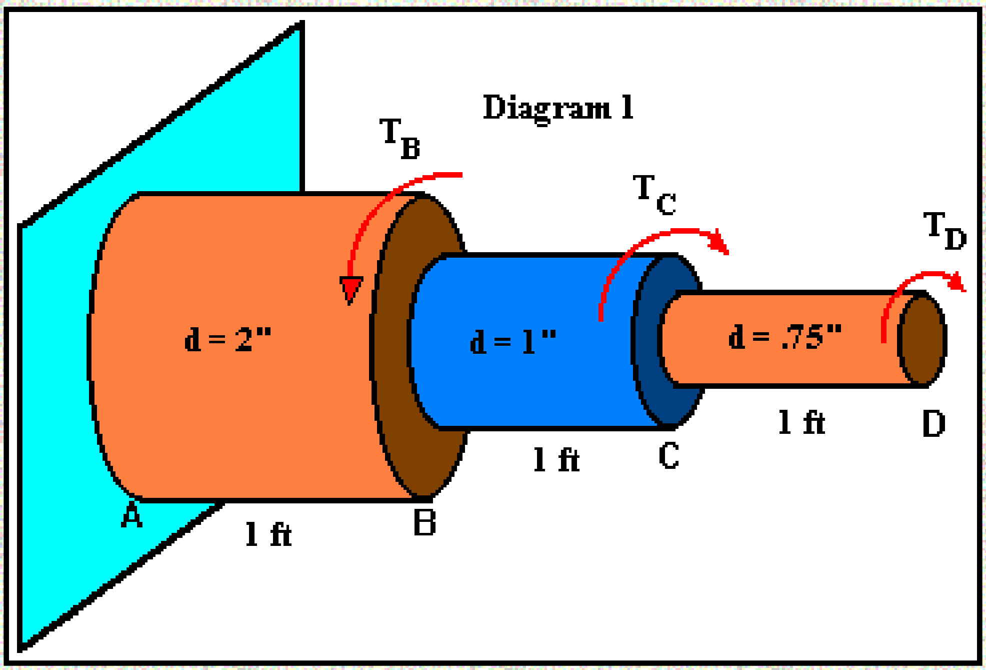
[Strength of Materials Home Page]

Topic 6.9: Torsion, Rivets & Welds - Topic Examination

Torsion - Topic Exam

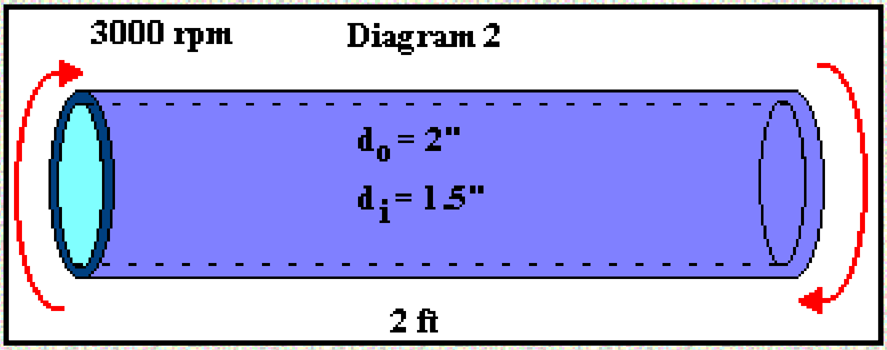
Modulus of Rigidity for several materials:
Steel = 12 x 10^6 lb/in^2.; Brass = 6 x 10^6 lb/in^2.; Aluminum = 4 x 10^6 lb/in^2.

1. A compound shaft is attached to a fixed wall as shown in Diagram 1. Section AB brass, section BC is steel, and section CD is brass. If the allowable shear stress for steel is 18,000 lb/in^2, and for brass is 16,000 lb/in^2.



A. Determine the maximum torque which could be applied at points D, C, & B (T_D, T_C, & T_B) without exceeding the allowable shear stress in any section of the shaft.
B. Using the torque found in part A, determine the resultant angle of twist of end D with respect to end A.
(For Solution Select: Torsion - Solution Problem 1)

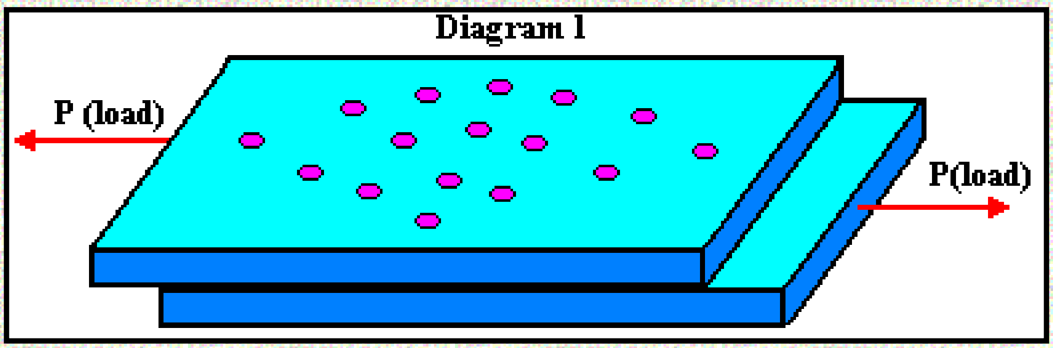
2. A 2 foot long hollow steel shaft with an outer diameter of 2 inches and an inner diameter of 1.5 inches is to transmit power while being driven a 3000 rpm.



A.) If the allowable shear stress in the shaft is 15,000 lb/in^2., what is the maximum horsepower which can be transmitted down the shaft ?
B.) If we were not given the outer diameter of the shaft, but were told that the inner diameter was to be four-tenths of the outer diameter, what would be the minimum outer diameter of the shaft which could safely transmit the horse power found in part A? (The allowable shear stress in the shaft is 15,000 lb/in^2)
(For Solution Select: Torsion - Solution Problem 2**)**

Rivets & Welds - Topic Exam

1. A riveted lap joint is shown in Diagram 1. The diameter of the rivets is 1 inch. The width of the plates is 12 inches, and the thickness of the plates is 5/8 inch. The allowable stresses are as follows:

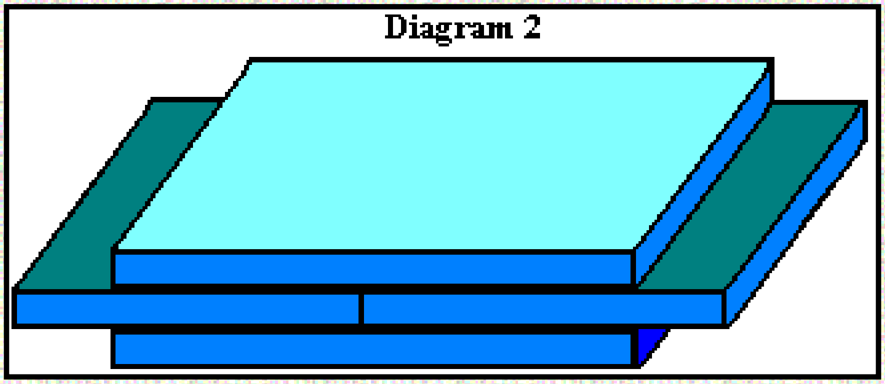


Rivets: τ = 20, 000 lb/in^2, σ_t = 26, 000 lb/in^2, σ_c = 28, 000 lb/in^2

Plate: τ = 17,000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 24, 000 lb/in^2

Determine the **Strength of the Joint**, and the **Efficiency of the Joint.**
(For Solution Select: Rivets & Welds - Solution Problem 1**)**

2. A double cover plate butt joint (Diagram 2) is to connect two steel plates both with a width of 9 inches and a thickness of 7/8 inch. The rivets to be used have a diameter of 3/4 inch.

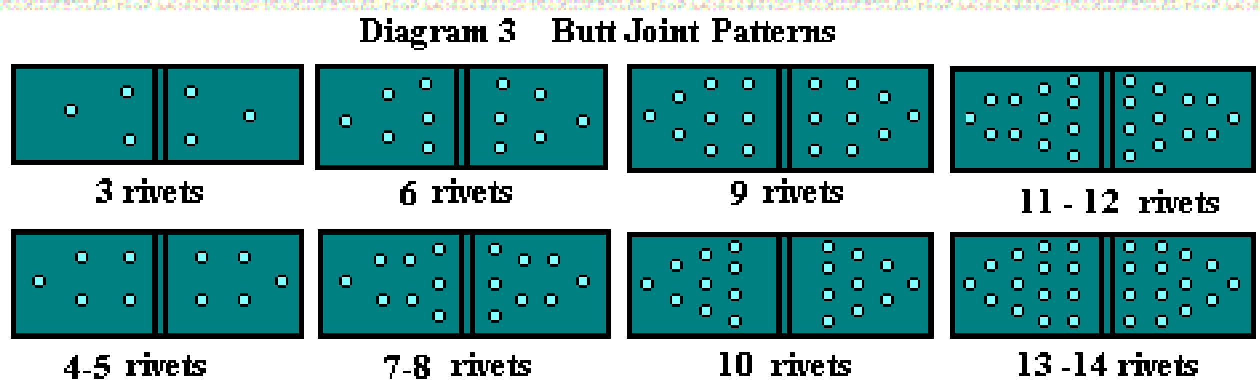


The maximum allowable stresses for the rivet and plate materials are as follows:

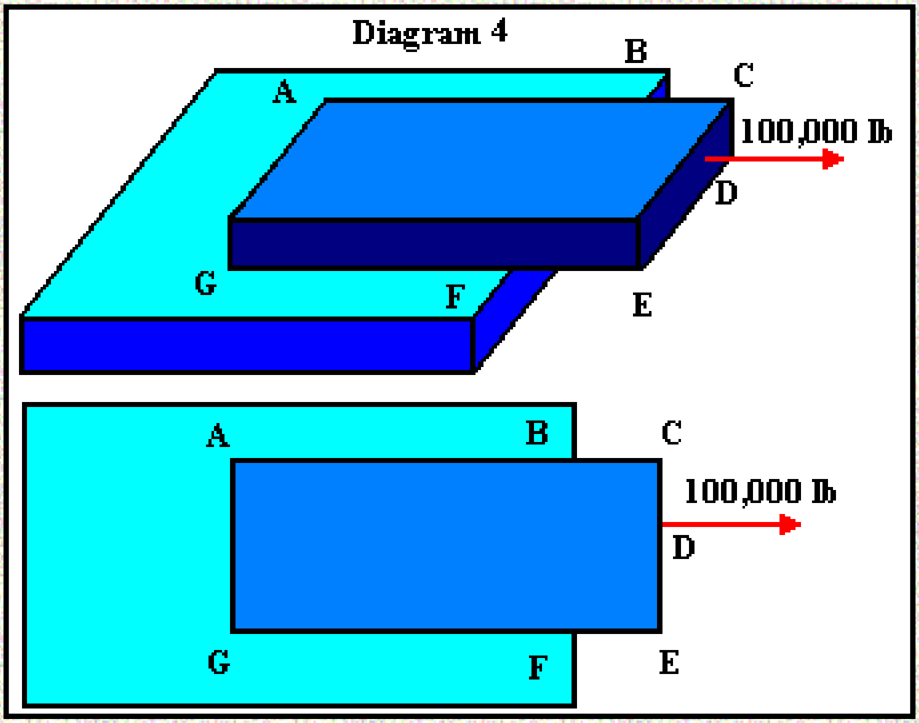
Rivets: τ = 17, 000 lb/in^2, σ_t = 22, 000 lb/in^2, σ_c = 23, 000 lb/in^2

Plate: τ = 18,000 lb/in^2, σ_t = 24, 000 lb/in^2, σ_c = 26, 000 lb/in^2

A. Determine the number of rivets for the most efficient joint.
B. Select the best pattern from the patterns in Diagram 3 .
C. Calculate the strength and efficiency of the joint.
(For Solution Select: Rivets & Welds - Solution Problem 2)



3. Two metal plates are shown below. The top plate is 3/4 inch thick and 9 inches wide, and is to be welded to the bottom plate with a 45º fillet weld. The top plate is to be weld completely across end AG and partially along sides AB and FG.



A.) Determine the minimum inches of weld need to carry the 100,000 lb. load and specify how many inches of weld should be placed along each side AB and FG.

B.) Determine the number of inches of weld needed to make the weld strength as great as the plate strength.

The allowable stresses are as follows;
Weld Material: τ = 12,000 lb/in^2 ; σ_t = 24, 000 lb/in^2
Plate Material: τ = 13,000 lb/in^2 ; σ_t = 22,000 lb/in^2
Dimensions: AB = GF = 20 inches; CD = 3 inches; DE = 6 inches
(For Solution Select: Rivets & Welds - Solution Problem 3)

Select:
Topic 6: Torsion, Rivets & Welds - Table of Contents
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Topic 7.1a: Euler Buckling - Example 1

An 16 ft. long ASTM-A36 steel, W10x29 I-Beam is to be used as a column with pinned ends. For this column, determine the slenderness ratio, the load that will result in Euler buckling, and the associated Euler buckling stress. The beam characteristics may be found in the I-Beam Table, and are also listed below.

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A-in^2	d - in	w_f - in	t_f - in	t_w - in	I - in^4	S -in^3	r - in	I - in^4	S -in^3	r - in
W 10x29	8.54	10.22	5.799	0.500	0.289	158.0	30.8	4.30	16.30	5.61	1.38

The slenderness ratio = L_e / r = 16 ft. * 12 in./ft./ 1.38 in = 139

Notice that we must use the smallest radius of gyration, with respect to the y-y axis, as that is the axis about which buckling will occur. We also notice that the slenderness ratio is large enough to apply Euler's buckling formula to this beam. To verify this we use the relationship for the minimum slenderness ratio for Euler's equation to be valid.

$$\frac{L_e}{r} = \sqrt{\frac{\pi^2 E}{\sigma_p}}$$

Or, after finding for ASTM-A36 Steel, E = 29 x 10^6 lb/in^2, and yield stress = 36,000 lb/in^2, we can solve and determine that **L_e/r = 89**.

The Euler Buckling Load is then give by: $P_{cr} = \frac{\pi^2 EI}{L_e^2}$, and after substituting values, we obtain:

Pcr = [$(3.14)^2$*29x10^6 lb/in^2 * 16.30 in^4/$(16'x12"/ft)^2$] = 126,428 lb

c) **The Euler Stress** is then easily found by **Stress = Force/Area = 126,428 lb/8.54 in^2 = 14,800 lb/in^2**. Notice that this stress which will produce buckling is much less than the yield stress of the material. This means that the column will fail in buckling before axial compressive failure.

Return to: Topic 7.1: Columns & Buckling - I
Continue to: Topic 7.1b: Euler Buckling - Example 2

or Select:

[Topic 7: Columns & Pressure Vessels - Table of Content]
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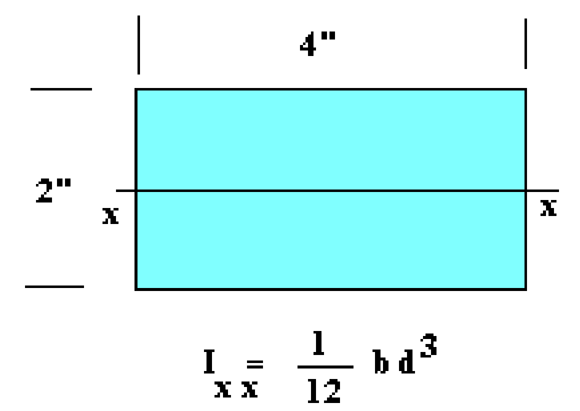
Topic 7.1b: Euler Buckling - Example 2

A 8 ft. long southern pine 2" x 4" is to be used as a column. The yield stress for the wood is 6,500 lb/in^2, and Young's modulus is 1.9 x 10^6 lb/in^2. For this column, determine the slenderness ratio, the load that will produce Euler buckling, and the associated Euler buckling stress.

The slenderness ratio = L_e / r . To determine the slenderness ratio in this problem, we first have to find the radius of gryration (smallest), which we may do from the relationship: **Radius of Gyration: $r_{xx} = (I_{xx}/A)^{1/2}$** , where this is the radius of gyration about an x-x axis, and where I = (1/12)bd^3 for a rectangular cross section. Or **$r_{xx} = [(1/12)bd^3/bd]^{1/2}$** , where we have substituted A = bd. We now simplify and obtain:

$r_{xx} = [(1/12)d^2]^{1/2} = .5774(d/2)$ We want the smallest radius of gyration, so we use d =2". That is, buckling will first occur about the x-x axis shown is the diagram, and **r = .5774 in**.



Then Slenderness ratio is given by: **L_e / r = (8 ft x 12"/ft)/.5774" = 166** which puts the beam in the long slender category.

The Euler Buckling Load is then give by: $P_{cr} = \frac{\pi^2 EI}{L_e^2}$, and after substituting values, we obtain:

Pcr = [(3.14)^2*1.9x106 lb/in^2 * (4*2^3/12) in^4/(8'x12"/ft)^2] = **5,420 lb.**

c) **The Euler Stress** is then easily found by **Stress = Force / Area = 5,420 lb / (2"*4") in^2 = 678 lb / in^2** . Notice that this stress which will produce buckling is much less than the yield stress of the material.

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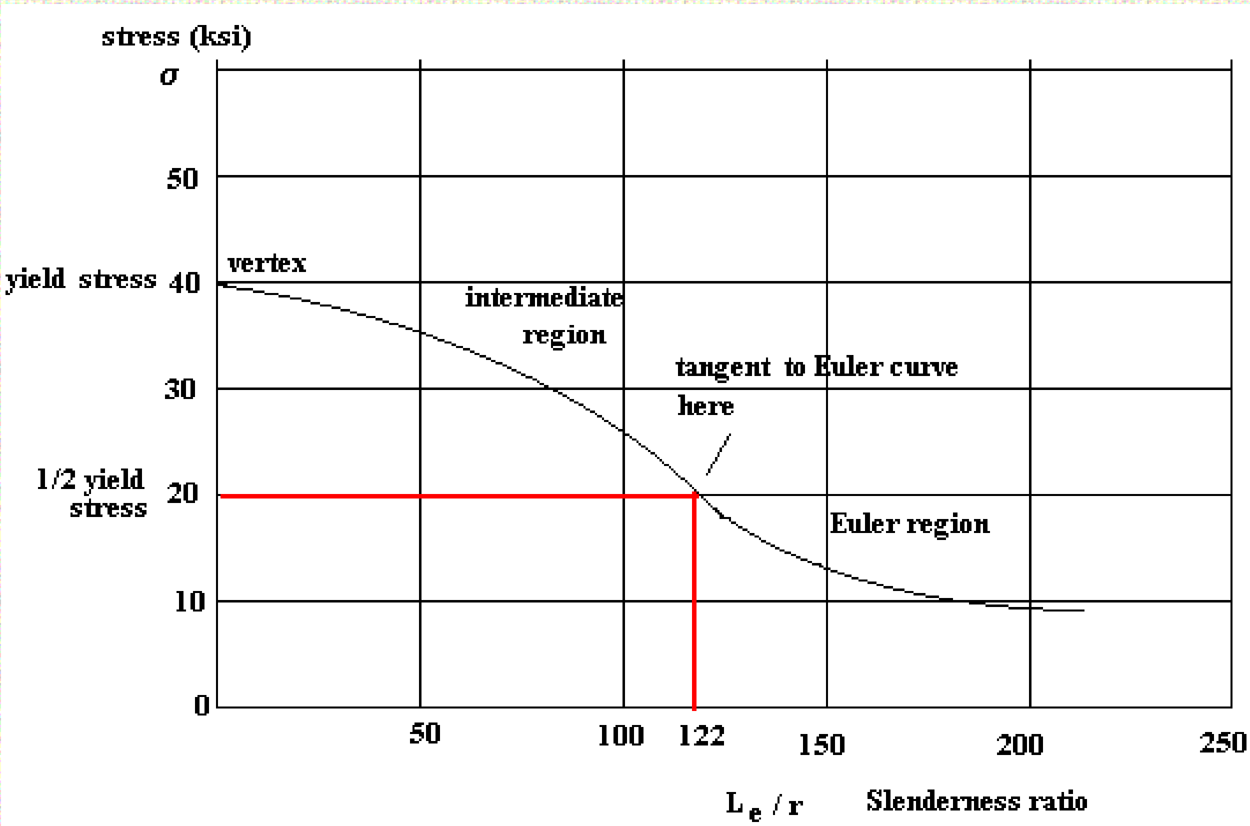
Topic 7.2: Columns & Buckling - II

III. Intermediate Columns

There are a number of semi-empirical formulas for buckling in columns in the intermediate length range. One of these is the J.B. Johnson Formula. We will not derive this formula, but make several comments. The J.B. Johnson formula is the equation of a parabola with the following characteristics. For a graph of stress versus slenderness ratio, the parabola has its vertex at the value of the yield stress on the y-axis. Additionally, the parabola is tangent to the Euler curve at a value of the slenderness ratio, such that the corresponding stress is one-half of the yield stress.

In the diagram below, we have a steel member with a yield stress of 40,000 psi. Notice the parabolic curve beginning at the yield stress and arriving tangent to the Euler curve at 1/2 the yield stress.



We first note that at the point where the Johnson formula and Euler's formula are tangent, we can relate the stress to Euler's formula as follows (where C represents the slenderness ratio when the stress is 1/2 the yield stress):

$$\sigma_{cr} = \frac{1}{2}\sigma_y = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 E}{C^2}$$

From this we find an expression for C (critical slenderness ratio) of: $C = \sqrt{\dfrac{2\pi^2 E}{\sigma_y}}$

For our particular case, where we have a steel member with a yield stress of 40,000 psi, and a Young's modulus of 30 x

10^6 psi., we find **C = sqrt(2 * 3.14^2 * 30 x 10^6 / 40,000 psi) = 122**. If our actual beam has a slenderness ratio greater than the critical slenderness ratio we may use Euler's formula. If on the other hand our actual slenderness ratio is smaller than the critical slenderness ratio, we may use the J.B. Johnson Formula.

Example: As an example let us now take a 20 foot long W12 x 58 steel column (made of same steel as above), and calculate the critical stress using the J.B. Johnson formula. (Beam information and Johnson formula shown below.)

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick						
-	A-in^2	d - in	w_f - in	t_f - in	t_w - in	I - in^4	S -in^3	r - in	I - in^4	S -in^3	r - in
W 12x58	17.10	12.19	10.014	0.641	0.359	476.0	78.1	5.28	107.00	21.40	2.51

J.B. Johnson's formula:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \left[1 - \frac{(L_e / r)^2}{2 C^2} \right] \sigma_y$$

For our beam the slenderness ratio = (20 ft * 12 in/ft)/2.51in = 95.6 (where 2.51 in. is the smallest radius of gyration, about y-y axis). Inserting values we find:

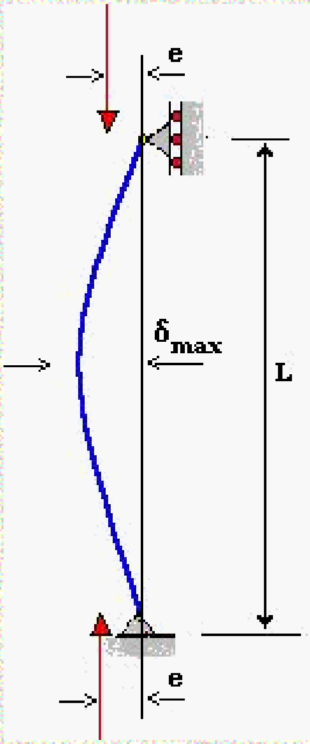
Critical Stress = [1 - (95.6^2/2* 122^2)]*40,000 psi. = 27,720 psi. This is the critical stress that would produce buckling. Note we did not have a safety factor in this problem. As a result we really would not want to load the column to near the critical stress, but at a lower 'allowable' stress.

The Critical Load will equal the product of the critical stress and the area, or **P_{cr} = 27,720 psi. * 17.10 in2 = 474,012 lb**.

VI. The Secant Formula:

Another useful formula is known as the Secant formula. We will not go through the derivation of this relationship, but focus on its application.

The Secant formula may be used for both axially loaded and eccentrically loaded columns. It may be used with pinned-pinned (L_e = L), and with fixed-free (Le = 2L) end columns, but not with other end conditions.



Secant Formula

$$\boxed{\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right]}$$

where

e = eccentricity of loading (shown in diagram)

c = maximum distance from centroid to outer edge of column

$\frac{ec}{r^2}$ = eccentricity ratio

L_e = effective length

r = radius of gyration (smallest)

E = Young's modulus of material

The Secant formula gives the maximum compressive stress in the column as a function of the average axial stress (P/A), the slenderness ratio (L/r), the eccentricity ratio (ec/r^2), and Young's Modulus for the material.

If, for a given column, the load, P, and eccentricity of the load, e, are known, then the maximum compressive stress can be calculated. Once we have the maximum compressive stress due to the load, we can compare this stress with the allowable stress for the material and decide if the column will be able to carry the load.

On the other hand, if we know the allowable compressive stress for the column, we may use the Secant formula to determine the maximum load we can safely apply to the column. In this case we will be solving for P, and we take note that the equation is a transcendental equation when solved for P. Thus, the easiest method of solution is to simply try different values of P, until we find a satisfactory fit. See following example.

Please Select 7.2a: Secant Formula - Example 1

The eccentricity ratio has a normal range from 0 to 3, with most values being less than 1. When the eccentricity value is zero (corresponding to an axial loading) we have the special case that the maximum load is the critical load:

$P_{cr} = \frac{\pi^2 EI}{L^2}$ and the corresponding stress is the critical stress $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{L_e^2 A}$ or $\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$

This is one way to look at axial loads. On the other hand a common practice with axially loaded structural steel columns is to use an eccentricity ratio of .25 to account for the effects of imperfections, etc. Then the allowable stress does not have to be reduced to account for column imperfections, etc., as this is taken into account in eccentricity ratio.

V. Empirical Design Formulas for Columns:

A number of empirical design formulas have been developed for materials such as structural steel, aluminum and wood, and may be found in such publications as the *Manual of Steel Construction*, *Mechanic*, *Specifications for Aluminum Structures*, *Aluminum Construction Manuel*, *Timber Construction Manual*, and *National Design Specifications for Wood Constuction*.

1. Structural Steel:

Structural Steel with a yield stress of σ_y	
Short and/or Intermediate Columns	**Long Columns**
Critical Slenderness ratio: $C_c^2 = \dfrac{2\pi^2 E}{\sigma_y}$	Range Limits: $\dfrac{L_e}{r} > C_c$
Range Limits: $0 \le \dfrac{L_e}{r} \le C_c$	Allowable Stress: $\sigma_{all} = \dfrac{\pi^2 E}{1.92\left(\dfrac{L_e}{r}\right)^2}$
Allowable Stress: $\sigma_{all} = \dfrac{\sigma_y}{FS}\left[1 - \dfrac{1}{2}\left(\dfrac{L_e/r}{C_c}\right)^2\right]$	
Factor of Safety: $FS = \dfrac{5}{3} + \dfrac{3}{8}\left(\dfrac{L_e/r}{C_c}\right) - \dfrac{1}{8}\left(\dfrac{L_e/r}{C_c}\right)^3$	

Please Select 7.2b: Structural Steel - Example 2 for structural steel example.
Please Select 7.2c: Structural Steel Column Selection - Example 3 for structural steel example.

2. Aluminum

Aluminum with a yield stress of σ_y	
Short and/or Intermediate Columns Range Limits: $0 \le \frac{L_e}{r} \le C_1$ Allowable Stress: $\sigma_{all} = \frac{\sigma_y}{k_c\,(FS)}$ Range Limits: $C_2 \le \frac{L_e}{r} \le C_3$ Allowable Stress: $\sigma_{all} = \frac{1}{FS'}\left(B_c - D_c\,\frac{L_e}{r}\right)$	**Long Columns** Range Limits: $\frac{L_e}{r} > C_3$ Allowable Stress: $\sigma_{all} = \frac{\pi^2 E}{FS'\left(\frac{L_e}{r}\right)^2}$ Where FS = Safety Factor based on yield stress. Where FS'= Safety Factor based on ultimate stress. Where C_1, C_2, C_3, B_c, and D_c are appropriate constants (see following table)

Aluminum alloy 2014-Ty (Alclad)	
Short and/or Intermediate Columns Range Limits: $0 \le \frac{L_e}{r} \le 12$ Allowable Stress: $\sigma_{all} = 28{,}000$ psi Range Limits: $12 \le \frac{L_e}{r} \le 55$ Allowable Stress: $\sigma_{all} = \left(30.7 - 0.23\,\frac{L_e}{r}\right)$ ksi	**Long Columns** Range Limits: $\frac{L_e}{r} > 55$ Allowable Stress: $\sigma_{all} = \frac{54{,}000}{\left(\frac{L_e}{r}\right)^2}$ ksi ksi = 1000 psi $\frac{L_e}{r}$ = Slenderness ratio

Please Select 7.2d: Aluminum Column - Example 4 for Aluminum example.

<table>
<tr><th colspan="2">Aluminum alloy 6061 - T6</th></tr>
<tr><td>

Short and/or Intermediate Columns

Range Limits: $0 \le \frac{L_e}{r} \le 9.5$

Allowable Stress: $\sigma_{all} = 19{,}000$ psi

Range Limits: $9.5 \le \frac{L_e}{r} \le 66$

Allowable Stress: $\sigma_{all} = \left(20.2 - 0.126 \frac{L_e}{r}\right)$ ksi

</td><td>

Long Columns

Range Limits: $\frac{L_e}{r} > 66$

Allowable Stress: $\sigma_{all} = \frac{51{,}000}{\left(\frac{L_e}{r}\right)^2}$ ksi

ksi = 1000 psi

$\frac{L_e}{r}$ = Slenderness ratio

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</table>

3. Wood Columns

<table>
<tr><th colspan="2">Wood columns with a rectangular cross section b x d, where b > d</th></tr>
<tr><td>

Short and/or Intermediate Columns

Range Limits: $0 \le \frac{L_e}{d} \le 11$

Allowable Stress: $\sigma_{all} = \sigma_c$

Range Limits: $11 \le \frac{L_e}{d} \le k$

Allowable Stress: $\sigma_{all} = \sigma_c \left[1 - \frac{1}{3}\left(\frac{L/d}{k}\right)^4\right]$

$k = 0.671\sqrt{E/\sigma_c}$

</td><td>

Long Columns

Range Limits: $k \le \frac{L_e}{d} \le 50$

Allowable Stress: $\sigma_{all} = \frac{0.3\,E}{\left(\frac{L_e}{d}\right)^2}$

σ_c = allowable compress stress parallel to grain

E = Young's Modulus

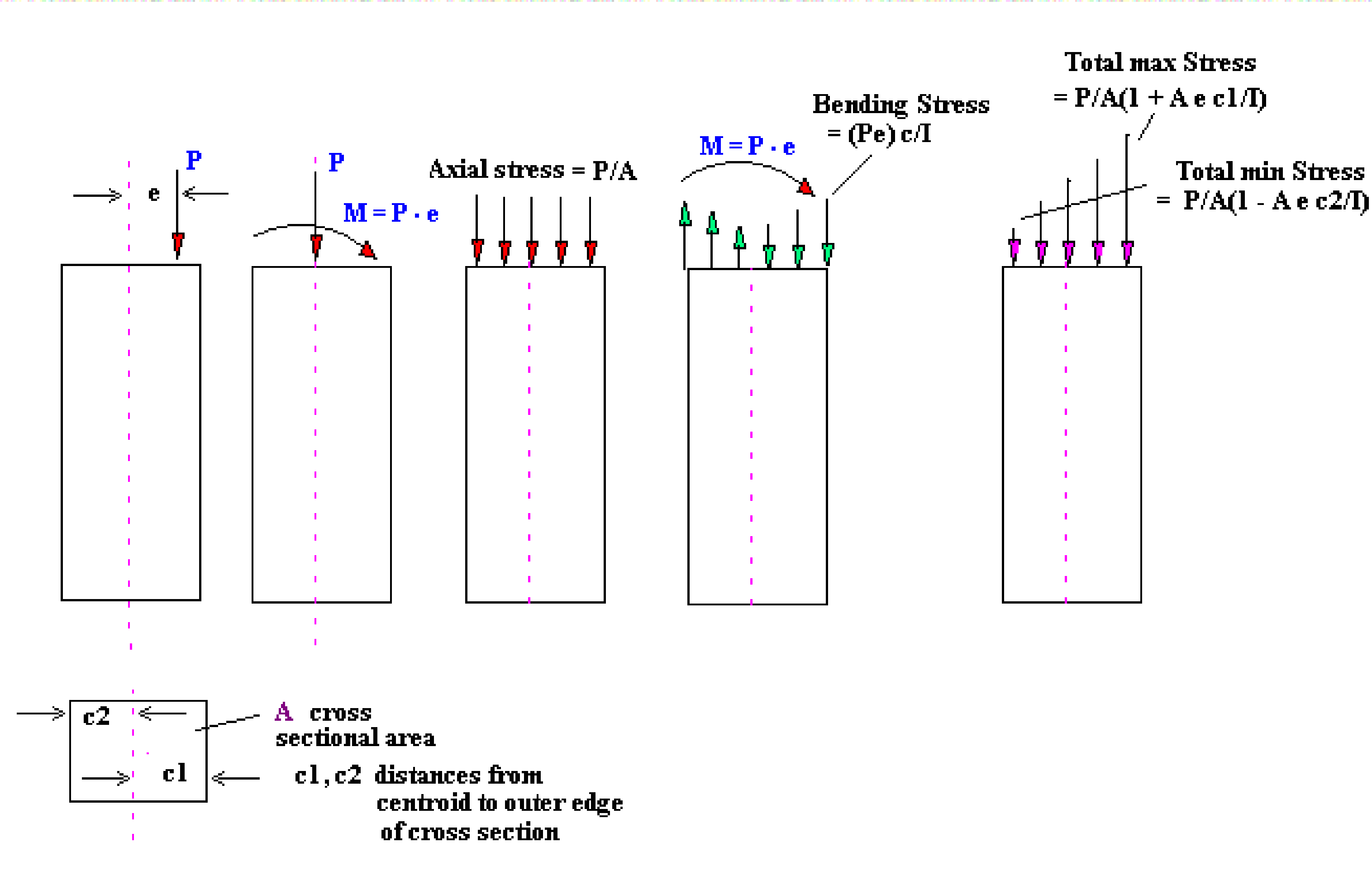
</td></tr>
</table>

Please Select 7.2e: Wood Column - Example 5 for wood column example.

VI Short Eccentrically Loaded Columns

An eccentrically loaded short column is shown in the diagram, with the force, P, acting a distance, e, from the centroid of the column cross sectional area. We may replace the eccentrically acting force, P , with an axial force, P, plus a moment whose value will be M = P x e. Next we calculate the compressive stress due to the axial force, P, which will simply be P/A. Then we calculate the bending stress due to the moment P x e, which gives (Pe)c/I where the bending stress will be a compressive maximum on the right side of the column and a tensile maximum on the left side of the column (and zero at the centroid). Finally, we add the two stress and obtain **Total Maximum Compressive Stress = $(P/A)(1 + A\,e\,c_1/I)$**

(right side of column), and the **Total Minimum Compressive Stress = (P/A)(1 - A e c_1/I) (left side of column).** And in fact, if the bending stress is large enough, the left side on the column may be in tension.



Continue to: [Topic 7.3: Columns & Buckling - Problem Assignment]
or Select:
[Topic 7: Columns & Pressure Vessels - Table of Content]
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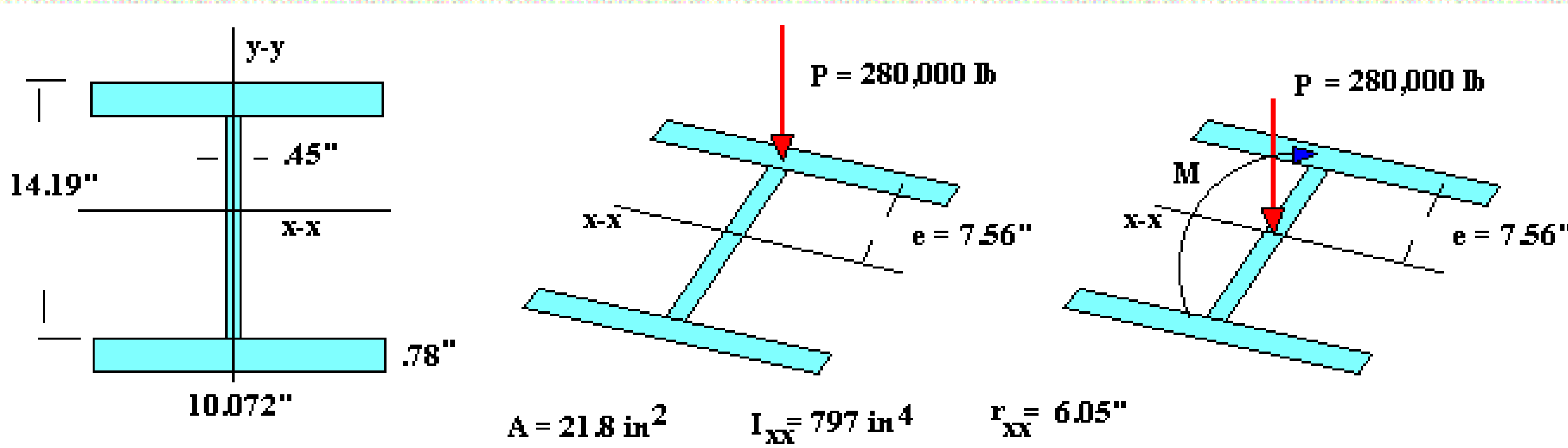
Topic 7.2a: Secant Formula - Example 1

A steel I-beam (W 14 x 74) is used as a column. The beam is 20 ft. long and pinned on both ends. An eccentrically applied load of 280,000 lb. acts at the center of one flange as shown in the diagram. Young's Modulus for steel is 30 x 10^6 psi., and the yield stress for the steel is 40,000 psi. Let us first calculate the maximum compressive stress using the Secant Formula.

Beam Data

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick						
-	A-in^2	d - in	w_f - in	t_f - in	t_w - in	I - in^4	S -in^3	r - in	I - in^4	S -in^3	r - in
W 14x74	21.80	14.19	10.072	0.783	0.450	797.0	112.0	6.05	133.00	26.50	2.48



We replace the eccentric loading with an axial load plus a moment:
P =280,000 lb., M = P x e = 2,116,800 in-lb.

Next we calculate some constants:
Slenderness ratio = L/r = 20 ft x 12 in/ft / 6.05 in = 39.7
Eccentricity ratio = ec/r^2 = (7.56")(14.19"/2)/(6.05")^2 = 1.47
Axial Stress = P/A = 280,000 lb/ 21.8 in^2 = 12, 844 psi

Now we apply the secant formula: $$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{E\,A}}\right)\right]$$

or **Maximum Compressive Stress = (12, 844 lb/in^2)[1 + 1.47 sec {(39.7/2)*sqrt (12,844 lb/in^2 / 30x 10^6 lb/in^2)}] = (12, 844 lb/in^2)[1 + 1.47 (1.09)] = 33,440 lb/in^2**

We observe that the maximum compressive stress is less than the yield stress, 40,000 psi., thus this column is safe.

A second interesting question is what is the maximum load, P, which would result in a maximum compressive stress equal to the yield stress. For this we replace the maximum stress with the yield stress in the secant formula, and then solve the secant formula for the value of P.

Or **40,000 lb/in^2 = (P/21.8 in^2)[1 + 1.47 sec{(39.7/2)sqrt(P/(30 x106 lb/in^2 * 21.8 in^2))}]**
Or simplifying a bit we have: **872,000 lb = P[1 + 1.47 sec {(19.85)sqrt(P/6.54 x 10^8 lb)}]**

This is a transcendental equation, and perhaps the most effective way to solve at this level is to guess solutions until we find an acceptable one.
We first try a value of P somewhat greater than the original. Let **P = 350,000 lb**.
Then we have: **872,000 lb. = 350,000 lb. [1 + 1.47 sec {(19.85)sqrt(350,000 lb./6.54 x 10^8 lb.)}] or**
872,000 lb. = 923,960 lb. (which is clearly not correct, our value of P is somewhat high, so we try again.

Let **P = 330,000 lb**., then **872,000 lb. = 330,000 lb. [1 + 1.47 sec {(19.85)sqrt(330,000 lb./6.54 x 10^8 lb.)}],**
or 872,000 lb. = 867,700 lb. Here we see we are quite close. So a load of 330,000 pounds will be very close to causing buckling to occur in the column.

Return to: Topic 7.2: Columns & Buckling - II
Continue to: Topic 7.2b: Structural Steel - Example 2
or Select:
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Topic 7.2b: Structural Steel - Example 2

A structural steel T-beam (WT 15 x 66) is used as a column. The beam is 12 ft. long and is fixed on one end and free on the other. Young's Modulus for the steel is 30 x 10^6 psi., and the yield stress for the steel is 38,000 psi. Determine the maximum allowable axial load. (Note that since this is a fixed-free column, the effective length is equal to 2 * L.). Repeat the problem if the column is 20 ft long.

-	-	Depth	Flange			-	Cross	Section	Info.	-
Designation	Area	of T	Width	thick	thick	-	x-x axis	x-x axis	x-x axis	x-x axis
-	A-in^2	d - in	w_f - in	t_f - in	t_w - in	d/t_w	I - in^4	S -in^3	r - in	y - in
WT15x66	19.40	15.15	10.551	1.000				37.400		

We will first do this problem assuming that radius of gyration is lowest about the X-X Axis, and buckling will occur first with respect to that axis. Calculating the slenderness ratio = L_e/r = (2*12ft * 12"/ft)/4.65 = 61.94

Next we calculate the critical slenderness ratio **C_c^2 = (2 * 3.14^2 * 30 x 10^6 lb/ in^2 /38,000 lb/in^2) = 15568; and C = 124.8.** Since the slenderness ratio of the column is less than the critical slenderness ratio, we use the intermediate formula to find the allowable stress.

Structural Steel with a yield stress of σ_y

Short and/or Intermediate Columns

Critical Slenderness ratio: $C_c^2 = \dfrac{2\pi^2 E}{\sigma_y}$

Range Limits: $0 \le \dfrac{L_e}{r} \le C_c$

Allowable Stress: $\sigma_{all} = \dfrac{\sigma_y}{FS}\left[1 - \dfrac{1}{2}\left(\dfrac{L_e/r}{C_c}\right)^2\right]$

Factor of Safety: $FS = \dfrac{5}{3} + \dfrac{3}{8}\left(\dfrac{L_e/r}{C_c}\right) - \dfrac{1}{8}\left(\dfrac{L_e/r}{C_c}\right)^3$

Long Columns

Range Limits: $\dfrac{L_e}{r} > C_c$

Allowable Stress: $\sigma_{all} = \dfrac{\pi^2 E}{1.92\left(\dfrac{L_e}{r}\right)^2}$

Before we can determine the allowable stress, we first calculate the factor of safety.

FS = (5/3) + (3/8)(61.94/124.8) - (1/8)(61.94/124.8)^3 = 1.84.

Then Allowable Stress = (38,000/1.84)[1 - (1/2)(61.94/124.8)^2] = 18,110 lb/in^2.

Finally the allowable load = Stress * Area = 18,110 lb/in^2 * 19.4 in^2 = 351,334 lb.

PART 2: Repeat if the column is 20 ft long.

We again assume that radius of gyration is lowest about the X-X Axis, and buckling will occur first with respect to that axis. **Calculating the slenderness ratio = L_e/r = (2*20ft * 12"/ft)/4.65 in = 103.2**

Next we calculate the critical slenderness ratio C_c^2 = (2 * 3.14^2 * 30 x 10^6 lb/in^2 /38,000 lb/in^2) = 15568; and **C = 124.8**. Since the slenderness ratio of the column is less than the critical slenderness ratio, we use the formula for intermediate columns to find the allowable stress. However, before we can determine the allowable stress, we must first calculate the factor of safety.

$FS = (5/3) + (3/8)(103.2/124.8) - (1/8)(103.2/124.8)^3 = 1.91$

Then Allowable Stress = $(38{,}000/1.91)[1 -(1/2)(103.2/124.8)^2] = 13{,}090$ lb/in^2

Finally the Allowable Load = Stress * Area = 13,090 lb/in^2 * 19.4 in^2 = 253,946 lb.

Notice how much the change in length has reduced the allowable load.

Return to: Topic 7.2: Columns & Buckling - II
Continue to: Topic 7.2c: Structural Steel Column Selection - Example 3
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Topic 7.2c: Structural Steel Column Selection - Example 3

Select the best (safe & lightest) I-beam to be used as a 16 foot vertical column, with one end fixed and the other end pinned, and which is to carry an axial load of 120,000 lb. Young's Modulus for the steel is 30 x 10^6 psi., and the yield stress for the steel is 34,000 psi. (Note that since this is a fixed-pinned column, the effective length is equal to .7 * L.) Use the Table of I-Beams to select from.

One method of determining the best I-beam to use is an iterative type process. We first find the minimum cross sectional area of the beam by assuming the slenderness ratio, **$L_e/r = 0$**. Then the factor of safety, **FS = 5/3 = 1.667**, and the **allowable stress = (yield stress/safety factor) = 34,000 lb/in^2/1.667 = 20,400 lb/in^2**. Since the allowable stress is also Force/Area, we can solve for the minimum area: **Area = 120,000 lb/20,400 lb/in^2 = 5.88 in^2**.

Structural Steel with a yield stress of σ_y

Short and/or Intermediate Columns

Critical Slenderness ratio: $C_c^2 = \dfrac{2\pi^2 E}{\sigma_y}$

Range Limits: $0 \le \dfrac{L_e}{r} \le C_c$

Allowable Stress: $\sigma_{all} = \dfrac{\sigma_y}{FS}\left[1 - \dfrac{1}{2}\left(\dfrac{L_e/r}{C_c}\right)^2\right]$

Factor of Safety: $FS = \dfrac{5}{3} + \dfrac{3}{8}\left(\dfrac{L_e/r}{C_c}\right) - \dfrac{1}{8}\left(\dfrac{L_e/r}{C_c}\right)^3$

Long Columns

Range Limits: $\dfrac{L_e}{r} > C_c$

Allowable Stress: $\sigma_{all} = \dfrac{\pi^2 E}{1.92\left(\dfrac{L_e}{r}\right)^2}$

Next we choose (guess) a beam with an area greater than the minimum area found in step 1. From our table we will try a **W10 x 21,** which has an **area of 6.2 in^2** and a **minimum radius of gyration of 1.32 in**. The **Slenderness ratio for this column = L_e/r = (.7*16' * 12"/ft)/ 1.32 in. = 101.8**. We also calculate the critical slenderness ratio **C_c^2 = (2 * 3.14^2 * 30 x 10^6 lb/in^2 /34,000 lb/in^2) = 17417; and C = 132** Since the slenderness ratio of the column is less than the critical slenderness ratio, we use formula for intermediate columns to find the allowable stress.

Before we can determine the allowable stress, we first calculate the factor of safety. **FS = (5/3) + (3/8)(101.8/132) - (1/8)$(101.8/132)^3$ = 1.9.** Then the **Allowable Stress = (34,000/1.9)[1 - (1/2)$(101.8/132)^2$] = 10,994 lb/in^2**

Finally the Allowable Load = Stress * Area = 10,994 lb/in^2 * 6.2 in^2 = 68,165 lb.

We notice that the allowable load is about half of the load we would like to apply. This means we need an I-beam with a larger area and/or a larger radius of gyration. So we now select a larger cross section beam, and repeat steps 2, 3, and 4.

From our table we will try a **W10 x 33**, which has an **area of 9.71 in^2** and a **minimum radius of gyration of 1.94 in**. The **Slenderness ratio for this column = L_e/r = (.7*16' * 12"/ft)/ 1.94 in. = 69.3.** The critical slenderness ratio remains the same [C_c^2 = (2 * 3.14^2 * 30 x 10^6 lb/in^2 /34,000 lb/in^2) = 17417; and **C = 132**] Since the slenderness ratio of the column is less than the critical slenderness ratio, we use the formula for intermediate columns to find the allowable stress.

Before we can determine the allowable stress, we first calculate the factor of safety. **FS = (5/3) + (3/8)(69.3/132) - (1/8)$(69.3/132)^3$ = 1.85.** Then the **Allowable Stress = (34,000/1.85)[1 - (1/2)$(69.3/132)^2$] = 15,845 lb/in^2**.

Finally the **Allowable Load = Stress * Area = 15,845 lb/in^2 * 9.71 in^2 = 153,855 lb**. We notice that the allowable load is somewhat higher than the load we would like to apply. This means, of course, that this I-beam would work, but it is probably not the best (lightest). So we make one more guess/estimate. We need a beam with a slightly lower area and/or radius of gyration.

From our table we will try a **W12 x 31**, which has **area of 9.13 in^2** and a **minimum radius of gyration of 1.54 in**. The **Slenderness ratio for this column = L_e/r = (.7*16' * 12"/ft)/ 1.54 in. = 87.3**. The critical slenderness ratio remains the same [C_c^2 = (2 * 3.14^2 * 30 x 10^6 lb/in^2 /34,000 lb/in^2) = 17417; and **C = 132**] Since the slenderness ratio of the column is less than the critical slenderness ratio, we use the formula for intermediate columns to find the allowable stress.

Before we can determine the allowable stress, we first calculate the factor of safety. **FS = (5/3) + (3/8)(87.3/132) - (1/8)$(87.3/132)^3$ = 1.88**. Then the **Allowable Stress = (34,000/1.88)[1 - (1/2)$(87.3/132)^2$] = 14,130 lb/in^2**

Finally the **Allowable Load = Stress * Area = 14,130 lb/in^2 * 9.13 in^2 = 129,000 lb**. Here we see our maximum allowable load is just slightly above the load we would like to apply, (120,000 lb.). This beam is a very good candidate for the best beam. It might be possible to find a slightly better beam, but we will end here.

There are other design procedures for selecting columns than the one above. Most of these also utilize a trial and error type of procedure. However, with computers and spreadsheets today, it is not a hard process to develop a program to try a number of different columns and arrive at the best one in a short time.

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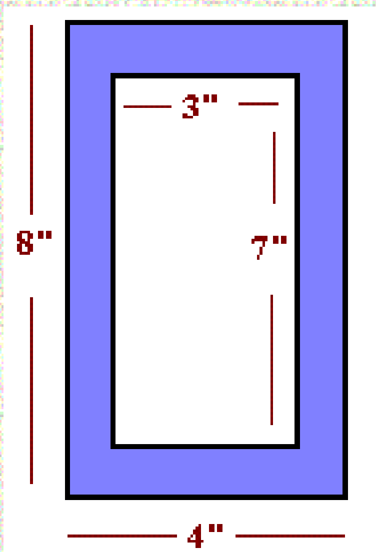
Topic 7.2d: Aluminum Columns - Example 4

A hollow rectangular tube is to be used as a 10-foot long vertical, fixed-fixed, column. The tube material is the aluminum alloy 2014-Ty (Alclad). The rectangular cross section has outer dimension of 4" x 8", and inner dimensions of 3" x 7", as shown in the diagram. Determine the maximum axial load that may be applied to the column before the allowable stress for buckling will be exceeded.



We first wish to determine the slenderness ratio, however we need the minimum radius of gyration, which may be calculated from $r = (I/A)^{1/2}$, where I is taken about the axis which gives the smallest value.

Aluminum alloy 2014-Ty (Alclad)
Short and/or Intermediate Columns
Range Limits: $0 \leq \frac{L_e}{r} \leq 12$
Allowable Stress: $\sigma_{all} = 28,000$ psi
Range Limits: $12 \leq \frac{L_e}{r} \leq 55$
Allowable Stress: $\sigma_{all} = \left(30.7 - 0.23 \frac{L_e}{r}\right)$ ksi
Long Columns
Range Limits: $\frac{L_e}{r} > 55$
Allowable Stress: $\sigma_{all} = \frac{54,000}{\left(\frac{L_e}{r}\right)^2}$ ksi

In this case that would be about a vertical y-y axis through the center of the cross section, resulting in the following value: **$I = (1/12)[b_o d_o^3 - b_i d_i^3] = (1/12)[8"*4"^3 - 7"*3"^3] = 26.9$ in^4**. Additionally, the **area = $b_o d_o - b_i d_i = 32$ in^2 – 21 in^2 = 11 in^2**. Then **$r = (I/A)^{1/2} = 1.56$ in**. And **slenderness ratio = L_e/r = (.5*10' * 12"/ft)/1.56in. = 38.4**

Since the slenderness ratio is greater than 12 and less then 55, we use the appropriate formula (for Alclad) for intermediate columns. The **Allowable Stress = [30.7- .23(L_e/r)] ksi. = 21.87 ksi. = 21,870 lb/in^2**.

Finally the **Allowable Load = Stress * Area = 21,870 lb/in^2 * 11 in^2 = 240,570 lb.**

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Topic 7.2e: Wood Columns - Example 5

Four twenty five foot long Douglas Fir columns are to support a observation tower. The columns are rought cut and exactly 5" by 5". How heavy can the observation platform be before buckling will occur? (You should assume the wood columns are fixed on both ends.)

Wood columns with a rectangular cross section b x d, where b > d

Short and/or Intermediate Columns

Range Limits: $0 \leq \frac{L_e}{d} \leq 11$

Allowable Stress: $\sigma_{all} = \sigma_c$

Range Limits: $11 \leq \frac{L_e}{d} \leq k$

Allowable Stress: $\sigma_{all} = \sigma_c \left[1 - \frac{1}{3} \left(\frac{L/d}{k} \right)^4 \right]$

$$k = 0.671 \sqrt{E/\sigma_c}$$

Long Columns

Range Limits: $k \leq \frac{L_e}{d} \leq 50$

Allowable Stress: $\sigma_{all} = \frac{0.3\,E}{\left(\frac{L_e}{d}\right)^2}$

We first wish to determine the slenderness ratio we use **slenderness ratio = L_e / d = (.5*25' * 12"/ft) / 5 in. = 30.**

We next need to calculate the transition slenderness ratio, **k = .671 sqrt(E / allowable compressive stress along grain) = .671 sqrt(1.3 x 10^6 lb/in^2 / 6 x 10^3 lb/in^2) = 9.88**

Since the column's slenderness ratio is greater than k, we use the appropriate formula (for wood) for long slender columns. The **Allowable Stress = [.3E / (L_e /**

$d)^2] = .3 * 1.3 \times 10^6\ lb/in^2 / (30)^2 = 433\ lb/in^2$.

Finally the **Allowable Load = Stress * Area = 433 lb/in^2 * 25 in^2 = 10,825 lb./per column**. And since there are four columns, the **platform weight could be 4 * 10,825 lb. = 44,300 lb.**

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Topic 6.3: Columns & Buckling - Problem Assignment

1. A 10 foot long wood beam, nominally 2" by 4" (actual 1.5" by 3.5"), is used as a pinned-pinned column. Determine the slenderness ratio, and the Euler Buckling load for this column. Young's modulus is 1.8×10^{6} lb/in^2, and the allowable yield stress is 6000 psi.
(277, 1220 lb.)

2. A 16 foot long wood beam, nominally 4" by 8" (actual 3.5" by 7.25"), is used as a fixed - pinned column. Determine the slenderness ratio, and the Euler Buckling load for this column. Young's modulus is 2×10^{6} lb/in^2, and the allowable yield stress is 6600 psi.
(133, 28,360 lb.)

3. A 16 foot long WT 6 x 46 steel T-beam is used as a fixed - free column. Determine the slenderness ratio, the Euler Buckling load, and the axial stress when the Euler Buckling load is applied. Young's modulus is 30×10^{6} lb/in^2, and the allowable yield stress is 38×10^{3} lb/in^2.
(254, 61,810 lb., 4580 psi.)

4. A wood yard stick has dimensions, length = 36", width = 2", thickness = 3/8". If one end of the year stick is held fixed and a person pushes on the other end, what force will cause the yard stick to fail in Euler buckling. Assume both fixed-free, and fixed - pinned columns and compare your answers. Use a Young's modulus of 2×10^{6} lb/in^2, and the allowable yield stress of 6000 psi.
[fixed-free: 34 lb. (45psi.); fixed-pinned: 274 lb. (365 psi.)]

5. A 40 foot long W8 x 40 structural steel I-beam is used as a column. Young's modulus is 30×10^{6} lb/in^2 , and the allowable yield stress is 35×10^{3} lb/in^2 . Determine the maximum safe load assuming:
a) pinned-pinned; b) pinned-fixed; c) fixed - free; d) fixed - fixed conditions for ends of the beam.
[a. 32,950 lb (2790 psi), b. 67100 lb. (5680 psi), c. 8200 lb. (695 psi), d. 127,200 lb. (10,780 psi)]

6. Select the best (safe & lightest) structural steel wide flange I-beam (from web table) to be used as a 16 foot long, pinned-pinned, column to support an axial load of 120,000 lb. Young's modulus is 30×10^{6} lb/in^2 , and the allowable yield

stress is 36×10^{3} lb/in^2.
(W8x31)

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Topic 7.4: Columns & Buckling - Topic Examination

1. A large steel pipe is used as a pinned-pinned column. The pipe is 18 feet long with an outer diameter of 6 inches and an inner diameter of 5 inches. Young's modulus is 30 x 10^6 lb/in^2. Determine a) the slenderness ratio; b) the Euler buckling load; c) the axial stress in the column when the Euler load is applied.
The radius of gyration of hollow circular area: r = [sqrt ($d_o^2 + d_i^2$)] / 4 ; Moment of Inertia: I = [pi * ($d_o^4 - d_i^4$)] / 64
[a) 111; b) 209,030 lb; c) 24,200 psi.]

2. A 20 foot long WT 12 x 42 steel T-beam is used as a fixed - free column. Determine the slenderness ratio, the Euler Buckling load, and the axial stress when the Euler Buckling load is applied. Young's modulus is 30 x 10^6 lb/in^2. [radius of gryration about y-y axis = 1.95"; moment of inertia about y-y axis = 47.2 in^4; area = 12.4 in^2]
[a) 246; b) 60,660 lb.; c) 4,890 psi.]

3. A W 8x40 aluminum 6061 -T6 alloy I-beam is used as a pinned-pinned column. Determine the maximum allowable length permittted if a load of a) 50,000 lb is applied., and if a load of b) 100,000 lb is applied. See table below. [a) 18.65 ft.; b) 13.2 ft.]

-	A-in^2	d - in	w_f - in	t_f - in	t_w - in	I_{xx} - in^4	S_{xx} - in^3	r_{xx}- in	I_{yy} - in^4	S_{yy} -in^3	r_{yy} - in
W8x40	11.80	8.25	8.077	0.558	0.365	146.0	35.5	3.53	49.00	12.10	2.04

<table>
<tr><th colspan="2">Aluminum alloy 6061 - T6</th></tr>
<tr><td>Short and/or Intermediate Columns

Range Limits: $0 \le \frac{L_e}{r} \le 9.5$

Allowable Stress: $\sigma_{all} = 19{,}000$ psi

Range Limits: $9.5 \le \frac{L_e}{r} \le 66$

Allowable Stress: $\sigma_{all} = \left(20.2 - 0.126 \frac{L_e}{r}\right)$ ksi</td><td>Long Columns

Range Limits: $\frac{L_e}{r} > 66$

Allowable Stress: $\sigma_{all} = \frac{51{,}000}{\left(\frac{L_e}{r}\right)^2}$ ksi

ksi = 1000 psi

$\frac{L_e}{r}$ = Slenderness ratio</td></tr>
</table>

4. Select the best (safe and lightest) structural streel I-beam to be used as a pinned-pinned column of length 20 feet which is to carry a load of 60,000 lb. The yield stress of the steel is 36,000 psi. (See tables below)
[More than one beam may work well. The W12 x 36 gives a maximum load of 68,000 lb, with an allowable buckling stress of 6,420 psi for the 20 ft beam with a allow steel stress of 36,000 psi.]

Structural Steel with a yield stress of σ_y

Short and/or Intermediate Columns

Critical Slenderness ratio: $$C_c^{\,2} = \frac{2\,\pi^2 E}{\sigma_y}$$

Range Limits: $$0 \le \frac{L_e}{r} \le C_c$$

Allowable Stress: $$\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right]$$

Factor of Safety: $$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L_e/r}{C_c}\right) - \frac{1}{8}\left(\frac{L_e/r}{C_c}\right)^3$$

Long Columns

Range Limits: $$\frac{L_e}{r} > C_c$$

Allowable Stress: $$\sigma_{all} = \frac{\pi^2 E}{1.92\left(\frac{L_e}{r}\right)^2}$$

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	w_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 5x18.5	5.43	5.12	5.025	0.420	0.265	25.4	9.9	2.16	8.89	3.54	1.28
W 6x16	4.72	6.25	4.030	0.404	0.260	31.7	10.2	2.59	4.42	2.19	0.97
W 6x25	7.35	6.37	6.080	0.456	0.320	53.3	16.7	2.69	17.10	5.62	1.53
W 8x67	19.70	9.00	8.287	0.933	0.575	272.0	60.4	3.71	88.60	21.40	2.12
W 10x29	8.54	10.22	5.799	0.500	0.289	158.0	30.8	4.30	16.30	5.61	1.38
W 10x45	13.20	10.12	8.022	0.618	0.350	249.0	49.1	4.33	53.20	13.30	2.00
W 12x22	6.47	12.31	4.030	0.424	0.260	156.0	25.3	4.91	4.64	2.31	0.85
W 12x36	10.60	12.24	6.565	0.540	0.305	281.0	46.0	5.15	25.50	7.77	1.55
-	A-in^2	d - in	w_f - in	t_f - in	t_w - in	I - in^4	S -in^3	r - in	I - in^4	S -in^3	r - in
W 14x38	11.20	14.12	6.776	0.513	0.313	386.0	54.7	5.88	26.60	7.86	1.54
W 14x74	21.80	14.19	10.072	0.783	0.450	797.0	112.0	6.05	133.00	26.50	2.48
W 14x136	40.00	14.75	14.740	1.063	0.660	1590.0	216.0	6.31	568.00	77.00	3.77
W 14x426	125.00	18.69	16.695	3.033	1.875	6610.0	707.0	7.26	2360.00	283.00	4.34
W 16x50	14.70	16.25	7.073	0.628	0.380	657.0	80.8	6.68	37.10	10.50	1.59
W 16x96	28.20	16.32	11.533	0.875	0.535	1360.0	166.0	6.93	224.00	38.80	2.82
W 18x60	17.70	18.25	7.558	0.695	0.416	986.0	108.0	7.47	50.10	13.30	1.68
W 21x73	21.50	21.24	8.295	0.740	0.455	1600.0	151.0	8.64	70.60	17.00	1.81

W 24x94	27.70	24.29	9.061	0.872	0.516	2690.0	221.0	9.86	108.00	23.90	1.98
W 27x114	33.60	27.28	10.070	0.932	0.570	4090.0	300.0	11.00	159.00	31.60	2.18
W 33x240	70.60	33.50	15.865	1.400	0.830	13600.0	813.0	13.90	933.00	118.00	3.64
W 36x300	88.30	36.72	16.655	1.680	0.945	20300.0	1110.0		1300.00	156.00	

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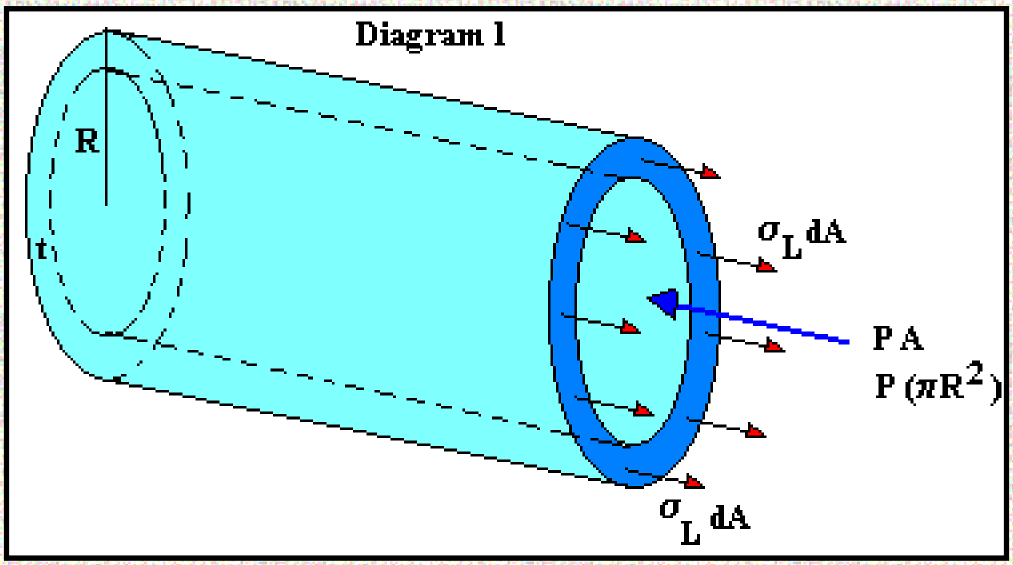
Topic 7.5:Pressure Vessels - Thin Wall Pressure Vessels

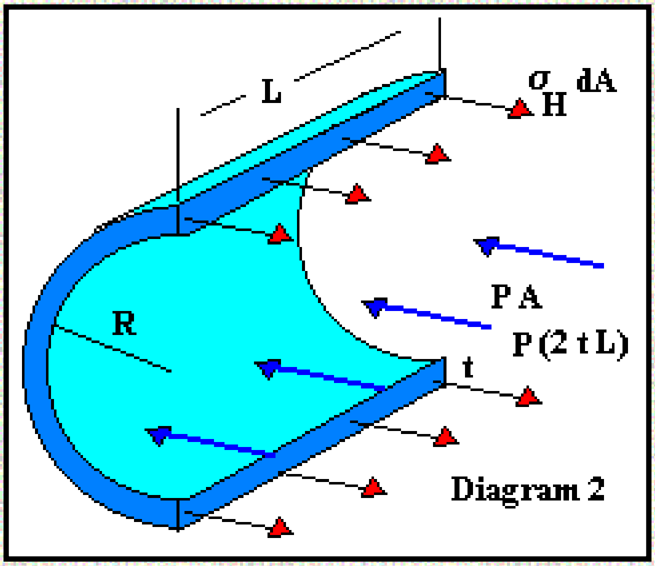
Thin wall pressure vessels are in fairly common use. We would like to consider two specific types. Cylindrical pressure vessels, and spherical pressure vessels. By thin wall pressure vessel we will mean a container whose wall thickness is less than 1/10 of the radius of the container. Under this condition, the stress in the wall may be considered uniform.

We first look at a cylindrical pressure vessel shown in Diagram 1, where we have cut a cross section of the vessel, and have shown the forces due to the internal pressure, and the balancing forces due to the longitudinal stress which develops in the vessel walls. (There is also a transverse or circumferential stress which develops, and which we will consider next.)



The longitudinal stress may be found by equating the force due to internal gas/ fluid pressure with the force due to the longitudinal stress as follows:
P(A) = σ_L(A'); or P(3.1416 * R^2) = σ_L (2 * 3.1416 * R * t), then canceling terms and solving for the longitudinal stress, we have:
σ_L = P R / 2 t ; where
P = internal pressure in cylinder; R = radius of cylinder, t = wall thickness

To determine the relationship for the transverse stress, often called the hoop stress, we use the same approach, but first cut the cylinder lengthwise as shown in Diagram 2.



We once again equate the force on the cylinder section wall due to the internal pressure with the resistive force which develops in walls and may be expressed in terms of the hoop stress, σ_H. The effective area the internal pressure acts on may be consider to be the flat cross section given by (2*R*L). So we may write:

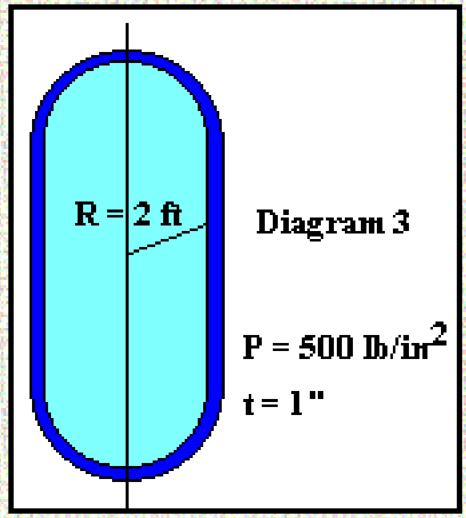
$P(A) = \sigma_H(A')$; or $P(2*R*L) = \sigma_H (2*t*L)$, then canceling terms and solving for the hoop stress, we have:

$\sigma_H = P R / t$; where

P = internal pressure in cylinder; R = radius of cylinder, t = wall thickness

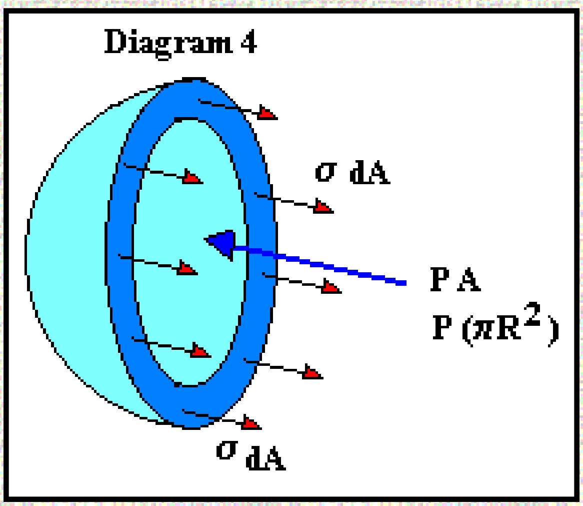
We note that the hoop stress is twice the value of the longitudinal stress, and is normally the limiting factor. The vessel does not have to be a perfect cylinder. In any thin wall pressure vessel in which the pressure is uniform and which has a cylindrical section, the stress in the cylindrical section is given by the relationships above.

Example A: A thin wall pressure vessel is shown in Diagram 3. It's cylindrical section has a radius of 2 feet, and a wall thickness of the 1". The internal pressure is 500 lb/in^2. Determine the longitudinal and hoop stresses in the cylindrical region.



Solution: We apply the relationships developed for stress in cylindrical vessels.
σ_L **= P R / 2 t = 500 lb/in^2 * 24"/2 * 1" = 6000 lb/in^2.**
σ_H**= P R / t = 500 lb/in^2 * 24"/ 1" = 12, 000 lb/in^2.**

Next we consider the stress in thin wall spherical pressure vessels. Using the approach as in cylindrical vessels, in Diagram 4 we have shown a half section of a spherical vessel. If we once again equal the force due to the internal pressure with the resistive force expressed in term of the stress, we have:



P(A) = σ(A'); or P(3.1416 * R^2) = σ(2 * 3.1416 * R * t), then canceling terms and solving for the stress, we have:
σ= P R / 2 t ; where
P = internal pressure in cylinder; R = radius of cylinder, t = wall thickness

Note that we have not called this a longitudinal or hoop stress. We do not do so since the symmetry of the sphere means that the stress in equal in what we could consider a longitudinal and/or transverse direction.

Example B. In Example A, above, if the radius of the spherical section of the container is also 2 feet, determine the stress in the spherical region.

Solution: We apply our spherical relationship:
σ= P R / 2 t = 500 lb/in^2 * 24"/2 * 1" = 6000 lb/in^2.
Thus in the container in Example A, the limiting (maximum) stress occurs in the in the cylindrical region and has a value of 12,000 lb/in^2.

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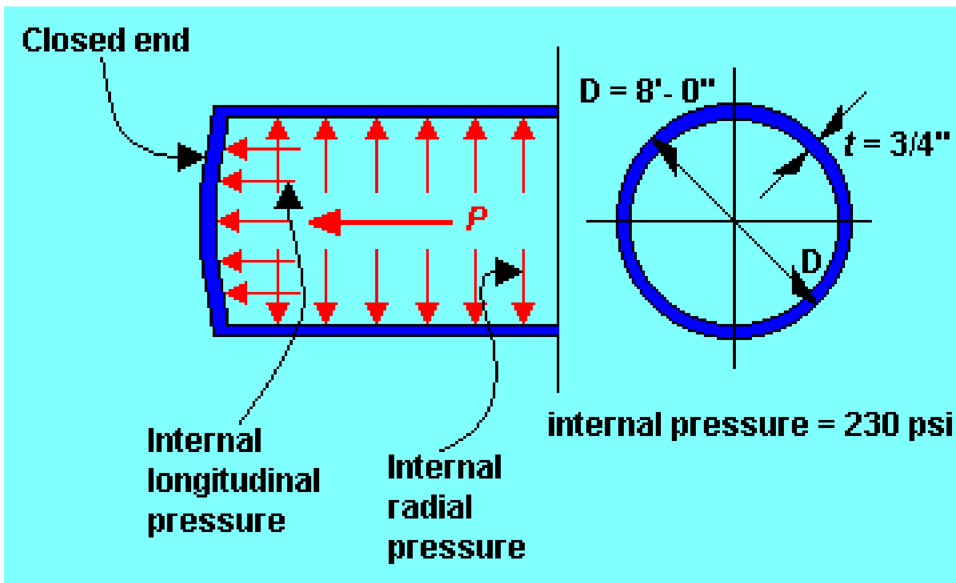
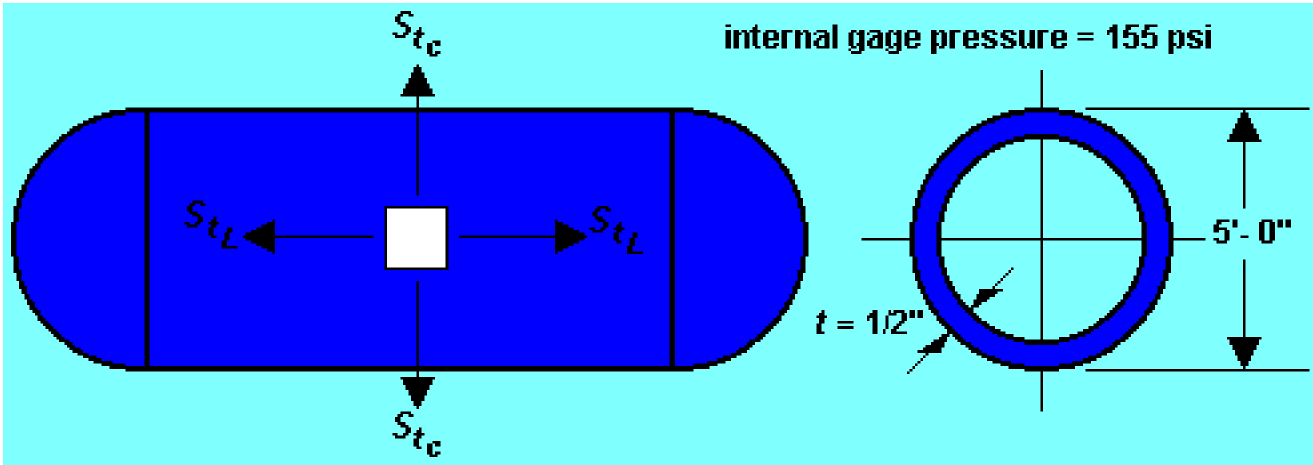
Topic 7.6a :Pressure Vessels - Problem Assignment 1

1. A welded water pipe has a diameter of 8 ft. and a wall of steel plate 3/4 in. thick. After fabrication, this pipe was tested under an internal pressure of 230 psi. Calculate the circumferential stress developed in the walls of the pipe. (14,700 psi)



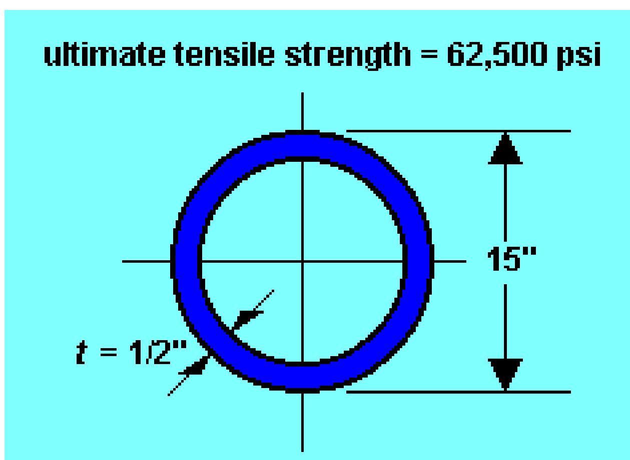
Problem #1

2. Calculate the tensile stresses developed (circumferential and longitudinal) in the walls of a cylindrical boiler 5 ft. in diameter with a wall thickness of 1/2 in. The boiler is subjected to an internal gage pressure of 155 psi. (9300 psi, 4650 psi)



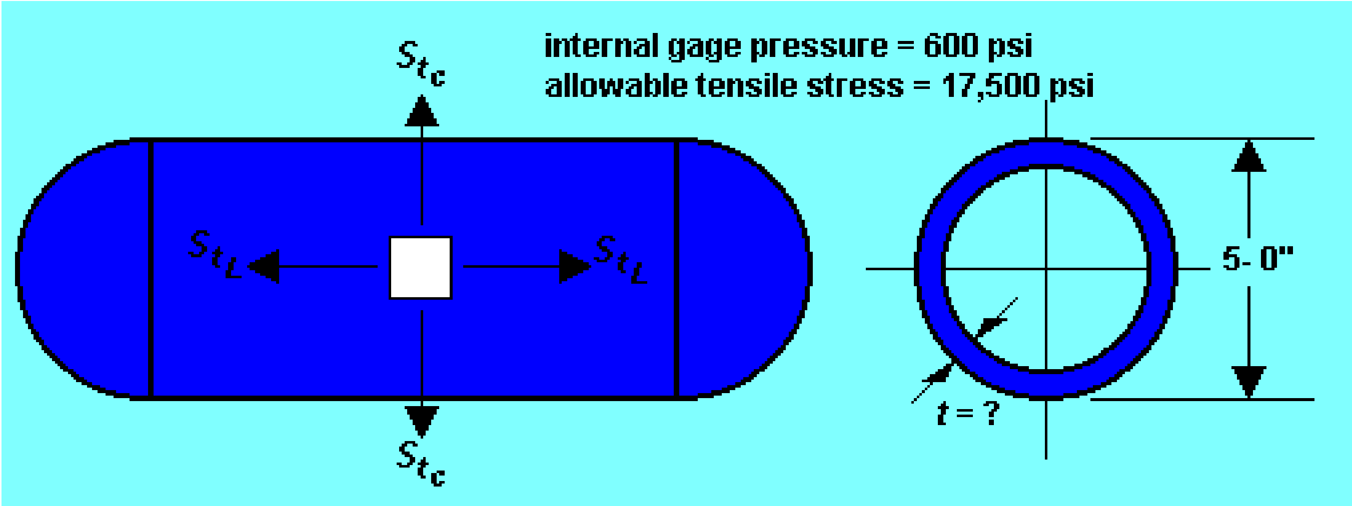
Problem #2

3. Calculate the internal water pressure that will burst a 15 in. diameter cast-iron water pipe if the wall thickness is 1/2 in. Use an ultimate tensile strength of 62,500 psi. for the pipe. (8333 psi)



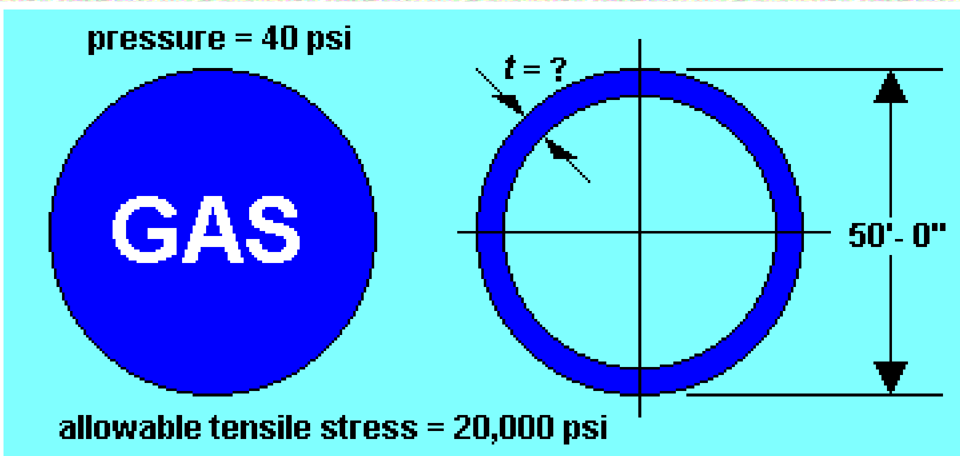
Problem #3

4. Calculate the wall thickness required for a 5 ft. diameter cylindrical steel tank containing gas at an internal gage pressure of 600 psi. The allowable tensile stress for the steel is 17,500 psi. (1.03")



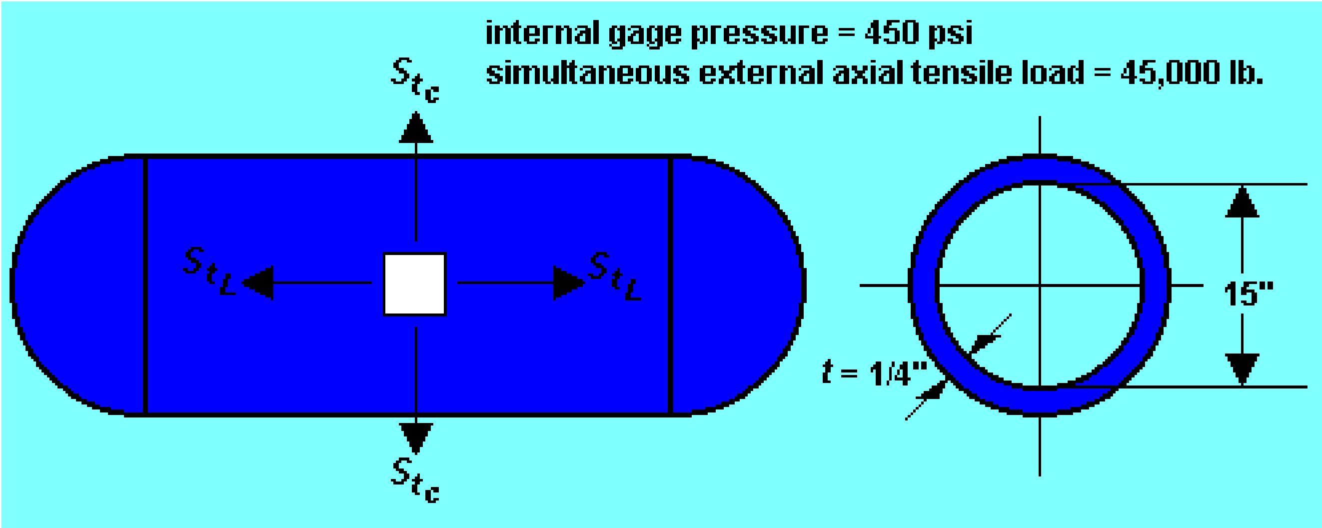
Problem #4

5. A spherical gas container, 50 ft. in diameter, is to hold gas at a pressure of 40 psi. Calculate the thickness of the steel wall required. The allowable tensile stress is 20,000 psi. (.3")



Problem #5

6. Calculate the tensile stresses (circumferential and longitudinal) developed in the walls of a cylindrical pressure vessel. The inside diameter is 15 in. The wall thickness is 1/4 in. The vessel is subjected to an internal gage pressure of 450 psi. and a simultaneous external axial tensile load of 45,000 lb. (13,500 psi, 10,570 psi)

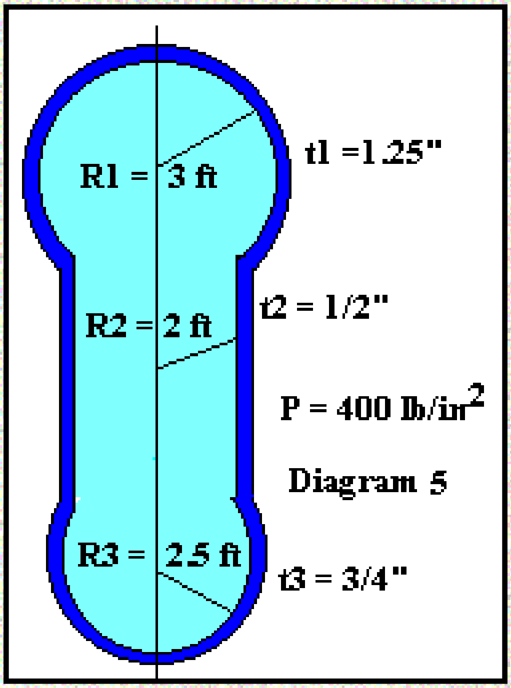


Problem #6

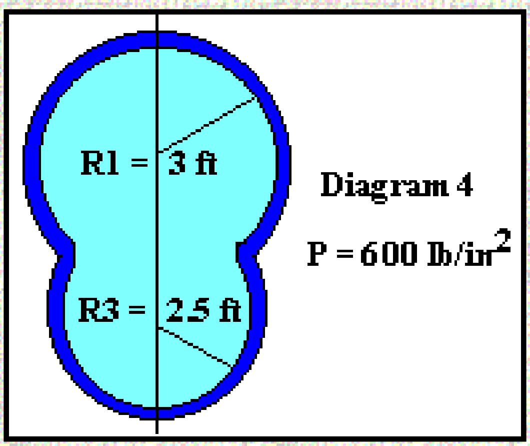
7. A thin wall pressure vessel is composed of two spherical regions and a cylindrical region is shown in Diagram 5.

The larger spherical region has a radius of 3 ft and a wall thickness of 1.25". The smaller spherical region has a radius of 2.5 ft and a wall thickness of 3/4". The cylindrical region has a radius of 2 ft and a wall thickness of 1/2".

Determine the axial and hoop stress in the cylindrical region, and the wall stresses in the two spherical regions. Which stress is the largest? (1: 5,760 psi, 2: 19,200 psi, 9600 psi, 3: 8,000 psi)



8. A thin wall pressure vessel is composed of two spherical regions and is shown in Diagram 4. The larger spherical region has a radius of 3 ft , and the smaller spherical region has a radius of 2.5 ft. The vessel is made of steel with an allowable tensile stress of 24,000 lb/in^2.



If we wish the maximum stress in both spherical sections to at the allowable stress, Determine wall thickness for each spherical region.(.45", .375")

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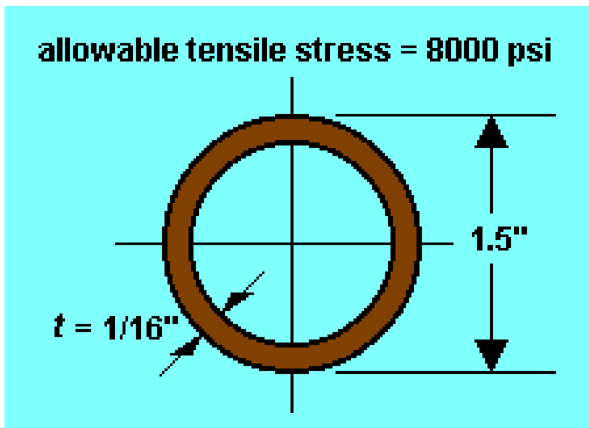
Statics & Strength of Materials

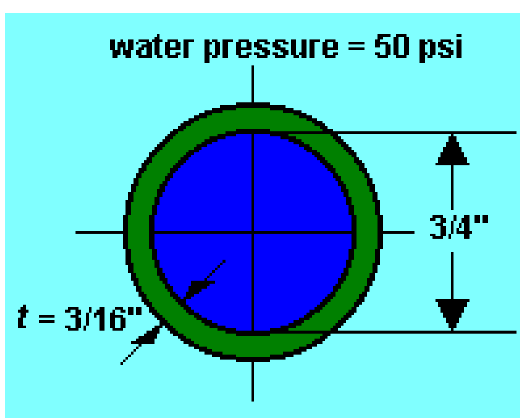
Topic 7.6b: Pressure Vessels - Problem Assignment 2

1. Copper water pipe in a house is 1.5 inches in diameter with 1/16 inch walls. The allowable tensile stress in the copper is 8000 psi. Determine the maximum pressure that the pipes can withstand. (667 psi)



Problem #1

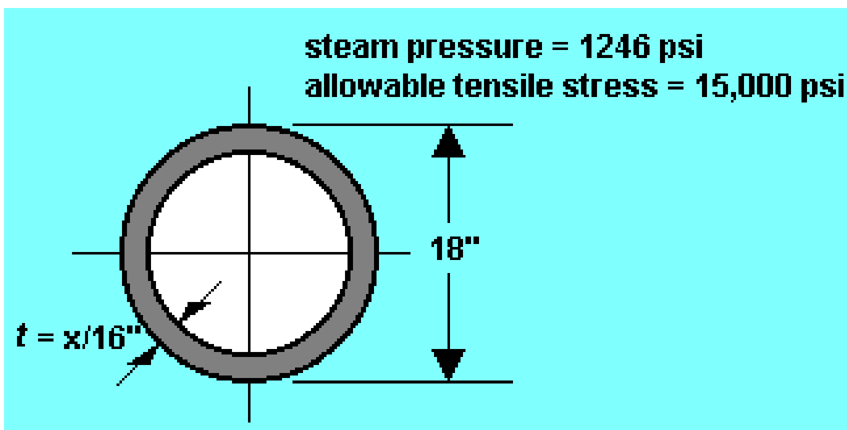
2. A plastic garden hose has an inside diameter of 3/4 inch and a wall thickness of 3/16 inch. Determine the stress in the plastic when the hose is filled with water at 50 psi. (200 psi.)



Problem #2

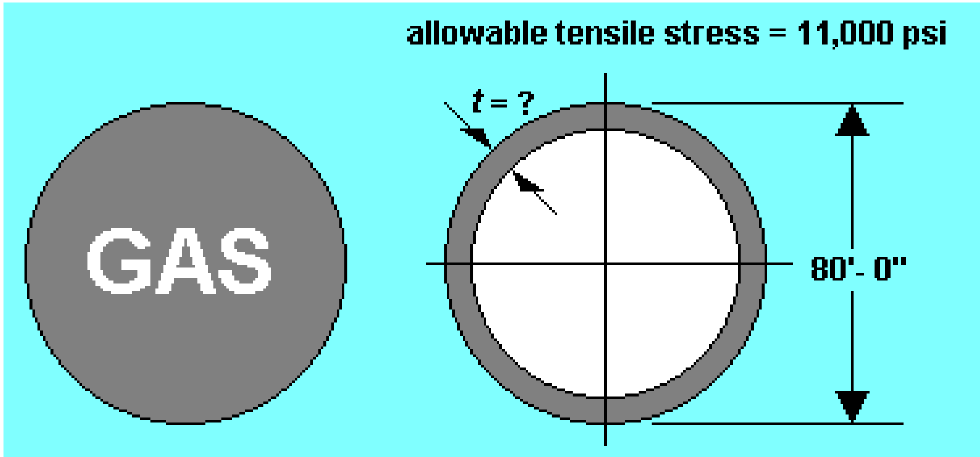
3. Determine the speed of the water in the hose in problem 2 if it is used to fill a five gallon pail in one minute. (one gallon is 231 cubic inches) Express the speed in feet per second and convert to miles per hour if you can. (3.65 ft/sec = 2.49 mi/hr)

4. Process steam is generated at a central plant and distributed to various buildings in a city. The steam is at a temperature of 300 Celsius which has a pressure of 1246 psi. The steam is transferred in an 18 inch SA-202 chrome-magnesium silicon steel pipe having an allowable tensile stress of 15,000 psi. Determine the minimum thickness of the pipe where wall thickness is given in sixteenths of an inch. (3/4 in.)



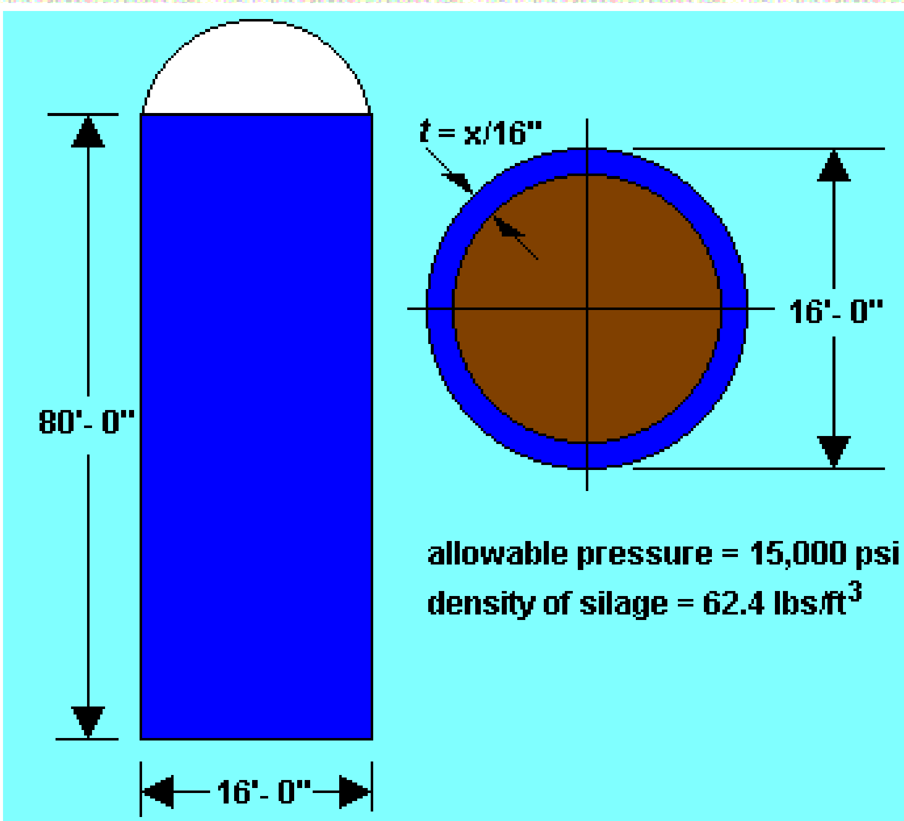
Problem #4

5. Determine the maximum pressure of gas that is stored in a spherical container 80 feet in diameter made of SA-515-55 carbon-silicon steel having an allowable tensile stress of 11,000 psi.[P = 45.8 lb/in^3 * (t in.)] (If t = 1", P = 45.8 psi)



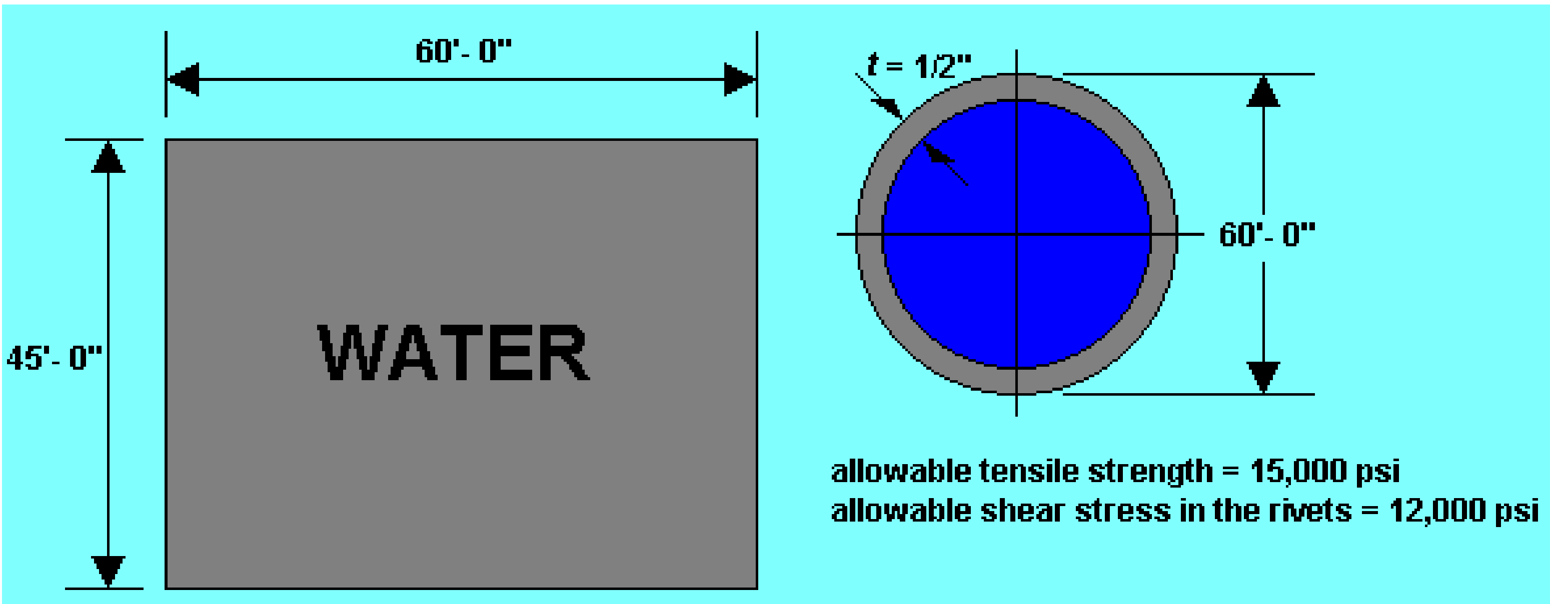
Problem #5

6. Determine the thickness necessary in a steel silo (Harvester or similar) that is 80 feet tall. The maximum allowable pressure in the steel is 15,000 psi. The silo is 16 feet in diameter. Silage has the same density as water (62.4 pounds per cubic foot) and the pressure in a liquid is equal to the weight density of the liquid times the depth of the liquid. Give the thickness to the closest sixteenth of an inch. (1/4 in.)



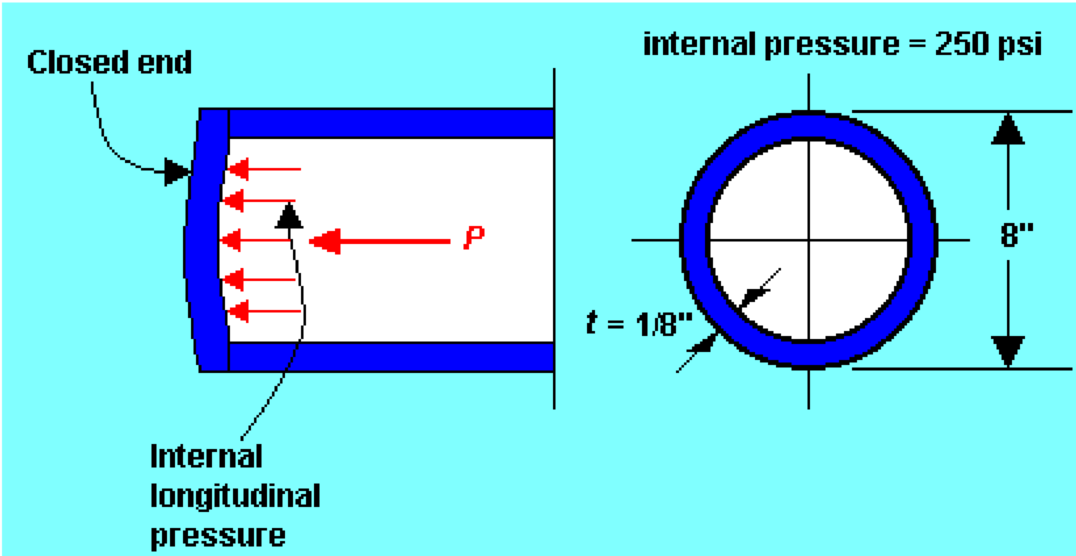
problem #1

7. A water tank is 60 feet in diameter and 45 feet high. It is made of 1/2 inch steel plate with single cover plate riveted joints. Determine the number of 3/4 inch diameter rivets needed to secure the joint. Assume that the allowable tensile strength of the steel is 15,000 psi. and that the allowable shear stress in the rivets is 12,000 psi. [Hoop stress in steel ~ 15,000 psi (depends a little on assumptions), F = Stress * A = 4.05 x 10^6 lb, f/rivet = 5300 lb/rivet; # rivet/vertical seam = 764 x 2 =1528]



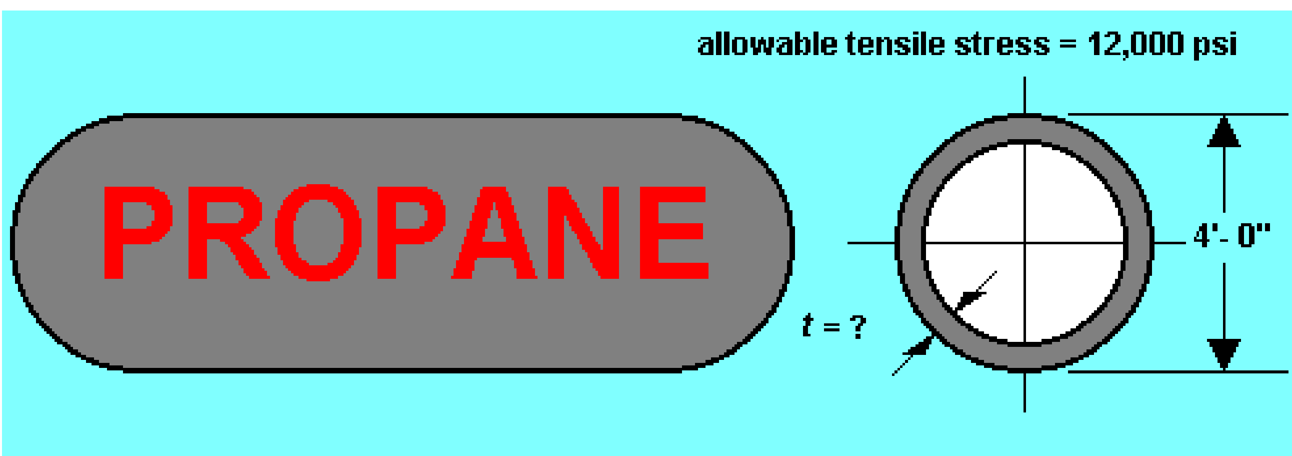
problem #2

8. An air compressor used for a nail gun has a diameter of 8 inches and is charged to a pressure of 250 psi. The tank is made of welded 1/8 inch steel. Determine the tensile stress in the tank. The ends of the tank are welded to the cylindrical body. Determine the force acting on the tank ends that the weld must withstand. [hoop = 8000 psi; Fend = 12, 570 lb]



Problem #3

9. A propane tank used for home heating has a diameter of 4 feet. The allowable tensile stress in the steel of the tank is 12,000 psi. Determine the required thickness of the tank. Propane has the following vapor pressures. [t_{1atm} = .0294 in; t_{40atm} = 1.176 in]



Problem #4

P	T
1 atm,	-40 F
2 atm,	-14 F
5 atm,	34.5 F
10 atm,	80 F
20 atm,	137 F

40 atm, 203 F

Select:
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7.6c Columns & Buckling - Topic Examination

1. A 16 foot long wood 2" by 6" is used as a fixed-fixed column. Young's modulus for the wood is 2×10^{6} lb/in^2. Determine: a) the slenderness ratio; b) the Euler buckling load; c) the axial stress in the column when the Euler load is applied.

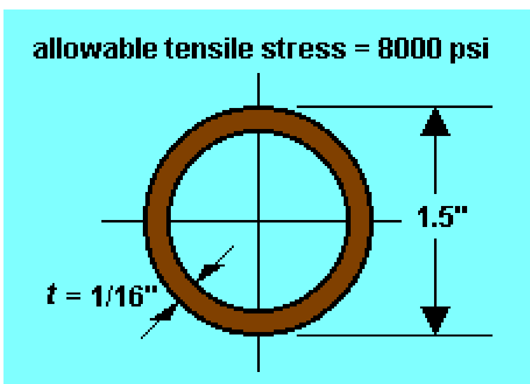
2. A W10x29 steel I-Beam, 18 ft long, is to be used as a column with pinned-pinned ends. Young's modulus for the steel is 30×10^{6} lb/in^2, and the allowable axial stress for the steel is 45,000 psi. Determine the load which will cause the beam to buckle. [Hint: Determine the slenderness ratio. Then use the Euler formula if it is a long column or the J.B. Johnson formula (see below) is it is an intermediate column.] The beam characteristics are listed below.

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick						
-	A-in^2	d - in	w_f - in	t_f - in	t_w - in	I - in^4	S -in^3	r - in	I - in^4	S -in^3	r - in
W 10x29	8.54	10.22	5.799	0.500	0.289	158.0	30.8	4.30	16.30	5.61	1.38

J.B. Johnson's formula:
$$\sigma_{cr} = \frac{P_{cr}}{A} = \left[1 - \frac{(L_e / r)^2}{2\,C^2} \right] \sigma_y$$

C (critical slenderness ratio) :
$$C = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$$

3. Copper water pipe in a house is 1.5 inches in diameter with 1/16 inch walls. The allowable tensile stress in the copper is 8000 psi. Determine the maximum pressure that the pipes can withstand.



Problem #1

4. Select the best (safe and lightest) structural steel I-beam to be used as a fixed-free column of length 16 feet which is to carry a load of 300,000 lb. The yield stress of the steel is 42,000

psi. (See tables below). $E_{steel} = 30 \times 10^6$ psi.

Structural Steel with a yield stress of σ_y	
Short and/or Intermediate Columns	**Long Columns**
Critical Slenderness ratio: $C_c^2 = \frac{2\pi^2 E}{\sigma_y}$	Range Limits: $\frac{L_e}{r} > C_c$
Range Limits: $0 \le \frac{L_e}{r} \le C_c$	Allowable Stress: $\sigma_{all} = \frac{\pi^2 E}{1.92\left(\frac{L_e}{r}\right)^2}$
Allowable Stress: $\sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right]$	
Factor of Safety: $FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L_e/r}{C_c}\right) - \frac{1}{8}\left(\frac{L_e/r}{C_c}\right)^3$	

-	-	-	Flange	Flange	Web	Cross	Section	Info.	Cross	Section	Info.
Designation	Area	Depth	Width	thick	thick	x-x axis	x-x axis	x-x axis	y-y axis	y-y axis	y-y axis
-	A	d	w_f	t_f	t_w	I	S	r	I	S	r
-	in^2	in	in	in	in	in^4	in^3	in	in^4	in^3	in
W 5x18.5	5.43	5.12	5.025	0.420	0.265	25.4	9.9	2.16	8.89	3.54	1.28
W 6x16	4.72	6.25	4.030	0.404	0.260	31.7	10.2	2.59	4.42	2.19	0.97
W 6x25	7.35	6.37	6.080	0.456	0.320	53.3	16.7	2.69	17.10	5.62	1.53
W 8x67	19.70	9.00	8.287	0.933	0.575	272.0	60.4	3.71	88.60	21.40	2.12
W 10x29	8.54	10.22	5.799	0.500	0.289	158.0	30.8	4.30	16.30	5.61	1.38
W 10x45	13.20	10.12	8.022	0.618	0.350	249.0	49.1	4.33	53.20	13.30	2.00
W 12x22	6.47	12.31	4.030	0.424	0.260	156.0	25.3	4.91	4.64	2.31	0.85
W 12x36	10.60	12.24	6.565	0.540	0.305	281.0	46.0	5.15	25.50	7.77	1.55
W 14x38	11.20	14.12	6.776	0.513	0.313	386.0	54.7	5.88	26.60	7.86	1.54
W 14x74	21.80	14.19	10.072	0.783	0.450	797.0	112.0	6.05	133.00	26.50	2.48
W 14x136	40.00	14.75	14.740	1.063	0.660	1590.0	216.0	6.31	568.00	77.00	3.77
W 14x426	125.00	18.69	16.695	3.033	1.875	6610.0	707.0	7.26	2360.00	283.00	4.34
W 16x50	14.70	16.25	7.073	0.628	0.380	657.0	80.8	6.68	37.10	10.50	1.59
W 16x96	28.20	16.32	11.533	0.875	0.535	1360.0	166.0	6.93	224.00	38.80	2.82
W 18x60	17.70	18.25	7.558	0.695	0.416	986.0	108.0	7.47	50.10	13.30	1.68
W 21x73	21.50	21.24	8.295	0.740	0.455	1600.0	151.0	8.64	70.60	17.00	1.81

W 24x94	27.70	24.29	9.061	0.872	0.516	2690.0	221.0	9.86	108.00	23.90	1.98
W 27x114	33.60	27.28	10.070	0.932	0.570	4090.0	300.0	11.00	159.00	31.60	2.18
W 33x240	70.60	33.50	15.865	1.400	0.830	13600.0	813.0	13.90	933.00	118.00	3.64
W 36x300	88.30	36.72	16.655	1.680	0.945	20300.0	1110.0	15.20	1300.00	156.00	3.83

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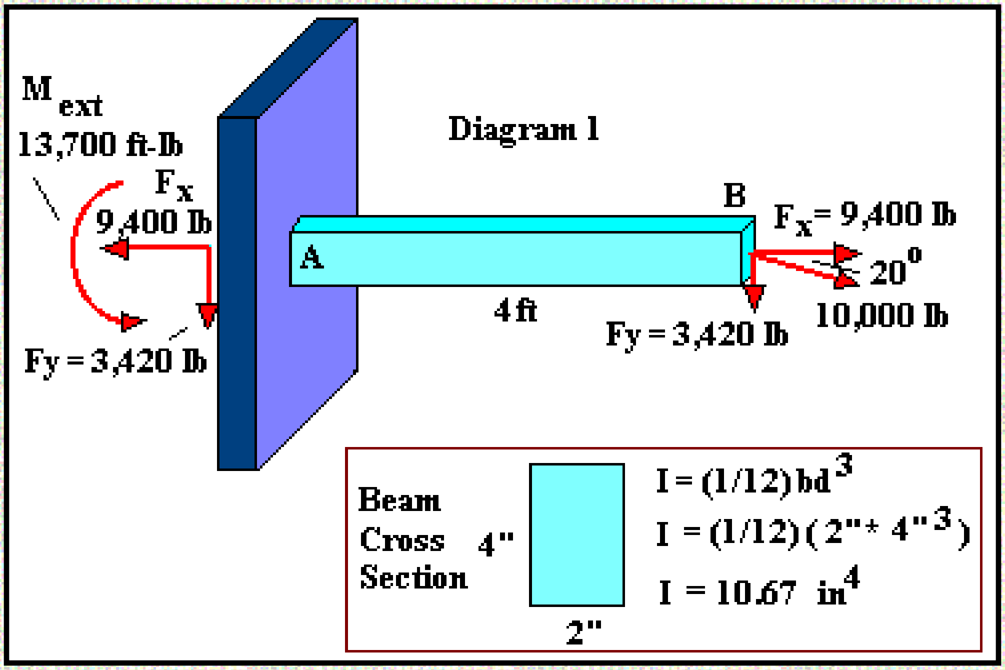
Topic 8 - Special Topics I

Topic 8.1: Special Topics I - Combined Stress

Up to this point we have considered only or mainly one type of applied stress acting on a structure, member of a structure, beam, shaft, rivet, or weld. Many situations involve more than one type of stress occurring simultaneously in a structure. These problems can become relatively complicated. We will look at several examples of relatively simple combined stress problems.

One of the principles we will apply in these examples is the principle of superposition, that is, the resultant stress will be the algebraic sum of the individual stresses - at least in the case of similar stresses acting along the same line, such as the axial stress due to an axial load and a bending stress. We look at this type of problem in the example below.

Example. A four foot long cantilever beam (shown in Diagram 1) is attached to the wall at point A, and has a load of 10, 000 lb. acting (at the centroid of the beam) at angle of 20º below the horizontal. We would like to determine the maximum axial stress acting in the beam cross section.

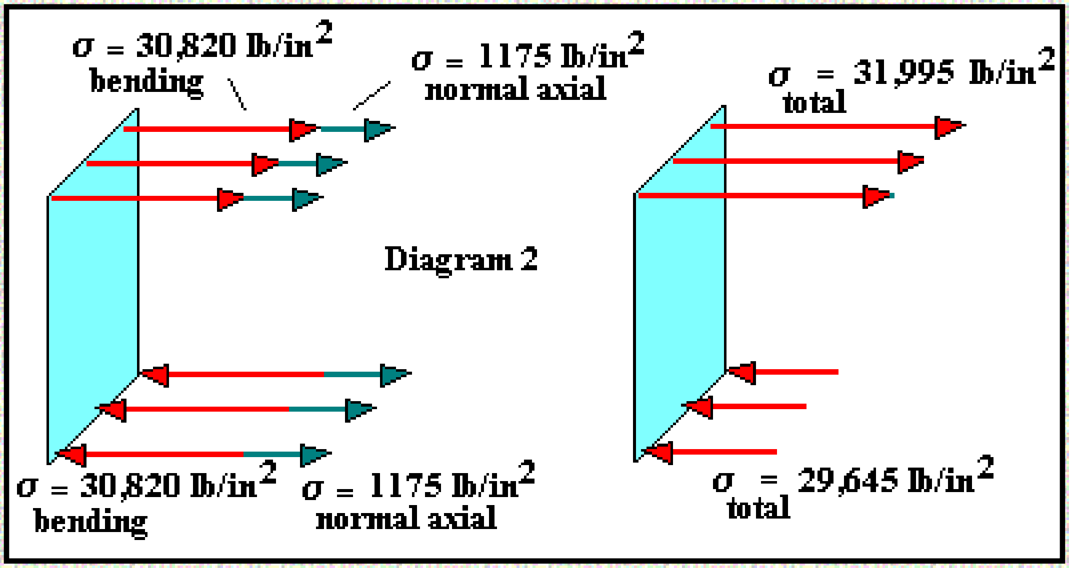


Solution:
We first apply static equilibrium conditions to the beam and determine the external support reactions, and the external moment acting on the beam at point A. Notice we have resolved the **10,000 lb**. load into its perpendicular x and y components. The horizontal component of the load (**9,400 lb**.) produces a normal horizontal axial stress in the beam. The vertical component of the load (**-3,420 lb**.) causes a torque about point A (**13,700 ft-lb**) to act on the beam (balanced by the external moment). The resulting internal bending moment(s) in the beam produces an axial bending stress. **The total axial stress at a point in the beam will be the sum of the normal axial stress and the axial bending stress**.

The Normal Axial Stress = Force / Area = 9,400 lb. / (2" x 4") = 1175 lb / in^2. We note that this stress will be tensile and constant through out the length of the beam. So the maximum normal Axial Stress is **1175 lb / in^2**, and is the same everywhere in the beam.

The maximum bending stress occurs at the outer edge of the beam, and at the point in the beam where the bending moment is a maximum. In the cantilever beam, the maximum bending moment occurs at the wall and is equal to the (negative of) external bending moment. (**M = -13,700 ft-lb. = -164,400 in-lb**.) We can then calculate the maximum bending moment by:
Maximum Bending Stress = M y / I = (164,400 in-lb.)(2") / (10.67 in^4) = 30,820 lb / in^2.
Since the bending moment was negative, this means that the top of the beam (above the centroid) is in tension, and the bottom on the beam is in compression.

We can now combine (sum) the axial stress at the very top and bottom of the beam to determine the maximum axial stress. We see in the beam section in Diagram 2, that the stresses at the top of the beam are both tensile, and so add to a **total tensile stress of 30,820 lb./in^2 + 1,175 lb./in^2 = 31,995 lb./in^2**. At the bottom of the beam, the bending stress is compressive and the normal axial stress in tensile so the resultant bottom **stress is - 30,820 lb./in^2 + 1,175 lb./in^2 = -29,645 lb./in^2 (compression).**



We will now look at several additional examples of combined stresses.

Select :

Topic 8.1a: Combined Stress - Example 1

Topic 8.1b: Combined Stress - Example 2

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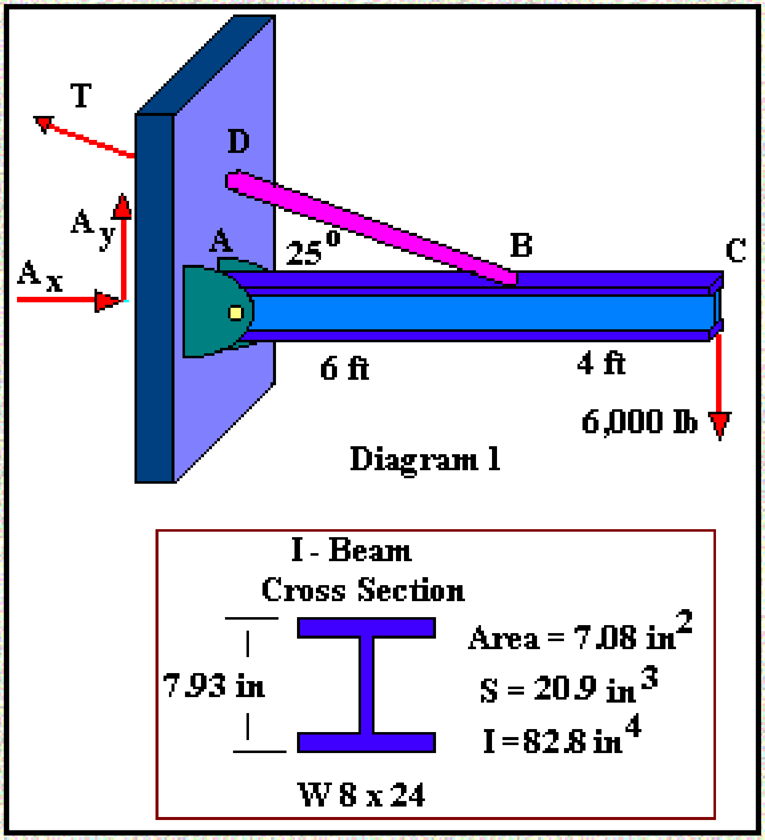
Topic 8.1a: Combined Stress - Example 1

A loaded beam (shown in Diagram 1) is pinned to the wall at point A, and is supported by a rod DB, attached to the wall at point D and to the beam at point B. The beam has a load of 6,000 lb. acting downward at point C. The supporting rod makes an angle of 25º with respect to the beam. The beam cross section is a W8 x 24 I-Beam, with the characteristics shown in Diagram 1. We would like to determine the maximum axial stress acting in the beam cross section.



Solution:

We first apply static equilibrium to the beam and determine the external support reactions acting on the beam at point A.

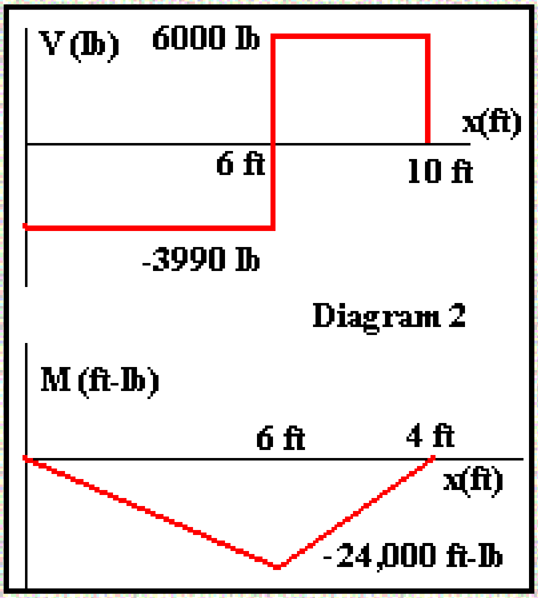
Sum of Force$_x$ = A_x - T cos 25º = 0

Sum of Force$_y$ = A_y + T sin 25º - 6,000 lb. = 0

Sum of Torque$_A$ = -6,000 lb (10ft) + T cos 25º (2.8 ft.) = 0 (where 2.8 ft. = distance from A to D)

Solving: **T = 23,640 lb.; A_x = 21,430 lb., and A_y = -3990 lb**. (A_y acts downward)

We next draw the Shear Force and Bending Moment Diagrams, and use the Bending Moment Diagram to determine the Maximum Bending Stress in the beam. (See Diagram 2.)

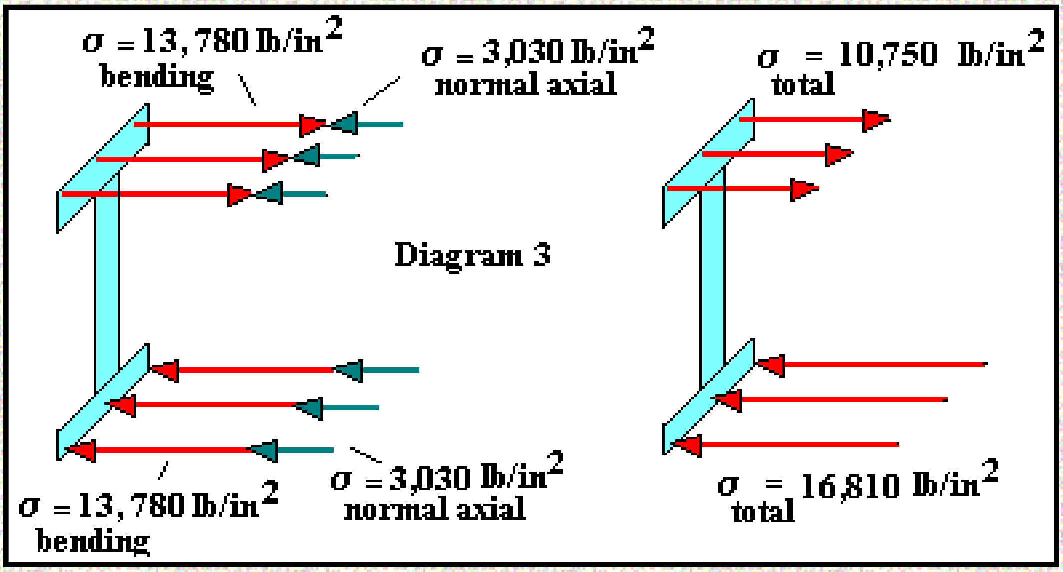


We next will consider the axial stress due to the horizontal force acting on the beam. In section AB the beam is in compression with horizontal axial force of 21,430 lb. (Due to the force A_x and the horizontal component of the force in rod DB.) For beam section BC, there is no horizontal axial force due to an external horizontal force. That is, section AB is in compression, but section BC is not experiencing normal horizontal stress, since it is to the right of where the support rod is attached. (However, there is a horizontal bending stress due to the bending moment, which is in turn due to the vertical loads being applied. This will be considered in a moment.)

The compressive horizontal axial stress in section AB is given simply by:
F/A = 21,430 lb. / 7.08 in^2 = 3,030 lb/in^2. (We have considered the force to act along the centroid of the beam.)
There is a bending stress also acting in the beam. The maximum bending stress occurs at the outer edge of the beam, and at the point in the beam where the bending moment is a maximum. From our bending moment diagram, we see that the maximum bending moment occurs at 6 feet from the left end, and has a value of **-24,000 ft-lb. = -288,000 in-lb.** (The negative sign indicating that the top of the beam is in tension and the bottom of the beam is in compression.) We can then calculate the maximum bending moment by: **Maximum Bending Stress = M / S** Where S is the section modulus for the beam. In this example S = 20.9 in^3. Then:
Maximum Bending Stress = (288,000 in-lb.)/20.9 in^3 = 13,780 lb/in^2.
Since the bending moment was negative, the top of the I-Beam will be in tension,

and the bottom of the beam will be in compression.
The total axial stress at a point in the beam will be the sum of the normal axial stress and the axial bending stress. (See Diagram 3)

We can now combine (sum) the axial stresses at the very top and bottom of the beam to determine the maximum axial stress. We see in the beam section (at 6 ft from left end) in Diagram 3, that the stresses at the bottom of the beam are both compressive, and so add to a **total compressive stress of 13,780 lb./in^2 + 3,030 lb./in^2 = 16, 810 lb./in^2**. At the top of the beam, the bending stress is tensile and the normal axial stress in compressive so the resultant bottom **stress is: +13,780 lb./in^2 - 3,030 lb./in^2 = 10,750 lb./in^2 (tension).**



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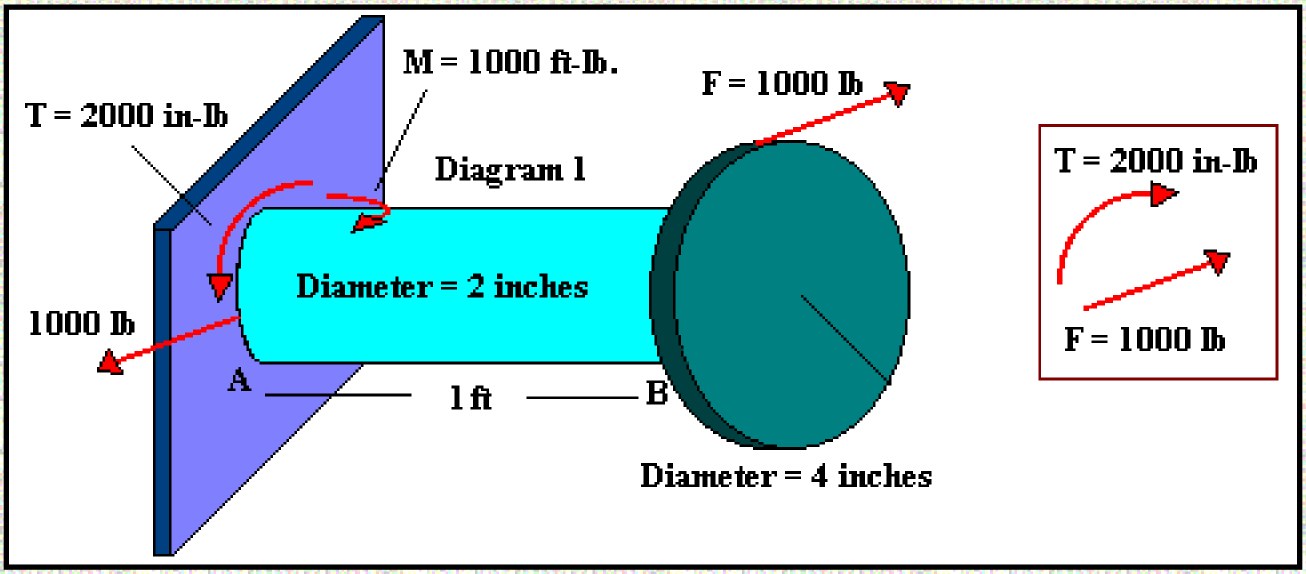
Topic 8.1b: Combined Stress - Example 2

A second example of a structure where stress may be added using the principle of superposition is shown in Diagram 1.



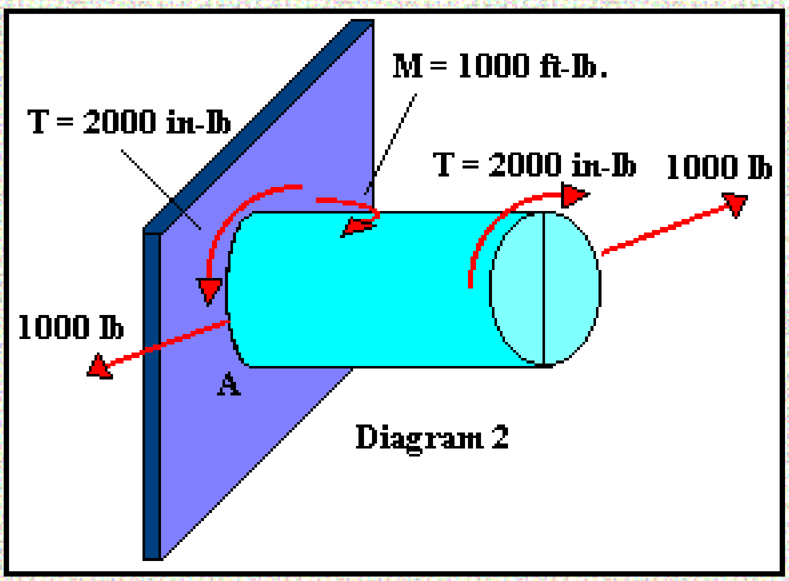
In this example, a solid 1 foot long shaft is attached to a wall at point A, and has a disk attached at end B. A force of 1000 lb. is applied to the outer edge of the disk, as shown. We would like to determine the maximum shear stress in the shaft.
We first consider the static equilibrium conditions. The 1000 lb. force acting on the outside edge of the disk produces a torque of 2000 in-lb. with respect to the center (centroid) of the shaft. Additionally it also effectively exerts a 1000 lb. force perpendicular to the shaft. It can be considered to produce a torque and a force acting at the centroid, as shown in the box in Diagram 1.
In response to this, a torque and perpendicular force develops in the supporting wall, such that the shaft is in translational and rotational equilibrium. (Notice also that an external moment develops at the wall, since the shaft is also acting as a cantilevered beam. An axial bending stress will also develop in the beam, but we will not consider this at this point, since we are concerned with the total shear stress in the beam.)

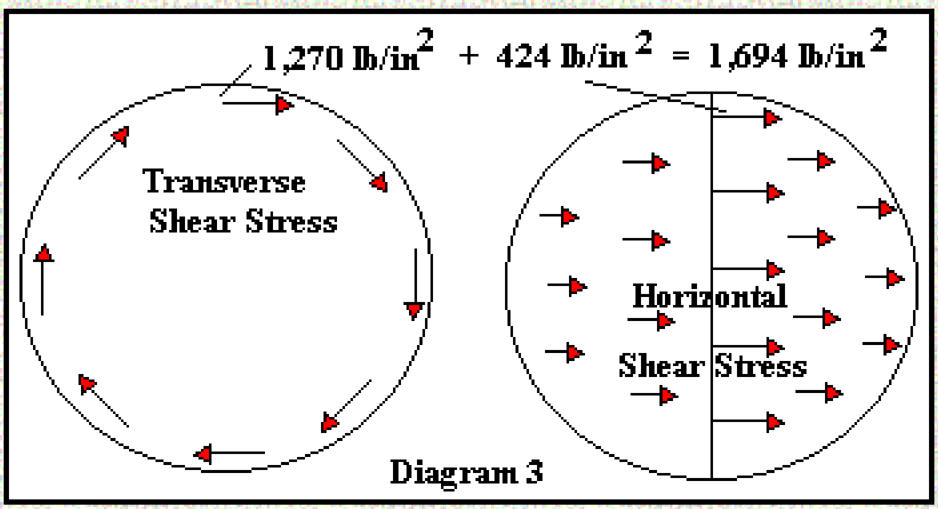
The internal torque in the shaft produces transverse shearing stress. The perpendicular forces produce an internal shearing force in the shaft, which also produces a shear stress. The two stresses may be summed to find the maximum shear stress in the shaft.
Maximum Transverse Shear Stress = Tr / J = (2000 in-lb.) * (1") / (pi * d^4 / 32) = (2000 in-lb.)*(1") / (3.1416 * $2"^4$ / 32) = 1,270 lb / in^2.

Maximum Horizontal Shear Stress = Vay' / Ib = (4 / 3) V / A This last is the expression for the maximum horizontal (and vertical) shear stress in a circular cross section beam. The derivation for this is a bit messy, so the result will simply be stated here. (The student is referred to a complete Strength of Materials Text for the derivation.)

Maximum Horizontal Shear Stress = (4 / 3)V / A = (4 / 3) (1000 lb.) / 3.1416 * 2"^{2} / 4) = 424 lb / in^2.

We can now sum the two shear stresses to determine the maximum shear stress in the shaft. In Diagram 2 and Diagram 3, we have shown a section of the shaft, and cross sectional views showing the directions of the two shear stresses.





As we can see from the diagram, at the top the cross sections the two stresses add giving the result of **1,270 lb / in^2 + 424 lb / in^2 = 1,694 lb / in^2.**
This is the maximum shear stress in the shaft.

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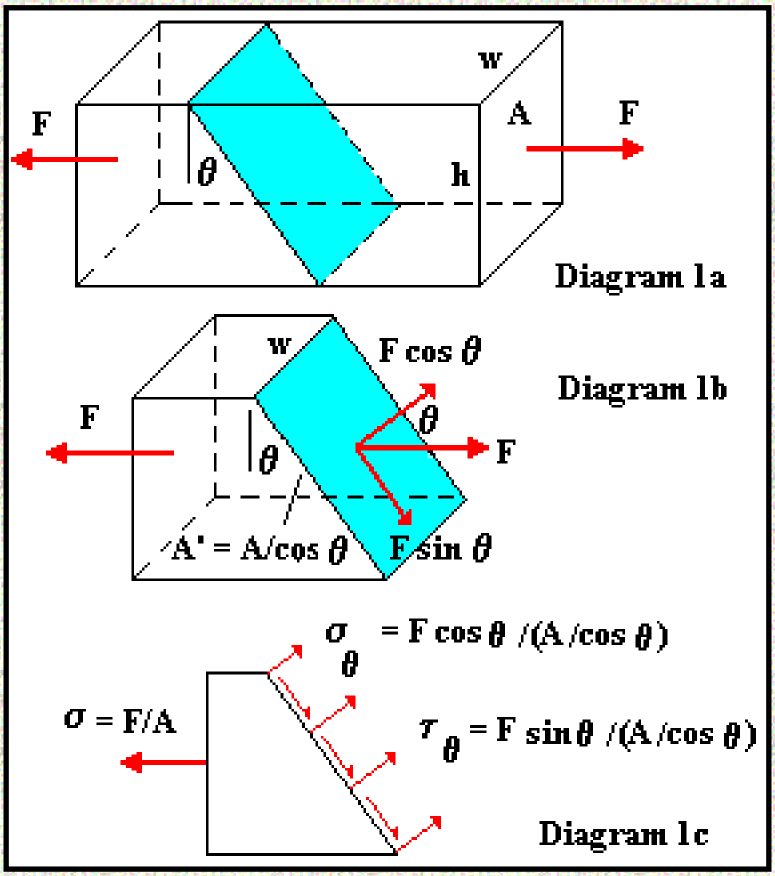
Topic 8.2: Special Topics I - Stresses on Incline Planes

In Diagram 1a we have shown a rectangular section in simple tension with an axial force F apply as shown to each end. The normal stress on the end on the rectangular section is simply given by **Normal Stress = F/A**. However if we cut the section at an angle theta as shown in Diagram 1b, the force F is no longer perpendicular to the inclined plane area. To determine the stresses on the inclined plane area we break the force into components perpendicular and parallel to the incline plane. (Again as shown in Diagram 1b.) The area of incline plane can be written as the cross sectional area of the rectangular section divided by the cosine of angle theta, or **A' = A/cos (theta).**

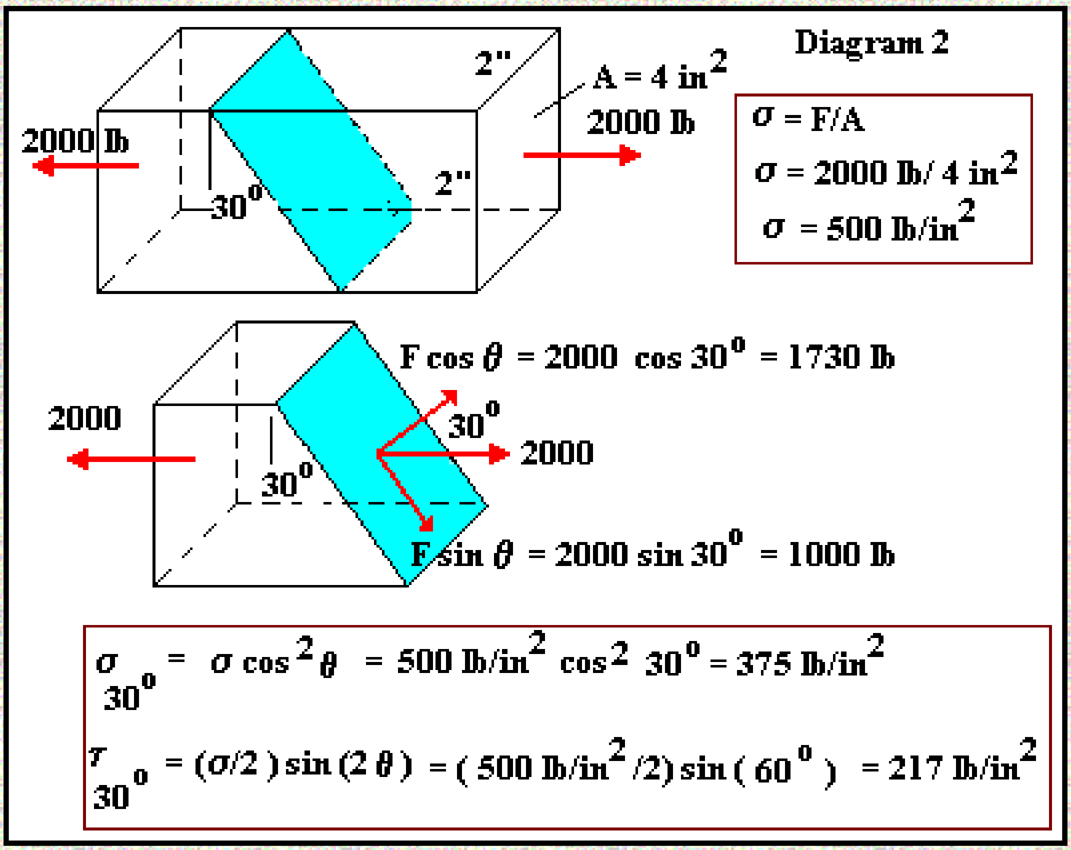


We can then write the axial and shear stress on the inclined area as follows:
Axial Stress = F cos (theta) / A /cos (theta) = (F/A) cos^2(theta)
Shear Stress = F sin (theta) / A /cos (theta) = (F/A) sin(theta) * cos (theta)
or, using a trigonometric identity, we can rewrite the shear stress as
Shear Stress = (F/2A)Sin (2*theta), and finally writing symbolically, and in terms of the normal stress on the rectangular area, we have:
$\sigma_\theta = \sigma\cos^2\theta$; and $\tau_\theta = (\sigma/2)\sin 2\theta$

where these relationships allow us to calculate the axial and shear stress on an incline plane section at an angle theta.

From our relationships we can determine the maximum stresses. The maximum axial stress is just the initial normal stress on the rectangular cross section = F/A. The maximum shear stress occurs at theta = 45º, and is equal to F/(2A), which is half of the maximum axial stress. We now look at a simple example of this application.

Example
In Diagram 2, we have shown a square, 2" by 2", section in tension with a normal force of 2000 lb. acting on each end. We would like to know the axial and shear stress on a 30º incline plane cut through the section.



As shown in Diagram 2, we first calculate the normal stress on the square cross section, and find **Normal Stress (sigma) = 500 lb/in^2**. We next apply the our formula for axial and shear stress on an incline plane (again, as shown in Diagram 2), and find that the normal axial stress on the 30º incline plane is **375 lb/in^2**, and the shear stress on the 30º incline plane is **217 lb/in^2**.

Up to this point we have considered a section with an axial stress only in one direction. We will shortly look at the general case where we have several axial and shear stresses acting. We will wish to determine what are called the principal stresses and the principal planes.

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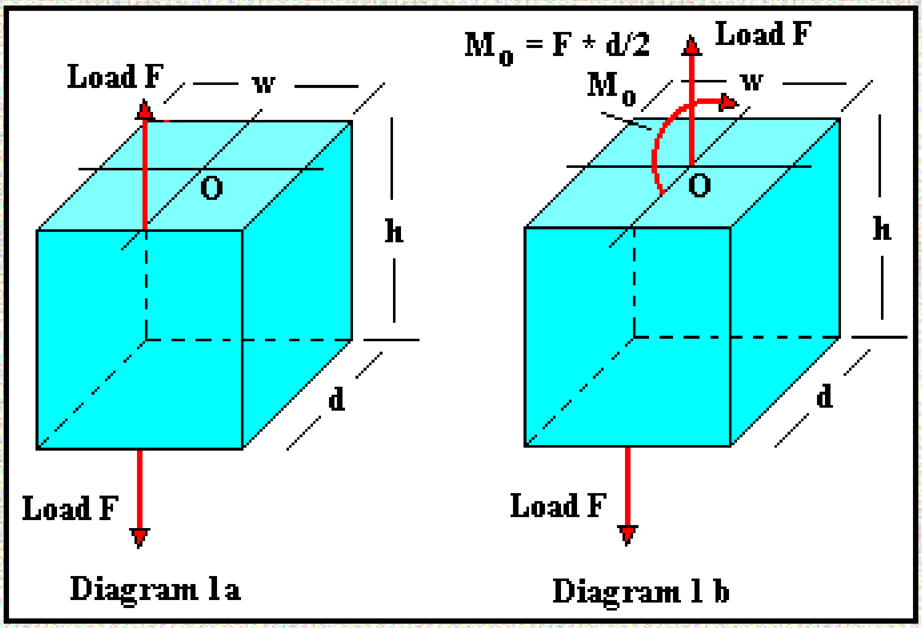
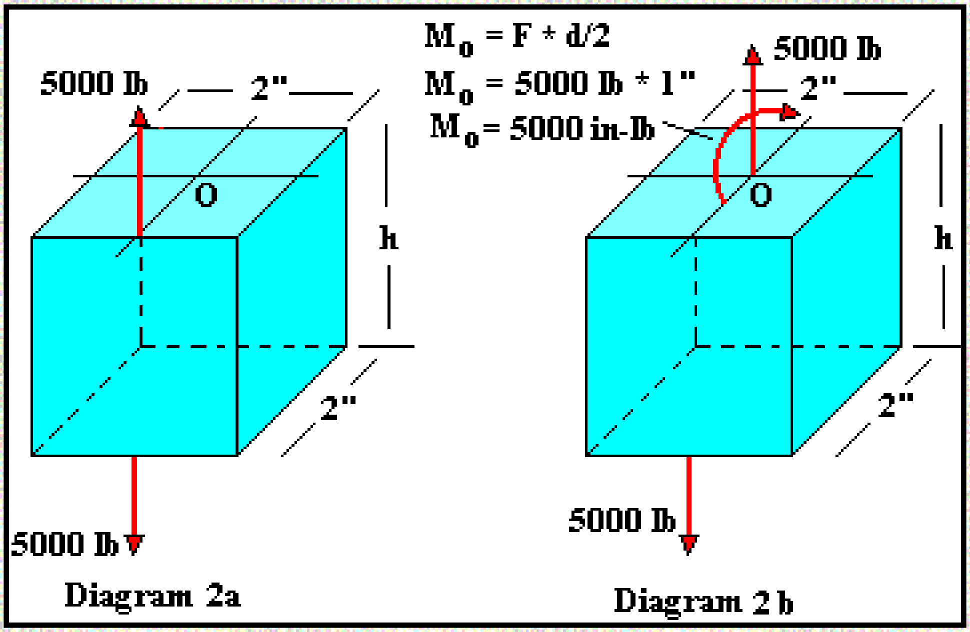
Topic 8.3: Special Topics I - Non-Axial Loads

We next look at the case where there is a non-axial loading on a structure or structural element.

In Diagram 1a, we have shown a structural element in tension with a load F acting at the edge of the element. The load force F is non-axial, it is not acting along the axis of the element, however we can represent the effect of this edge force, F, by an axial force, equal to F and acting at the centroid, and a moment (torque) about the centroid, as shown in Diagram 1b.
When we represent the structural element in this manner, we can determine the axial stress acting on the element by superposition of stresses, as shown in the following example.



Example: In Diagram 2a, we have shown a 2" by 2" structural element with a tensile force of 5000 lb. applied at the edge of the element as shown. In Diagram 2b, we have replaced the 5000 lb. edge force with a 5000 lb. force acting at the centroid of the element and a moment about the centroid of 5000 lb. * 1" = 5000 in-lb.



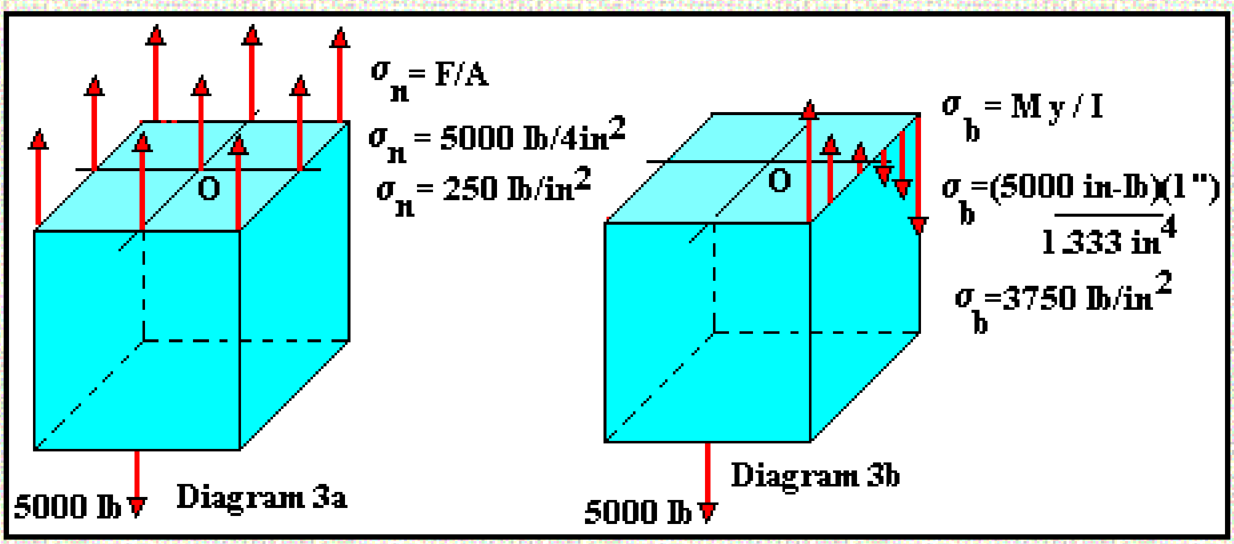
We next determine the normal axial stress and the bending stress on the top cross sectional area. The normal axial stress is given by:

σ_n= F / A = 5000 lb. / (2"x 2") = 250 lb / in^2.

And the bending stress is given by:

= My / I = 5000 in-lb.* 1" / [(1 / 12)2"*2"^{3}] = 3750 lb / in^2. Where for y we have used the maximum distance from the neutral axis to the outer edge of the area, giving us the maximum bending stress. In Diagram 3a we have shown the normal stress distribution on the area. In Diagram 3b we have shown the bending stress distribution along one edge (it is the same across rest of the area). From the direction of stresses we see that they add along the front side of the element face, and subtract along the back side the element. We can then find the maximum axial stress from:

Maximum Axial Stress = 250 lb / in^2 + 3750 lb / in^2 = 4000 lb / in^2.



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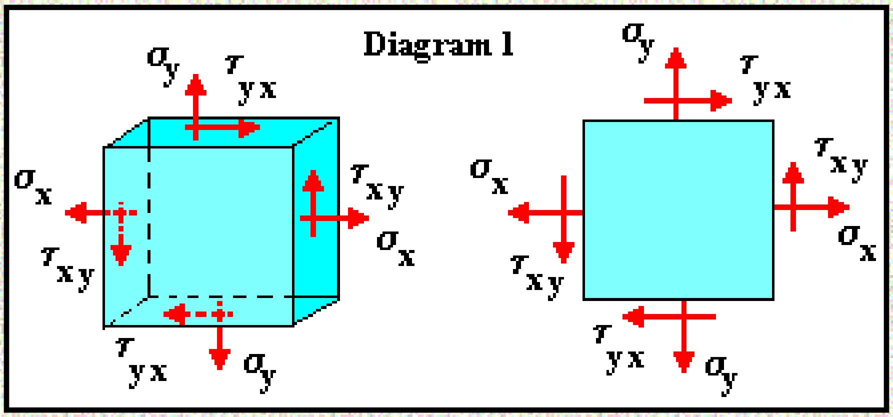
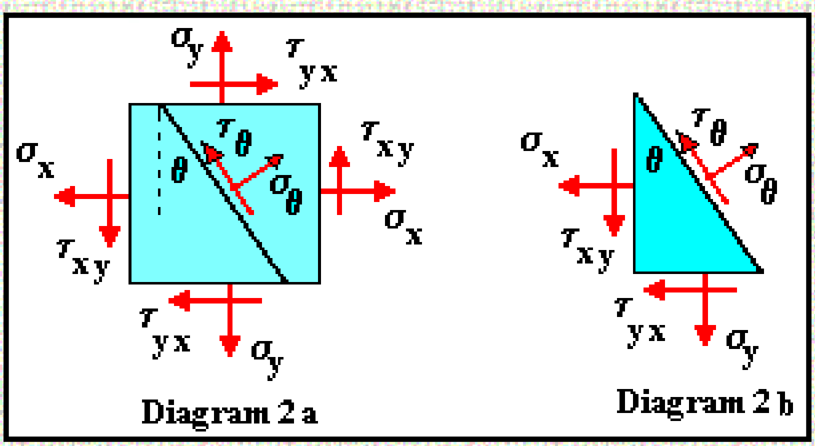
Topic 8.4: Special Topics I - Principal Stresses

We next look at the general case where we have several axial stresses and shear stresses acting. We will look at the plane stresses on an inclined plane section to determine what are called the principal stresses and the principal planes.

In Diagram 1 we have shown a structure element with both normal axial stresses and shear stresses acting on the element. We remember at this point that for static equilibrium the shear stresses Tau_{xy} and Tau_{yx} must be equal in magnitude.



In Diagram 2 we have shown the structure element with a plane cut through it at angle theta. Acting on this plane will be both an axial stress σ_θ and a shear stress τ_θ, as shown in Diagram 2a. We would like to write relationships which allow us to calculate the value of these two stress for any arbitrary plane section.
In Diagram 2b, we have shown a triangular element with axial and shear stresses shown. If we multiply these stresses by the appropriate areas, we have the forces on each surface. We may then apply static equilibrium conditions and write the equilibrium equations. Before we do so, we need to establish a sign convention as follows:



1. Tensile Stress will be considered positive, and Compressive Stresses will be considered negative.
2. The Shear Stress will be considered positive when a pair of shear stress acting on opposite sides of the element produce a counterclockwise torque (couple). (Some text use the opposite direction for the positive shear stress. This changes a sign in several equations, so we must be somewhat careful of signs when working problems and examples.)
3. The incline plane angle will be measure from the vertical, counterclockwise to the plane. This will be the positive direction for the angle.

We now write the following equilibrium equations:
Sum of Forces$_x$ $= -\sigma_x\,(A\cos\theta) - \tau_{xy}(A\sin\theta) - \tau_\theta\,A(\sin\theta) + \sigma_\theta\,A(\cos\theta) = 0$
Sum of Forces$_y$ $= -\sigma_y(A\sin\theta) - \tau_{xy}(A\cos\theta) - \tau_\theta\,A(\cos\theta) + \sigma_\theta\,A(\sin\theta) = 0$
where we have used that face that **magnitude of τ_{xy} = magnitude of τ_{yx}.**

The two equation above may be solved for two "unknowns". In this case we solve for σ_θ, and τ_θ; the stresses acting on the incline plane shown in Diagram 2. The details of solving these two simultaneous equations involve a number of trigonometric identities and some extended algebraic manipulations, and will not be presented. The results of this process are as follows:

$$\sigma_\theta = (\sigma_x + \sigma_y)/2 + [(\sigma_x - \sigma_y)/2](\cos 2\theta) + \tau_{xy}(\sin 2\theta)$$
$$\tau_\theta = -[(\sigma_x - \sigma_y)/2](\sin 2\theta) + \tau_{xy}(\cos 2\theta)$$

The equations may be referred to as **transformation equations.** In the above equations, it is clear that we will may have both maximum and minimum stress values. The maximum and minimum normal axial stresses are known as the **Principal Stresses**, and the planes at which they occur are known as the **Principal Planes**. At the principal planes, where the axial stress is a maximum or minimum, the shear stress will be zero. The value of the principal(maximum/minimum) stresses are given by:

$$\sigma_{max} = (\sigma_x + \sigma_y)/2 + \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$$
$$\sigma_{min} = (\sigma_x + \sigma_y)/2 - \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$$
$$\tau_{max} = +/- \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$$

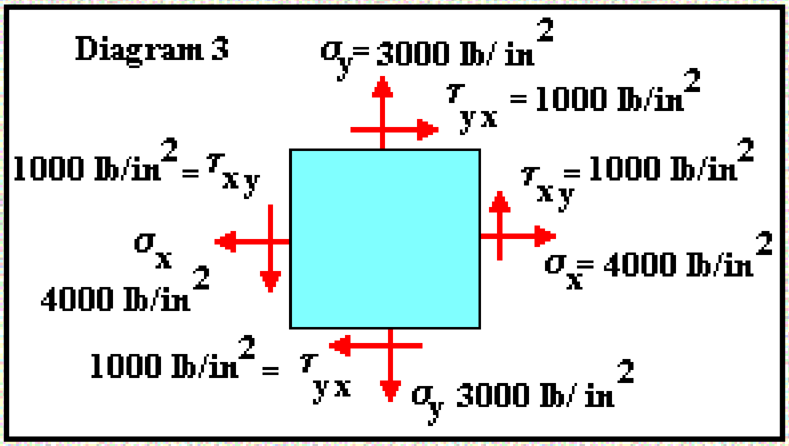
and
$\mathrm{Tan}2\theta = \tau_{xy}/[(\sigma_x - \sigma_y)/2]$ for the principal plane. The planes for maximum shear stress vary by 45^o from the principal planes.

The principal stresses may also be related as follows:

$$\sigma_{max} = (\sigma_x + \sigma_y)/2 + \tau_{max}$$

$\sigma_{min} = (\sigma_x + \sigma_y)/2 - \tau_{max}$
$\tau_{max} = (\sigma_{max} - \sigma_{min})/2$

Example. In Diagram 3 a structural element is shown with axial and shear stress given by: normal x-stress = 4000 lb/in^2, normal y-stress = 3000 lb/in^2, shear stress = 1000 lb/in^2. We would like to find the principal planes, the principal stresses, and the maximum shear stress.



We first apply the formula for determining the angle of the principal planes:

$\mathrm{Tan}2\theta = \tau_{xy}/[(\sigma_x - \sigma_y)/2]$ **= 1000 lb/in^2/[(4000 lb/in^2 - 3000 lb/in^2)/2] = 2**, then
2(theta) = 63.4^o, and 243.4^o, and theta = 31.7^o, and 121.7^o. These are the angles of the principal planes. We will calculate the principal stress two ways. First from the general formula for plane stresses
$\sigma_\theta = (\sigma_x + \sigma_y)/2 + [(\sigma_x - \sigma_y)/2](\cos 2\theta) + \tau_{xy}(\sin 2\theta)$**= (4000 lb/in^2 + 3000 lb/in^2)/2 + [(4000 lb/in^2 - 3000 lb/in^2)/2]cos(63.4^o) + 1000 lb/in^2*sin(63.4^o)**
= 3500 lb/in^2 + 224 lb/in^2 + 894 lb/in^2 = 4618 lb/in^2
and for 2nd principal plane
= (4000 lb/in^2 + 3000 lb/in^2)/2 + [(4000 lb/in^2 - 3000 lb/in^2)/2]cos(243.4^o) + 1000 lb/in^2*sin(243.4^o)
= 3500 lb/in^2 - 224 lb/in^2 - 894 lb/in^2 = 2382 lb/in^2

The second method is from specific formula for the maximum/minimum stresses:
$\sigma_{max/min} = (\sigma_x + \sigma_y)/2 +/- \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$ **= (4000 lb/in^2 + 3000 lb/in^2)/2 +/- Sqrt[{(4000 lb/in^2 - 3000 lb/in^2)/2}^{2} + (1000 lb/in^2)^2]**
= 3500 lb/in^2 + 1118 lb/in^2 = 4618 lb/in^2
= 3500 lb/in^2 - 1118 lb/in^2 = 2382 lb/in^2

Note that we arrive at the same result by both methods. And finally we calculate the maximum shear stress from:

$\tau_{max} = +/- \sqrt{[(\sigma_x - \sigma_y) / 2]^2 + \tau_{xy}^2}$ = **Sqrt[{(4000 lb/in^2 - 3000 lb/in^2)/2}^{2} + (1000 lb/in^2)^2] = +/- 1118 lb/in^2.**

Now please select: **Topic 8.4a: Principal Stresses - Example 1**
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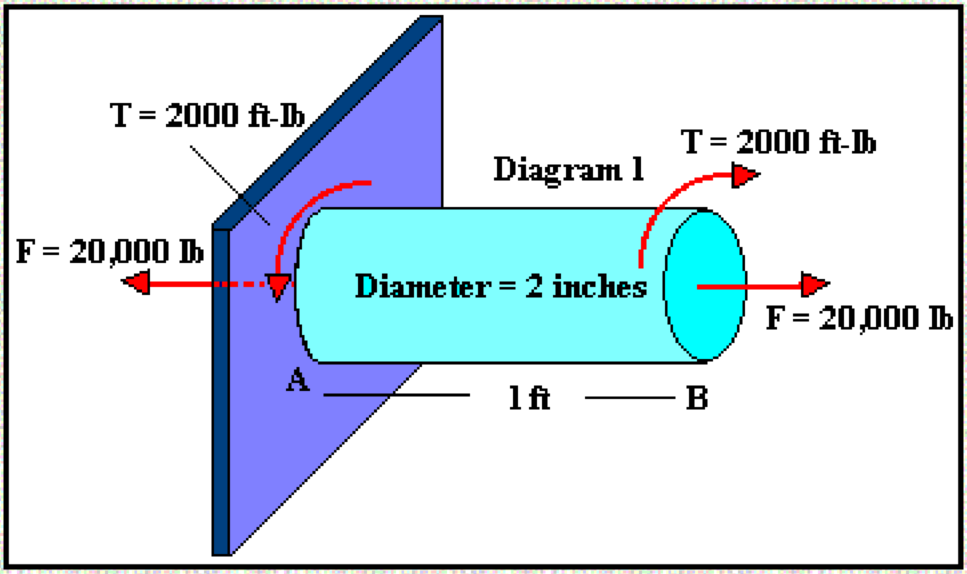
Topic 8.4a: Principal Stresses - Example 1

In Diagram 1 we have shown a shaft attached to a wall at end A with a torque of 2000 ft-lb. and an axial force of 20,000 lb. acting at end B. We wish to determine the principal planes, principal stresses and the maximum shear stress. (The principal, maximum, stress will occur at an element on the outer edge of the shaft as that is where the shear stress is a maximum, while the normal axial stress we will assume is uniform across the area of the shaft.)

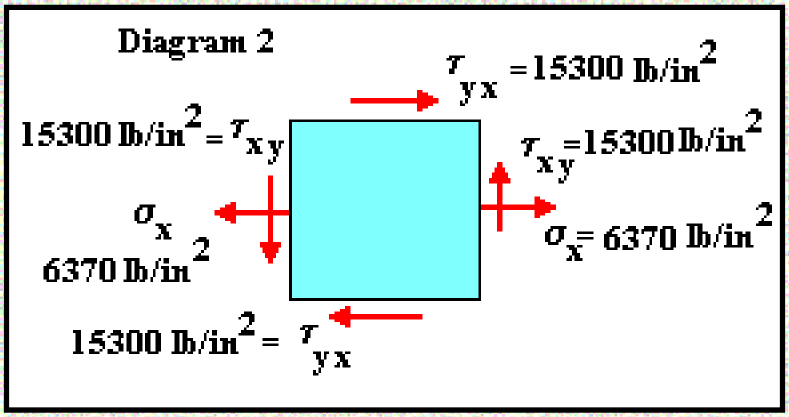


Solution:
We first review briefly the static equilibrium conditions for the shaft. The applied torque of 2000 lb/in^2 at end B is balance by an equal torque which develops in the wall acting on the shaft. The applied horizontal force of 20,000 lb. also applied at end B is balance by an equal 20,000 lb. force which the wall exerts on the shaft. Thus the shaft is in equilibrium. (We have ignored any weight of the shaft.)

We next determine the axial (normal) stress due to the applied 20,000 lb. force by:
σ_{axial} = **F / A = 20,000 lb. / (3.1416 * 1"^{2}) in^2 = 6,370 lb / in^2.** (We assume the stress in uniform across the circular cross section.)
We then determine the maximum transverse shear stress by:
Maximum Transverse Shear Stress = Tr / J = (2000 ft-lb * 12 in / ft) * (1") / (pi * d^4 / 32) = (24,000 in-lb.)*(1") / (3.1416 * 2"^{4} / 32) = 15,300 lb / in^2.

We next show a shaft element with these values indicated in Diagram 2. Also remembering our sign conventions, shown below.
1. Tensile Stress will be considered positive, and Compressive Stresses

will be considered negative.
2. The Shear Stress will be considered positive when a pair of shear stress acting on opposite sides of the element will produce a counterclockwise torque (couple). (Some text use the opposite direction for the positive shear stress. This changes a sign in several equations, so we must be somewhat careful of signs when working problems and examples.)
3. The incline plane angle will be measure from the vertical, counterclockwise to the plane. This will be the positive direction for the angle.



And finally we apply our plane stress and transformation equations.
We first apply the formula for determining the angle of the principal planes:
$\tan 2\theta = \tau_{xy} / [(\sigma_x - \sigma_y)/2]$ **= 15,300 lb/in^2 / [(6370 lb/in^2 - 0)/2] = 4.8, then 2(theta) = 78.4º, and 258.4º, and theta = 39.2º, and 129.2º.** These are the angles of the principal planes.

We calculate the principal stress from specific formula for the maximum/minimum stresses:
$\sigma_{max/min} = (\sigma_x + \sigma_y)/2 \;+/-\; \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$ **= (6370 lb/in^2 + 0)/2 +/- Sqrt[{(6370 lb/in^2 - 0)/2}^{2} + (15,300 lb/in^2)^2]**
= 3185 lb/in^2 + 15,630 lb/in^2 = 18,815 lb/in^2
= 3185 lb/in^2 - 15,630 lb/in^2 = -12445 lb/in^2
And finally we calculate the maximum shear stress from:
$\tau_{max} = +/- \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$ **= Sqrt[{(6370 lb/in^2 - 0)/2}^{2} + (15,300 lb/in^2)^2] = +/- 15,630 lb/in^2.**

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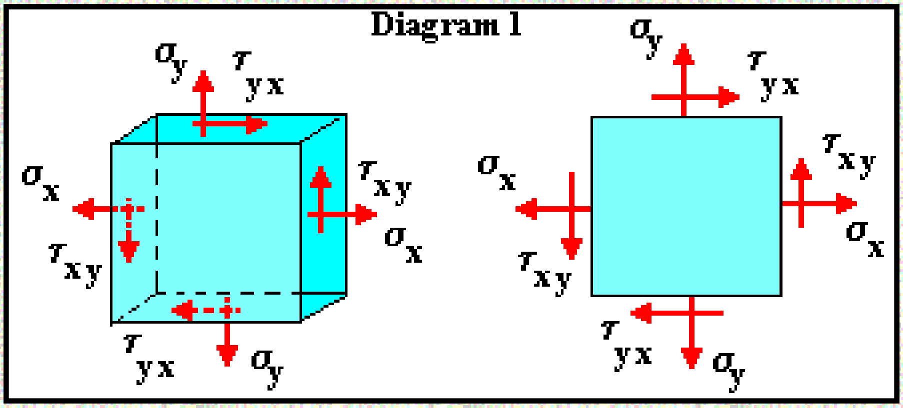
Topic 8.5: Special Topics I - Mohr's Circle

The equations for the axial and shear stress at any plane in a structural element, and the equations for the principal stress present in Topic 8.3 Plane Stresses are accurate and useful, however they are not easily remembered. A very useful way of expressing and visualizing the plane stresses in a loaded structural element is method known as Mohr's Circle, developed by a German engineer, Otto Mohr (1835 - 1918).

The method for drawing Mohr's Circle is as follows:

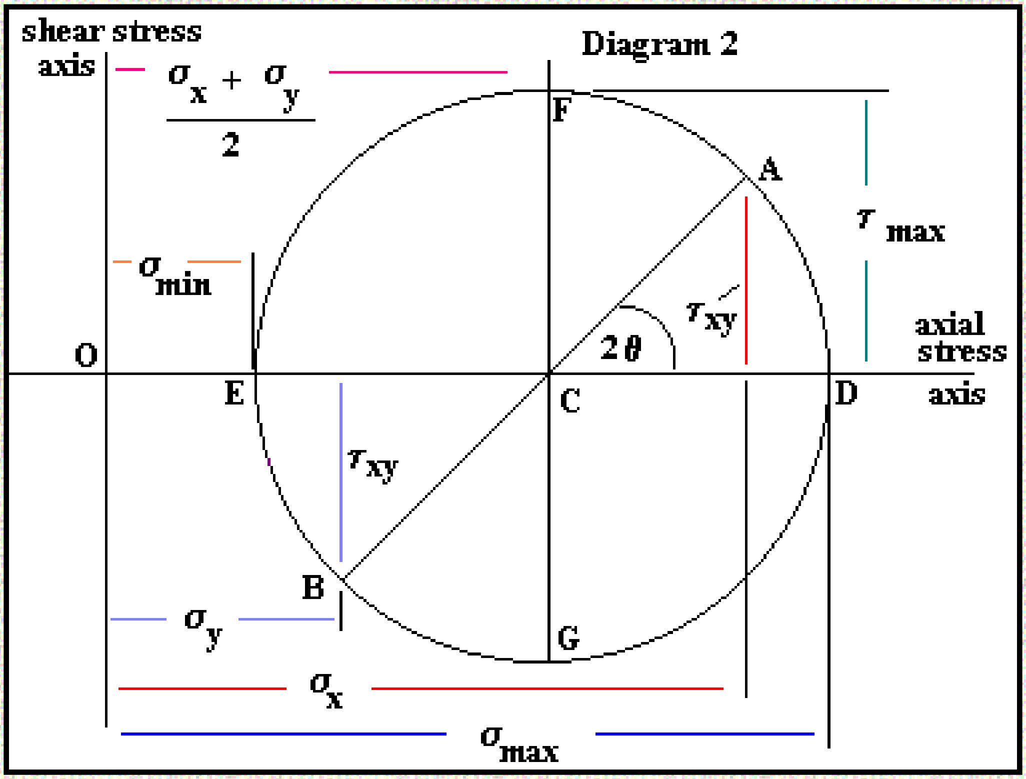
1. We draw a coordinate system with the x-axis representing the normal stresses, and the y-axis representing the shear stresses.

2. Using the values from a given structural element (Diagram 1), we graph two initial points. Point A with coordinates (σ_x, τ_{xy}), and Point B with coordinates (σ_y, τ_{yx}) as shown in Diagram 2. [According to our sign convention, the normal x-stress is positive (tension) ,as is the shear stress (counterclockwise) on the associated face. Thus Point A is above the x-axis as shown. The normal y-stress in Diagram 1 is also positive, but the shear stress on that face is negative, so Point B is below the x-axis in the Mohr Circle Diagram.}]



3. We now connect points A and B. The line connecting points A and B intersects the x-axis. This is the center of Mohr's Circle.

In Mohr's Circle, the principal plane is represented by the line ED, which has an angle of zero and zero shear stress. The distance from the origin to Points D and E are the values of the maximum and minimum principal stresses, as shown in Diagram 2.



The radius of the circle is given by

$R = \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$,
which is also equal to the maximum shear stress. In Diagram 2, the maximum shear stress is represented by line CF. We note that the plane represented by line FCG make an angle of 90º with respect to the principal plane (ACB). This Mohr's Circle angle however is twice the angle in real space, so the angle the plane of maximum shear stress makes is actually 45º different from the angle of the principal plane.

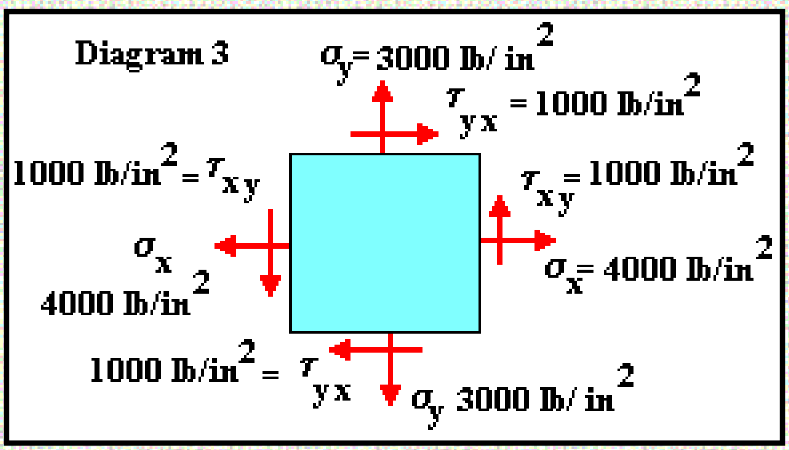
We also note that the location of the center of Mohr's Circle is $(\sigma_x + \sigma_y)/2$ from the origin.

Structural Element Sign Conventions
1. Tensile Stress will be considered positive, and Compressive Stresses will be considered negative.
2. The Shear Stress will be considered positive when a pair of shear stress acting on opposite sides of the element will produce a counterclockwise torque (couple). (Some text use the opposite direction for the positive shear stress. This changes a sign in several equations, so we must be somewhat careful of signs when working problems and examples.)
3. The incline plane angle will be measure from the vertical, counterclockwise to the plane. This will be the positive direction for the

angle.

We will now work a Mohr's Circle example.

Example. In Diagram 3 a structural element is shown with axial and shear stress of normal x-stress = 4000 lb/in^2, normal y-stress = 3000 lb/in^2, shear stress = 1000 lb/in^2. We would like to find the principal planes, principal stresses, and the maximum shear stress.

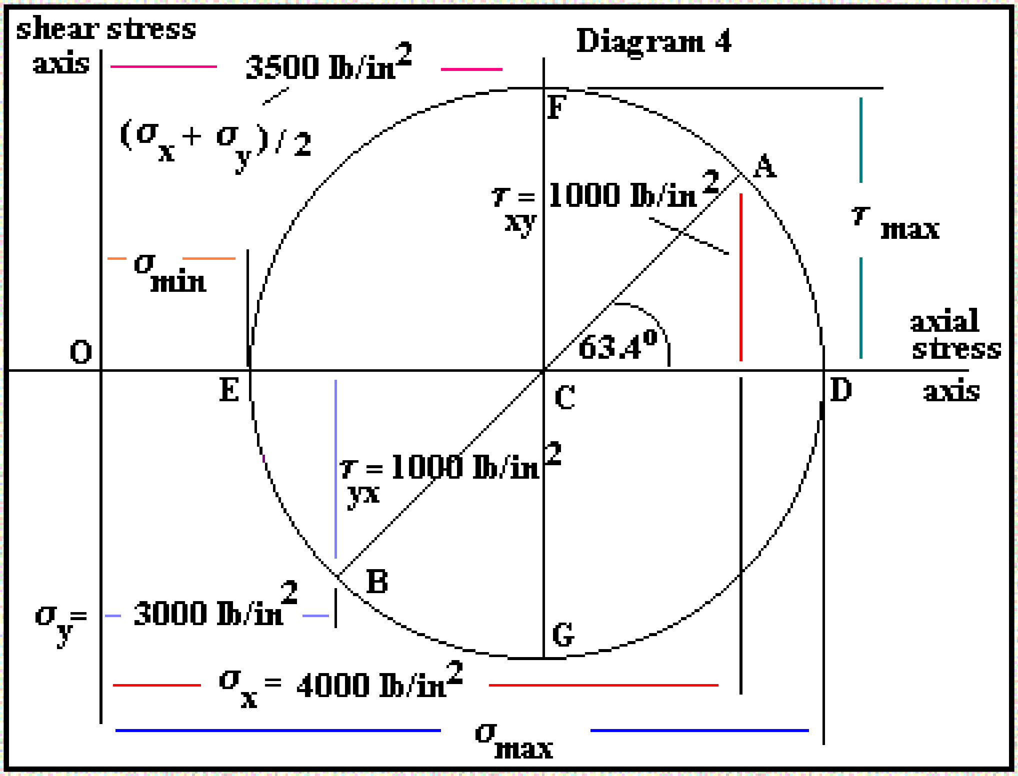


We begin by drawing Mohr's Circle for this problem. Point A (+4000 lb/in^2, +1000 lb/in^2), and Point B (3000 lb/in^2 , -1000 lb/in^2) are drawn and connected. We also calculate the radius of the circle from

R = $\sqrt{[\,(\sigma_x - \sigma_y)/2\,]^2 + \tau_{xy}^2}$ **= {[(4000 lb/in^2 -3000 lb/in^2)/2]^2 + (1000 lb/in^2)^2}$^{1/2}$**

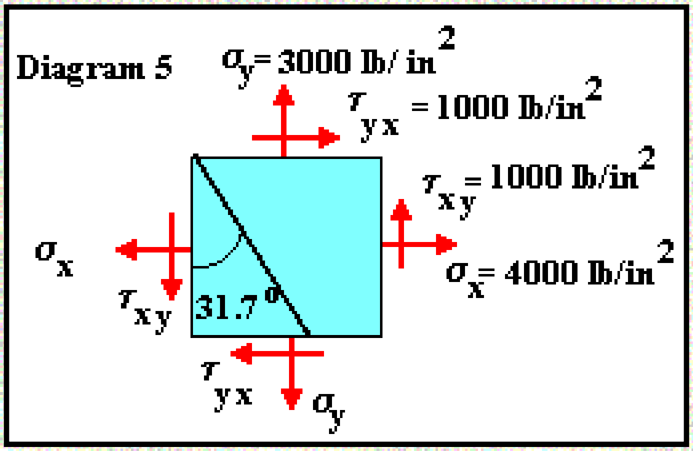
R = 1118 lb/in^2. This value is also the value of the maximum shear stress as we see from Mohr's Circle.

We next calculate the location of the center of Mohr's Circle = **(4000 lb/in^2 +3000 lb/in^2)/2 = 3500 lb/in^2**. We then use the value of the radius and the location of the center of the circle to find the principal stresses. From Mohr's Circle we see that:



Maximum Stress = Location of Center + Radius = 3500 lb/in^2 + 1118 lb/in^2 = 4618 lb/in^2
and
Minimum Stress = Location of Center - Radius = 3500 lb/in^2 - 1118 lb/in^2 = 2382 lb/in^2
And from the geometry of the circle we can determine the angle the principal axis makes with respect to the element from Tan (2*theta) = (1000 lb/in^2)/(4000 lb/in^2 - 3500 lb/in^2), and solving for(2 theta), we find (2 theta) = 63.4^o, and 243.4^o, then **theta = 31.7^o, and 121.7^o**. The Mohr Circle angles, 63.4^o, and 243.4^o , are double the angles in real space, so the actual angles the principal planes make are **31.7^o, and 121.7^o**.

The other important point with respect to the angles is that due to the way the initial points were chosen for Mohr's Circle, and due to the sign conventions used, the angles in Mohr's circle are clockwise from the structural element. **In real space they will be in the opposite direction, counterclockwise from the vertical.** (See Diagram 5.)



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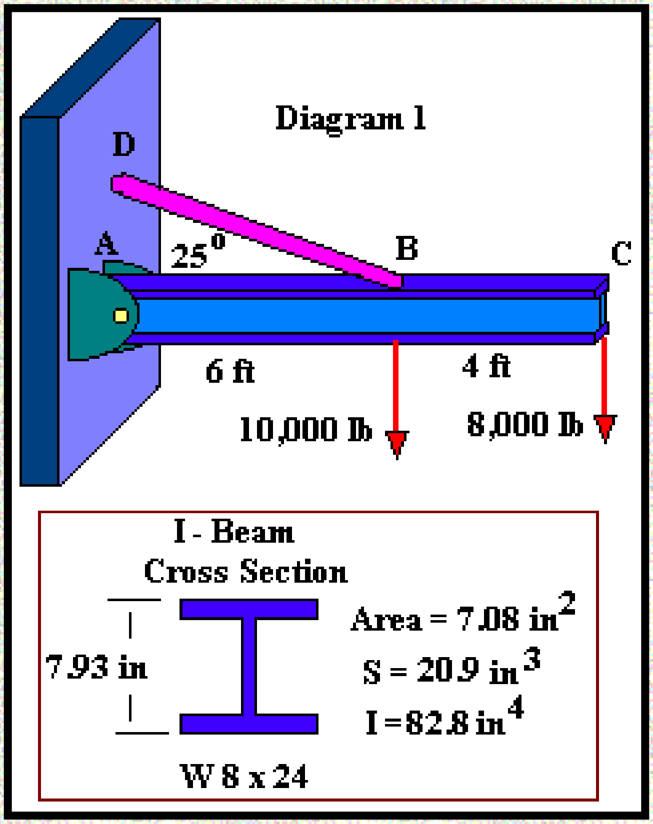
Topic 8.7a - Special Topics - Problem Assignment 1

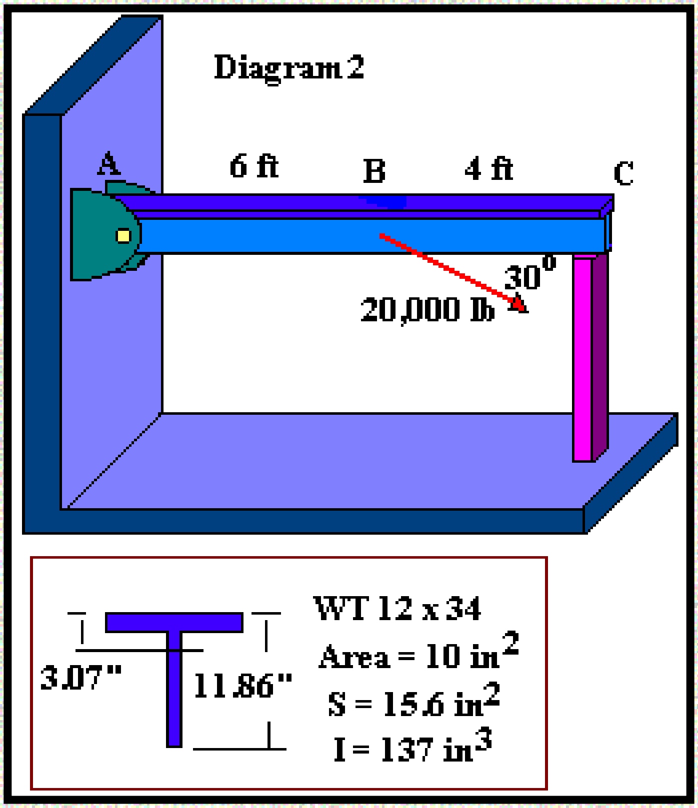
1. A loaded beam (shown in Diagram 1) is pinned to the wall at point A, and is supported by a rod DB, attached to the wall at point D and to the beam at point B.



The beam has a load of 8,000 lb. acting downward at point C and a second load of 10,000 lb. acting downward at point B. The supporting rod makes an angle of 25º with respect to the beam. The beam cross section is a W8 x 24 I-Beam, with the characteristics shown in Diagram 1.

Determine the maximum axial stress acting in the beam cross section and state where it occurs. [25,400 psi @ x = 6' at bottom of beam, compression]

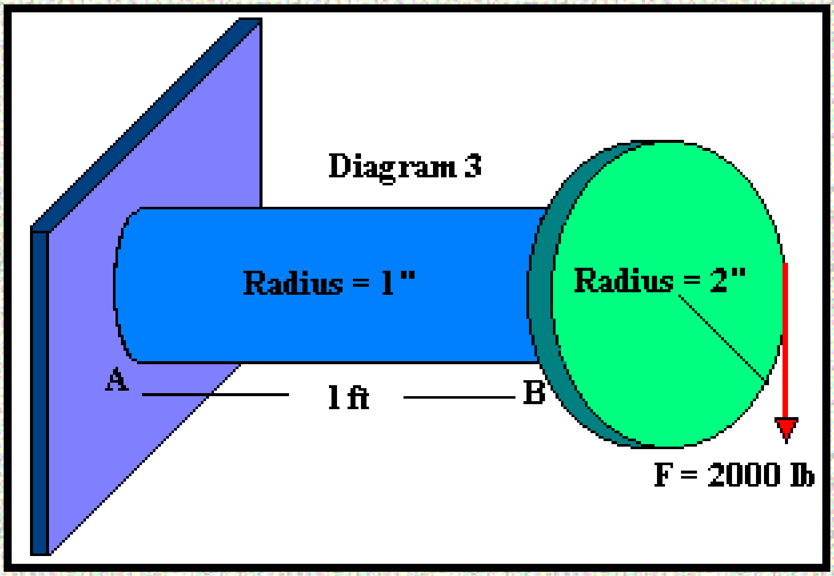
2. A loaded beam (shown in Diagram 2) is pinned to the wall at point A, and is supported a vertical support at point C.



A force of 20,000 lb. is applied at the centroid of the beam, at an angle of 30º with respect to the horizontal as shown in the Diagram. The beam cross section is a WT12 x 34 T-Beam, with the characteristics shown in Diagram 2.

Determine the maximum axial stress acting in the beam cross section and state where it occurs. [20,192 psi @ x = 6' at bottom of beam, tension]

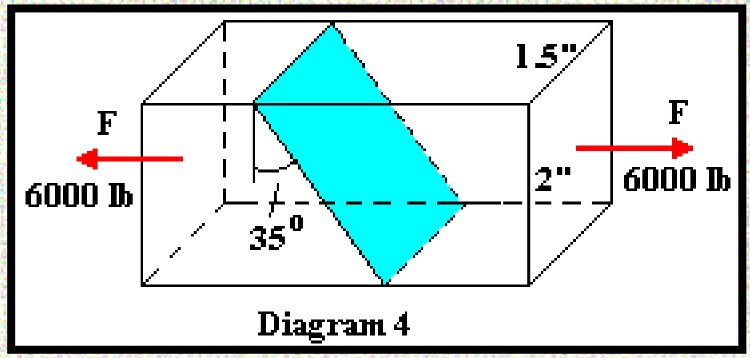
3. As shown in Diagram 3, a solid 1 foot long shaft with a radius of 1" is attached to a wall at point A, and has a disk with a radius of 2" attached at end B.



A force of 2000 lb. is applied to the outer edge of the disk, as shown.

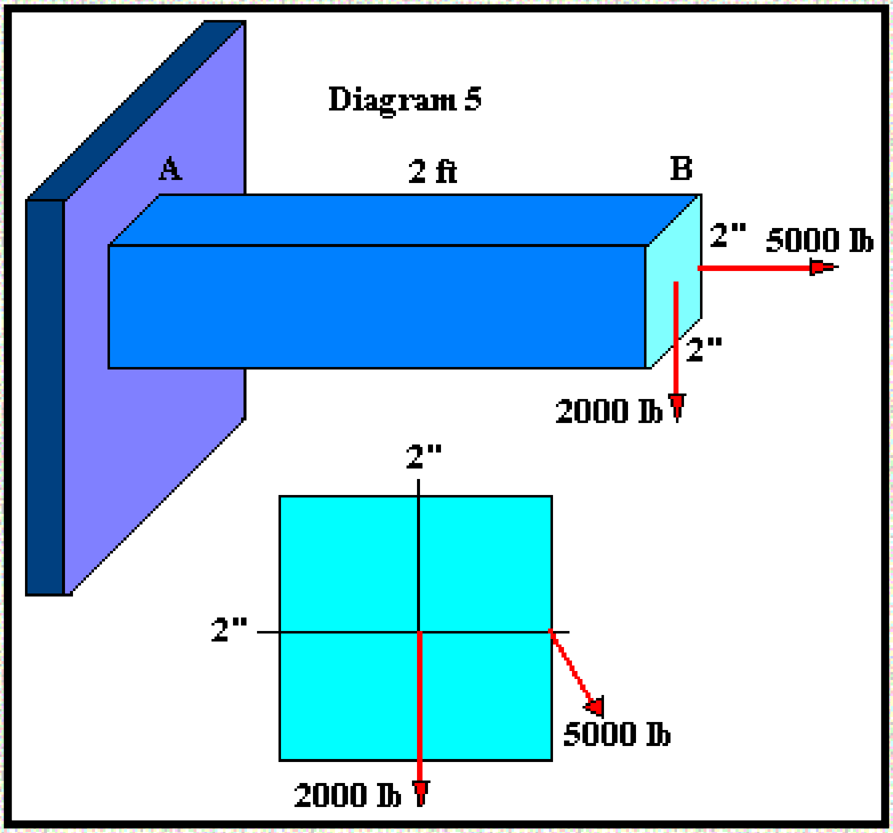
Determine the maximum shear stress in the shaft, and state where it acts.
[849 psi + 2,546 psi = 3,395 psi at top of beam]

4. In Diagram 4, we have shown a rectangular, 2" by 1.5", section in tension with a normal force of 6000 lb. acting on each end.



Determine the axial and shear stress on a 35º incline plane cut through the section.[axial 1,342 psi., shear 940 psi.]

5. A cantilever beam is shown in Diagram 5. At end B at 2000 lb. downward load is applied at the centroid of the beam, and a 5000 lb. horizontal force is applied in the vertical center, but at the far outside edge as shown in Diagram 5.



The beam cross section is a 2" by 2" square.

Determine the maximum axial stress in the beam, and state where is acts.[= 36,000 psi. + 3750 psi. + 1250 psi. = 41,000 psi @ wall and at top of beam on far edge]

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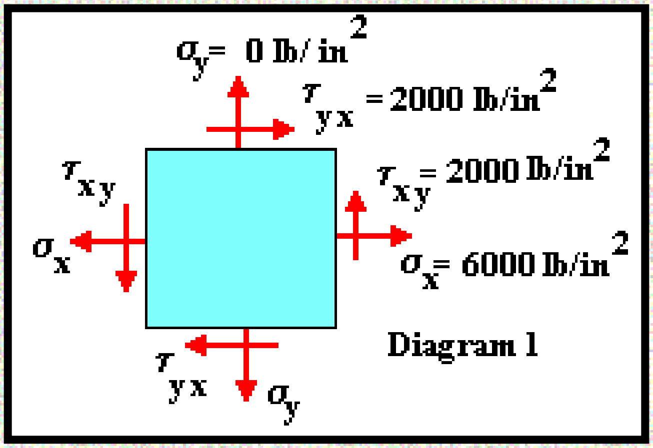
Topic 8.7b - Special Topics - Problem Assignment 2

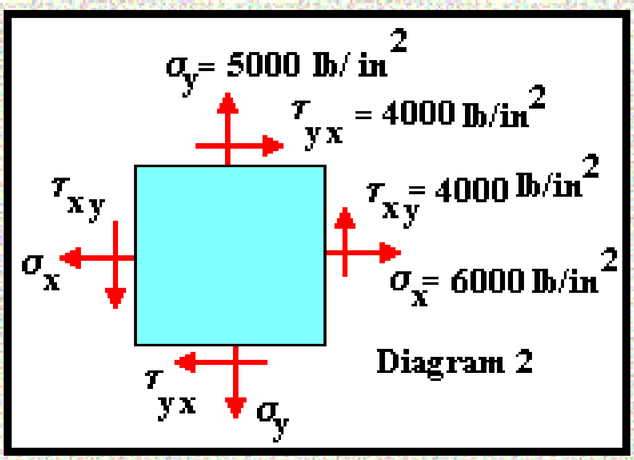
1. A structural element with given axial and shear stresses is shown in Diagram 1.



Determine principal planes, the principal stresses, and the maximum shear stress. [16.8º, 106.8º, axial max 6,606 psi.,axial min -606 psi., shear = 3606 psi.]

Also determine the axial and shear stresses on a plane which makes an angle of 37 degrees counterclockwise from the negative vertical axis in the given element. [axial 5750 psi., shear -2235 psi.]

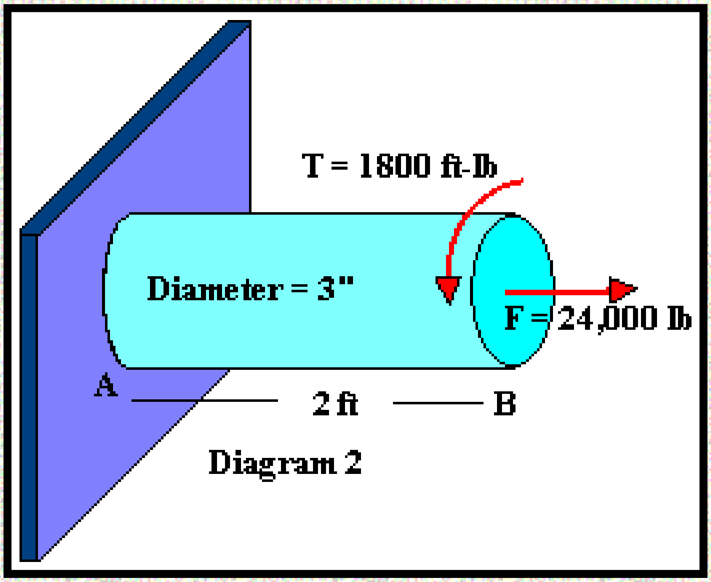
2. A structural element with given axial and shear stresses is shown in Diagram 2.



Determine principal planes, the principal stresses, and the maximum shear stress. [41.4º, 131.4º, axial max 9,530 psi.,axial min -1,470 psi., shear = 4030 psi.]

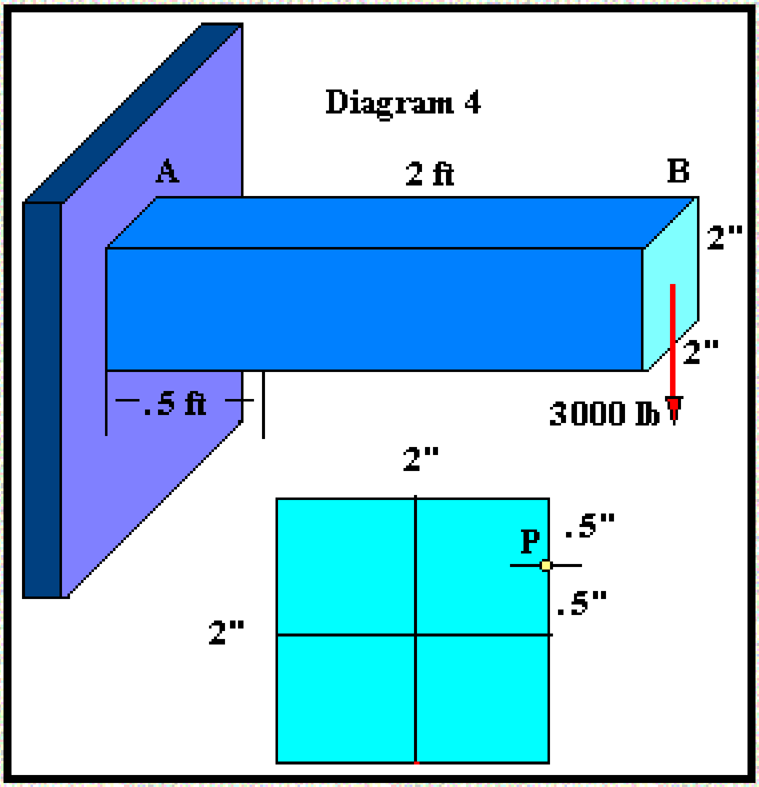
Also determine the axial and shear stresses on a plane which makes an angle of 60 degrees counterclockwise from the negative vertical axis in the given element. [axial 8714 psi., shear -2433 psi.]

3. In Diagram 3 we have shown a shaft attached to a wall at end A with a torque of 1800 ft-lb. and a horizontal axial force of 24,000 lb. acting at end B.



Determine the principal planes, principal stresses and the maximum shear stress for a structural element on the outer edge of the shaft. (We will assume that the normal axial stress is uniform across the area of the shaft.)
[56.3º, 146.3º, axial max 6,112 psi.,axial min -2,716 psi., shear = 4,414 psi.]

4. A loaded cantilever beam is shown in Diagram 4. The beam cross section is a 2" by 2" square.



Determine the principal planes, principal stresses, and maximum shear stress on a structural element 1/2 foot from the wall, and at point P in the cross sectional area, as shown in Diagram 4.
[2.4º, 92.4º, axial max 20,285 psi.,axial min -35 psi., shear = 10,160 psi.]

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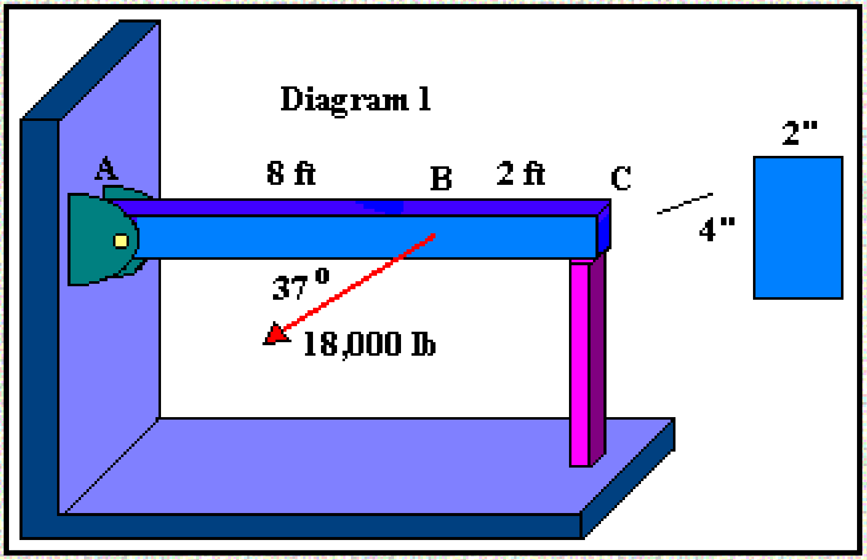
Topic 8.8: Special Topics I - Topic Examination

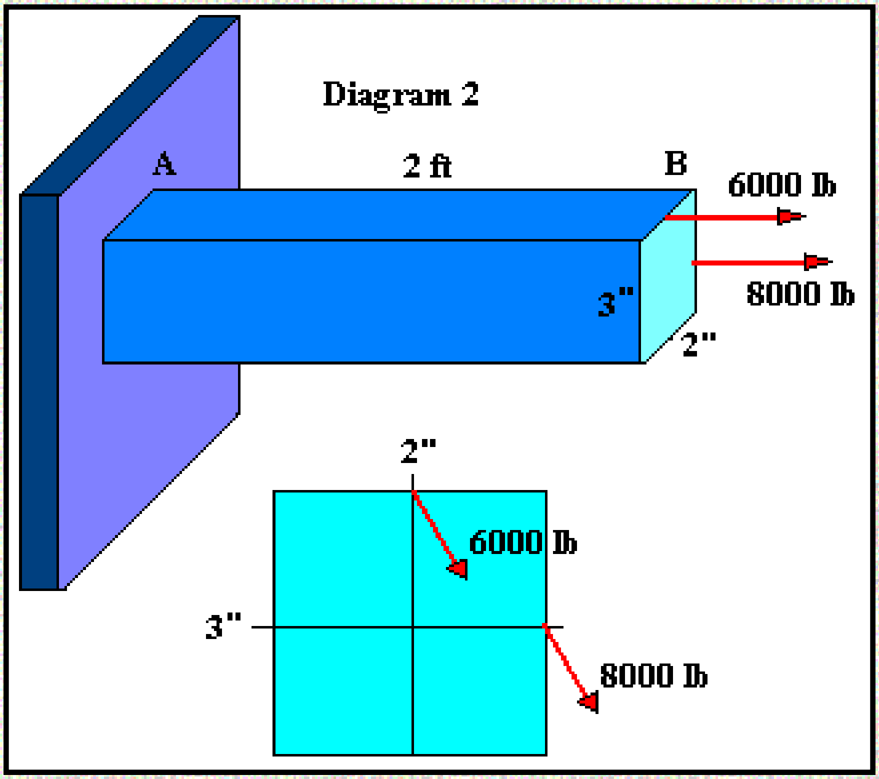
1. A loaded beam (shown in Diagram 1) is pinned to the wall at point A, and is supported a vertical support at point C.



A force of 18,000 lb. is applied at the centroid of the beam, at an angle of 37º with respect to the horizontal as shown in the Diagram. The beam cross section is a rectangular 2" by 4".

Determine the maximum axial stress acting in the beam cross section and state where it occurs.
[(38880 psi + 1800 psi = 40680 psi. compression, at top of beam 8 ft from left end) This is considering only sum of axial stress acting. If one takes a small section at point indicated and finds principle stresses, the principle axis stress in slightly greater, ~ 41,000 psi @ 4.6º angle off vertical plane.]

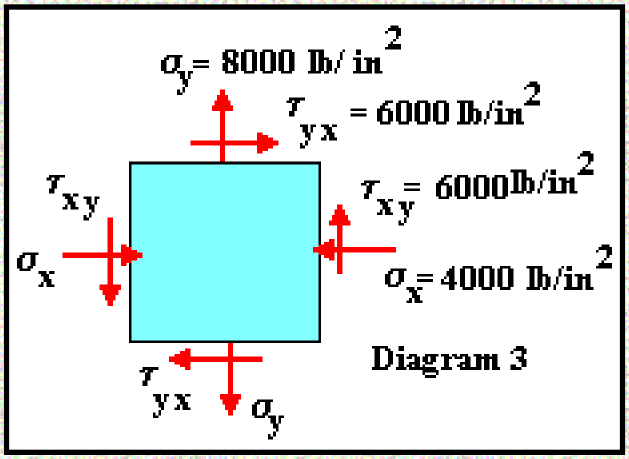
2. A cantilever beam is shown in Diagram 2.



At end B a 8,000 lb. horizontal force is applied in the vertical center, but at the far outside edge as shown in Diagram 5. A second 6,000 lb. horizontal force is applied at the horizontal center of the beam, but at the top edge of the area as shown in Diagram 5. The beam cross section is a 2" by 3" rectangle. Determine the maximum axial stress in the beam, and state where it acts.

[(2333 psi + 3000 psi + 4000 psi = 9333 psi, tension at top right corner of cross section. Again considering only combined axial stresses. There are two points of zero axial stress in cross section. One would be on horizontal neutral axis .583" to left of center point (2333 psi (t) + 0 + 2333 psi (c) = 0), and the other on the vertical axis 1.167" below the center point of cross section (2333 psi (t) + 2333 psi (c) + 0 = 0)]

3. A structural element with applied axial and shear stresses is shown in Diagram 3.



Determine principal planes, the principal stresses, and the maximum shear stress. (-22.5º, +67.5º, 10,485 psi, -6485 psi, +/- 8485 psi)

Also determine the axial and shear stresses on a plane which makes an angle of 40 degrees counterclockwise from the negative vertical axis in the given element. (6867 psi, 6951 psi)

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